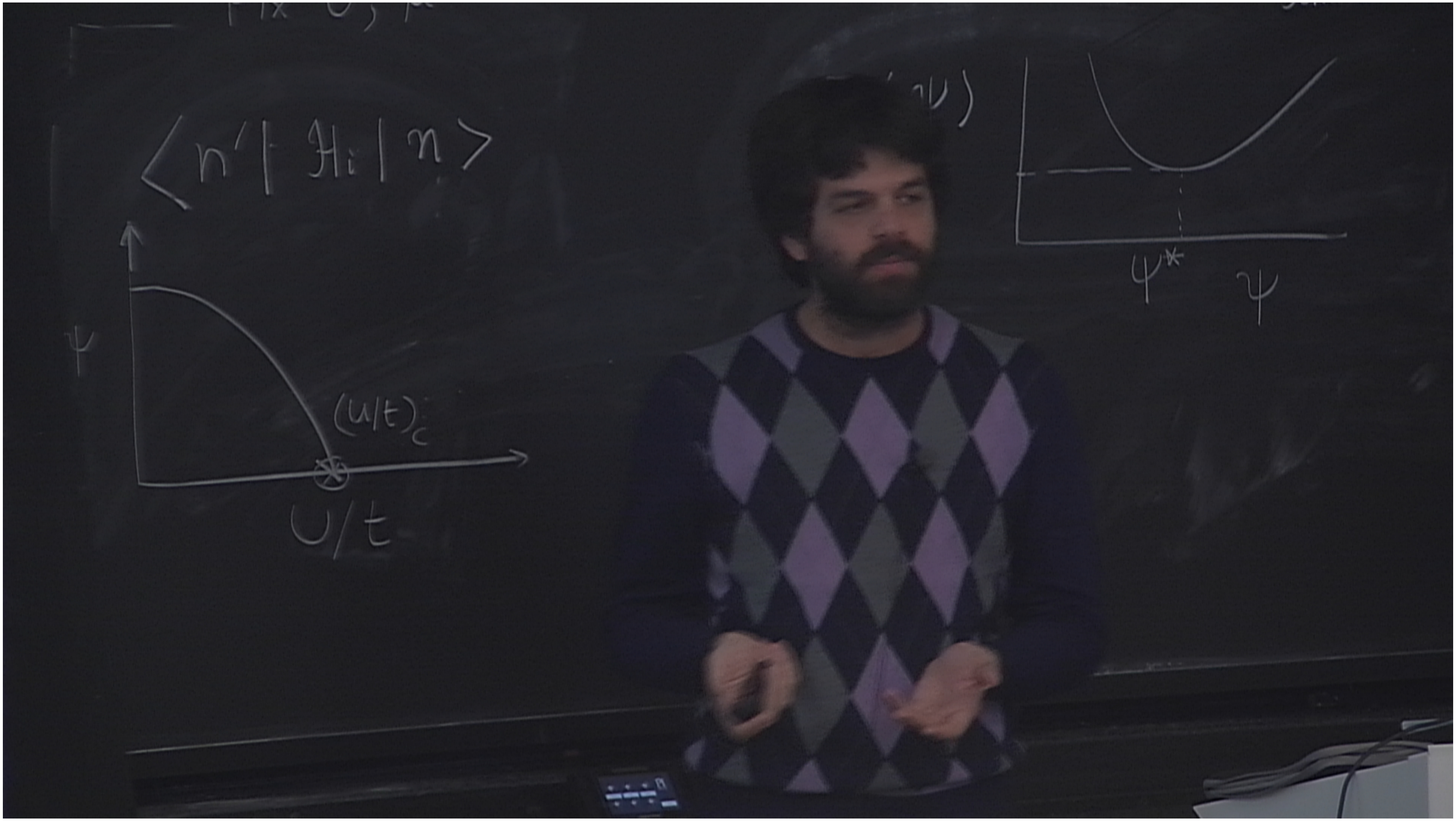
Title: Higher-Spin Interactions: Three-point Functions and Beyond

Date: Nov 10, 2011 02:00 PM

URL: http://pirsa.org/11110074

Abstract: Taking String Theory as a ``theoretical laboratory", I will present handy expressions for bosonic and fermionic (SUSY) higher-spin Noether currents. I will also describe a class of non-local higher-spin Lagrangian couplings that are generically required by the Noether procedure starting from four-points. The construction clarifies the origin of old problems for these systems and links String Theory to some aspects of Field Theory that go beyond its conventional low energy limit. I will finally discuss how the extension of these results to (A)dS brings about the emergence of minimal-like couplings from higher-derivative ones.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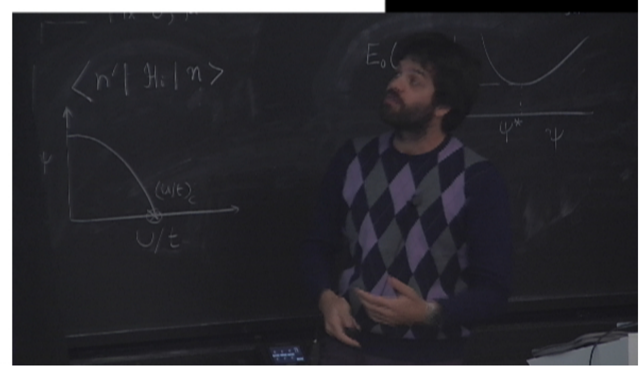
Problems with String Theory

✓It is a scheme based on the mechanical model of a vibrating relativistic string

✓Although very natural it raises several questions:
Background (in)dependence(?) ....

Key ingredient for consistency: Infinite tower of HS excitations

Soft UV behavior, open-closed duality, planar duality, modular invariance, etc...



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The difficulty ⟹ The mechanical model hides the geometry

Very general fact whenever an S-matrix theory is considered!

Global symmetries (Ward Identities) on the asymptotic (boundary) states are dual to bulk Geometry

String Theory is a consistent Higher-Spin Theory!

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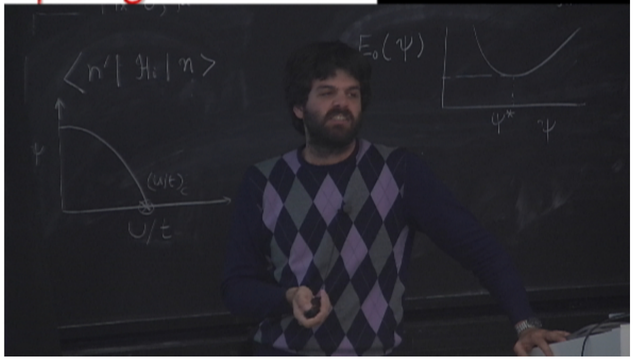
Plan

- String HS cubic couplings
  - Limiting couplings
  - Higher-Spin cubic interactions
- (A)dS by radial reduction!
- Noether procedure
  - A toy-model
  - S-matrix amplitudes or Lagrangians?
- HS Four-point functions and corresponding couplings
  - Colored spin-2 or gravity?
  - Lagrangian non-localities and non-local Geometry

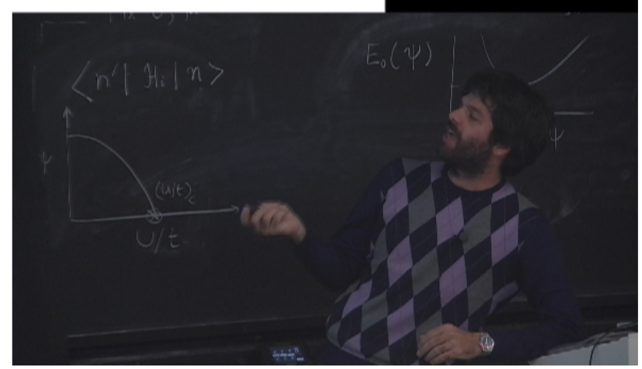


(Open) Bosonic-String S-matrix

Gauge fixed version of the Polyakov path integral

$$S^{open}_{j_1 \cdots j_n} = \int_{\mathbb{R}^{n-3}} dy_4 \cdots dy_n \, |y_{12} y_{13} y_{23}|$$
$$\times \langle \mathcal{V}_{j_1}(\hat{y}_1) \mathcal{V}_{j_2}(\hat{y}_2) \mathcal{V}_{j_3}(\hat{y}_3) \cdots \mathcal{V}_{j_n}(y_n) \rangle Tr(\Lambda^{a_1} \cdots \Lambda^{a_n}) + (1 \leftrightarrow 2)$$

$$y_{ij} = y_i - y_j$$



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Chan-Paton factors

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Vertex operators associated to asymptotic states via the state-operator isomorphism

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$$(L_0 - 1)\,|\phi\rangle = 0 \qquad L_1\,|\phi\rangle = 0 \qquad L_2\,|\phi\rangle = 0$$

Generalized form of the Fierz-Pauli equations for massive fields!

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$$(\Box - m_s^2)\,\phi_{\mu_1 \ldots \mu_s} = 0 \qquad \partial^{\mu_1} \phi_{\mu_1 \ldots \mu_s} = 0 \qquad \phi^{\alpha}{}_{\alpha \mu_3 \ldots \mu_s} = 0$$

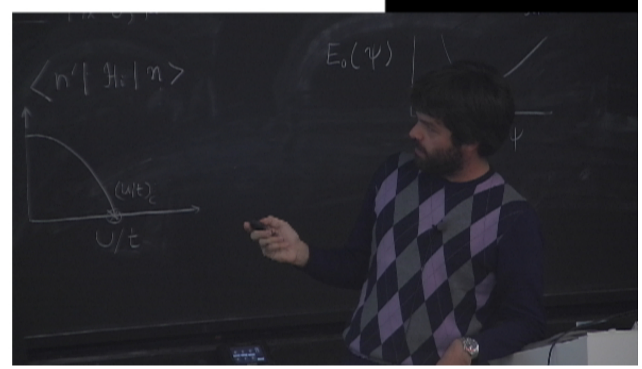
4

Generating Functions

For external states of the first Regge trajectory of the open bosonic string

$$\phi_i(p_i, \xi_i) = \sum_n \frac{1}{n!} \phi_{i\mu_1 \cdots \mu_n} \xi_i^{\mu_1} \ldots \xi_i^{\mu_n} = \phi_i + \xi_i^{\mu} \phi_{i\mu} + \frac{1}{2} \xi_i^{\mu_1} \xi_i^{\mu_2} \phi_{\mu_1 \mu_2} + \ldots$$

Convenient simplification: $\alpha_{-1}^{\mu} \to \xi_{\mu}$



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Commuting variables

$$\mathcal{A}_n = [\phi_1(\xi_1) \ldots \phi_n(\xi_n)] \star_{1\ldots n} Z_n(\xi_i')$$

Generating Function

$$\phi(\xi_1) \star_1 \psi(\xi_1') = \exp\left(\frac{\partial}{\partial \xi_1} \cdot \frac{\partial}{\partial \xi_1'}\right) \phi(\xi_1)\psi(\xi_1')\Big|_{\xi=0} = \phi\,\psi + \phi^{\mu}\,\psi_{\mu} + \frac{1}{2}\,\phi^{\mu_1\mu_2}\,\psi_{\mu_1\mu_2} + \ldots$$

Weyl-Wigner calculus!

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One gets:

$$Z \sim \int dy_4\dots dy_n \exp\left[-\frac{1}{2}\sum_{i\neq j}^{n}\alpha'\,p_i\cdot p_j\,\ln|y_{ij}| - \sqrt{2\alpha'}\,\frac{\xi_i\cdot p_j}{y_{ij}} + \frac{1}{2}\frac{\xi_i\cdot\xi_j}{y_{ij}^2}\right]$$

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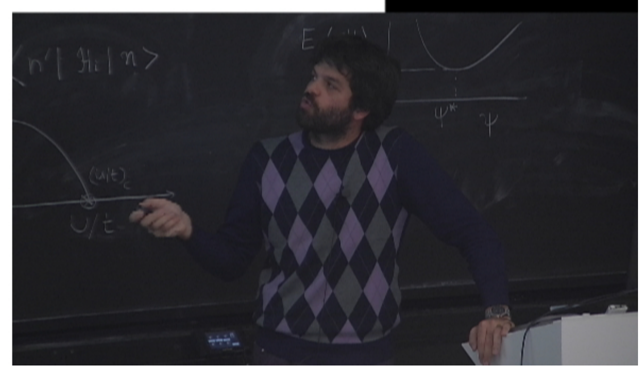
Unphysical dependence on the unintegrated y_i's

5

Three-point Amplitudes

Eliminate the unphysical dependence imposing the Virasoro constraints at the level of the generating function Z

$$-p_1^2 = \frac{s_1 - 1}{\alpha'} \quad - p_2^2 = \frac{s_2 - 1}{\alpha'} \quad - p_3^2 = \frac{s_3 - 1}{\alpha'}$$



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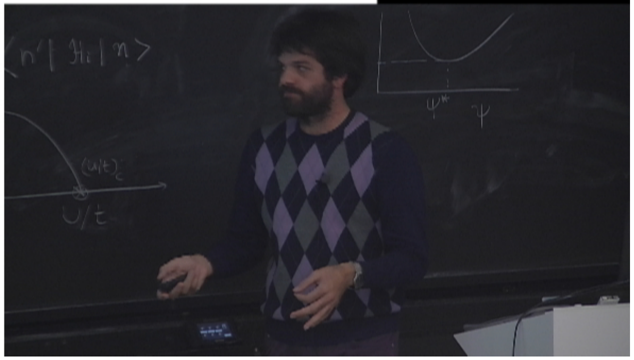
As in any S-matrix theory one recover on-shell results (Geometry is hidden...)

$$\mathbf{Z}_{phys} \sim exp\left\{\sqrt{\frac{\alpha'}{2}}\left(\xi_1 \cdot p_{23}\left\langle\frac{y_{23}}{y_{12}y_{13}}\right\rangle + \xi_2 \cdot p_{31}\left\langle\frac{y_{13}}{y_{12}y_{23}}\right\rangle + \xi_3 \cdot p_{12}\left\langle\frac{y_{12}}{y_{13}y_{23}}\right\rangle\right) + (\xi_1 \cdot \xi_2 + \xi_1 \cdot \xi_3 + \xi_2 \cdot \xi_3)\right\}$$



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On-shell Couplings: Star product with generating functions of fields

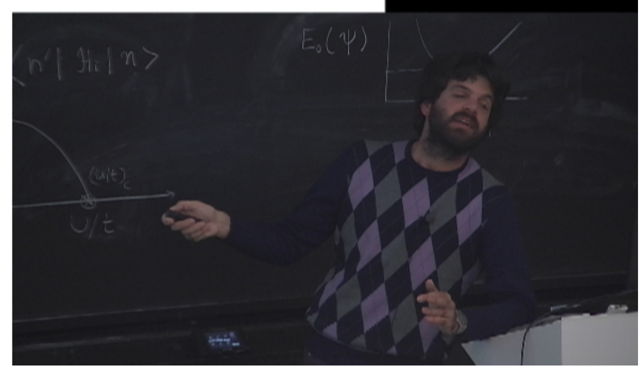
$$\mathcal{A} = \phi_1\left(p_1, \frac{\partial}{\partial\xi} + \sqrt{\frac{\alpha'}{2}}p_{31}\right)\phi_2\left(p_2, \xi + \frac{\partial}{\partial\xi} + \sqrt{\frac{\alpha'}{2}}p_{23}\right)\phi_3\left(p_3, \xi + \sqrt{\frac{\alpha'}{2}}p_{12}\right)\Bigg|_{\xi=0}$$

Master Thesis (2009) [hep-th/1005.3061]

Examples

0-0-s:

$$\mathcal{A}_{0-0-s} = \left(\sqrt{\frac{\alpha'}{2}}\right)^s \phi_1(p_1)\,\phi_2(p_2)\,\phi_3(p_3)\cdot p_{12}^s$$



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Induced by a conserved current:

$$J(x,\xi) = \Phi\left(x + i\sqrt{\frac{\alpha'}{2}}\,\xi\right)\,\Phi\left(x - i\sqrt{\frac{\alpha'}{2}}\,\xi\right)$$

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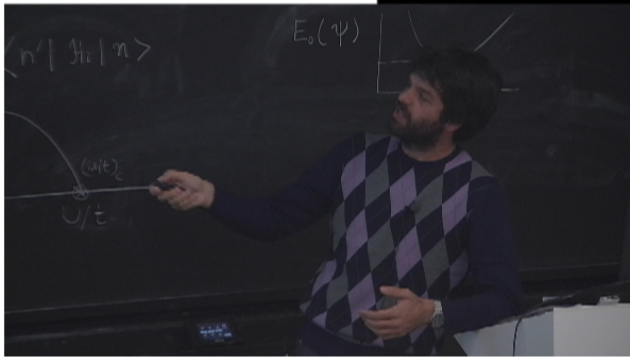
The complex scalar

$$= \phi(x)^{*}\phi(x) + i\sqrt{\frac{\alpha'}{2}}\,\xi^{\mu}\left[\phi^{*}(x)\partial_{\mu}\phi(x) - \phi(x)\partial_{\mu}\phi^{*}(x)\right] + \dots$$

HS currents



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1-1-s:

$$\begin{aligned}
\mathcal{A}_{s-1-1} = {} & \left(\sqrt{\frac{\alpha'}{2}}\right)^{s-2} s(s-1)\,A_{1\mu}\,A_{2\nu}\,\phi^{\mu\nu\cdots}p_{12}^{s-2} \\
& + \left(\sqrt{\frac{\alpha'}{2}}\right)^{s} \left[A_1\cdot A_2\,\phi\cdot p_{12}^{s} + sA_1\cdot p_{23}\,A_{2\nu}\,\phi^{\nu\cdots}p_{12}^{s-1} + sA_2\cdot p_{31}\,A_{1\nu}\,\phi^{\nu\cdots}p_{12}^{s-1}\right] \\
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The amplitudes can contain extra "stuff" that drops out in the massless limit where genuine Noether interactions ought to be recovered! Similar to a scaling limit

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HS cubic couplings

A gauge invariant pattern is recovered!

hep-th/1006.5242: A. Sagnotti and M.T.

The result is:

$$\mathcal{A} = \phi_1(p_1;\xi_1)\,\phi_2(p_2;\xi_2)\,\phi_3(p_3;\xi_3)$$
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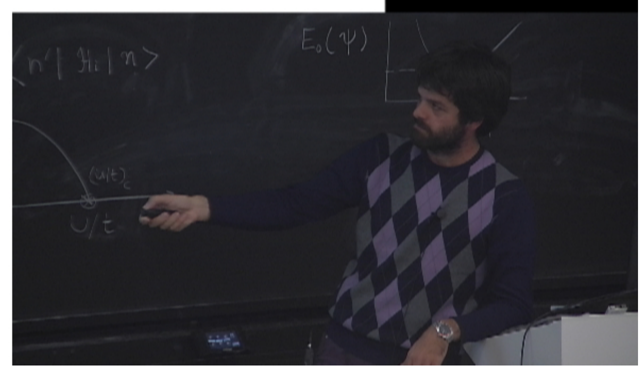

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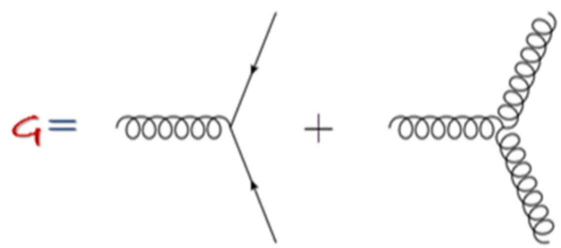

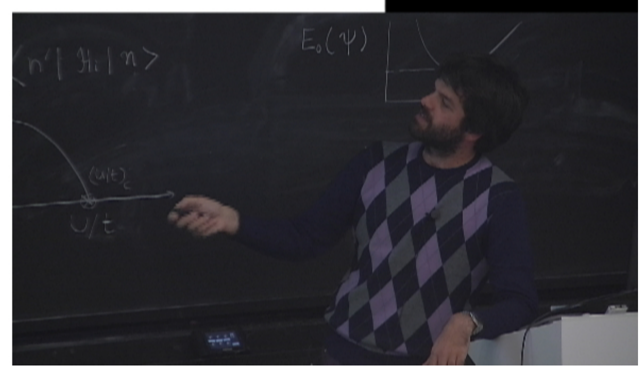

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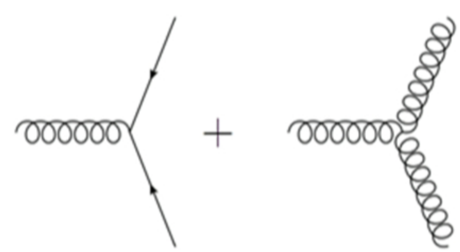
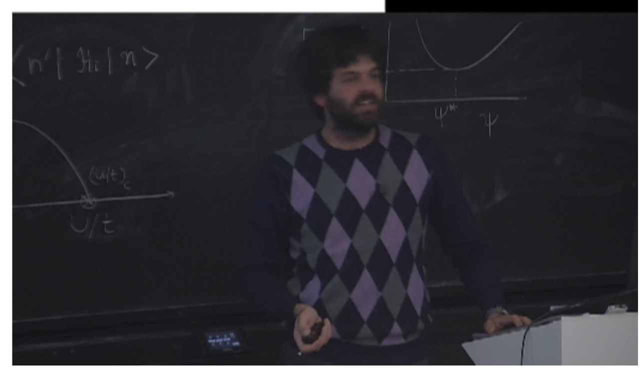
G =





HS cubic couplings

A gauge invariant pattern is recovered!

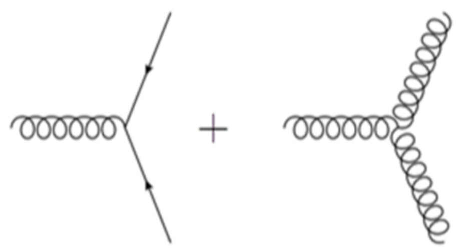
hep-th/1006.5242: A.Sagnotti and M.T.

The result is:

$$\mathcal{A} = \phi_1(p_1;\xi_1)\,\phi_2(p_2;\xi_2)\,\phi_3(p_3;\xi_3)$$
$$\star_{123}\ \exp\left\{\sqrt{\frac{\alpha'}{2}}\left[\xi_1\cdot p_{23} + \xi_2\cdot p_{31} + \xi_3\cdot p_{12} + \xi_1\cdot\xi_2\,\xi_3\cdot p_{12} + \xi_2\cdot\xi_3\,\xi_1\cdot p_{23} + \xi_3\cdot\xi_1\,\xi_2\cdot p_{31}\right]\right\}$$

Gauge invariant up to the linearized massless Eom's. Off-shell completion uniquely fixed up to partial integrations and field redefinitions.

Non-abelian deformation of gauge symmetry!

G = 

Lego bricks of any scattering amplitude!

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The result is:

Coupling function: completely arbitrary in Field Theory (?)

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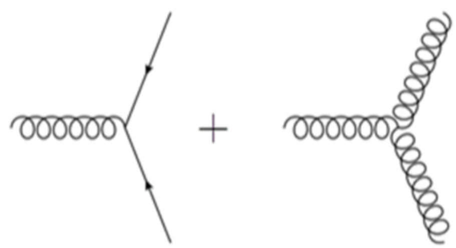
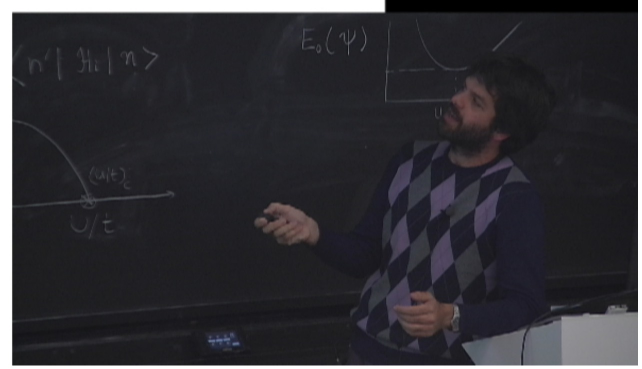


Lego bricks of any scattering amplitude!

Cubic vertices

s_1-s_2-s_3 couplings

$$\mathcal{V}_3 \sim \partial^{k_1}\phi_{s_1}\,\partial^{k_2}\phi_{s_2}\,\partial^{k_3}\phi_{s_3}$$



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$$\mathcal{V}_3 \sim \partial^{k_1} \phi_{s_1} \partial^{k_2} \phi_{s_2} \partial^{k_3} \phi_{s_3}$$

$$\mathcal{V}_3 \sim (\mathcal{G})^0 \phi_1 \phi_2 \phi_3$$

$$\mathcal{G} \sim p\, \partial_{\xi_1} \partial_{\xi_2} \partial_{\xi_3}$$

Cubic vertices

$p_{ij} = p_i - p_j$

s_1-s_2-s_3 couplings

$$\mathcal{V}_3 \sim \partial^{k_1} \phi_{s_1} \partial^{k_2} \phi_{s_2} \partial^{k_3} \phi_{s_3}$$

Number of derivatives

$\mathcal{V}_3 \sim (\mathcal{G})^0 \phi_1 \phi_2 \phi_3$ $s_1+s_2+s_3$ $A_1 \cdot p_{23} A_2 \cdot p_{31} A_3 \cdot p_{12}$

$\mathcal{G} \sim p\, \partial_{\xi_1} \partial_{\xi_2} \partial_{\xi_3}$

9

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$\mathcal{V}_3 \sim (\mathcal{G})^1 \phi_1 \phi_2 \phi_3$ $\quad s_1+s_2+s_3-2 \quad$ $A_1 \cdot A_2 A_3 \cdot p_{12} + \text{cyclic}$

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Cubic vertices

$s_1 \geq s_2 \geq s_3$

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$\mathcal{V}_3 \sim (\mathcal{G})^{s_3} \phi_1 \phi_2 \phi_3$ | $s_1+s_2-s_3$ → Minimum number of derivatives

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$\ldots$ — $\ldots$

$\mathcal{V}_3 \sim (\mathcal{G})^{s_3} \phi_1 \phi_2 \phi_3$ — $s_1+s_2-s_3$ → Minimum number of derivatives

No minimal coupling of HS with gravity, but multipolar couplings are possible! Generalizes Weinberg-Witten Theorem...

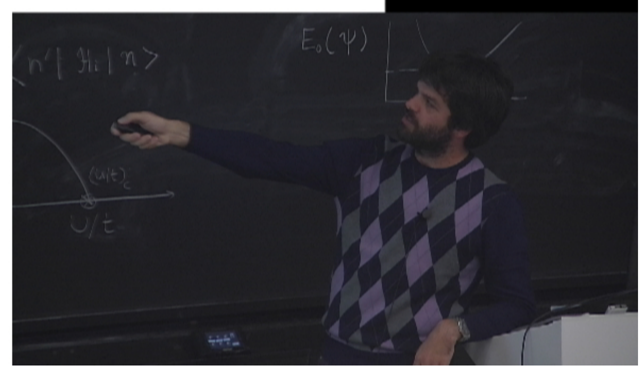
(A)dS cubic vertices

Remarkably from the flat space result one can extract all (A)dS vertices

Trick: ambient space formulation

General isomorphism between ambient space fields and (A)dS fields

$$X \cdot \partial_{\Xi} \Phi(X, \Xi) = 0$$



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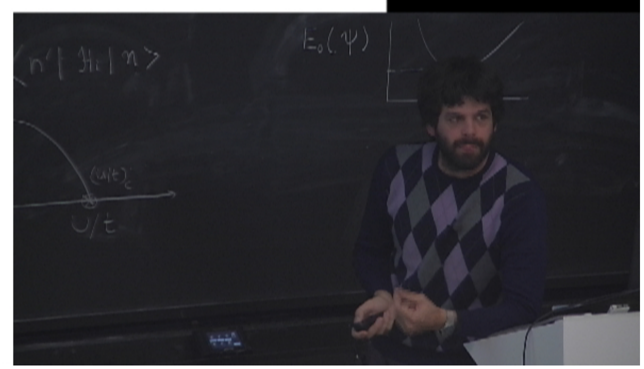
$$(X \cdot \partial_X - \Delta_h) \Phi(X, \Xi) = 0$$

One can write (A)dS Lagrangians using the measure $\int d^{d+1}X \, \delta\left(\frac{\sqrt{X^2}}{L} - 1\right)$



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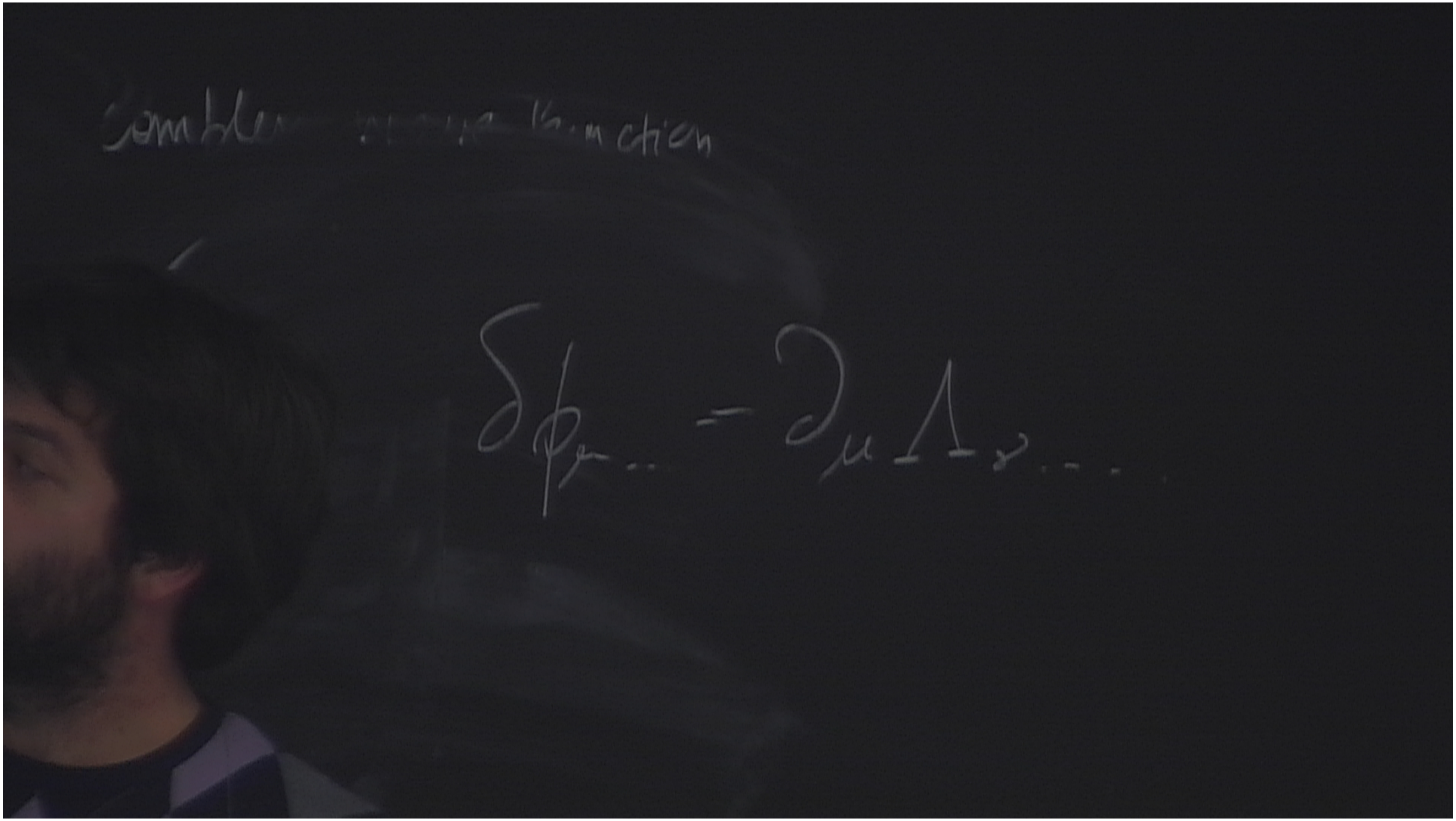
$\delta \phi_{\mu \dots} = \partial_\mu \Lambda_{\nu \dots}$

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Cubic (A)dS HS couplings can be encoded within the flat ones modulo a fixed boundary term! (hep-th/1110.5918: E. Joung and M.T.)

$$\mathcal{G}_{123}^{(A)dS} \,=\, \mathcal{G}_{123}^{\text{flat}} \,+\, \partial_X \cdot (\alpha_1\, \Xi_1 \,+\, \alpha_2\, \Xi_2 \,+\, \alpha_3\, \Xi_3 \,+\, \ldots)$$

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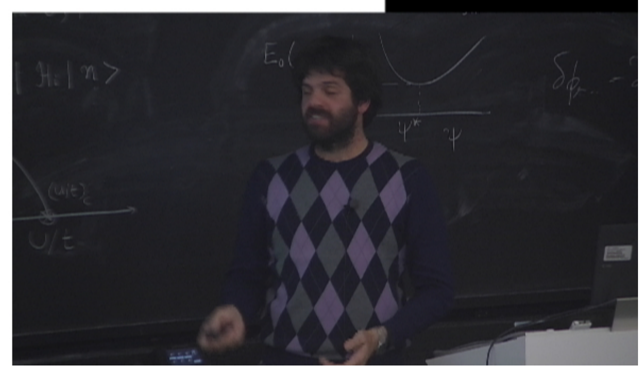
(A)dS cubic vertices

Lower derivative tail appears automatically
in terms of intrinsic (A)dS coordinates
after the reduction:



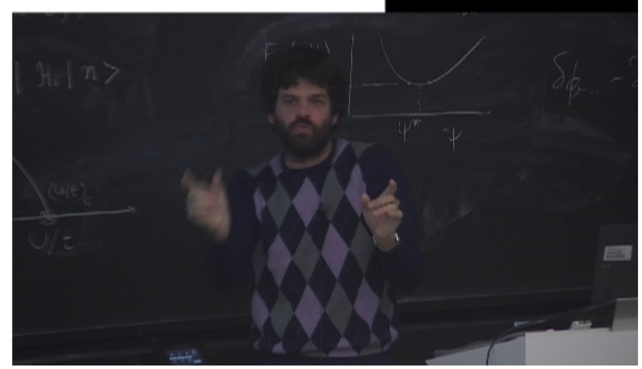
(A)dS cubic vertices

Lower derivative tail appears automatically in terms of intrinsic (A)dS coordinates after the reduction:

$$X^M = R\,\hat{X}^M(x)$$

$$\partial_{X^M} = \hat{X}_M\,\partial_R + \frac{L^2}{R}\frac{\partial \hat{X}_M}{\partial x_\mu}\left[D_\mu + \frac{1}{L}(\ldots)\right]$$

$$\Xi^M = \hat{X}_M\,\zeta + L\,\frac{\partial \hat{X}_M}{\partial x_\mu}\,\xi\cdot e_\mu$$



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The minimal coupling is recovered but it is completely encoded within the non-minimal ambient space coupling and requires by consistency a whole tail of higher derivative contributions

$$\mathcal{L}^{(3)} \simeq A_0 + \frac{1}{\Lambda}A_1 + \ldots + \frac{1}{\Lambda^n}A_n$$

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Similar logic to the massless limit of the ST vertex! The lower derivative tail disappears whenever Λ goes to zero while the G operator is disentangled!

This kind of approach can shed some new light on Vasiliev's system!

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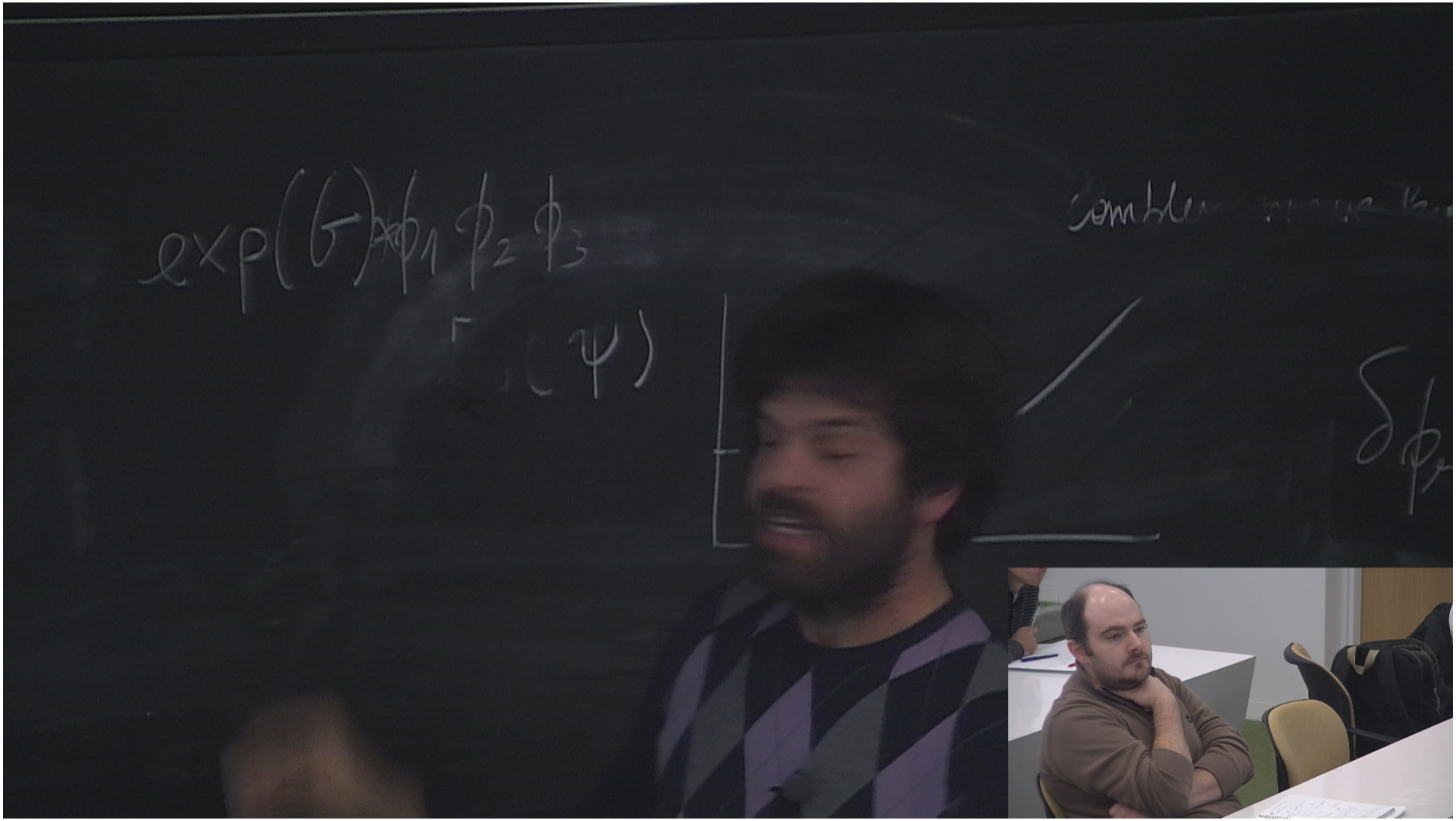
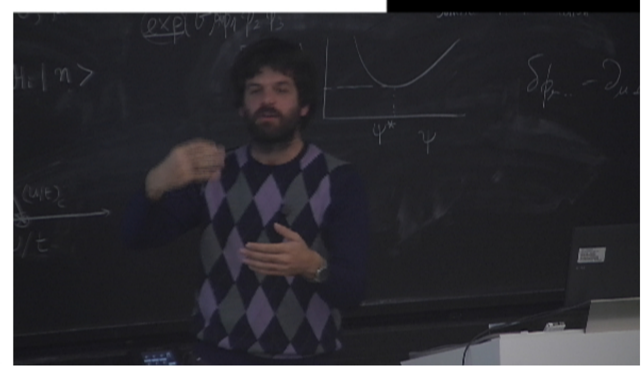

exp(G) φ1 φ2 φ3

Noether Procedure

Perturbative approach to Geometry (non-linear gauge symmetry)

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} + \dots$$

$$\delta\Phi = \delta^{(0)}\Phi + \delta^{(1)}\Phi + \delta^{(2)}\Phi + \dots$$



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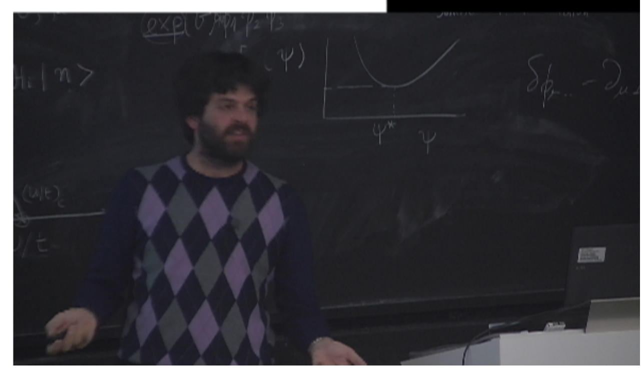
Usual linearized gauge transformations: $\delta^{(0)}\Phi = \partial\Lambda$



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Fully non-linear deformation that sums up to what we call Geometry

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Finding a solution order by order!

$$\delta^{(1)}\mathcal{L}^{(2)} + \delta^{(0)}\mathcal{L}^{(3)} = 0$$

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......

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......

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Noether Procedure

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Non-homogeneous contributions!

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......

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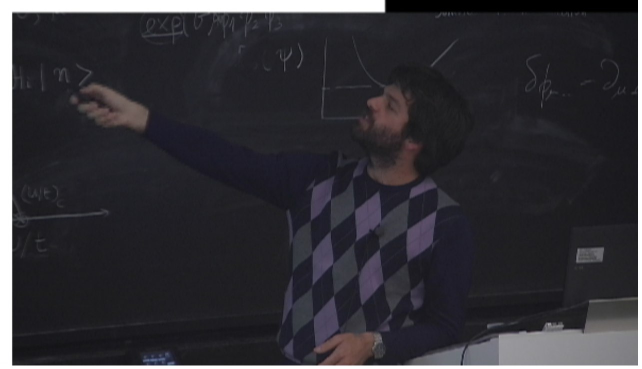
Symbol Calculus

Convenient simplification: → $A_\mu \to \xi_\mu$

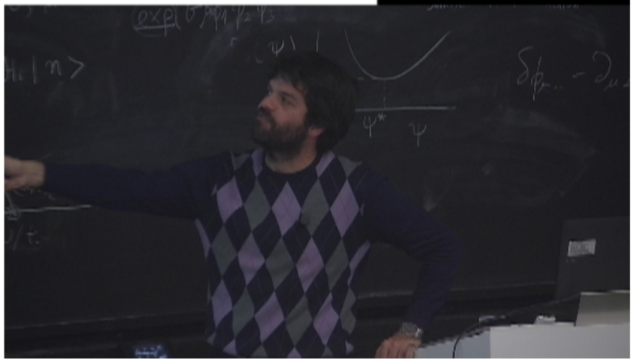


Symbol Calculus

Commuting auxiliary variables

Convenient simplification: $A_\mu \to \xi_\mu$

YM: $$\mathcal{A}_{123}(p_i) = \prod_{i=1}^{3} \left[A_i(p_i)\cdot \xi_i + \phi_i(p_i)\right] \star_{123} Z_{123}(p_i, \xi'_i)$$



Symbol Calculus

Convenient simplification: $A_\mu \to \xi_\mu$ — Commuting auxiliary variables

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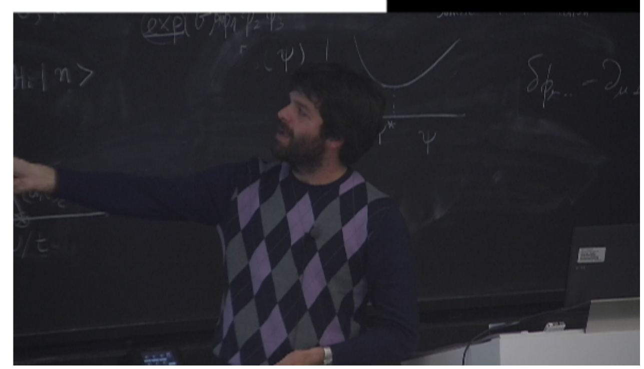
Formal sum of all amplitudes

$$p_{1\alpha} A_1^\mu A_{2\mu} A_3^{\alpha_2} = \prod_{i=1}^{3} [A_i(p_i) \cdot \xi_i + \phi_i(p_i)] \star_{123} (p_1 \cdot \xi_3 \, \xi_1 \cdot \xi_2)$$



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Weyl-Wigner calculus!

Symbol Calculus

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YM: $\mathcal{A}_{123}(p_i) = \prod_{i=1}^{3} [A_i(p_i)\cdot\xi_i + \phi_i(p_i)] \star_{123} Z_{123}(p_i, \xi'_i)$

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Weyl-Wigner calculus!

Straightforward to generalize to HS (most general QFT)

$$\mathcal{A}_n = [\phi_1(\xi_1)\dots\phi_n(\xi_n)] \star_{1\dots n} Z_n(\xi'_i)$$

Generating Function

$$\phi_i(p_i,\xi_i) = \sum_n \frac{1}{n!}\,\phi_{i\,\mu_1\cdots\mu_n}\,\xi_i^{\mu_1}\dots\xi_i^{\mu_n} = \phi_i + \xi_i^\mu\,\phi_{i\,\mu} + \frac{1}{2}\,\xi_i^{\mu_1}\,\xi_i^{\mu_2}\phi_{\mu_1\mu_2} + \dots$$

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A toy model: Scalar Yang-Mills

We already know the answer!

$$\mathcal{L} = \frac{1}{4}\left(F^{a}_{\mu\nu}\right)^2 - \frac{1}{2}\left(D_\mu \Phi^a\right)^2 + \ldots$$

$$\boxed{p \cdot A = 0}$$

$$\mathcal{L}^{(2)} = \frac{1}{2}\,\mathrm{Tr}\left(A_\mu p^2 A^\mu + \Phi p^2 \Phi\right)$$

$$A_\mu = A_\mu T^a$$

$$\delta^{(1)} \mathcal{L}^{(2)} + \delta^{(0)} \mathcal{L}^{(3)} = 0$$

$$\sim p \cdot \partial_\xi$$

A toy model: Scalar Yang-Mills

We already know the answer!

$$\mathcal{L} = \frac{1}{4}\left(F^{a}_{\mu\nu}\right)^2 - \frac{1}{2}\left(D_\mu \Phi^a\right)^2 + \ldots$$

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$$\delta^{(0)} \sim p \cdot \partial_\xi$$

The cubic coupling is the solution of simple linear homogeneous differential equations

Convenient simplification: the solution is a function of simple building blocks...

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$$\mathcal{G}^{(1)}_{123} = \xi_1 \cdot \xi_2\, \xi_3 \cdot p_{12} + \xi_2 \cdot \xi_3\, \xi_1 \cdot p_{23} + \xi_3 \cdot \xi_1\, \xi_2 \cdot p_{31}$$

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Starting from the quartic order the differential equation is non-homogeneous

In general: not easy to find a solution...

...but there is a clear logic!

Split it into non-local contributions $\mathcal{L}^{(4)} = \boxed{\mathcal{L}^{(4)}_{\text{part}}} + \mathcal{L}^{(4)}_{\text{homo}}$

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15

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$$p_i \cdot \partial_{\xi_i}\,\mathcal{L}^{(4)}_{\rm part}(\xi_1, \ldots) \approx -\,\delta^{(1)}\,\mathcal{L}^{(3)}$$

The particular solution to the non-homogeneous equation is always given by minus the current exchange contribution and is entirely specified by the lower-point couplings!

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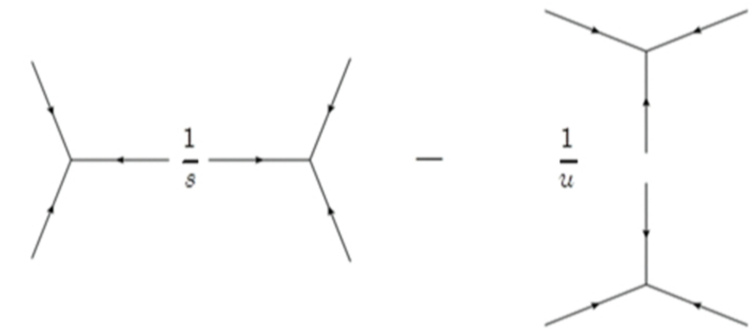
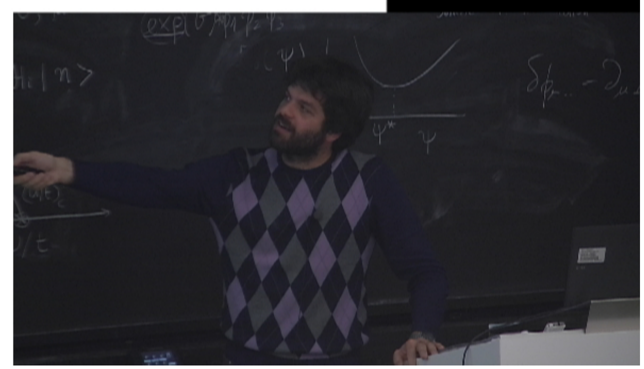
A toy model: Scalar Yang-Mills

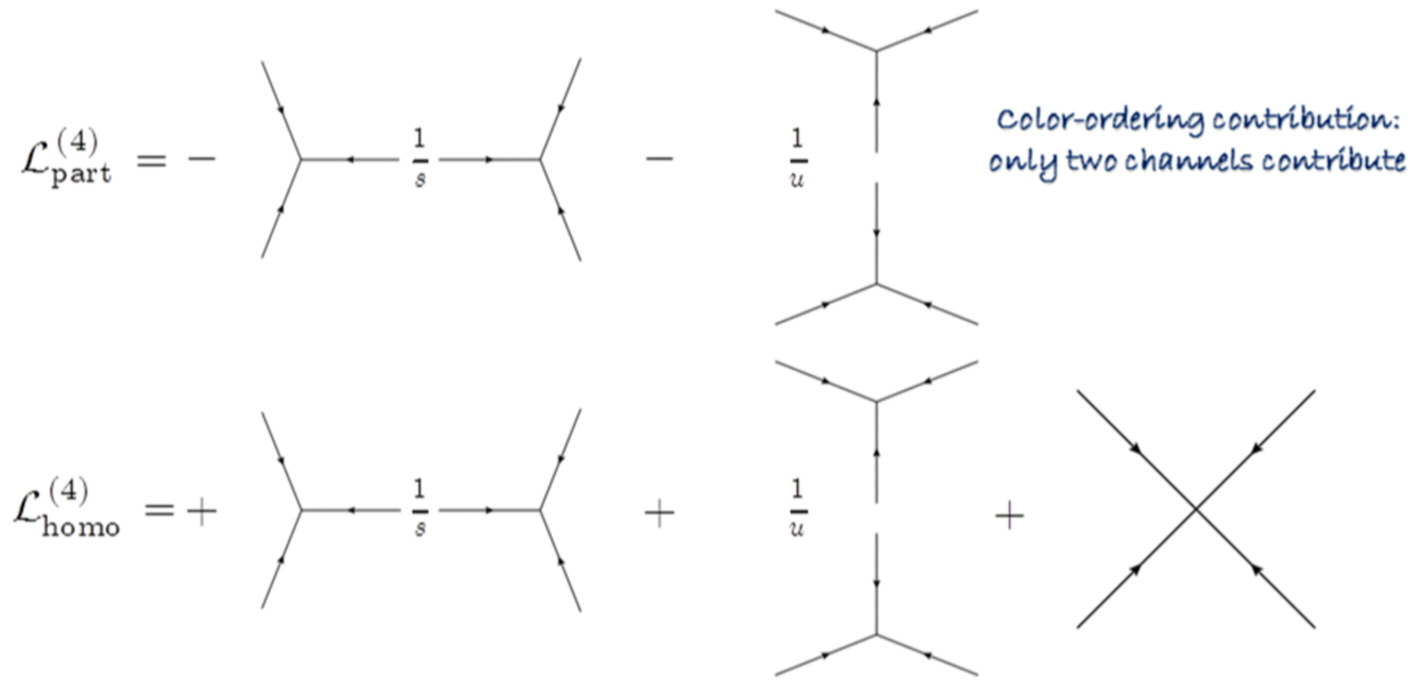
$$\mathcal{L}^{(4)}_{\rm part} = -$$



Color-ordering contribution:
only two channels contribute



A toy model: Scalar Yang-Mills



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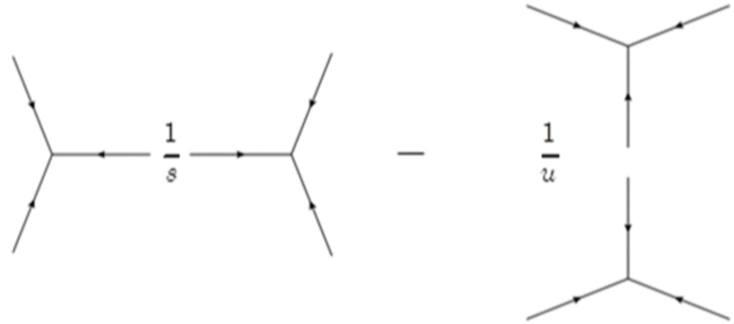
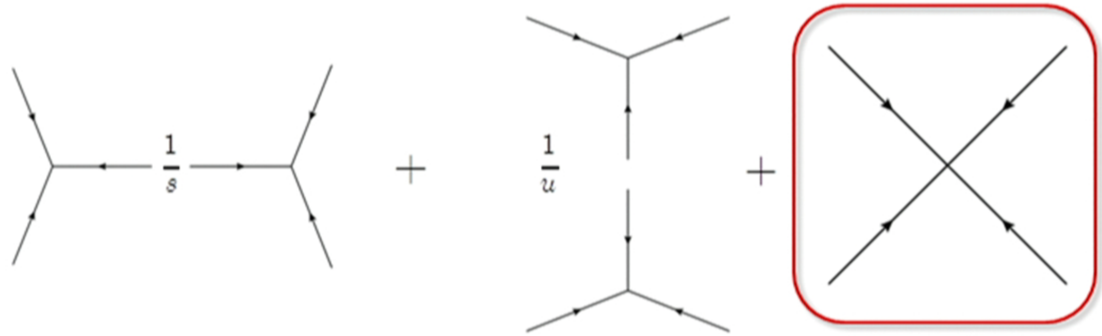
A toy model: Scalar Yang-Mills

$$\mathcal{L}^{(4)}_{\text{part}} = -$$



Color-ordering contribution: only two channels contribute

$$\mathcal{L}^{(4)}_{\text{homo}} = +$$

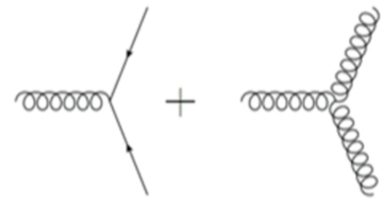


We can characterize any contact Lagrangian quartic coupling as the counterterm compensating the violation of the linearized gauge invariance of the current exchange part hep-th/1107.5843: M.T.

HS four-point functions

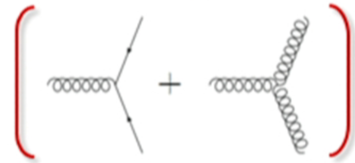
ST suggests how to provide an answer to all orders:

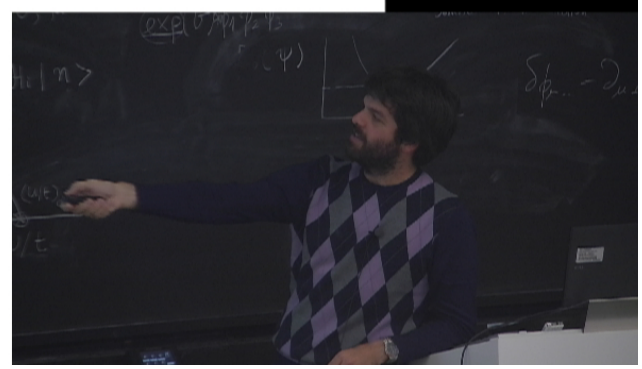
YM: Lego bricks of any scattering amplitude!

$G_{123} =$ 

Noether procedure indeed is solved by any generating function satisfying:

$$p_i \cdot \partial_{\xi_i} \mathcal{L}^{(3)}(\xi_1, \xi_2, \xi_3) \approx 0 \longrightarrow \mathcal{L}^{(3)} = \exp\left(\quad \right)$$





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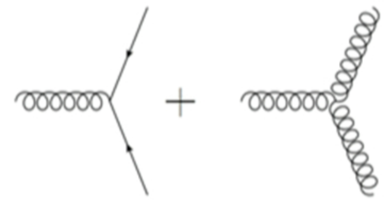

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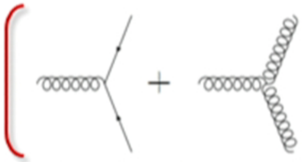
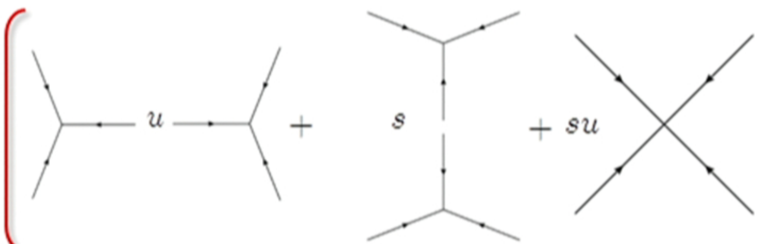


Something analogous happens at the quartic order but not at the level of couplings!

$$p_i \cdot \partial_{\xi_i} \mathcal{A}^{(4)}(\xi_1, \xi_2, \xi_3, \xi_4) \approx 0$$

hep-th/1107.5843: M.T.

$$\mathcal{A}^{(4)}(\xi_i) = \frac{1}{su} \exp\left(\quad \right)$$



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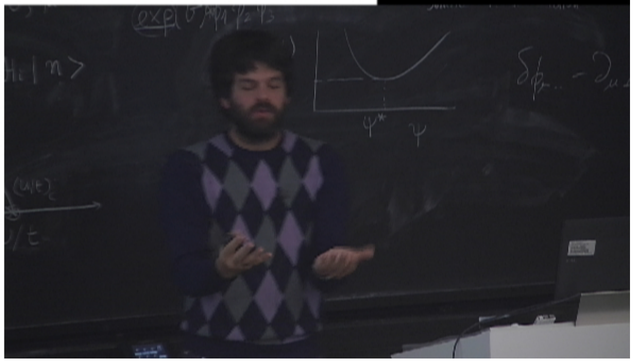
Colored spin-2 or Gravity?

Four spin-2 case: two different options!

→ At the fourth order two different color orderings are independent!



Colored spin-2 or Gravity?

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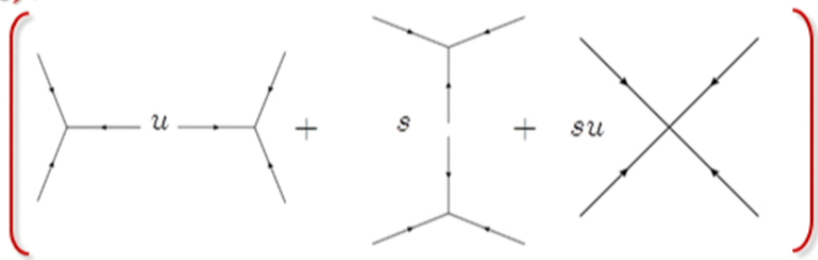
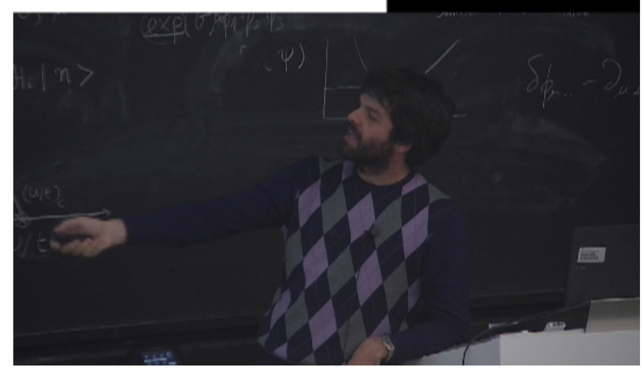
Planar! (Open-string like):

$$A_{1234} = \frac{a(s,t,u)}{su} \left(\quad \right)^2$$



Non-planar (closed-string like):



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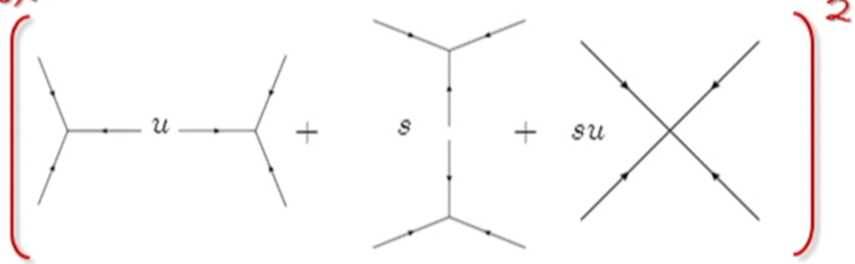
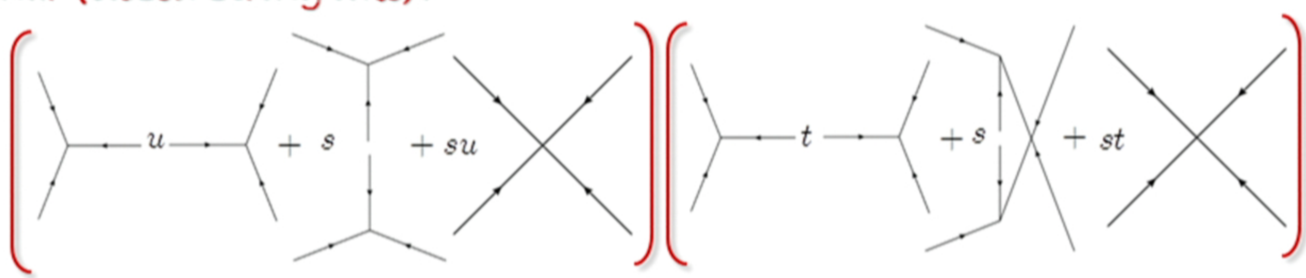


Non-planar (closed-string like):

$$A = \frac{a(s,t,u)}{stu}\left(\; \cdots \; \right)\left(\; \cdots \; \right)$$



+ cyclic

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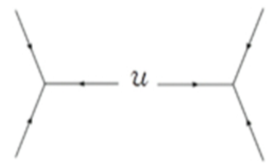
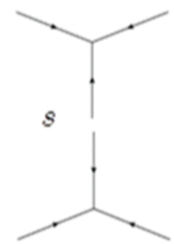
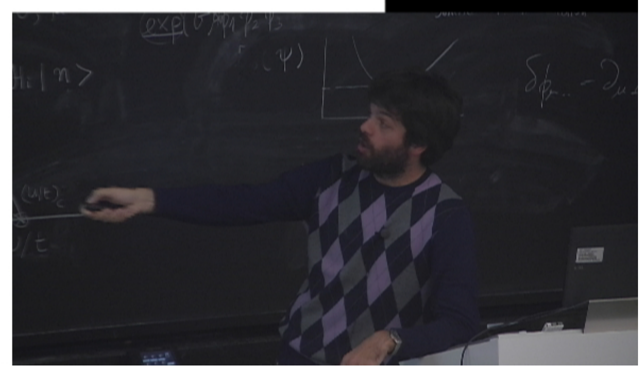
Colored spin-2 or Gravity?

Let us expand the result looking at the current exchange part!

$A_{1234} \sim \quad \frac{1}{su}\Bigg($ 

$\Bigg)^2 + \frac{1}{su}\Bigg($ 

$\Bigg)^2$ + Local terms



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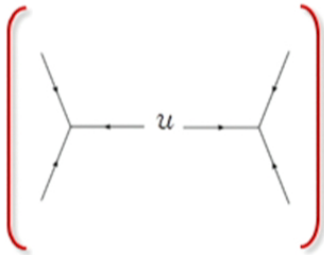
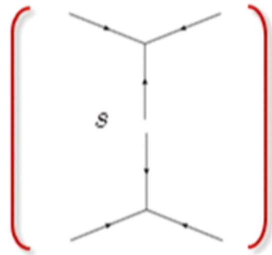
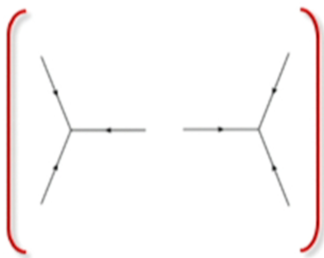
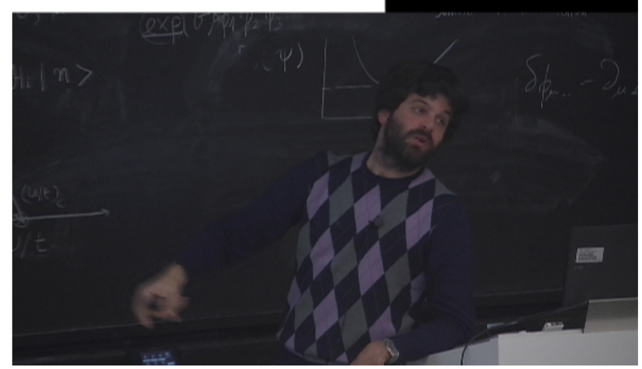
$A_{1234} \sim \frac{1}{su}$  $^2 + \frac{1}{su}$  ^{2} + Local terms

$A \sim \frac{1}{s}$  ^{2} + Local terms + cyclic channels



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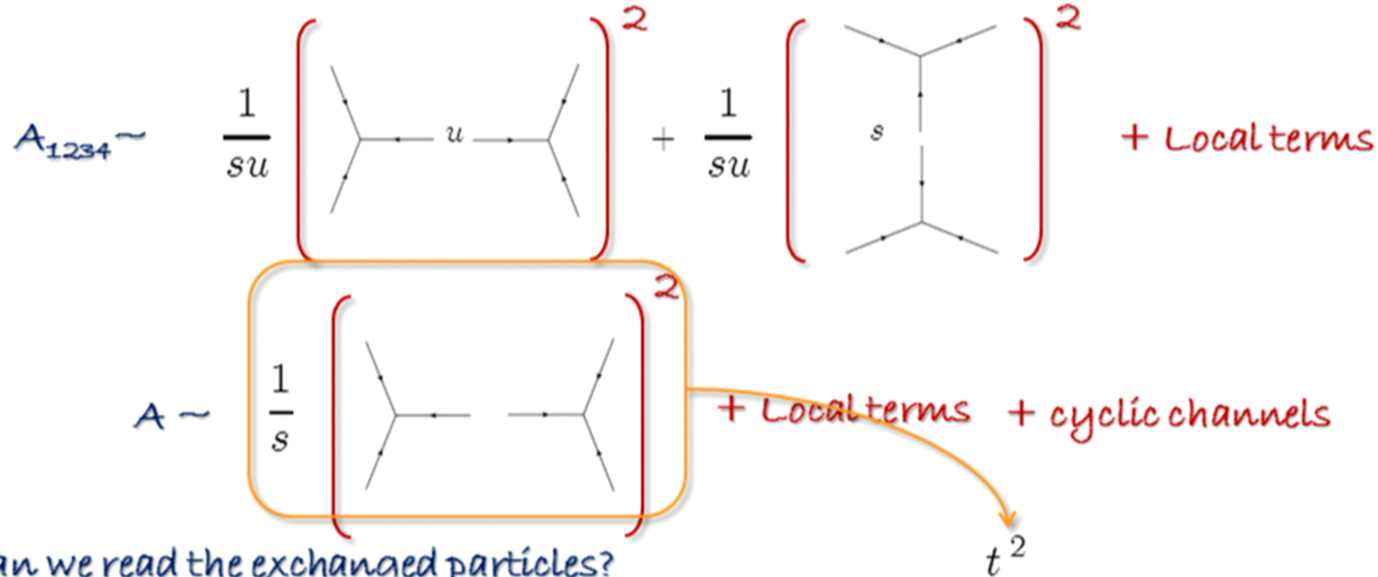

Can we read the exchanged particles?

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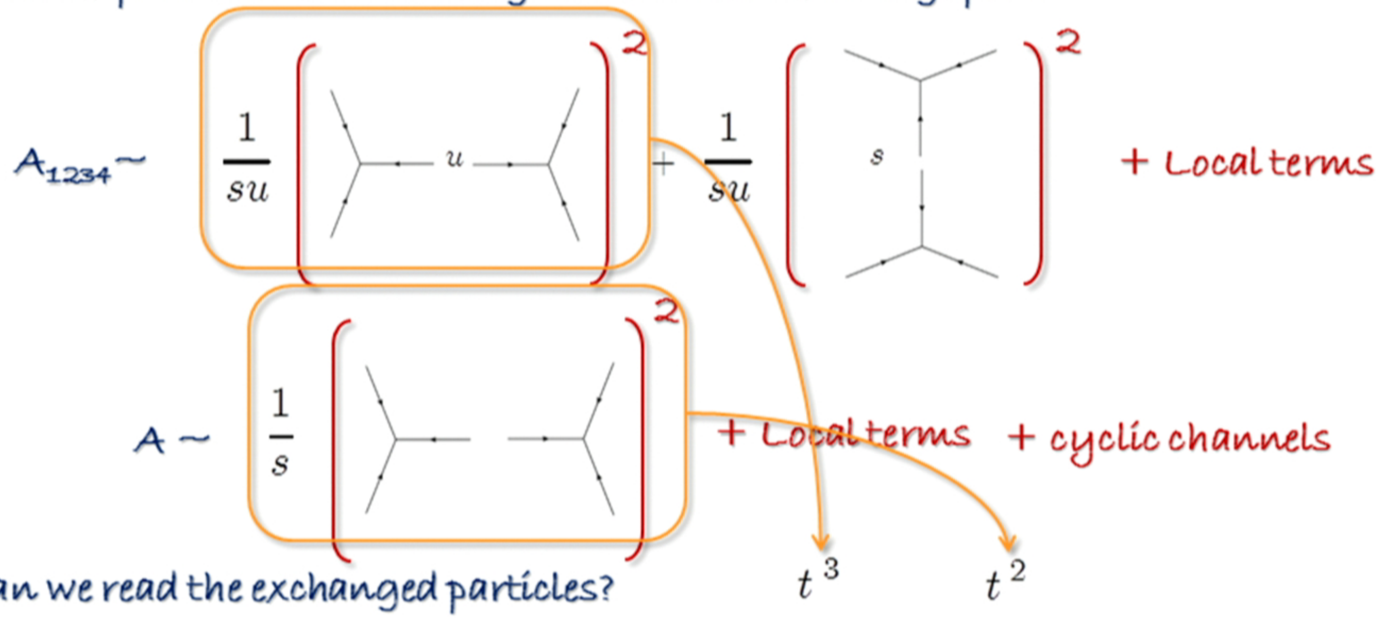

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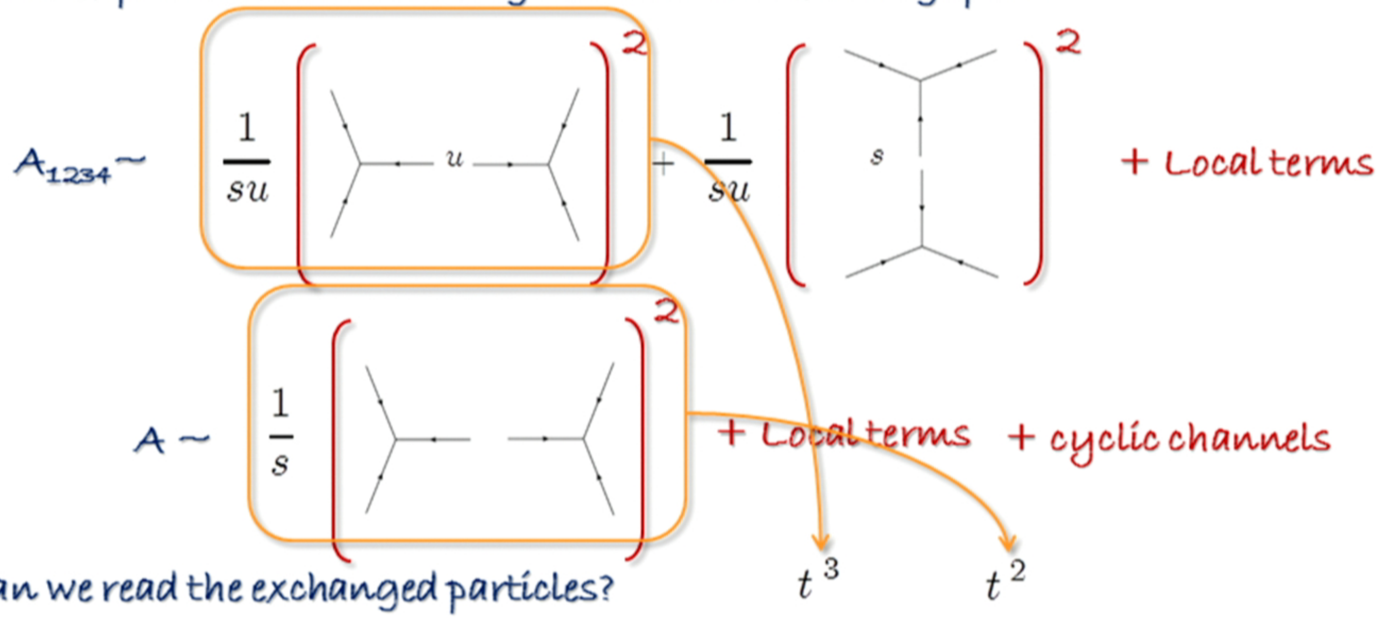

Can we read the exchanged particles?

The second amplitude is the Gravitational one...

...but the first is not!

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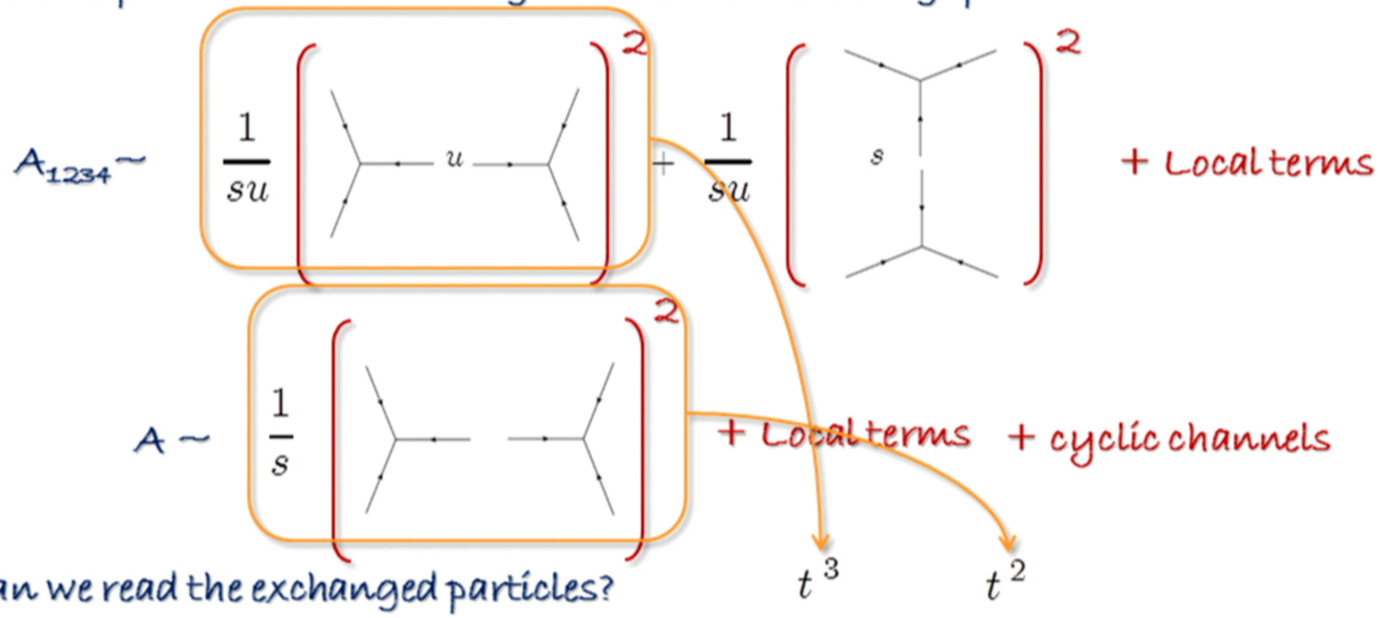

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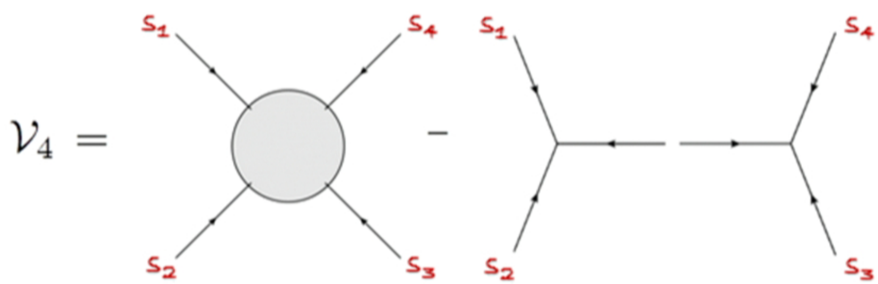
Massless spin-2 not necessarily Gravity in HS theories (mixing?) But non-localities are needed on the Lagrangian side (see Vasiliev's system...) 19

Why non-localities?

From the S-matrix perspective everything is standard...

...but if we extract the Lagrangian couplings explicit non-localities can arise!

Crucial observation (as in the YM case):



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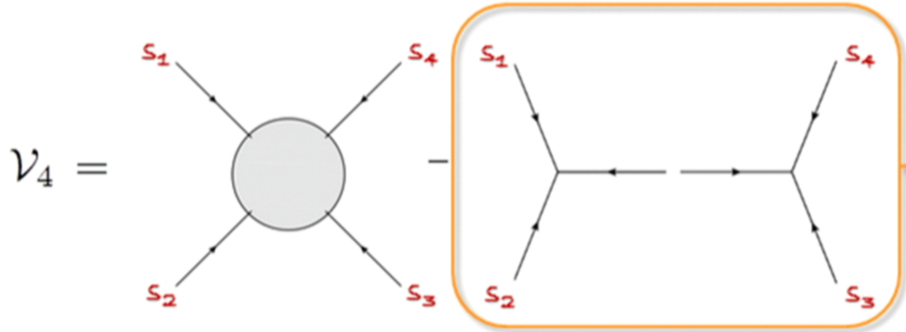

The Noether procedure requires the full current exchange!

Non local 4-p couplings If the first term does not factorize on all available exchanges!

Only possible way out to bypass Weinberg argument!

NON-LOCAL Geometry!

…but this kind of structure forces infinitely many spins propagating!

$$s_{\min} = \frac{1}{2}(s_1 + s_2 + s_3 + s_4) - 1\,(2)$$

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Outlook

- All three-point couplings (abelian and non-abelian!)
- Starting from ST all reference to the mechanical model completely eliminated
- Four-point functions and couplings by Noether procedure via Ward Identities
- Systematics of (A)dS couplings from the flat space analysis

BEHIND THE CORNER:

Classifying the consistent coupling functions

Full systematics of HS theories (work in progress)

For the near future:

- Compute n-point functions in (A)dS by radial reduction (fix boundary term)
- Compute the coupling function of Vasiliev's system with AdS/CFT
- Loop computations can shed light on mass-generation!
- Non-local HS Geometry!

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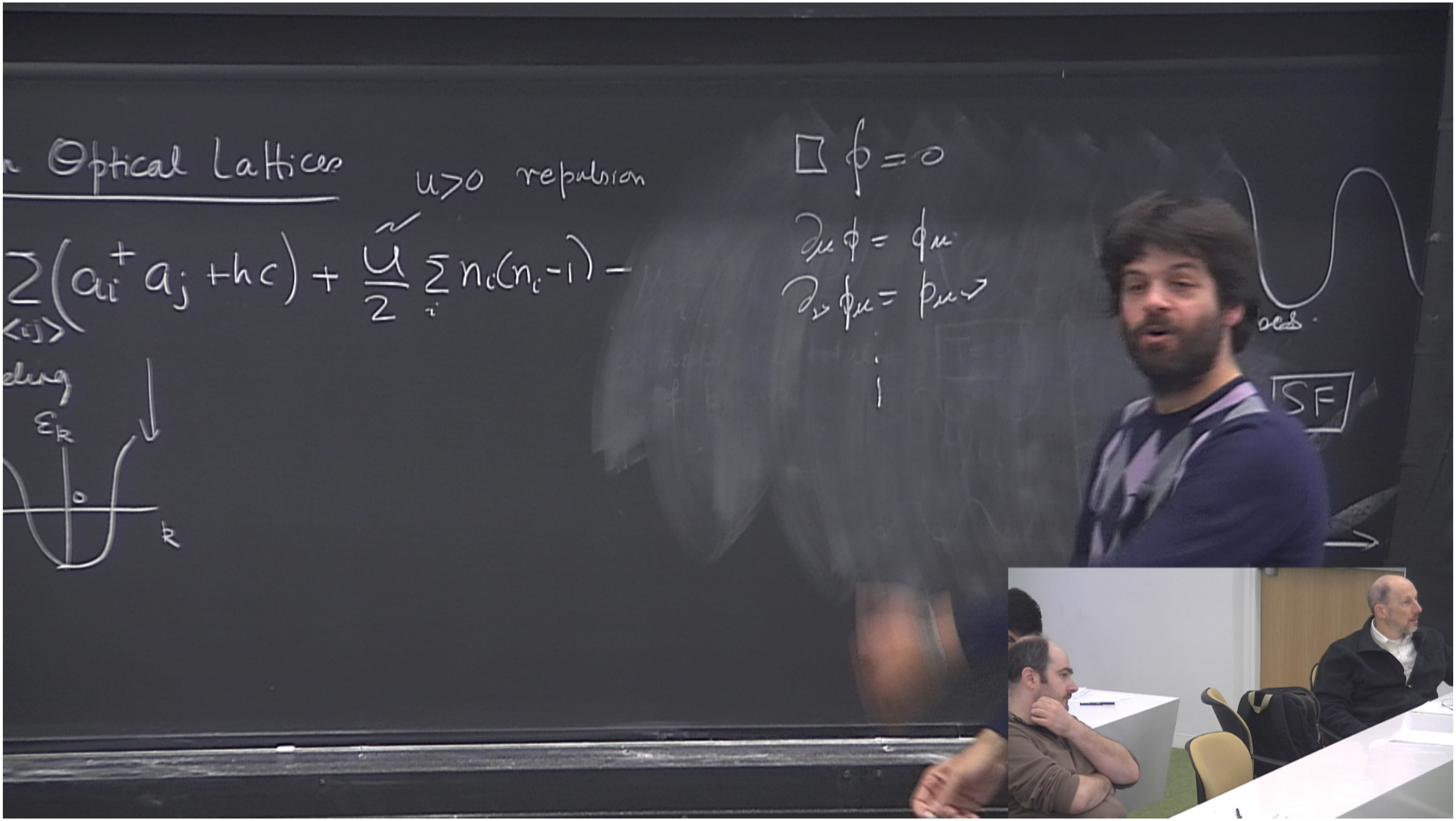
Optical Lattices
U>0 repulsion
$\sum_{\langle ij \rangle} (a_i^+ a_j + hc) + \frac{U}{2} \sum_i n_i(n_i - 1) -$
ε_k
k
$\Box \phi = 0$
$\partial_\mu \phi = \phi_\mu$
$\partial_\nu \phi_\mu = \phi_{\mu\nu}$
SF