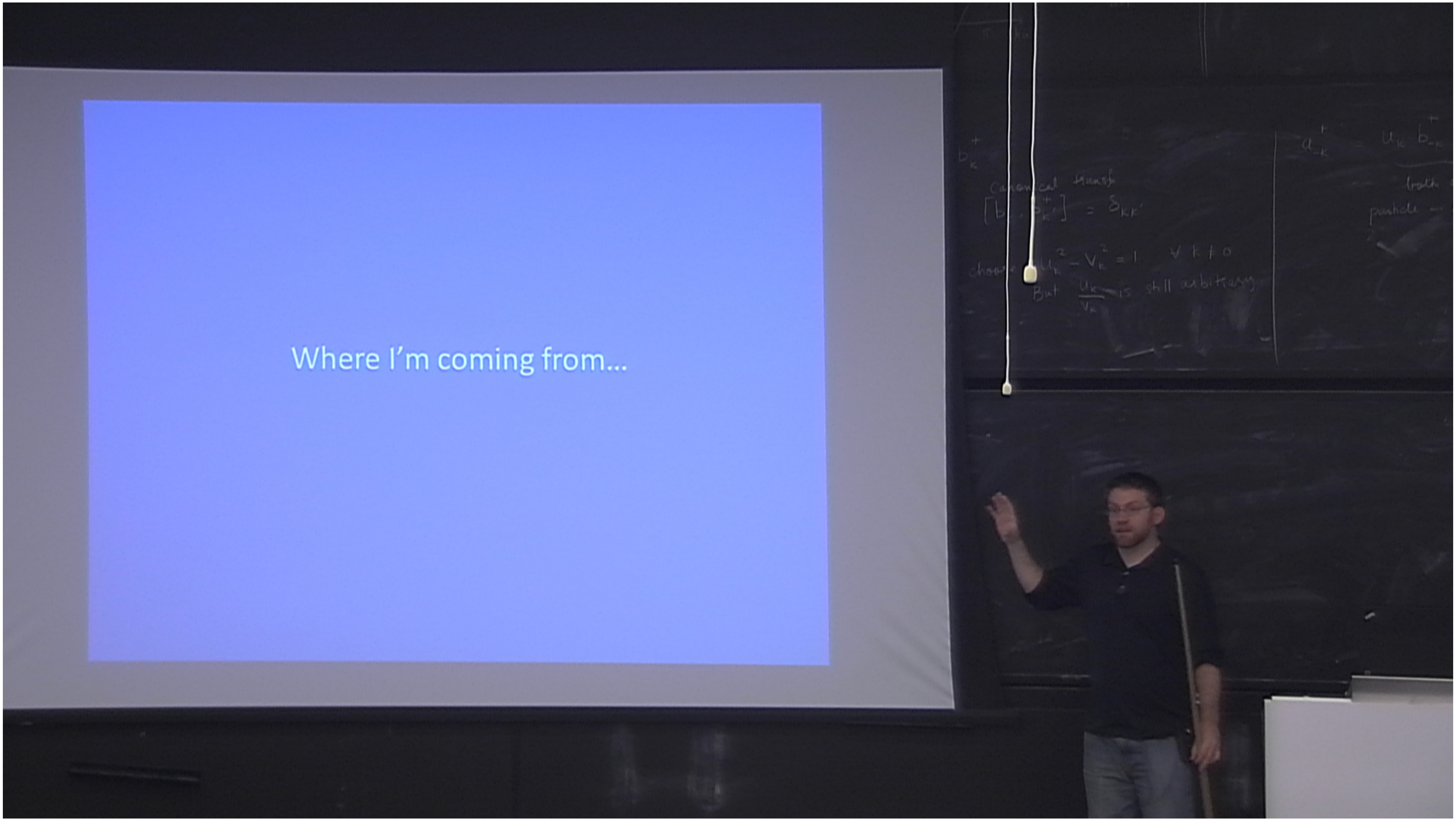
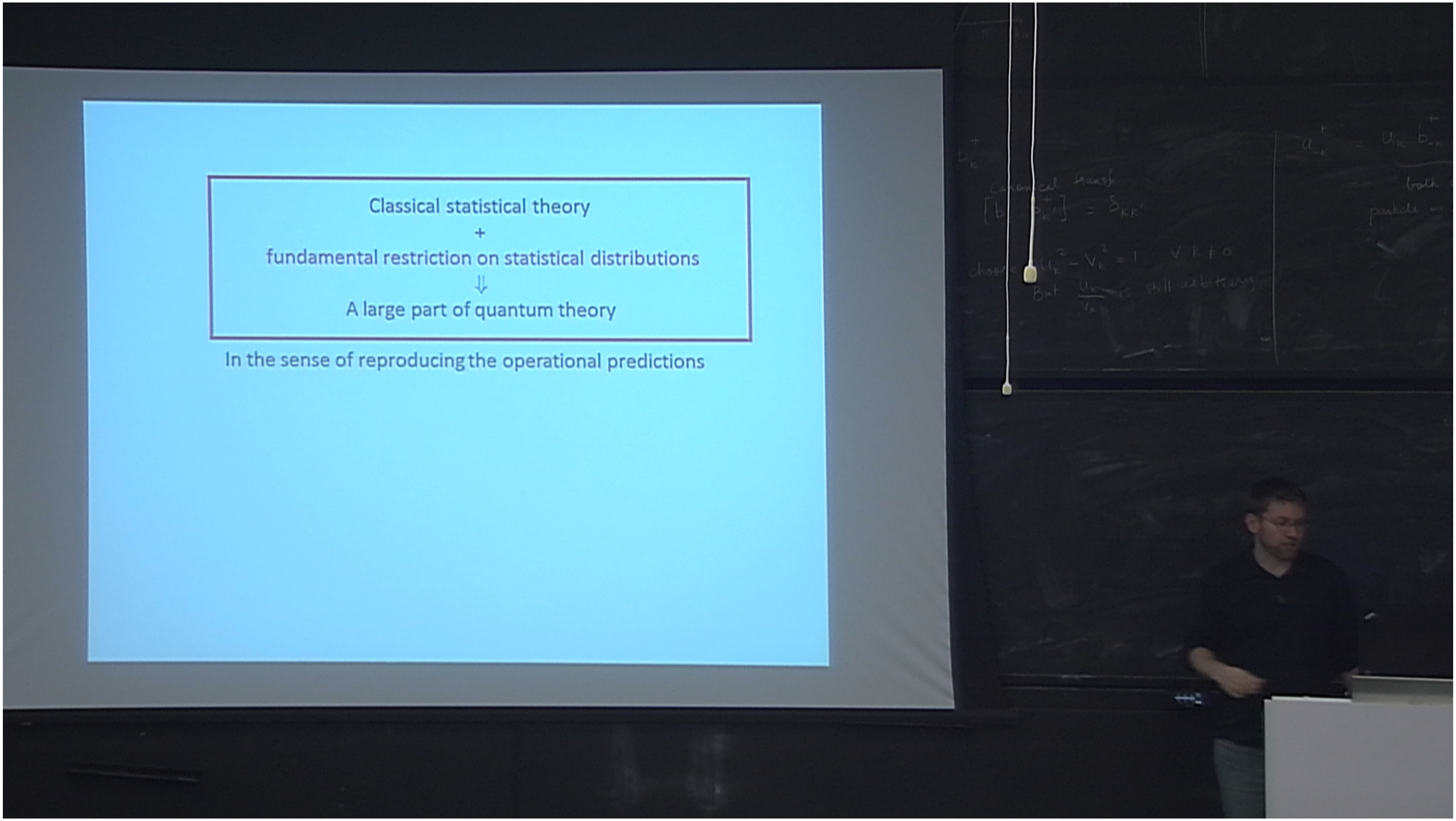
Title: Formulating Quantum Theory as a Causally Neutral Theory of Bayesian Inference

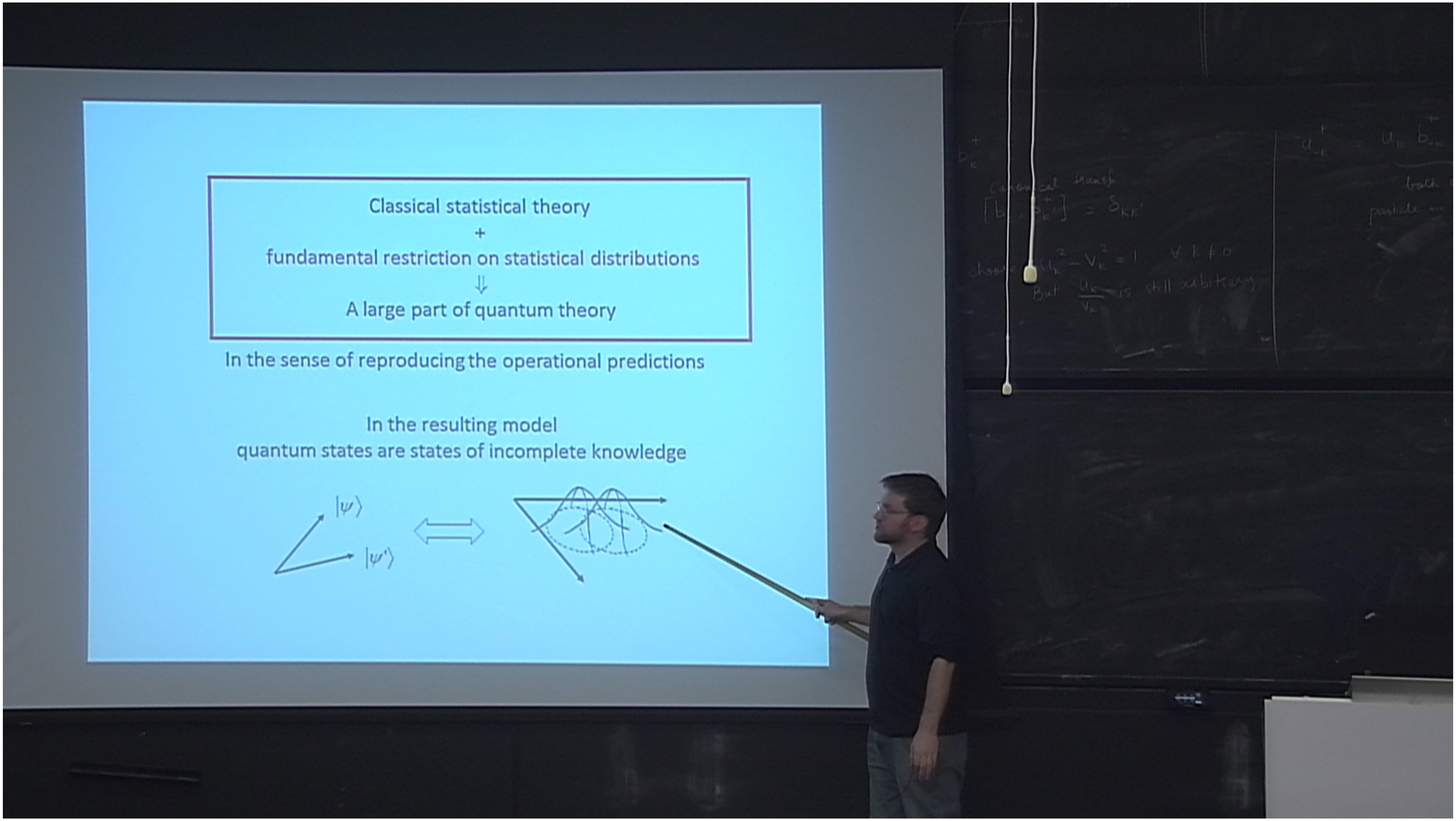
Date: Nov 08, 2011 03:30 PM

URL: http://www.pirsa.org/11110114

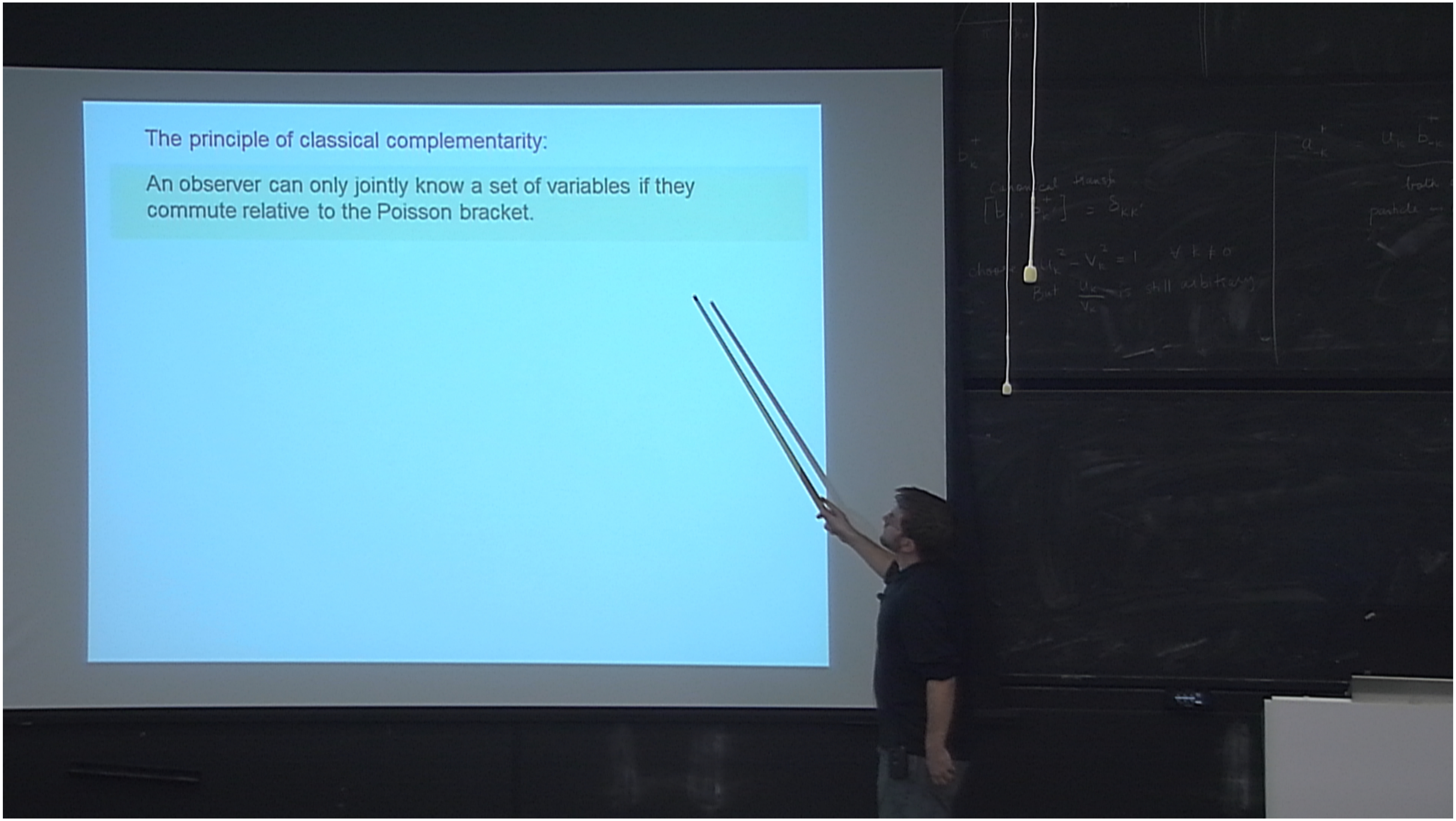
Abstract: Quantum theory can be thought of as a noncommutative generalization of Bayesian probability theory, but for the analogy to be convincing, it should be possible to describe inferences among quantum systems in a manner that is independent of the causal relationship between those systems. In particular, it should be possible to unify the treatment of two kinds of inferences: (i) from beliefs about one system to beliefs about another, for instance, in the Einstein-Podolsky-Rosen or "quantum steering" phenomenon, and (ii) from beliefs about a system at one time to beliefs about that same system at another time, for instance, in predictions or retrodictions about a system undergoing dynamical evolution or undergoing a measurement. I will present a formalism that achieves such a unification by making use of "conditional quantum states", a noncommutative generalization of conditional probabilities. I argue for causal neutrality by drawing a comparison with a classical statistical theory with an epistemic restriction. (Joint work with Matthew Leifer).

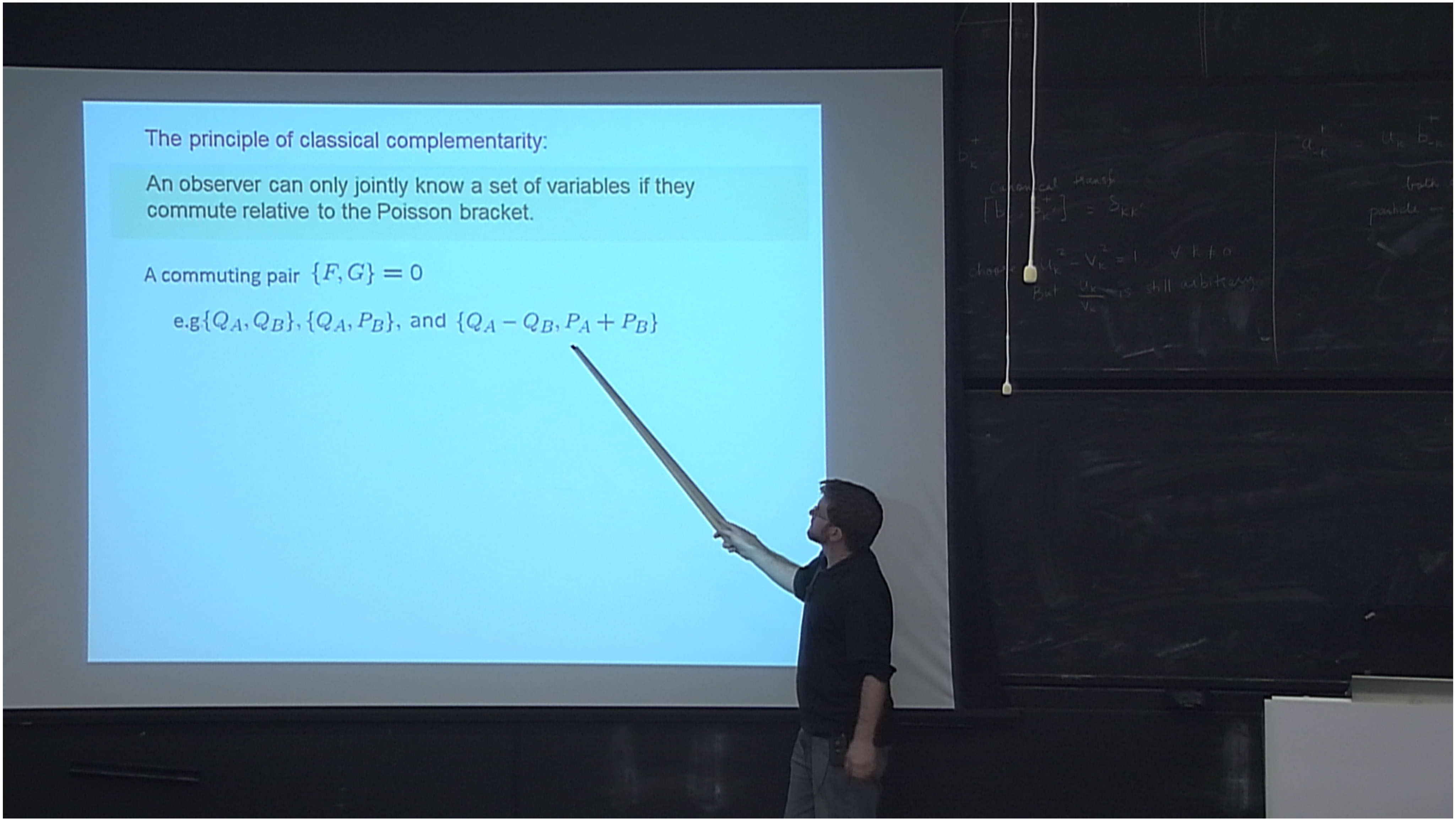

Where I'm coming from…

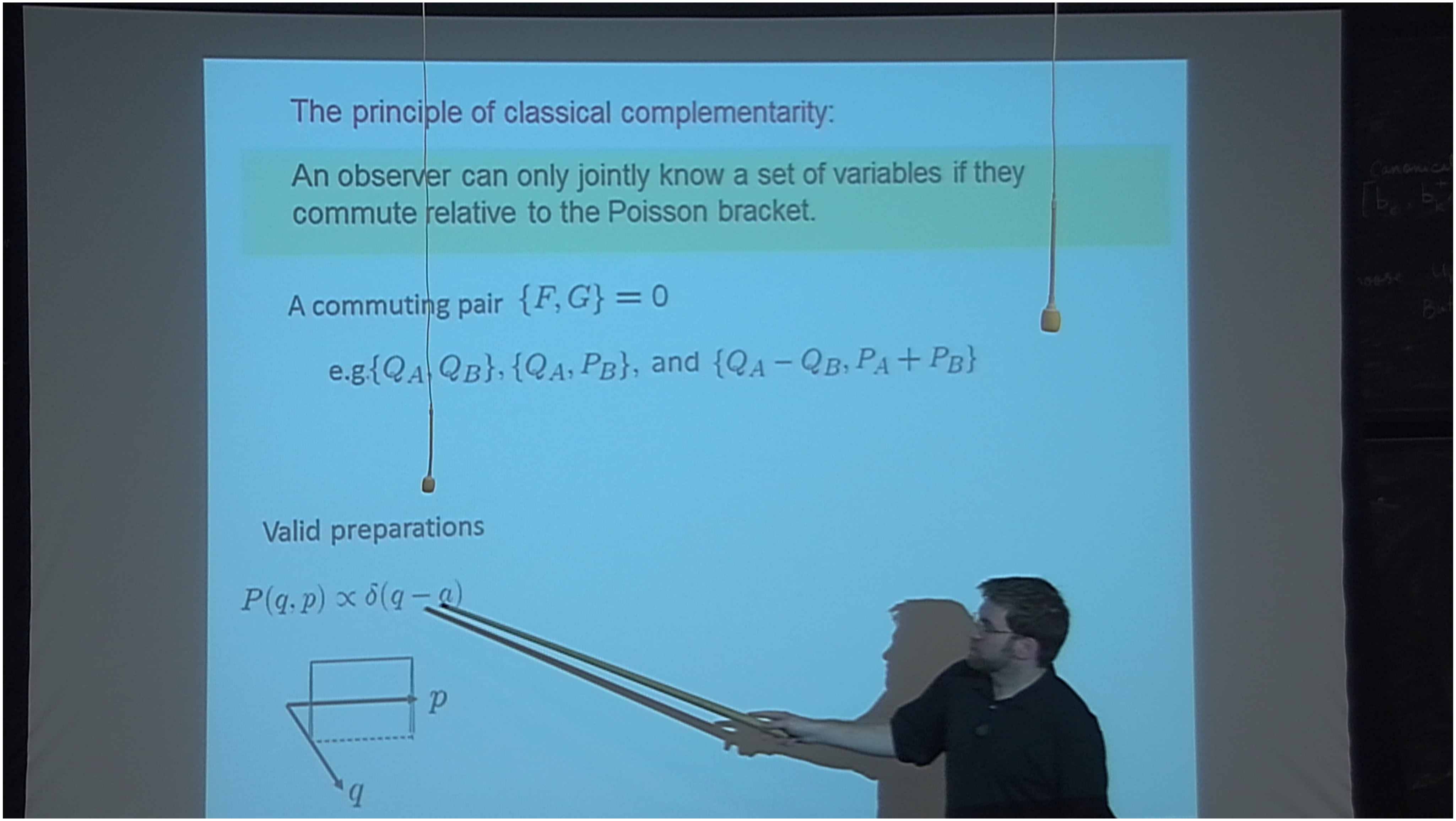

Classical statistical theory
+
fundamental restriction on statistical distributions
⇓
A large part of quantum theory
In the sense of reproducing the operational predictions

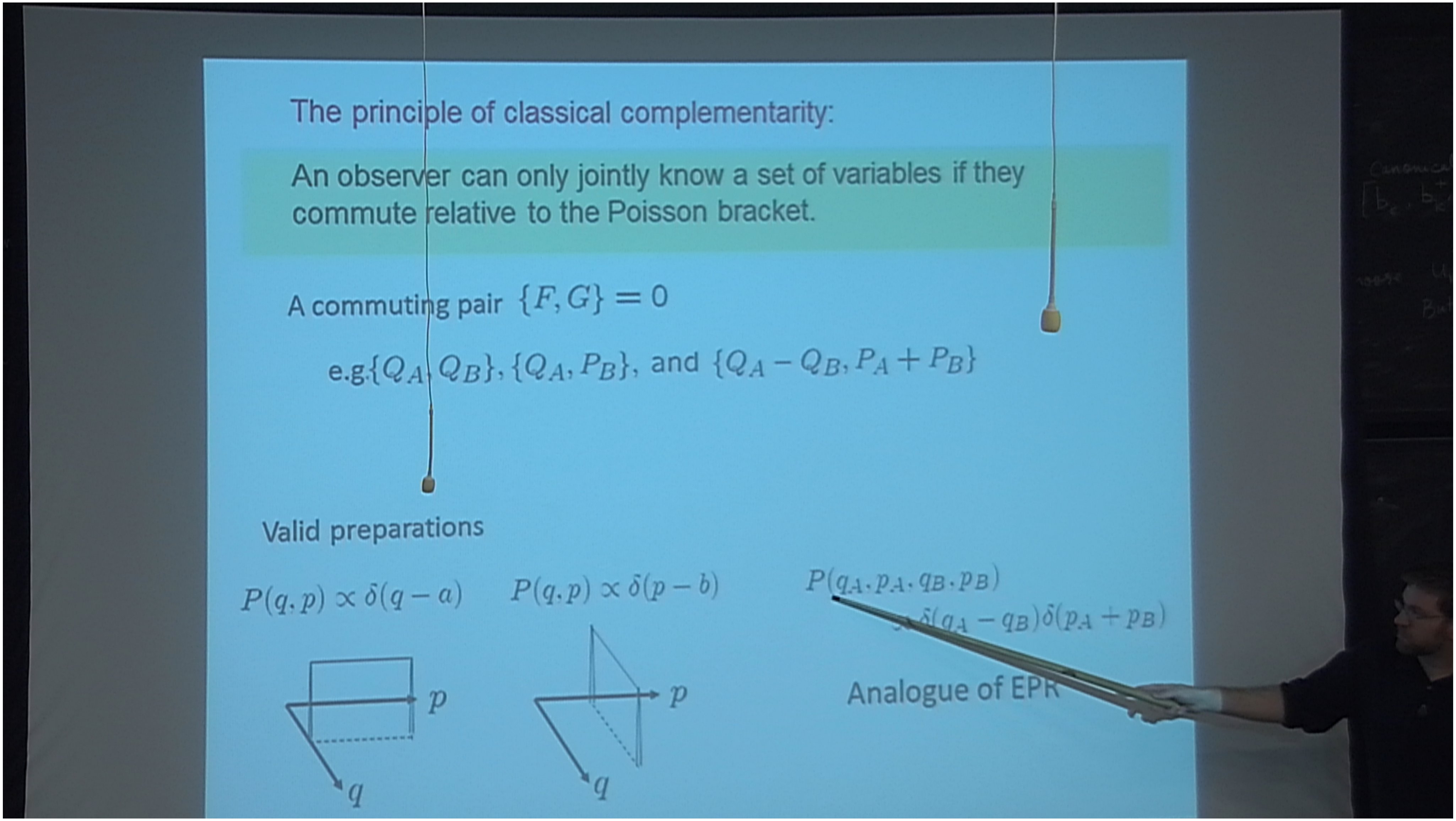


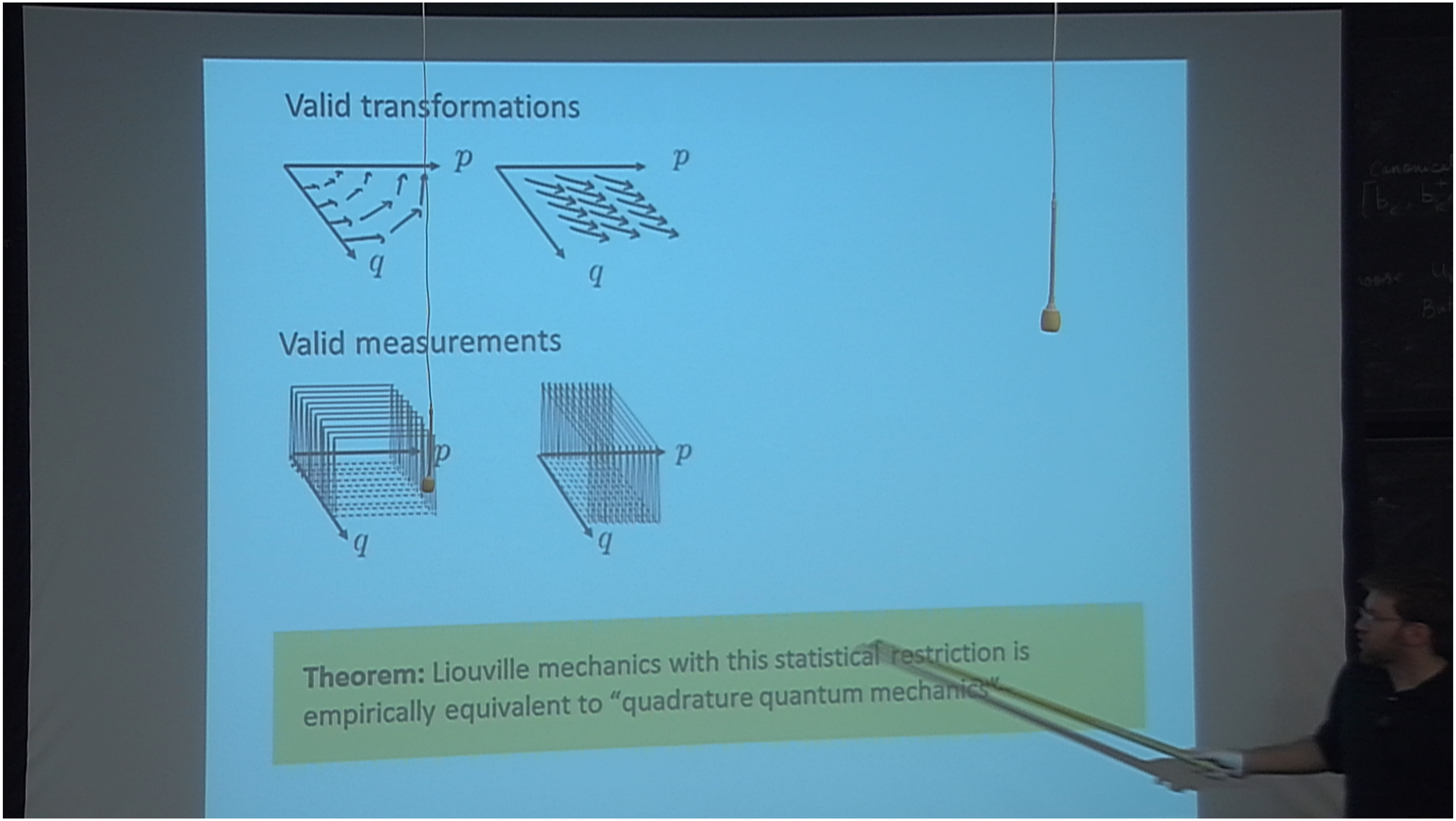
Classical statistical theory
+
fundamental restriction on statistical distributions
⇓
A large part of quantum theory
In the sense of reproducing the operational predictions
In the resulting model
quantum states are states of incomplete knowledge
$|\psi\rangle$
$|\psi'\rangle$

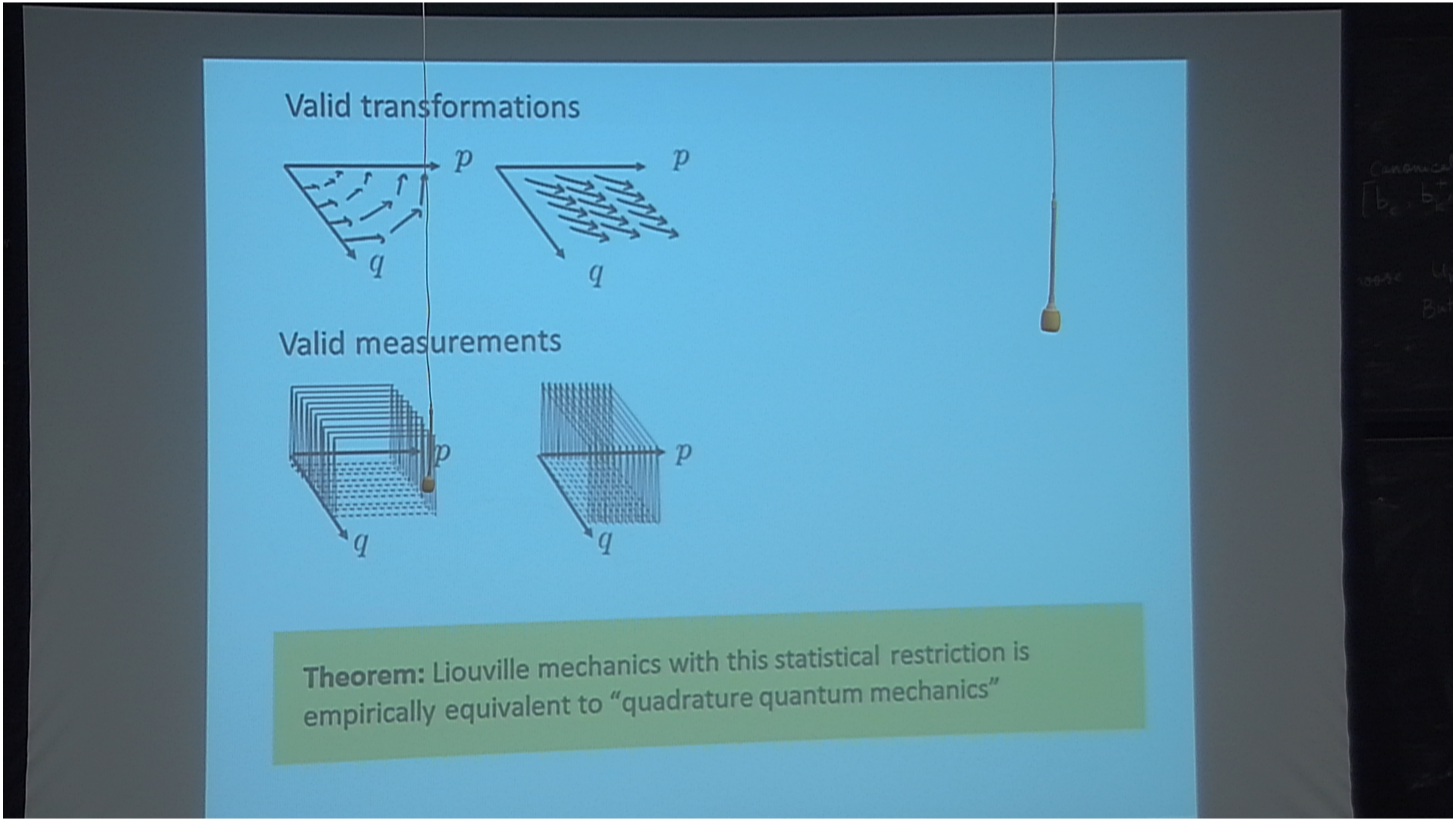

The principle of classical complementarity:
An observer can only jointly know a set of variables if they commute relative to the Poisson bracket.

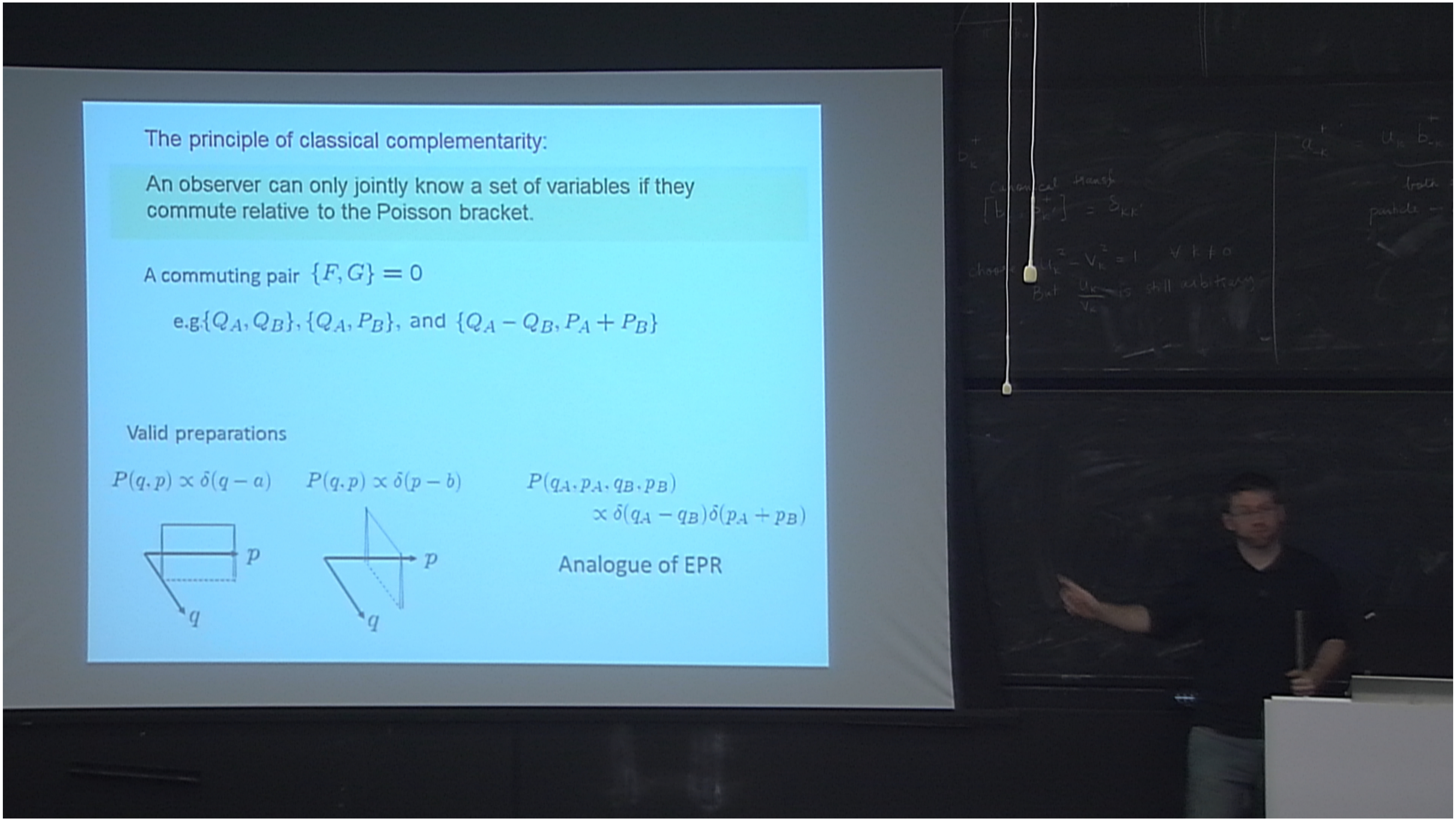

The principle of classical complementarity:
An observer can only jointly know a set of variables if they commute relative to the Poisson bracket.
A commuting pair $\{F, G\} = 0$
e.g $\{Q_A, Q_B\}, \{Q_A, P_B\}$, and $\{Q_A - Q_B, P_A + P_B\}$

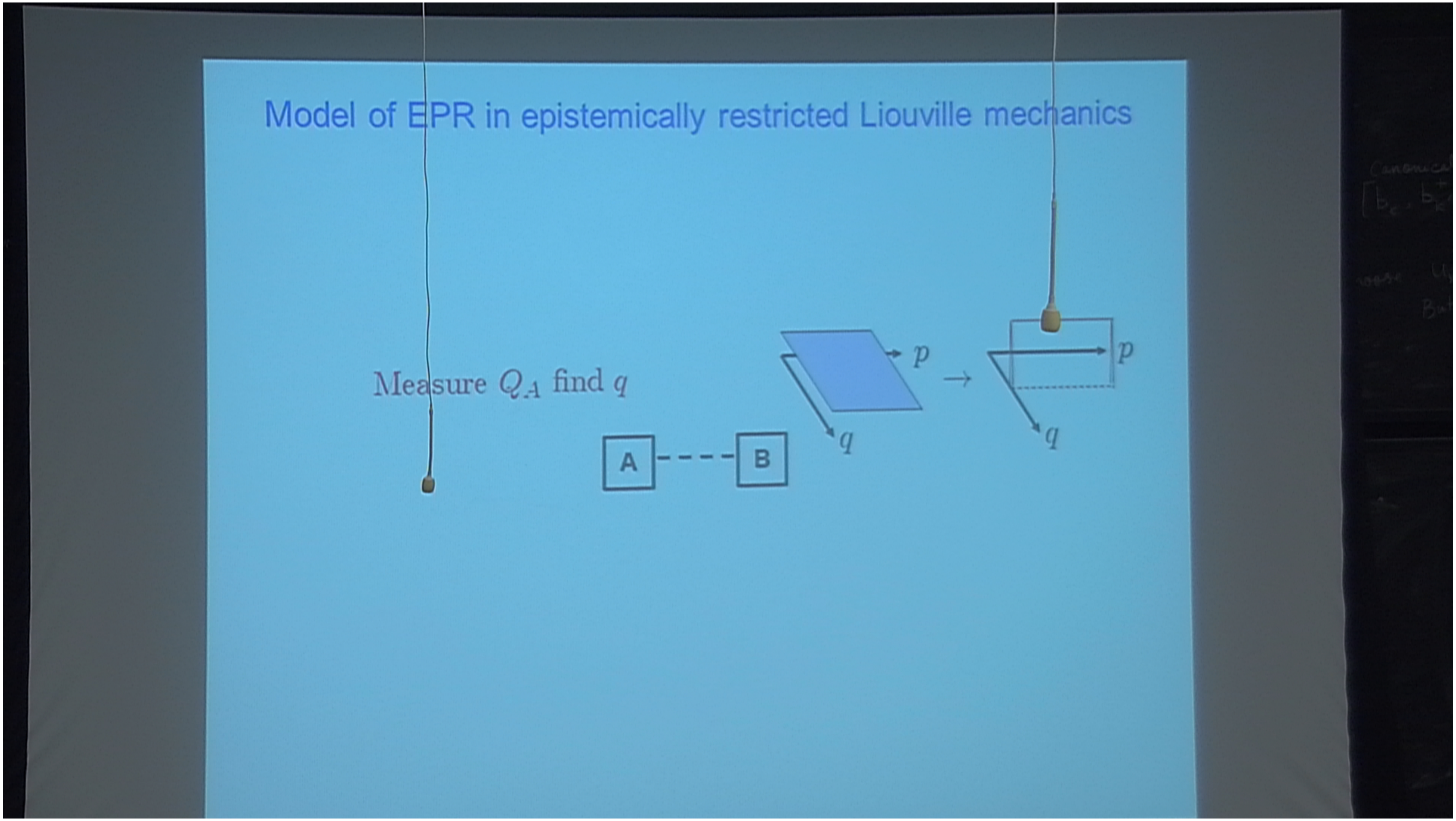

The principle of classical complementarity:
An observer can only jointly know a set of variables if they commute relative to the Poisson bracket.
A commuting pair $\{F, G\} = 0$
e.g. $\{Q_A, Q_B\}, \{Q_A, P_B\}$, and $\{Q_A - Q_B, P_A + P_B\}$
Valid preparations
$P(q, p) \propto \delta(q - a)$
p
q


The principle of classical complementarity:
An observer can only jointly know a set of variables if they commute relative to the Poisson bracket.
A commuting pair $\{F, G\} = 0$
e.g. $\{Q_A, Q_B\}$, $\{Q_A, P_B\}$, and $\{Q_A - Q_B, P_A + P_B\}$
Valid preparations
$P(q,p) \propto \delta(q-a)$
$P(q,p) \propto \delta(p-b)$
$P(q_A, p_A, q_B, p_B)$
$\delta(q_A - q_B)\delta(p_A + p_B)$
p
q
p
q
Analogue of EPR


Valid transformations
p
q
p
q
Valid measurements
p
q
p
q
Theorem: Liouville mechanics with this statistical restriction is empirically equivalent to “quadrature quantum mechanics”

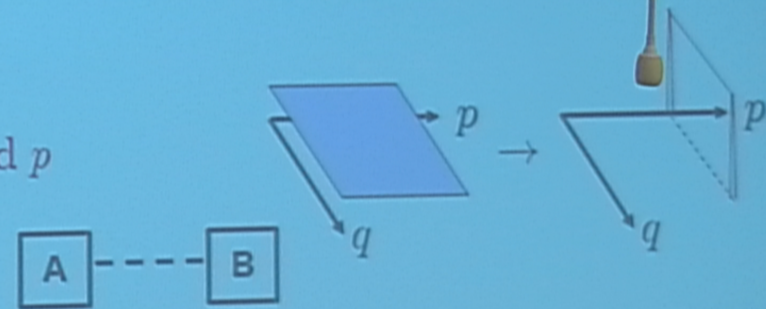

Valid transformations
p
q
p
q
Valid measurements
p
q
p
q
Theorem: Liouville mechanics with this statistical restriction is empirically equivalent to "quadrature quantum mechanics"

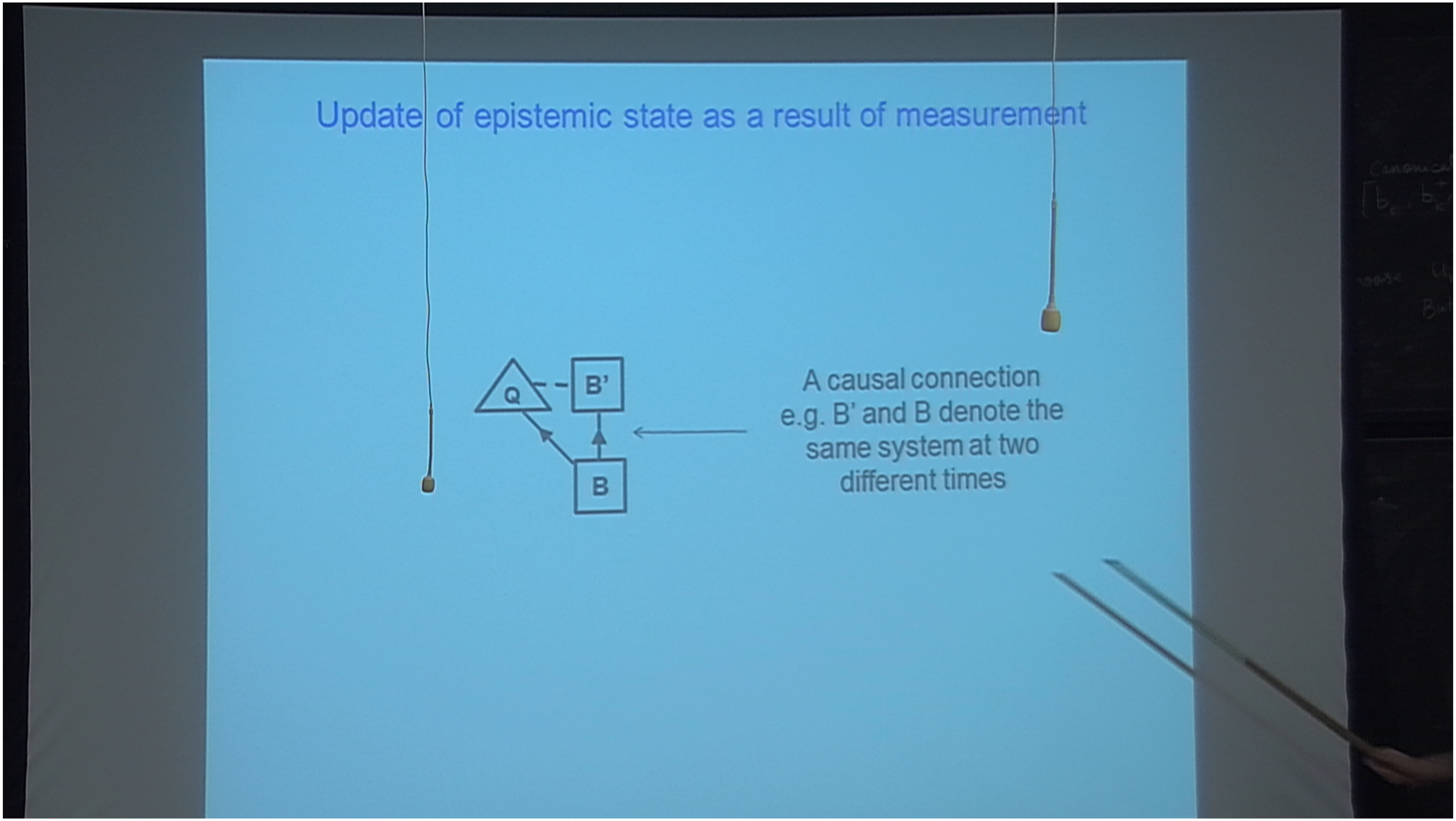

The principle of classical complementarity:
An observer can only jointly know a set of variables if they commute relative to the Poisson bracket.
A commuting pair $\{F, G\} = 0$
e.g. $\{Q_A, Q_B\}$, $\{Q_A, P_B\}$, and $\{Q_A - Q_B, P_A + P_B\}$
Valid preparations
$P(q,p) \propto \delta(q-a)$
$P(q,p) \propto \delta(p-b)$
$P(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$
p
q
p
q
Analogue of EPR

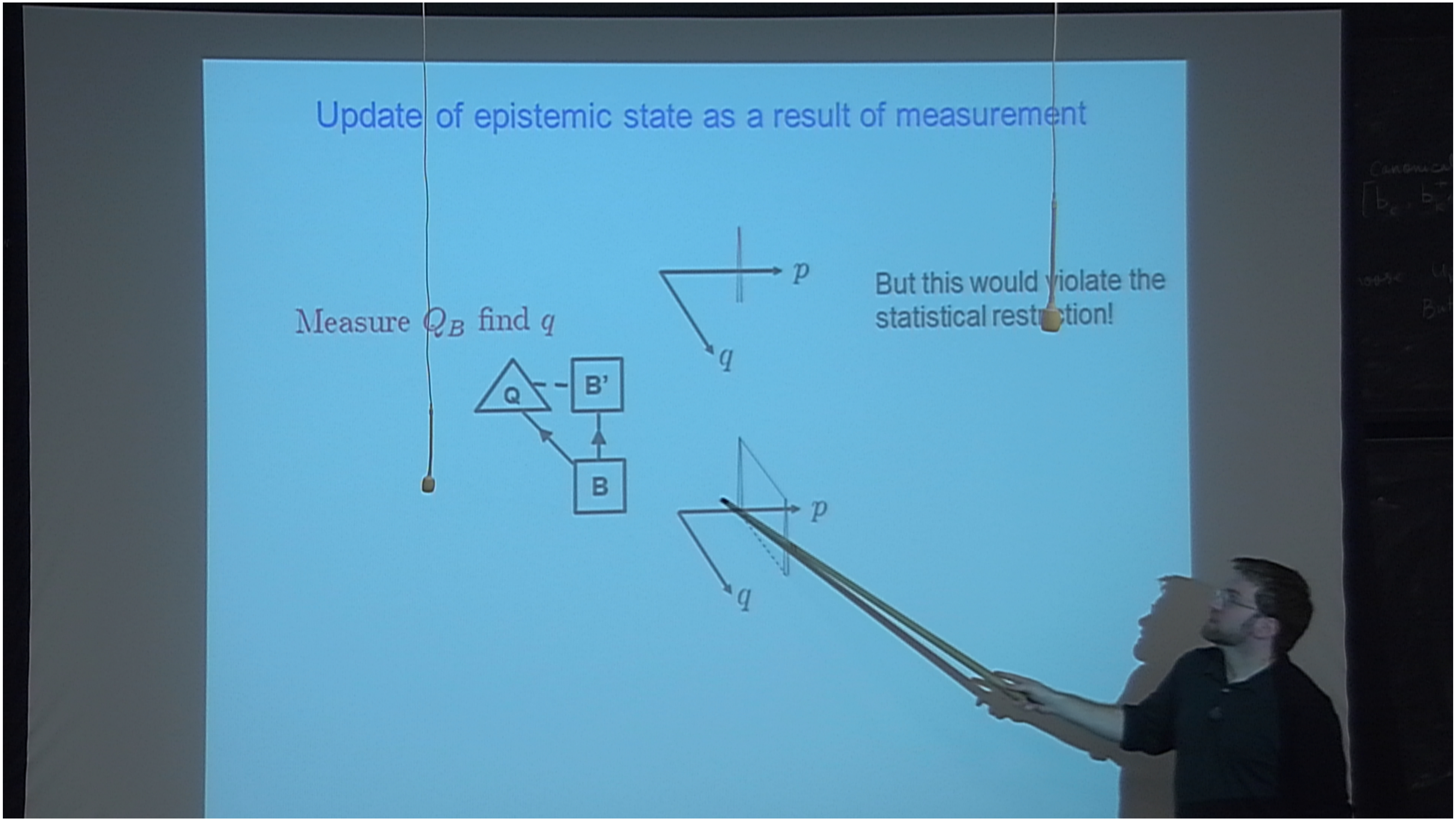

Model of EPR in epistemically restricted Liouville mechanics
Measure Q_A find q
A
B
p
q
→
p
q

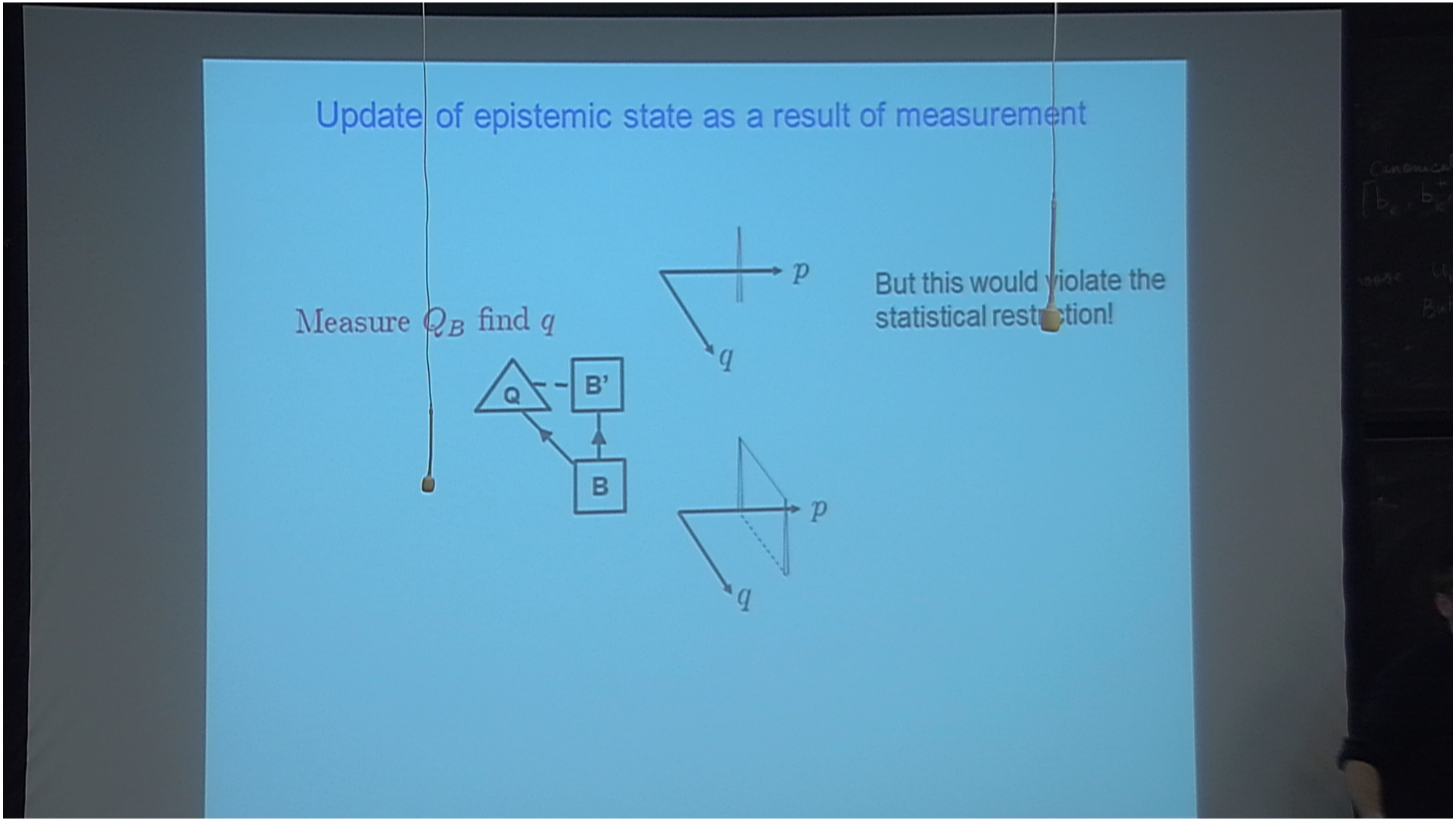
Model of EPR in epistemically restricted Liouville mechanics

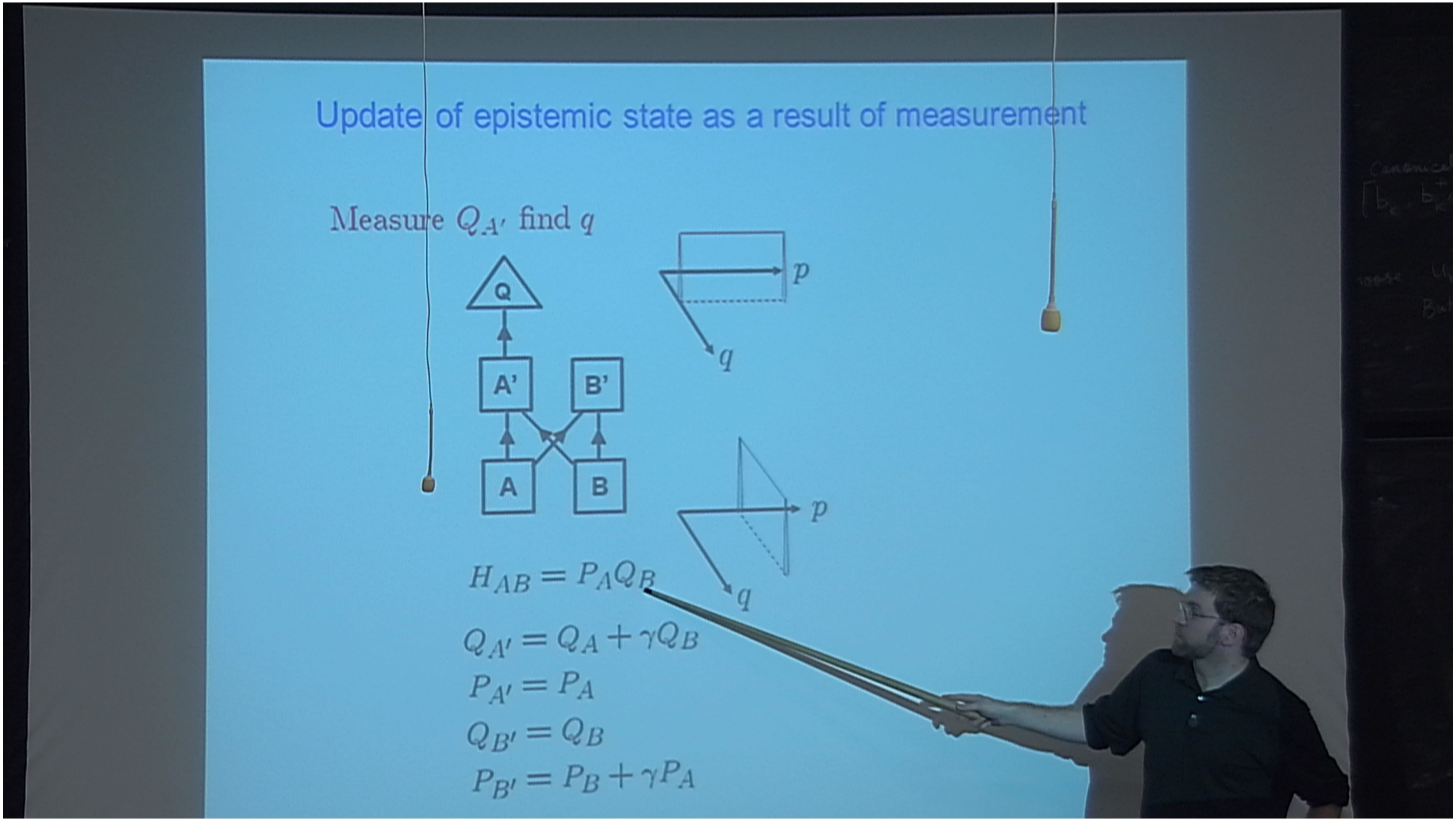
Measure P_A find p

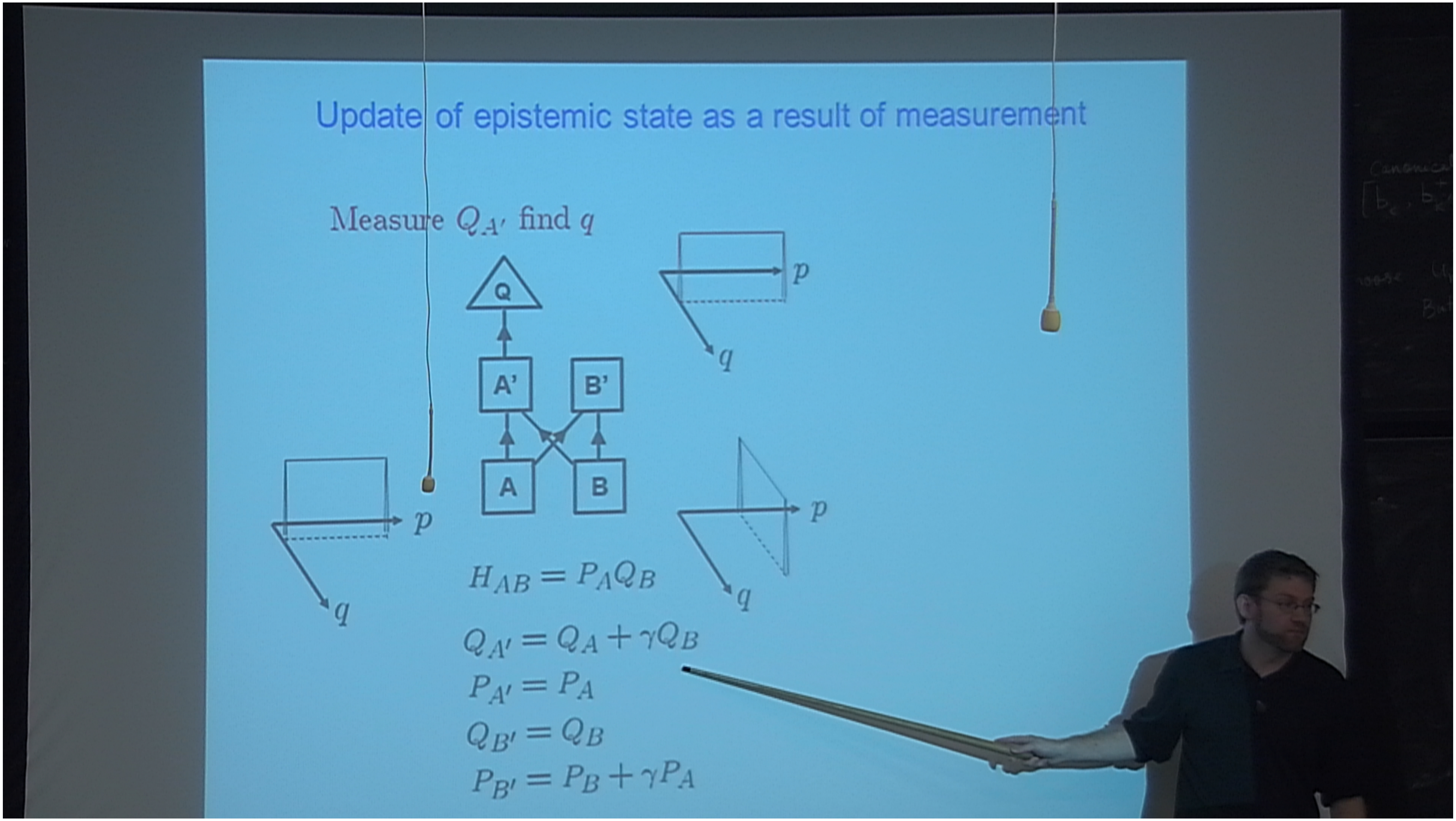


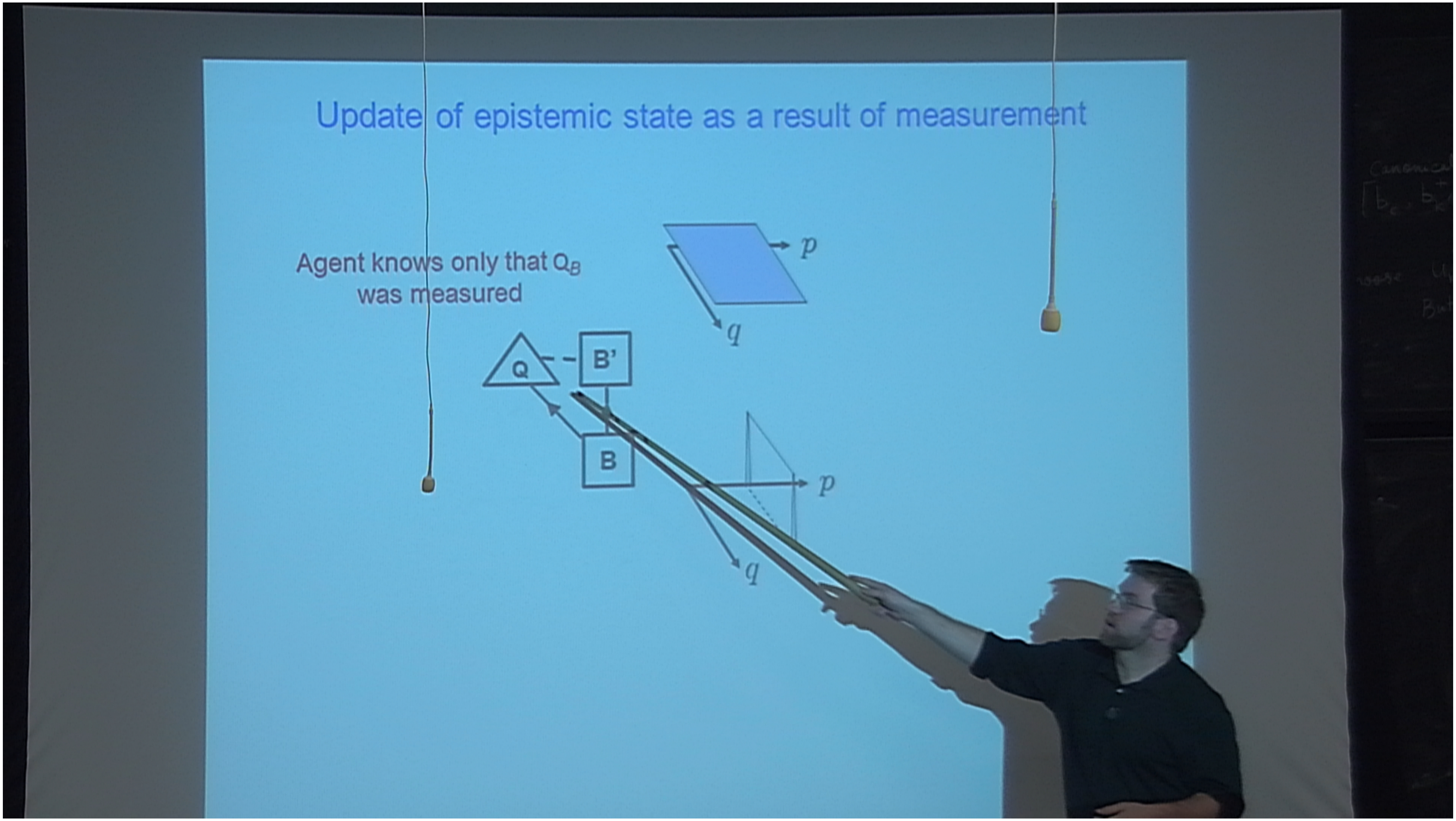

Update of epistemic state as a result of measurement
Q
B'
B
A causal connection
e.g. B' and B denote the
same system at two
different times


Update of epistemic state as a result of measurement
Measure Q_B find q
Q
B'
B
p
q
p
q
But this would violate the
statistical restriction!


Update of epistemic state as a result of measurement
Measure Q_B find q
Q
B'
B
p
q
But this would violate the statistical restriction!
p
q

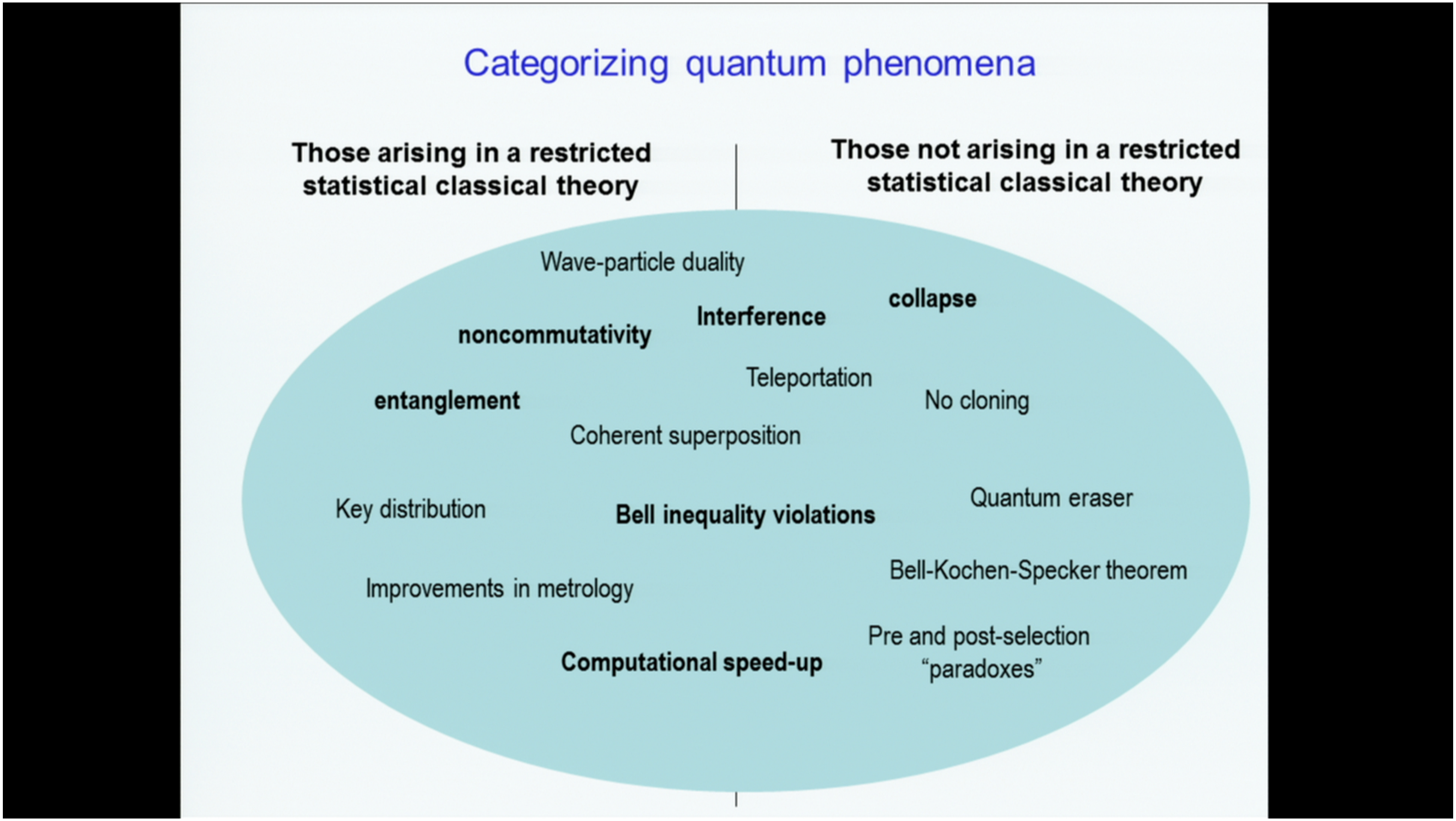

Update of epistemic state as a result of measurement
Measure $Q_{A'}$ find q
Q
A'
B'
A
B
p
q
p
q
$H_{AB} = P_A Q_B$
$Q_{A'} = Q_A + \gamma Q_B$
$P_{A'} = P_A$
$Q_{B'} = Q_B$
$P_{B'} = P_B + \gamma P_A$


Update of epistemic state as a result of measurement
Measure $Q_{A'}$ find q
Q
A'
B'
A
B
p
q
p
q
p
q
$H_{AB} = P_A Q_B$
$Q_{A'} = Q_A + \gamma Q_B$
$P_{A'} = P_A$
$Q_{B'} = Q_B$
$P_{B'} = P_B + \gamma P_A$


Update of epistemic state as a result of measurement
Agent knows only that Q_B was measured
p
q
Q
B'
B
p
q

Categorizing quantum phenomena

Those arising in a restricted statistical classical theory	Those not arising in a restricted statistical classical theory


Categorizing quantum phenomena
Those arising in a restricted statistical classical theory
Those not arising in a restricted statistical classical theory
Wave-particle duality
collapse
Interference
noncommutativity
Teleportation
entanglement
No cloning
Coherent superposition
Key distribution
Bell inequality violations
Quantum eraser
Bell-Kochen-Specker theorem
Improvements in metrology
Pre and post-selection "paradoxes"
Computational speed-up

Categorizing quantum phenomena

Those arising in a restricted statistical classical theory	Those not arising in a restricted statistical classical theory
Interference	**Bell inequality violations**
Noncommutativity	**Computational speed-up (if it exists)**
Entanglement	Bell-Kochen-Specker theorem
Collapse	Certain aspects of items on the left
Wave-particle duality	Others…
Teleportation	
No cloning	
Key distribution	
Improvements in metrology	
Quantum eraser	
Coherent superposition	
Pre and post-selection "paradoxes"	
Others…	

Classical		Quantum	
R	phase-space coordinates of a canonical system	A	label for a quantum system
$P(R)$	Probability distribution over R	ρ_A	Operator on Hilbert space of A
$P(R=r)$	probability that $R=r$		No obvious analogue (yet!)
$\sum_R P(R) = 1$		$\mathrm{Tr}_A \rho_A = 1$	

Classical		Quantum	
R	phase-space coordinates of a canonical system	*A*	label for a quantum system
$P(R)$	Probability distribution over R	ρ_A	Operator on Hilbert space of A
$P(R=r)$	probability that *R=r*		No obvious analogue (yet!)
$\sum_R P(R) = 1$		$\mathrm{Tr}_A \rho_A = 1$	

	Classical	Quantum
State of knowledge	$P(R)$	ρ_A
Normalization	$\sum_R P(R) = 1$	$\mathrm{Tr}_A \rho_A = 1$
Joint state	$P(R,S)$	ρ_{AB}
Marginalization	$P(S) = \sum_R P(R,S)$	$\rho_B = \mathrm{Tr}_A \rho_{AB}$

The Three Pillars of Bayesian Inference

- Belief propagation

$$P(S) = \sum_R P(S|R)P(R)$$

- Bayes' theorem

$$P(R|S) = \frac{P(S|R)P(R)}{\sum_R P(S|R)P(R)}$$

- Bayesian conditioning

$$P(R) \to P(R|X = x)$$

Conditional probability

$P(S|R)$

Conditional state

$\rho_{B|A}$

Conditional probability

$$P(S|R)$$

Normalization condition

$$\sum_S P(S|R) = 1$$

Conditional state

$$\rho_{B|A}$$

Normalization condition

$$\mathrm{Tr}_B(\rho_{B|A}) = I_A$$

Conditional probability

$P(S|R)$

Normalization condition

$\sum_S P(S|R) = 1$

Conditional state

$\rho_{B|A}$

Normalization condition

$\mathrm{Tr}_B(\rho_{B|A}) = I_A$

See: Leifer, PRA 74, 042310 (2006)

Conditional probability

$P(S|R)$

Normalization condition

$\sum_S P(S|R) = 1$

Relation of conditional to joint

$P(S|R) = \frac{P(R,S)}{P(R)}$

Conditional state

$\rho_{B|A}$

Normalization condition

$\mathrm{Tr}_B(\rho_{B|A}) = I_A$

Relation of conditional to joint

$\rho_{B|A} = (\rho_A^{-1/2} \otimes I_B)\rho_{AB}(\rho_A^{-1/2} \otimes I_B)$

Conditional probability

$$P(S|R)$$

Normalization condition

$$\sum_S P(S|R) = 1$$

Relation of conditional to joint

$$P(S|R) = \frac{P(R,S)}{P(R)}$$

$$P(R,S) = P(S|R)P(R)$$

Conditional state

$$\rho_{B|A}$$

Normalization condition

$$\mathrm{Tr}_B(\rho_{B|A}) = I_A$$

Relation of conditional to joint

$$\rho_{B|A} = \rho_A^{-1/2} \rho_{AB} \rho_A^{-1/2}$$

$$\rho_{AB} = \rho_A^{1/2} \rho_{B|A} \rho_A^{1/2}$$

Conditional probability

$$P(S|R)$$

Normalization condition

$$\sum_S P(S|R) = 1$$

Relation of conditional to joint

$$P(S|R) = \frac{P(R,S)}{P(R)}$$

$$P(R,S) = P(S|R)P(R)$$

Classical belief propagation

$$P(S) = \sum_R P(S|R)P(R)$$

Conditional state

$$\rho_{B|A}$$

Normalization condition

$$\mathrm{Tr}_B(\rho_{B|A}) = I_A$$

Relation of conditional to joint

$$\rho_{B|A} = \rho_{AB} * \rho_A^{-1}$$

$$\rho_{AB} = \rho_{B|A} * \rho_A$$

Quantum belief propagation

$$\rho_B = \mathrm{Tr}_A(\rho_{B|A}\rho_A)$$

Two formulas for the joint probability

$$P(R,S) = P(S|R)P(R)$$
$$= P(R|S)P(S)$$

Two formulas for the joint state

$$\rho_{BA} = \rho_A^{1/2}\rho_{B|A}\rho_A^{1/2}$$
$$= \rho_B^{1/2}\rho_{A|B}\rho_B^{1/2}$$

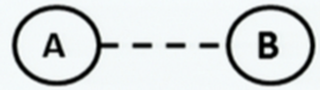
Classical Bayes' theorem

$$P(S|R) = \frac{P(R|S)P(S)}{P(R)}$$

Quantum Bayes' theorem

$$\rho_{B|A} = \rho_A^{-1/2}\rho_B^{1/2}\rho_{A|B}\rho_A^{-1/2}\rho_B^{1/2}$$

$P(R,S)$

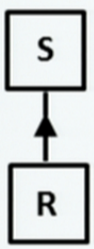


ρ_{AB}

$P(S|R) = P(R,S)/P(R)$????

$P(S) = \sum_R P(S|R)P(R)$????

$P(R,S)$????



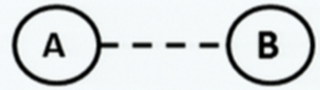
$P(S|R) = P(R,S)/P(R)$

????

$P(S) = \sum_R P(S|R)P(R)$????

$$\rho_B = \mathcal{E}_{A\to B}(\rho_A)$$

$P(R,S)$



ρ_{AB}

$$P(S|R) = P(R,S)/P(R)$$

$$P(S) = \sum_R P(S|R)P(R)$$

$$P(S) = \Gamma_{R\to S}[P(R)]$$

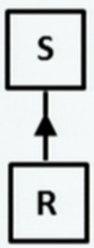
$$\rho_{B|A} = \rho_A^{-1/2}\rho_{AB}\rho_A^{-1/2}$$

$$\rho_B = \mathrm{Tr}_A(\rho_{B|A}\rho_A)$$

$$\rho_B = \mathfrak{E}_{A\to B}(\rho_A)$$

$$\rho_{B|A} \geq 0$$

$\mathfrak{E}_{A\to B}\circ T_A$ is CP



$P(R,S)$

ϱ_{AB}

$$P(S|R) = P(R,S)/P(R)$$

$$P(S) = \sum_R P(S|R)P(R)$$

$$P(S) = \Gamma_{R\to S}[P(R)]$$

$$\varrho_{B|A} = \varrho_A^{-1/2}\varrho_{AB}\rho_A^{-1/2}$$

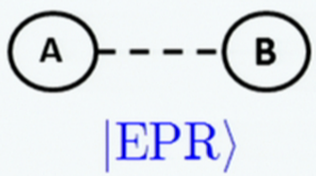
$$\rho_B = \mathrm{Tr}_A(\varrho_{B|A}\rho_A)$$

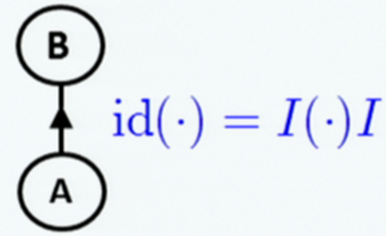
$$\rho_B = \mathcal{E}_{A\to B}(\rho_A)$$

$$\varrho_{B|A}^{T_A} \geq 0$$

$\mathcal{E}_{A\to B}$ is CP

Comparing causal and acausal correlations in Quantum Mechanics



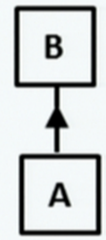


	$\hat{Q}_A, \hat{Q}_B$	$\hat{P}_A, \hat{P}_B$
|EPR⟩	C	A

	$\hat{Q}_A, \hat{Q}_B$	$\hat{P}_A, \hat{P}_B$
id	C	C

Comparing causal and acausal correlations in Epistemically Restricted Liouville mechanics

$$P_{\text{id}}(q_B, p_B | q_A, p_A)$$
$$\propto \delta(q_A - q_B)\delta(p_A - p_B)$$



$$Q_B = Q_A$$
$$P_B = P_A$$

$$P_{\text{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$
$$P_B + P_A = 0$$

	Q_A, Q_B	P_A, P_B
$\mathbf{P_{EPR}}$	C	A

	Q_A, Q_B	P_A, P_B
$\mathbf{P_{id}}$	C	C

Jeffrey conditioning

Suppose $P(S) = \sum_R P(S|R)P(R)$

If $P(R) \to P^{\text{post}}(R)$

then $P(S) \to P^{\text{post}}(S)$

where $P^{\text{post}}(S) = \sum_R P(S|R)P^{\text{post}}(R)$

Suppose $\rho_B = \text{Tr}_A(\rho_{B|A}\rho_A)$

If $\rho_A \to \rho_A^{\text{post}}$

then $\rho_B \to \rho_B^{\text{post}}$

where $\rho_B^{\text{post}} = \text{Tr}_A(\rho_{B|A}\rho_A^{\text{post}})$

Bayesian conditioning

Suppose $P(S) = \sum_X P(S|X)P(X)$

If $P(X) \to P^{\text{post}}(X) = \delta_{X,x}$

then $P(S) \to P^{\text{post}}(S)$

where $P^{\text{post}}(S) = \sum_X P(S|X)P^{\text{post}}(X)$
$= P(S|X=x)$

$P(S) \to P(S|X=x)$

Suppose $\rho_B = \text{Tr}_X(\rho_{B|X}\rho_X)$

If $\rho_X \to \rho_X^{\text{post}} = |x\rangle\langle x|_X$

then $\rho_B \to \rho_B^{\text{post}}$

where $\rho_B^{\text{post}} = \text{Tr}_X(\rho_{B|X}\rho_X^{\text{post}})$
$= \rho_{B|X=x}$

$\rho_B \to \rho_{B|X=x}$

Other applications of the formalism

- Identify analogies between multi-time and multi-system scenarios

e.g. no-broadcasting theorem $\leftrightarrow$ monogamy of entanglement
BB84 key distribution $\leftrightarrow$ Ekert key distribution

- Accommodate Aharonov *et al.* two-time and multi-time states

(pre and post selection is an instance of Bayesian inference)

- Obtain quantum analogues of key notions of Bayesian statistics

e.g. sufficient statistics, conditional independence, etc.

- Multiple observers: state compatibility criteria, state pooling rules, etc.

- Quantum analogues of belief propagation algorithms are important for quantum error correction