Title: Parity and the Immirzi Parameter in Lorentzian Spinfoams

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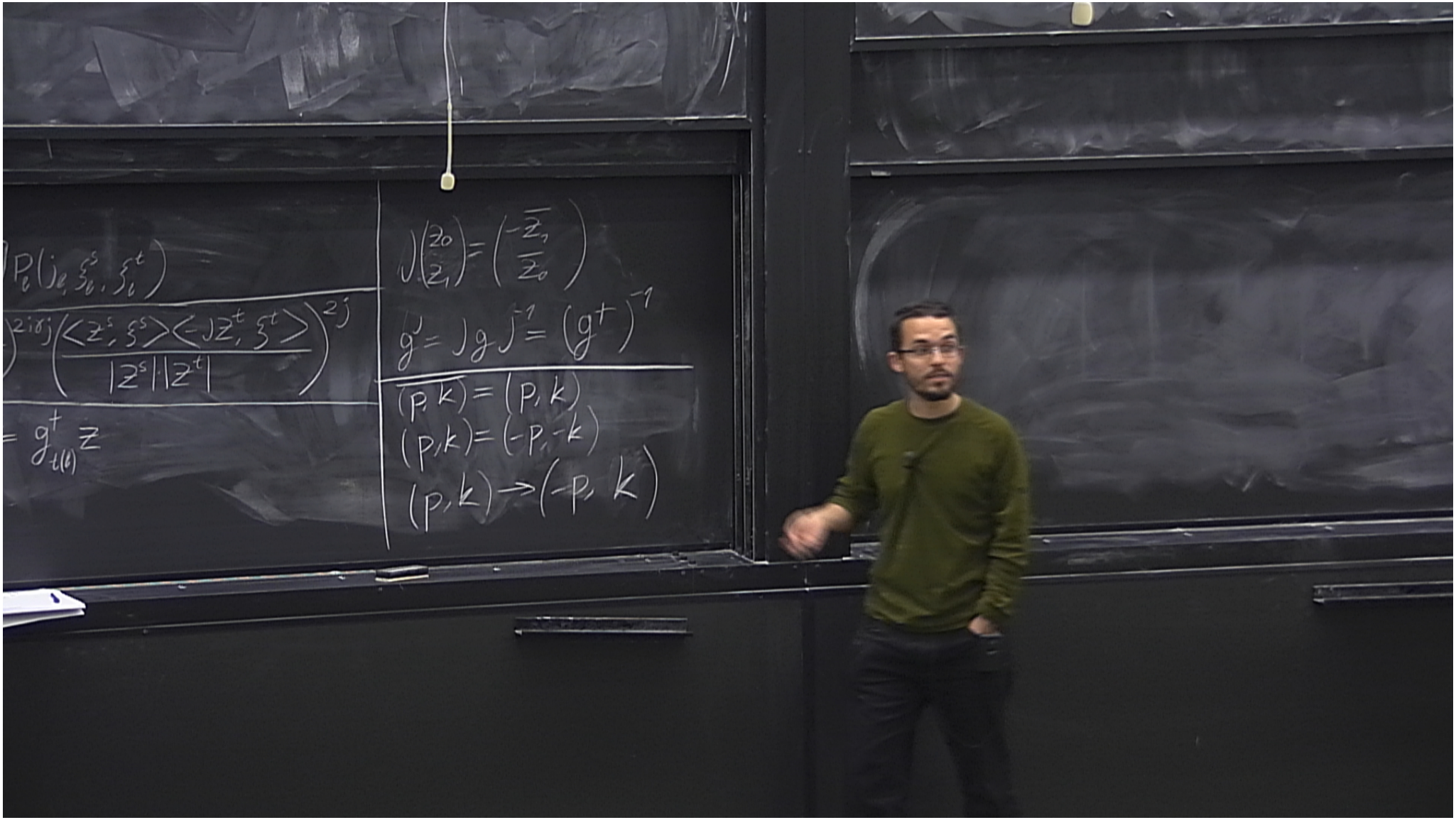
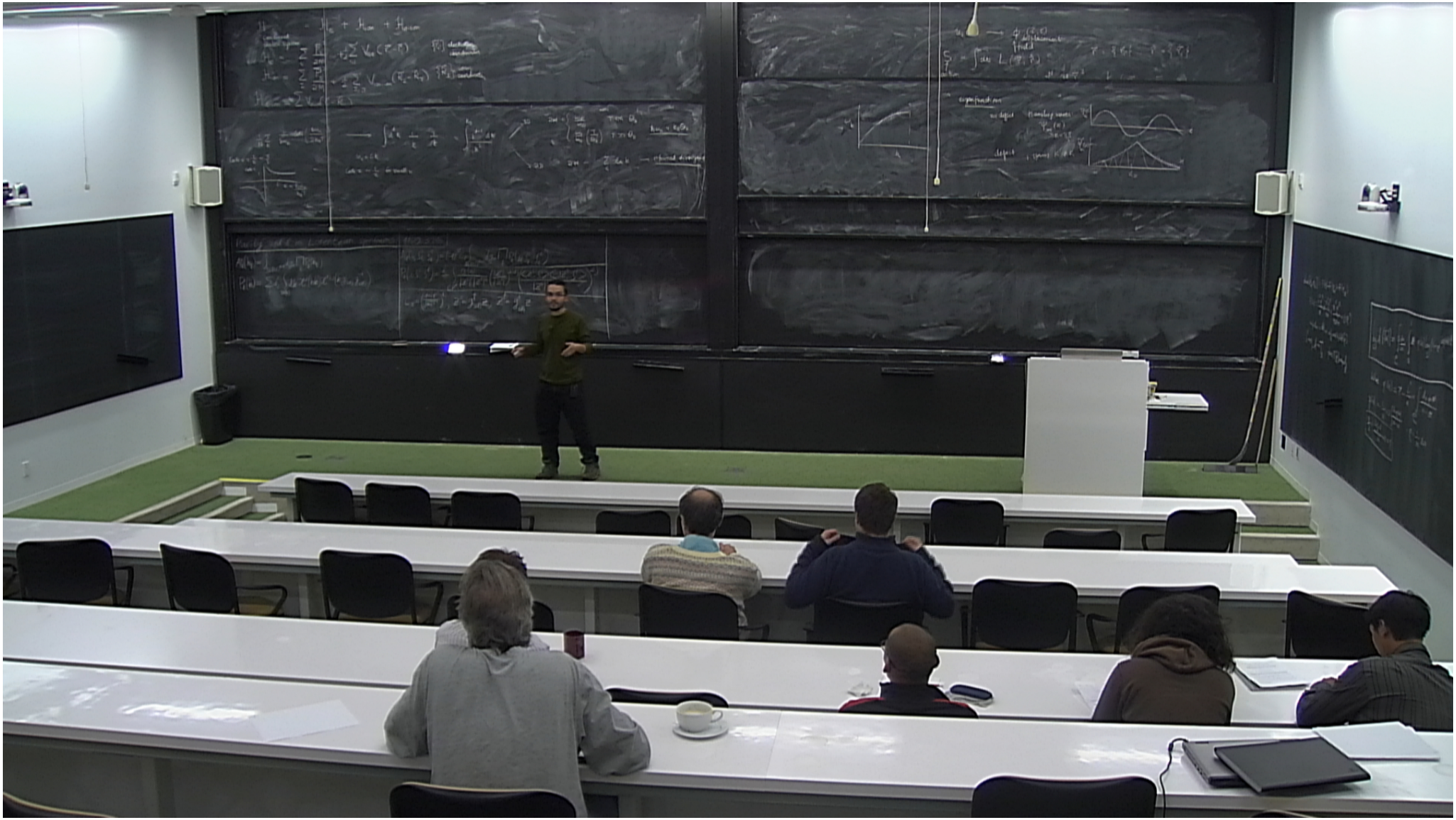
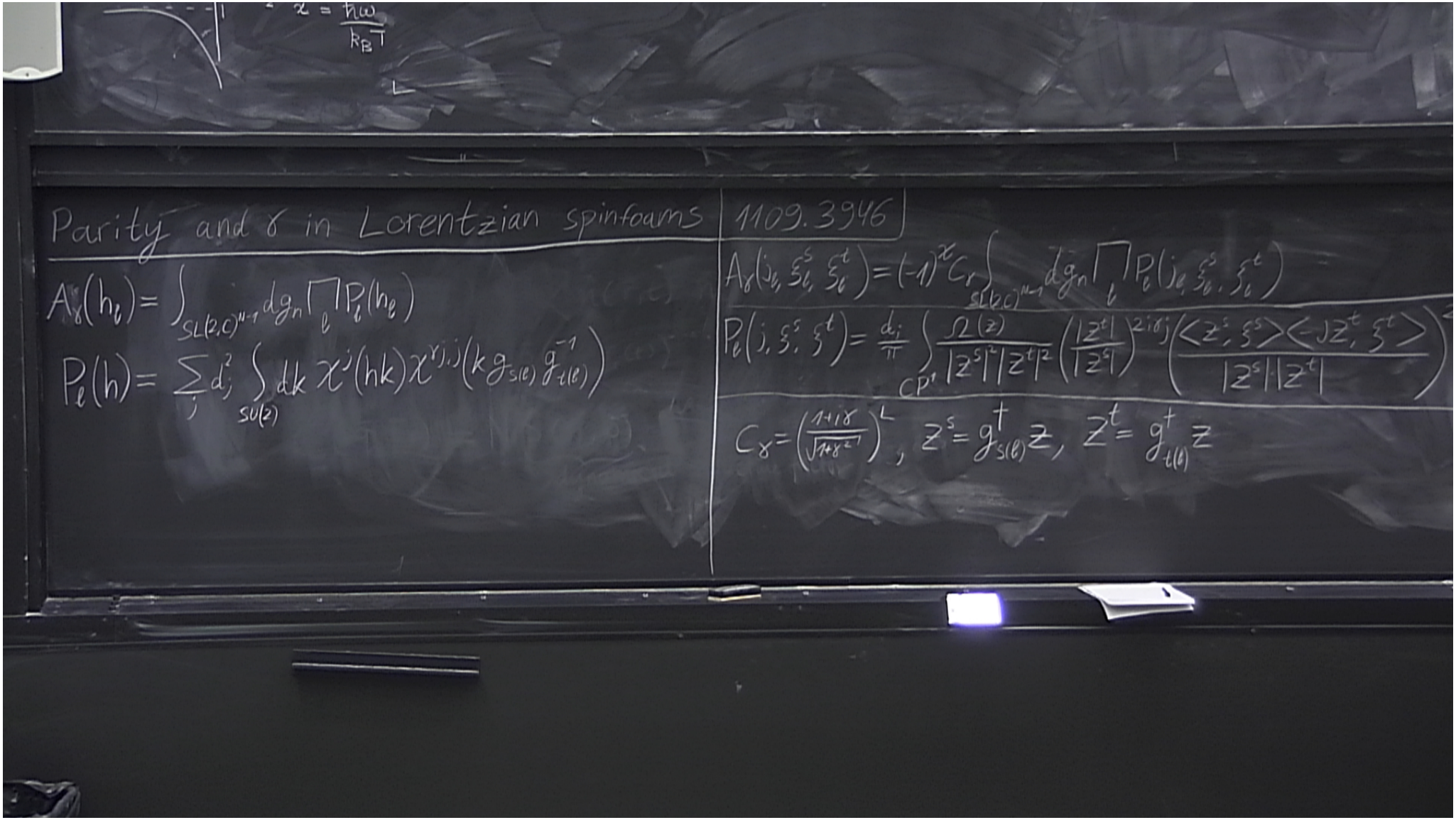
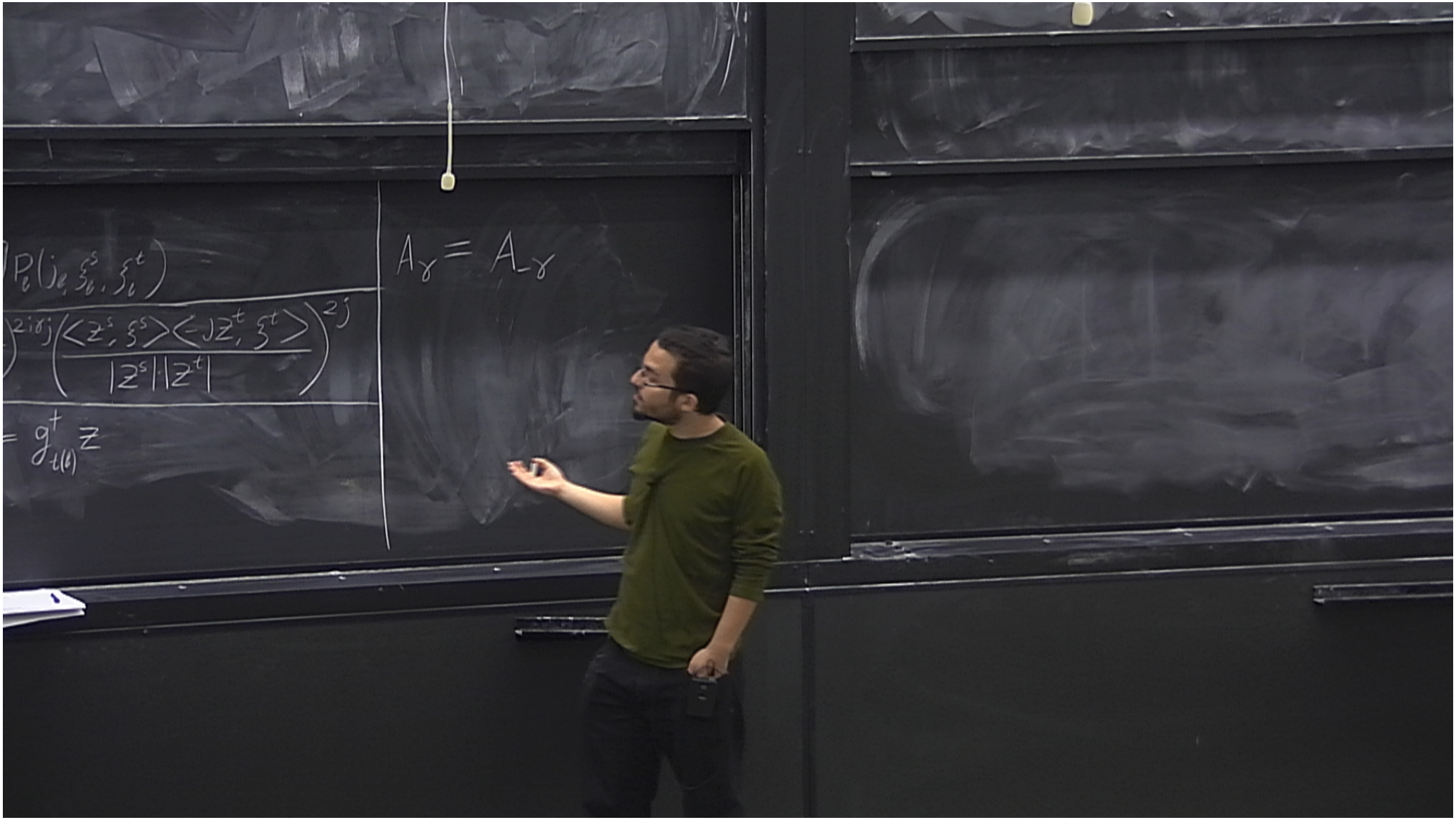
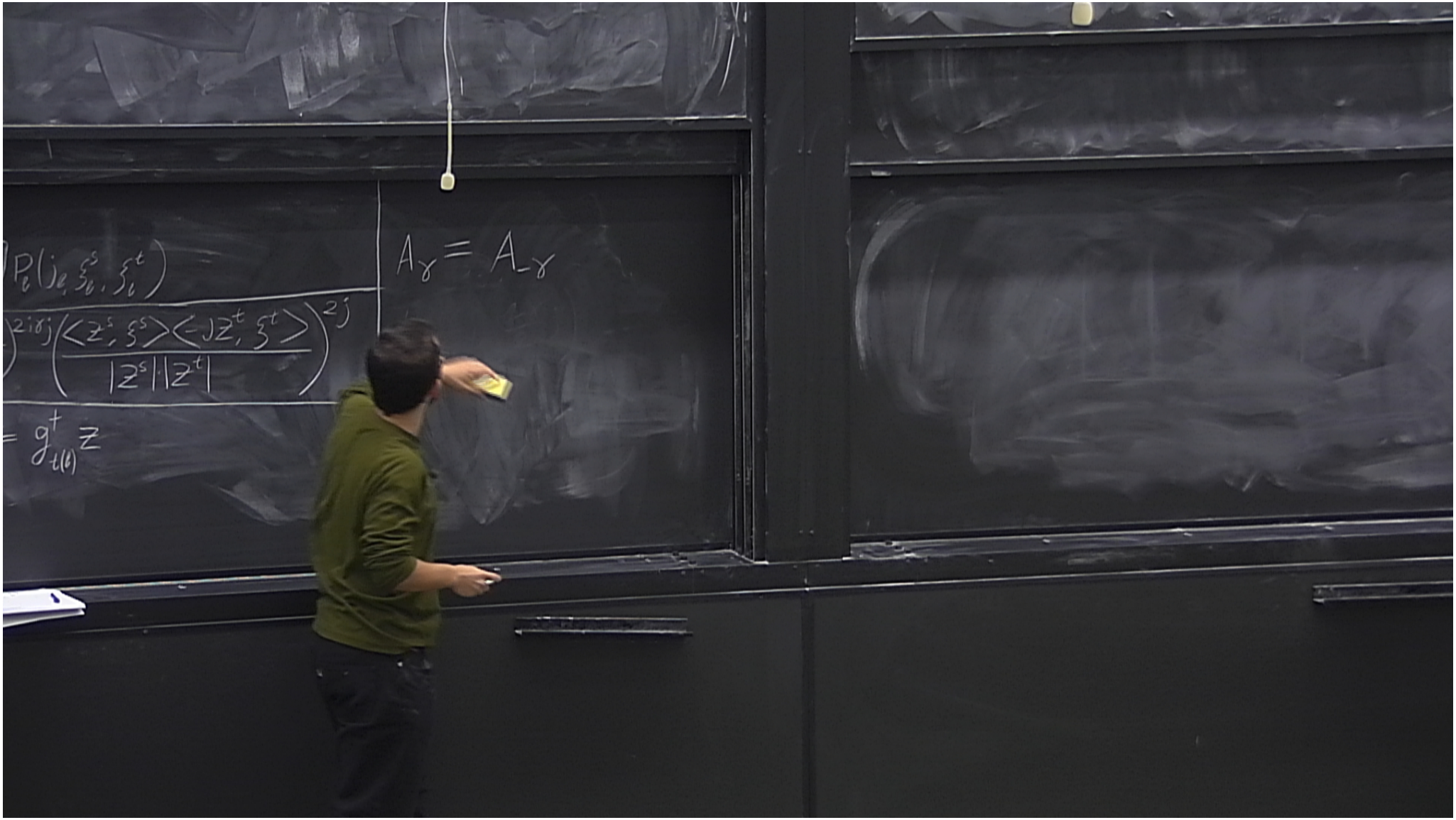
Abstract: The parity invariance of spinfoam gravity is an open question. Naively, parity breaking should reside in the sign of the Immirzi parameter. I show that the new Lorentzian vertex formula is in fact independent of this sign, suggesting that the dynamics is parity-invariant. The situation with boundary states and operators is more complicated. I discuss parity-related pieces of the transition amplitude and graviton propagator in the large-spin 4-simplex limit. Numerical results indicate patterns similar to those in the Euclidean case. In particular, parity-related components of the graviton propagator differ by a phase. I discuss possible resolutions of this issue.


$P_t(j_t, \xi_0^s, \xi_t^t)$
$\left(\frac{\langle z^s, \xi^s\rangle\langle -Jz^t, \xi^t\rangle}{|z^s||z^t|}\right)^{2j}$
$= g^+_{t(t)} z$
$J\begin{pmatrix} z_0 \\ z_1 \end{pmatrix} = \begin{pmatrix} -\bar{z}_1 \\ \bar{z}_0 \end{pmatrix}$
$g^J = J g J^{-1} = (g^+)^{-1}$
$\overline{(p,k)} = (p,k)$
$(p,k) = (-p,-k)$
$(p,k) \to (-p,k)$




$x = \frac{\hbar\omega}{k_B T}$
Parity and γ in Lorentzian spinfoams
1109.3946
$A_\gamma(h_e) = \int_{SL(2,C)^{N-1}} dg_n \prod_e P_e(h_e)$
$P_e(h) = \sum_j d_j^2 \int_{SU(2)} dk\, \chi^j(hk)\chi^{(j,j)}(k\, g_{s(e)}\, g_{t(e)}^{-1})$
$A_\gamma(j_e, \xi_e^s, \xi_e^t) = (-1)^{2j} C_\gamma \int_{SL(2,C)^{N-1}} dg_n \prod_e P_e(j_e, \xi_e^s, \xi_e^t)$
$P_e(j, \xi^s, \xi^t) = \frac{d_j}{\pi} \int_{CP^1} \frac{\Omega(z)}{|z^s|^2 |z^t|^2} \left(\frac{|z^t|}{|z^s|}\right)^{2i\gamma j} \left(\frac{\langle z^s, \xi^s\rangle \langle -Jz^t, \xi^t\rangle}{|z^s||z^t|}\right)^{2j}$
$C_\gamma = \left(\frac{1+i\gamma}{\sqrt{1+\gamma^2}}\right)^L$, $z^s = g_{s(e)}^\dagger z$, $z^t = g_{t(e)}^\dagger z$


$A_\gamma = A_{-\gamma}$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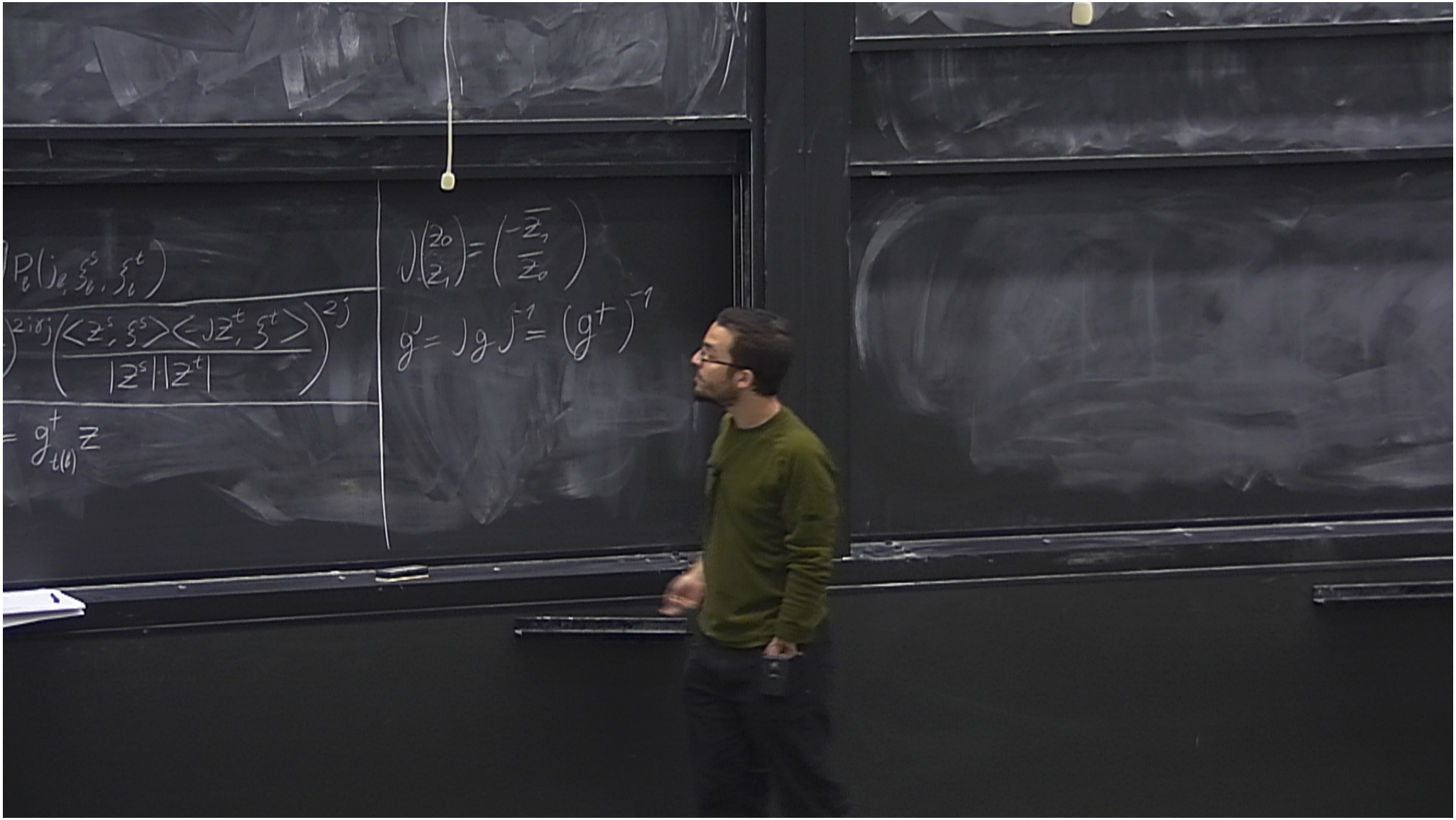
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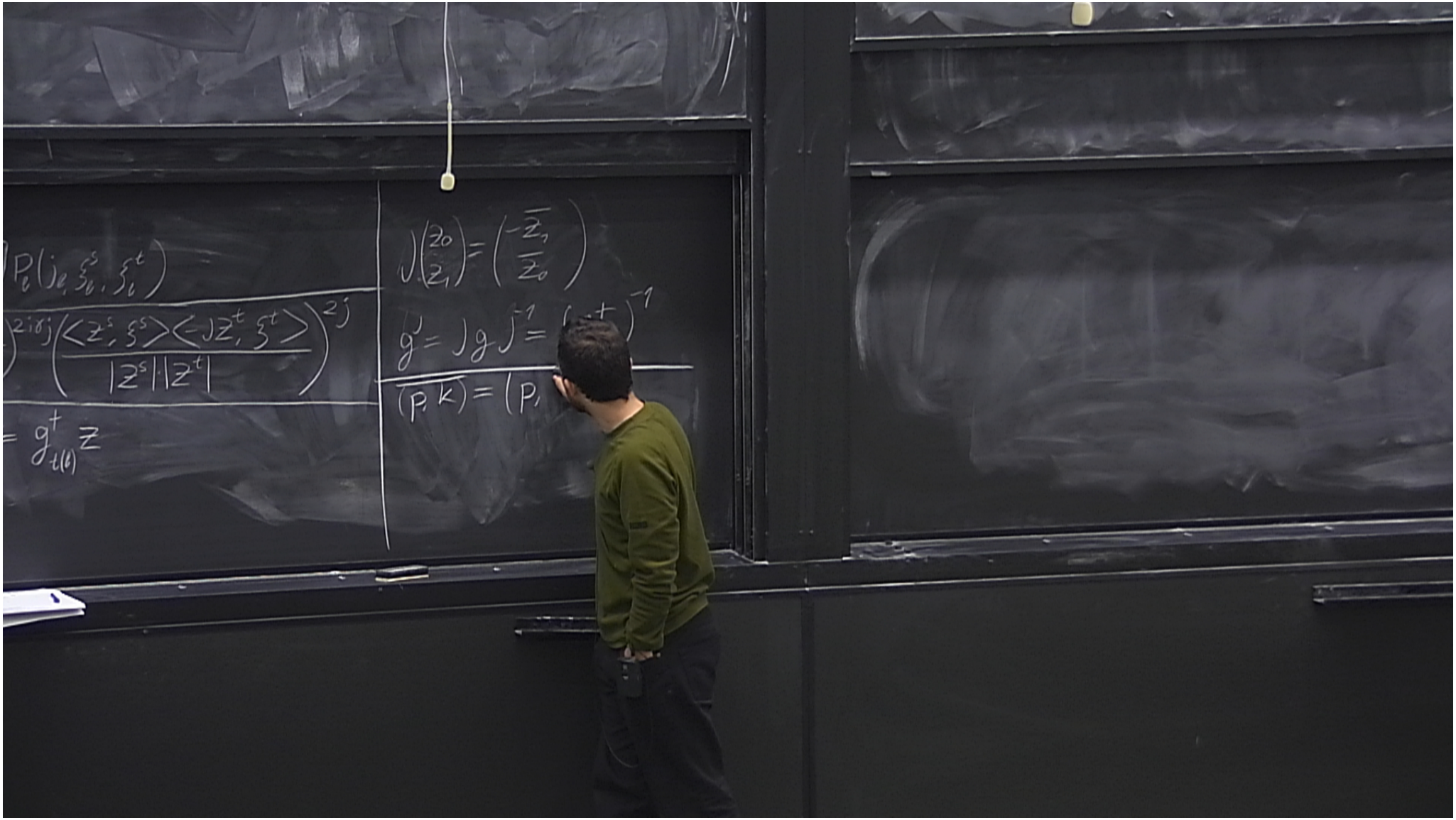

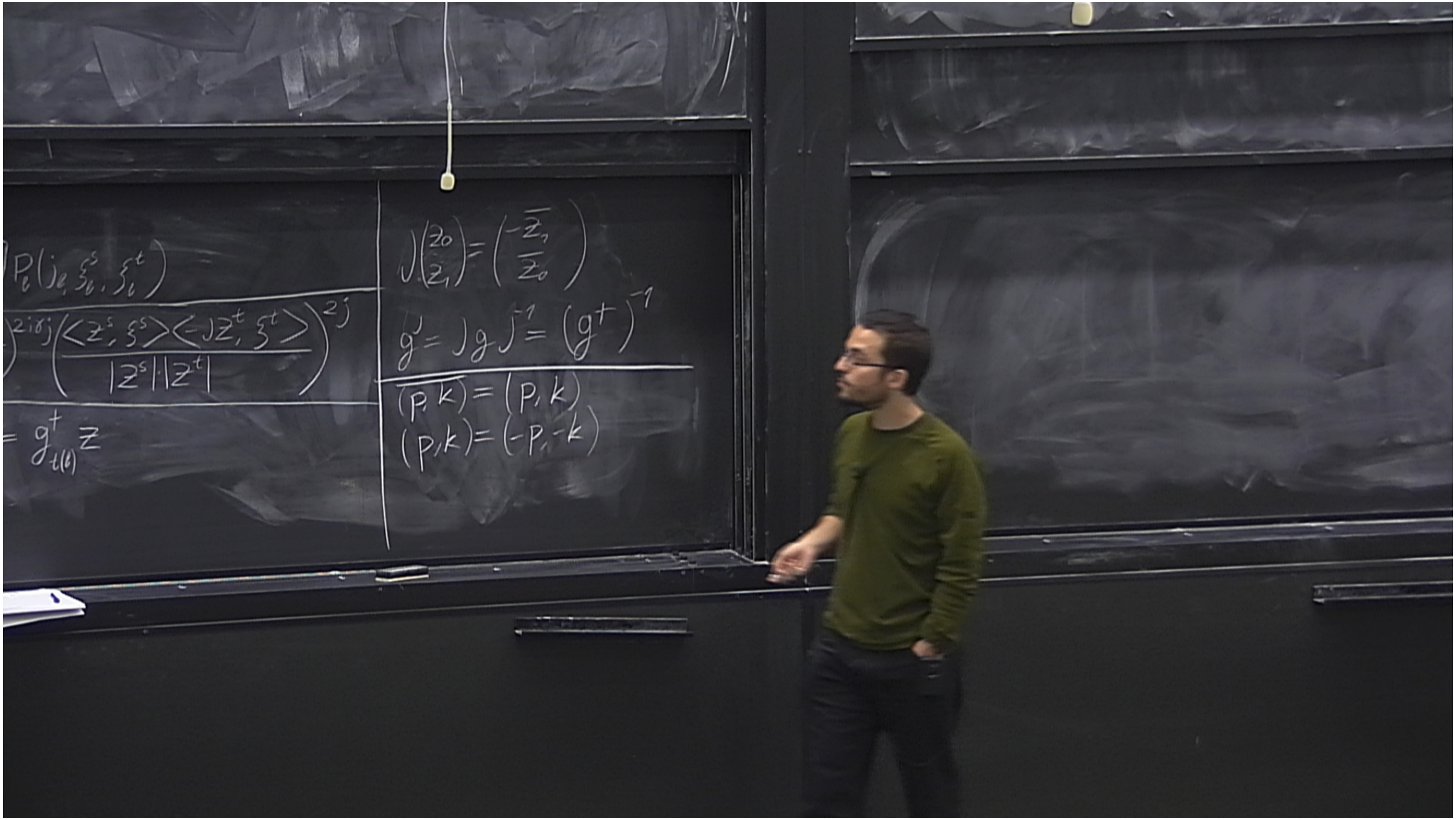
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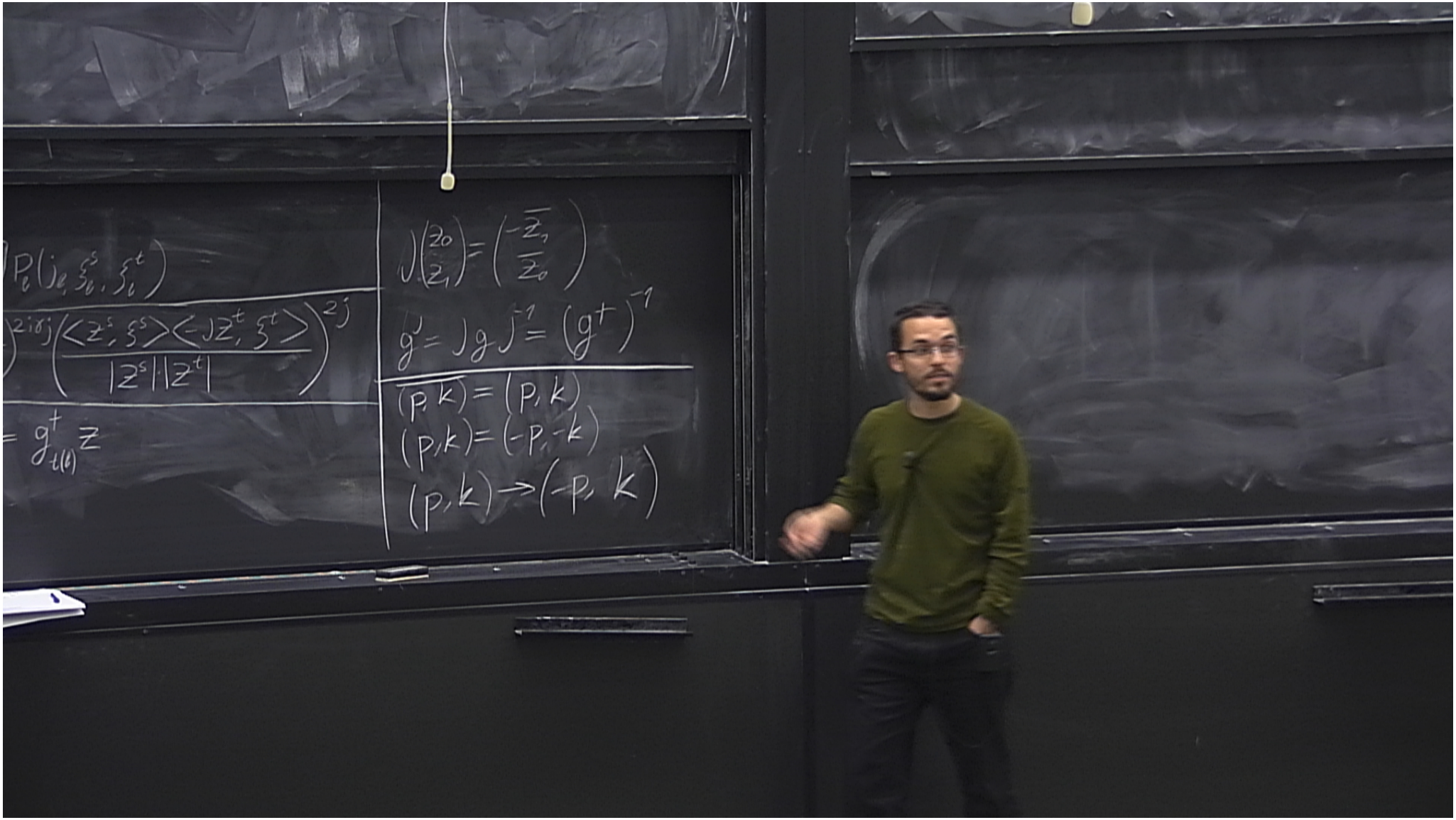
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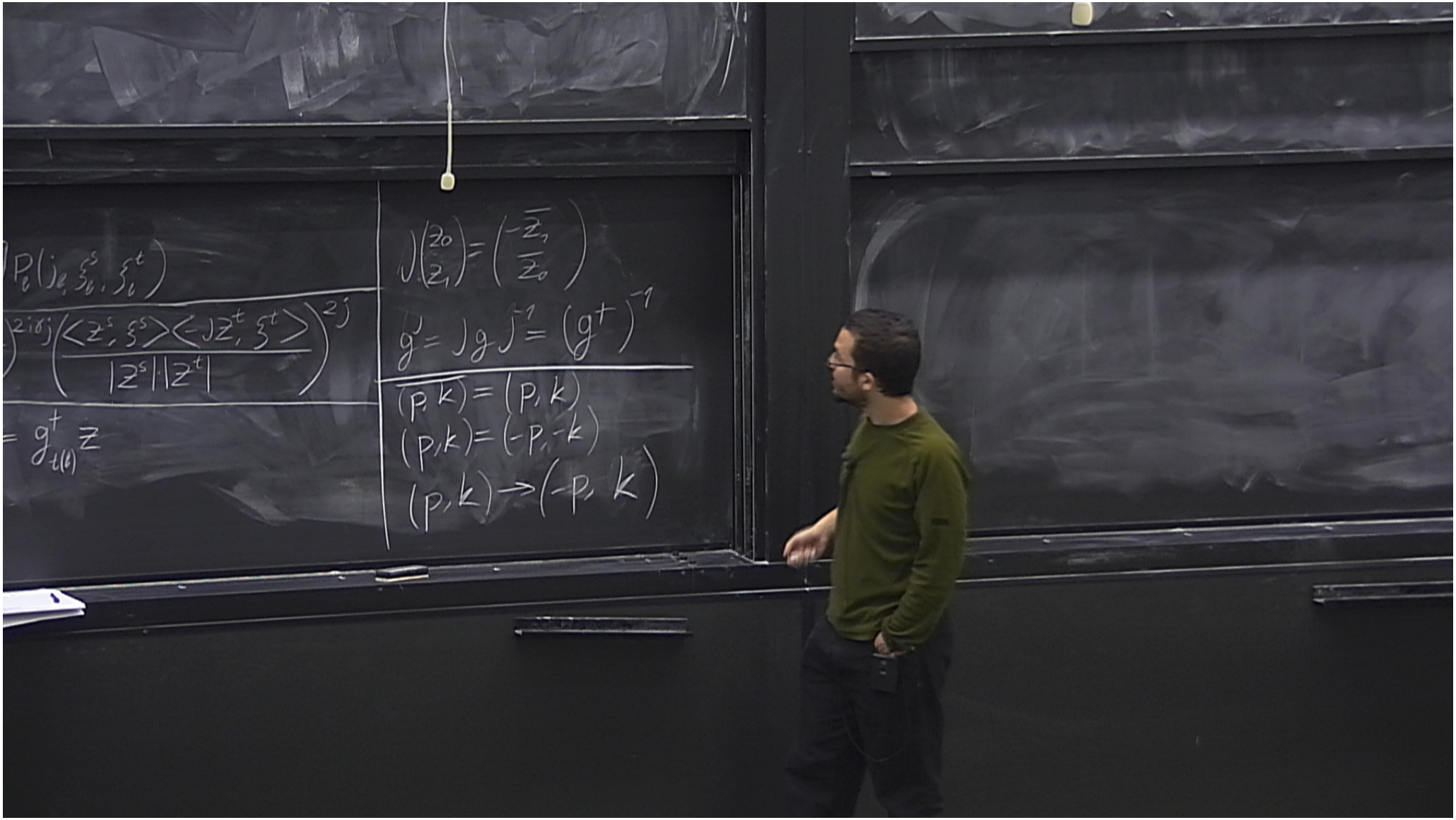
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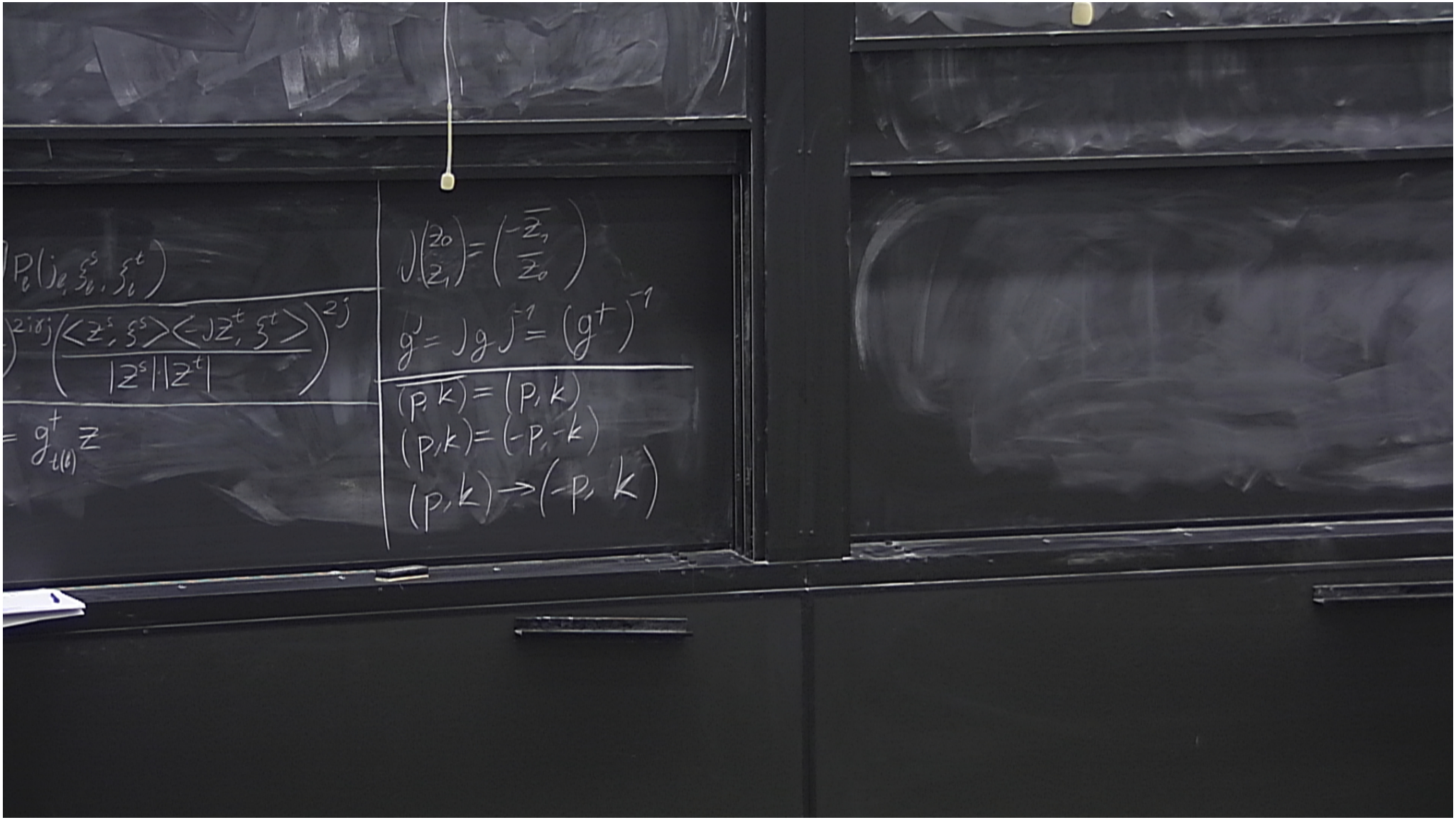
$P_i(j_i, \xi_i^s, \xi_i^t)$
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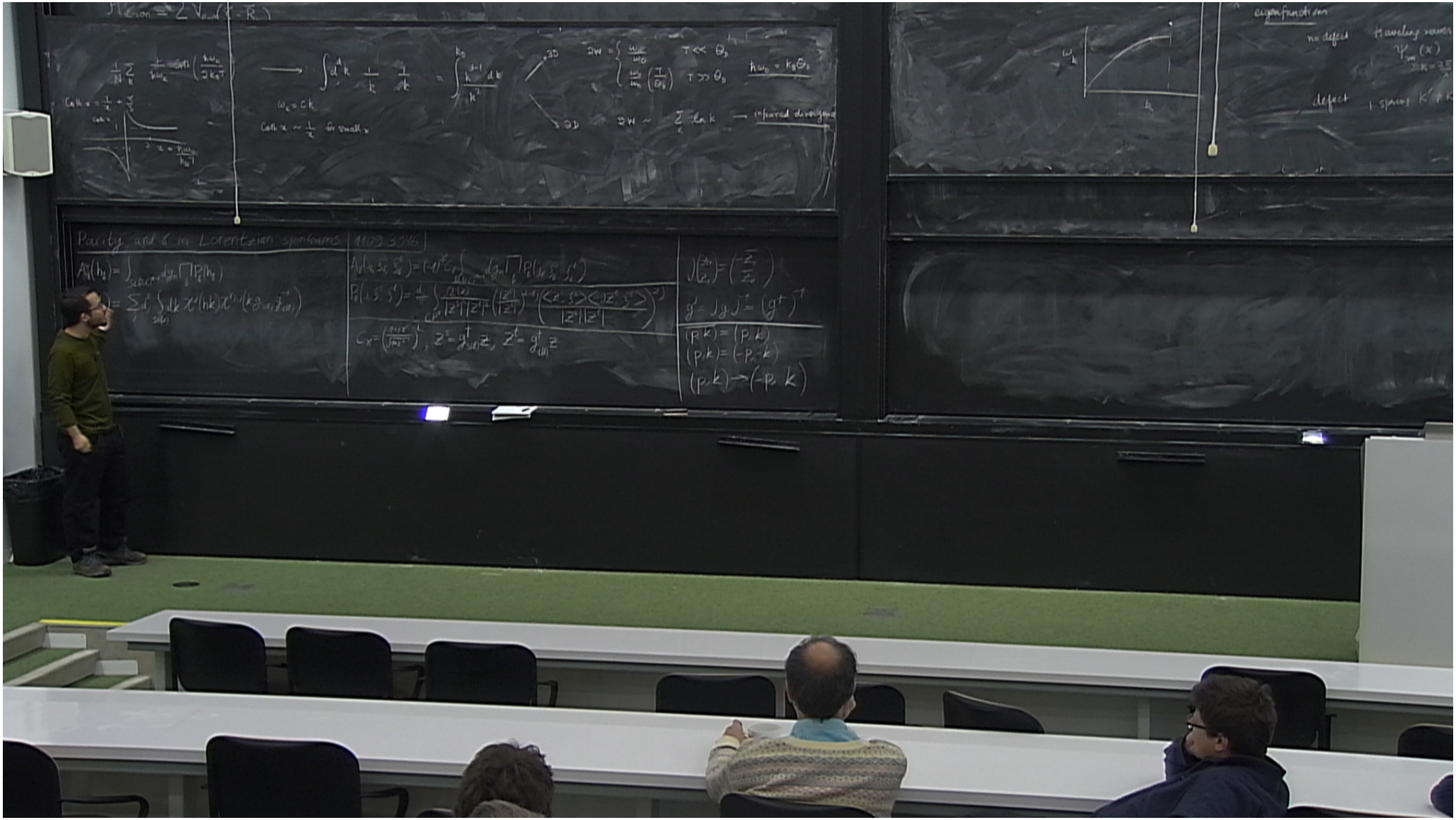
Parity and σ in Lorentzian spinforms
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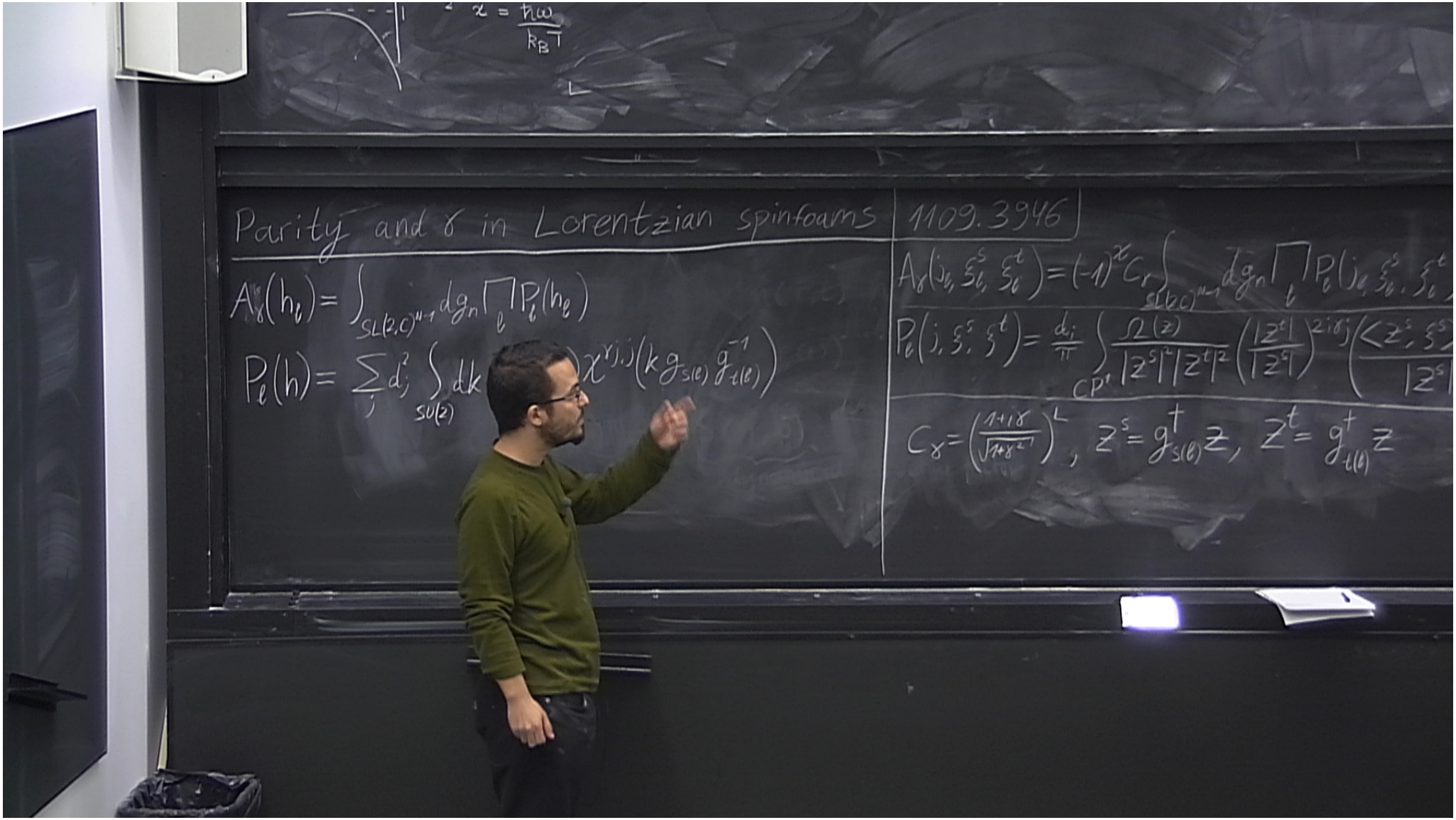
Parity and γ in Lorentzian spinfoams
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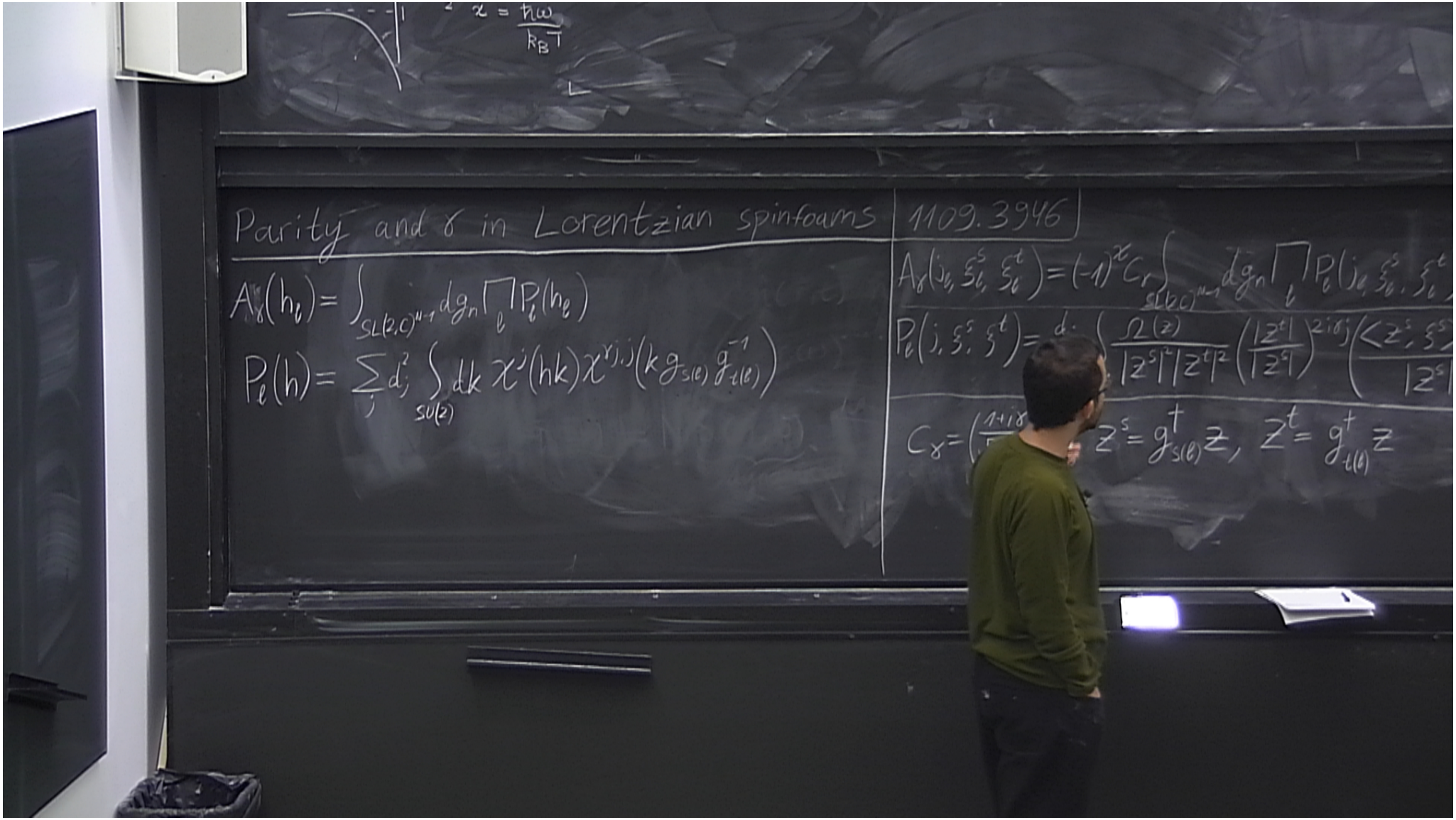
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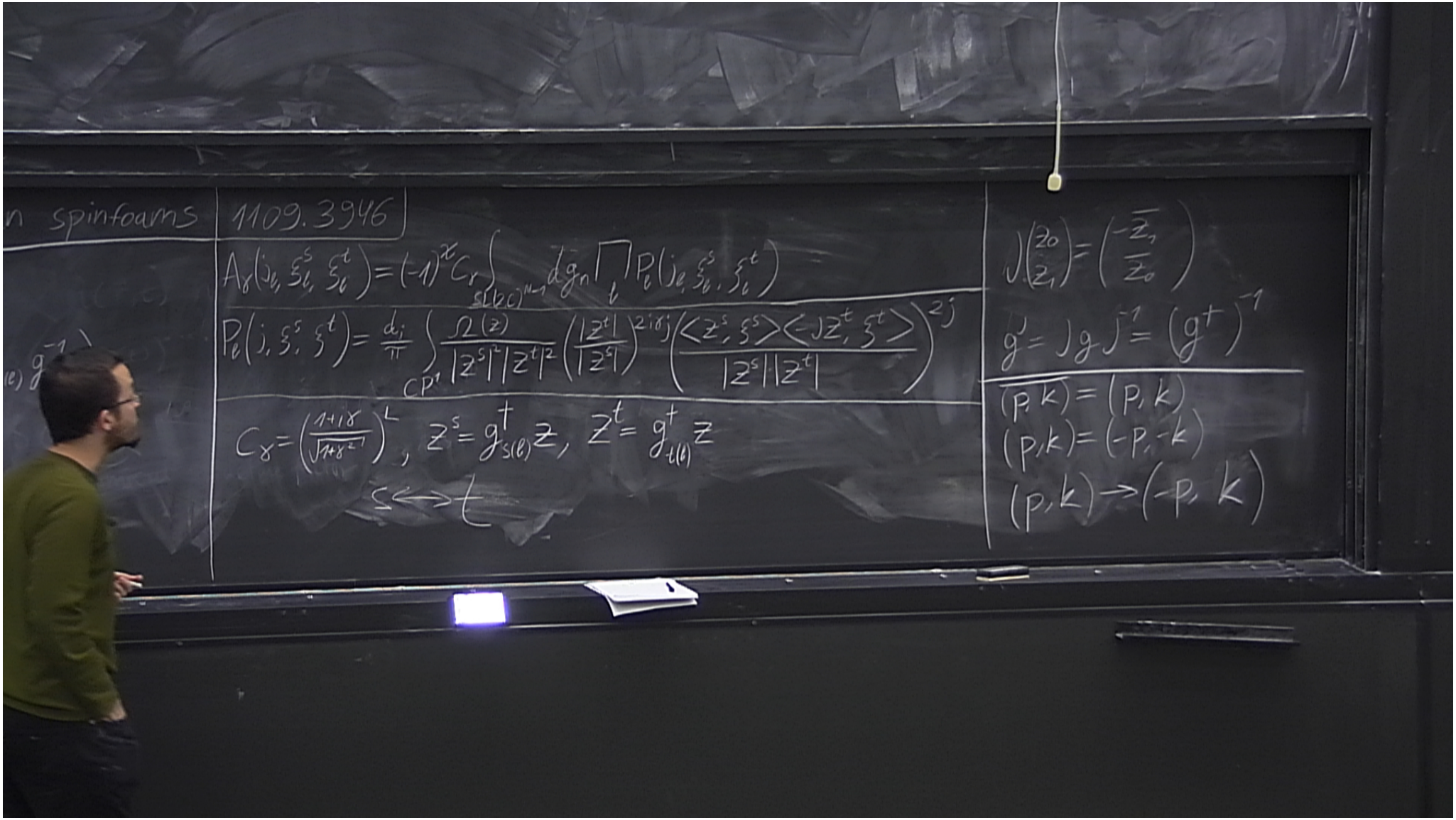
n spinfoams
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$A_\gamma(j_e, \xi_e^s, \xi_e^t) = (-1)^\chi C_\gamma \int_{SL(2,\mathbb{C})^{N-1}} dg_n \prod_e P_e(j_e, \xi_e^s, \xi_e^t)$
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$C_\gamma = \left(\frac{1+i\gamma}{\sqrt{1+\gamma^2}}\right)^L$, $z^s = g^\dagger_{s(e)} z$, $z^t = g^\dagger_{t(e)} z$
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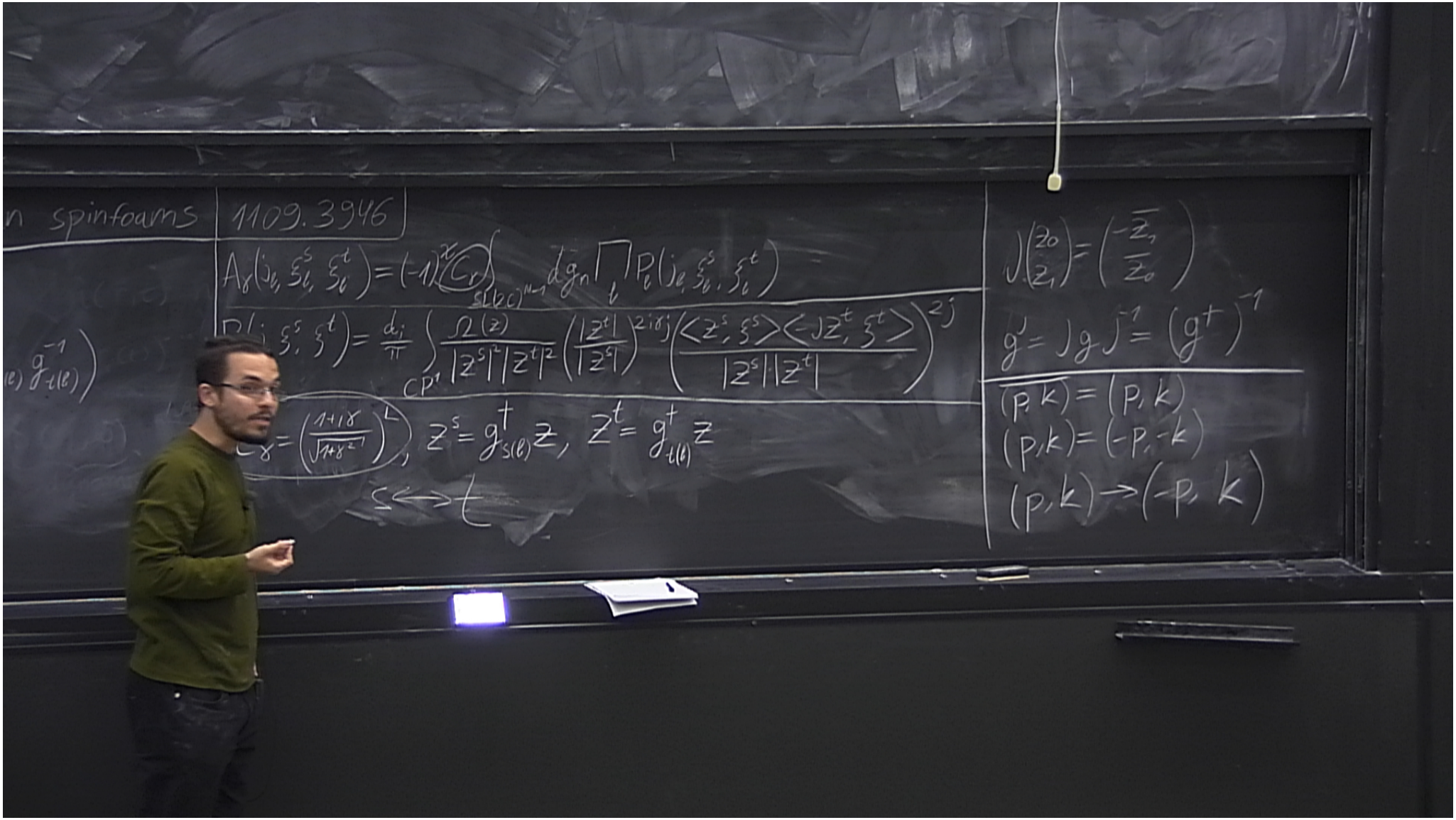
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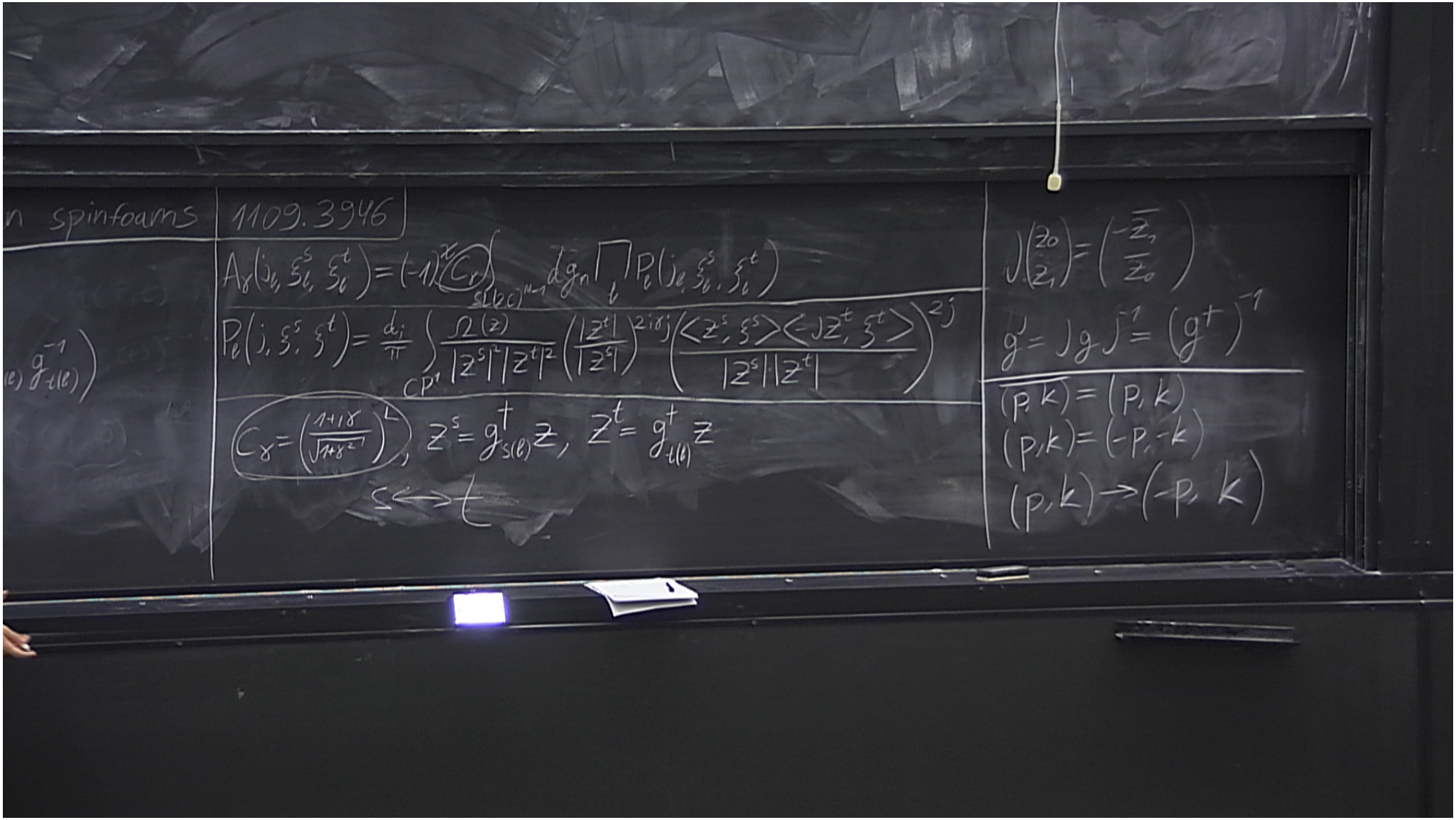
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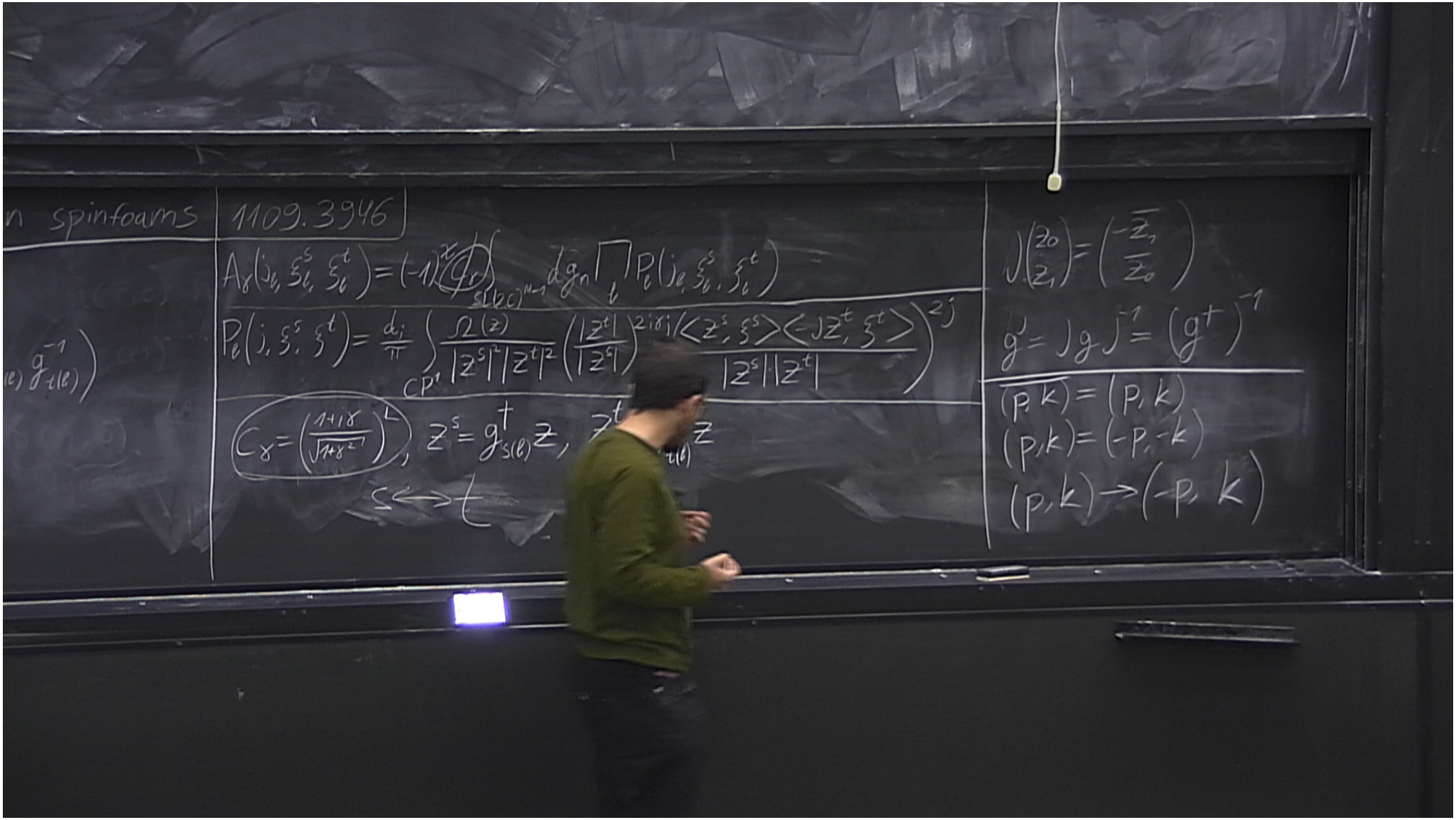
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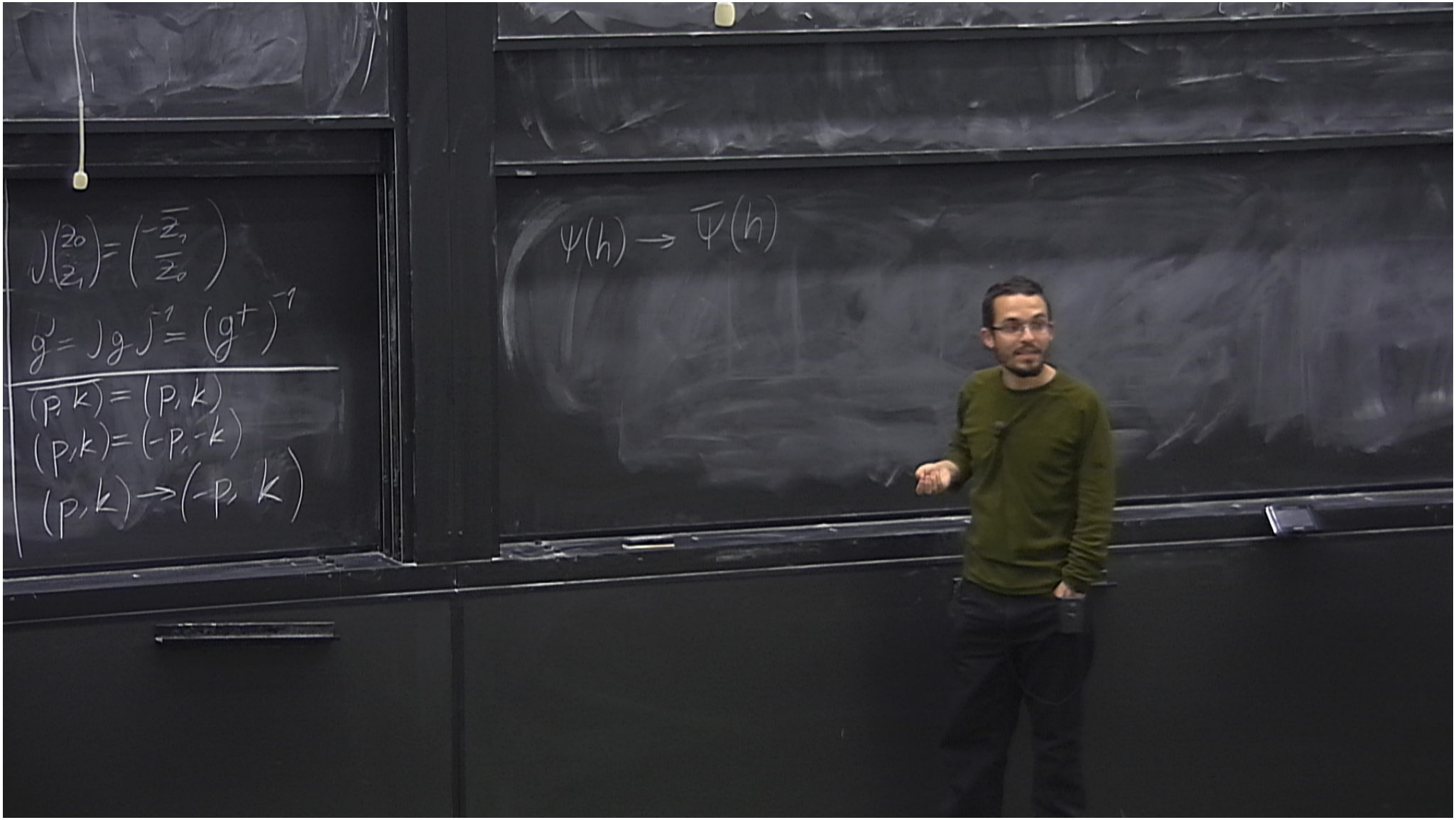
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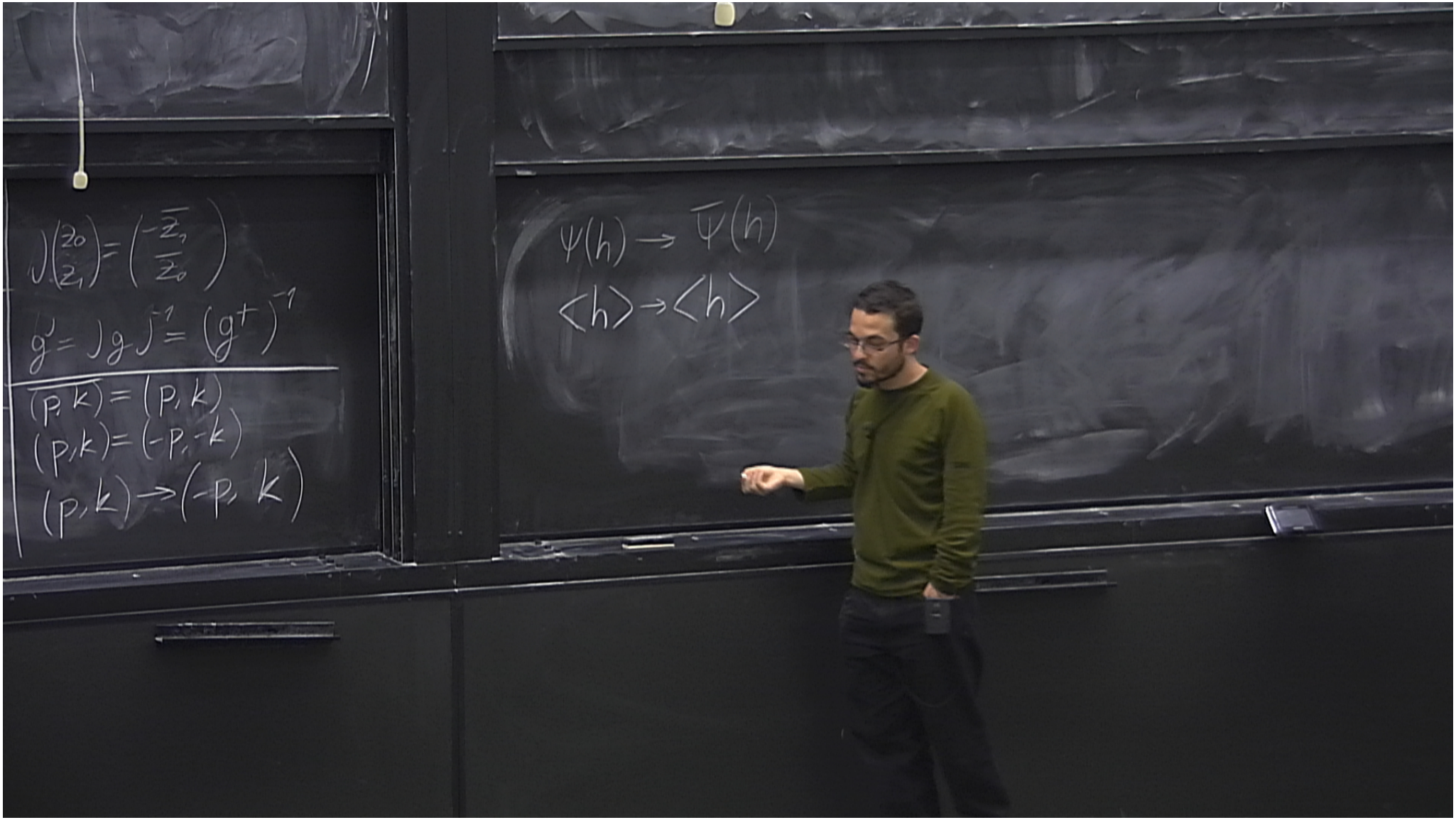
$\psi(h) \to \bar{\psi}(h)$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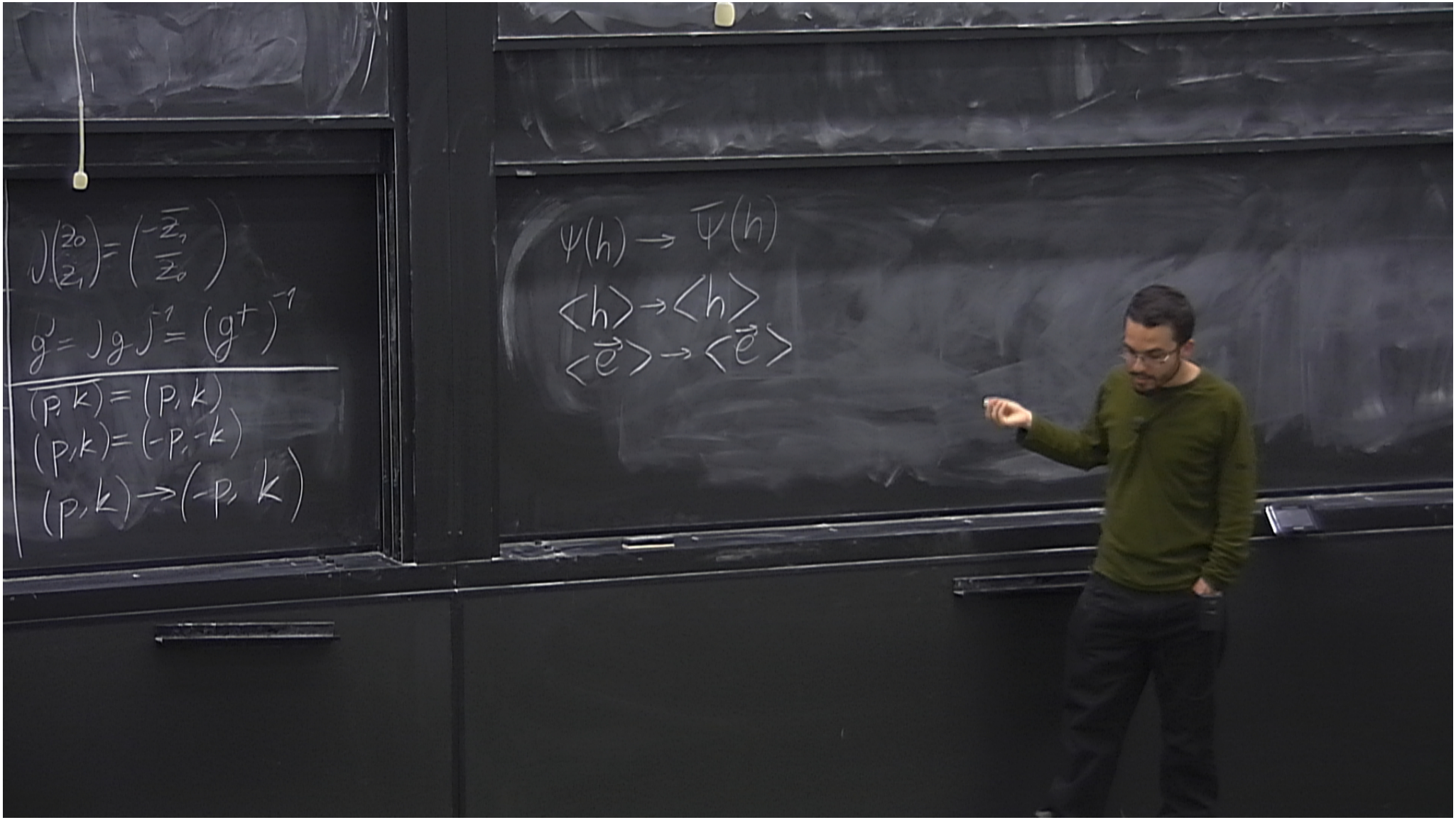
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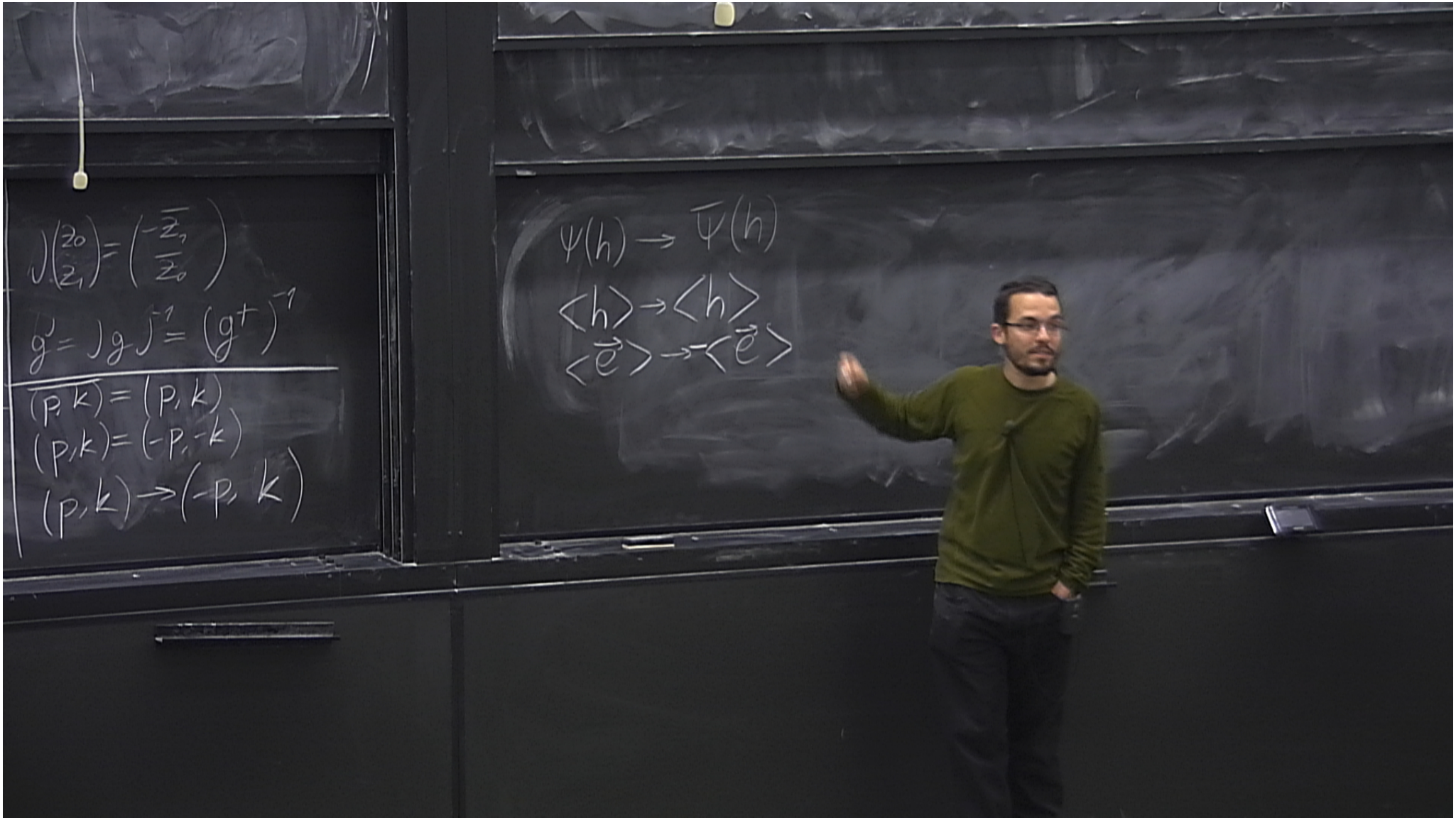
$\langle \vec{e} \rangle \to \langle \vec{e} \rangle$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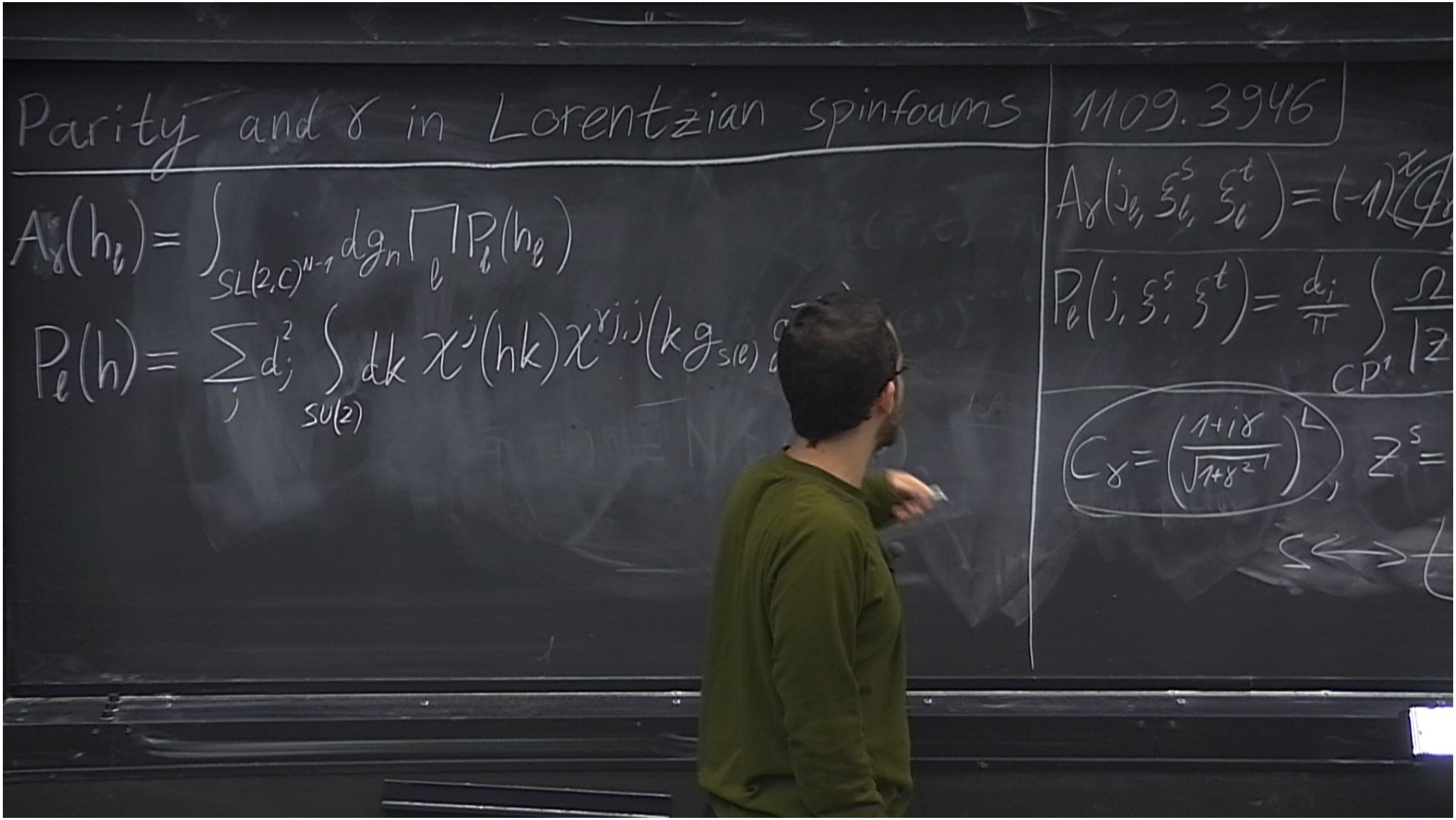
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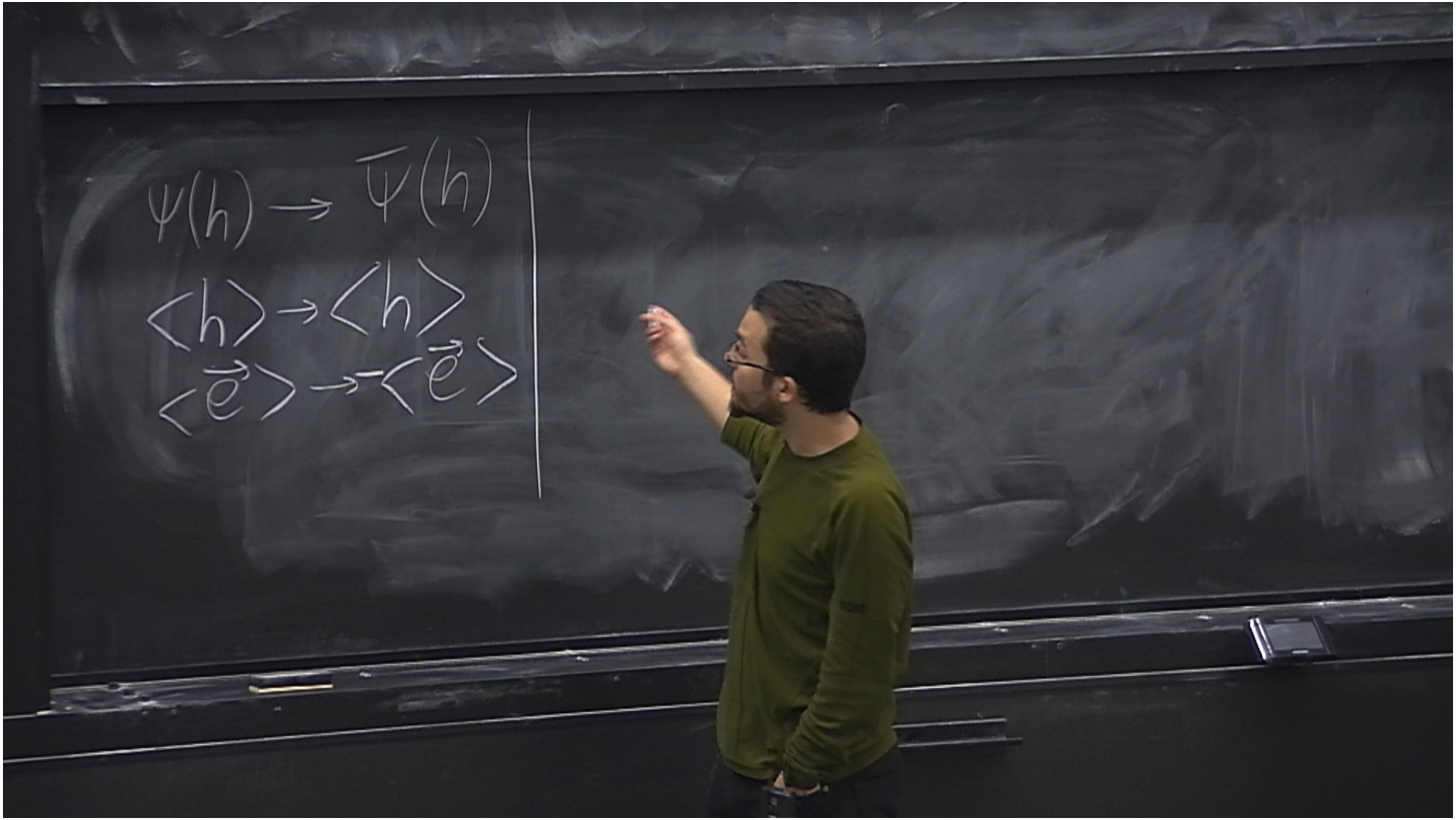
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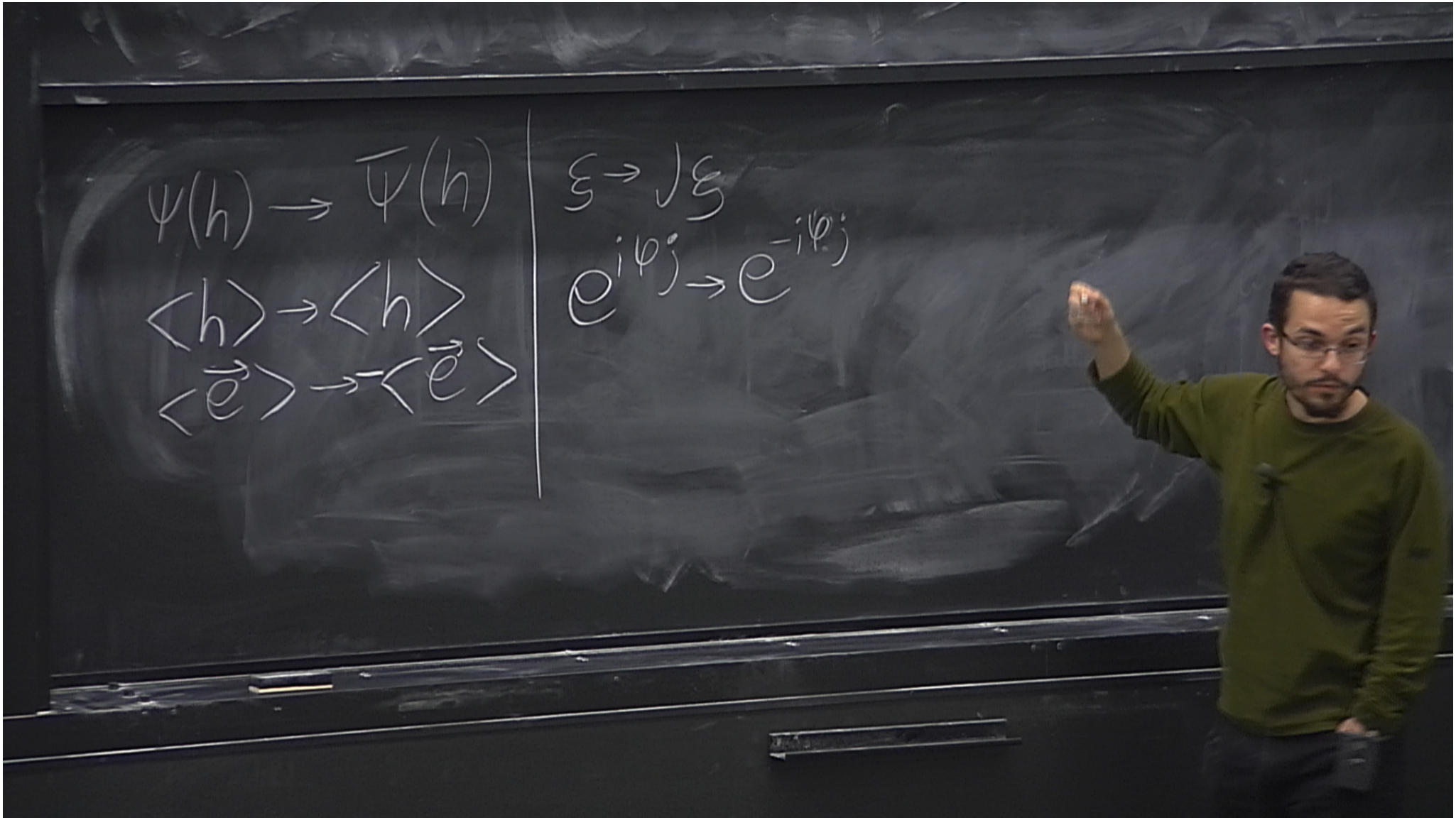
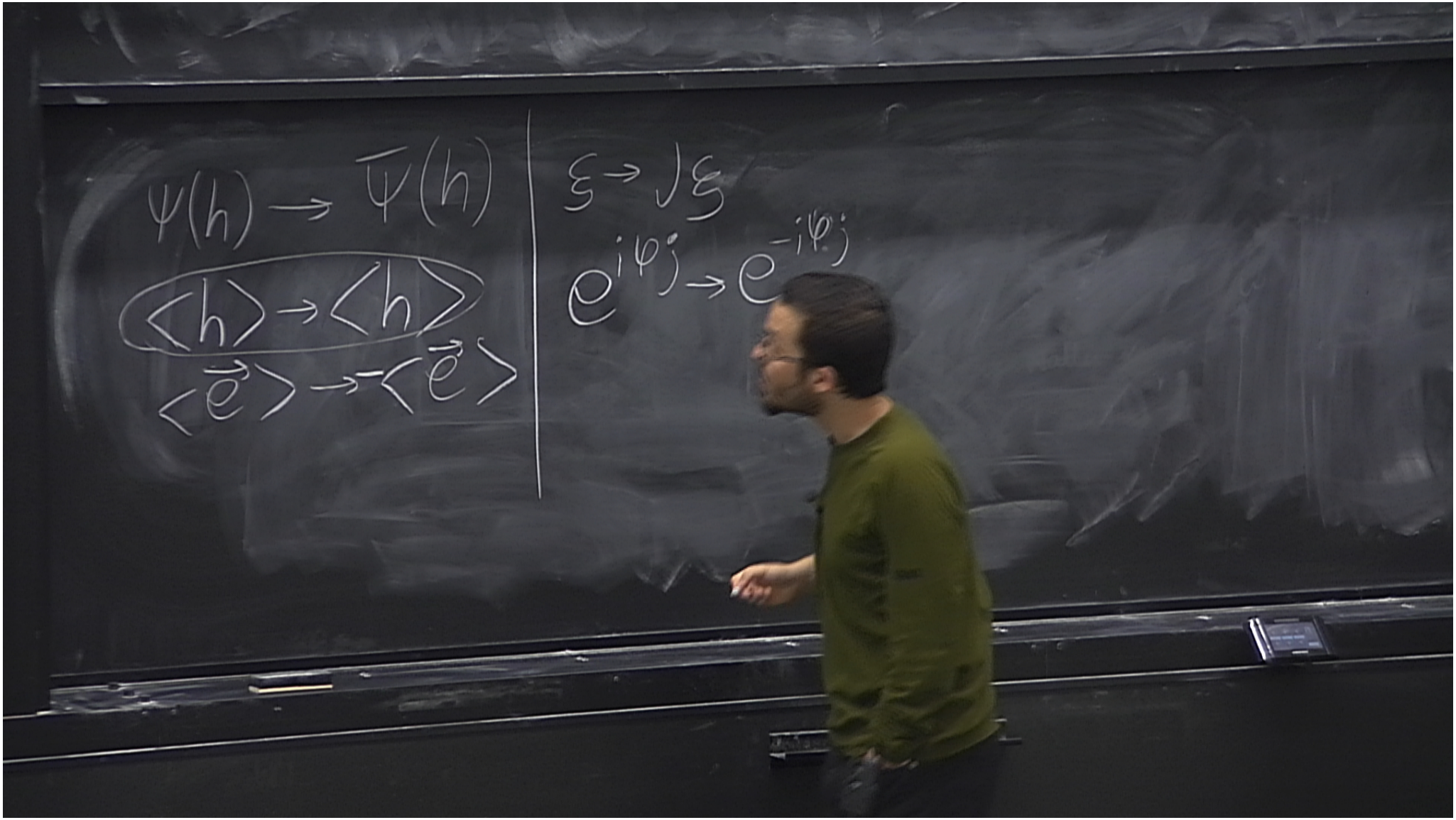

$\psi(h) \to \bar{\psi}(h)$
$\langle h \rangle \to \langle h \rangle$
$\langle \vec{e} \rangle \to -\langle \vec{e} \rangle$
$\xi \to J\xi$
$e^{i\phi_j} \to e^{-i\phi_j}$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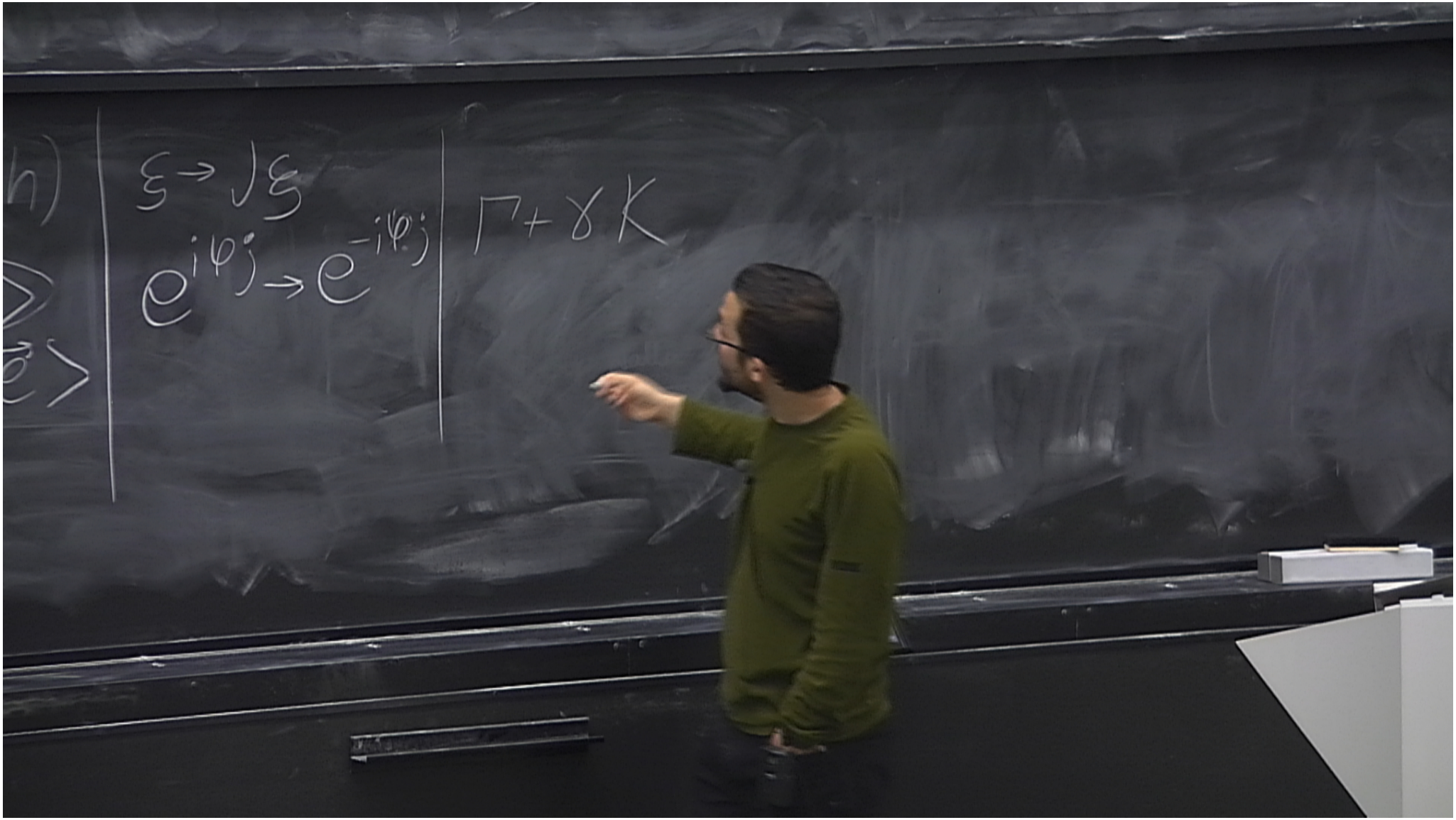
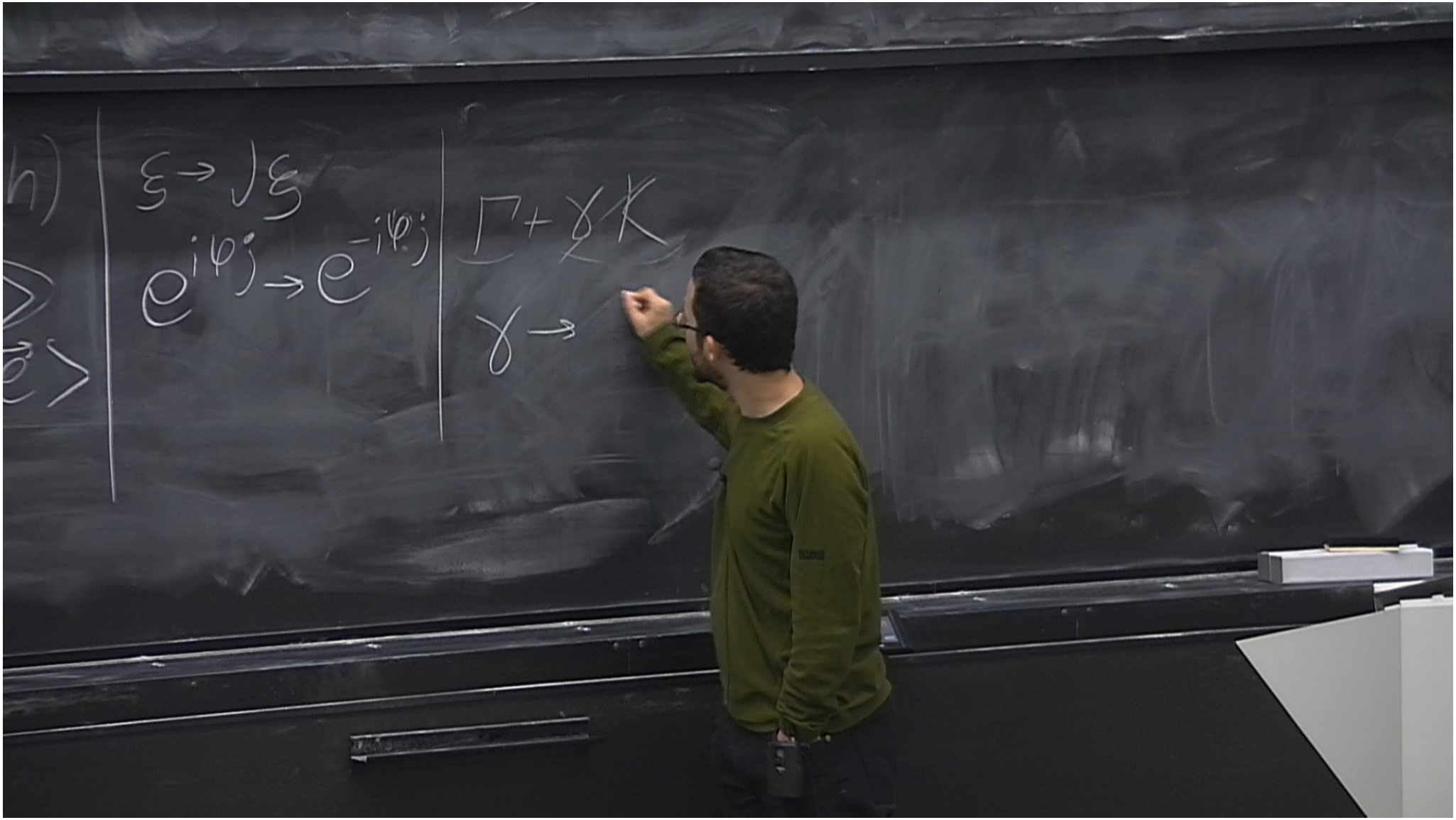
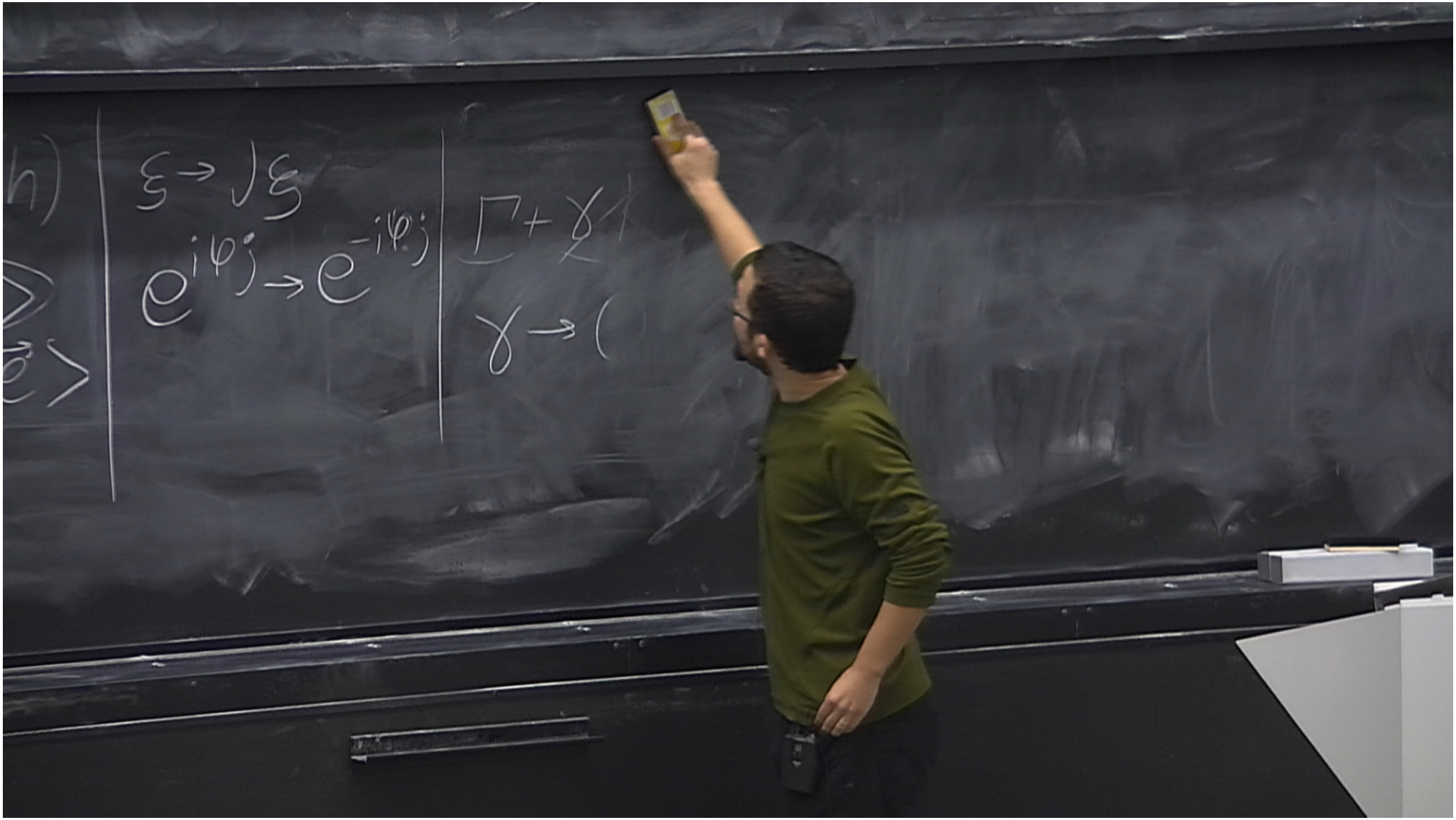
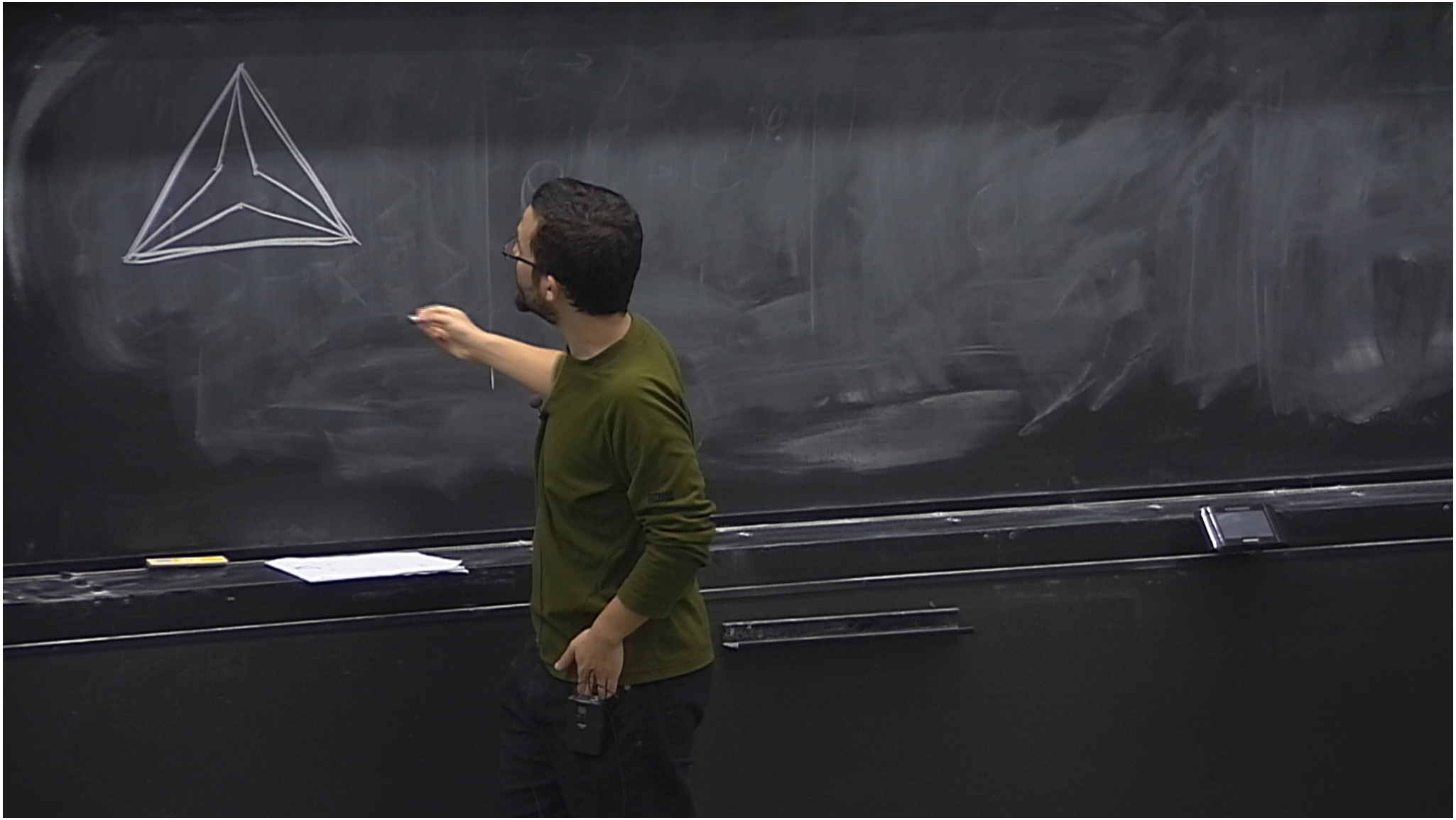
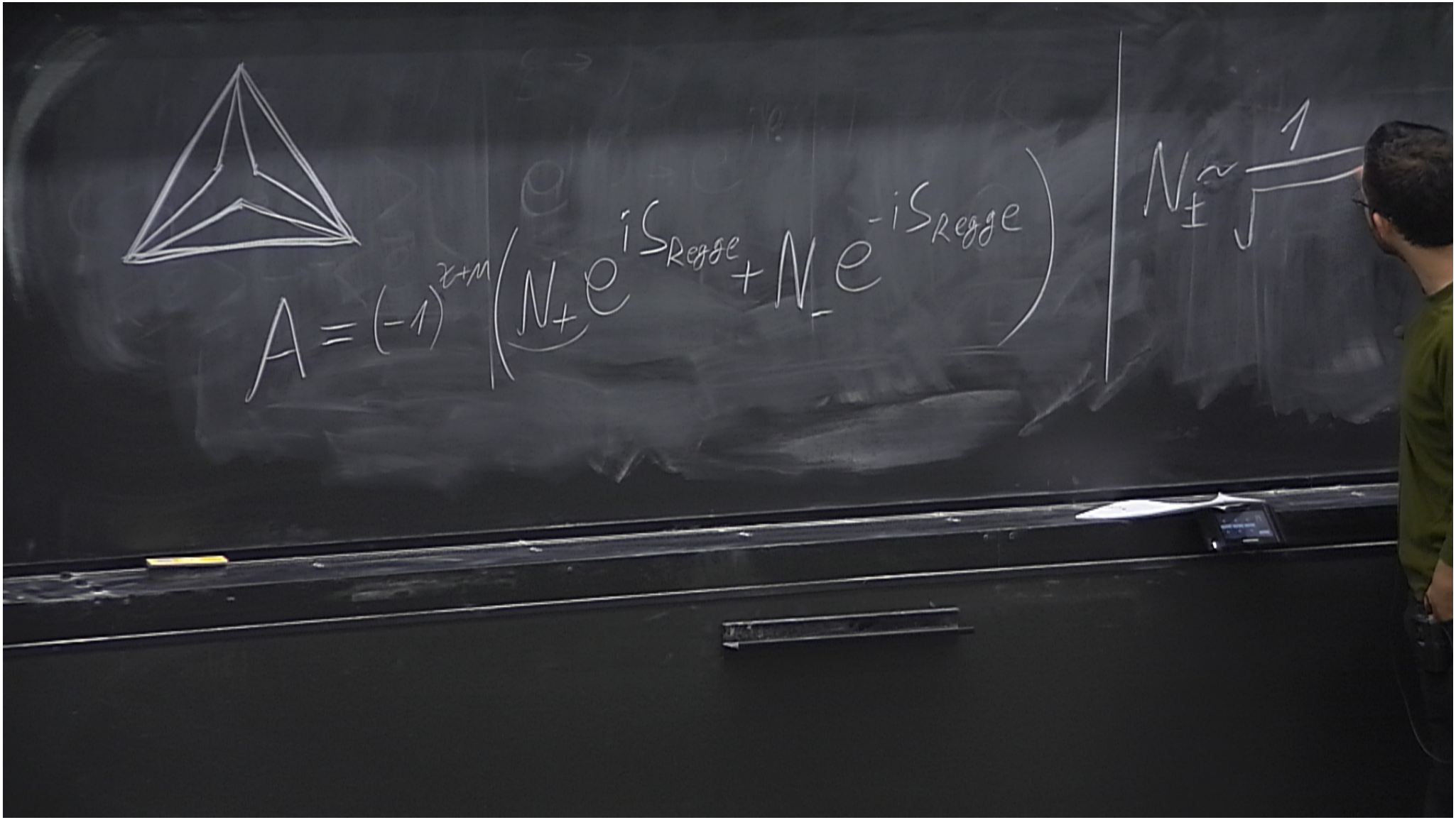
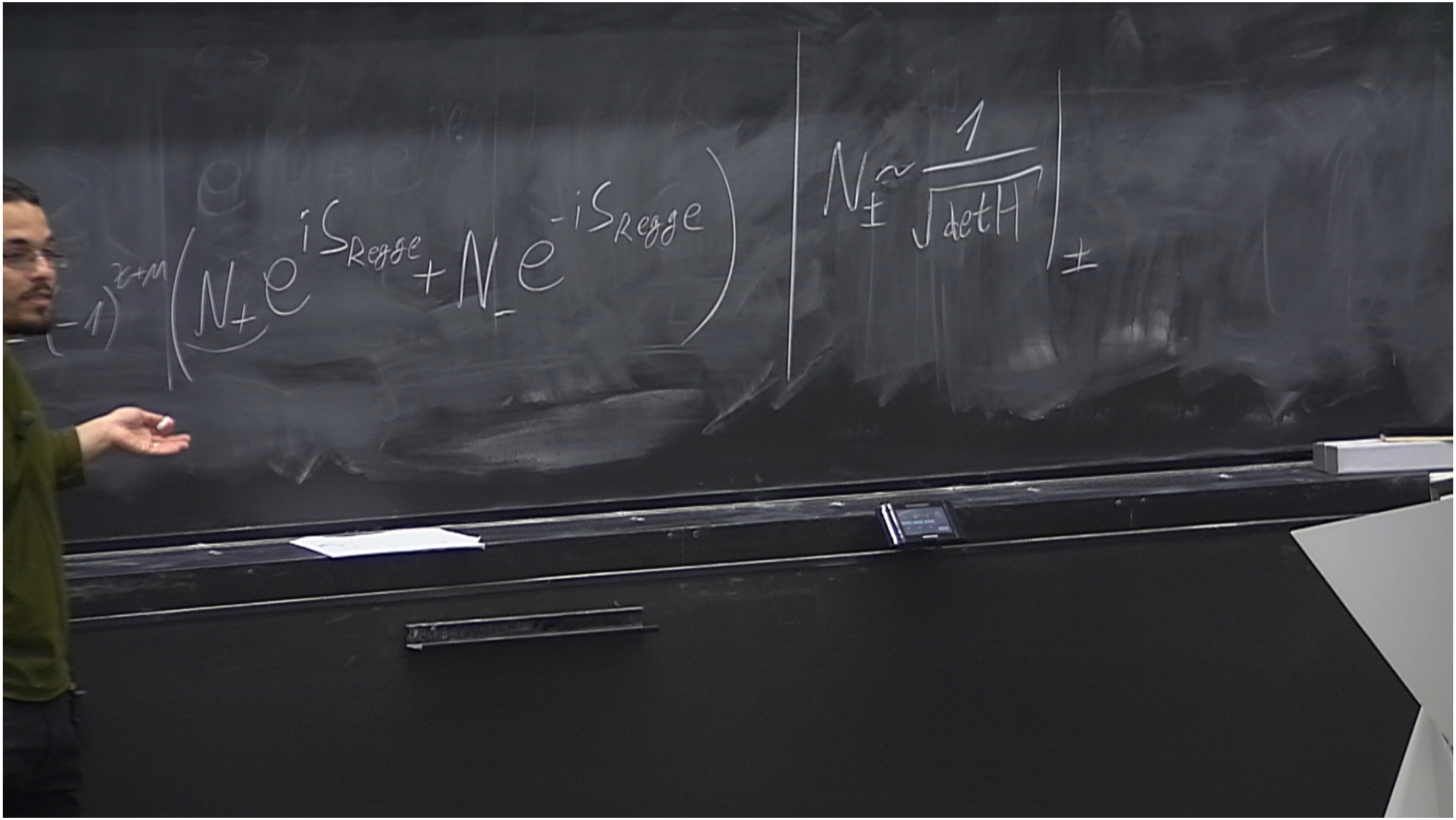

$\varepsilon \to J\varepsilon$
$e^{i\phi_j} \to e^{-i\phi_j}$
$\Gamma + \gamma K$

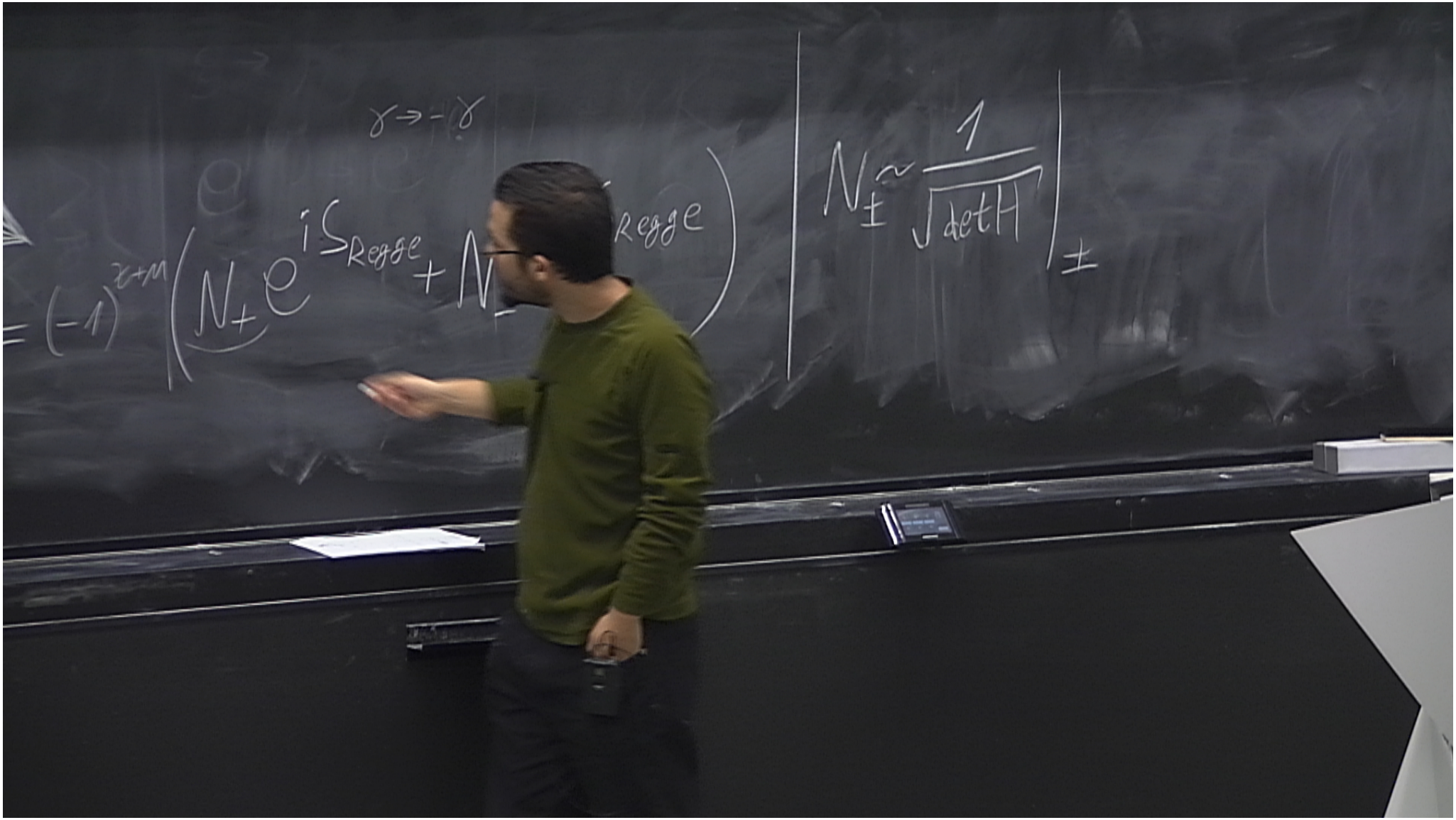

$\xi \to J\xi$
$e^{i\phi_j} \to e^{-i\phi_j}$
$\Gamma + \gamma K$
$\gamma \to$


ξ → Jξ
e^{iφj} → e^{-iφj}
Γ + γ
γ → (

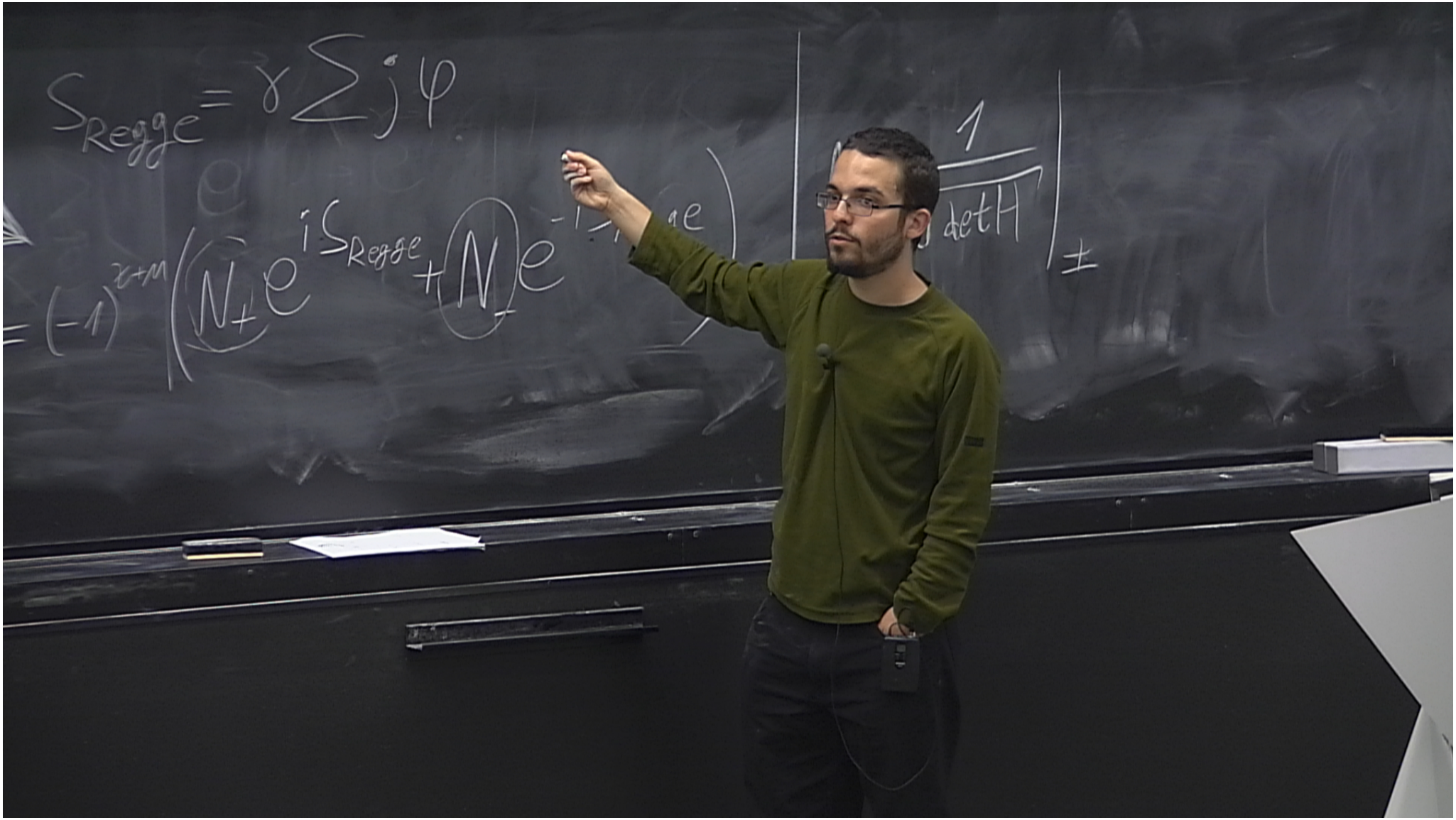


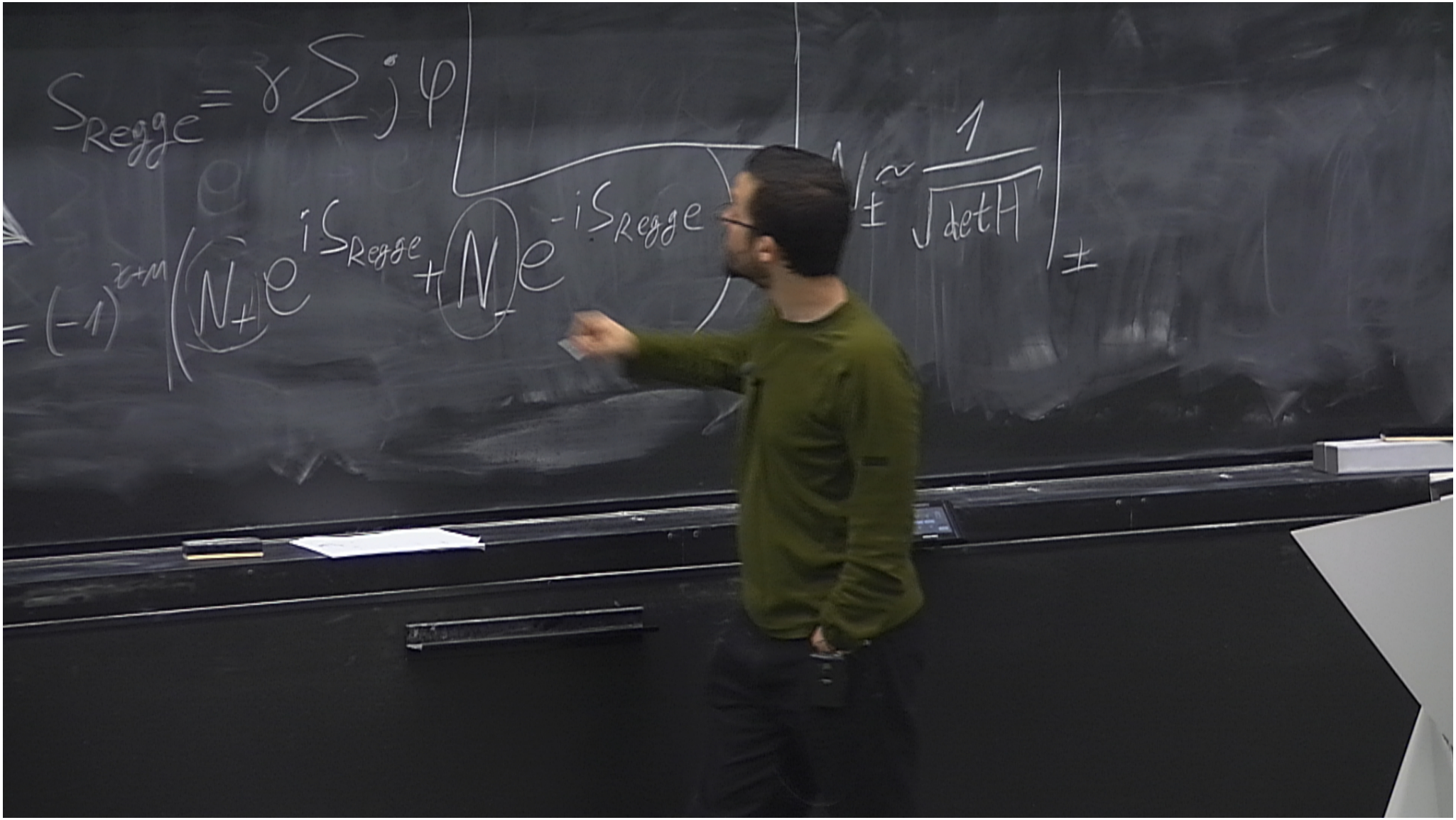

$A = (-1)^{z+M} \left(N_+ e^{i S_{Regge}} + N_- e^{-i S_{Regge}} \right)$
$N_\pm \sim \frac{1}{\sqrt{\quad}}$

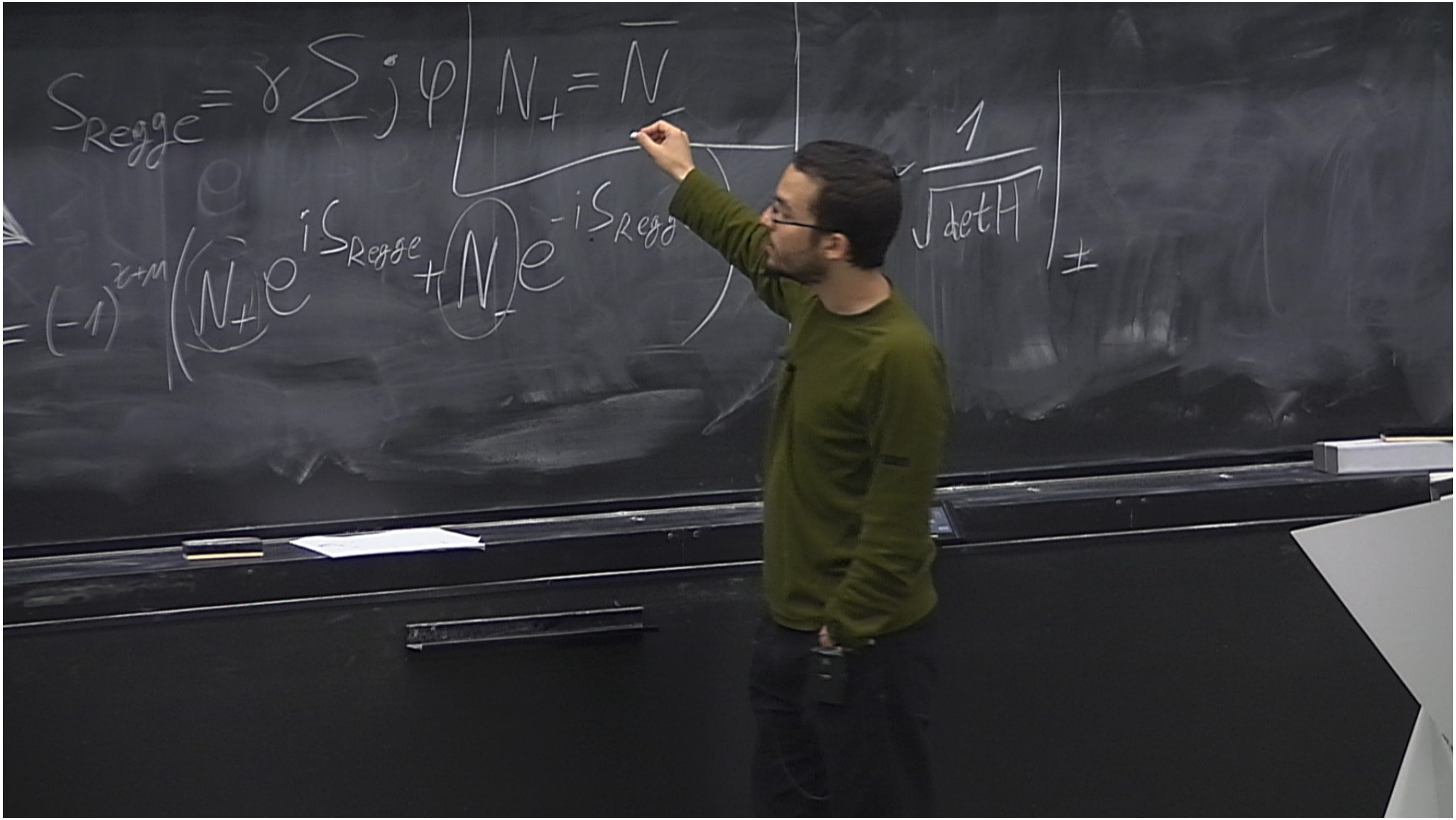

$(-1)^{2+m}\left(N_+ e^{iS_{Regge}} + N_- e^{-iS_{Regge}}\right)$
$N_\pm \sim \frac{1}{\sqrt{\det H}}$
$\pm$

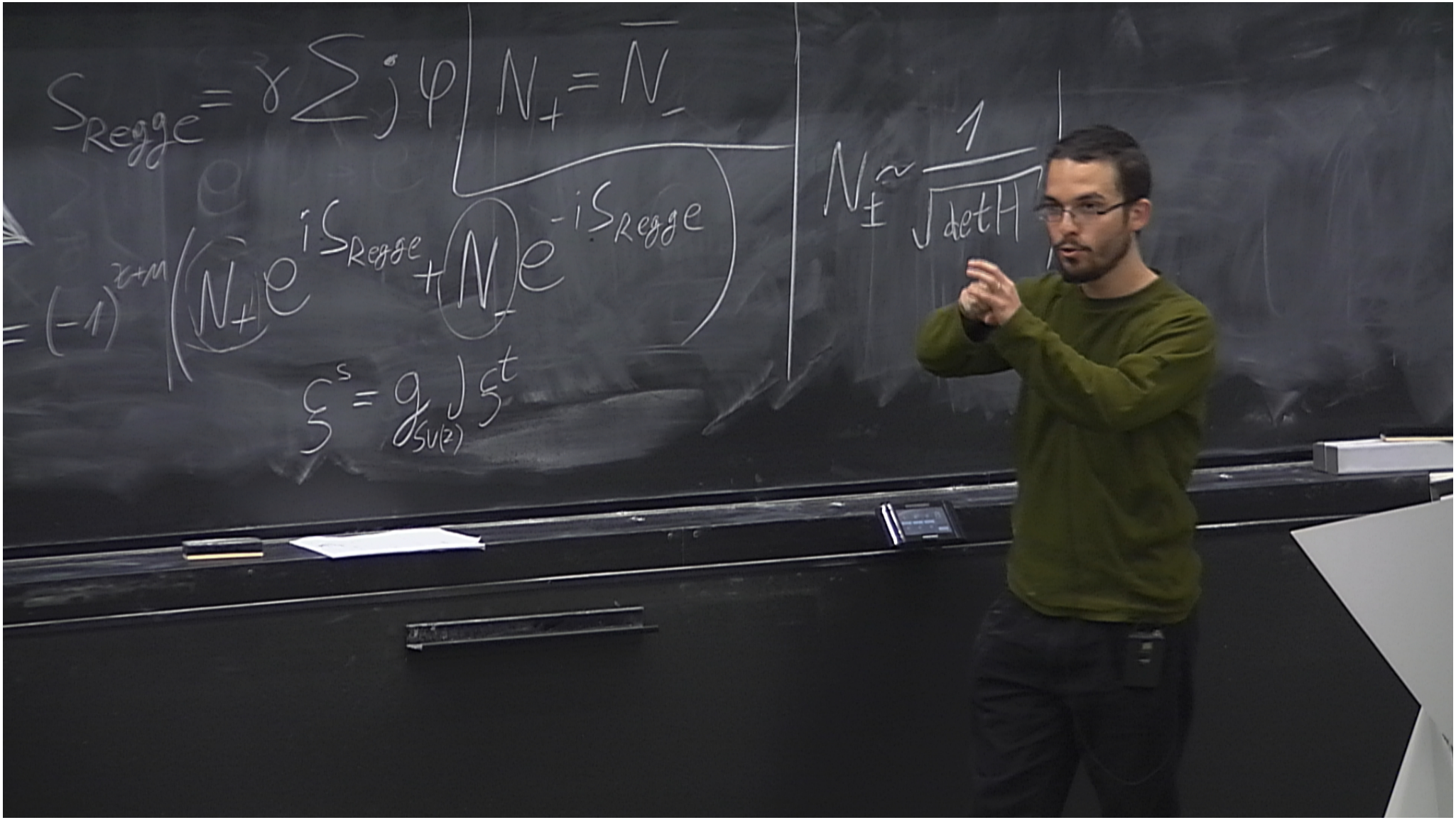

γ → -γ
N± ~ 1/√detH |±

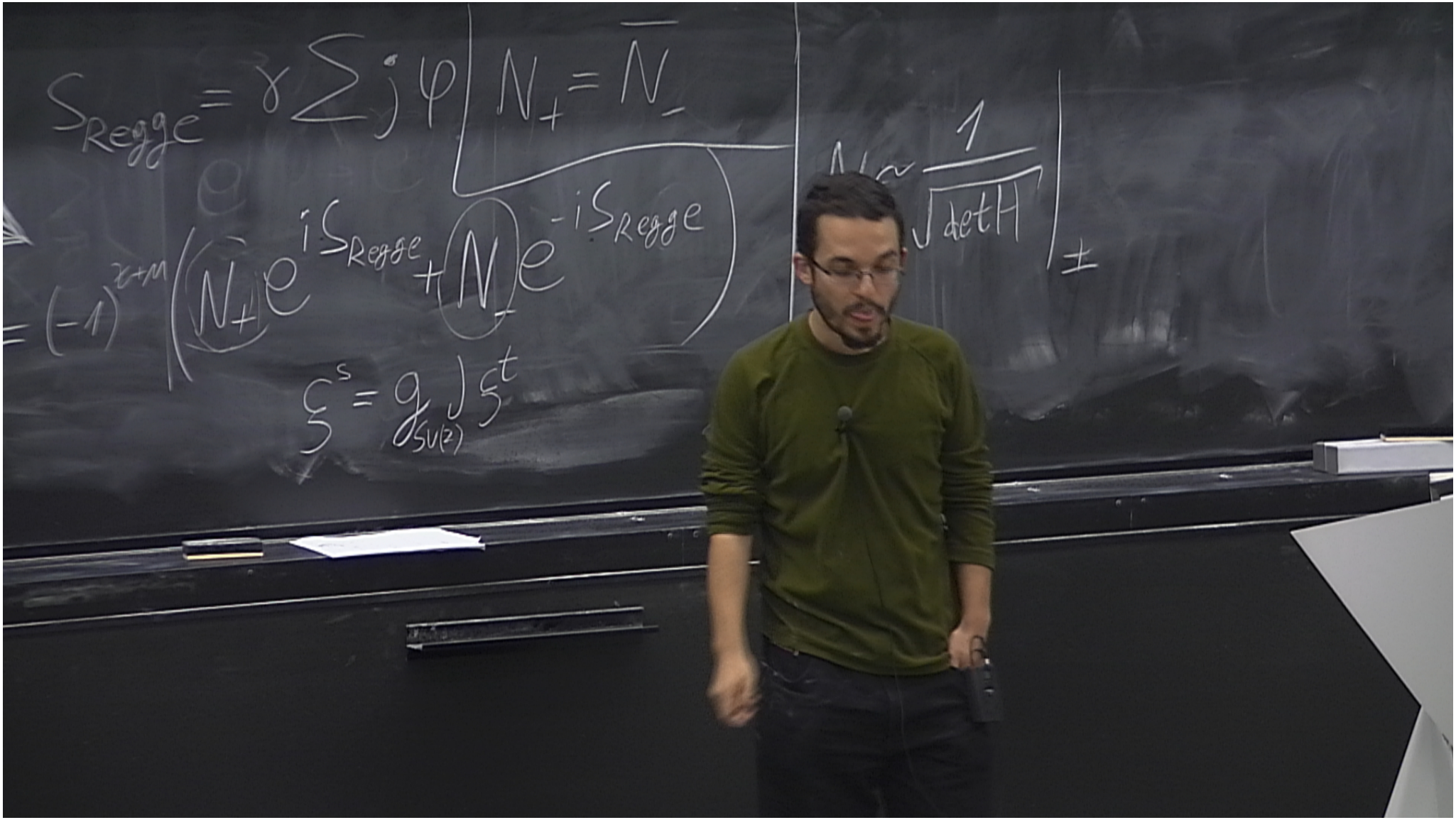
$\gamma \to -\gamma$
$= (-1)^{2+m} \left(N_+ e^{iS_{Regge}} + N_- e^{-iS_{Regge}} \right)$
$\frac{1}{\sqrt{\det H}}$
$\pm$

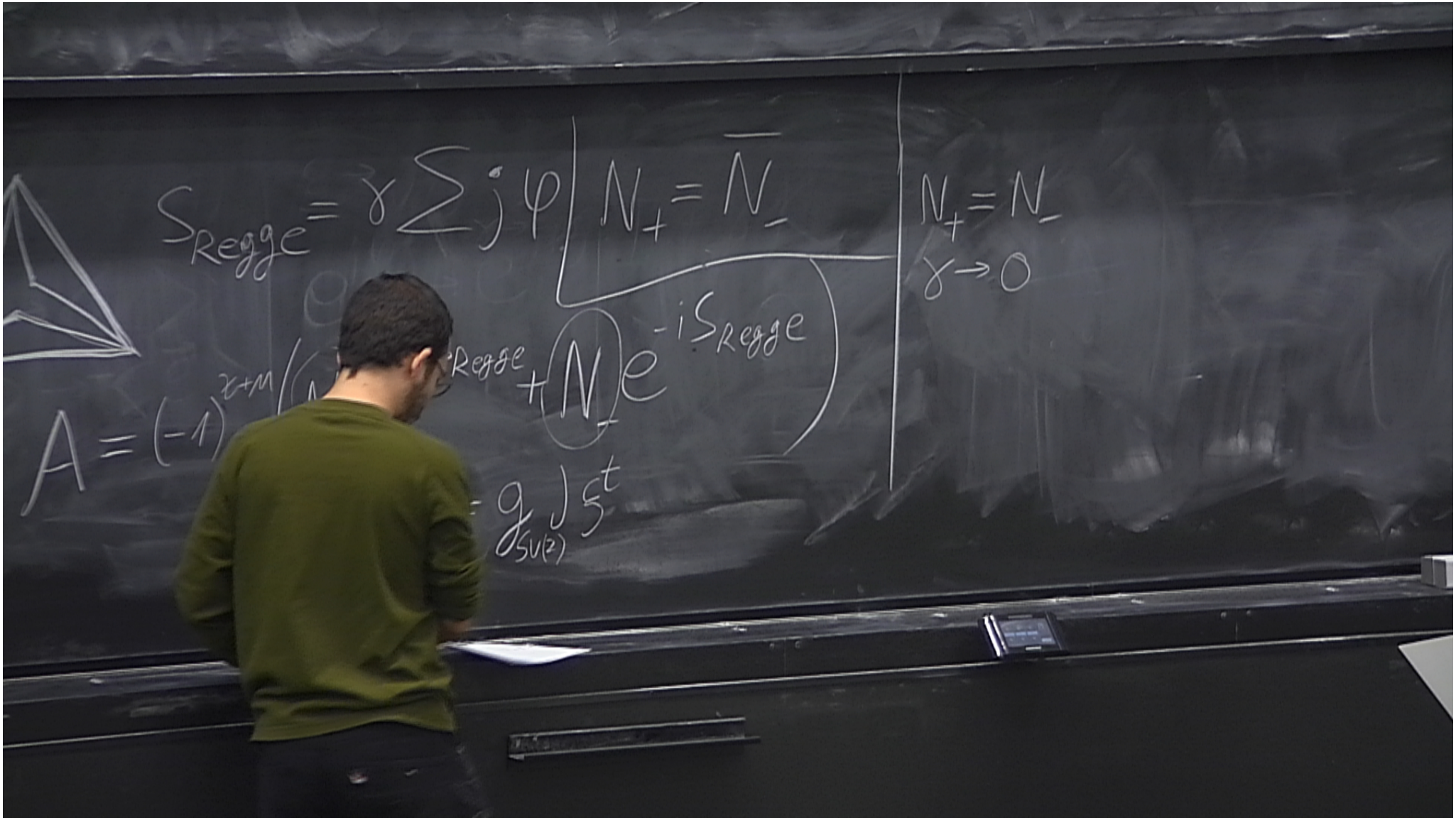

$S_{Regge} = \gamma \sum_j \varphi$
$= (-1)^{2+m} \left(N_+ e^{i S_{Regge}} + N_- e^{-i S_{Regge}} \right)$
$\frac{1}{\det H}$
$\pm$

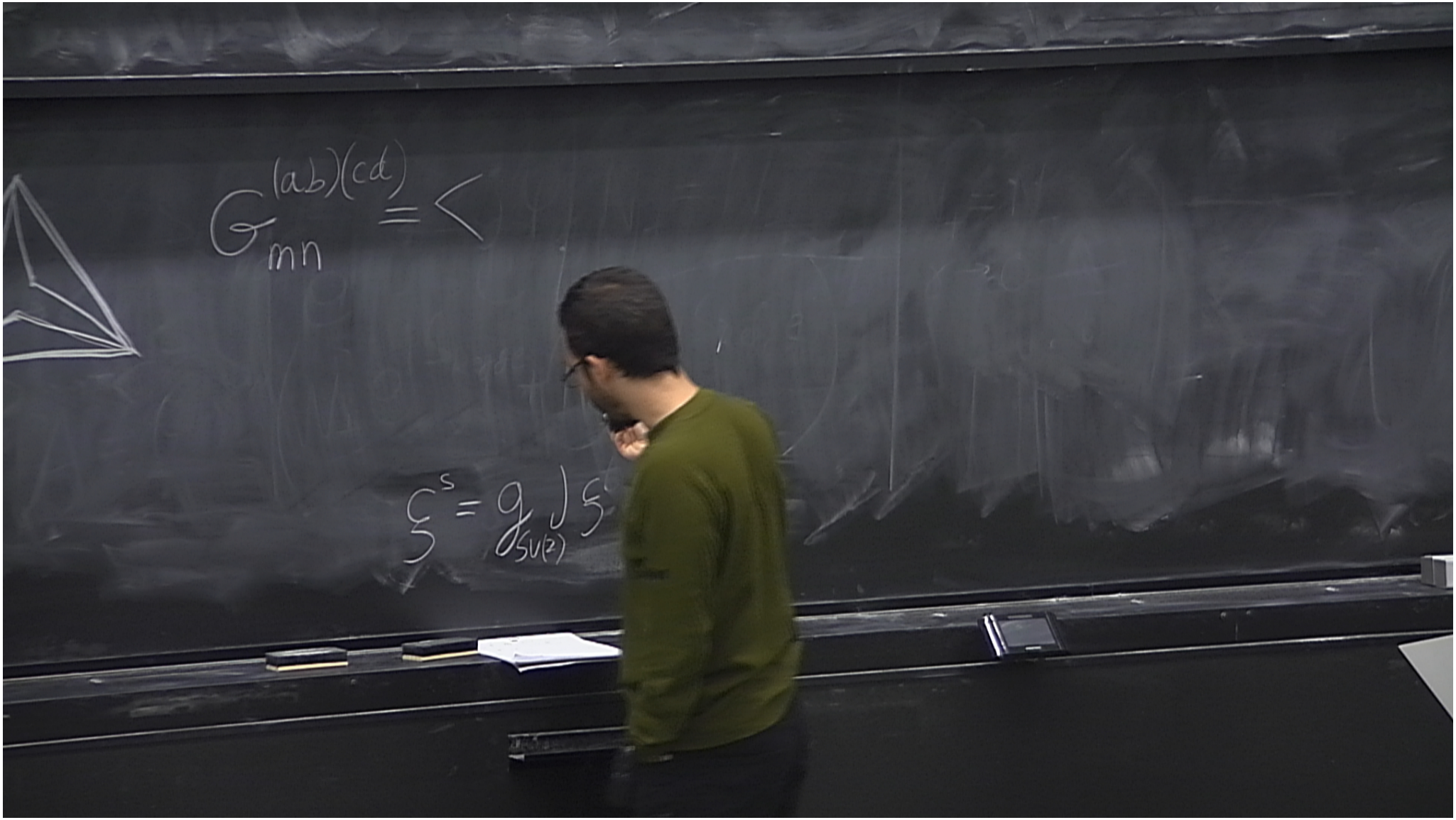

$S_{Regge} = \gamma \sum_j \varphi$
$= (-1)^{2+m} \left(N_+ e^{iS_{Regge}} + N_- e^{-iS_{Regge}} \right)$
$\simeq \frac{1}{\sqrt{\det H}} \Big|_\pm \Big|_\pm$

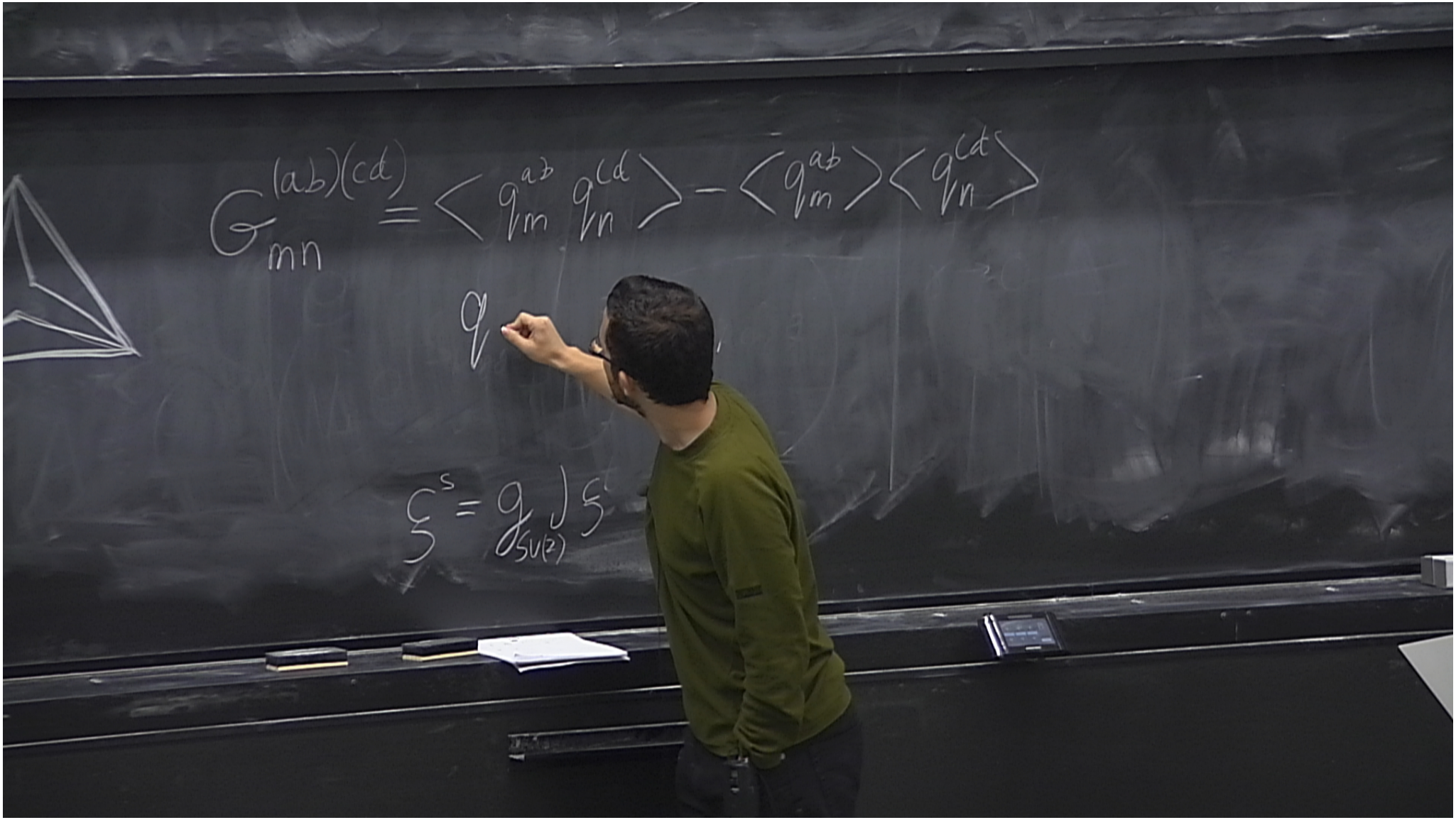

$S_{Regge} = \gamma \sum_j \varphi$
$N_+ = \bar{N}_-$
$= (-1)^{2+m}$
$N_+ e^{iS_{Regge}} + N_- e^{-iS_{Regge}}$
$\frac{1}{\sqrt{\det H}}$
$\pm$

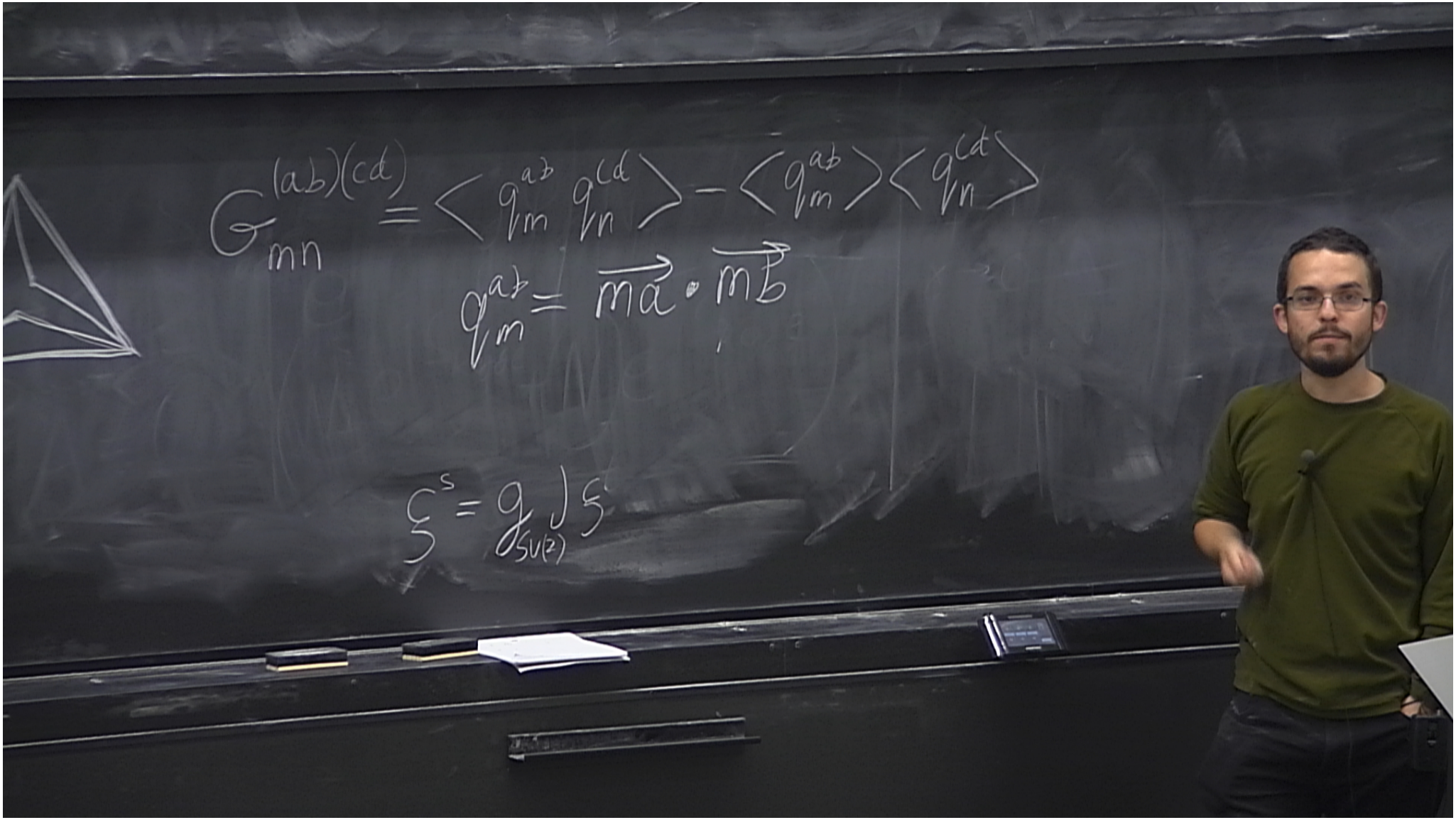

$S_{Regge} = \gamma \sum_j \varphi$
$N_+ = \bar{N}_-$
$= (-1)^{z+m} \left(N_+ e^{iS_{Regge}} + N_- e^{-iS_{Regge}} \right)$
$\xi^s = g^J_{SU(2)} \xi^t$
$N_\pm \sim \frac{1}{\sqrt{\det H}}$

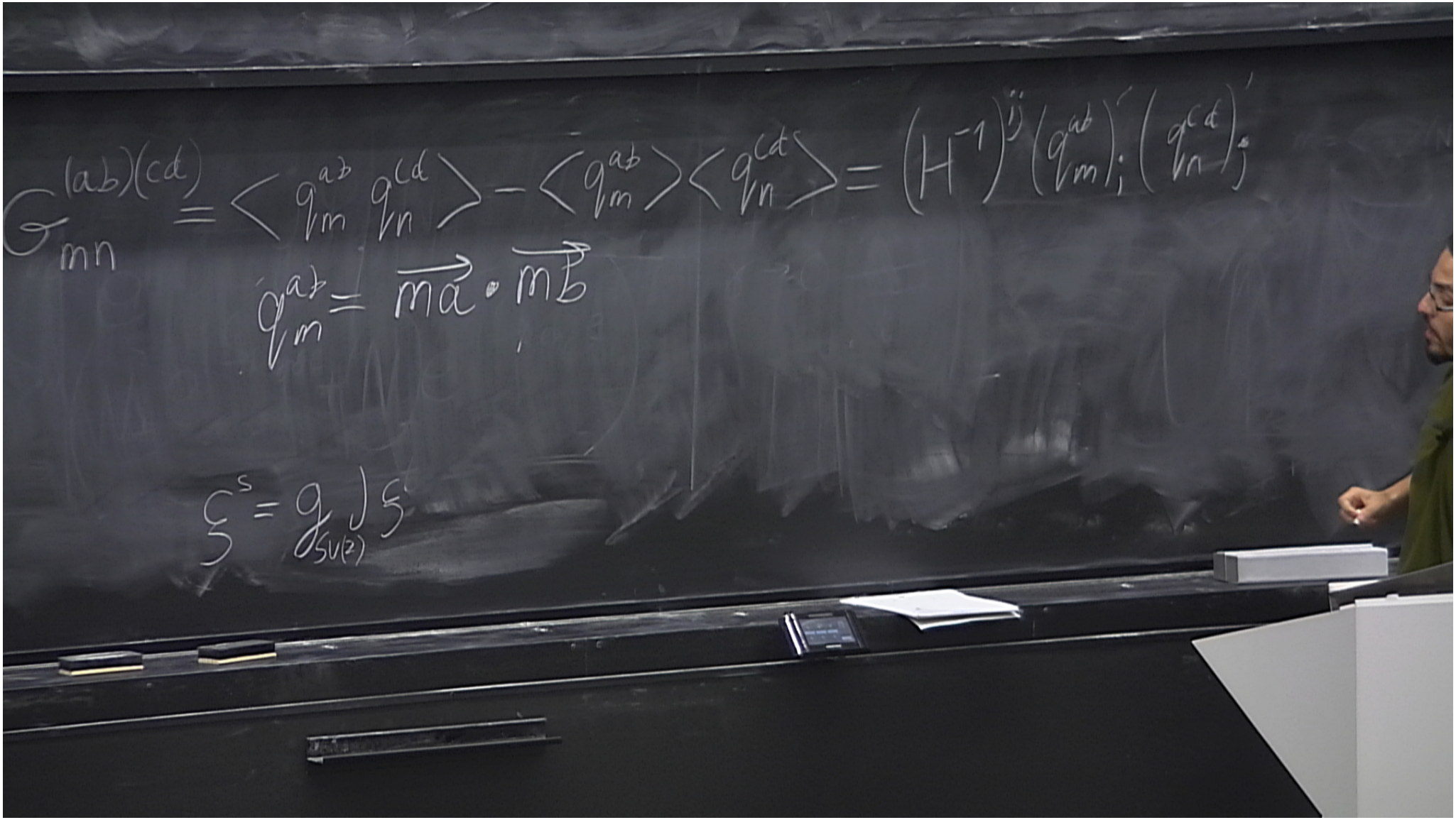

$S_{Regge} = \gamma \sum_j \varphi \Big|_{N_+ = \bar{N}_-}$
$= (-1)^{2+m} \left(N_+ e^{iS_{Regge}} + N_- e^{-iS_{Regge}} \right)$
$\xi^s = g^J_{SU(2)} \xi^t$
$\sim \frac{1}{\sqrt{\det H}} \Big|_{\pm}$

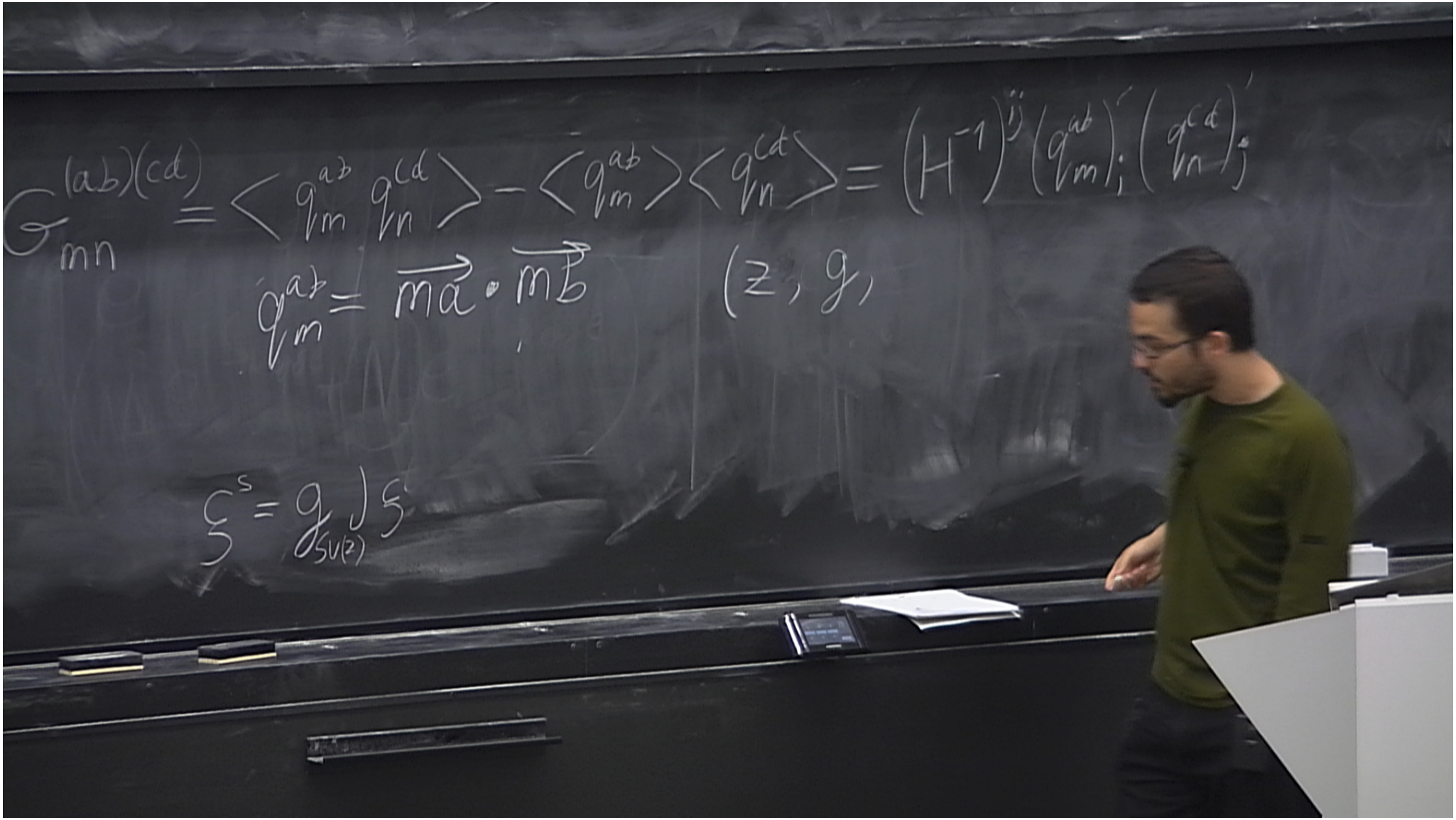

$S_{Regge} = \gamma \sum j \varphi$
$N_+ = \bar{N}_-$
$N_+ = N_-$
$\gamma \to 0$
$A = (-1)^{z+M}$
$+ N_- e^{-iS_{Regge}}$
$g_{SU(2)}$

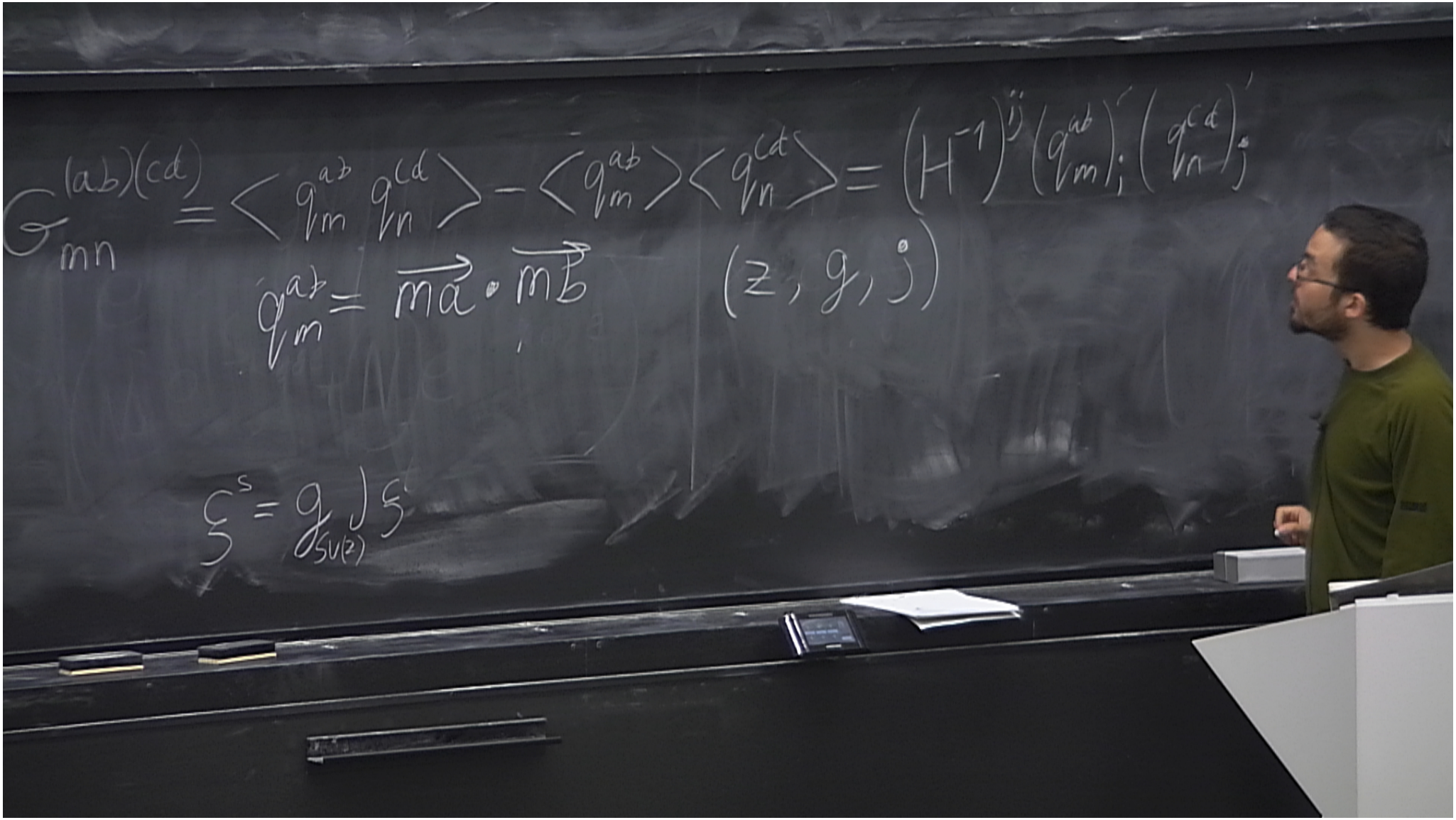

$G_{mn}^{(ab)(cd)} = <$
$\xi^s = g_{SU(2)}^{J} \xi$

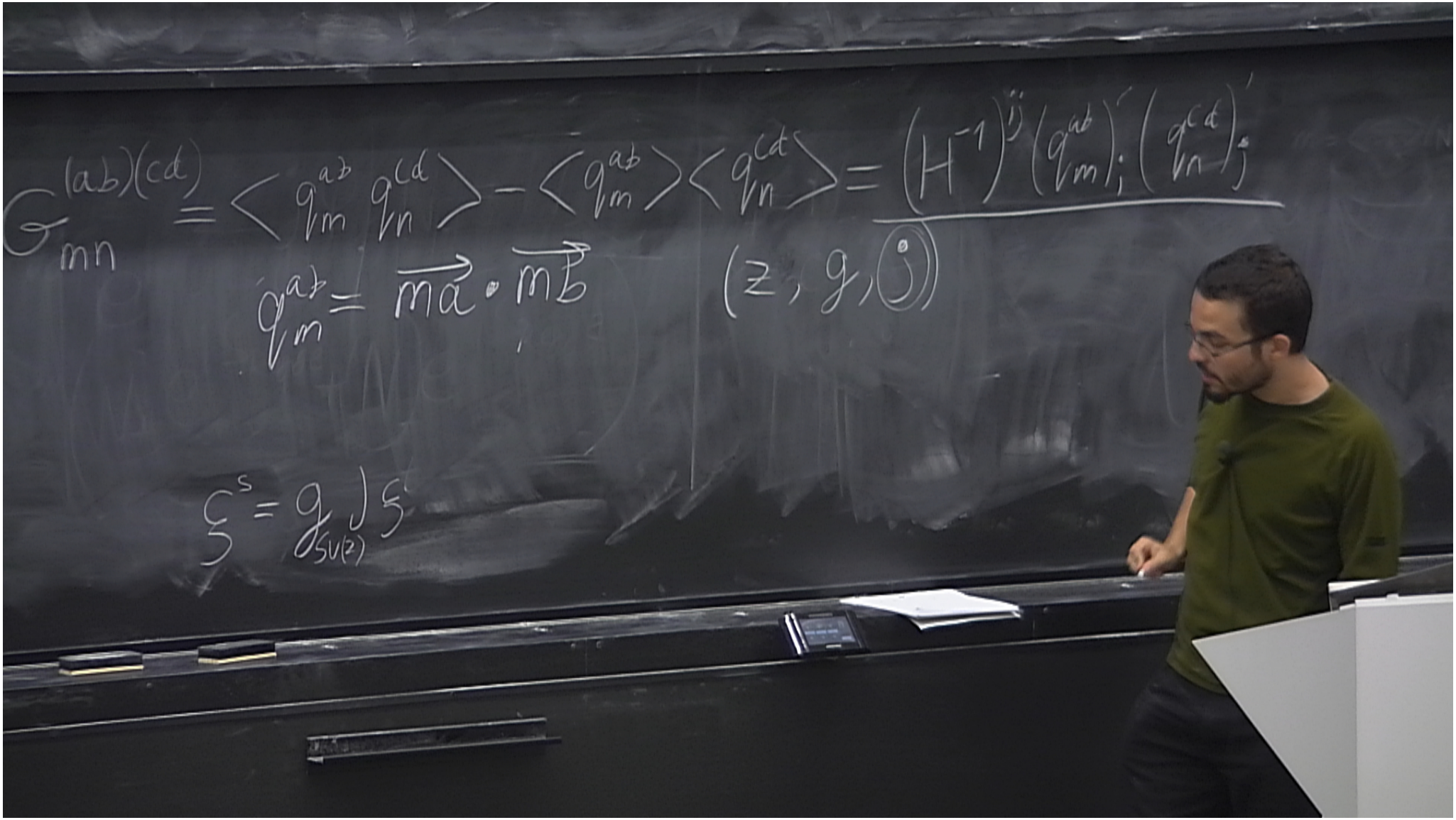

$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle$
$\xi^s = g_{SU(2)}^{J} \xi$

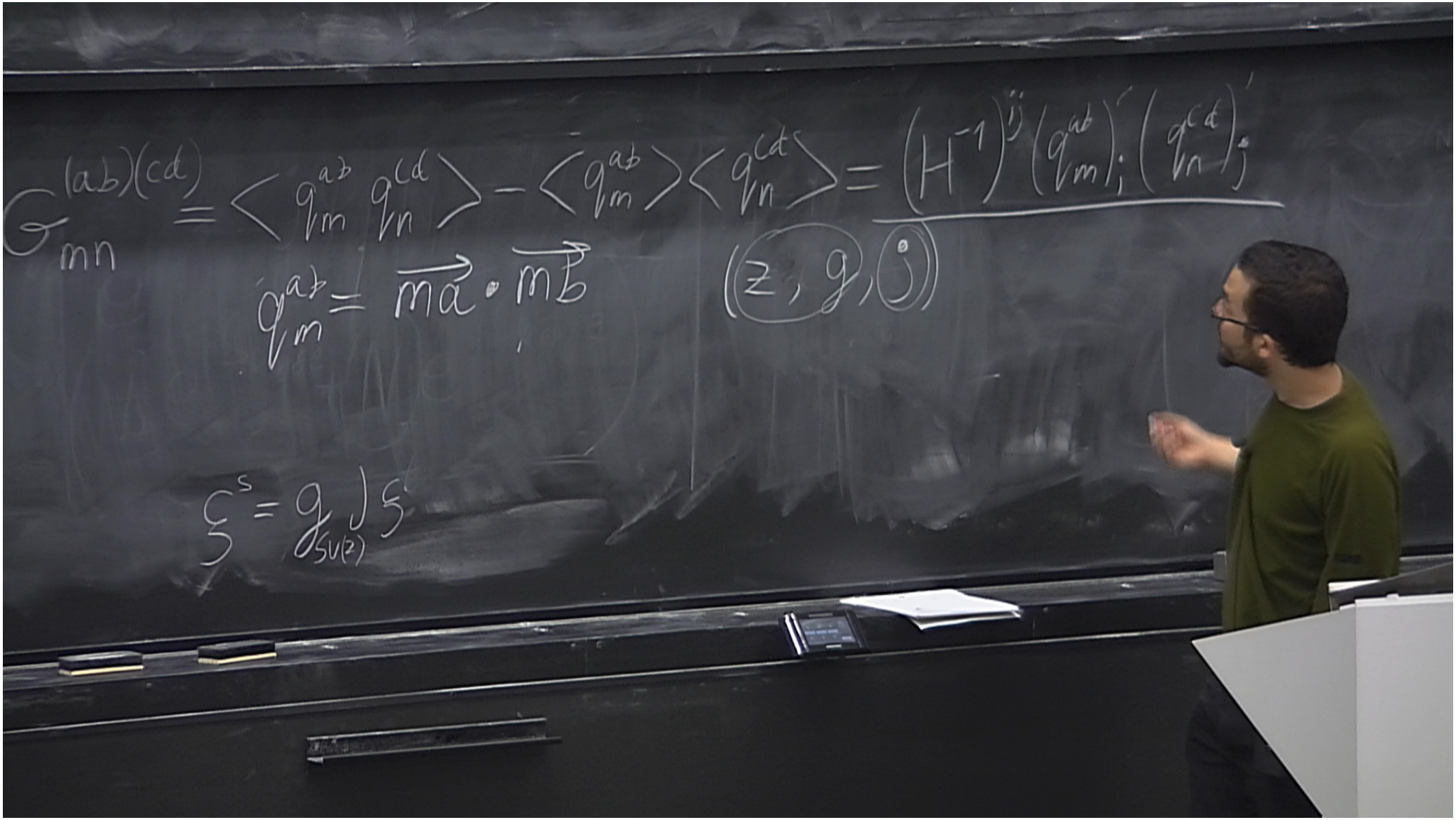

$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle$
$q_m^{ab} = \vec{ma} \cdot \vec{mb}$
$\xi^s = g_{SU(2)}^{J} \xi$

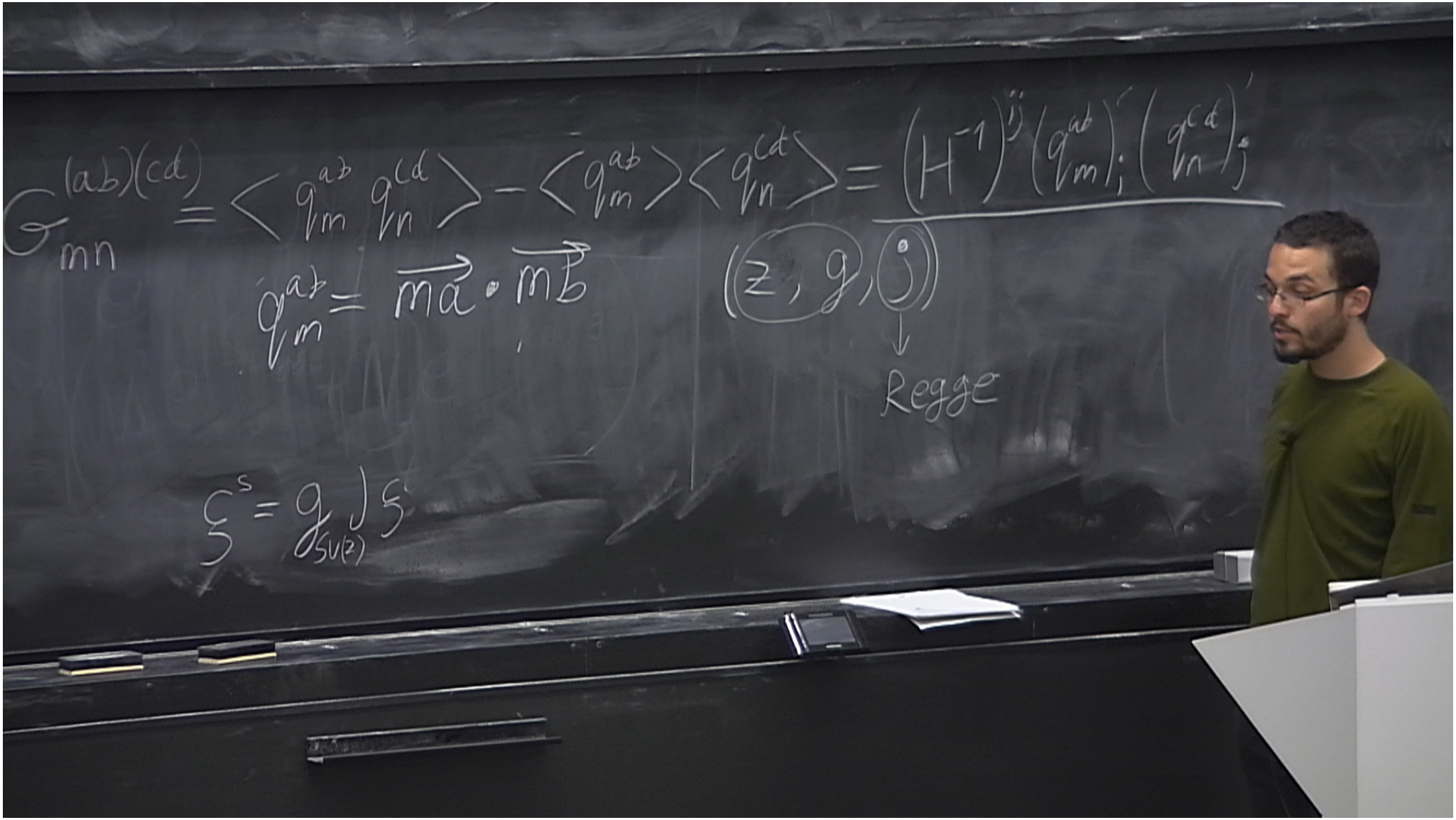

$\sigma^{(ab)(cd)}_{mn} = \langle q^{ab}_m q^{cd}_n \rangle - \langle q^{ab}_m \rangle \langle q^{cd}_n \rangle = (H^{-1})^{ij} (q^{ab}_m)_i (q^{cd}_n)_j$
$q^{ab}_m = \vec{ma} \cdot \vec{mb}$
$\xi^s = g_{SU(2)} J^s$

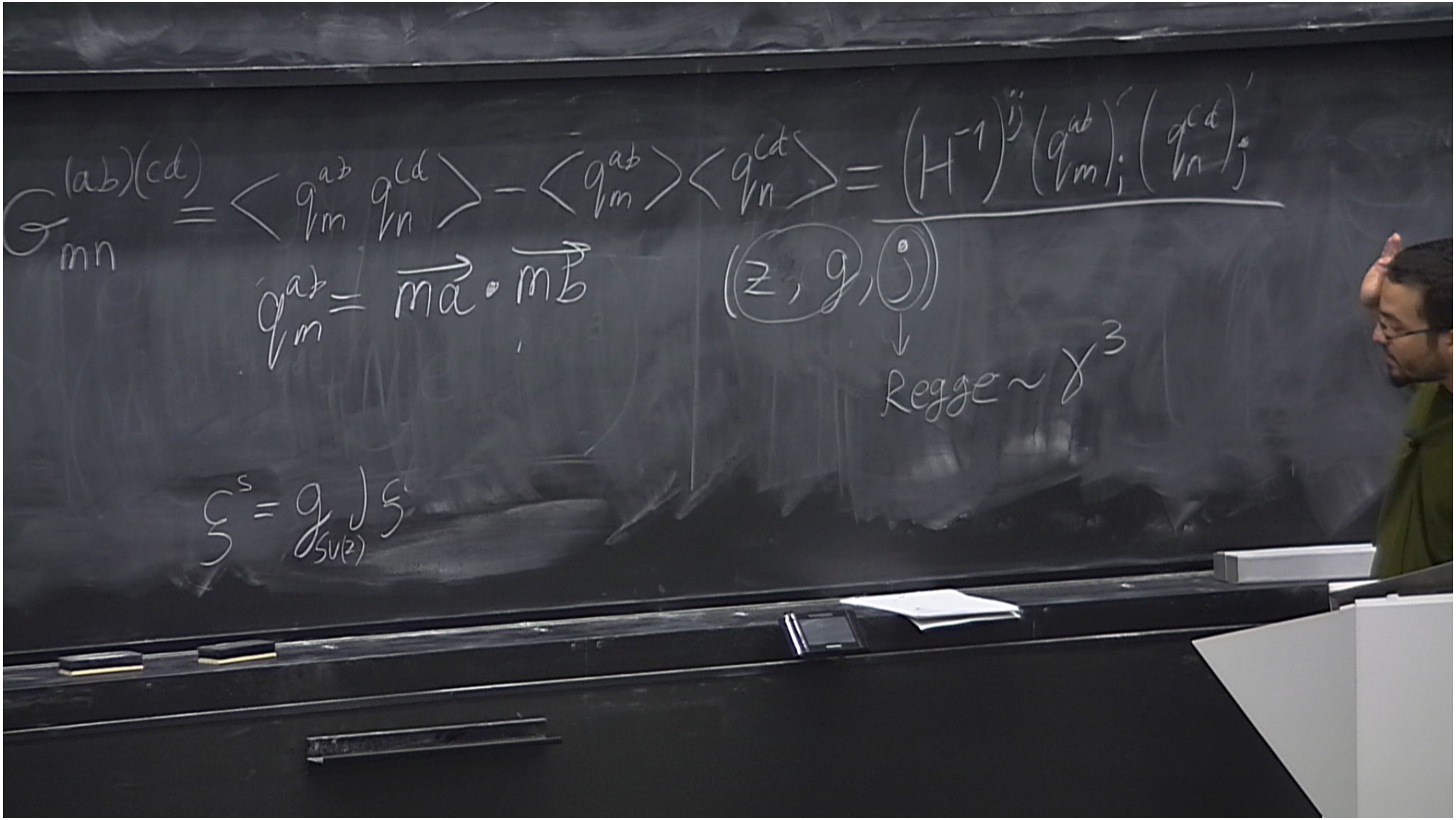

$\sigma_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij} (q_m^{ab})_i (q_n^{cd})_j$
$q_m^{ab} = \vec{ma} \cdot \vec{mb}$
(z, g)
$\xi^s = g_{SU(2)} J^s$

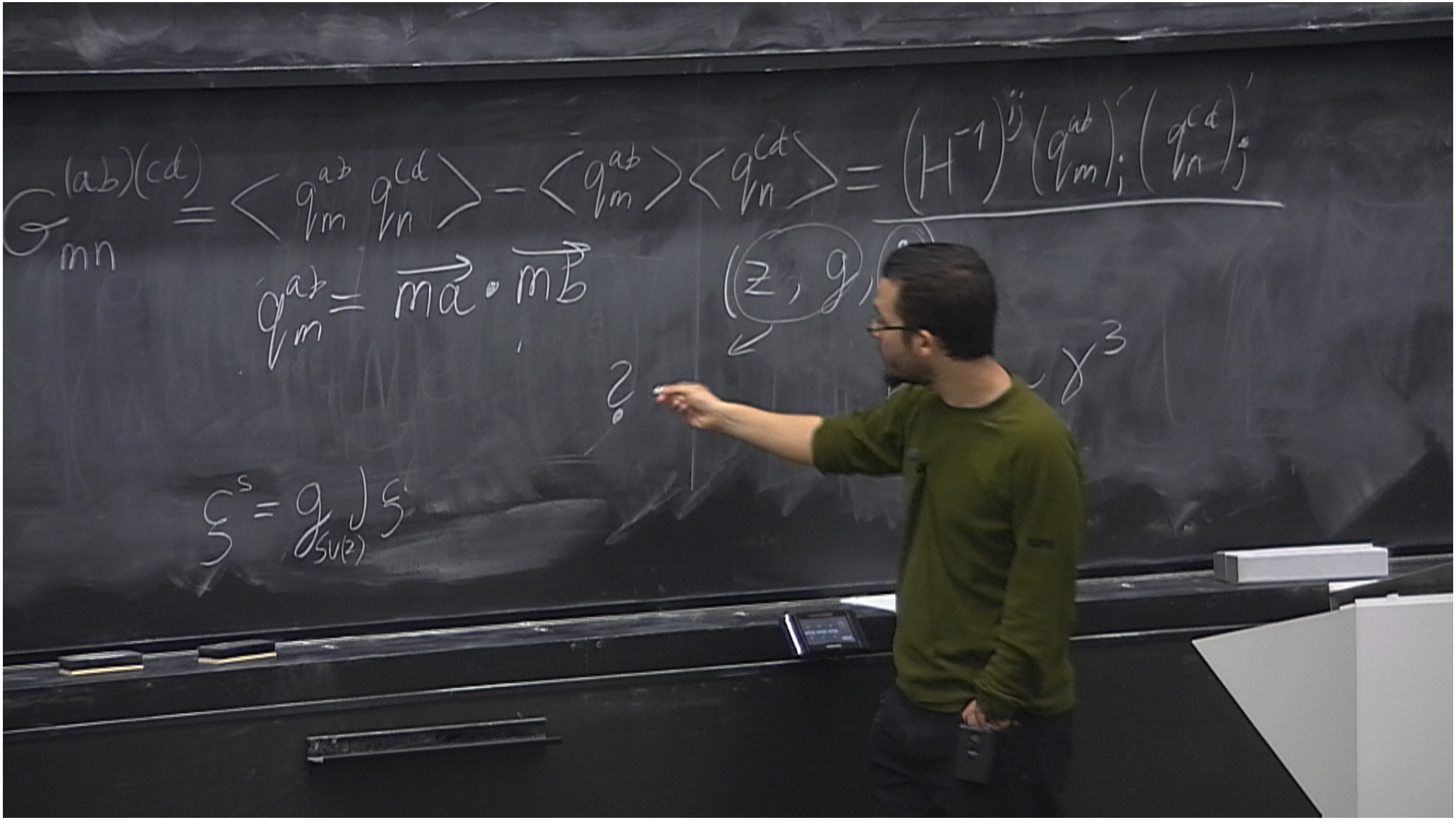

$\sigma_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij} (q_m^{ab})_i (q_n^{cd})_j$
$q_m^{ab} = \vec{ma} \cdot \vec{mb}$
(z, g, j)
$\xi^s = g_{SU(2)}^{ij} s^i$

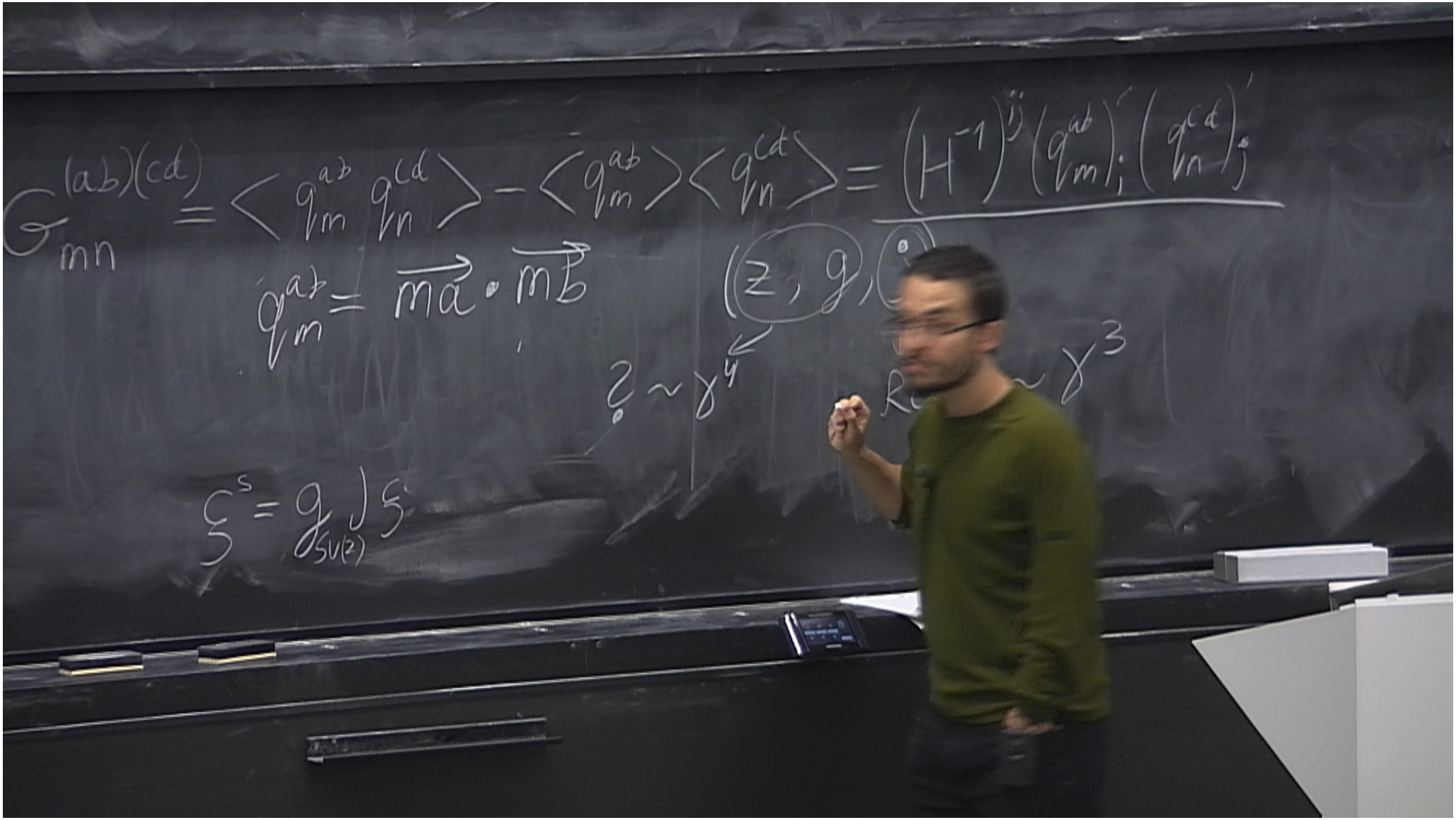

$G^{(ab)(cd)}_{mn} = \langle q^{ab}_m q^{cd}_n \rangle - \langle q^{ab}_m \rangle \langle q^{cd}_n \rangle = (H^{-1})^{ij} (q^{ab}_m)_i (q^{cd}_n)_j$
$q^{ab}_m = \vec{ma} \cdot \vec{mb}$
(z, g, j)
$\xi^s = g_{SU(2)} J^s$

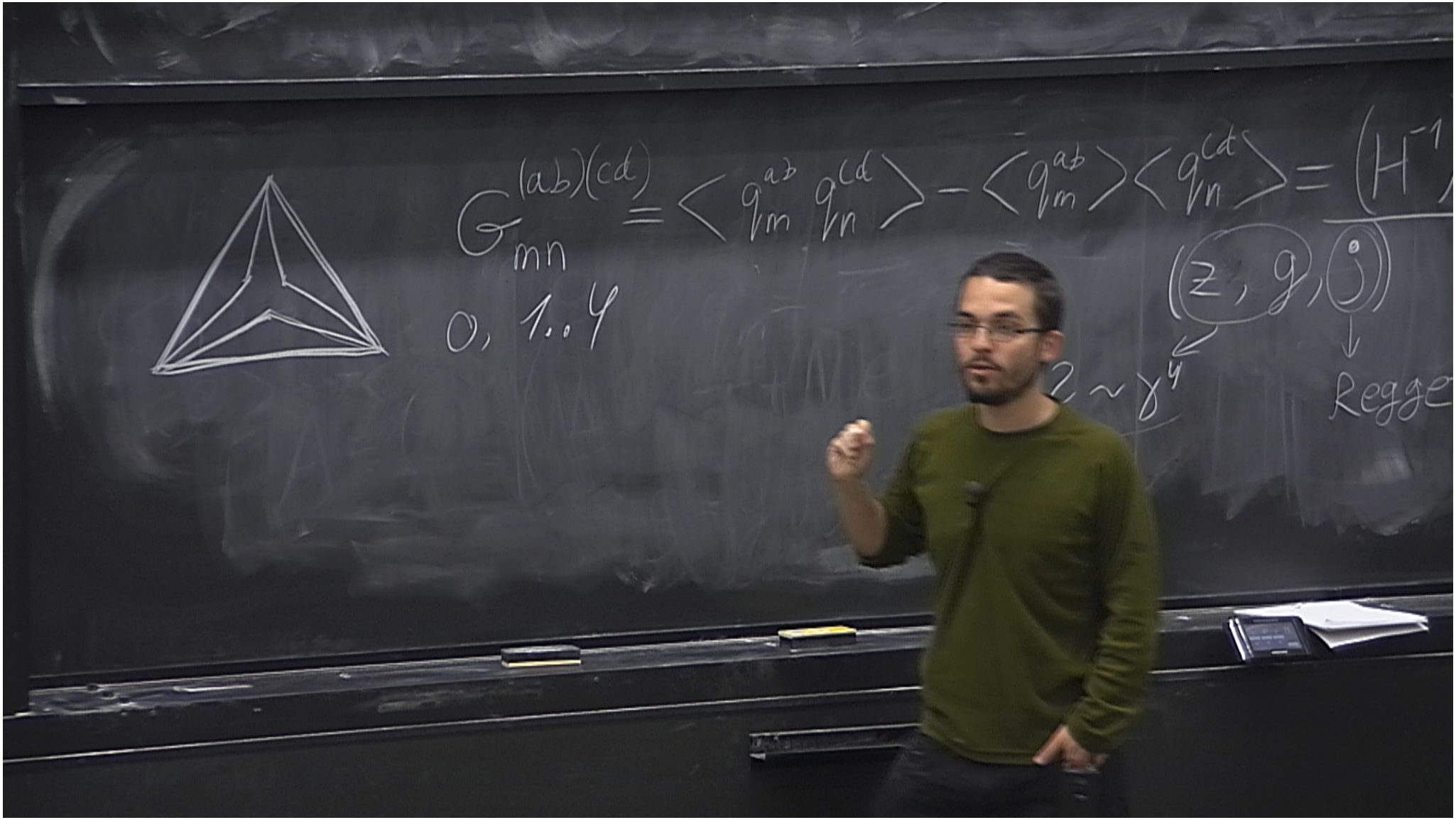

$G^{(ab)(cd)}_{mn} = \langle q^{ab}_m q^{cd}_n \rangle - \langle q^{ab}_m \rangle \langle q^{cd}_n \rangle = (H^{-1})^{ij} (q^{ab}_m)_i (q^{cd}_n)_j$
$q^{ab}_m = \vec{ma} \cdot \vec{mb}$
(z, g, j)
$\xi^s = g_{SU(2)} J \xi$

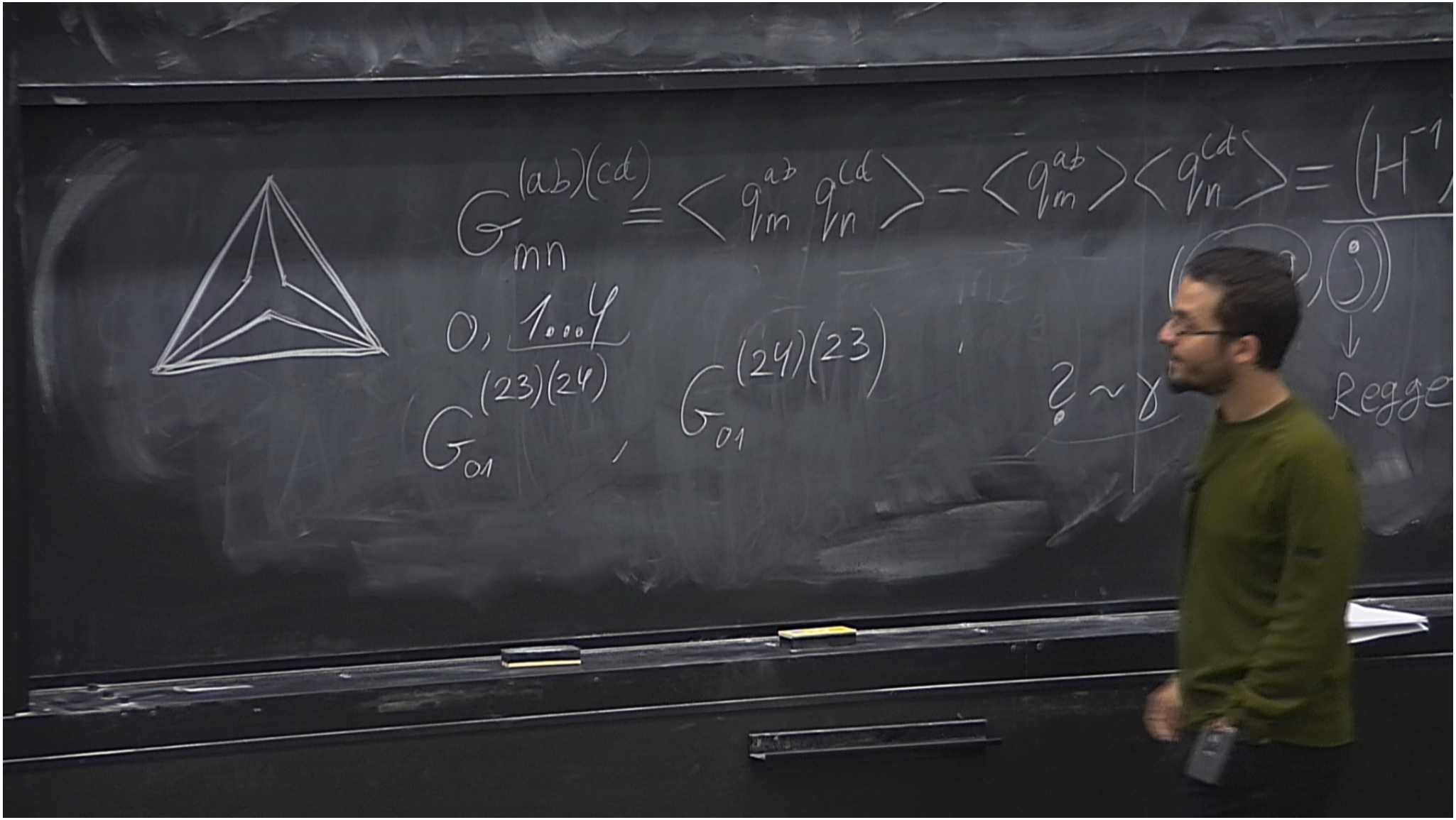

$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij} (q_m^{ab})_i (q_n^{cd})_j$
$q_m^{ab} = \vec{ma} \cdot \vec{mb}$
(z, g, 0)
Regge
$\xi^s = g_{SU(2)}^{ij} \xi^s$

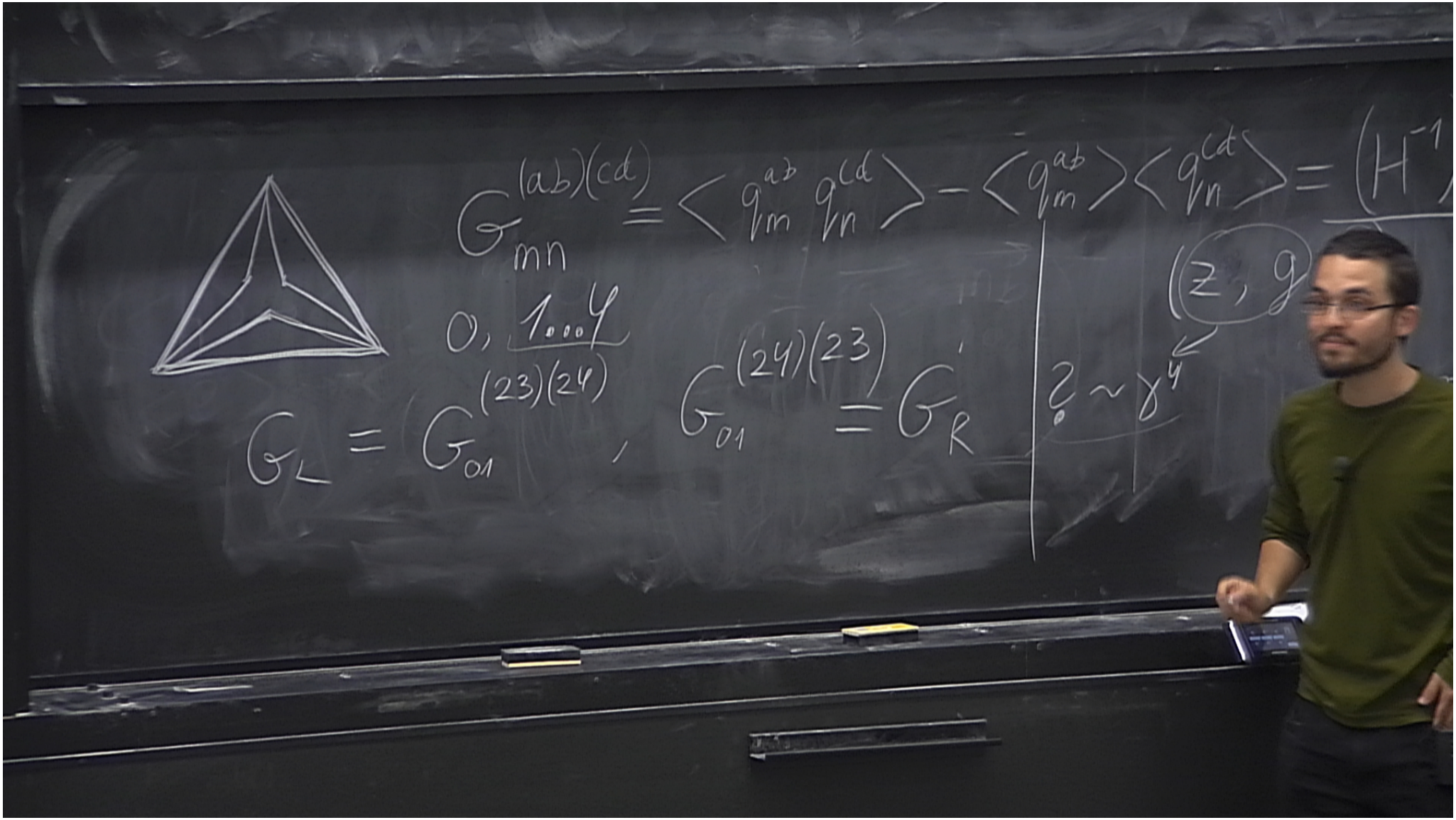

$G^{(ab)(cd)}_{mn} = \langle q^{ab}_m q^{cd}_n \rangle - \langle q^{ab}_m \rangle \langle q^{cd}_n \rangle = (H^{-1})^{ij} (q^{ab}_m)_i (q^{cd}_n)_j$
$q^{ab}_m = \vec{ma} \cdot \vec{mb}$
$(z, g, 0)$
Regge $\sim \gamma^3$
$\xi^s = g_{SU(2)} J^s$

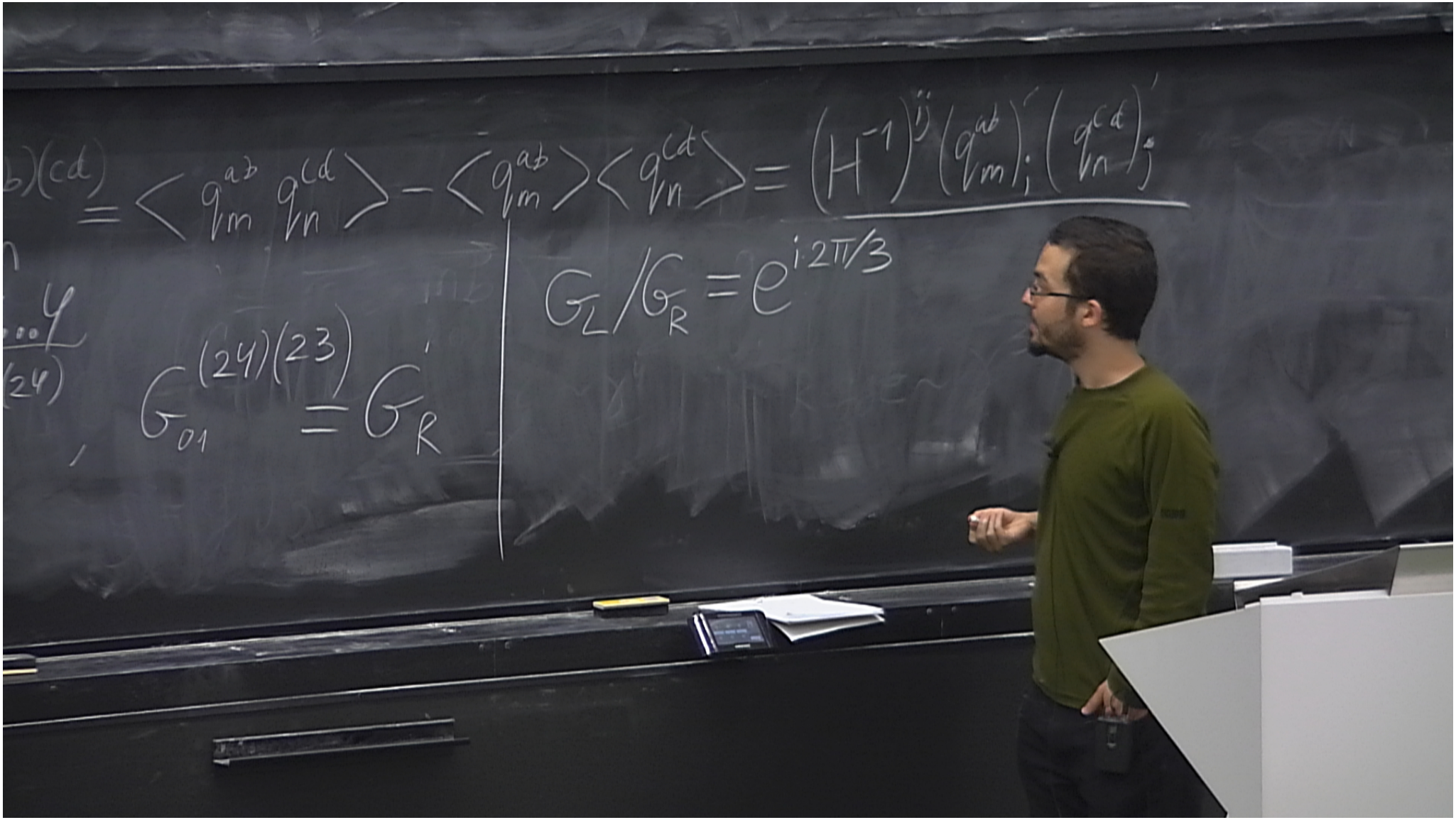

$G^{(ab)(cd)}_{mn} = \langle q^{ab}_m q^{cd}_n \rangle - \langle q^{ab}_m \rangle \langle q^{cd}_n \rangle = (H^{-1})^{ij} (q^{ab}_m)_i (q^{cd}_n)_j$
$q^{ab}_m = \vec{ma} \cdot \vec{mb}$
(z, g,
?
γ^3
$\xi^s = g_{SU(2)} J s$

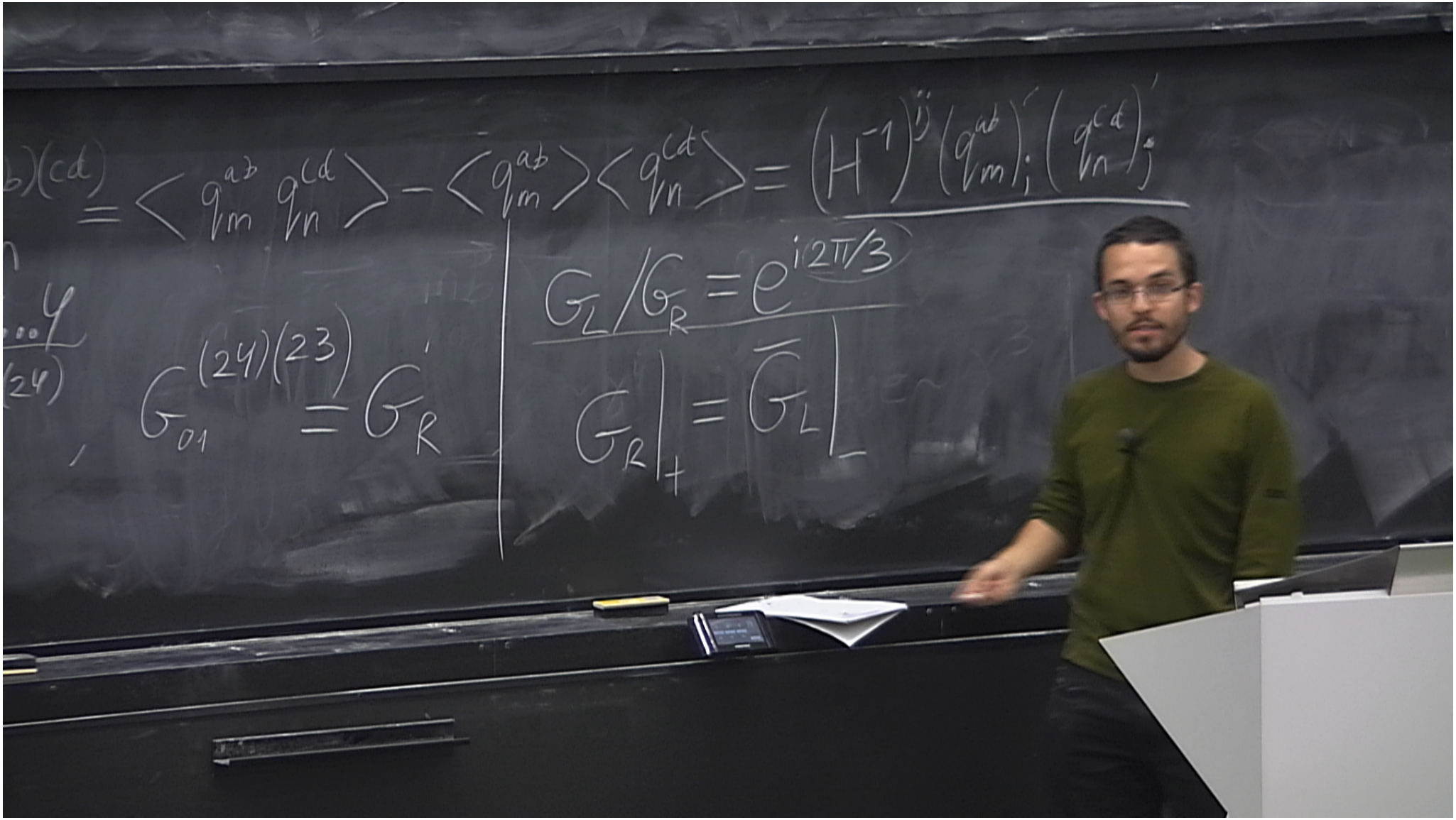

$G^{(ab)(cd)}_{mn} = \langle q^{ab}_m q^{cd}_n \rangle - \langle q^{ab}_m \rangle \langle q^{cd}_n \rangle = (H^{-1})^{ij} (q^{ab}_m)_i (q^{cd}_n)_j$
$q^{ab}_m = \vec{ma} \cdot \vec{mb}$
$(z, g,$
$? \sim \gamma^4$
$R \sim \gamma^3$
$\xi^s = g_{SU(2)} J^s$

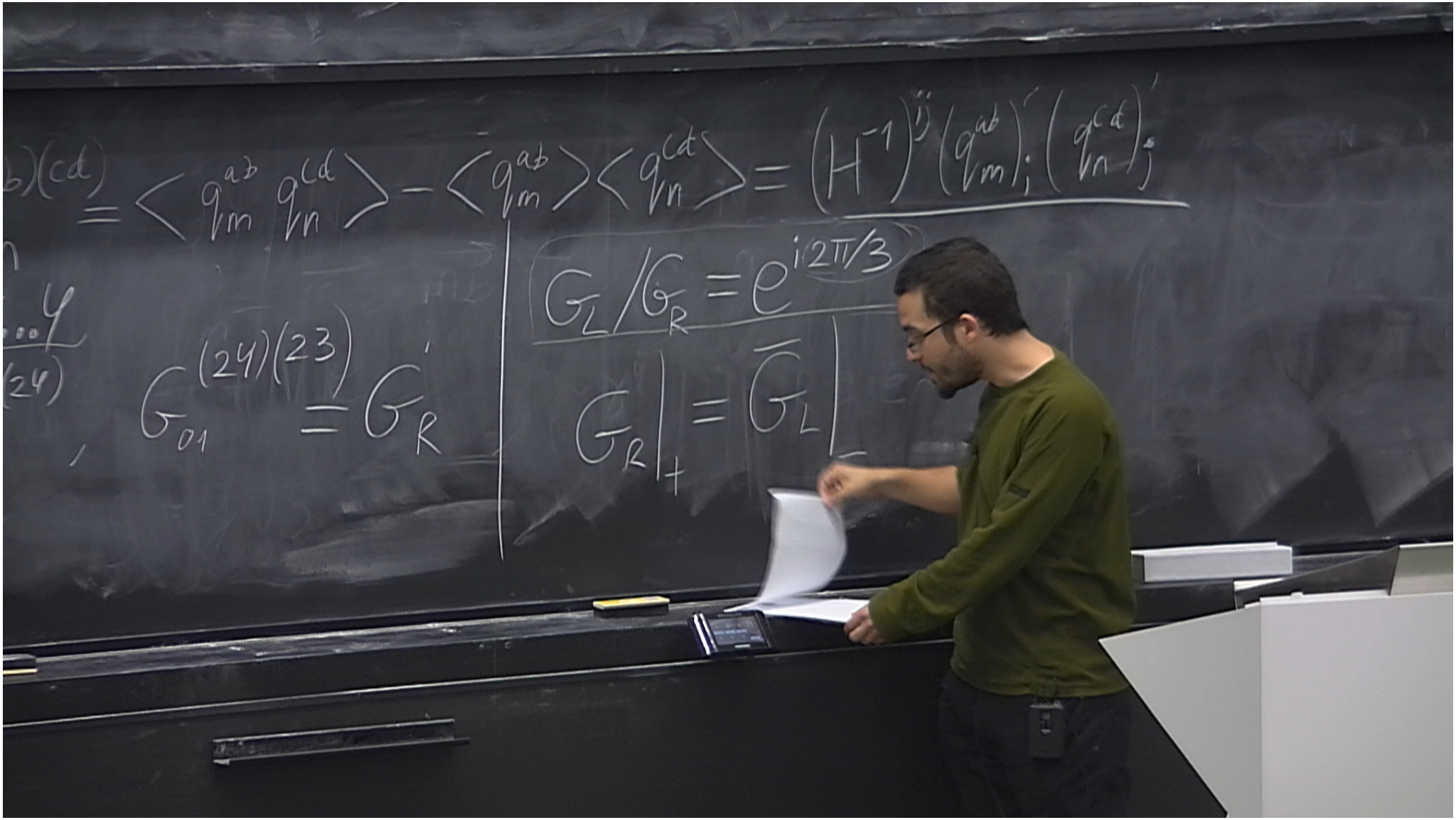

$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1}$
0, 1..4
(z, g, 0)
$z \sim \gamma^4$
Regge

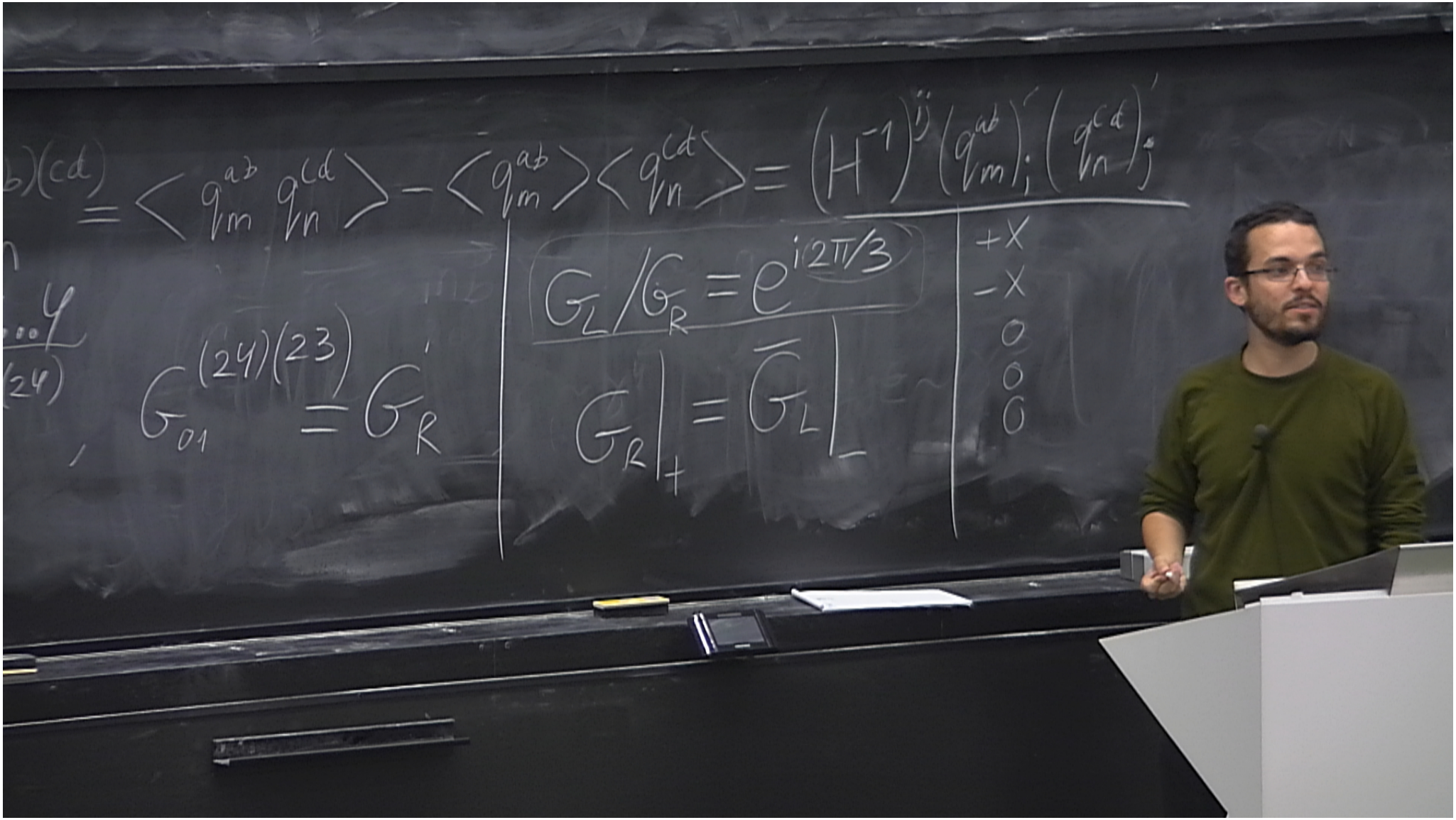

$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1}$
$0, \underline{1000}$
$G_{01}^{(23)(24)}$
$G_{01}^{(24)(23)}$
$\xi \sim \gamma$
Regge

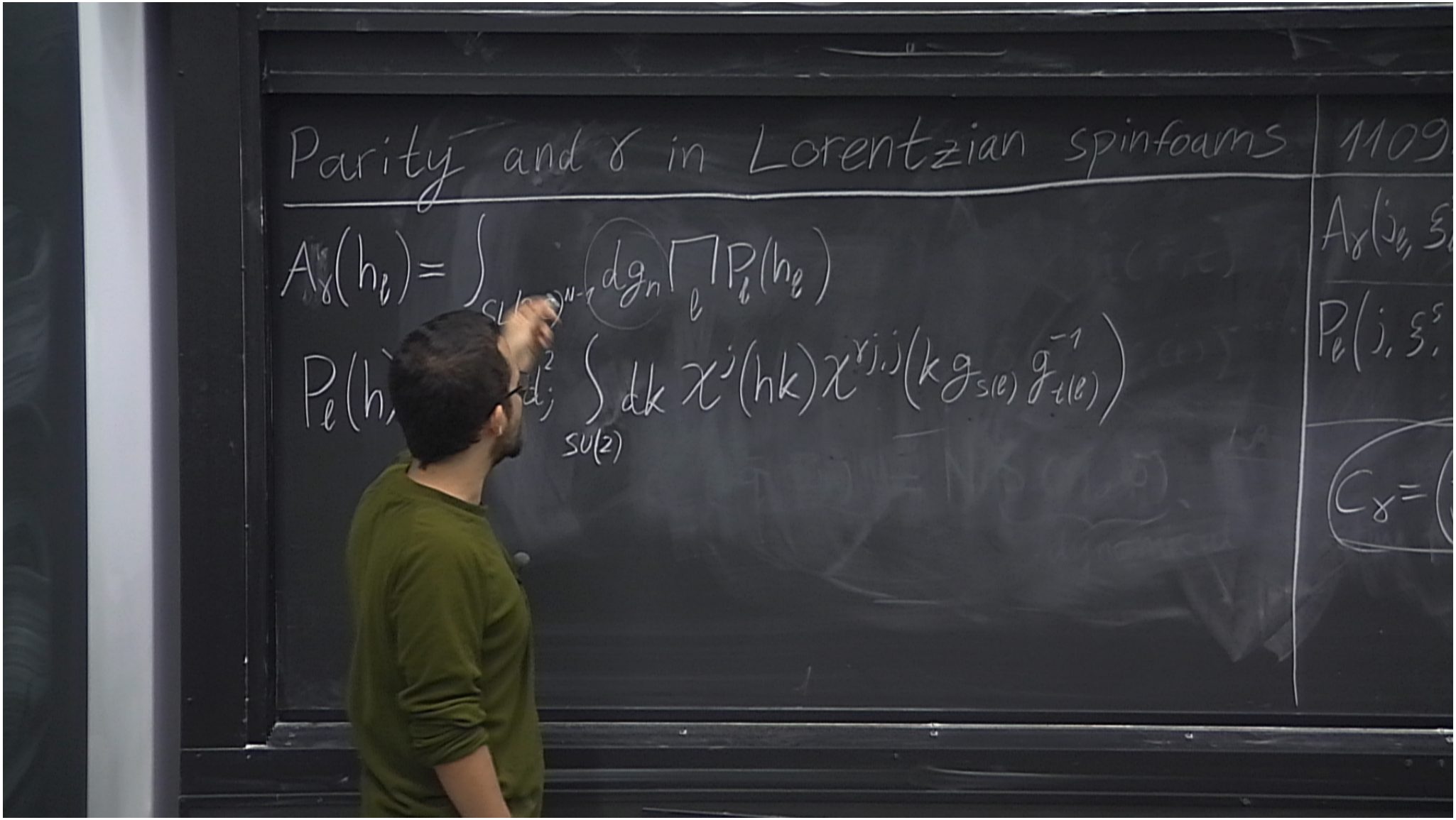

$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1}$
$0, \underline{1 \ldots 4}$
$G_L = G_{01}^{(23)(24)}$, $G_{01}^{(24)(23)} = G_R$
$(z, g$
$z \sim \gamma^4$

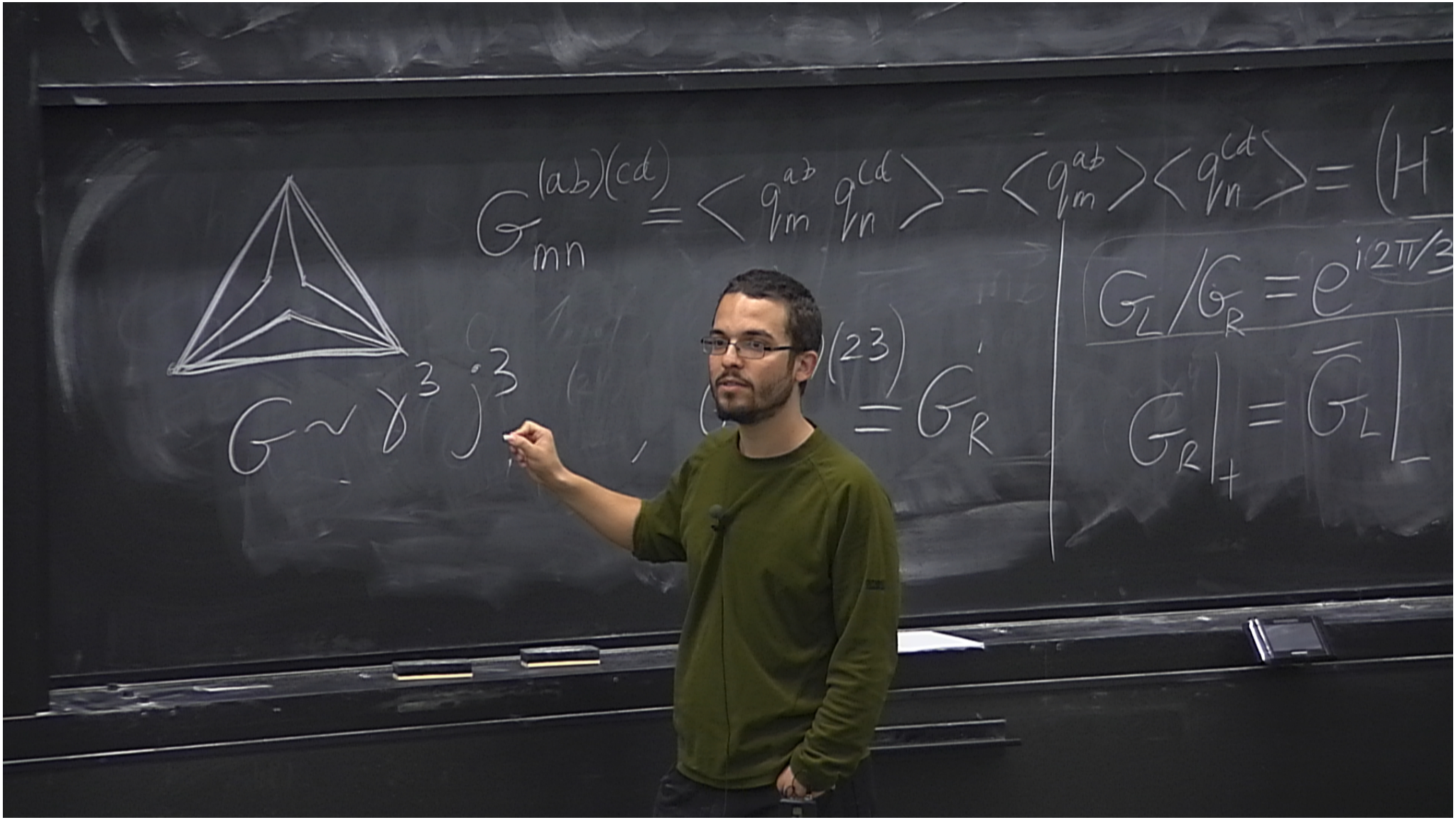

$\langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij} (q_m^{ab})_i (q_n^{cd})_j$
$G_L / G_R = e^{i 2\pi/3}$
$G_{01}^{(24)(23)} = G_R$

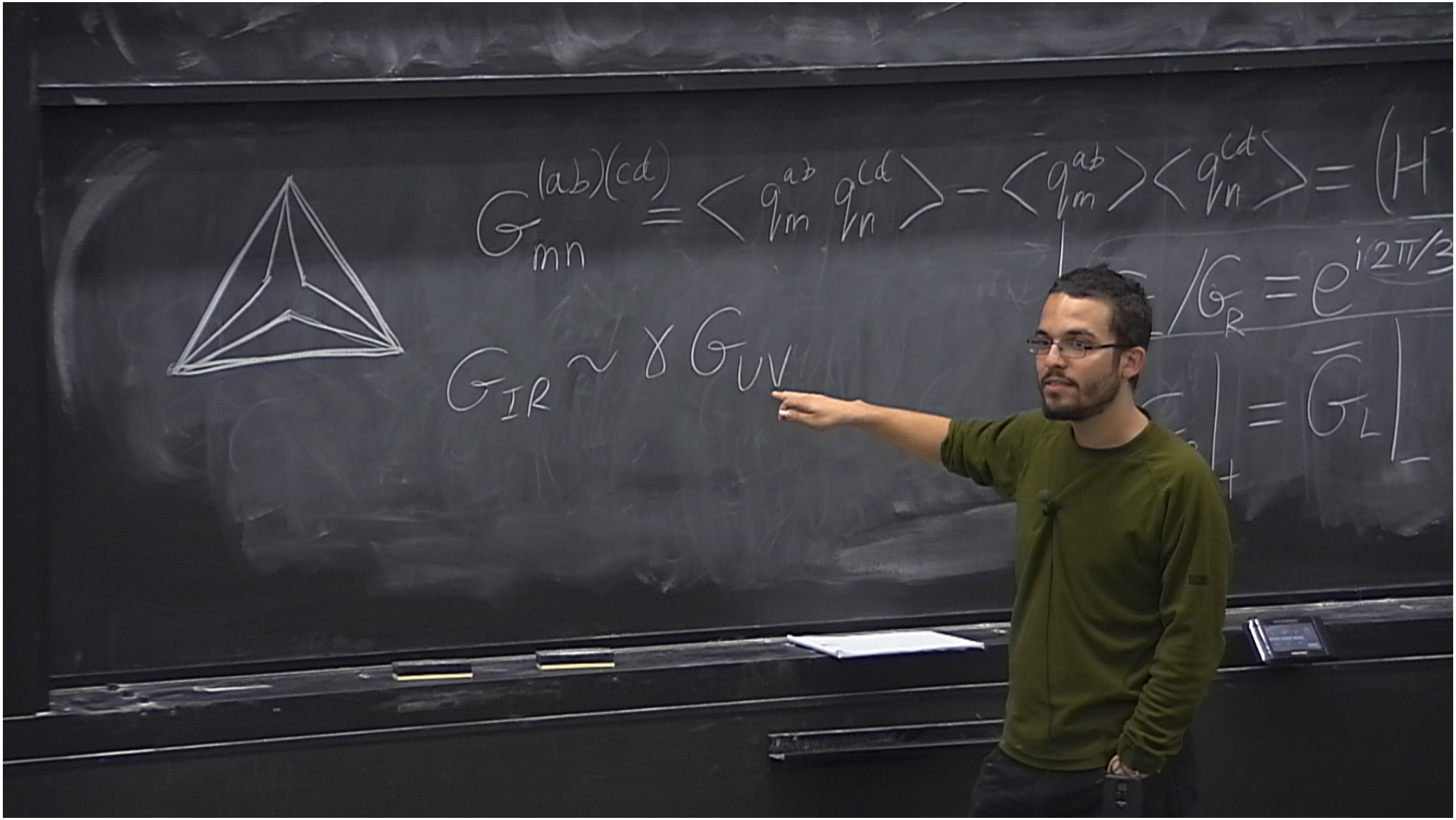

$\langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij} (q_m^{ab})_i (q_n^{cd})_j$
$G_{01}^{(24)(23)} = G_R$
$G_L / G_R = e^{i 2\pi/3}$
$G_R|_+ = \bar{G}_L|_-$

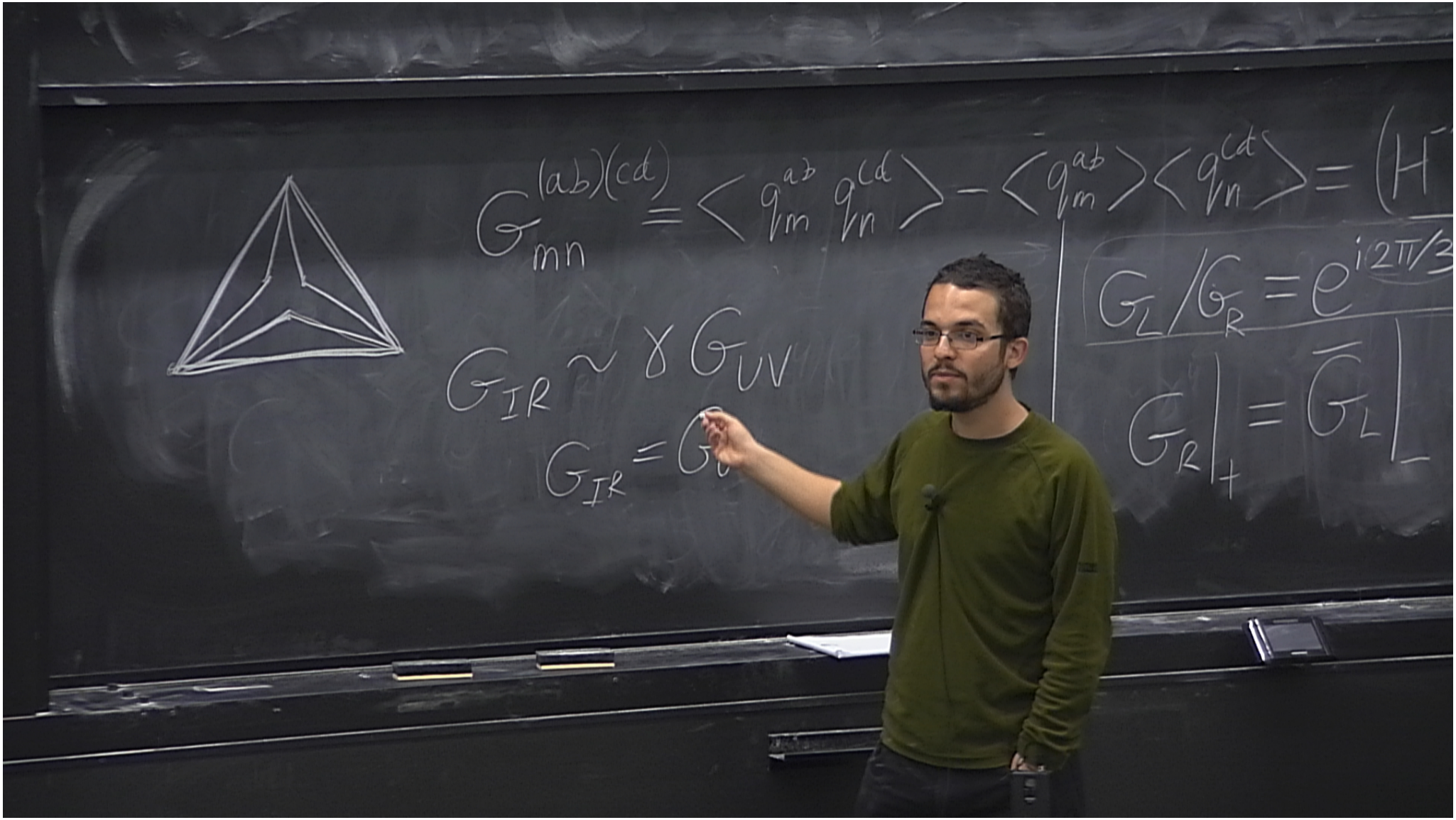

$= \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij} (q_m^{ab})_i (q_n^{cd})_j$
$G_{01}^{(24)(23)} = G_R$
$G_L / G_R = e^{i 2\pi/3}$
$G_R|_+ = \bar{G}_L|_-$

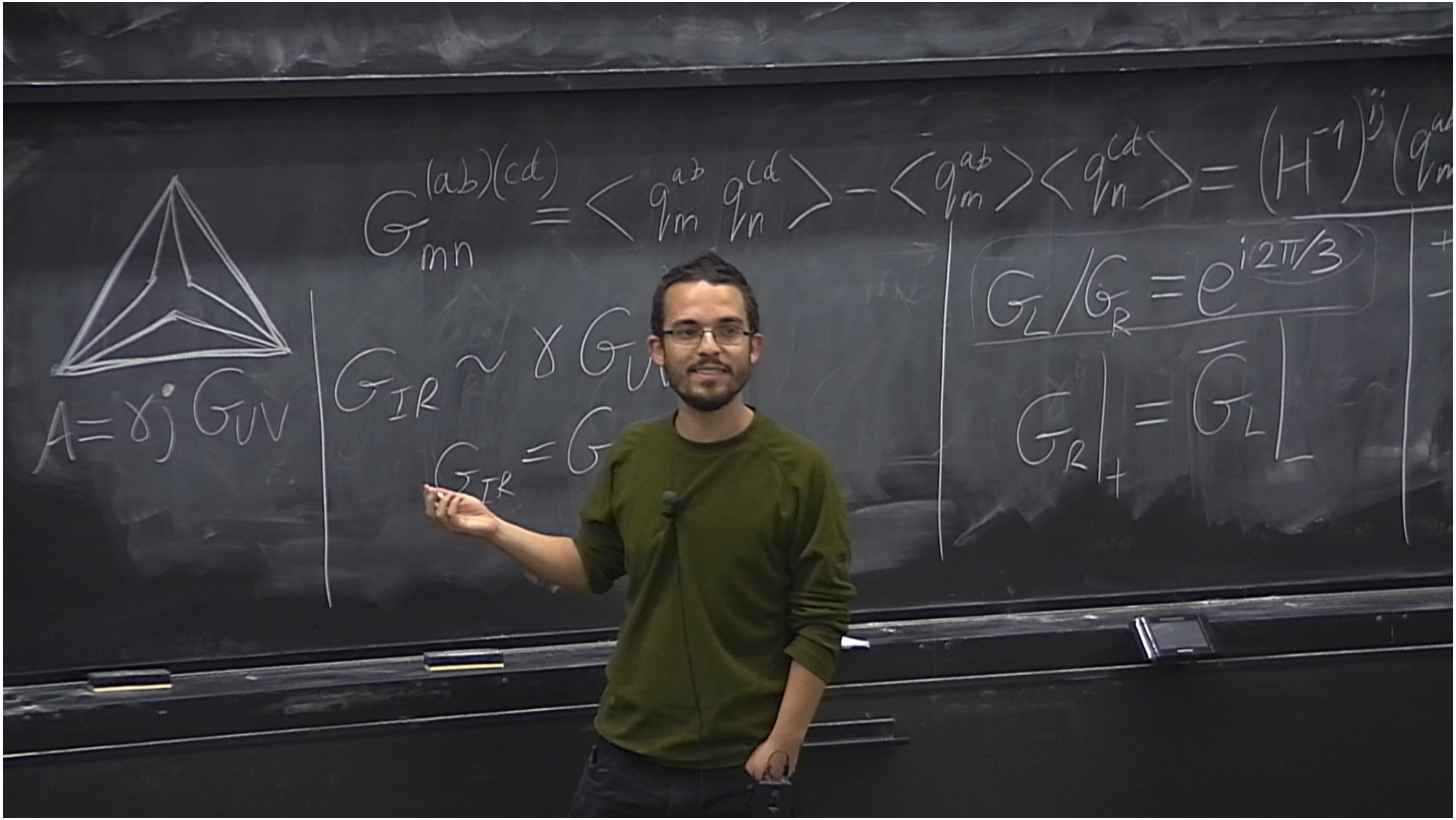

$= \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij} (q_m^{ab})_i (q_n^{cd})_j$
$G_{01}^{(24)(23)} = G_R$
$G_L / G_R = e^{i 2\pi/3}$
$G_R|_+ = \bar{G}_L|_-$
+X
−X
0
0
0

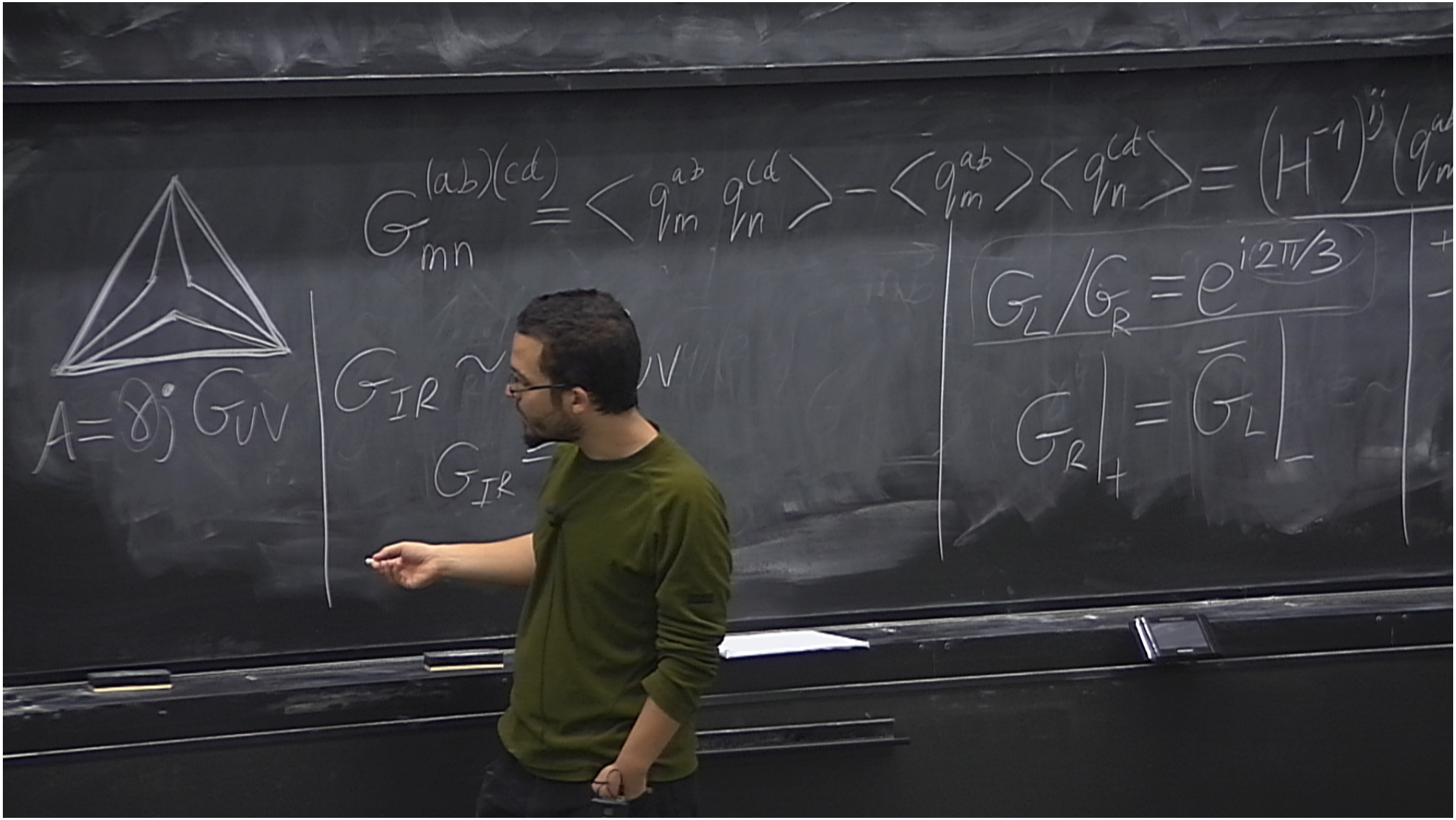

Parity and γ in Lorentzian spinfoams
1109
$A_\gamma(h_\ell) = \int_{SU(2)^{N-1}} dg_n \prod_\ell P_\ell(h_\ell)$
$P_\ell(h_\ell) = d_j^2 \int_{SU(2)} dk\, \chi^j(hk)\, \chi^{\gamma j, j}(k g_{s(\ell)} g_{t(\ell)}^{-1})$
$A_\gamma(j_\ell, \xi$
$P_\ell(j, \xi^$
$C_\gamma =$


$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-}$
$G_L / G_R = e^{i 2\pi/3}$
$G \sim \gamma^3 j^3$
$= G_R$
$G_R|_{+} = \bar{G}_L|_{-}$


$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-}$
$/G_R = e^{i2\pi/3}$
$= \bar{G}_L$
$G_{IR} \sim \gamma G_{UV}$


$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-}$
$G_{IR} \sim \gamma G_{UV}$
$G_{IR} = G_{U}$
$G_L / G_R = e^{i 2\pi/3}$
$G_R|_{+} = \bar{G}_L|_{-}$


$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij}$
$A = \gamma^j G_{UV}$
$G_{IR} \sim \gamma G_{UV}$
$G_{IR} = G$
$G_L / G_R = e^{i 2\pi/3}$
$G_R|_+ = \bar{G}_L|_-$


$G_{mn}^{(ab)(cd)} = \langle q_m^{ab} q_n^{cd} \rangle - \langle q_m^{ab} \rangle \langle q_n^{cd} \rangle = (H^{-1})^{ij}$
$A = \delta_{ij} G_{UV}$
$G_{IR} \sim$
$G_{IR} =$
$G_L / G_R = e^{i 2\pi/3}$
$G_R|_+ = \bar{G}_L|_-$