Title: Effective field theories for hydrodynamical systems

Date: Nov 30, 2011 09:30 AM

URL: http://pirsa.org/11110090

Abstract: <span>

<p>I
will review EFT techniques that have been developed recently for dealing with the infrared dynamics of ordinary fluids and of superfluids. Gravity does not play an essential role in the construction (though it can be added straightforwardly to the system), yet certain applications resemble very closely the EFT approach to gravity wave emission by binary systems. I will describe in some detail one such application, as well as a possible application to cosmology.</p></span>

# Motivation

Fluids are everywhere:

- Around us
- In the lab
- In space

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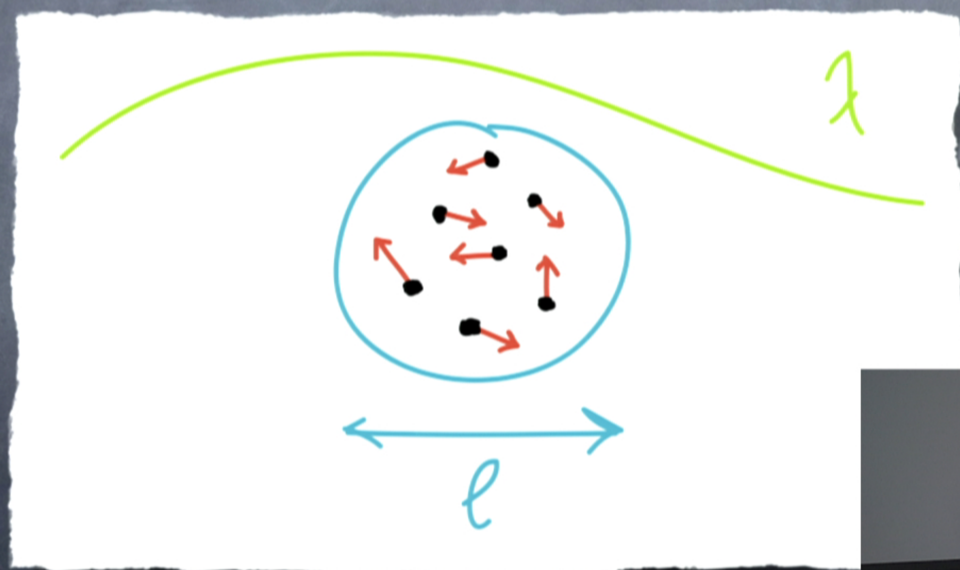
# Hydrodynamics

$\lambda \gg \ell \qquad \omega \ll 1/\tau$




3

Continuity + Euler equations:

$$\partial_\mu T^{\mu\nu} = 0$$

"Constitutive" relations:

$$T^{\mu\nu} = f(u^\mu, \rho, p, \ldots, \partial)$$

Derivative expansion:

$$(\ell \cdot \vec{\nabla})^n \,, \qquad (\tau \cdot \partial_t)^n$$

4

Continuity + Euler equations:

$$\partial_\mu T^{\mu\nu} = 0$$

"Constitutive" relations:

$$T^{\mu\nu} = f(u^\mu, \rho, p, \ldots, \partial)$$

highly non-trivial

Derivative expansion:

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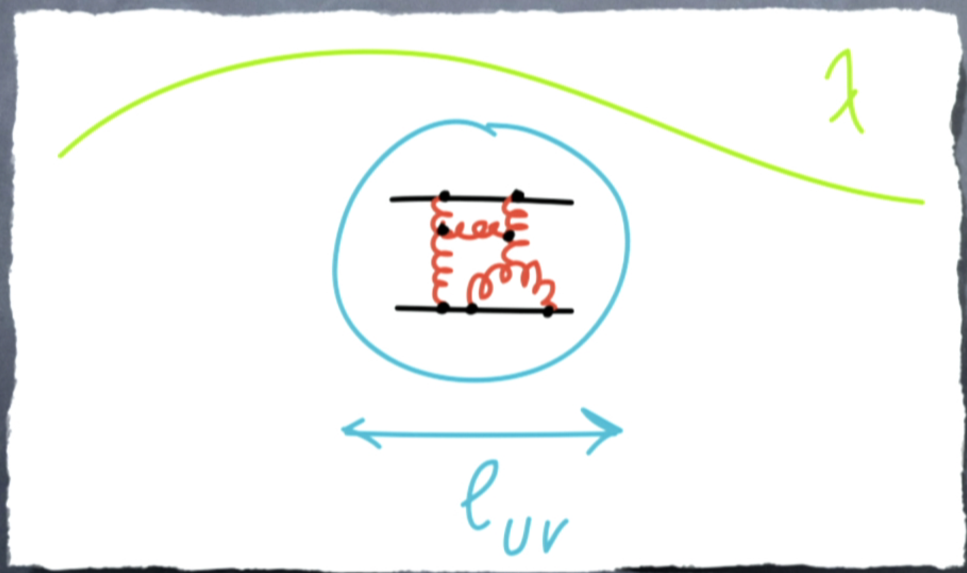
4

# Effective field theory

$\lambda \gg \ell_{UV} \qquad \omega \ll \omega_{UV}$




5

IR degrees of freedom + symmetries:

$$\Phi \to G[\Phi]$$

Lagrangian:

$$\mathcal{L} = f(\Phi, \partial)$$

Derivative expansion:

$$(\ell_{UV} \cdot \vec{\nabla})^n \,, \qquad (1/\omega_{UV} \cdot \partial_t)^n$$

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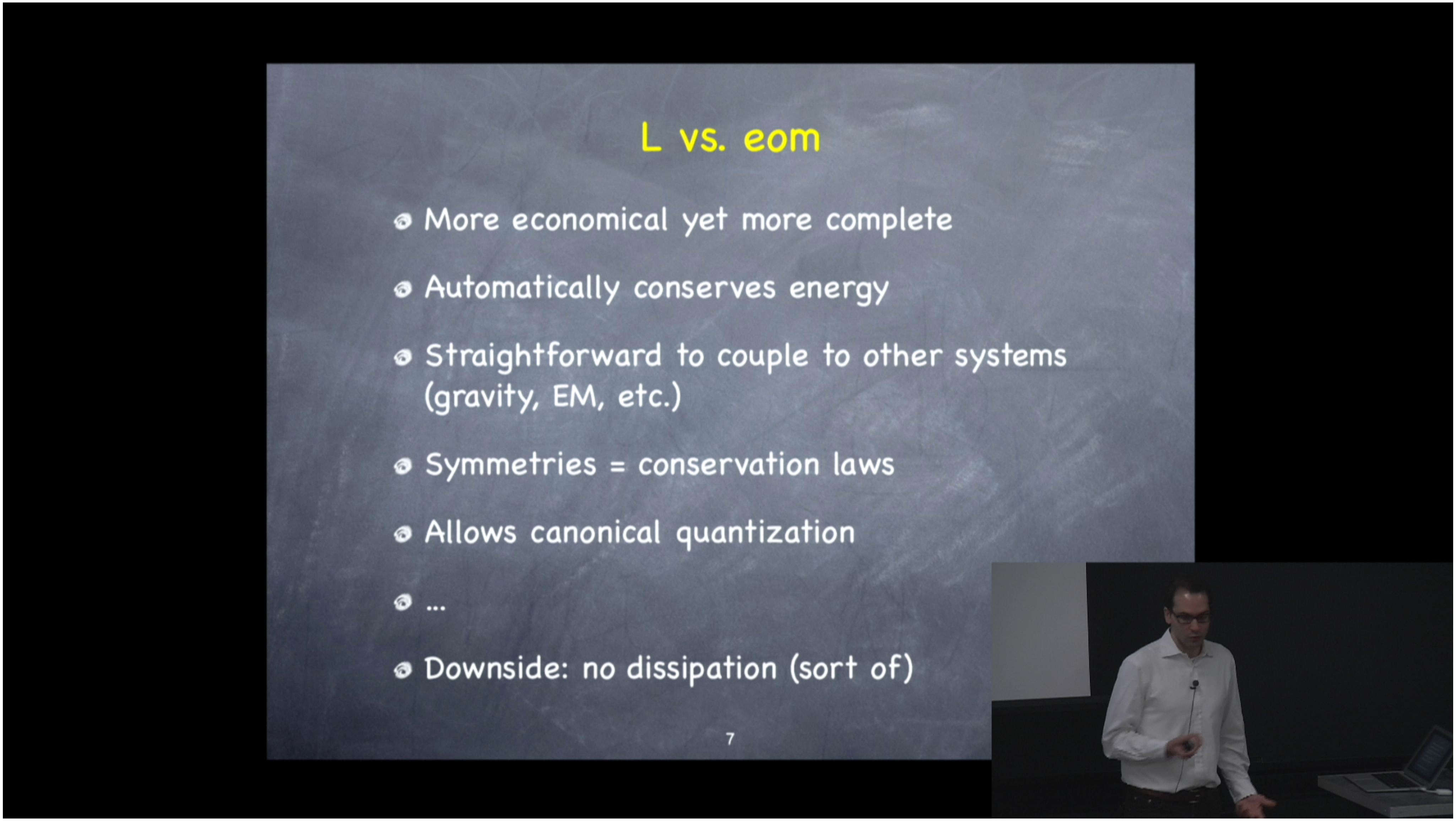
# L vs. eom

- More economical yet more complete
- Automatically conserves energy
- Straightforward to couple to other systems (gravity, EM, etc.)
- Symmetries = conservation laws
- Allows canonical quantization
- ...
- Downside: no dissipation (sort of)

7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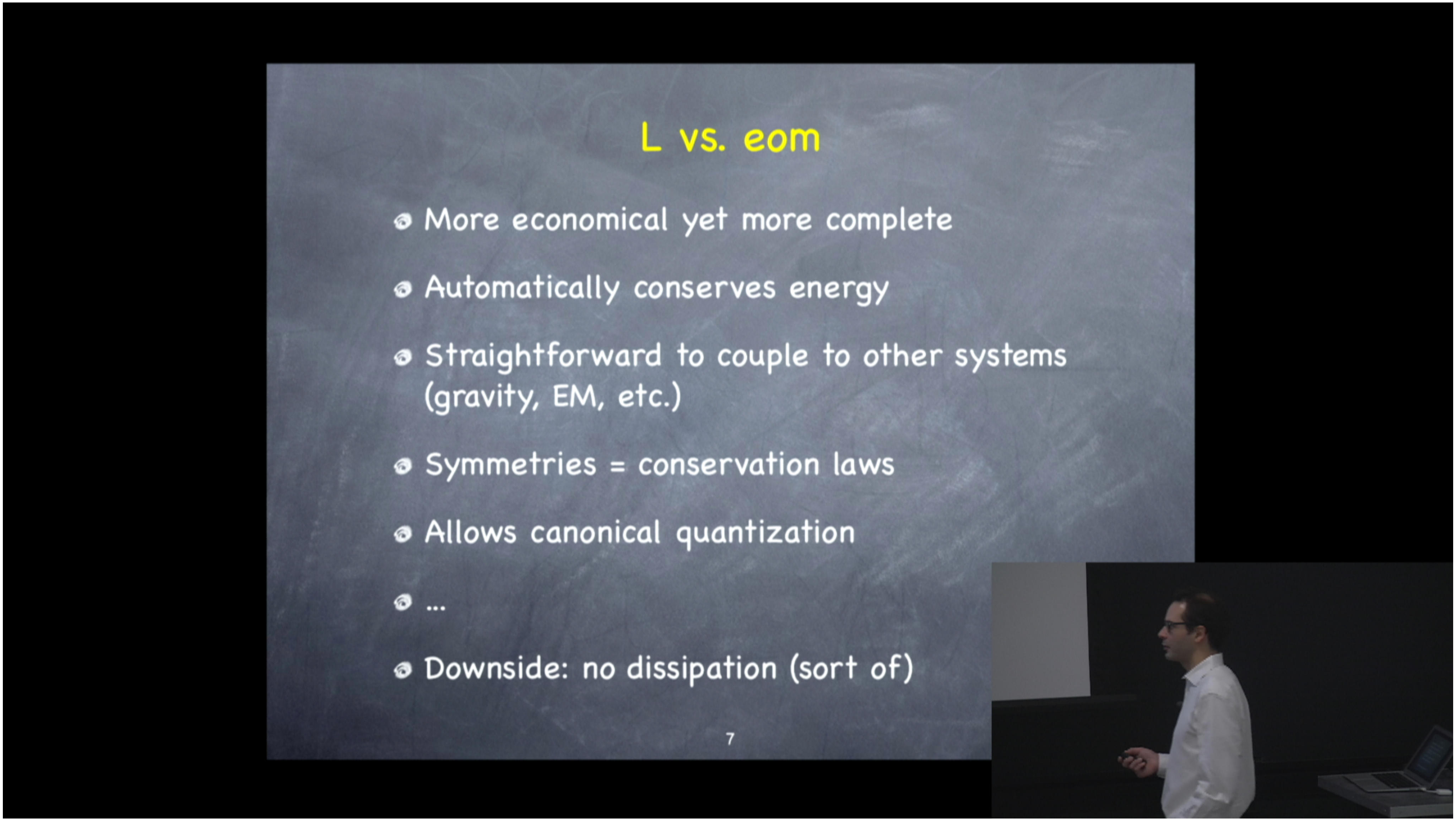
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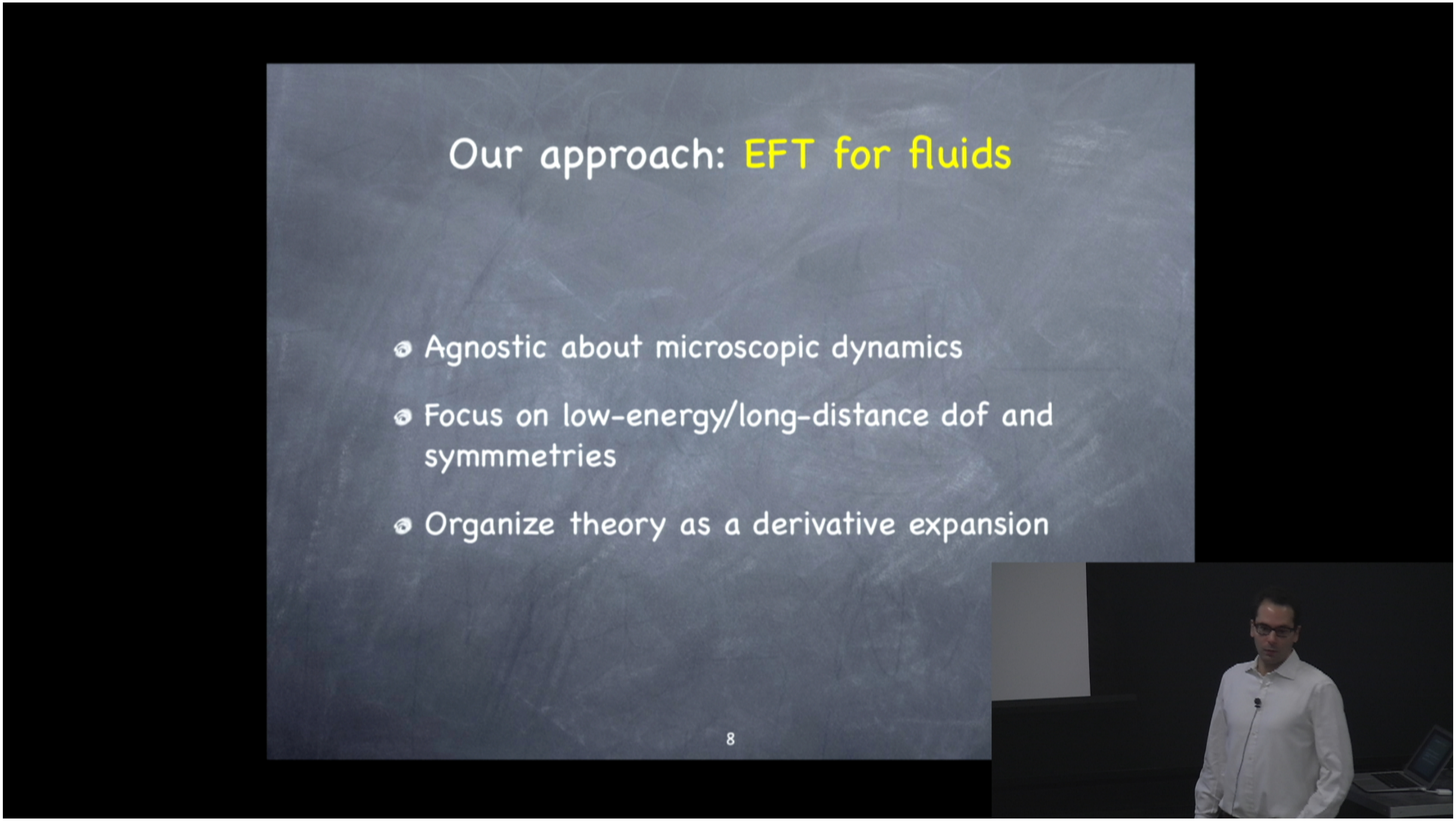
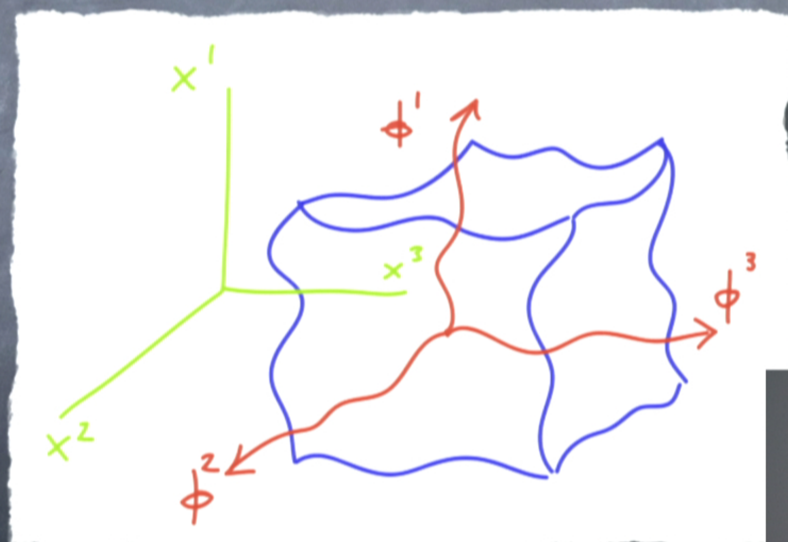

Our approach: EFT for fluids
Agnostic about microscopic dynamics
Focus on low-energy/long-distance dof and symmmetries
Organize theory as a derivative expansion
8

**Dof:** mapping internal $\longleftrightarrow$ target spaces

$$\phi^I(\vec{x}, t) \qquad I = 1, 2, 3$$




9

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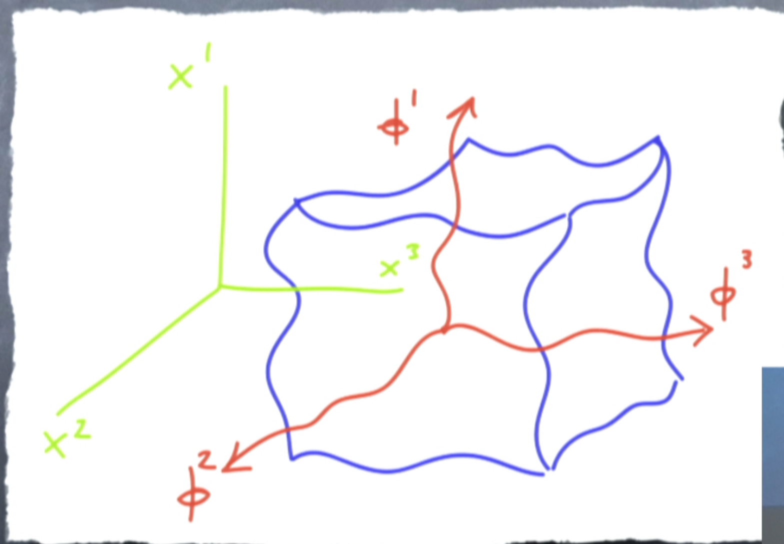


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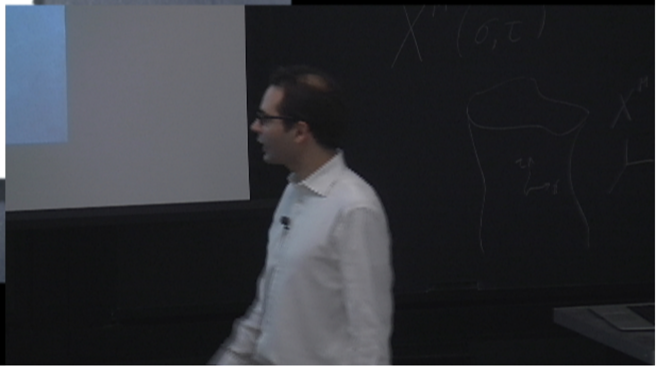

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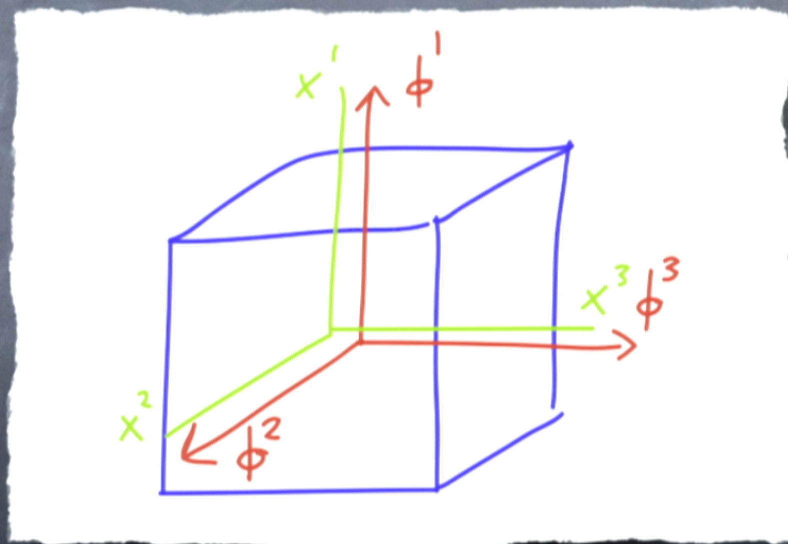

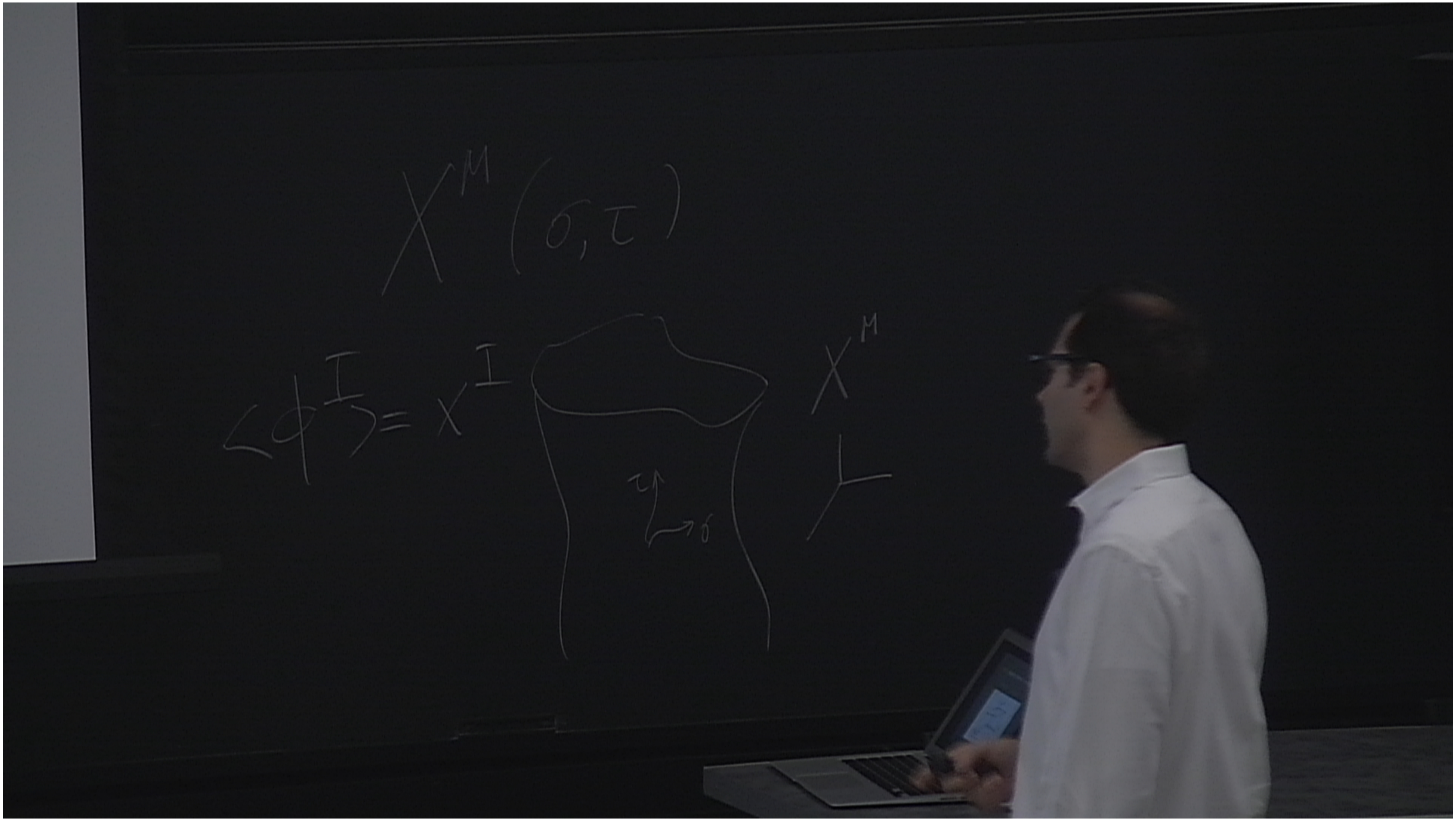
X^M(σ, τ)
<φ^I> = x^I
X^M
τ
σ

## Symmetries: Poincaré + internal

$$\phi^I \to \phi^I + a^I$$

$$\phi^I \to SO(3)\,\phi^I$$

$$\phi^I \to \xi^I(\phi) \qquad \det \frac{\partial \xi^I}{\partial \phi^J} = 1$$

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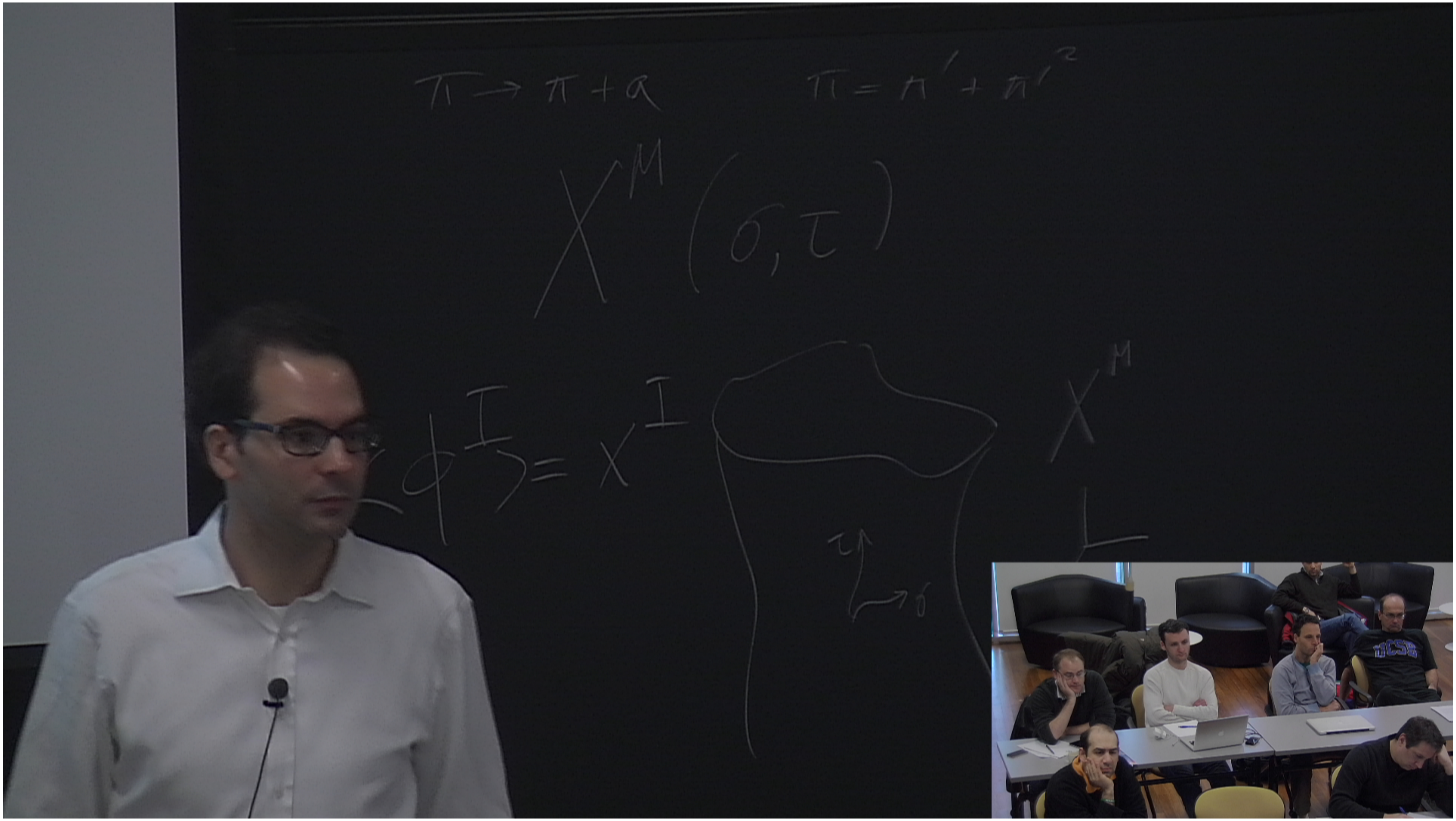

π → π + a
π = π' + π'²
X^M (σ, τ)
⟨φ^I⟩ = x^I
τ
σ
X^M
⊥

**Action:** $S = \int d^4x \, F(B)$ $\quad B = \det \partial_\mu \phi^I \partial^\mu \phi^J$

Correct hydrodynamics ( $T_{\mu\nu}$ + eom)

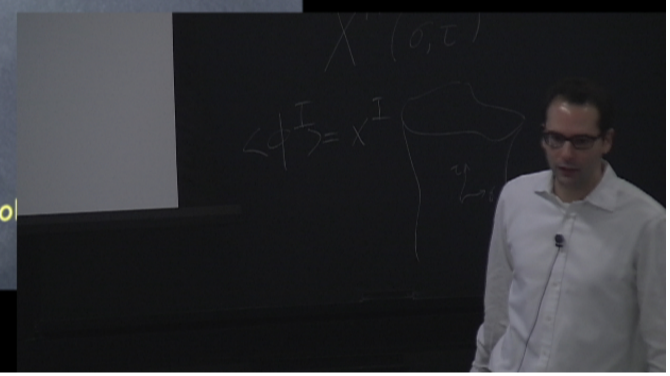
with

$$\rho = -F$$
$$p = F - 2F'B$$
$$u^\mu = \frac{1}{6\sqrt{B}} \epsilon\epsilon \, \partial\phi\partial\phi\partial\phi$$

Relativistic, non-linear

(Dubovsky, Gregoire, Nicol

11



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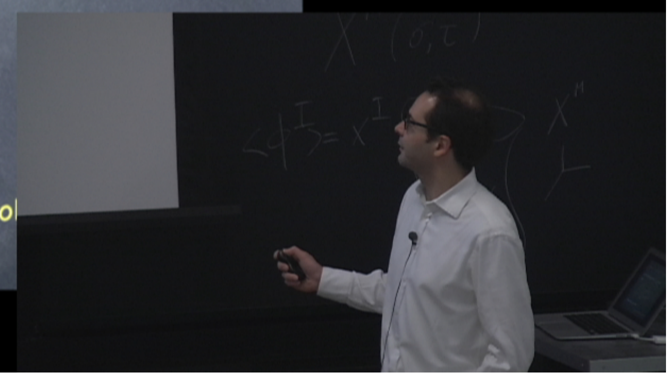
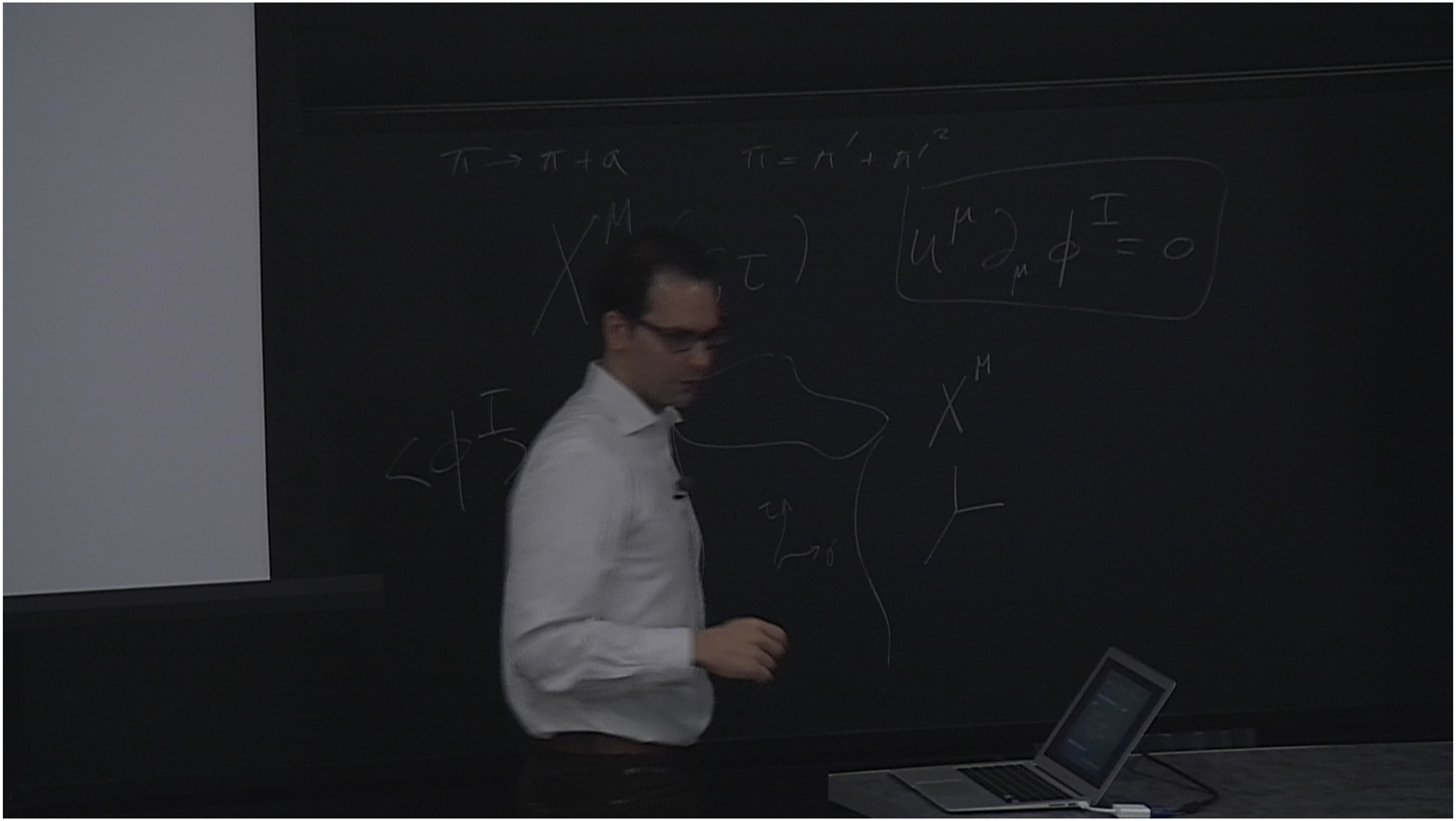
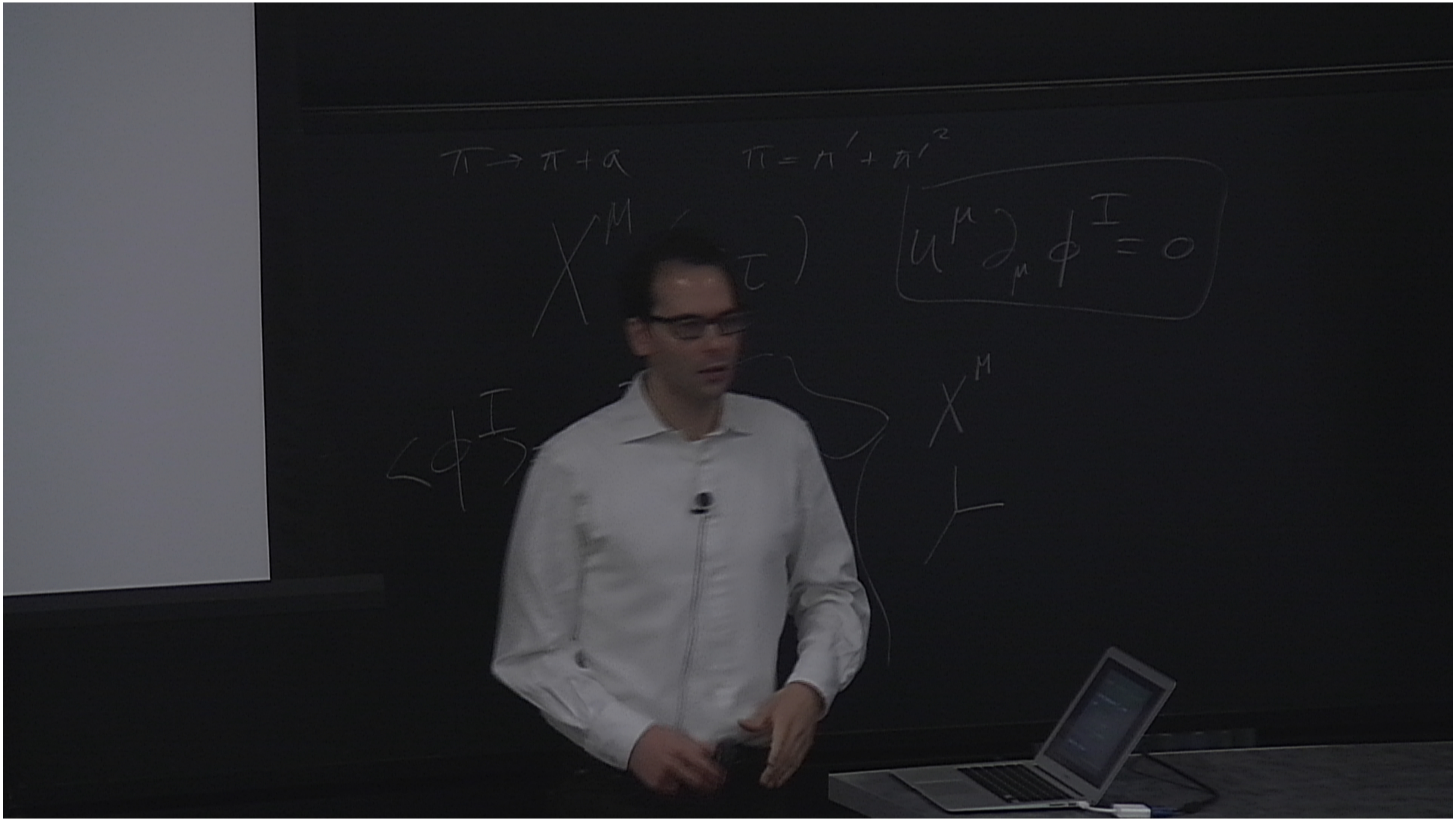



π → π + a
π = π' + π'²
u^μ ∂_μ φ^I = 0
X^M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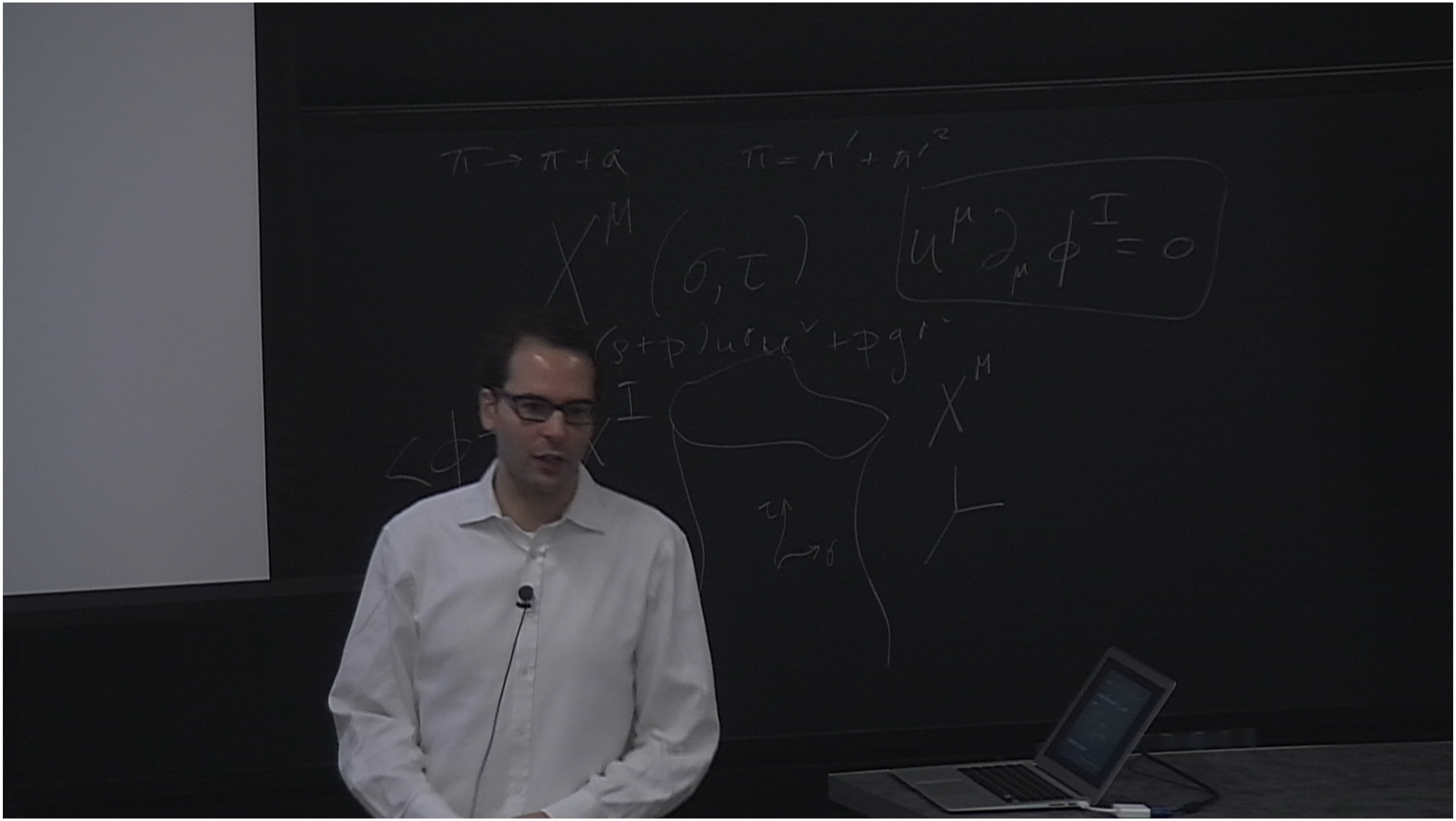
$\pi = \pi' + \pi'^2$
$u^\mu \partial_\mu \phi^I = 0$


π → π + a
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X^M (σ, τ)
u^μ ∂_μ φ^I = 0
X^M

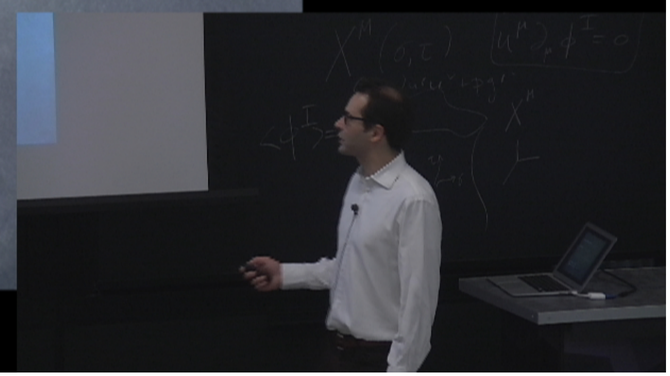
ground state (at given p): $\phi^I = x^I$

Goldstones: $\phi^I = x^I + \pi^I$

$$\mathcal{L} \to \left(\dot{\pi}^I\right)^2 - c_s^2\left(\partial_I \pi^I\right)^2 + \text{interactions}$$

longitudinal = sound $\omega = c_s k$

transverse = vortices $\omega = 0$



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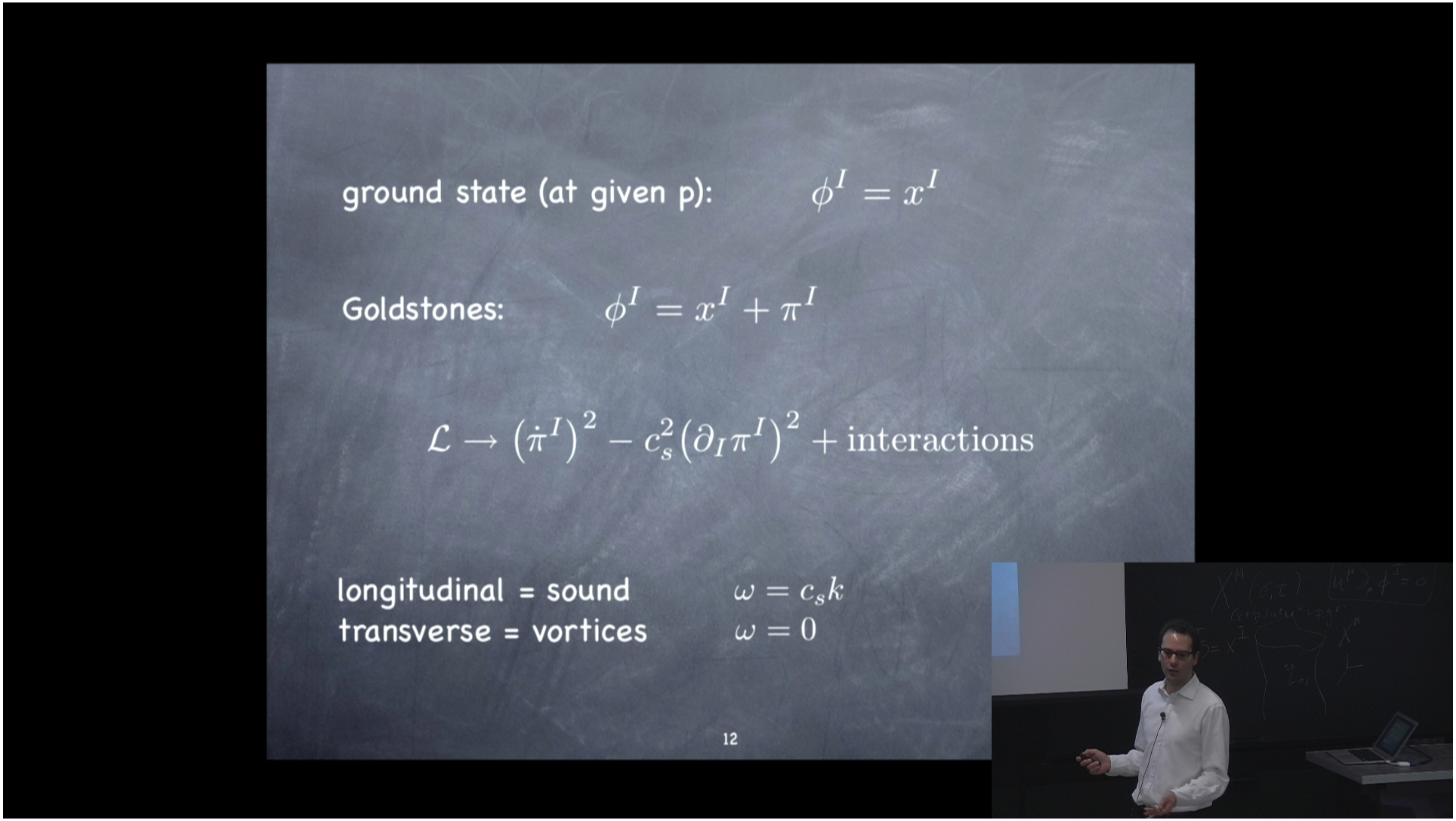
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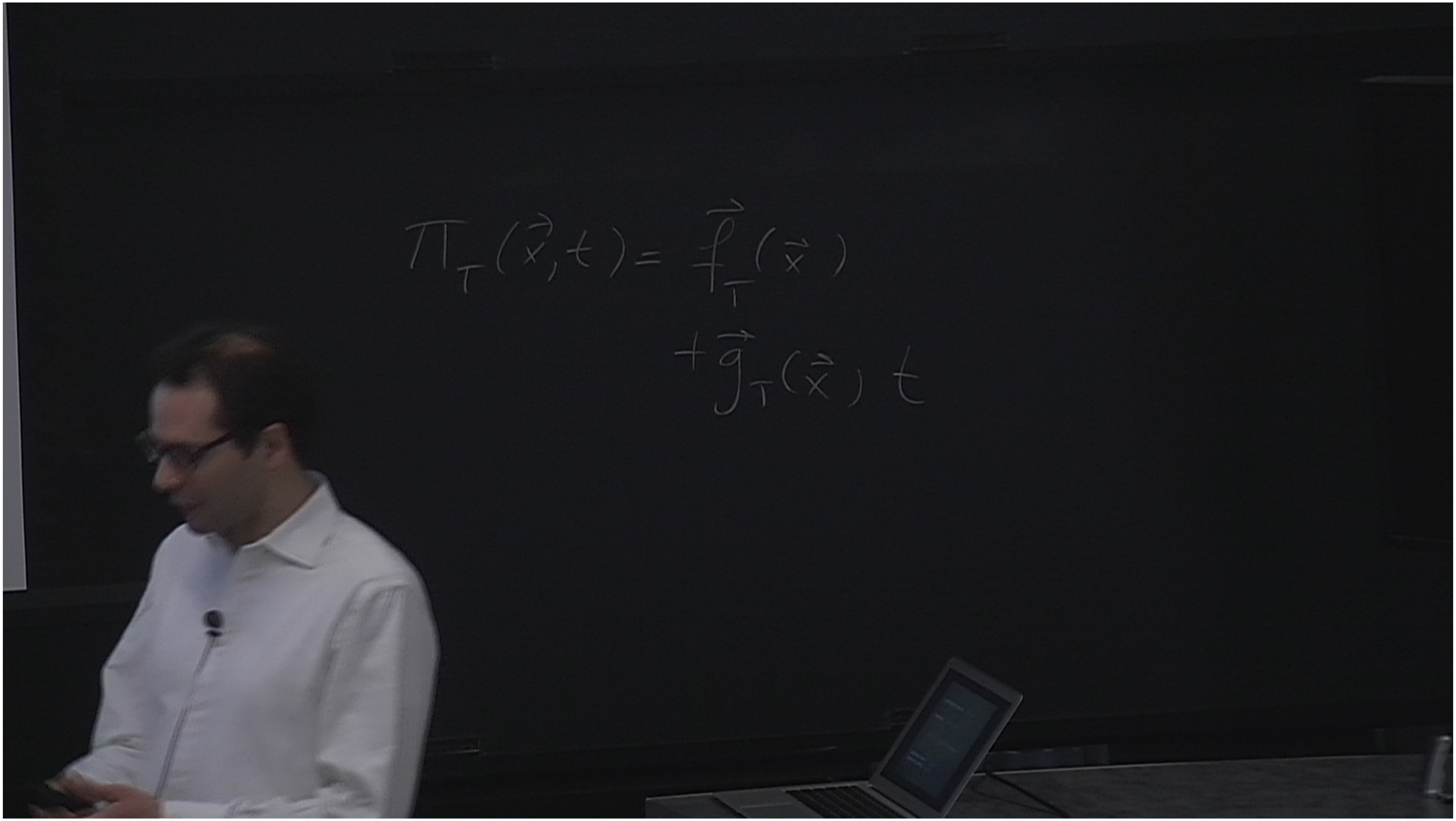
$\pi_T(\vec{x}, t) = \vec{f}_T(\vec{x})$
$+ \vec{g}_T(\vec{x})\, t$

# Applications

- Sound-vortex interactions
- Hall viscosity in 2+1
- Fluids with quantum anomalies
- Finite T relativistic superfluids
- Quantum hydrodynamics
- Dissipative hydrodynamics
- "Backreaction" in cosmology

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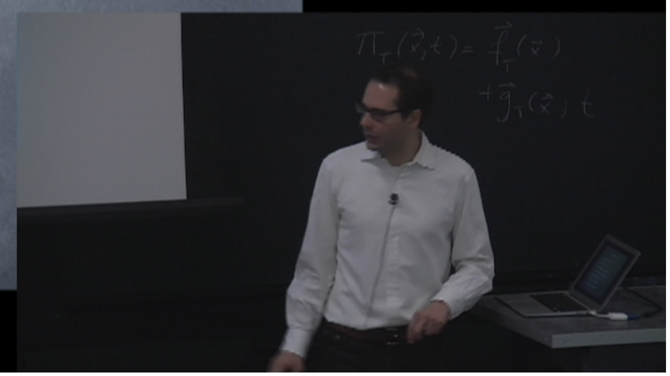

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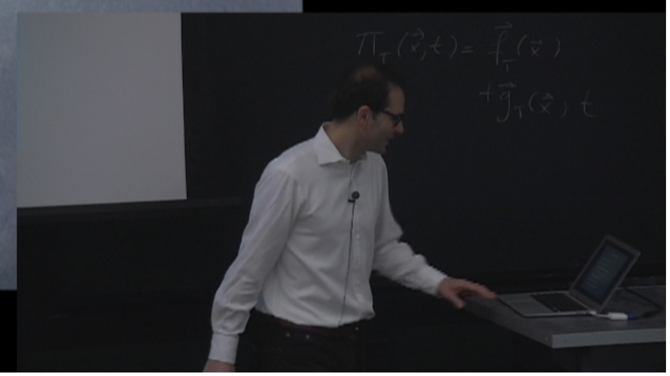
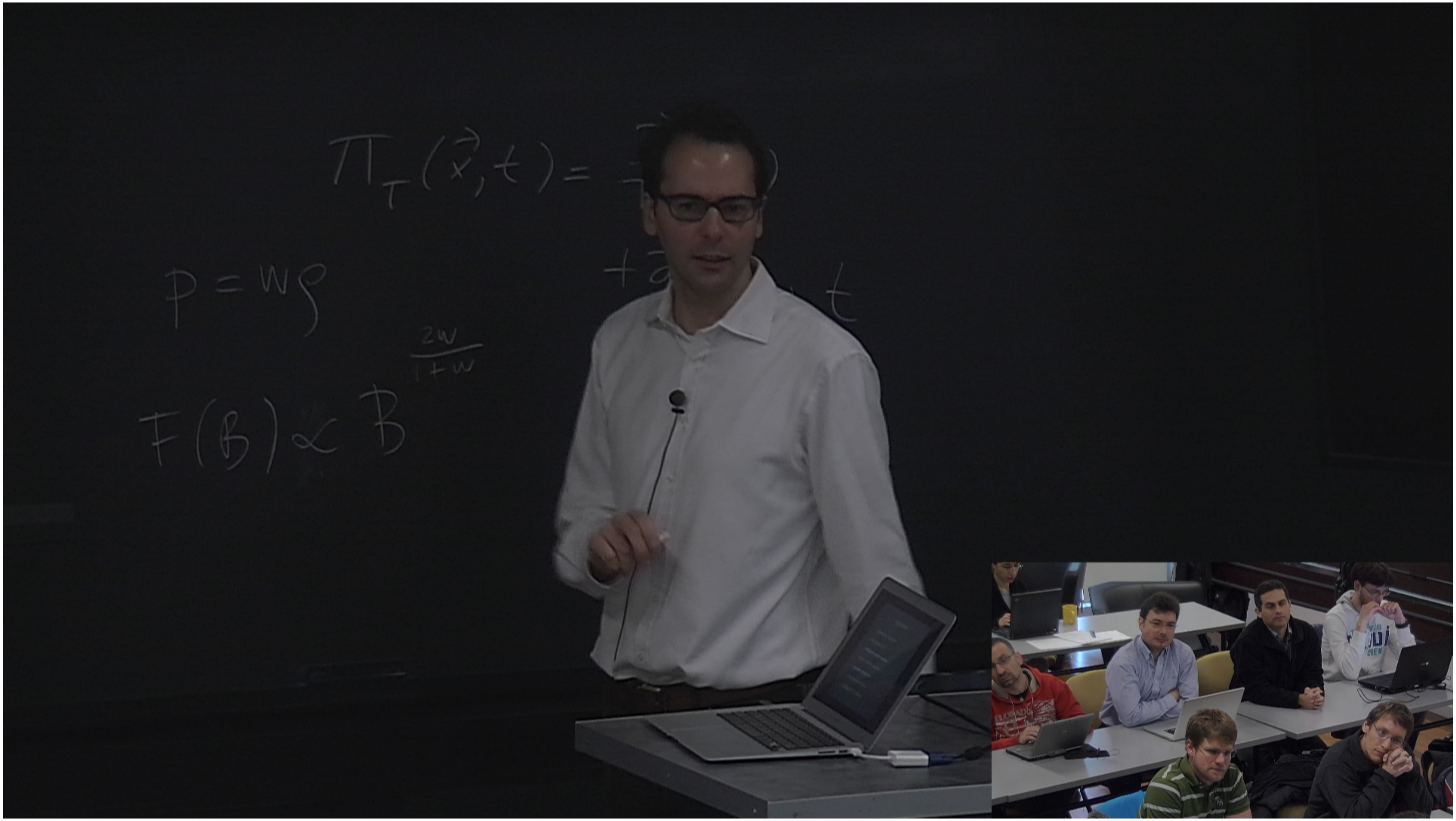
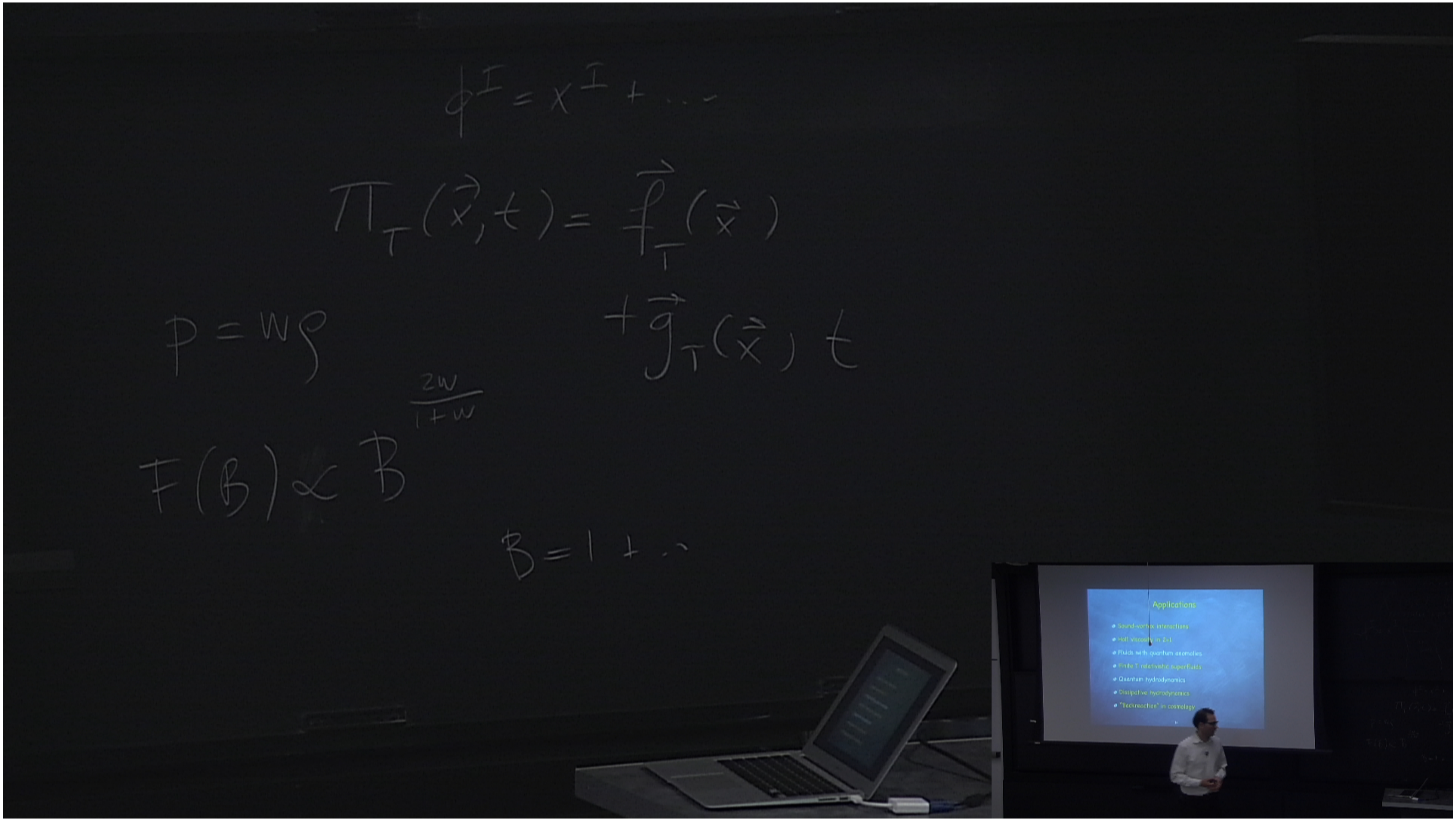
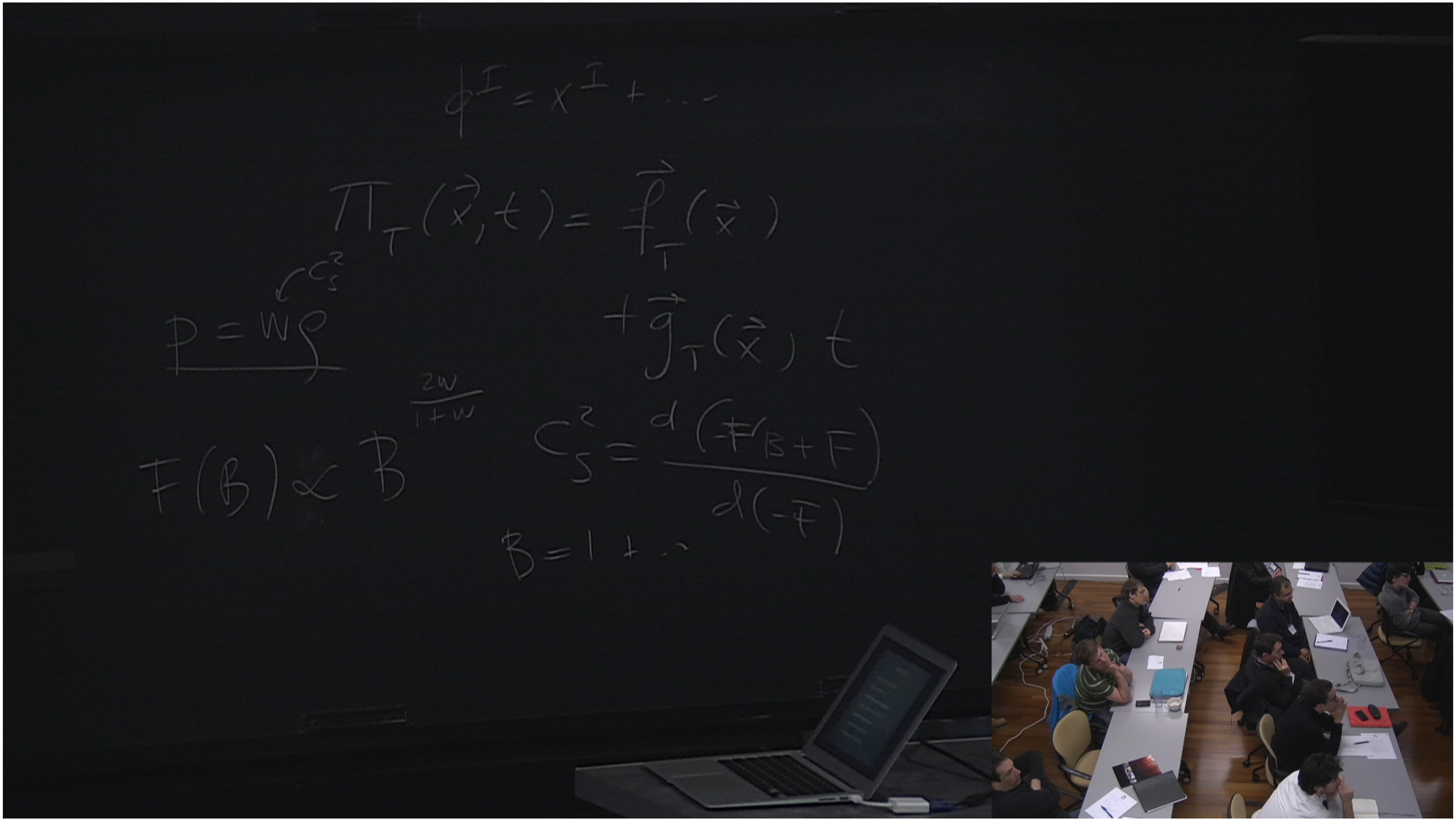



$\pi_T(\vec{x}, t) =$
$p = w\rho$
$F(B) \propto B^{\frac{2w}{1+w}}$


φ^I = x^I + ...
π_T(x⃗, t) = f⃗_T(x⃗)
+ g⃗_T(x⃗) t
p = wρ
F(B) ∝ B^(2w/(1+w))
B = 1 + ...
Applications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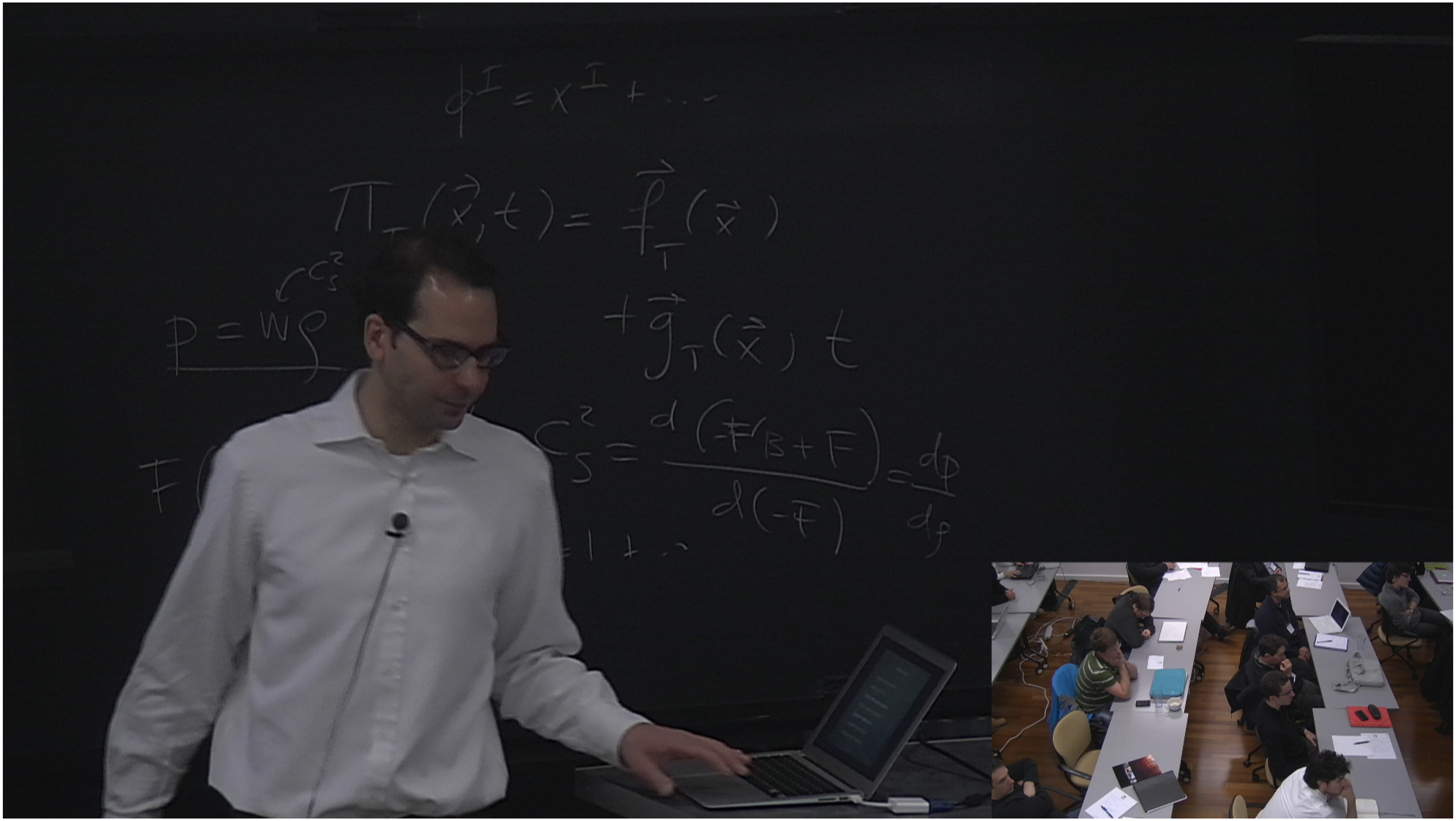
+ g⃗_T(x⃗) t
c_s^2
P = wρ
2w/(1+w)
F(B) ∝ B
c_s^2 = d(-F'B + F) / d(-F)
B = 1 + ...



$\phi^I = x^I + \ldots$
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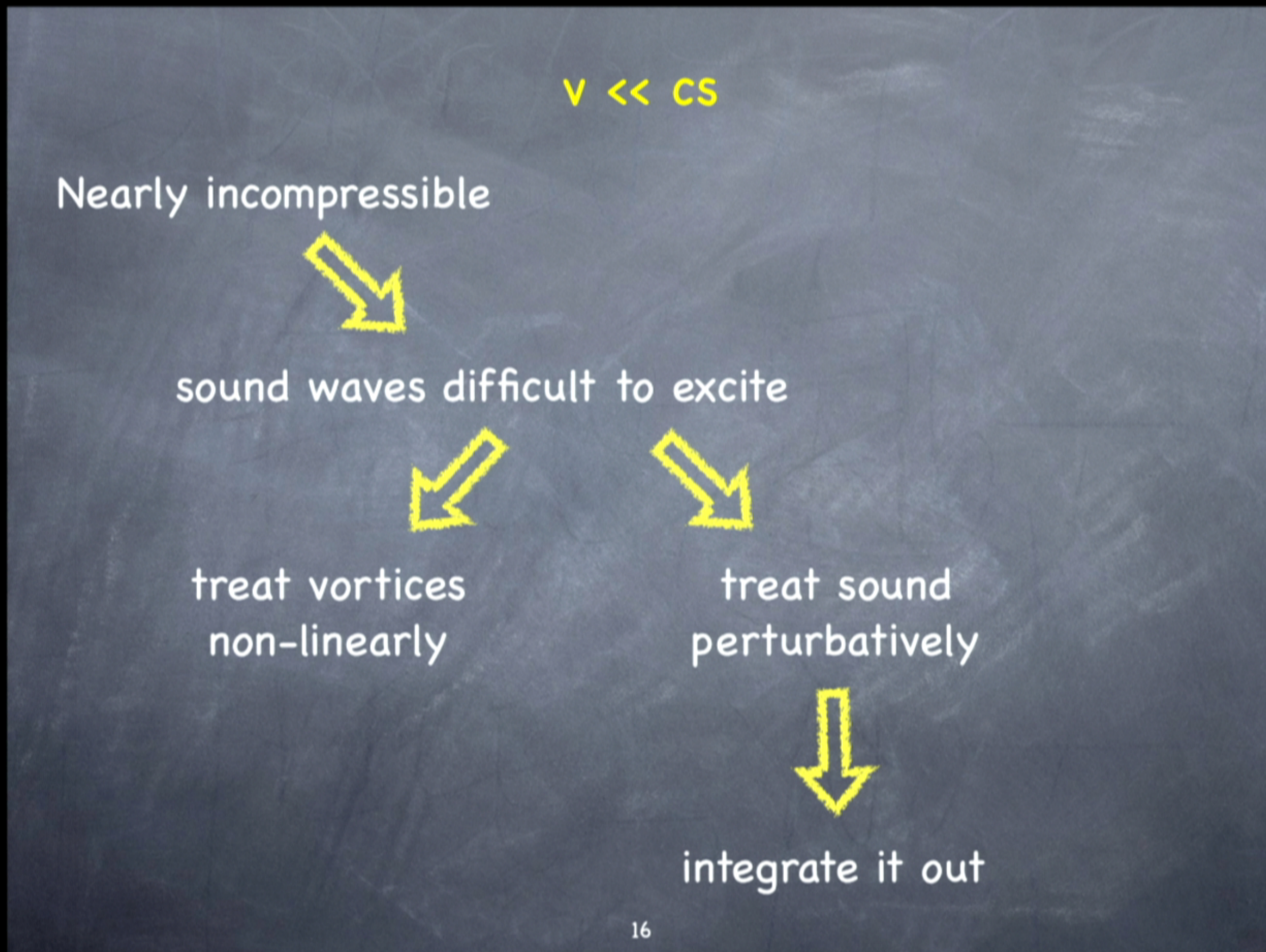
v << cs
Nearly incompressible
sound waves difficult to excite
treat vortices non-linearly
treat sound perturbatively
integrate it out
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# 1. Sound-vortex interactions

(Endlich, Nicolis, soon)

# Vortex-sound decomposition

$$\vec{x}(\vec{\phi},t) = \vec{x}_0(\vec{\phi},t) + \delta\vec{x}(\vec{\phi},t)$$

$$(J_0)^i{}_j \equiv \frac{\partial x_0^i}{\partial \phi^j} \,, \qquad \det J_0 = 1$$

$$\delta\vec{x} = \vec{\nabla}_0 \psi(\vec{x}_0,t) \,, \qquad \vec{\nabla}_0 \equiv \frac{\partial}{\partial \vec{x}_0} = (J_0^T)^{-1} \cdot \frac{\partial}{\partial \vec{\phi}}$$

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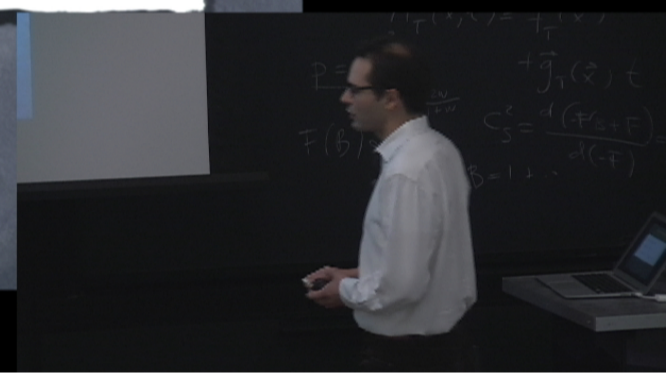

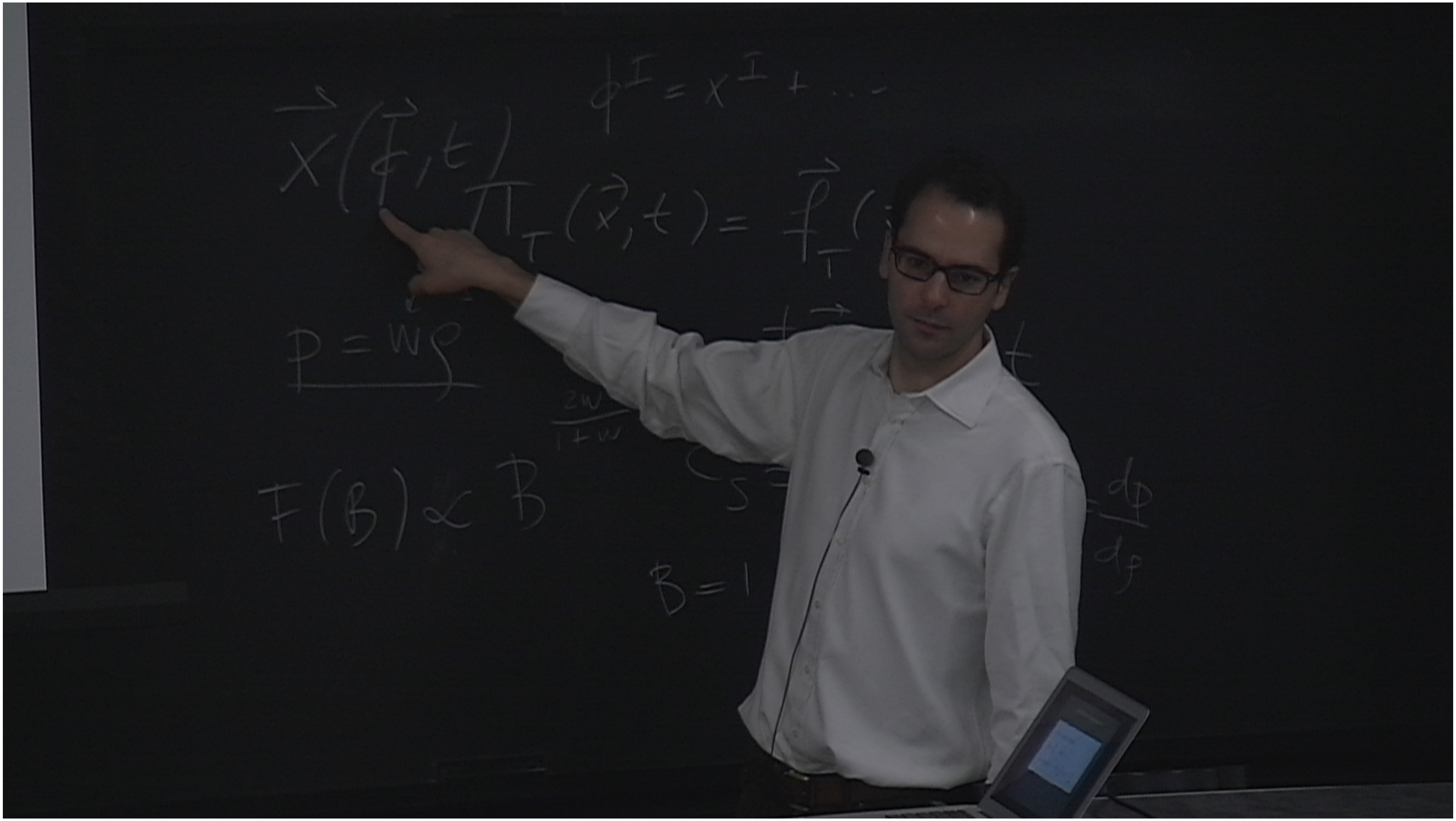
$\phi^I = x^I + \ldots$
$p = w\rho$
$F(B) \propto B$
$B = 1$

# The action, expanded

$$S = S_{x_0} + S_\psi + S_{\rm int}$$

$$S_{x_0} = (\rho + p) \int d^3\phi dt \big[ \tfrac{1}{2} v_0^2 + \tfrac{1}{8} v_0^4 \big( 1/c^2 - c_s^2/c^4 \big) + \ldots \big]$$

$$S_\psi = (\rho + p) \int d^3 x_0 dt \big[ \tfrac{1}{2} \big( \nabla_0 \dot\psi \big)^2 - \tfrac{1}{2} c_s^2 \big( \nabla_0^2 \psi \big)^2 + \ldots \big]$$

$$S_{\rm int} = (\rho + p) \int d^3 x_0 dt \big[ - \tfrac{1}{2} c_s^2/c^2 \, (\nabla_0^2 \psi) \, v_0^2 - \vec\nabla_0 \psi \cdot \big( \vec v_0 \cdot \vec\nabla_0 \big) \vec v_0 + \ldots \big]$$

$$v_0 \equiv \partial_t x_0(\vec\phi, t)$$

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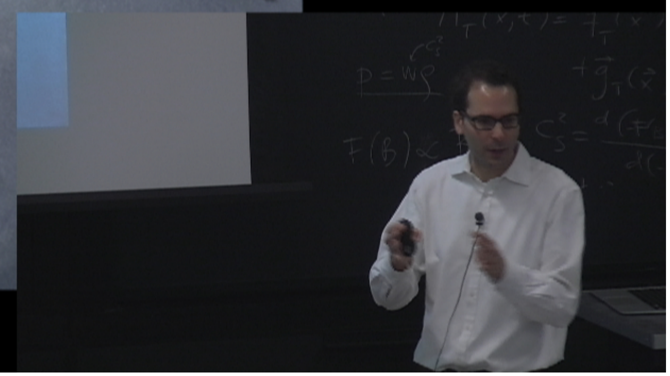
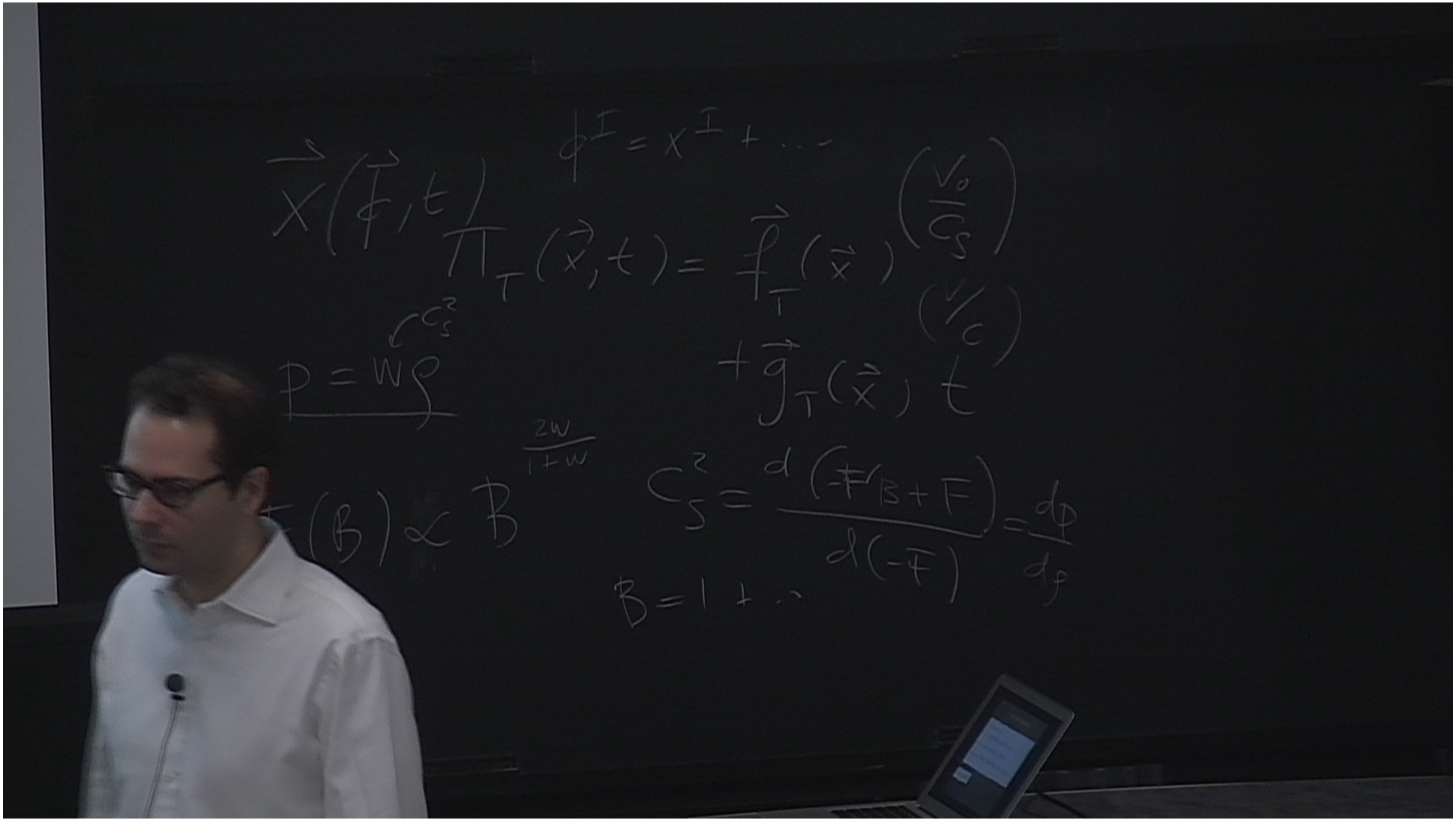
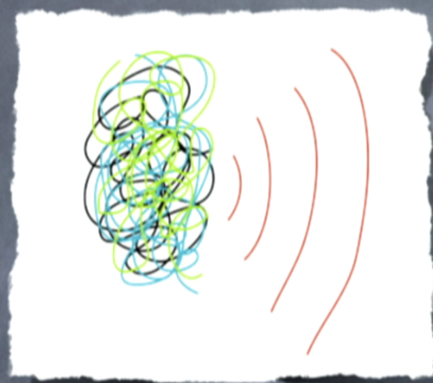




ϕ^I = x^I + ...
p = wρ
B = 1 + ...

# The sound of turbulence



= Im (  )

$$P = \frac{\rho + p}{c_s^5} \langle \ddot{Q} \ddot{Q} \rangle$$

$$Q_{ij} \equiv \int d^3x \left( v_i v_j - \frac{c_s^2}{c^2} v^2 \, \delta_{ij} \right)$$

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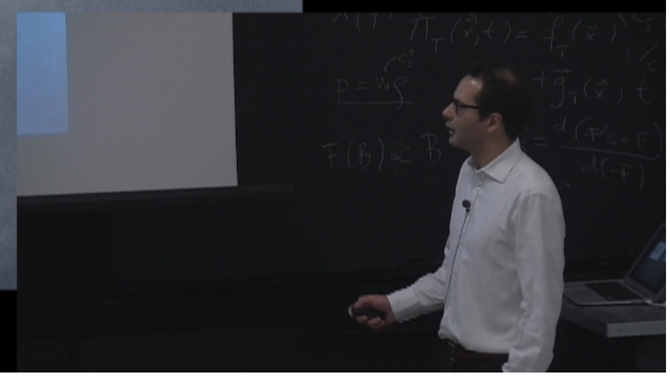

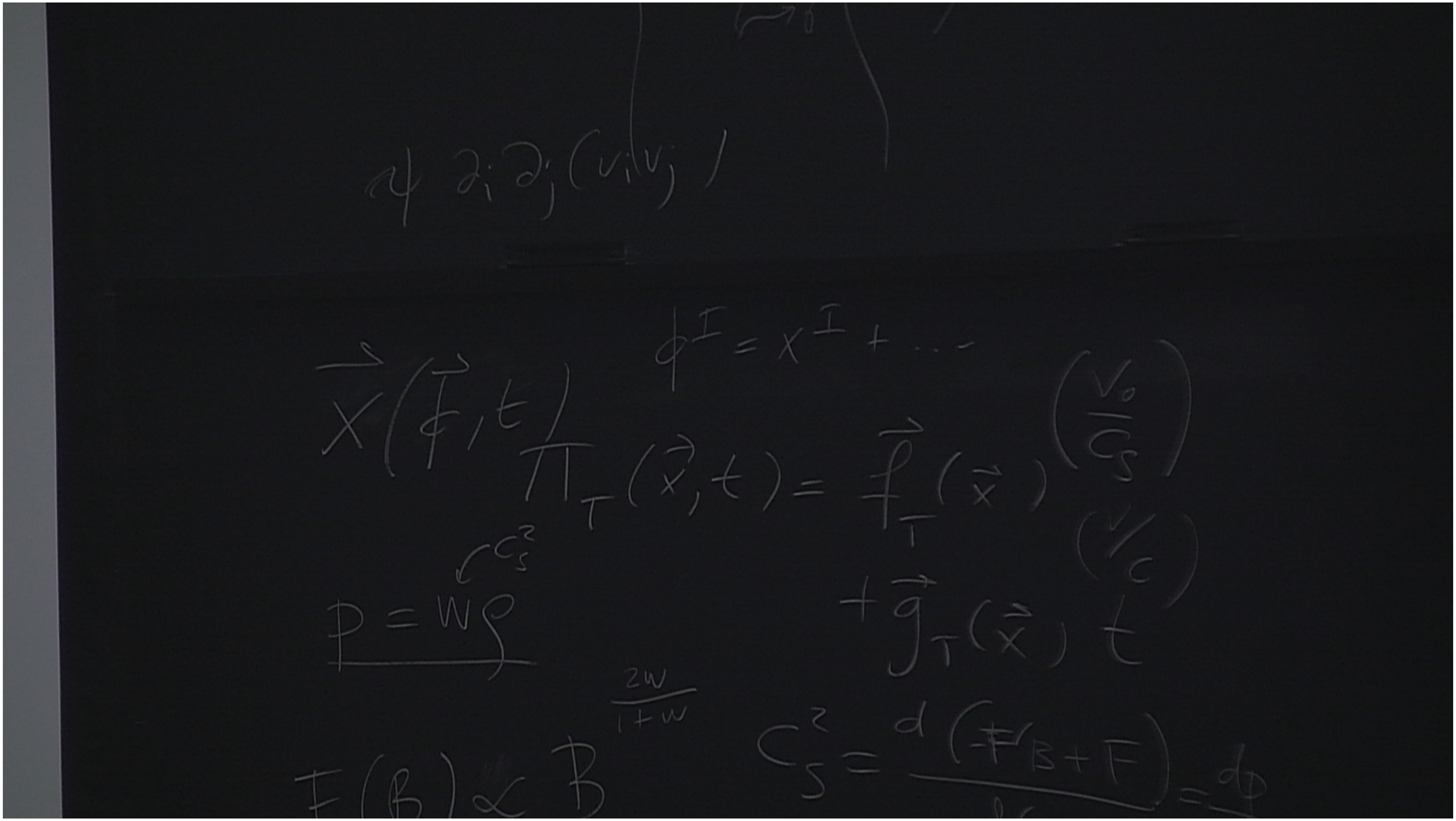
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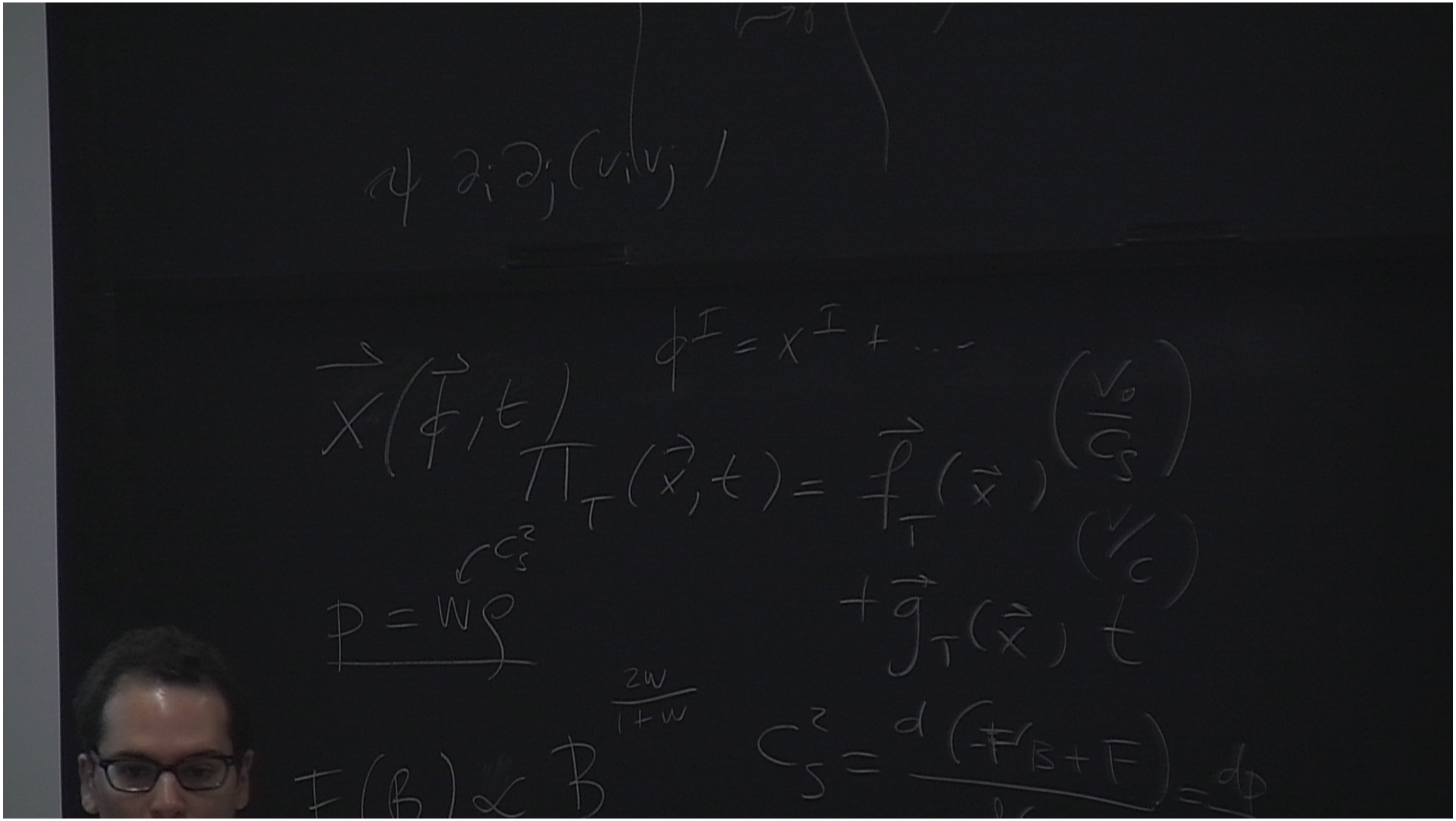

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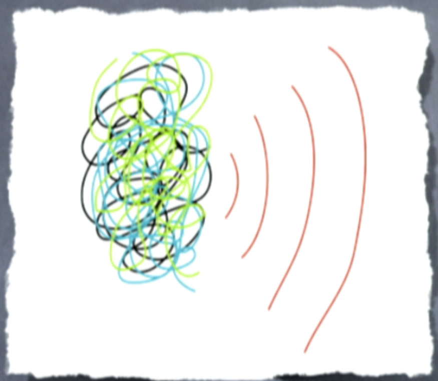

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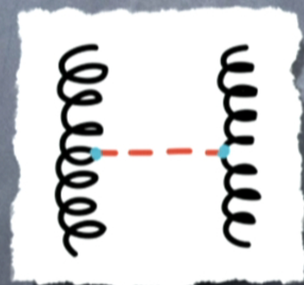
## Sound mediated vortex-vortex potential

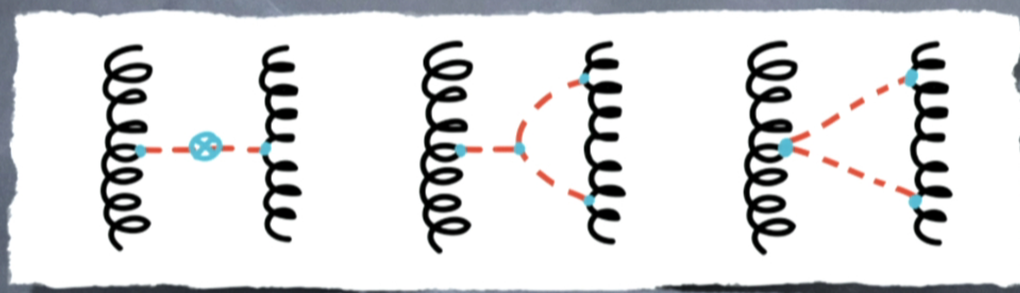
$$\frac{1}{\omega^2 - c_s^2 k^2} \to -\frac{1}{c_s^2 k^2} + \frac{\omega^2}{c_s^4 k^4} + \dots , \qquad \omega \ll c_s k$$

Vanishes at leading order



= 0

# Next to leading contributions



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$$\mathcal{O}(1)\frac{iw_0}{c_s^4}\frac{1}{12}\int dt\left(\dot{T}_1^{ik}\dot{T}_2^{jn}+T_1^{ik}T_2^{jn}V_1^aV_2^b\partial_{12}^a\partial_{12}^b\right)\partial_{12}^i\partial_{12}^k\partial_{12}^j\partial_{12}^n\left(\frac{1}{4\pi}\left|\vec{X}_{12}\right|^3\right)$$

$$\mathcal{O}(1)\frac{iw_0}{c_s^4}\int dt\Big[2T_1^{jB}P_2^l\left(\partial_{12}^j\partial_{12}^B\partial_{12}^s\partial_{12}^l\left(\frac{-1}{8\pi}\left|\vec{X}_{12}\right|\right)\right)\left(\dot{T}_1^{iA}+T_1^{iA}V_1^a\partial_{12}^a)\right)\left(\partial_{12}^i\partial_{12}^A\partial_{12}^s\left(\frac{-1}{8\pi}\left|\vec{X}_{12}\right|\right)\right) \quad (14)$$

$$+T_1^{iA}T_1^{jB}T_2^{nm}\left(\partial_{12}^i\partial_{12}^A\partial_{12}^n\partial_{12}^s\left(\frac{-1}{8\pi}\left|\vec{X}_{12}\right|\right)\right)\left(\partial_{12}^j\partial_{12}^B\partial_{12}^m\partial_{12}^s\left(\frac{-1}{8\pi}\left|\vec{X}_{12}\right|\right)\right) \quad (15)$$

$$+\Big(1\leftrightarrow 2\Big)\Big] \quad (16)$$

$$\mathcal{O}(1)\frac{iw_0}{c_s^4}\int dt\Big[3T_1^{iA}T_1^{nB}T_2^{mC}\frac{1}{2^{11}}\frac{1}{\pi^2}\left(\partial_{12}^C\partial_{12}^m\partial_{12}^r\partial_{12}^s\left(\frac{1}{\left|\vec{X}_{12}\right|}\right)\right)$$

$$*\left(\delta^{Bn}\delta^{Ai}\delta^{rs}+14 \text{ permutations}\right)$$

$$+\Big(1\leftrightarrow 2\Big)\Big]$$

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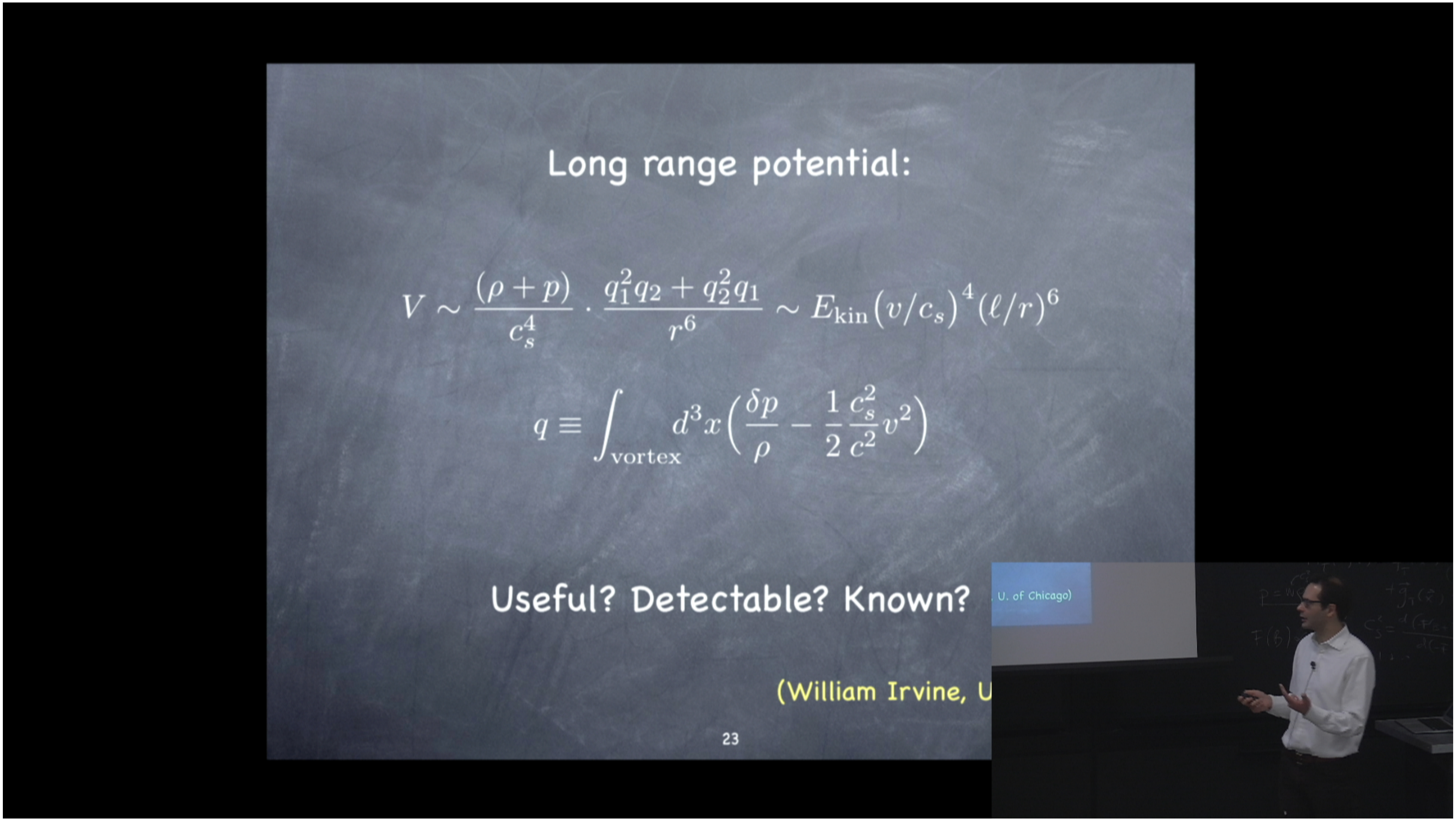
Long range potential:
$$V \sim \frac{(\rho + p)}{c_s^4} \cdot \frac{q_1^2 q_2 + q_2^2 q_1}{r^6} \sim E_{\rm kin} (v/c_s)^4 (\ell/r)^6$$
$$q \equiv \int_{\rm vortex} d^3x \left( \frac{\delta p}{\rho} - \frac{1}{2} \frac{c_s^2}{c^2} v^2 \right)$$
Useful? Detectable? Known?
(William Irvine, U
U. of Chicago)
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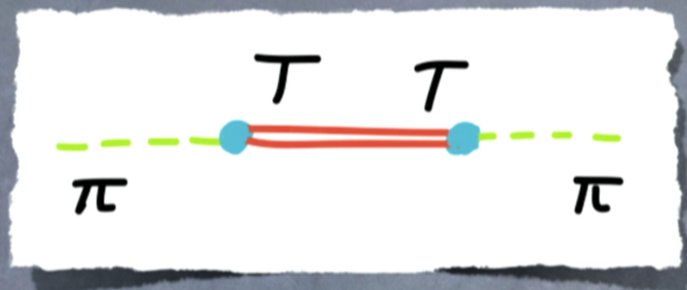
- for t-independent perturbations, these extra dof should follow (= operational def of "living in the fluid")

$$S_{\rm int} = \int T_{ij}\, \partial^i \pi^j (\vec{x})$$

- Everything works if we postulate the same coupling for t-dependent as well

- Still very confused about how to make this more systematic (e.g., non-linear order)

- If you buy that coupling, then




$$\eta, \zeta \propto \lim_{\omega \to 0} \lim_{k \to 0} \frac{1}{\omega} \mathrm{Im}\langle T_{ij} T_{kl} \rangle$$

- matches precisely "Kubo relations" for viscosity coefficients

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## Conclusions

- Compact and systematic EFT for hydrodynamical systems
- Reproduces standard properties
- Still unclear how to deal with dissipation systematically
- Suggests--and answers--new questions (perturbation theory)
- So far, mostly playing. Important applications? (e.g.: $\eta/s$)

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