Title: Fractional Quantum Hall Effect in the Absence of Landau Levels

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Abstract: It has been well-known that topological phenomena with fractional excitations, i.e., the fractional quantum Hall effect (FQHE) will emerge when electrons move in Landau levels. In this talk, I will show FQHE can emergy even in the absence of Landau levels in interacting fermion models and boson models. The non-interacting part of our Hamiltonian contains topologically nontrivial flat band.

The Topology of Dirac Cones

A simple Dirac model: $H = k_x\sigma^x + k_y\sigma^y + m\sigma^z$

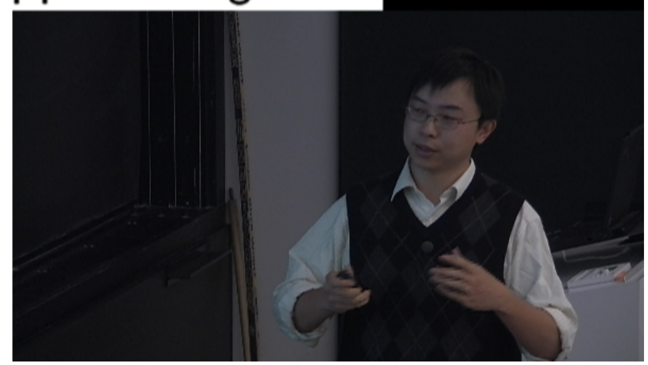
m=0: massless Dirac cone

Dirac cone as half a skyrmion

$$\frac{1}{4\pi}\int d^2k\hat{\mathbf{d}}\cdot\partial_x\hat{\mathbf{d}}\times\partial_y\hat{\mathbf{d}} = \frac{1}{4\pi}\int d^2k\frac{m}{E(\boldsymbol{k})^3} = \frac{1}{2}\mathrm{sgn}(m); \quad E(\boldsymbol{k}) = \sqrt{k^2+m^2}$$

Dirac cone must appear in pairs

- Chern number =1 if two Dirac masses have the same sign.
- Chern number =0 if two Dirac masses have opposite sign.



Fractional Quantum Hall Effect in the absence of Landau levels

Zheng-Cheng Gu (KITP)

Collaborators:

Dr. Kai Sun(University of Maryland)
Prof. D. N. Sheng(California State)
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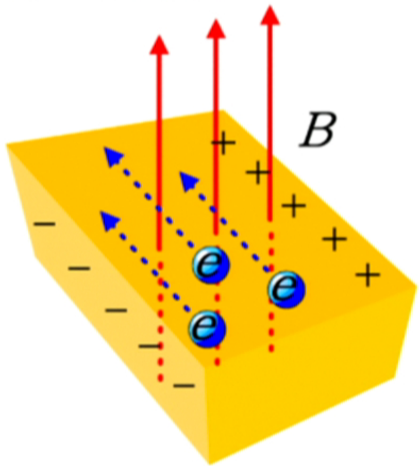
Outline

- Integer and fractional quantum hall effect(QHE)
- IQHE in a lattice model without Landau levels
- Lattice models with topological flat band
- FQHE in the absence of Landau levels
- Summary and outlook

Quantum hall effect

Hall effect



$$\boldsymbol{f}_L = -e\boldsymbol{v} \times \boldsymbol{B}$$

$$R_{xy} = \frac{V_y}{I_x} = \frac{B_z}{ne}$$

Quantum hall effect

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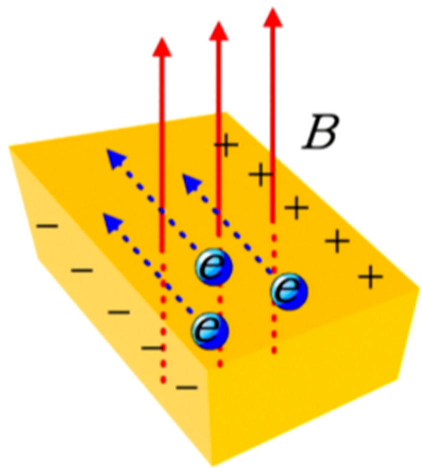

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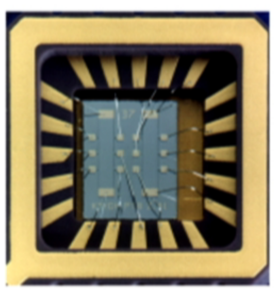
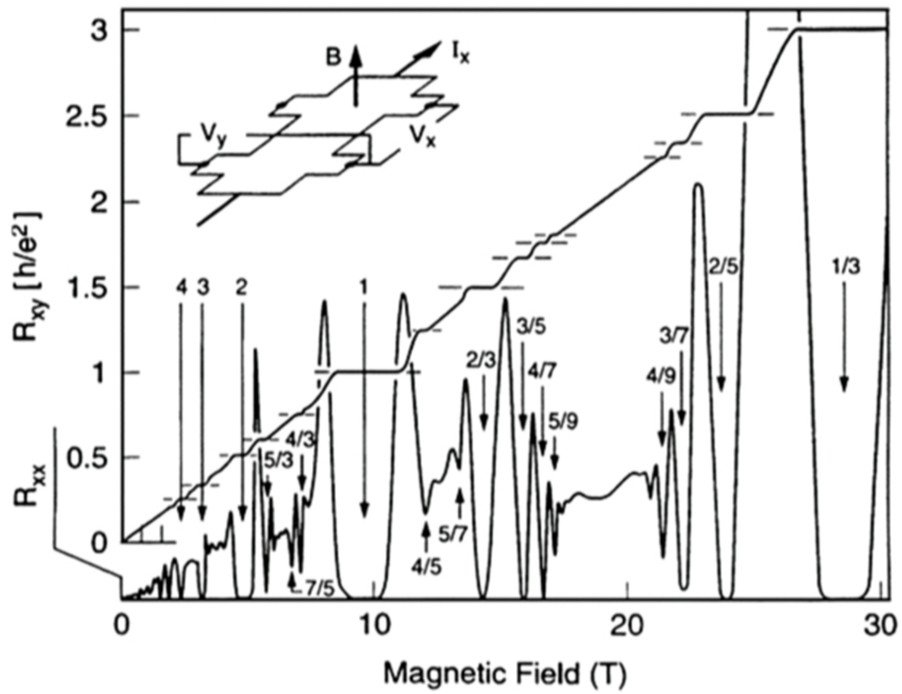
(D.C.Tsui, *etal* 1982)

Quantum Hall effect





The nature of quantization

Landua levels and Integer Quantum Hall Effect(IQHE)

$$H = \frac{p_y^2}{2m} + \frac{(p_x - eyB/c)^2}{2m} \qquad E_n = (n + 1/2)\hbar\omega_c$$

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Fractional Quantum Hall Effect(FQHE)

v=1/3 Laughlin State: $\Psi_3 = \prod_{i<j}(z_i - z_j)^3 \mathrm{e}^{-\frac{1}{4}\sum_i |z_i|^2}$

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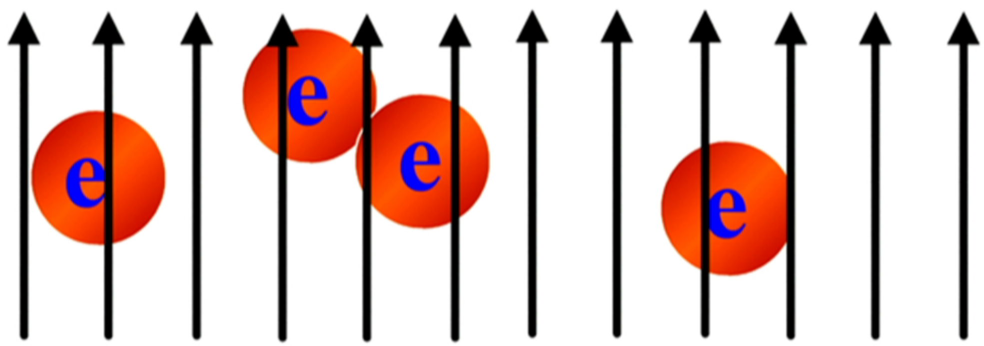

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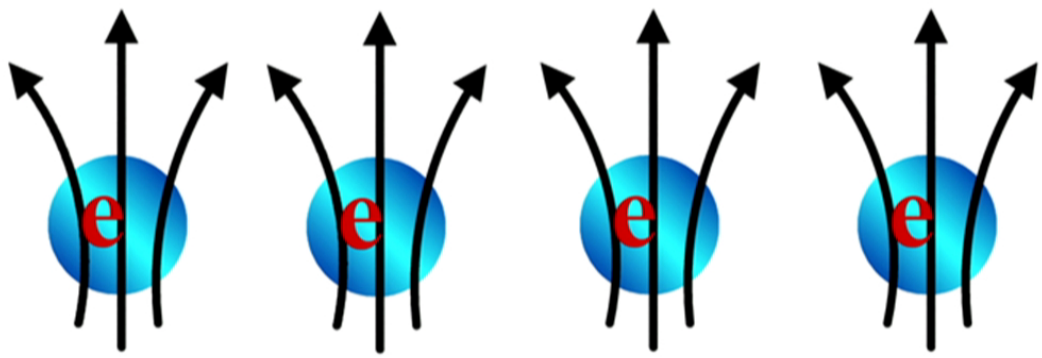
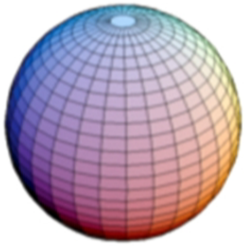
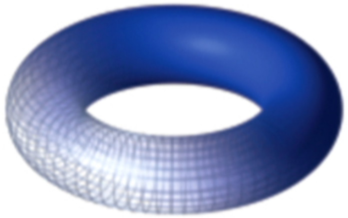


The topological quantization

Gauss-Bonnet Theorem

$$\int_M \kappa \, dA = 2\pi\chi = 2\pi(2-2g) \quad \kappa = (r_1 r_2)^{-1}$$





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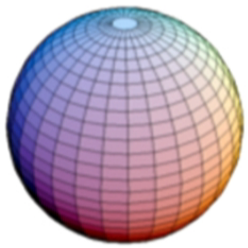

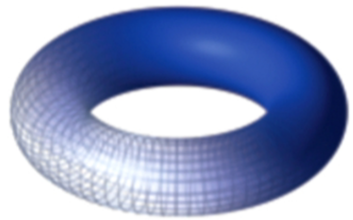

"Gauss-Bonnet" in quantum system: Berry Phase

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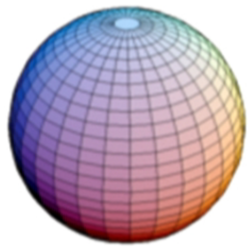

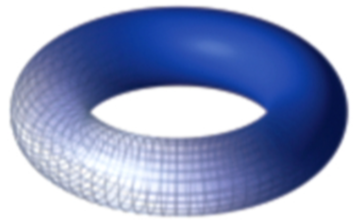

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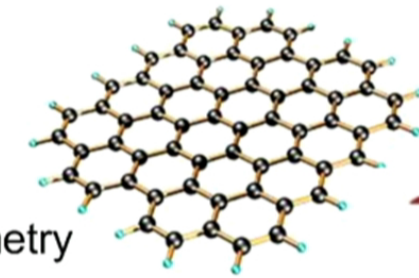
$$\phi = \oint \mathcal{A} \cdot d\mathbf{k}, \quad \mathcal{A} = \langle \psi_k | -i\nabla_k | \psi_k \rangle$$

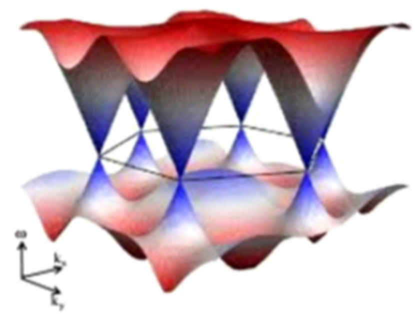
$$\psi_k \rightarrow e^{i\chi(k)}\psi_k \qquad \mathcal{A} \rightarrow \mathcal{A} + \nabla_k \chi$$

Berry phase in solid physics

Bloch's theorem:

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$





- The lattice translational symmetry implies crystal momentum k is a good quantum number.

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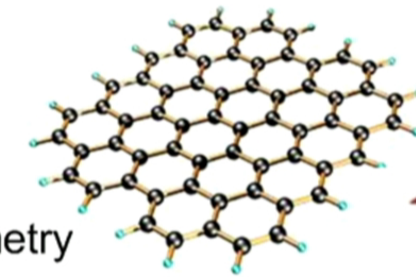

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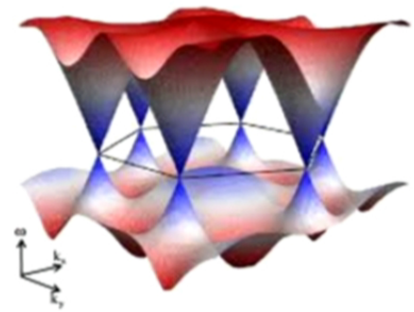

"Gauss-Bonnet" in solid physics

$$\mathcal{A} = \langle u_{\mathbf{k}} | -i\nabla_k | u_{\mathbf{k}} \rangle \qquad \mathcal{F} = \nabla \times \mathcal{A}$$

$$n = \sum_{bands} \frac{i}{2\pi} \int d^2k \left(\left\langle \frac{\partial u}{\partial k_1} \middle| \frac{\partial u}{\partial k_2} \right\rangle - \left\langle \frac{\partial u}{\partial k_2} \middle| \frac{\partial u}{\partial k_1} \right\rangle \right)$$

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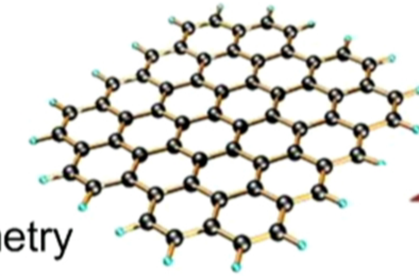

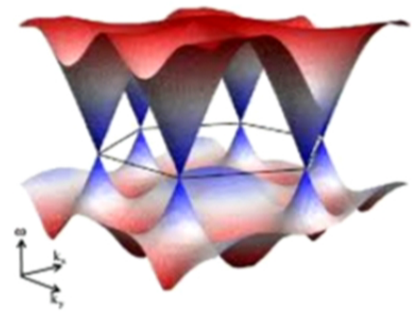

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$$\sigma_{xy} = n\frac{e^2}{h}$$

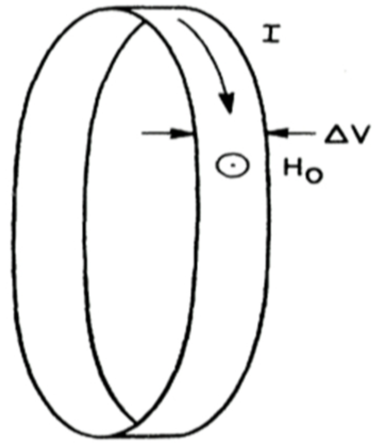
(TKNN 1982) "First Chern number"

With disorder and interactions

Laughlin's gauge argument

$$I = c\frac{\partial U}{\partial \phi} = \frac{c\,\partial U}{L\,\partial A}$$

$$I = c\frac{neV}{\Delta\phi} = \frac{ne^2 V}{h}$$



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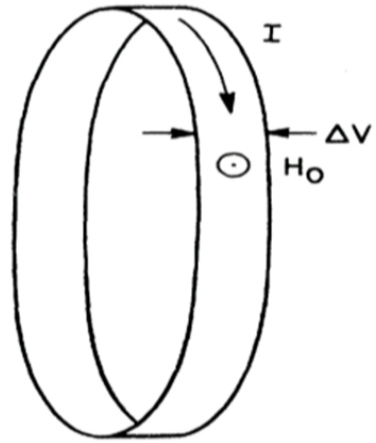
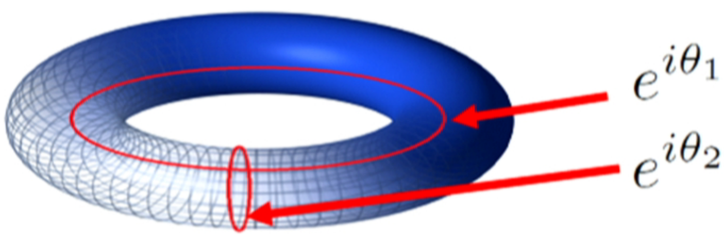


Hall conductance from a many-body wave function

$$\sigma = \bar{\sigma} = \frac{e^2}{h}\int_0^{2\pi}\int_0^{2\pi} \mathrm{d}\theta_1 \mathrm{d}\theta_2 \frac{1}{2\pi i}\left(\left\langle \frac{\partial \Psi}{\partial \theta_2}\middle|\frac{\partial \Psi}{\partial \theta_1}\right\rangle - \left\langle \frac{\partial \Psi}{\partial \theta_1}\middle|\frac{\partial \Psi}{\partial \theta_2}\right\rangle\right)$$

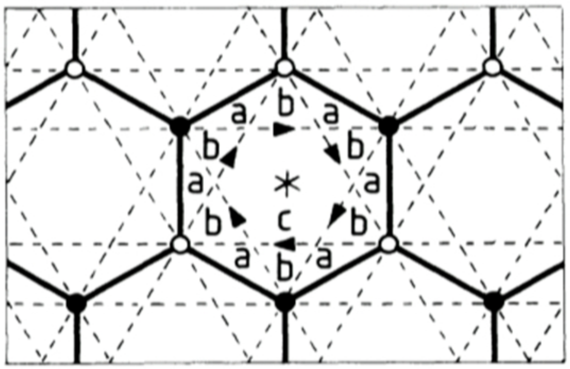


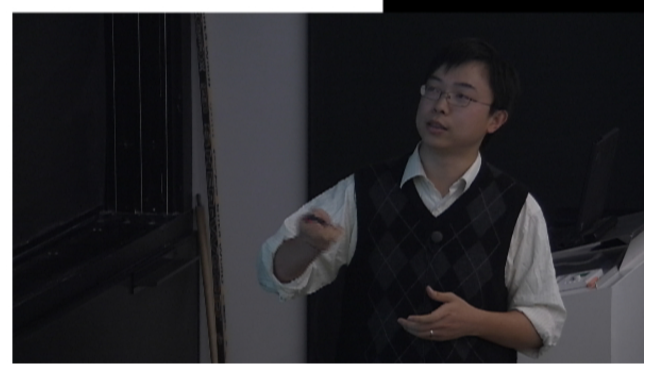
(Qian Niu, etal, 1985)

Does Landau levels necessary?

Haldane model: (FDM Haldane, 1988)

$$H_{\text{Haldane}} = t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + h.c. \right) + t' \sum_{\langle\langle ij \rangle\rangle} \left(e^{i\phi} c_i^\dagger c_j + h.c. \right)$$





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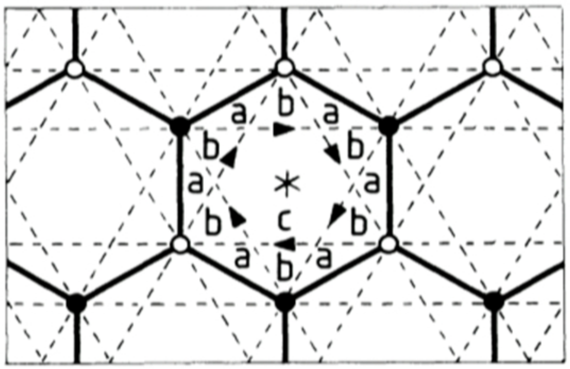

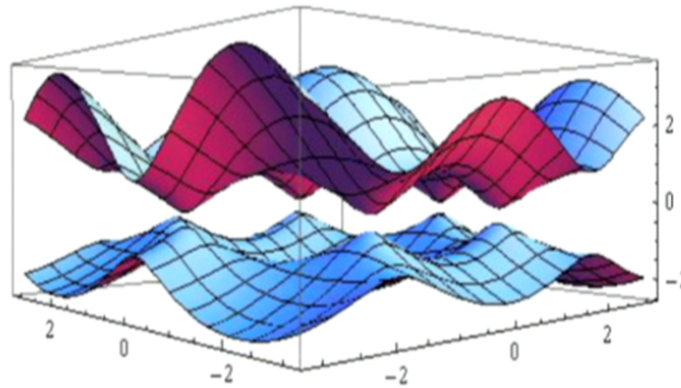

Nonzero Chern number:

$$t' = 0.1t; \quad \phi = 0.1\pi$$

$$c = \frac{1}{2\pi}\int_{BZ} d^2k F_{12}(k) \qquad A_\mu(k) = -i\langle n_{\mathbf{k}}|\frac{\partial}{\partial k_\mu}|n_{\mathbf{k}}\rangle$$

$$\sigma_H = \frac{e^2}{h}c; \quad c = \pm 1 \qquad F_{12}(k) = \frac{\partial}{\partial k_1}A_2(k) - \frac{\partial}{\partial k_2}A_1(k)$$

The topological nature

A generic two band model: $H = \varepsilon(\boldsymbol{k}) + d_\alpha \sigma^\alpha$

$$E_\pm(\boldsymbol{k}) = \varepsilon(\boldsymbol{k}) \pm d(\boldsymbol{k}); \quad \min_{\boldsymbol{k}} E_+(\boldsymbol{k}) > \max_{\boldsymbol{k}} E_+(\boldsymbol{k})$$

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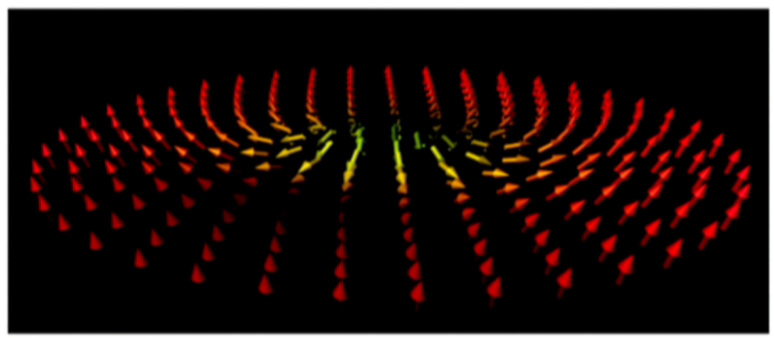
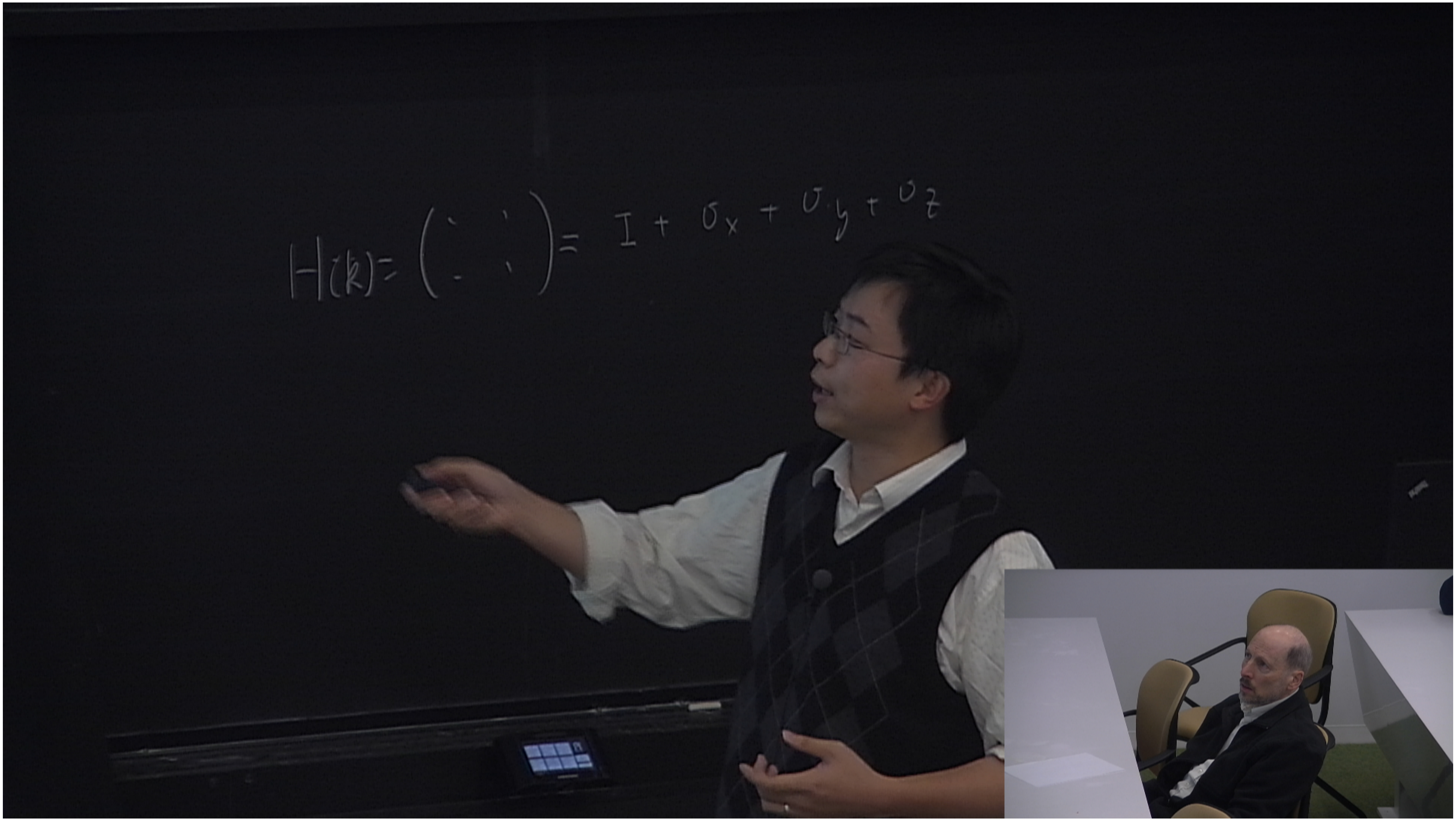
$4\pi n$

Chern number $$\sigma_{xy} \;=\; -\frac{1}{8\pi^2} \int\!\!\int_{\mathrm{FBZ}} dk_x dk_y \hat{\mathbf{d}} \cdot \partial_{\mathbf{x}} \hat{\mathbf{d}} \times \partial_{\mathbf{y}} \hat{\mathbf{d}}$$

Skyrmion number

$$T^2 \rightarrow S^2$$




H(k) = ( ) = I + σx + σy + σz

Fractional quantum hall effect?

- Bands with non-trivial topology mimic the physics of IQHE

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Construct topological flat band model in k space

$$H(\boldsymbol{k}) = U(\boldsymbol{k})\Lambda(\boldsymbol{k})U(\boldsymbol{k})^{\dagger}$$

$$\Downarrow$$

$$H_{\text{flat}}(\boldsymbol{k}) = U(\boldsymbol{k})\Lambda U(\boldsymbol{k})^{\dagger}$$

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Highly non-local in real space!

How to construct a local model with topological flat band?

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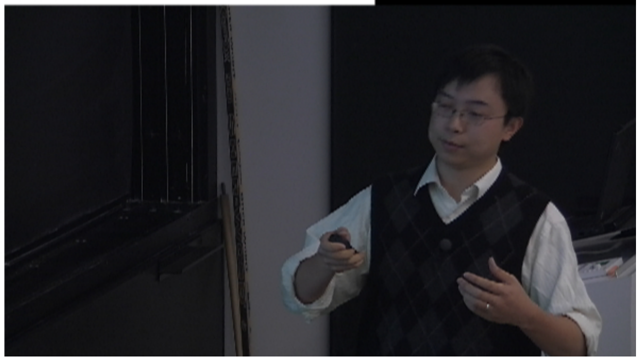
Topological bands can be constructed by gapping Dirac cones, but with uncertainty.

Do we have better method?

Quadratic band touching

$$H = (k_x^2 - k_y^2)\sigma^x + 2k_x k_y \sigma^y \qquad E_\pm(\boldsymbol{k}) = \pm k^2$$



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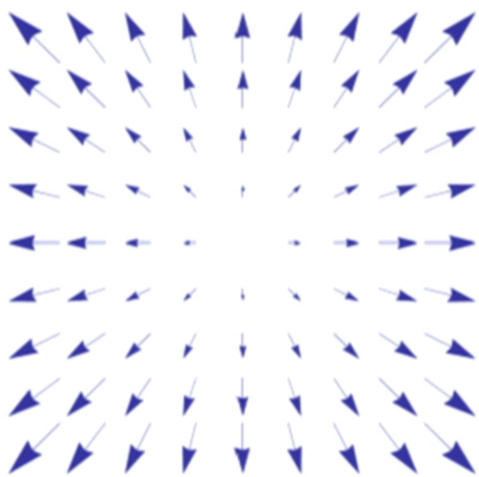
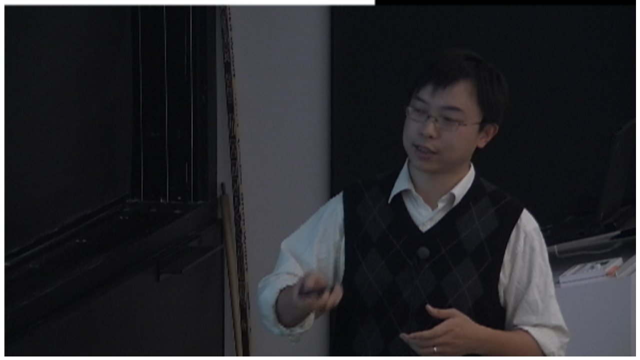
Dirac cone as a vortex in k space with

winding number = 1

$$\boldsymbol{d} = (k_x, k_y, 0)$$





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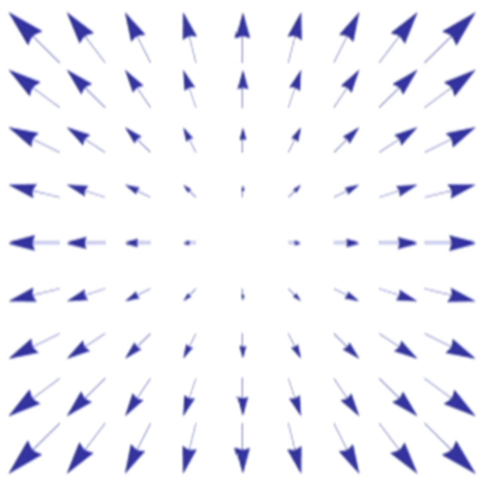
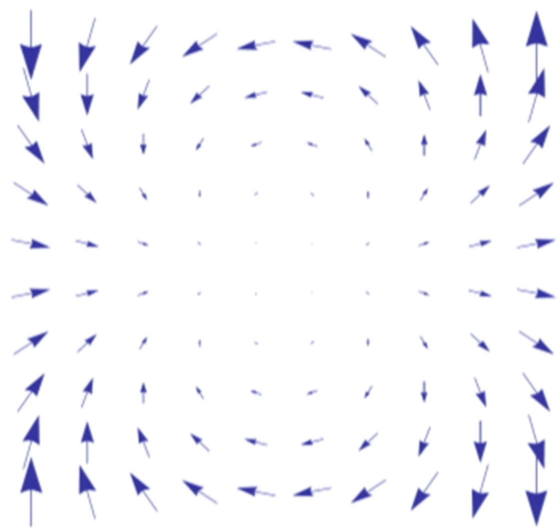


A quadratic touching as a vortex in k space with

winding number = 2

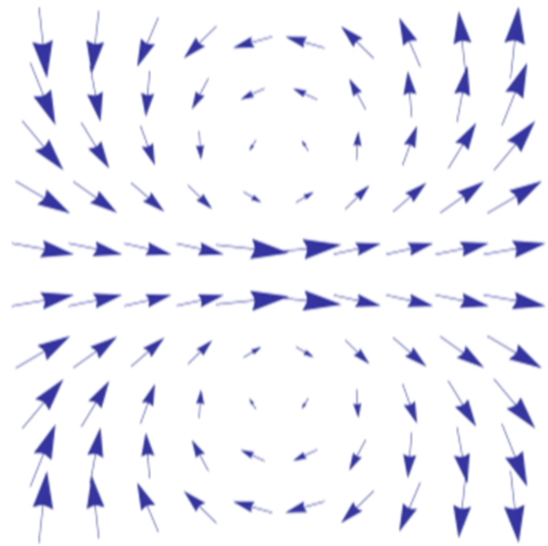
$$\boldsymbol{d} = (k_x^2 - k_y^2, 2k_x k_y, 0)$$



Symmetry protection of quadratic band touching

A quadratic touching is protected by 4-fold/6-fold rotational symmetry.

$$H = (k_x^2 - k_y^2 + M)\sigma^x + 2k_x k_y \sigma^y$$
$$\boldsymbol{d} = (k_x^2 - k_y^2 + M, 2k_x k_y, 0)$$



A quadratic touching can be regarded as merging of two Dirac cones with the same winding number.

- A single quadratic touching is allowed.
- Gapping a quadratic touching by breaking time reversal symmetry will produce topologically non-trivial bands.

Topological flat band model

A three band model with quadratic band touching

$$H_0 = -t_{dd}\sum_{\vec{r}} d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_x} + d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c.$$
$$+ t_{pp}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_y} p_{y,\vec{r}} + h.c. - t'_{pp}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_y} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_x} p_{y,\vec{r}} + h.c.$$
$$+ t_{pd}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} d_{\vec{r}} - p^\dagger_{x,\vec{r}} d_{\vec{r}+\vec{a}_x} + p^\dagger_{y,\vec{r}+\vec{a}_y} d_{\vec{r}} - p^\dagger_{y,\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c.$$

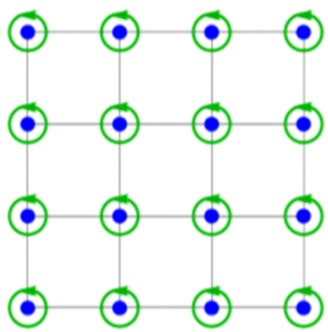
Topological flat band model

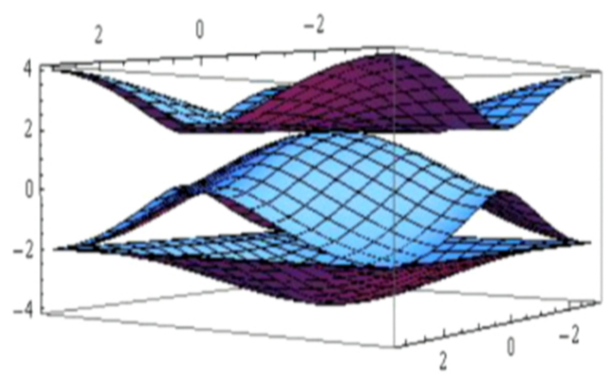
A three band model with quadratic band touching

$$\begin{aligned} H_0 = &- t_{dd} \sum_{\vec{r}} d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_x} + d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c. \\ &+ t_{pp} \sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_y} p_{y,\vec{r}} + h.c. - t'_{pp} \sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_y} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_x} p_{y,\vec{r}} + h.c. \\ &+ t_{pd} \sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} d_{\vec{r}} - p^\dagger_{x,\vec{r}} d_{\vec{r}+\vec{a}_x} + p^\dagger_{y,\vec{r}+\vec{a}_y} d_{\vec{r}} - p^\dagger_{y,\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c. \end{aligned}$$

Gap opens after breaking T

$$H_1 = \sum_{\vec{r}} i\Delta p^\dagger_{x,\vec{r}} p_{y,\vec{r}} - i\Delta p^\dagger_{y,\vec{r}} p_{x,\vec{r}}$$





Topological flat band model

A three band model with quadratic band touching

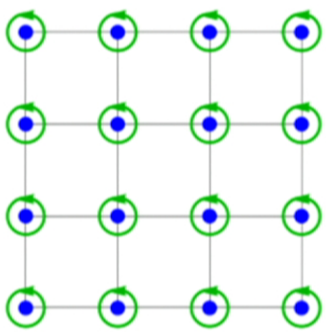
$$H_0 = -t_{dd}\sum_{\vec{r}} d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_x} + d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c.$$
$$+ t_{pp}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_y} p_{y,\vec{r}} + h.c. - t'_{pp}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_y} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_x} p_{y,\vec{r}} + h.c.$$
$$+ t_{pd}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} d_{\vec{r}} - p^\dagger_{x,\vec{r}} d_{\vec{r}+\vec{a}_x} + p^\dagger_{y,\vec{r}+\vec{a}_y} d_{\vec{r}} - p^\dagger_{y,\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c.$$

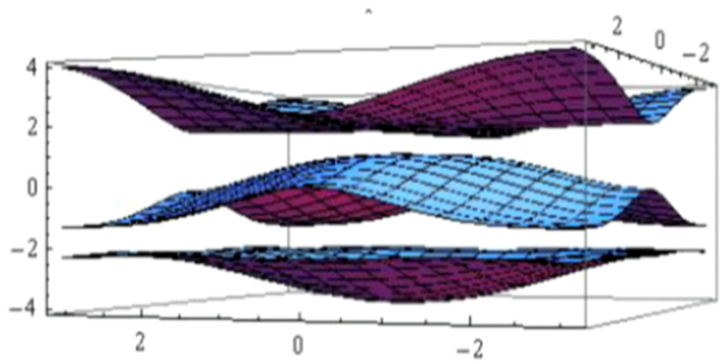
Gap opens after breaking T

$$t_{dd} = t_{pd} = t_{pp} = 1$$

$$H_1 = \sum_{\vec{r}} i\Delta p^\dagger_{x,\vec{r}} p_{y,\vec{r}} - i\Delta p^\dagger_{y,\vec{r}} p_{x,\vec{r}}$$

$$t'_{pp} = \frac{\Delta t_{pp}}{4t_{pp}+\Delta}; \quad \Delta = 2.8$$





c=1

c=0

c=-1

Topological flat band model

A three band model with quadratic band touching

$$H_0 = -\,t_{dd}\sum_{\vec{r}} d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_x} + d^\dagger_{\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c.$$

$$+\,t_{pp}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_y} p_{y,\vec{r}} + h.c. - t'_{pp}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_y} p_{x,\vec{r}} + p^\dagger_{y,\vec{r}+\vec{a}_x} p_{y,\vec{r}} + h.c.$$

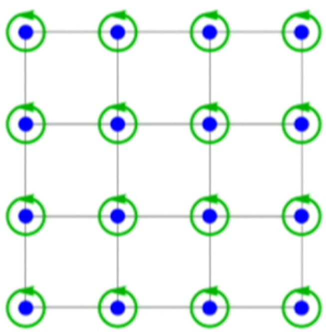
$$+\,t_{pd}\sum_{\vec{r}} p^\dagger_{x,\vec{r}+\vec{a}_x} d_{\vec{r}} - p^\dagger_{x,\vec{r}} d_{\vec{r}+\vec{a}_x} + p^\dagger_{y,\vec{r}+\vec{a}_y} d_{\vec{r}} - p^\dagger_{y,\vec{r}} d_{\vec{r}+\vec{a}_y} + h.c.$$

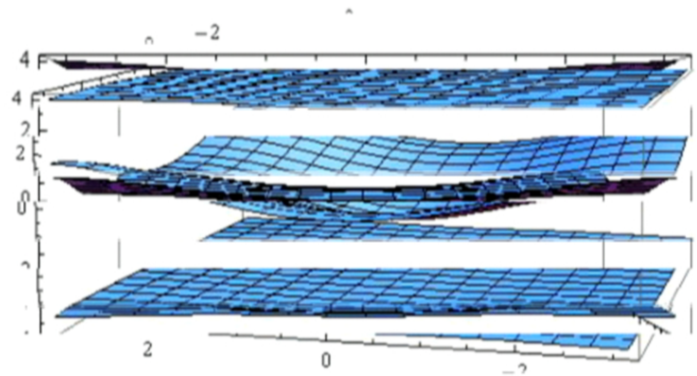
Gap opens after breaking T

$$t_{dd} = t_{pd} = t_{pp} = 1$$

$$H_1 = \sum_{\vec{r}} i\Delta p^\dagger_{x,\vec{r}} p_{y,\vec{r}} - i\Delta p^\dagger_{y,\vec{r}} p_{x,\vec{r}}$$

$$t'_{pp} = \frac{\Delta t_{pp}}{4t_{pp}+\Delta}; \quad \Delta = 2.8$$





c=1

c=0

c=-1

Topological flat band model

A three band model with quadratic band touching

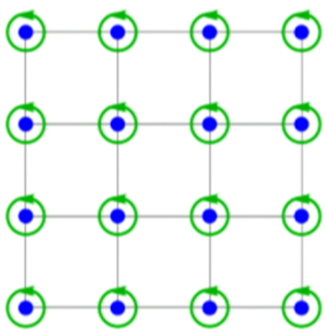
$$H_0 = -t_{dd}\sum_{\vec r} d^\dagger_{\vec r} d_{\vec r+\vec a_x} + d^\dagger_{\vec r} d_{\vec r+\vec a_y} + h.c.$$
$$+ t_{pp}\sum_{\vec r} p^\dagger_{x,\vec r+\vec a_x} p_{x,\vec r} + p^\dagger_{y,\vec r+\vec a_y} p_{y,\vec r} + h.c. - t'_{pp}\sum_{\vec r} p^\dagger_{x,\vec r+\vec a_y} p_{x,\vec r} + p^\dagger_{y,\vec r+\vec a_x} p_{y,\vec r} + h.c.$$
$$+ t_{pd}\sum_{\vec r} p^\dagger_{x,\vec r+\vec a_x} d_{\vec r} - p^\dagger_{x,\vec r} d_{\vec r+\vec a_x} + p^\dagger_{y,\vec r+\vec a_y} d_{\vec r} - p^\dagger_{y,\vec r} d_{\vec r+\vec a_y} + h.c.$$

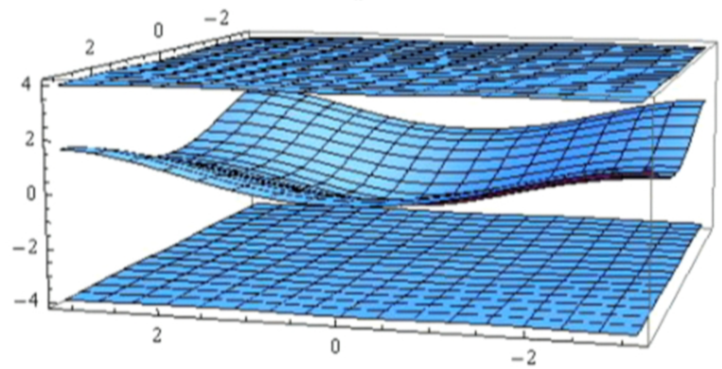
Gap opens after breaking T

$$t_{dd} = t_{pd} = t_{pp} = 1$$

$$H_1 = \sum_{\vec r} i\Delta p^\dagger_{x,\vec r} p_{y,\vec r} - i\Delta p^\dagger_{y,\vec r} p_{x,\vec r}$$

$$t'_{pp} = \frac{\Delta t_{pp}}{4t_{pp}+\Delta}; \quad \Delta = 2.8$$





Topological flat band model

A three band model with quadratic band touching

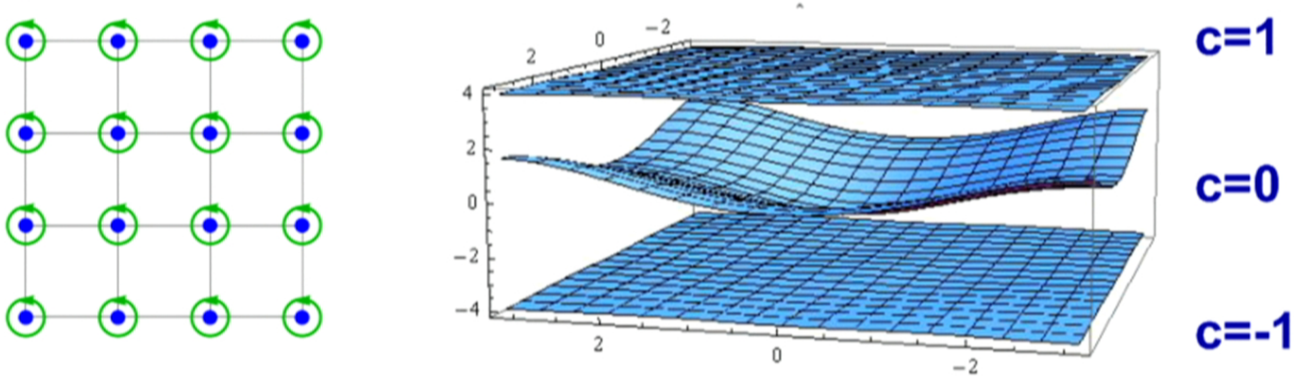
$$H_0 = -\,t_{dd}\sum_{\vec r} d^\dagger_{\vec r} d_{\vec r+\vec a_x} + d^\dagger_{\vec r} d_{\vec r+\vec a_y} + h.c.$$
$$+\,t_{pp}\sum_{\vec r} p^\dagger_{x,\vec r+\vec a_x} p_{x,\vec r} + p^\dagger_{y,\vec r+\vec a_y} p_{y,\vec r} + h.c. - t'_{pp}\sum_{\vec r} p^\dagger_{x,\vec r+\vec a_y} p_{x,\vec r} + p^\dagger_{y,\vec r+\vec a_x} p_{y,\vec r} + h.c.$$
$$+\,t_{pd}\sum_{\vec r} p^\dagger_{x,\vec r+\vec a_x} d_{\vec r} - p^\dagger_{x,\vec r} d_{\vec r+\vec a_x} + p^\dagger_{y,\vec r+\vec a_y} d_{\vec r} - p^\dagger_{y,\vec r} d_{\vec r+\vec a_y} + h.c.$$

Gap opens after breaking T

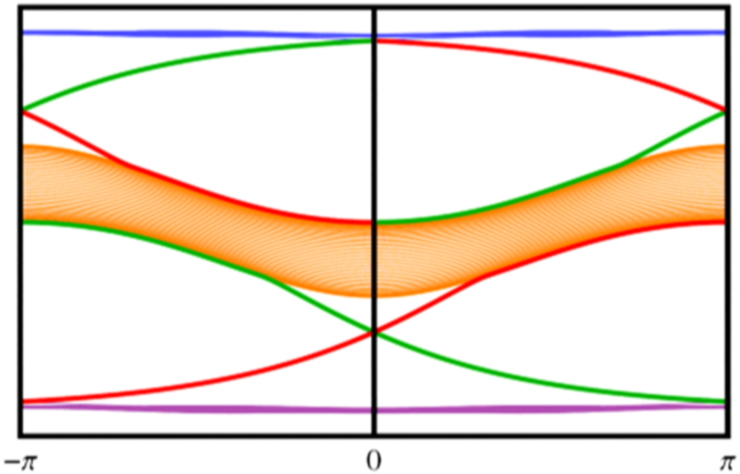
$$t_{dd} = t_{pd} = t_{pp} = 1$$

$$H_1 = \sum_{\vec r} i\Delta p^\dagger_{x,\vec r} p_{y,\vec r} - i\Delta p^\dagger_{y,\vec r} p_{x,\vec r}$$

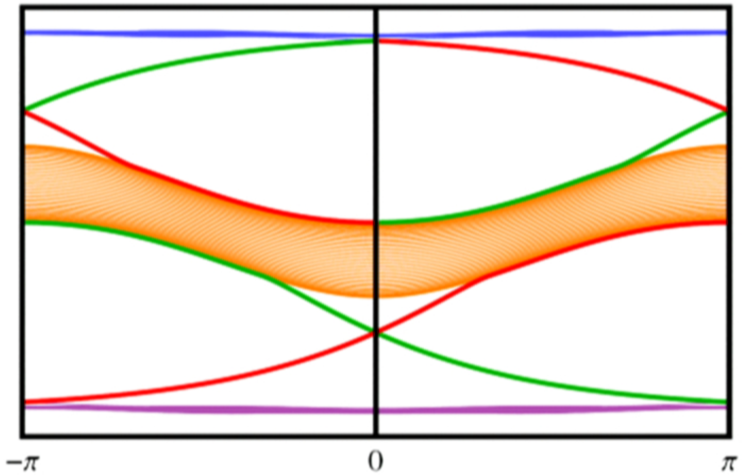
$$t'_{pp} = \frac{\Delta t_{pp}}{4t_{pp}+\Delta}; \quad \Delta = 2.8$$



Edge modes on cylinder

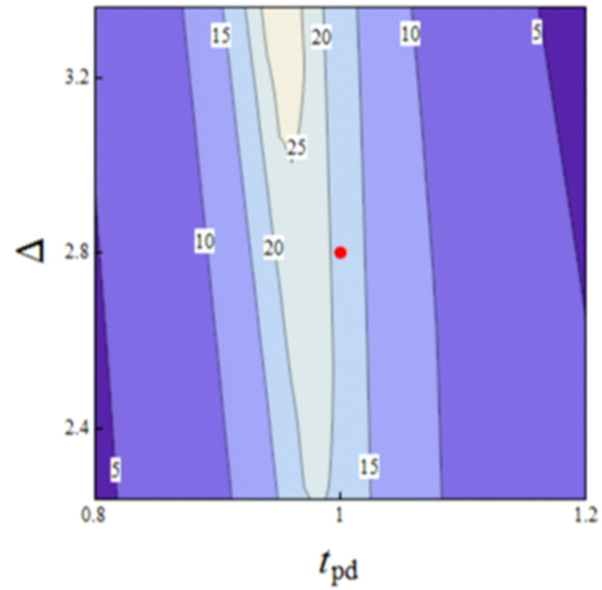


Edge modes on cylinder

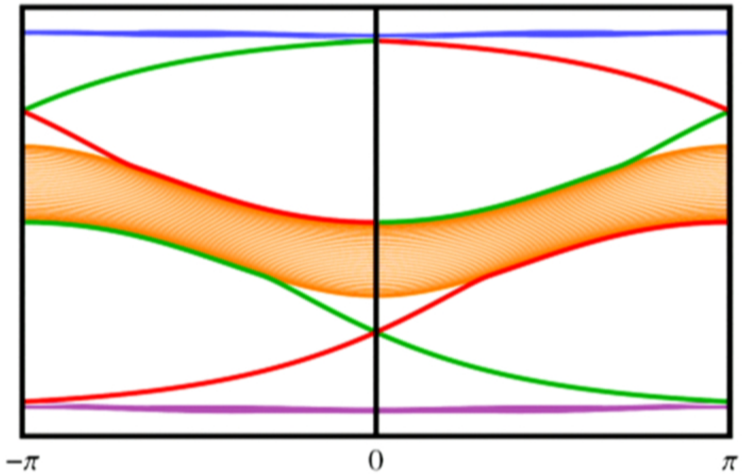


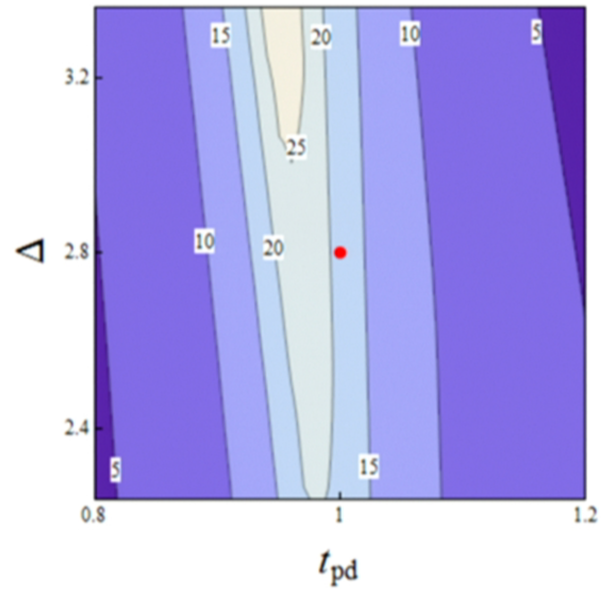
Flatness ratio

= band gap/band width =20



Edge modes on cylinder





Flatness ratio

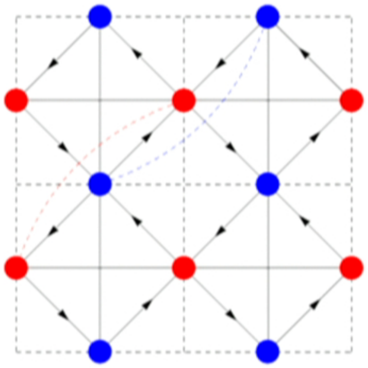
= band gap/band width =20

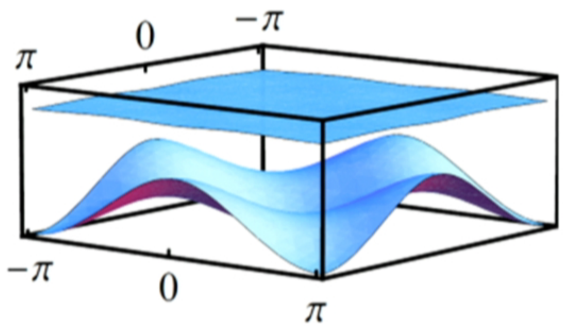
Possible realizations

- Cold atoms in optical lattice
- Imaginary hopping can be realized by rotating lattice, spin orbit coupling or anisotropic p-band Feshbach resonance

A two band model

$$H_0 = -t \sum_{\langle i,j \rangle} e^{i\phi_{ij}} (c_i^\dagger c_j + h.c.) - \sum_{\langle\langle i,j \rangle\rangle} t'_{ij} (c_i^\dagger c_j + h.c.) - t'' \sum_{\langle\langle\langle i,j \rangle\rangle\rangle} (c_i^\dagger c_j + h.c.)$$

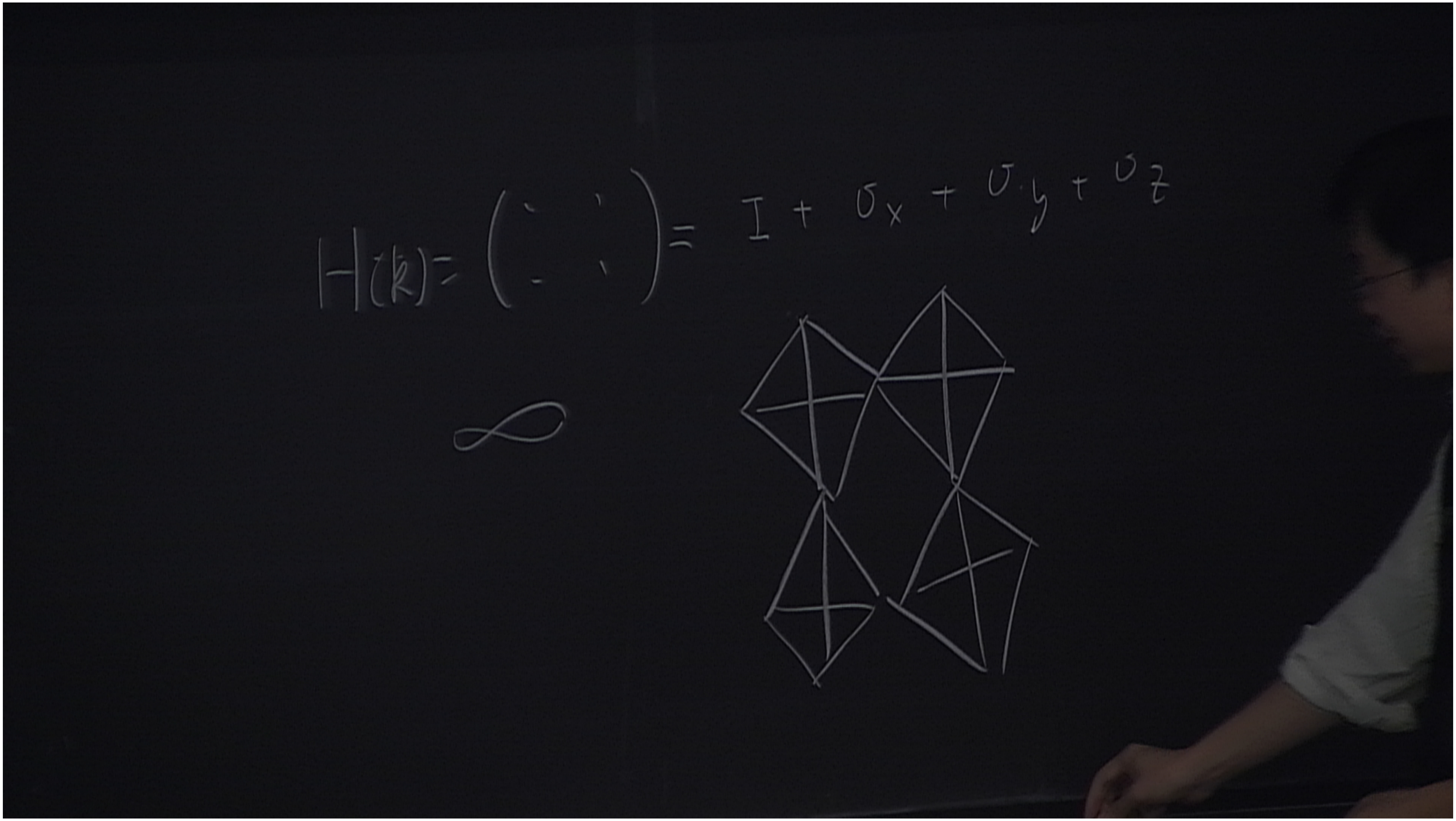




$$t'' = 1/(2 + 2\sqrt{2})$$

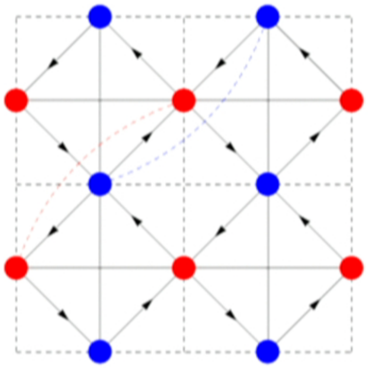
$$t'_1 = -t'_2 = 1/(2 + \sqrt{2})$$

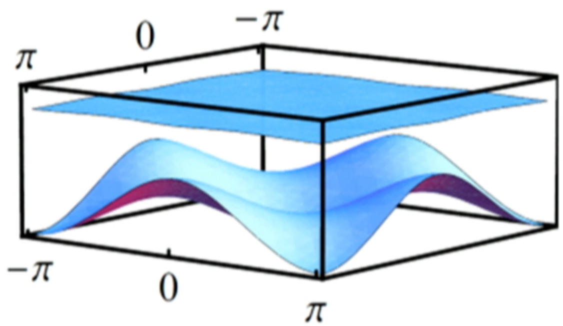
$$\phi = \pi/4$$


H(k) = ( ) = I + σx + σy + σz

A two band model

$$H_0 = -t\sum_{\langle i,j\rangle} e^{i\phi_{ij}}(c_i^\dagger c_j + h.c.) - \sum_{\langle\langle i,j\rangle\rangle} t'_{ij}(c_i^\dagger c_j + h.c.) - t''\sum_{\langle\langle\langle i,j\rangle\rangle\rangle}(c_i^\dagger c_j + h.c.)$$





$$t'' = 1/(2+2\sqrt{2})$$

$$t'_1 = -t'_2 = 1/(2+\sqrt{2})$$

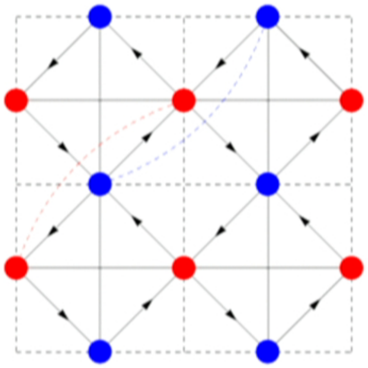
$$\phi = \pi/4$$

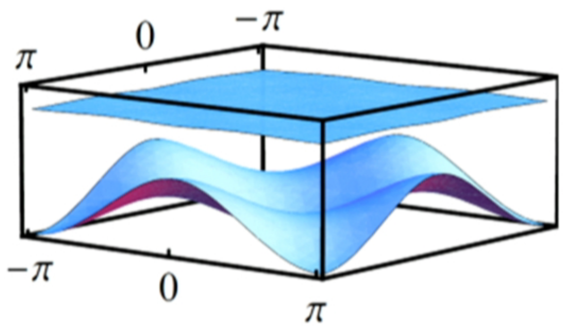
Chern number = 1,-1 for two bands

Flatness ratio = 30

A two band model

$$H_0 = -t\sum_{\langle i,j\rangle} e^{i\phi_{ij}}(c_i^\dagger c_j + h.c.) - \sum_{\langle\langle i,j\rangle\rangle} t'_{ij}(c_i^\dagger c_j + h.c.) - t''\sum_{\langle\langle\langle i,j\rangle\rangle\rangle}(c_i^\dagger c_j + h.c.)$$





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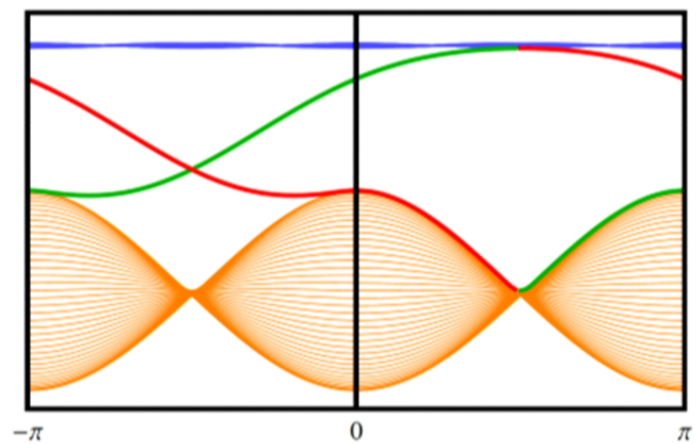
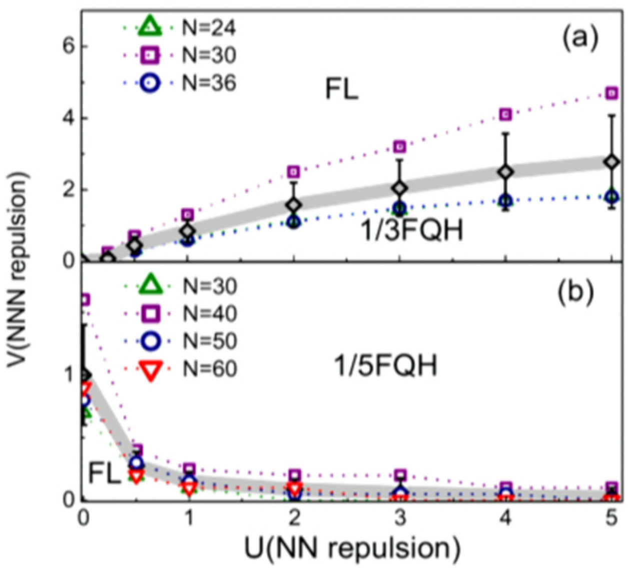
Edge modes on cylinder



Fractional quantum hall effect in the absence of magnetic field

FQHE appear after turning on interactions

$$H = -H_0 + U \sum_{\langle i,j \rangle} n_i n_j + V \sum_{\langle\langle i,j \rangle\rangle} n_i n_j$$



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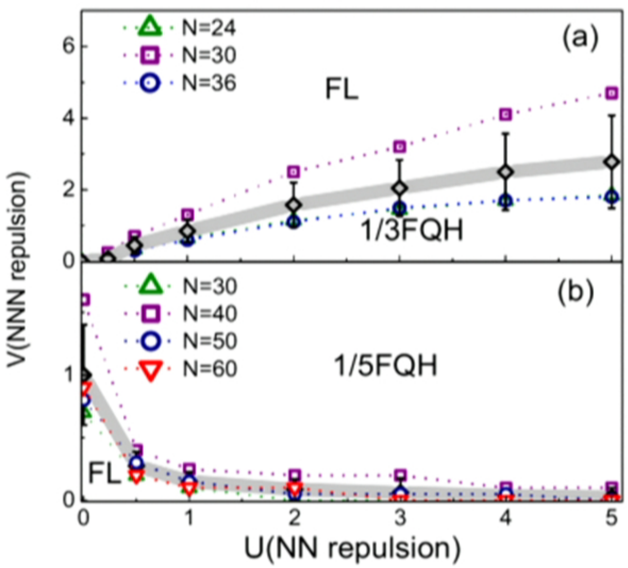
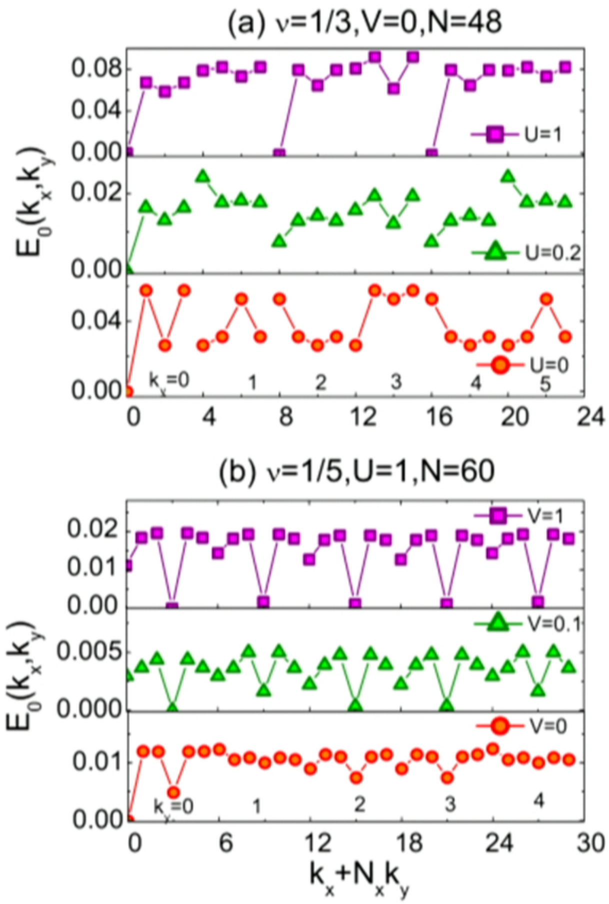
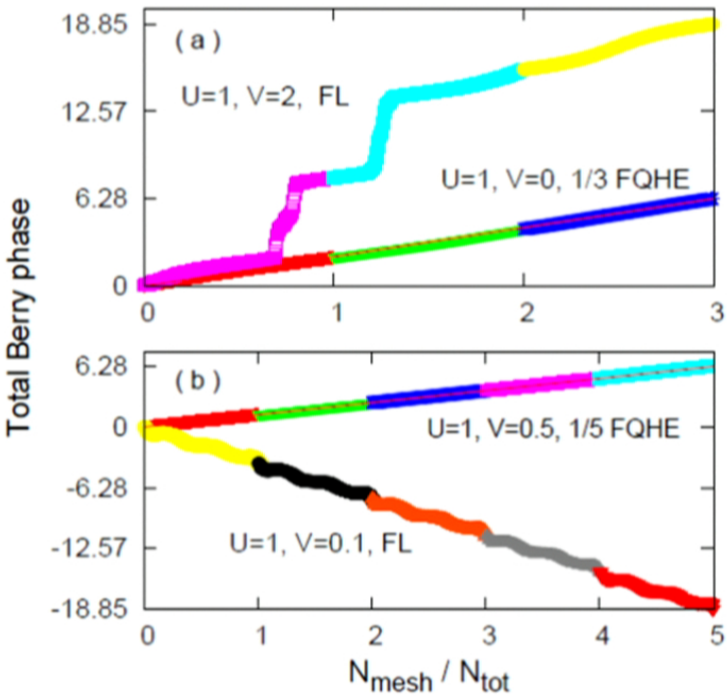




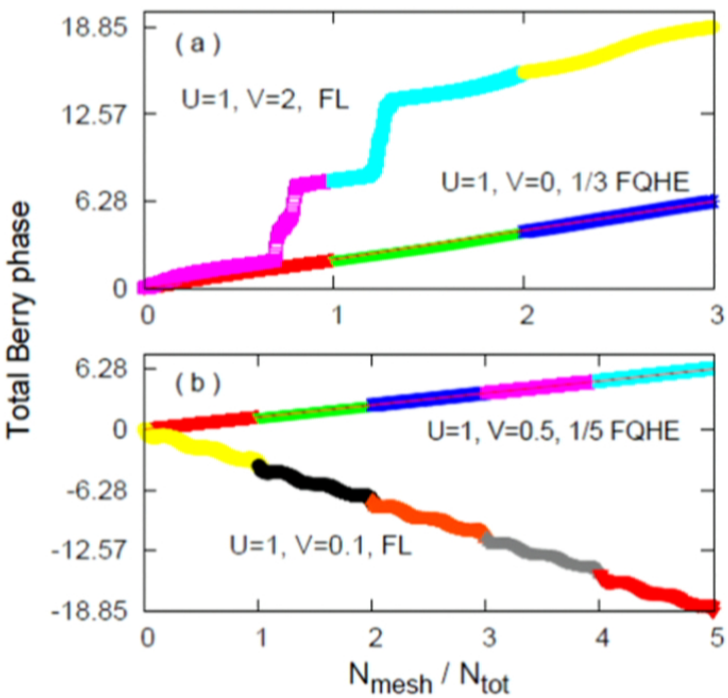
Confirmed by Chern number

$$\sigma = \bar{\sigma} = \frac{e^2}{h} \int_0^{2\pi} \int_0^{2\pi} \mathrm{d}\theta_1 \mathrm{d}\theta_2 \frac{1}{2\pi i} \left(\left\langle \frac{\partial \Psi}{\partial \theta_2} \middle| \frac{\partial \Psi}{\partial \theta_1} \right\rangle - \left\langle \frac{\partial \Psi}{\partial \theta_1} \middle| \frac{\partial \Psi}{\partial \theta_2} \right\rangle \right)$$

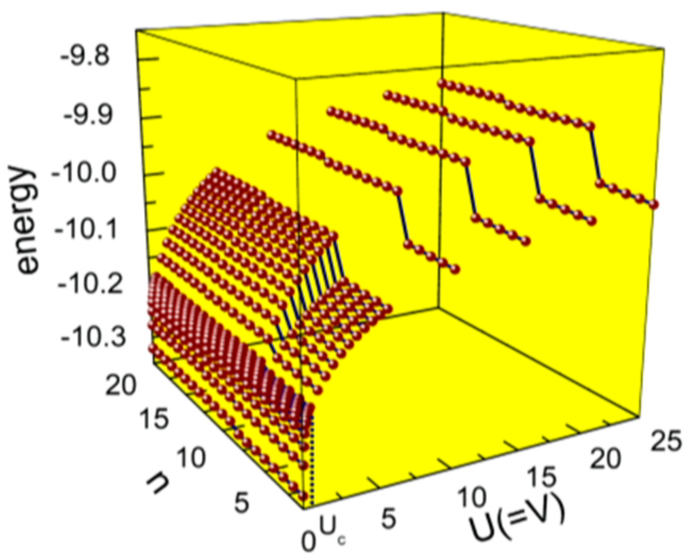


Confirmed by Chern number

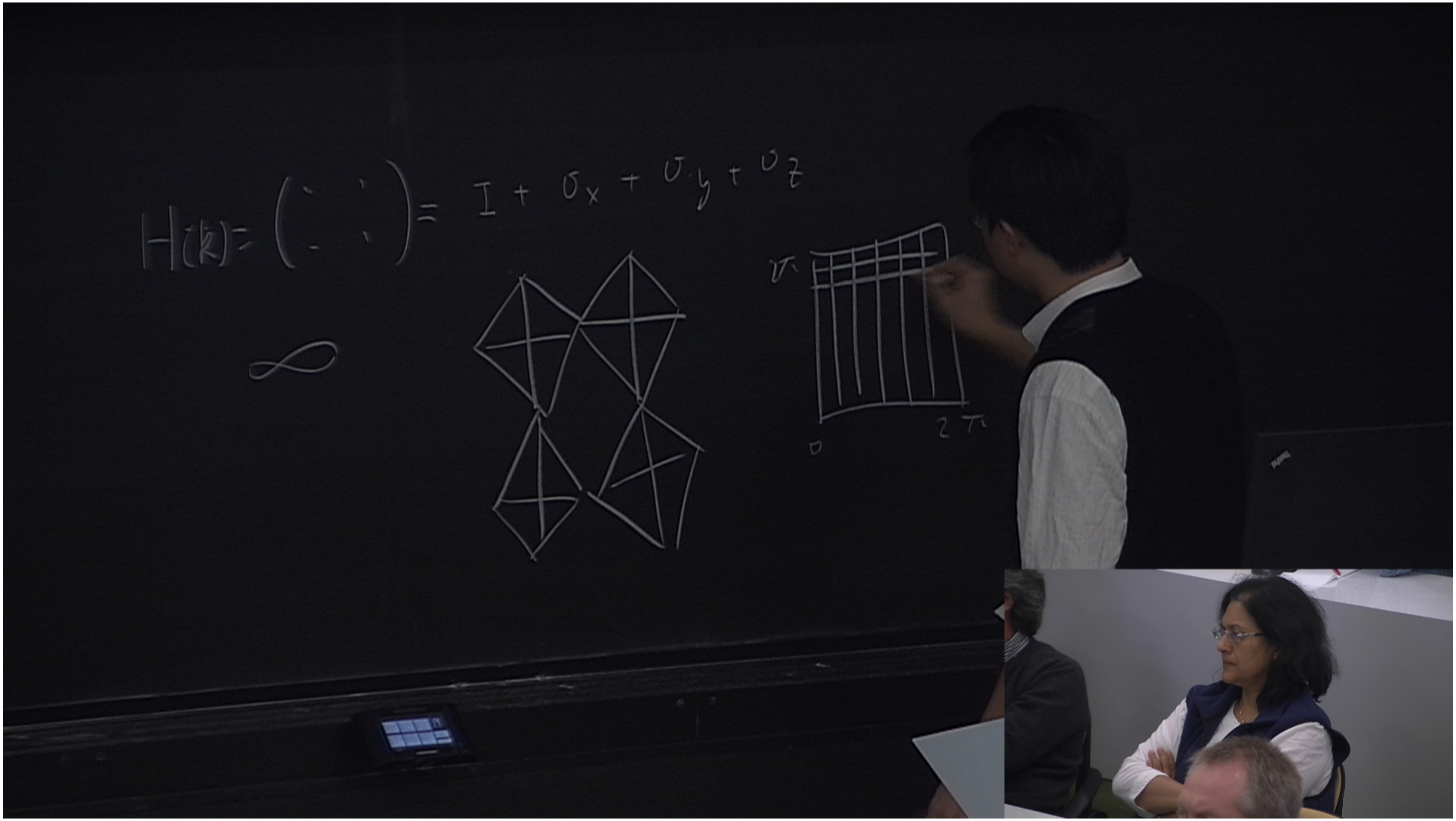
$$\sigma = \bar{\sigma} = \frac{e^2}{h}\int_0^{2\pi}\int_0^{2\pi} \mathrm{d}\theta_1 \mathrm{d}\theta_2 \frac{1}{2\pi i}\left(\left\langle \frac{\partial \Psi}{\partial \theta_2} \middle| \frac{\partial \Psi}{\partial \theta_1}\right\rangle - \left\langle \frac{\partial \Psi}{\partial \theta_1} \middle| \frac{\partial \Psi}{\partial \theta_2}\right\rangle\right)$$



Stable at strong interaction

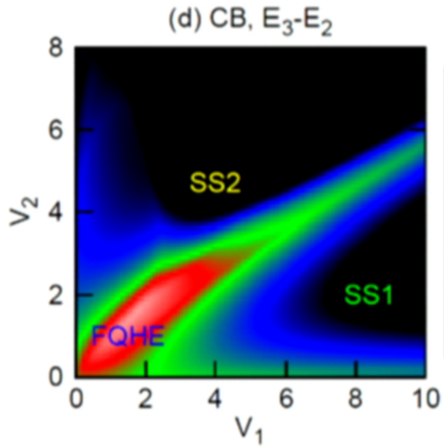


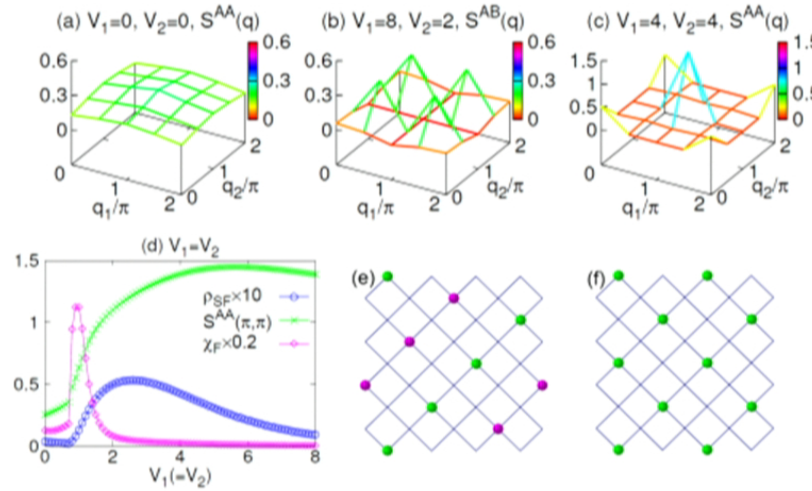
1/5 filling factor

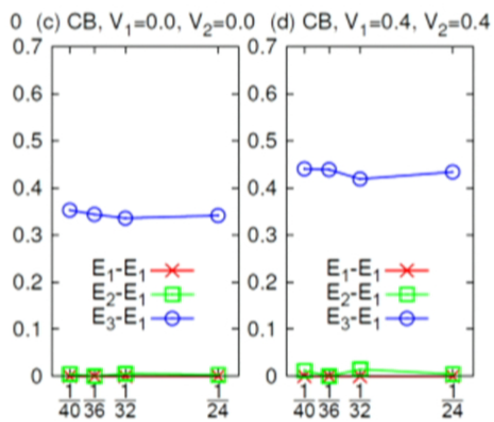

H(k) =
= I + σx + σy + σz
π
0
2π

Bosonic FQHE

Filling factor 1/2

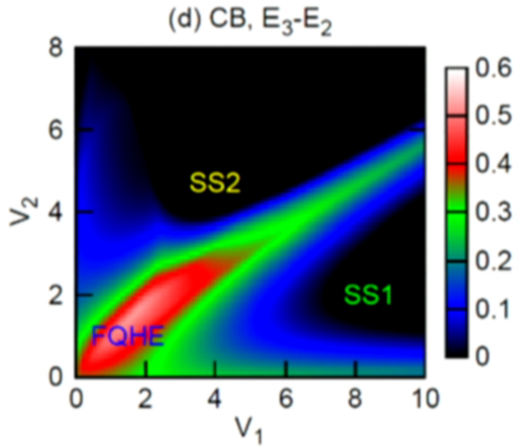


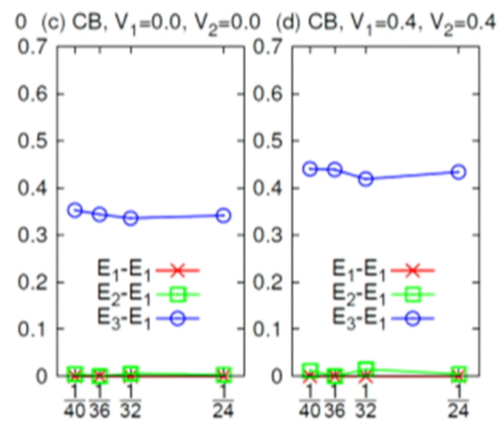


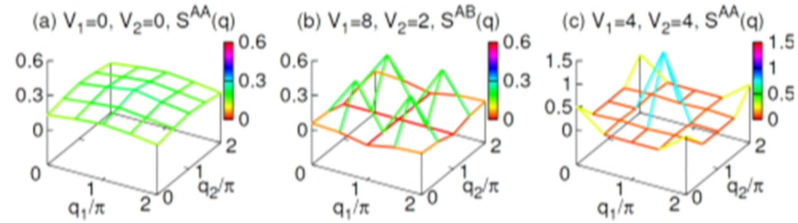


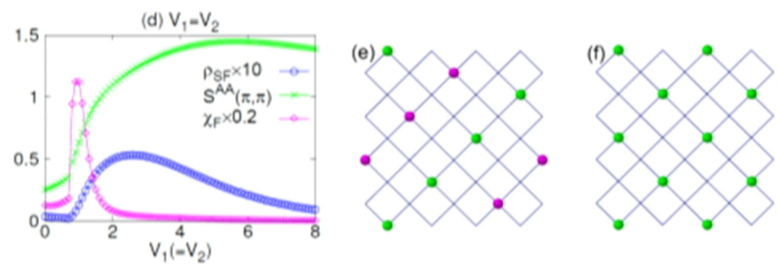
Bosonic FQHE

Filling factor 1/2

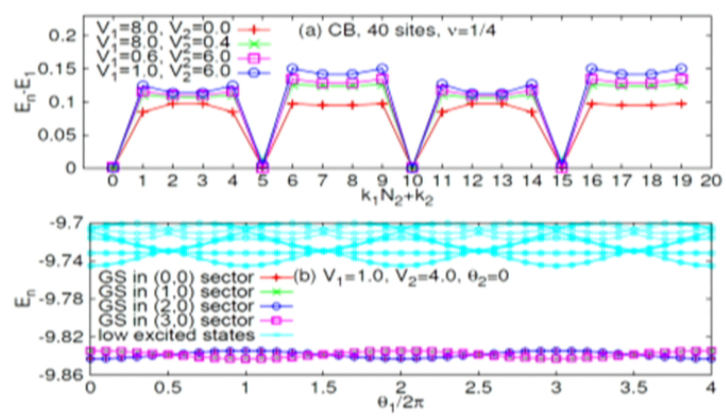






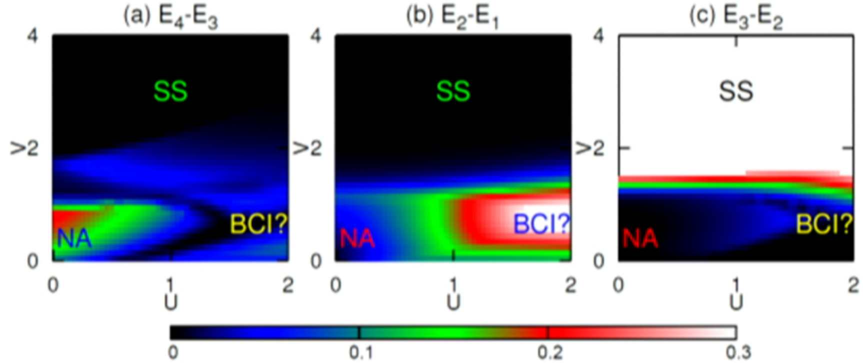


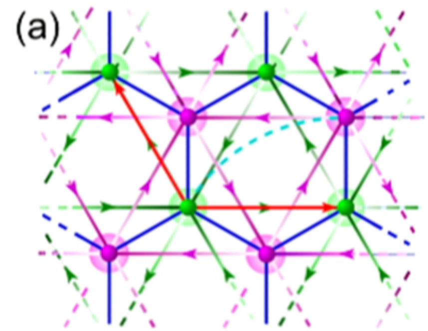
Filling factor 1/4



Non-Abelian State

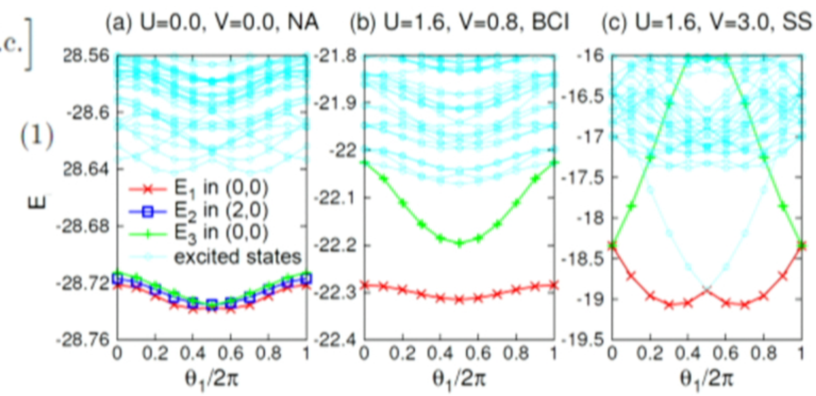
Generalized Haldane model





$$
\begin{aligned}
H = & - t' \sum_{\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle} \left[b^{\dagger}_{\mathbf{r}'} b_{\mathbf{r}} \exp\left(i\phi_{\mathbf{r}'\mathbf{r}}\right) + \mathrm{H.c.} \right] \\
& - t \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \left[b^{\dagger}_{\mathbf{r}'} b_{\mathbf{r}} + \mathrm{H.c.} \right] - t'' \sum_{\langle\langle\langle \mathbf{r}\mathbf{r}' \rangle\rangle\rangle} \left[b^{\dagger}_{\mathbf{r}'} b_{\mathbf{r}} + \mathrm{H.c.} \right] \\
& + \frac{U}{2} \sum_{\mathbf{r}} n_{\mathbf{r}} \left(n_{\mathbf{r}} - 1\right) + V \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} n_{\mathbf{r}} n_{\mathbf{r}'} \qquad (1)
\end{aligned}
$$

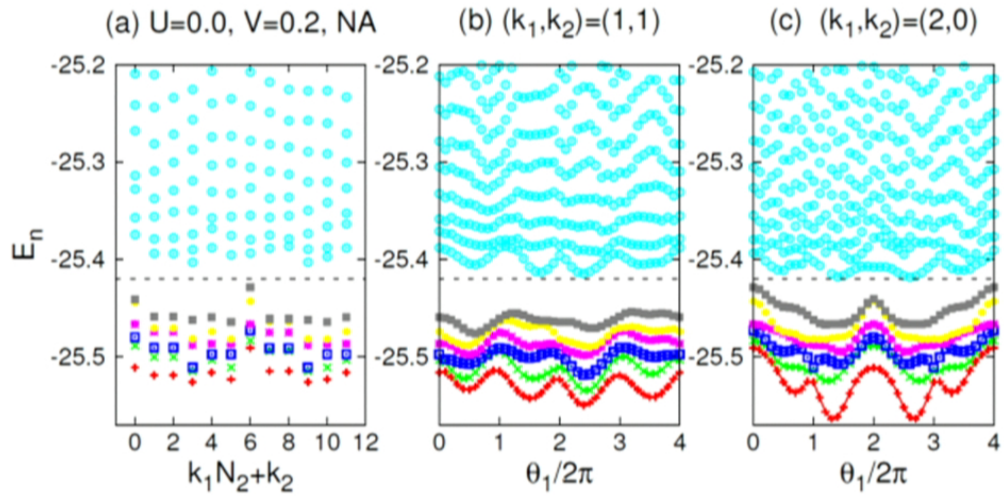
Filling factor 1



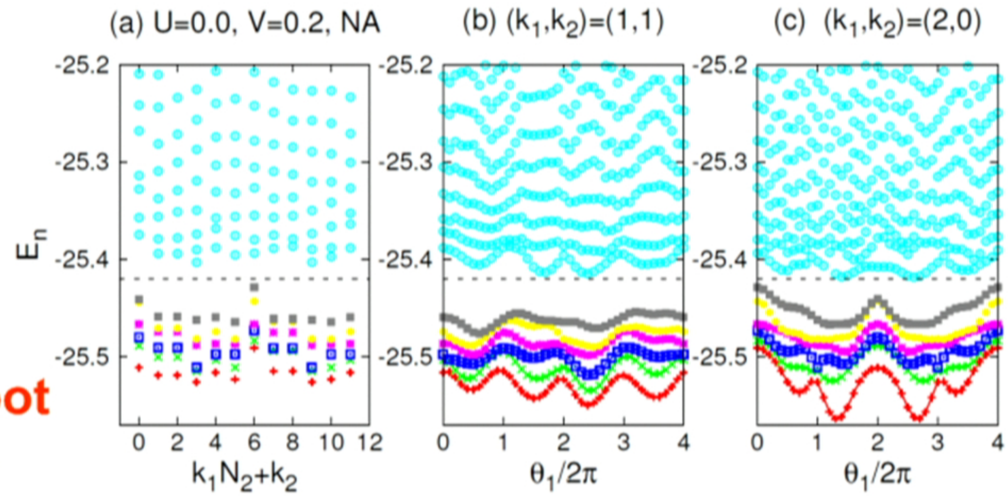
Three-hardcore boson

$$
\left(b^{\dagger}_{\mathbf{r}}\right)^3 = 0 \text{ and } \left(b_{\mathbf{r}}\right)^3 = 0
$$

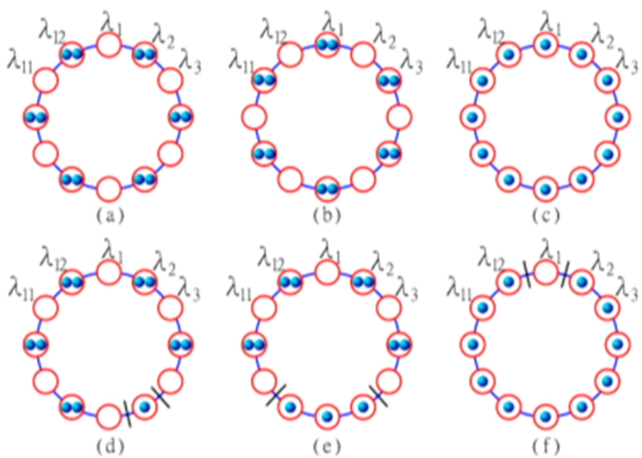
Quasihole spectrum



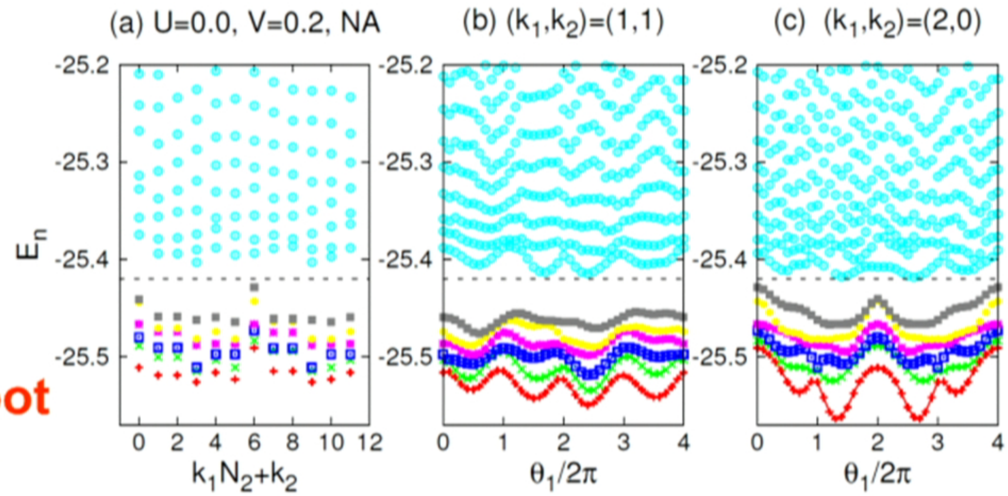
Quasihole spectrum



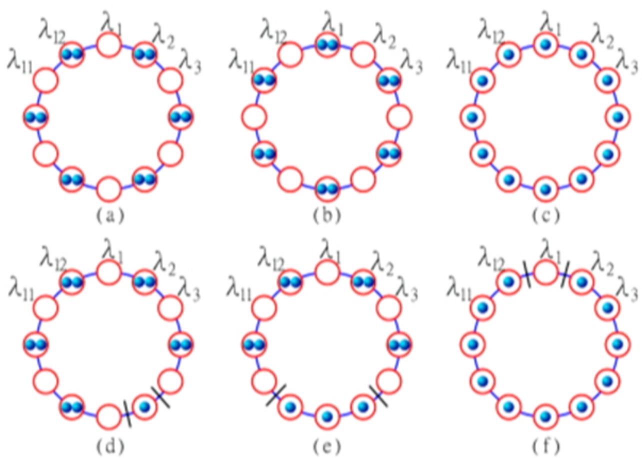
Quasihole counting & root configuration



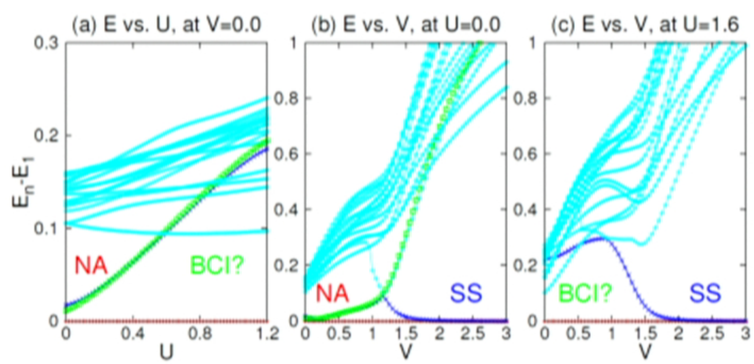
Quasihole spectrum



Quasihole counting & root configuration



Quantum phase transition



Summary and outlook

- Topological flat band models are constructed based on the quadratic touching mechanism.
- FQHE is observed after turning on interactions.
- Possible realization in optical lattice.
- ν=1 bosonic Non-abelian state.
- Can be realized at room temperature!

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- Mechanism.(Varational wave functions.)
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