Title: Dissipative effects during inflation: An EFT approach

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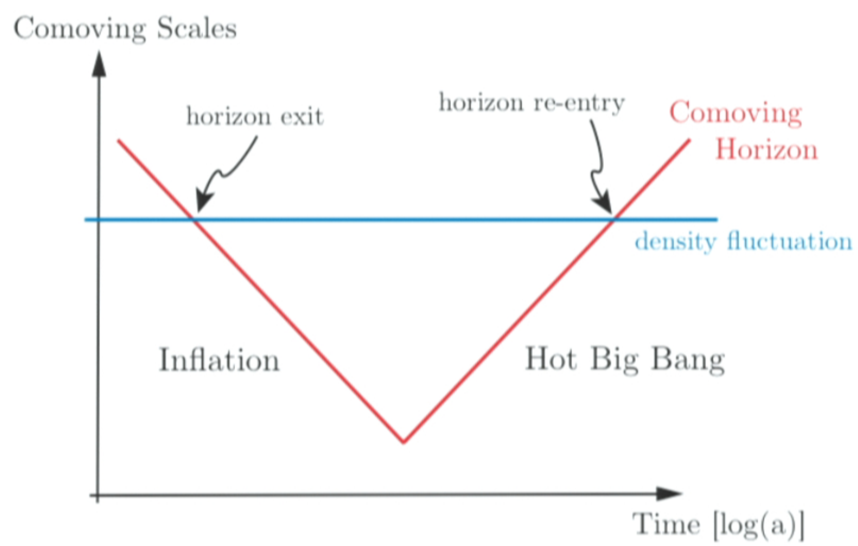
Abstract: Using an approach originally developed to study gravitational wave absorption in black hole binary systems, we generalize the EFT of single clock inflation to include dissipative effects. We restrict ourselves to situations where the degrees of freedom responsible for dissipation do no contribute to the density perturbations at late time, and moreover they are predominately sensitive to the field whose fluctuations control the end of inflation. The dynamics of the perturbations is then modified by the appearance of `friction' and noise terms, and assuming certain locality properties we show that there is a regime, characterized by a large friction coefficient \gamma >> H, in which the power spectrum is dominated by the noise and it is significantly modified with respect to the Bunch-Davies result. Furthermore, the non-linear realization of the symmetries implies non-gaussianties which are enhanced with respect to single clock models without dissipation by a factor of \gamma/H, and whose shape functions can in principle be distinguished from those obtained in the Bunch-Davies vacuum. We also discuss the matching of the EFT with a few key examples such as trapped and warm inflation.

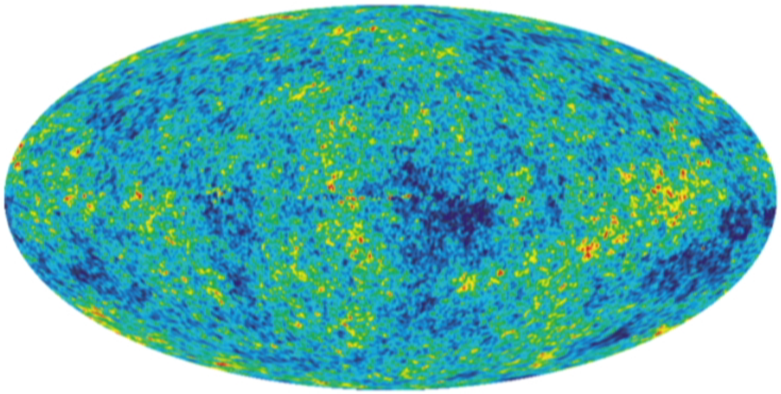
Motivation & Introduction:

We are all familiar with what zero-th order Inflation *does*

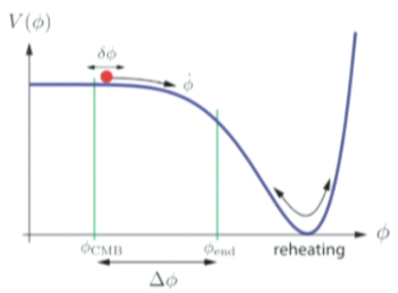


a given scale *re-enter* the horizon in the past
(CMB scales exit way before end of inflation)

moreover, it provides the seed for the observed (scale invariant) density perturbations



The process is rather simple:



Different parts of the universe terminate inflating at slightly different *times*, when they damp the energy stored in V(\phi) into radiation. Therefore, we go from constant density to a 1/a^4 law.

If this is the only mechanism, different regions will red-shift slightly different producing density perturbations (that's the reason of the 1/epsilon enhancement!)

The power spectrum is also simple to estimate, since it is produced by quantum effects in the vacuum!

Recall the action for the scalar perturbations $\zeta \sim -H\delta\phi/\dot{\phi}$

$$\mathcal{S}_2 = \int d^4x(\partial\phi)^2 \rightarrow \frac{\dot{\phi}^2}{H^2}\int d^4x(\partial\zeta)^2 \sim \epsilon_\star M_p^2 \frac{\zeta^2}{\omega_\star^2} \sim 1 \quad \rightarrow \quad \zeta \sim \frac{\omega_\star}{\sqrt{2\epsilon_\star}M_p}.$$

since freezeout occurs for w*,k* ~ H (cs=1)

$$\langle\zeta_k\zeta_q\rangle_{\mathrm{BD}} \simeq (2\pi)^3\frac{H_\star^2}{4\epsilon_\star M_p^2 k^3}\delta^{(3)}(\mathbf{q}+\mathbf{k}),$$

(This result changes for low speed of sound models, namely non-canonical kinetic terms.)

However, this is just one number! (still depends upon H* and epsilon*)
It i not obvious density perturbations are quantum in nature!

More information: Non-Gaussianities (NG)

Non-linearities can potentially teach us about the mechanism of inflation via the bispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle_{\rm BD} = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

- Single field inflation with canonical kinetic term (cs=1) leads to negligible non-gaussianities! (Maldacena)
- Some models (with non-standard kinetic term) can enhance non-gaussianities (e.g. DBI, P(X),...)

The reason for small NG in vanilla inflation is due to the flatness of V(\phi).
On the other hand, in P(X) for instance, for small speed of sound non-linearities are enhanced

Can we make model-independent predictions?

EFT of Inflation in a small nutshell (see Leonardo's talk)

Similarly to EWchL we can construct the EFT noticing that $t+\pi$ remains invariant

For the kinetic term we have: $\partial_\mu(t+\pi)\partial^\mu(t+\pi)$

We can also write functions of t+\pi: $f(t+\pi)$

The overall normalization is determined by the symmetry breaking scale, similar to EWchL, i.e. $f_\pi \leftrightarrow M_p^2 \dot{H}, \text{ e.g. } \dot{\phi}^2$

The tadpoles are canceled by adding

$$-\int d^4x\sqrt{-g}M_p^2(3H^2(t+\pi)+\dot{H}(t+\pi))$$

To compute curvature perturbation we need the relationship between \pi and \zeta $\zeta \simeq -H\pi$

this means \pi cannot be exactly massless (\zeta is massless, constant outside the horizon) $\left(\text{“ } \pi \sim \delta\phi/\dot{\phi} \text{ ”}\right)$

Scale invariance <=> dS + approx shift symmetry *new physics* appears via higher dimensional operators

$$M_2^4 \left(1 + \partial_\mu (t+\pi) \partial^\mu (t+\pi)\right)^2$$

and we generate terms such as

$$\dot{\pi}^2, \ \dot{\pi}^3, \dot{\pi}(\partial_j \pi)^2, \ldots$$

Therefore, just from the symmetries, one concludes that a modification of the dispersion relation, e.g. non-trivial speed of sound:

$$(-M_p^2 \dot{H} + 2M_2^4)\dot{\pi}^2 \rightarrow 1/c_s^2 = 1 - \frac{2M_2^4}{M_p^2 \dot{H}}$$

Also induces non-linearities of order (for cs<<1, also k/a*~H/cs)

$$\frac{M_p^2 \dot{H}}{c_s^2}\left(\dot{\pi}^3 - \dot{\pi}\frac{(\partial_j \pi)^2}{a^2}\right) \quad \rightarrow f_{NL} \equiv \frac{\mathcal{L}_3}{\mathcal{L}_2} \simeq \frac{1}{c_s^2}$$

Hence models of single clock inflation predict large NG, of order 1/cs^2

Going beyond the *standard model*

- What happens if we add more fields?
- Could the spectrum of fluctuations be produced by physics unrelated to the BD vacuum? For example, thermal fluctuations, noise?
- What type of different dynamics shall we expect?
- What kind of observational signatures would this new paradigm predict?
- Can we construct an EFT approach ?!

L. Senatore & M. Zaldarriaga studied adding new *light* fields (to preserve scale invariance) in an EFT of multifield inflation. One gets new contributions to the *local* shape. (Related to consistency conditions, ns-1, etc...)

We analyzed a different setup, with new degress of freedom (dof) but only *one clock*
New dof do not contribute to \zeta

Note we cannot integrate them out. Even though they are not necessarily 'light', they are produced during inflation (not in vacuum) and act upon the clock at low(er) frequencies (w~H)

Paradigmatic examples include: Warm inflation (Berera et al.) and Trapped Inflation (Green et al.)

(Adiabaticity is violated for instance during particle production)

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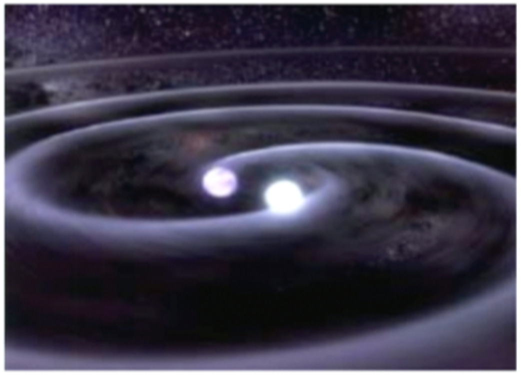
An EFT approach to dissipation (other than in-in)

(Rothstein & Goldberger)

Case of study:
Absorption in binary
BH-BH inspirals



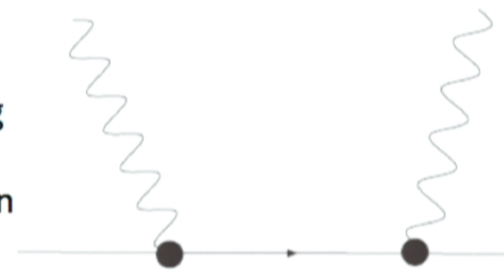
If we assume there is a long wavelength derivative expansion, then we can include dissipation by adding new couplings into the worldline action that describes the BHs

$$-m \int d\tau + \int d\tau Q^E_{ab} E^{ab} + \int d\tau Q^B_{ab} B^{ab}$$

The quadrupole-like additional degrees of freedom (ADOF) characterizes the response of the system to external perturbations

This form of the extra terms in the action is dictated by general covariance

Lesson: Even though we don't know how to compute the Green functions from first principles, we can obtain them from matching with the full theory. These are universal, and we can use them later gain predictive power in more complicated scenarios.



$$\sigma^{eft}_{abs}(\omega) = 2\,\mathrm{Im}\,\frac{i\omega}{8m_p^2}\int dx^0 e^{-i\omega x^0}\left[\omega^2\epsilon^*_{ab}\epsilon_{cd}\langle T(Q^E_{ab}(0)Q^E_{cd}(x^0))\rangle + (\mathbf{k}\times\epsilon^*)_{ab}(\mathbf{k}\times\epsilon)_{cd}\langle T(Q^B_{ab}(0)Q^B_{cd}(x^0))\rangle\right],$$

$$\int dx^0 e^{-i\omega x^0}\langle 0|TQ^E_{ab}(0)Q^E_{cd}(x^0)|0\rangle = -\frac{i}{2}Q_{abcd}F(\omega)$$

For example for spinning BHs (RAP)

$$\langle Q^{E(B)}_{ab}Q^{E(B)}_{cd}\rangle_{spin} \sim \omega G_N^3 m^3 (G_N m^2 a_*)(1+3a_*^2)$$

Universality

Let's use our dual EFT to predict the power of absorption in binary systems

In the NR limit we have w~v/r.

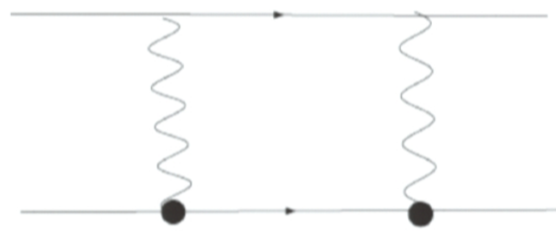
Therefore we can power count Q^E(B)

$$\left(Q_{ab}^{E(B)}\right)_{spin} \sim \sqrt{a_*(1+3a_*^2)}Lv^{5/2}/m_p$$

$$\int d\tau \left(Q_{ab}^{E}\right)_{spin} E^{ab}[H] \sim \sqrt{a_*(1+3a_*^2)}v^5$$

This tells us right away at what order absorption enters in the dynamics

For the binary system



This diagram scales as v^10, contrary to v^13 for spinless. This represents an enhancement of v^3!

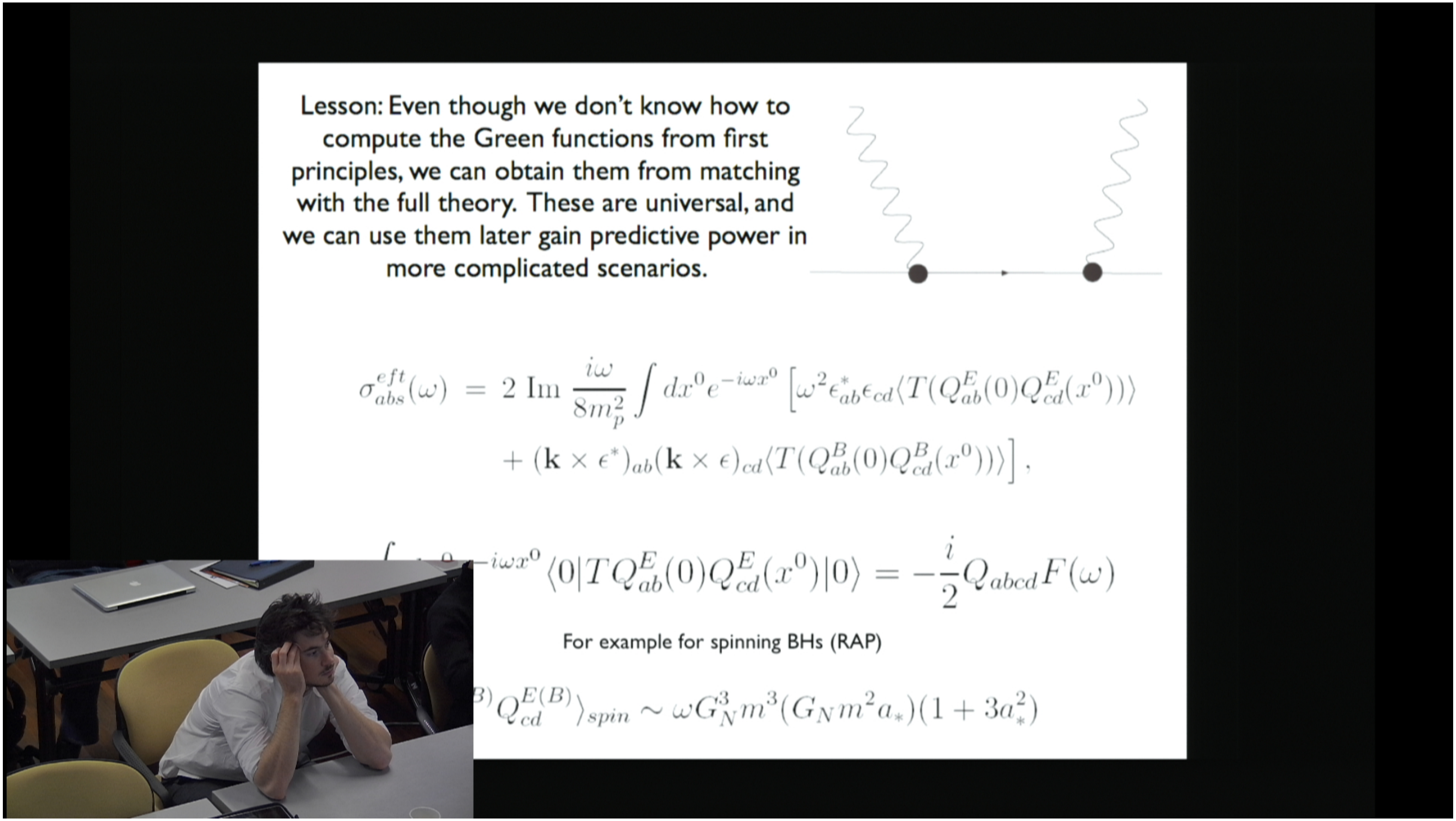
from here we then predict the absorption power for spinning BH-BH binary systems (RAP)

$$P_{abs}^{spin} = \frac{8}{45} G_N^6 \left\langle \sum_{a \neq b} m_a^5 m_b^2 \left(\dot{q}_{ij}^a q_{il}^a s_{jl}^a \right) (a_* + 3a_*^3) \right\rangle$$

$$= -\frac{8}{5} G_N^6 m_1^2 m_2^2 \left\langle \frac{1 \cdot \xi}{r^8} (a_* + 3a_*^3) \right\rangle$$

Agrees with E. Poisson's result

For the case of inflation we will use the power spectrum to match and then make a prediction about the size of the non-linear contributions and NG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$-i\omega x^0 \langle 0|TQ^E_{ab}(0)Q^E_{cd}(x^0)|0\rangle = -\frac{i}{2}Q_{abcd}F(\omega)$
For example for spinning BHs (RAP)
$B)\, Q^{E(B)}_{cd}\rangle_{spin} \sim \omega G_N^3 m^3 (G_N m^2 a_*)(1+3a_*^2)$

Dissipation: an illustrative example

$\gamma\dot{\phi}$ in slow roll

We know dissipation is associated with velocity dependent forces.

However, general covariance requires the following combination:

$$\gamma\dot{\phi} \rightarrow n^{\mu}\partial_{\mu}\phi$$

Where n is the unit vector perpendicular to the equal time surfaces
In the unperturbed background: n=(1,0,0,0)

Since \phi is 'the clock' then we also have

$$n^{\mu} = g^{\mu\nu}\partial_{\nu}\phi/\sqrt{(\partial\phi)^2}$$

Therefore, introducing the perturbations around \phi(t) we generate a term in the EOM

$$\frac{\gamma}{\dot{\phi}}(\partial_i\delta\phi)^2 \quad \text{(Properly normalized)} \quad \zeta \simeq -H\delta\phi/\dot{\phi}$$

So that $$\frac{\gamma}{\dot{\phi}}\frac{(\partial_i\delta\phi)^2}{c_s^2\partial_i^2\delta\phi} \sim f_{\rm NL}\zeta \quad \rightarrow \quad |f_{\rm NL}| \sim \frac{\gamma}{c_s^2 H},$$

The story of O

Let's start at linear order

Given a theory for \pi we can add the following operators in the action

$$f(t+\pi)\mathcal{O} \rightarrow \quad S_{\rm int} = -\int d^4x \mathcal{O}(x)\pi(x)$$

If we had a shift symmetry, \pi to \pi +c, then we'd have

$$\partial_\mu(t+\pi)\partial^\mu(t+\pi)\tilde{\mathcal{O}} \rightarrow \quad \tilde{S}_{\rm int} = \int d^4x \tilde{\mathcal{O}}(x)\dot{\pi}(x)$$

but also vector couplings, e.g. $\partial^\mu \pi \ \mathcal{O}_\mu$ and tensors, etc.

However, at linear order we IBP and consider in general

$$S_{\rm int} = -\int d^4x \mathcal{O}(x)\pi(x)$$

where the composite operator O describes the coupling to the perturbations at linear order

I will return to the general procedure in more detail later on.

Now we define $\delta\mathcal{O} = \mathcal{O} - \bar{\mathcal{O}}$

The vev goes into the background (more later). We then split into two parts

$\delta\mathcal{O} = \delta\mathcal{O}_R + \delta\mathcal{O}_S$ R stands for 'response' and S for 'stochastic'

$$\delta\mathcal{O}_R(x) = -\int d^4y G^{\mathcal{O}}_{\rm ret}(x,y)\pi(y),$$

where

$$G^{\mathcal{O}}_{\rm ret}(x,y) = i\langle[\delta\mathcal{O}(x), \delta\mathcal{O}(y)]\rangle\theta(t_x - t_y).$$

If we denote as D\pi=0 the EOM w/out O's then

$$D_\pi\pi - \int d^4y G^{\mathcal{O}}_{\rm ret}(x-y)\pi(y) = -\delta\mathcal{O}_S + O(\pi^2),$$

or in Fourier space

$$\left(D_\pi(\mathbf{q},\omega) - G^{\mathcal{O}}_{\rm ret}(\mathbf{q},\omega)\right)\pi_q(\omega) = -\delta\mathcal{O}_S(\mathbf{q},\omega) + \ldots.$$

(Formally the stochastic part enters with anti-commutators)

The main objective is to understand the analytic properties of G^O_ret

Say we're dealing with an harmonic oscillator. In general the EOM takes the Langevin form:

$$\ddot{\pi} + \omega_0^2 \pi + \int dt' \tilde{\gamma}(t - t') \pi(t') = J(t)$$

The more familiar (local) form $\gamma \dot{\pi}$ emerges when (Ohmic)

$$\mathrm{Im} \tilde{\gamma}(\omega) \simeq \gamma \omega,$$

or in terms of the O's, when

$$\mathrm{Im} G^{\mathcal{O}}_{\mathrm{ret}}(\omega, \mathbf{q}) \simeq \mathrm{Im} G^{\mathcal{O}}_{\mathrm{ret}}(\omega, \mathbf{0}) \simeq \gamma \omega,$$

up to corrections of order $(\mathbf{q}/M_{\mathcal{O}}, \omega/\Gamma_{\mathcal{O}})$ where the non-local effects start to play a role

Notice that for the case of a shift symmetry we'd require $\mathrm{Im} \tilde{G}^{\mathcal{O}}_{\mathrm{ret}}(\omega) \sim 1/\omega$

In more realistic setting we expect (this may require some tuning)

$$\omega^2 \mathrm{Im} \tilde{G}^{\mathcal{O}}_{\mathrm{ret}}(\omega) \simeq \gamma \frac{\omega^3}{\omega^2 + \mu_{\mathcal{O}}^2} + O(\omega/\Gamma_{\mathcal{O}}) \simeq \gamma \omega + O(\mu_{\mathcal{O}}/\omega, \omega/\Gamma_{\mathcal{O}})$$

The condition on the Green's function may be more difficult to achieve
for vector or tensor couplings

Fluctuation/Dissipation theorem

In statistical mechanics we're familiar with the high temperature limit leading to local Kernels

$$\langle \delta\mathcal{O}_S(\mathbf{k},t)\delta\mathcal{O}_S(\mathbf{q},t')\rangle = (2\pi)^3\nu_{\mathcal{O}}\delta(t-t')\delta^{(3)}(\mathbf{k}+\mathbf{q})$$

If we think of the O's as representing a 'bath' at temperature T then one can show

$$\frac{\mathrm{Im}G^{\mathcal{O}}_{\mathrm{ret}}(\omega)}{\omega} \equiv \gamma = \frac{\nu_{\mathcal{O}}}{T}$$

Algebraically this follows using the commutator form of the Green's function
Intuitively it is a consequence of the fact that given a perturbation dH the system relaxes to the equilibrium distribution with the new Hamiltonian H0+dH.

Therefore we can relate the LRT to the power spectrum

The validity of the FD theorem is subtle for an expanding universe (more later)

For us the local approximation will result from the dynamics (and in the EFT as an assumption)

Key hypothesis:
Emergent shift symmetry

Notice that in principle (just from analyticity) we could generate a large $ReG^{\mathcal{O}}_{\mathrm{ret}}$

This would imply that the O's respond directly to values of \pi inducing a 'mass term'

This is not forbidden by any symmetry, however,
it may lead to an evolution for \zeta outside the horizon, which we want to preclude.

Therefore, we will assume there's an *emergent* shift symmetry such that
the O's only respond to derivatives of \pi

The shift symmetry may not in general be realized at the level of the action,
but it will be a consequence of the dynamics

The emergence of the shift symmetry will also guarantee scale invariance

Hence, throughout our analysis we'll assume $ReG^{\mathcal{O}}_{\mathrm{ret}}(\omega \to 0) \to 0$

Homogeneous solution: intuitive picture

After we solve for O_R we end up with an equation of the type (\gamma>>H)

$$\ddot{\pi}_k + \gamma\dot{\pi}_k + c_s^2\frac{k^2}{a^2}\pi_k = 0.$$

To gain intuition let's start solving it for constant *a* (w0 = cs^2k^2/a^2)
(This is a good approx provided, \dot w0/w0^2<1, or k/a*>H)

It is trivial to show there're two independent solution (here we run time [-|t0|,0])

$$f_{\mp}(t) = A_{\mp}\exp\left[\frac{-\gamma t}{2}\left(1 \mp \sqrt{1 - 4\frac{\omega_0^2}{\gamma^2}}\right)\right]$$

Assuming we start in the BD vacuum at a late enough time $f_{\rm BD}^{\pm} = \frac{1}{\sqrt{2\omega_0}}e^{\pm i\omega_0 t_0}.$

fixes the normalization $A_{\pm} \simeq \frac{e^{\frac{-\gamma|t_0|}{2}}}{\sqrt{2\omega_0}} + O(\gamma/\omega_0)$

It is then clear the homogenous solution decays exponentially as t0->infty

$$f_- \sim \frac{e^{-\frac{\gamma|t_0|}{2}}}{\sqrt{\omega_0^\star}}e^{\frac{-(\omega_0^\star)^2 t}{\gamma}}.$$

notice this already suggests freezeout at (w0 goes to zero) $c_s k/a^* \simeq \sqrt{\gamma H}$

The power spectrum is dominated by the noise.
(unrelated to BD state)

we have to deal now with the EOM
(Nc is the normalization of \pi_c)

$$\ddot{\pi}_k + \gamma \dot{\pi}_k + \omega_0^2 \pi_k = -N_c^{-1} \delta \mathcal{O}_S$$

The solution is given in terms of a Green's function (again we take w0 constant to start with)

$$\pi_k(t) = -N_c^{-1} \int_0^\infty dt' G_\gamma^k(t-t') \delta \mathcal{O}_S(\mathbf{k}, t')$$

$$G_\gamma^k(t-t') = \frac{1}{\gamma} e^{-\frac{\omega_0^2}{\gamma}(t-t')} \theta(t-t')$$

Assuming: $\langle \delta \mathcal{O}_S(\mathbf{k}, t') \delta \mathcal{O}_S(\mathbf{q}, t) \rangle \simeq (2\pi)^3 \nu_{\mathcal{O}} \delta(t-t') \delta^{(3)}(\mathbf{q}+\mathbf{k})$

$$\langle \pi_k(t) \pi_q(t) \rangle \simeq \frac{\nu_{\mathcal{O}} (2\pi)^3}{N_c^2 \omega_0^2 \gamma} \left(1 - e^{-\frac{2\omega_0^2}{\gamma} t}\right) \delta^{(3)}(\mathbf{k}+\mathbf{q}) \quad \rightarrow \quad \frac{\nu_{\mathcal{O}} (2\pi)^3}{N_c^2 \gamma \omega_0^2} \delta^{(3)}(\mathbf{k}+\mathbf{q})$$

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This simple analysis allows us to understand the behavior of the Green's functions in the expanding universe. Notice there's an equilibration time

$$\tau_{\rm eq}^{-1} \simeq \omega_0^2/\gamma$$

Our approximation applies as long as 1/H>> \tau.

However, as \tau ~ 1/H the expansion of the universe becomes important and the system does not equilibrate, hence it freezes out (before w0->0). This is the case because the EOM only contain derivatives of \pi.

Therefore we conclude that freezeout occurs when

$$\frac{\omega_0^2}{\gamma} \sim H \quad \Rightarrow \quad \omega_0 \sim \sqrt{\gamma H} \quad \Rightarrow \quad k_\star \sim \sqrt{\frac{\gamma H}{c_s^2}}$$

We can insert factors of *a* to read off the (scale invariant) power spectrum

$$\langle \pi_k \pi_q \rangle (t_\star) \sim \frac{\nu_{\mathcal{O}} (2\pi)^3}{c_s^2 N_c^2 \gamma (k/a_\star)^2} \frac{1}{a_\star^3} \delta^{(3)}(\mathbf{k}+\mathbf{q}) \ \rightarrow \ \frac{1}{a_\star} \frac{\nu_{\mathcal{O}} (2\pi)^3}{N_c^2 \gamma c_s^2 k^2} \delta^{(3)}(\mathbf{k}+\mathbf{q}) \sim \frac{\sqrt{H_\star/\gamma}\,\nu_{\mathcal{O}}}{N_c^2 (c_s^\star k)^3} (2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{q})$$

For example, using the FD theorem (for the O's) one gets

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$$\langle \pi_k \pi_q \rangle (t_\star) \sim \frac{\nu_{\mathcal{O}} (2\pi)^3}{c_s^2 N_c^2 \gamma (k/a_\star)^2} \frac{1}{a_\star^3} \delta^{(3)}(\mathbf{k}+\mathbf{q}) \;\rightarrow\; \frac{1}{a_\star} \frac{\nu_{\mathcal{O}} (2\pi)^3}{N_c^2 \gamma c_s^2 k^2} \delta^{(3)}(\mathbf{k}+\mathbf{q}) \sim \frac{\sqrt{H_\star/\gamma}\,\nu_{\mathcal{O}}}{N_c^2 (c_s^\star k)^3} (2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{q})$$

For example, using the FD theorem (for the O's) one gets

$$\langle \pi_k \pi_q \rangle (t_\star) \simeq (2\pi)^3 \frac{\sqrt{\gamma H_\star} T}{N_c (c_s^\star k)^3} \delta^{(3)}(\mathbf{k}+\mathbf{q})$$

(more on Nc later)

This simple analysis allows us to understand the behavior of the Green's functions in the expanding universe. Notice there's an equilibration time

$$\tau_{\rm eq}^{-1} \simeq \omega_0^2/\gamma$$

Our approximation applies as long as 1/H>> \tau.

However, as \tau ~ 1/H the expansion of the universe becomes important and the system does not equilibrate, hence it freezes out (before w0->0). This is the case because the EOM only contain derivatives of \pi.

Therefore we conclude that freezeout occurs when

$$\frac{\omega_0^2}{\gamma} \sim H \quad \Rightarrow \quad \omega_0 \sim \sqrt{\gamma H} \quad \Rightarrow \quad k_\star \sim \sqrt{\frac{\gamma H}{c_s^2}}$$

We can insert factors of *a* to read off the (scale invariant) power spectrum

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(more on Nc later)

Before we move on let's have a word or two on the local approximation in expanding universe.

The EOM takes the form:

$$\ddot{\pi}_k(t) + 3H\dot{\pi}_k(t) + \frac{c_s^2\mathbf{k}^2}{a^2}\pi_k - \frac{1}{N_c}\int dt' a^3(t') G_{\rm ret}^{\mathcal{O}}(t,t',\mathbf{k})\pi_k(t') = -\frac{1}{N_c}\delta\mathcal{O}_S(t,\mathbf{k})$$

First we assume locality in space:

$$G_{\rm ret}^{\mathcal{O}}(t,t',\mathbf{k}) = \frac{G_{\rm ret}^{\mathcal{O}}(t,t')}{a^{3/2}(t)a^{3/2}(t')} + O(|\mathbf{k}|/M_{\mathcal{O}})$$

with M>>k*. In other words, there's a gap in momentum space where the 'free path' is much smaller than 1/k*. The factors of a^3/2 are related to co-moving coordinates.

Something similar applies for the correlation of the noise.

$$\langle \delta\mathcal{O}_S(t,\mathbf{k})\delta\mathcal{O}_S(t',\mathbf{q})\rangle \simeq \frac{\tilde{\nu}_{\mathcal{O}}(t,t')}{a^{3/2}(t)a^{3/2}(t')}(2\pi)^3\delta^{(3)}(\mathbf{q}+\mathbf{k})$$

For locality in time we assume the scale of dissipation is such $\Gamma_{\mathcal{O}}^{-1} \ll 1/H$

Then we can expand the time variation of \pi in the EOM $\pi_k(t-\tau) \simeq \pi_k(t) - \dot{\pi}_k(t)\tau + \ldots$

The requirement of an emergent shift symmetry imposes (from the first term)

$$\int \frac{a^{3/2}(t-\tau)}{a^{3/2}(t)} G^{\mathcal{O}}_{\rm ret}(t, t-\tau) d\tau = 0.$$

For the friction coefficient we thus get $N_c \gamma \simeq -\int \frac{a^{3/2}(t-\tau)}{a^{3/2}(t)} \cdot \tau \cdot G^{\mathcal{O}}_{\rm ret}(t, t-\tau) d\tau$

which is equivalent to the flat space condition $G^{\mathcal{O}}_{\rm ret}(t,t') \simeq -\gamma N_c \partial_t \delta(t-t') + \ldots$

something similar applies to the noise, so that $\tilde{\nu}_{\mathcal{O}}(t,t') \simeq \nu_{\mathcal{O}} \delta(t-t')$

at the end of the day the EOM becomes

$$\ddot{\pi}_k(t) + (3H+\gamma)\dot{\pi}_k(t) + \frac{c_s^2 \mathbf{k}^2}{a^2} \pi_k = -\frac{1}{N_c} \delta\mathcal{O}_S(t, \mathbf{k})$$

with $\langle \delta\mathcal{O}_S(t,\mathbf{k}) \delta\mathcal{O}_S(t',\mathbf{q}) \rangle \simeq \frac{\nu_{\mathcal{O}} \delta(t-t')}{a^3(t)} (2\pi)^3 \delta^{(3)}(\mathbf{q}+\mathbf{k}).$

The power spectrum: exact results

The solution is given in terms of the exact Green's function as

$$\pi_k(\eta) = \frac{kc_s}{N_c H^2} \int_{\eta_0}^{\eta} d\eta' G_\gamma(kc_s|\eta|, kc_s|\eta'|) \frac{\delta \mathcal{O}_S}{(kc_s\eta')^2}$$

the power spectrum for \pi then becomes

$$P_\pi(k) \equiv \langle \pi_k \pi_k \rangle_{\mathcal{O}} = \frac{\nu_{\mathcal{O}}}{N_c^2 (kc_s)^3} \int_z^{z_0} dz' (G_\gamma(z, z'))^2$$

and performing the integral we get, as we anticipated, (\zeta~-H\pi)

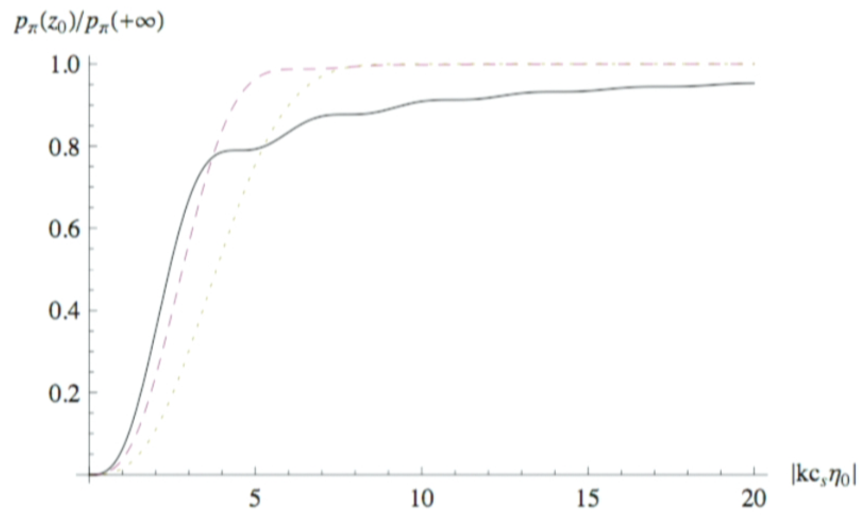
$$\Delta_\zeta \equiv k^3 P_\zeta(k) \simeq \nu_{\mathcal{O}}^\star \sqrt{\pi H_\star / \gamma_\star} \frac{H_\star^2}{2c_s^\star \left(c_s^\star N_c\right)^2}$$

which looks like (if we assume FD for the O's at high temperature)

$$k^3 \langle \zeta_k \zeta_k \rangle_T \simeq \sqrt{\pi \gamma_\star H_\star} \frac{T H_\star^2}{2c_s^\star \left(c_s^{\star 2} N_c\right)}$$

This is one of the key results of warm inflation

Let me show you a plot



which shows how the memory of the initial conditions is rapidly lost.

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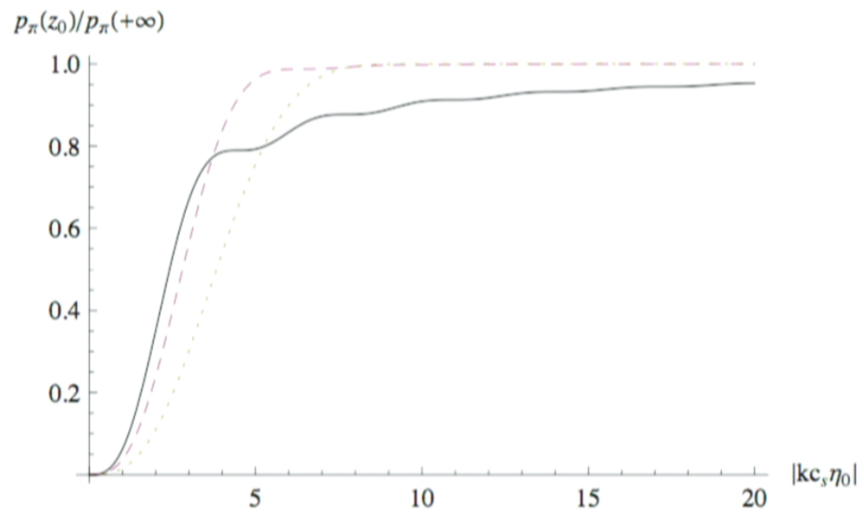


which shows how the memory of the initial conditions is rapidly lost.

Non Linearities I: Exact shift symmetry (scalar coupling)

$$\partial_\mu(t+\pi)\partial^\mu(t+\pi)\tilde{\mathcal{O}} \to \qquad -\frac{1}{2}\tilde{\mathcal{O}}(\partial_i\pi)^2$$

then the EOM becomes (the dot comes from the linear term O\dot\pi and IBP)

$$\ddot{\pi}_k + \gamma\left(\dot{\pi}_k - \frac{1}{2}[\partial_i\pi\partial_i\pi]_k\right) + \omega_0^2(k)\pi_k = -N_c^{-1}\left(\delta\dot{\mathcal{O}}_k^S - [\partial_i(\tilde{\mathcal{O}}\partial_i\pi)]_k\right)$$

assuming Gaussian noise we get

$$\langle\pi_k\pi_k\pi_k\rangle_{(\gamma)} = -\frac{\gamma\nu_{\mathcal{O}}^2 k^2}{N_c^4}\int dt'dt''dt'''\left(G_\gamma^k(t-t''')\right)^2 G_\gamma^k(t-t')\left(G_\gamma^k(t'-t'')\right)^2$$

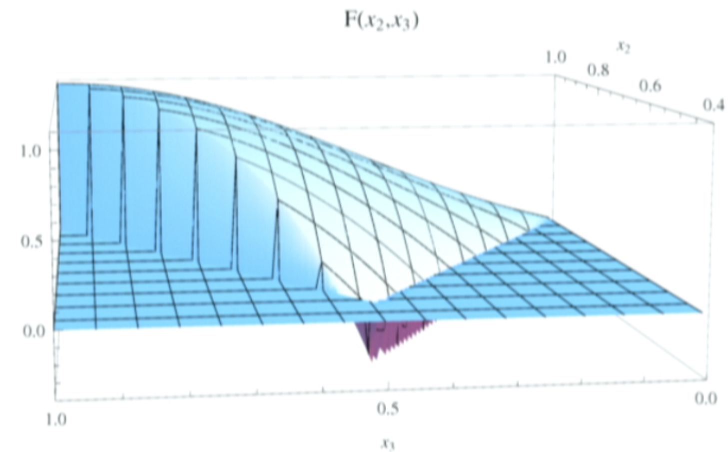
so that $\qquad f_{\rm NL}^{\rm eq} \simeq \dfrac{H^3\langle\pi_k\pi_k\pi_k\rangle_{(\gamma)}}{P_\zeta^2} \simeq \dfrac{\gamma}{Hc_s^2}$

also from

$$\left.\frac{\mathcal{O}(\partial_i\pi)^2}{\mathcal{O}\dot{\pi}}\right|_{k_\star\sim\sqrt{\gamma H/c_s^2},\ \omega_\star\sim H} \sim \frac{k_\star^2\zeta^2}{H^2\zeta} \sim \frac{\gamma}{c_s^2 H}\zeta \to |f_{\rm NL}| \sim \frac{\gamma}{c_s^2 H}$$

Sample shape ($\gamma \simeq 40H$)

$$F(x_2, x_3) = x_2^2 x_3^2 \frac{F(1,x_2,x_3)}{F(1,1,1)}$$



Notice also a second pick at the folded configuration (1/2,1/2). This follows from terms 1/(-k1+k2+k3) which cancel out in the BD compuation ~1/(k1+k2+k3). This is the smoking gun!

Non-linearities II: Approx shift symmetry

first of all it is clear that NL are small if we have a softly broken shift symmetry

at linear order we have $-\dot{f}(t)\delta\mathcal{O}\pi$ from $f(t+\pi)\mathcal{O}$

Therefore

$$\frac{\ddot{f}(t)\mathcal{O}\pi^2}{\dot{f}(t)\mathcal{O}\pi} \sim -\frac{\ddot{f}(t)}{\dot{f}(t)H}\zeta \rightarrow f_{\rm NL} \sim -\frac{\ddot{f}(t)}{\dot{f}(t)H} \sim O(\epsilon)$$

Caveat: The power spectrum depends also on the noise part and they might cancel out! (more later)

Comment on local shape

$$\delta\mathcal{O}_R \simeq \frac{\gamma N_c}{\dot{f}}\left(\dot{\pi} + \frac{\ddot{f}}{\dot{f}}\dot{\pi}\pi\right)$$ and also a source term $\ddot{f}\pi\delta\mathcal{O}_S$.

$$f_{\rm NL}^{\rm sq} = \lim_{x_3\to 0,\ x_2\to 1} \frac{5}{6}\frac{F(1,x_2,x_3)}{(P_\zeta(1)P_\zeta(x_2)+P_\zeta(1)P_\zeta(x_3)+P_\zeta(x_2)P_\zeta(x_3))} \sim \frac{\ddot{f}}{\dot{f}H} \sim \epsilon$$

which is slow roll suppressed. However recall the power spectrum depends upon $\dot{f}^2\nu_{\mathcal{O}}$

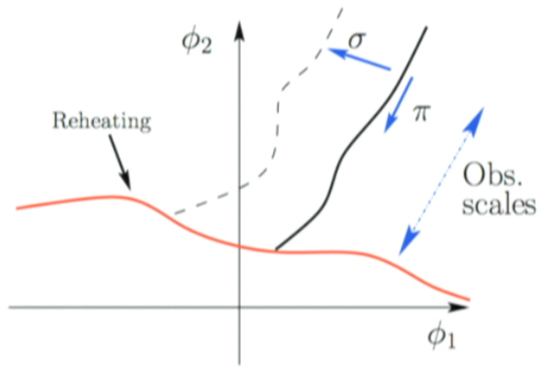
At the end of the day one can show that the consistency relation DOES hold

$$\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle_{k_1\to 0} = -(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(n_s-1)P_{k_1}P_{k_3}$$

and moreover, the next term (after the ns-1) does not grow faster than (kL/kS)^2
(Creminelli et al.)

One slide (or two) on the more formal stuff

Say we have a bunch of fields, we can always define the 'time' perturbation \pi as:
(Senatore & Zaldarriaga)



where \sigma is the 'orthogonal' direction.
In other words, we 'diagonalize' the time diffs
and the \sigma's don't pick vevs

This means that the quadratic Lagrangian for \pi
defined this way is set by the symmetries

$$\int \sqrt{-\tilde{g}}\left(-M_p^2(3H^2(t+\tilde{\pi})+\dot{H}(t+\tilde{\pi}))+M_p^2\dot{H}(t+\tilde{\pi})\tilde{g}^{00}(\tilde{\pi})\right)+\ldots$$

This is not the pion we're using! Basically
because it also contains \delta O.

$$\tilde{\pi}\sim\pi+\delta O$$

We assumed \delta O does not contribute to
\zeta, hence we work with \pi to keep

$$\zeta\sim -H\pi$$

Therefore the normalization for \pi is open

$$S_\pi=\int d^4x a^3\frac{N_c}{2}\left\{\dot{\pi}^2-c_s^2\frac{(\partial_i\pi)^2}{a^2}\right\}$$

Models

(local) Trapped Inflation (D. Green et al.)

$$S_{\rm trap} = \int d^4x\sqrt{-g}\left\{\sum_i\left[-\frac{1}{2}\partial_\mu\chi_i\partial^\mu\chi_i - \frac{g^2(\phi-\phi_i)^2}{2}\chi_i^2\right] - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\}$$

but we add a decaying channel for \chi to make it local $S_{\rm int} = \sum_i \frac{1}{\Lambda_\varphi}\partial^2\chi_i\varphi^2$

the EOM reads $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + g\sum_i n_{\chi_i}(t,t_i) = 0$

$$n_{\chi_i}(t_i+\delta t_c) = M_{\chi_i}\bar{\chi}_i^2 \simeq \frac{\kappa^3(t_i)}{(2\pi)^3} \quad |\beta_k|^2 = e^{-\frac{\pi k^2}{a^2(t_i)\kappa^2(t_i)}} \quad \left(\kappa(t_i)\sim\sqrt{g\dot{\phi}(t_i)}\right)$$

In the local approximation:

$$g\sum_i n_{\chi_i}(t,t_i) \simeq \frac{g|\dot{\phi}(t)|}{\Gamma_\chi \Delta}\frac{\kappa^3(t)}{(2\pi)^3} \simeq \frac{(g|\dot{\phi}|)^{5/2}}{\Gamma_\chi|\Delta|(2\pi)^3}$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{c_s^2}{a^2}\partial_i^2\phi + V'(\phi) + \frac{(g|\dot{\phi}|)^{5/2}}{\Gamma_\chi|\Delta|(2\pi)^3} = 0$$

and for the perturbations:

$$\ddot{\delta\phi} + \frac{c_s^2 k^2}{a^2}\delta\phi + 3H\dot{\delta\phi} + V''(\phi)\delta\phi + \frac{5}{2}\frac{g^{5/2}|\dot{\phi}|^{3/2}}{\Gamma_\chi|\Delta|(2\pi)^3}\dot{\delta\phi} = -g\Delta n_\chi$$

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Matching

$$f(t)\mathcal{O} \equiv \sum_i f_i(t)\mathcal{O}_i \qquad f_i(t) = \frac{g^2}{2}(\phi(t)-\phi_i)^2, \quad \mathcal{O}_i = \chi_i^2$$

$$\sum_i f_i(t)\bar{\mathcal{O}}_i = \frac{g^2}{2}\sum_i(\phi(t)-\phi_i)^2\bar{\chi}_i^2 = \frac{1}{2}\sum_i M_{\chi_i} n_{\chi_i}$$

$$g\sum_i \delta n_{\chi_i} \simeq \frac{5}{2}\frac{g^{5/2}|\dot{\phi}|^{3/2}}{\Gamma_\chi|\Delta|(2\pi)^3}\delta\dot{\phi} + \ldots \quad \rightarrow \quad \sum_i \dot{f}_i \delta\mathcal{O}_i^R = \dot{\hat{f}}\sum_i \delta n_{\chi_i} \simeq N_c\gamma\dot{\pi} + \ldots$$

$$\frac{1}{N_c}\sum_i \dot{f}_i\delta\mathcal{O}_i^S \quad \rightarrow \quad \frac{1}{N_c}\dot{\hat{f}}\Delta n_\chi$$

$$\langle \Delta n_\chi(t,\mathbf{k})\Delta n_\chi(t',\mathbf{k}')\rangle \simeq (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\delta(t-t')}{a^3(t)}\frac{\kappa^3 N_{\rm hits}}{2\Gamma_\chi} \equiv \nu_\chi(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\delta(t-t')}{a^3(t)}$$

$$\gamma = \frac{5}{2}\frac{g^{5/2}|\dot{\phi}|^{3/2}}{\Gamma_\chi|\Delta|(2\pi)^3} \qquad \nu_\chi \equiv \frac{\kappa^3 N_{\rm hits}}{2\Gamma_\chi}$$

Non-Gaussianities

$$\dot{\phi} \to n^\mu \partial_\mu \phi \text{ with } n_\mu \sim \partial_\mu \phi \qquad \frac{(g|\dot{\phi}|)^{5/2}}{\Gamma_\chi |\Delta| (2\pi)^3} \to \frac{g^{5/2}\left(\sqrt{-(\partial\phi)^2}\right)^{5/2}}{\Gamma_\chi |\Delta| (2\pi)^3}$$

and again we generate our old friend (missed in trapped inflation paper)

$$\ddot{\pi}_2 - c_s^2 \partial_i^2 \pi_2 + (3H + \gamma)\dot{\pi}_2 + \gamma\left(\alpha \dot{\pi}_1^2 - \frac{1}{2}(\partial_i \pi_1)^2\right) + \ldots = \text{Noise}$$

One might worry about the local approximation, since we're assuming that the result for the unperturbed background still applies in the gauge \delta\phi=0

However, this is the case as long as 'extrinsic curvature' effects for the equal time surfaces are suppressed, in other words

$$n_\chi(\dot{\phi}) \to n_\chi\left(\partial_{\tilde{t}} \phi(\tilde{t})\right) \quad \tilde{t} = t + \pi$$

up to effects suppressed by $\quad k_\star/\kappa \sim k_\star/\sqrt{g|\dot{\phi}|} \simeq \sqrt{\frac{\gamma H}{g\dot{\phi}}} \ll 1$

\kappa controls the k dependence of the state
(This is equivalent to Schwinger pair production where the wavelength of the particles is much shorter than the scale of variation of the electric field)

$$|\beta_k|^2 = e^{-\frac{\pi k^2}{a^2(t_i)\kappa^2(t_i)}}$$

$$M_\chi^2 \simeq g\dot{\phi}$$

A comment on the emergence of shift symmetry

first important result: $$\bar{\chi}_i^2 \simeq \frac{n_{\chi_i}(\dot{\phi}(t_i))}{g|\phi - \phi_i|}$$

which cancels the factor of \phi-\phi_i in the EOM, and appears to give us a shift invariant EOM

not so fast, because t_i depends upon \phi

This dependence is ultimately erased by the presence of the sum

$$S^{\rm int}_{\rm trap} = \int d^4x\sqrt{-g}\sum_i \frac{g^2(\phi-\phi_i)^2}{2}\chi_i^2 - V(\phi)$$

is invariant under \phi -> \phi+ c as long as we sum over large number of periods
(we can absorb c into a redefinition of \phi_i)

Therefore, the dynamics only dependence on the velocity of the clock, as we required

Incidentally, the same happened in the original model (also for scale invariance!)

$$\hat{m}^2\delta\phi + \int^t dt'\hat{m}^2\left(\frac{5}{2}\delta\dot{\phi}(t') - 3H\delta\phi(t')\right)\frac{a^3(t')}{a^3(t)}$$

Warm inflation

(Berera et al.)

main idea: $H^4 \ll T^4 \ll V(\phi)$ $\qquad \dot{\rho}_r = 0 \rightarrow \rho_r \sim \frac{\gamma}{H}\dot{\phi}^2$

The inflaton can slow roll w/out stringent conditions on V

Most promising model: two-staged dissipation, though not fully under control.
(similar to local trapped inflation, g^2\chi^2\phi^2, but fluctuations induced by thermal noise)

$$\mathcal{L}_{\chi\varphi} = hm\chi\varphi^2$$

$$m_\chi \gg T$$

$$\gamma(T) \simeq g^2 h^4 \left(\frac{m}{m_\chi}\right)^4 \frac{T^3}{m_\chi^2}$$

Despite the appealing solution of the \eta-problem, this is a very inefficient mechanism, needs 'boost factor' of O(10^6) for \gamma >H.

Nevertheless, this model also suggests NG of order

$$f_{\rm NL} \sim \gamma(T)/H$$

(The story is slightly different, because of the existence of a 'second clock', e. g. the radiation fluid
However, since m_\chi >>T, the particular are created at rest, then u_\chi ~ n
(u_\chi enters in the computation of \gamma, 'rest frame of the fluid')

Dissipative effects in inflation: An EFT approach

Summary of ideas/results:

- Introduce new scalar, vector, tensor ADOF into EFT of single clock inflation, like in the GW case.
- Write the most general action. Use symmetries to couple to \pi (the fluctuations of the clock)
- Basic hypothesis on the dynamics of O: Preferred clock + emergent symmetry.
- Find relevant scales that permit a derivative expansion, e.g. dissipative time scale much faster than Hubble expansion.
- Perform matching to determine Green's functions and correlation functions.
- Compute power spectrum (dominated by the noise for large friction)
- Obtain non-linear terms from the enforcement of the symmetries and single clock hypothesis
- Compute NG and shapes - All models reduced to a few new parameters