

Title: New Paths to Unconventional Topological Superconductivity

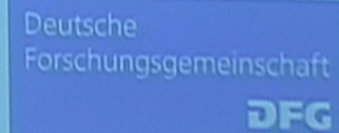
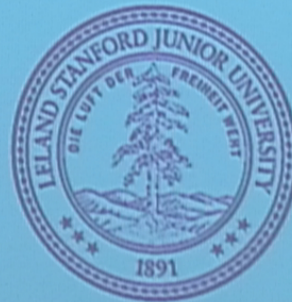
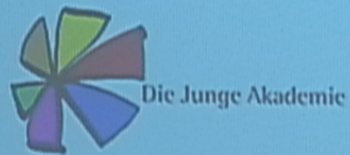
Date: Nov 07, 2011 02:00 PM

URL: <http://pirsa.org/11110078>

Abstract: We report on our recent progress to investigate materials classes exhibiting d+id superconductivity, where topologically nontrivial pairing phases can emerge. Specifically, motivated by recent experimental progress, we show that graphene doped to the van Hove regime can give rise to a plethora of interesting ordering instabilities such as spin density wave and superconductivity. As a function of system parameters such as doping and range of Coulomb interaction, we explain which instability is favored by the system, and analyze the effect of long-range interactions on superconductivity giving rise to a competition between singlet d+id and triplet f wave. We also outline our work in progress for other materials classes which we believe are promising to stabilize such interesting topological superconducting states of matter.

# New paths to unconventional topological superconductivity

Ronny Thomale  
Stanford University

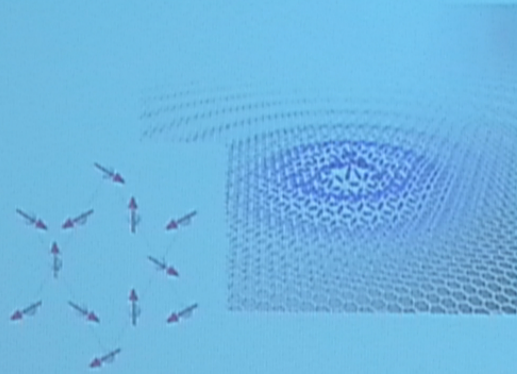


Perimeter Institute, Nov 7<sup>th</sup> 2011

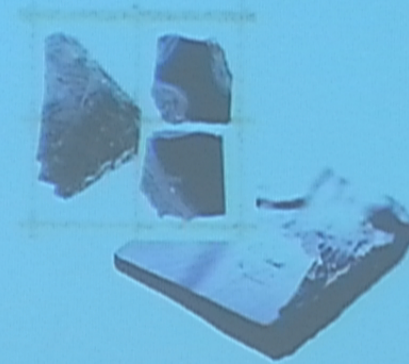


# My current areas of research

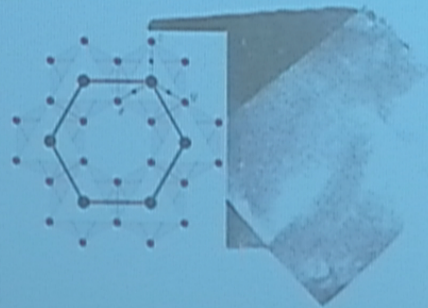
## Frustrated Magnetism



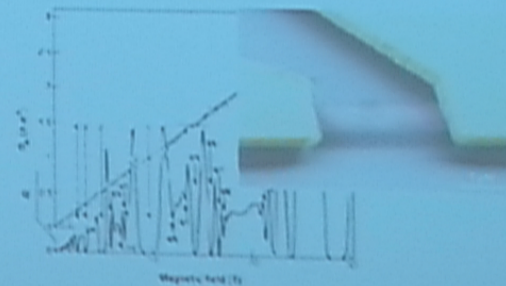
## Superconductivity



## Spin-orbit Phenomena

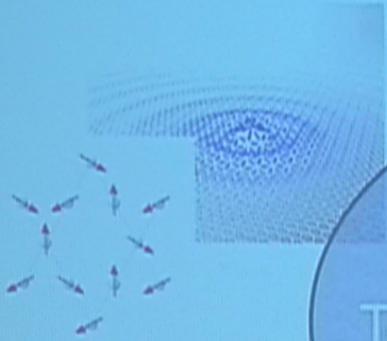


## Quantum Hall Effect

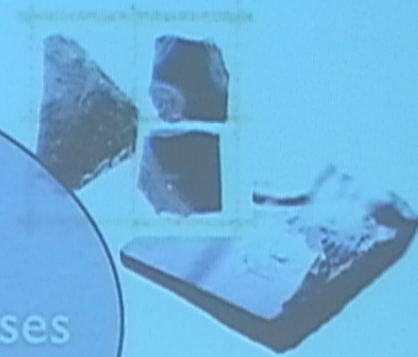




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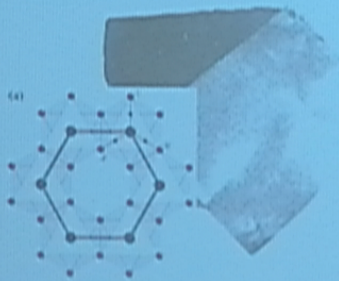


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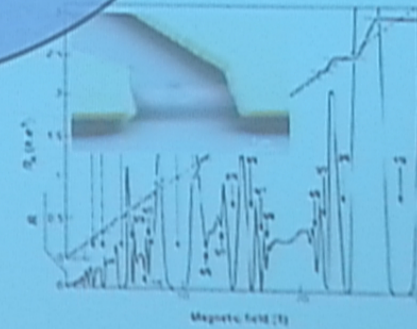


Topological Quantum Phases

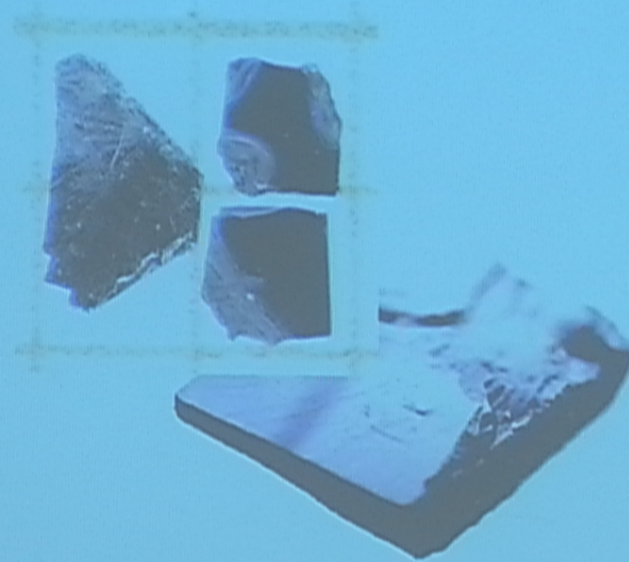
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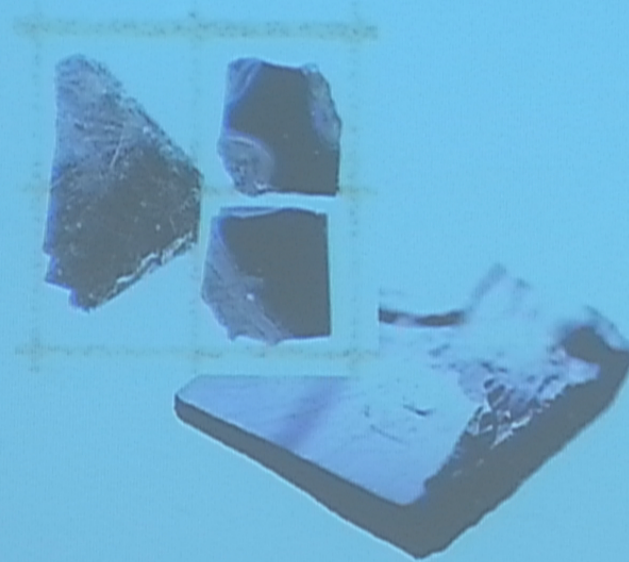


This talk:





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How to find topological superconductivity?

# Collaborators

- C. Platt, M. Kiesel, W. Hanke (Würzburg)
- D.A. Abanin (Harvard, Perimeter)

Thanks for discussions:  
G. Baskaran, S. Kivelson, S. Raghu, X.-L. Qi, S. C. Zhang,  
J. Analytis, I. Fisher



# Outline

- Topological superconductivity – order parameter, edge modes, vortex cores, phase transitions
- Functional renormalization group – from a microscopic material description to collective many-body phases
- Graphene – d+id superconductivity and helical magnetic order at van Hove filling
- Cobaltates – d+id superconductivity and multi-orbital anisotropy



## Superconductivity formalism in a nutshell

Electronic model with a pairing instability mean field:

$$H_{\text{BCS}}^{\text{MF}} = \sum_{\mathbf{k}, \sigma} \xi_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \Delta_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} - \sum_{\mathbf{k}} \Delta_{\mathbf{k}}^* c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}$$

Order parameter of broken  $U(1)/\mathbb{Z}_2$  symmetry:  $\Delta_{\mathbf{k}} \sim \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$

Ground state wave function:  $|\Phi_0\rangle = \prod_{\mathbf{k}} \left( u_{\mathbf{k}} + v_{\mathbf{k}} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} \right) |0\rangle$

Quasiparticle description:

$$H_{\text{BCS}}^{\text{MF}} = \sum_{\mathbf{k}} E_{\mathbf{k}} \gamma_{\mathbf{k}}^{\dagger} \gamma_{\mathbf{k}} \quad E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2} \quad Q_{\gamma, \mathbf{k}} = -e \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}$$







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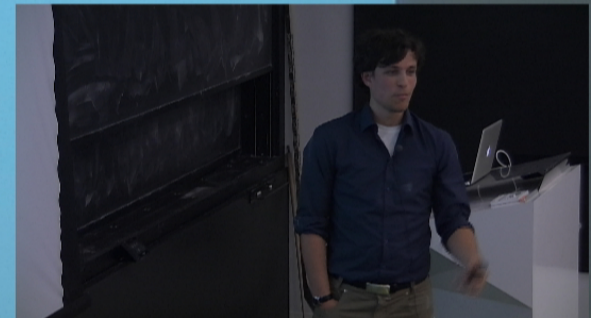
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Charged superfluids possess in general non-local order parameters and are described by topological field theories

$$\phi(\mathbf{r}) := \langle \psi_{\downarrow}(\mathbf{r}) \psi_{\uparrow}(\mathbf{r}) \rangle$$

Hansson, Oganessian, Sondhi, 2004

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Proximity pairing leading to topological phenomena at interfaces

Fu, Kane '09, Sau, Lutchyn et al. '10, Alicea '10



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The band structure  $\xi_{\mathbf{k}}$  in which pairing is induced can be topologically nontrivial

Fu, Berg 2010

The pairing  $\Delta_{\mathbf{k}}$  constitutes a topological phase

Volovik 1976



p+ip superconductivity

$$\Delta_{\mathbf{k}} \sim \hat{z}(k_x \pm ik_y)$$

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Spin triplet pairing order parameter breaks time reversal T and parity P

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Suggested realization in the ruthenate  $\text{Sr}_2\text{RuO}_4$

$$\Phi_0(r_1, \dots, r_N) = \mathcal{A}_{[r_i, r_{i+1}]} f(r_i - r_{i+1})$$

$$f(r) \sim 1/r$$



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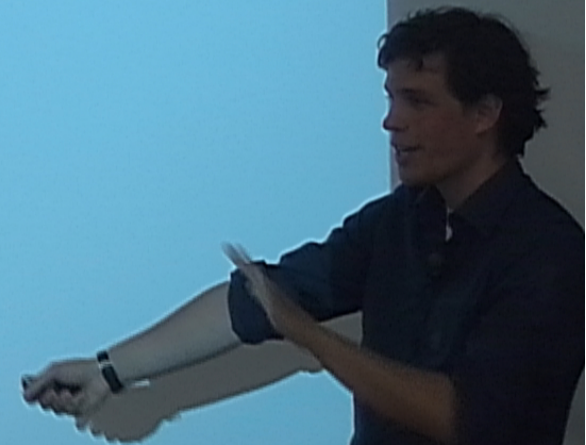
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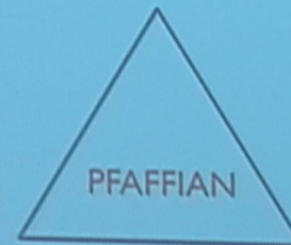
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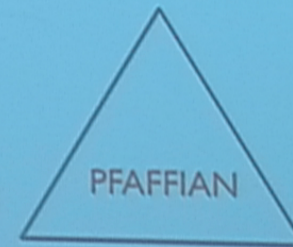
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p+ip SC

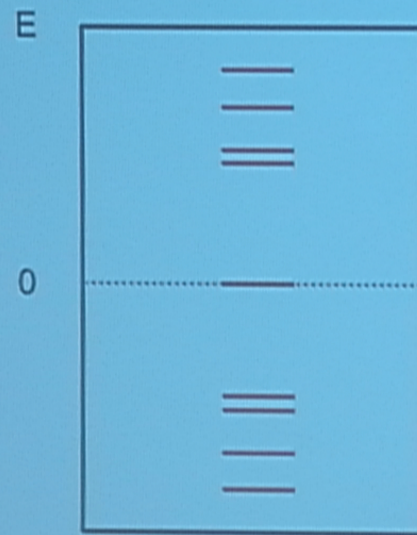
Moore, Read '92, Greiter Wen Wilczek '92

Read, Green '00



# Non-Abelian statistics in the $p+ip$ vortex core

D.A. Ivanov '01



$$[H, \gamma] = E\gamma$$

$$[H, \gamma^\dagger] = -E\gamma^\dagger$$

$$\gamma = \gamma^\dagger$$

Majorana zero mode  
in the vortex core

Braiding of vortices induces non-commutative rotations in the internal Hilbert space of Majorana zero modes



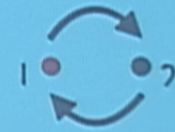
# Topological quantum computation

Non-Abelian statistics: action on topological ground state manifold  $|\psi_a\rangle$

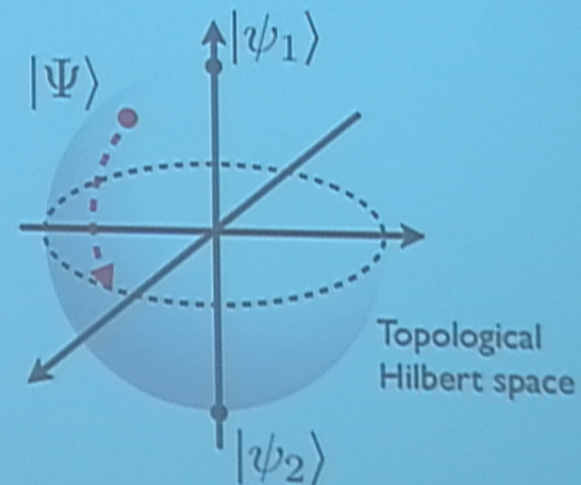
$$|\psi_a\rangle = \mathcal{W}_{ab}^{12} |\psi_b\rangle$$

$$[\mathcal{W}^{ij}, \mathcal{W}^{kl}] \neq 0$$

Freedman, 2003



Braiding of particles: rotation in the ground state sector (topological qubit)





## d+id superconductivity

$$\Delta_{\mathbf{k}} = \cos k_x - \cos k_y \pm i \sin k_x \sin k_y$$

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Two edge mode branches giving rise to quantized thermal Hall conductance and quantized spin Hall conductance

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Connection between d+id and S=1/2 chiral spin liquids and spin singlet quantum Hall states

Affleck, Zou, Hsu, Anderson '88

Wu, Wen, Hatsugai '94

Thomale, Schroeter, Kapit, Greiter '08



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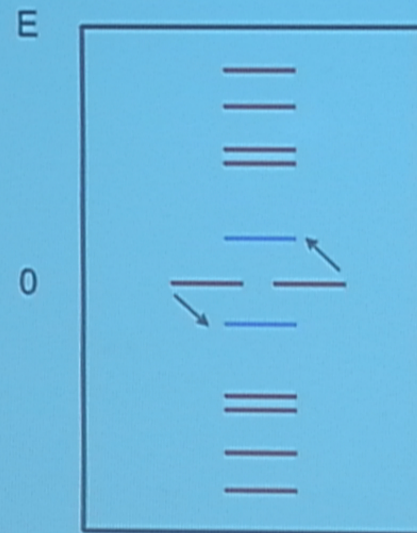
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# Non-Abelian statistics in the $d+id$ vortices?

Sato, Takahashi, Fujimoto 2010



even number of modes  
in the vortex core: no  
topological protection

Claim: Addition of Rashba coupling and Zeeman field can drive a topological phase transition from TKNN 2 to 1

Advantage  $d+id$ : only a small Zeeman field needed



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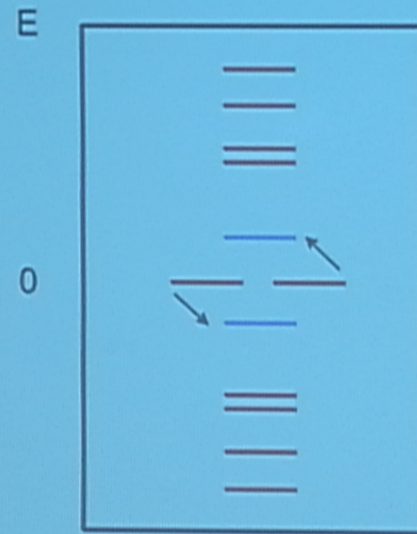
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# Recipe for finding X+iX SC order

Platt, Thomale, in prep.

The favored real space pairing function  $\Delta_{i,j} \sim \langle c_i^\dagger c_j^\dagger \rangle$   
 transforms under irreducible point group representations

Layered square lattice (tetragonal)

$C_{4v}$	$E$	$C_2$	$2C_4$	$2\sigma_v$	$2\sigma_d$
$A_1$	1	1	1	1	1
$A_2$	1	1	1	-1	-1
$B_1$	1	1	-1	1	-1
$B_2$	1	1	-1	-1	1
$E$	2	-2	0	0	0

px, py

Layered honeycomb lattice (hexagonal)

$C_{6v}$	$E$	$C_2$	$2C_3$	$2C_6$	$3\sigma_v$	$3\sigma_d$
$A_1$	1	1	1	1	1	1
$A_2$	1	1	1	1	-1	-1
$B_1$	1	-1	1	-1	1	-1
$B_2$	1	-1	1	-1	-1	1
$E_1$	2	-2	-1	1	0	0
$E_2$	2	2	-1	-1	0	0

d(x<sup>2</sup>-y<sup>2</sup>), d(xy)

When different separately nodal SC solutions are degenerate, the system gains condensation energy by forming X+iX



# Recipe for finding $d+id$ SC order

Kiesel, Platt, Hanke, Thomale, in prep.

Choose a system with a triangular or honeycomb lattice and a sufficiently large single pocket Fermi surface

Achieve an electronically driven pairing mechanism through spin fluctuations

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Dope the system away from magnetically favorable nesting or adjust the interaction/bandwidth ratio (pressure)

# Functional renormalization group



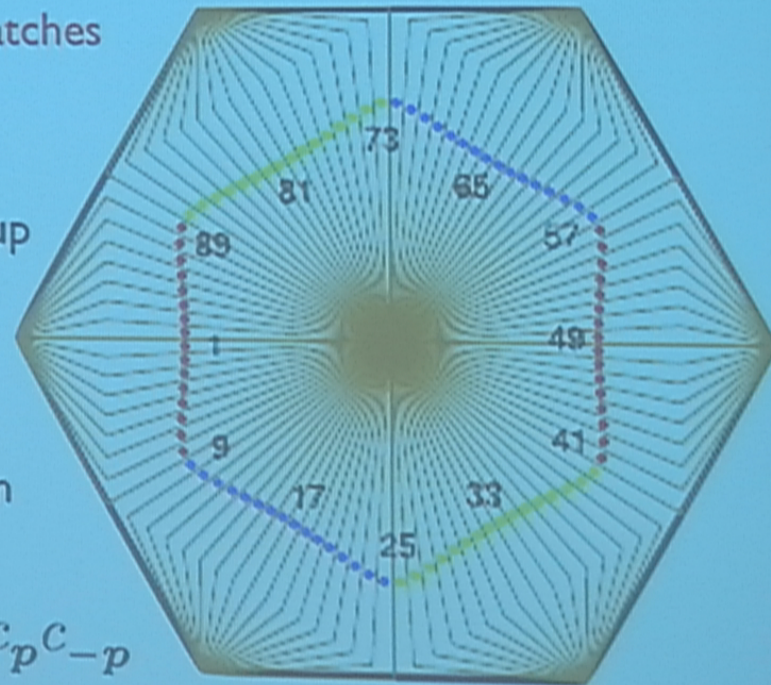
# SC instability in the FRG

Zanchi Schulz '99; Honerkamp, Salmhofer, Rice '01; Wang et al '09; Thomale et al '09

Discretize the BZ into different patches along the Fermi surface

Integrate out high energy modes up to  $\Lambda$  and approach the FS

If the pairing channel diverges, extract the leading SC gap function



$$V(\mathbf{k}, -\mathbf{k}, \mathbf{p}, -\mathbf{p}, \Lambda) c_{\mathbf{k}}^{\dagger} c_{-\mathbf{k}}^{\dagger} c_{\mathbf{p}} c_{-\mathbf{p}}$$

$$V(\mathbf{k}, \mathbf{p}, \Lambda) = \sum_n c_n(\Lambda) \Delta_{n,\mathbf{k}} \Delta_{n,\mathbf{p}}$$



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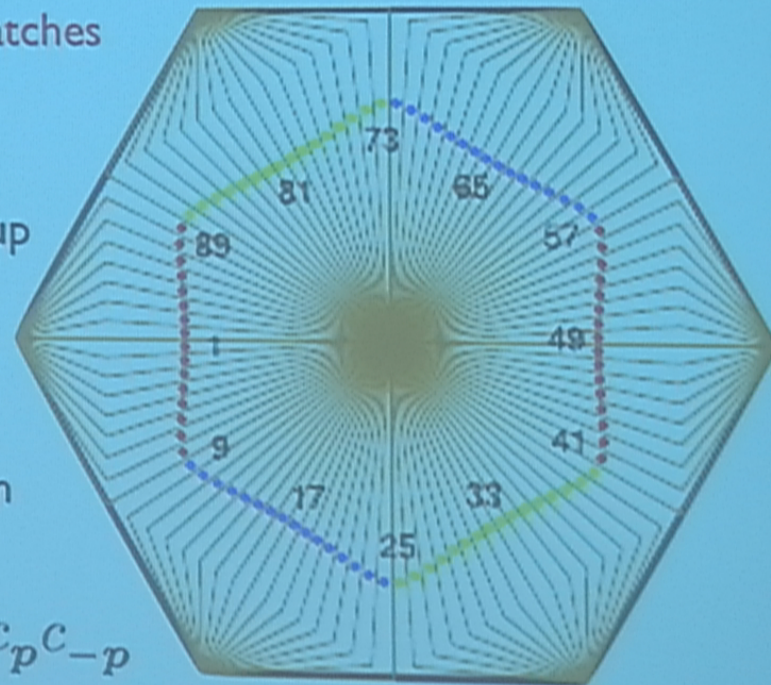
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# Superconductivity in Graphene

Graphene is doped to a van Hove singularity where many body interactions are enhanced due to divergent DOS

Baskaran '02, Gonzalez '08, Nandkishore et al. '11, Kiesel, et al. '11, Wang et al. '11

Disorder-reduced doping is experimentally accomplished by a potassium superlattice

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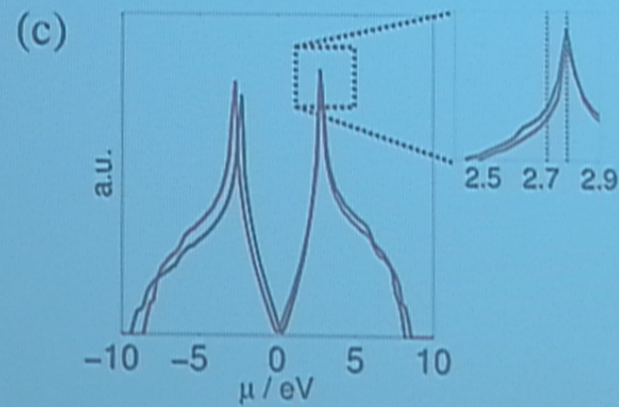
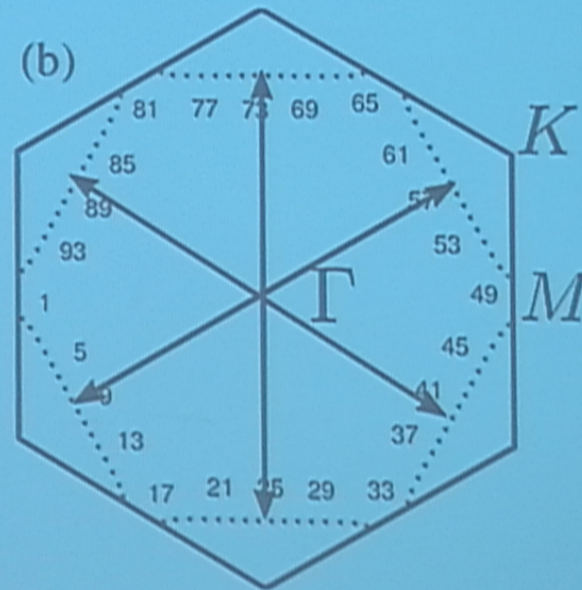
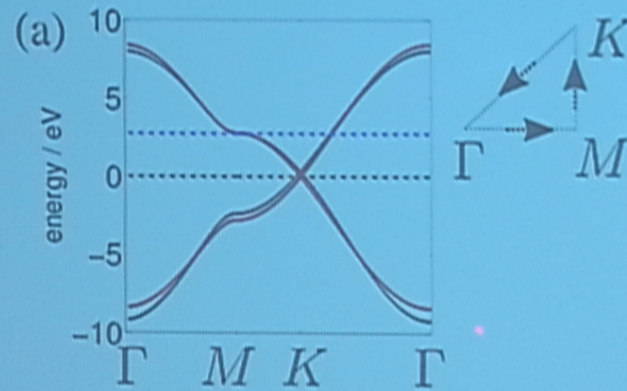
A variety of competing many body phases is found with high tunability via kinetic and interaction parameters

Kiesel, et al. '11



# Graphene - band structure

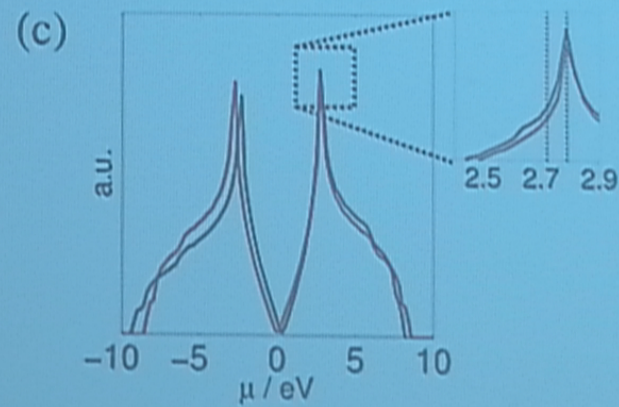
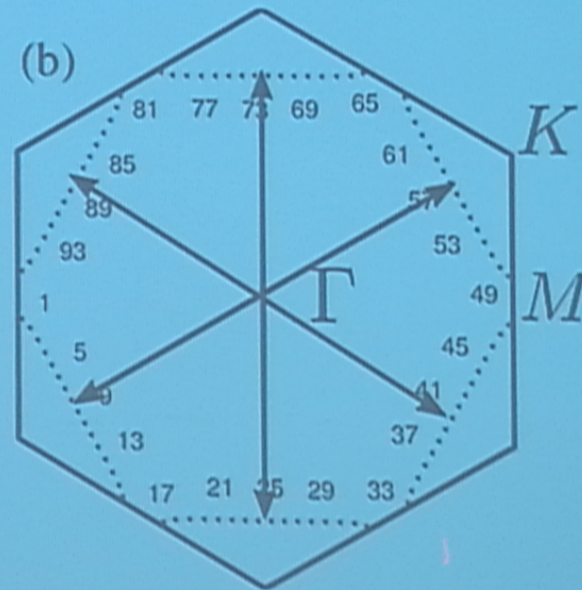
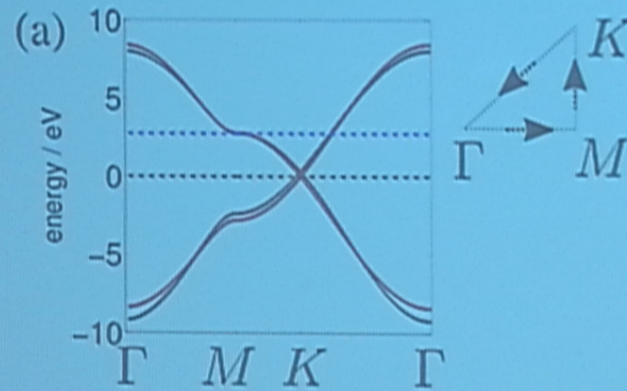
Kiesel, Platt, Hanke, Abanin, Thomale '11





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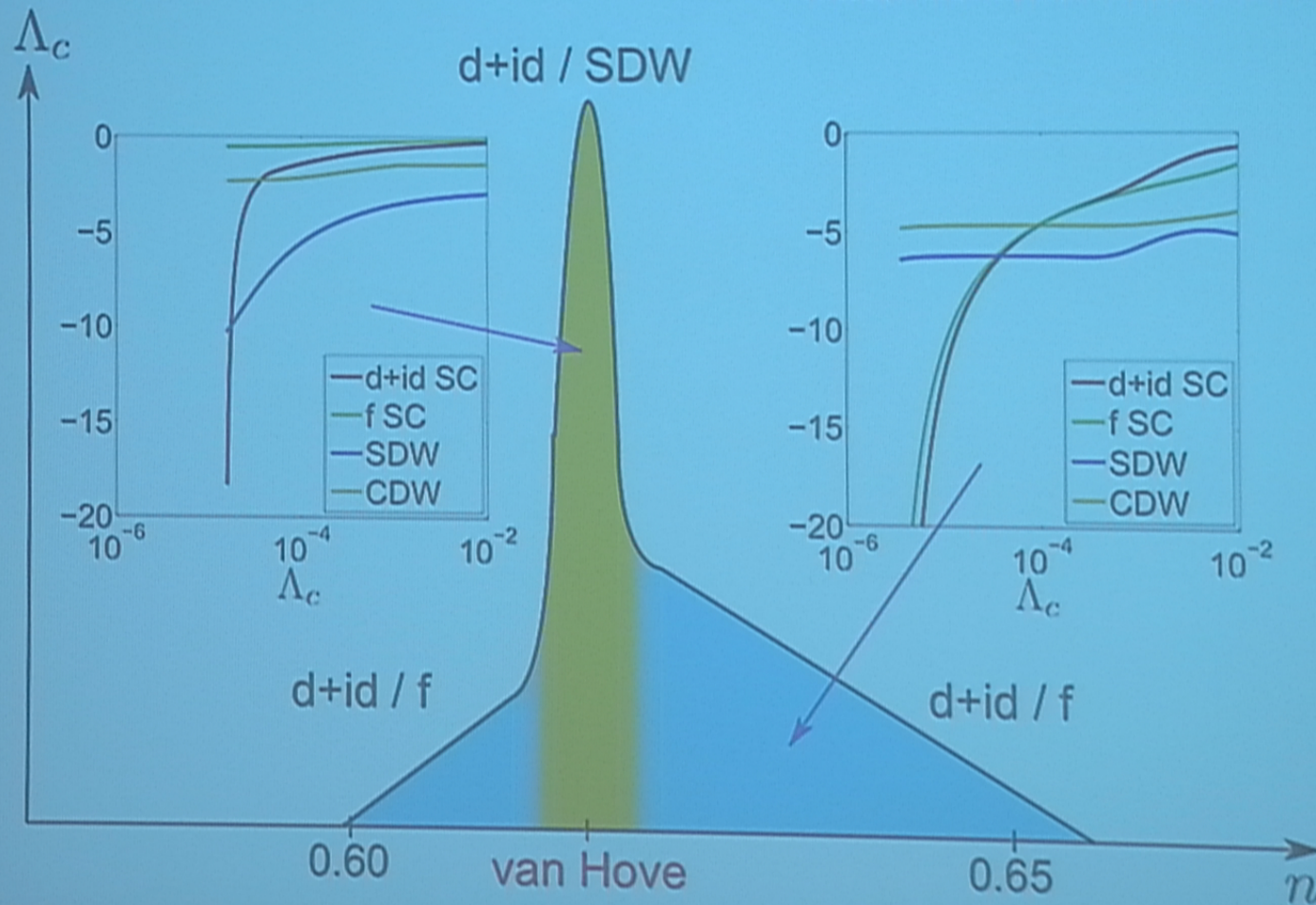
Kiesel, Platt, Hanke, Abanin, Thomale '11





# Graphene - phase diagram

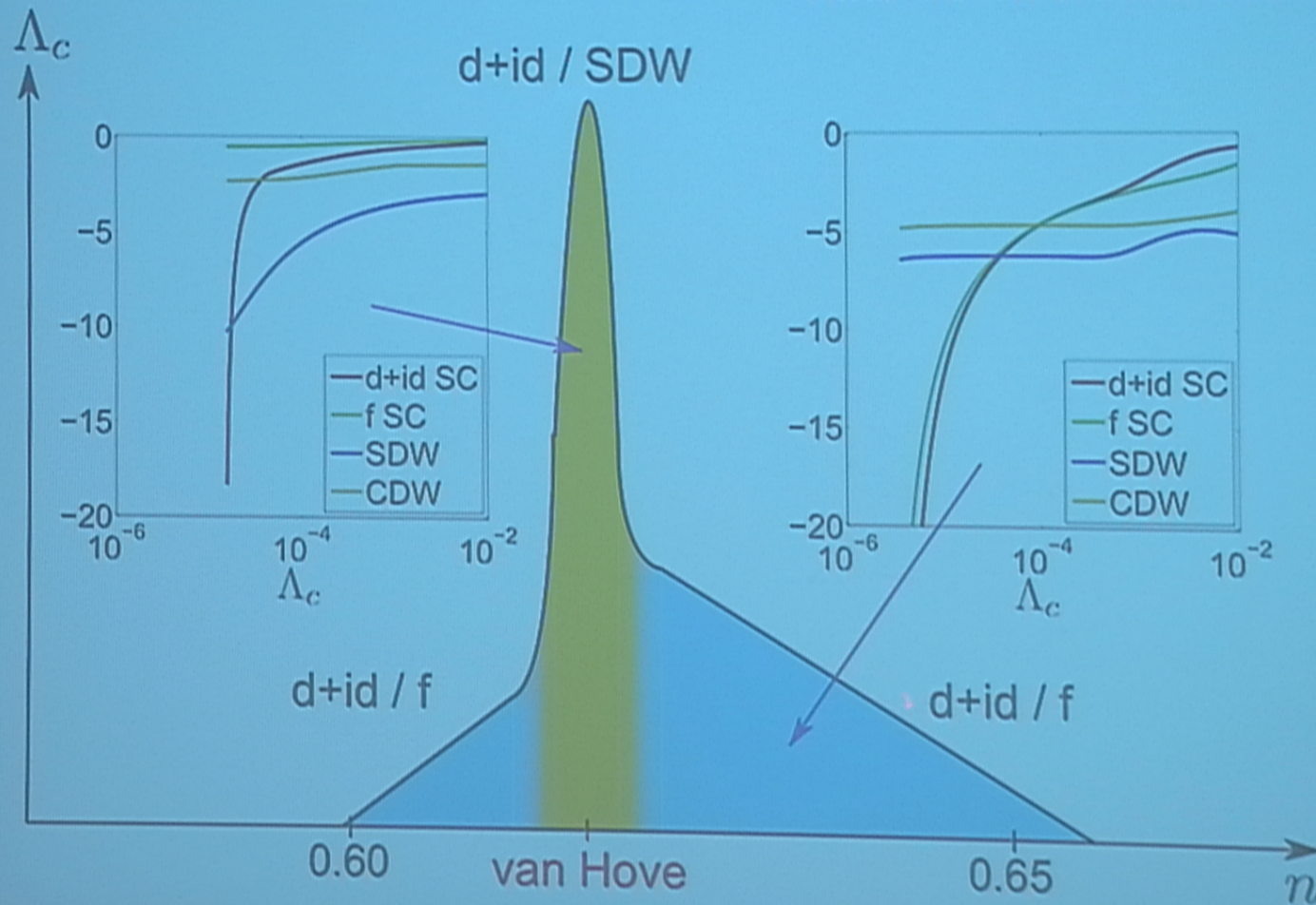
Kiesel, Platt, Hanke, Abanin, Thomale '11





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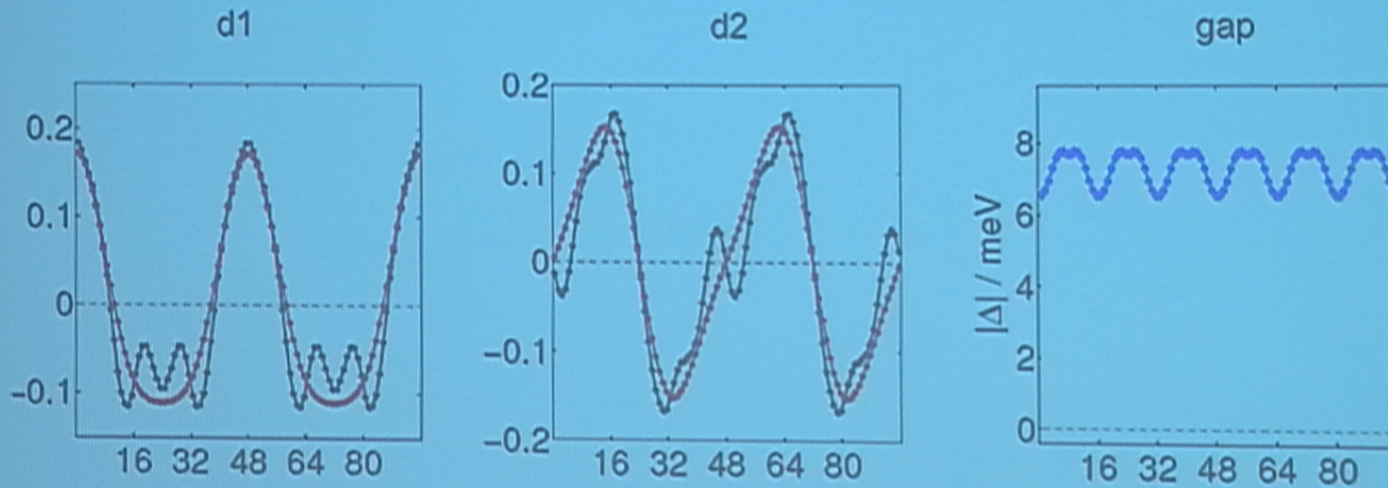
Kiesel, Platt, Hanke, Abanin, Thomale '11





# Graphene - d+id superconductivity

Kiesel, Platt, Hanke, Abanin, Thomale '11



Free energy functional analysis: d+id is favored by the system, as also seen by a fully gapped Fermi surface

Gap anisotropy and competition to SDW and f-wave depends on details of doping, interaction, and kinetic parameters



# Superconductivity in the Cobaltates

Kiesel, Platt, Hanke, Thomale, in prep.

$T_c=5\text{K}$  superconductivity is found at  $x=0.3$  in the Cobaltates  
with water intercalation to separate the triangular Co layers

Takada et al. '03; Baskaran '03



# Superconductivity in the Cobaltates

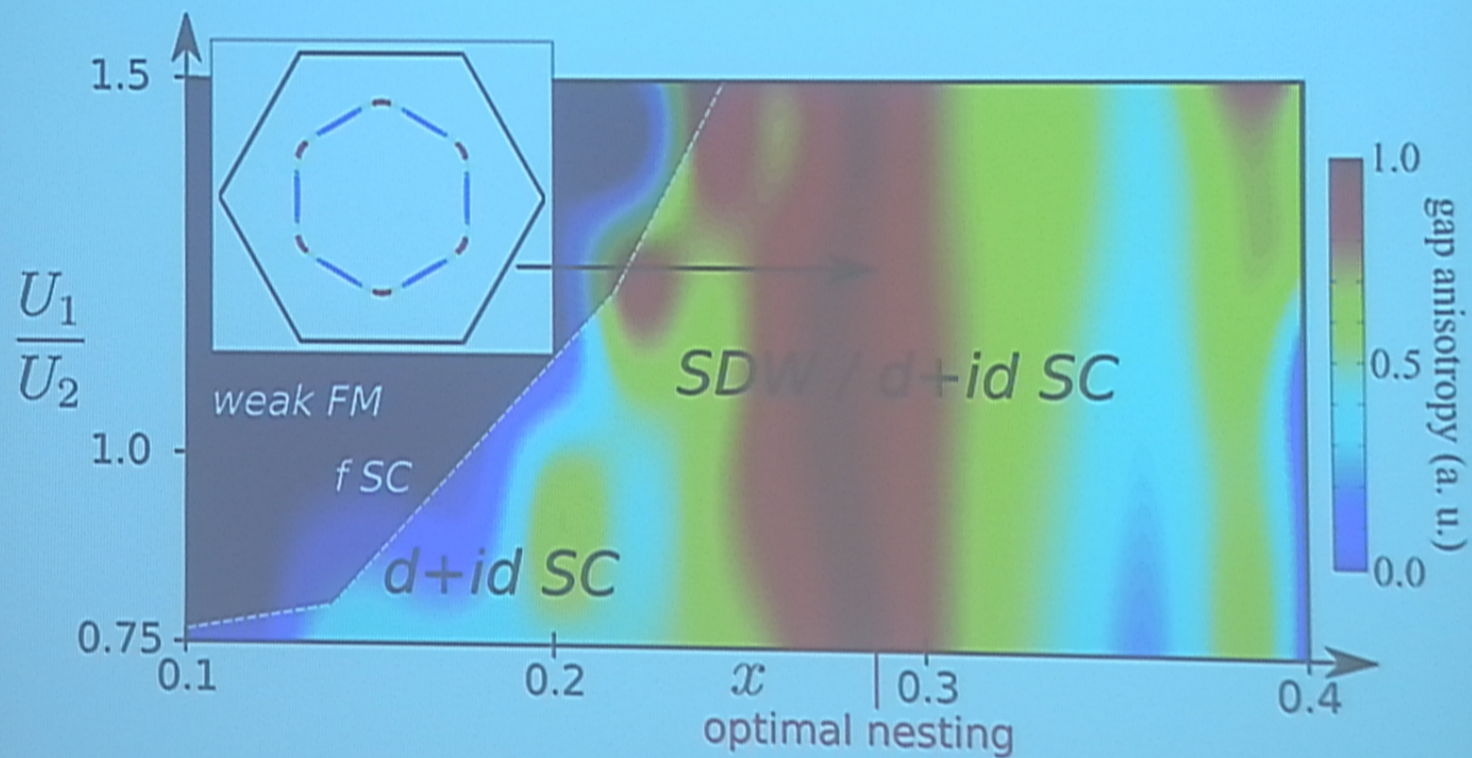
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## Anisotropy profile of the d+id phase





# Summary

Detailed knowledge about the topological phase helps to design a material investigation plan for topological superconductors



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Detailed knowledge about the topological phase helps to design a material investigation plan for topological superconductors

Aside from chiral triplet SC, chiral singlet SC promises intricate topological properties that can also be driven into the triplet SC universality class

$d+id$  superconductivity is predicted for (i) graphene doped to van Hove filling and (ii) water-intercalated cobaltates where the controversy on the nodal gap is resolved by a large order parameter anisotropy

Exotic singlet superconductivity can be realized in a large class of materials and promises new discoveries of potentially topological unconventional superconductors



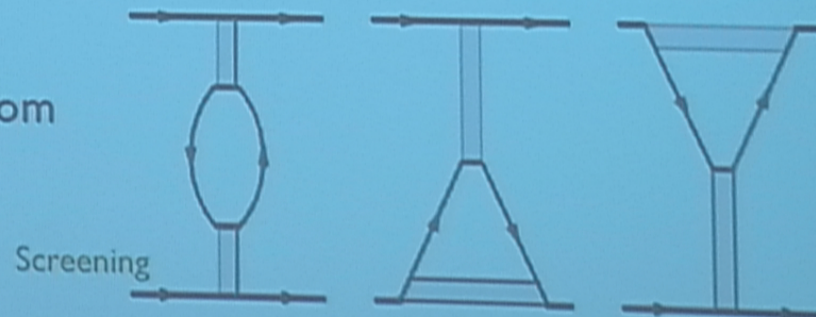
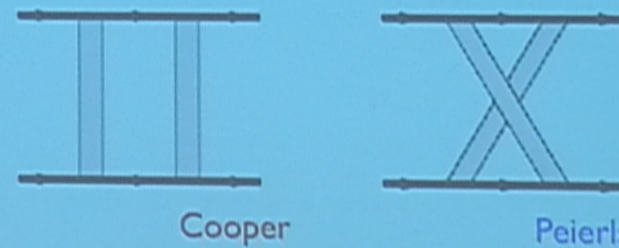
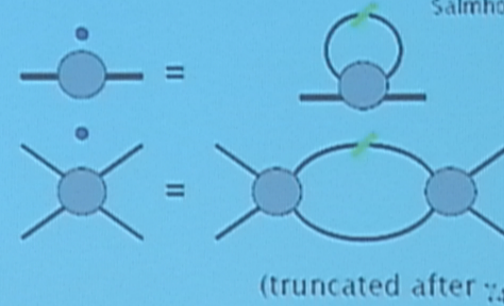
# Investigate Fermi instabilities in electronic systems

- Polchinski's flow equations: hierarchy of m-point functions
- Infinitesimal steps of mode integration via cutoff flow  $\Lambda$ : FRG solution of fermionic action

$$Z = \int \mathcal{D}\bar{\Psi} \mathcal{D}\Psi e^{S\{\Psi\}}$$

- Consider diagrams contributing to 4 point function  $V(k_1, k_2, k_3, k_4)$ : 2<sup>nd</sup> order plaquet terms
- Asymptotically exact starting from bare weak coupling
- Kohn-Luttinger physics

Wetterich 1993  
Salmhofer 1998





## d+id superconductivity

$$\Delta_{\mathbf{k}} = \cos k_x - \cos k_y \pm i \sin k_x \sin k_y$$

Spin singlet pairing order parameter breaks time reversal T and parity P

Two edge mode branches giving rise to quantized thermal Hall conductance and quantized spin Hall conductance

Fisher, Senthil, Ludwig '98



