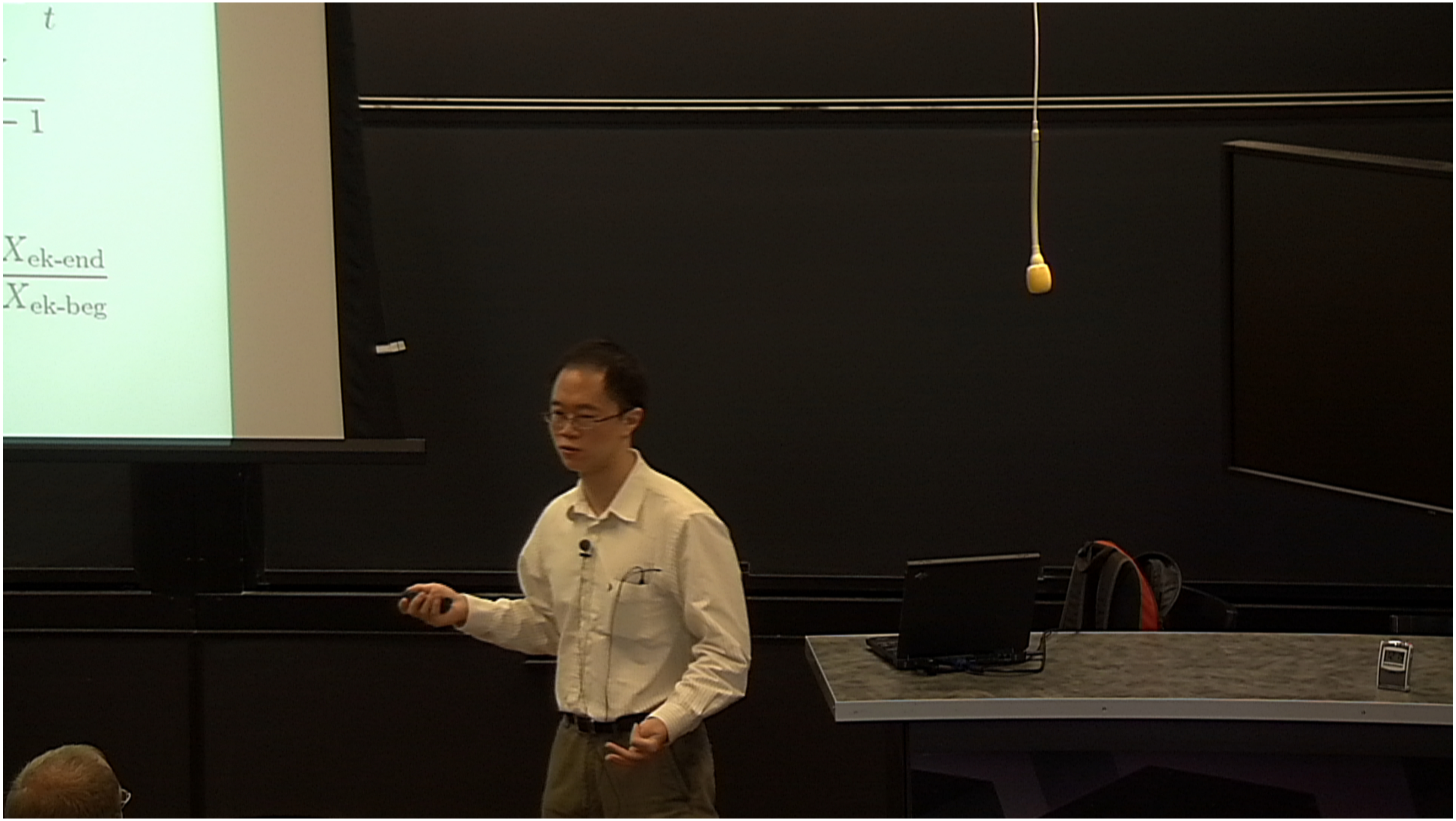
Title: Curvature and Anisotropy Near a Nonsingular Bounce

Date: Nov 10, 2011 11:00 AM

URL: http://pirsa.org/11110119

Abstract: Problematic growths of curvature and anisotropy are found in nonsingular bouncing cosmologies that include both an ekpyrotic phase and a bouncing phase. Classically, initial curvature and anisotropy that are suppressed during the ekpyrotic phase will grow back exponentially during the nonsingular bouncing phase. Besides, curvature and shear perturbations are generated by quantum fluctuations during the ekpyrotic phase. In the bouncing phase, an adiabatic curvature perturbation grows to dominate and gives rise to a blue spectrum that spoils the scale-invariance. Meanwhile, a scalar shear perturbation grows nonlinear and creates an overwhelming anisotropy that disrupts the nonsingular bounce altogether. We examine the common origin of these problems and discuss possible ways to avoid them.



t
$- 1$
$X_{\text{ek-end}}$
$X_{\text{ek-beg}}$


non-singular bouncing cosmologies
BingKan Xue
Princeton University
Nov. 10, 2011
B.K. Xue and P. J. Steinhardt, Phys. Rev. Lett. 105 (2010) 261301
B.K. Xue and P. J. Steinhardt, Phys. Rev. D 84 (2011) 083520

Bouncing cosmologies

Model building

Curiosity:

- state of Universe
- evolution of Universe

Observations:

- homogeneity, flatness and isotropy
- nearly scale-invariant power spectrum

Models:

- cosmic expansion, (inflation), big bang.
- big bounce, cosmic contraction ?

Bouncing cosmologies

Contracting Universe

Friedmann equation:

$$H^2 = \frac{1}{3}\Big(- \frac{3\mathrm{k}}{a^2} + \frac{\sigma_0^2}{a^6} + \frac{\rho_{\phi_0}}{a^{3(1+w)}} + \cdots \Big)$$

Essential ingredients:

- curvature: $w = -\frac{1}{3}$
- anisotropy: $w = 1$
- Smooth contraction needs $w > 1$!

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Ekpyrotic phase: $w \gg 1$

✓ homogenizes, flattens, isotropizes

✓ generates scale-invariant perturbations

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Bouncing Universe

Intuition:

large scale properties **decouple** from high energy bounce

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- singular bounce: quantum treatment ...
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Ekpyrotic Non-singular Bouncing Model

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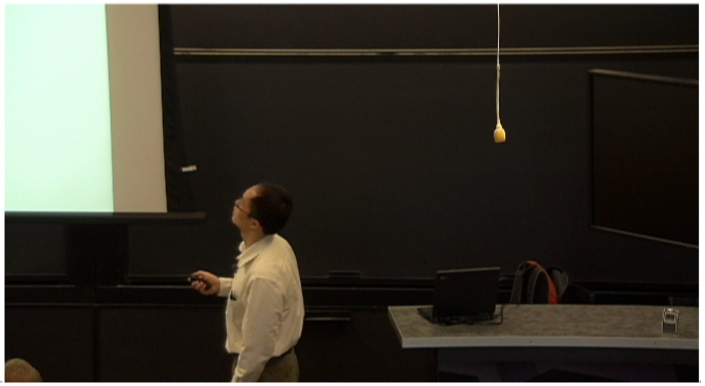
New ekpyrotic model

Effective Lagrangian: $\mathcal{L} = \sqrt{-g}\left[P(X) - V(\phi)\right], \; X \equiv -\frac{1}{2}\left(\partial\phi\right)^2$



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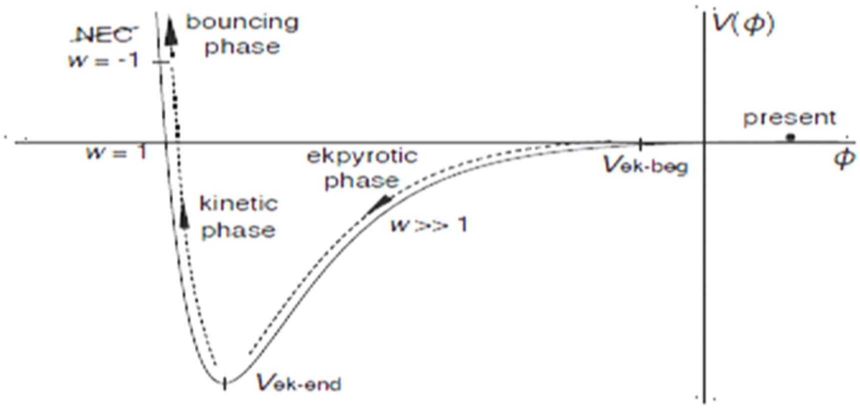

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$$V(\phi) \approx -V_0\ e^{-\sqrt{2/p}\,\phi}$$

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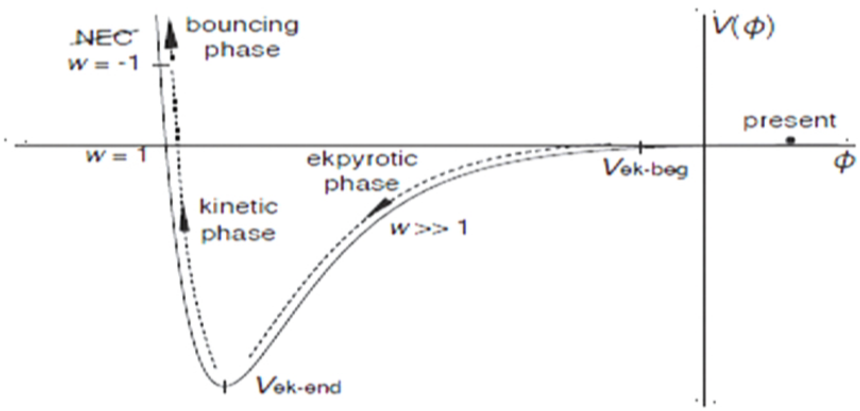

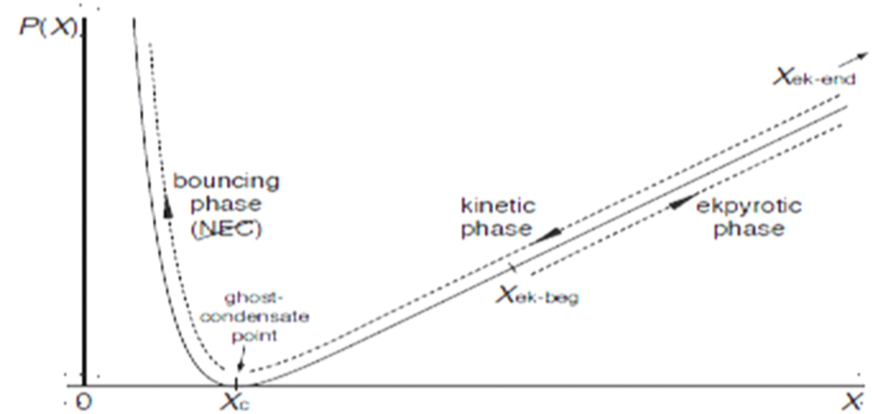

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Violating NEC:

$$\dot{H} = -X P_{,X} > 0$$

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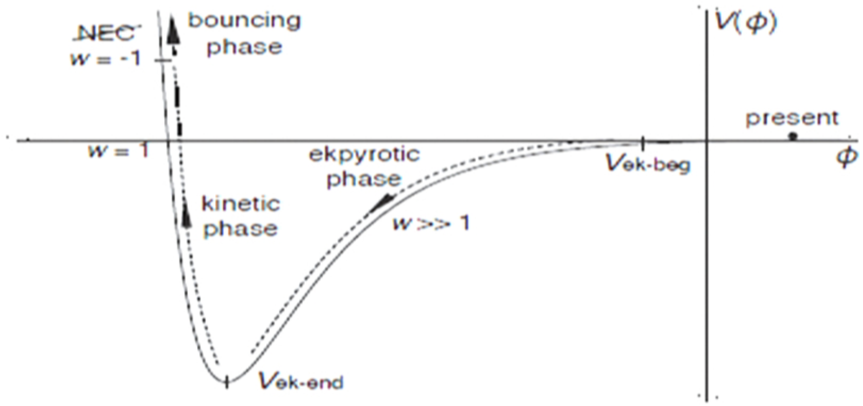

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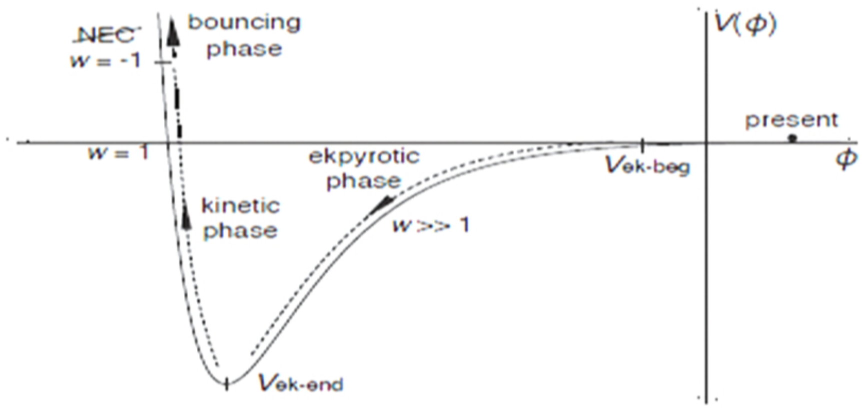

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$$w = \tfrac{2}{3p} - 1 \gg 1, \text{ if } p \ll 1$$

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$$a \sim (-t)^p, \quad H = \frac{p}{t}$$

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number of e-folds:

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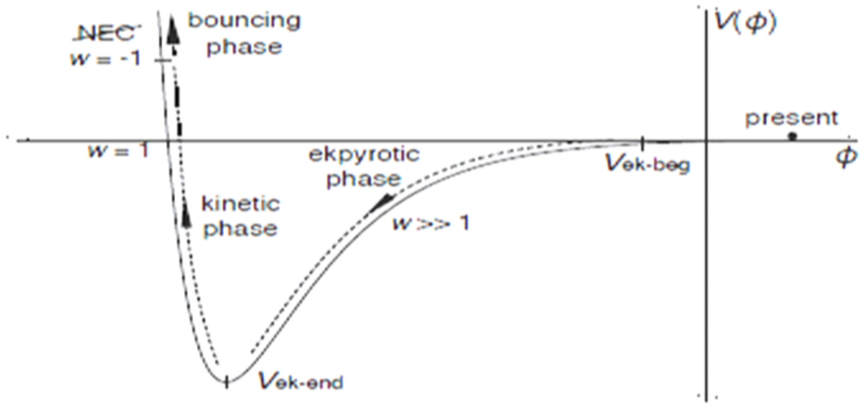

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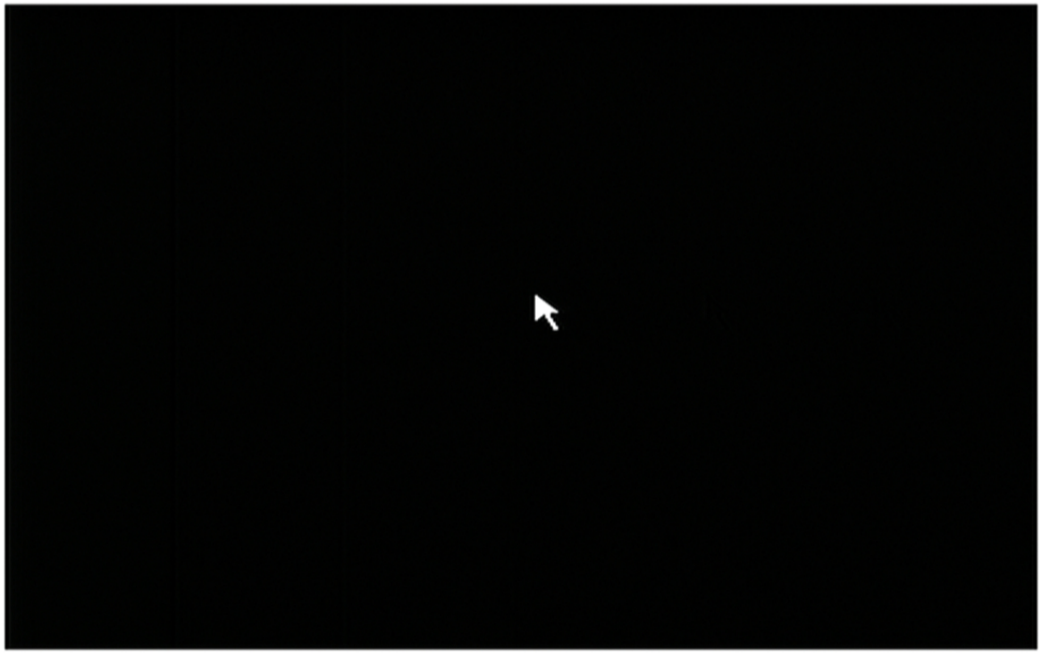

Figure: Normalized energy density in matter Ω_m (yellow), curvature Ω_k (pink), and shear Ω_s (blue).

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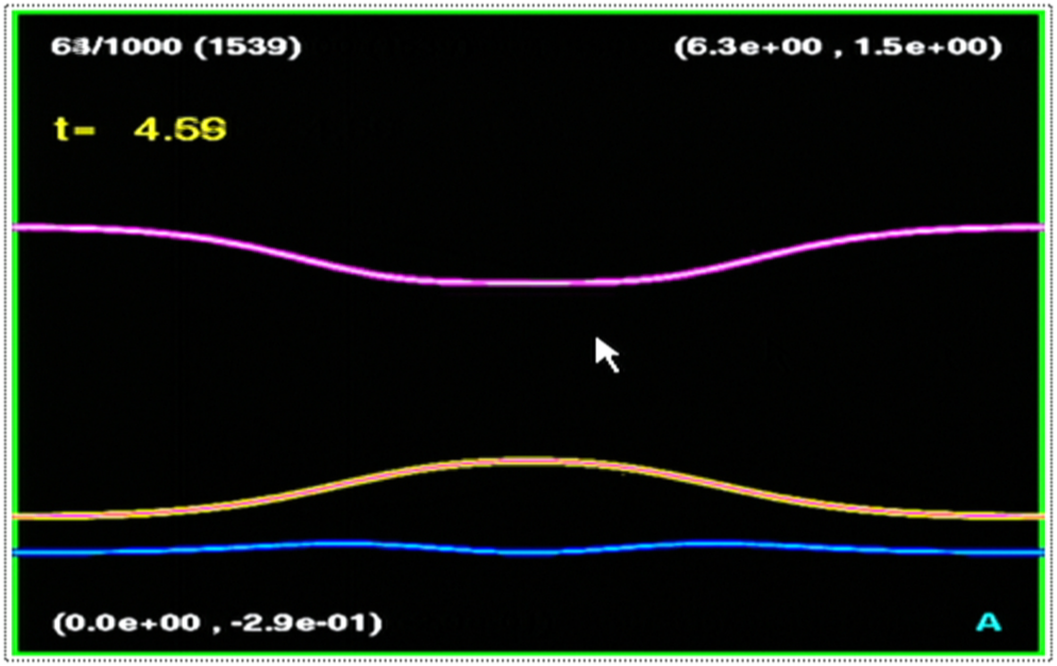

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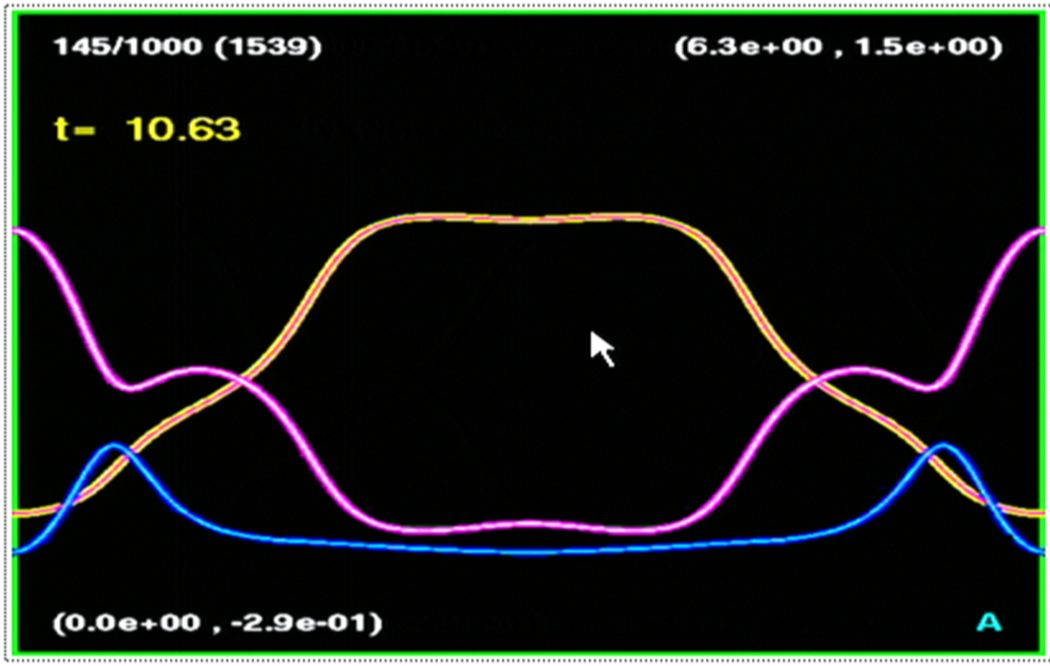

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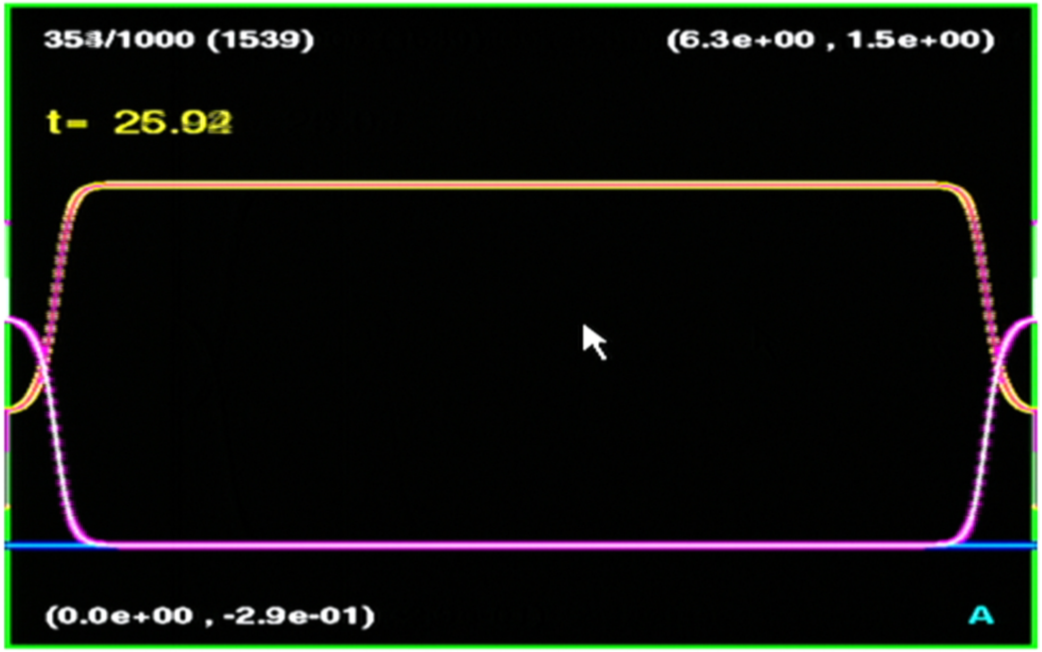

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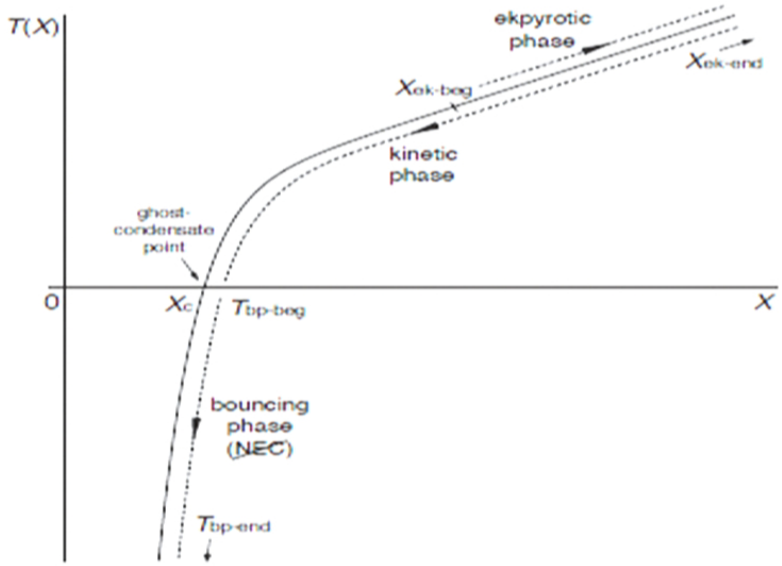
Bouncing phase: $w < -1$

kinetic energy $T = 2XP_{,X} - P$



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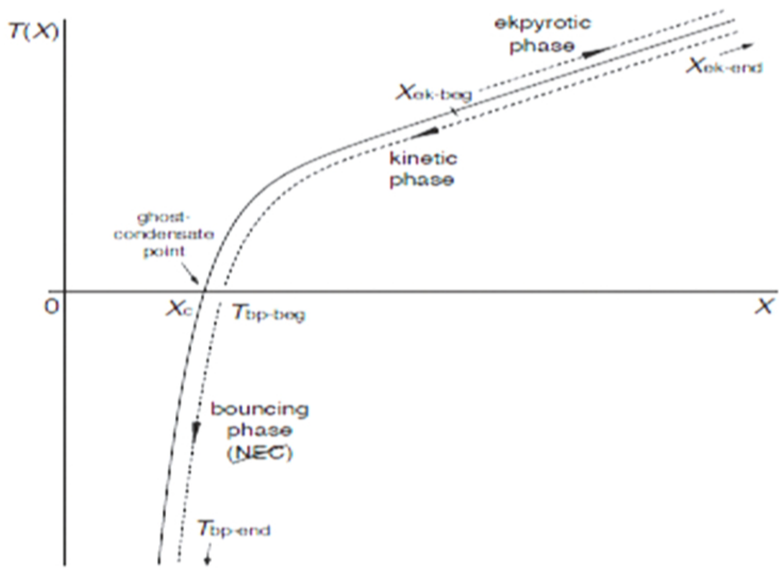

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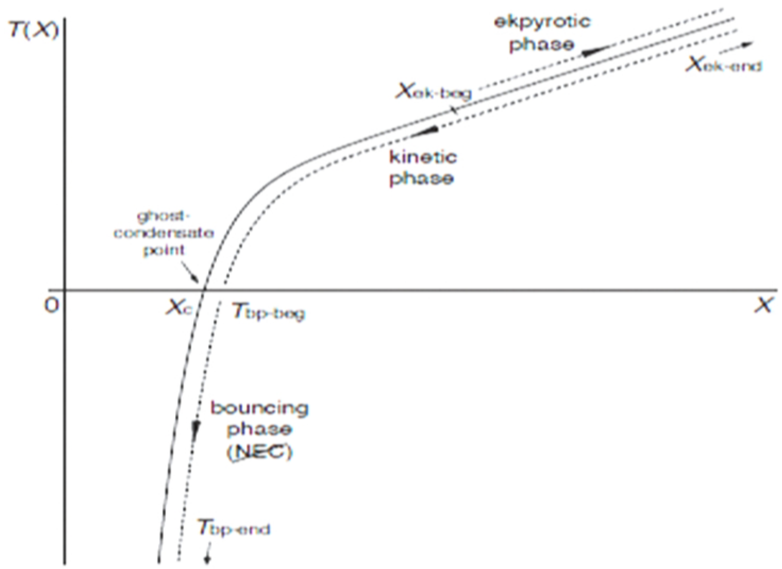

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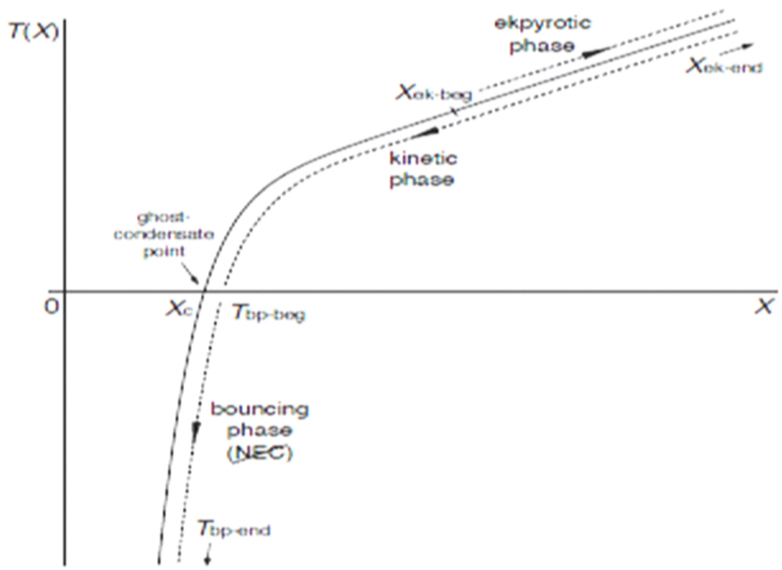

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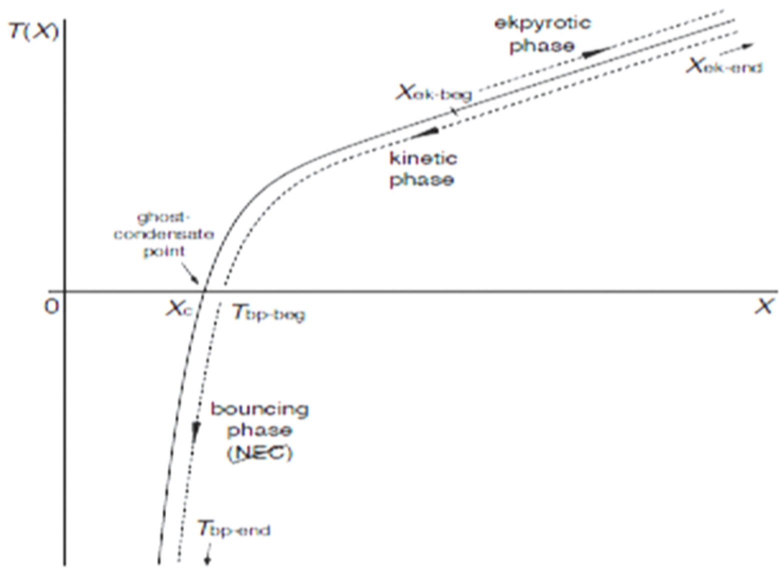

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duration of bouncing phase:

$$\Delta t_{bp} \sim \frac{N}{3|H_c|}, \quad \frac{a_{\text{bp-end}}}{a_{\text{bp-beg}}} \sim e^{-N/3}$$

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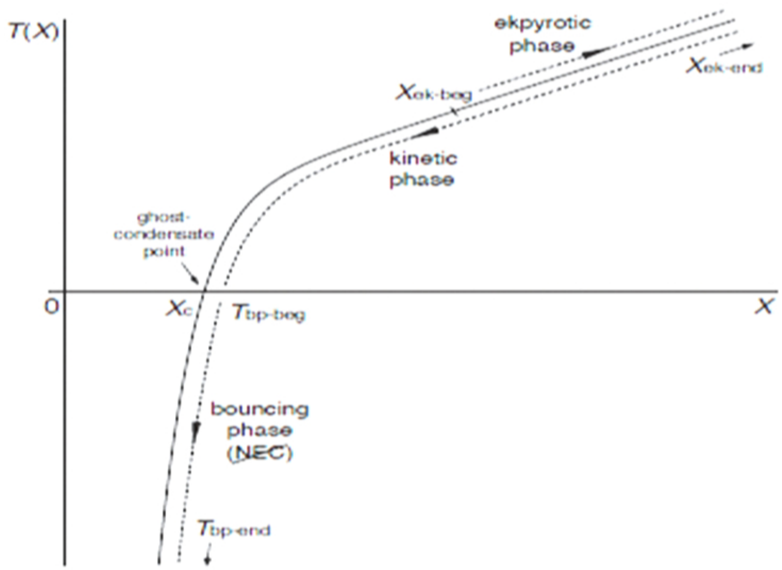

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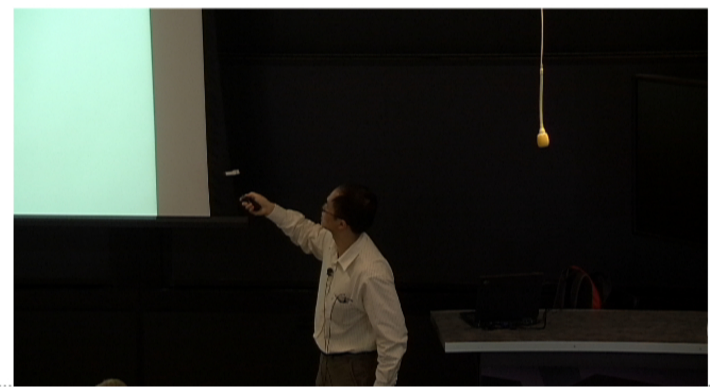

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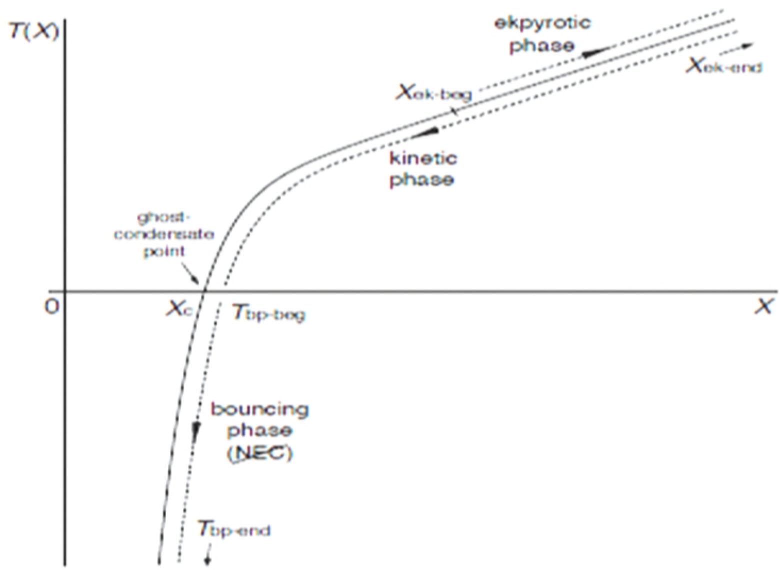

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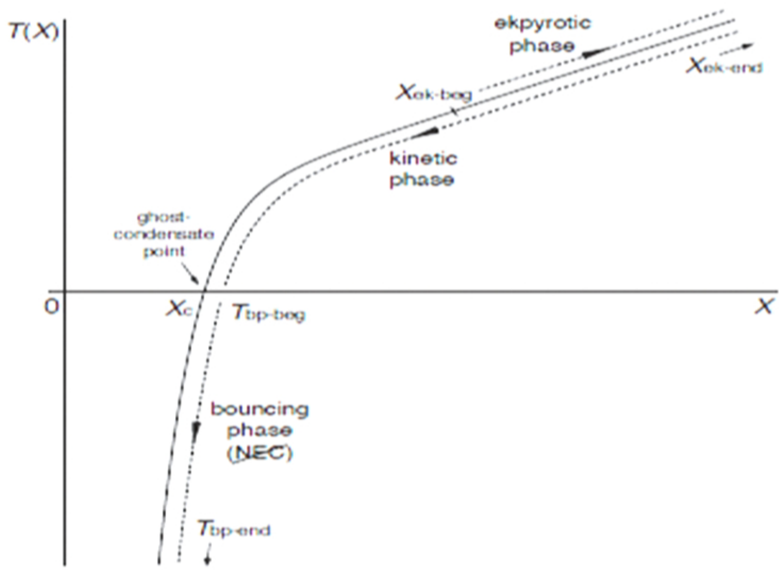

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Simulation of the nonsingular bounce:

upcoming...

Curvature and Shear Perturbations

Perturbative inhomogeneity, curvature, and anisotropy

Linear perturbations about FRW:

$$ds^2 = a(\tau)^2\Big[-(1+2A)d\tau^2 + 2(B_{,i} + S_i)d\tau dx^i + \big((1-2\psi)\delta_{ij} + 2E_{,ij} + 2F_{(i,j)} + 2h_{ij}\big)dx^i dx^j\Big]$$

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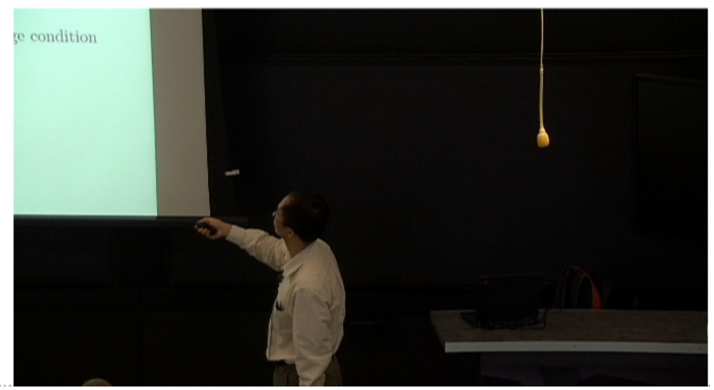
Scalar perturbations: A, B, ψ, E, $\delta\phi$

Einstein equations: δT^0_0, δT^0_i, δT^i_i, δT^i_j, plus 1 gauge condition



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Perturbations on constant time surface:

- curvature perturbation ψ : $\delta^{(3)}R = \frac{4}{a^2}\nabla^2\psi$
- shear perturbation $\sigma = E' - B$: $\sigma_{ij} = a(\sigma_{,ij} - \frac{1}{3}\delta_{ij}\nabla^2\sigma)$

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Curvature Perturbations

Comoving curvature perturbation

Gauge-invariant variable $\mathcal{R} \equiv \psi + H\frac{\delta\phi}{\dot{\phi}}$

represents curvature perturbation in comoving gauge $\delta\phi = 0$

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + c_s^2 k^2 \mathcal{R}_k = 0, \quad z \equiv a\sqrt{\frac{-\dot{H}}{c_s^2 H^2}}$$

On large scales, adiabatic modes

$$\mathcal{R}_k^{(0)} = \frac{C_1}{\sqrt{k}} + \frac{C_2}{\sqrt{k}} \int \frac{k\, d\tau}{z^2} \equiv \mathcal{R}_k^{\mathrm{const}} + \mathcal{R}_k^{\mathrm{int}}$$

Curvature Perturbations

Comoving curvature perturbation

Gauge-invariant variable $\mathcal{R} \equiv \psi + H\frac{\delta\phi}{\dot{\phi}}$

represents curvature perturbation in comoving gauge $\delta\phi = 0$

$$\mathcal{R}''_k + 2\frac{z'}{z}\mathcal{R}'_k + c_s^2 k^2 \mathcal{R}_k = 0, \quad z \equiv a\sqrt{\frac{-\dot{H}}{c_s^2 H^2}}$$

On large scales, adiabatic modes

$$\mathcal{R}_k^{(0)} = \frac{C_1}{\sqrt{k}} + \frac{C_2}{\sqrt{k}} \int \frac{k\, d\tau}{z^2} \equiv \mathcal{R}_k^{\mathrm{const}} + \mathcal{R}_k^{\mathrm{int}}$$

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In ekpyrotic phase:

- Constant term (blue) is conserved outside horizon

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In kinetic phase:

integral term grows exponentially as $w \to -1$

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Related to comoving curvature perturbation $\mathcal{R}$ through

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Anisotropy in uniform Hubble gauge

Shear perturbation in uniform Hubble gauge $\delta H = 0$

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Problems for ekpyrotic non-singular bounces:

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Origin of the problems

Running of energy scale X:

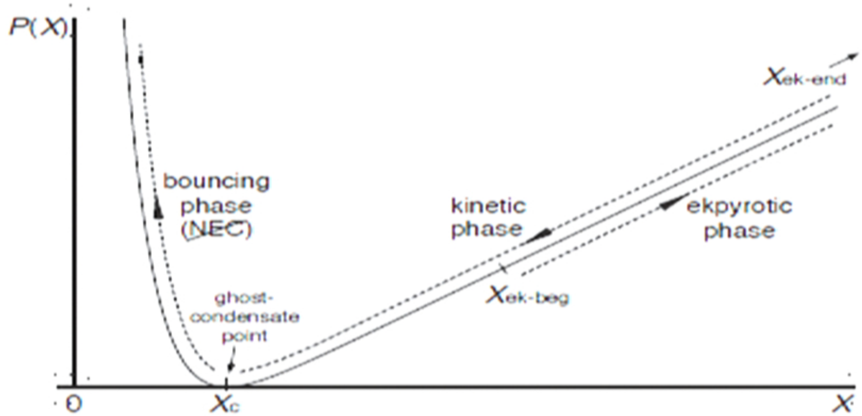
exponential increase in ekpyrotic phase

$$\frac{X_{\text{ek-end}}}{X_{\text{ek-beg}}} \sim e^{2N}$$

dramatic decrease before bouncing phase

$$X_{\text{c}} < X_{\text{ek-beg}}$$

$\Rightarrow$ Large ratio $\frac{X_{\text{ek-end}}}{X_{\text{c}}}$ emerges



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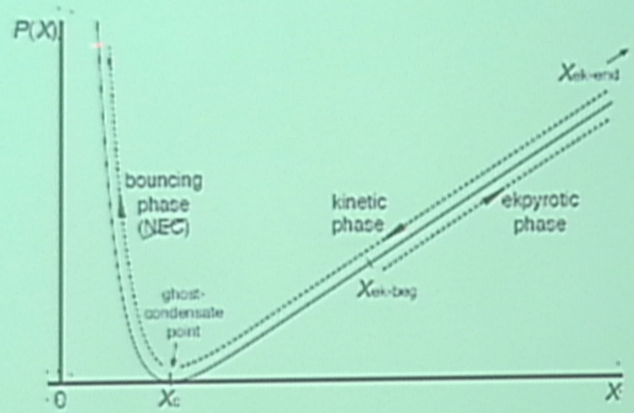

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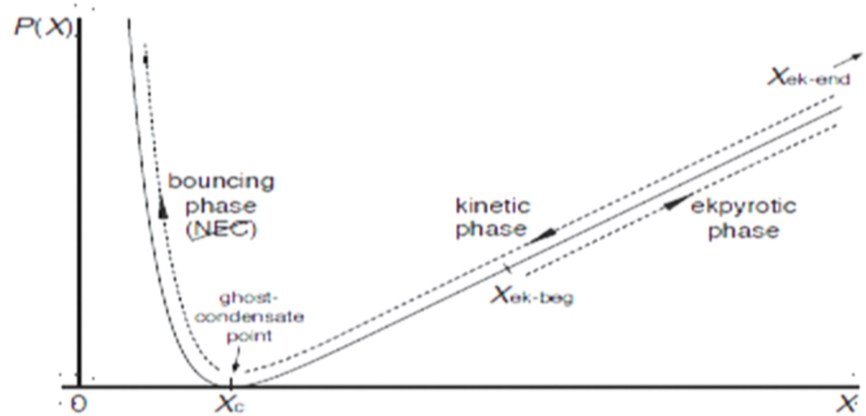

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Avoiding the problems:

- High energy bounce – singular bounces ?
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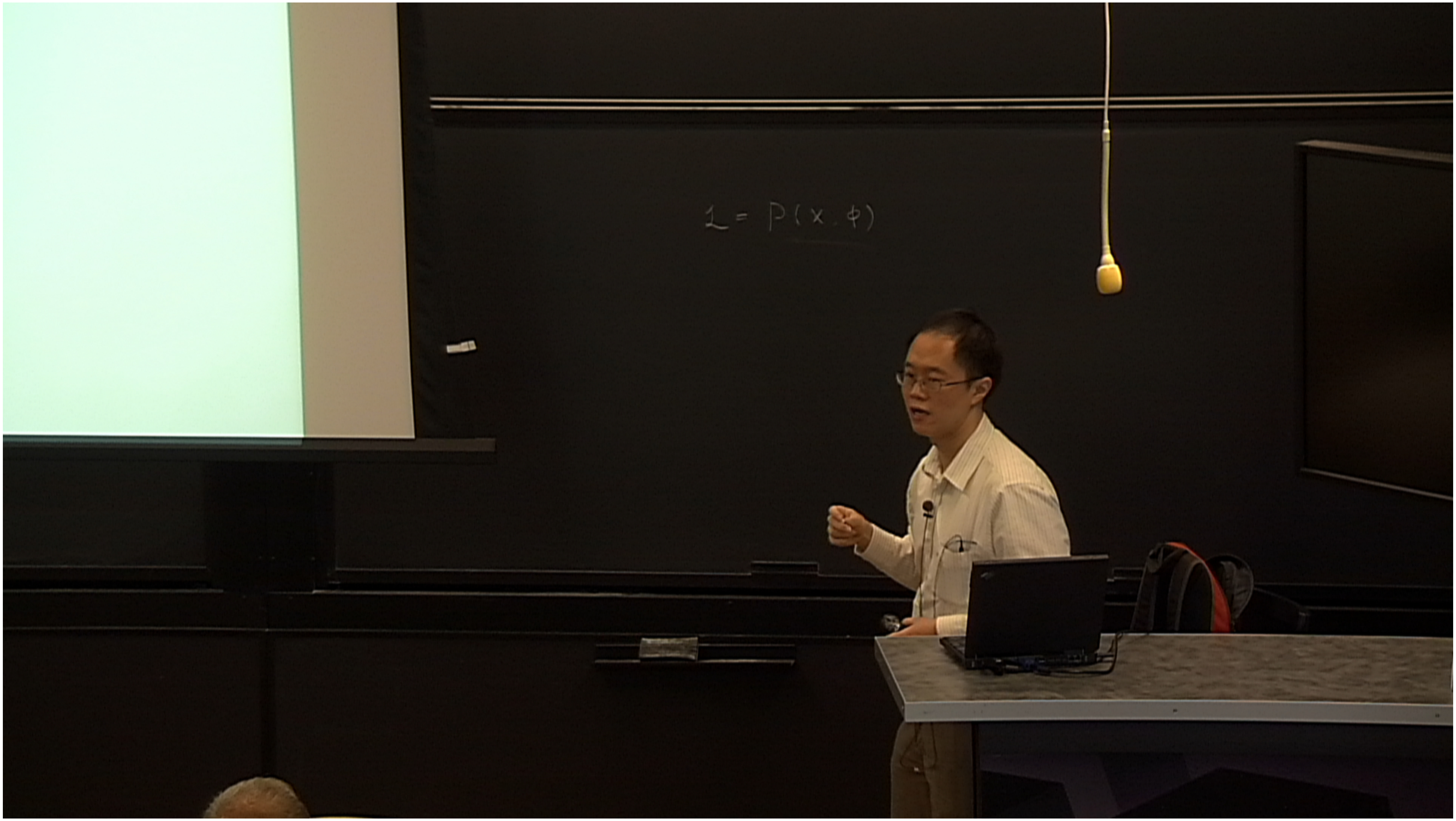
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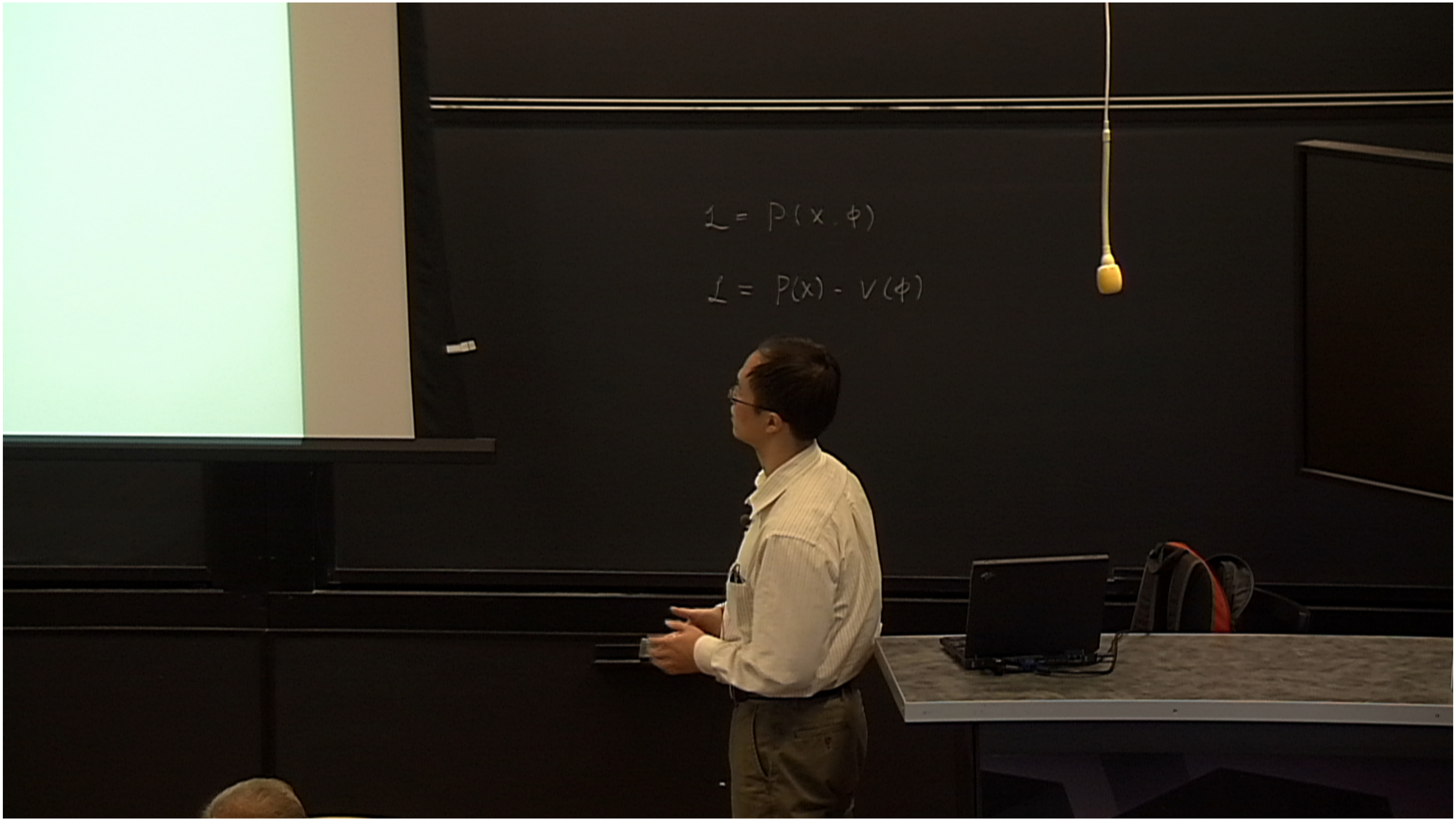
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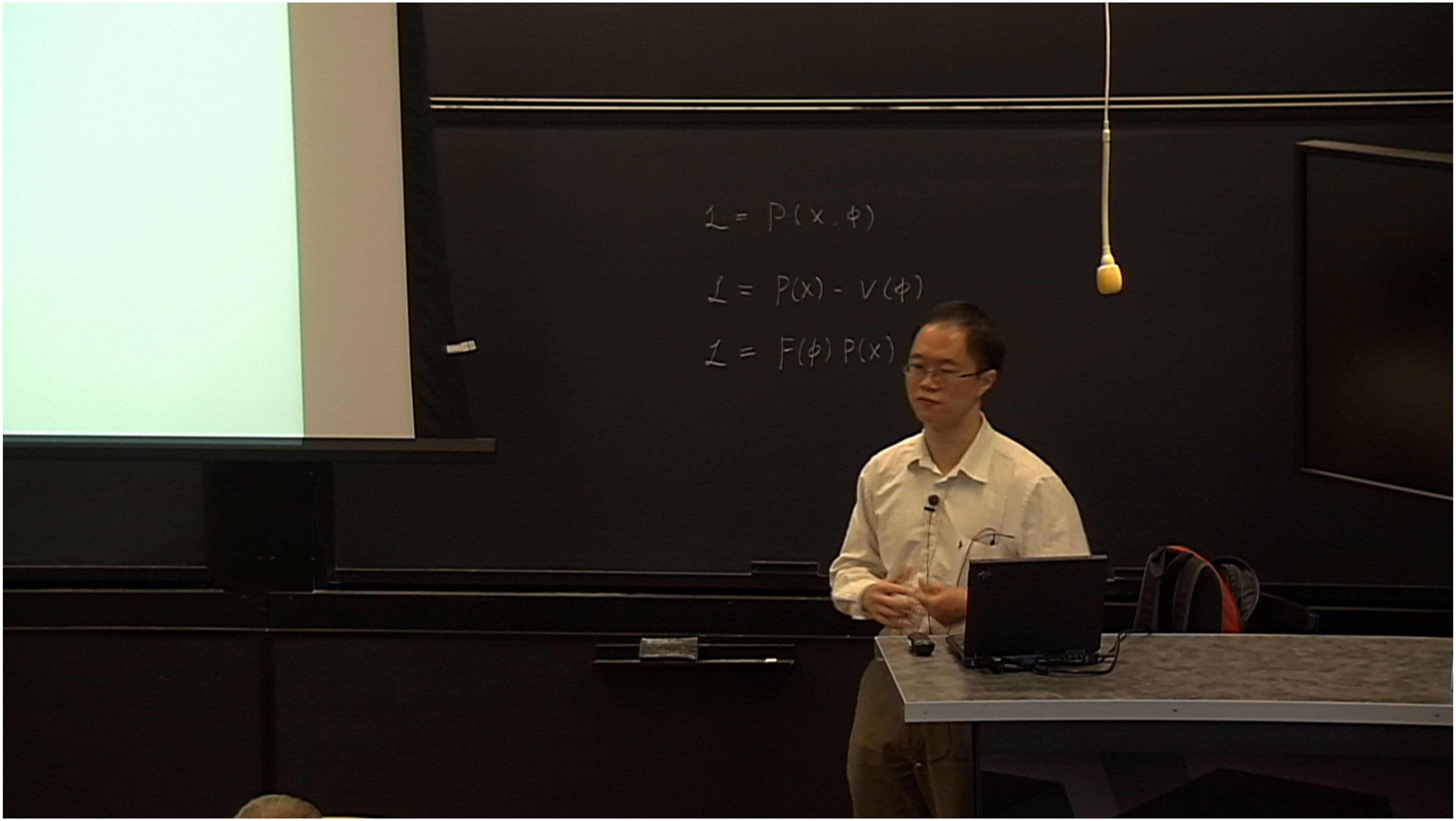
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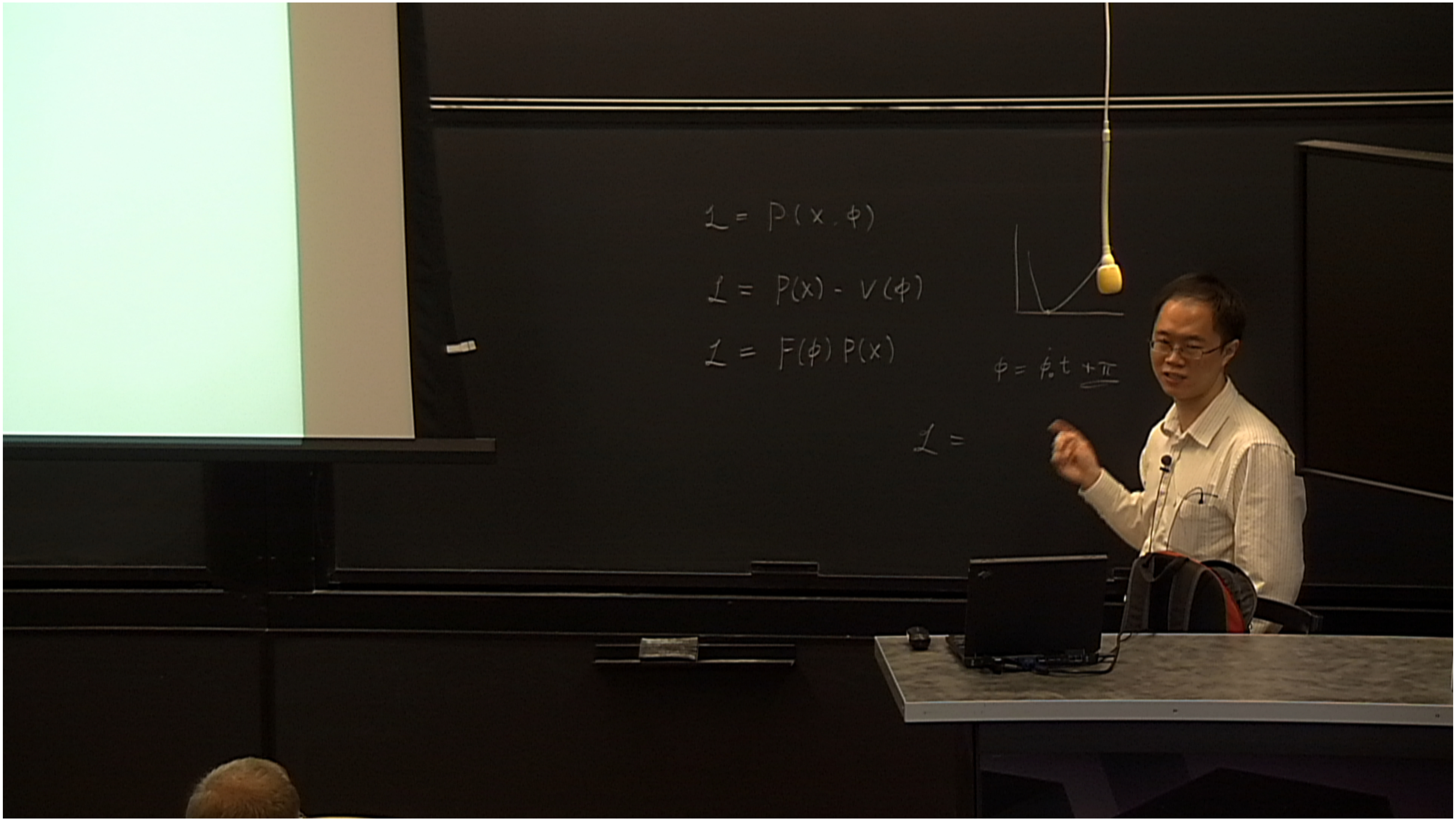
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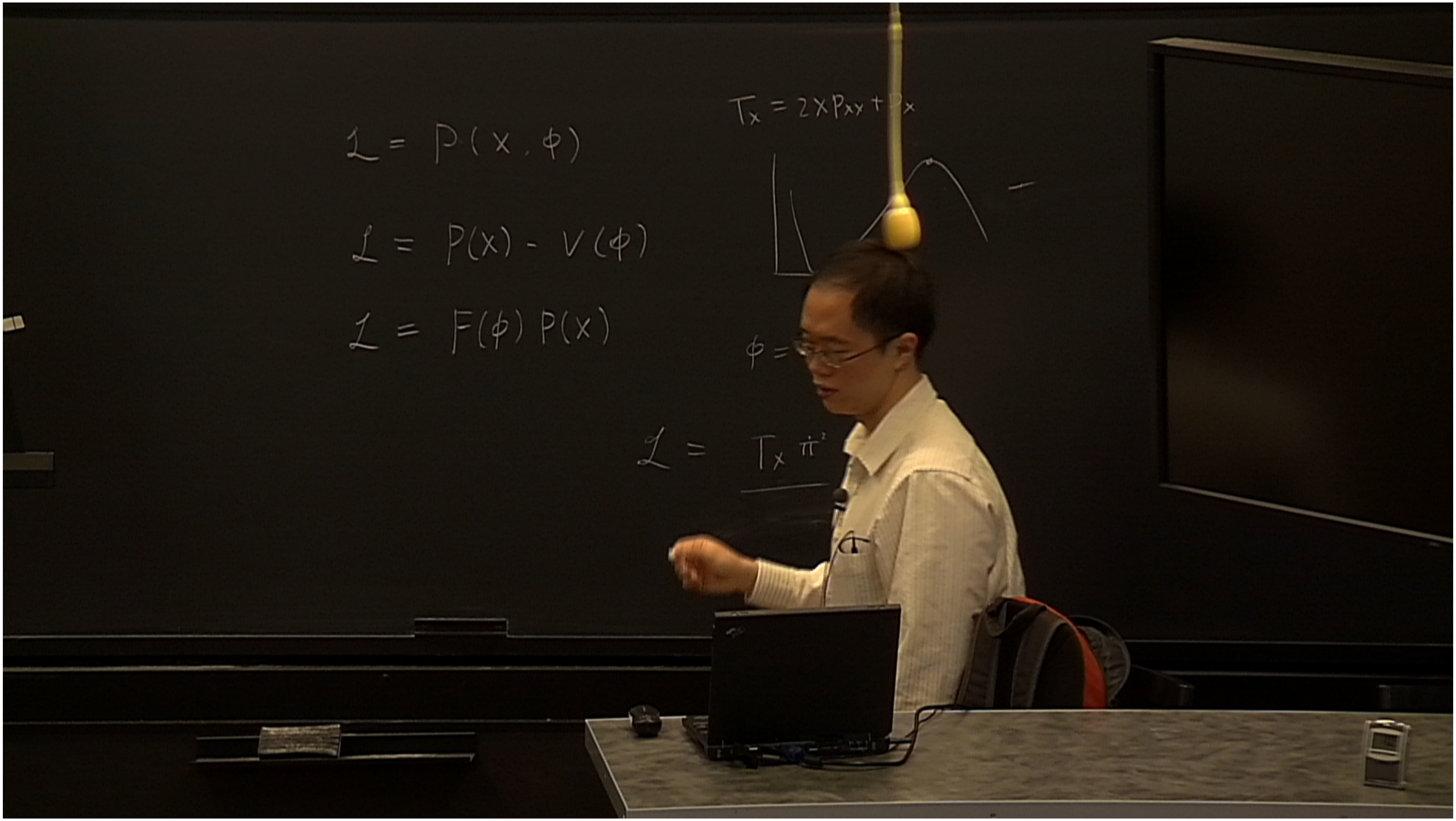
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φ =
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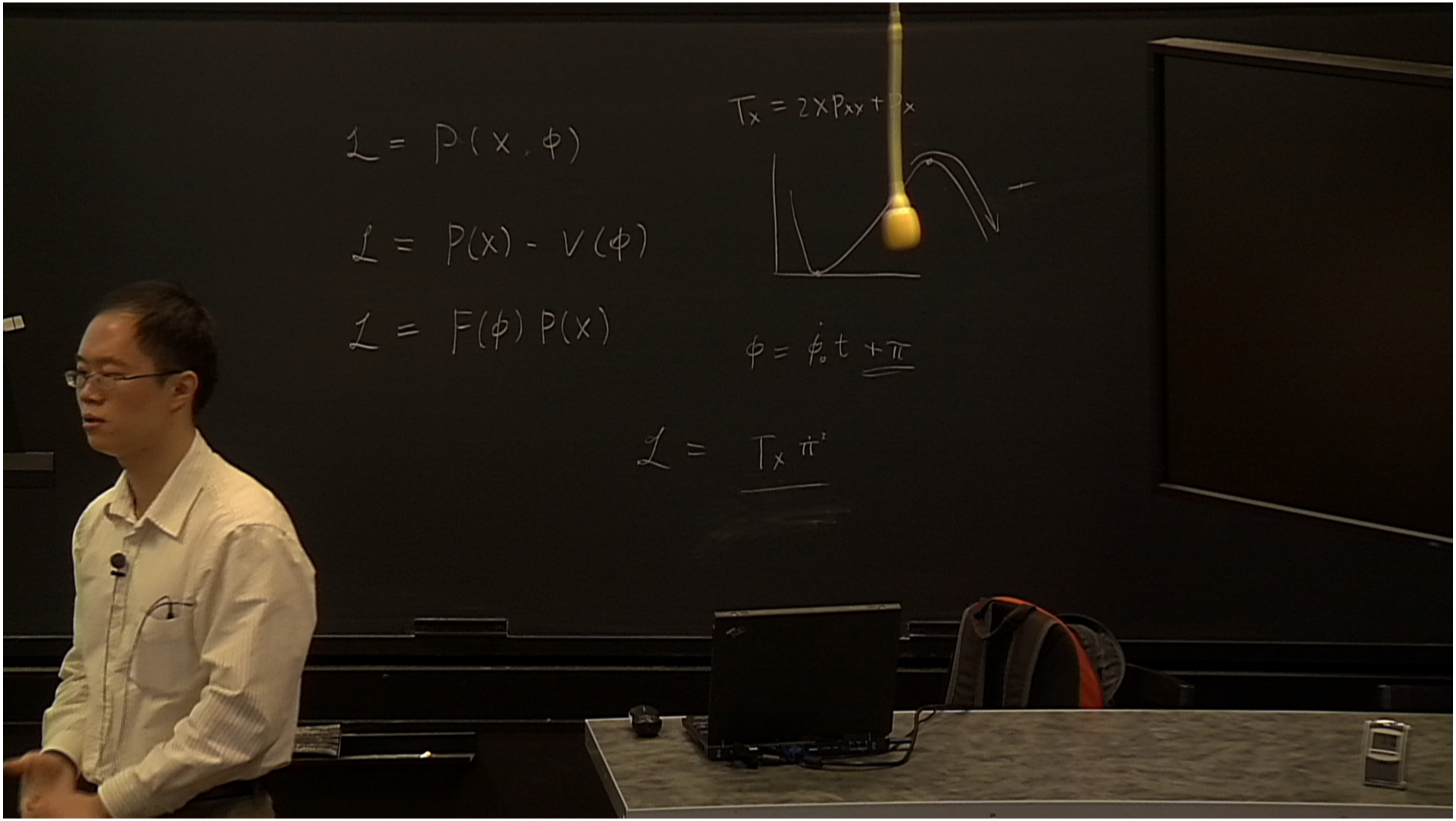

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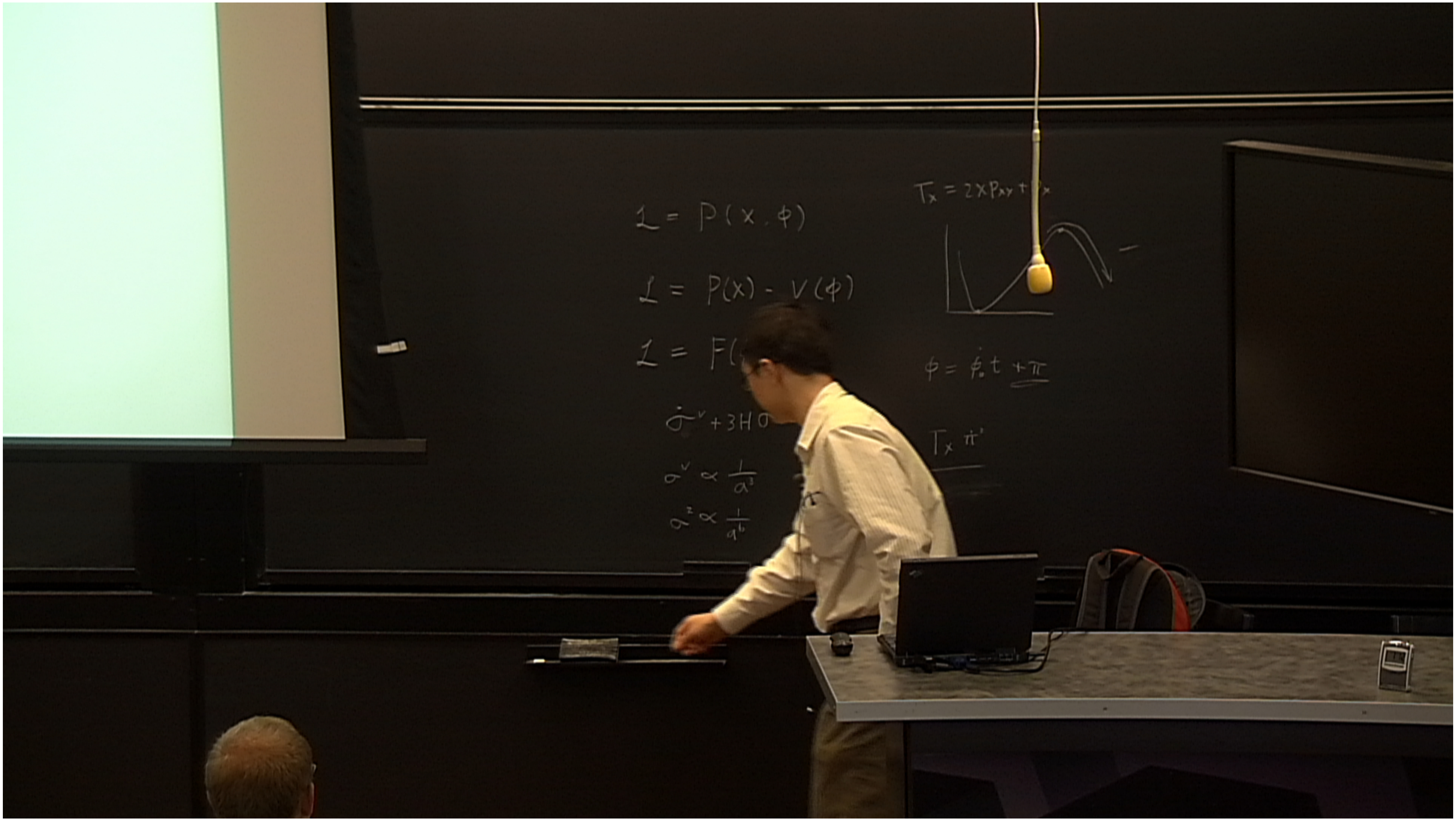
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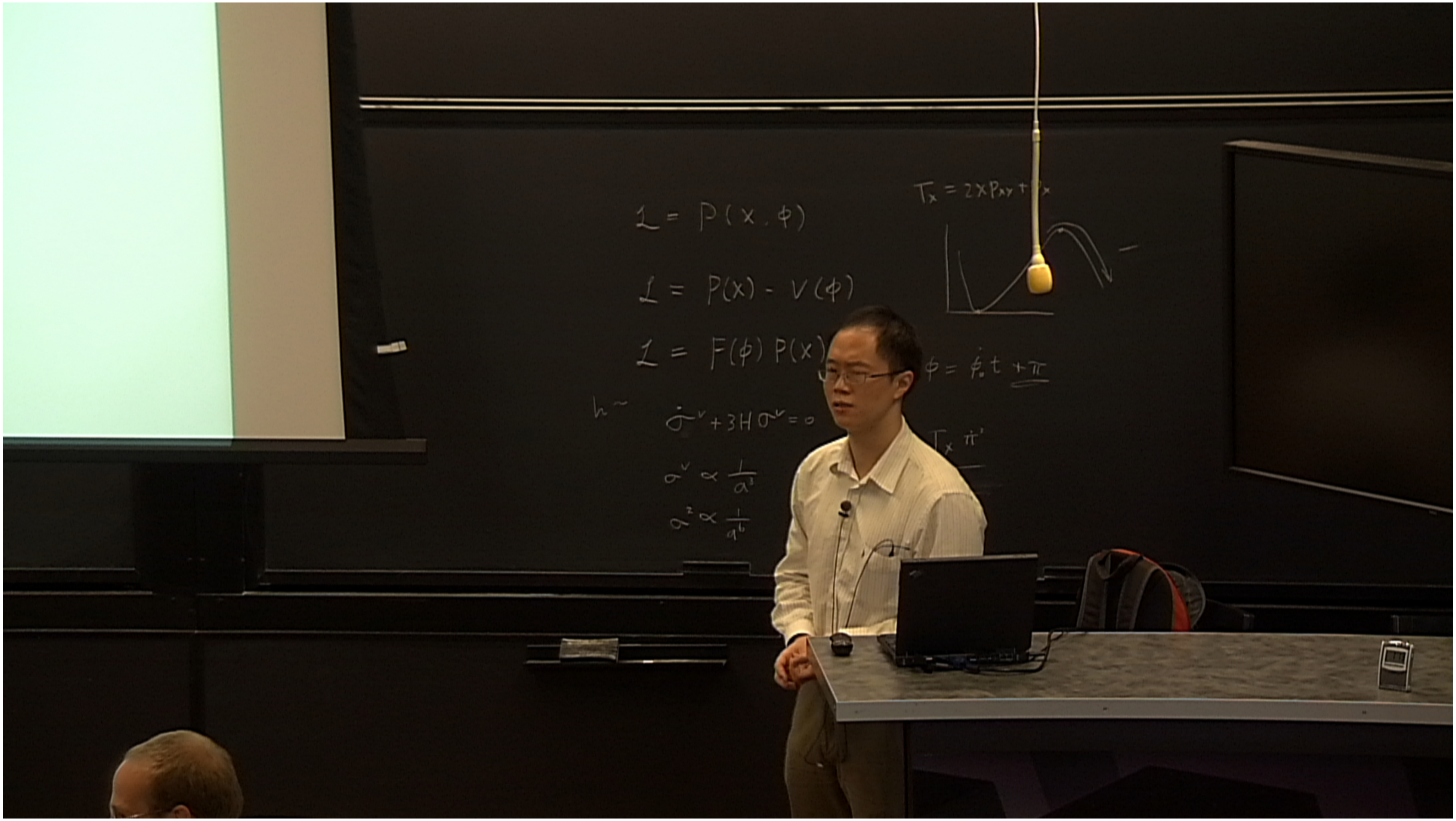
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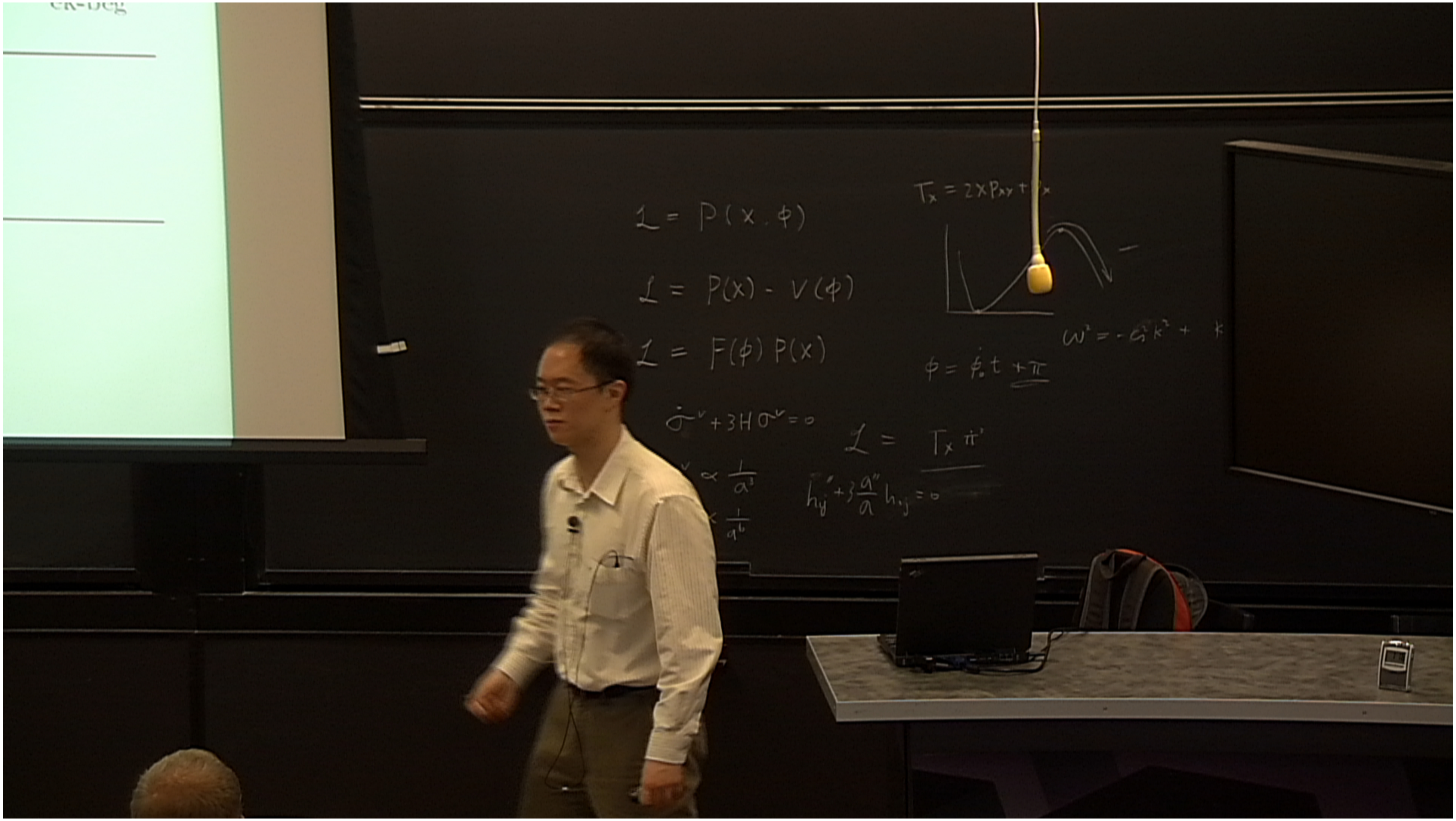
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