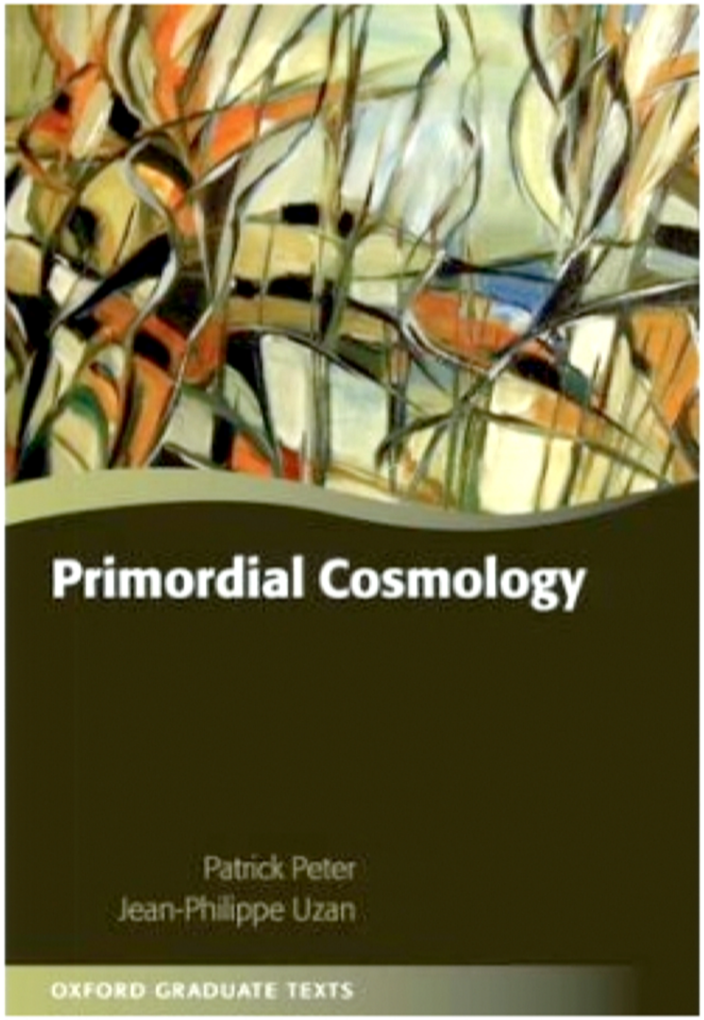
Title: Currents in Cosmic Strings and Associated Cosmology

Date: Nov 01, 2011 03:00 PM

URL: http://pirsa.org/11110112

Abstract: Cosmic strings are a generic prediction of Grand Unified Theories that can leave a sufficient imprint in the Cosmic Microwave Background anisotropies to open an observational window into an otherwise unreachable high energy domain. Being formed as topological defects of a Higgs field, they are also naturally coupled to various other fields, that can lead to superconducting-like currents, hence radically changing their structure and properties. After having summarized the standard string network behaviour and its cosmological effects, I will concentrate on the current-carrying properties and show how those can modify drastically the overall picture. In particular, I will exhibit the many current case, including the special non abelian situation that requires more care to be fully understood.


Primordial Cosmology
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OXFORD GRADUATE TEXTS


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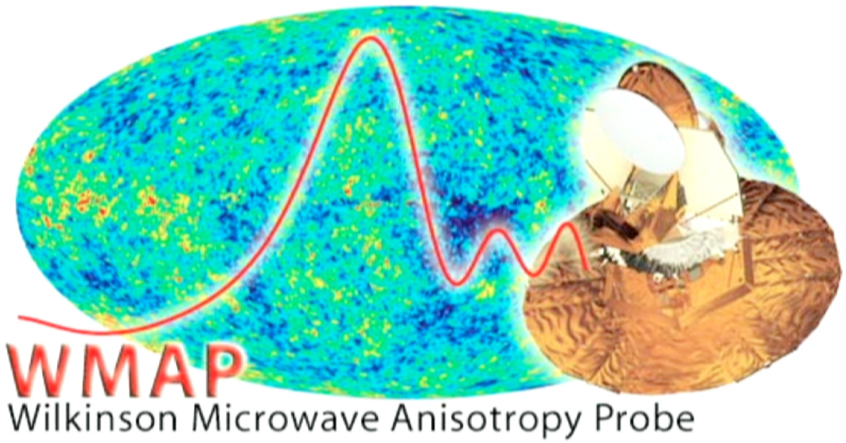
Currents in cosmic strings and associated cosmology

Why cosmic strings all over again???

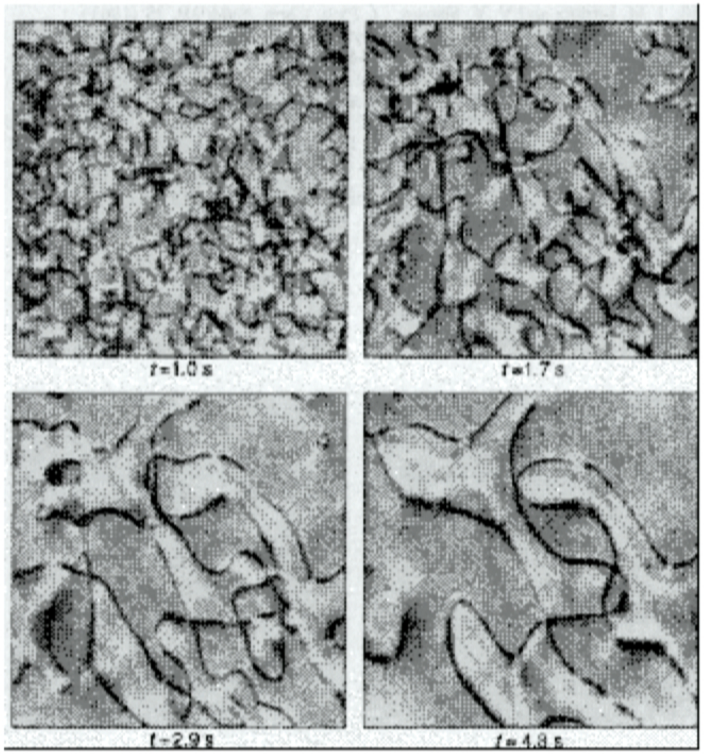


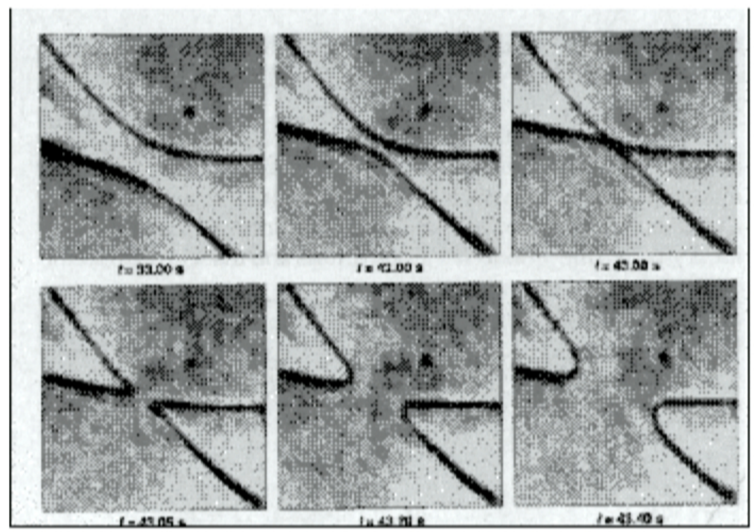
Adiabatic perturbations

Inflation is fine!!

No cosmological need for defects

Not so exotic objects ...





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General symmetry breaking

$G \rightarrow H$ (T. Kibble)

Vacuum manifold: *G/H*

Defect classification

$\pi_0(G/H)$	**Domain walls**	$\Omega \sim 10^8$ for $E \sim 100$ GeV
$\pi_1(G/H)$	**Cosmic strings**	$\Omega \sim 10^{-6}$ for $E \sim E_{\rm GUT}$
$\pi_2(G/H)$	**Monopoles**	**Inflation ...**
$\pi_3(G/H)$	**Textures**	$\Delta T/T$ ...

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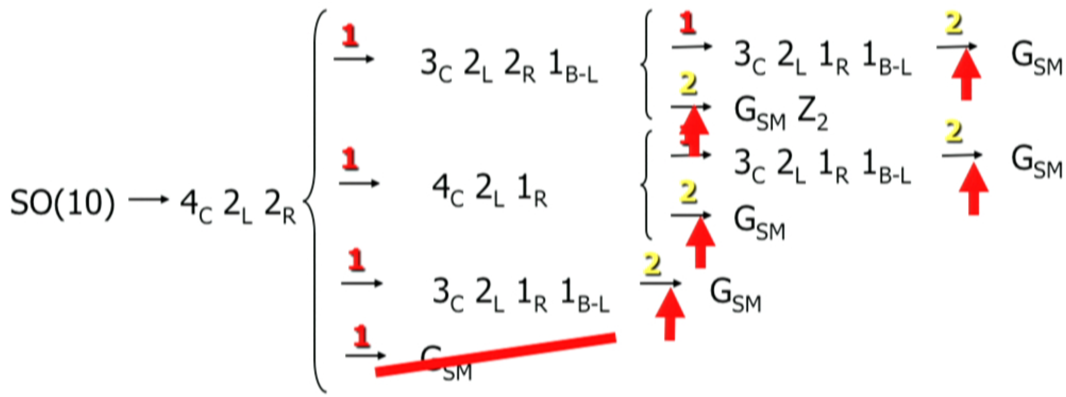
A simple example: *SO(10)*



1: Monopoles   2: Cosmic strings   ↑INFLATION

SO(10) : 34 possible schemes

E_6 : 1024 …

Hybrid Inflation ...

R. Jeannerot, J. Rocher & M. Sakellariadou, PR**D 68**, 104514 (2003)

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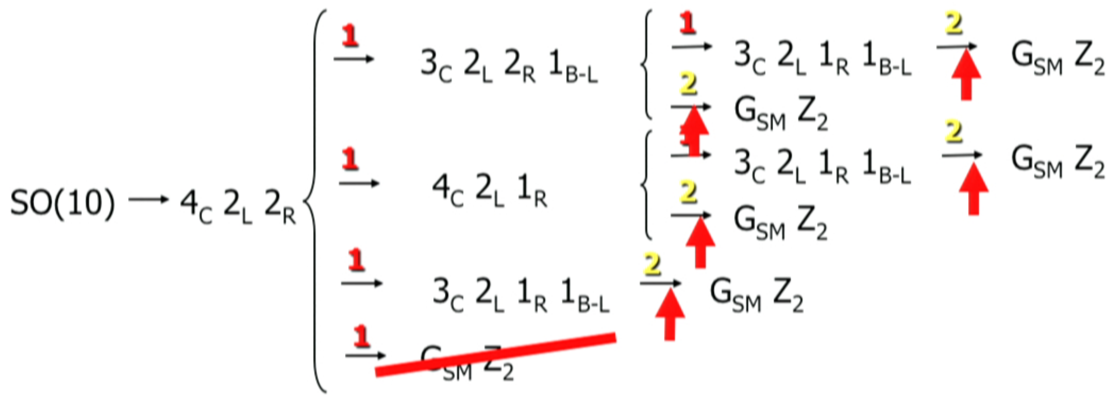

1: Monopoles **2**: Cosmic strings ↑INFLATION

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+ SUSY breaking and R-parity

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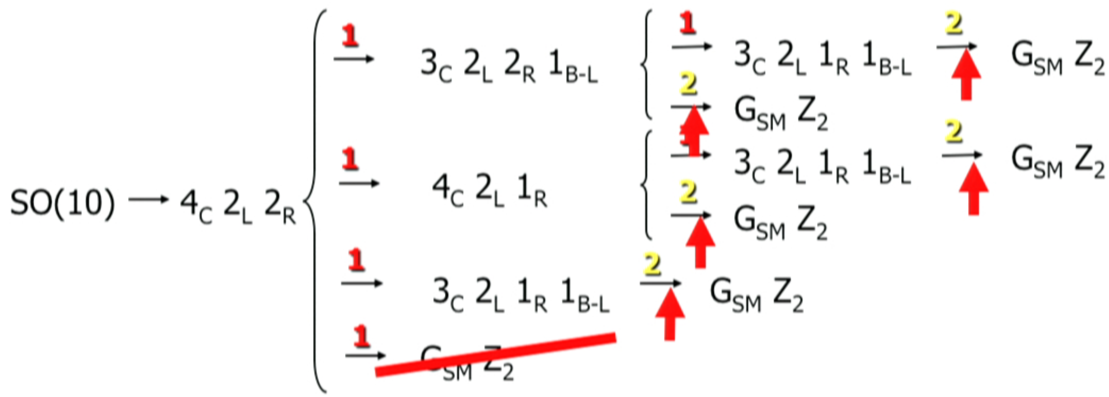

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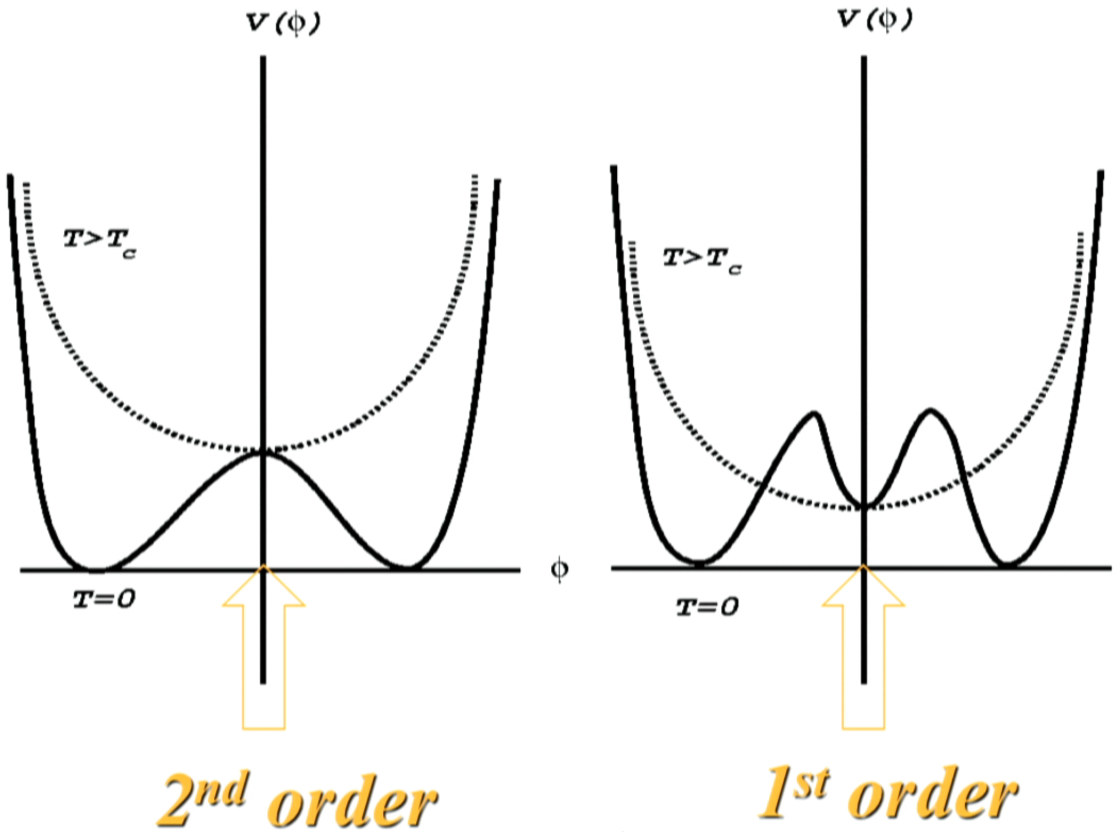
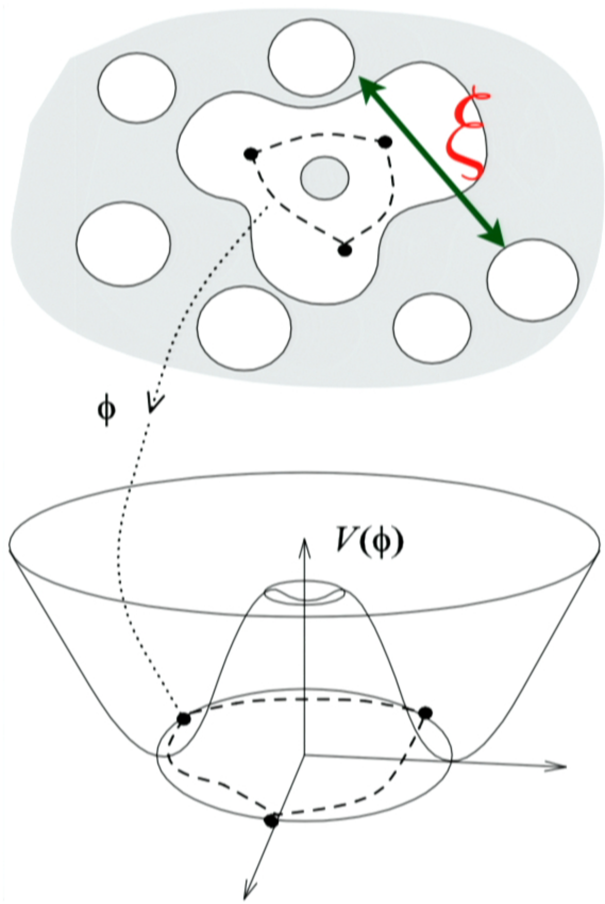
Formation

Symmetry breaking ⇨ Phase transition



Phase transition



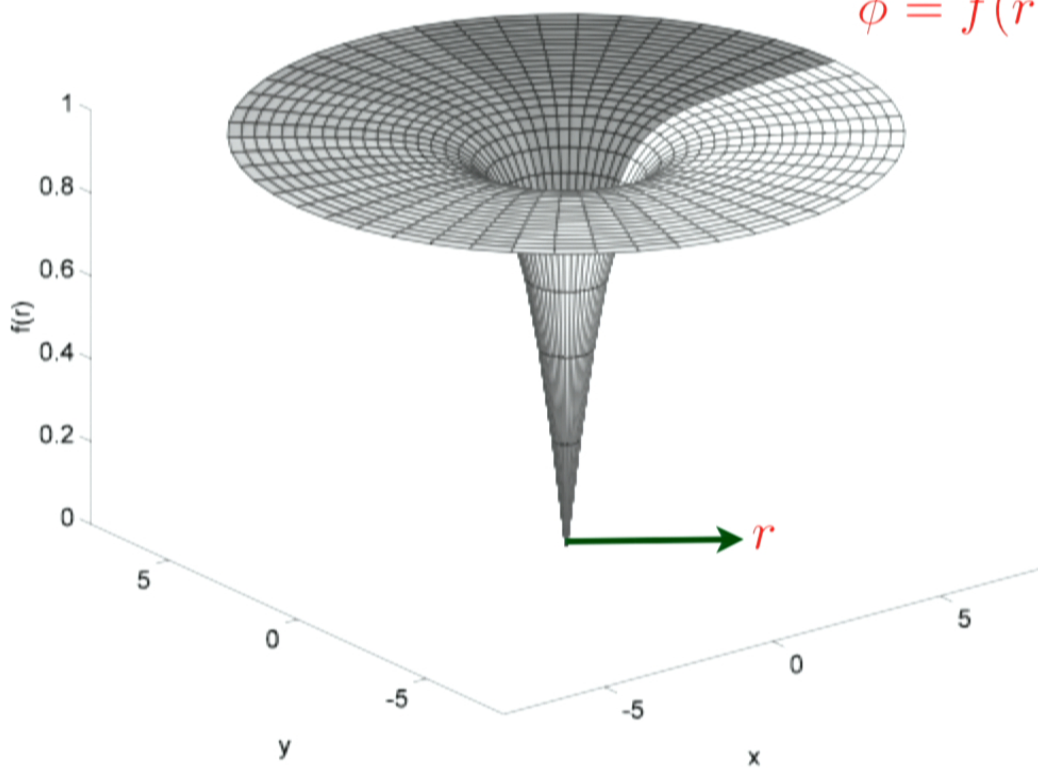
Correlation length

$$\langle 0|\phi(\vec{x})\phi(\vec{x}+\vec{r})|0\rangle \propto \mathrm{e}^{-|\vec{r}|/\xi}$$

Abelian Higgs model

$$\mathcal{L}_{\rm a.H} = -D_\mu\phi D^\mu\phi - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} - V(\phi)$$

$$\phi = f(r)\mathrm{e}^{in\theta}$$



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Negligible thickness ⇨ $\delta\,[x^{\alpha} - X^{\alpha}(\xi^{a})]$

Goto-Nambu action : $\mathcal{S} = -m^2 \int \mathrm{d}^2\xi \, \sqrt{-\gamma}$

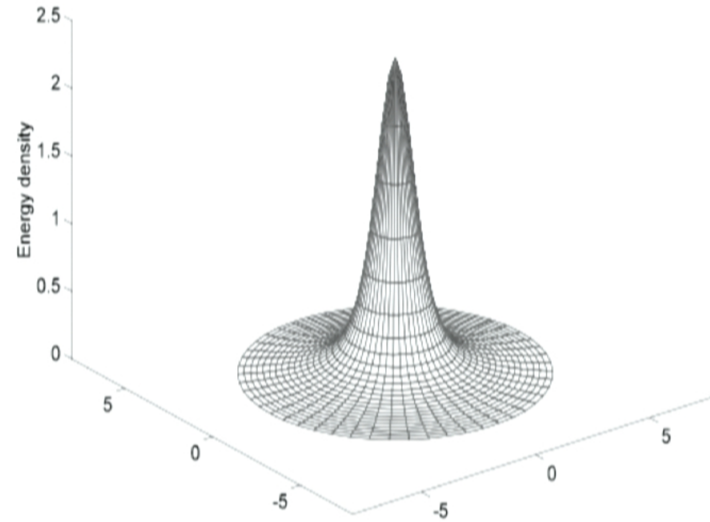
Area spanned by the worldsheet

Induced metric $\gamma_{ab} \equiv g_{\mu\nu} \frac{\partial X^{\mu}}{\partial \xi^{a}} \frac{\partial X^{\nu}}{\partial \xi^{b}}$

Background metric

Equation of State : $U = T = m^2$

Localized energy / axis



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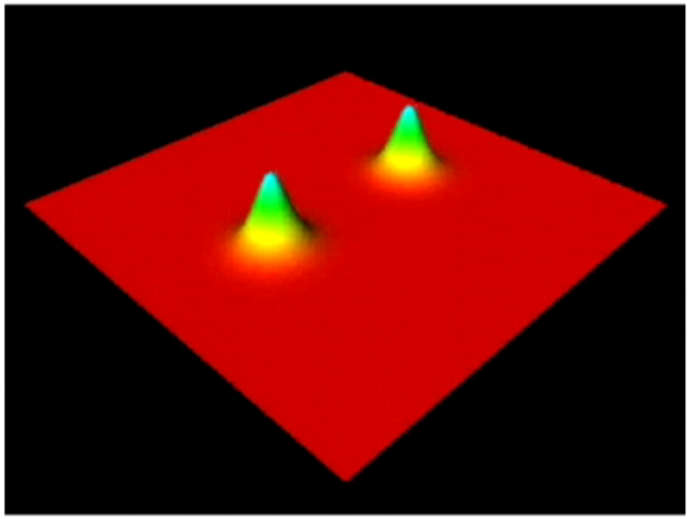
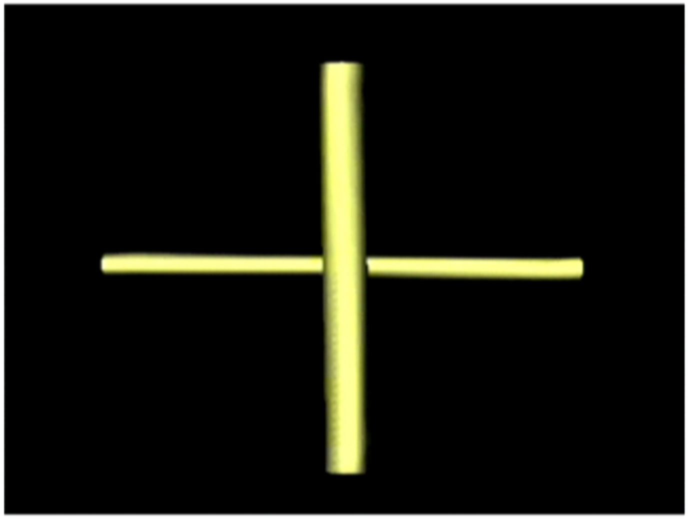
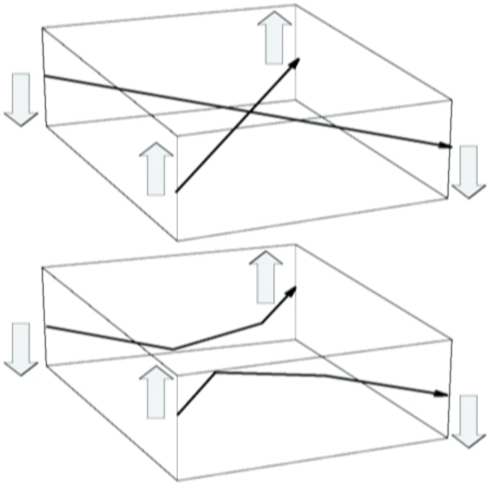
Interactions and evolution

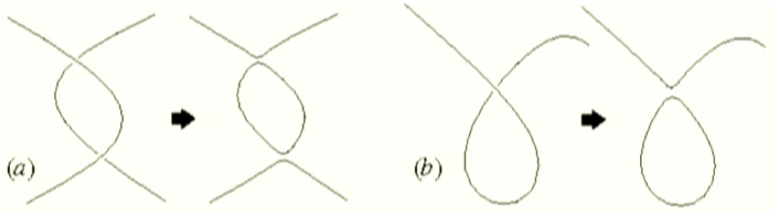
Interaction ...

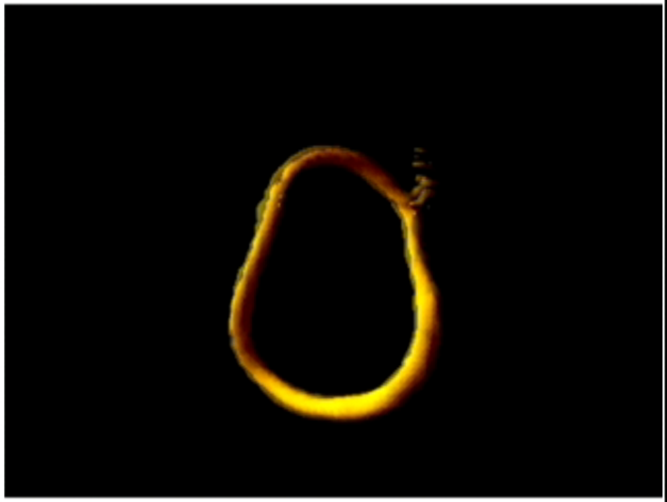


Reconnexion (intercommutation)









Simulations : P. Shellard (DAMTP - Cambridge)

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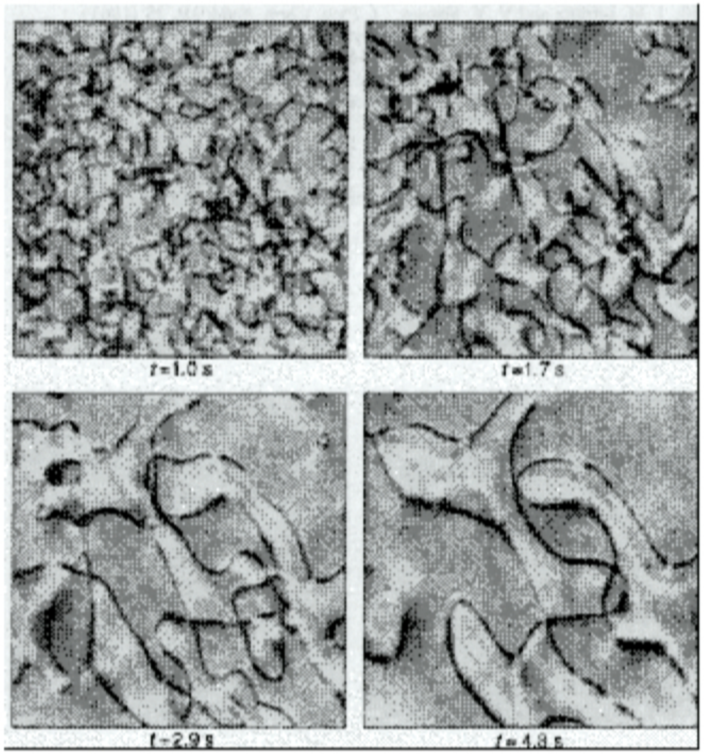
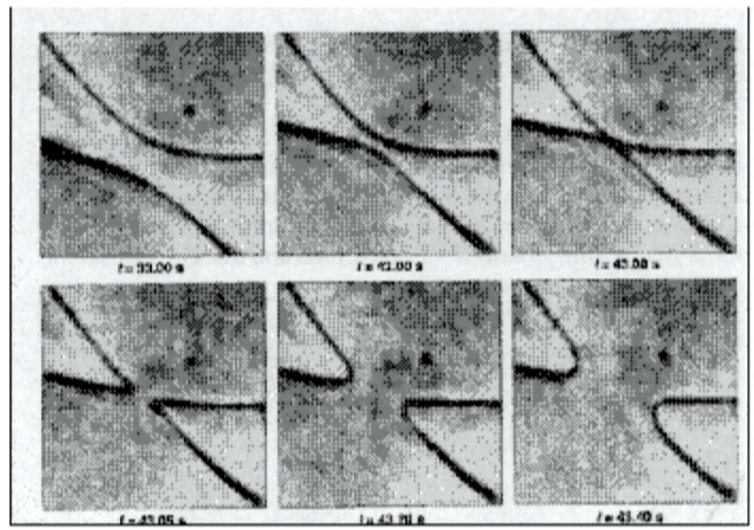
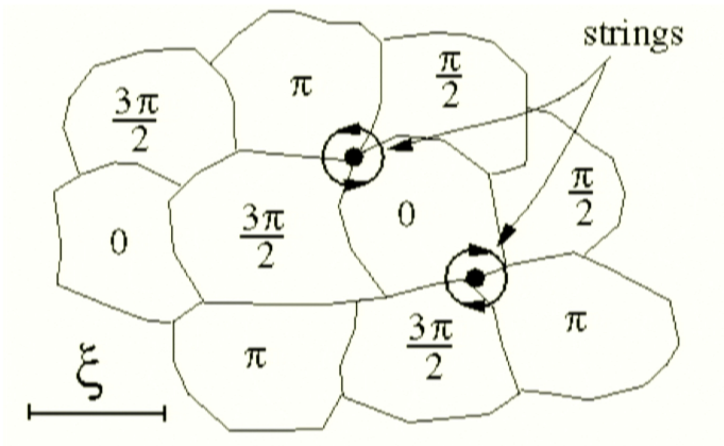
Not so exotic objects ...





Initial conditions: random phases



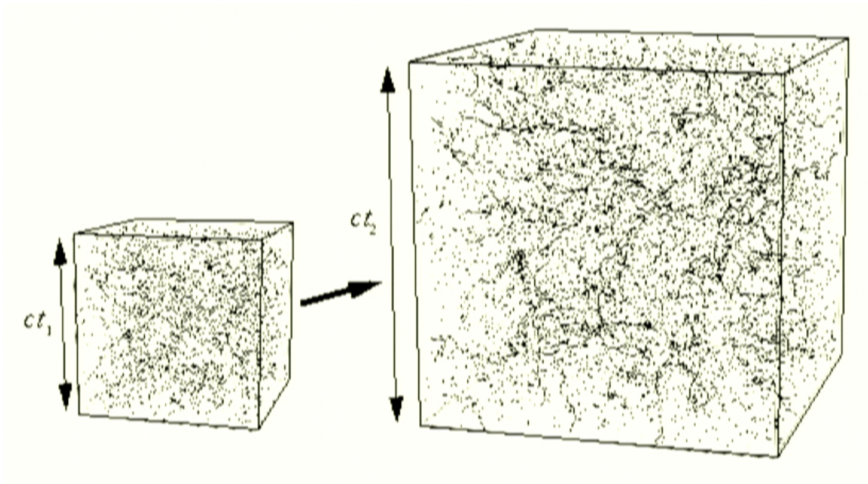
+ Evolution ...

⇨ Scaling!

$$\langle \ell \rangle \simeq \alpha t$$

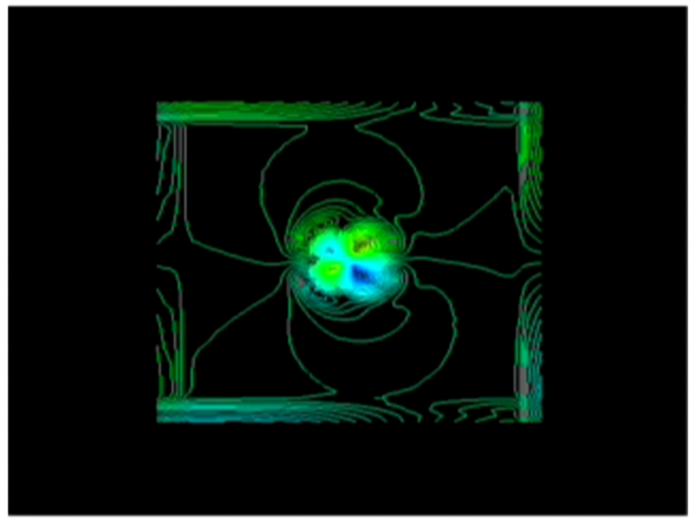
$$\rho_\infty = \zeta \frac{U}{t^2}$$

$$\rho_\ell = \sqrt{\frac{\alpha}{\Gamma G_N U}} \rho_\infty$$



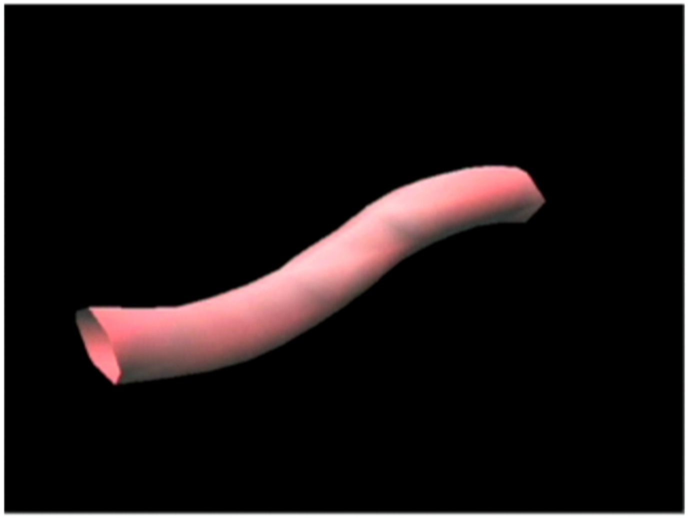
Radiation

Oscillations

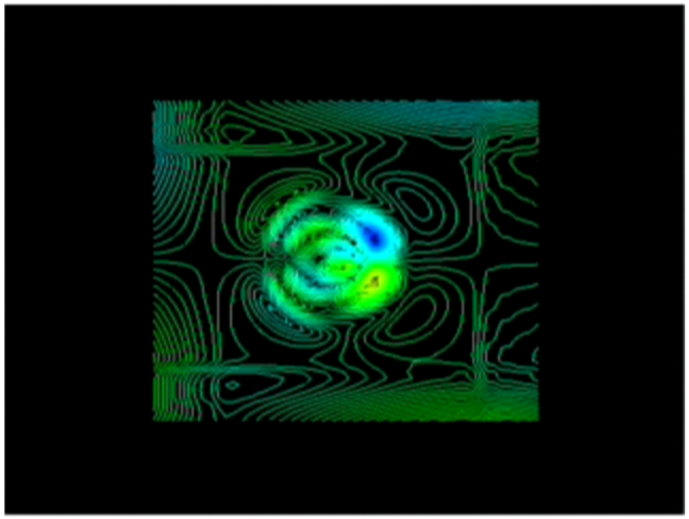


Axions, $h_+, ... h_\times$

Radiation



Oscillations



Axions, $h_+, ... h_\times$

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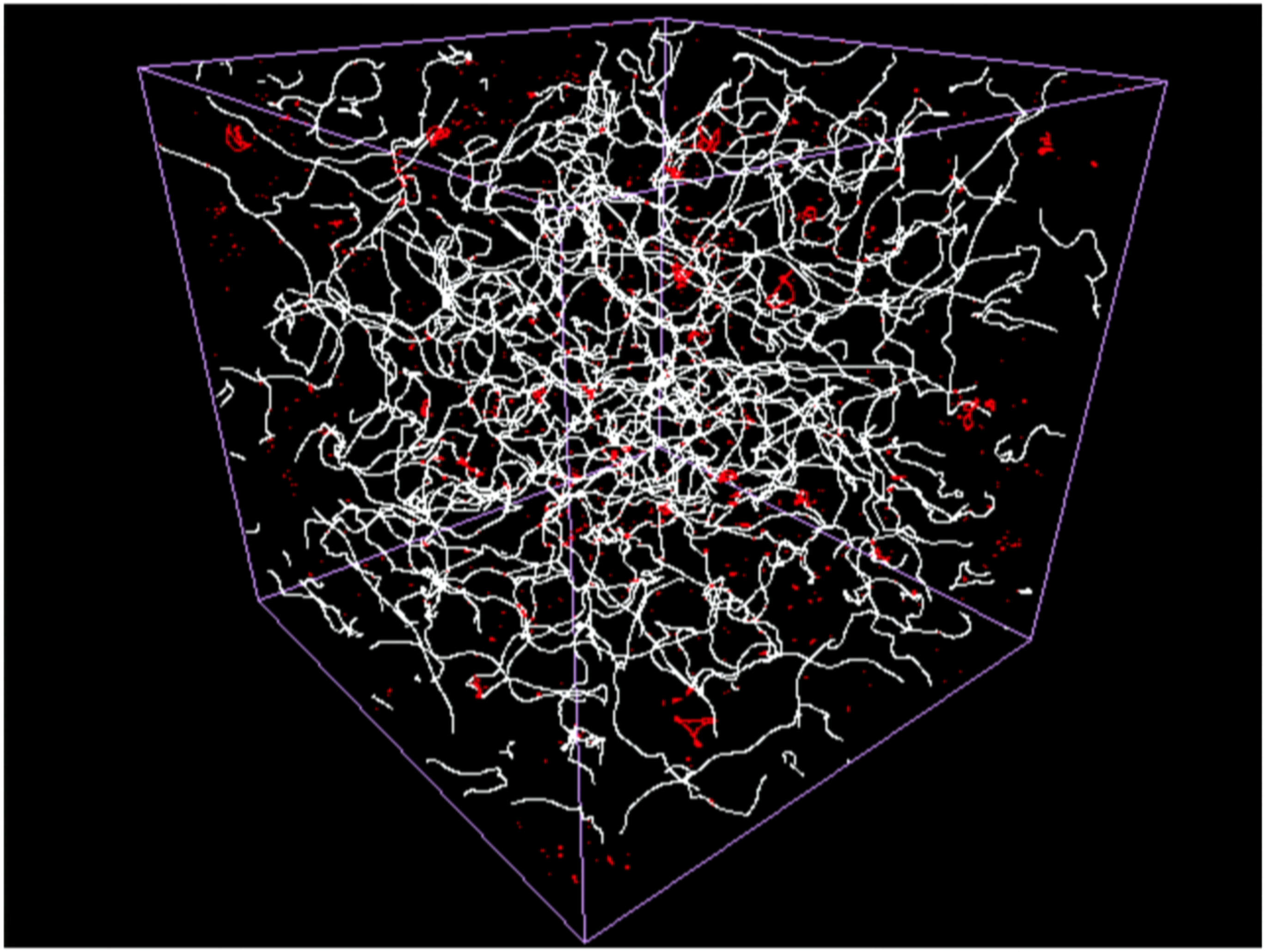
String simulation

D. Bennett, F. R. Bouchet & C. Ringeval



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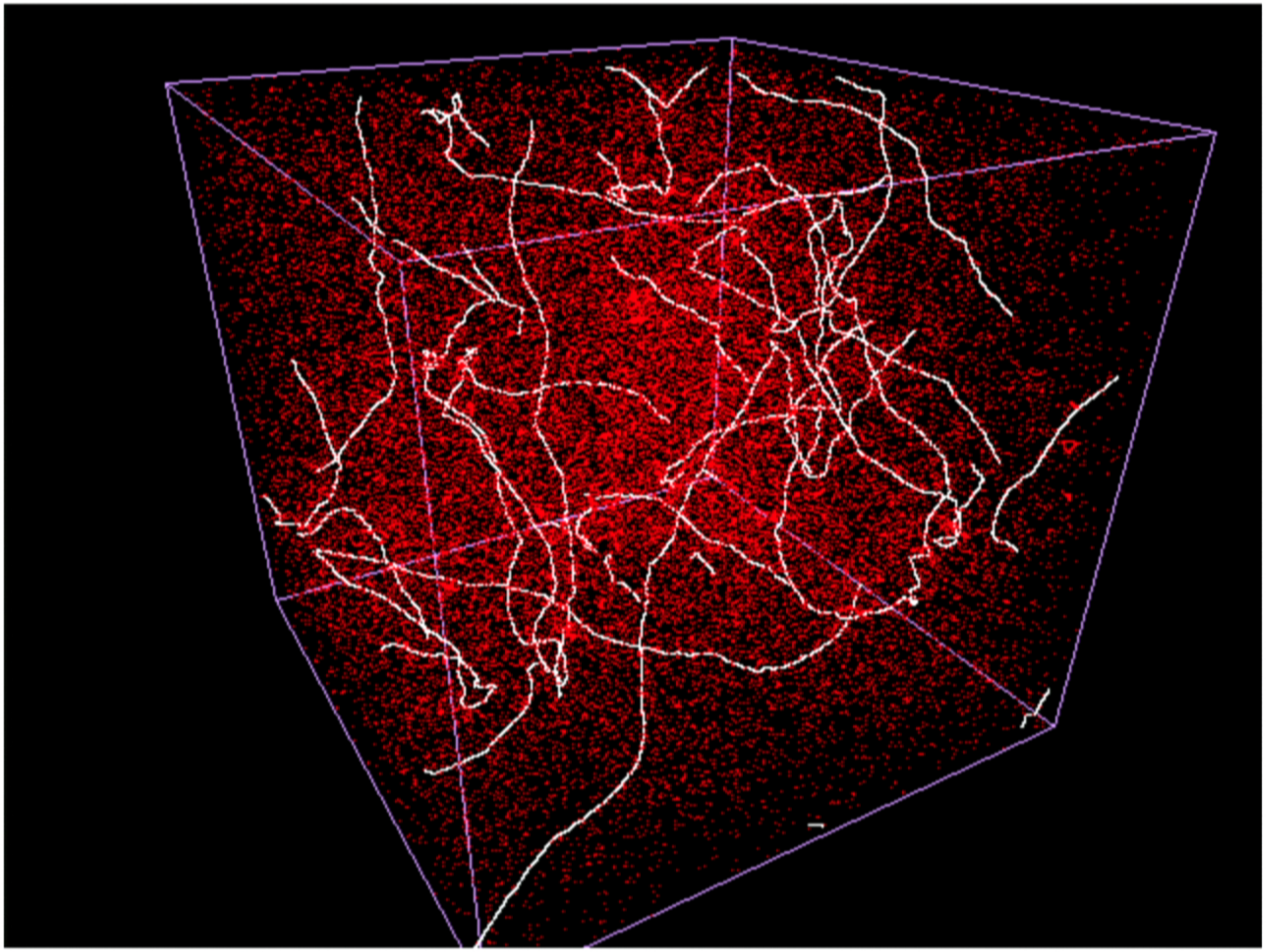

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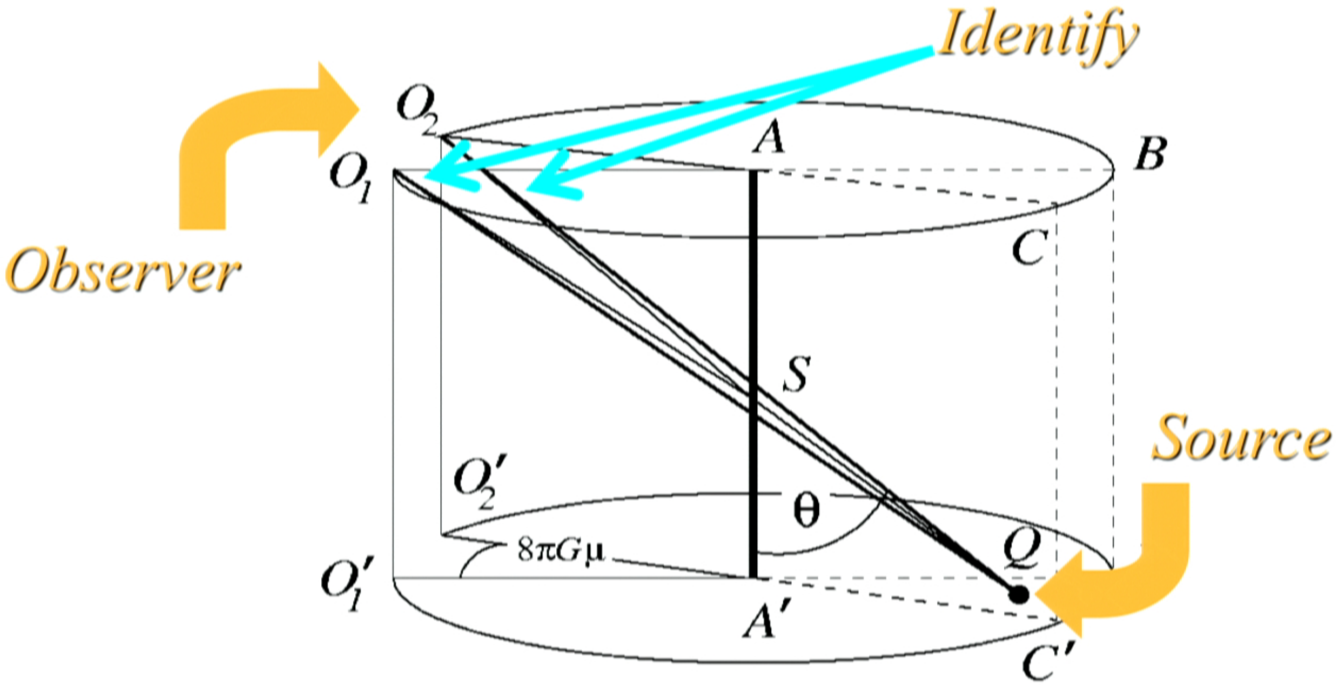
Observing cosmic strings?

Gravitation: conical metric $ds^2 = -dt^2 + dr^2 + dz^2 + r^2 (1 - 4\pi G_N U) d\theta^2$



Kaiser-Stebbins effect: "Doppler" $\frac{\Delta\nu}{\nu} \propto \frac{\Delta T}{T} \propto G_N U$

$\Theta_{\mu\nu}{}^{s}(\eta, x^{i})$ = *10 statistically isotropic and spatially homogeneous random fields*

Correlators $\left\langle \hat{\Theta}^{s}_{\mu\nu}(\mathbf{k},\eta)\hat{\Theta}^{s}_{\rho\sigma}(\mathbf{k}',\eta')\right\rangle = \delta(\mathbf{k}-\mathbf{k}')\hat{C}_{\mu\nu\rho\sigma}(\mathbf{k},\eta,\eta')$

(Ensemble average over a large number of realisations)

(In-)Coherence Hypothesis $\hat{C}_{\mu\nu\rho\sigma}(\mathbf{k},\eta,\eta') = \sum_{(i)} \lambda^{(i)}\hat{c}^{(i)}_{\mu\nu}(\mathbf{k},\eta)\hat{c}^{(i)}_{\rho\sigma}(\mathbf{k},\eta')$

Ansatz:

$$\hat{P}^{s} = \eta^{-\frac{1}{2}}\exp(-k^2\eta^2)e(\mathbf{k}),$$

$$\hat{\Pi}^{s} = -4\eta^{-\frac{1}{2}}(k^2\eta^2)\exp(-k^2\eta^2)e(\mathbf{k})$$

Random variable ***e(k)***

Loop decay

$$D_\mu\Theta^{\mu\nu}_{s} = F^{\nu},$$

$$\hat{F}^{\nu} = -\dot{\mu}\hat{\Theta}^{0\nu}_{s} \quad \text{with} \quad \dot{\mu} = \Gamma Y(k-k_c)$$

Heaviside

Transfered into the other fluids
Branching ratios x_f

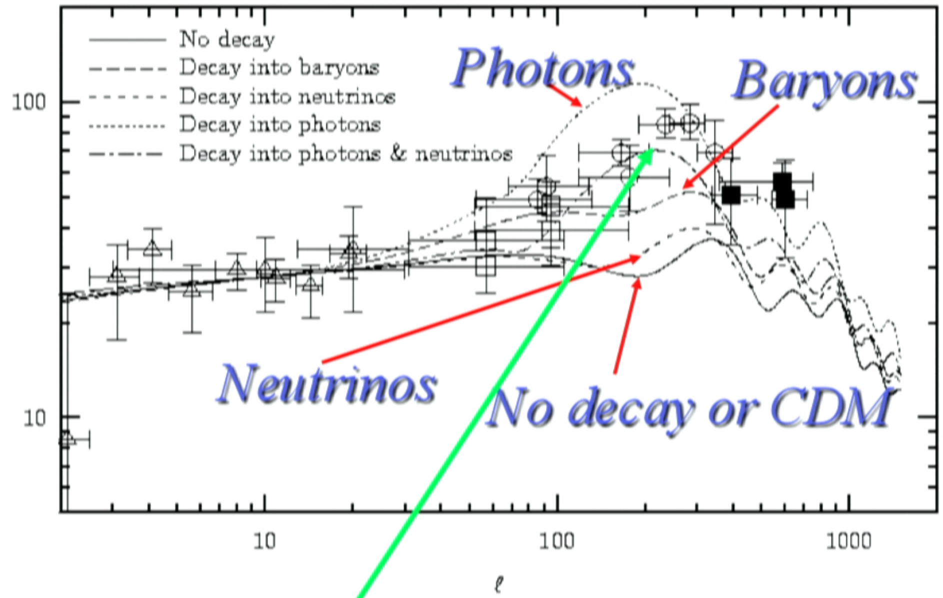
$$\sum_f x_f = 1$$

Scale beyond which loops decay

Decay width

RESULTS

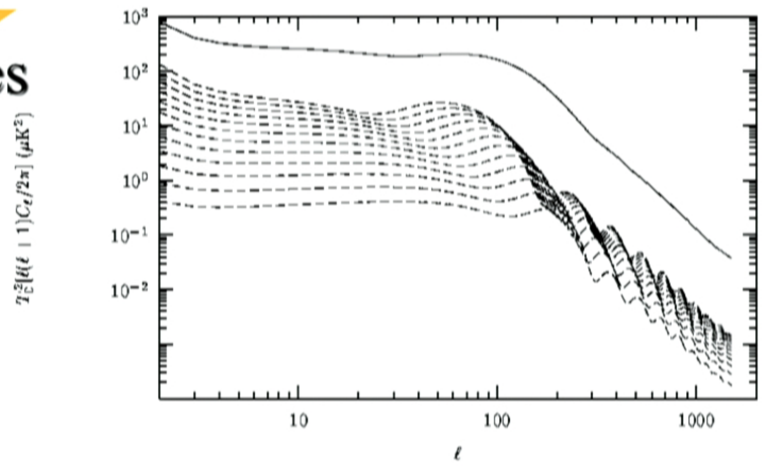
One coherent mode ...



Best fit: $x_{\nu} = x_{\gamma} = 0.5$

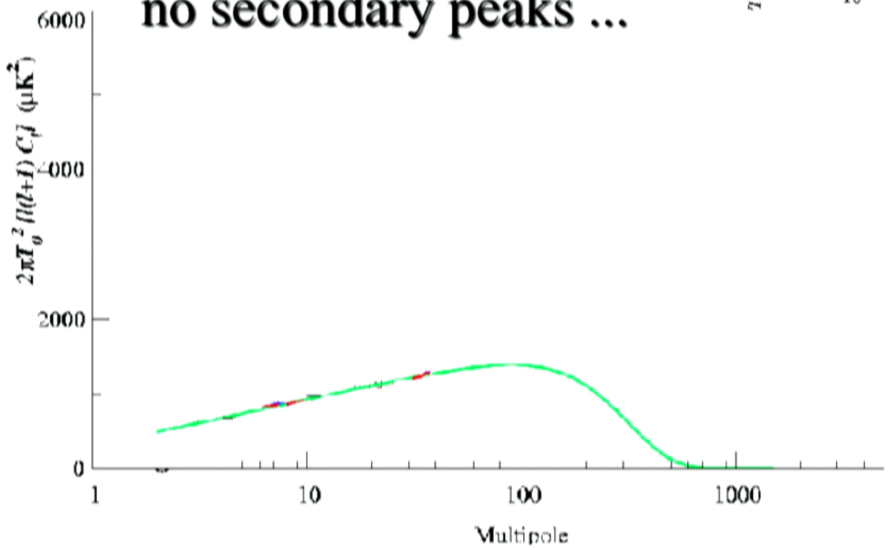
30

Coherent ⇨ Incoherent



Sum over coherent modes

no secondary peaks ...



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⇨ Inflation + Strings!

Mandatory
(peaks, $\Omega \sim 1$*, ...)*

Predicted
(GUT ...)

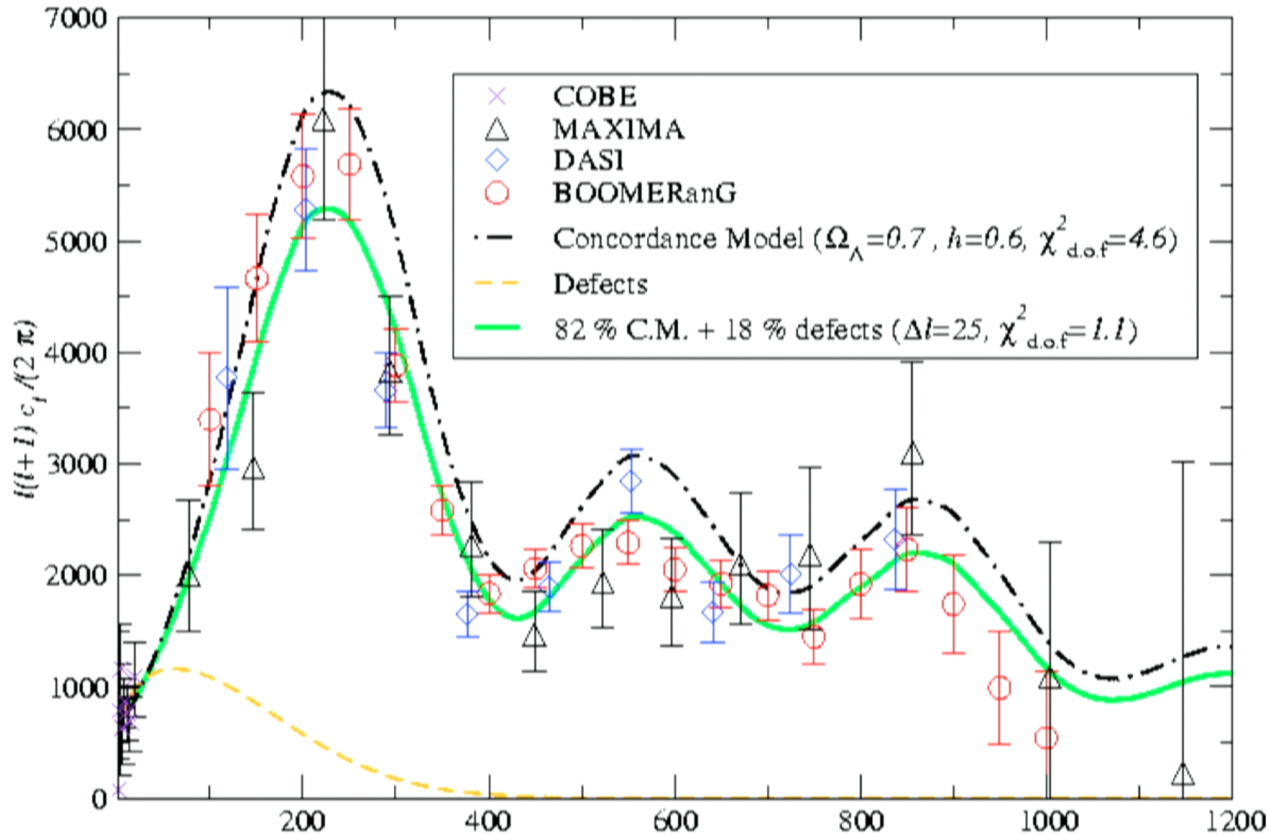
Linearized gravity

$$C_\ell = \alpha C_\ell^{\rm inf} + (1-\alpha) C_\ell^{\rm CS}$$

Best fit

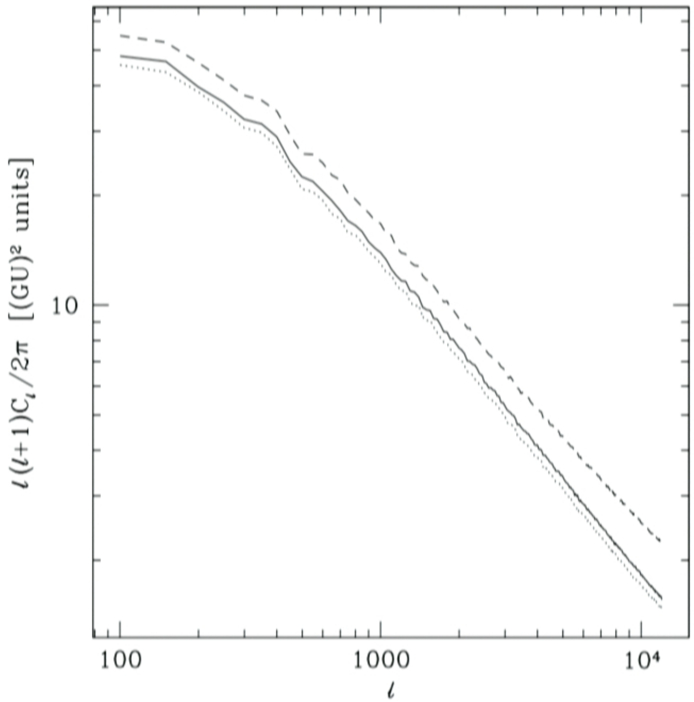
$$\alpha \lesssim 0.1$$

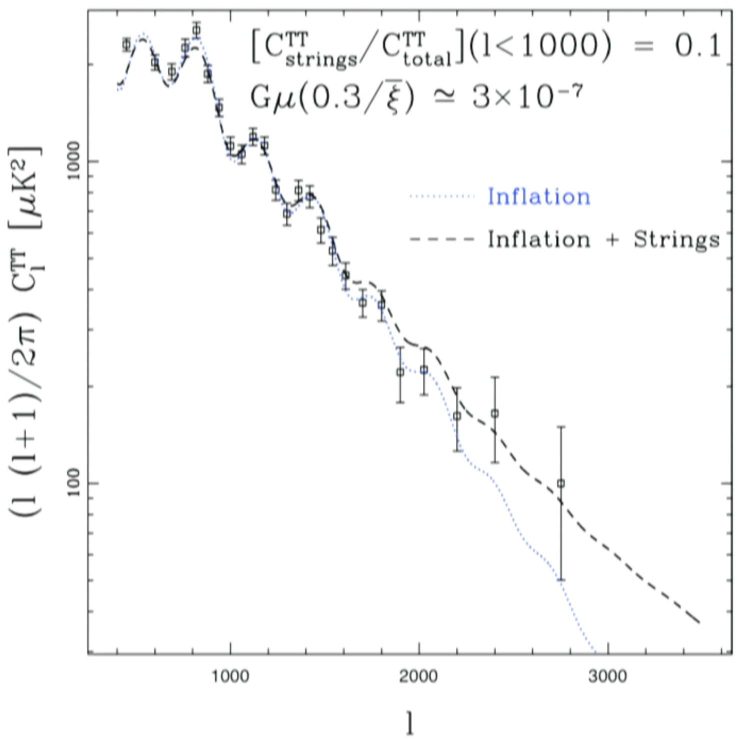
(Old) CMB data ...



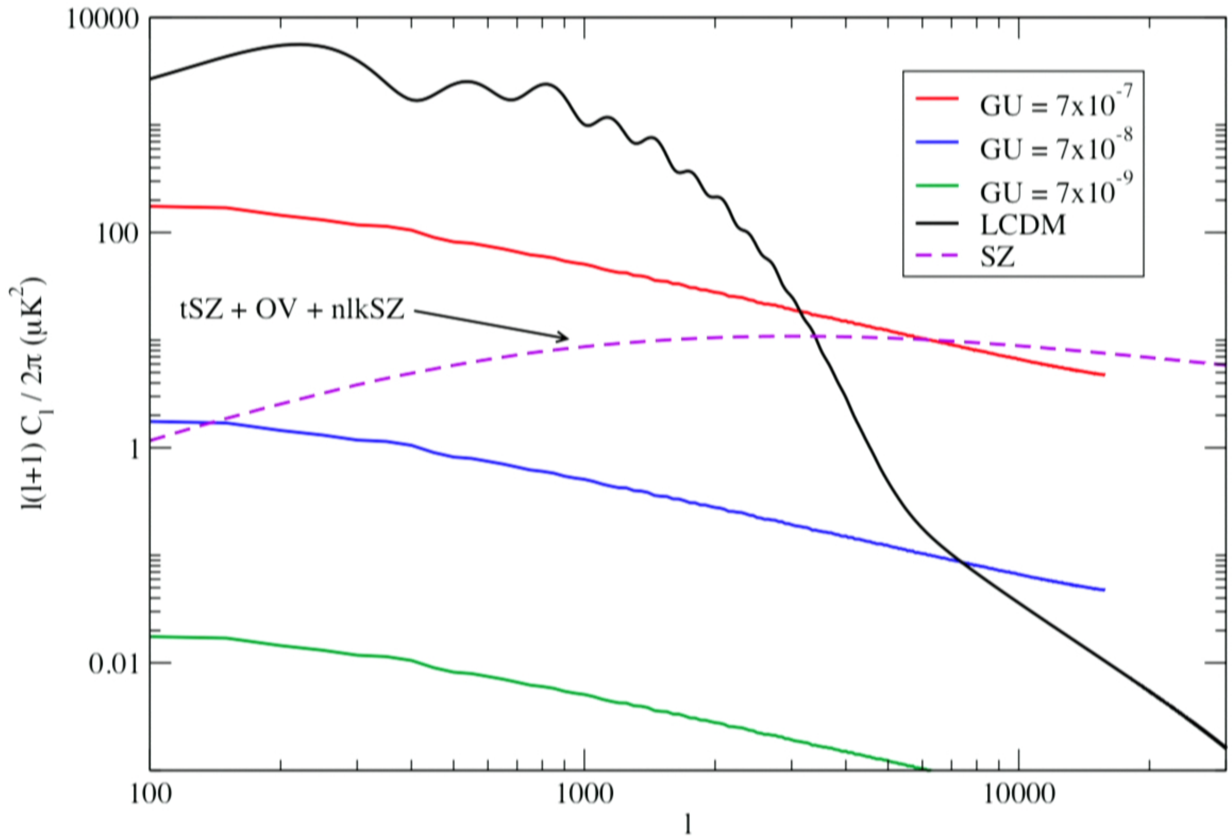
Bouchet et al. (2000)

Fraisse, Ringeval, Spergel & Bouchet (2008)

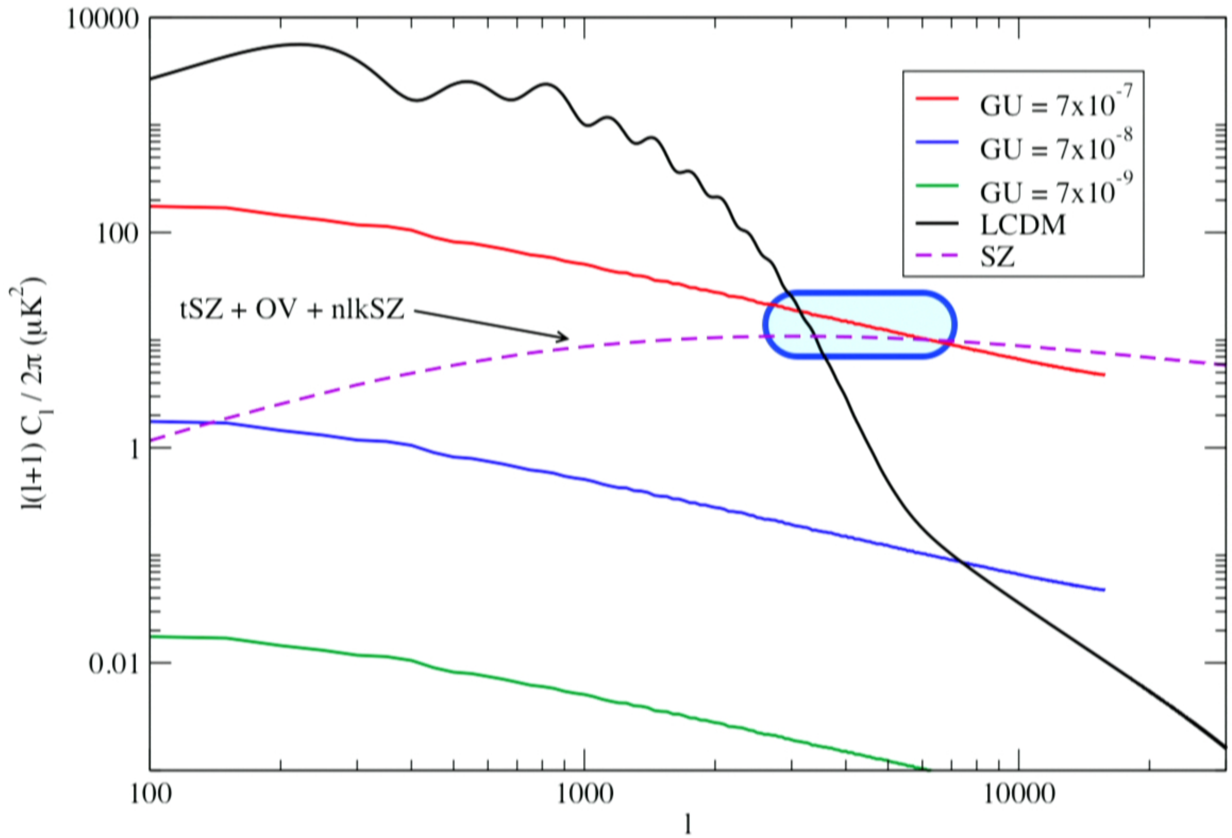




Pogosian et al. (2008)


10000
100
1
0.01
l(l+1) C_l / 2π (μK²)
100
1000
10000
l
tSZ + OV + nlkSZ
GU = 7x10^-7
GU = 7x10^-8
GU = 7x10^-9
LCDM
SZ

Possible window? dominates ...

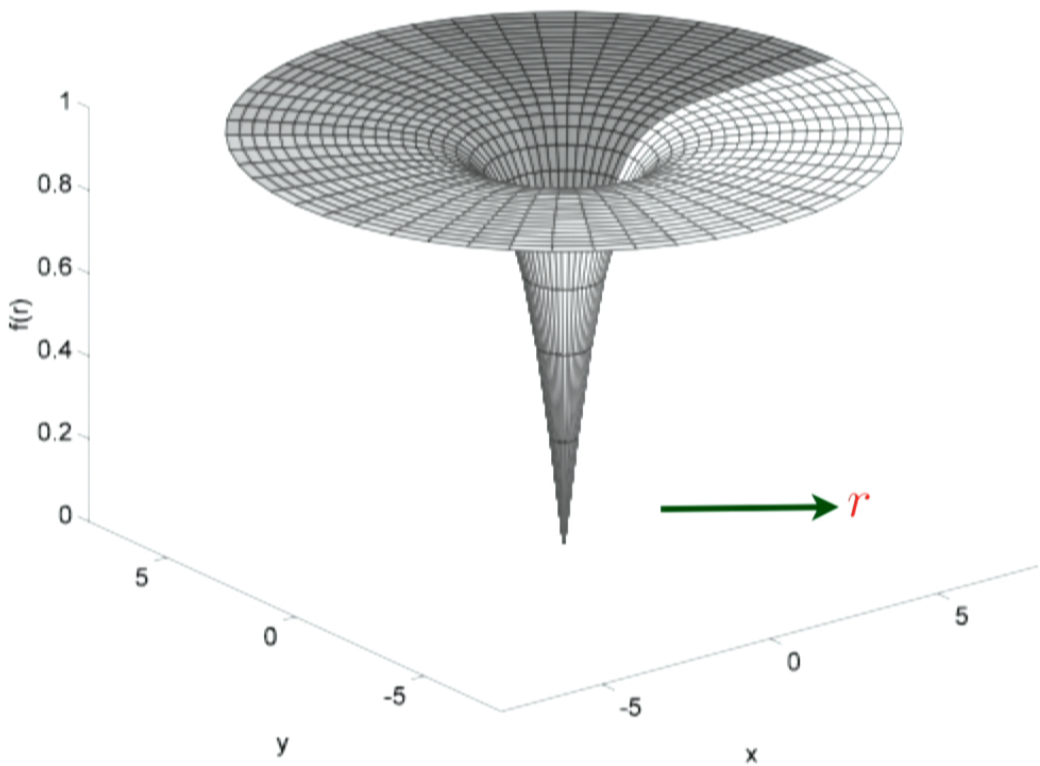


Structure & Models?

Abelian Higgs model

$$\mathcal{L}_{\rm a.H} = -D_\mu \phi D^\mu \phi - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - V(\phi)$$

$$\phi = f(r) \mathrm{e}^{in\theta}$$



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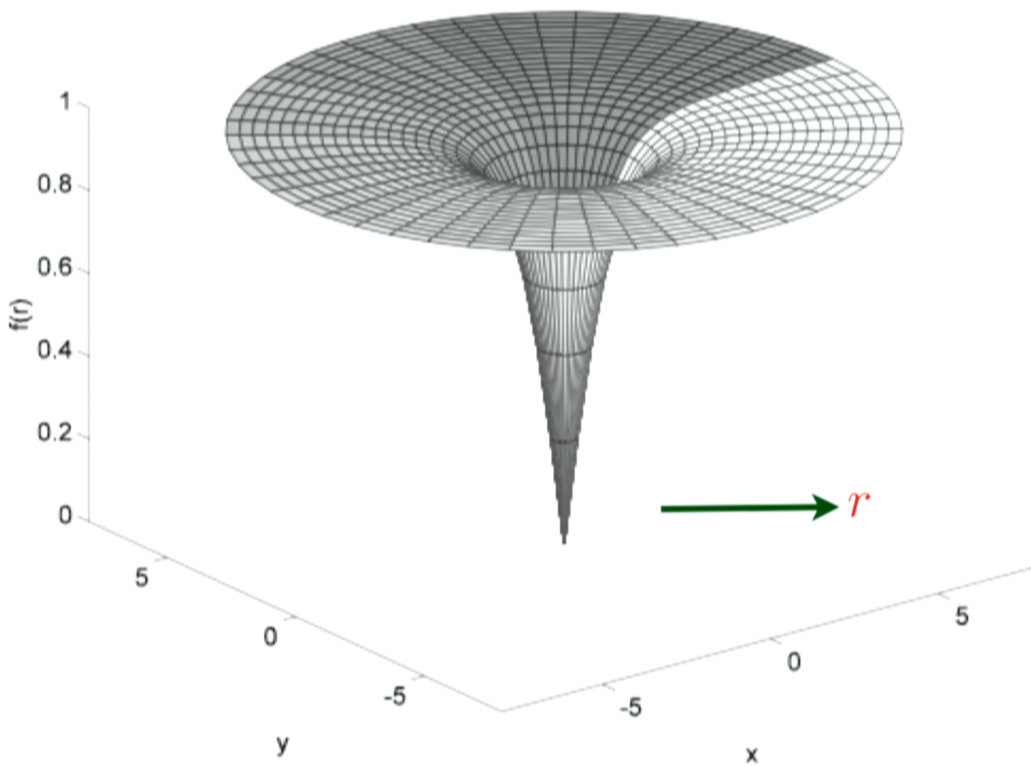


Sufficient???

Witten Superconducting String Model :

(E. Witten)

Bosonic carrier

$$\mathcal{L} = \mathcal{L}_{AH}(\Phi, B_\mu) - \frac{1}{2}(D_\mu\Sigma)^\star D^\mu\Sigma - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\Phi,\Sigma)$$

e, A_μ

$$V(\Phi,\Sigma) = f(|\Phi|^2 - \eta^2)|\Sigma|^2 + \frac{m_\sigma^2}{2}|\Sigma|^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4$$

Fermionic carrier

$$\mathcal{L} = \mathcal{L}_{AH}(\Phi, B_\mu) + \frac{i}{2}\left[\overline{\Psi}_R\gamma^\mu D_\mu\Psi_R + \overline{\Psi}_L\gamma^\mu D_\mu\Psi_L\right] - g\overline{\Psi}_L\Psi_R\Phi + \text{h.c.}$$

e, A_μ and q, B_μ

Vortex configuration

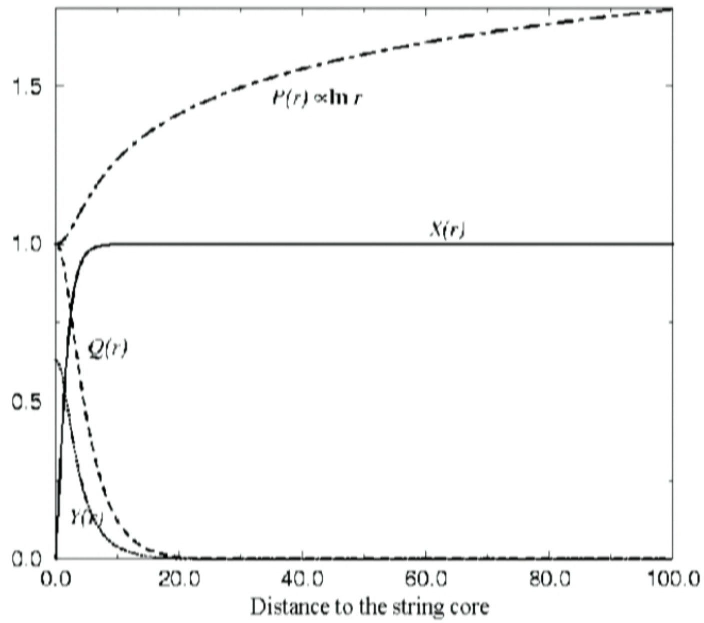
$$\Sigma(x^\alpha) = \sigma(r)\, e^{i(\omega t - kz)}$$

$$P_t = \omega + eA_t$$

$$P_z = -k + eA_z$$

$$wP_*^2 = P_z^2 - P_t^2$$

State Parameter



Stress-Energy Tensor : $T^{\mu}_{\nu} = -2g^{\mu\alpha}\frac{\delta\mathcal{L}}{\delta g^{\alpha\nu}} + \delta^{\mu}_{\nu}\mathcal{L}$

$$U(w) = -2\pi \int r\mathrm{d}\, r\, T^{t}_{t}[\varphi, Q, \sigma, P_{\star}]$$

Energy per unit length

$$T(w) = -2\pi \int r\mathrm{d}\, r\, T^{z}_{z}[\varphi, Q, \sigma, P_{\star}]$$

Tension

Current : $\mathcal{J}^{\mu} = \sigma^2[\nabla^{\mu}(\omega t - kz) + eA^{\mu}] \propto \frac{\delta\mathcal{L}}{\delta A_{\mu}}$

$$\mathcal{C}(w) = 2\pi \int r\mathrm{d}\, r \sqrt{|\mathcal{J}^{\mu}\mathcal{J}_{\mu}|}$$

Charge per unit length or current

Dual Formalism

(B. Carter)

State Parameter w ⇨ Lagrangian function $\mathcal{L}\{w\}$

$$\mathcal{S}_{\mathcal{L}} = -m^2 \int \mathrm{d}^2\xi \sqrt{-\gamma}\mathcal{L}\{w\} \iff \mathcal{S}_{\Lambda} = -m^2 \int \mathrm{d}^2\xi \sqrt{-\gamma}\Lambda\{\chi\}$$

2 formulations: w, χ ↔ Master function $\Lambda\{\chi\}$

$$w = \kappa_0 \gamma^{ab}\nabla_a\varphi\nabla_b\varphi \iff \chi = \tilde{\kappa}_0\gamma^{ab}\nabla_a\psi\nabla_b\psi$$

Orthogonal phase gradients

Opposite signs

$$\gamma^{ab}\nabla_a\varphi\nabla_b\psi = 0$$

2 Conserved currents

$$n^a = -\frac{\partial\Lambda}{\partial\nabla_a\psi} \iff z^a = -\frac{\partial\mathcal{L}}{\partial\nabla_a\varphi}$$

Current Amplitudes $\tilde{\mathcal{K}} n^a = \tilde{\kappa}_0 \nabla^a \psi \Longleftrightarrow \mathcal{K} z^a = \kappa_0 \nabla^a \varphi$

$$\tilde{\mathcal{K}}^{-1} = -2\frac{\mathrm{d}\Lambda}{\mathrm{d}\chi} \Longleftrightarrow \mathcal{K}^{-1} = -2\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}w}$$

Equivalent formulations ⇨ $\tilde{\mathcal{K}} = -\mathcal{K}^{-1}$ ⇨ $w = \mathcal{K}^2 \chi$

Legendre Transformation

$$\Lambda = \mathcal{L} + \mathcal{K}\chi$$

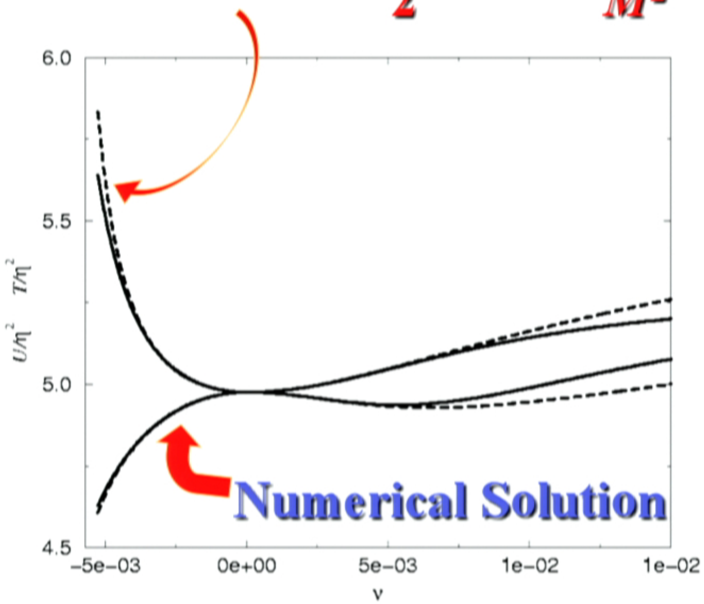
Stress-Energy tensor $T^{\mu\nu} = U u^\mu u^\nu - T v^\mu v^\nu$

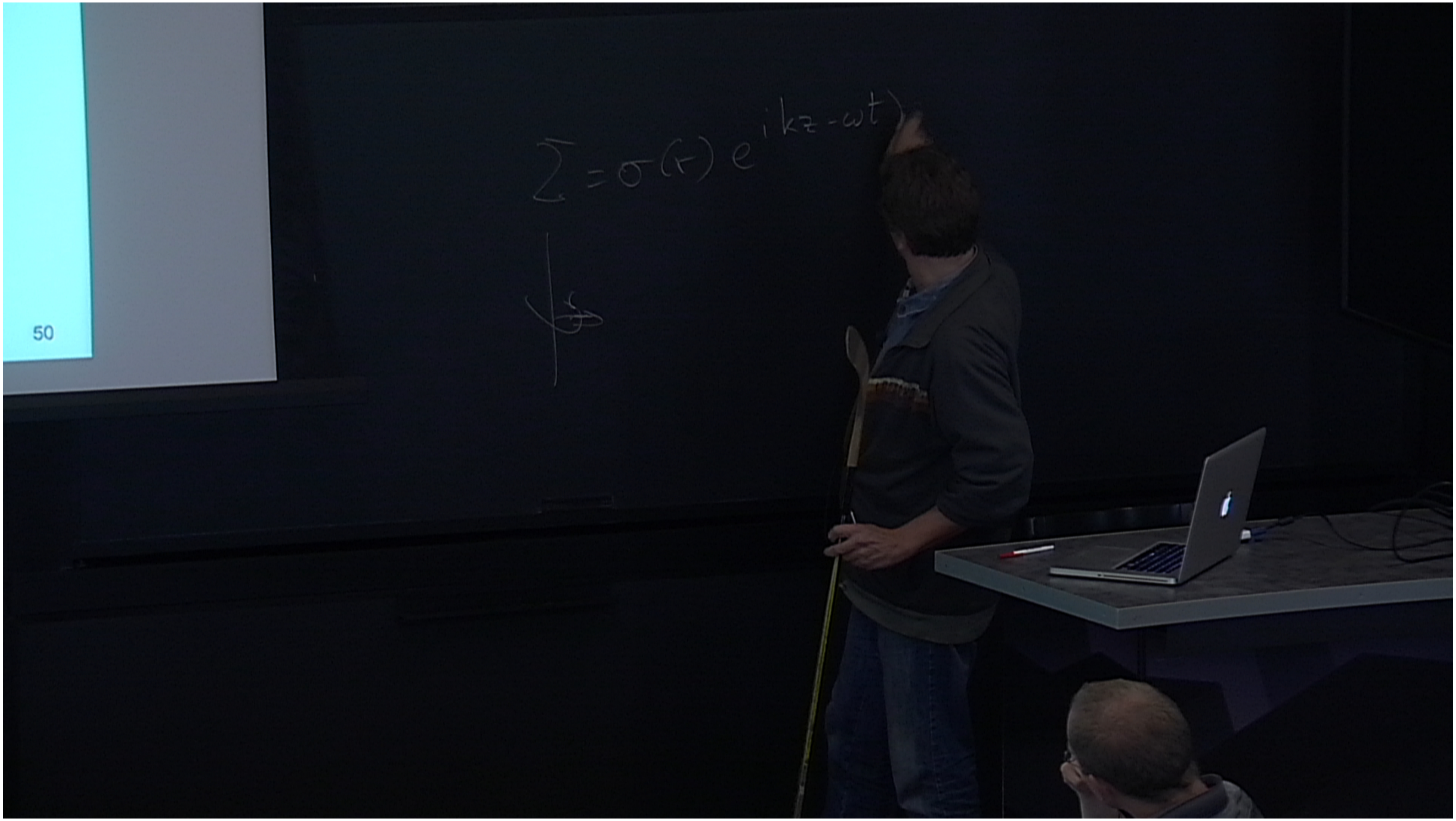
Regime	U	T	w and χ	Current
Electric	$-\Lambda$	$-\mathcal{L}$	<0	Timelike
Magnetic	$-\mathcal{L}$	$-\Lambda$	>0	Spacelike

Equations of State

Goto-Nambu : $\mathcal{L} = -m^2 \Rightarrow U = T = m^2$

Bosonic Carrier : $\mathcal{L} = -m^2 - \frac{M^2}{2} \ln \{1 + \frac{w}{M^2}\}$

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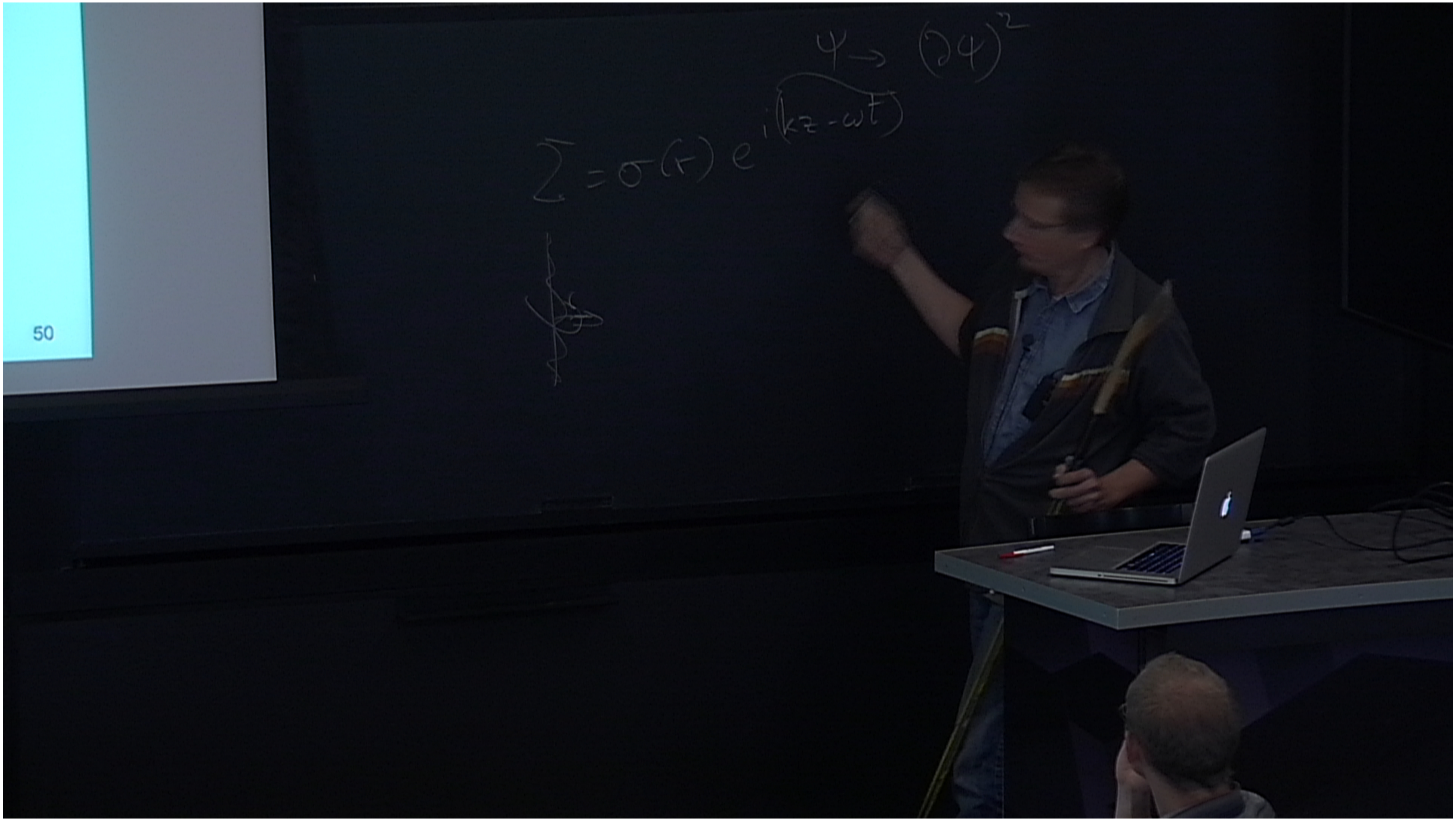
$\Sigma = \sigma(r)\, e^{i(kz-\omega t)}$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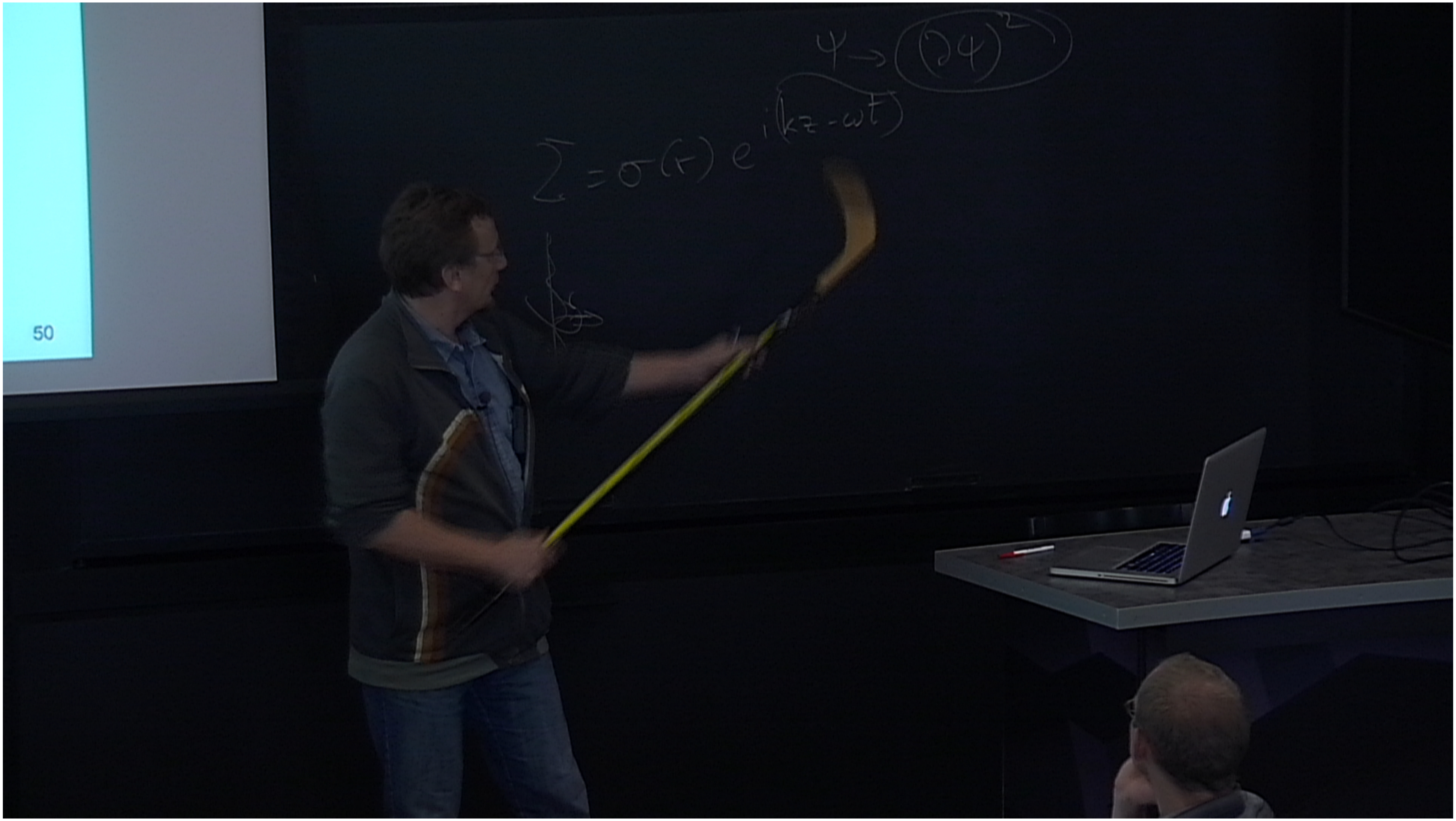
$\Psi \to (\partial\varphi)^2$
$\Sigma = \sigma(r)\, e^{i(kz-\omega t)}$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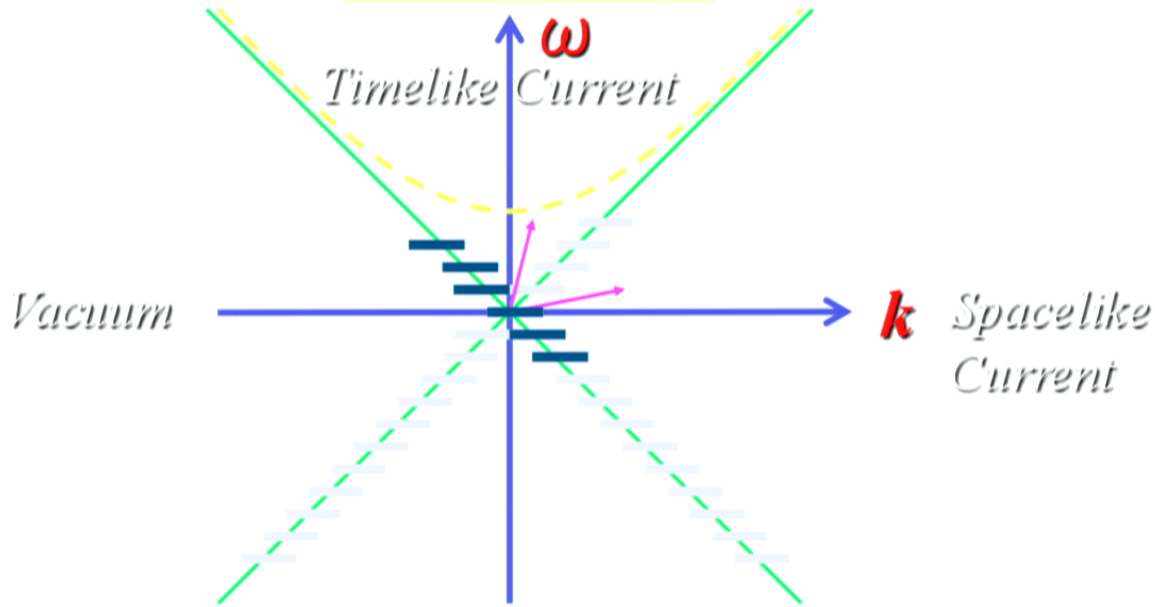
ψ → (∂φ)²
Σ = σ(r) e^{i(kz-ωt)}

Fermions ⇨ Quantization

Zero Modes



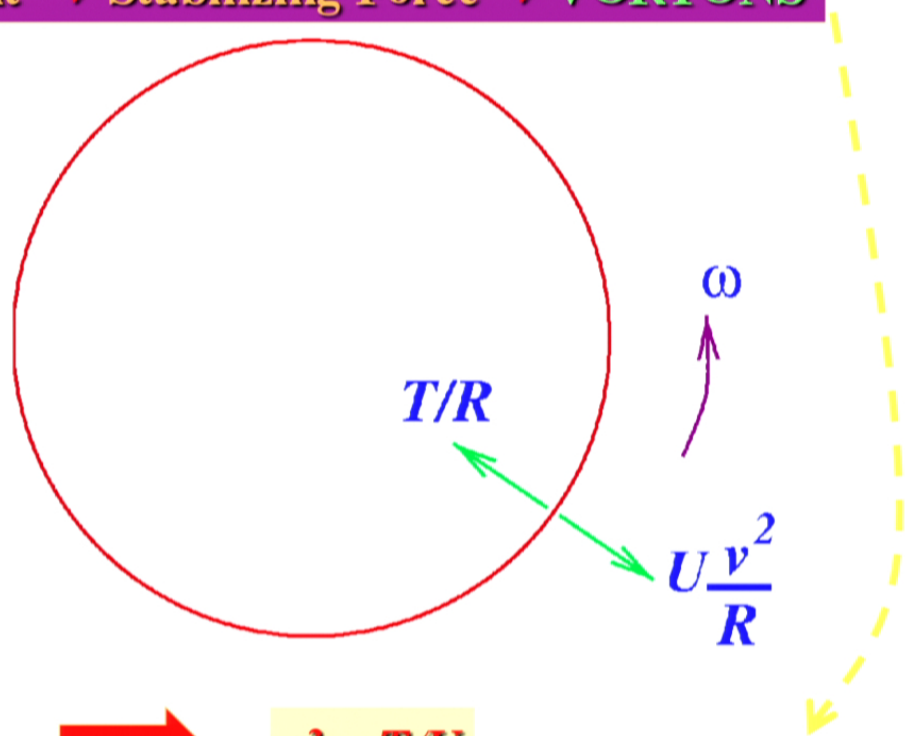
Fermionic Carrier : $\mathcal{L} = -m^2 +$???? **But** $U + T = 2\, m^2$

Chiral Current Model : $\mathcal{L} = -m^2 - \frac{1}{2}\psi^2\gamma^{ab}\nabla_a\varphi\nabla_b\varphi$

Arbitrary Current from Fermions = Sum of C. Cs

$$\mathcal{L}_W = -m^2 - \sum_i \frac{1}{2}\psi_i^2\gamma^{ab}\nabla_a\varphi_i\nabla_b\varphi_i$$

Current ⇨ Stabilizing Force ⇨ VORTONS

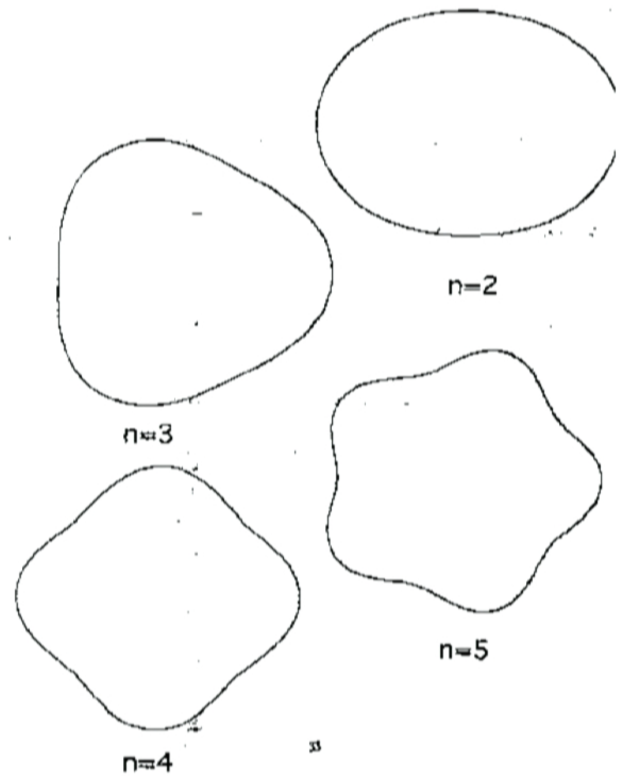


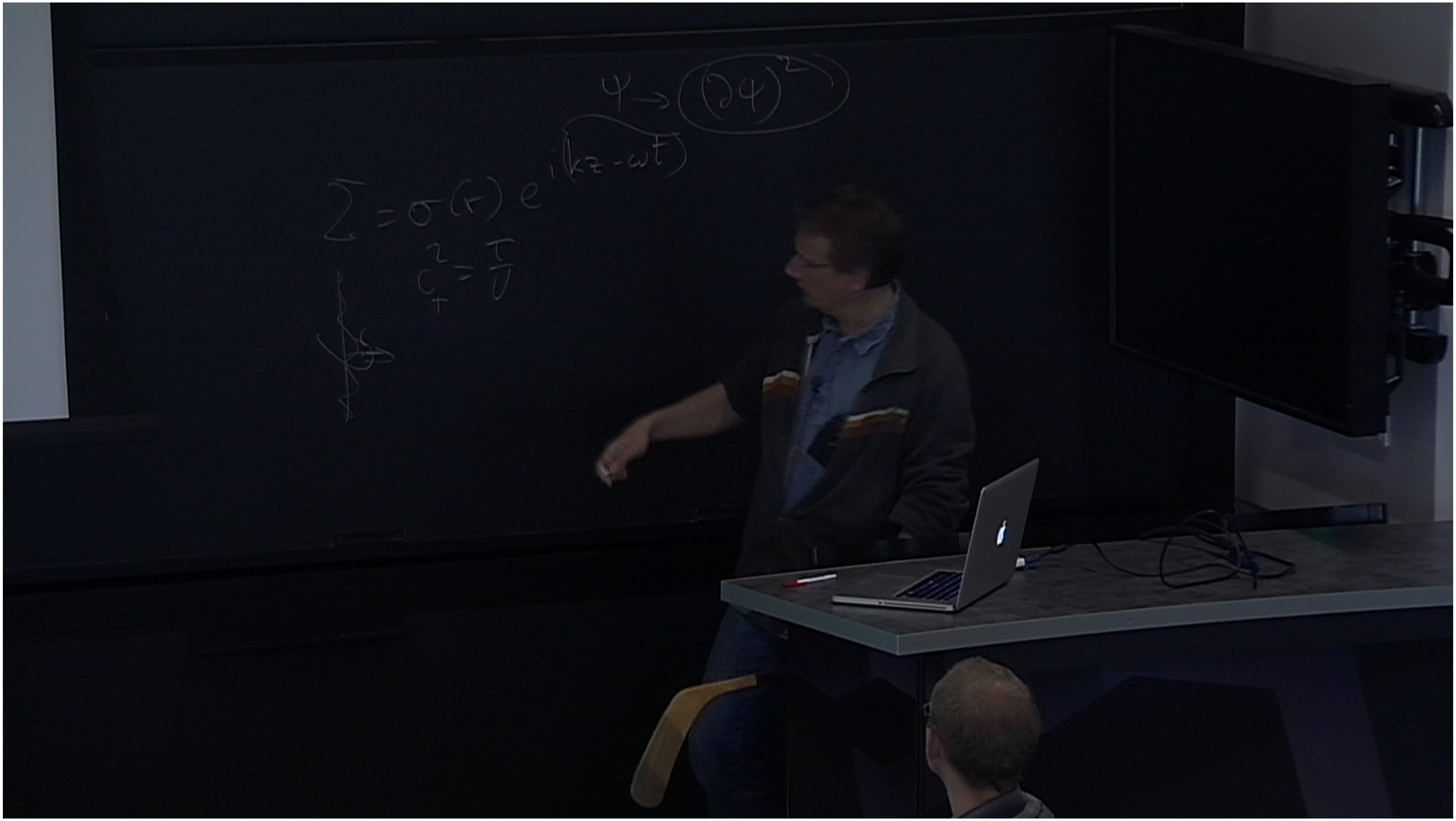
$$v^2 = T/U$$

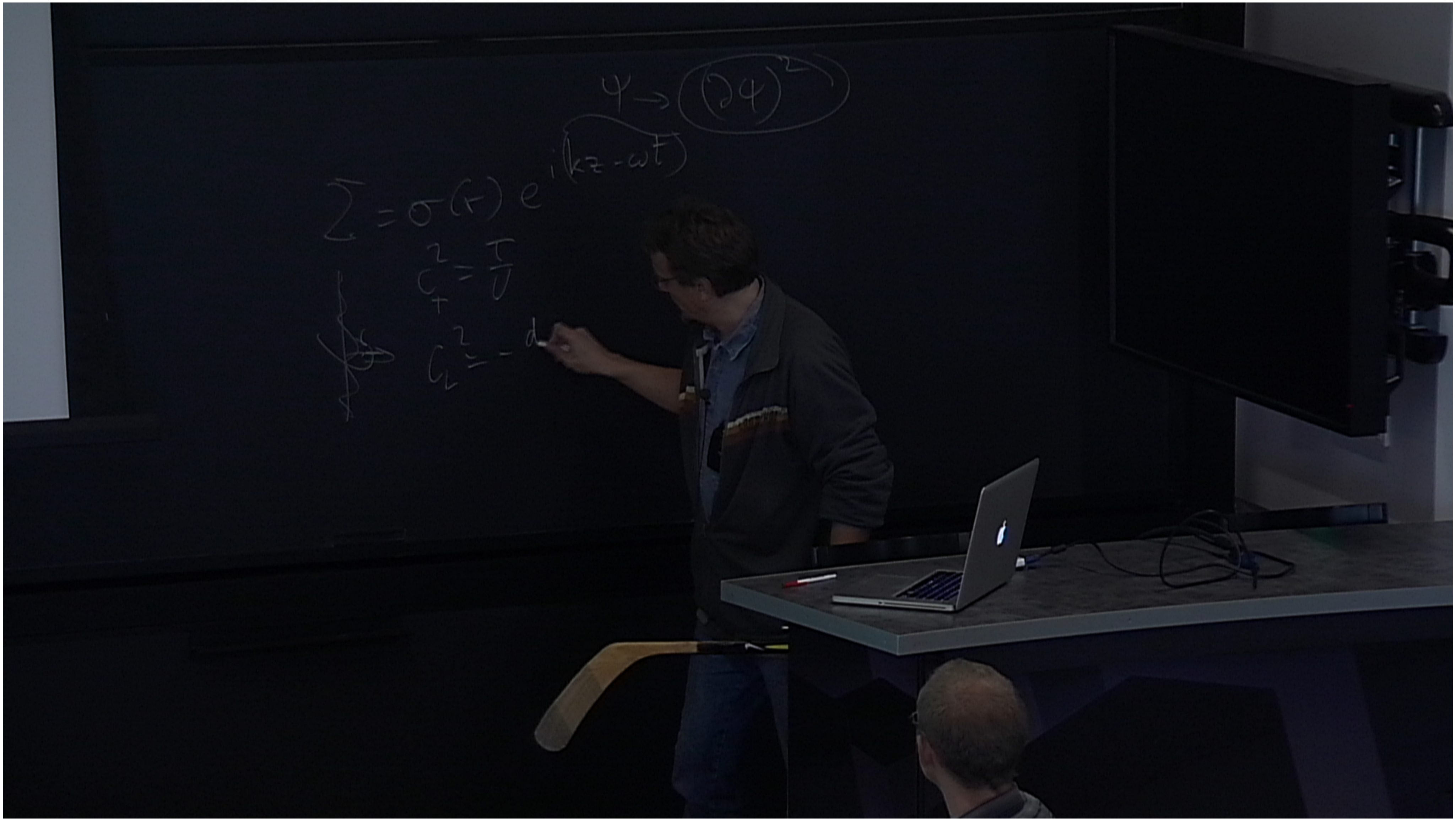
STABILITY …

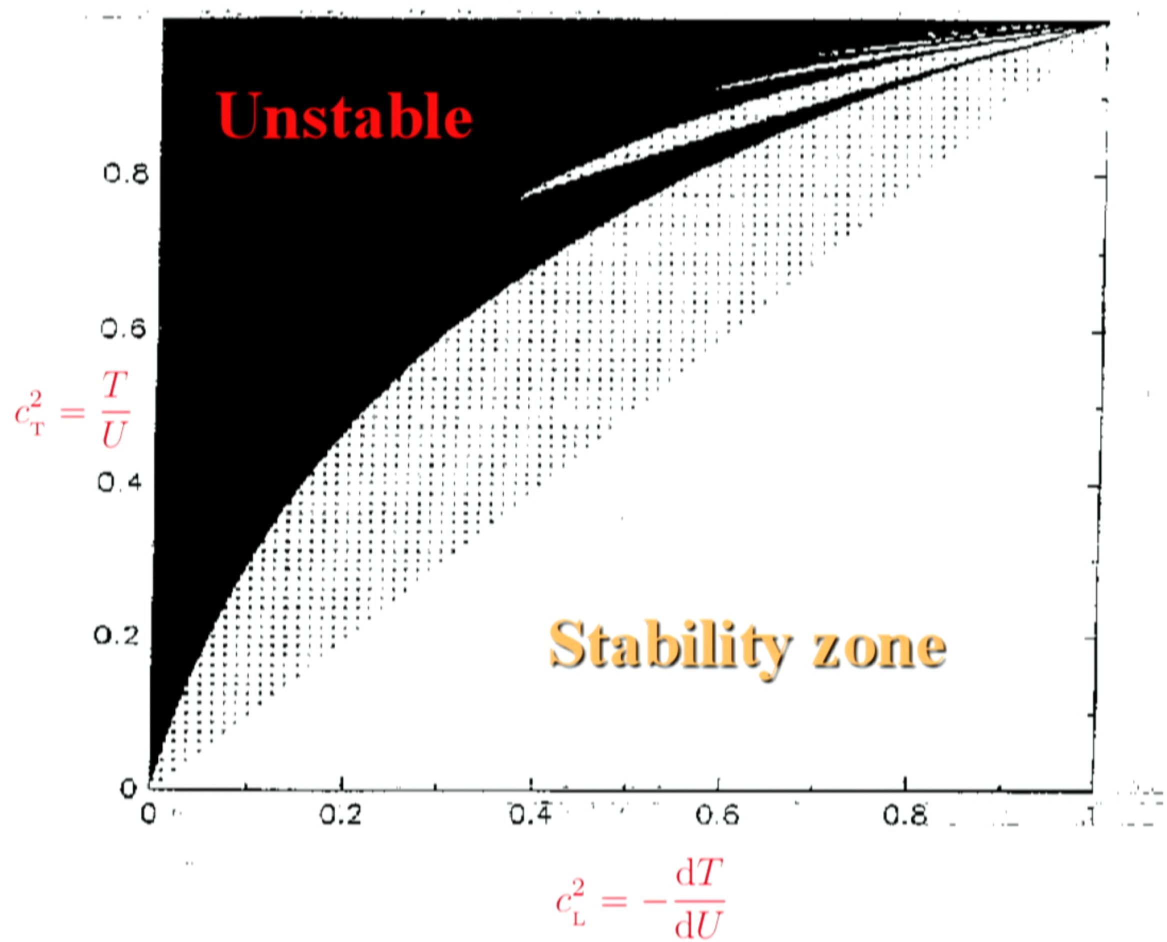
$$\Omega_{\rm vortons} \gg 1$$

Potentially unstable configurations

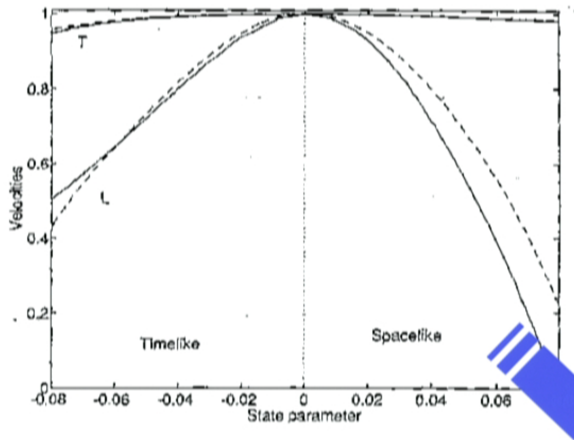


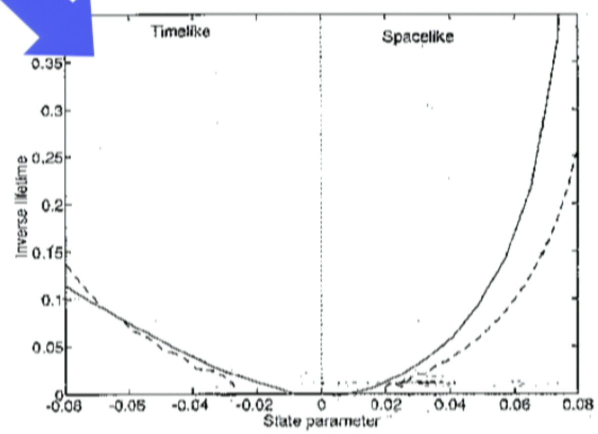

ψ → (∂ψ)²
Σ = σ(r) e^{i(kz-ωt)}
c_T² = T/U




Unstable
Stability zone
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
$c_{\mathrm{T}}^2 = \frac{T}{U}$
$c_{\mathrm{L}}^2 = -\frac{\mathrm{d}T}{\mathrm{d}U}$

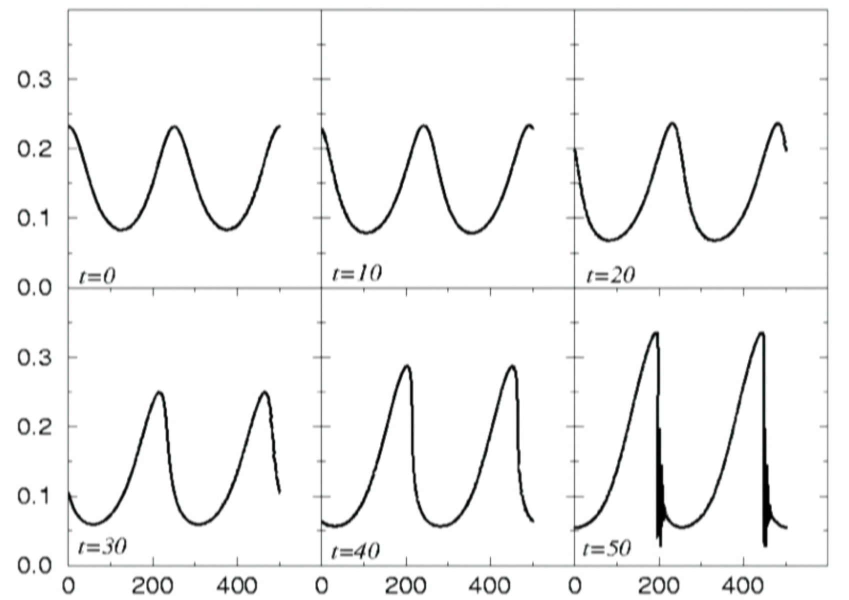
Witten Model

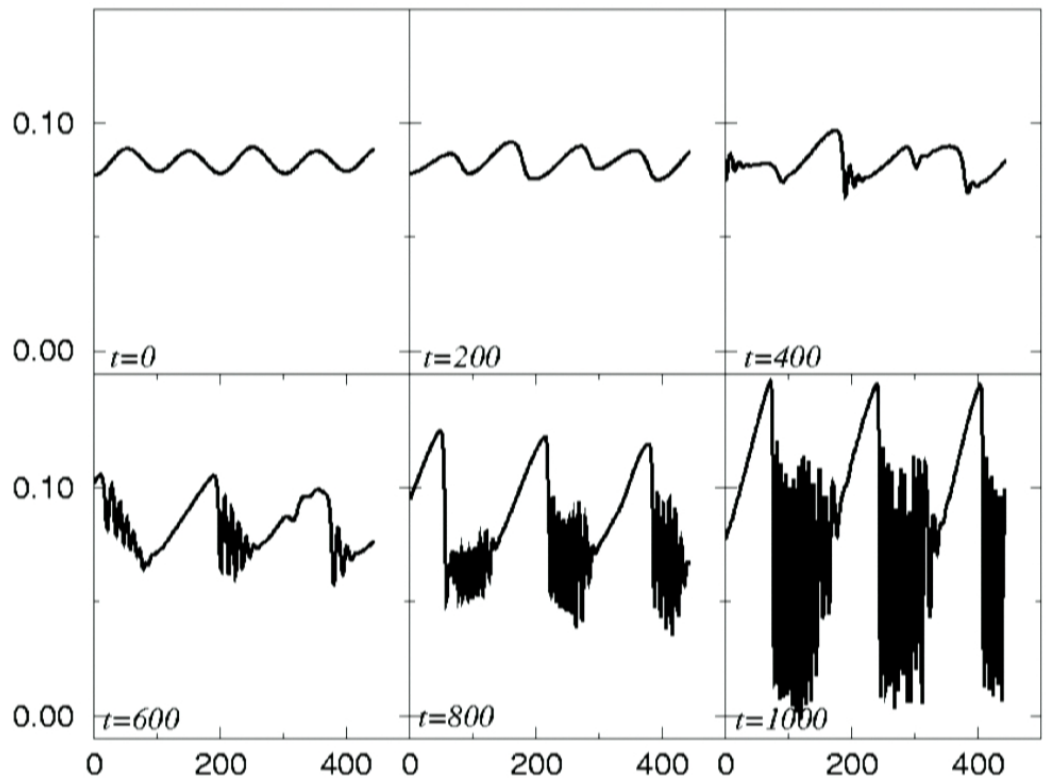




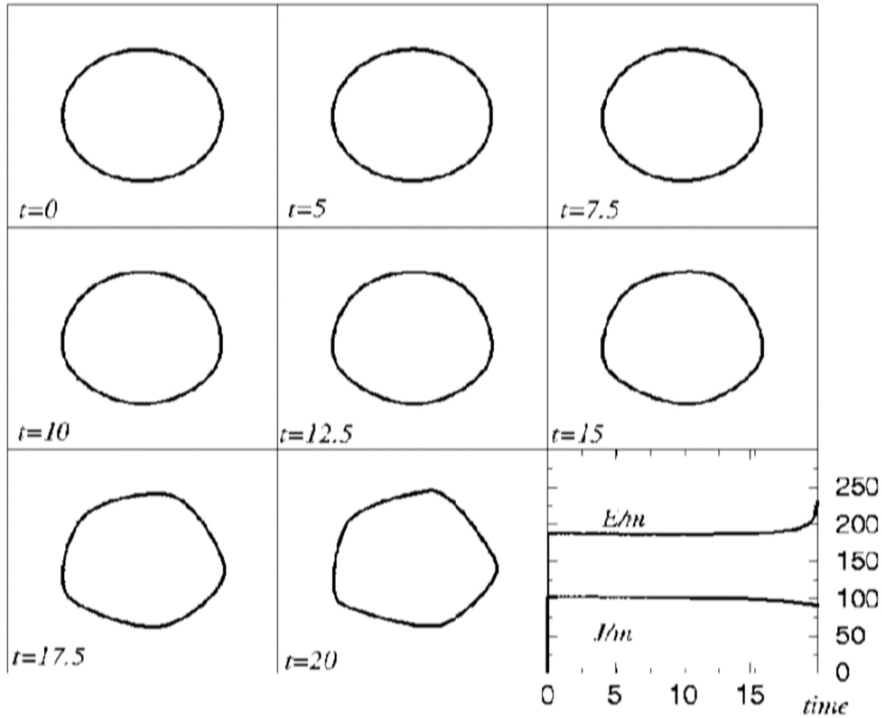
Numerical loop simulation

Shocks …




0.10
0.00
t=0
t=200
t=400
t=600
t=800
t=1000
0
200
400

Kinks ...



Particle Radiation ...

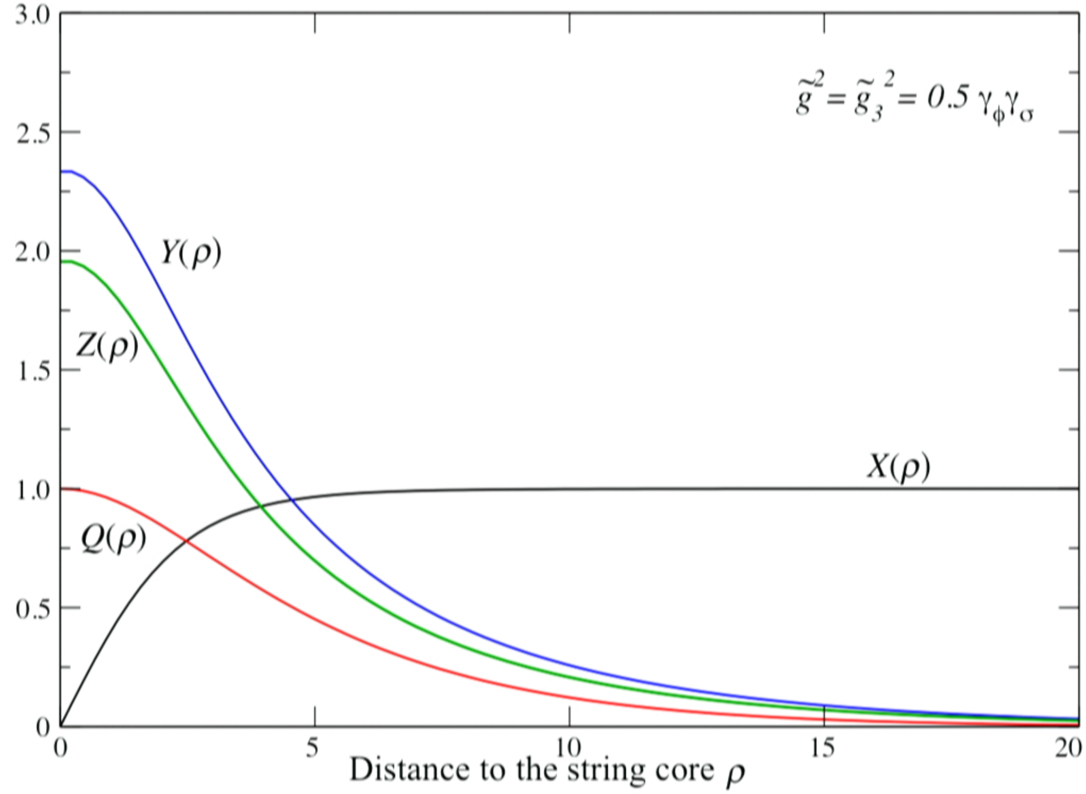
Currents: Witten bosonic model

$$\mathcal{L}_{\mathrm{W}} = \mathcal{L}_{\mathrm{a.H}}\left(\phi, C_\mu; q_\phi, m_\phi, \lambda_\phi\right) + \mathcal{L}\left(\Sigma, A_\mu; q_\sigma, m_\sigma, \lambda_\sigma\right) + V\left(\phi, \Sigma\right)$$

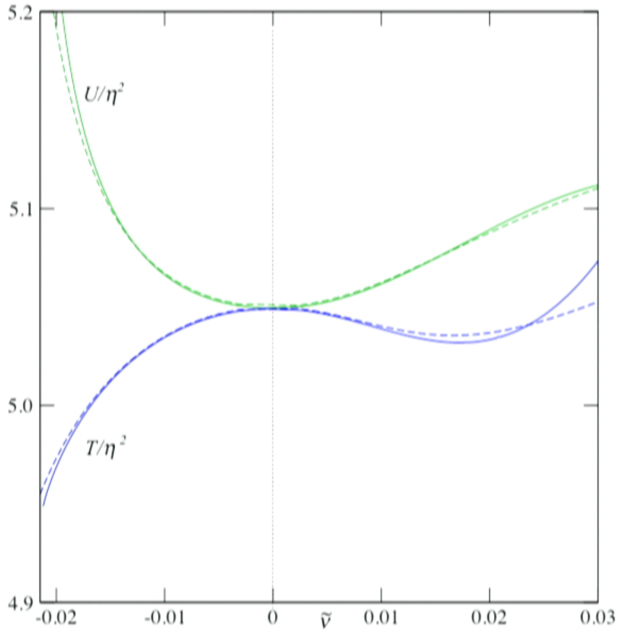
→ One (abelian) current

$$\Sigma = \mathrm{e}^{i\psi(\xi_a)}\sigma(x^\perp)$$

Typical string configuration (neutral limit)



Equation of state (B. Carter)



$$T^{\mu\nu} = g^{\mu\nu}\mathcal{L} - 2\frac{\delta\mathcal{L}}{\delta g_{\mu\nu}}$$

Timelike u^μ and spacelike v^μ eigenvectors

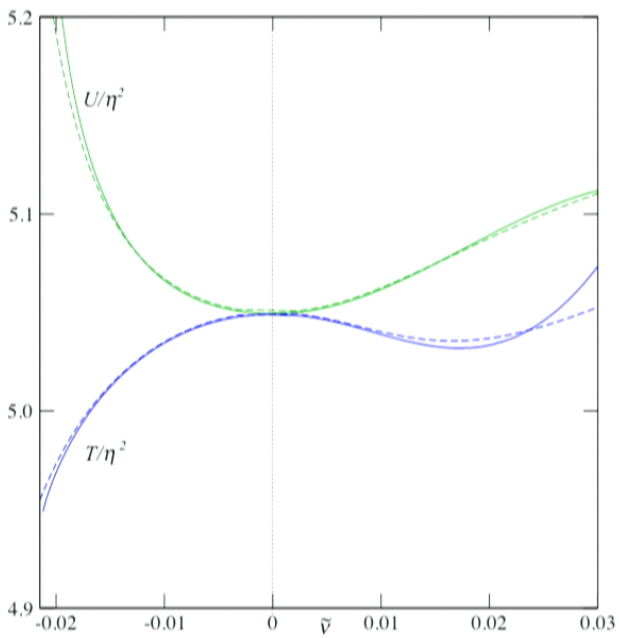
$$\bar{T}^{\mu\nu} = \int \mathrm{d}^2x^\perp \, T^{\mu\nu} = U u^\mu u^\nu - T v^\mu v^\nu$$

↓

Diagonalisation & Integration

State parameter $w \equiv \eta^{\mu\nu}\bar{\nabla}_\mu\psi\bar{\nabla}_\nu\psi$

Equation of state (B. Carter)



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Diagonalisation & Integration

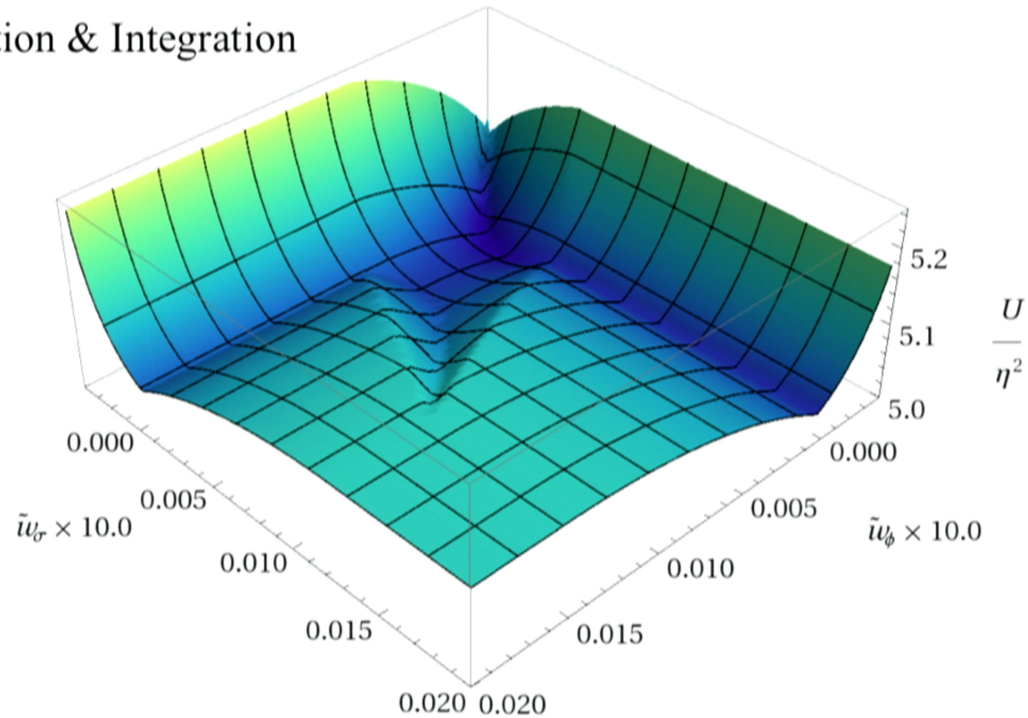
State parameter $w \equiv \eta^{\mu\nu}\bar{\nabla}_\mu\psi\bar{\nabla}_\nu\psi$

State parameter**s**

$$\chi_{ij} \equiv \eta^{\mu\nu}\bar{\nabla}_\mu\psi_i\bar{\nabla}_\nu\psi_j$$

symmetric matrix of all possible Lorentz invariants (scalars in the worldsheet)

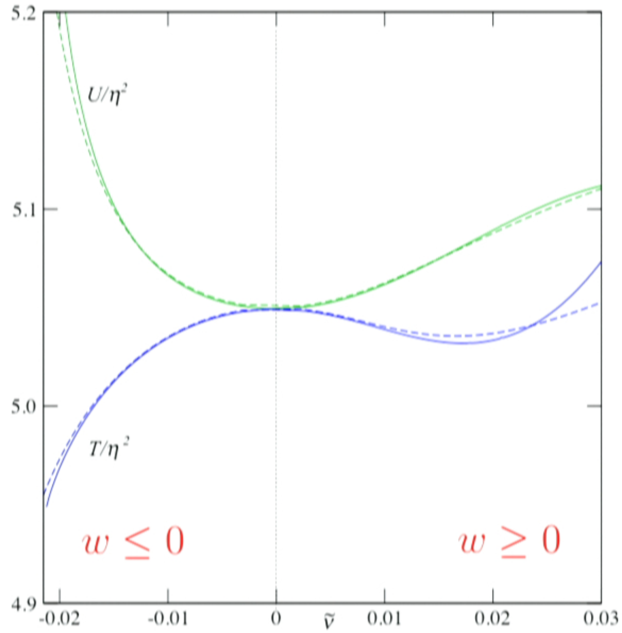
+ Diagonalisation & Integration



Macroscopic modelling

Single current:

$$\begin{cases} \mathcal{L}_{\rm magn}(w) = -m^2 - \frac{1}{2} w \left(1 + \frac{w}{m_*^2}\right)^{-1} & w \geq 0 \\ \mathcal{L}_{\rm elec}(w) = -m^2 - \frac{1}{2} m_*^2 \ln\left(1 + \frac{w}{m_*^2}\right) & w \leq 0 \end{cases}$$



Many currents

$$V(\phi, \Sigma_i) \supset \left(|\phi|^2 - \eta^2\right) \sum_i f^{(i)} |\Sigma_i|^2$$

$$+ \sum_{i,j} \lambda^{(i,j)} |\Sigma_i|^2 |\Sigma_j|^2$$

↓

each condensate acts as a positive mass term for all the others

→ $\lambda^{(i,j)}|_{i \neq j} \ll \lambda^{(i,i)}$

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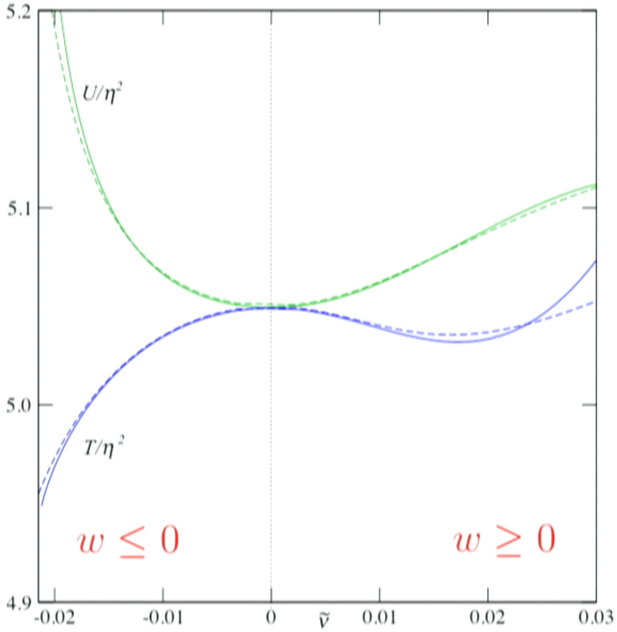

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Small coupling additive model

$$\mathcal{L}(\chi) = -m^2 + \sum_i \mathcal{L}_i(w_i)$$

→ $T^{\mu\nu} = m^2\eta^{\mu\nu} + \sum_i T^{\mu\nu}_{(i)}$ with $T^{\mu\nu}_{(i)} = U_i u_i^\mu u_i^\nu - T_i v_i^\mu v_i^\nu$

→ The dynamics essentially depends on w_i but not on x

Non abelian currents $\Sigma \in n$ Arbitrary gauge group G

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- Parameters such that there exists a condensate

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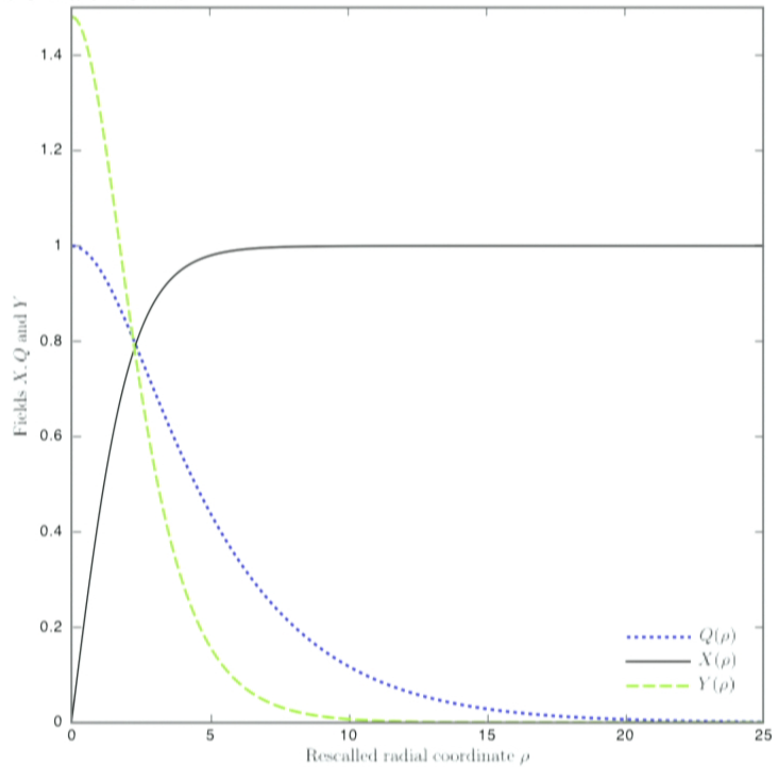
- Minimum energy state $\sigma_0\left(x^\perp\right)$



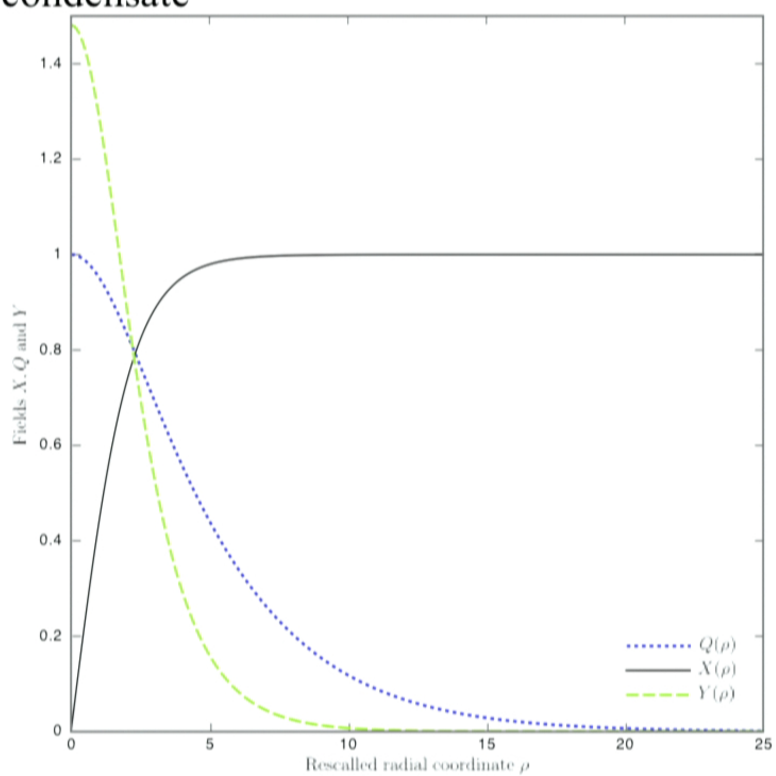
Non abelian currents

$\Sigma \in n$ Arbitrary gauge group G

- Parameters such that there exists a condensate
- Minimum energy state $\sigma_0\left(x^{\perp}\right)$
- G exitations in the worldsheet

$$\Sigma = e^{i\psi^a(\xi)T_a}\sigma_0$$

Generator for $\pounds_G$



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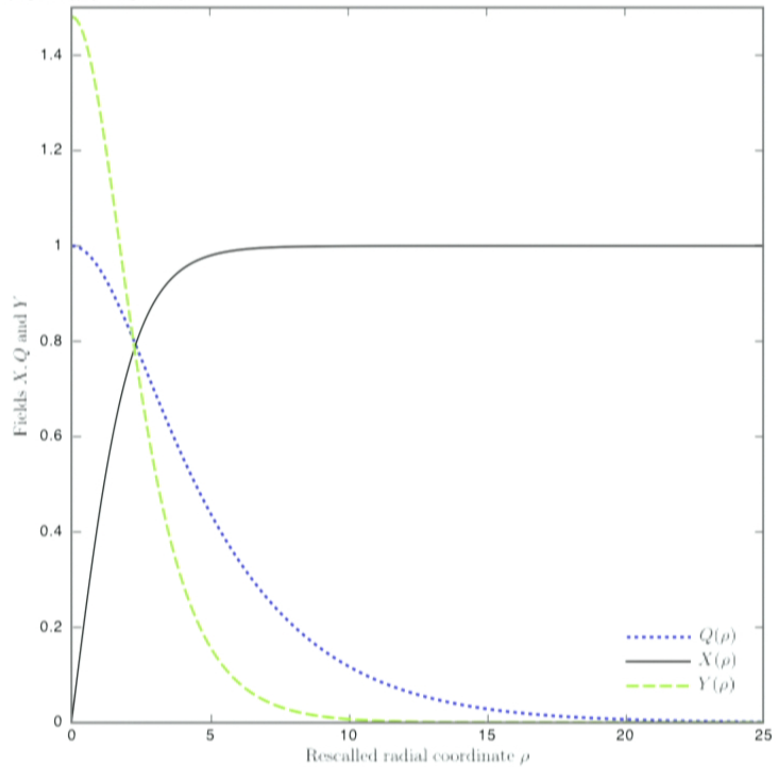

Non abelian currents $\quad \Sigma \in n \quad$ Arbitrary gauge group G

- Parameters such that there exists a condensate
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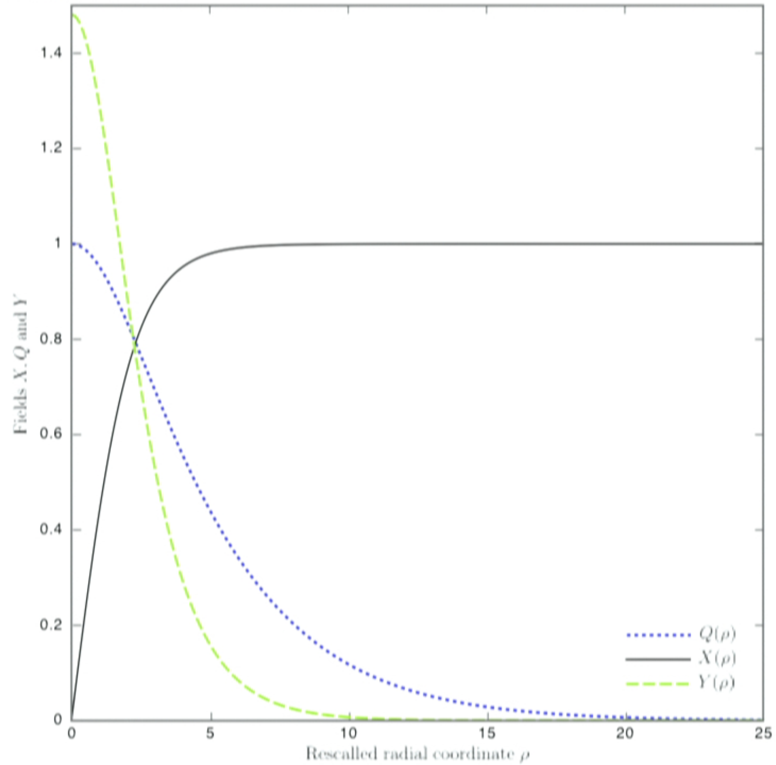
- G exitations in the worldsheet

$$\Sigma = e^{i\psi^a(\xi)T_a}\sigma_0$$

Generator for $\pounds_G$

- currents:

$$\mathcal{J}^a_\mu = \frac{\delta\mathcal{L}}{\delta\partial_\mu\psi^a}$$



+ quantization $\Longrightarrow$ current algebra

$$\mathcal{J}_a^\mu = \boldsymbol{\sigma}_0^\dagger \{T_a, T_b\} \boldsymbol{\sigma}_0 \nabla_\mu \psi^b + i \left(\nabla^\mu \boldsymbol{\sigma}_0^\dagger T_a \boldsymbol{\sigma}_0 - \boldsymbol{\sigma}_0^\dagger T_a \nabla^\mu \boldsymbol{\sigma}_0 \right) \rightarrow 0$$

$$\downarrow$$

$$Q_a = \int \mathrm{d}^2 x^\perp \pi_a \qquad \text{where} \qquad \pi_a = \frac{\delta \mathcal{L}^{(2)}}{\delta \dot{\psi}^a}$$

+ quantization $\Longrightarrow$ current algebra

$$\mathcal{J}_a^\mu = \boldsymbol{\sigma}_0^\dagger \{T_a, T_b\} \boldsymbol{\sigma}_0 \nabla_\mu \psi^b + i \left(\nabla^\mu \boldsymbol{\sigma}_0^\dagger T_a \boldsymbol{\sigma}_0 - \boldsymbol{\sigma}_0^\dagger T_a \nabla^\mu \boldsymbol{\sigma}_0 \right) \to 0$$

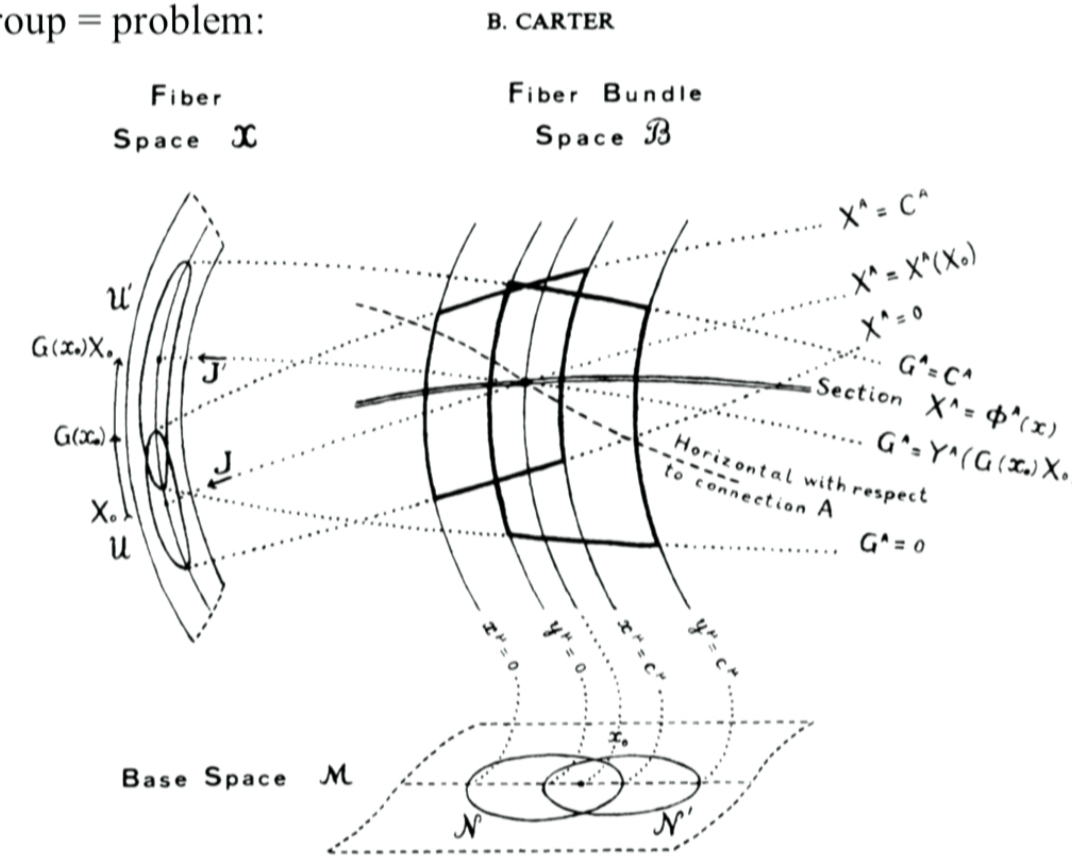
$$Q_a = \int \mathrm{d}^2 x^\perp \pi_a \qquad \text{where} \qquad \pi_a = \frac{\delta \mathcal{L}^{(2)}}{\delta \dot{\psi}^a}$$

ETCR $\quad [\psi^a(\ell_1, t), \pi_b(\ell_2, t)] = i\delta_b^a \delta(\ell_1 - \ell_2)$ (worldsheet quantization)

$\Longrightarrow \quad [Q_a, Q_b] = 0$

String (wall, brane, ...) current $\Leftrightarrow [\mathrm{U}(1)]^n$

General nonabelian group = problem:



a sphere cannot be projected on a plane ...

General nonabelian group = problem:

$$\frac{\partial}{\partial \xi^i} \mathrm{e}^{i\boldsymbol{\psi}(\xi)\cdot \boldsymbol{T}} = i\int_0^1 \mathrm{e}^{i(1-s)\boldsymbol{\psi}(\xi)\cdot \boldsymbol{T}} \frac{\partial \boldsymbol{\psi}}{\partial \xi^i}\cdot \boldsymbol{T} \mathrm{e}^{is\boldsymbol{\psi}(\xi)\cdot \boldsymbol{T}} \mathrm{d}s \neq i \frac{\partial \boldsymbol{\psi}}{\partial \xi^i}\cdot \boldsymbol{T} \mathrm{e}^{i\boldsymbol{\psi}(\xi)\cdot \boldsymbol{T}}$$

A simple case: SU(2) τ^a Pauli matrices $\mathrm{e}^{i\boldsymbol{\psi}(\xi)\cdot\boldsymbol{\tau}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\tau}\sin\alpha$

Angular variables

$$\delta_{ab}n^a n^b = 1 \longrightarrow \begin{aligned} n^1 &= \sin\beta\sin\gamma \\ n^2 &= \sin\beta\cos\gamma \\ n^3 &= \cos\beta \end{aligned}$$

$$n_a \Box n^a = -(\partial\beta)^2 - \sin^2\beta\,(\partial\gamma)^2$$

Eqs of motion

$$\Delta\sigma - \left[(\partial\alpha)^2 + \tan\alpha\,\Box\alpha\right]\sigma - 2\tan\alpha\,\partial\alpha\cdot\partial\sigma = \frac{1}{2}\frac{\partial V}{\partial\sigma} \qquad \text{Id}$$

$$n^a\left\{\Delta\sigma + \left[\frac{\Box\alpha}{\tan\alpha} - (\partial\alpha)^2\right]\sigma + 2\frac{\partial\alpha\cdot\partial\sigma}{\tan\alpha}\right\} + 2\left(\partial\sigma + \frac{\sigma\partial\alpha}{\tan\alpha}\right)\cdot\partial n^a + \sigma\Box n^a = \frac{1}{2}\frac{\partial V}{\partial\sigma}n^a \qquad \tau_a$$

$$\Delta\sigma - \left[(\partial\alpha)^2 - (n_a\Box n^a)\sin^2\alpha\right]\sigma = \frac{1}{2}\frac{\partial V}{\partial\sigma}$$

profile function (condensate)

$$\Box\alpha + \frac{2}{\sigma}\partial\sigma\cdot\partial\alpha + \sin\alpha\cos\alpha\,(n_a\Box n^a) = 0$$

$$\Box\beta + 2\left(\frac{\partial\sigma}{\sigma} + \frac{\partial\alpha}{\tan\alpha}\right)\cdot\partial\beta = \cos\beta\sin\beta\,(\partial\gamma)^2$$

Phases

$$\Box\gamma + 2\left(\frac{\partial\sigma}{\sigma} + \frac{\partial\alpha}{\tan\alpha} + \frac{\partial\beta}{\tan\beta}\right)\partial\gamma = 0$$

Special cases:

$$\Sigma = \frac{\sigma}{\sqrt{2}} \begin{pmatrix} e^{i\gamma} \sin\alpha \sin\beta \\ \cos\alpha - i \sin\alpha \cos\beta \end{pmatrix}$$

abelian sub-group $\mathrm{U}(1) \in \mathrm{SU}(2)$

$$\Sigma = e^{i\psi} \begin{pmatrix} f \\ g \end{pmatrix} \qquad \alpha = \beta = \frac{\pi}{2}, \quad \psi = \gamma, \quad f = \frac{\sigma}{\sqrt{2}}, \quad g = 0,$$

$$\Delta\sigma - (\partial\psi)^2 \sigma = \frac{1}{2} \frac{\partial V}{\partial \sigma}$$ amplitude of the condensate

$$\Box\psi + \frac{2}{\sigma} \frac{d\sigma}{dr} \partial_r \psi = 0$$ phase

biabelian $U(1) \times U(1) \not\subset SU(2)$

$$\Sigma \equiv \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma_1 e^{i\psi_1} \\ \sigma_2 e^{i\psi_2} \end{pmatrix}$$

no possible identification

$$\sigma_1^2 = \sigma^2 \sin^2\alpha \sin^2\beta$$

$$\psi_1 = \gamma, \quad \psi_2 = -\tan^{-1}(\cos\beta \tan\alpha)$$

$$\sigma_2^2 = \sigma^2 \left(\cos^2\alpha + \sin^2\alpha \cos^2\beta\right)$$

Ultralocal hypothesis

Fields are to be evaluated at a single worldsheet point

$$\partial_z \alpha \to k_\alpha \qquad \partial_t \alpha \to -\omega_\alpha$$

$$K = \frac{1}{2}\left(\sigma_1'^2 + \sigma'^2 + w_1\sigma_1^2 + w_2\sigma_2^2\right) = |\partial\Sigma_1|^2 + |\partial\Sigma_2|^2$$

trichiral case

$$\alpha_{\text{chiral}} = \alpha\,(t + \varepsilon z)$$

$$\beta_{\text{chiral}} = \beta\,(t + \varepsilon z)$$

$$\gamma_{\text{chiral}} = \gamma\,(t + \varepsilon z)$$

surface action over the wordsheet

$$S = \int d^2\xi\,\sqrt{-h}\,\mathcal{L}^{(2)}(\xi_i)$$

$$\mathcal{L}^{(2)} = -m^2 - \frac{1}{2}\mathcal{M}^{AB} h^{ij}\partial_i\psi_A\partial_j\psi_B$$

→ matrix Lagrange multiplier

ULTRALOCAL CROOKED STRING ultralocal analysis

$$\frac{d^2\alpha}{dr^2} + \frac{1}{r}\frac{d\alpha}{dr} + 2\frac{d\sigma}{dr}\frac{d\alpha}{dr} + \alpha^0_{,zz} - \alpha^0_{,tt} - \sin\alpha\cos\alpha\left[\left(\frac{d\beta}{dr}\right)^2 + k_\beta^2 - \omega_\beta^2\right] - \sin\alpha\cos\alpha\sin^2\beta\left[\left(\frac{d\gamma}{dr}\right)^2 + k_\gamma^2 - \omega_\gamma^2\right] = 0$$

$$\frac{d^2\beta}{dr^2} + \frac{1}{r}\frac{d\beta}{dr} + 2\frac{d\sigma}{dr}\frac{d\beta}{dr} + \beta^0_{,zz} - \beta^0_{,tt} + \frac{2}{\tan\alpha}\left(\frac{d\alpha}{dr}\frac{d\beta}{dr} + k_\alpha k_\beta - \omega_\alpha\omega_\beta\right) - \sin\beta\cos\beta\left[\left(\frac{d\gamma}{dr}\right)^2 + k_\gamma^2 - \omega_\gamma^2\right] = 0$$

$$\frac{d^2\gamma}{dr^2} + \frac{1}{r}\frac{d\gamma}{dr} + 2\frac{d\sigma}{dr}\frac{d\gamma}{dr} + \gamma^0_{,zz} - \gamma^0_{,tt} + \frac{2}{\tan\alpha}\left(\frac{d\alpha}{dr}\frac{d\gamma}{dr} + k_\alpha k_\gamma - \omega_\alpha\omega_\gamma\right) + \frac{2}{\tan\beta}\left(\frac{d\beta}{dr}\frac{d\gamma}{dr} + k_\beta k_\gamma - \omega_\beta\omega_\gamma\right) = 0$$

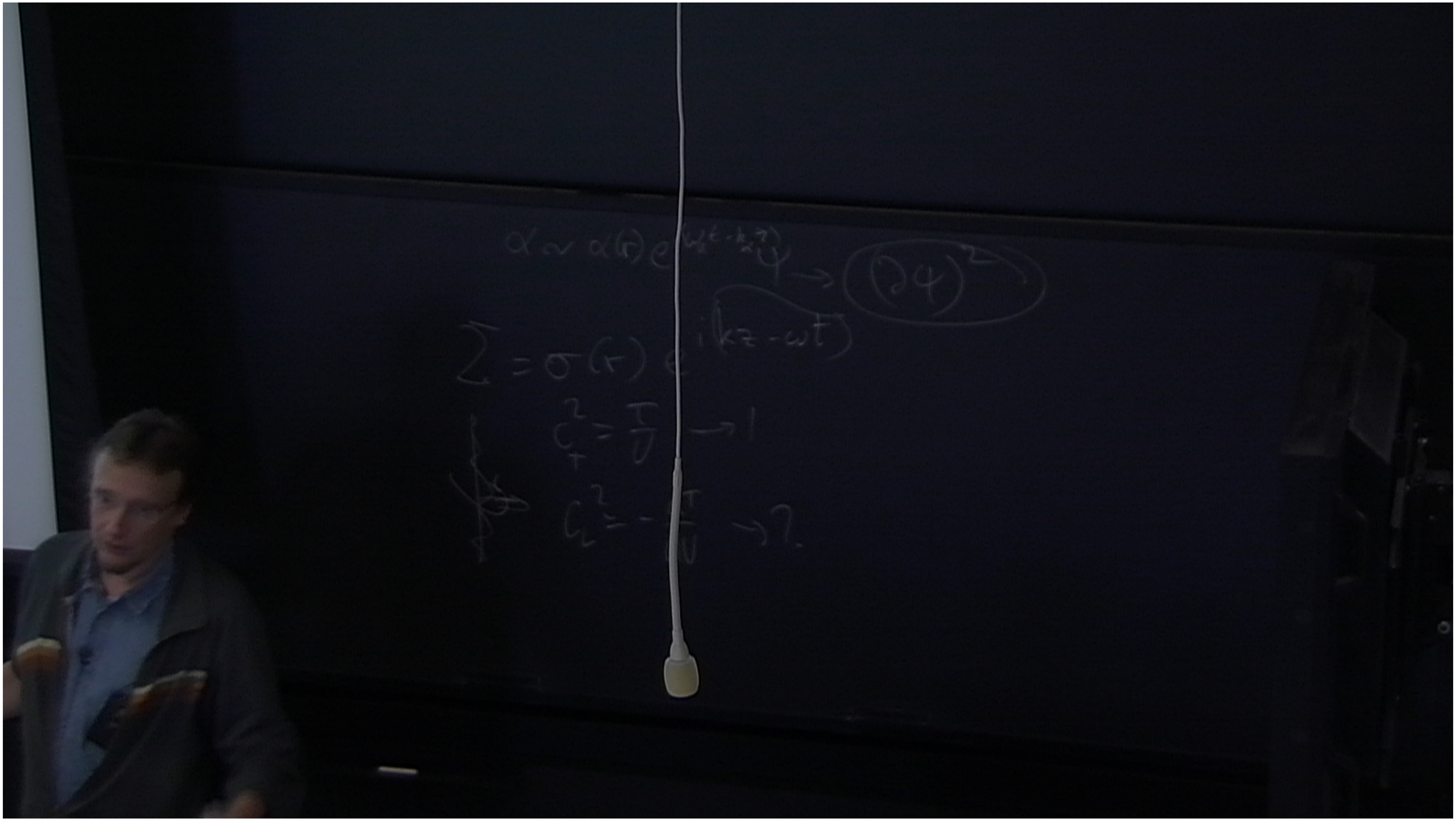
field equations depend on all Lorentz invariant parameters **up to 2nd order**

Surface stress energy tensor

$$\tilde{T}_{ab} \equiv \int r\, dr d\theta\, T_{ab}$$

$$\tilde{T}^a{}_b = \begin{pmatrix} T^t{}_t & T^t{}_z \\ T^z{}_t & T^z{}_z \end{pmatrix} = \begin{pmatrix} -A+B & C \\ -C & -A-B \end{pmatrix}$$

parameter matrix $w_{ij} = k_i k_j - \omega_i \omega_j$



Conclusions

- Cosmic strings are a generic prediction of GUT/string/high energy theories

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- Cosmic strings are a generic prediction of GUT/string/high energy theories
- Various cosmological consequences
- Often "superconducting"
- One current = well-defined one parameter worldsheet model
- Many current = Sum over one-current models
- Non abelian current?
- Cosmological consequences to be derived ...

Thank you for your attention!

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