Title: The Fluid/Gravity Correspondence, Quantum Anomalies and a Generalized Entropy Current

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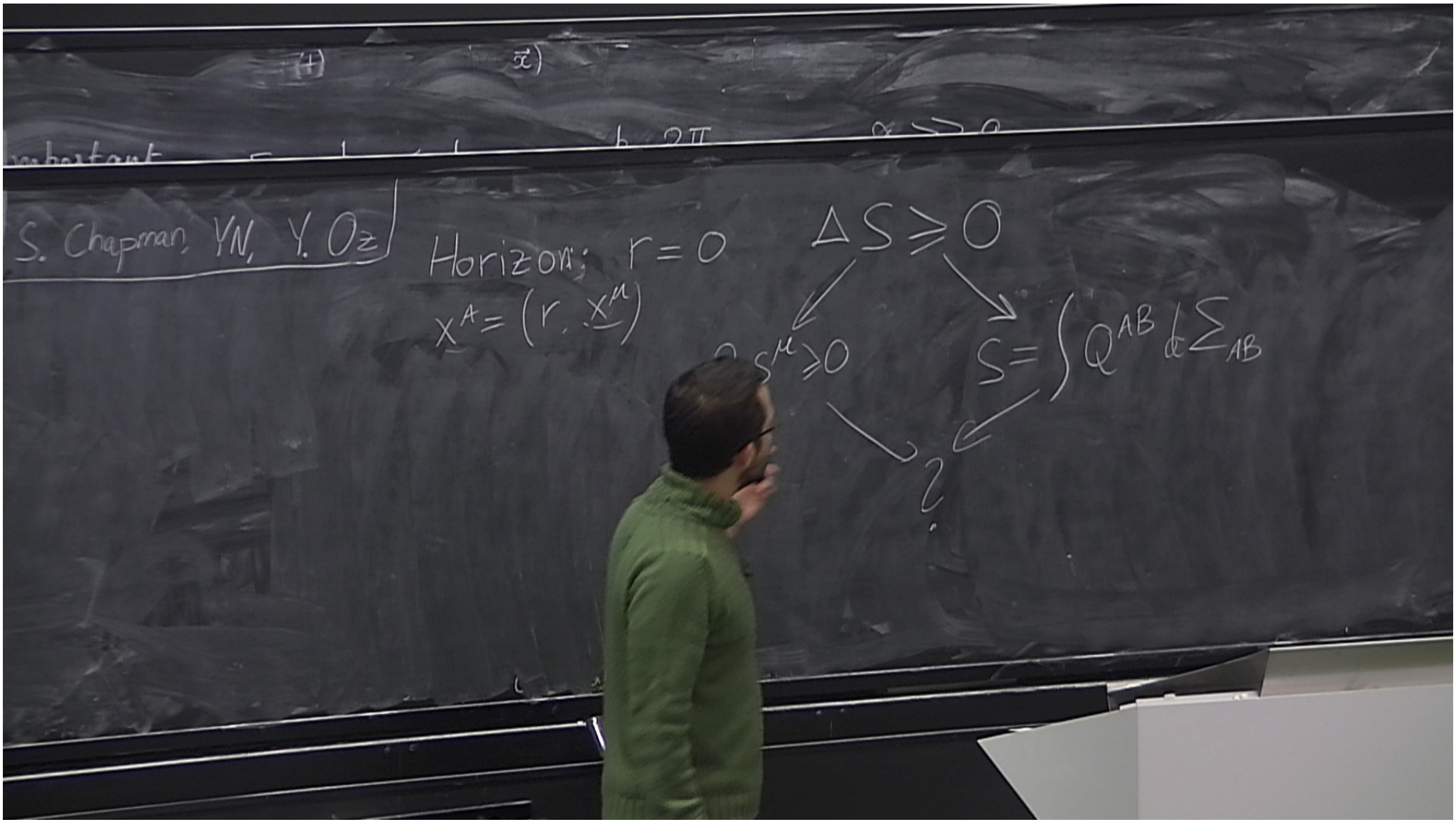
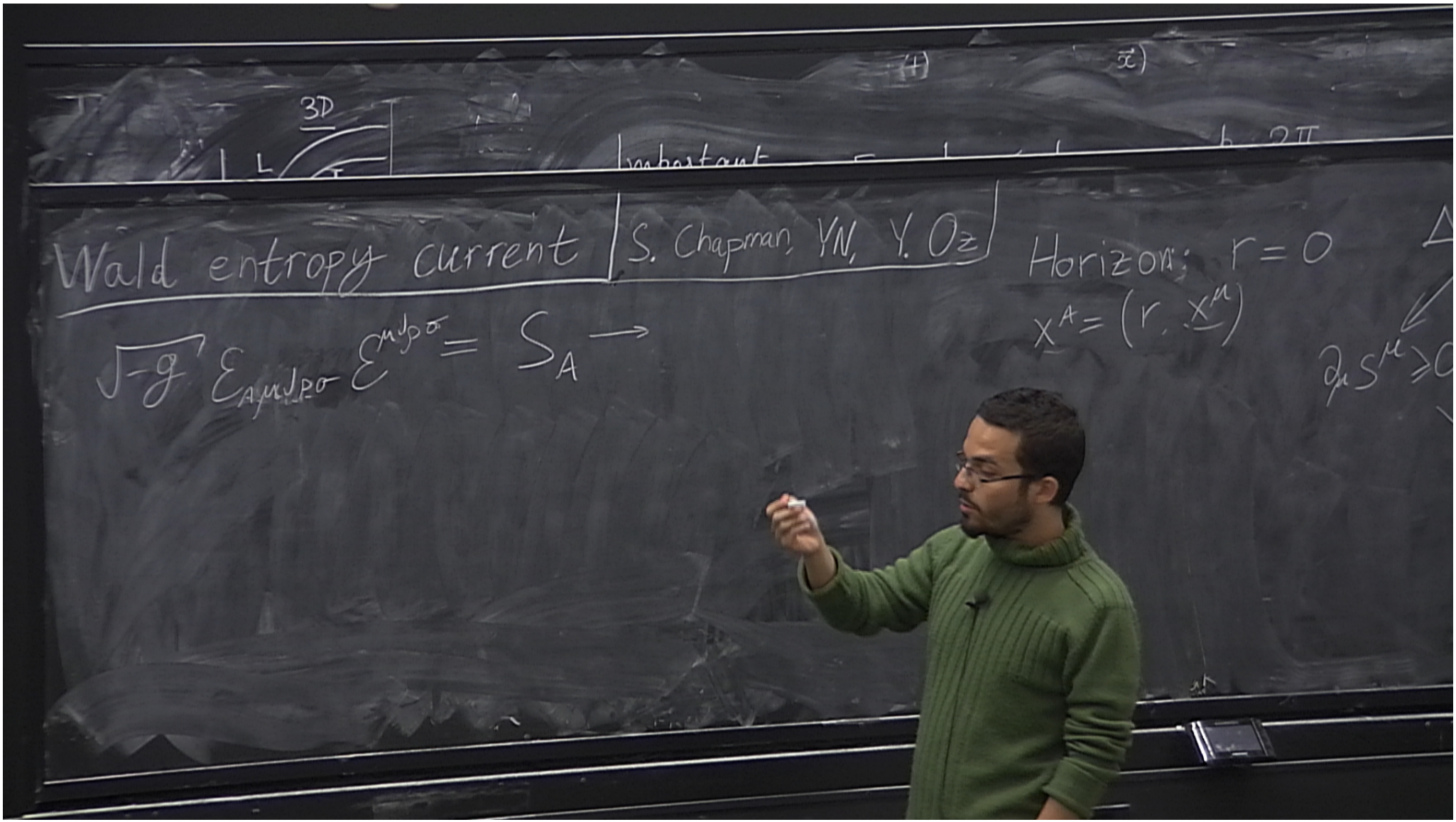
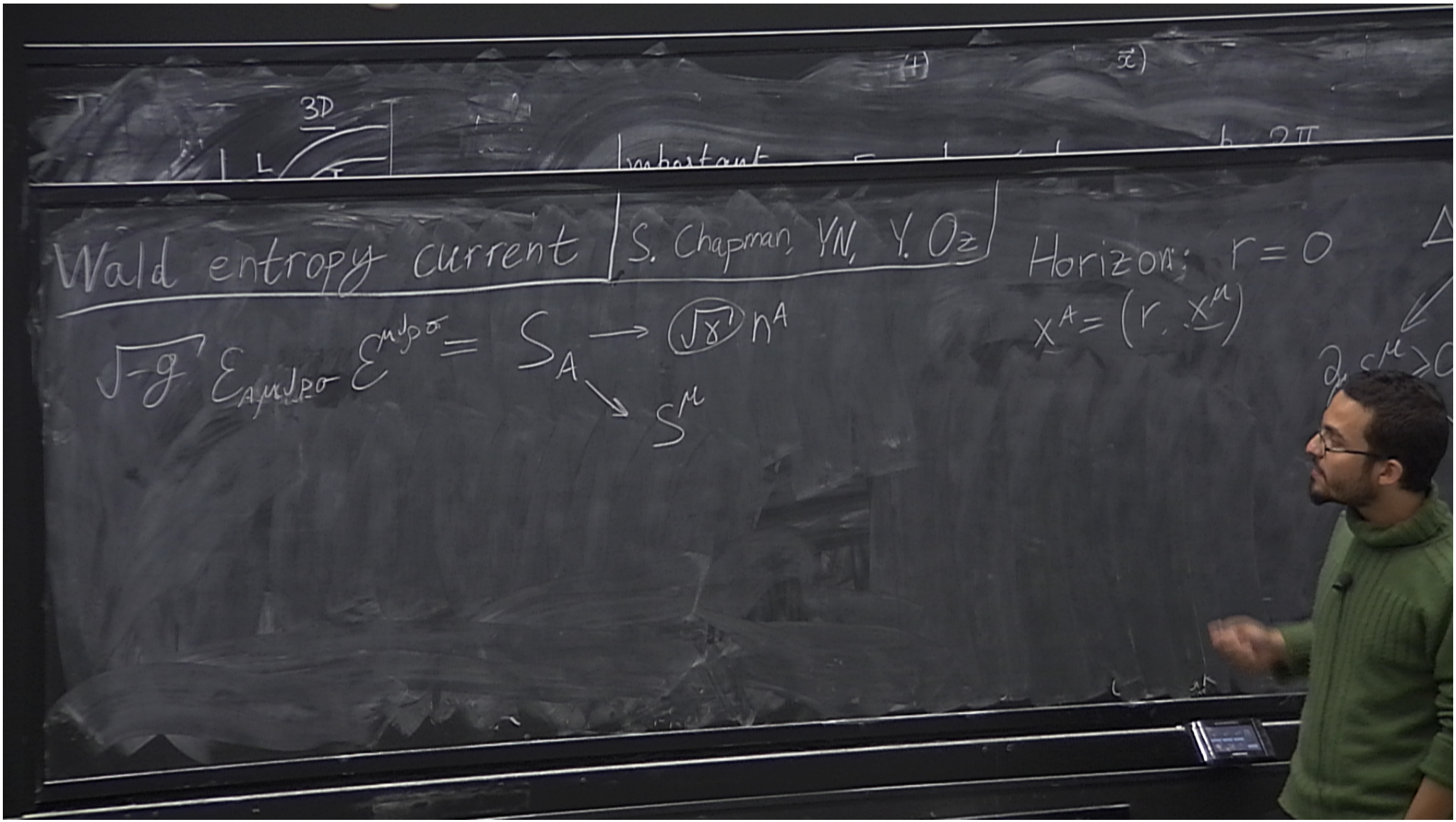
URL: http://pirsa.org/11110080

Abstract: The fluid/gravity correspondence relates hydrodynamic flows in D spacetime dimensions to black hole dynamics in D+1 dimensions. It is an extension of black hole thermodynamics, where charges are upgraded into local currents, and the Bekenstein-Hawking entropy - into a local entropy current. I will propose a definition for a generalized local current of Wald entropy relevant to higher-curvature gravity. As an example, I will consider 5d gravity with a U(1) gauge field and a gravitational Chern-Simons term. This corresponds to a charged fluid in 4d spacetime, whose charge current has a gravitational anomaly. I will discuss the effects of this anomaly on the generalized entropy curent and the flow equations.


S. Chapman, YN, Y. Oz
Horizon: r = 0
$x^A = (r, x^\mu)$
$\Delta S \geq 0$
$\partial_\mu S^\mu \geq 0$
$S = \int Q^{AB} d\Sigma_{AB}$
?


3D
Wald entropy current
S. Chapman, YN, Y. Oz
Horizon: r = 0
$x^A = (r, x^\mu)$
$\sqrt{-g}\, \varepsilon_{A\mu\nu\rho\sigma} \mathcal{E}^{\mu\nu\rho\sigma} = S_A$
$\partial_\mu S^\mu \geq 0$


3D
Wald entropy current
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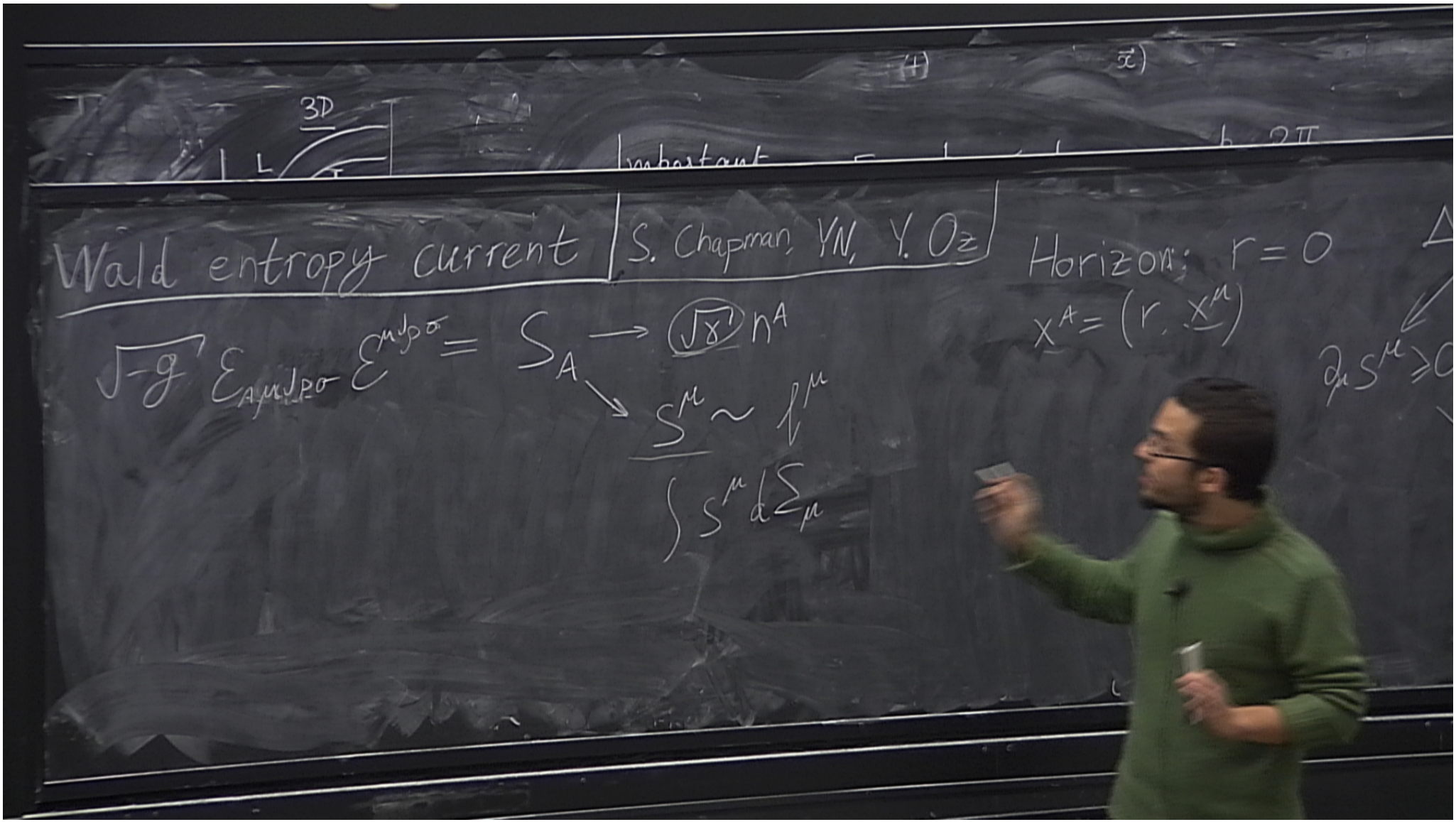
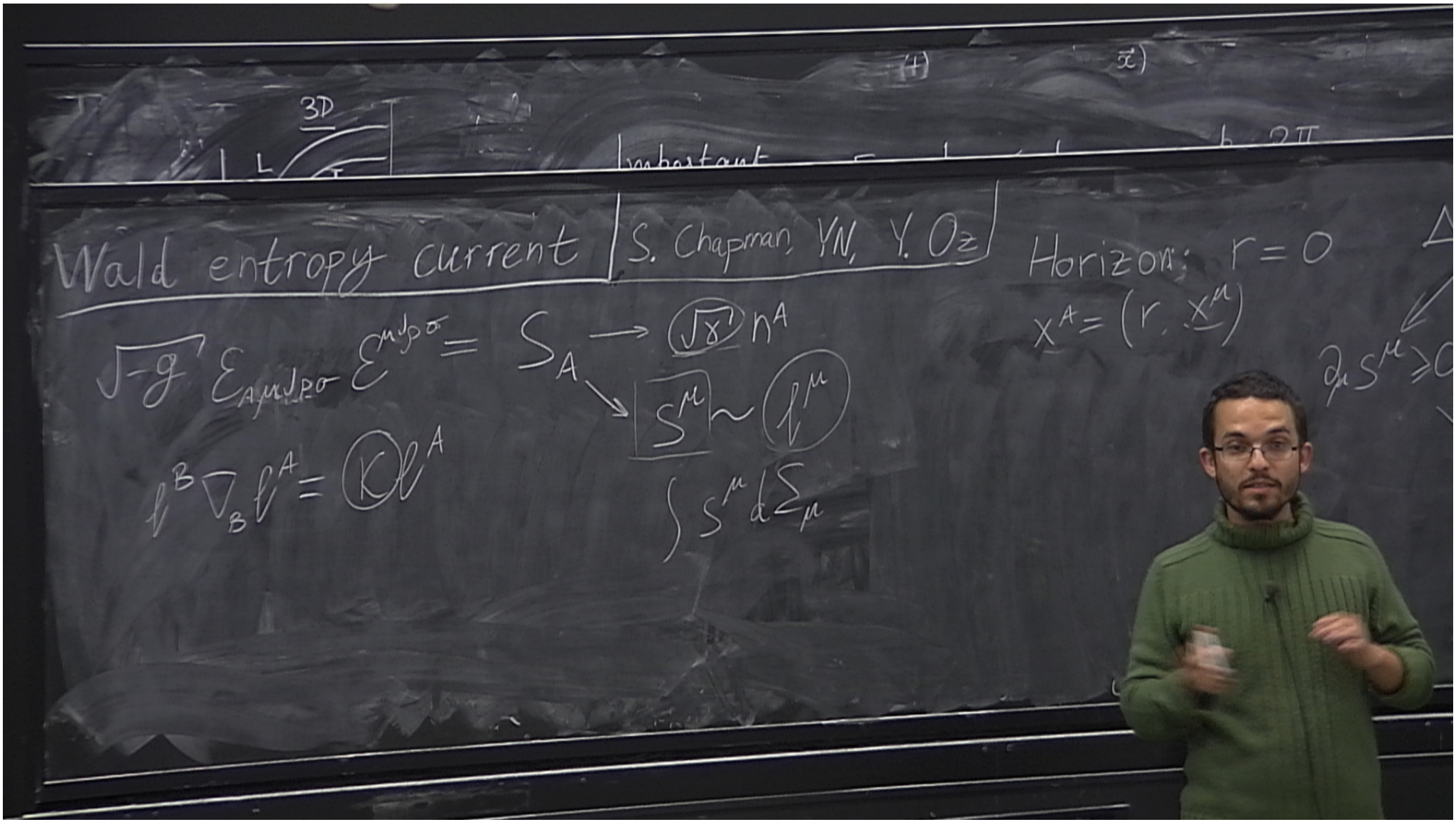
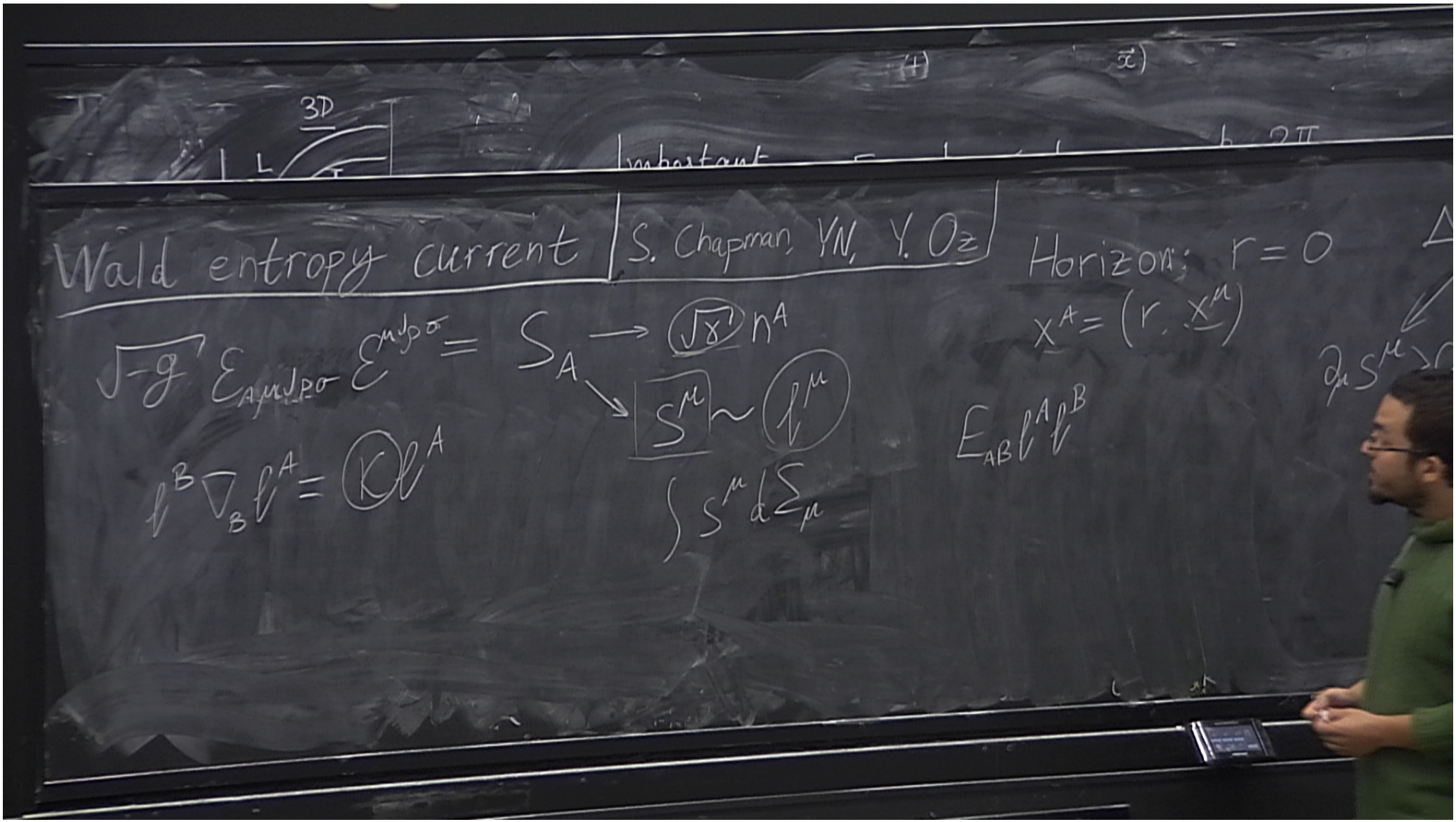
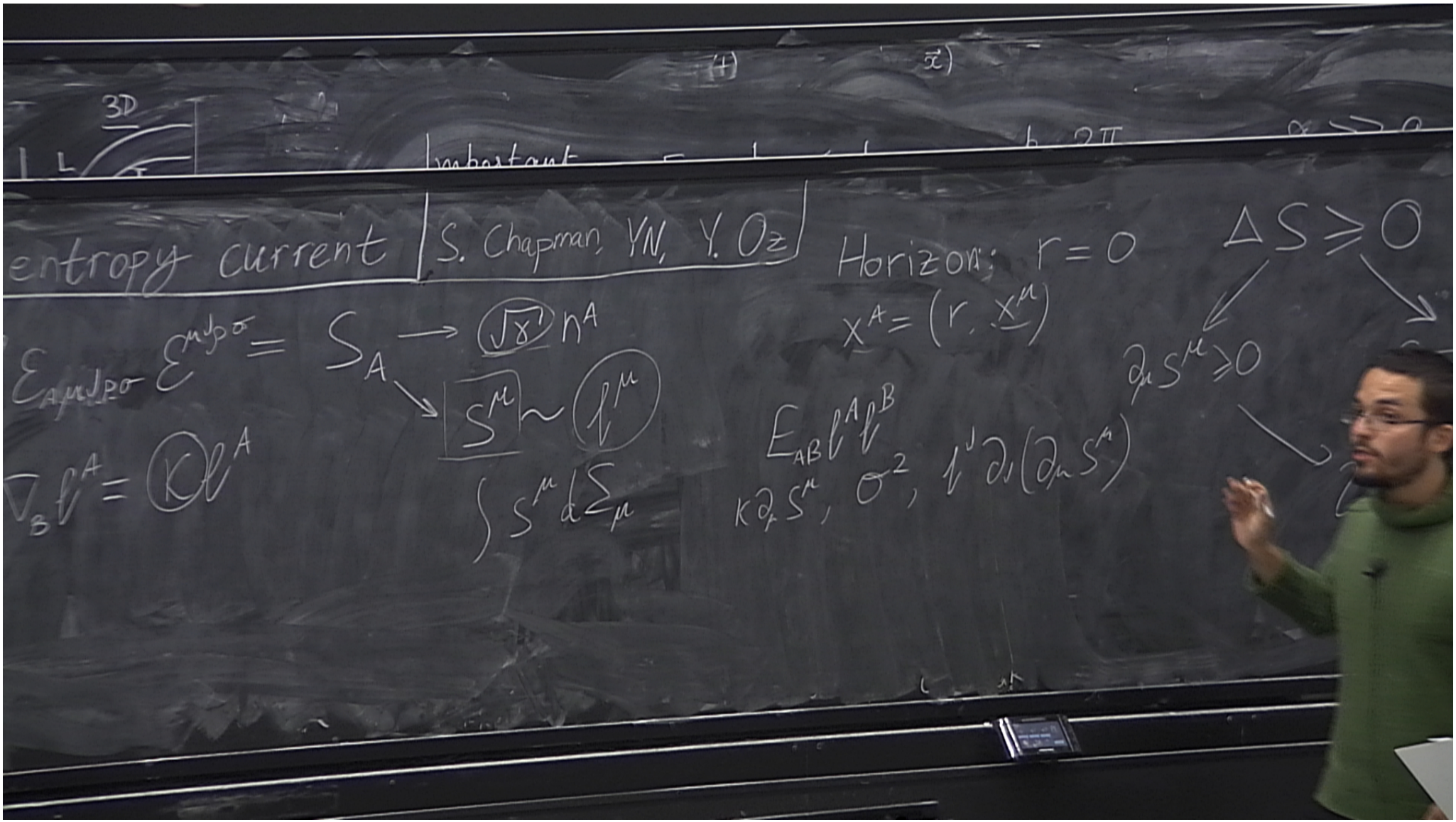
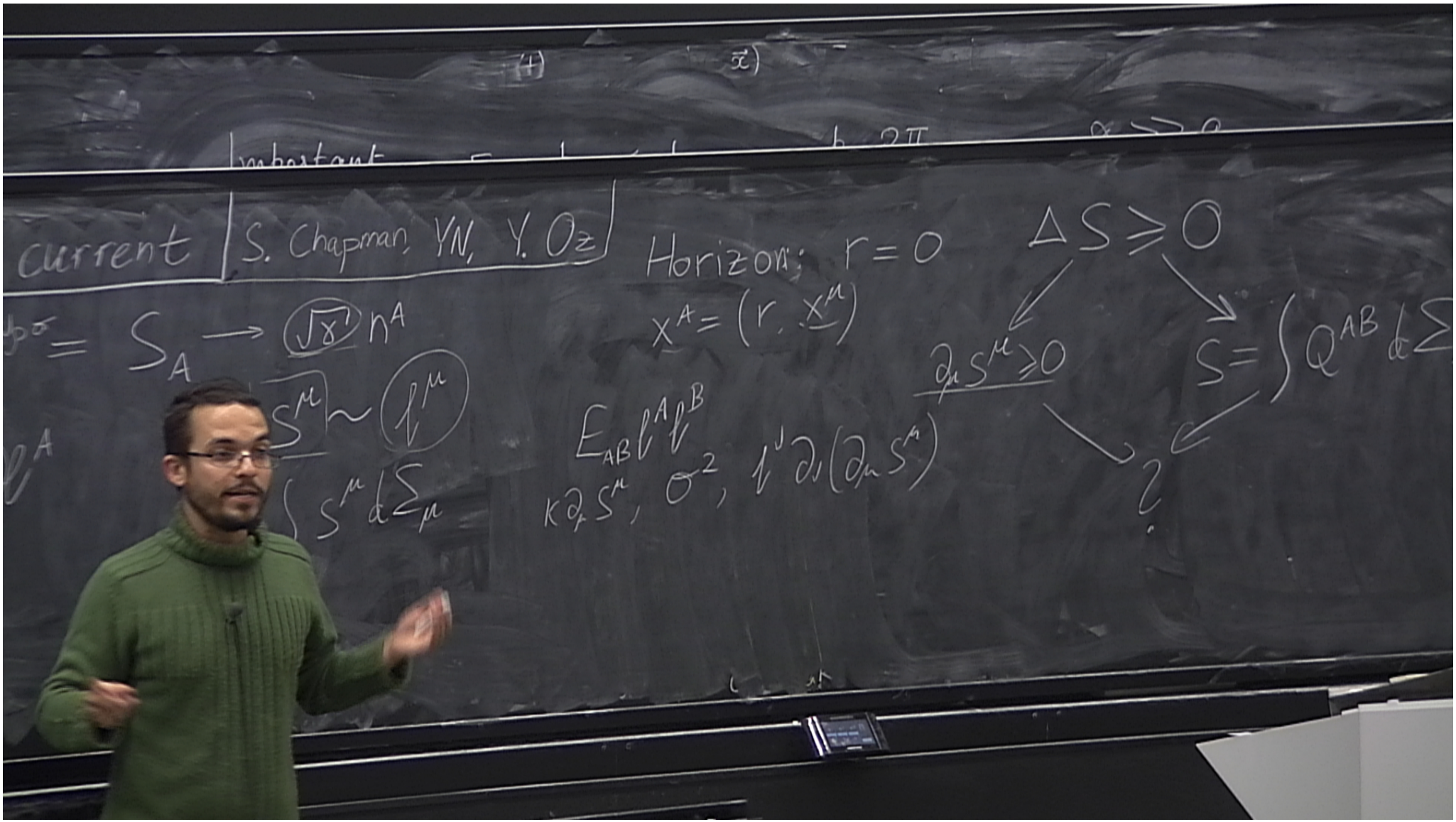
Horizon: r = 0
$x^A = (r, x^\mu)$
$\sqrt{-g}\, \varepsilon_{A\mu\nu\rho\sigma}\, \mathcal{E}^{\mu\nu\rho\sigma} = S_A$
$\sqrt{\gamma}\, n^A$
S^μ


3D
Important
Wald entropy current
S. Chapman, YN, Y. Oz
Horizon: r = 0
$x^A = (r, x^\mu)$
$\sqrt{-g}\, \varepsilon_{A\mu\nu\rho\sigma} \mathcal{E}^{\mu\nu\rho\sigma} = S_A \to \sqrt{\gamma}\, n^A$
$S^\mu \sim f^\mu$
$\int S^\mu d\Sigma_\mu$
$\partial_\mu S^\mu \geq 0$


3D
Important
Wald entropy current
S. Chapman, YN, Y. Oz
Horizon: r = 0
$\sqrt{-g}\,\varepsilon_{A\mu\nu\rho\sigma}\,\varepsilon^{\mu\nu\rho\sigma} = S_A \to \sqrt{\gamma}\, n^A$
$S_A \to S^\mu \sim \ell^\mu$
$\ell^B \nabla_B \ell^A = \kappa \ell^A$
$\int S^\mu d\Sigma_\mu$
$x^A = (r, x^\mu)$
$\partial_\mu S^\mu \geq 0$


3D
Important
Wald entropy current
S. Chapman, YN, Y. Oz
Horizon: r = 0
$\sqrt{-g}\,\epsilon_{A\mu\nu\rho\sigma}\,\epsilon^{\mu\nu\rho\sigma} = S_A \to \sqrt{\gamma}\, n^A$
$S_A \to \boxed{S^\mu} \sim \ell^\mu$
$\int S^\mu d\Sigma_\mu$
$\ell^B \nabla_B \ell^A = \kappa \ell^A$
$x^A = (r, x^\mu)$
$\epsilon_{AB}\ell^A \ell^B$
$\partial_\mu S^\mu$


3D
entropy current
S. Chapman, YN, Y. Oz
Horizon: r = 0
ΔS ≥ 0
∂μ S^μ ≥ 0
E_AB l^A l^B
K ∂_A S^μ, σ^2, l^ν ∂_ν (∂_μ S^μ)


current
S. Chapman, YN, Y. Oz
Horizon: r = 0
ΔS ≥ 0
∂μ S^μ ≥ 0
S = ∫ Q^AB dΣ
?

$u^\mu, \quad \eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$

$g_{rr} = 0, \quad g_{r\mu} = -u_\mu, \quad g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$

$r = 0 : \quad k = 0$

$s \sim f^{3/2}, \quad k' \sim K, T$

u^μ, $\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$

$g_{rr} = 0$, $g_{r\mu} = -u_\mu$, $g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$

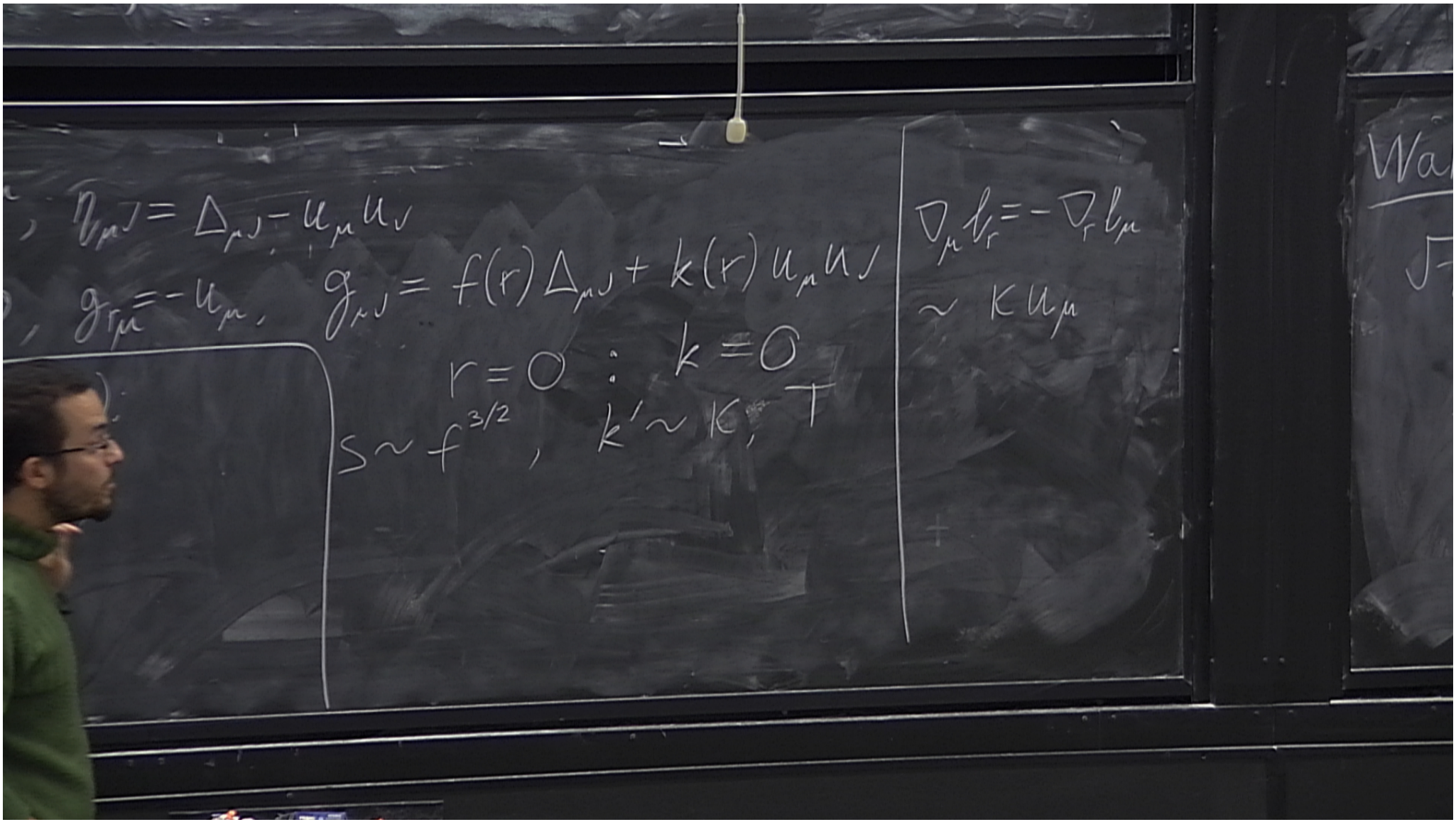
$r = 0$: $k = 0$

$s \sim f^{3/2}$, $k' \sim K$, T

$\ell^A = (0, u^\mu)$:

$\ell_A = (1, 0_\mu)$

∇


$\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$
$g_{r\mu} = -u_\mu$, $g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$
$r = 0 : k = 0$
$s \sim f^{3/2}$, $k' \sim K$, T
$\nabla_\mu b_r = -\nabla_r b_\mu$
$\sim K u_\mu$

$\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$

$g_{r\mu} = -u_\mu$, $\quad g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$

$r = 0 : \quad k = 0$

$s \sim f^{3/2}, \quad k' \sim K, \quad T$

∂_μ

$\nabla_\mu b_r = -\nabla_r b_\mu$

$\sim K u_\mu$

u^μ):

a_μ)

Wa

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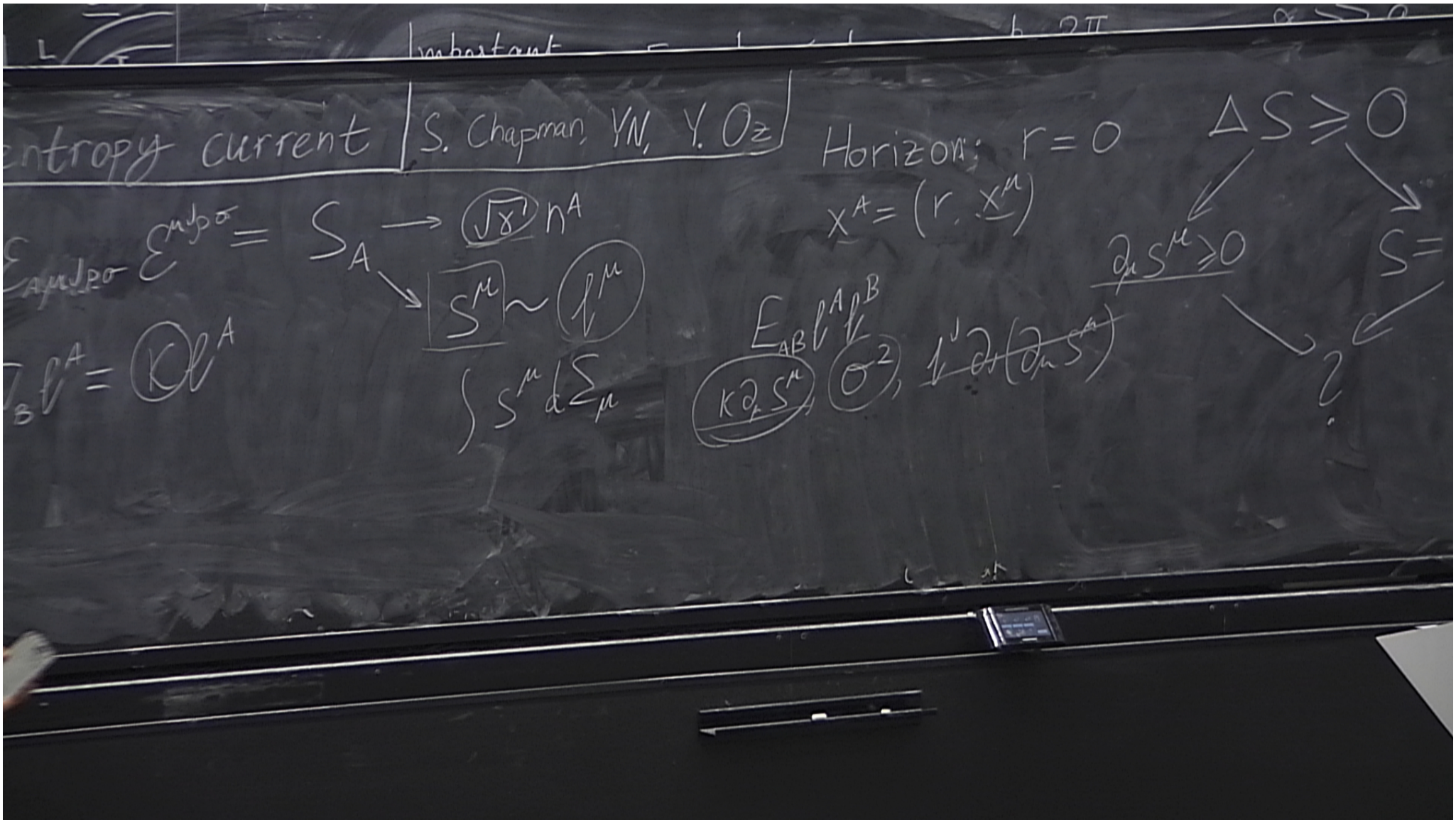

entropy current
S. Chapman, YN, Y. Oz
Horizon: r = 0
ΔS ≥ 0
S =
?

$$L(R_{ABCD})$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}}$$

$$Q^{AB} = \frac{2\pi}{K}\left(-2P^{ABCD}\nabla_C l_D + 4 l_D \nabla_C P^{ABCD}\right)$$

$$L(R_{ABCD})$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}}$$

$$Q^{AB} = \frac{2\pi}{\kappa}\left(-2P^{ABCD}\nabla_C l_D + 4 l_D \nabla_C P^{ABCD}\right)$$

$$S = \int Q^{AB} \, \epsilon_{AB}$$

$$L(R_{ABCD})$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}}$$

$$\nabla_{[\mu} l_{\nu]}$$

$$Q^{AB} = \frac{2\pi}{\kappa}\left(-2P^{ABCD}\underline{\nabla_C l_D} + 4 l_D \nabla_C P^{ABCD}\right)$$

$$Q^{\nu\mu} = \frac{2\pi}{\kappa}\left(-2P^{\nu\mu}\right.$$

$$L(R_{ABCD})$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}} \qquad \nabla_{[\mu} l_{\nu]} = 0$$

$$Q^{AB} = \frac{2\pi}{K}\left(-2P^{AB\underline{CD}}\underline{\nabla_C l_D} + 4 l_D \nabla_C P^{ABCD}\right)$$

$$Q^{r\mu} = \frac{2\pi}{K}\left(-2P^{r\mu r\nu}(\nabla_r l_\nu - \nabla_\nu l_r) + \nabla_\nu P^{r\mu\nu r}\right.$$

$$L(R_{ABCD})$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}} \qquad \nabla_{[\mu} l_{\nu]} = 0$$

$$Q^{AB} = \frac{2\pi}{\kappa}\left(-2P^{ABC\underline{D}}\underline{\nabla_C l_D} + 4 l_D \nabla_C P^{ABCD}\right)$$

$$Q^{\mu\nu} = \frac{2\pi}{\kappa}\left(-2P^{\mu\nu r\nu}(\nabla_r l_\nu - \nabla_\nu l_r) + \nabla_\nu P^{\mu\nu\nu r}\right)$$

$$L(R_{ABCD})$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}} \qquad \nabla_{[\mu} l_{\nu]} = 0$$

$$Q^{AB} = \frac{2\pi}{\kappa}\left(-2P^{ABCD}\underline{\nabla_C l_D} + 4 l_D \nabla_C P^{ABCD}\right)$$

$$Q^{r\mu} = \frac{2\pi}{\kappa}\left(-2P^{r\mu r\nu}\left(\nabla_r l_\nu - \nabla_\nu l_r\right) + \nabla_\nu P^{r\mu\nu r}\right)$$

$$L(R_{ABCD})$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}} \qquad \nabla_{[\mu} l_{\nu]} = 0$$

$$Q^{AB} = \frac{2\pi}{\kappa}\left(-2P^{ABCD}\underline{\nabla_C l_D} + 4 l_D \nabla_C P^{ABCD}\right)$$

$$Q^{r\mu} = \frac{2\pi}{\kappa}\left(-2P^{r\mu r\nu}\left(\nabla_r l_\nu - \nabla_\nu l_r\right) + \nabla_\nu P^{r\mu\nu r}\right)$$

$\Box = 0$

$-4 b_D \nabla_C P^{ABCD})$

$-\nabla_a b_r)$

$\nabla_r)$

u^μ, $\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$

$g_{rr} = 0$, $g_{r\mu} = -u_\mu$, $g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$

$r = 0$: $k = 0$

$s^{\mu(0)} \sim u^\mu$

$s^{\mu(1)}$?

$s \sim f^{3/2}$, $k' \sim K$, T

∂_μ

∇_A

$\sim$

$+$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}} \qquad \nabla_{[\mu} l_{\nu]} = 0$$

$$Q^{AB} = \frac{2\pi}{K}\left(-2P^{ABCD}\underline{\nabla_C l_D}^{(0)} + 4 l_D \nabla_C P^{ABCD}\right)$$

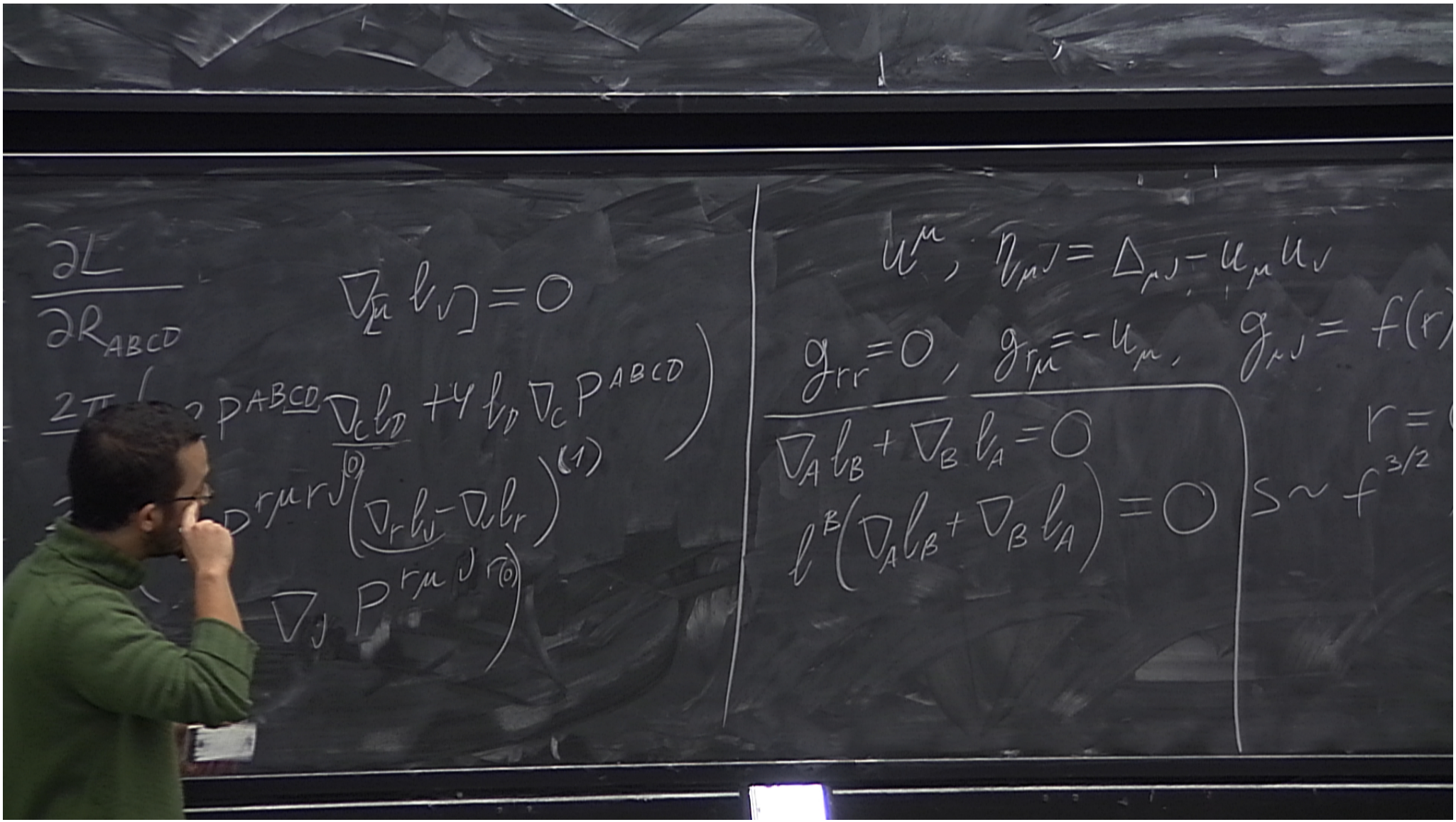
$$Q^{r\mu} = \frac{2\pi}{K^{(1)}}\left(-2P^{r\mu r\nu}\left(\nabla_r l_\nu - \nabla_\nu l_r\right)^{(1)} + \nabla_\nu P^{r\mu\nu r}{}^{(0)}\right)$$

u^μ, $\eta_{\mu\nu} = \Delta$

$g_{rr} = 0$, $g_{r\mu} = -u$

$S^{\mu(0)} \sim u^\mu$

$S^{\mu(1)}$?


$\frac{\partial \mathcal{L}}{\partial R_{ABCD}}$
$\nabla_{[\mu} l_{\nu]} = 0$
$P^{ABCD} \nabla_C l_D + 4 l_D \nabla_C P^{ABCD}$
$P^{r\mu r\nu}$
$(\nabla_r l_\nu - \nabla_\nu l_r)$
$\nabla_\nu P^{r\mu\nu r}$
u^μ, $\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$
$g_{rr} = 0$, $g_{r\mu} = -u_\mu$, $g_{\mu\nu} = f(r)$
$\nabla_A l_B + \nabla_B l_A = 0$
$l^B (\nabla_A l_B + \nabla_B l_A) = 0$
$S \sim f^{3/2}$

$$\frac{\partial \mathcal{L}}{\partial R_{ABCD}} \qquad \nabla_{[\mu} l_{\nu]} = 0$$

$$\frac{2\pi}{K}\left(-2P^{ABCD}\underbrace{\nabla_C l_D}_{(0)} + 4 l_D \nabla_C P^{ABCD}\right)^{(1)}$$

$$\frac{2\pi}{K^{(1)}}\left(-2\underbrace{P^{r\mu r\nu}}\left(\nabla_\nu l_r\right) + \nabla_\nu P^{r\mu\nu r}{}^{(0)}\right)$$

$$u^\mu, \quad \eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$$

$$g_{rr} = 0, \quad g_{r\mu} = -u_\mu, \quad g_{\mu\nu} = f(r)$$

$$\nabla_A l_B + \nabla_B l_A = 0$$

$$l^B\left(\nabla_A l_B + \nabla_B l_A\right) = 0$$

$$u^\mu, \quad \eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$$

$$g_{rr} = 0, \quad g_{r\mu} = -u_\mu, \quad g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r)\, u_\mu u_\nu$$

$$\nabla_A l_B + \nabla_B l_A = 0$$

$$l^B\left(\nabla_A l_B + \nabla_B l_A\right) = 0$$

$$P^{r\mu r\nu}(0) = \alpha\, l^\mu l^\nu + \beta\, \Delta^{\mu\nu}$$

$$P^{ABCD} = \sqrt{-g}\left(g^{AC} g^{BD} - g^{AD} g^{BC}\right)$$

$$\nabla_\mu l_r = -\nabla_r l_\mu$$

$$\sim K u_\mu$$

$$P^{ABCD} = \frac{\partial L}{\partial R_{ABCD}} \qquad \nabla_{[\mu} l_{\nu]} = 0$$

$$Q^{AB} = \frac{2\pi}{K}\left(-2P^{ABCD}\underbrace{\nabla_C l_D}_{(0)} + 4 l_D \nabla_C P^{ABCD}\right)^{(1)}$$

$$Q^{r\mu} = \frac{2\pi}{K^{(1)}}\left(-2P^{r\mu r\nu}\left(\nabla_\nu l_r\right)^{(0)} + \nabla_\nu P^{r\mu\nu r}\right)$$

$$u^\mu, \quad \eta_{\mu\nu} = \Delta$$

$$g_{rr} = 0, \quad g_{r\mu} = -u$$

$$\nabla_A l_B + \nabla_B l_A = 0$$

$$l^B\left(\nabla_A l_B + \nabla_B l_A\right) =$$

$ABCD)$

$u^\mu, \quad \eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$

$g_{rr} = 0, \quad g_{r\mu} = -u_\mu, \quad g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r)\, u_\mu u_\nu$

$\nabla_A l_B + \nabla_B l_A = 0$

$l^B(\nabla_A l_B + \nabla_B l_A) = 0$

$P^{r\mu r\nu}(0) = \alpha\, l^\mu l^\nu + \beta\, \Delta^{\mu\nu}$

$l^\nu \nabla_\nu l_r = K$

$\nabla_\mu l_r = -\nabla_r l$

$\sim K u_\mu$

Important

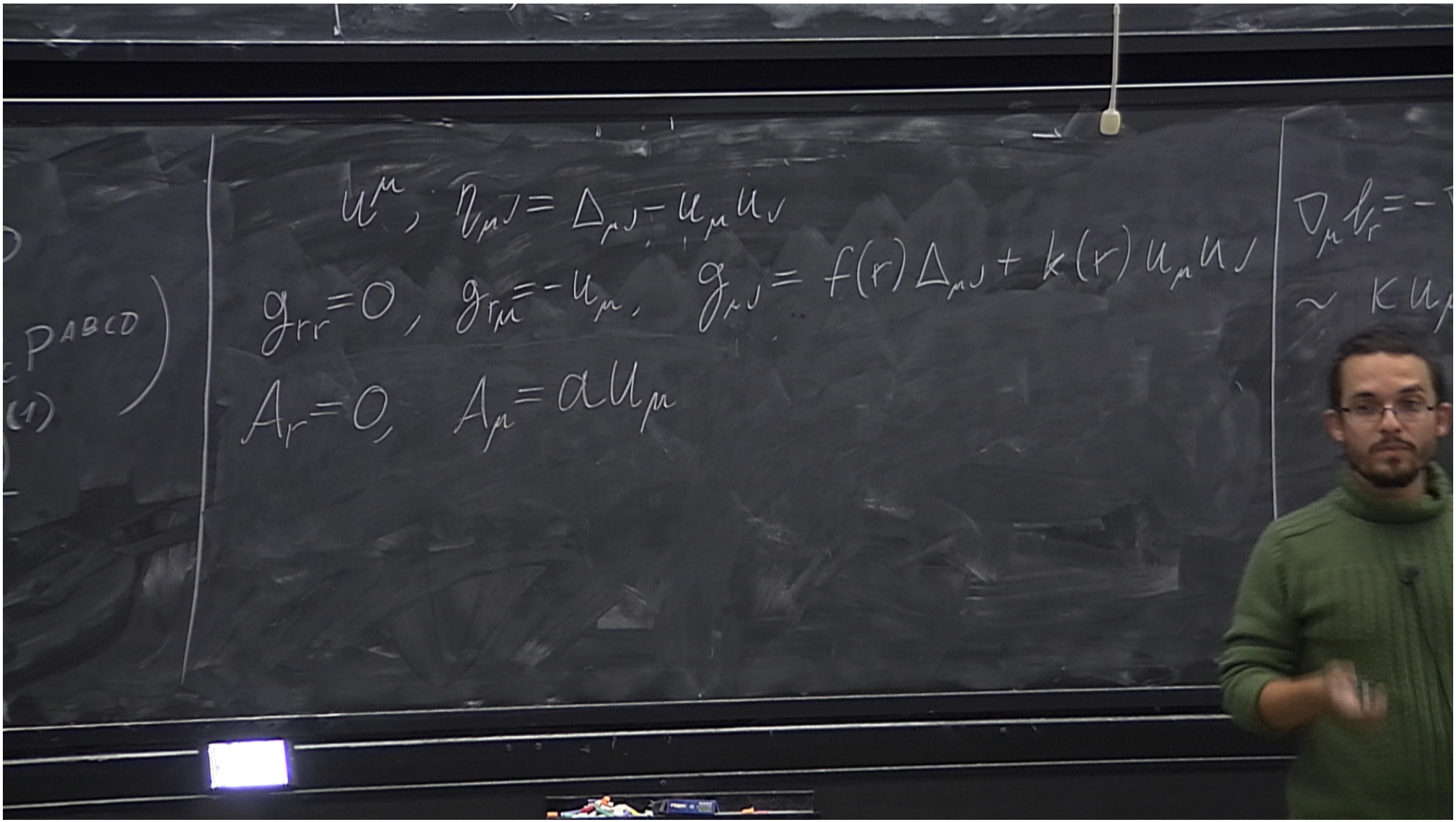
Wald entropy current | S. Chapman, YN, Y. Oz

Horizon: $r = 0$

$A = (r, x^\mu)$

If $\frac{\partial L}{\partial R_{r\mu r\nu}} \sim \ell^\mu \ell^\nu$

We have $S^{\mu(r)}$


u^μ, $\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$
$g_{rr} = 0$, $g_{r\mu} = -u_\mu$, $g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$
$A_r = 0$, $A_\mu = a u_\mu$
P^{ABCD}
(1)
$\nabla_\mu b_r = -$
$\sim K u_\mu$

$$u^\mu, \quad \eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$$

$$g_{rr} = 0, \quad g_{r\mu} = -u_\mu, \quad g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r)\, u_\mu u_\nu$$

$$A_r = 0, \quad A_\mu = a u_\mu$$

$$L = \varepsilon^{ABCDE} A_A R_{BCKL} R_{DE}{}^{KL}$$

$$\nabla_\mu b_r = -$$

$$\sim K u_\mu$$

$$u^\mu, \quad \eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$$

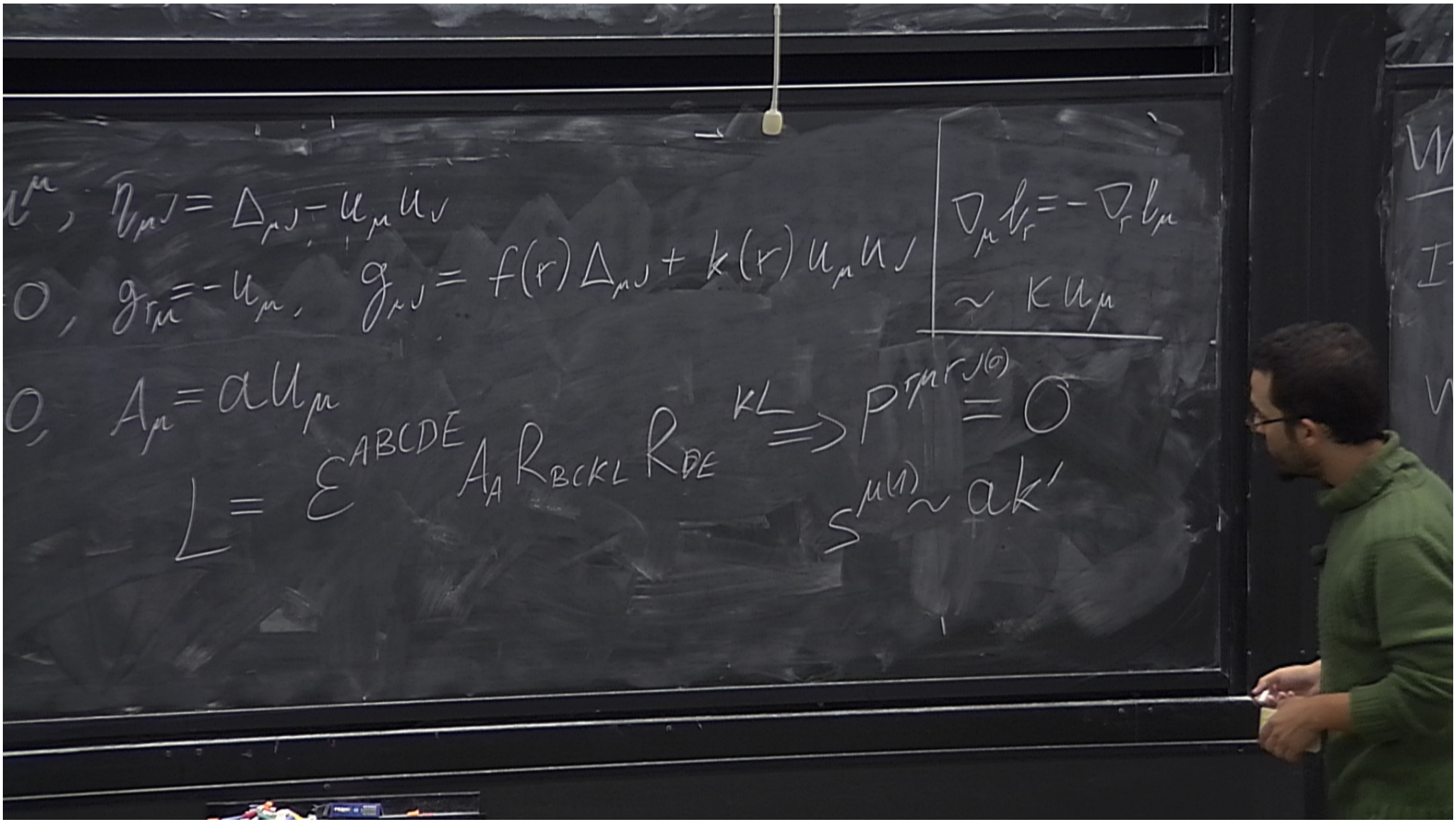
$$g_{rr} = 0, \quad g_{r\mu} = -u_\mu, \quad g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r)\, u_\mu u_\nu$$

$$A_r = 0, \quad A_\mu = a u_\mu$$

$$L = \varepsilon^{ABCDE} A_A R_{BCKL} R_{DE}{}^{KL}$$

$$\nabla_\mu b_r = -$$

$$\sim K u_\mu$$


u^μ, $\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$
0, $g_{r\mu} = -u_\mu$, $g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$
$\nabla_\mu l_r = -\nabla_r l_\mu$
$\sim \kappa u_\mu$
0, $A_\mu = a u_\mu$
$L = \varepsilon^{ABCDE} A_A R_{BCKL} R_{DE}{}^{KL} \Longrightarrow P^{\mu r r \nu(0)} = 0$
$S^{\mu(1)} \sim a k'$

u^μ, $\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$

0, $g_{\tau\mu} = -u_\mu$, $g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r) u_\mu u_\nu$

$\nabla_\mu b_\tau = -\nabla_\tau b_\mu$

$\sim K u_\mu$

0, $A_\mu = a u_\mu$

$$L = \varepsilon^{ABCDE} A_A R_{BCKL} R_{DE}{}^{KL} \Longrightarrow P^{\tau\mu\tau\nu(0)} = 0$$

$$S^{\mu(1)} \sim a k' \cdot \omega^\mu$$

$\mu \cdot T$

u^μ, $\eta_{\mu\nu} = \Delta_{\mu\nu} - u_\mu u_\nu$

0, $g_{r\mu} = -u_\mu$, $g_{\mu\nu} = f(r)\Delta_{\mu\nu} + k(r)\, u_\mu u_\nu$

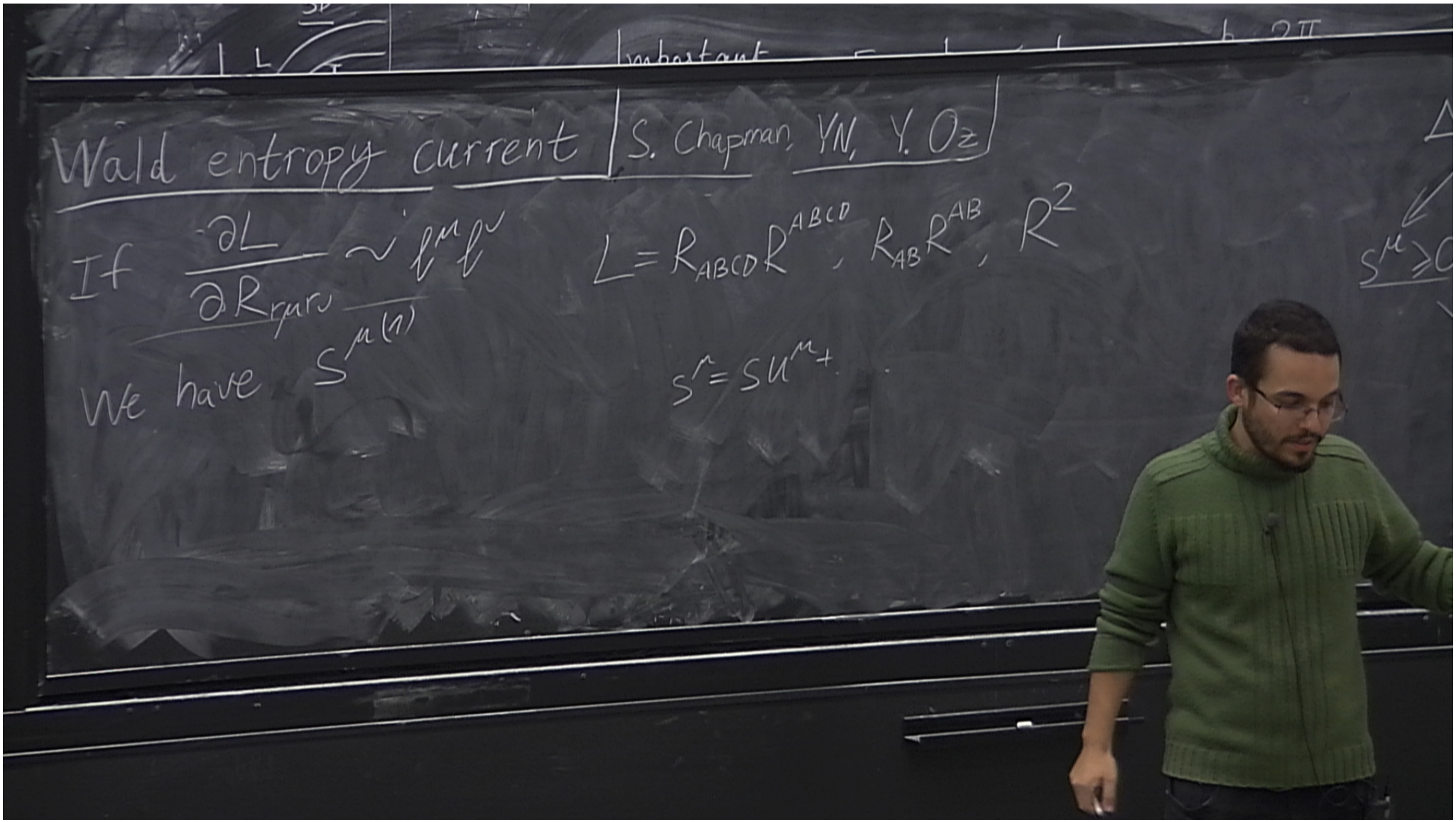
$\nabla_\mu b_r = -\nabla_r b_\mu$

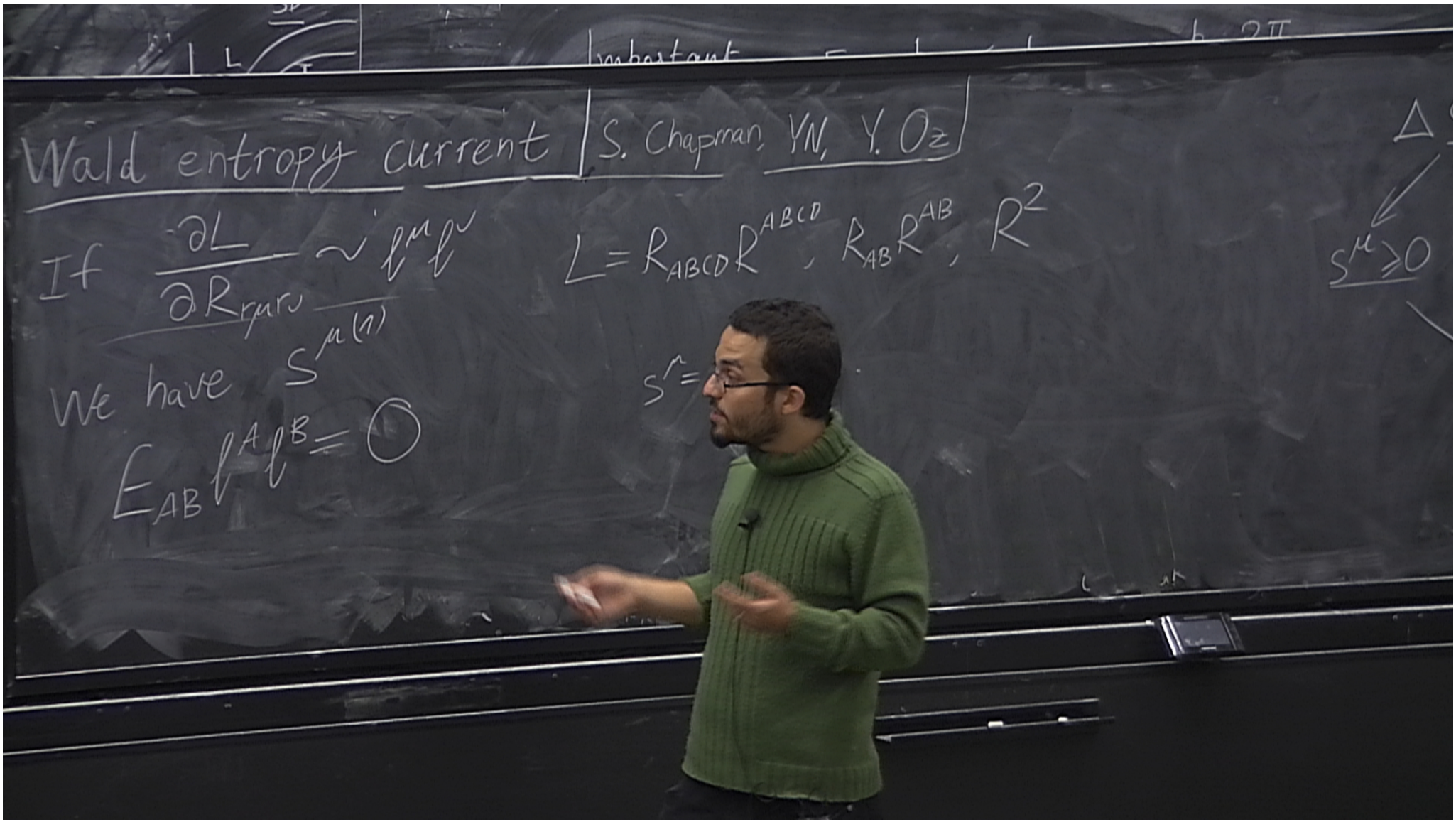
$\sim K u_\mu$

0, $A_\mu = a u_\mu$

$$L = \frac{\varepsilon^{ABCDE} A_A R_{BCKL} R_{DE}{}^{KL}}{\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma} \Longrightarrow P^{r\mu r\nu(0)} = 0$$

$$S^{\mu(1)} \sim \frac{a k'}{\mu \cdot T} \cdot \omega^\mu$$


Wald entropy current
S. Chapman, YN, Y. Oz
If $\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}} \sim \ell^\mu \ell^\nu$
$L = R_{ABCD}R^{ABCD}, R_{AB}R^{AB}, R^2$
We have $S^{\mu(n)}$
$S^\mu = S u^\mu +$


Wald entropy current
S. Chapman, YN, Y. Oz
If $\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}} \sim l^\mu l^\nu$
$L = R_{ABCD}R^{ABCD}, R_{AB}R^{AB}, R^2$
$S^\mu \geq 0$
We have $S^{\mu(n)}$
$E_{AB} l^A l^B = 0$

Wald entropy current | S. Chapman, YN, Y. Oz

If $\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}} \sim \ell^\mu \ell^\nu$ $\quad L = R_{ABCD}R^{ABCD},\ R_{AB}R^{AB},\ R^2$

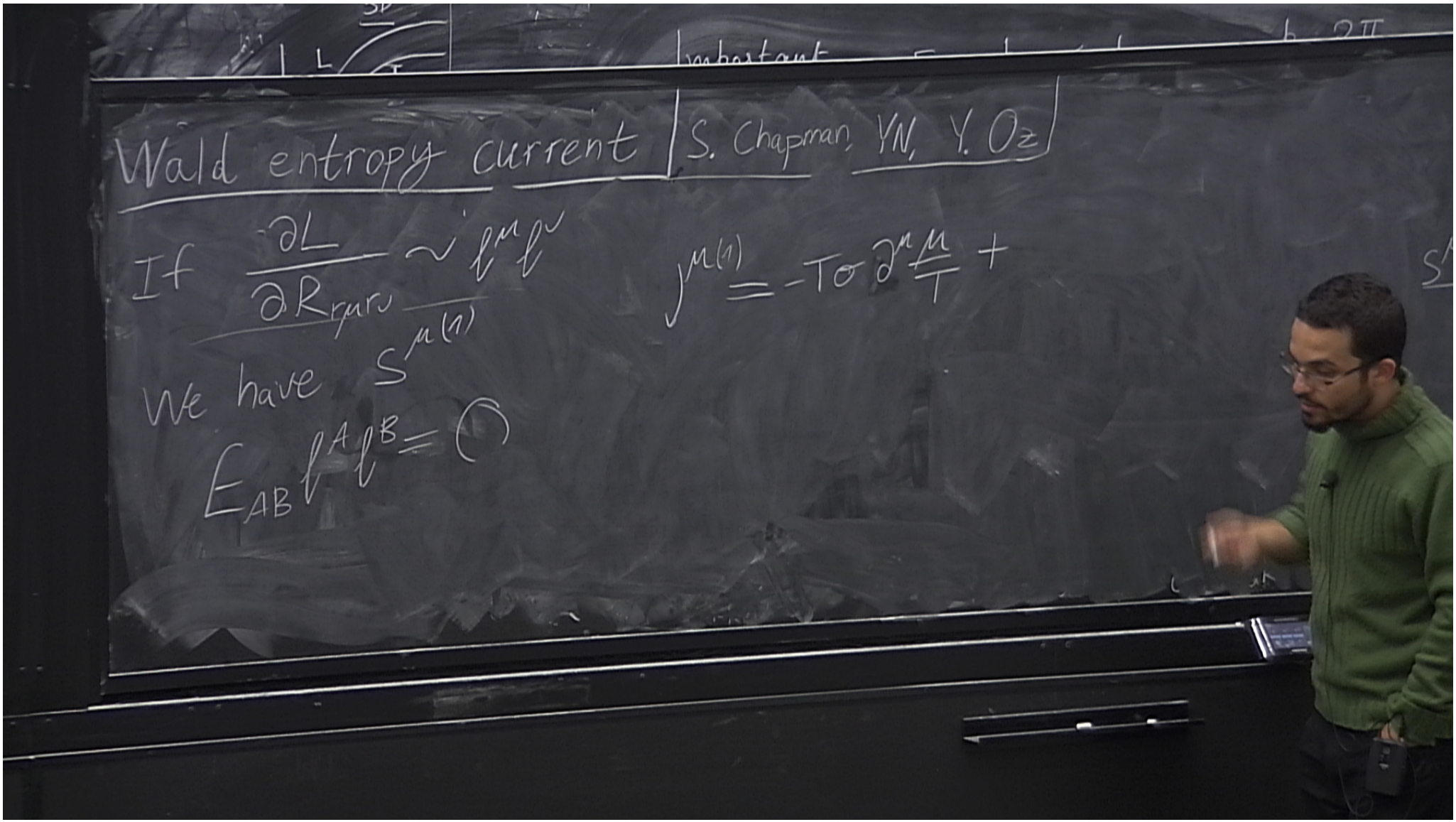
We have $S^{\mu(n)}$ $\quad S^\mu = S u^\mu +$

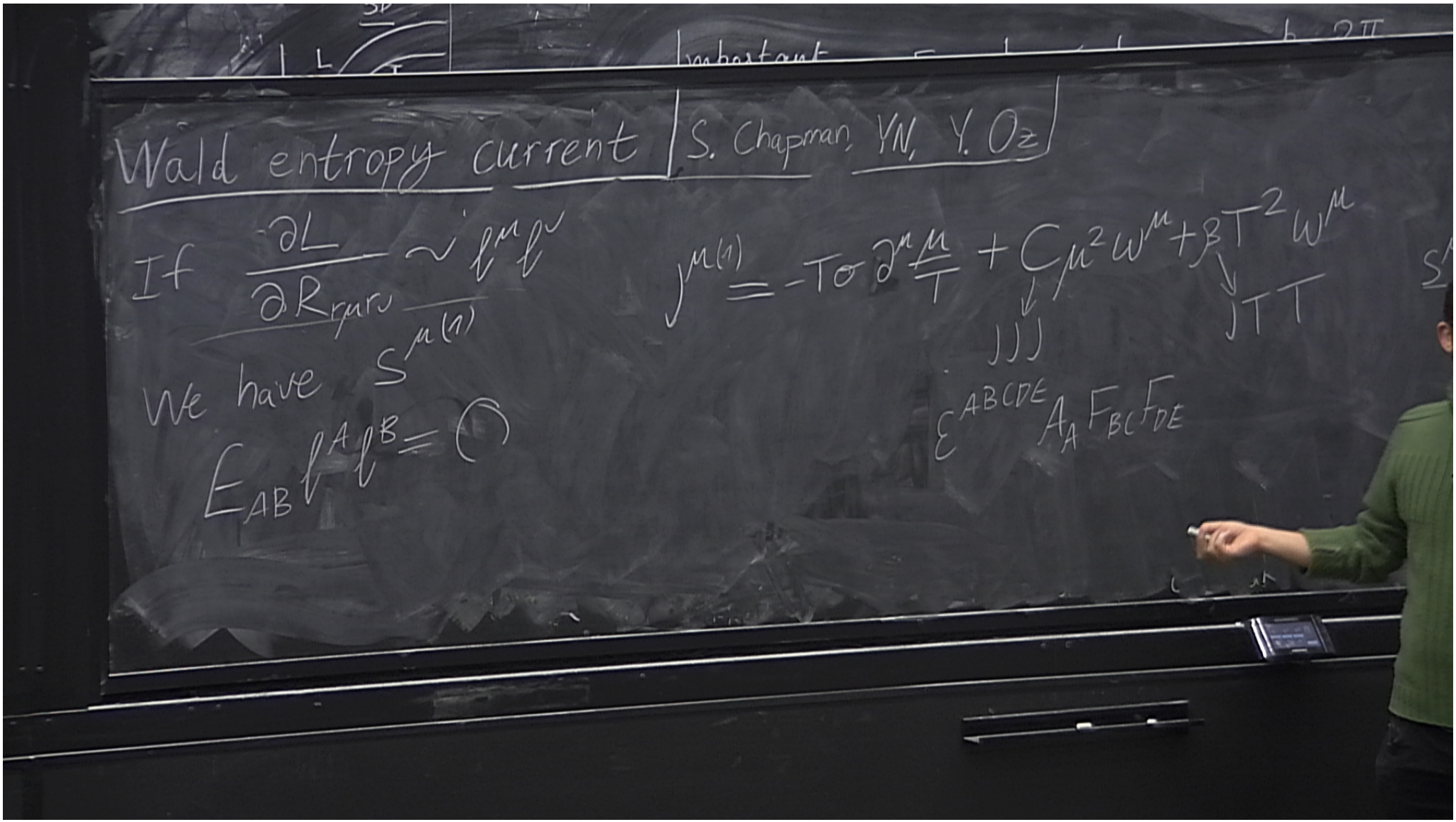
$$E_{AB}\ell^A \ell^B = 0$$

$$\partial_\mu S^\mu \sim \theta^2 + \ldots \geq 0$$

$$\Delta$$

$$S^\mu \geq 0$$


Wald entropy current
S. Chapman, YN, Y. Oz
If $\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}} \sim \ell^\mu \ell^\nu$
$j^{\mu(1)} = -T\sigma \partial^\mu \frac{\mu}{T} +$
We have $S^{\mu(1)}$
$\epsilon_{AB} \ell^A \ell^B = 0$


Wald entropy current
S. Chapman, YN, Y. Oz
If $\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}} \sim \ell^\mu \ell^\nu$
We have $S^{\mu(1)}$
$E_{AB}\ell^A \ell^B = 0$
$j^{\mu(1)} = -T\sigma\,\partial^\mu \frac{\mu}{T} + C\mu^2 \omega^\mu + \beta T^2 \omega^\mu$
$\varepsilon^{ABCDE} A_A F_{BC} F_{DE}$
TT

S. Chapman, YN, Y. Oz

$$j^{\mu(1)} = -T\sigma\,\partial^\mu \frac{\mu}{T} + C\mu^2 \omega^\mu + \beta T^2 \omega^\mu$$

$$\mathcal{E}^{ABCDE} A_A F_{BC} F_{DE} \qquad JTT$$

$$\Delta S \geq 0$$

$$S^\mu \geq 0 \qquad S = \int Q^{AB}\, d\Sigma_{AB}$$

?