

Title: The Radiation Sector of NRGR

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Abstract: This talk will review the description of gravitational radiation in the effective field theory framework NRGR and report some recent results obtained in the radiation sector. In the matching to the radiation theory one needs to perform a multipole expansion which we present to all order. Furthermore, we will show how non-linear radiative corrections (such as tail effects) are handled in the EFT, how different kinds of divergences arise and how the renormalization group can be used to resum logarithmic terms in the PN expansion of the energy flux. Finally, we present results for the spin components of the multipole moments which are sufficient to compute the phase to 3PN including all spin effects and the waveform to 2.5PN.



The radiation sector of NRGR

Andreas Ross
Carnegie Mellon University

November 29, 2011

Effective Field Theory and Gravitational Physics
Workshop at Perimeter Institute

W. Goldberger, I. Rothstein hep-th/0409156

W. Goldberger, AR arXiv:0912.4254 [gr-qc]

R. Porto, AR, I. Rothstein arXiv:1007.1312 [gr-qc]

Outline

1. EFT setup & radiation sector

scales in binary inspiral, potential and radiation gravitons, integrating out potentials, multipole expansion, observables

2. Radiative corrections

tail effects, divergences, renormalization, RG running and resummation

3. Multipole expansion & matching

all orders multipole expansion, matching to 1PN spinless

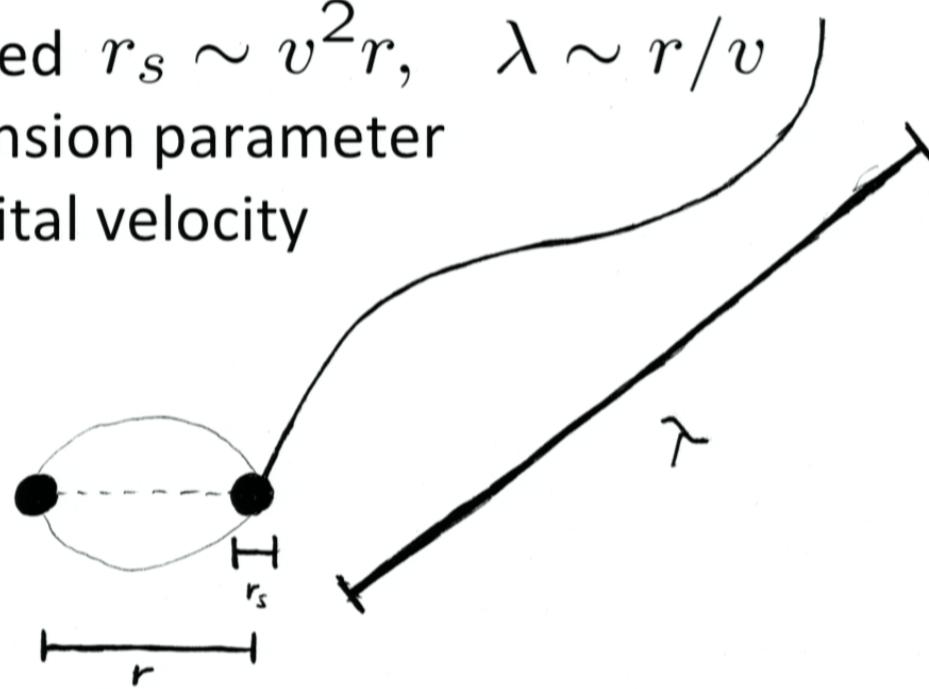
4. Spin effects

multipole moments for phase to 3PN and amplitude to 2.5PN

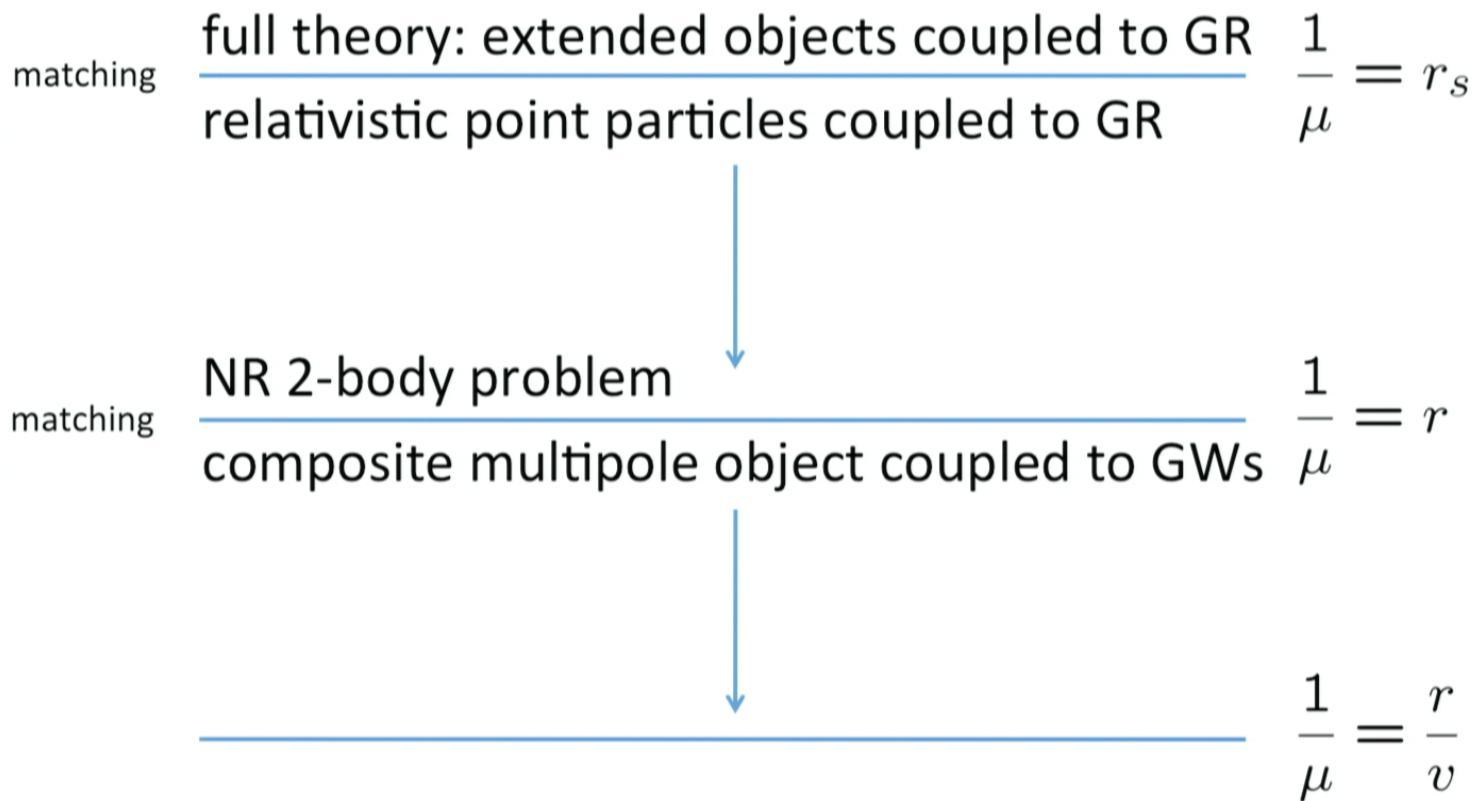
5. Conclusions

EFT Setup

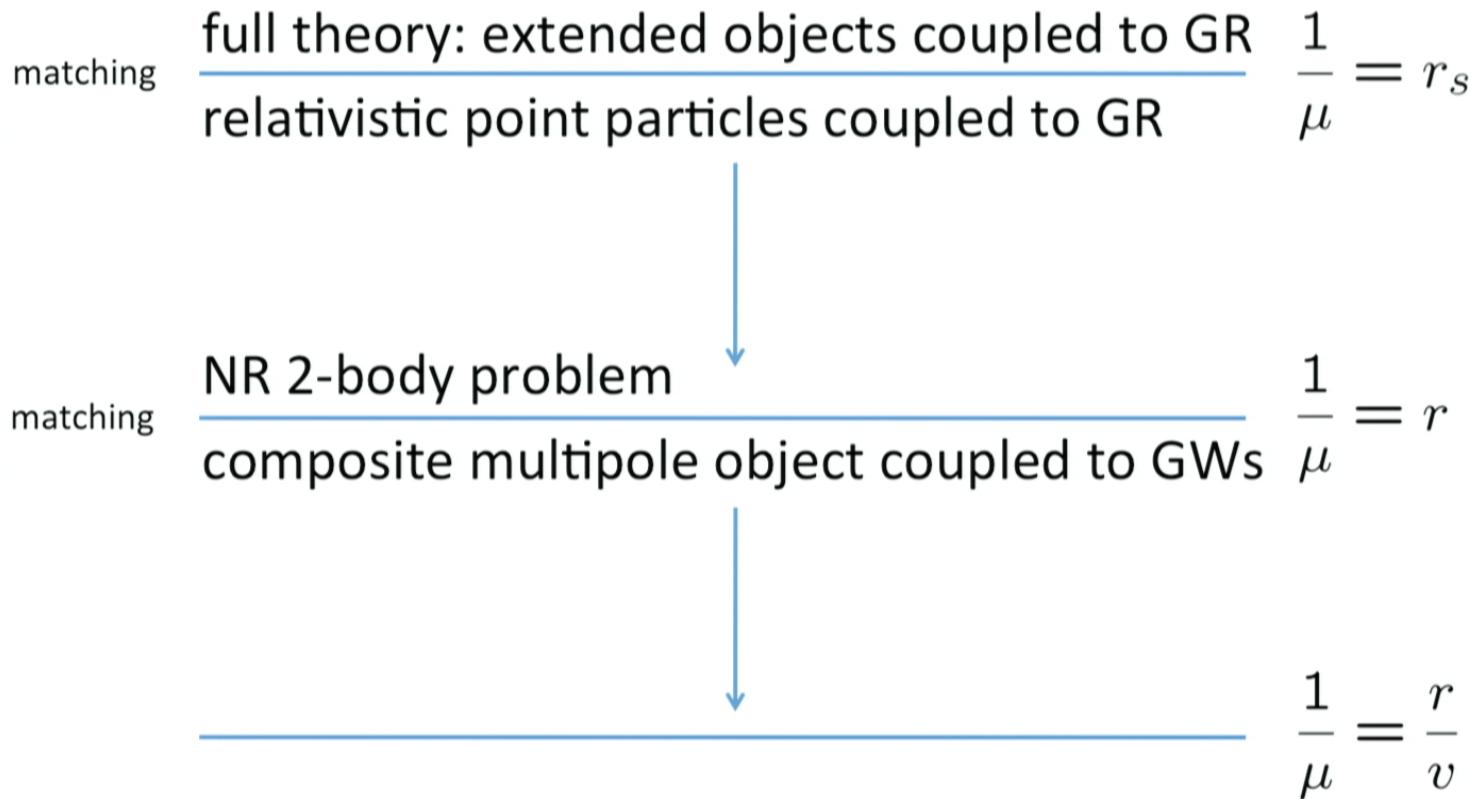
- build effective theories based on hierarchy of scales in binary $r_s \ll r \ll \lambda$
- scales correlated $r_s \sim v^2 r$, $\lambda \sim r/v$ by single expansion parameter v which is orbital velocity



EFT Setup



EFT Setup



EFT Setup

- action: point particle coupled to gravity

$$S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{-g} R$$

$$S_{GF} = m_{Pl}^2 \int d^4x \sqrt{-g} \Gamma^\mu \Gamma^\nu g_{\mu\nu} \quad \text{harmonic gauge}$$

$$S_{pp} = - \sum_{N=1}^2 m_N \int d\tau_N + \dots$$

$$d\tau_N = \sqrt{g_{\mu\nu}(x_N) dx_N^\mu dx_N^\nu}$$

finite size effects

EFT Setup

Potential modes $H_{\mu\nu}$

- yield binding dynamics of binary
- 4-momenta $p^\mu \sim (v/r, 1/r)$
- cannot be on-shell, so integrate out

Radiation modes $\bar{h}_{\mu\nu}$

- GWs which propagate out to detector, on-shell
- 4-momenta $k^\mu \sim (v/r, v/r)$
- treat as background field

$$\rightarrow g_{\mu\nu} = \bar{g}_{\mu\nu} + H_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} + H_{\mu\nu}$$

EFT Setup

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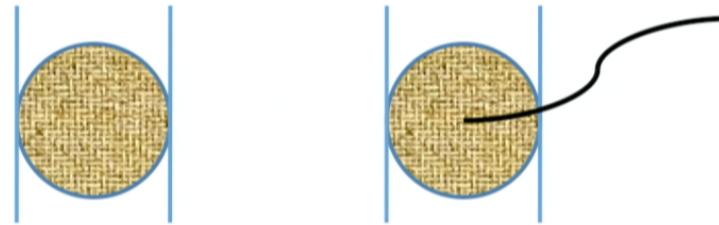


EFT Setup

- for complete NRGR power counting rules, see Goldberger & Rothstein, hep-th/0409156
- integrate out potential modes

$$e^{iS_{eff}[x_N, \bar{h}]} = \int \mathcal{D}H_{\mu\nu} e^{iS[x_N, \bar{h}+H]}$$

$$iS_{eff}[x_N, \bar{h}] = iS_0[x_N] + iS_1[x_N, \bar{h}] + \dots$$



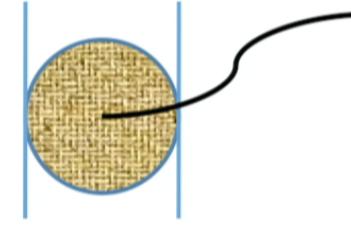
EFT Setup

$$iS_{eff}[x_N, \bar{h}] = iS_0[x_N] + \textcircled{i} S_1[x_N, \bar{h}] + \dots$$

$$S_1[x_N, \bar{h}] = -\frac{1}{2m_{pl}} \int d^4x T^{\mu\nu}(x) \bar{h}_{\mu\nu}(x)$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

- power counting forces us to Taylor expand $\bar{h}_{\mu\nu}$ in action S_1 around a single point
- yields a single worldline EFT with action S_1 in form of a multipole expansion
- form fixed by gauge and reparam. invariance



Radiation Sector

$$\begin{aligned} S = & -m \int d\tau - \frac{1}{2} \int dx^\mu L_{ab} \omega_\mu^{ab}(\tau) \\ & + \frac{1}{2} \sum_{n=0}^{\infty} \int d\tau c_n^{(I)} I^{aba_1 \dots a_n}(\tau) \nabla_{a_1} \dots \nabla_{a_n} E_{ab}(x) \\ & + \frac{1}{2} \sum_{n=0}^{\infty} \int d\tau c_n^{(J)} J^{aba_1 \dots a_n}(\tau) \nabla_{a_1} \dots \nabla_{a_n} B_{ab}(x) \end{aligned}$$

- describe an arbitrary source of gravitational radiation in the long wavelength limit
- single worldline EFT endowed with multipoles

Radiation Sector

- two distinct expansions:
 1. multipole expansion in $a/\lambda \ll 1$
 2. post-Minkowskian exp. in $\eta = Gm/\lambda \ll 1$
- in PN regime $a/\lambda \sim v$ and $\eta \sim v^3$
- multipole moments are Wilson coefficients which encode short distance physics
- given a description of the short distance physics we can perform a matching calculation to determine these Wilson coefficients

Radiation Sector

Calculating observables – instantaneous fluxes

- start from single graviton emission amplitude

$$i\mathcal{A}_h(\mathbf{k}) = \text{---} \circlearrowleft$$

- graviton emission rate $d\Gamma_h(\mathbf{k}) = \frac{1}{T} \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} |\mathcal{A}_h(\mathbf{k})|^2$
- 4-momentum flux $\dot{P}^\mu \Big|_{h=\pm 2} = \int k^\mu d\Gamma_h(\mathbf{k})$
- for total flux sum over all helicities

Radiation Sector

Calculating observables - waveform

- need to use retarded propagators!
- linear order trivial, hit source multipole moment with retarded propagator
- non-linear terms in waveform is work in progress

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Radiation Sector

- at order η^0 the amplitudes are trivial and give rise to standard results

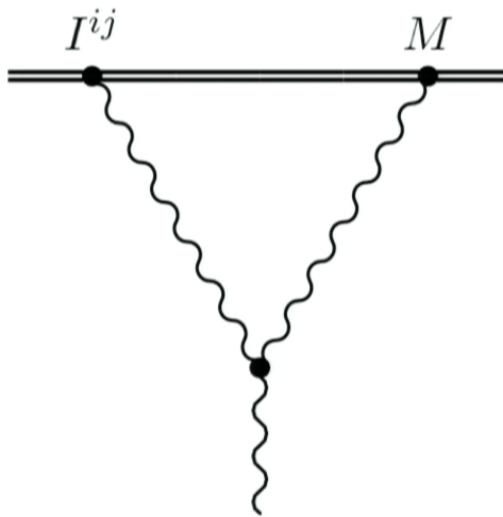
$$i\mathcal{A}_h(\mathbf{k}) = \overbrace{\text{---}}^{I^{ij}} + \overbrace{\text{---}}^{J^{ij}} + \overbrace{\text{---}}^{I^{ijk}} + \dots$$
$$= \frac{i}{4m_{Pl}} \epsilon_{ij}^*(\mathbf{k}, h) \left[\mathbf{k}^2 I^{ij}(k) + \frac{4}{3} |\mathbf{k}| \mathbf{k}^l \epsilon^{ikl} J^{jk}(k) - \frac{i}{3} \mathbf{k}^2 \mathbf{k}^l I^{ijl}(k) + \dots \right]$$

$$\dot{P}^0 = \frac{G_N}{\pi T} \int_0^\infty dk \left[\frac{k^6}{5} |I^{ij}(k)|^2 + \frac{16}{45} k^6 |J^{ij}(k)|^2 + \frac{k^8}{189} |I^{ijk}(k)|^2 + \dots \right]$$

Radiative Corrections

Leading Tail Effect at order η

- simplest case: quadrupole + monopole

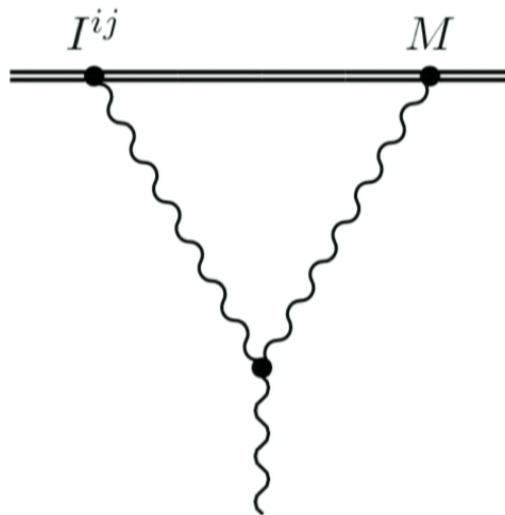


- effect of $1/r$ potential on GW propagation

Radiative Corrections

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Radiation Sector

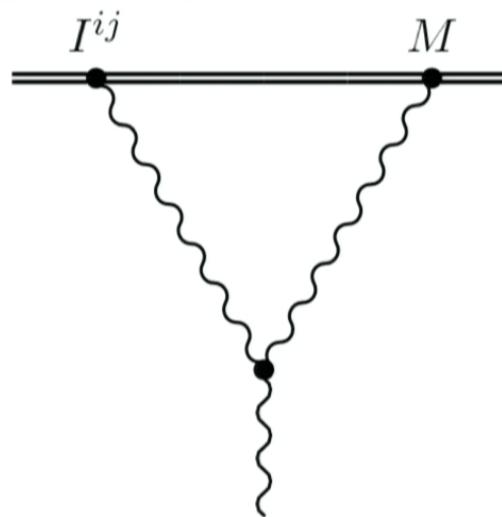
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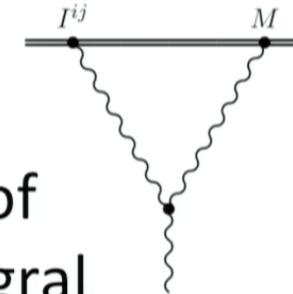
Leading Tail Effect at order η

- amplitude includes an integration of the form of a 1-loop Feynman integral
- integral part linear combination of integrals

$$\left(\frac{1}{\mathbf{k}^2}\right)^n \int \frac{d^{d-1}\mathbf{q}}{(2\pi)^{d-1}} \left(\frac{1}{\mathbf{q}^2}\right)^{1-n} \frac{1}{\mathbf{k}^2 - (\mathbf{k} + \mathbf{q})^2 + i\epsilon}$$

- For $n=0$, long distance behavior ($\mathbf{q} \rightarrow 0$) is

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{\mathbf{q}^2} \frac{1}{\mathbf{k} \cdot \mathbf{q}} \rightarrow \text{logarithmic IR divergence}$$







Radiative Corrections

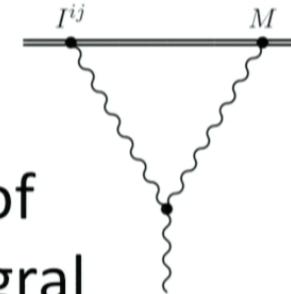
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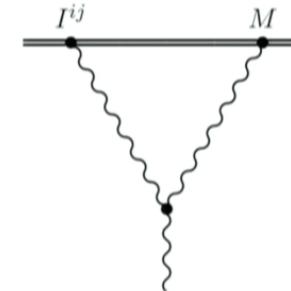
Radiative Corrections

Leading Tail Effect at order η

- amplitude reads

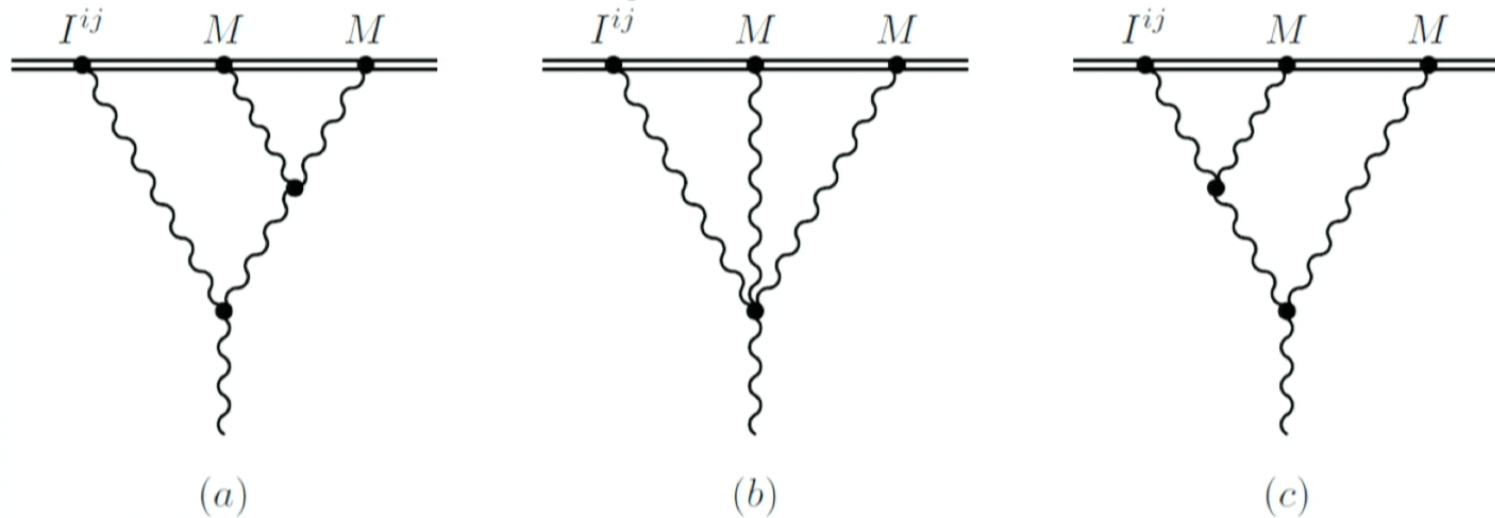
$$\begin{aligned} i\mathcal{A}_{\eta^1} &= i\mathcal{A}_{\eta^0} \times iGM|\mathbf{k}| \frac{\Gamma[(1-d)/2]}{(4\pi)^{(d-3)/2}} \frac{d^4 - 8d^3 + 23d^2 - 28d + 24}{d^3 - 6d^2 + 8d} \left(-\frac{\mathbf{k}^2 + i\epsilon}{\mu^2}\right)^{(d-4)/2} \\ &= i\mathcal{A}_{\eta^0} \times iGM|\mathbf{k}| \left[\frac{2}{d-4} + \log \frac{-\mathbf{k}^2 - i\epsilon}{\pi\mu^2} + \gamma_E - \frac{11}{6} + \mathcal{O}(d-4) \right] \end{aligned}$$

- IR divergence from long-ranged $1/r$ potential
- leading tail effect enters observables from
$$\left| \frac{\mathcal{A}}{\mathcal{A}_{\eta^0}} \right|^2 = 1 + 2\text{Re} \frac{\mathcal{A}_{\eta^1}}{\mathcal{A}_{\eta^0}} + \mathcal{O}(\eta^2) = 1 + 2\pi G_N m |\mathbf{k}| + \mathcal{O}(\eta^2)$$
- universal factor, independent of rad. multipole



Radiative Corrections

Tail-of-Tail & Tail-Squared Effects at order η^2



- calculations challenging, with integrals corresponding to 2-loop Feynman integrals
- IR and UV divergences

Radiative Corrections

Treating the Divergences

- IR divergences cancel in any observable, and to all orders they exponentiate to a phase

$$\frac{|\mathcal{A}|^2}{|\mathcal{A}_{\eta^0}|^2} = 1 + 2\pi GM|\mathbf{k}| + (GM|\mathbf{k}|)^2 \left[-\frac{214}{105} \left(\frac{1}{\epsilon_{UV}} + \log \frac{\mathbf{k}^2}{\pi\mu^2} + \gamma_E \right) + \frac{4}{3}\pi^2 + \frac{634913}{44100} \right]$$

- UV divergence and arbitrary scale μ **must** be canceled by renormalization of quadrupole

moment $i\mathcal{A}_{\eta^0} = \frac{i}{4m_{Pl}} \epsilon_{ij}^*(\mathbf{k}, h) \mathbf{k}^2 I^{ij}(k)$

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Radiative Corrections

Renormalization and the RG

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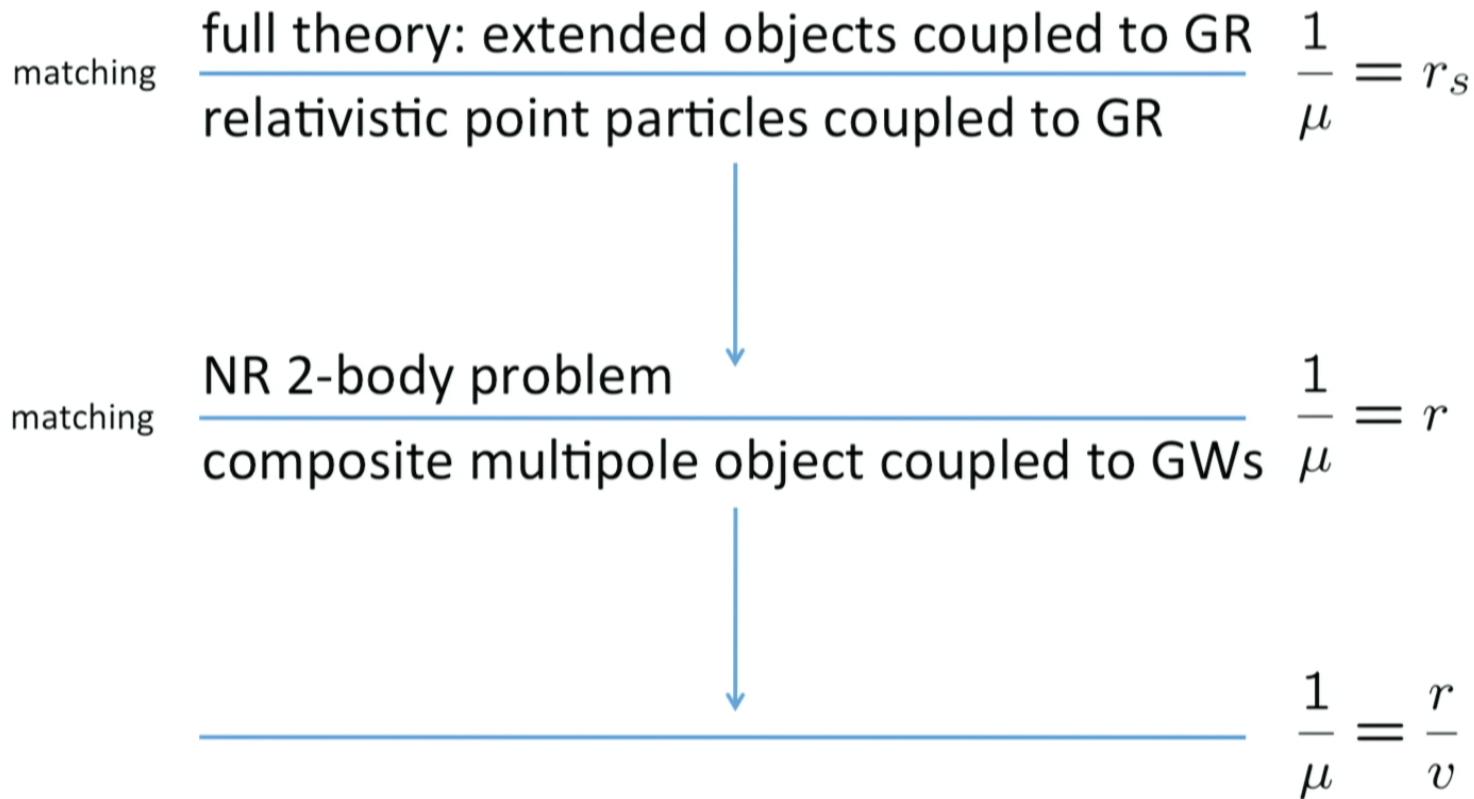
- from requirement of μ independence of physics

$$\mu \frac{d}{d\mu} I^{ij}(|\mathbf{k}|, \mu) = -\frac{214}{105} (GM|\mathbf{k}|)^2 I^{ij}(|\mathbf{k}|, \mu)$$

$$I^{ij}(|\mathbf{k}|, \mu) = \exp \left[-\frac{214}{105} (GM|\mathbf{k}|)^2 \log \frac{\mu}{\mu_0} \right] I^{ij}(|\mathbf{k}|, \mu_0)$$

- typically choose $\mu = |\mathbf{k}|$ and $\mu_0 \sim 1/a$

Radiative Corrections



Radiative Corrections

Renormalization and the RG

- EFTs are useful tool to resum large logs via RG, where large logs means a correction of the form

$$(1 + \alpha \log \mu/\mu_0) \sim (1 + \mathcal{O}(1))$$

- many examples where this is essential, e.g. QCD corrections to weak decays
- unfortunately in gravitational wave physics the logarithms cannot become large

$$(1 + \eta^2 \log a/\lambda) \quad \text{where} \quad \eta \sim r_s/\lambda$$

Radiative Corrections

Renormalization and the RG

- nevertheless RG yields interesting information about the dynamics since it constrains the pattern of logs in amplitude squared

$$\left| \frac{\mathcal{A}(\omega)}{\mathcal{A}_{\eta^0}(\omega, \mu_0)} \right|^2_{\text{leading log}} = 1 - \frac{428}{105} (G_N m \omega)^2 \ln \frac{\omega}{\mu_0} + \frac{91592}{11025} (G_N m \omega)^4 \ln^2 \frac{\omega}{\mu_0} - \frac{39201376}{3472875} (G_N m \omega)^6 \ln^3 \frac{\omega}{\mu_0} + \dots .$$

- independent of short distance physics
- in PN case, predicts $\log^n(n)$ term at $(3n)\text{PN}$

Radiative Corrections

Renormalization and the RG

- beyond leading log, new logarithmic UV divergences will appear at every even order in η
- since running is a UV effect, it is dependent on which multipole radiated gravitational wave, and we only calculated running of quadrupole
- RG resummation could be incorporated into resummed waveforms

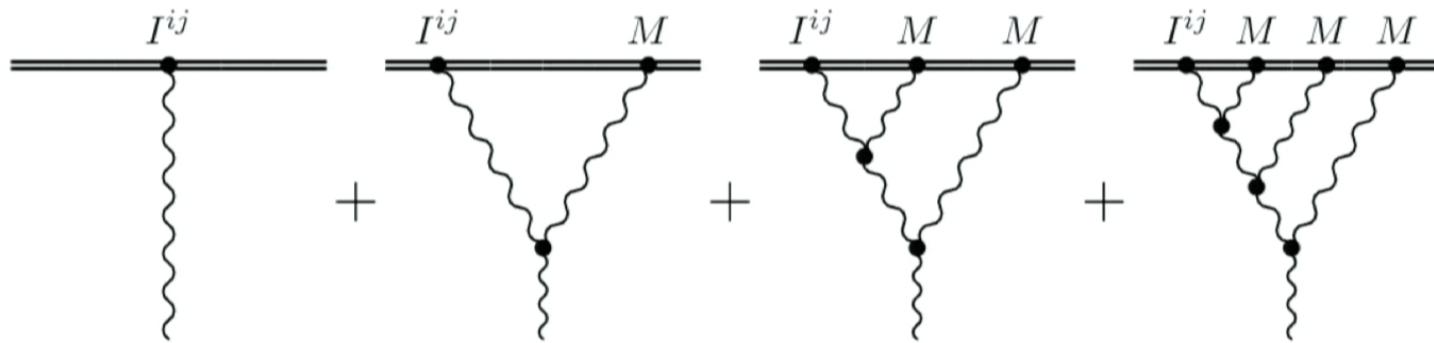
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Radiative Corrections

Resumming the leading IR tail effect



- summing ladder diagrams corresponds to solving wave equation with Coulomb potential yields a Sommerfeld factor
$$\frac{4\pi GM|\mathbf{k}|}{1 - \exp(-4\pi GM|\mathbf{k}|)}$$
Khriplovich et al.(1997), Damour et al.(2007), Asada et al.(1997)
- factorization of IR & UV resummations

Matching

$$\begin{aligned} S = & -m \int d\tau - \frac{1}{2} \int dx^\mu L_{ab} \omega_\mu^{ab}(\tau) \\ & + \frac{1}{2} \sum_{n=0}^{\infty} \int d\tau c_n^{(I)} I^{aba_1 \dots a_n}(\tau) \nabla_{a_1} \dots \nabla_{a_n} E_{ab}(x) \\ & + \frac{1}{2} \sum_{n=0}^{\infty} \int d\tau c_n^{(J)} J^{aba_1 \dots a_n}(\tau) \nabla_{a_1} \dots \nabla_{a_n} B_{ab}(x) \end{aligned}$$

- How do we determine the multipole moments given a short distance description at scales smaller than a ?

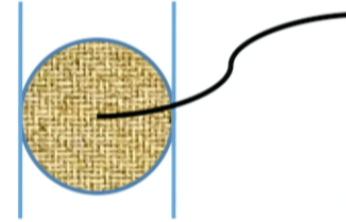
Matching

$$iS_{eff}[x_N, \bar{h}] = iS_0[x_N] + \textcircled{i}S_1[x_N, \bar{h}] + \dots$$

$$S_1[x_N, \bar{h}] = -\frac{1}{2m_{pl}} \int d^4x T^{\mu\nu}(x) \bar{h}_{\mu\nu}(x)$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

- compute one-graviton emission amplitudes to define $T^{\mu\nu}$
- Matching consists of two steps:
 1. relate multipoles to Feynman diagrams
 2. compute Feynman diagrams



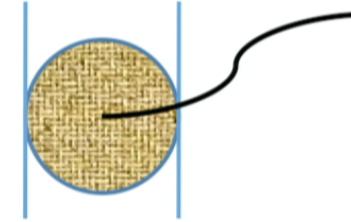
Matching

1st step: Multipole expansion of the action S_1

$$S_1[x_N, \bar{h}] = -\frac{1}{2m_{pl}} \int d^4x T^{\mu\nu}(x) \bar{h}_{\mu\nu}(x)$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

- Taylor expand field around a point
- bring into gauge invariant form
- decompose coefficients in Taylor expansion into irreducible representations of $SO(3) \rightarrow STF$
- performed multipole expansion to all orders



Matching

$$\begin{aligned}
S_{source}^{rad} = & \int dt \sum_{\ell=2}^{\infty} \sum_{p=0}^{\infty} d_p^{(\ell+2p)} \frac{\ell(\ell-1)}{(\ell+2p)(\ell+2p-1)} \left[\int d^3x \partial_t^{2p} T^{00} r^{2p} x^L \right]^{\text{STF}} \partial_{L-2} E_{k_{\ell-1} k_\ell} \\
& + \int dt \sum_{\ell=2}^{\infty} \sum_{p=0}^{\infty} \frac{\ell(\ell-1)}{(\ell+2p+1)!} \left(\sum_{j=0}^p \frac{c_j^{(\ell+2j)} ((\ell+2j)^2 - \ell)}{(\ell+2j)^2 (\ell+2j-1)} \right) \left[\int d^3x \partial_t^{2p} T^{aa} r^{2p} x^L \right]^{\text{STF}} \partial_{L-2} E_{k_{\ell-1} k_\ell} \\
& + \int dt \sum_{\ell=2}^{\infty} \sum_{p=0}^{\infty} \frac{2\ell(\ell-1)}{(\ell+2p+3)!} \left(\sum_{j=0}^{p+1} \frac{c_j^{(\ell+2j)} [\ell(\ell+2j-1)(p-j+1) - 2j(\ell+j)]}{(\ell+2j)^2 (\ell+2j-1)} \right) \\
& \quad \times \left[\int d^3x \partial_t^{2p+2} T^{ab} r^{2p} x^{abL} \right]^{\text{STF}} \partial_{L-2} E_{k_{\ell-1} k_\ell} \\
& - \int dt \sum_{\ell=2}^{\infty} \sum_{p=0}^{\infty} \frac{2\ell^2(\ell-1)}{(\ell+2p+2)!} \left(\sum_{j=0}^p \frac{c_j^{(\ell+2j)} (p-j+1)}{(\ell+2j)^2} \right) \left[\int d^3x \partial_t^{2p+1} T^{0a} r^{2p} x^{aL} \right]^{\text{STF}} \partial_{L-2} E_{k_{\ell-1} k_\ell} \\
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& \quad \times \left[\int d^3x \partial_t^{2p} T^{ak_\ell} r^{2p} x^{aL-1} \right]^{\text{STF}} \partial_{L-2} E_{k_{\ell-1} k_\ell} \\
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\end{aligned}$$

Matching

1st step: Multipole expansion of the action S_1

- usually Taylor expansion performed when solving field equations, taking limit $x \gg a$
- different here: expansion in the action for long wavelength EFT $\lambda \gg a$
- worked consistently at the level of the action which is manifestly gauge invariant
- also did multipole expansion for scalar theory and E&M

Matching

- multipoles needed for 1PN energy flux

$$I^{ij} = \int d^3\mathbf{x} \left(T^{00} + T^{kk} - \frac{4}{3}\dot{T}^{0k}\mathbf{x}^k + \frac{11}{42}\ddot{T}^{00}\mathbf{x}^2 \right) [\mathbf{x}^i\mathbf{x}^j]^{TF} + \dots$$

$$I^{ijk} = \int d^3\mathbf{x} T^{00} [\mathbf{x}^i\mathbf{x}^j\mathbf{x}^k]^{TF} + \dots$$

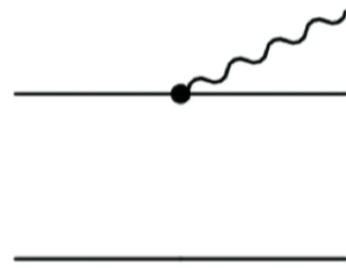
$$J^{ij} = -\frac{1}{2} \int d^3\mathbf{x} (\epsilon^{ikl}T^{0k}\mathbf{x}^j\mathbf{x}^l + \epsilon^{jkl}T^{0k}\mathbf{x}^i\mathbf{x}^l) + \dots$$

- use partial FT $T^{\mu\nu}(x^0, \mathbf{k}) = \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} T^{\mu\nu}(x^0, \mathbf{x})$
and Taylor expand in \mathbf{k} to read off moments

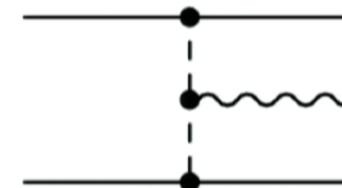
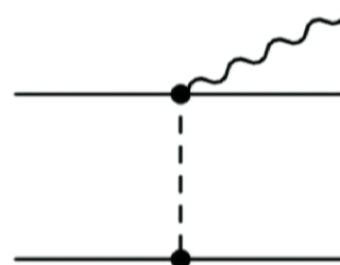
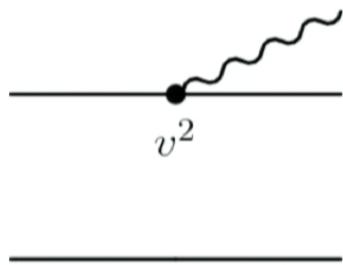
$$T^{\mu\nu}(x^0, \mathbf{k}) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \left(\int d^3\mathbf{x} T^{\mu\nu}(x^0, \mathbf{x}) \mathbf{x}^{i_1} \cdots \mathbf{x}^{i_n} \right) \mathbf{k}_{i_1} \cdots \mathbf{k}_{i_n}$$

Matching for PN

- $T^{\mu\nu}(x^0, \mathbf{k}) = \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} T^{\mu\nu}(x^0, \mathbf{x})$ from Feynman diagrams, e.g.



$$\rightarrow T^{00}(x^0, \mathbf{k}) = \sum_a m_a e^{-i\mathbf{k}\cdot\mathbf{x}_a}$$



Matching for PN

- simple Feynman diagrams yielding

$$I^{ij} = \sum_a m_a \left(1 + \frac{3}{2} \mathbf{v}_a^2 - \sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|} \right) [\mathbf{x}_a^i \mathbf{x}_a^j]^{TF}$$

$$+ \frac{11}{42} \sum_a m_a \frac{d^2}{dt^2} \left(\mathbf{x}_a^2 [\mathbf{x}_a^i \mathbf{x}_a^j]^{TF} \right)$$

$$- \frac{4}{3} \sum_a m_a \frac{d}{dt} \left(\mathbf{x}_a \cdot \mathbf{v}_a [\mathbf{x}_a^i \mathbf{x}_a^j]^{TF} \right)$$

$$J^{ij} = \frac{1}{2} \sum_a m_a \left((\mathbf{x}_a \times \mathbf{v}_a)^i \mathbf{x}_a^j + (\mathbf{x}_a \times \mathbf{v}_a)^j \mathbf{x}_a^i \right)$$

$$I^{ijk} = \sum_a m_a [\mathbf{x}_a^i \mathbf{x}_a^j \mathbf{x}_a^k]^{TF}$$

- for 1PN circ. orbit $P = P_{LO} \left[1 + \left(-\frac{1247}{336} - \frac{35}{12} \nu \right) x \right]$

Matching for PN

Reality check – comparing with known results

$$\begin{aligned} P = & \frac{32c^5}{5G}\nu^2x^5 \left\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu \right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2 \right)x^2 \right. \\ & + \left(-\frac{8191}{672} - \frac{583}{24}\nu \right)\pi x^{5/2} \\ & + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16x) \right. \\ & \quad \left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2 \right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3 \right]x^3 \\ & \left. + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2 \right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right) \right\} \end{aligned}$$

from Blanchet, Living Rev. Relativity, 9, (2006), 4

Matching for PN

Reality check – comparing with known results

$$\begin{aligned}\left\langle \frac{dE}{dt} \right\rangle = \left(\frac{dE}{dt} \right)_N \times & \left[1 - \frac{1247}{336}x^2 + 4\pi x^3 - \frac{44711}{9072}x^4 - \frac{8191}{672}\pi x^5 \right. \\ & + \left(\frac{6643739519}{69854400} - \frac{1712}{105}\gamma + \frac{16}{3}\pi^2 - \frac{3424}{105}\ln 2 - \frac{1712}{105}\ln x \right)x^6 - \frac{16285}{504}\pi x^7 \\ & + \left(-\frac{323105549467}{3178375200} + \frac{232597}{4410}\gamma - \frac{1369}{126}\pi^2 \right. \\ & \quad \left. + \frac{39931}{294}\ln 2 - \frac{47385}{1568}\ln 3 + \frac{232597}{4410}\ln x \right)x^8 \\ & + \left(\frac{265978667519}{745113600}\pi - \frac{6848}{105}\pi\gamma - \frac{13696}{105}\pi\ln 2 - \frac{6848}{105}\pi\ln x \right)x^9 \\ & + \left(-\frac{2500861660823683}{2831932303200} + \frac{916628467}{7858620}\gamma - \frac{424223}{6804}\pi^2 \right. \\ & \quad \left. - \frac{83217611}{1122660}\ln 2 + \frac{47385}{196}\ln 3 + \frac{916628467}{7858620}\ln x \right)x^{10} \\ & + \left(\frac{8399309750401}{101708006400}\pi + \frac{177293}{1176}\pi\gamma \right. \\ & \quad \left. \left. + \frac{8521283}{17640}\pi\ln 2 - \frac{142155}{784}\pi\ln 3 + \frac{177293}{1176}\pi\ln x \right)x^{11} \right],\end{aligned}$$

- too bad that 6PN not available to see \log^2

Matching for PN

Reality check – comparing with known results

$$\begin{aligned} \frac{dE^{(12)}}{dt} = & \frac{2067586193789233570693}{602387400044430000} - \frac{246137536815857}{157329572400} \gamma - \frac{271272899815409}{157329572400} \ln(2) - \frac{246137536815857}{157329572400} \ln(v) \\ & - \frac{27392}{105} \zeta(3) - \frac{54784}{315} \ln(2) \pi^2 - \frac{27392}{315} \pi^2 \ln(v) + \frac{5861888}{11025} \ln(2) \ln(v) + \frac{3803225263}{10478160} \pi^2 - \frac{256}{45} \pi^4 \\ & + \frac{5861888}{11025} (\ln(2))^2 + \frac{1465472}{11025} (\ln(v))^2 - \frac{27392}{315} \pi^2 \gamma + \frac{2930944}{11025} \ln(v) \gamma + \frac{5861888}{11025} \ln(2) \gamma + \frac{1465472}{11025} \gamma^2 \\ & - \frac{437114506833}{789268480} \ln(3) - \frac{37744140625}{260941824} \ln(5), \end{aligned} \tag{4}$$

Fujita, arXiv:1104.5615v1 [gr-qc]

Matching for PN

Reality check – comparing with known results

$$\begin{aligned} \frac{dE^{(12)}}{dt} = & \frac{2067586193789233570693}{602387400044430000} - \frac{246137536815857}{157329572400} \gamma - \frac{271272899815409}{157329572400} \ln(2) - \frac{246137536815857}{157329572400} \ln(v) \\ & - \frac{27392}{105} \zeta(3) - \frac{54784}{315} \ln(2) \pi^2 - \frac{27392}{315} \pi^2 \ln(v) + \frac{5861888}{11025} \ln(2) \ln(v) + \frac{3803225263}{10478160} \pi^2 - \frac{256}{45} \pi^4 \\ & + \frac{5861888}{11025} (\ln(2))^2 + \frac{1465472}{11025} (\ln(v))^2 - \frac{27392}{315} \pi^2 \gamma + \frac{2930944}{11025} \ln(v) \gamma + \frac{5861888}{11025} \ln(2) \gamma + \frac{1465472}{11025} \gamma^2 \\ & - \frac{437114506833}{789268480} \ln(3) - \frac{37744140625}{260941824} \ln(5), \end{aligned} \quad (4)$$

Fujita, arXiv:1104.5615v1 [gr-qc]



Matching for PN - spin

Spin in NRGR Porto, gr-qc/0511061, arXiv:0710.5150 [hep-th]

- introduced as additional degrees of freedom on the worldline
- complications arise since rotations comprise only 3 out of 6 dofs of Lorentz transformations
→ need to impose constraints to project out spin
- spin formalism gives effectively new vertices of gravitational field coupling to worldline
- finite size effects for spinning objects arise already at 2PN and are quadratic in spin



Matching for PN - spin

Towards the 3PN phase with full spin-dependence

- conservative dynamics obtained to 3PN
Porto et al., arXiv:0802.0720, arXiv:0804.0260, arXiv:1005.5730
- computed matching of all multipole moments for
3PN energy flux/phase & 2.5PN amplitude
R. Porto, AR, I. Rothstein arXiv:1007.1312 & in preparation
- work in progress: computing observables and
bringing them into useable form

Matching for PN - spin

Multipole moments for 3PN phase

- need to account for contributions both linear and quadratic in spin
- at which order do spin effects enter multipoles?

$$K_\ell^{\mu\nu} \equiv \int d^3x T^{\mu\nu} \mathbf{x}^{i_1} \dots \mathbf{x}^{i_\ell}$$

	$\mathcal{O}(\mathbf{S})$	$\mathcal{O}(\mathbf{S}_A)$	$\mathcal{O}(\mathbf{S}_A^2)$
K_ℓ^{00}	mr^ℓ	$mr^\ell v^3$	$mr^\ell v^4$
K_ℓ^{0i}	$mr^\ell v$	$mr^\ell v^2$	$mr^\ell v^5$
K_ℓ^{ij}	$mr^\ell v^2$	$mr^\ell v^3$	$mr^\ell v^6$

$$\dot{P}^0 = \frac{G_N}{\pi T} \int_0^\infty dk \left[\frac{k^6}{5} |I^{ij}(k)|^2 + \frac{16}{45} k^6 |J^{ij}(k)|^2 + \frac{k^8}{189} |I^{ijk}(k)|^2 + \dots \right]$$

Matching for PN - spin

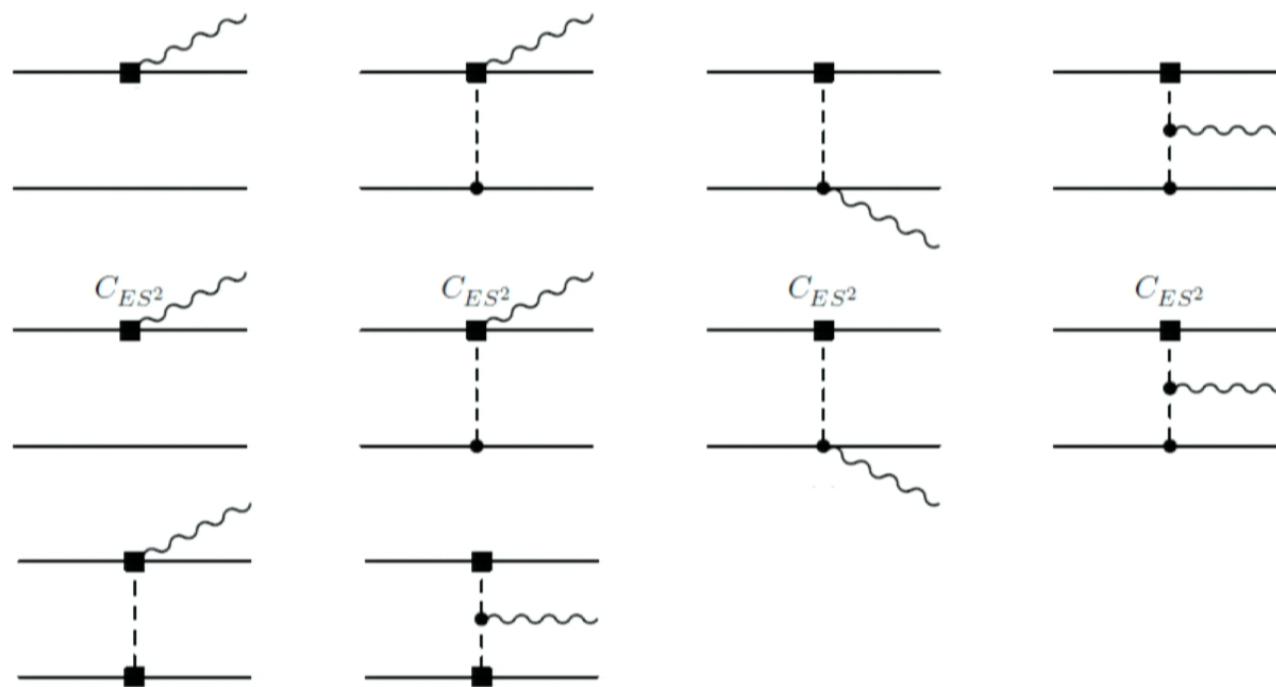
Multipole moments for 3PN phase

- mass quadrupole with $\mathcal{O}(\mathbf{S}_A)$ and $\mathcal{O}(\mathbf{S}_A^2)$ components to NLO & leading $\mathcal{O}(\mathbf{S}_A \mathbf{S}_B)$ terms
- current quadrupole with leading $\mathcal{O}(\mathbf{S}_A^2)$ and up to NLO $\mathcal{O}(\mathbf{S}_A)$ contributions
- mass octupole with leading $\mathcal{O}(\mathbf{S}_A)$ and $\mathcal{O}(\mathbf{S}_A^2)$
- current octupole with leading $\mathcal{O}(\mathbf{S}_A)$
- the spin-independent 1PN corrections to the mass and current quadrupole moments

Matching for PN - spin

Multipole moments for 3PN phase

- straightforward from simple Feynman diagrams



Matching for PN - spin

$$I_{\mathbf{S}_A, \mathbf{S}_A^2, \mathbf{S}_A \mathbf{S}_B}^{ij} = \sum_A \left[\frac{8}{3} (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j - \frac{4}{3} (\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{v}_A^j - \frac{4}{3} (\mathbf{x}_A \times \dot{\mathbf{S}}_A)^i \mathbf{x}_A^j \right. \quad (79)$$

$$\begin{aligned} & - \frac{4}{3} \frac{d}{dt} \left\{ \mathbf{v}_A \cdot \mathbf{x}_A (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j \right\} + \frac{1}{7} \frac{d^2}{dt^2} \left\{ \frac{1}{3} \mathbf{x}_A \cdot \mathbf{v}_A (\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j \right. \\ & \left. + 4 \mathbf{x}_A^2 (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j + \mathbf{x}_A^2 (\mathbf{S}_A \times \mathbf{x}_A)^i \mathbf{v}_A^j - \frac{5}{6} (\mathbf{v}_A \times \mathbf{S}_A) \cdot \mathbf{x}_A \mathbf{x}_A^i \mathbf{x}_A^j \right\} \Big|_{\text{STF}} \\ & + \sum_{A,B} \frac{2Gm_B}{r^3} \left[(\mathbf{v}_B \times \mathbf{S}_A) \cdot \mathbf{r} (\mathbf{x}_B^i \mathbf{x}_B^j - 2\mathbf{x}_A^i \mathbf{x}_A^j) + (\mathbf{v}_A \times \mathbf{S}_A) \cdot \mathbf{r} (\mathbf{x}_A^i \mathbf{x}_A^j + \mathbf{x}_B^i \mathbf{x}_B^j) \right. \\ & \left. + 2r^2 \left\{ (\mathbf{v}_B \times \mathbf{S}_A)^i (\mathbf{x}_B^j - \mathbf{x}_A^j) + (\mathbf{r} \times \mathbf{S}_A)^i \left(\mathbf{v}_B^j - \mathbf{v}_A^j - \frac{\mathbf{v}_B \cdot \mathbf{r}}{r^2} (\mathbf{x}_A^j + \mathbf{x}_B^j) \right) \right\} \right] \Big|_{\text{STF}} \\ & - \frac{2}{3} \sum_{A,B} \frac{d}{dt} \left[\frac{Gm_B}{r^3} \left\{ r^2 \left((\mathbf{x}_B \times \mathbf{S}_A)^i \mathbf{x}_A^j - 3(\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j + 3(\mathbf{x}_B \times \mathbf{S}_A)^i \mathbf{x}_B^j - (\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}_B^j \right) \right. \right. \\ & \left. \left. - 2\mathbf{r} \cdot \mathbf{x}_B (\mathbf{r} \times \mathbf{S}_A)^i (\mathbf{x}_A^j + \mathbf{x}_B^j) + (\mathbf{x}_A \times \mathbf{S}_A) \cdot \mathbf{x}_B (\mathbf{x}_A^i \mathbf{x}_A^j - 2\mathbf{x}_B^i \mathbf{x}_B^j) \right\} \right] \Big|_{\text{STF}} \\ & + \sum_A \frac{C_{ES^2}^{(A)}}{m_A} \left[\mathbf{S}_A^i \mathbf{S}_A^j \left(-1 + \frac{13}{42} \mathbf{v}_A^2 + \frac{17}{21} \mathbf{a}_A \cdot \mathbf{x}_A \right) + \mathbf{S}_A^2 \left(-\frac{11}{21} \mathbf{v}_A^i \mathbf{v}_A^j + \frac{10}{21} \mathbf{a}_A^i \mathbf{x}_A^j \right) \right. \\ & \left. - \frac{8}{21} \mathbf{x}_A^i \mathbf{S}_A^j \mathbf{a}_A \cdot \mathbf{S}_A + \frac{4}{7} \mathbf{v}_A^i \mathbf{S}_A^j \mathbf{S}_A \cdot \mathbf{v}_A - \frac{22}{21} \mathbf{a}_A^i \mathbf{S}_A^j \mathbf{S}_A \cdot \mathbf{x}_A \right] \Big|_{\text{STF}} \end{aligned}$$

$$\begin{aligned} & + \sum_{A,B} \frac{G}{2r^3} \left[\frac{C_{ES^2}^{(B)} m_A}{m_B} (\mathbf{S}_B^2 + 9(\mathbf{S}_B \cdot \mathbf{n})^2) \mathbf{x}_B^i \mathbf{x}_B^j + 6 \frac{C_{ES^2}^{(B)} m_A}{m_B} r^2 \mathbf{S}_B^i \mathbf{S}_B^j \right. \\ & + \left(\frac{C_{ES^2}^{(B)} m_A}{m_B} (3(\mathbf{S}_B \cdot \mathbf{n})^2 - \mathbf{S}_B^2) + 12 \mathbf{S}_A \cdot \mathbf{n} \mathbf{S}_B \cdot \mathbf{n} - 4 \mathbf{S}_A \cdot \mathbf{S}_B \right) \mathbf{x}_A^i \mathbf{x}_A^j \\ & \left. - 4 \frac{C_{ES^2}^{(B)} m_A}{m_B} \mathbf{S}_B^2 \mathbf{x}_A^i \mathbf{x}_B^j + 4 \left(3 \frac{C_{ES^2}^{(B)} m_A}{m_B} \mathbf{S}_B \cdot \mathbf{r} + 2 \mathbf{S}_A \cdot \mathbf{r} \right) \mathbf{S}_B^i \mathbf{x}_B^j \right] \Big|_{\text{STF}} \end{aligned}$$

moments for
phase to 3PN

$$\begin{aligned} J_{\mathbf{S}_A, \mathbf{S}_A^2}^{ij} = & \sum_A \left[\frac{C_{ES^2}^{(A)}}{m_A} (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{S}_A^j + \mathbf{S}_A^i \mathbf{x}_A^j \left(\frac{3}{2} + \frac{2}{7} \mathbf{v}_A^2 - \frac{5}{7} \mathbf{a}_A \cdot \mathbf{x}_A \right) - \frac{3}{7} \mathbf{v}_A^i \mathbf{S}_A^j \mathbf{v}_A \cdot \mathbf{x}_A \right. \\ & \left. + \frac{11}{28} \mathbf{S}_A^i \mathbf{a}_A^j \mathbf{x}_A^2 + \frac{2}{7} \mathbf{S}_A \cdot \mathbf{x}_A \mathbf{a}_A^i \mathbf{x}_A^j + \frac{1}{7} \mathbf{x}_A^i \mathbf{x}_A^j \mathbf{a}_A \cdot \mathbf{S}_A - \frac{3}{7} \mathbf{S}_A \cdot \mathbf{v}_A \mathbf{v}_A^i \mathbf{x}_A^j + \frac{11}{14} \mathbf{S}_A \cdot \mathbf{x}_A \mathbf{v}_A^i \mathbf{v}_A^j \right] \Big|_{\text{STF}} \\ & + \sum_{A,B} \frac{Gm_B}{2r^3} \left[3 \mathbf{S}_A \cdot \mathbf{x}_B (\mathbf{x}_B^i \mathbf{x}_B^j - \mathbf{x}_A^i \mathbf{x}_A^j) + \mathbf{S}_A \cdot \mathbf{x}_A (2\mathbf{x}_A^i \mathbf{x}_A^j + \mathbf{x}_A^i \mathbf{x}_B^j - 3\mathbf{x}_B^i \mathbf{x}_B^j) \right. \\ & \left. + \mathbf{S}_A^i \mathbf{x}_A^j (\mathbf{x}_A \cdot \mathbf{r} - 6r^2) \right] \Big|_{\text{STF}} \end{aligned} \quad (80)$$

$$J_{\mathbf{S}_A}^{ijk} = 2 \sum_A \left[\mathbf{x}_A^i \mathbf{x}_A^j \mathbf{S}_A^k \right] \Big|_{\text{STF}} \quad (81)$$

$$I_{\mathbf{S}_A, \mathbf{S}_A^2}^{ijk} = \sum_A \left[\frac{9}{2} (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}_A^j \mathbf{x}_A^k - 3(\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{v}_A^j \mathbf{x}_A^k - 3 \frac{C_{ES^2}^{(A)}}{m_A} \mathbf{S}_A^i \mathbf{S}_A^j \mathbf{x}_A^k \right] \Big|_{\text{STF}}, \quad (82)$$

Matching for PN - spin

Multipole moments for 2.5PN amplitude

- most multipole moments already included in set for phase to 3PN
- additional moments computed:

LO $\mathcal{O}(\mathbf{S}_A)$: J^{ijkl} , J^{ijklm} , I^{ijkl} and NLO $\mathcal{O}(\mathbf{S}_A)$: J^{ijk}

Conclusions

- NRGR works well in radiation sector
- novel resummation from RG
- multipole expansion performed to all orders
- new results for spin components of multipole moments for phase to 3PN and amplitude to 2.5PN

Outlook

- catching up slowly for spinless binary systems
- full spin-dependence in phase to 3PN
- eventually reach 3.5PN precision for binary systems with spin
- RG running for arbitrary moments
- waveform with non-linearities







