Title: Quantum Discord and the Quantum Advantage

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Abstract: Entanglement is a paradigmatic example of quantum correlations, a presumed reason for the superior performance of quantum computation and an obvious divider of states and processes into classical and quantum. In the last decade all these notions were challenged. Entanglement does not capture the totality of non-classical behavior. Quantum discord (in its different versions) is a more general measure of quantum correlations. It can be related to the advantage in some tasks like the extraction of work from a Szilrad heat engine using Maxwell's demons with various resources. The discord turns out to be the difference between the work extracted from a given bipartite system using a a global and a local strategy. Different strategies relate to different definitions of discord, but all definitions agree on zero, so "classical" systems are universal to all heat engines. One way of identifying a task as quantum or classical is by examining the quantum resources required to implement it. This can be done by examining the entanglement required for a LOCC implementation of the task. Creation (or non-creation) of entanglement as a result of its implementation is not enough to identify these resources. An example is a bi-local implementation of an entangling quantum gate (C-NOT) by LOCC with unentagled input and output states. This lack of entanglement does not guarantee the LOCC implementability. A method to determine if entanglement resources are required is to track the change in quantum discord during the process.

Quantum Discord and the Quantum Advantage

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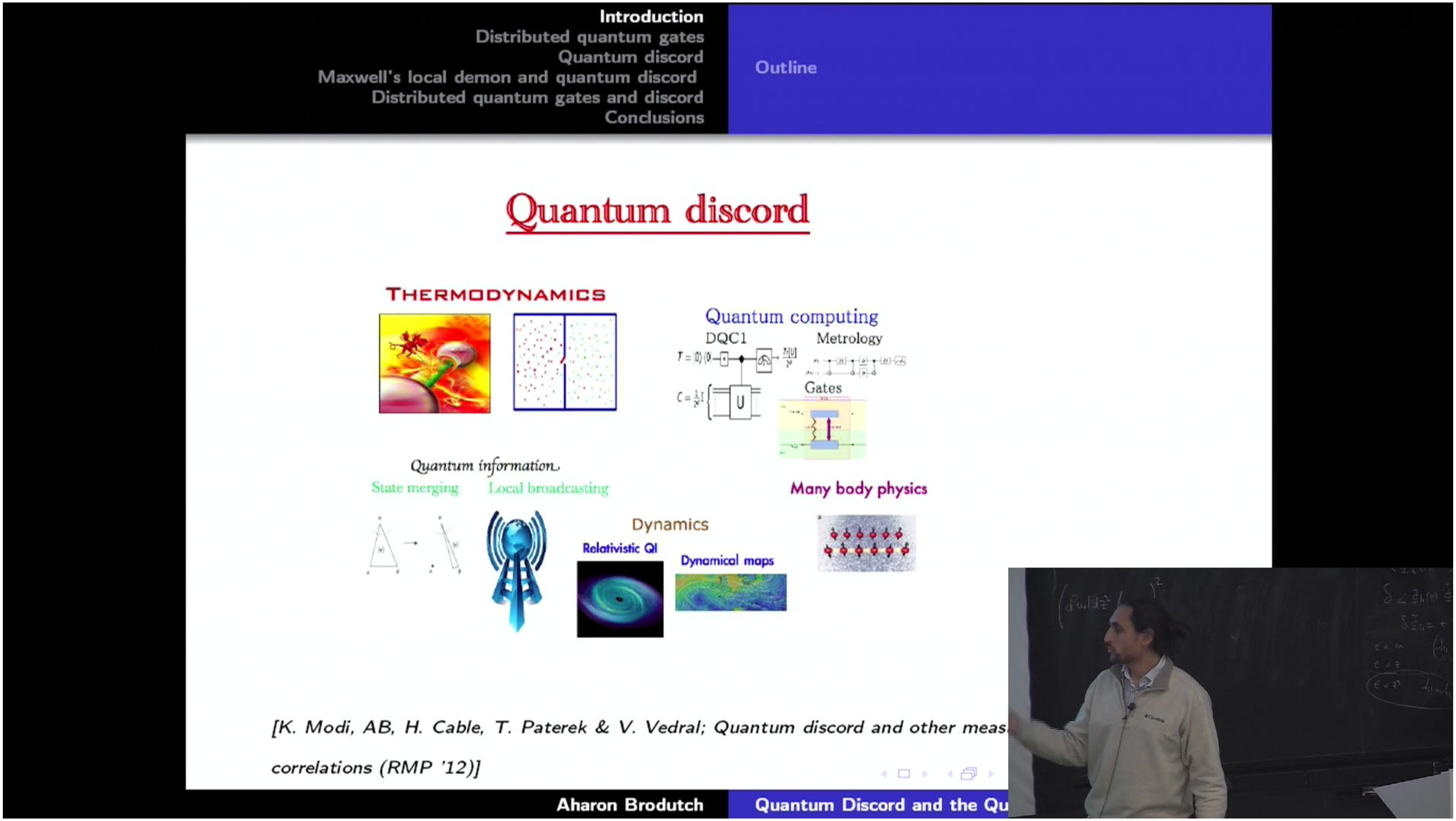
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[K. Modi, AB, H. Cable, T. Paterek & V. Vedral; Quantum discord and other meas
correlations (RMP '12)]
Aharon Brodutch
Quantum Discord and the Qu

- Our *quantum* resource in the LOCC paradigm is *entanglement*
- Separable states usually contain some quantum correlations
- Quantum correlations are an indication of a *quantum advantage*
- Intuition: adding noise to a maximally entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right] \longrightarrow \frac{1-z}{4}\mathbb{I} + z\,|\psi\rangle\langle\psi|$$

Always quantum correlated but separable for $z < 1/3$.

Outline

- Distributed quantum gates
- Quantum discord
- Quantum discord and Maxwell's demons
- Discord in quantum information: distributed gates.

Discord as an indicator of quantumness

- In pure state quantum computing entanglement must scale with the problem for a *quantum advantage*. However mixed state quantum computation may not require entanglement *[R. Jozsa and N. Linden PRS A '03]*

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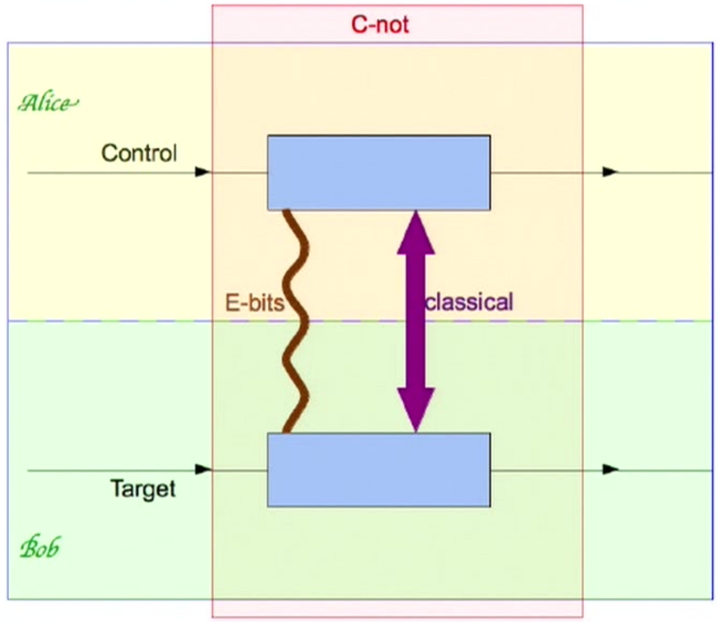
- Discord is indeed present in DQC1 where entangl vanishingly small. It is probably the reason behinc *[A. Datta, A. Shaji & C. Caves. PRL '08]*

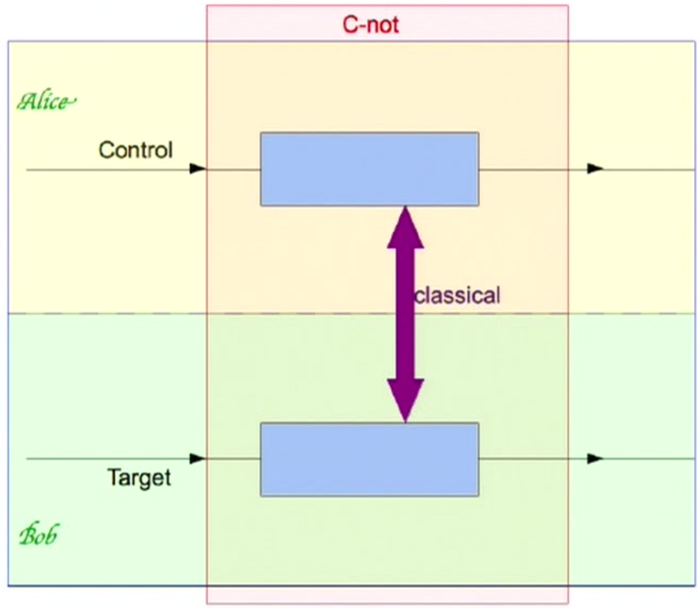
The classical c-not gate (control,target)	The quantum c-not gate (control,target)
Input $\to$ *output*	*Input* $\to$ *output*
$00 \to 00$	$\lvert 0\psi\rangle \to \lvert 0\psi\rangle$
$01 \to 01$	$\lvert X_+ 0\rangle \to [\lvert 11\rangle + \lvert 00\rangle]/\sqrt{2}$
$10 \to 11$	$\lvert \psi X_+\rangle \to \lvert \psi X_+\rangle$
$11 \to 10$	$\lvert 1X_-\rangle \to \lvert 0X_-\rangle$

The distributed c-not



2 classical bits and 1 e-bit

The distributed c-not



What can we do without entanglement?

We try to restrict the input to separable states.

$$|0\rangle\,|\psi\rangle \rightarrow |0\rangle\,|\psi\rangle$$
$$|1\rangle\,|\psi\rangle \rightarrow |1\rangle\,\sigma_x\,|\psi\rangle$$

$$|\psi\rangle\,|X_+\rangle \rightarrow |\psi\rangle\,|X_+\rangle$$
$$|\psi\rangle\,|X_-\rangle \rightarrow \sigma_z\,|\psi\rangle\,|X_-\rangle$$

#	State	#	State
a	$\|1\rangle\|Y_+\rangle \rightarrow \|1\rangle\|Y_-\rangle$	c	$\|Y_+\rangle\|X_-\rangle \rightarrow \|Y_-$
b	$\|0\rangle\|Y_+\rangle \rightarrow \|0\rangle\|Y_+\rangle$	d	$\|Y_+\rangle\|X_+\rangle \rightarrow \|Y_+$

#	State
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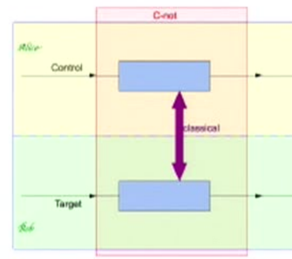


- In an LOCC protocol Alice and Bob can always know what operation they performed.
- The gate should either "flip" or "not flip" a Y_+state on either Alice's or Bob's side.
- But these operations are incompatible, so Alice and Bob can know what the input state was.

example

#	State
a	$\|1\rangle\|Y_+\rangle \rightarrow \|1\rangle\|Y_-\rangle$
b	$\|0\rangle\|Y_+\rangle \rightarrow \|0\rangle\|Y_+\rangle$
c	$\|Y_+\rangle\|X_-\rangle \rightarrow \|Y_-\rangle\|X_-\rangle$
d	$\|Y_+\rangle\|X_+\rangle \rightarrow \|Y_+\rangle\|X_+\rangle$

Alice	Bob F	Bob N
F	$\{a, c\}$	$\{c, b\}$
N	$\{a, d\}$	$\{b, d\}$

We are now assured that the next operation would be a "flip-flip"

example

#	State
a	$\|1\rangle\|Y_+\rangle \rightarrow \|1\rangle\|Y_-\rangle$
b	$\|0\rangle\|Y_+\rangle \rightarrow \|0\rangle\|Y_+\rangle$
c	$\|Y_+\rangle\|X_-\rangle \rightarrow \|Y_-\rangle\|X_-\rangle$
d	$\|Y_+\rangle\|X_+\rangle \rightarrow \|Y_+\rangle\|X_+\rangle$

Alice	Bob	
	F	N
F	$\{a \quad c\}$	$\{b \quad c\}$
N	$\{a \quad d\}$	$\{b \quad d\}$

- A protocol which would allow Alice and Bob to implement this gate without entanglement will allow discrimination between the 4 non orthogonal states.
- We can see that a restricted version of the c-not gate with separable input-output states cannot be implemented without entanglement. *[AB & D. Terno PRA 2011]*

Quantum discord

Before a measurement

After a measurement

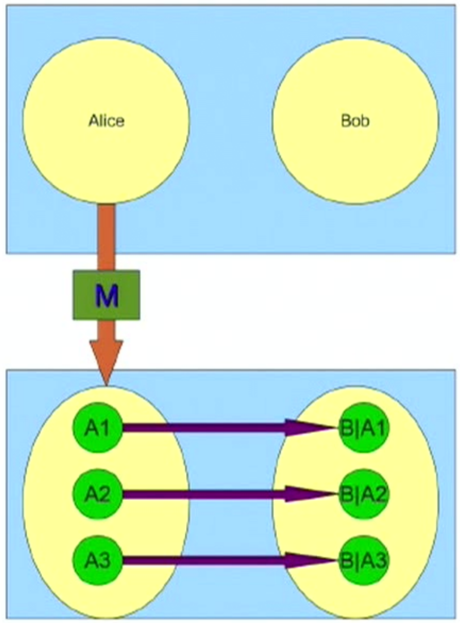
Quantum discord







$S(AB)$

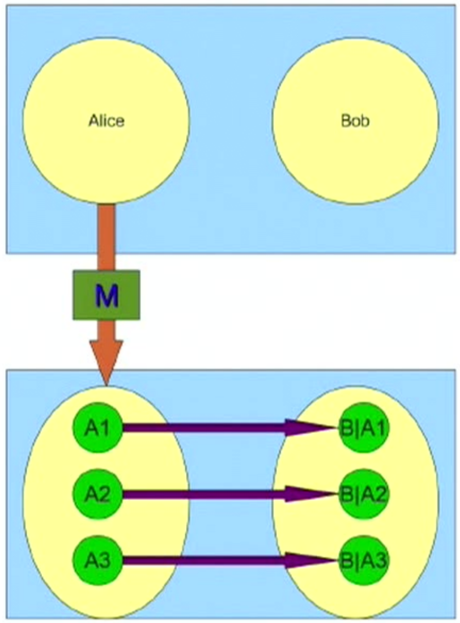
$S(A)$  $S(B)$

$S(B|A) = \sum_i p_i S(B|Ai)$

$$J(B|A) = S(B) - S(B|A) = S(A) + S(B) - S(AB)$$

$S(AB)$

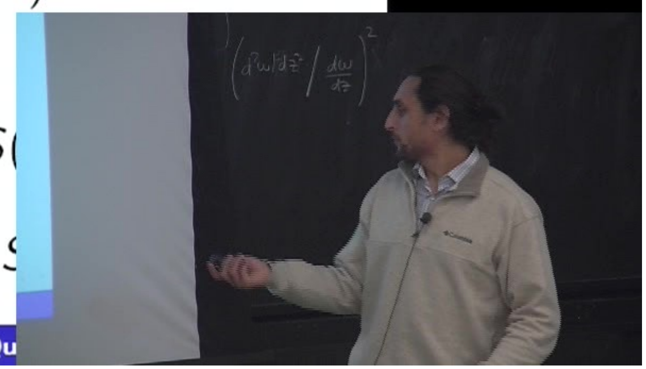
$S(A)$ $S(B)$



$S(B|A) = \sum_i p_i S(B|Ai)$

$J(B|A) = S(B) - S(B|A) = S(A) + S(B) - S($

$J(B|M_A) = S(B) - S(B|M_A) \neq S(A) + S(B) - S$



- Mutual information:

$$I(A:B) := S(A) + S(B) - S(AB)$$

- Classical correlations:

$$J(B|A) := S(B) - \min_{M_A} S(B|M_a)$$

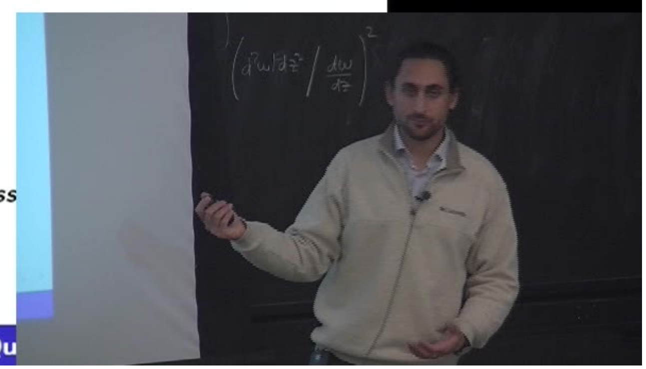
[L. Henderson & V. Vedral "Classical, quantum and total correlations" JPA 2001]

- Quantum Discord:

$$D(B|A) := I(A:B) - J(B|A)$$

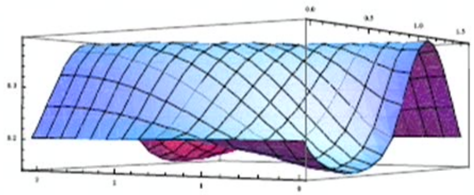
[H. Ollivier & W. Zurek "Quantum Discord: A Measure of the Quantumness

PRL 2001]



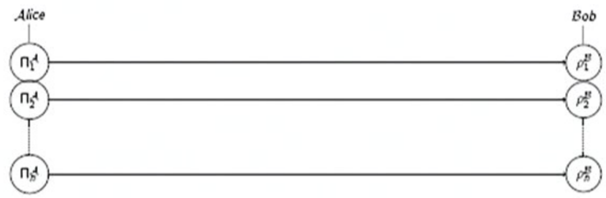
Examples of discord

1. $\boxed{\frac{1}{2}[|00\rangle\langle 00| + |1+\rangle\langle 1+|]}$ $D(A|B) \approx 0.2$; $D(B|A) = 0$
2. $\boxed{\frac{1}{2}[|00\rangle\langle 00| + |++\rangle\langle ++|]}$ $D(A|B) = D(B|A) \approx 0.14$



Discord is not symmetric & Separable states can have non zero discord

A (zero discord) *classical* state is a sum of orthogonal states pointing from Alice to Bob: $\chi_{AB} = \sum_j a_j \Pi_j^A \otimes \rho_j^B$



After a (von Neumann) measurement

$$\rho_{AB} \to \mathcal{M}(\rho_{AB}) = \chi_{AB}$$

For a classical state

$$\mathcal{M}_A(\chi_{AB}) = \sum_i \Pi_i \chi_{AB} \Pi_i = \chi_{AB}$$

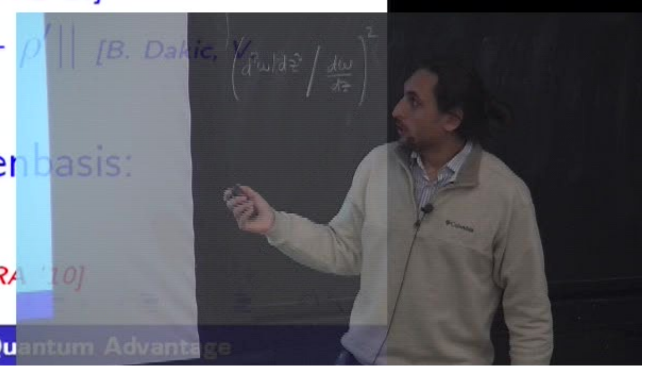
Almost all states are not classical

Types of discord

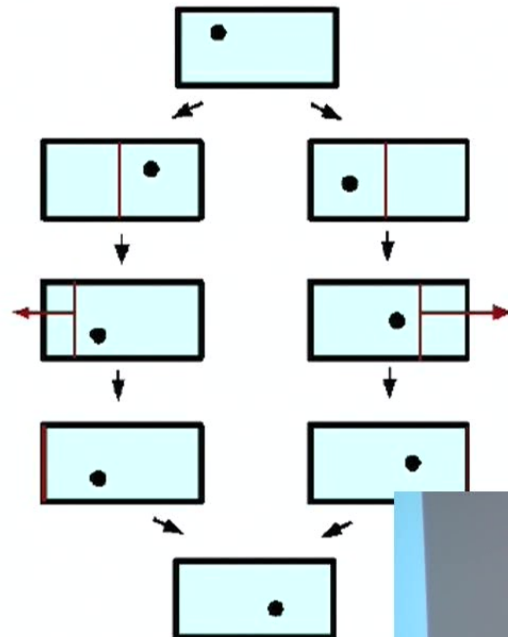
We can redefine discord using $\rho \rightarrow \mathcal{M}_A(\rho) = \rho'$

- The change in MI: $D(B|A) = I(\rho) - \max_{\mathcal{M}_A}[I(\rho')]$ *[W. Zurek AdP '00]*
- The change in entropy: $D_{th}(B|A) = \min_{\mathcal{M}_A} S(\rho') - S(\rho)$ *[W. Zurek PRA '03]*
- The relative entropy from a classical state: $D_R(B|A) = \min_\chi S(\rho|\chi) = D_{th}(B|A)$ *[K. Modi et al. PRL '10]*
- The geometric distance $D_G(B|A) = \min_{\mathcal{M}_A} ||\rho - \rho'||$ *[B. Dakic, V. Vedral, & C. Brukner PRL '10)]*
- The change when the measurement is in the eigenbasis: $\mathcal{M}_{\mathcal{A}}(\rho^A) = \rho^A$:

All vanish simultaneously! *[AB & D.Terno PRA '10]*



Maxwell's demon and the Szilard engine



Resetting the measurement device costs work proporti
entropy of the measurement.

Alice, Bob and Charlie

The players

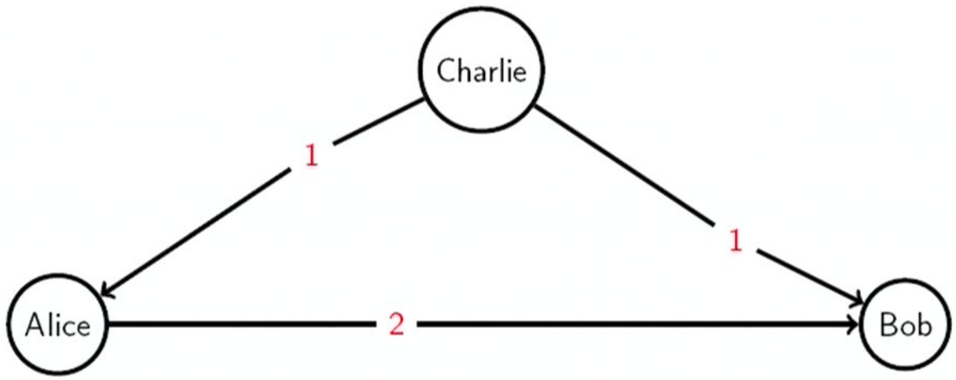
- Alice the goblin - can implement a Szilard engine on her side.
- Bob the goblin - can implement a Szilard engine on her side .
- Charlie the all-knowing demon - knows the initial density matrix and can send information to both Alice and Bob.

The rules

1. Classical (local) strategy - Charlie sends information to Alice and Bob. Alice can send information to Charlie.
2. Quantum (non-local) rules - Charlie can implement a Szilard engine on both system.

Alice and Bob start your engines!

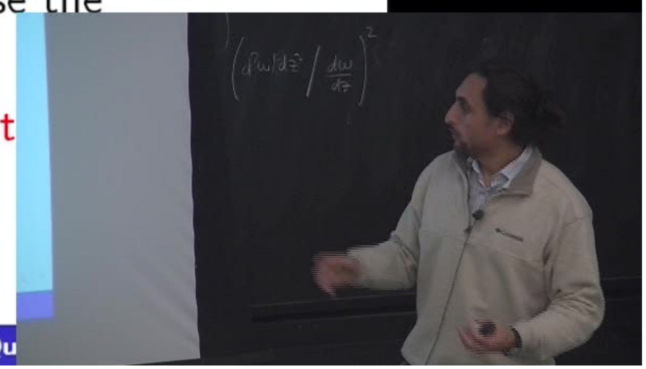
Local Rules.



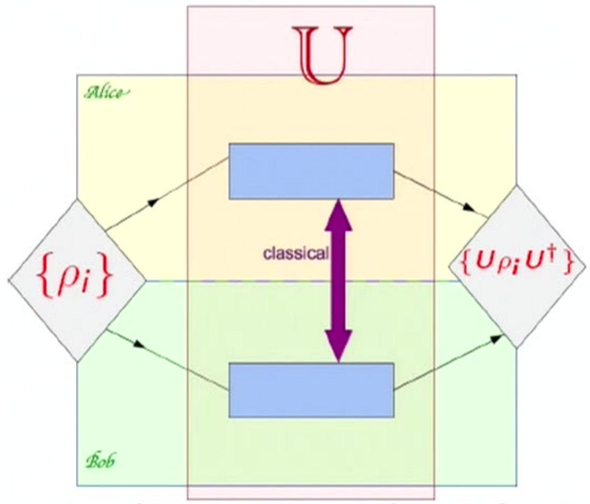
The net work is given by $k_b T[\log(d_A d_B) - S(M_A) - S(B|M_A)]$ where $S(M_A)$ is the entropy of Alice's measurement results

Charlie is a non-local demon

- If Charlie the is a non-local demon he can use a non-local measurement and do more work.
- The net work he can do is $k_b T[\log(d_A d_B) - S(AB)]$
- The difference between these quantum and classical strategies is given by the discord. When Alice has all the information she can optimize and the deficit is $D_{th}(B|A)$
- When Alice has only local information she will use the measurement $\mathcal{M}(\rho_a) = \rho_a$.
- Discord quantifies the advantage of a *quantum* st
- For classical states there is no advantage



Distributed quantum gates

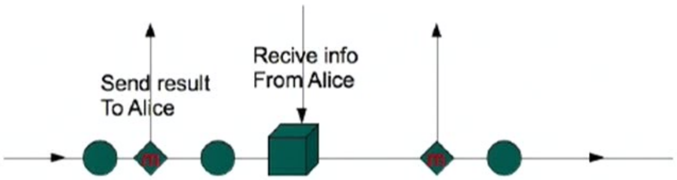


Impossible to implement on a discordant set of pu

if discord changes

[AB & D. Terno PRA '11]

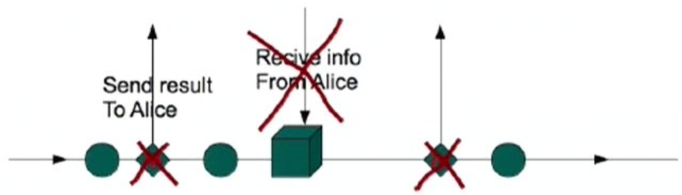
Gates and discord



What can Bob do?

- Perform some operation
- Make a measurement and send information to Ali
- Perform some operation which depends on inform received from Alice

Gates and discord



What can Bob do?

- Perform some operation
- ~~Make a measurement and send information to Ali~~
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~~received from Alice~~

conclusions

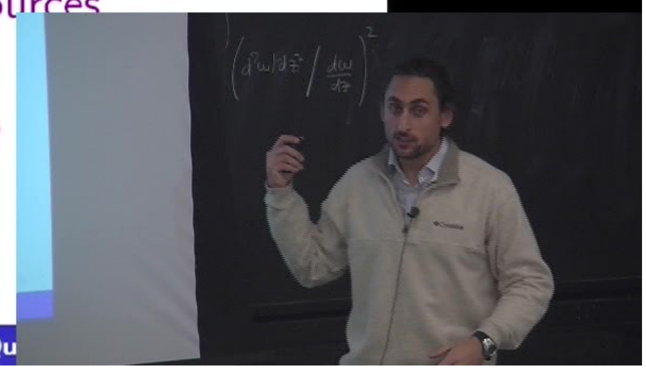
Almost all bi-partite quantum states contain quantum correlations.
The state

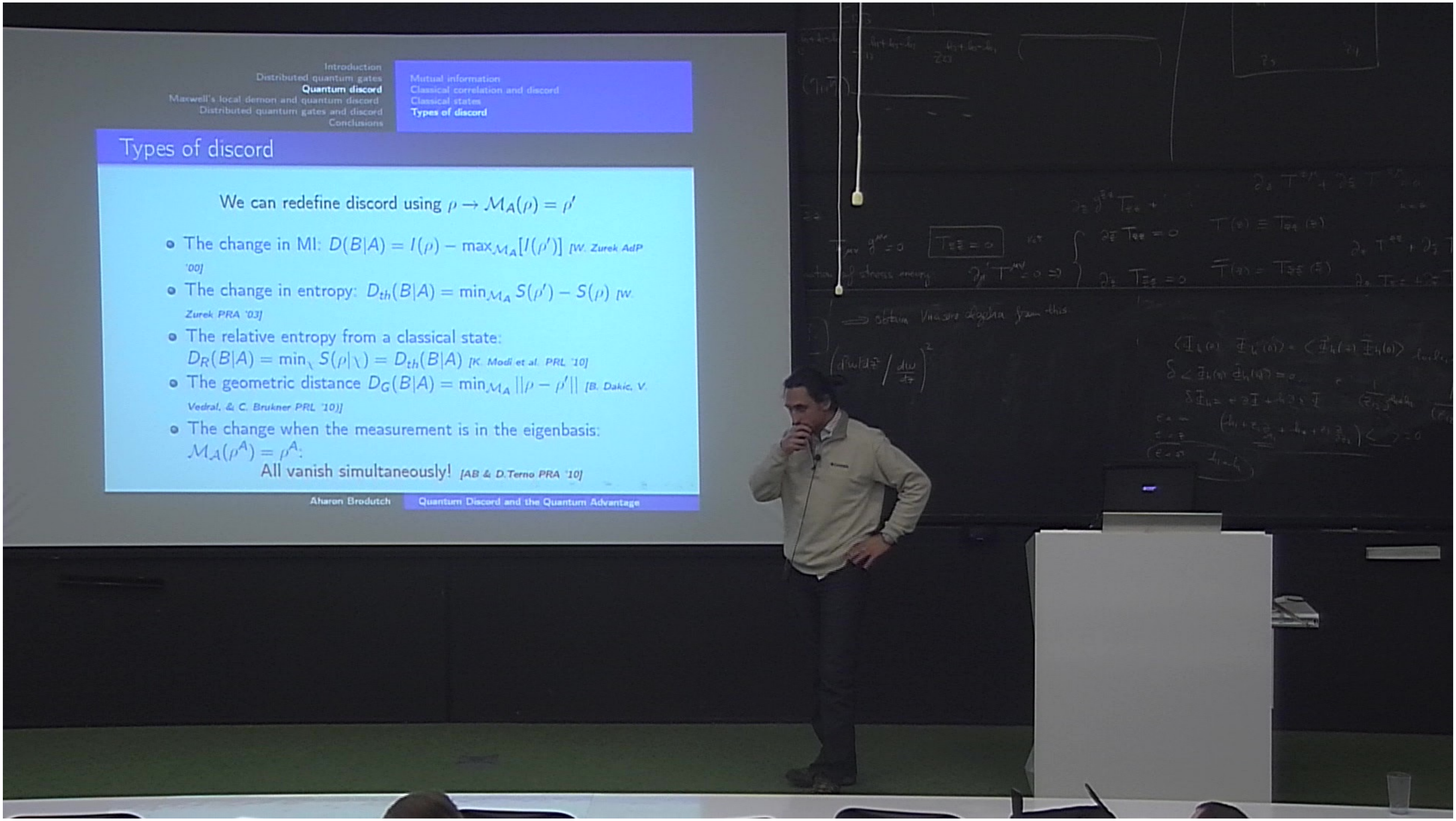
$$\frac{1-z}{4}\mathbb{I} + z\left|\psi\right\rangle\left\langle\psi\right|$$

has non zero discord for all values of $z > 0$.

Quantum correlations are an indication that an operation on the system cannot be implemented without quantum resources (entanglement).

Can this be used to identify the *quantum advantage?*

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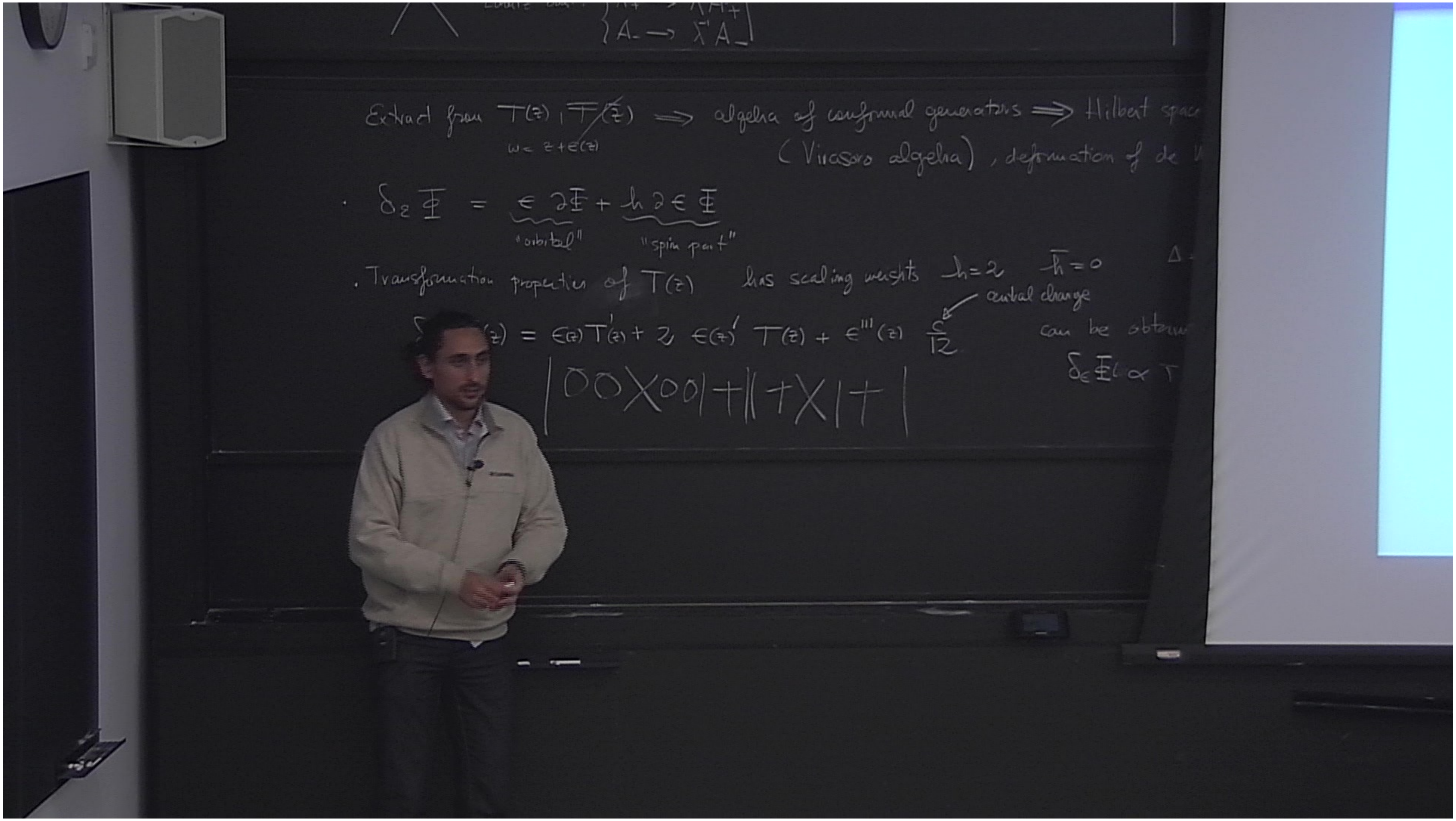
Extract from T(z), T̄(z̄) ⟹ algebra of conformal generators ⟹ Hilbert space
w = z + ε(z)
(Virasoro algebra), deformation of de
"orbital"
"spin part"
Transformation properties of T(z) has scaling weights h = 2 h̄ = 0
central charge
can be obtained

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$$I(A:B) := S(A) + S(B) - S(AB)$$

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