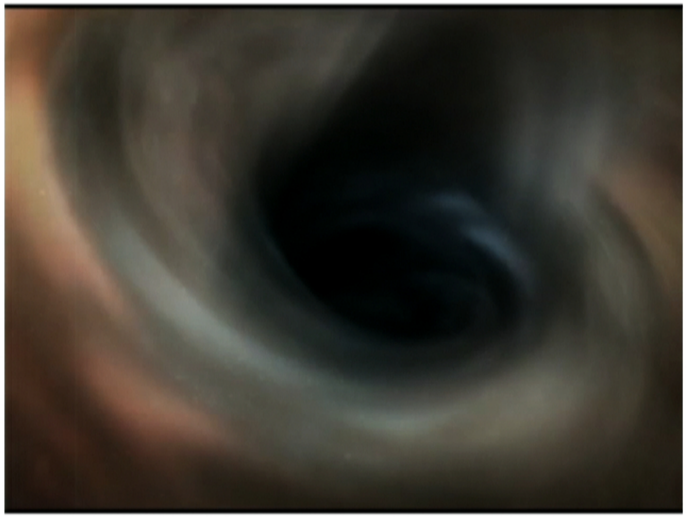
Title: How Good is "good enough" for Gravitational-wave Templates?

Date: Nov 28, 2011 09:30 AM

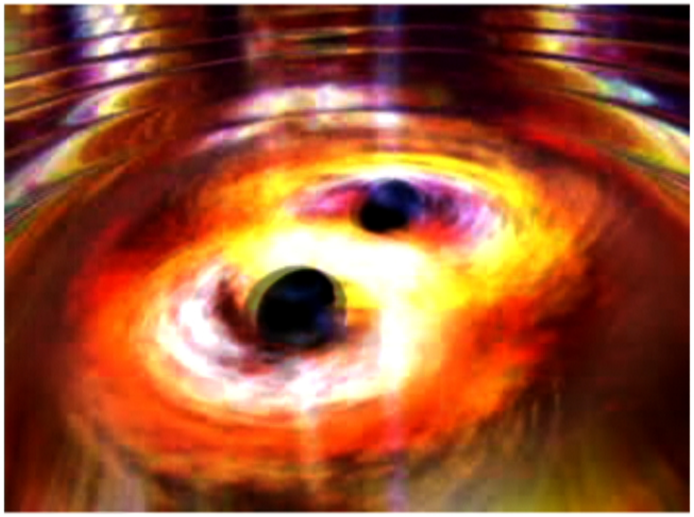
URL: http://pirsa.org/11110084

Abstract: After the first landmark gravitational-wave (GW) detection, GW astronomy will turn to the study of detector data to identify the physical properties of GW sources. The science payoff of GW observations must therefore depend critically on the accurate knowledge of the shapes of waveforms as functions of the source parameters. Effective-field-theory techniques have advanced and continue to advance the state of the art for the modeling of inspiraling-binary dynamics. But how far do we have to push our calculations to satisfy the needs of observations? I review current attempts to answer this question for different detectors and sources, and for the delicate problem of matching post-Newtonian and numerical-relativity waveforms.

Gravitational waves are propagating fluctuations of spacetime curvature, emitted by massive bodies in rapidly accelerated motion...

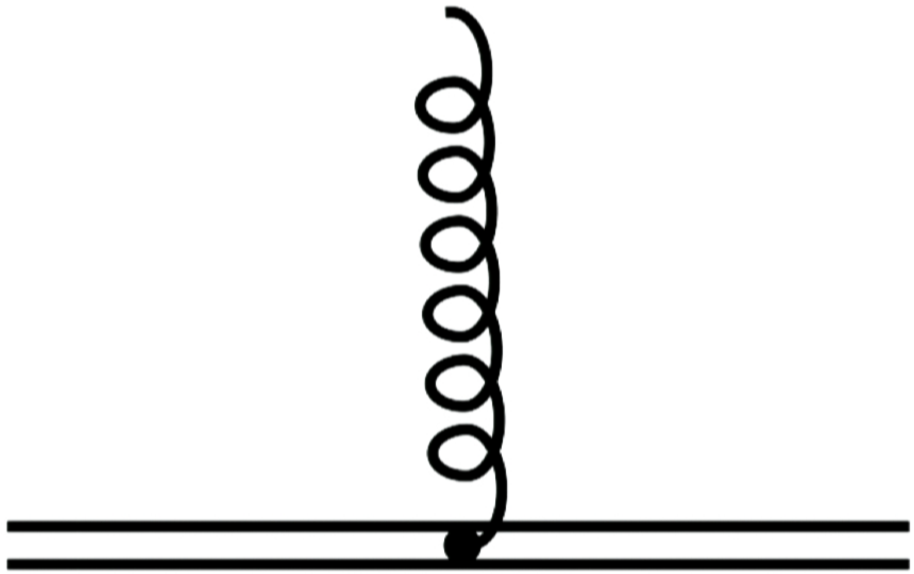


...such as black holes...

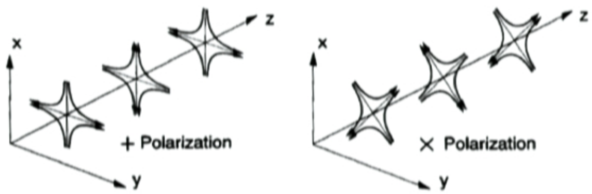


...that form in-spiraling binaries.

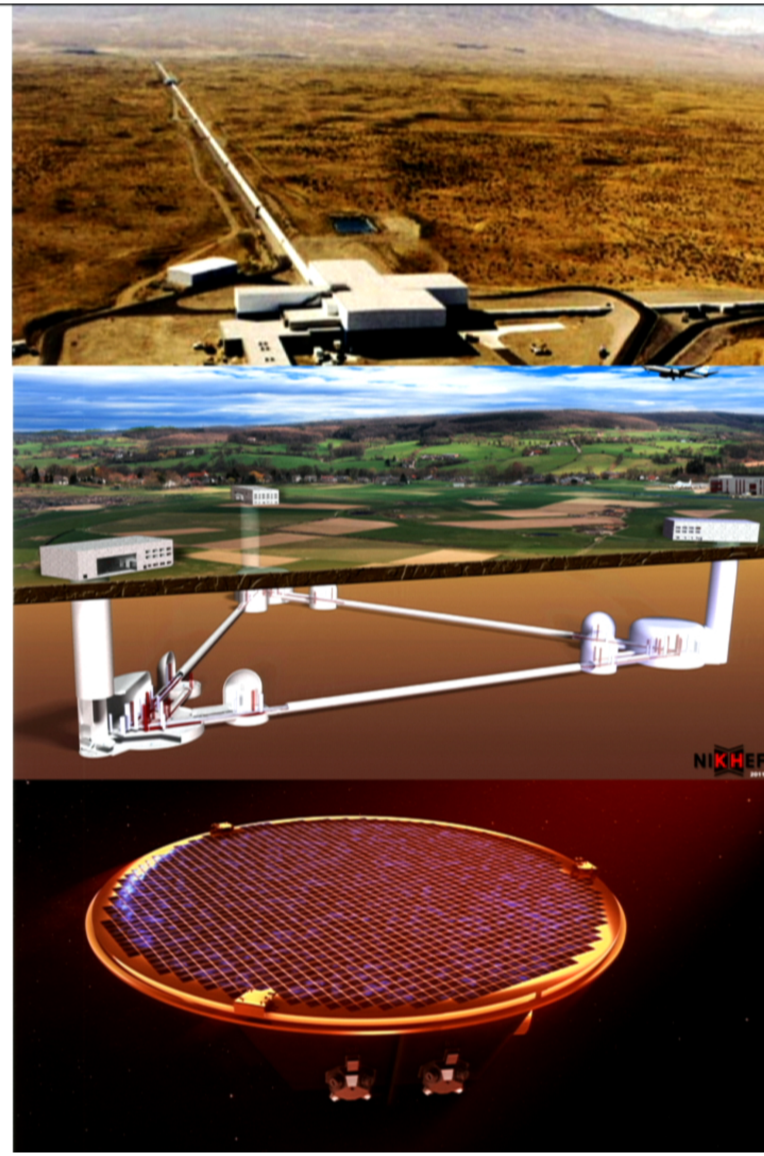
Gravitational waves are propagating fluctuations of spacetime curvature, emitted by massive bodies in rapidly accelerated motion...

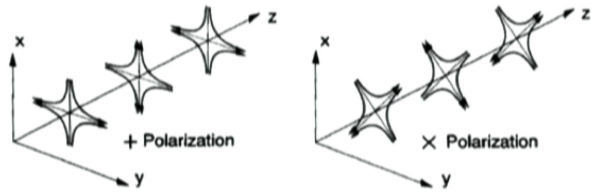


[courtesy C. Galley]



...and detected as transverse oscillations in the distance between test masses.





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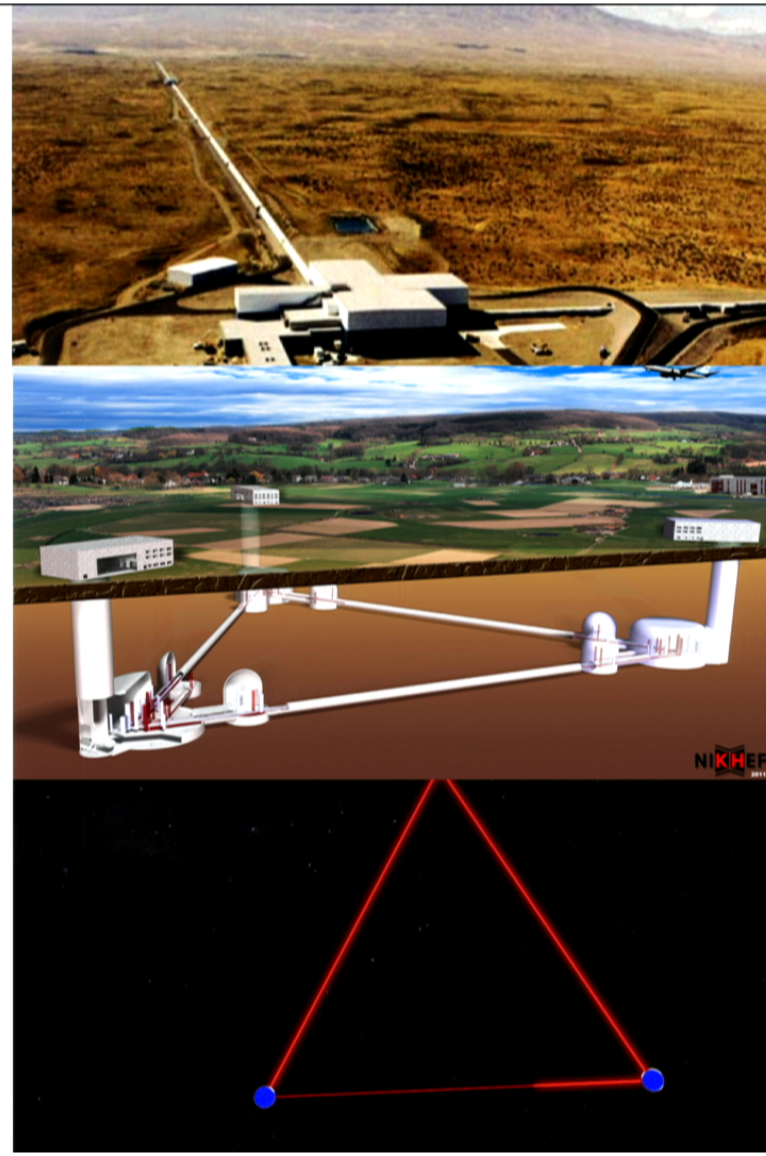

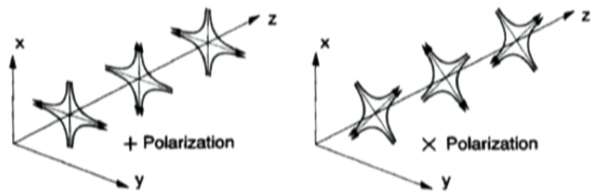

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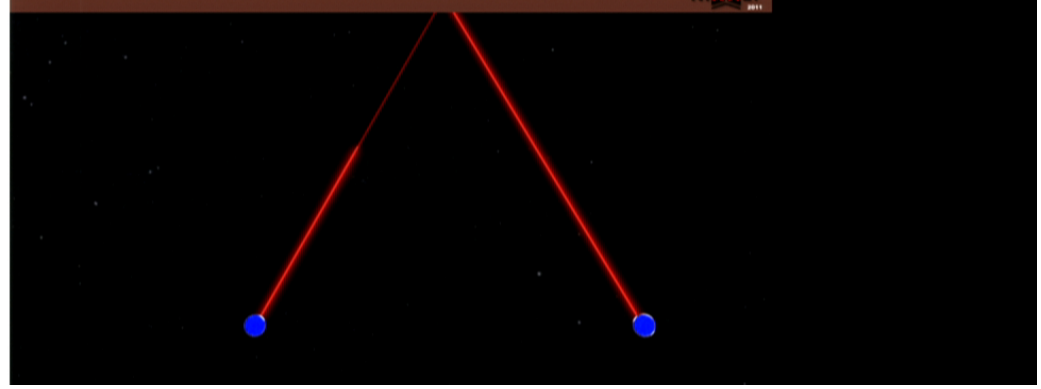
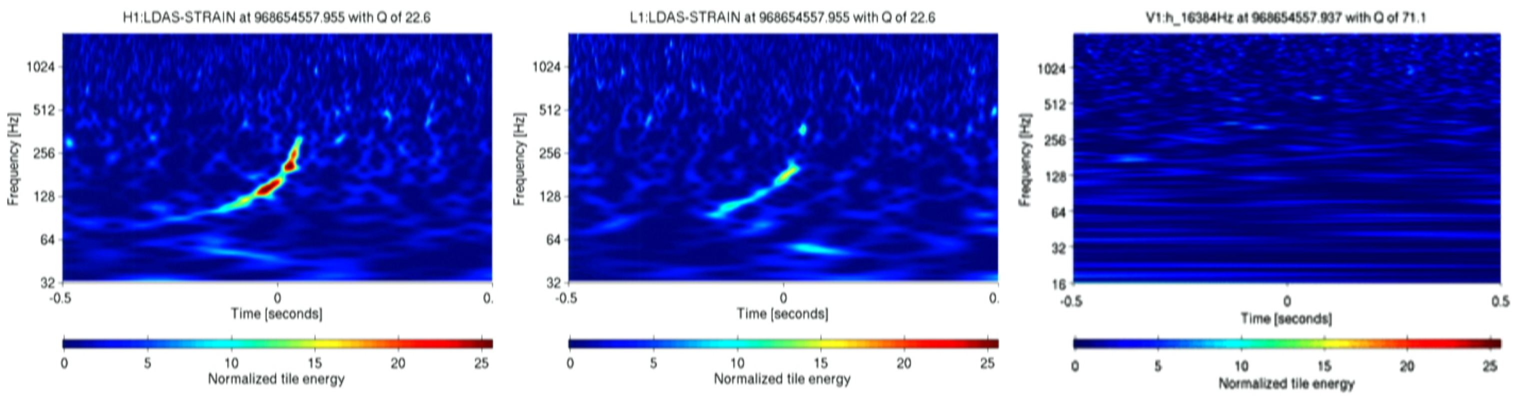
Gravitational waves...

- have typical strength 10^{-21}
- interact weakly with matter
- are emitted by bulk motions
- are phase coherent
- are measured by omnidirectional detectors and do not form images



a detection! GW100916, the Big Dog



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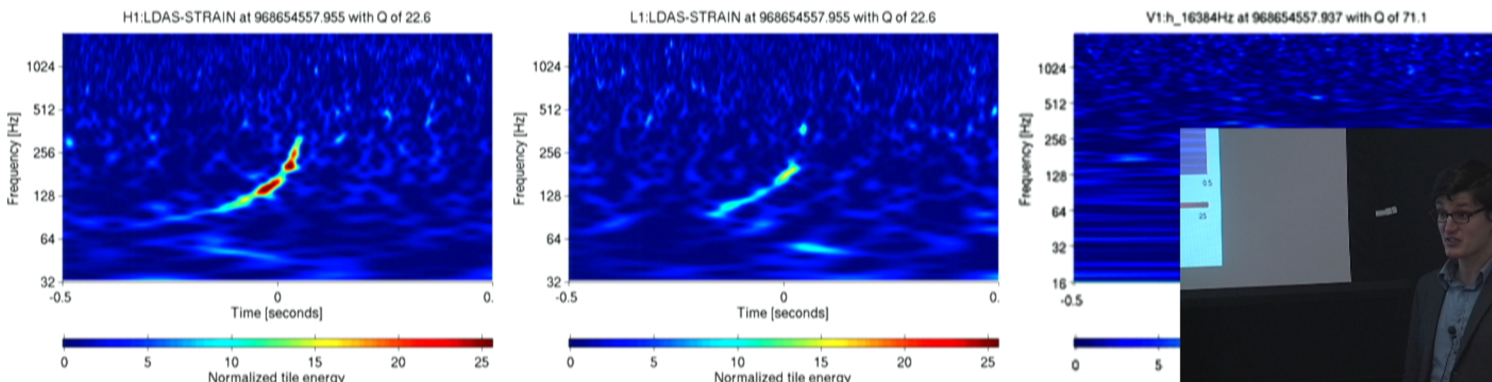
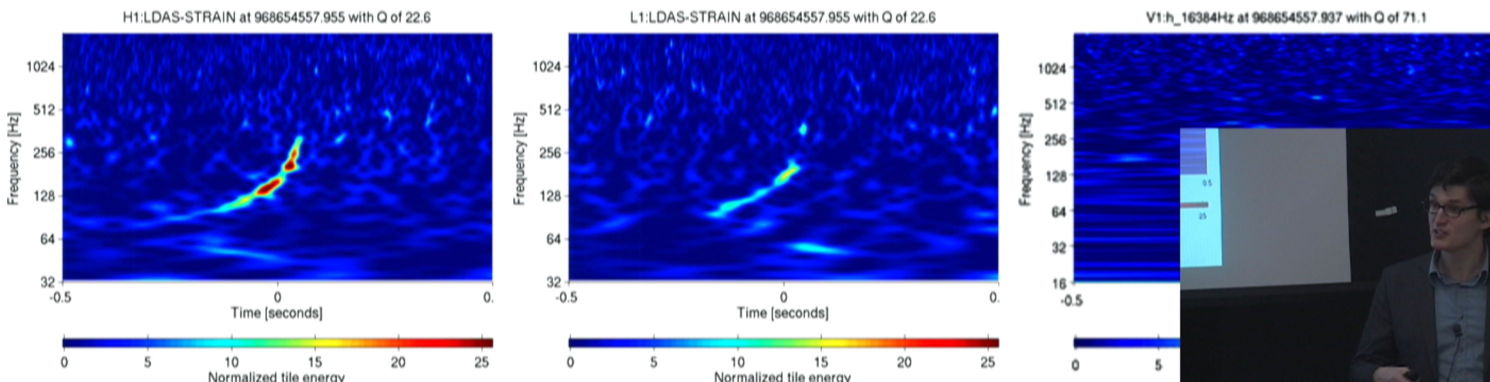
- opening the envelope
- unfortunately, a blind injection

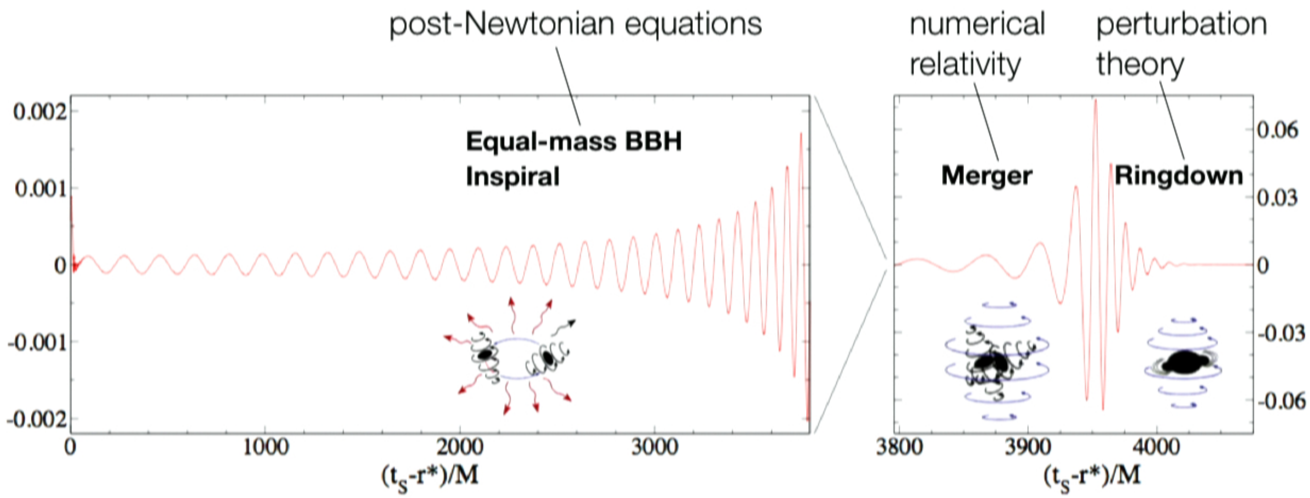


a detection! GW100916, the Big Dog

- opening the envelope
- unfortunately, a blind injection
- but found!
- testing methods, protocols, people



GW science in a nutshell: what's in a waveform?



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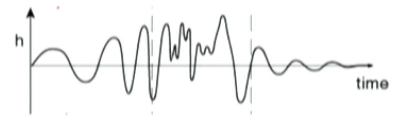

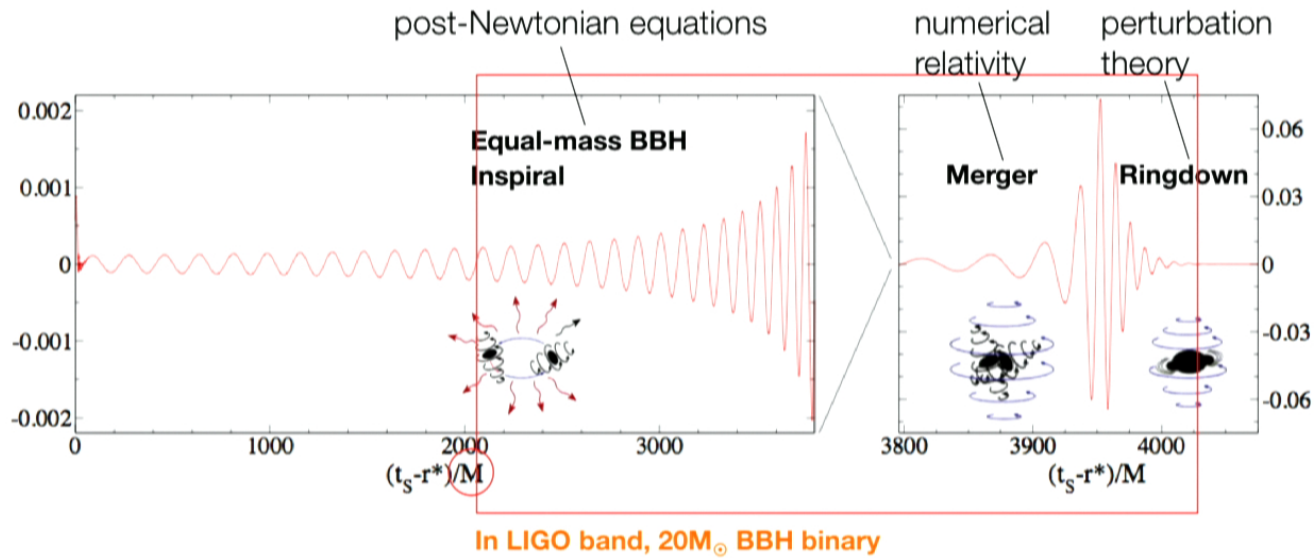

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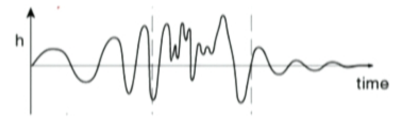

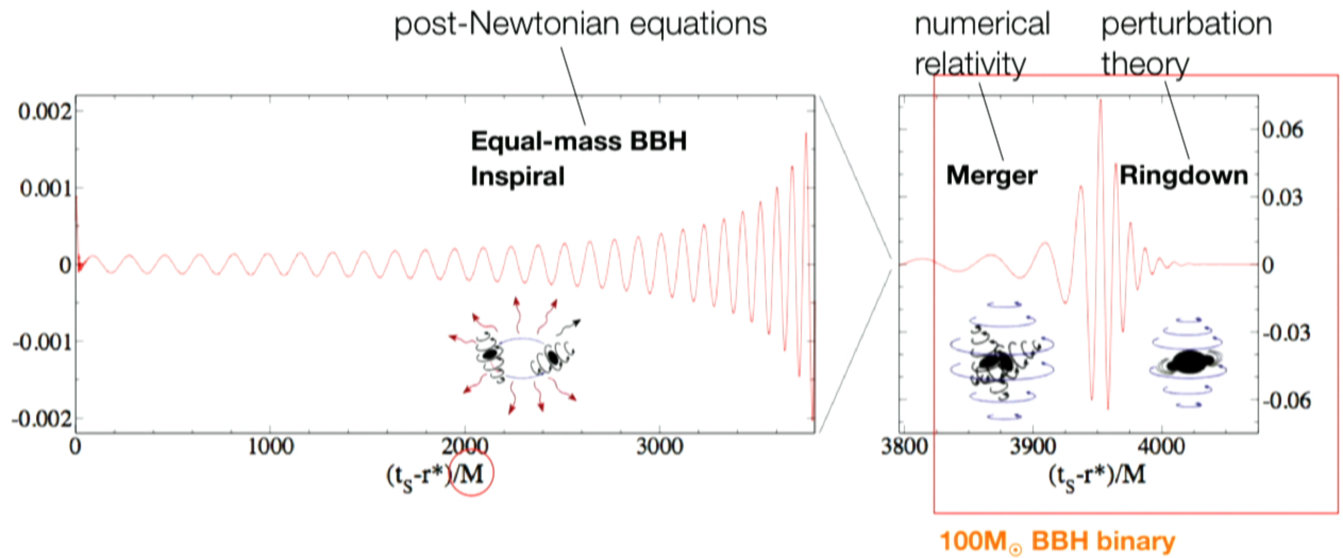

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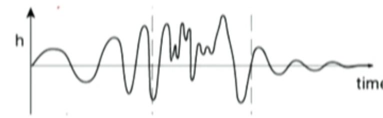

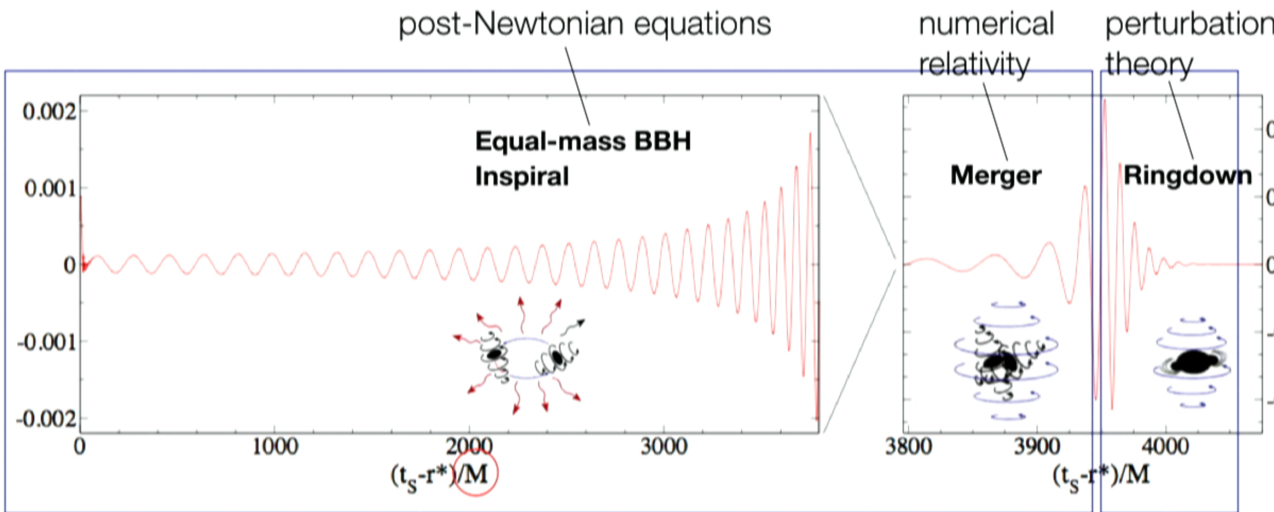

Depends on binary masses, spins, orientations

Depends on mass, spin, and orientation of final BH

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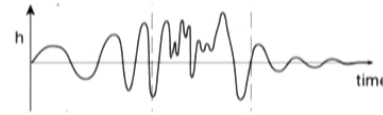

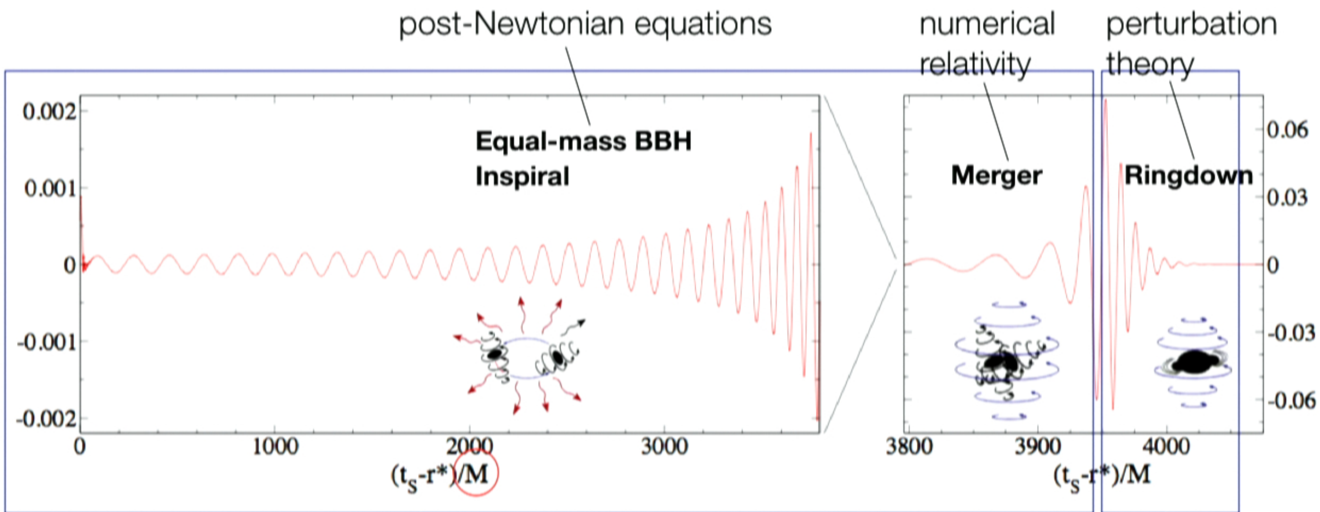

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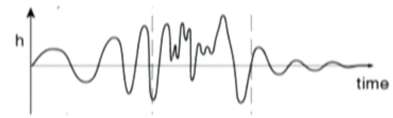
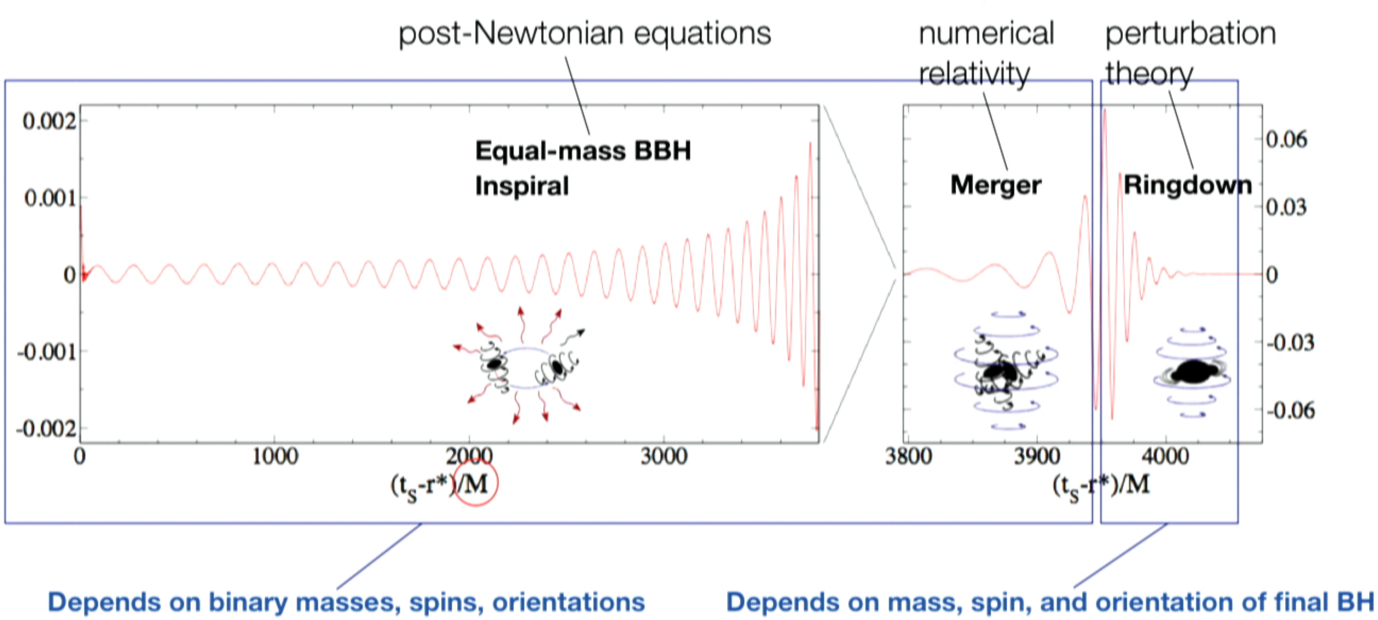
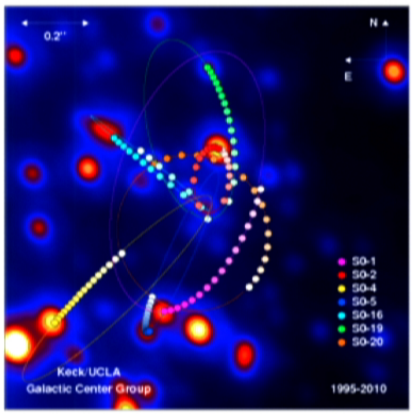




- astrophysics: compact-object physics; SN and GRB progenitors; origin, merger, and accretion history of massive galactic BHs
- cosmology: standard sirens (D_L from GWs, z from counterparts)
- strong-field GR, no-hair theorem: extreme-mass ratio inspirals, ringdown modes
- nuclear physics: late NS–NS inspiral, NS–BH tidal disruption
- alternative theories of gravity: strong field, radiation sector

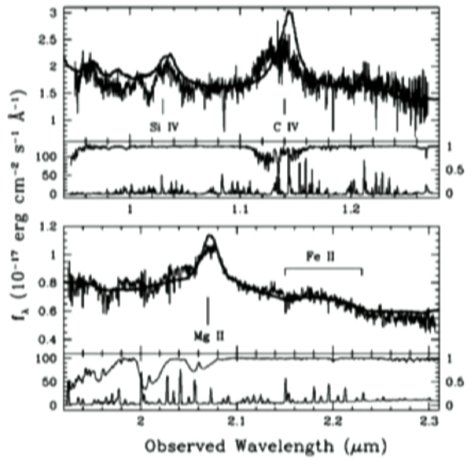
LISA science highlight: massive black-hole coalescences

Massive BHs (10^{6-9} $M_\odot$) populate galaxy centers...



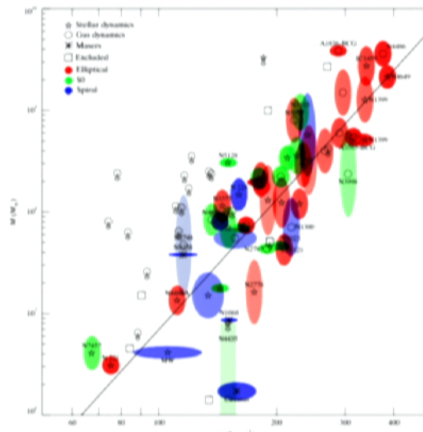
Ghez group 2010

...shone as quasars in the past, as early as z ~ 6...



SDSS J1148 (Barth et al. 2003)

...and co-evolved closely with their galaxies.



Gültekin et al. 2009

- How did the original seed BHs form? From pop III stars, or direct gas collapse?
- How did the MBHs grow? Through efficient disk accretion or chaotic episodes?
- How often do paired BHs shrink their orbits down to GW-led inspirals?

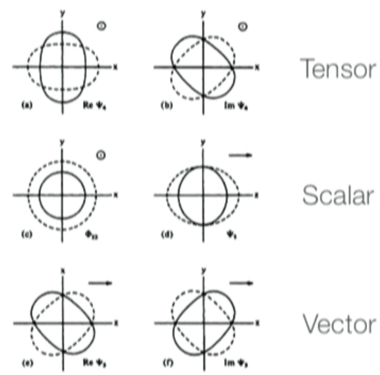
Testing GR with GWs—prospects and hopes

- Quadrupole formula

$$\left(\frac{dE}{dt}\right)_{\rm quadrupole} = -\frac{8}{15}\frac{\eta^2 m^4}{r^4}(12v^2 - 11\dot{r}^2)$$

$$\left(\frac{dE}{dt}\right)_{\rm dipole} = -\frac{2}{3}\frac{\eta^2 m^4}{r^4}\left(\frac{\mathcal{S}^2}{\omega}\right)$$

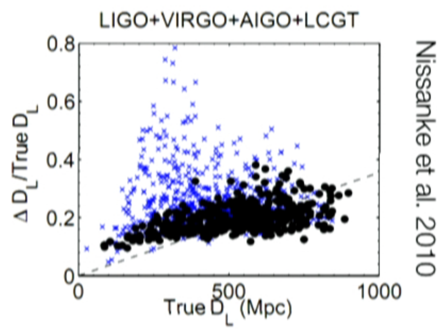
- GW polarizations



- Speed of GWs

$$\frac{v_{\rm g}^2}{c^2} = 1 - \frac{m_{\rm g}^2 c^4}{E^2}$$

- H_0, dark-energy EOS

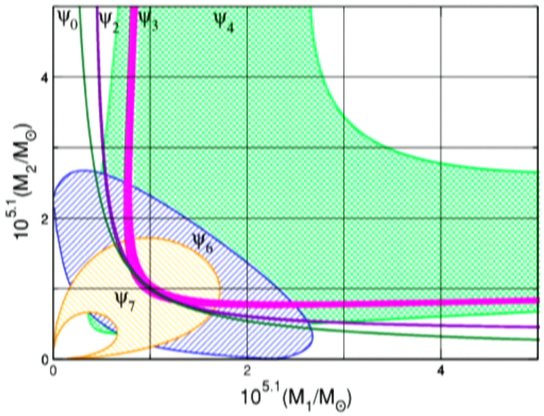


- Black-hole hair

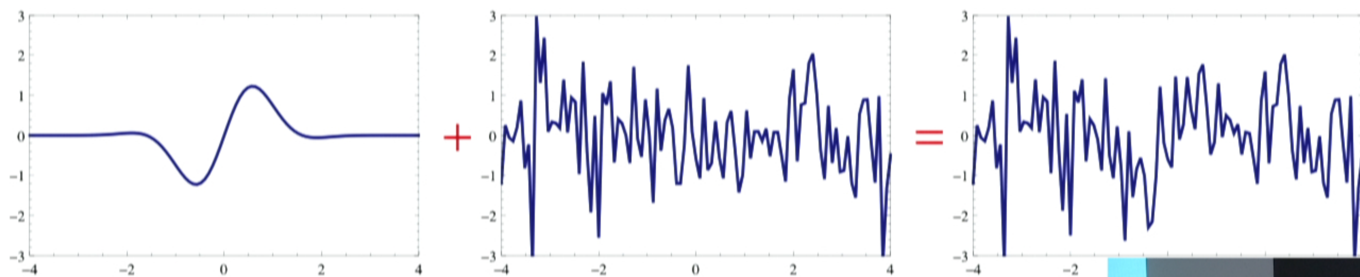
$$M_l + iJ_l = M(ia)^l$$

Testing GR with GWs—some skepticism

- Standard tests of GR are based on Will et al.'s "standard model" (equivalence principle + metric theories + parametrized post-Newtonian), which leaves little room for detecting non-standard radiation
- No simple principled framework exists for radiative systems or systems containing strong internal fields. So we must consider individual alternative theories (but the Hulse-Taylor pulsar already killed the best ones!)
- It will be hard to distinguish non-GR from non-BHs from (un)known astrophysics
- Conversely, the matched-filtering dilemma suggests that truly different gravity may go completely undetected
- Null tests, such as PN-coefficient consistency, are conceptually most robust

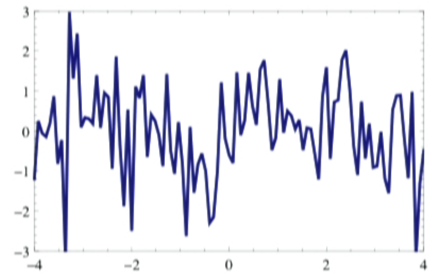


data = signal + noise



(SNR = 5)

therefore: noise = data – signal; to assess detection, we ask which noise is more probable?



(no signal)

or

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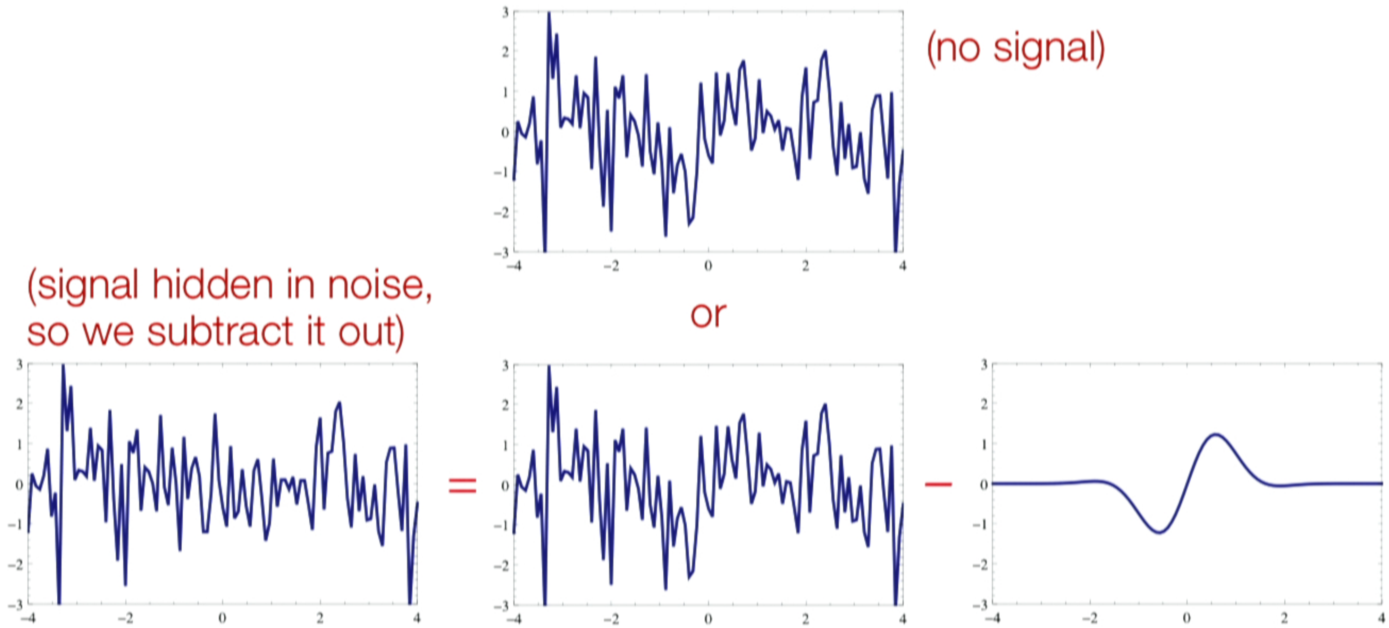

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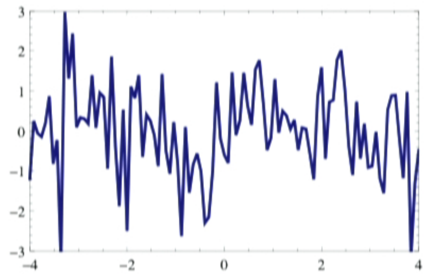
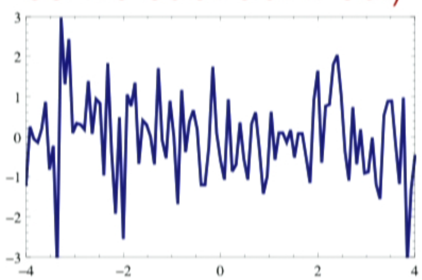


(no signal)

or

(signal hidden in noise, so we subtract it out)



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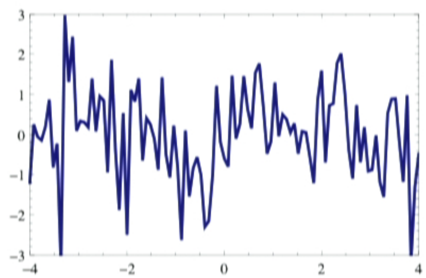

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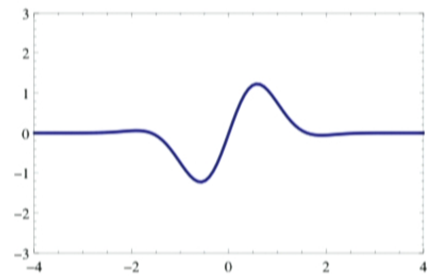
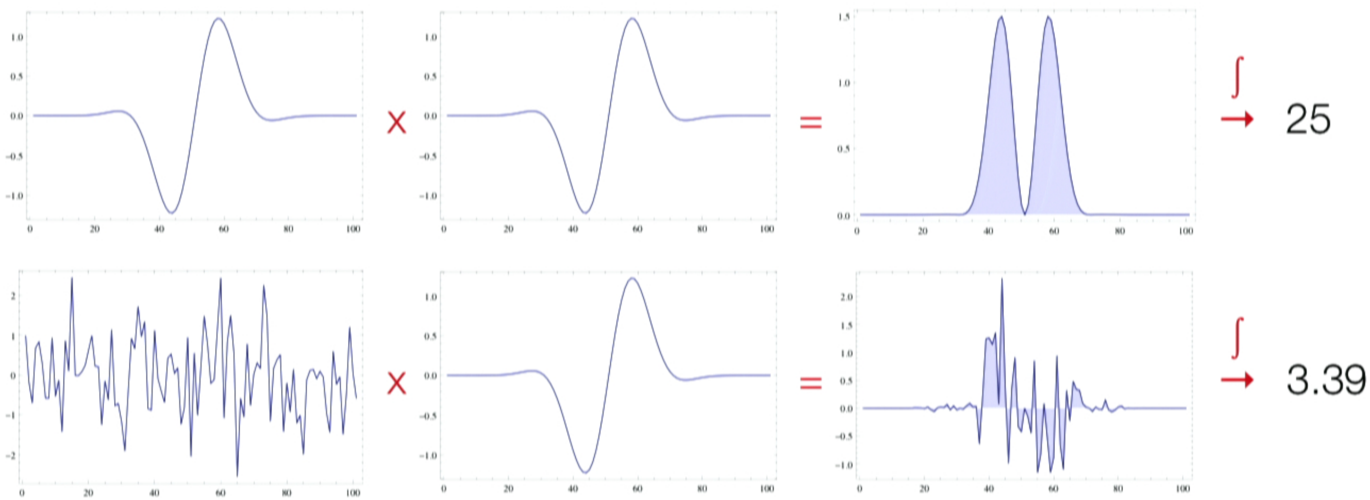


the ratio of probabilities is ~ exp $SNR^2/2$,
(here ~ 270,000)

a better interpretation of this process
is in terms of correlation products/matched filtering



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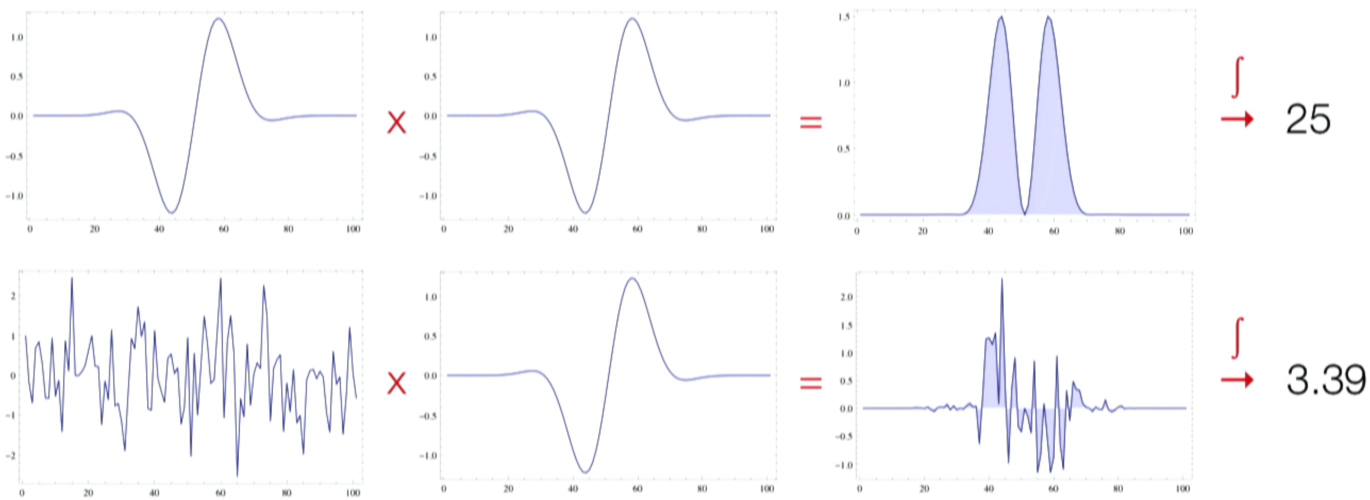
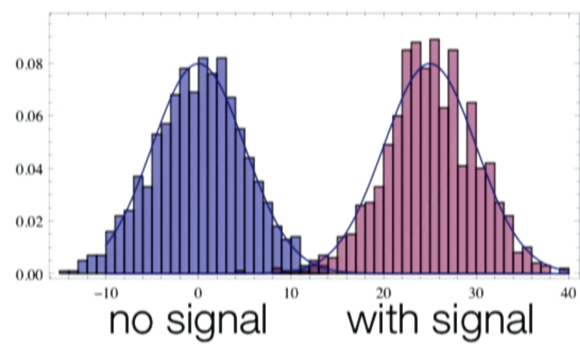


$$\int_0^T s(t) \times h(t)\, \mathrm{d}t$$



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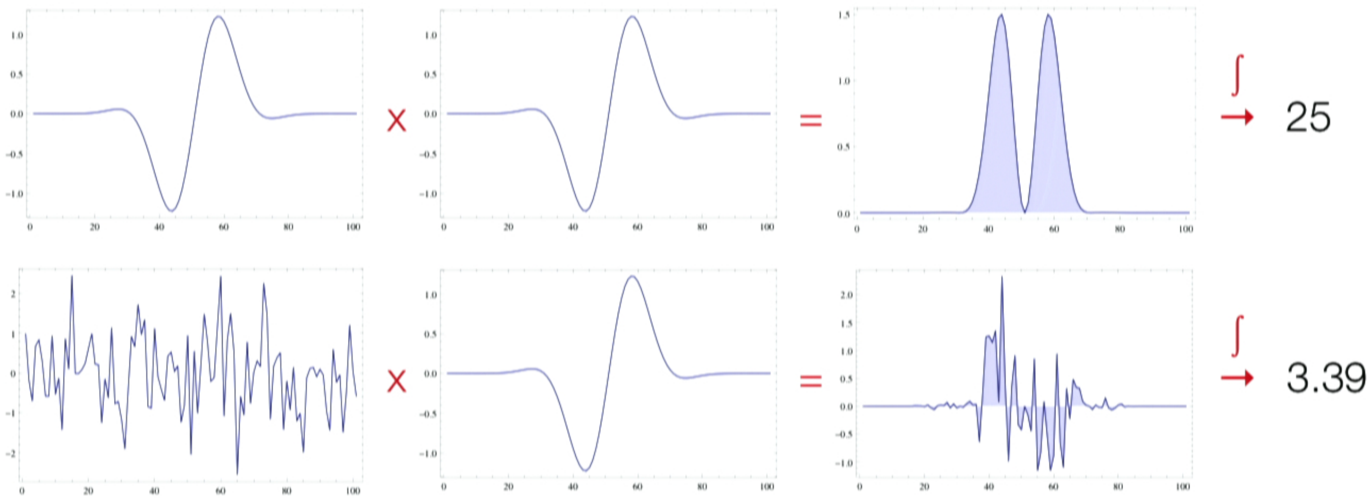
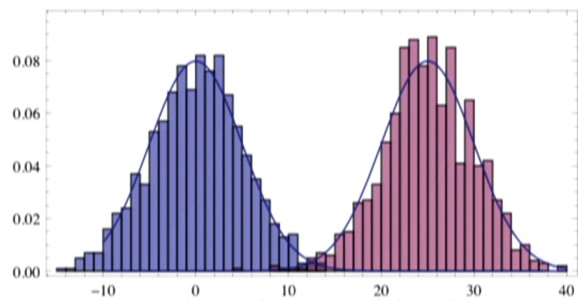


detector data

$$\int_0^T s(t) \times h(t)\,\mathrm{d}t$$

signal "template"



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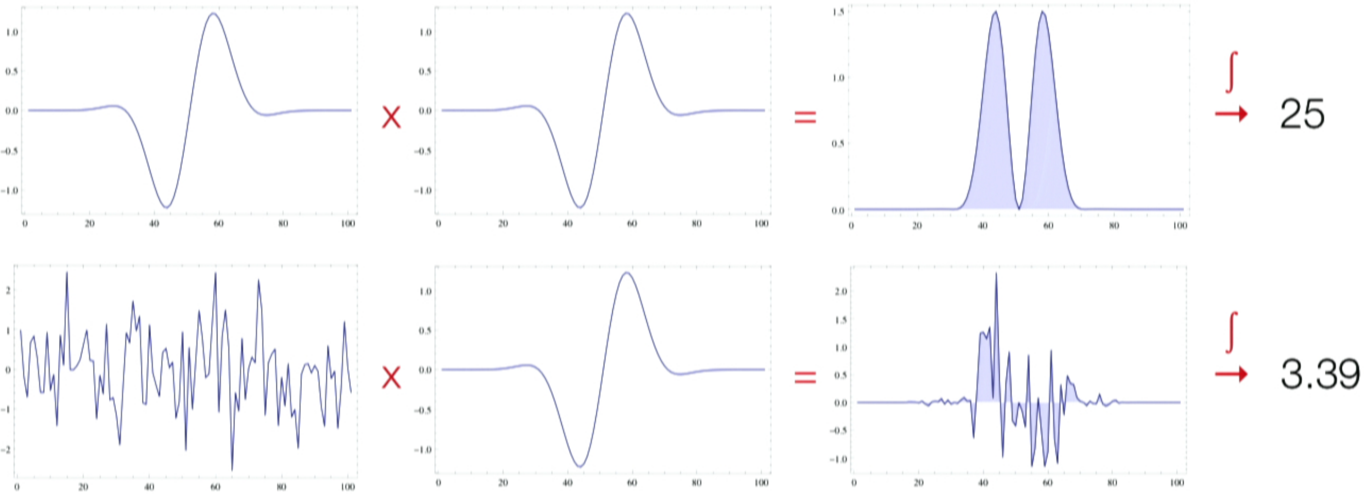

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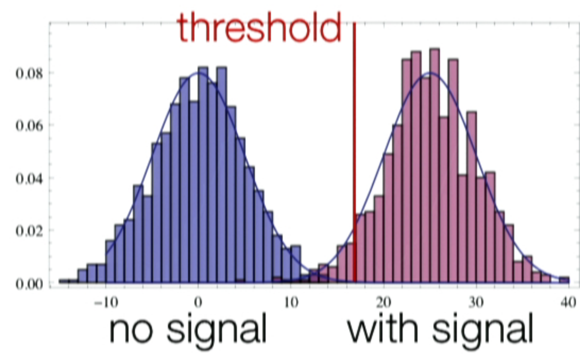
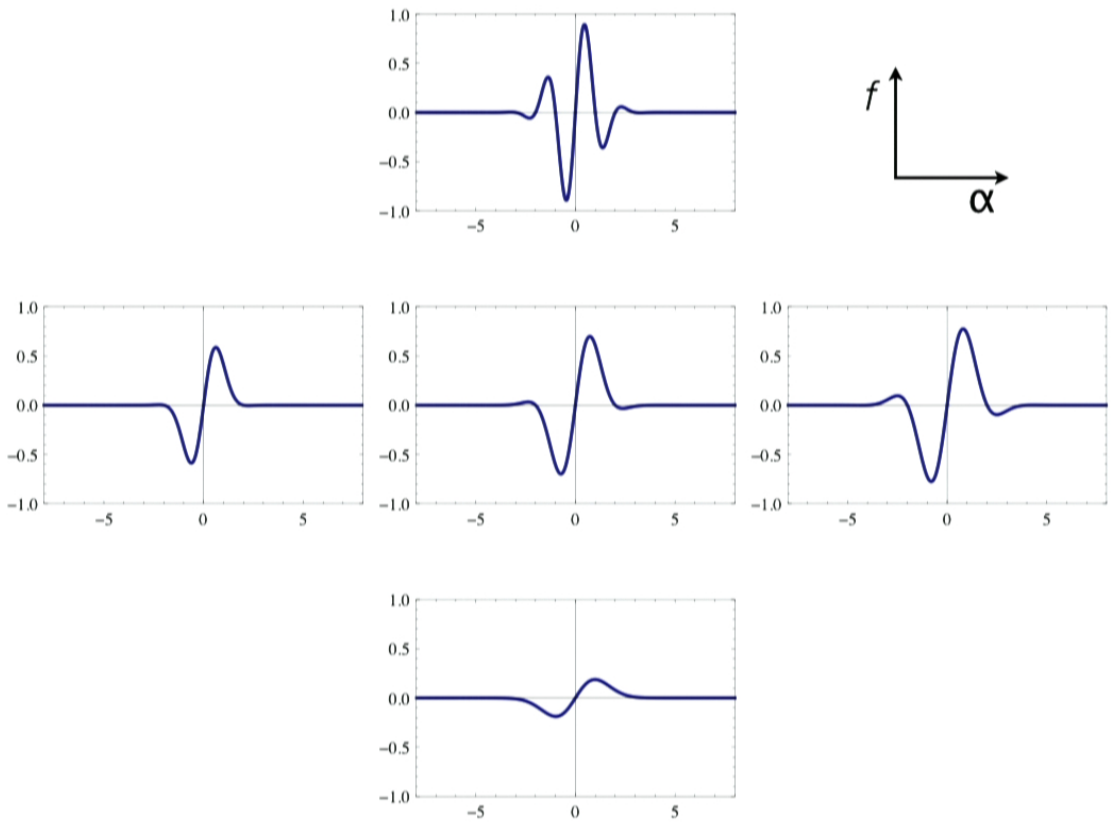


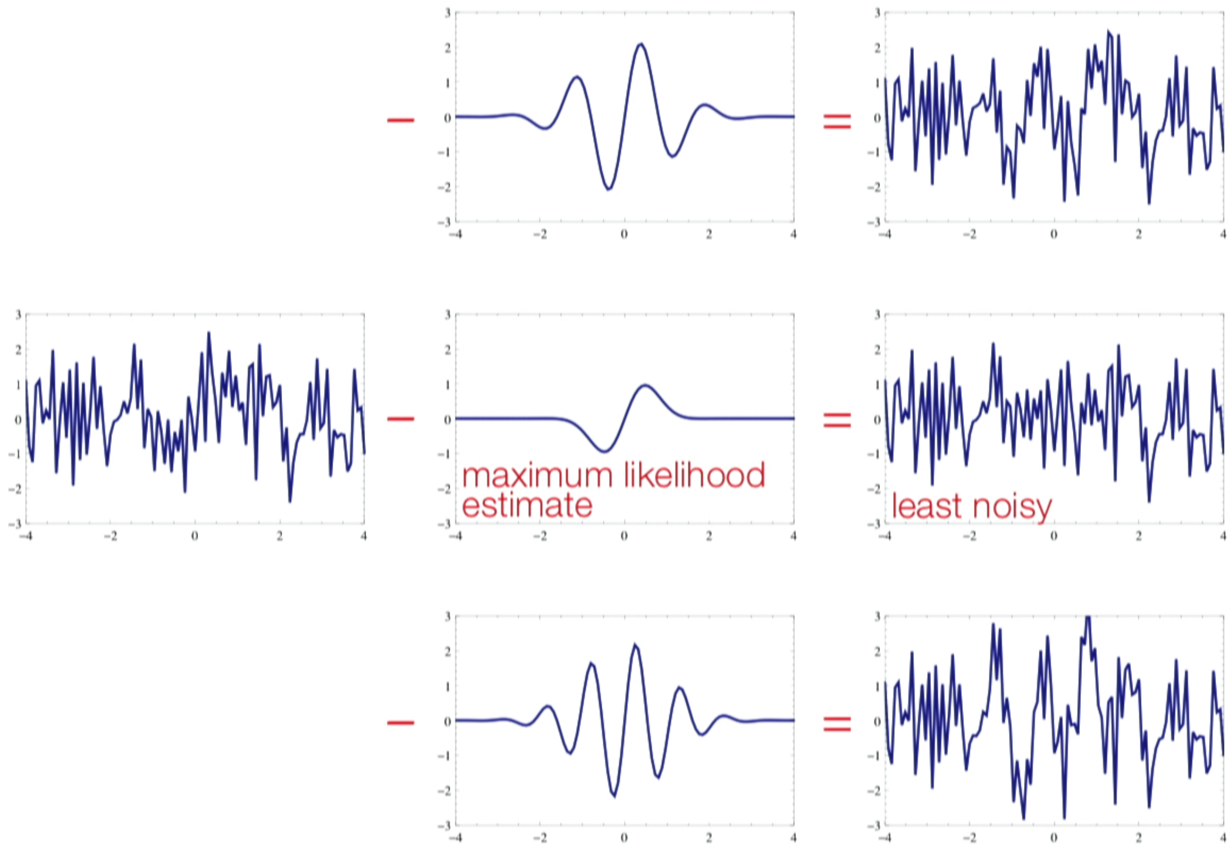
“science” ~ signal(parameters)

$$h(t; A, \alpha, f) = A\, e^{-\frac{t^2}{2\alpha^2}} \sin(2\pi f t)$$

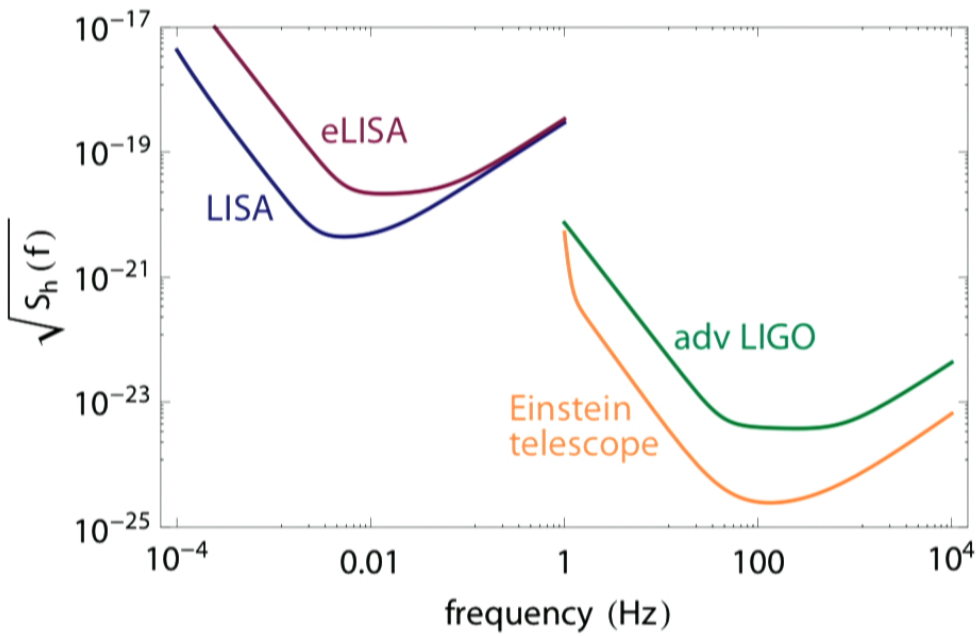


noise = data – signal

hence *p*(signal parameters) = *p*(noise residual)



because of colored detector noise, detection and parameter estimation are sensitive to the frequency content of waveforms...



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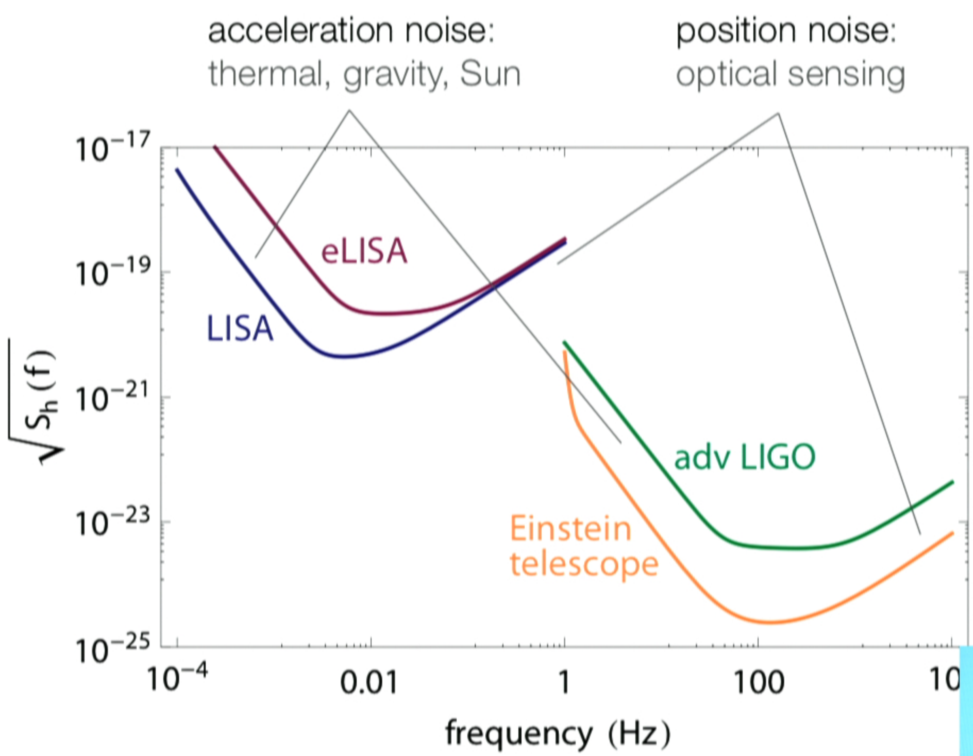

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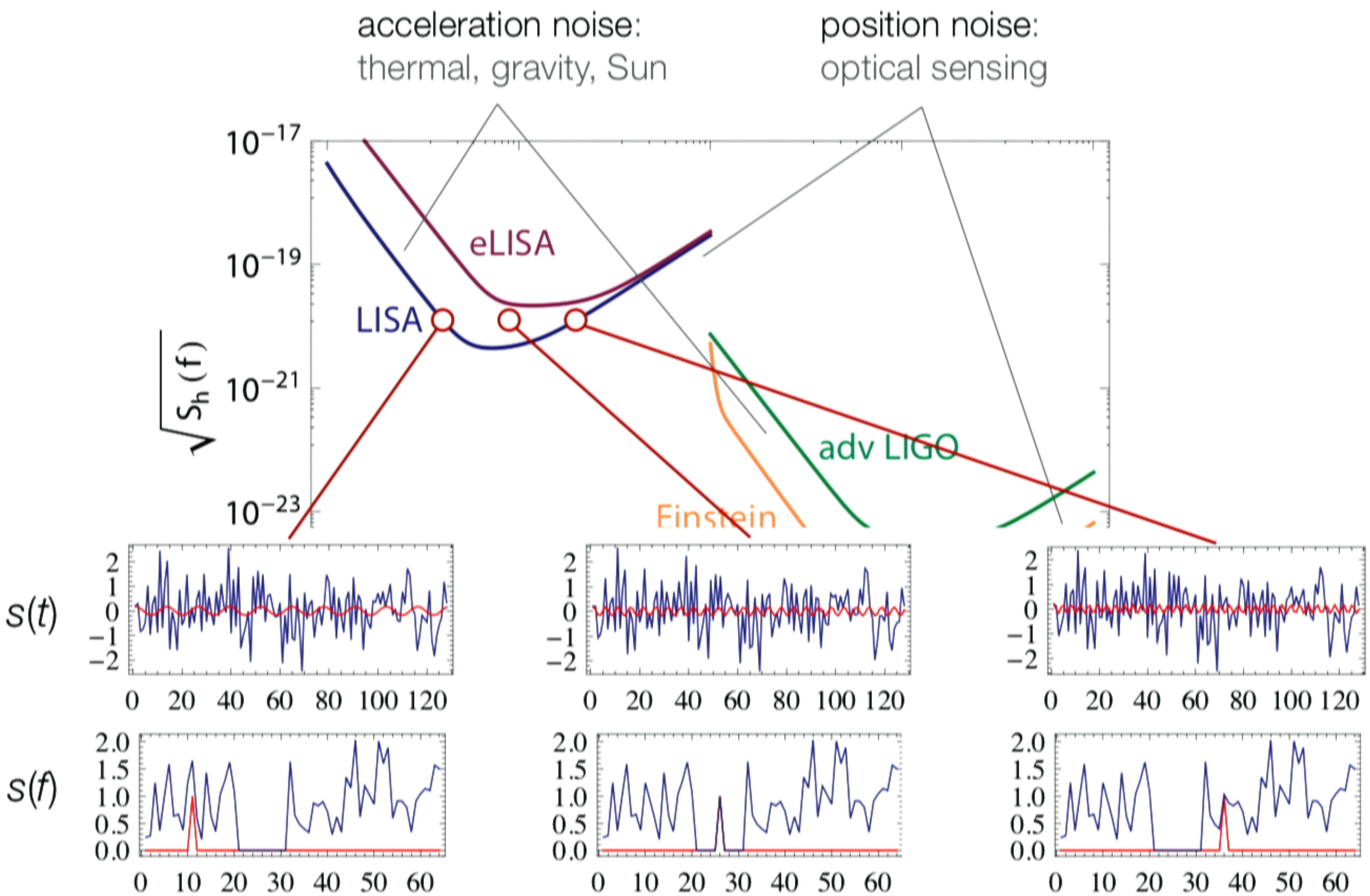


...hence the most famous formula of GW data analysis,
the noise-weighted signal power/correlation product

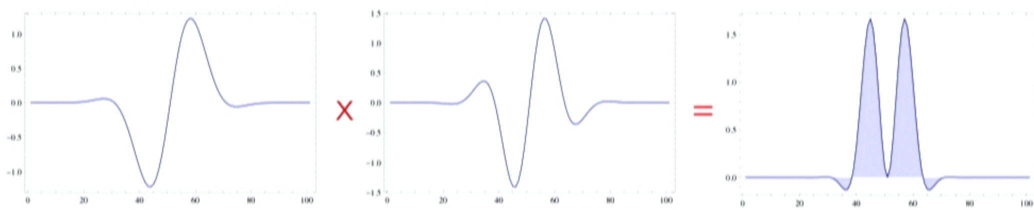
$$(h_1, h_2) = 4\,\mathrm{Re} \int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f)}{S_n(f)}\,\mathrm{d}f$$

yields a natural distance
in signal space

(remember that $\int_{-\infty}^{\infty} h_1(t) h_2(t)\,\mathrm{d}t = \int_{-\infty}^{\infty} \tilde{h}_1^*(f) h_2(f)\,\mathrm{d}f$)

waveform error has different effects on detection...

- by reducing the correlation integral,



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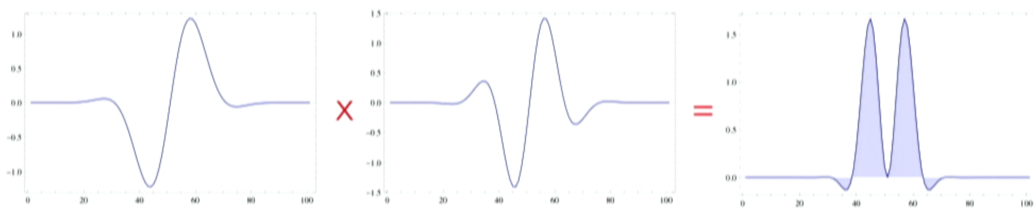
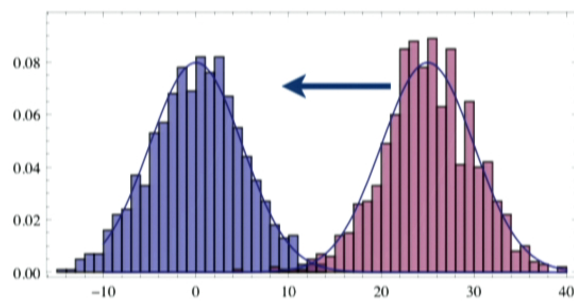


- we reduce noise vs. signal discrimination



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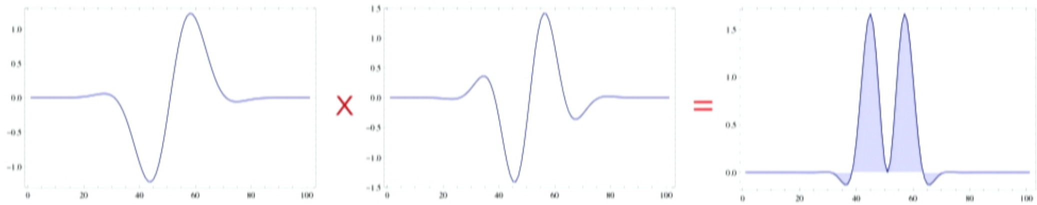

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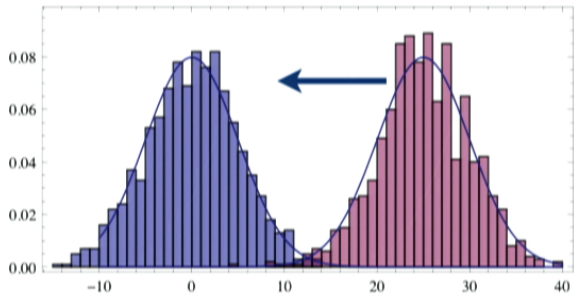
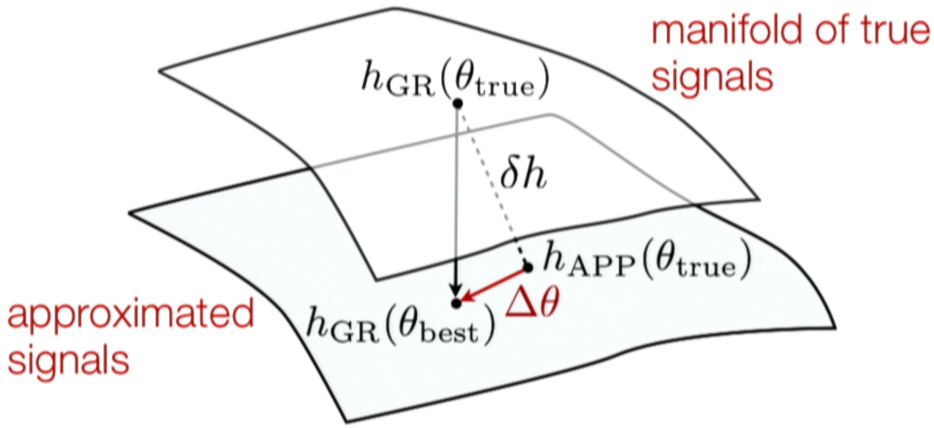


- requiring higher thresholds, reducing horizon distances and event rates

$$\mathrm{FF} = \frac{(h_{\mathrm{true}}, h_{\mathrm{best}})}{|h_{\mathrm{true}}| \times |h_{\mathrm{best}}|}; \quad \text{horizon} \propto \mathrm{FF}, \quad \text{rate} \propto \mathrm{FF}^3$$

...and parameter estimation

- if model signals differ from true signals with the same parameters, the best-fitting template will be displaced by an SNR-independent theoretical error



- by contrast, statistical error (due to noise) is ~1/SNR

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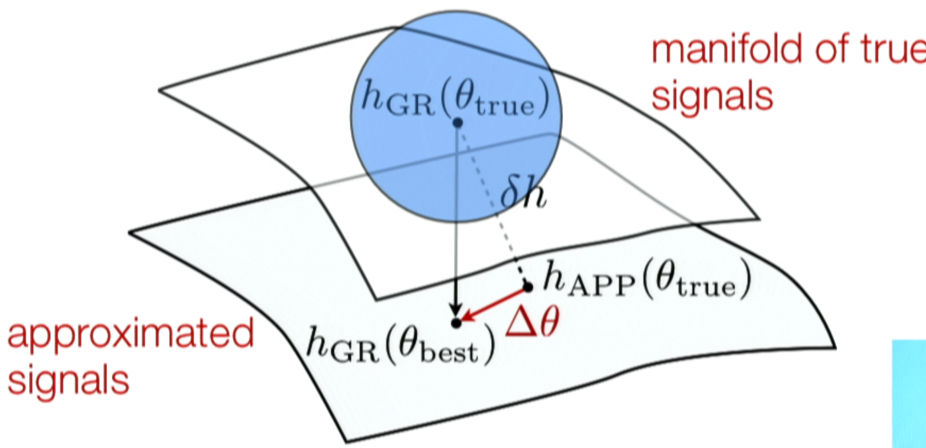

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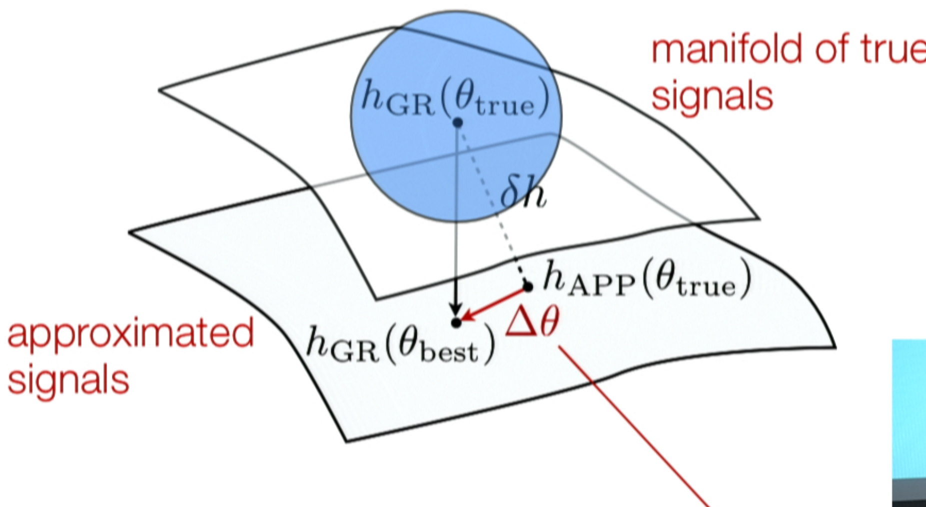

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Hence Lindblom's accuracy criteria (2008)
[here as reworked by MacDonald et al. 2011]

- for detection

$$\frac{|\delta h|}{|h|} < \sqrt{2\epsilon}$$

~ 0.005, limited by detection calibration and template banks

- for parameter estimation

$$\frac{|\delta h|}{|h|} < \frac{1}{\mathrm{SNR}}$$

~ 10 for initial adv. LIGO detections
~ 100 for best detections
~ 1,000 for space-based MBBH

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~ 1,000 for space-based MBBH

- the inaccuracy functional can be related to the waveform-and-noise weighted amplitude and phasing errors

$$\frac{|\delta h|}{|h|} \simeq \sqrt{\overline{\delta\chi^2} + \overline{\delta\phi^2}}$$

Hence Lindblom's accuracy criteria (2008)

[here as reworked by MacDonald et al. 2011]

- for detection

$$\frac{|\delta h|}{|h|} < \sqrt{2\epsilon}$$

~ 0.005, limited by detection calibration and template banks

- for parameter estimation

$$\frac{|\delta h|}{|h|} < \frac{1}{\mathrm{SNR}}$$

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$$\frac{|\delta h|}{|h|} \simeq \sqrt{\overline{\delta\chi^2} + \overline{\delta\phi^2}}$$

unfortunately quite difficult to get from PN expressions without comparing full waveforms!

$$h(t) \to \tilde{h}(f) \equiv \mathrm{e}^{\chi(f)+\mathrm{i}\phi(f)}$$

$$\overline{\delta\phi^2}(f) = (\delta\phi\, h, \delta\phi\, h)/(h, h)$$

MacDonald et al. (2011) analyze the accuracy of a hybrid inspiral-merger-ringdown waveform (PN + 15 orbit NR)

[see also Boyle 2011]

- they study the PN-approximant error by comparing waveforms hybridized at different frequencies; they conclude that NR waveforms are too short (or the PN approximants are too imprecise) to achieve the Lindblom criteria

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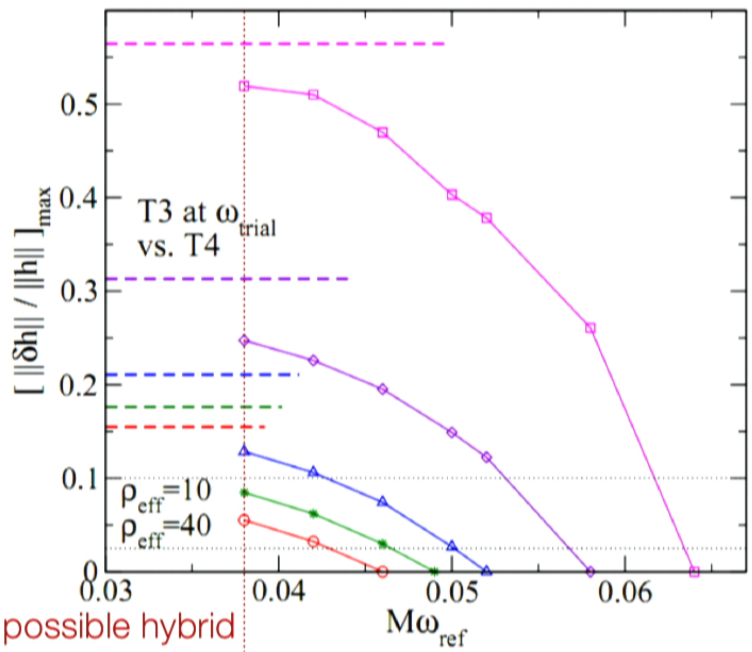

TaylorT3 PN inspirals, equal masses, no spin

earliest possible hybrid

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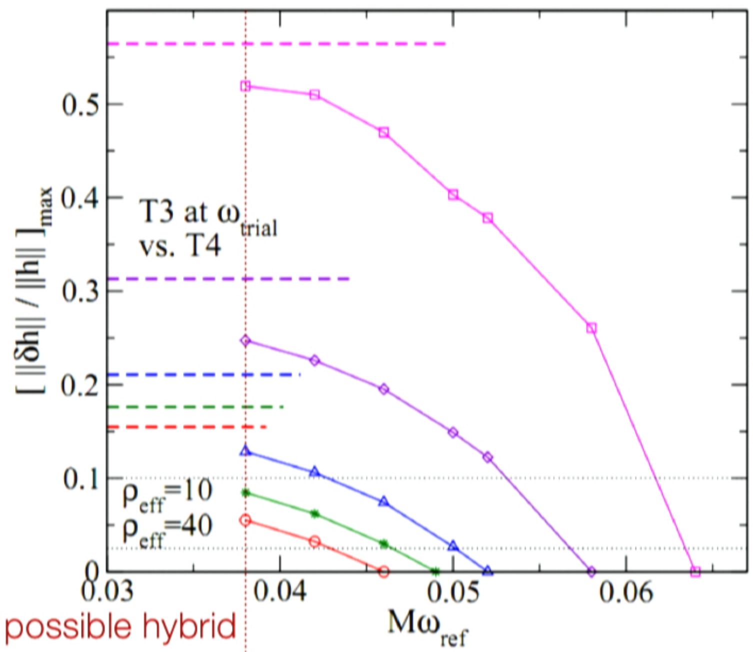

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$$\dot{\phi}_{\rm orb} = \omega_{\rm orb}$$
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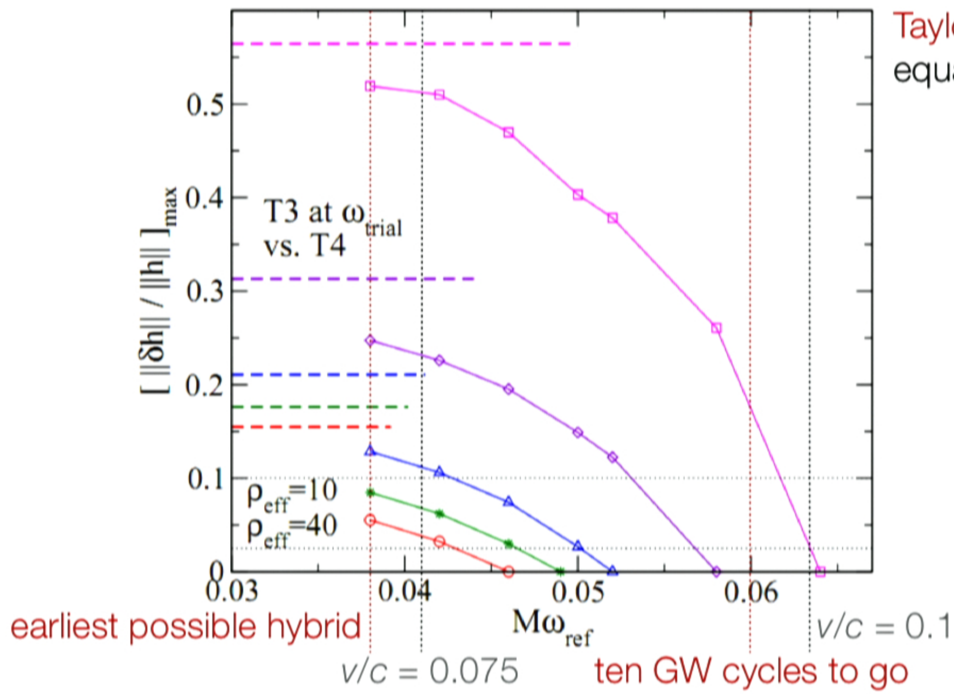

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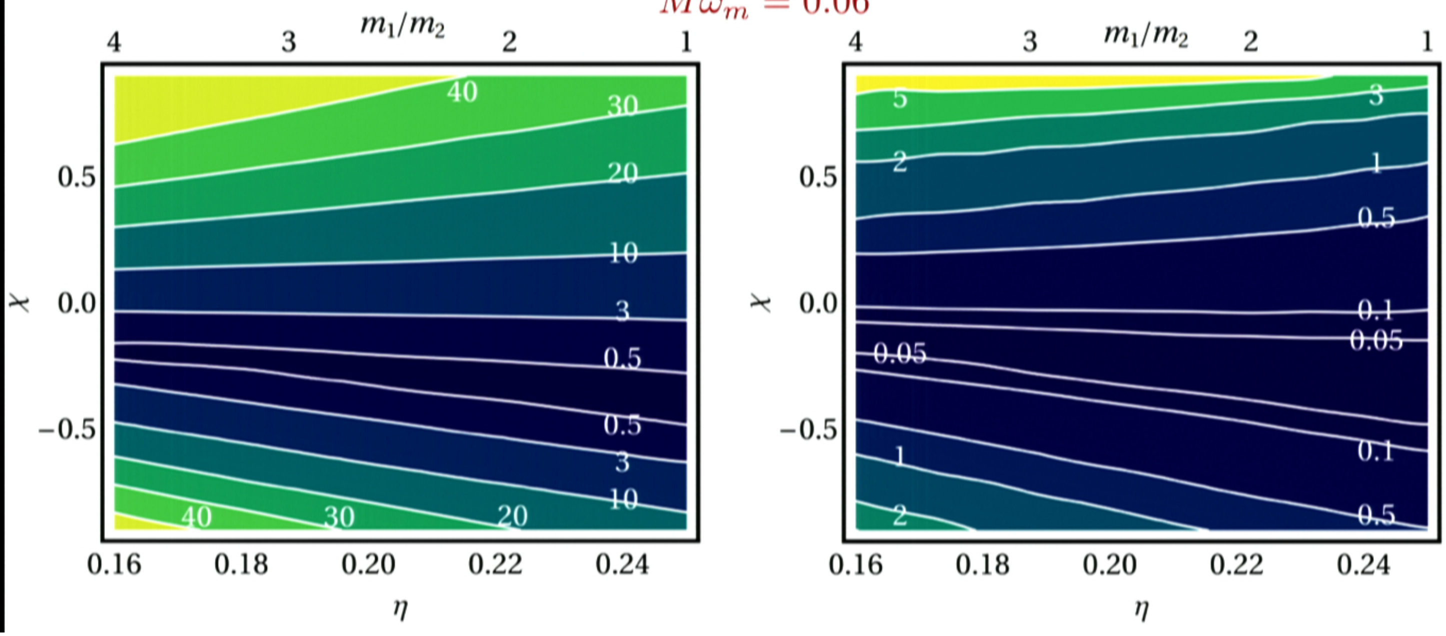
$$\text{mismatch}\,[\%] = \tfrac{1}{2}|\delta h|^2/|h|^2$$
$$M\omega_m = 0.06$$



Would higher-order PN corrections really help?
Asymptotic analysis is barely possible (and inconclusive) for EMRI inspirals, where exact energy flux is known numerically

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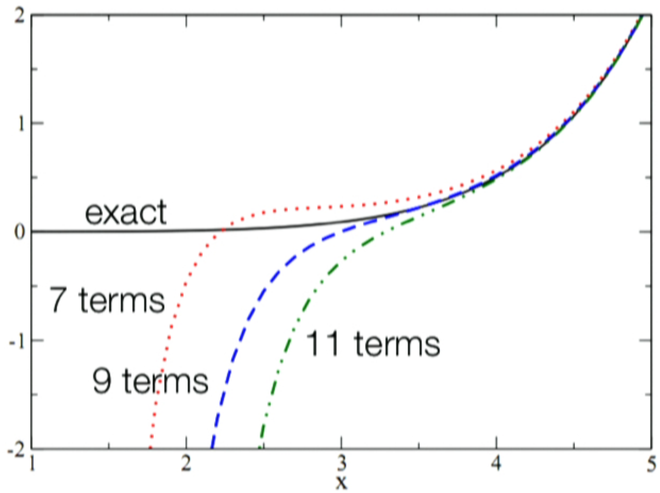
asymptotic $1/x^n$ expansion of modified Bessel function $I_5(x)$



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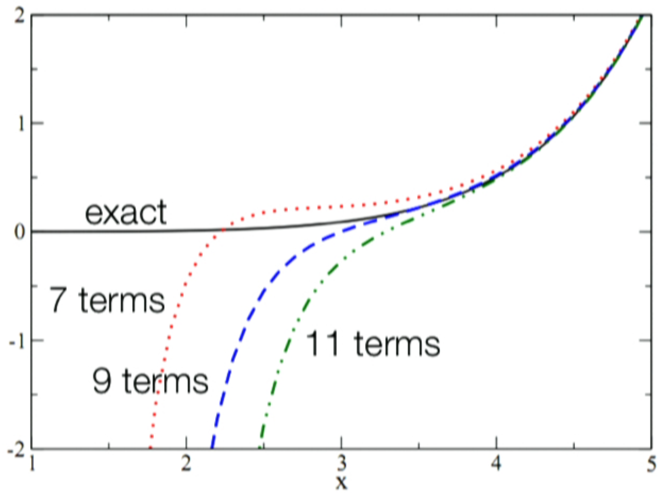
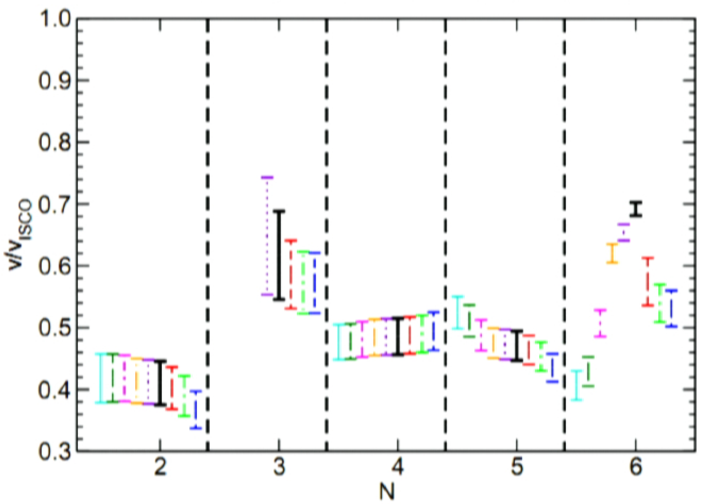
limited by boundary where next term ~ remainder

asymptotic $1/x^n$ expansion of modified Bessel function $I_5(x)$



region of validity for PN flux expansion, a = −0.99, −0.9, −0.6, −0.2, −0.1, **0**, 0.3, 0.6, 0.9



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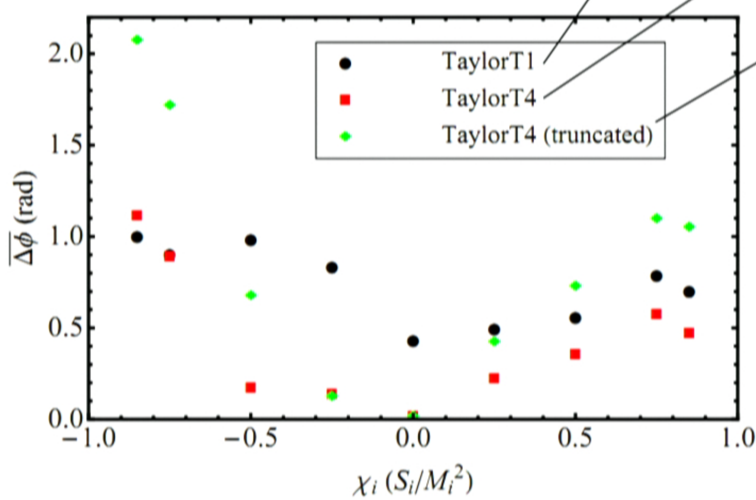
$\mathcal{F}/\mathcal{E}'$ Taylor expanded...

...with no (formal) 3PN and 3.5PN spin terms



integrated phase error for equal-mass, aligned-spin inspirals

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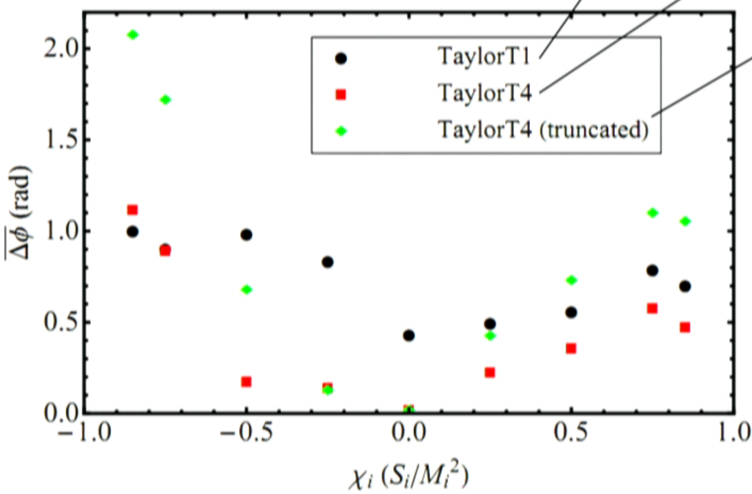

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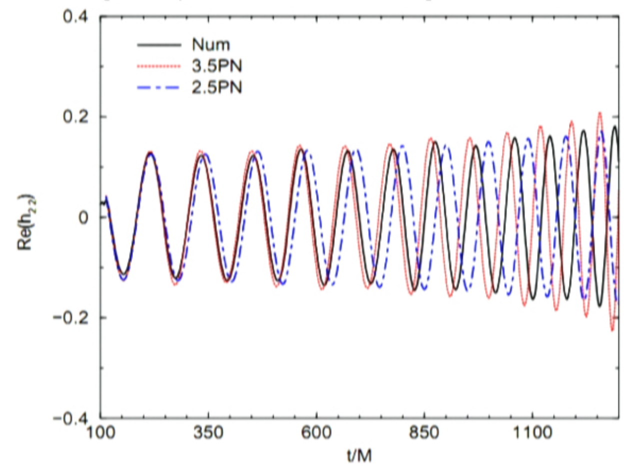
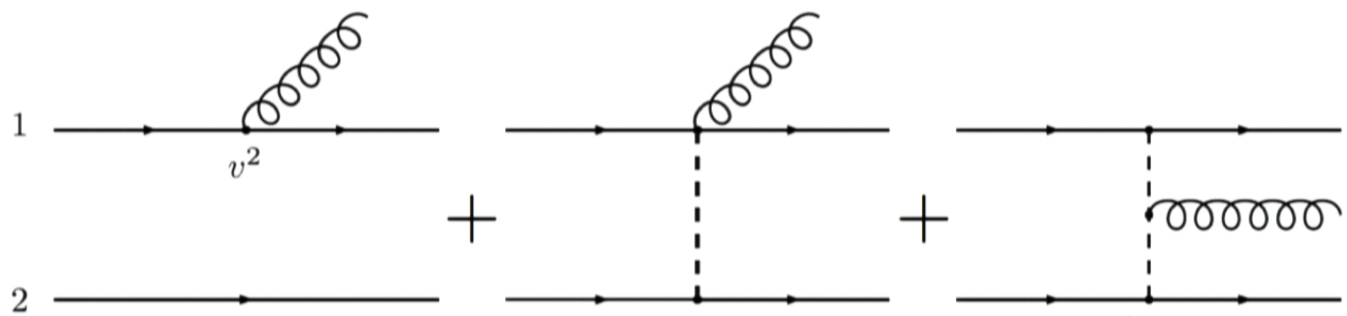
2.5PN and 3.5PN nonadiabatic waveforms for generic spins

[Campanelli et al. 2009]



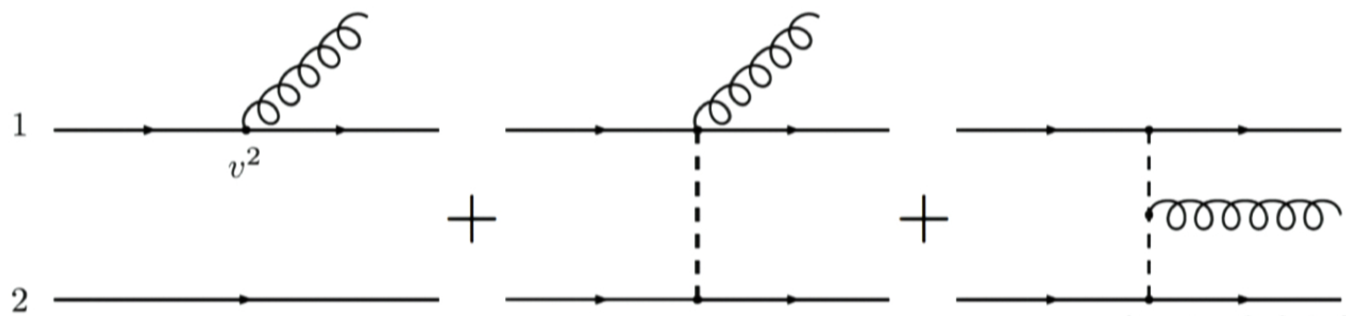
So how good is good enough? (Raise your hands to vote.)



[courtesy C. Galley]

So how good is good enough? (Raise your hands to vote.)

1. 10PN.
2. It’s never good enough.
3. We’ll know it when we get there.



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