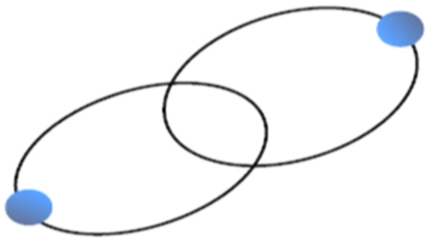
Title: Conservative binary dynamics at 3PN order and beyond via effective field theory methods

Date: Nov 29, 2011 03:30 PM

URL: http://pirsa.org/11110087

Abstract: The Effective Field Theory (EFT) approach can be employed to perform high PN order calculations of the Hamiltonian of a binary system. We show how we reproduced the 3PN dynamics by means of an algorithm implemented in Mathematica and our progress towards the computation of the 4PN Hamiltonian. We also show the EFT computation of the tail term affecting the conservative dynamics at 4PN order, first derived using traditional methods by Blanchet and Damour.



$$E(\vec{r}, \vec{v}, \vec{a})$$

no SPIN
no GW radiation

PN expansion parameter: $v^2 = \frac{GM}{r}$

needed to compute the phase $\varphi_{GW}(t)$: $\dot{E}(f_{GW}) = -P(f_{GW})$

3PN at least needed for O(1) phase determination

state of the art: 3.5PN

advanced LIGO-Virgo possibly sensitive to 4PN

EFT: Integrating out potential degrees of freedom

$$S[g,x_1,x_2] = S_{EH}[g] + \sum_{a=1,2} S_{pp}[g,x_a] = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,(R - \Gamma_\mu\Gamma^\mu) - \sum_{a=1,2} m_a \int \sqrt{-g_{\mu\nu}\mathrm{d}x_a^\mu \mathrm{d}x_a^\nu}$$

$$iS_{eff}[x_1,x_2] = \int [\mathcal{D}h]\mathrm{e}^{iS[h,x_1,x_2]} \qquad (g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu})$$

- Perturbative expansion: Feynman graphs
- Explicit v and G power counting:

$$k_h^\mu \sim (v/r, 1/r)$$

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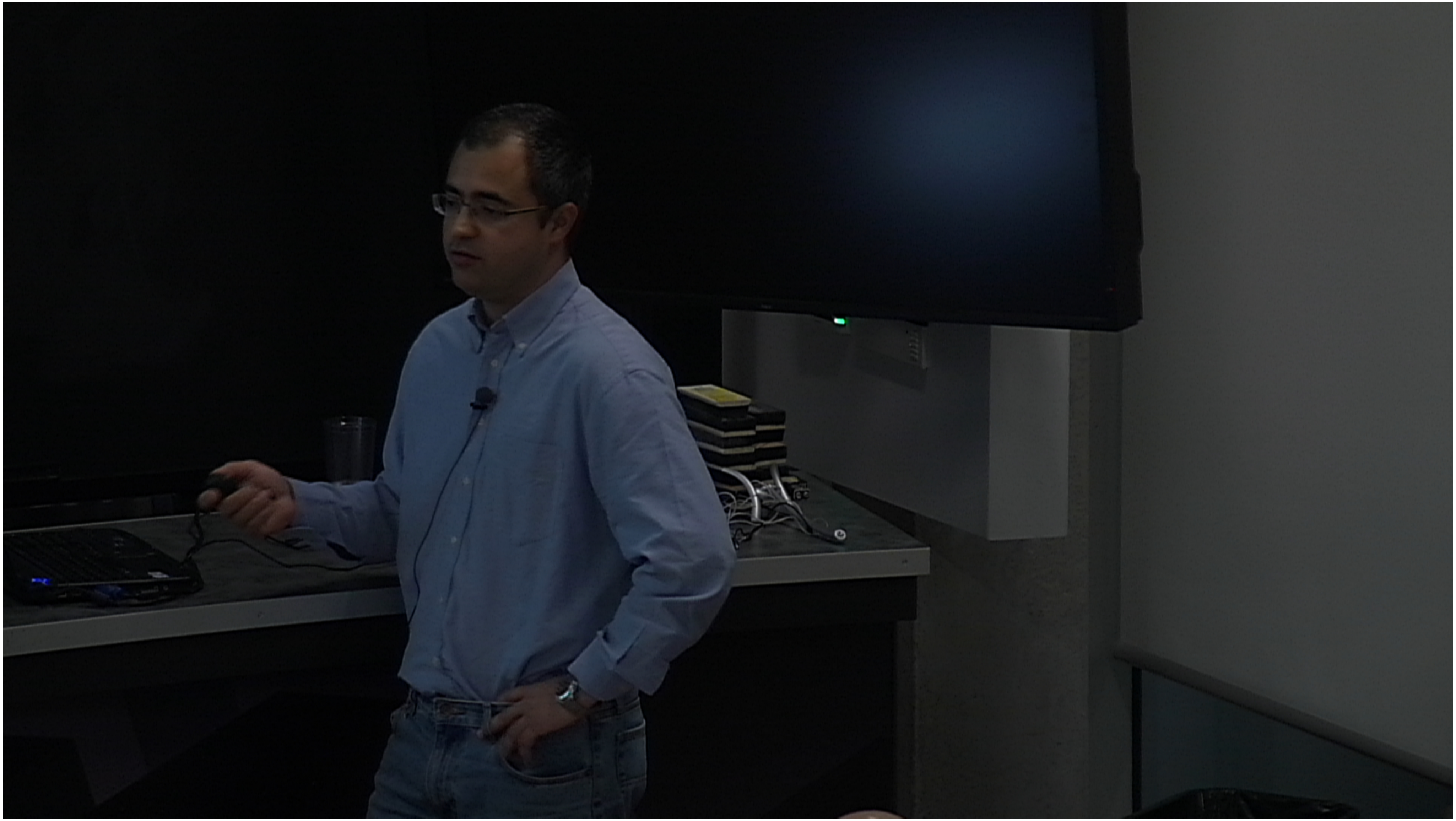

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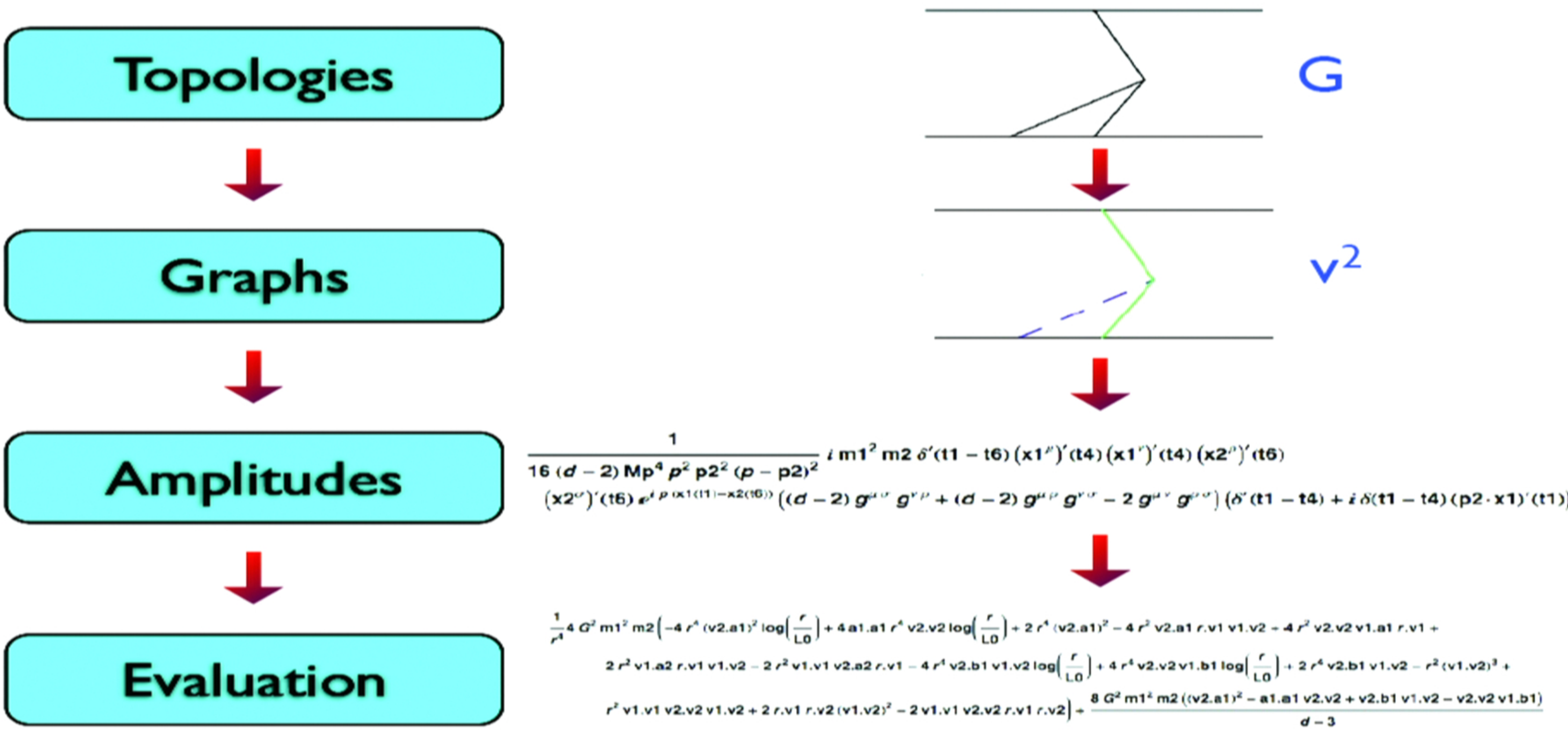
Algorithm



Topologies
Graphs
Amplitudes
Evaluation
G
v²
1 / (16 (d − 2) Mp⁴ p² p2² (p − p2)²) i m1² m2 δ′(t1 − t6) (x1^μ)′(t4) (x1^ν)′(t4) (x2^ρ)′(t6) (x2^σ)′(t6) e^{i p (x1(t1)−x2(t6))} ((d − 2) g^{μσ} g^{νρ} + (d − 2) g^{μρ} g^{νσ} − 2 g^{μν} g^{ρσ}) (δ′(t1 − t4) + i δ(t1 − t4) (p2·x1)′(t1))
(1/r⁴) 4 G² m1² m2 (−4 r⁴ (v2.a1)² log(r/L0) + 4 a1.a1 r⁴ v2.v2 log(r/L0) + 2 r⁴ (v2.a1)² − 4 r² v2.a1 r.v1 v1.v2 − 4 r² v2.v2 v1.a1 r.v1 +
2 r² v1.a2 r.v1 v1.v2 − 2 r² v1.v1 v2.a2 r.v1 − 4 r⁴ v2.b1 v1.v2 log(r/L0) + 4 r⁴ v2.v2 v1.b1 log(r/L0) + 2 r⁴ v2.b1 v1.v2 − r² (v1.v2)³ +
r² v1.v1 v2.v2 v1.v2 + 2 r.v1 r.v2 (v1.v2)² − 2 v1.v1 v2.v2 r.v1 r.v2) + 8 G² m1² m2 ((v2.a1)² − a1.a1 v2.v2 + v2.b1 v1.v2 − v2.v2 v1.b1) / (d − 3)

Topologies

propagator

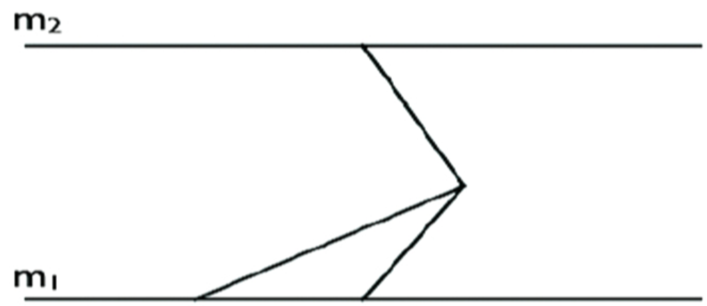
vertex

$$\begin{pmatrix} 111 & 331 \\ 112 & 331 \\ 211 & 331 \end{pmatrix} :$$

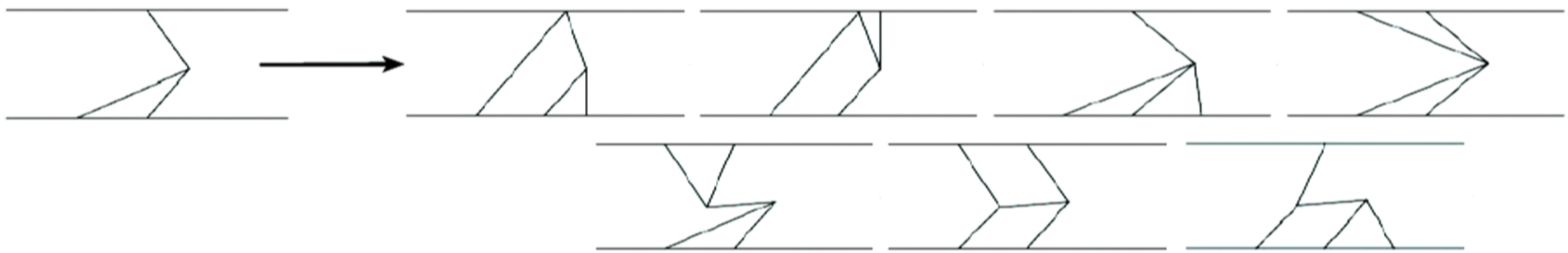
position

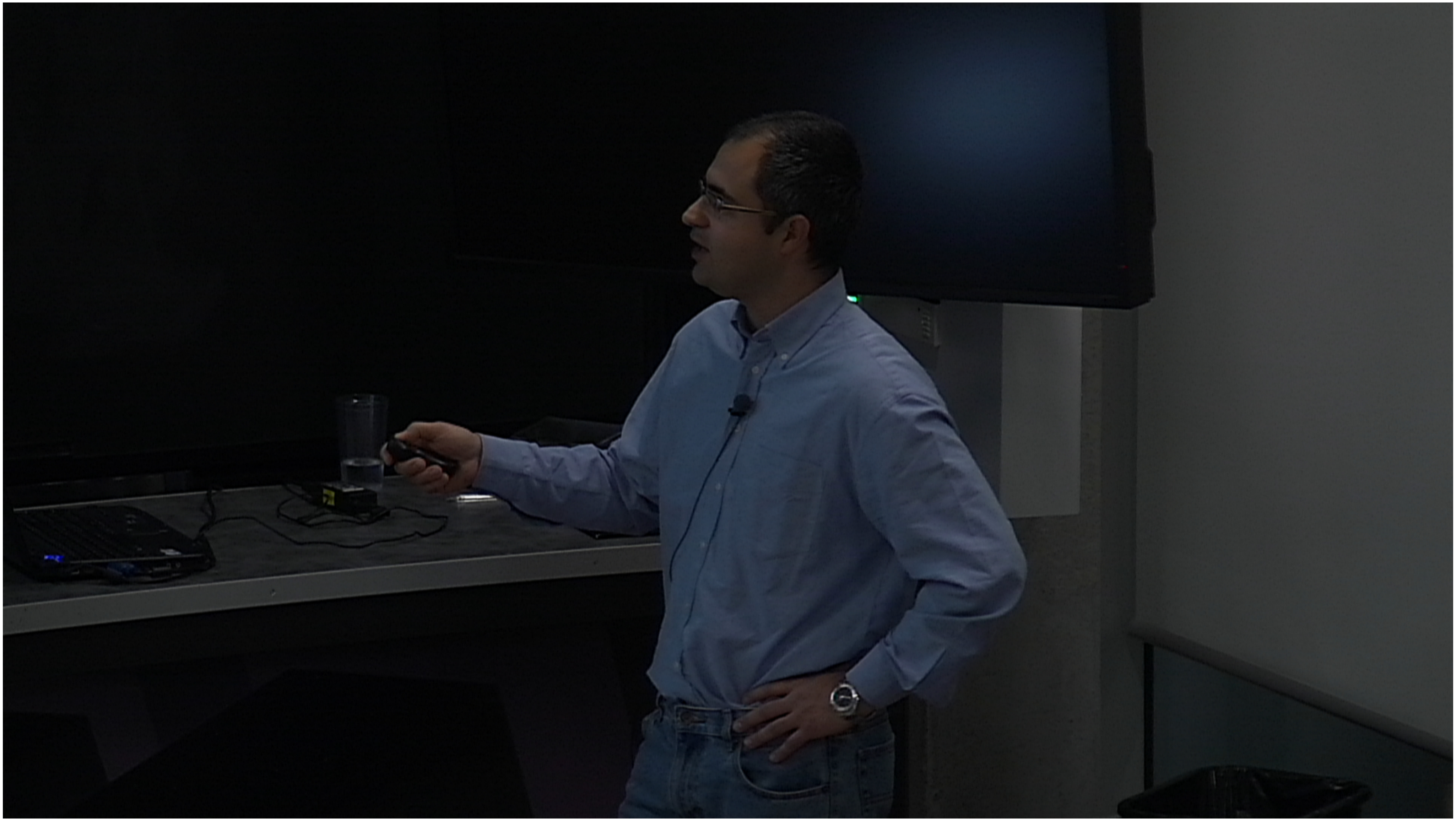
# of legs

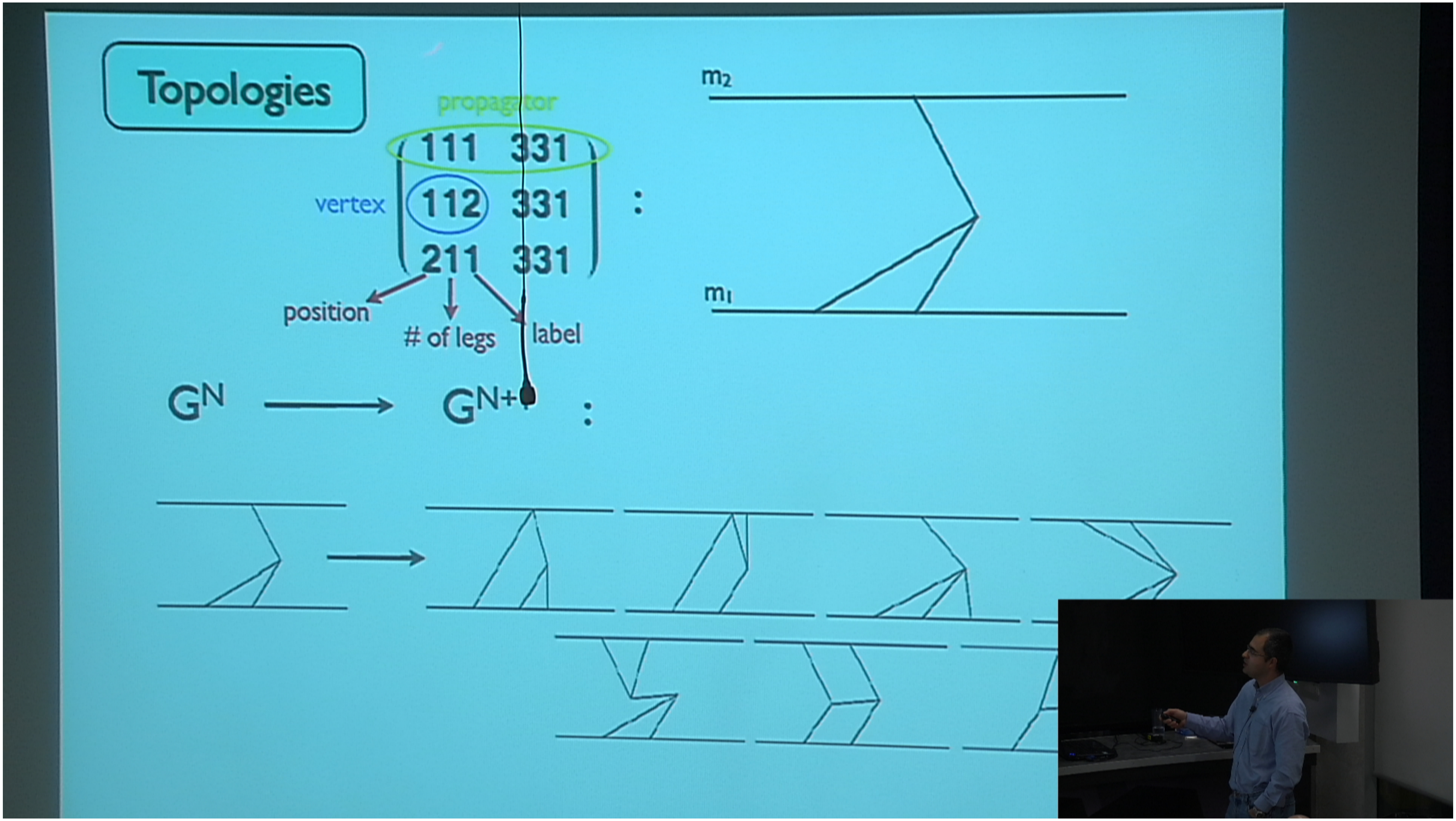
label

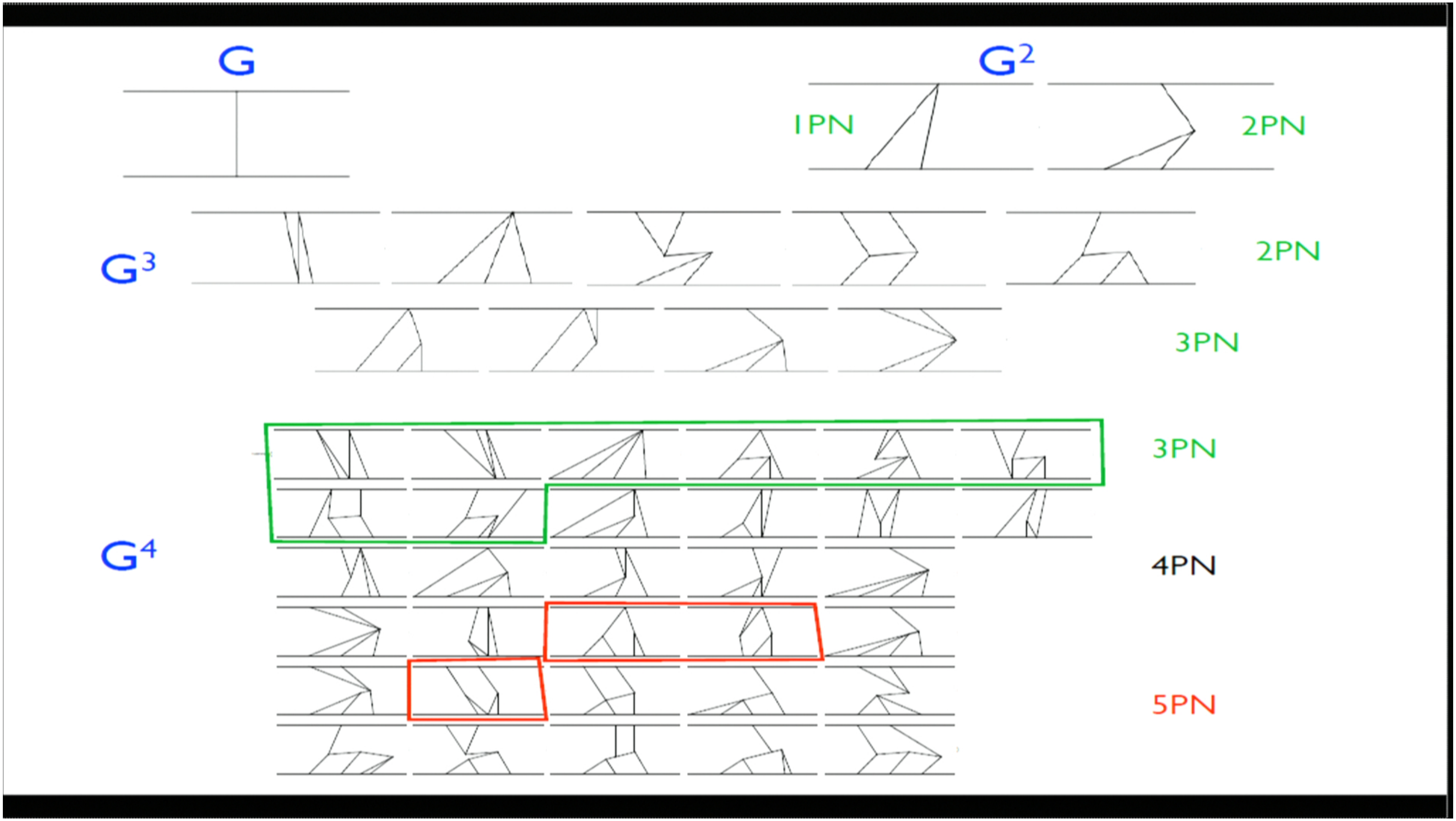


$$G^N \longrightarrow G^{N+1} :$$

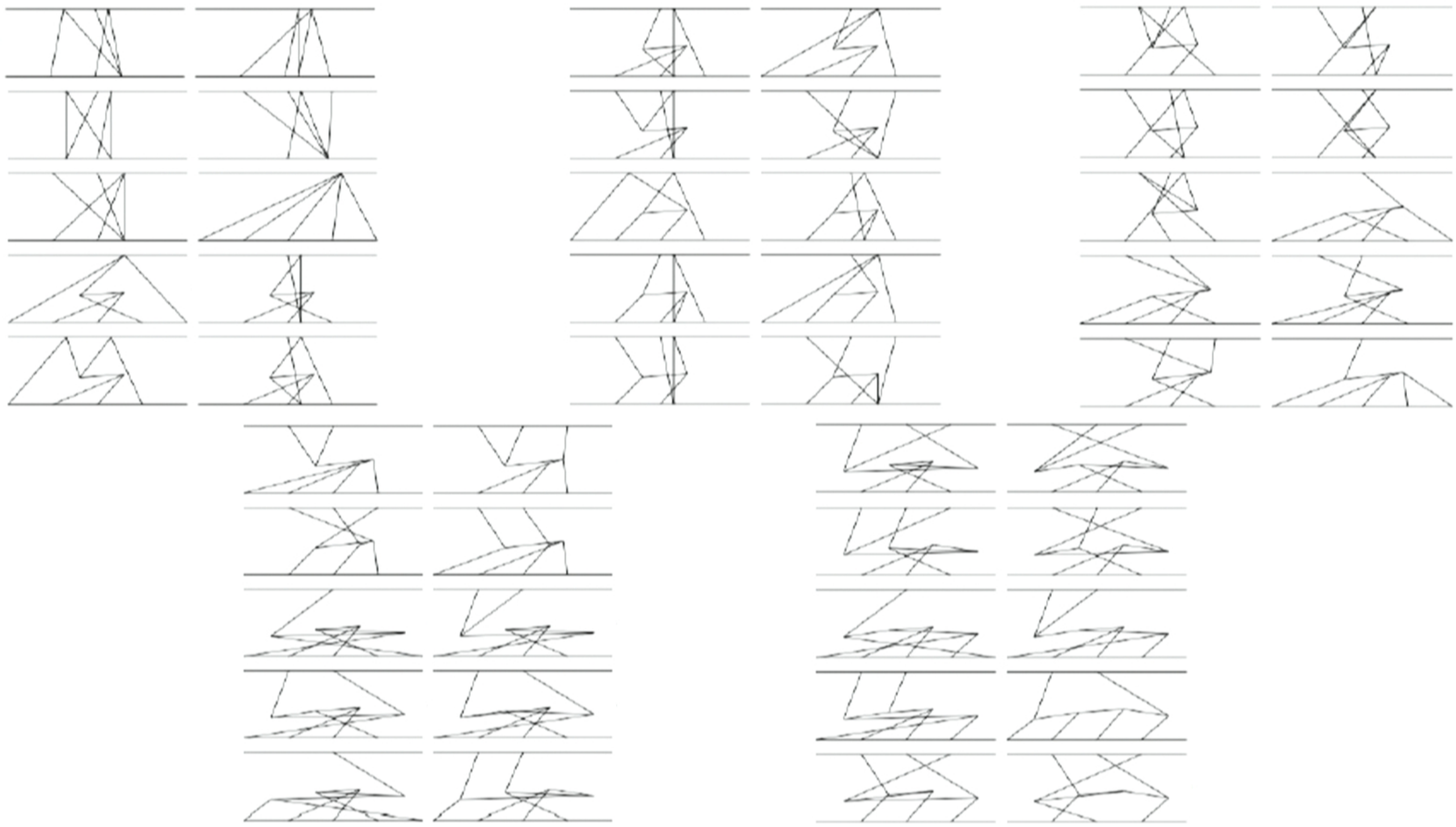





Topologies
propagator
111 331
vertex
112 331
211 331
:
position
# of legs
label
m2
m1
G^N
G^N+
:


G
G²
1PN
2PN
G³
2PN
3PN
G⁴
3PN
4PN
5PN

G^5 (50 relevant at 4PN, out of 164)

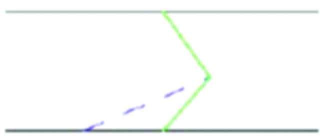


Graphs

Kol-Smolkin variables

$$g_{\mu\nu} = e^{2\phi} \begin{pmatrix} -1 & A_j \\ A_i & e^{-c_d\phi}(\delta_{ij} + \sigma_{ij}) - A_i A_j \end{pmatrix}$$

$$\begin{pmatrix} 111 & 331 \\ 112 & 331 \\ 211 & 331 \end{pmatrix}$$



$$\begin{pmatrix} 11001 & 31021 & 1 \\ 11002 & 31021 & 3 \\ 20011 & 31021 & 3 \end{pmatrix}$$

$$\begin{aligned} S^{4PN}_{bulk} \simeq \int \mathrm{d}^{d+1}x\sqrt{-\gamma}\Bigg\{ & \frac{1}{4}\left[(\vec{\nabla}\sigma)^2 - 2(\vec{\nabla}\sigma_{ij})^2 - (\dot{\sigma}^2 - 2(\dot{\sigma}_{ij})^2)\,\mathrm{e}^{\frac{-c_d\phi}{m_{Pl}}}\right] - c_d\left[(\vec{\nabla}\phi)^2 - \dot{\phi}^2\mathrm{e}^{-\frac{c_d\phi}{m_{Pl}}}\right] \\ & + \left[\frac{F_{ij}^2}{2} + \left(\vec{\nabla}\cdot\vec{A}\right)^2 - \dot{\vec{A}}^2\mathrm{e}^{-\frac{c_d\phi}{m_{Pl}}}\right]\mathrm{e}^{\frac{c_d\phi}{m_{Pl}}} + 2\frac{\left[F_{ij}A^i\dot{A}^j + \vec{A}\dot{\vec{A}}(\vec{\nabla}\cdot\vec{A})\right]\mathrm{e}^{\frac{c_d\phi}{m_{Pl}}} - c_d\dot{\phi}\vec{A}\vec{\nabla}\phi}{m_{Pl}} \\ & + 2c_d\left(\dot{\phi}\vec{\nabla}\cdot\vec{A} - \dot{\vec{A}}\vec{\nabla}\phi\right) + \frac{1}{m_{Pl}}\left[-\dot{\sigma}A_i\hat{\Gamma}^i_{jj} + 2\dot{\sigma}_{ij}\left(A_k\hat{\Gamma}^k_{ij} - A_i\hat{\Gamma}^j_{kk}\right)\right] - c_d\frac{\dot{\phi}^2\vec{A}^2}{m_{Pl}^2} \\ & + \frac{1}{m_{Pl}}\sigma^{ij}\left(\frac{1}{2}\sigma_{kl,i}\sigma^{kl}_{,j} + \sigma_{ik,l}\sigma^{k,l}_{j} + \sigma_{ik,l}\sigma^{l,k}_{j} - \sigma^k_{i,k}\sigma^l_{j,l} + \sigma_{,i}\sigma^k_{j,k} - \frac{1}{2}\sigma_{ij,k}\sigma^{,k} - \sigma_{ik,j}\sigma^{,k} - \frac{1}{4}\sigma_{,i}\sigma_{,j}\right) \\ & + \frac{1}{2m_{Pl}}\sigma\left(\frac{1}{4}\sigma_k\sigma k + \sigma^{ki}_{,i}\sigma^{,j}_{kj} - \sigma_{ki,j}\sigma^{kj,i} - \frac{1}{2}\sigma_{ki,j}\sigma^{ki,j}\right)\Bigg\} \end{aligned}$$

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$$S_{pp} = -m\int \mathrm{d}\tau = -m\int \mathrm{d}t\; e^{\phi/m_{Pl}}\sqrt{\left(1 - \frac{A_i}{m_{Pl}}v^i\right)^2 - e^{-c_d\phi/m_{Pl}}\left(v^2 + \frac{\sigma_{ij}}{m_{Pl}}v^iv^j\right)}$$

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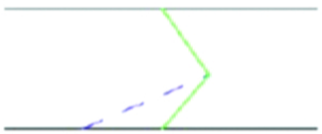

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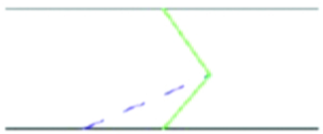
$\longrightarrow$



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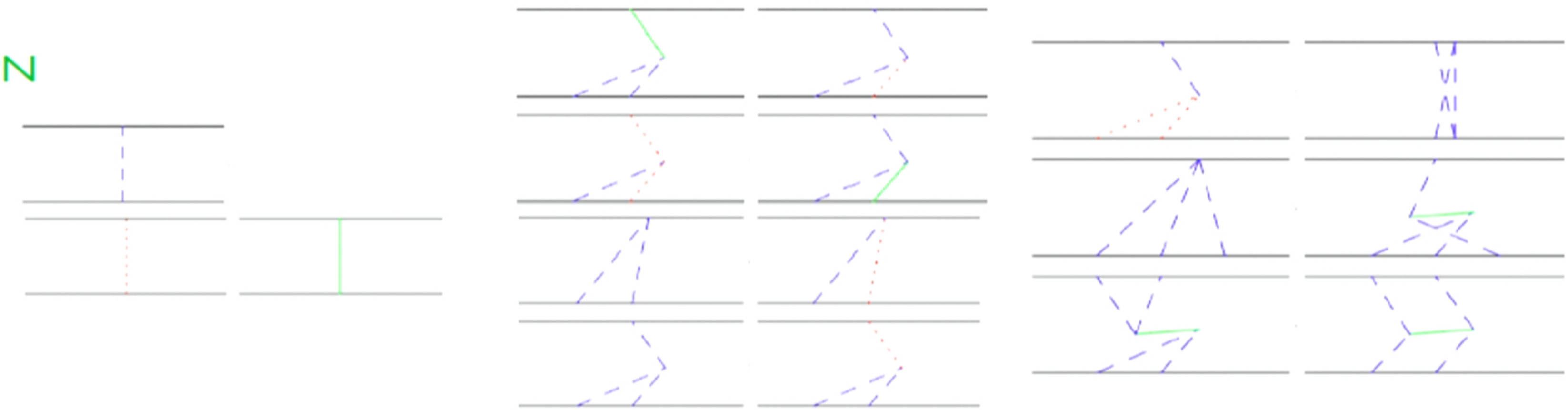
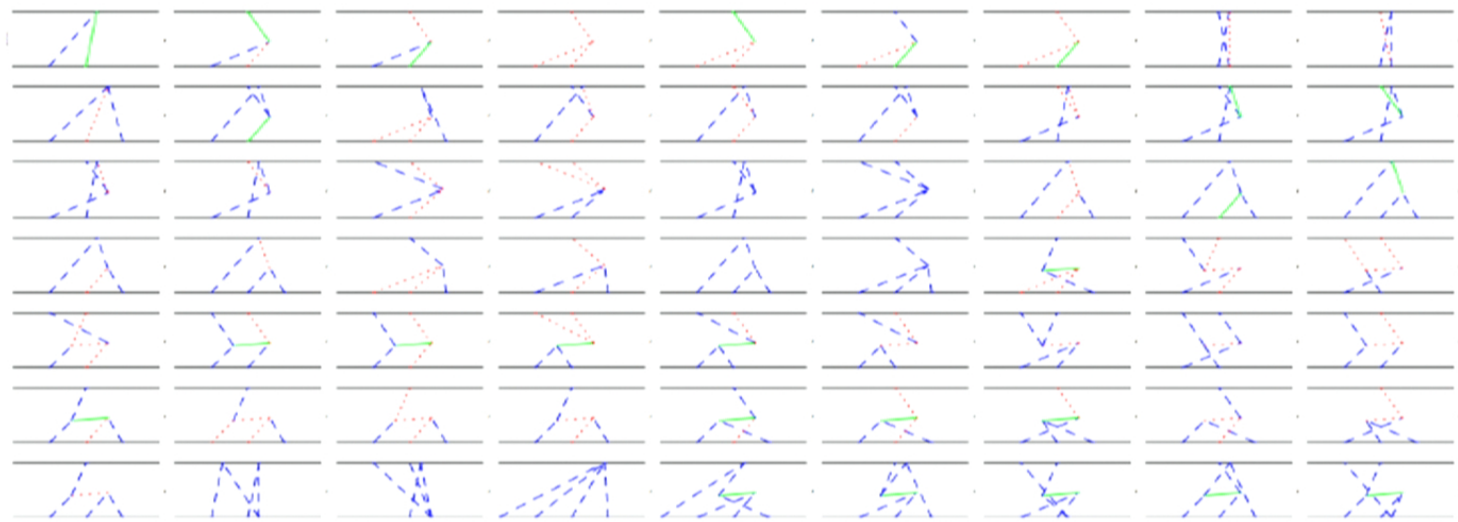
0-1-2PN



3PN



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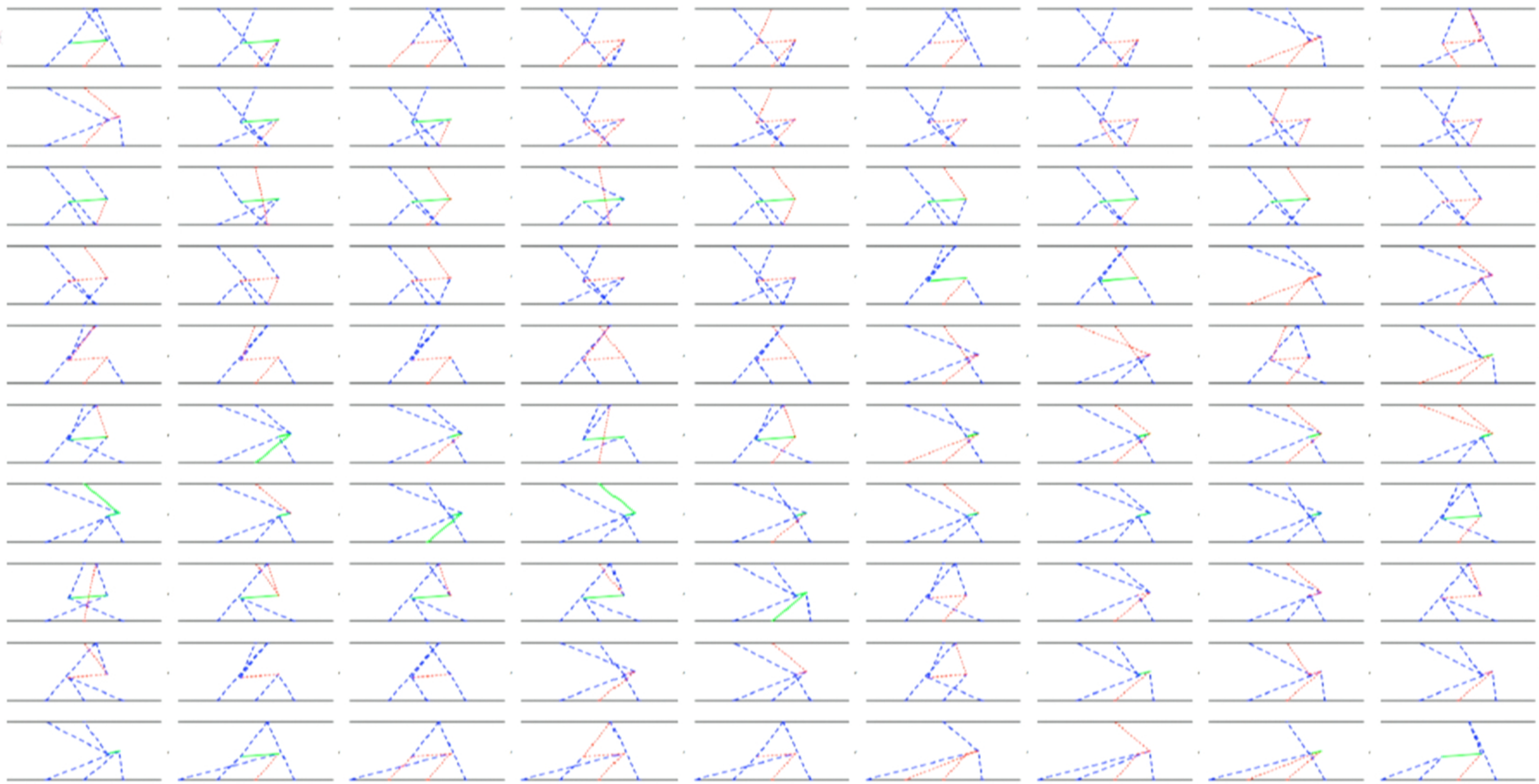

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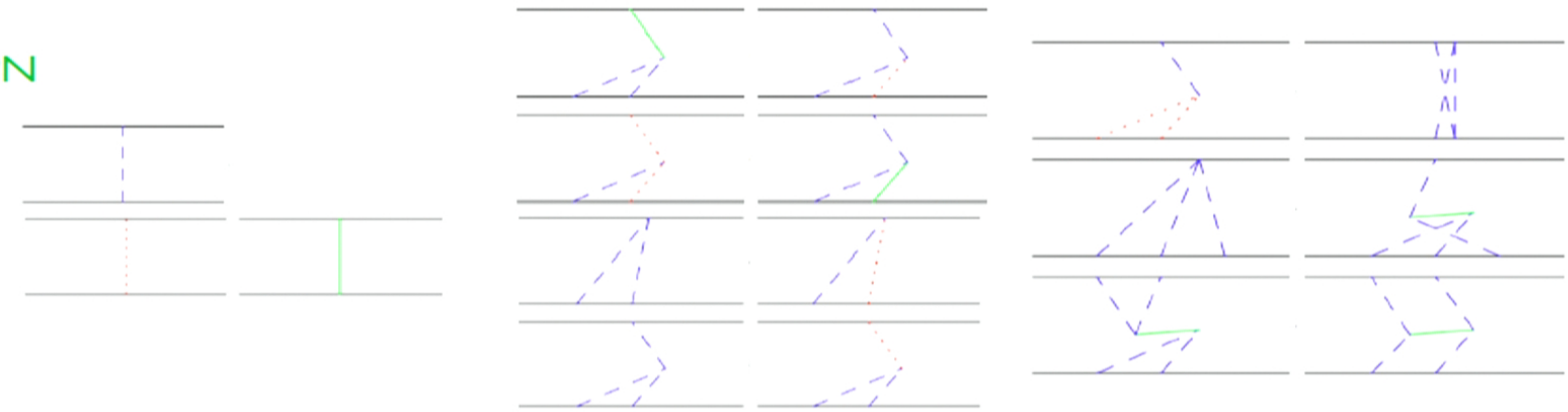
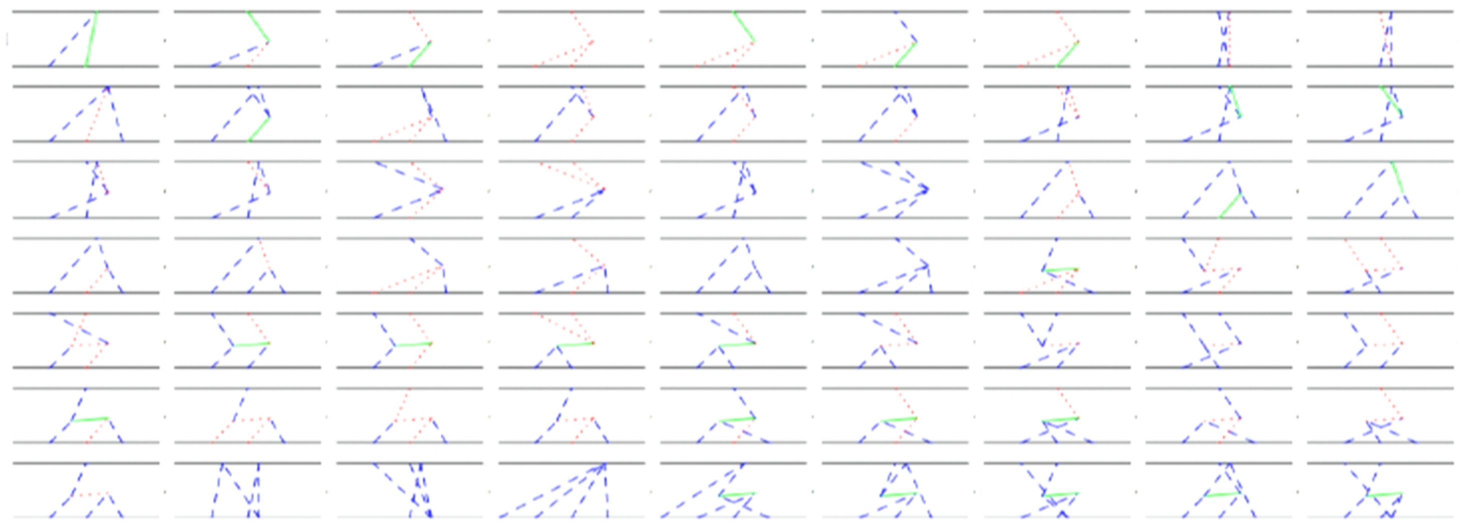
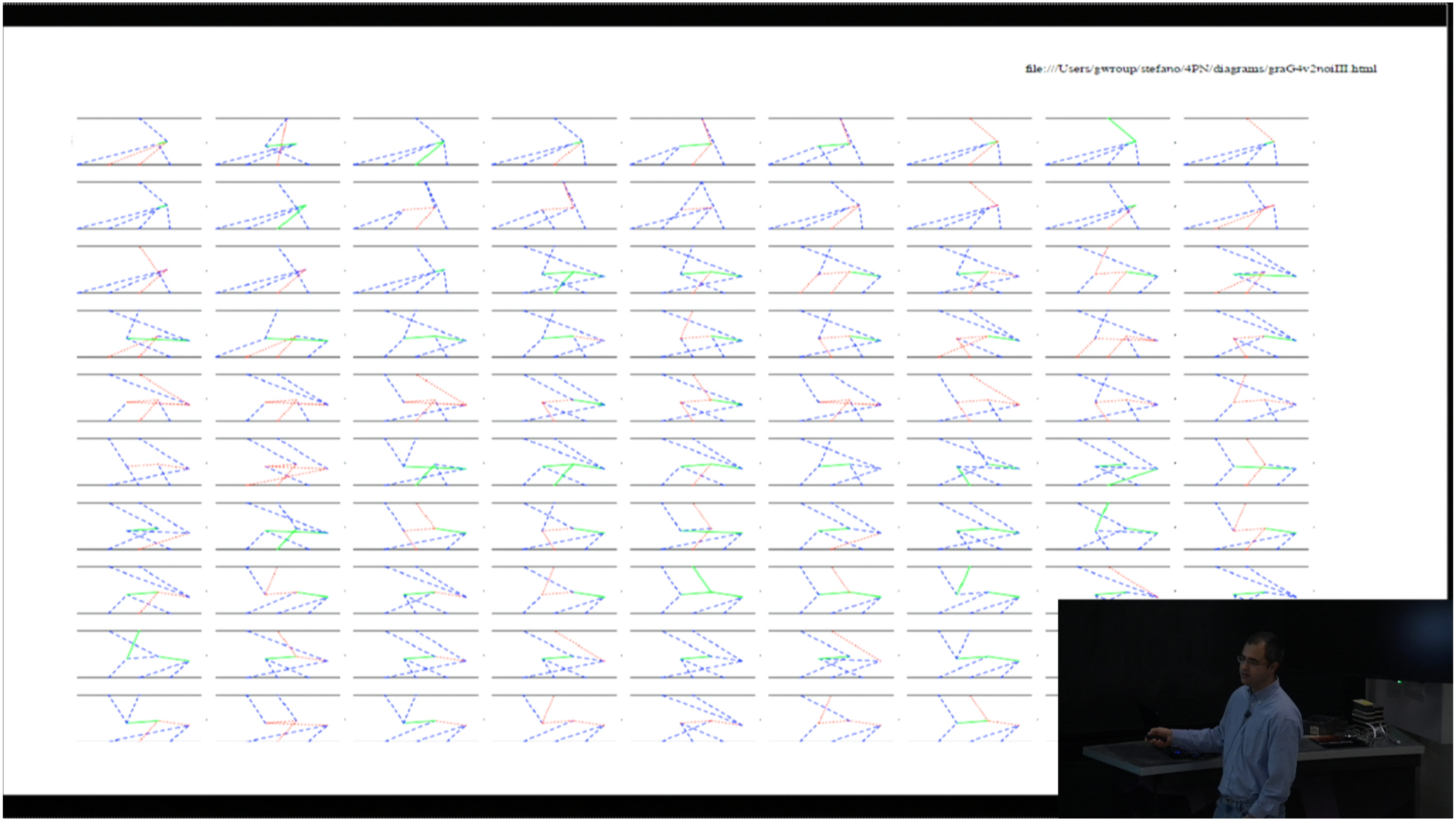
0-1-2PN



3PN

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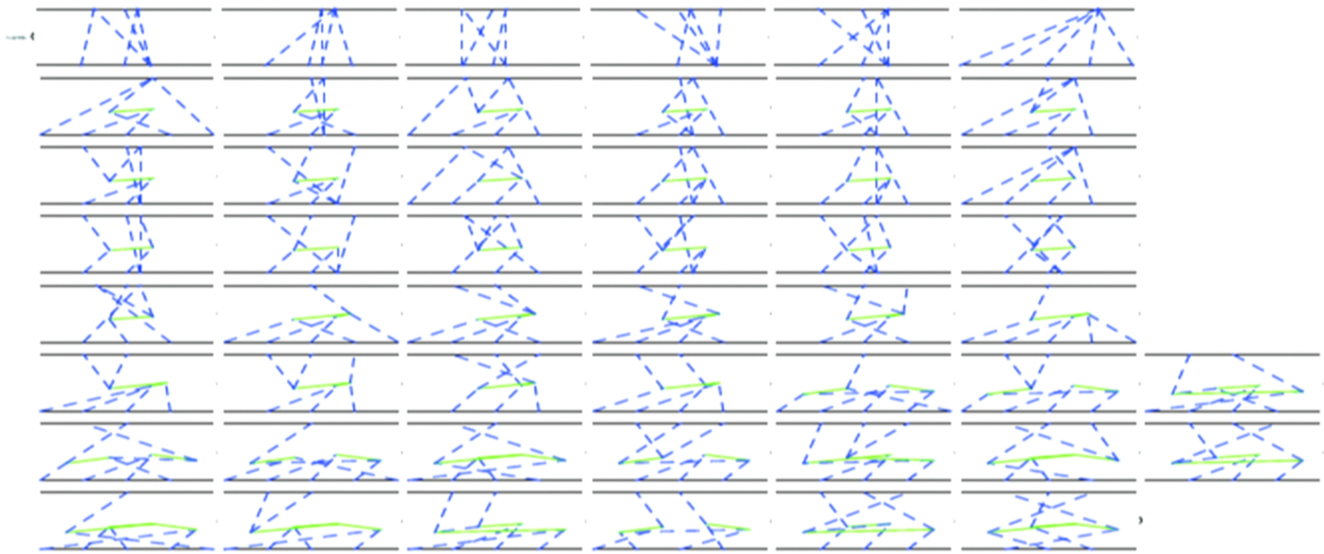
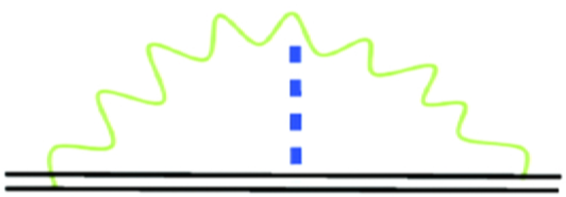


595 Feynman graphs in total

+ tail terms: Re 

(see PARTII)

Amplitudes

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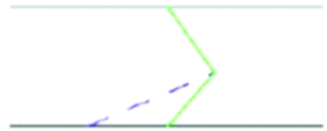


$$\frac{\delta'(t1-t6)\, e^{i\,p\,(x1(t1)-x2(t6))}\left((d-2)\, g^{\mu\sigma} g^{\nu\rho} + (d-2)\, g^{\mu\rho} g^{\nu\sigma} - 2\, g^{\mu\nu} g^{\rho\sigma}\right)\left(\delta'(t1-t4) + i\,\delta(t1-t4)\,(p2\cdot x1)'(t1)\right)}{4\,(d-2)\,Mp\,p^2\,p2^2\,(p-p2)^2} \quad \frac{i\, m1^2\, m2\, (x1^\mu)'(t4)\, (x1^\nu)'(t4)\, (x2^\rho)'(t6)\, (x2^\sigma)'(t6)}{4\, Mp^3}$$

Feynmann rules:

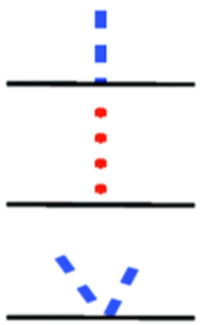
$S_{EH}[g]$: $\quad (\nabla\phi)^2 - \dot{\phi}^2$

(similarly for A_i , σ_{ij})

$$\frac{1}{k^2 - k_0^2} = \frac{1}{k^2}\left(1 + \frac{k_0^2}{k^2} + \left(\frac{k_0^2}{k^2}\right)^2 + \dots\right)$$

+interactions (Φ^3,Φ^2A,ΦA^2,$\Phi^2\sigma$,...,Φ^4,..):

$S_{pp}[g,x_a]$:



$$m_a\sqrt{G}\phi\frac{1+v^2}{\sqrt{1-v^2}}$$

$$-m_a\sqrt{G}A_i\frac{v^i}{\sqrt{1-v^2}}$$

$$m_a\sqrt{G}\phi^2\frac{1-6v^2+v^4}{2\sqrt{1-v^2}^3}$$

+.....

Amplitudes

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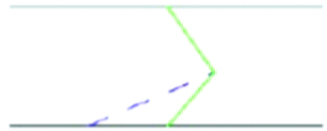

$$\frac{\delta'(t1-t6)\, e^{i\,p\,(x1(t1)-x2(t6))}\left((d-2)\, g^{\mu\sigma} g^{\nu\rho} + (d-2)\, g^{\mu\rho} g^{\nu\sigma} - 2\, g^{\mu\nu} g^{\rho\sigma}\right)\left(\delta'(t1-t4) + i\,\delta(t1-t4)\,(p2\cdot x1)'(t1)\right)}{4\,(d-2)\,Mp\, p^2\, p2^2\,(p-p2)^2} \quad \frac{i\, m1^2\, m2\, (x1^{\mu})'(t4)\,(x1^{\nu})'(t4)\,(x2^{\rho})'(t6)\,(x2^{\sigma})'(t6)}{4\, Mp^3}$$

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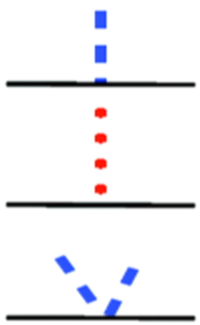
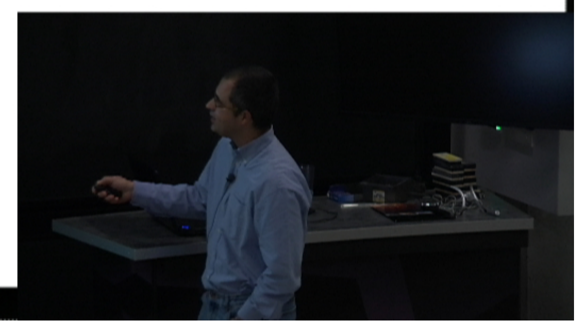


$$m_a\sqrt{G}\phi\frac{1+v^2}{\sqrt{1-v^2}}$$

$$-m_a\sqrt{G}A_i\frac{v^i}{\sqrt{1-v^2}}$$

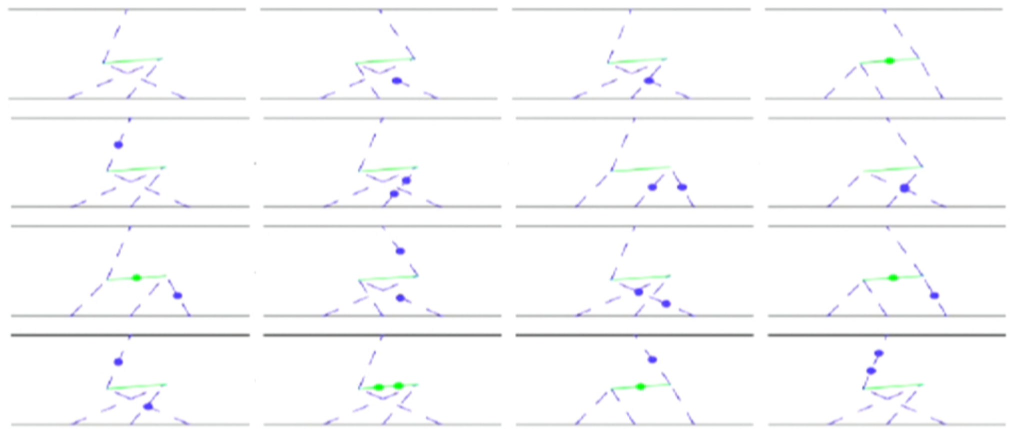
$$m_a\sqrt{G}\phi^2\frac{1-6v^2+v^4}{2\sqrt{1-v^2}^3}$$

+.....

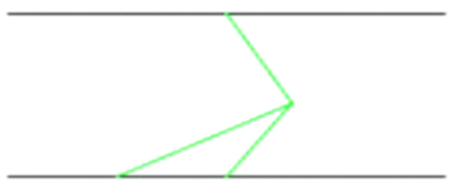


Long amplitudes:

propagator insertions:

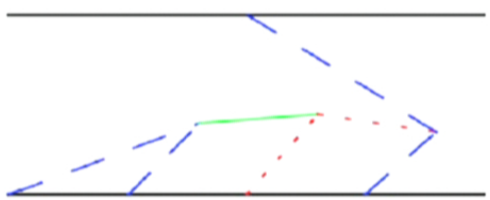


complex vertex:

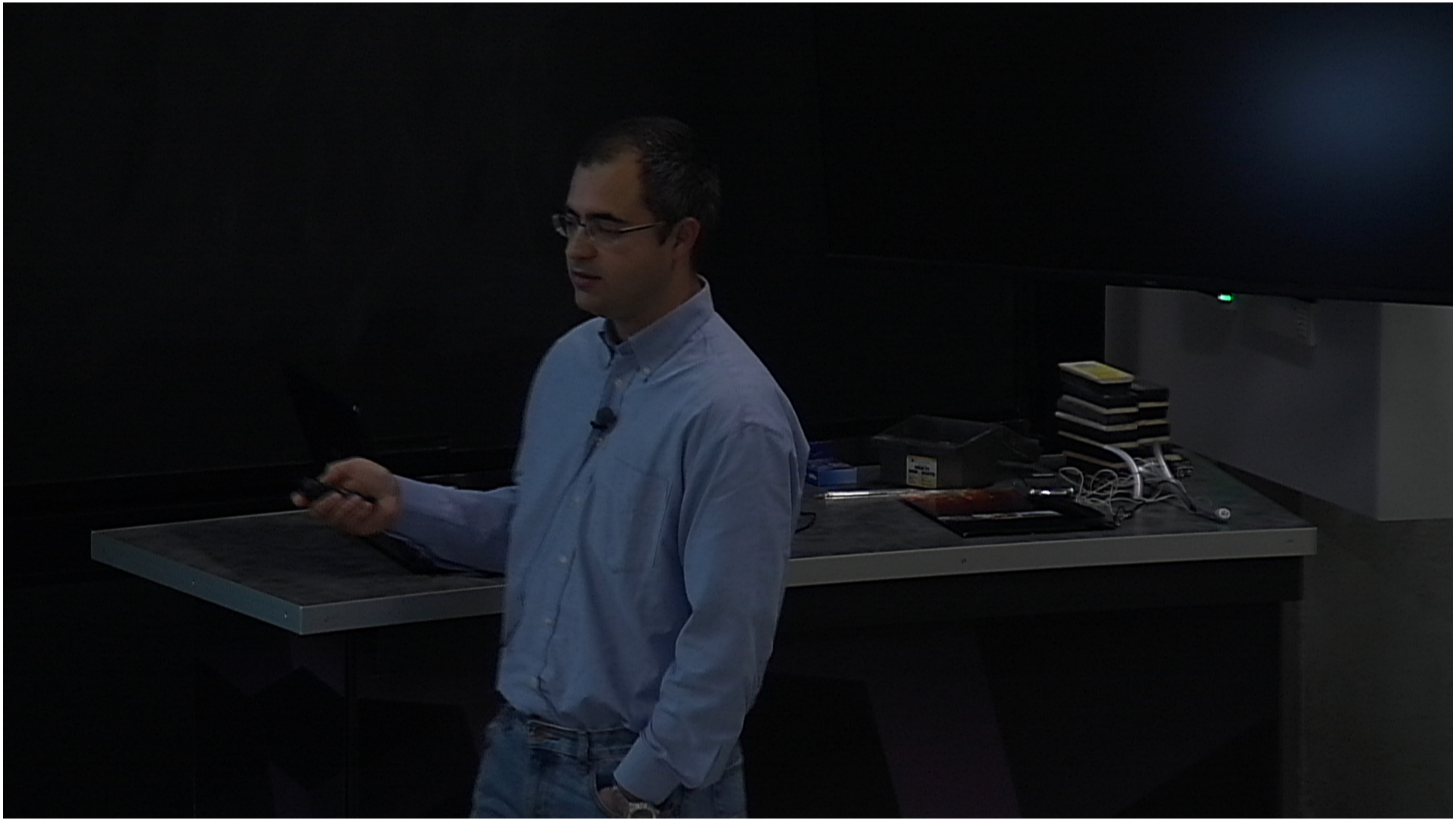


$$+\frac{1}{m_{Pl}}\sigma^{ij}\left(\frac{1}{2}\sigma_{kl,i}\sigma^{kl}_{,j}+\sigma_{ik,l}\sigma^{k,l}_{j}+\sigma_{ik,l}\sigma^{l,k}_{j}-\sigma^{k}_{i,k}\sigma^{l}_{j,l}+\sigma_{,i}\sigma^{k}_{j,k}-\frac{1}{2}\sigma_{ij,k}\sigma^{,k}-\sigma_{ik,j}\sigma^{,k}-\frac{1}{4}\sigma_{,i}\sigma_{,j}\right)$$
$$+\frac{1}{2m_{Pl}}\sigma\left(\frac{1}{4}\sigma_{,k}\sigma^{,k}+\sigma^{ki}_{,i}\sigma^{j}_{k,j}-\sigma_{ki,j}\sigma^{kj,i}-\frac{1}{2}\sigma_{ki,j}\sigma^{ki,j}\right)$$

many vertices:



Some diagrams contain several hundreds terms



Evaluation

$$\frac{1}{16\,(d-2)\,\mathrm{Mp}^4\,p^2\,\mathrm{p2}^2\,(p-\mathrm{p2})^2}\, i\,\mathrm{m1}^2\,\mathrm{m2}\,\delta'(\mathrm{t1}-\mathrm{t6})\,(\mathrm{x1}^{\mu})'(\mathrm{t4})\,(\mathrm{x1}^{\nu})'(\mathrm{t4})\,(\mathrm{x2}^{\rho})'(\mathrm{t6})\,(\mathrm{x2}^{\sigma})'(\mathrm{t6})\,e^{i\,p\cdot(\mathrm{x1}(\mathrm{t1})-\mathrm{x2}(\mathrm{t6}))}\left((d-2)\,g^{\mu\sigma}\,g^{\nu\rho}+(d-2)\,g^{\mu\rho}\,g^{\nu\sigma}-2\,g^{\mu\nu}\,g^{\rho\sigma}\right)\left(\delta'(\mathrm{t1}-\mathrm{t4})+i\,\delta(\mathrm{t1}-\mathrm{t4})\,(\mathrm{p2}\cdot\mathrm{x1})'(\mathrm{t1})\right)$$

↓

$$\frac{1}{r^4}4\,G^2\,\mathrm{m1}^2\,\mathrm{m2}\Big(-4\,r^4\,(\mathrm{v2.a1})^2\log\left(\frac{r}{\mathrm{L0}}\right)+4\,\mathrm{a1.a1}\,r^4\,\mathrm{v2.v2}\log\left(\frac{r}{\mathrm{L0}}\right)+2\,r^4\,(\mathrm{v2.a1})^2-4\,r^2\,\mathrm{v2.a1}\,r.\mathrm{v1}\,\mathrm{v1.v2}+4\,r^2\,\mathrm{v2.v2}\,\mathrm{v1.a1}\,r.\mathrm{v1}+$$
$$2\,r^2\,\mathrm{v1.a2}\,r.\mathrm{v1}\,\mathrm{v1.v2}-2\,r^2\,\mathrm{v1.v1}\,\mathrm{v2.a2}\,r.\mathrm{v1}-4\,r^4\,\mathrm{v2.b1}\,\mathrm{v1.v2}\log\left(\frac{r}{\mathrm{L0}}\right)+4\,r^4\,\mathrm{v2.v2}\,\mathrm{v1.b1}\log\left(\frac{r}{\mathrm{L0}}\right)+2\,r^4\,\mathrm{v2.b1}\,\mathrm{v1.v2}-r^2\,(\mathrm{v1.v2})^3+$$
$$r^2\,\mathrm{v1.v1}\,\mathrm{v2.v2}\,\mathrm{v1.v2}+2\,r.\mathrm{v1}\,r.\mathrm{v2}\,(\mathrm{v1.v2})^2-2\,\mathrm{v1.v1}\,\mathrm{v2.v2}\,r.\mathrm{v1}\,r.\mathrm{v2}\Big)+\frac{8\,G^2\,\mathrm{m1}^2\,\mathrm{m2}\left((\mathrm{v2.a1})^2-\mathrm{a1.a1}\,\mathrm{v2.v2}+\mathrm{v2.b1}\,\mathrm{v1.v2}-\mathrm{v2.v2}\,\mathrm{v1.b1}\right)}{d-3}$$

Reiterated use of

$$\int_{\mathbf{p}}\frac{e^{-i\mathbf{p}\mathbf{r}}}{p^{2\alpha}}=\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(d/2-\alpha)}{\Gamma(\alpha)}\left(\frac{2}{r}\right)^{d-2\alpha}$$

$$\int_{\mathbf{p}_1}\frac{1}{p_1^{2\alpha}(\mathbf{p}-\mathbf{p}_1)^{2\beta}}=\frac{1}{(4\pi)^{d/2}}\frac{\Gamma(d/2-\beta)\Gamma(d/2-\alpha)\Gamma(\alpha+\beta-d/2)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(d-\alpha-\beta)}p^{d-2\alpha-2\beta}$$

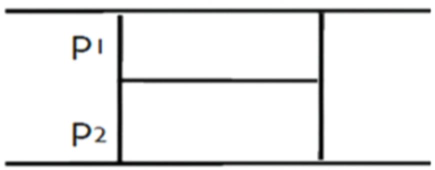
e.g.:

$$\frac{1}{p^{2\alpha}p1^{2\beta}(p-p1)^{2\gamma}(p1-p3)^{2\delta}p3^{2\epsilon}}$$

and of their generalization with nontrivial numerators (up to 6 free indices needed)

The result of each integral is saved to be used in other diagrams

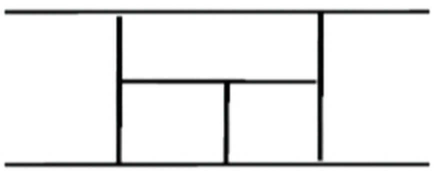
Tricky topologies:



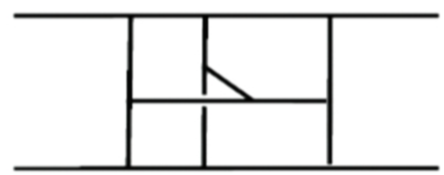
$$
\begin{aligned}
I(\alpha,\beta,\gamma,\delta,\epsilon) &\equiv \int_{\mathbf{p_1},\mathbf{p_2}} \frac{1}{p_1^{2\alpha}(\mathbf{p}-\mathbf{p_1})^{2\beta} p_2^{2\gamma}(\mathbf{p}-\mathbf{p_2})^{2\delta}(\mathbf{p_1}-\mathbf{p_2})^{2\epsilon}} \\
&= [\gamma\,(I(\alpha-1,\beta,\gamma+1,\delta,\epsilon) - I(\alpha,\beta,\gamma+1,\delta,\epsilon-1)) + \\
&\quad \delta\,(I(\alpha,\beta-1,\gamma,\delta+1,\epsilon) - I(\alpha,\beta,\gamma,\delta+1,\epsilon-1))]\,/\,(2\epsilon+\gamma+\delta-d)
\end{aligned}
$$

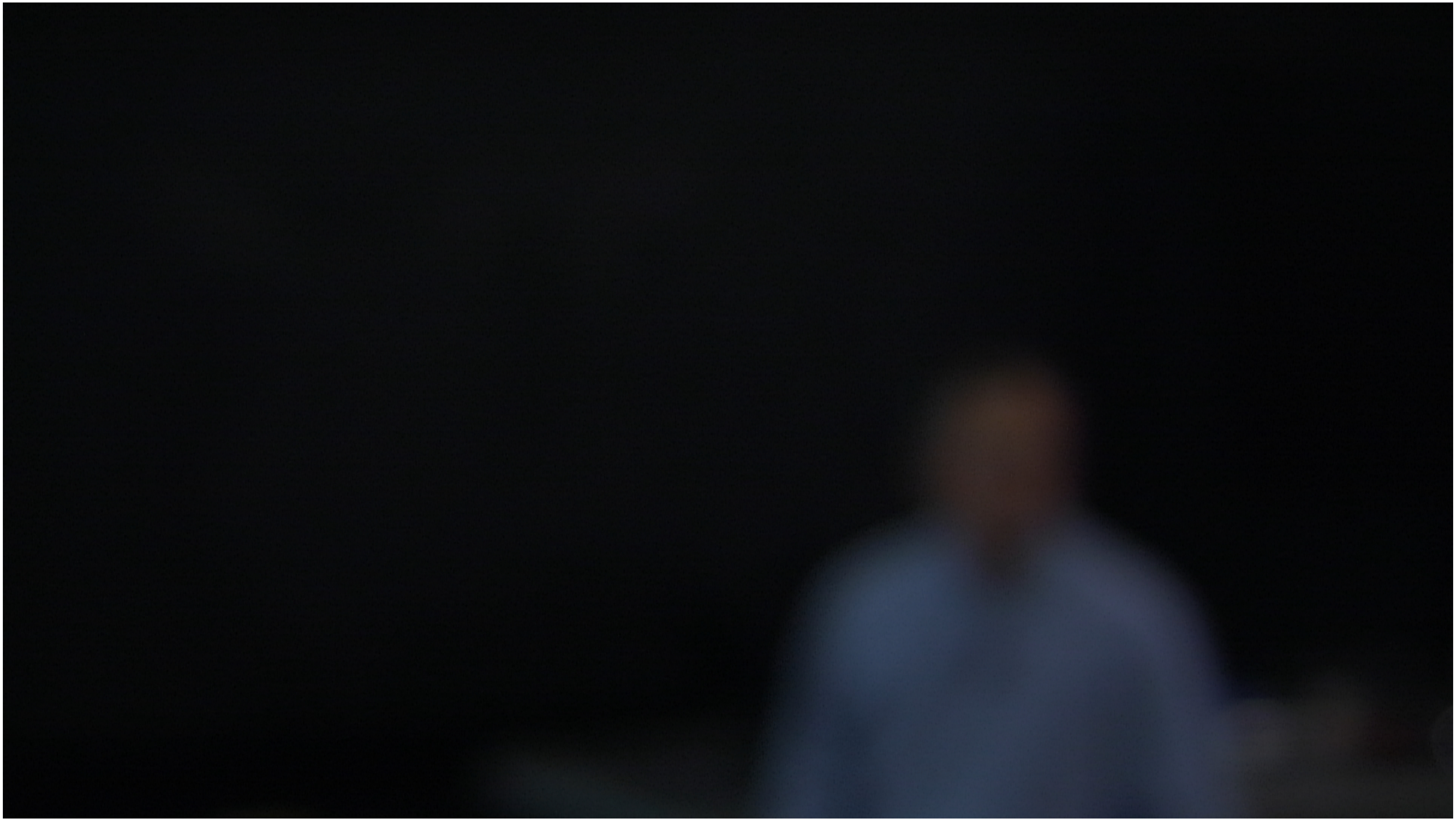
IBP

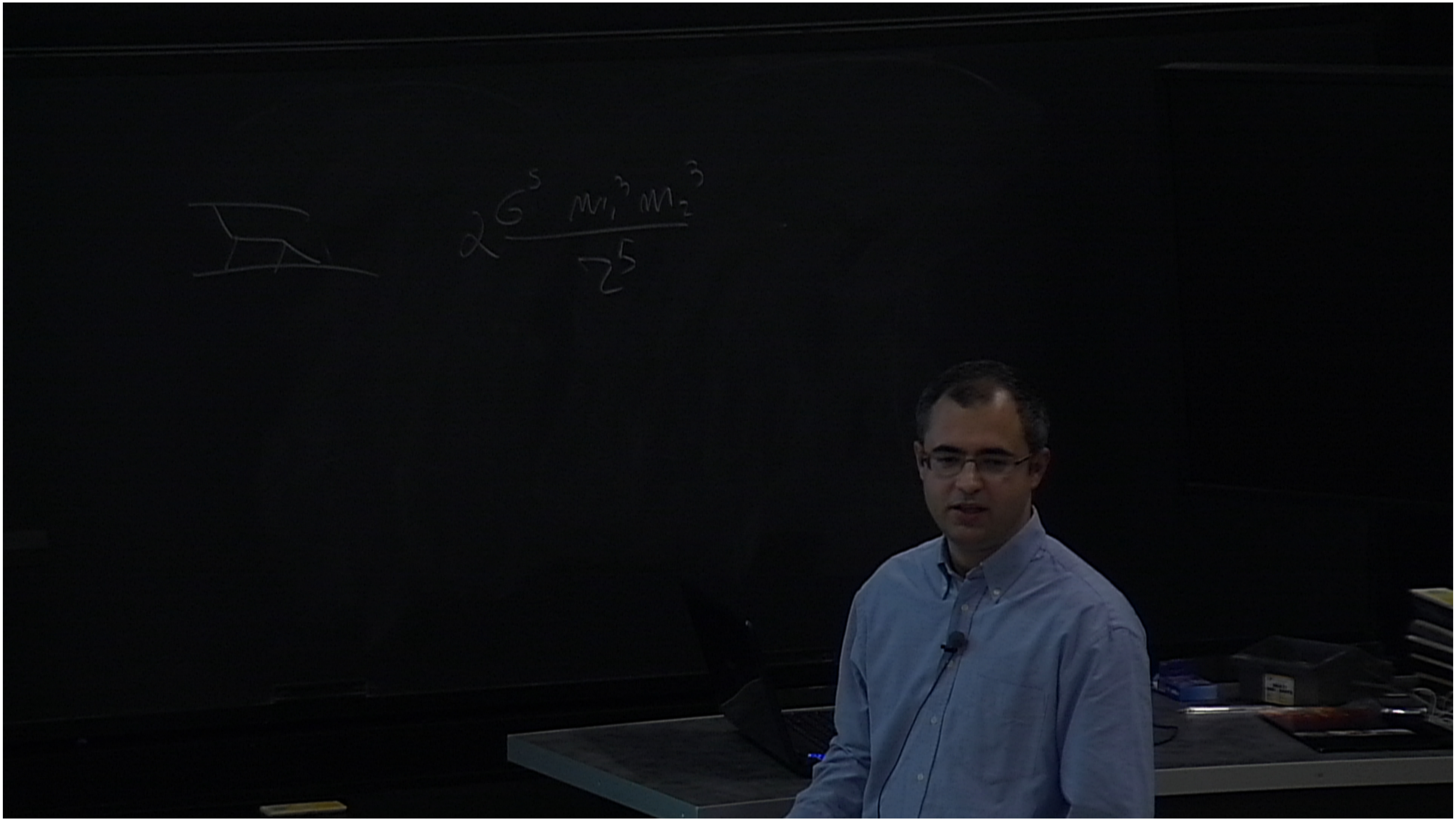
and similarly for



but there are a few nasty G^5 ones left, like:






$\propto G^5 \frac{m_1^3 m_2^3}{r^5}$

Conservative part of the self force

Conservative part of the self-force

EFT scale separation

Radiation reaction and gravity waves

Result

- At leading order the SF affects e.o.m. at 2.5PN order

Burke-Thorne radiation reaction

$$\Delta^{(SF)}\ddot{x}_{ai}(t) = \frac{2G_N}{5}x_{aj}(t)Q^{(5)}_{ij}(t) - \frac{8}{5}G_N^2 M x_{aj}\int_{-\infty}^{t} dt'\, Q^{(7)}_{ij}(t')\log\left[\frac{(t-t')}{T}\right]$$

relative 1.5PN tail correction

- Conservative part associated with tail integral

$$\Delta^{(SF)}\ddot{x}_{ai}(t) = \frac{8G_N^2 M}{5}x_{aj}(t)Q^{(6)}_{ij}(t)\log\left(\frac{r}{\lambda}\right)$$

Gravitational radiation emitted, scattered, and absorbed.

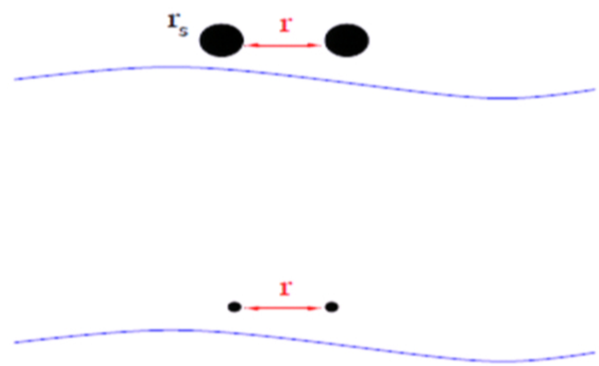
L. Blanchet and T. Damour PRD ’88

L. Blanchet, S.L. Detweiler, A. Le Tiec, B. F. Whiting PRD ’10

Different scales in EFT

- Very short distance $\lesssim r_s$ negligible up to 5PN (effacement principle)
- Short distance: potential gravitons $k_\mu \sim (v/r, 1/r)$

Log-divergences from at 3PN



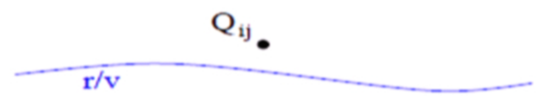
e.g. 

$$A \sim \int d^3k_1\, d^3k_2\, d^3k_3 \frac{f(k_1, k_2, k_3)}{k_1^2 k_2^2 k_3^2 (k_1 - k_2)^2 (k_2 - k_3)^2} e^{ik_1 r}$$

gives logarithmic divergences

Scale separation and radiation effects

- Long distance: GW's $k_\mu \sim (v/r, v/r)$ coupled to point particles with moments Responsible for dissipative and conservative effects



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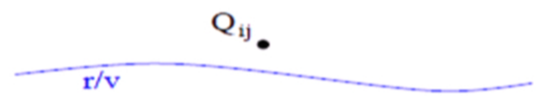
Scale separation and radiation effects

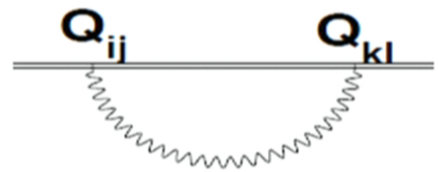
- Long distance: GW's

$k_\mu \sim (v/r, v/r)$ coupled to point particles with moments
Responsible for dissipative and conservative effects



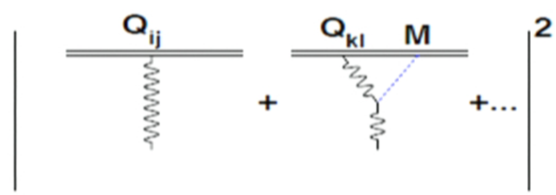
Radiation reaction and conservative dynamics

Radiation emitted and absorbed



Effective action modified:

Imaginary part $\longrightarrow$ power loss



Real part $\longrightarrow$ modifies e.o.m.

(Galley a Tiglio PRD '09)

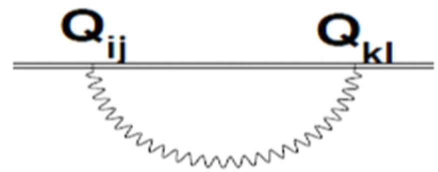
Optical theorem

(Goldberger and Ross PRD '10)


$\propto \frac{G^5 m_1^3 m_2^3}{r^5}$

Radiation reaction and conservative dynamics

Radiation emitted and absorbed



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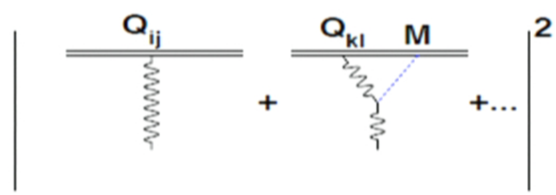

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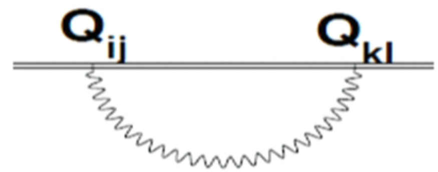
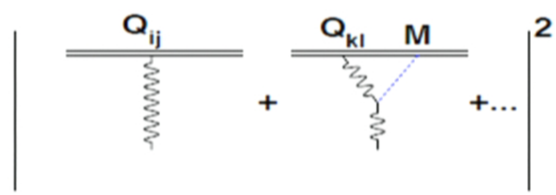
Radiation reaction and conservative dynamics

Radiation emitted and absorbed



Effective action modified:

Imaginary part $\longrightarrow$ power loss



Real part $\longrightarrow$ modifies e.o.m.
(Galley a Tiglio PRD '09)

Optical theorem
(Goldberger and Ross PRD '10)

The in-in formalism

- In-Out

$$e^{iW[J+Q]} = \int \mathcal{D}\phi_1 \exp\left\{iS[\phi+Q] + i\int d^4x\, J\phi\right\}$$
$$= \langle 0| U_{J+Q}(+\infty, -\infty)|0\rangle$$

$$\downarrow$$

$$\langle \phi_H \rangle = \langle 0| U_{J+Q}(\infty, t)\phi_I(t) U_{J+Q}(t, -\infty)|0\rangle$$

- In-In

$$e^{iW[J_1, J_2]} = \langle 0| U_{J_2+Q_2}(-\infty, +\infty) U_{J_1+Q_1}(+\infty, -\infty)|0\rangle$$

$$\downarrow$$

$$\langle \phi_{1H} \rangle = \langle 0| U_{J_2+Q_2}(-\infty, \infty)\times$$
$$U_{J_1+Q_1}(\infty, t)\phi_{1I}(t) U_{J_1+Q_1}(t, -\infty)|0\rangle$$

C. Galley and M. Tiglio PRD '09

In-In formalism in Keldysh representation

Conservative part of the self-force

EFT scale separation

Radiation reaction and gravity waves

Result

Specific causal structure:

$$
\begin{aligned}
W[J_1, J_2] &= \frac{i}{2}\int (J_1, J_2)\begin{pmatrix} G_F & -G_- \\ -G_+ & G_D \end{pmatrix}\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} \\
&= \frac{i}{2}\int (J_+, J_-)\begin{pmatrix} 0 & -iG_R \\ -iG_A & G_H \end{pmatrix}\begin{pmatrix} J_+ \\ J_- \end{pmatrix}
\end{aligned}
$$

with $J_\pm \propto J_1 \pm J_2$ and equations of motion:

$$
\ddot{x}_{ai} = \left.\frac{\partial S_{eff}}{\partial x_{-ai}}\right|_{\substack{x_{-ai} = 0 \\ x_{+ai} = x_{ai}}}
$$

Hence relevant for the e.o.m are linear in Q_-


$\propto \frac{G^5 m_1^3 m_2^3}{r^5}$

In-In formalism in Keldysh representation

Conservative part of the self-force

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Result

Specific causal structure:

$$W[J_1, J_2] = \frac{i}{2}\int (J_1, J_2)\begin{pmatrix} G_F & -G_- \\ -G_+ & G_D \end{pmatrix}\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \frac{i}{2}\int (J_+, J_-)\begin{pmatrix} 0 & -iG_R \\ -iG_A & G_H \end{pmatrix}\begin{pmatrix} J_+ \\ J_- \end{pmatrix}$$

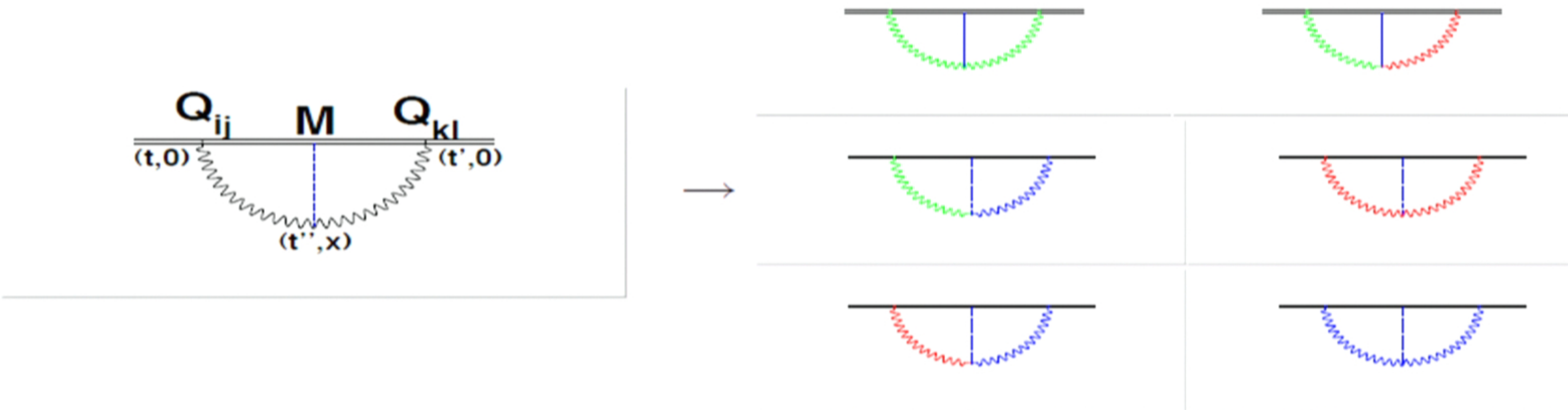
with $J_\pm \propto J_1 \pm J_2$ and equations of motion:

$$\ddot{x}_{ai} = \left.\frac{\partial S_{eff}}{\partial x_{-ai}}\right|_{\substack{x_{-ai} = 0 \\ x_{+ai} = x_{ai}}}$$

Hence relevant for the e.o.m are linear in Q_-

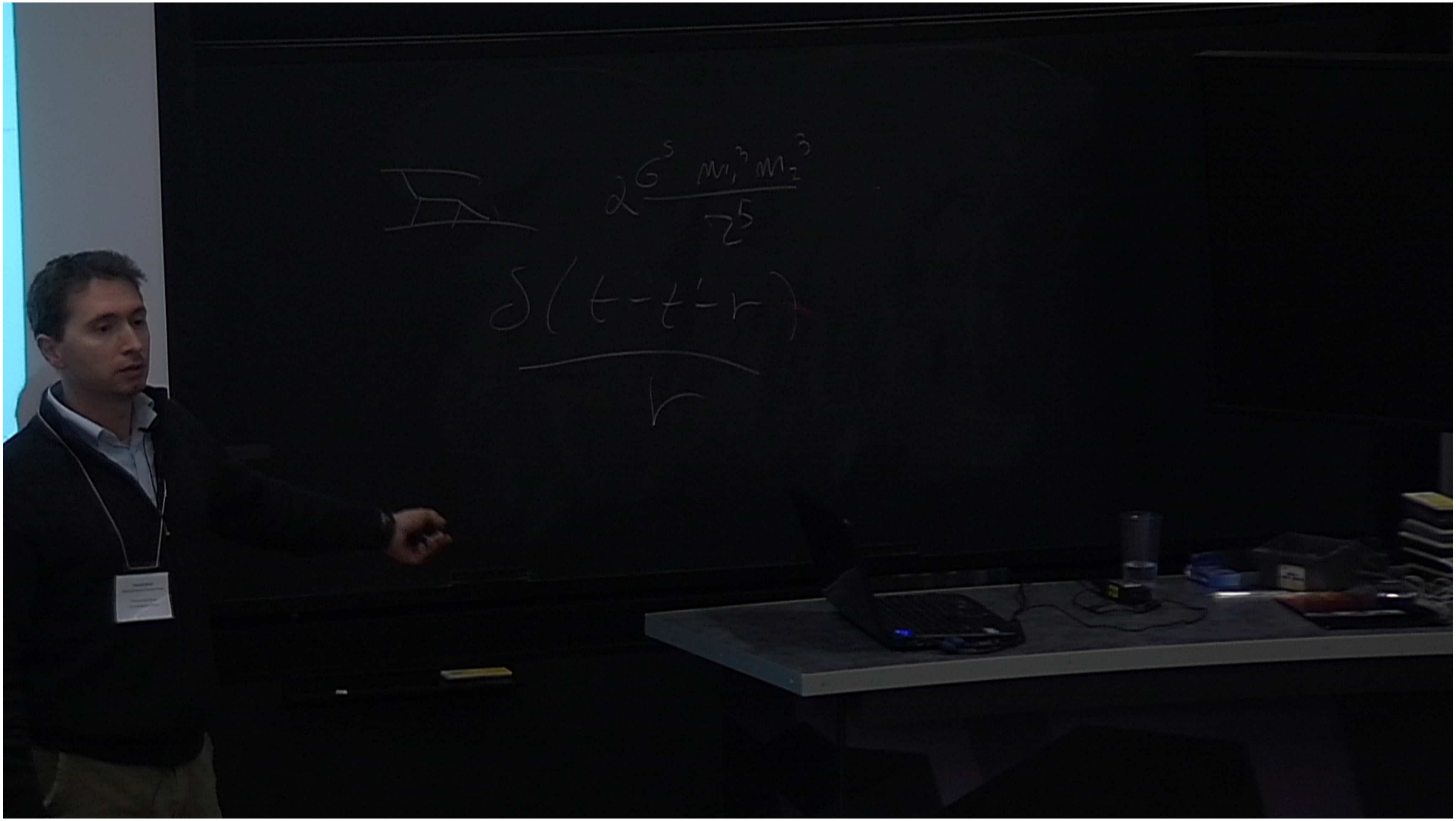
Structure of the result

Radiation emitted, scattered and absorbed



$$iS_{eff} \propto G_N^2 M \int dt\, Q^{(2)}_{-ij}(t) \int dt'\, Q^{(2)}_{+ij}(t') \times$$

$$\int dt''\, d^3x\, \partial_t^2 G_R(t-t'',x) G_R(t''-t',x) \frac{1}{r}$$


G^5 m_1^3 m_2^3 / r^5
δ(t − t' − r) / r

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Detailed calculation

$$
\begin{aligned}
iS_{eff} \propto\; & G_N^2 M \int dt \left(Q_{-ij}(t) R^0_{+i0j} + Q_{+ij}(t) R^0_{-i0j} \right) \\
& \int dt' \left(Q_{+ij}(t') R^0_{-i0j} + Q_{-ij}(t') R^0_{+i0j} \right) \\
& \int dt'' \phi_-(t'',0) \times \int d^3x \\
& \Big[2\dot{\sigma}_{+ar}\phi_+\dot{\sigma}_{-bp} \left(\delta_{ab}\delta_{rp} + \delta_{ap}\delta_{br} - \delta_{ar}\delta_{bp} \right) \\
& + \sigma_+ \partial\phi_+ \dot{A}_- + \sigma_- \partial\phi_+ \dot{A}_- \\
& + \sigma_+ \partial\phi_+ \partial\phi_- + \sigma_- \partial\phi_+ \partial\phi_+ \\
& + \partial A_+ \phi_+ \partial A_- \\
& + \partial A_+ \partial\phi_+ \dot{\phi}_- + \partial A_- \partial\phi_+ \dot{\phi}_+ \\
& + \dot{\phi}_+ \dot{\phi}_+ \dot{\phi}_- \Big] (t'', x)
\end{aligned}
$$

Renormalization in Fourier space

Conservative part of the self-force

EFT scale separation

Radiation reaction and gravity waves

Result

Classical renormalization from UV effect to real part of S_{eff}

$$S^{(R)}_{eff} = -\frac{G_N}{5}\int_{-\infty}^{\infty} dk\, Q_{-ij}(k) Q_{+ij}(-k) \left[(-ik)^5 + 4 G_N M (-ik)^6 \left(\log^{(UV)}\left(\frac{k^2}{\mu^2}\right) + c\right) + \ldots\right]$$

vs. imaginary part computed via optical theorem

$$S^{(I)}_{eff} = \frac{G_N}{10}\int_0^{\infty} dk\, Q_{ij}(k) Q_{ij}(-k)(-ik)^5 \left\{1 + 2\pi G_N M k + (G_N M k)^2 \left[-\frac{214}{105}\log^{(UV)}\left(\frac{k^2}{\mu^2}\right) + c'\right] + \ldots\right\}$$

found in Goldberger and Ross PRD '10

Regularization and renormalization

Conservative part of the self-force
EFT scale separation
Radiation reaction and gravity waves
Result

$$iS_{eff} = -i\frac{4G_N^2M}{5}\int dt\, Q_{-ij}(t)\times$$
$$\left\{-Q^{(6)}_{+ij}(t')\log(t-t')\Big|_{-\infty}^{t} + \int_{-\infty}^{t} dt'\, Q^{(7)}_{+ij}(t')\log\left[(t-t')\mu\right]\right\}$$

Arbitrary scale μ, renormalized multipole

$$Q_{ij}^{(Bare)}(\omega) = Z(\omega,\mu)Q_{ij}^{(Ren)}(\omega,\mu)$$

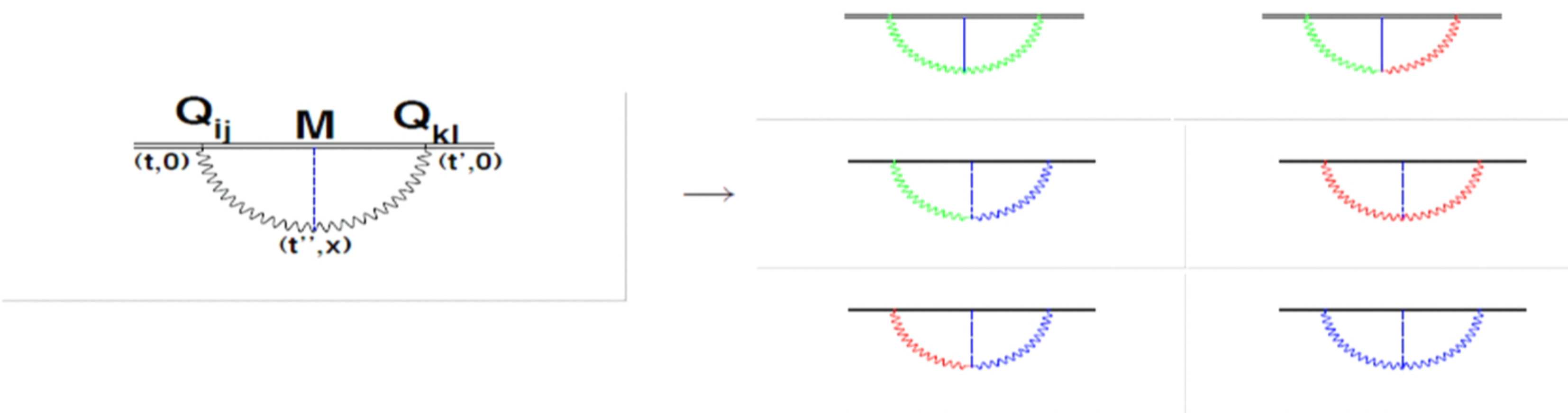
With $\mu \to 1/r$ and using

$$\log[(t-t')\mu] = \log\left(\frac{t-t'}{\lambda}\right) - \log\left(\frac{r}{\lambda}\right)$$

$$\Delta^{(QQM)}_{cons}\ddot{x}_{ai} = \frac{8}{5}x_{aj}Q^{(6)}_{ij}\log(r/\lambda)$$

Structure of the result

Radiation emitted, scattered and absorbed



$$iS_{eff} \propto G_N^2 M \int dt\, Q^{(2)}_{-ij}(t) \int dt'\, Q^{(2)}_{+ij}(t') \times$$
$$\int dt''\, d^3x\, \partial_t^2 G_R(t-t'',x)\, G_R(t''-t',x)\frac{1}{r}$$

Detailed calculation

$$
\begin{aligned}
iS_{eff} \propto\; & G_N^2 M \int dt \left(Q_{-ij}(t) R^0_{+i0j} + Q_{+ij}(t) R^0_{-i0j} \right) \\
& \int dt' \left(Q_{+ij}(t') R^0_{-i0j} + Q_{-ij}(t') R^0_{+i0j} \right) \\
& \int dt'' \, \phi_-(t'', 0) \times \int d^3x \\
& \Big[2\dot{\sigma}_{+ar} \phi_+ \dot{\sigma}_{-bp} \left(\delta_{ab}\delta_{rp} + \delta_{ap}\delta_{br} - \delta_{ar}\delta_{bp} \right) \\
& + \sigma_+ \partial\phi_+ \dot{A}_- + \sigma_- \partial\phi_+ \dot{A}_- \\
& + \sigma_+ \partial\phi_+ \partial\phi_- + \sigma_- \partial\phi_+ \partial\phi_+ \\
& + \partial A_+ \phi_+ \partial A_- \\
& + \partial A_+ \partial\phi_+ \dot{\phi}_- + \partial A_- \partial\phi_+ \dot{\phi}_+ \\
& + \dot{\phi}_+ \dot{\phi}_+ \dot{\phi}_- \Big] (t'', x)
\end{aligned}
$$

Renormalization in Fourier space

Classical renormalization from UV effect to real part of S_{eff}

$$S_{eff}^{(R)} = -\frac{G_N}{5}\int_{-\infty}^{\infty} dk\, Q_{-ij}(k)Q_{+ij}(-k)\left[(-ik)^5 + 4G_N M(-ik)^6\left(\log^{(UV)}\left(\frac{k^2}{\mu^2}\right) + c\right) + \dots\right]$$

vs. imaginary part computed via optical theorem

$$S_{eff}^{(I)} = \frac{G_N}{10}\int_{0}^{\infty} dk\, Q_{ij}(k)Q_{ij}(-k)(-ik)^5\left\{1 + 2\pi G_N Mk + (G_N Mk)^2\left[-\frac{214}{105}\log^{(UV)}\left(\frac{k^2}{\mu^2}\right) + c'\right] + \dots\right\}$$

found in Goldberger and Ross PRD '10

★ Where are we now:

- we have the results of 505 graphs
- most of the remaining 90 graphs are (long but) straightforward: the code is presently running on them
- a few graphs are not obvious to solve
- also the tail term has been reproduced

★ When all the diagrams will be done:

- some manipulations to make the Lagrangian linear in accelerations and to move to center-of-mass frame
- consistency checks: poles cancellation, independence of observables from subtraction scale
- validation with known 4PN terms: extreme mass ratio, log terms, numerical evaluations
- .....almost there: still a bit of patience!