Title: Black Holes from Within Loop Quantum Gravity

Date: Nov 30, 2011 04:00 PM

URL: http://pirsa.org/11110097

Abstract: In general relativity, the fields on a black hole horizon are obtained from those in the bulk by pullback and restriction. In quantum gravity, it would be natural to obtain them in the same manner. This is not fully realized in the quantum theory of isolated horizons in loop quantum gravity, in which a Chern-Simons phase space on the horizon is quantized separately from the bulk. I will outline an approach in which the quantum horizon degrees of freedom are simply components of the quantized bulk degrees of freedom. A condition is imposed on the quantum states to encode the existence of a horizon. I will present evidence that solutions to this condition have properties on the horizon that are remarkably similar to those of Chern-Simons theory. Instrumental in formulating the horizon condition are novel flux operators that use the Duflo isomorphism and seem to represent some type of quantum deformed SU(2). I will review their definition and summarize what I know about their properties.

Black holes from within loop quantum gravity

Hanno Sahlmann (APCTP & Postech)

Perimeter Institute, 30.11.2011

Introduction

BH entropy in LQG due to Chern-Simons theory on the horizon.

long story: Smolin ('95), Rovelli , Krasnov ('96)
Ashtekar + Baez + Corichi + Krasnov ('98)
Kaul+Majumdar ('00)
Domagala + Lewandowski, Meissner ('04)
Engle + Noui + Perez ('10)

Dreyer + Markopoulou + Smolin ('04)
Corichi + Diaz-Polo +Fernandez-Borja ('06)
Bianchi ('10)
Barbero + Villasenor ('11)
Frodden + Ghosh +Perez ('11)

and many more.

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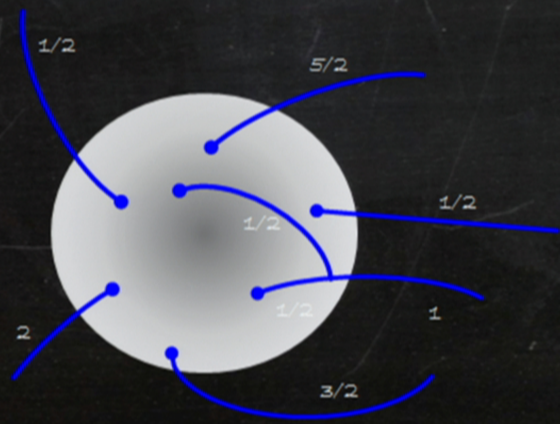
BH entropy in LQG due to Chern-Simons theory on the horizon.

- IH boundary condition $\underset{\Leftarrow}{F}(A) \propto *\underset{\Leftarrow}{E}$ adds CS boundary term to symplectic structure
- CS theory gets quantized separately, but fits beautifully to LQG bulk:



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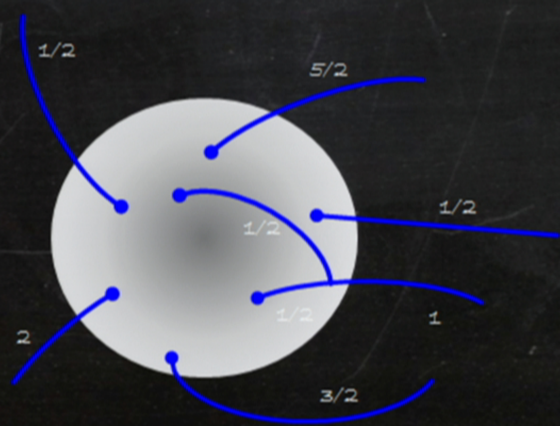

CS	LQG
singular curvature	singular flux
particle DOF	Flux components
$\mathcal{H}_{CS} \subset \mathrm{Inv}(j_1, j_2, \ldots)$	spin net with part hidden

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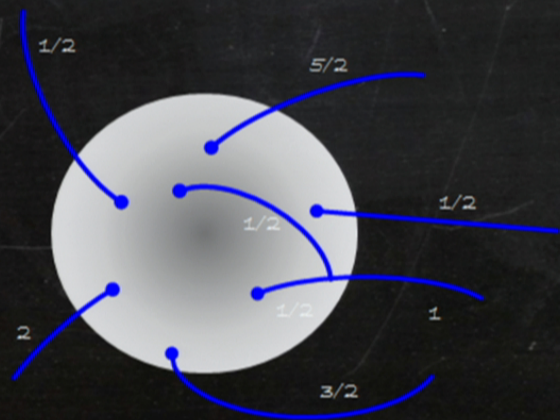

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How come?

Morally:

Same field in bulk and boundary, thus descriptions match!

Technically:

Not obvious from the way the system is treated.

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Aim of this talk:

Unified description of bulk and boundary from within LQG

- All degrees of freedom contained in holonomy-flux algebra
- Quantum boundary condition
- CS structures emerge

[H.S. Phys.Rev. D84 (2011) 044049]

LQG in a nutshell

variables: $\{A^I_a(x), E^b_J(y)\} = 8\pi\beta G \delta^b_a \delta^I_J \delta(x,y)$

$$W^{(j)}_\alpha := \mathrm{tr}_j(h_\alpha[A]) \equiv \mathrm{tr}_j\left(P \exp \oint_\alpha A\right)$$

$$E_{S,f} := \int_S f^I E^a_I \epsilon_{abc}\, dx^b dx^c$$

Holonomy flux algebra:

Relations among the $W^{(j)}_\alpha$

$$[E_{S,f}, \bigcirc] = 8\pi\beta G\, f(p)^I \;\bigcirc_I$$

$$[[E_{S,f}, E_{S',g}], \bigcirc] = (8\pi\beta l_P)^2 f(p)^I g(p)^J \;\bigcirc_{IJ}$$

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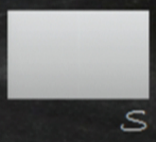

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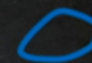

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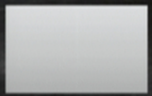

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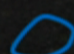
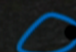
$[[E$  $_{,f}, E$  $_{,g}],$  $] = (8\pi\beta l_P)^2 f(p)^I g(p)^J$  $_{IJ}$

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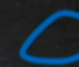

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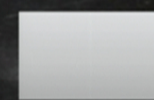

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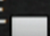
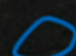
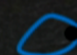
$[E$  $,f,$  $] = 8\pi\beta G\, f(p)^I$  $_I$

$[[E$  $,f, E$  $,g],$  $] = (8\pi\beta l_P)^2\, f(p)^I g(p)^J$  $_{IJ}$

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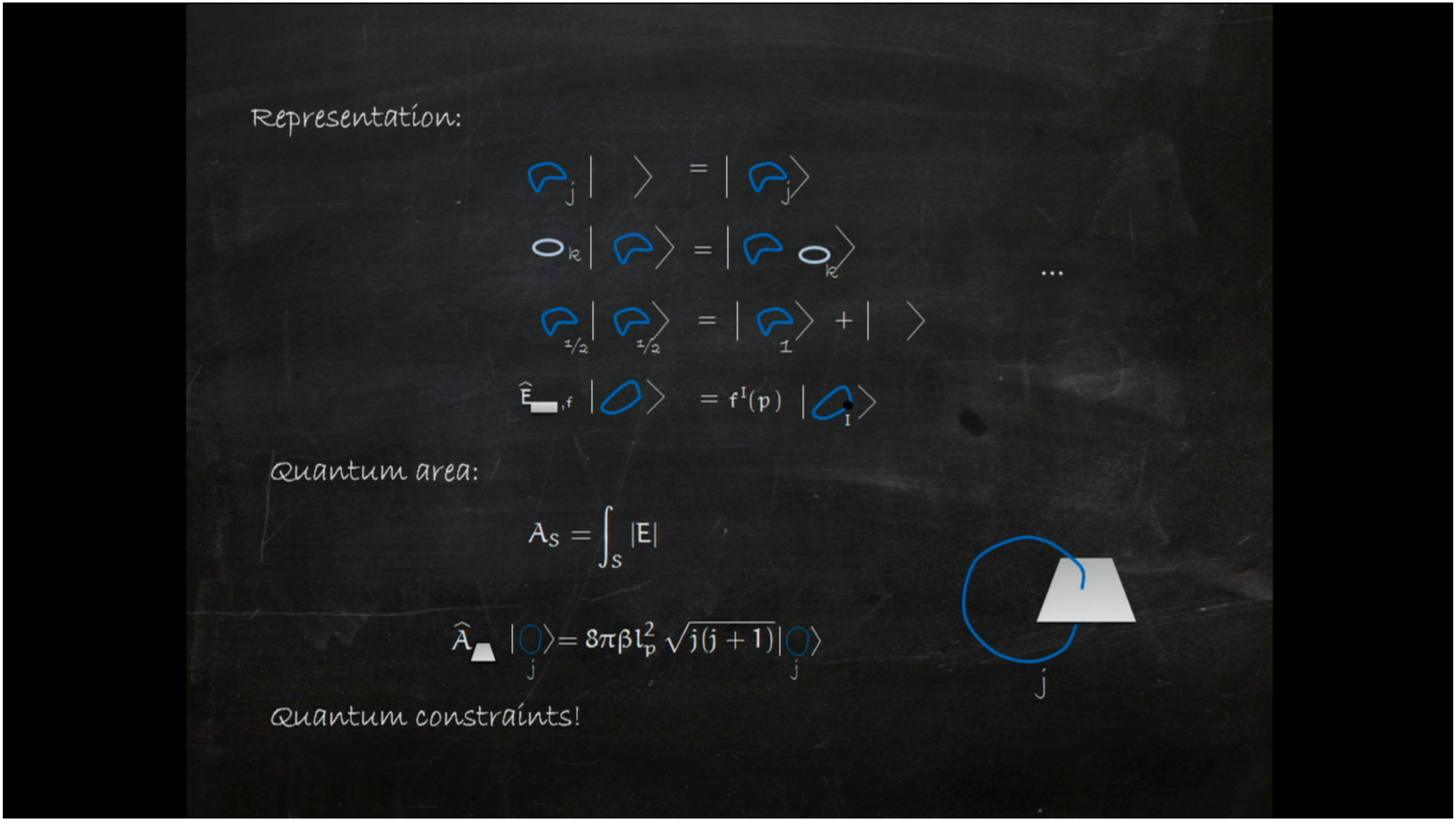
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Quantum area:
$A_S = \int_S |E|$
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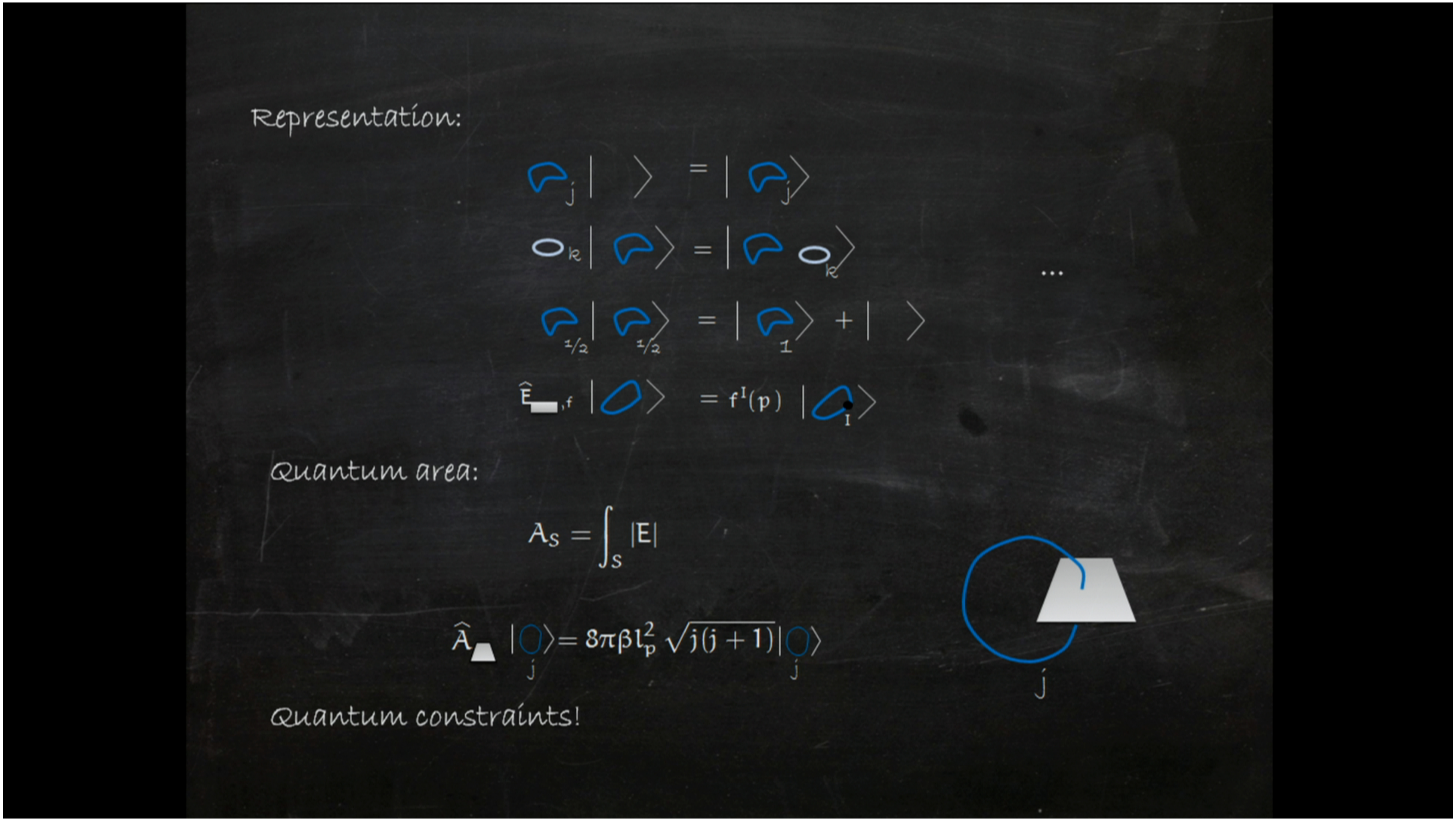
Quantum constraints!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Heuristics

From now on H connected boundaryless surface in 3-space, without self-intersections.

Components of fields on H:

Pullbacks:

- $\underset{\Leftarrow}{A}$: in W_α with α in H
- $\underset{\Leftarrow}{E}$: in transversal fluxes

Transversal components:

- $A^\perp$: in transversal holonomies
- $\underset{\Leftarrow}{*E}$: in $E_{S,f}$ with S in H

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Now ask for solutions of quantum boundary conditions (type I)

$$\underset{\Leftarrow}{\widehat{F}}^I(A)\Psi = \frac{(1-\beta^2)\pi}{a_H} \underset{\Leftarrow}{*\widehat{E}}^I \Psi$$

Can this work?

It can, but:

Since $\widehat{F}$ not well defined in LQG, exponentiate, using non-abelian Stokes' theorem:

$$h_{\partial S}[A] = \mathcal{P}\exp \oint_S \mathcal{F}[A]\, d^2 s \qquad\qquad W_S := \mathcal{P}\exp \oint_S \mathcal{E}[A,E]\, d^2 s$$

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$$\mathrm{tr}(h_{\partial S})\,\Psi = \mathrm{tr}(W_S)\,\Psi$$

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Also, will not be able to find solutions Ψ in standard representation of HF-algebra.

U(1) toy model

All structures as in LQG, only

SU(2) -----> U(1)

Quantum boundary condition:

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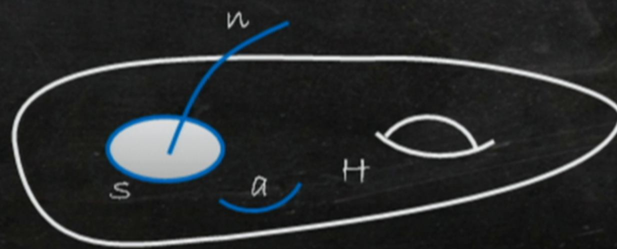
For example:

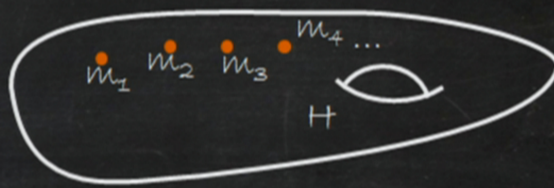
$$h_{\partial S}\Psi \overset{!}{=} e^{4\pi\beta\ell_P^2 c n}\psi$$

but h_a is completely free.



More generally: Specify H, puncture data $\mathcal{P}$:

$$\mathcal{P} = \{(p_1, m_1), (p_2, m_2), \ldots\}$$



Note:

1) Correspondence:

charge nets in H <---> 1-Chains in $H' = H - \{p_1, p_2, \ldots\}$

$$W_{\underline{n}} \equiv h_{e_1}^{n_1} h_{e_2}^{n_2} \ldots \longleftrightarrow \underline{n} = \{(n_1, e_1), \ldots\} \longleftrightarrow \underline{n} = \sum_i n_i e_i$$

Gauge invariance: $\partial \underline{n} = 0$

Evaluation: $W_{\underline{n}}[A] = \exp i \langle \underline{n} | A \rangle$

2) Decomposition of $\underline{n}$ in part $\underline{n}^H$ in H, part $\underline{n}^\dagger$ transversal to H

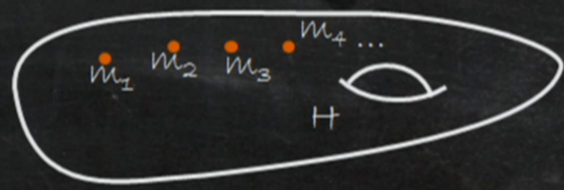
Using this, easy to write down a state on holonomies:

$$\omega(W_{\underline{n}}) = \omega_{\mathcal{P}}(W_{\underline{n}^{H}})\, \delta(\partial \underline{n}^{H})\, \omega_{AL}(W_{\underline{n}^{\perp}})$$

with

$$\omega_{\mathcal{P}}(W_{\underline{n}}) = \int_0^{2\pi} \ldots \int_0^{2\pi} e^{\langle \underline{n} | a(\phi) \rangle} \prod_{p_i} 2\pi\delta(\phi_i + 2\pi c n_i) \prod_j \frac{d\phi_j}{2\pi}$$

$$a(\phi) = \sum_i a_i \phi_i \text{ in } H^1(H', \mathbb{R})$$



Properties of the resulting representation?

All holonomies not contained in H are as in AL rep.

For charge nets in H:

- $$W_\alpha|0\rangle_{\mathcal{P}} = \begin{cases} 1 & \alpha = \partial S \\ \exp(-2\pi i/k\, n_i) & \alpha \text{ around } p_i \end{cases}, \qquad k = a_H/(2\pi\beta^2\ell_P^2)$$
- $|\underline{n}\rangle = |\underline{n}'\rangle$ if $n_i = n_i' \mod k, e_i = e_i'$
- $\Sigma_i m_i \overset{!}{=} 0 \mod k$
- for diffeo fixing punctures and $\partial\underline{n} : |\underline{n}\rangle = |\phi(\underline{n})\rangle$
- for diffeos that move punctures: path-dependent phases

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For the fluxes:

Not all of them well defined: E_S ok for

- $S \cap H = \emptyset$
- $S \cap H = S$
- $S \cap H$ is a cycle

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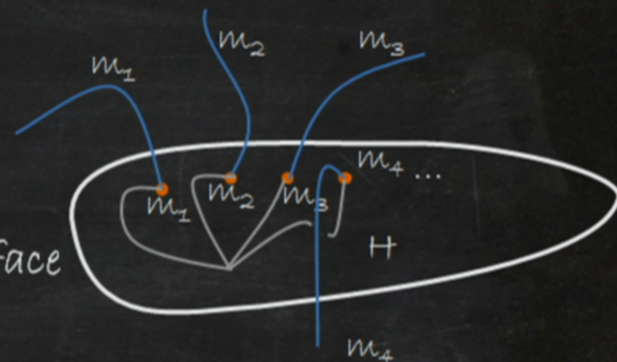
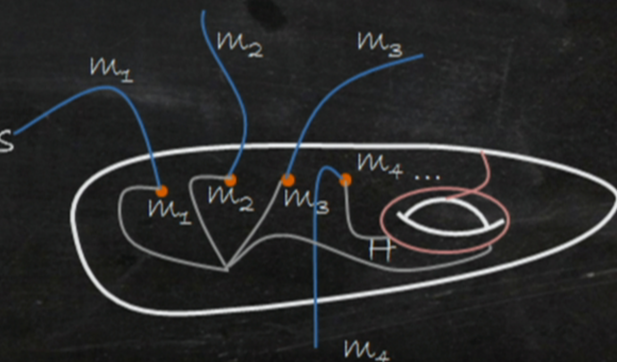
Gauge invariant solutions:



For S^2 topology:

- Gauge inv coupling at punctures
- for given bulk conf. just one surface state Note: H need not be boundary



For nontrivial topology:

- Gauge inv coupling at punctures
- for given bulk conf. multiple surface states Note: H need not be boundary

Recover all the structures of the U(1) treatment of IH (Ashtekar et al)

Theory on H has all features of quantum abelian CS.

Back to SU(2)

Key object:
$$\mathbf{W}_S := \mathcal{P}\exp \oint_S \mathcal{E}[A,E](s)\, d^2 s$$
$$= \mathbb{I}_2 + \int_S \mathcal{E}[A,E](s)$$
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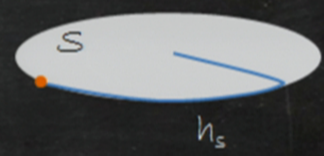

$$\mathcal{E}[A,E](s) = \frac{c}{4\pi\beta\ell_P^2} h_s * \widehat{E}(s) h_s^{-1}, \qquad *\widehat{E}(s) = *\widehat{E}^I T_I$$

Can we make it well defined? [HS+Thiemann]

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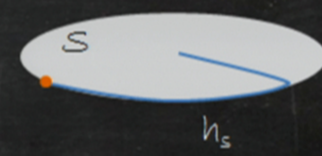

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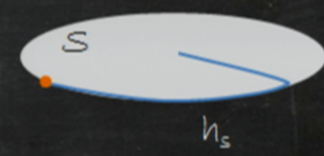

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Can we make it well defined? [HS+Thiemann]

First step: LQG E is operator (matrix) valued distribution, factorizes:

$$\widehat{E}^a_I(s) = \widehat{E}^a(s)\widehat{E}_I(s)$$

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Solution: Standard procedure gives factor 1/n!

2) Ordering problem: How to order the E_I ?
Solution: Harish-Chandra/Duflo isomorphism

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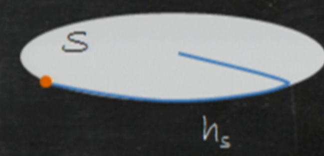
Back to SU(2)

Key object: $$\mathbf{W}_S := \mathcal{P}\exp\oint_S \mathcal{E}[A,E](s)\, d^2 s$$
$$= \mathbb{I}_2 + \int_S \mathcal{E}[A,E](s)$$
$$+ \int_{S^2} K_{s,s'}\mathcal{E}[A,E](s)\mathcal{E}[A,E](s')$$
$$+ \ldots$$



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Can we make it well defined? [HS+Thiemann]

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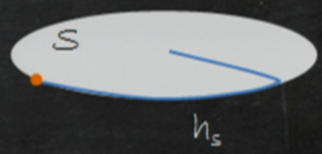

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Harish-Chandra/Duflo map

Given semisimple Lie algebra $\mathfrak{g}$.

Quantization map

$$\Upsilon : \mathrm{Sym}(\mathfrak{g}) \longrightarrow \mathrm{U}(\mathfrak{g})$$

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It's a refinement of symmetric quantization

$$\Upsilon = \chi \circ j^{\frac{1}{2}}(\partial)$$

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$$j^{\frac{1}{2}}(x) = \det{}^{\frac{1}{2}} \left(\frac{\sinh \frac{1}{2}\,\mathrm{ad}\,x}{\frac{1}{2}\,\mathrm{ad}\,x} \right) = 1 + \frac{1}{48} \|x\|^2 + \dots$$

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Example: For SU(2) $\quad \Upsilon(\|E\|^2) = \Delta_{SU(2)} + \frac{1}{8}\mathbb{I}$

This makes W_S well defined (albeit hard to determine explicitly)

General properties:

For suitably chosen path systems

$$W_{S_1+S_2} = W_{S_1} W_{S_2}, \qquad W_S^\dagger = W_{-S}$$

Under gauge transformations

$$U_g W_S U_g^{-1} = g(b) W_S g(b)^{-1}$$

Note:

These eigenvalues can be written in terms of quantum integers,

$$\lambda_{j,j'} = \frac{[(2j+1)(2j'+1)]_q}{[2j'+1]_q} \qquad [x]_q := \frac{q^x - q^{-x}}{q - q^{-1}} \qquad q = e^{ic}$$

They are related to

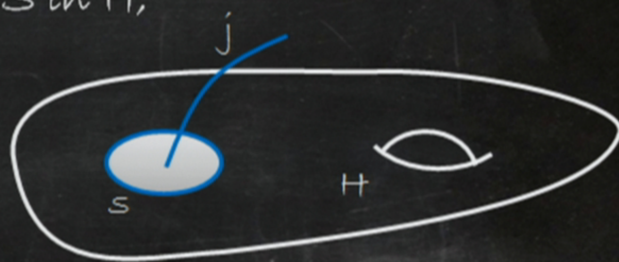
- Verlinde coefficients of $SU(2)_k$ rational CFT ($k=1/c$)
- Trace of the square of the R-matrix of $U_q(su(2))$ on $j \otimes j'$

And they are precisely what to expect for a holonomy around a particle in SU(2) CS [Witten]

Back to black holes

Quantum boundary condition: For any S in H,

$$\mathrm{tr}(h_{\partial S})\,\Psi = \mathrm{tr}(W_S)\,\Psi$$



If we had a solution Ψ, it would follow

- formula for simple loops: $W^{(j)}_{\alpha}\,\Psi = \begin{cases} \lambda_{jj_i}\Psi & \alpha \text{ around } p_i \\ \Psi & \alpha \text{ trivial} \end{cases}$
- inv under diffeos fixing punctures: $W_{\gamma}\,\Psi = W_{\phi(\gamma)}\,\Psi$
- reps on H only different mod k $\qquad k = \dfrac{a_H}{4\pi\beta(1-\beta^2)\ell_P^2}$
- nontrivial monodromy of punctures
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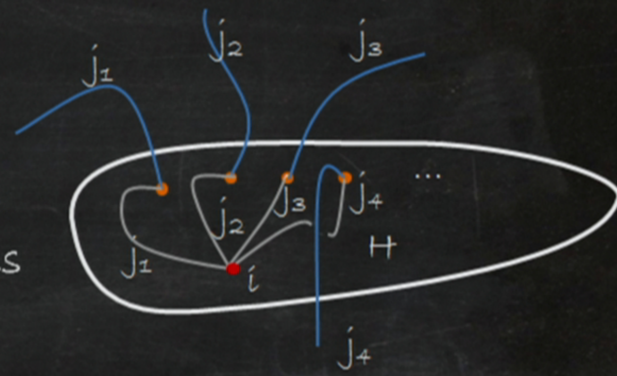
Gauge invariant solutions:

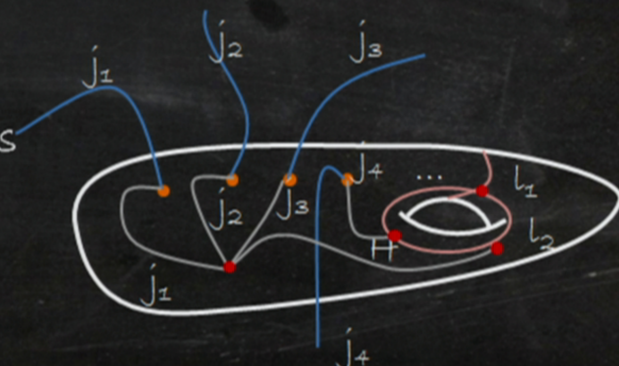
For S^2 topology:

- Gauge inv coupling at punctures
- for given bulk conf. surface states described by $i \in \mathrm{Inv}(j_1, j_2, j_3, \dots)$



For nontrivial topology:

- Gauge inv coupling at punctures
- for given bulk conf. ∞ many surface states



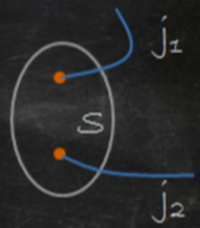
In particular: would recover all the structures of the SU(2) treatment of IH (Engle et al). Theory on H has all features of SU(2) CS.

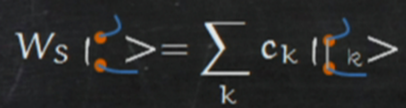
But: Do we have a representation?

Have a good representation for the bulk and simple loops in H.

Extension to non- simple loops? Difficult to answer due to

1) Action of W_S Complicated:

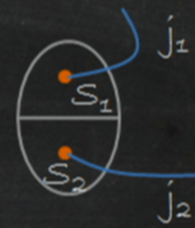




2) There are ∞ many Mandelstam identities to be satisfied, ex.

$$(\mathrm{tr}\,g)(\mathrm{tr}\,g') = \mathrm{tr}(gg')\,\mathrm{tr}(g(g')^{-1})$$

Have checked some things, for example



$$\mathrm{tr}_{\frac{1}{2}}\left(W_{S_1}\right)\mathrm{tr}_{\frac{1}{2}}\left(W_{S_2}\right)|j_1 j_2\rangle = \mathrm{tr}_{\frac{1}{2}}\left(W_{S_1+S_2}\right)\mathrm{tr}_{\frac{1}{2}}\left(W_{S_1}W_{-S_2}\right)|j_1 j_2\rangle$$

as it must be, but don't yet have all the necessary results.

Further question: Do we have SU(2) CS?

- DOF remaining on horizon may point to ISU(2)
- would be nice: 3d Euclidean quantum gravity on horizon

Summary

Quantum description of Type I IH fully in terms of holonomy-flux algebra of LQG, modulo some loose ends.

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Quantum description of Type I IH fully in terms of holonomy-flux algebra of LQG, modulo some loose ends.

In toy model:

- emergence of abelian CS on the horizon
- for S^2 topology fully equivalent to (Ashtekar et al)
- for nontrivial topology: 2 DOF per nontrivial cycle

In full theory:

- new flux operators W_S with quantum group structures in spec
- emergence of non-abelian CS on horizon
- for S^2 topology seems fully equivalent to [Engle et al]
- for nontrivial topology: 2 DOF per nontrivial cycle -> ISU(2)?

Outlook

A number of things to clarify, and to think about:

1) Do we have a rep in the SU(2) case?
2) Nature of horizon theory In SU(2) case: SU(2) vs. ISU(2)
3) Entropy counting when H is not a boundary
4) Non-spherically symmetric case
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Thanks for your attention!

Jones polynomial from W_S

[HS+Thiemann]

using

$$-\frac{8\pi i}{k}\epsilon_{abc}\frac{\delta}{\delta A_c}e^{iS_{CS}[A]} = F_{ab}e^{iS_{CS}[A]}$$

can replace holonomies under the CS path integral by W_S, obtaining relations among expectation values:

$$\begin{aligned}\langle L\,\mathrm{tr}_\rho(h_{\partial_S})\rangle &= \int_{\overline{\mathcal{A}}} L[A]\,\mathrm{tr}_\rho(h_{\partial_S})[A]e^{iS_{CS}[A]}\,d\mu[A]\\ &= \int_{\overline{\mathcal{A}}} L[A]\,\mathrm{tr}_\rho(W_S)e^{iS_{CS}[A]}\,d\mu[A]\\ &= \int_{\overline{\mathcal{A}}} (\mathrm{tr}_\rho(W_{-S})L)[A]e^{iS_{CS}[A]}\,d\mu[A]\\ &= \langle(\mathrm{tr}_\rho(W_{-S})L)\rangle.\end{aligned}$$

Enough to define linear functional in cases.