Title: Information Flow in Computation, Logic and Physics

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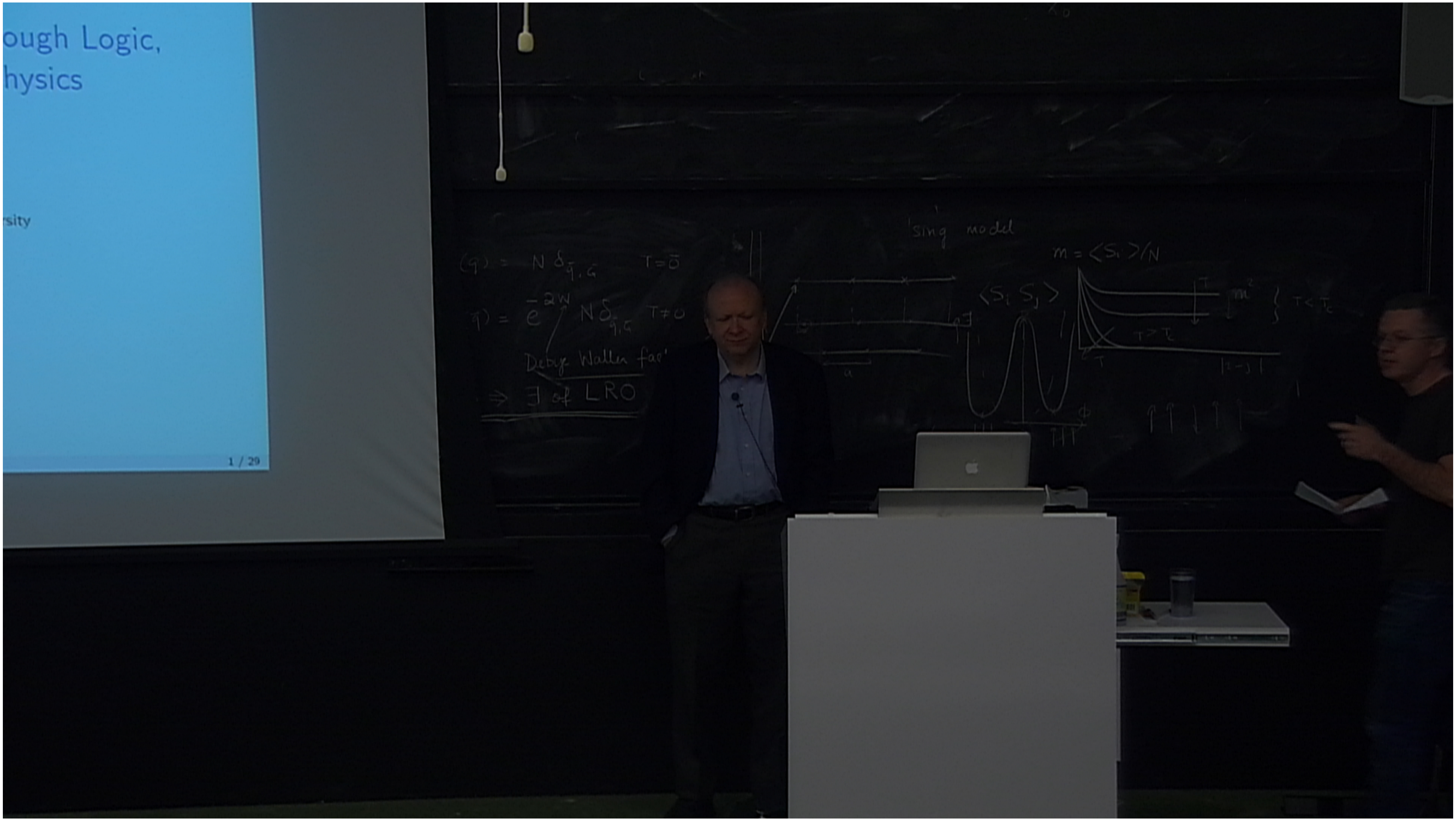
Abstract: Ideas about information are pervasive, yet the fundamental nature and structure of information - if indeed it has one! - remains elusive. Work done from many different perspectives, including those of physics, biology, logic, computer science, statistics, and game and decision theory, has yielded insights into various aspects of information. Could there be a comprehensive, unified theory? We shall chart a particular path, focusing on the idea of *information flow*, and show how common mathematical structures arise in the description of information flow in computer science, logic and quantum information.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Information Flow: tracing a path through Logic, Computation, Topology and Physics

Samson Abramsky

Department of Computer Science, Oxford University

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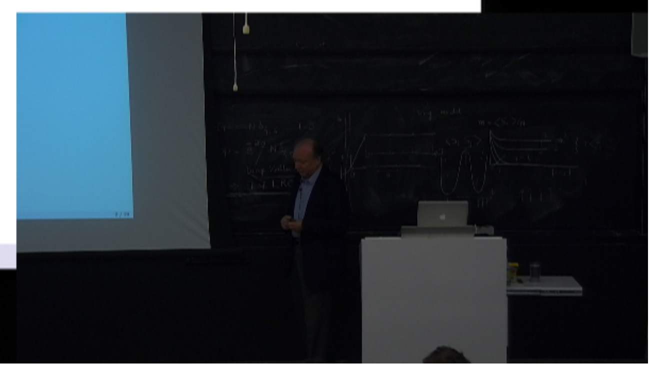

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(Sceptics: If it has these things!)

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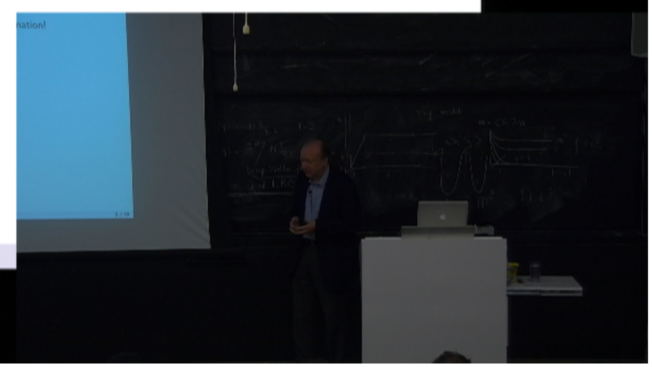

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Sceptic: didn't Shannon give the definitive approach?

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- Information gain is relative to *subsystems* (or *agents*).
- The dynamics of computation (and language, cognition, etc.) is about *information flow* to and from these sub-systems/agents.

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k-th approximation gives us the first k values. The *limit of the increasing sequence of approximations* gives us the whole thing — which is the *least fixpoint* of the associated functional.

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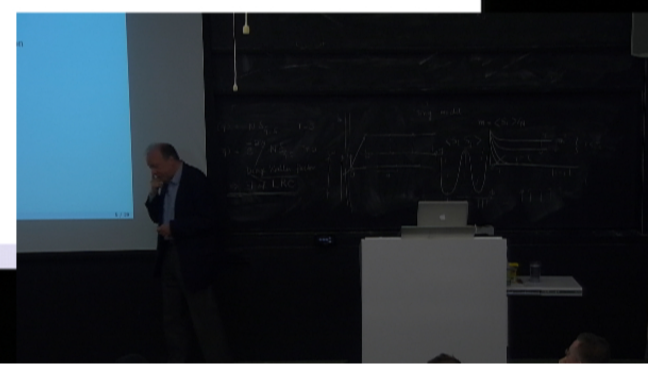

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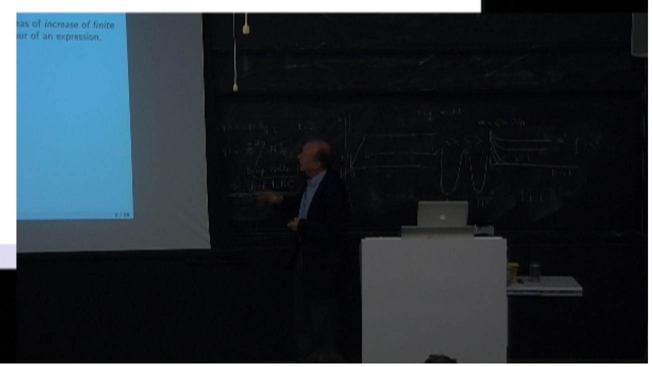

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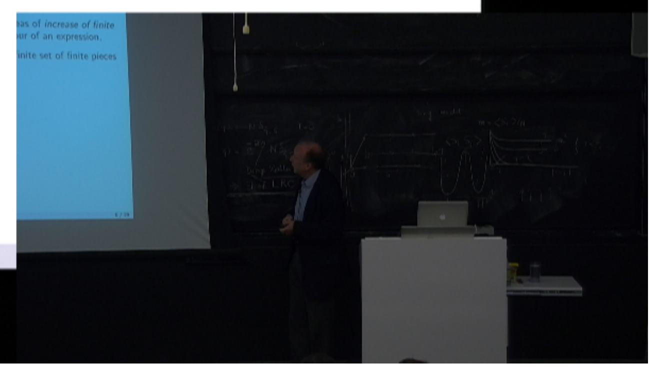

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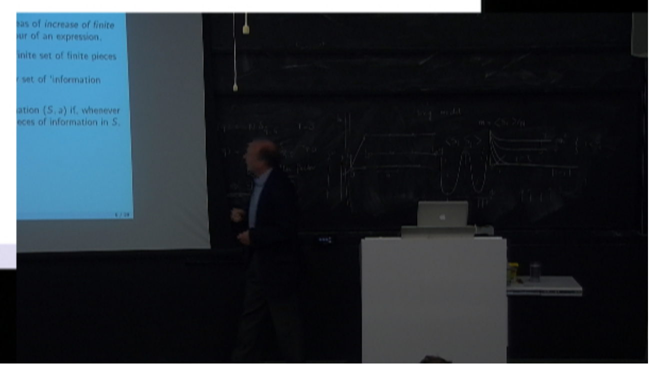

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It works! Gives a consistent model of the calculus, self-application, fixpoints, etc.

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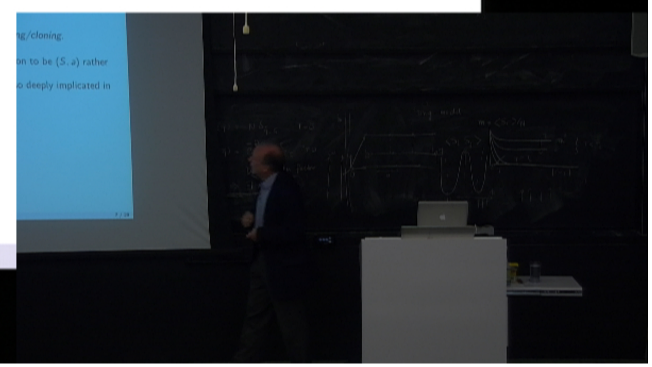

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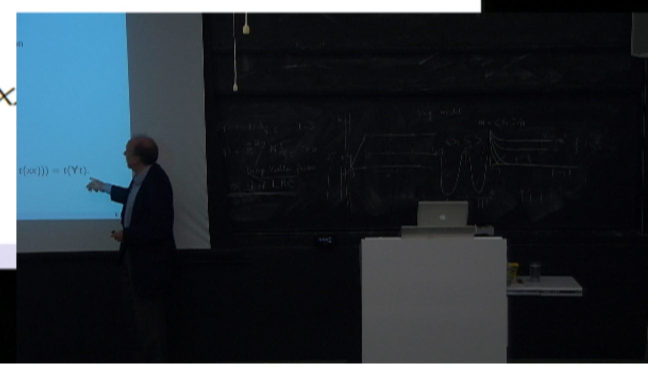

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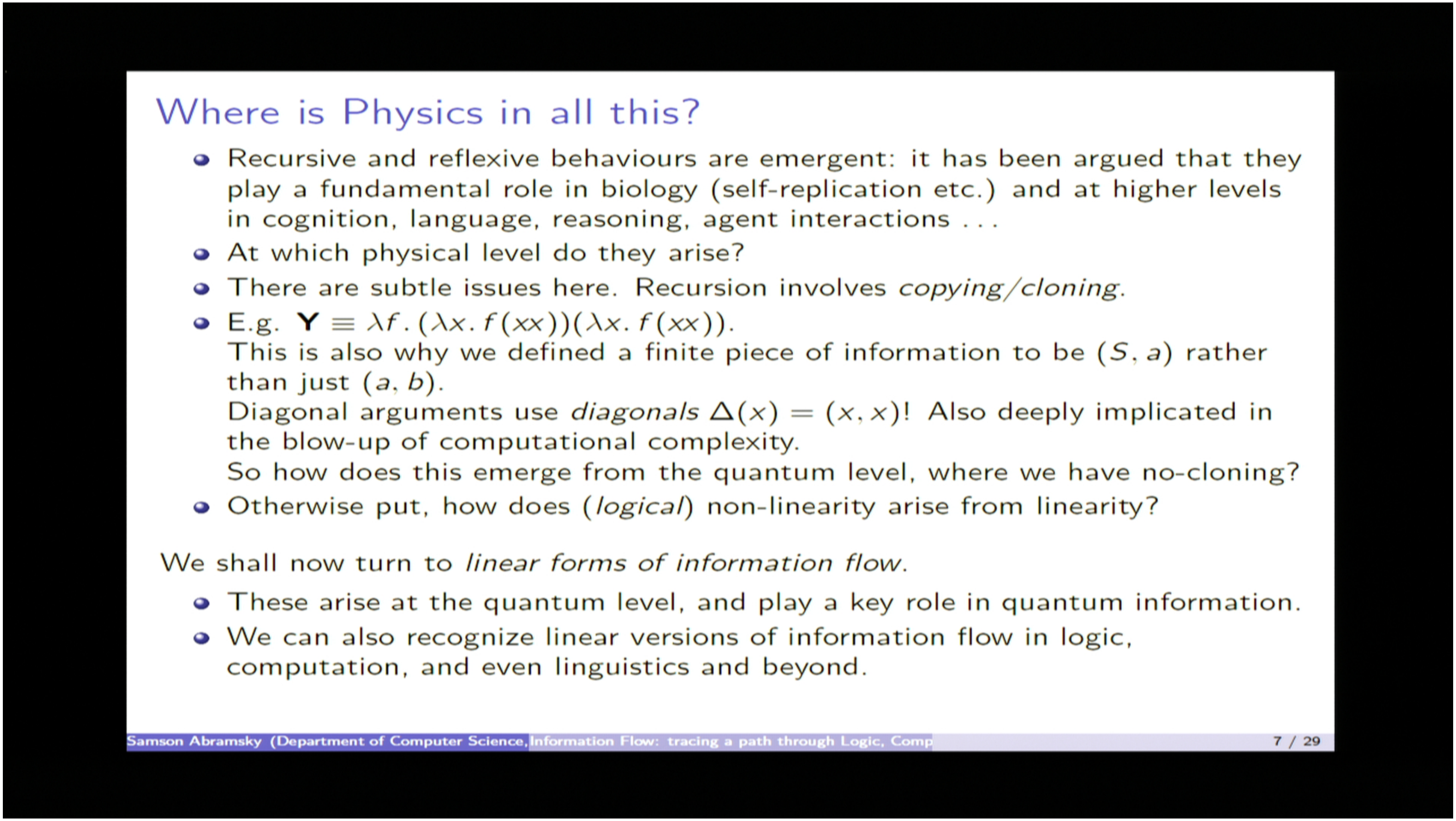
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At which physical level do they arise?
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Diagonal arguments use diagonals $\Delta(x) = (x, x)$! Also deeply implicated in the blow-up of computational complexity.
So how does this emerge from the quantum level, where we have no-cloning?
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These arise at the quantum level, and play a key role in quantum information.
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Samson Abramsky (Department of Computer Science, Information Flow: tracing a path through Logic, Comp
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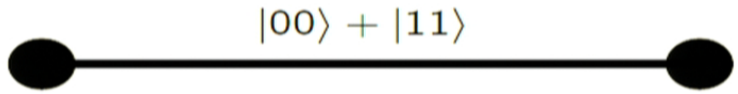
Quantum Entanglement

Bell state:



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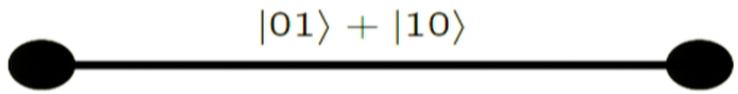

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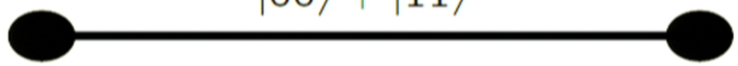

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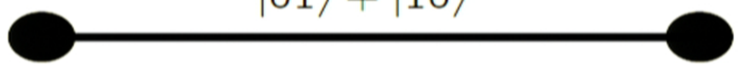

Compound systems are represented by *tensor product*: $\mathcal{H}_1 \otimes \mathcal{H}_2$. Typical element:

$$\sum_i \lambda_i \cdot \phi_i \otimes \psi_i$$

Superposition encodes *correlation*.

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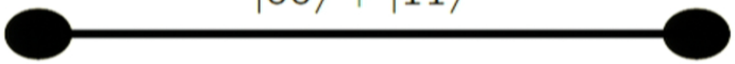

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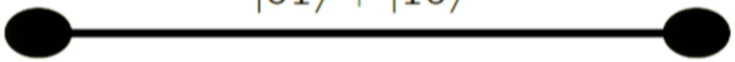

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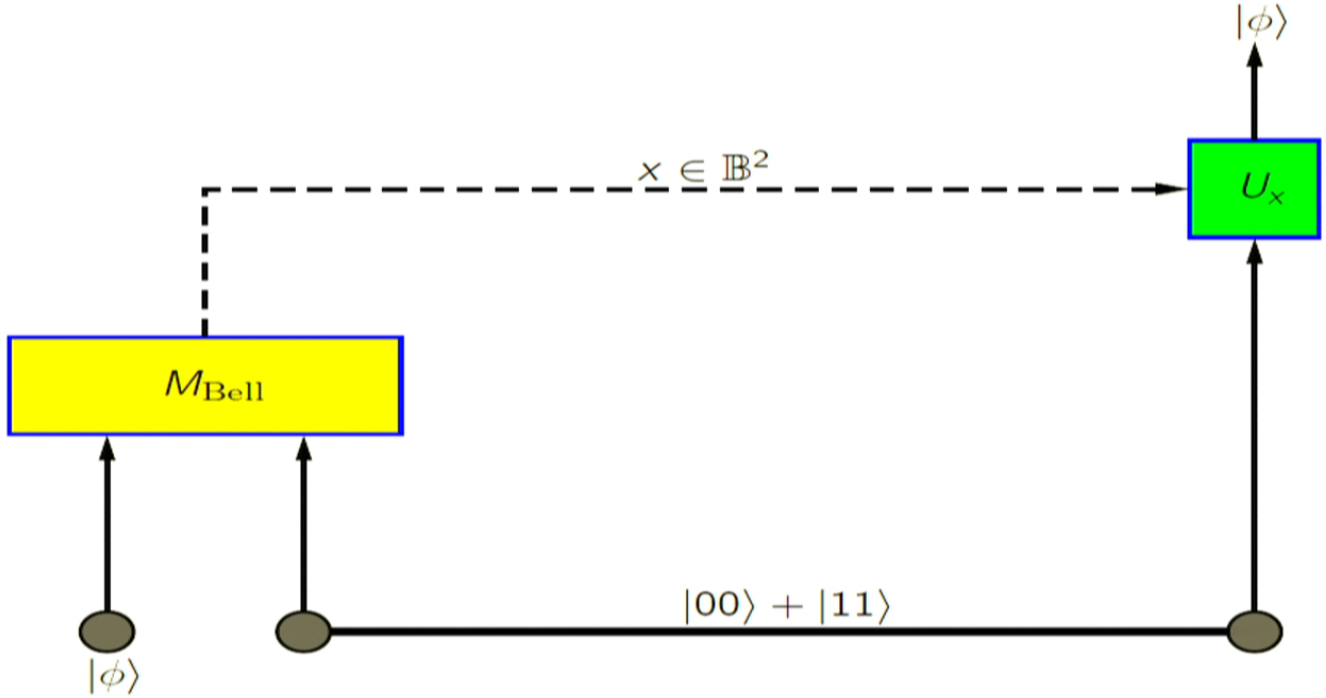
Einstein's 'spooky action at a distance'. Even if the particles are spatially separated, measuring one has an effect on the state of the other.

Bell's theorem: QM is *essentially non-local*.

From 'paradox' to 'feature': Teleportation



Entangled states as linear maps

$\mathcal{H}_1 \otimes \mathcal{H}_2$ is spanned by

$$\begin{matrix} |11\rangle & \cdots & |1m\rangle \\ \vdots & \ddots & \vdots \\ |n1\rangle & \cdots & |nm\rangle \end{matrix}$$

hence

$$\sum_{i,j} \alpha_{ij} |ij\rangle \quad \longleftrightarrow \quad \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{1m} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{nm} \end{pmatrix} \quad \longleftrightarrow \quad |i\rangle \mapsto \sum_{j} \alpha_{ij} |j\rangle$$

Pairs $|\psi_1, \psi_2\rangle$ are a special case — $|ij\rangle$ in a well-chosen basis.

This is **Map-State Duality**.

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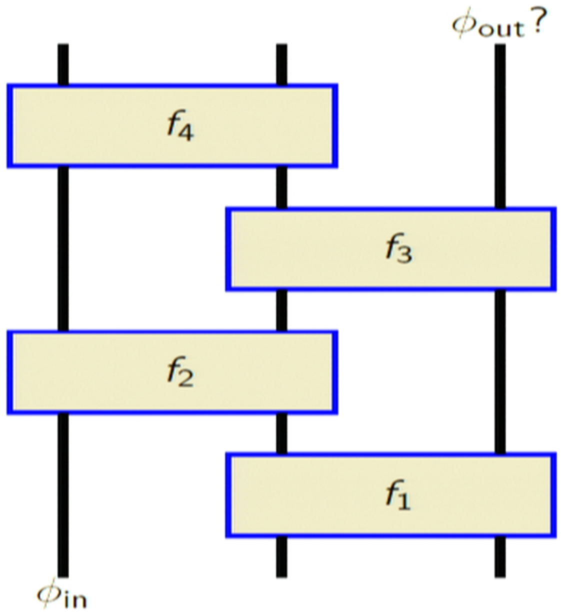
Notation. Given a linear map $f : \mathcal{H} \to \mathcal{H}$, we write $\mathbf{P}_f$ for the projector on $\mathcal{H} \otimes \mathcal{H}$ determined by the vector corresponding to f under Map-State duality.

What is the output?



$$(\mathbf{P}_{f_4} \otimes 1) \circ (1 \otimes \mathbf{P}_{f_3}) \circ (\mathbf{P}_{f_2} \otimes 1) \circ (1 \otimes \mathbf{P}_{f_1}) : \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \longrightarrow \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$$

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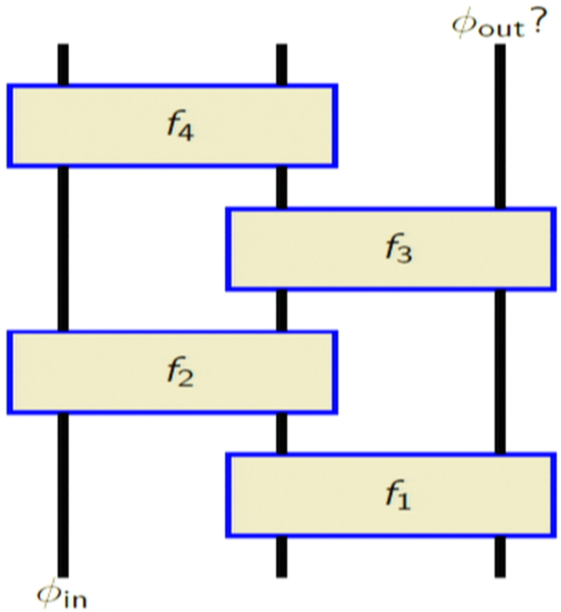

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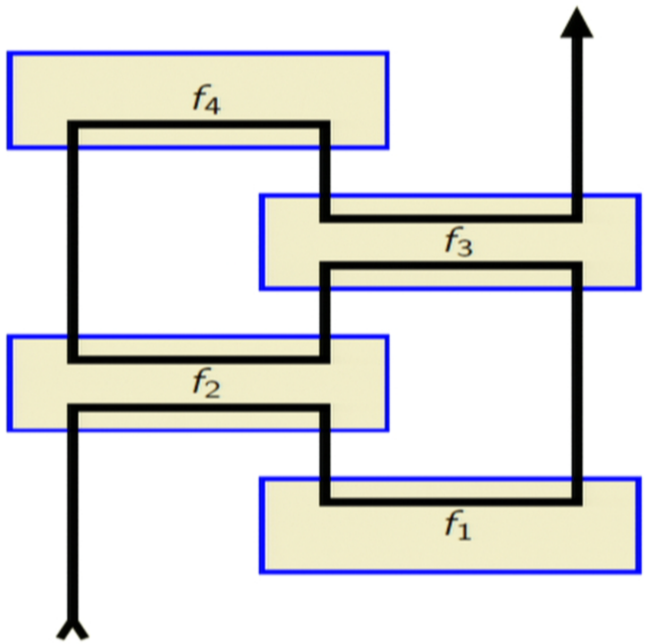
$$\phi_{out} = f_3 \circ f_4 \circ f_2^\dagger \circ f_3^\dagger \circ f_1 \circ f_2(\phi_{in})$$

Follow the line!



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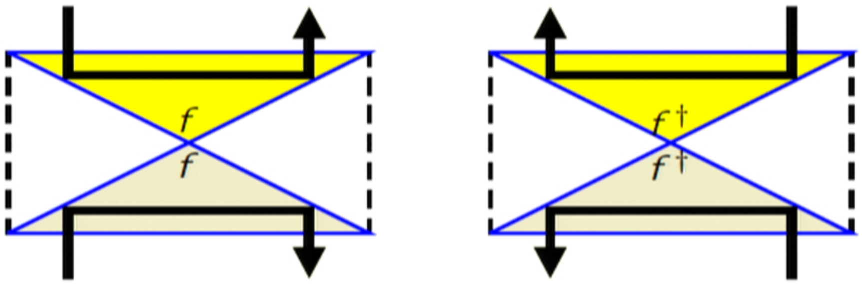
Bipartite Projectors

Information flow in entangled states can be captured mathematically by the isomorphism

$$\mathbf{Hom}(A, B) \cong A^* \otimes B.$$

This leads to a *decomposition* of bipartite projectors into "names" (preparations) and "conames" (measurements).

In graphical notation:



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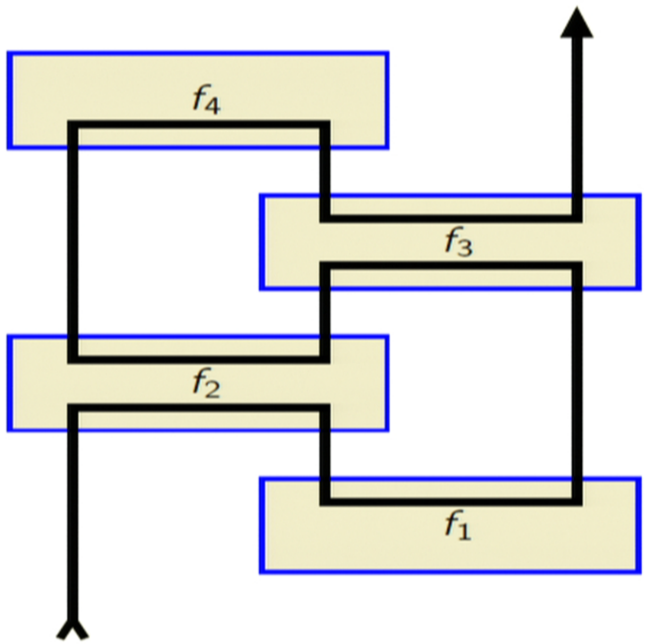

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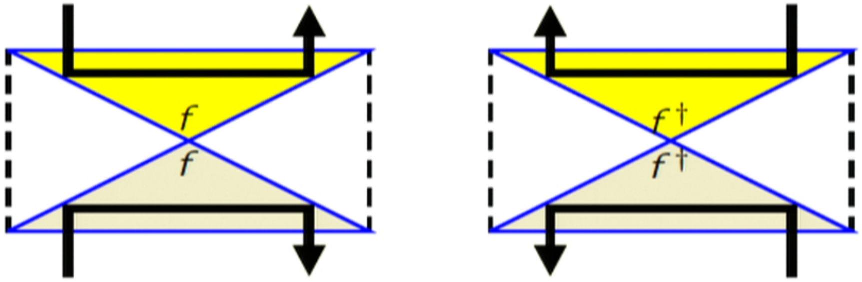

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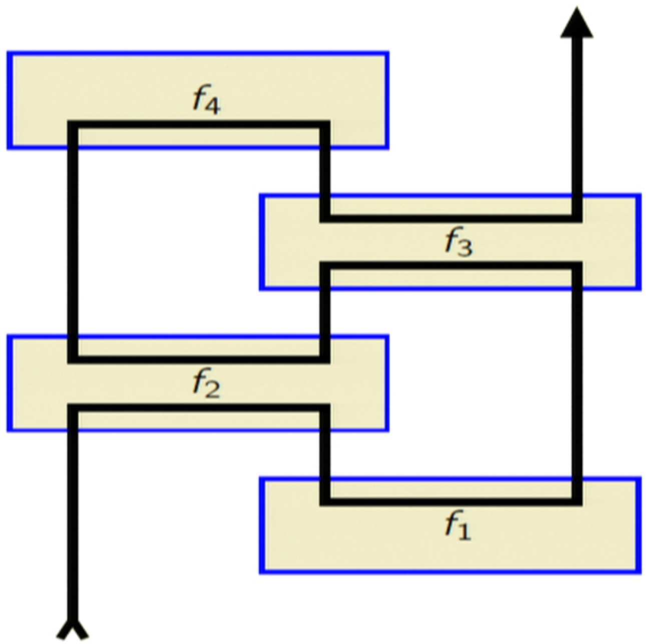

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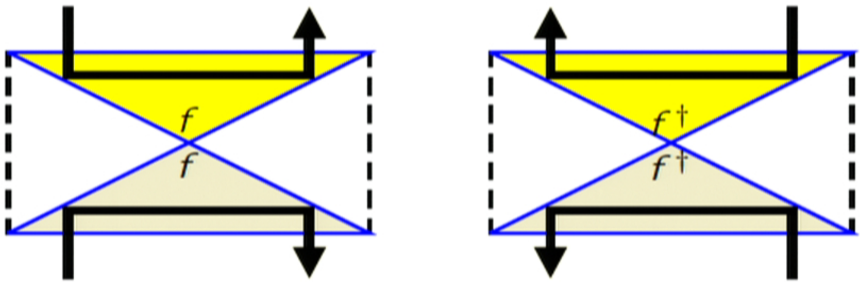

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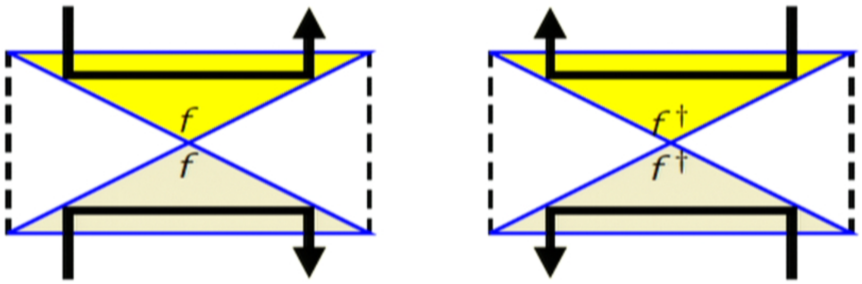
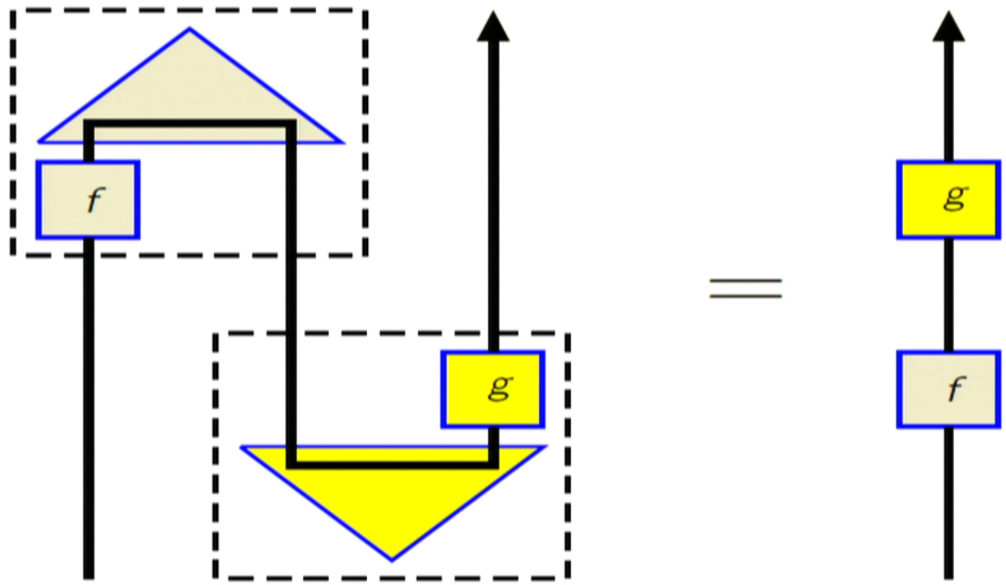


Compositionality

The key algebraic fact from which teleportation (and many other protocols) can be derived.



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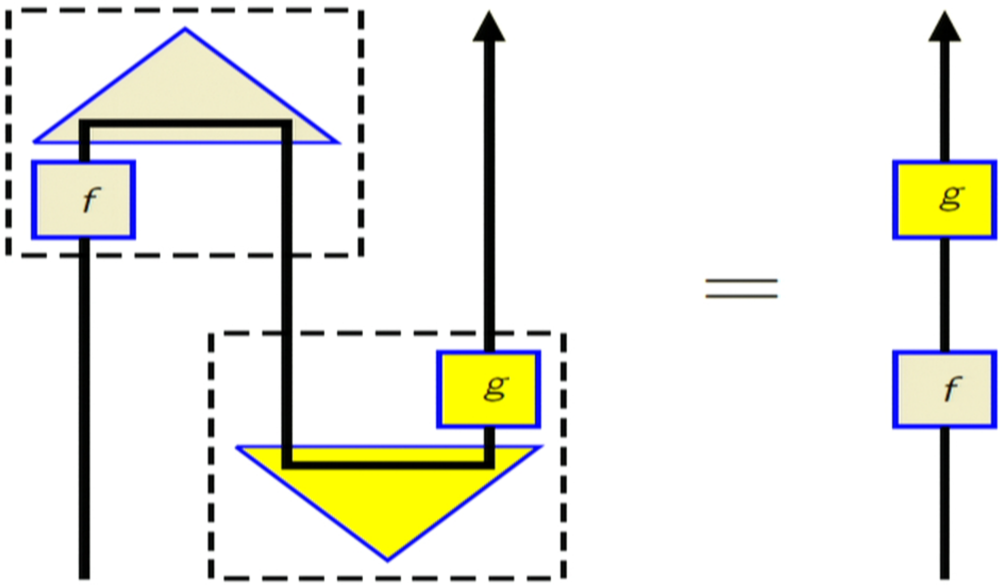

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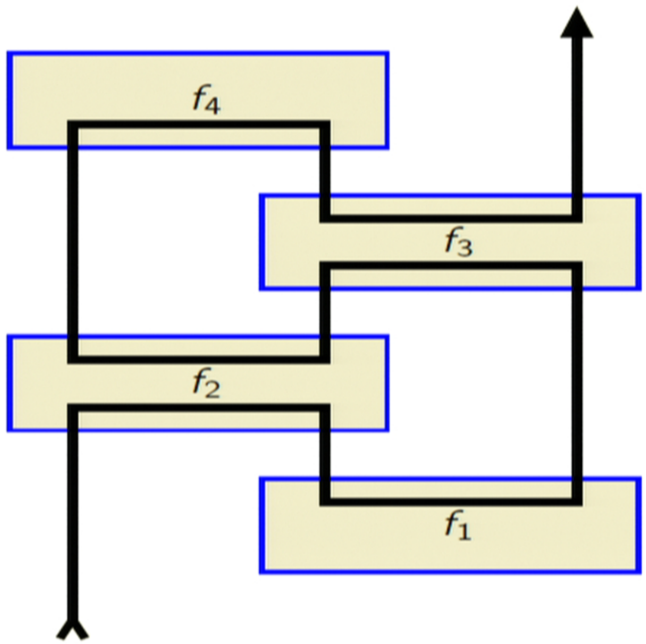

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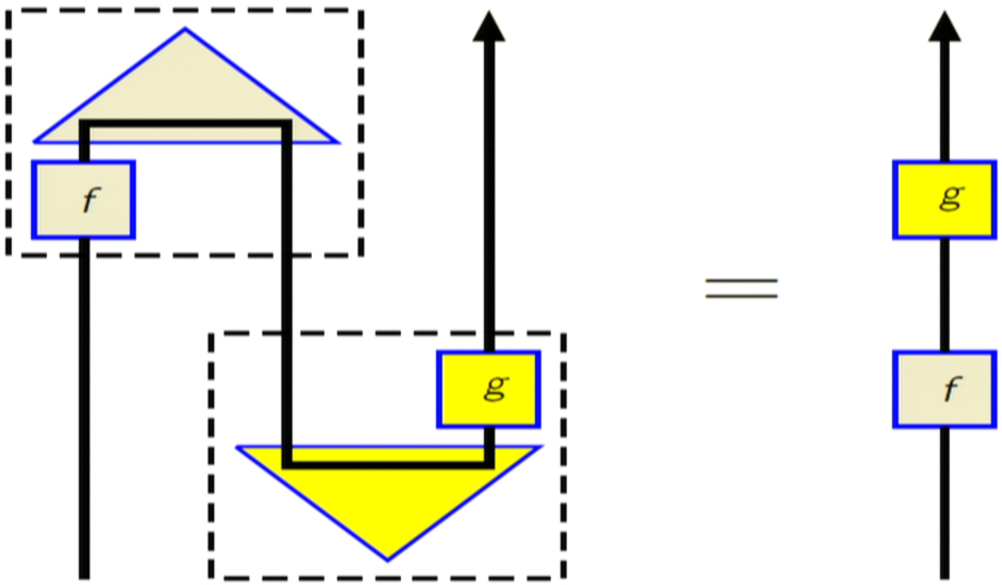

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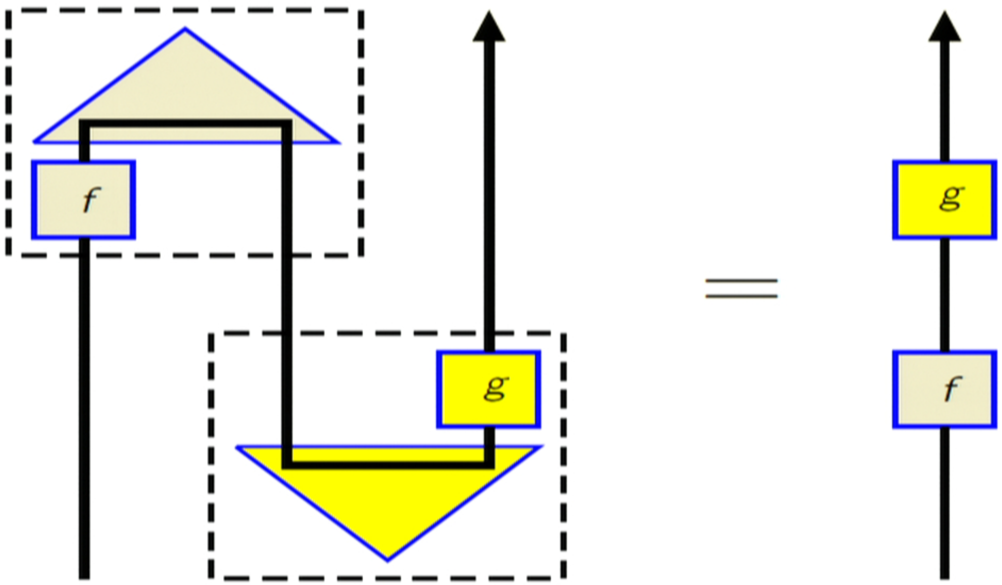

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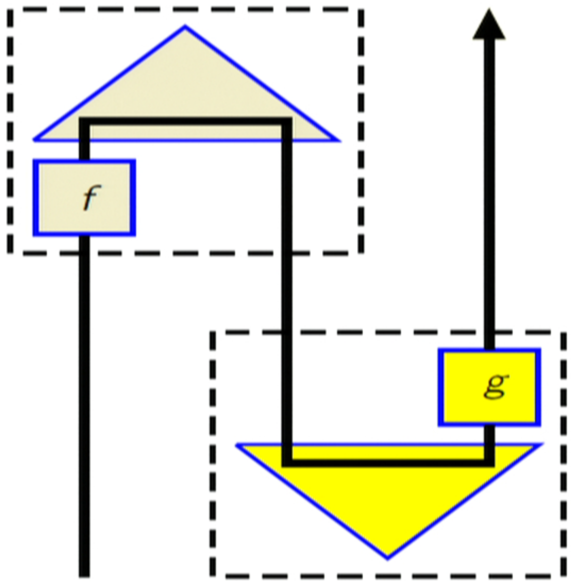

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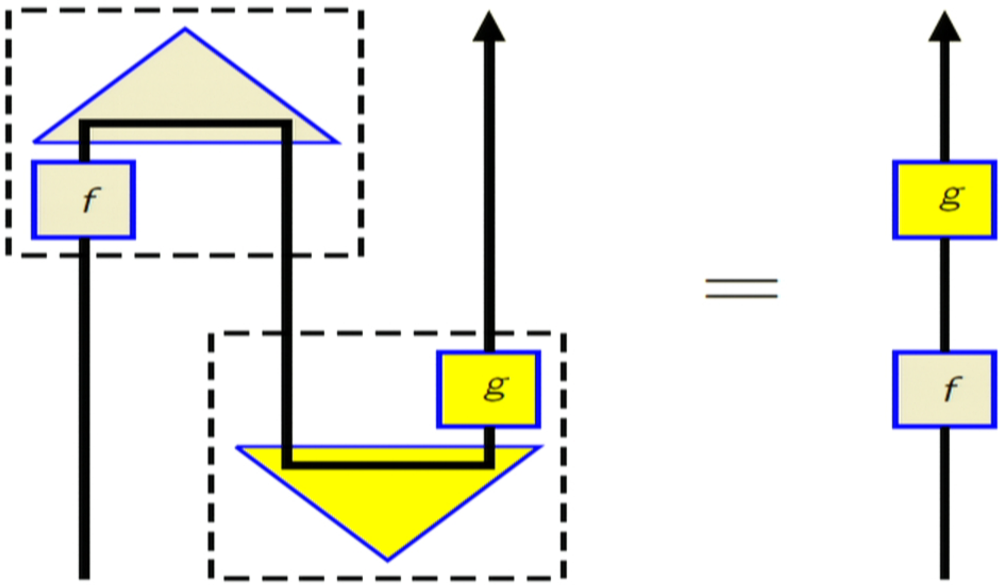

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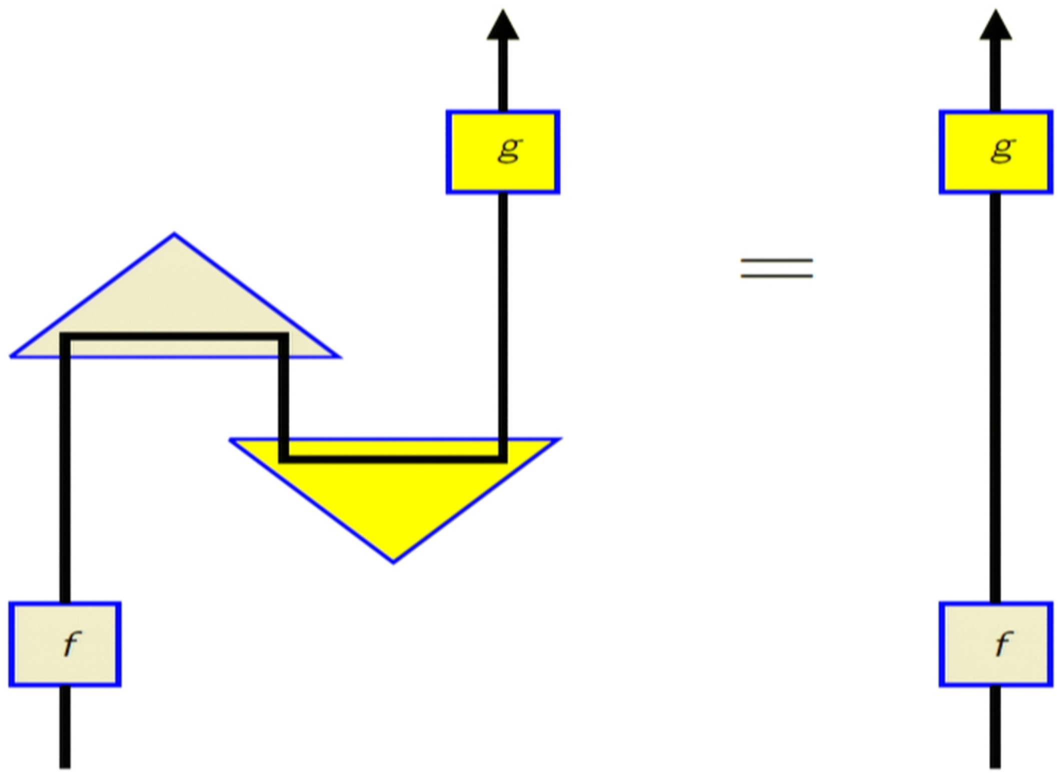

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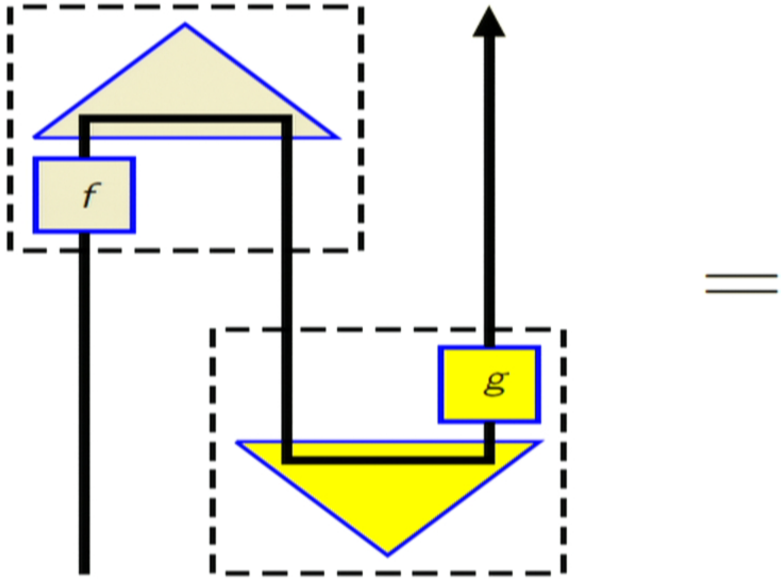

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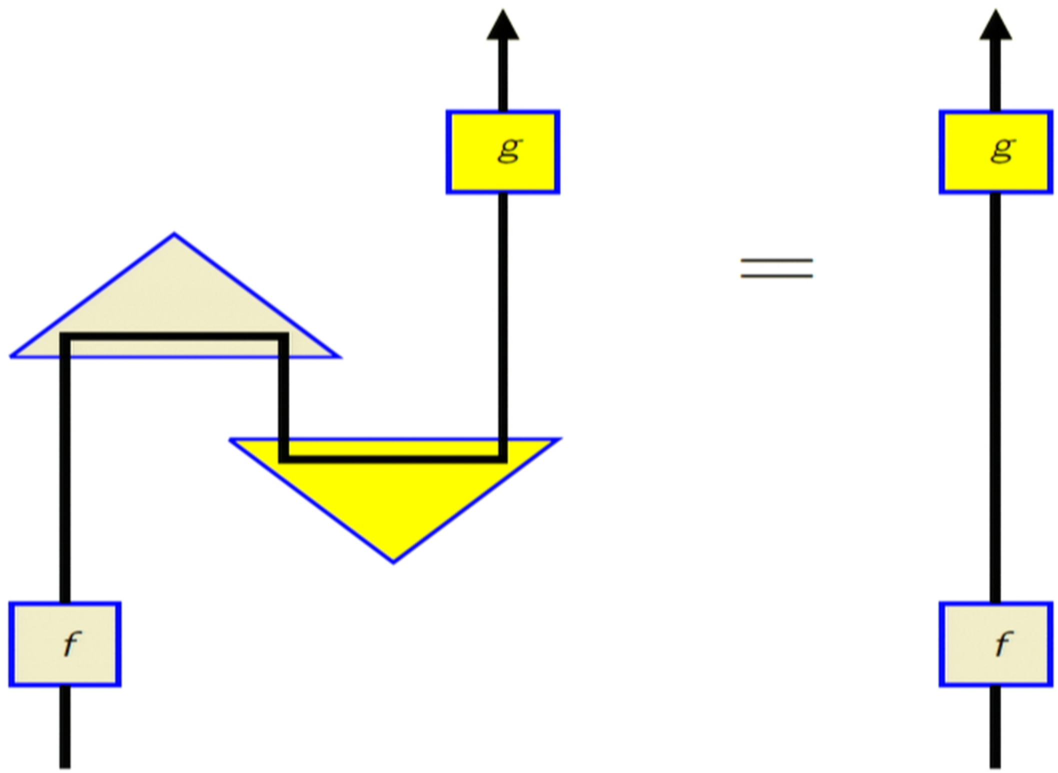
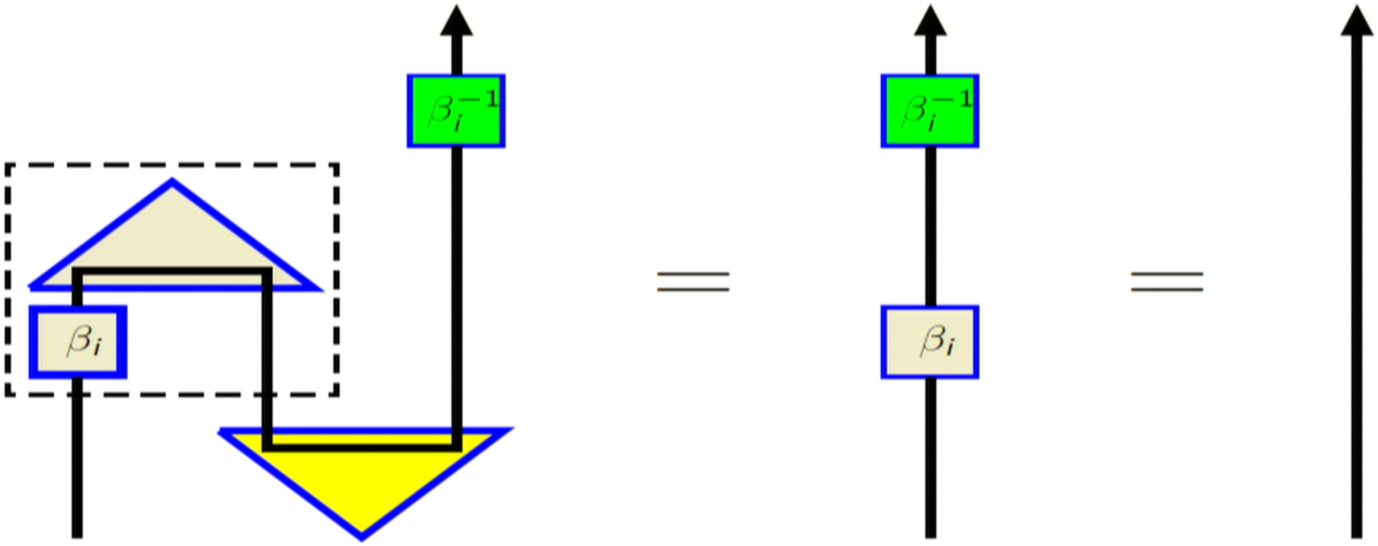


Teleportation diagrammatically



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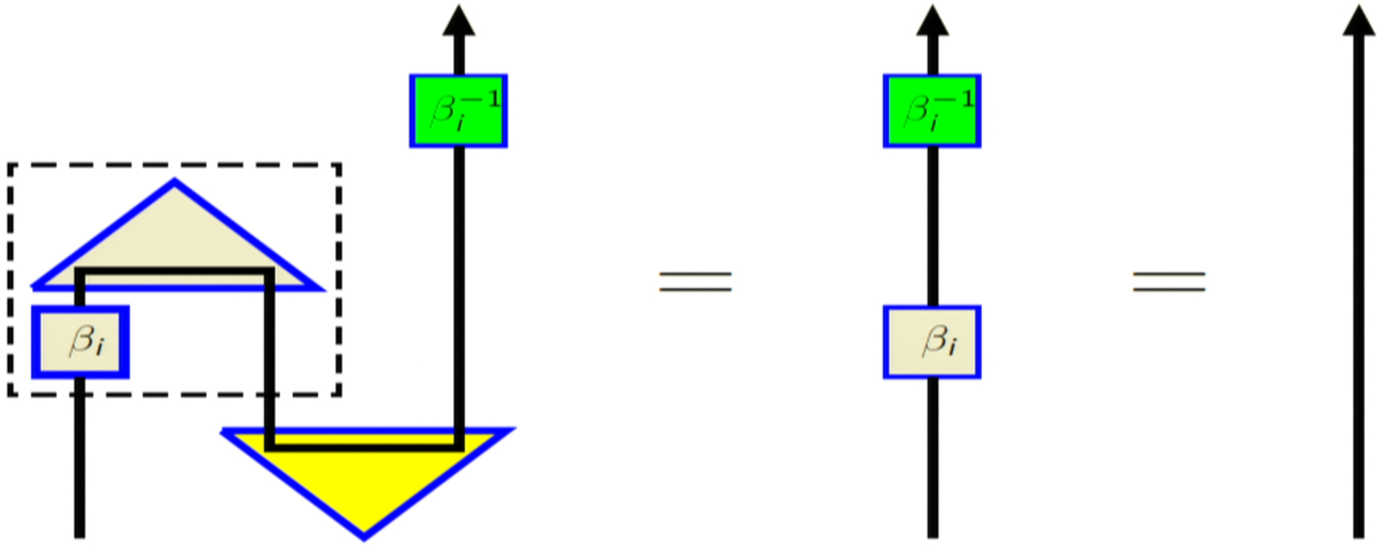

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Work of many people, both in the Quantum Group at Oxford CS Dept and elsewhere.

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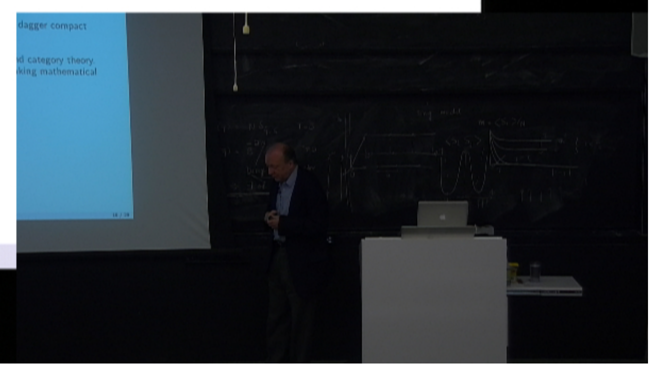

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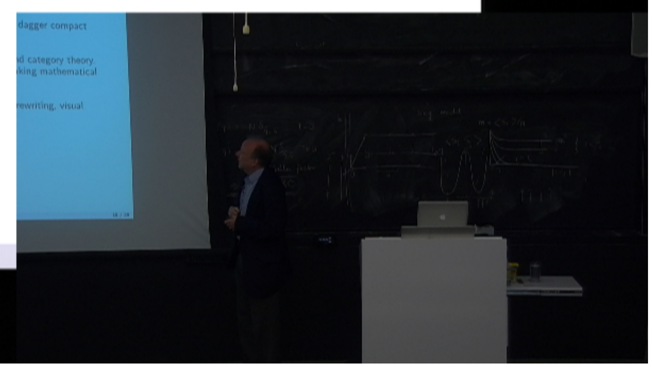

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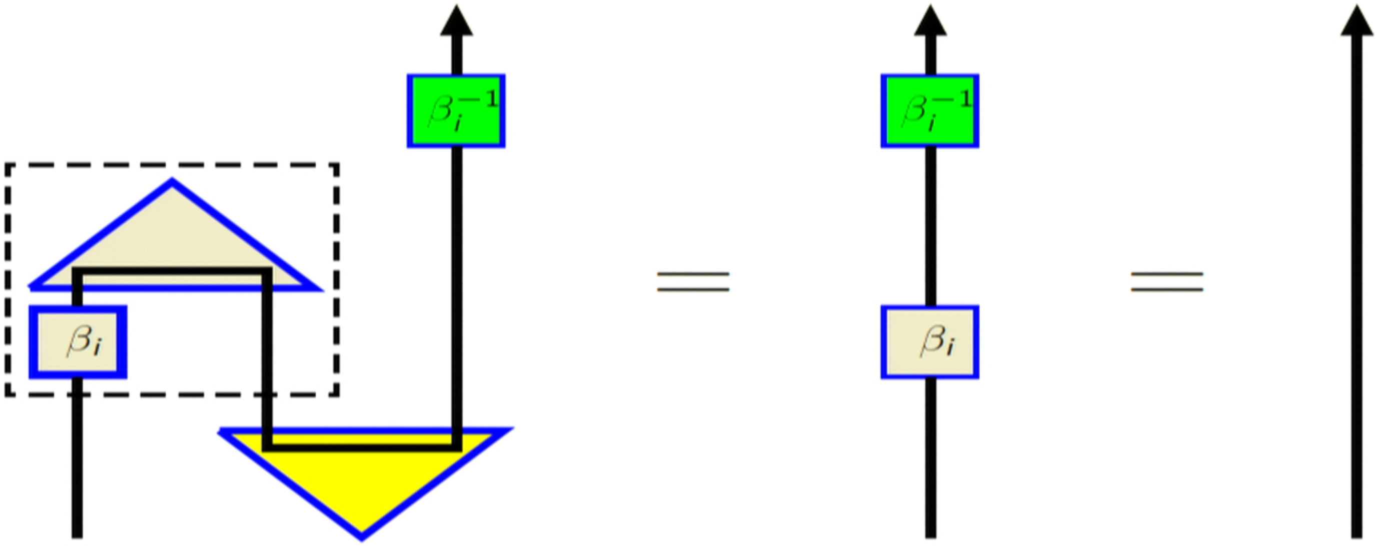

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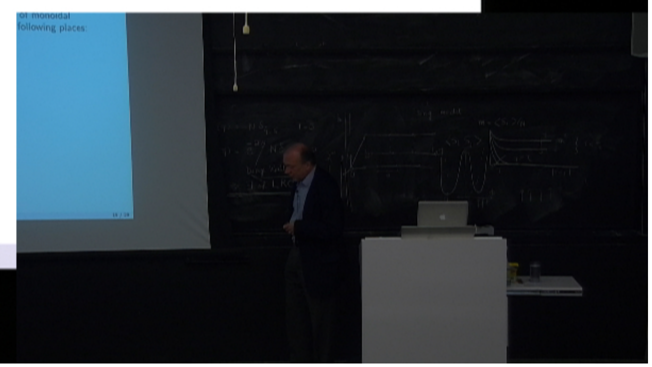

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- Logic: (linear version of) cut-elimination
- Computation: (linear version of) λ-calculus, feedback, processes.
- Linguistics: Lambek pregroup grammars, lifting vector space models of word meaning
- Topology, knot theory: Temperley-Lieb algebra, braided, pivotal and ribbon categories.

We will trace a path through some of these ...

The Temperley-Lieb Algebra

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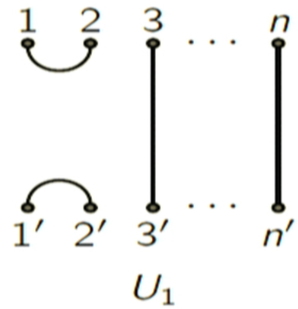
Generators:



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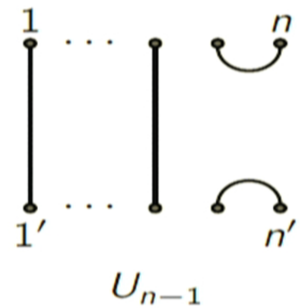
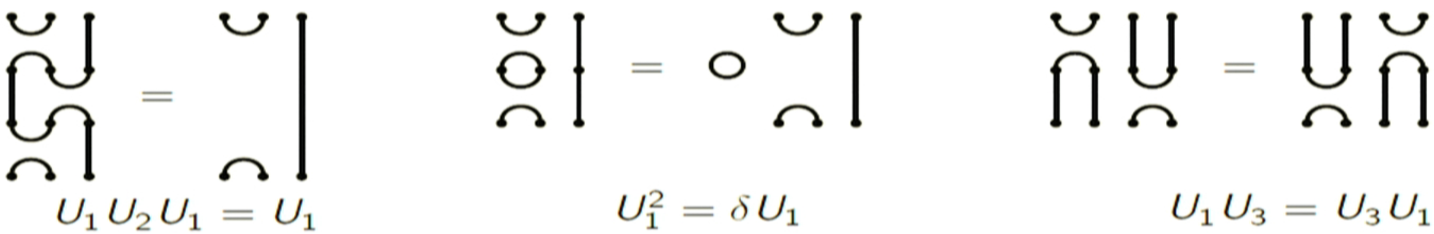


The Temperley-Lieb Algebra

Generators:

Relations:



Structure of Temperley-Lieb category

General form of composition:

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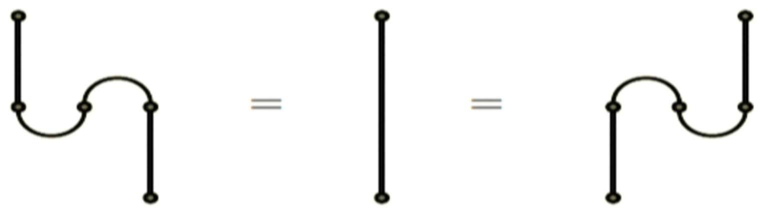
General form of composition:

Compact closure/rigidity:



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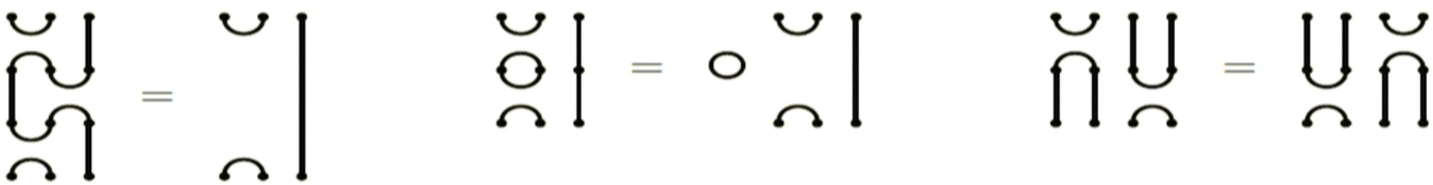
U_1 U_{n-1}

Relations:



$U_1 U_2 U_1 = U_1$ $U_1^2 = \delta U_1$ $U_1 U_3 = U_3 U_1$

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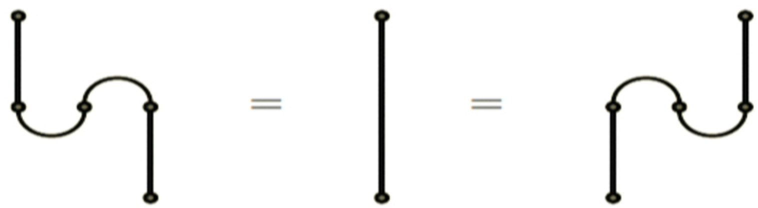

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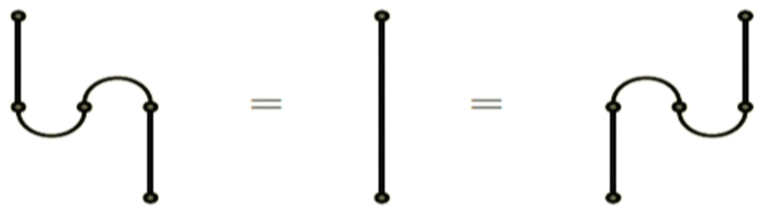
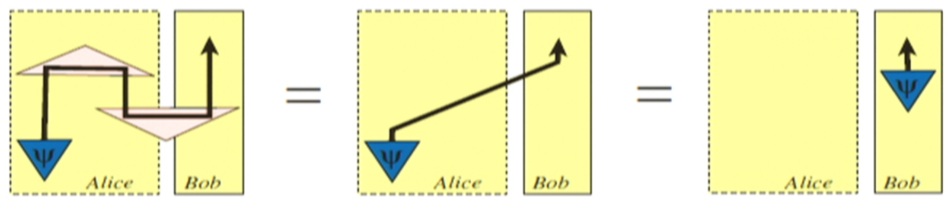


The same structure which accounts for teleportation:



Temperley-Lieb: expressiveness of the generators

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Diagrammatic trace:

The Ear is a Circle

Trace of Identity is the Dimension

The Connection to Knots

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The basic idea of the bracket polynomial is expressed by the following equation:

Each over-crossing in a knot or link is evaluated to a weighted sum of the two possible planar smoothings in the Temperley-Lieb algebra. With suitable choices for the coefficients A and B (as Laurent polynomials), this is invariant under the second and third Reidemeister moves. With an ingenious choice of normalizing factor, it becomes invariant under the first Reidemeister move — and yields the Jones polynomial!

Computation: back to the λ-calculus

We shall consider the *bracketing combinator*

$$\mathbf{B} \equiv \lambda x.\lambda y.\lambda z.\, x(yz) : (B \to C) \to (A \to B) \to (A \to C).$$

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We take $A = B = C = \mathbf{1}$ in **TL**. The interpretation of the open term

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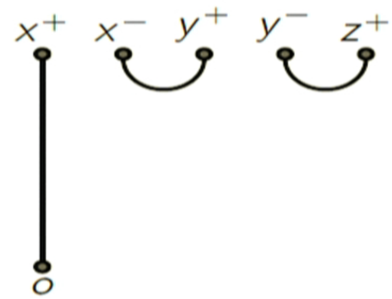

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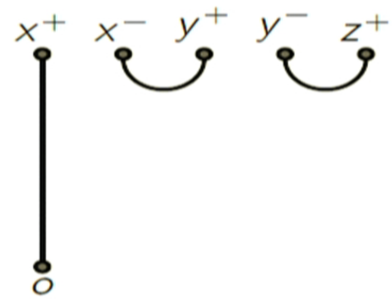

Diagrammatic Simplification as β-Reduction

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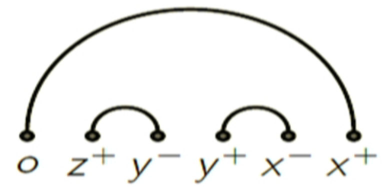

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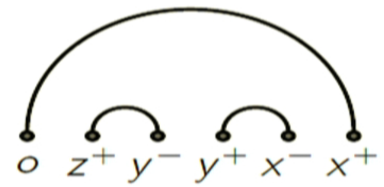

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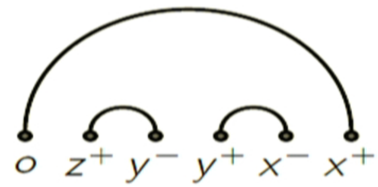
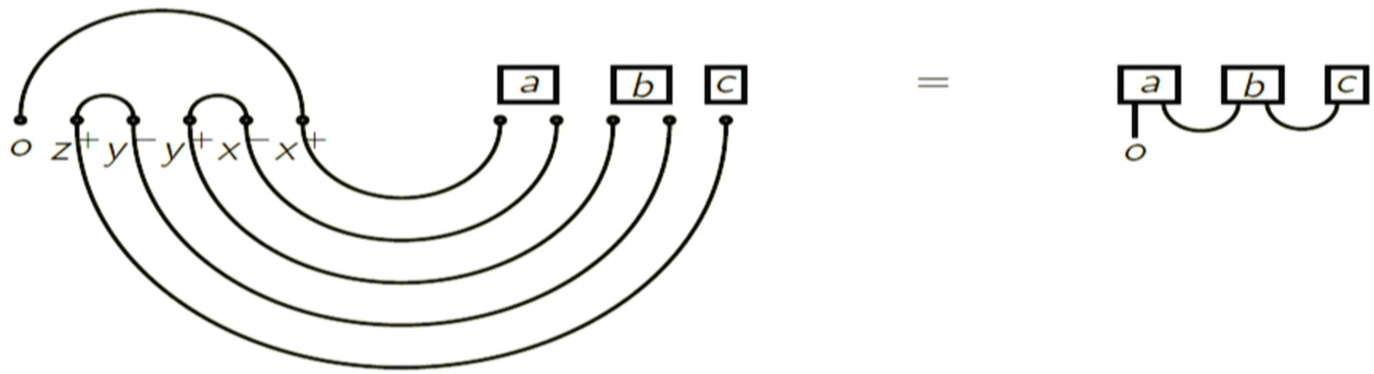


Now we consider an application $\mathbf{B}abc$ (where application is represented by cups):



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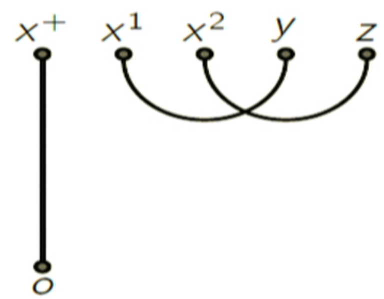

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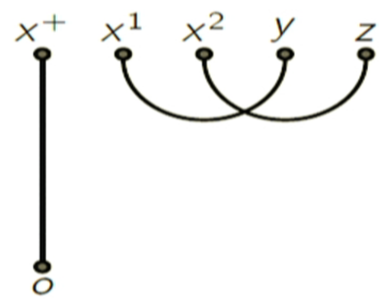

Here x^+ is the output of x, x^1 the first input, and x^2 the second input. The output of the whole expression is o.

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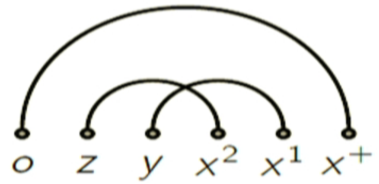

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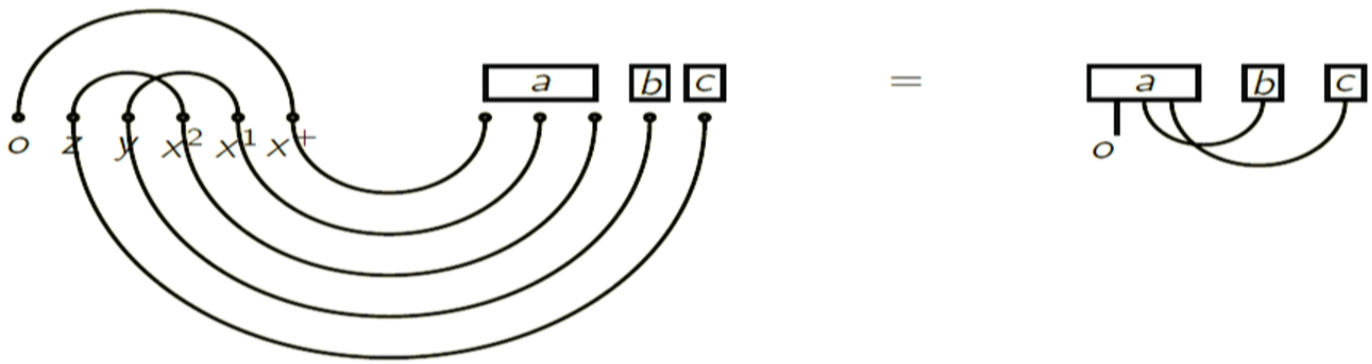

Linguistics

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Clark, Coecke and Sadrzadeh: Compositional Distributional Models of Meaning.

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Does	he	like	her?		question
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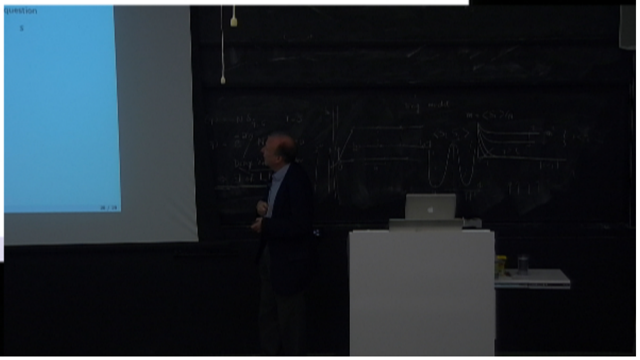

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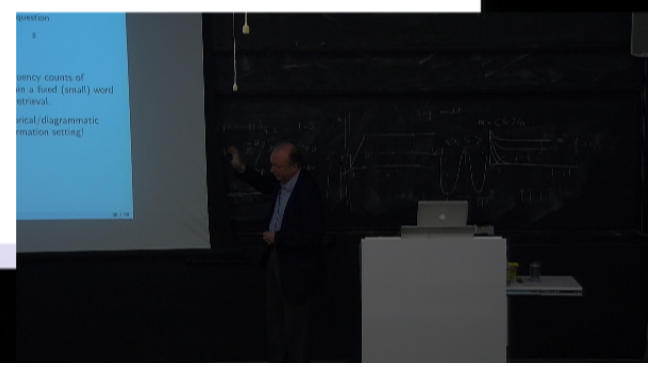

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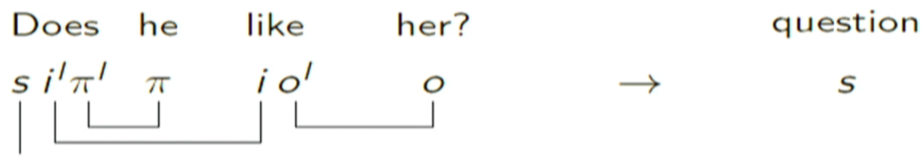

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So we can functorially map Lambek pregroup parses into vector spaces to lift the distributional word meanings compositionally to meanings for phrases and sentences.

Implementations and benchmarks look promising: see recent work by Sadrzadeh and Graefenstette.

Final Remarks

- Structures in monoidal categories, involving compact structure, trace etc., which support the diagrammatic calculus we have illustrated seem to provide a canonical setting for discussing *processes*. Have been widely used as such, implicitly or explicitly, in Computer Science. Recent work has emphasized their relevance in quantum information and quantum foundations. Significant links to work at PI in the Quantum Foundations group.

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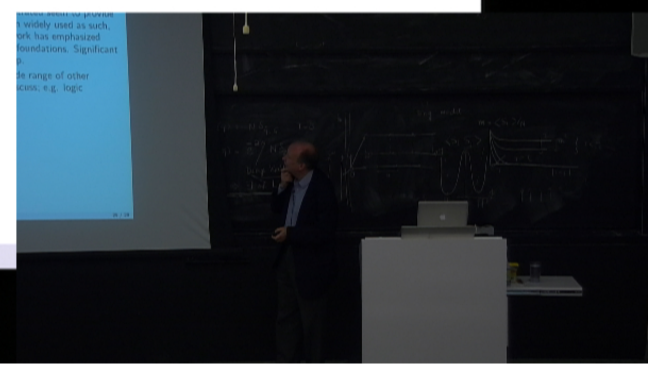

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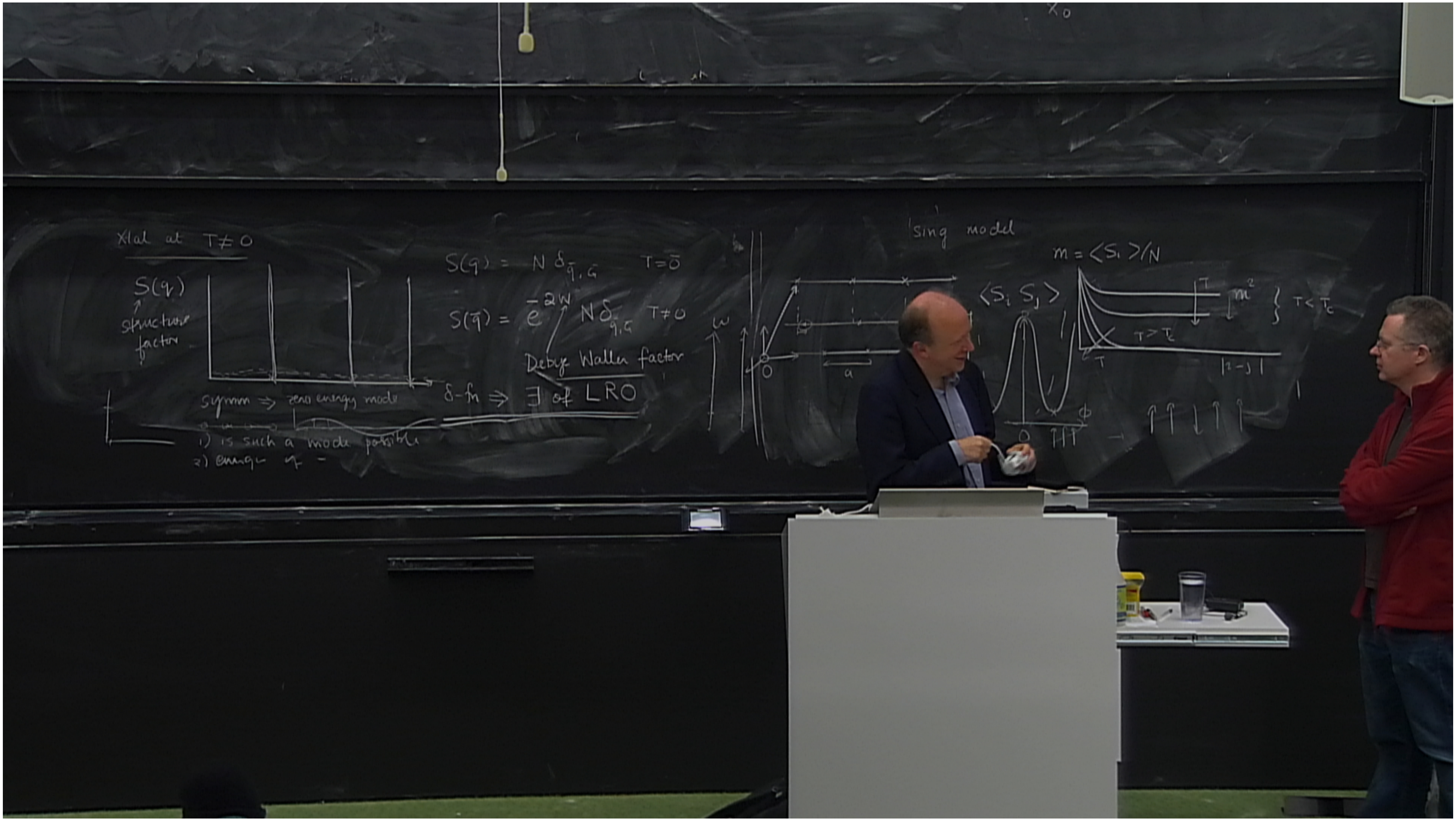
- The interface between physics and computer science is vibrant and fruitful. Long may this continue!


Xtal at T≠0
S(q)
structure factor
S(q) = N δ q,G T=0
S(q) = e^-2W N δ q,G T≠0
Debye Waller factor
δ-fn ⇒ ∃ of LRO
symm ⇒ zero energy mode
1) is such a mode possible
Ising model
m = <Si>/N
<Si Sj>
T < Tc
T > Tc