Title: Non-Gaussianity in Single Field Inflation Beyond Slow-roll

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Abstract: In inflationary theories, single field models are typically considered subject to slow-roll conditions. In this talk I will present current observational constraints on deviations from slow-roll, e.g. bounds coming from strong coupling considerations, scale-dependent non-Gaussianities and the tensor-to-scalar ratio. These constraints still allow significant violations of slow-roll conditions. Focusing on non-Gaussian signals, I will discuss a variety of intriguing observable signatures that can be found for fast-rolling single fields.

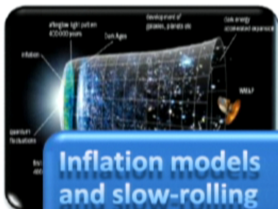

Non-Gaussianity
in single field inflation
beyond slow-roll
Imperial College London
Perimeter Institute, Strong Gravity Seminar, 10/11/2011

Non-Gaussianity in single field inflation beyond slow-roll

Based on: JN in progress; JN and Magueijo, **PRD 83;** Magueijo, JN, Piazza, **PRD 82**

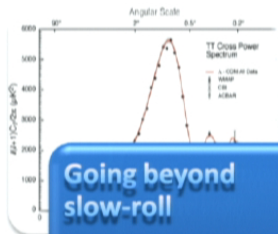
Johannes Noller, Imperial College London - **Perimeter Institute, Strong Gravity Seminar**, 10/11/2011

Outline



Inflation models and slow-rolling fields

- Model-space
- Observables
- Slow-roll signatures

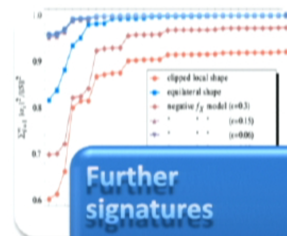


Going beyond slow-roll

- Setup
- Power spectra and scale invariance

Non-Gaussian signatures

- Suppressed amplitudes f_{NL}
- Blue scale-dependence n_{NG}
- Strong coupling considerations
- LSS observations
- Modified shapes of NG



Further signatures

- Tensor-to-scalar ratio r
- Modal expansions

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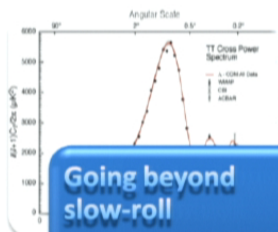

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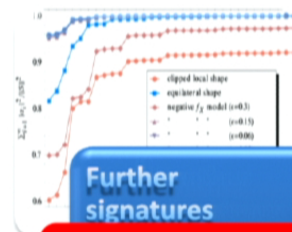

Further signatures
- Tensor-to-scalar ratio r
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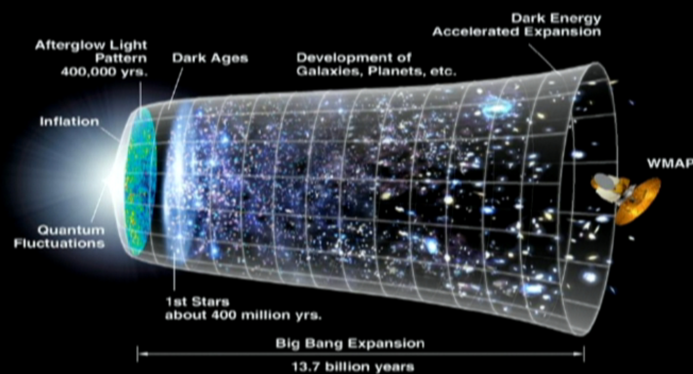
Slow-roll constraints

Single field inflation



Scalar field action: $S = \int d^4x\sqrt{-g}\left(\frac{M_{Pl}^2}{2}R - \frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi)\right)$

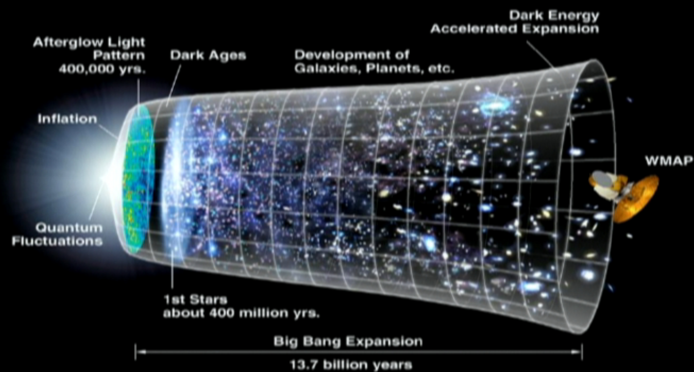
Higher derivatives: $S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + P(\phi, \partial\phi, \partial^2\phi, ..)\right]$

Effective theory: $S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2} + P(X,\phi)\right]$ $\qquad X = -\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$

cf. Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore '07, Armendariz-Picon, Damour, Mukhanov '99

The most general Lorentz invariant action for a single scalar field minimally coupled to gravity that contains at most first derivatives of the field

Single field inflation



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The most general Lorentz invariant action for a single scalar field minimally coupled to gravity that contains at most first derivatives of the field

A slow-rolling single field

Scalar field action: $S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{2} + P(X, \phi) \right]$

Slow-roll parameters: $\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H}, \ldots$ and $\epsilon_s \equiv \frac{\dot{c}_s}{c_s H}, \quad \eta_s \equiv \frac{\dot{\epsilon}_s}{\epsilon_s H}, \ldots$

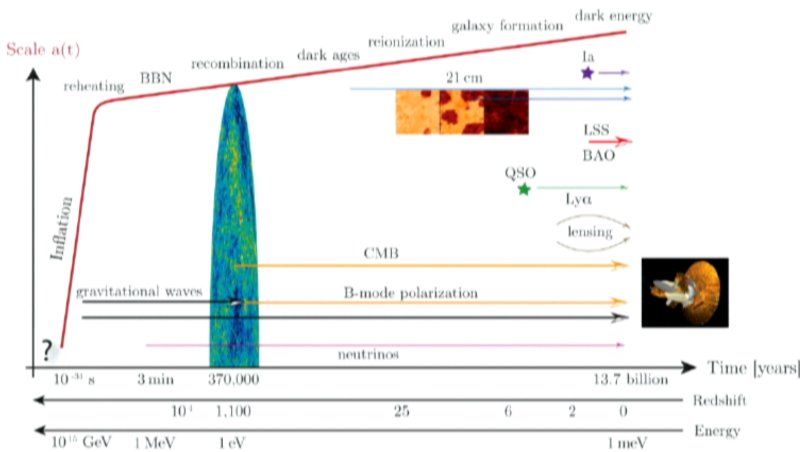
Inflation: $\epsilon < 1$

Slow-roll condition: $\epsilon_i \ll 1$ and $\eta_i \ll 1$

Primordial perturbations

Scalar perturbations in metric: $ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)a^2(t)\gamma_{ij}dx^i dx^j$

Gauge invariant curvature perturbation: $\zeta = \Phi \frac{5+3w}{3(1+w)} + \frac{2}{3(1+w)}\frac{\Phi'}{\mathcal{H}}$



Plot from: Baumann & Peiris '08

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^5\delta^3(\mathbf{k}_1+\mathbf{k}_2)P_\zeta\frac{1}{2k_1^3}$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta^{\,2}\frac{1}{\Pi_j k_j^3}\mathcal{A}$$

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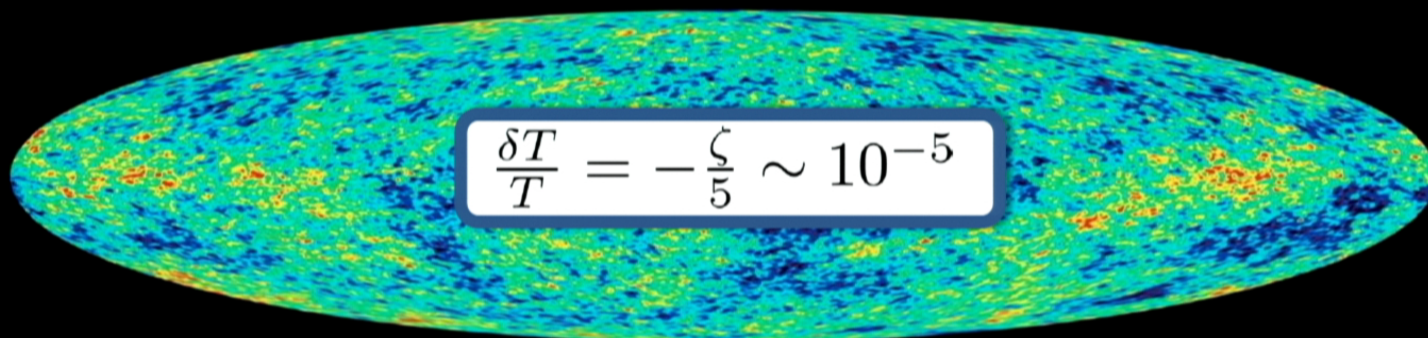

$$\frac{\delta T}{T} = -\frac{\zeta}{5} \sim 10^{-5}$$

Observables $\quad |\mathbf{P}_\zeta|, \mathbf{n_s}, \mathcal{A}, \mathbf{r}, \mathbf{n}_{NG} \ldots$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^5 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta \frac{1}{2k_1^3}$$

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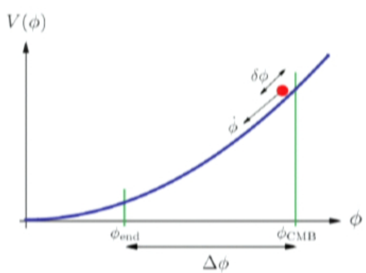
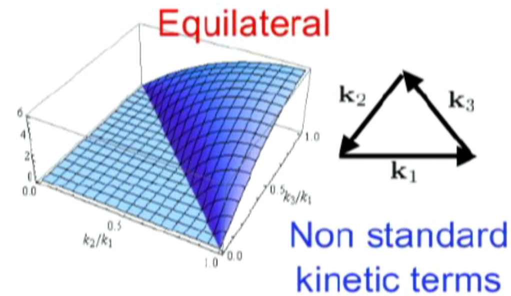
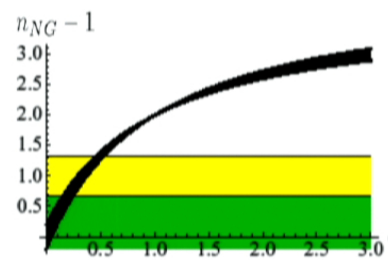
Predominantly equilateral non-Gaussianity, negligible running n_{NG}

cf. Khoury, Piazza '09 , Senatore, Smith, Zaldarriaga '09 , Chen et al. '08, Seery, Lidsey '05







Plots by: Baumann; Renaux-Petel, Chen; JN

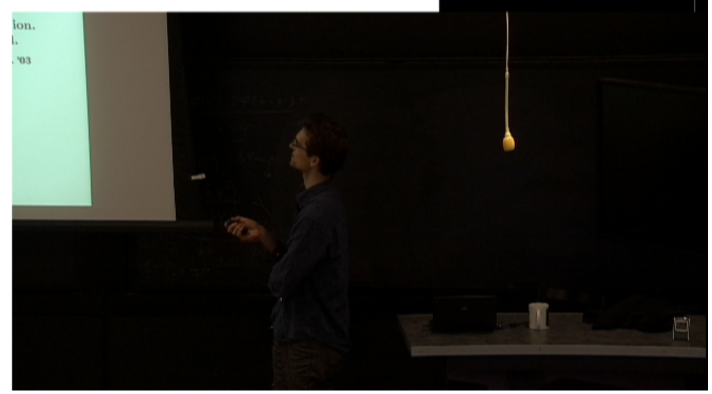
Relaxing slow-roll

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \epsilon_s \equiv \frac{\dot{c}_s}{c_s H}, \qquad \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} \sim 0, \quad \eta_s \equiv \frac{\dot{\epsilon_s}}{\epsilon_s H} \sim 0$$

No longer require $\epsilon_i \ll 1$, but retain slow-roll conditions for η_i.

Continuously modified dynamics for several e-folds of evolution.
Not **a temporary violation from e.g. features in the potential.**

cf. Dvorkin & Hu '09, Pahud et al. '08, Joy et al. '08, Covi et al. '06, WMAP Peiris et al. '03



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Two-point function: $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^5 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta \frac{1}{2k_1^3}$

Power spectrum: $P_\zeta \equiv \frac{1}{2\pi^2} k^3 |\zeta_k|^2 = \frac{(\epsilon_s + \epsilon - 1)^2\, 2^{2\nu-3}}{2(2\pi)^2 \epsilon} \frac{\bar{H}^2}{\bar{c}_s M_{\rm Pl}^2}$

Spectral index: $n_s - 1 \equiv \frac{d\ln P_\zeta}{d\ln k} = \frac{2\epsilon + \epsilon_s}{\epsilon + \epsilon_s - 1}$

cf. JN & Magueijo '11, Magueijo, JN, Piazza '10, Khoury & Piazza '09

Non-Gaussianities

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) P_\zeta^2 \frac{1}{\Pi_j k_j^3} \mathcal{A}$$

Non-linearity parameter f_{NL}:

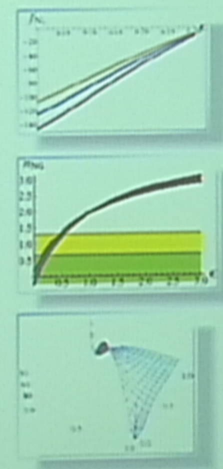
$$f_{NL}^{\rm equi} = 30 \left. \frac{\mathcal{A}}{K^3} \right|_{k_1=k_2=k_3}$$

The running n_{NG}:

$$n_{NG} - 1 = \frac{\mathrm{d}\log|f_{NL}|}{\mathrm{d}\log K}$$

The shape functions $\mathcal{A}$:

$$\mathcal{A} = \mathcal{A}_{\dot\zeta^3} + \mathcal{A}_{\zeta\dot\zeta^2} + \mathcal{A}_{\zeta(\partial\zeta)^2} + \mathcal{A}_{\dot\zeta\partial\zeta\partial\chi} + \mathcal{A}_{\epsilon^2}$$



cf. Chen et al. '08, Seery, Lidsey '05, Maldacena '03

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Non-linearity parameter $f_{\rm NL}$:

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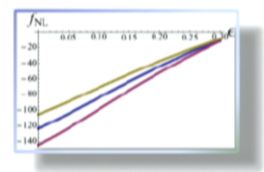
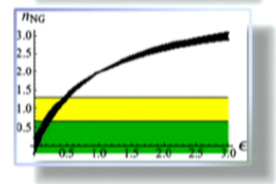
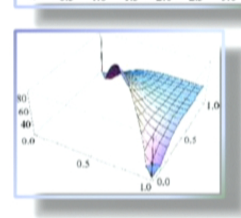






cf. Chen et al. '08, Seery, Lidsey '05, Maldacena '03

The cubic action

$$S_3 = M_{Pl}{}^2 \int \mathrm{d}t\mathrm{d}^3x \left\{ -a^3 \left[\Sigma \left(1 - \frac{1}{c_s^2} \right) + 2\lambda \right] \frac{\dot\zeta^3}{H^3} + \frac{a^3\epsilon}{c_s^4}(\epsilon - 3 + 3c_s^2)\zeta\dot\zeta^2 \right.$$
$$+ \frac{a\epsilon}{c_s^2}(\epsilon - 2\epsilon_s + 1 - c_s^2)\zeta(\partial\zeta)^2 - 2a\frac{\epsilon}{c_s^2}\dot\zeta(\partial\zeta)(\partial\chi)$$
$$\left. + \frac{a^3\epsilon}{2c_s^2}\frac{d}{dt}\left(\frac{\eta}{c_s^2}\right)\zeta^2\dot\zeta + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + 2f(\zeta)\left.\frac{\delta L}{\delta\zeta}\right|_1 \right\}$$

Seery, Lidsey '05

$$f_{\rm NL}^{\rm equi} = 30\frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3}$$

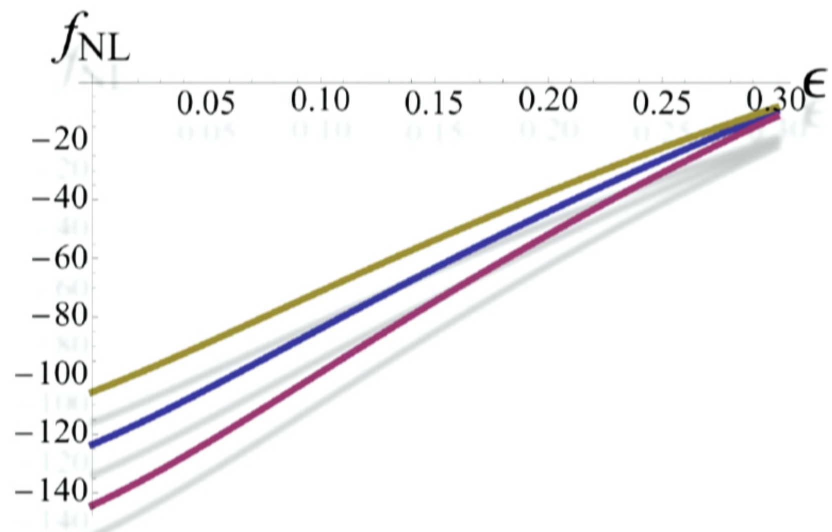
$$f_{\rm NL}^{\rm equi} \sim \mathcal{O}(c_s^{-2}) + \mathcal{O}(\tfrac{\lambda}{\Sigma})$$

$$c_s^2 = \frac{P_{,X}}{\rho_{,X}} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}},$$
$$\Sigma = XP_{,X} + 2X^2P_{,XX} = \frac{H^2\epsilon}{c_s^2},$$
$$\lambda = X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX},$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\tfrac{R}{2} + P(X,\phi)\right]$$

$\mathrm{f_{NL}}$ and fast-roll suppression

$$f_{NL}^{\frac{\lambda}{\Sigma}\gg 1} \sim 3\frac{1+\epsilon}{n_s-2}\frac{\lambda}{\Sigma}\cos\left(\alpha_2\frac{\pi}{2}\right)\Gamma(3+\alpha_2) \qquad \alpha_2 = \frac{2\epsilon-\epsilon_s}{\epsilon_s+\epsilon-1}$$



Fast-roll generically suppresses the amplitude $\mathcal{A}$

cf. JN & Magueijo '11, Magueijo, JN, Piazza '10, Khoury & Piazza '09

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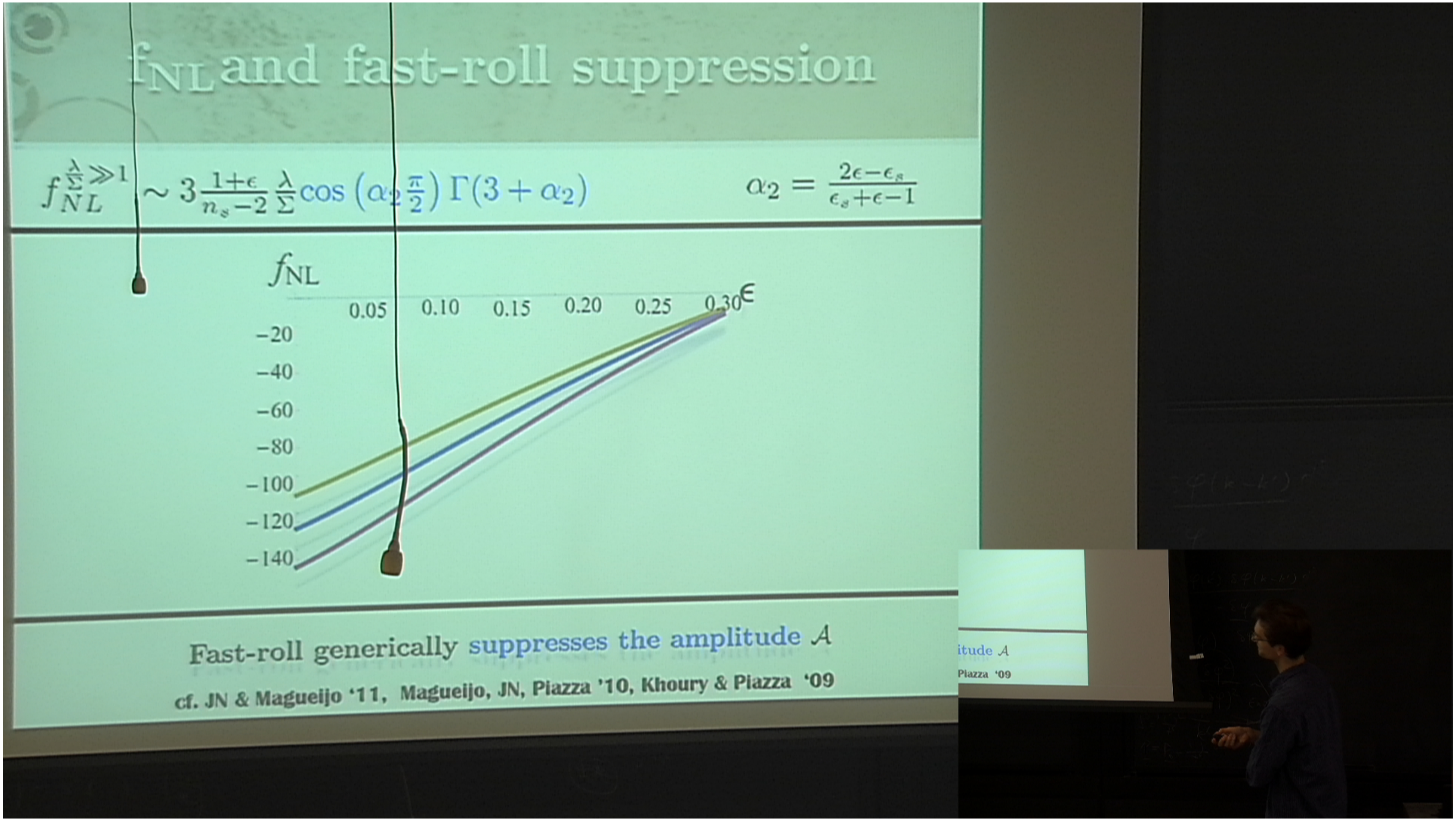

fNL and fast-roll suppression
$f_{NL}^{\frac{\lambda}{\Sigma}\gg 1} \sim 3\frac{1+\epsilon}{n_s-2}\frac{\lambda}{\Sigma}\cos\left(\alpha_2\frac{\pi}{2}\right)\Gamma(3+\alpha_2)$
$\alpha_2 = \frac{2\epsilon-\epsilon_s}{\epsilon_s+\epsilon-1}$
f_{NL}
0.05 0.10 0.15 0.20 0.25 0.30 ϵ
−20
−40
−60
−80
−100
−120
−140
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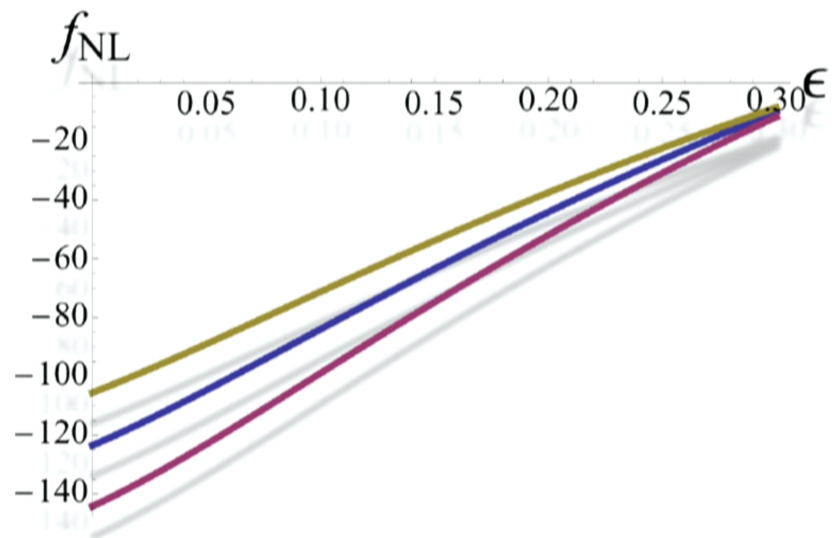

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f_{NL} and fast-roll suppression

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) {P_\zeta}^2 \frac{1}{\Pi_j k_j^3} \mathcal{A}$$

$$\mathcal{A} = \mathcal{A}_{\dot\zeta^3} + \mathcal{A}_{\zeta\dot\zeta^2} + \mathcal{A}_{\zeta(\partial\zeta)^2} + \mathcal{A}_{\dot\zeta\partial\zeta\partial\chi} + \mathcal{A}_{\epsilon^2}$$

$$\mathcal{I}_{\dot\zeta^3}(\alpha) = \cos\frac{\alpha\pi}{2}\Gamma(3+\alpha)\frac{k_1^2 k_2^2 k_3^2}{K} ;$$

$$\mathcal{I}_{\zeta\dot\zeta^2}(\alpha) = \cos\frac{\alpha\pi}{2}\Gamma(1+\alpha)\left[(2-\alpha)\frac{1}{K}\sum_{i<j} k_i^2 k_j^2 - (1+\alpha)\frac{1}{K^2}\sum_{i\neq j} k_i^2 k_j^3\right] ;$$

$$\mathcal{I}_{\zeta(\partial\zeta)^2}(\alpha) = -\cos\frac{\alpha\pi}{2}\Gamma(1+\alpha)\left(\sum_i k_i^2\right)\left[\frac{K}{\alpha-1} + \frac{1}{K}\sum_{i<j} k_i k_j + \frac{1+\alpha}{K^2} k_1 k_2 k_3\right]$$

$$= \cos\frac{\alpha\pi}{2}\Gamma(1+\alpha)\left[\frac{1}{1-\alpha}\sum_j k_j^3 + \frac{4+2\alpha}{K}\sum_{i<j} k_i^2 k_j^2 - \frac{2+2\alpha}{K^2}\sum_{i\neq j} k_i^2 k_j^3 + \frac{\alpha}{(1-\alpha)}\sum_{i\neq j} k_i k_j^2 - \alpha k_1 k_2 k_3\right] ;$$

$$\mathcal{I}_{\dot\zeta\partial\zeta\partial\chi}(\alpha) = \cos\frac{\alpha\pi}{2}\Gamma(1+\alpha)\left[\sum_j k_j^3 + \frac{\alpha-1}{2}\sum_{i\neq j} k_i k_j^2 - 2\frac{1+\alpha}{K^2}\sum_{i\neq j} k_i^2 k_j^3 - 2\alpha k_1 k_2 k_3\right] ;$$

$$\mathcal{I}_{\epsilon^2}(\alpha) = \cos\frac{\alpha\pi}{2}\Gamma(1+\alpha)(2-\alpha/2)\left[\sum_j k_j^3 - \sum_{i\neq j} k_i k_j^2 + 2k_1 k_2 k_3\right] ,$$

$\alpha_1 = \frac{2\epsilon+\epsilon_s}{\epsilon_s+\epsilon-1} = n_s - 1$ $\qquad$ $\alpha_2 = \frac{2\epsilon-\epsilon_s}{\epsilon_s+\epsilon-1}$

f_{NL} and fast-roll suppression

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle_{\zeta\dot{\zeta}^2} = i(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)u_{k_1}(0)u_{k_2}(0)u_{k_3}(0)$$

$$\times\int_{-\infty+i\epsilon}^{0} \mathrm{d}y\frac{c_s}{a}\frac{a^3\epsilon}{c_s^4}(\epsilon-3+3c_s^2)u_{k_1}^*(y)\frac{du_{k_2}^*(y)}{dy}\frac{du_{k_3}^*(y)}{dy} + \text{perm.} + \text{c.c.}$$

Conformal time behaviour: $\mathrm{d}y = c_s \mathrm{d}\tau$

$$a\sim(-\tau)^{\frac{1}{\epsilon-1}} \quad ; \quad c_s\sim(-\tau)^{\frac{\epsilon_s}{\epsilon-1}} \quad ; \quad H\sim(-\tau)^{\frac{-\epsilon}{\epsilon-1}},$$

Commutation relations:

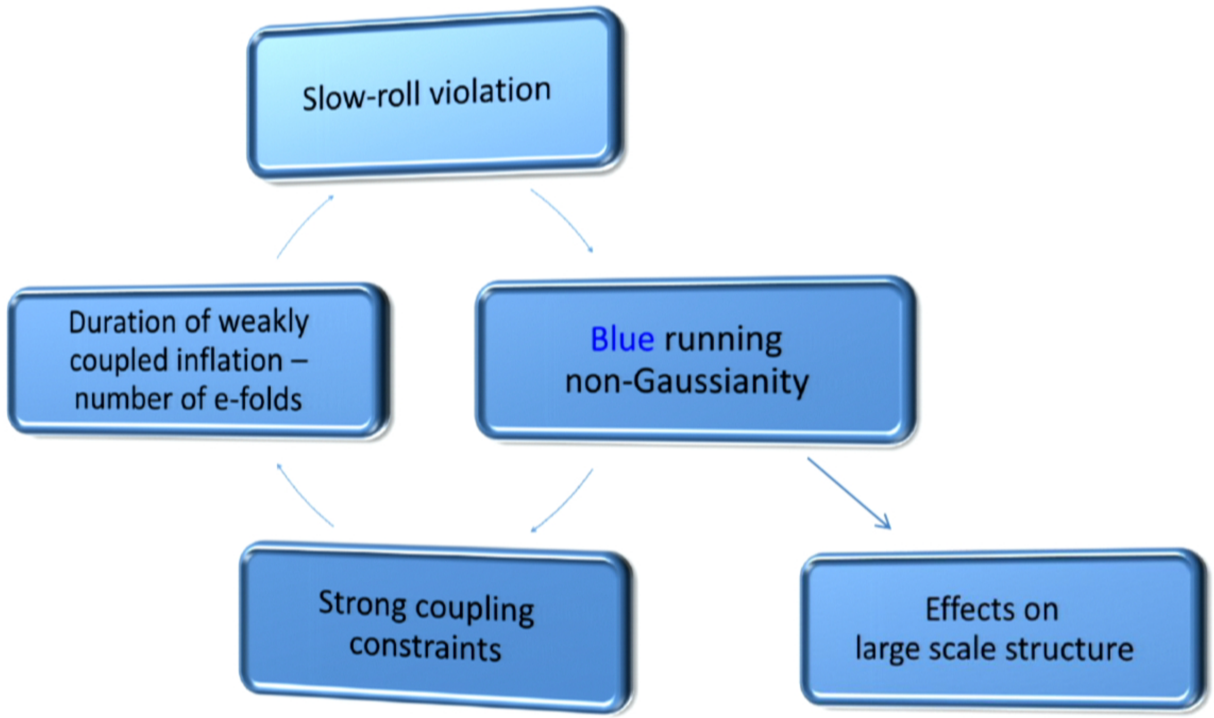
$$\zeta(y,\mathbf{k}) = u_k(y)a(\mathbf{k}) + u_k^*(y)a^\dagger(-\mathbf{k}) \quad , \quad [a(\mathbf{k}),a^\dagger(\mathbf{k}')] = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$$

The "propagator":

$$u_k(y) = \frac{c_s^{1/2}}{aM_{\rm Pl}2^{3/2}}\sqrt{\frac{\pi}{\epsilon}}\sqrt{-y}H_\nu^{(1)}(-ky) \qquad n_s-1 = 3-2\nu$$

cf. Mukhanov, Feldman, Brandenberger '90

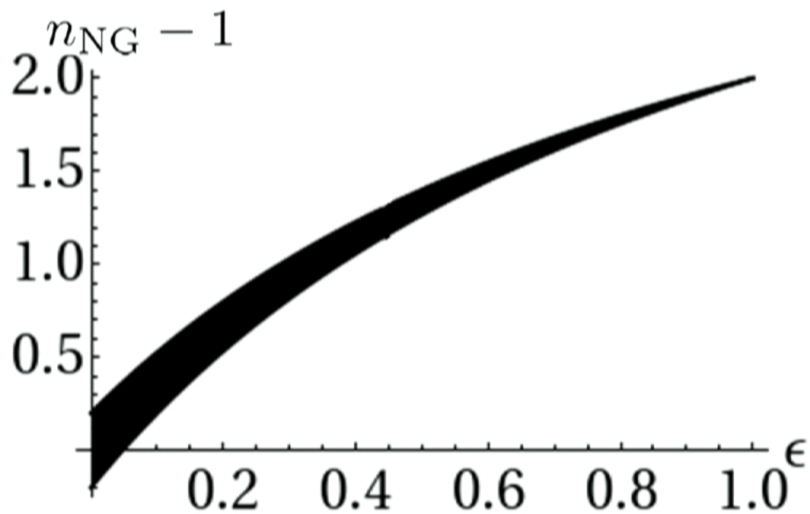
Scale-dependence



A blue running

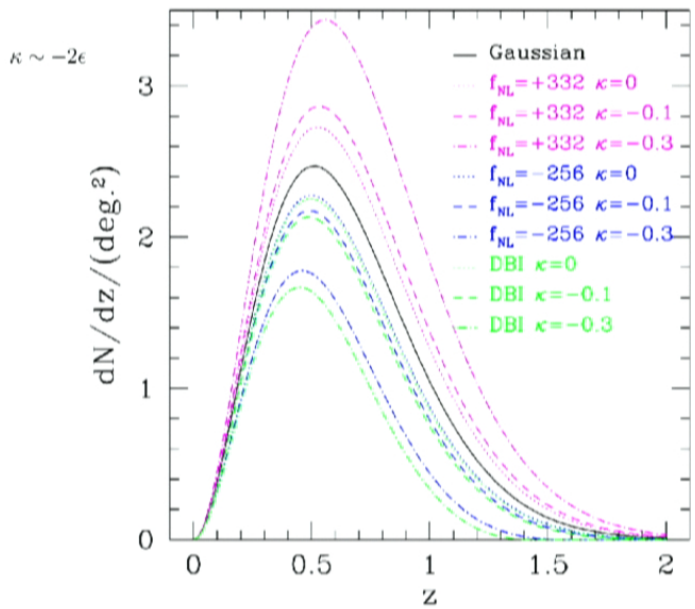
$$f_{\rm NL} = 30\frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3} \qquad n_{\rm NG} - 1 \equiv \frac{{\rm d}\ln|f_{\rm NL}|}{{\rm d}\ln K}$$

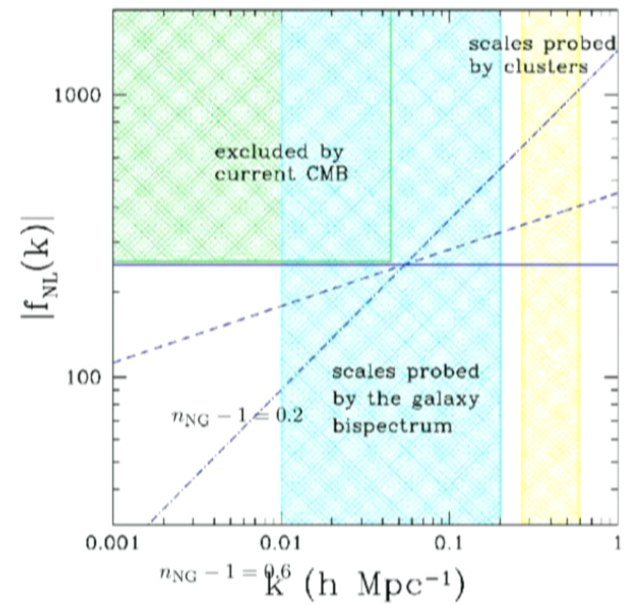
Fast-roll generically gives rise to a large blue running n_{NG}.



$$n_{\rm NG} - 1 \overset{\rm SI}{\sim} \frac{4\epsilon}{1+\epsilon}$$

Running non-Gaussianities





$$M > M_{lim} = 1.75 \cdot 10^{14} h^{-1} M_{sun}$$

Plots from: LoVerde, Miller, Shandera, Verde '07
cf. Desjacques & Seljak '10, Liguori et al. '10, Schmidt & Kamionkowski '10, Bartolo et al. '10, Afshordi & Tolley '08, Sefusatti et al '07, Matarrese et al. '00, Verde et al. '00,....

Strong coupling constraints

$$f_{\rm NL} = 30\frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3} \qquad n_{\rm NG} - 1 \equiv \frac{\mathrm{d}\ln|f_{\rm NL}|}{\mathrm{d}\ln K}$$

Fast-roll generically gives rise to a large blue running n_{NG}.

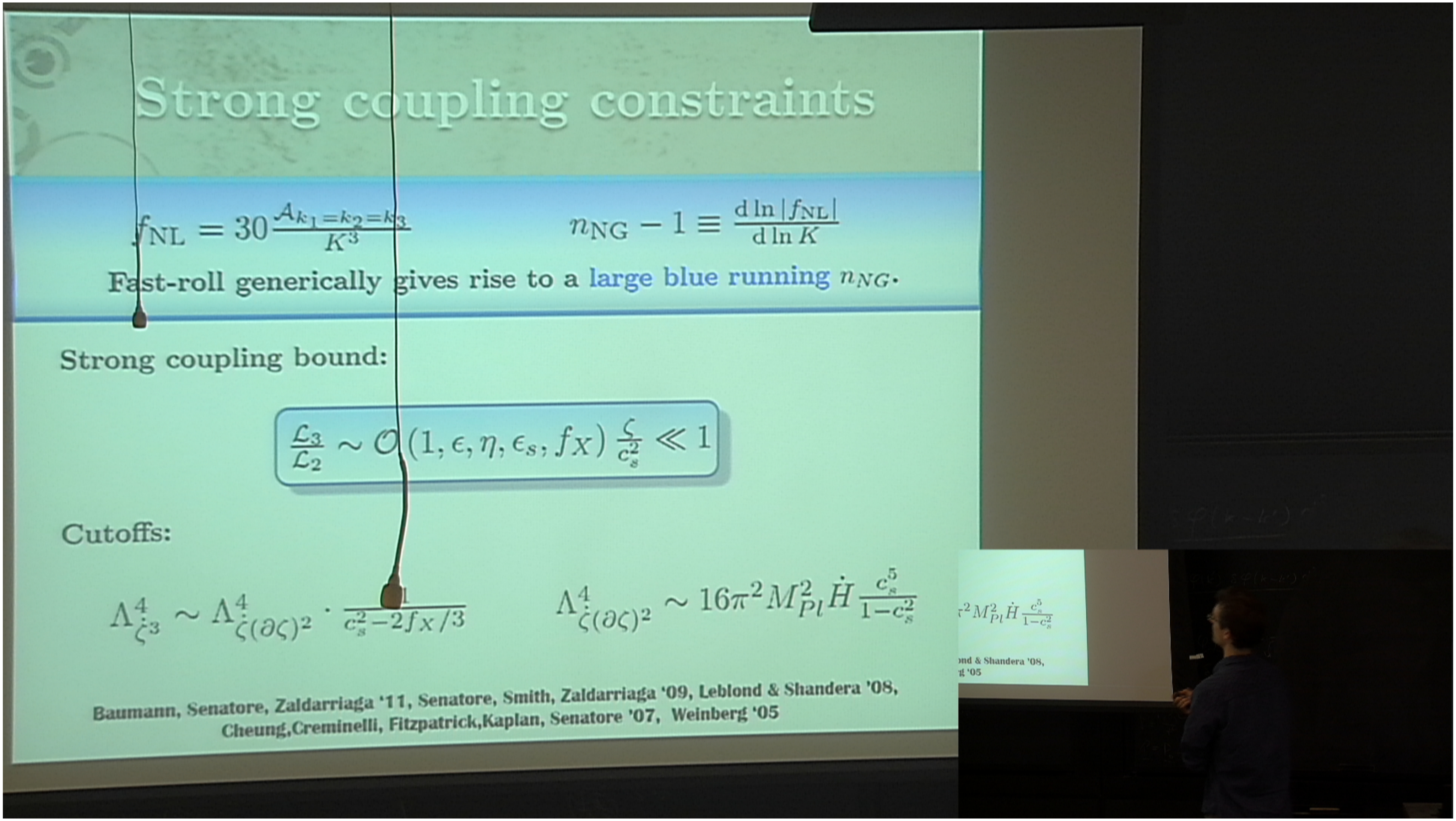
Strong coupling bound:

$$\frac{\mathcal{L}_3}{\mathcal{L}_2} \sim \mathcal{O}\left(1, \epsilon, \eta, \epsilon_s, f_X\right) \frac{\zeta}{c_s^2} \ll 1$$

Cutoffs:

$$\Lambda^4_{\dot{\zeta}^3} \sim \Lambda^4_{\dot{\zeta}(\partial\zeta)^2} \cdot \frac{1}{c_s^2 - 2f_X/3} \qquad \Lambda^4_{\dot{\zeta}(\partial\zeta)^2} \sim 16\pi^2 M_{Pl}^2 \dot{H} \frac{c_s^5}{1-c_s^2}$$

Baumann, Senatore, Zaldarriaga '11, Senatore, Smith, Zaldarriaga '09, Leblond & Shandera '08, Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore '07, Weinberg '05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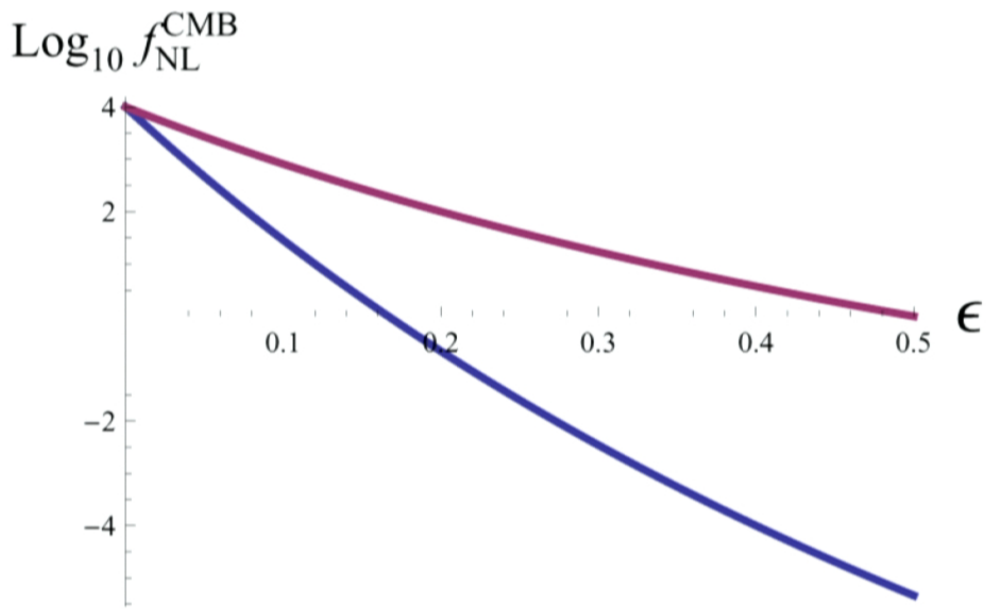
Duration of weak coupling regime

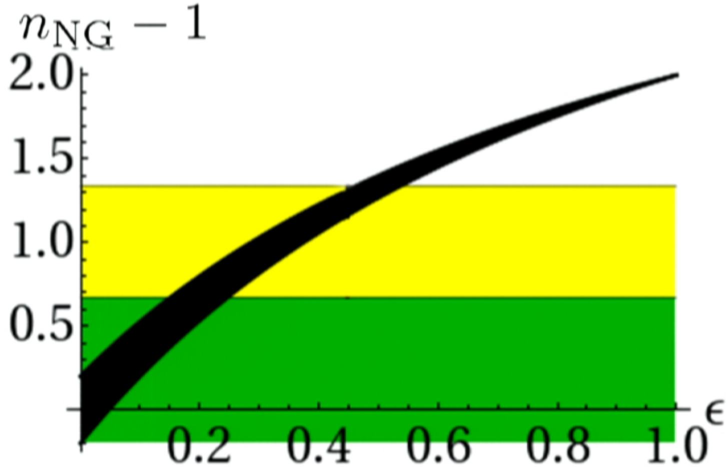


$$f_{\mathrm{NL}}(K_{\mathrm{CMB}}) \sim \left(\frac{K^*}{K_{\mathrm{CMB}}}\right)^{-(n_{\mathrm{NG}}-1)} f_{\mathrm{NL}}(K^*)$$

Running non-Gaussianities

$f_{\rm NL} = 30 \frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3}$ $\qquad$ $n_{\rm NG} - 1 \equiv \frac{{\rm d} \ln |f_{\rm NL}|}{{\rm d} \ln K}$

Fast-roll generically gives rise to a large blue running n_{NG}.



cf. JN & Magueijo '11 , Byrnes et al. '10, Khoury & Piazza '09

$\epsilon \lesssim 0.3$ $\qquad$ $f_{NL}(\text{CMB}) \lesssim \mathcal{O}(100)$

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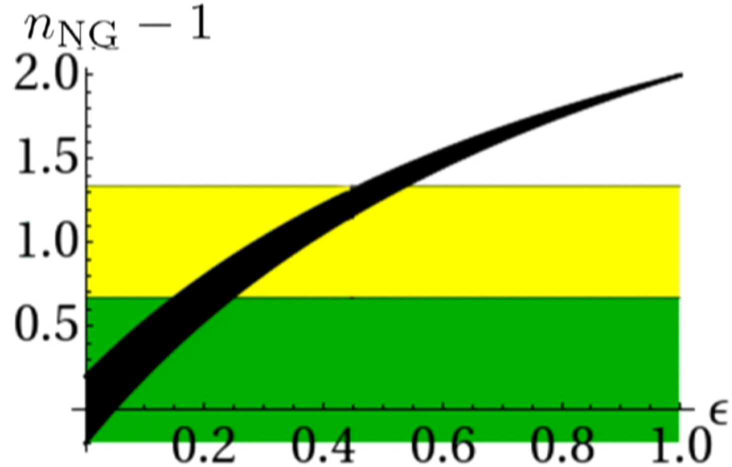

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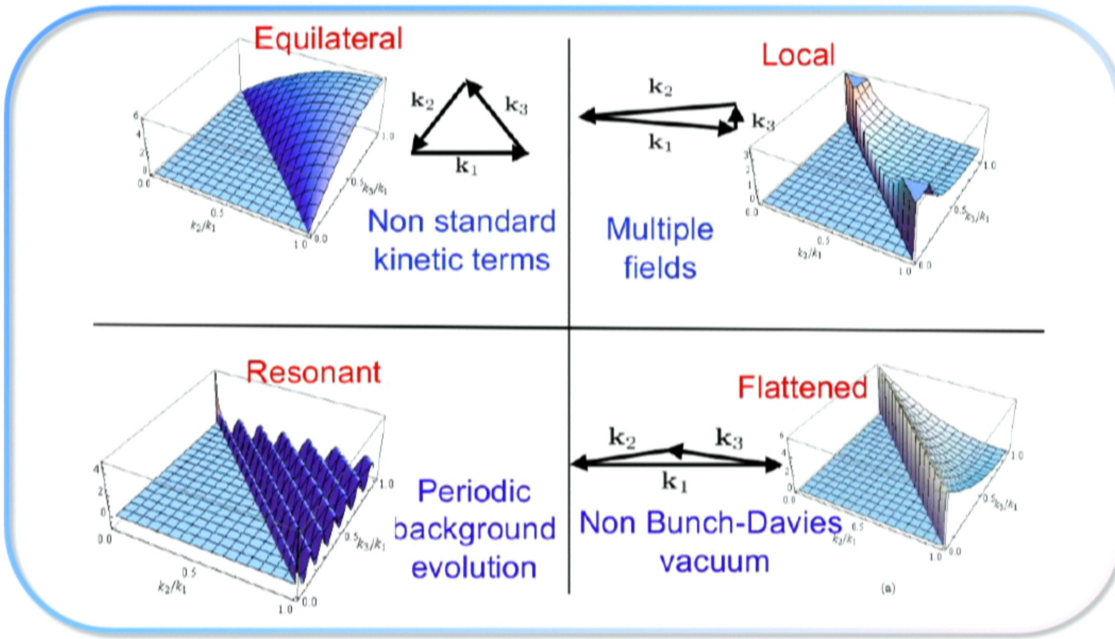
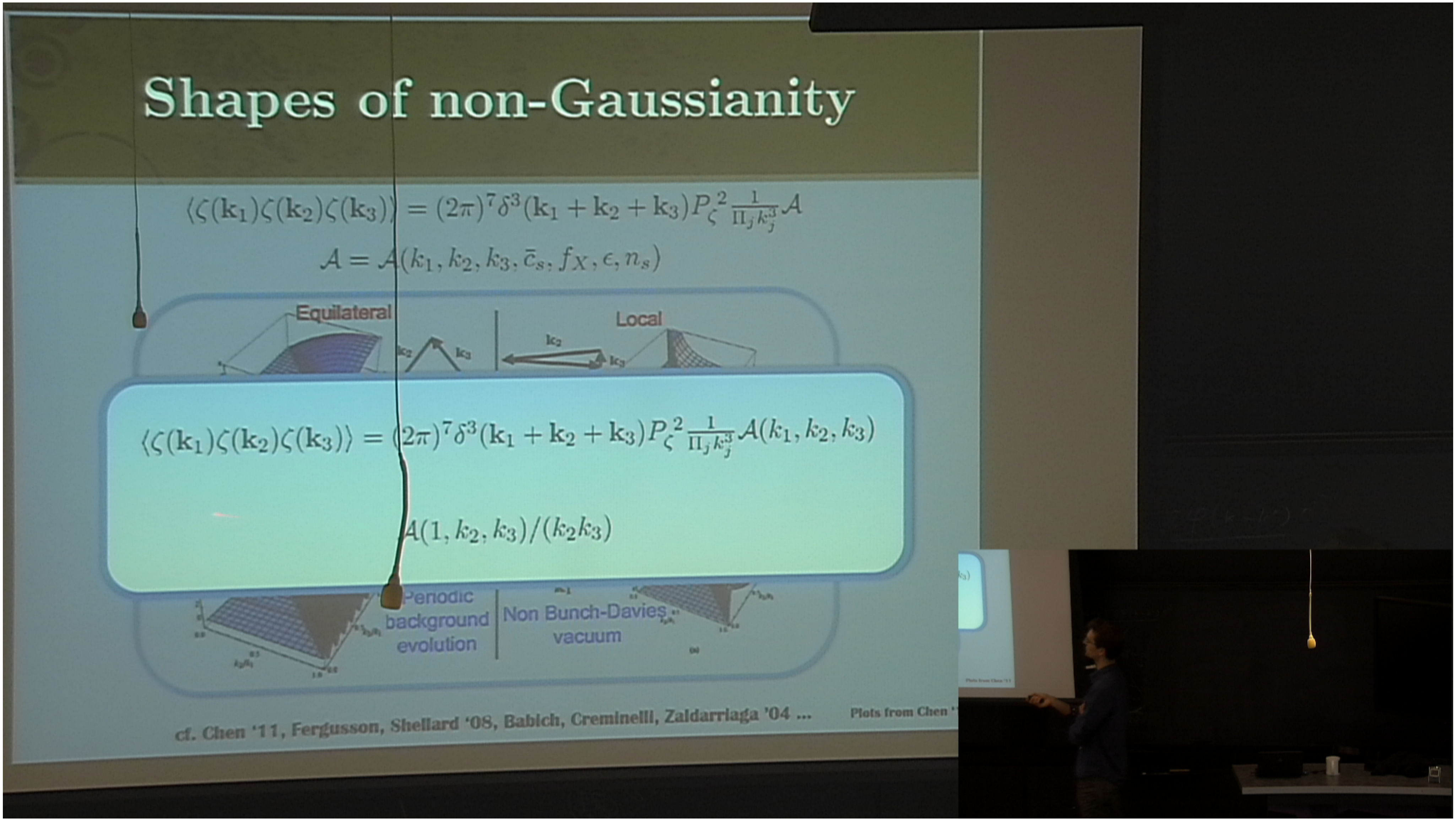
Shapes of non-Gaussianity

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta{}^2\frac{1}{\Pi_j k_j^3}\mathcal{A}$$

$$\mathcal{A} = \mathcal{A}(k_1, k_2, k_3, \bar{c}_s, f_X, \epsilon, n_s)$$



cf. Chen '11, Fergusson, Shellard '08, Babich, Creminelli, Zaldarriaga '04 ... **Plots from Chen '11**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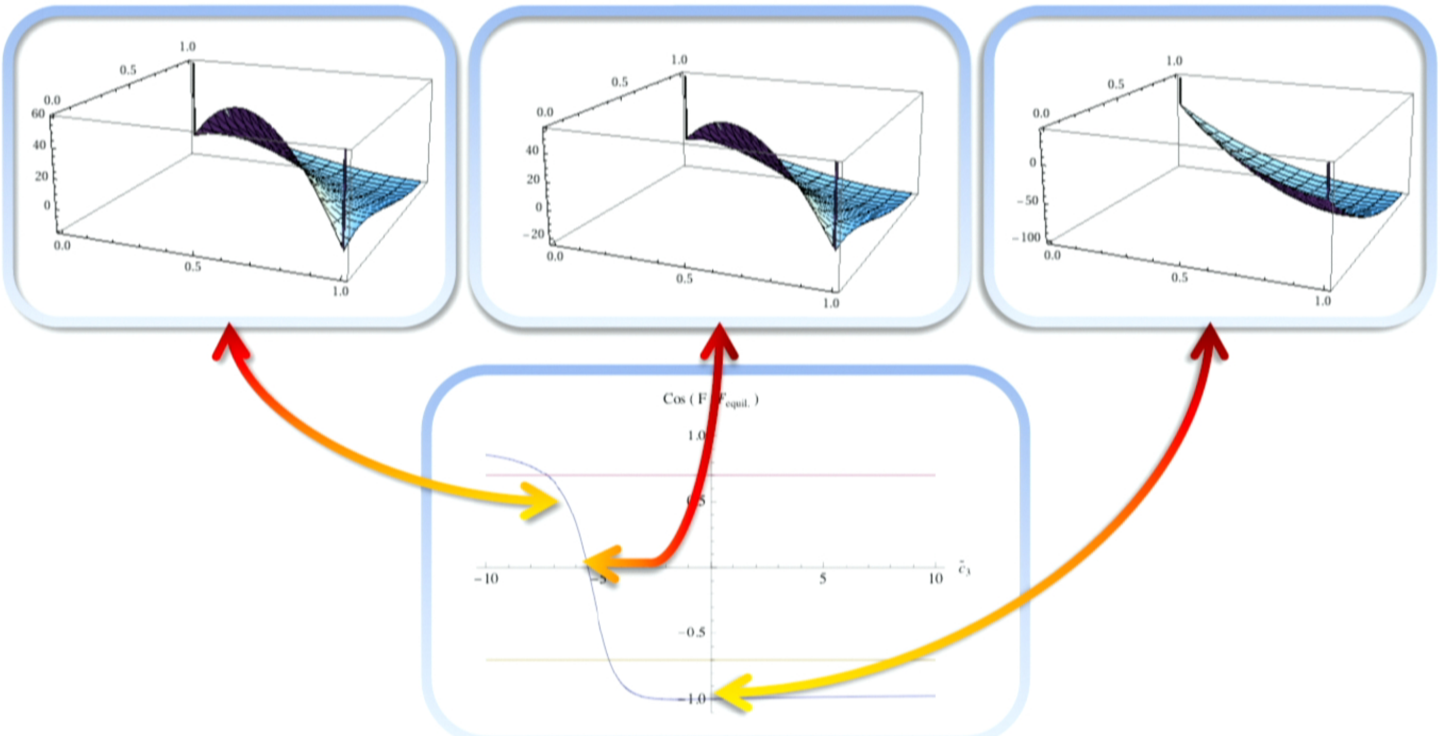
$\mathcal{A} = \mathcal{A}(k_1, k_2, k_3, \bar{c}_s, f_X, \epsilon, n_s)$
Equilateral
Local
$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta^2\frac{1}{\Pi_j k_j^3}\mathcal{A}(k_1, k_2, k_3)$
$\mathcal{A}(1, k_2, k_3)/(k_2 k_3)$
Periodic background evolution
Non Bunch-Davies vacuum
cf. Chen '11, Fergusson, Shellard '08, Babich, Creminelli, Zaldarriaga '04 ...
Plots from Chen '

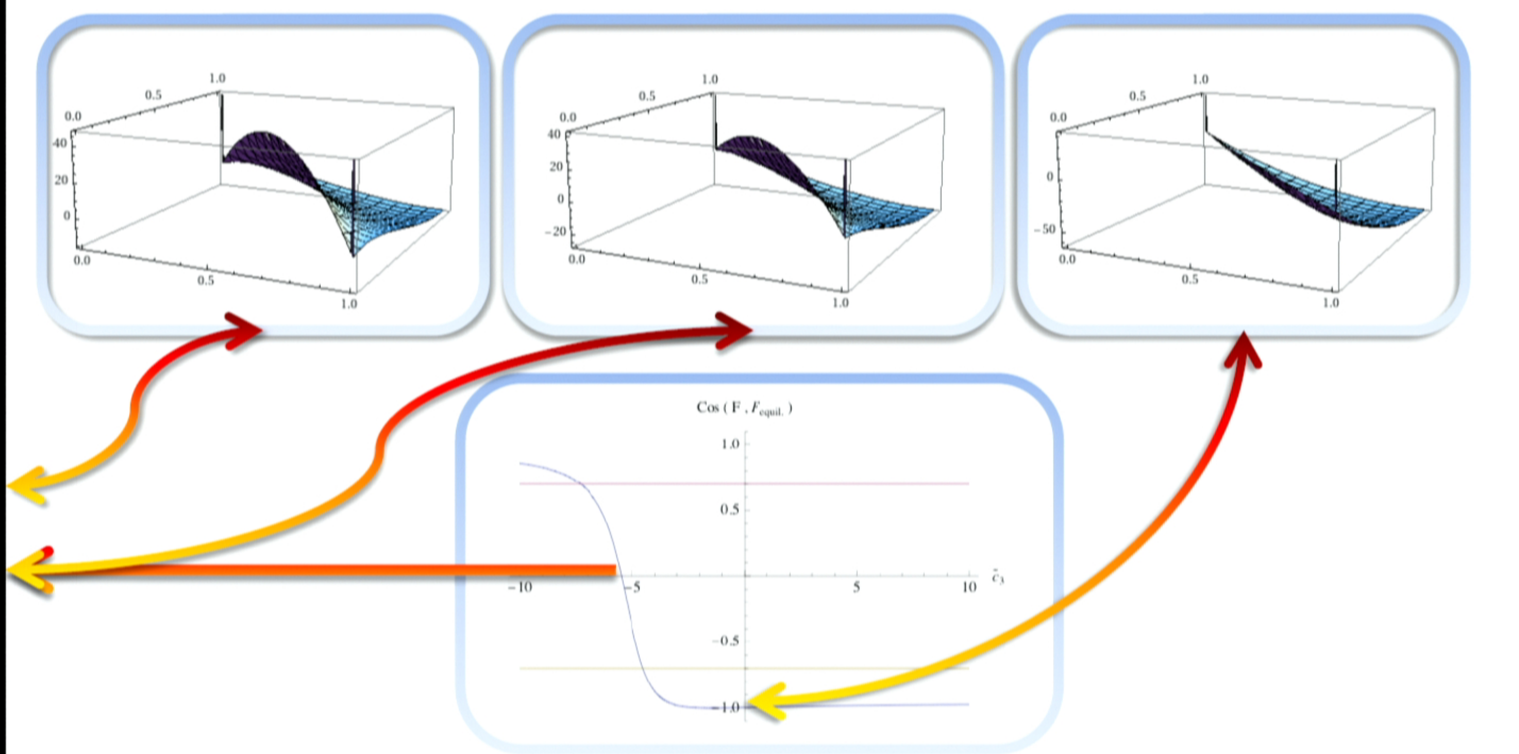
Shape-space with slow-roll



cf. Meerburg, van der Schaar, Corasaniti '09, Senatore, Smith, Zaldarriaga '09

Bottom plot from Senatore, Smith, Zaldarriaga '09

Shape-space with fast-roll



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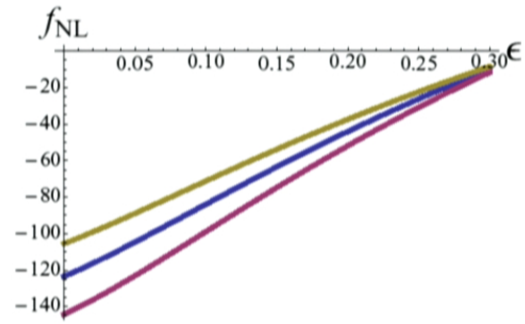
Bottom plot from Senatore, Smith, Zaldarriaga '09

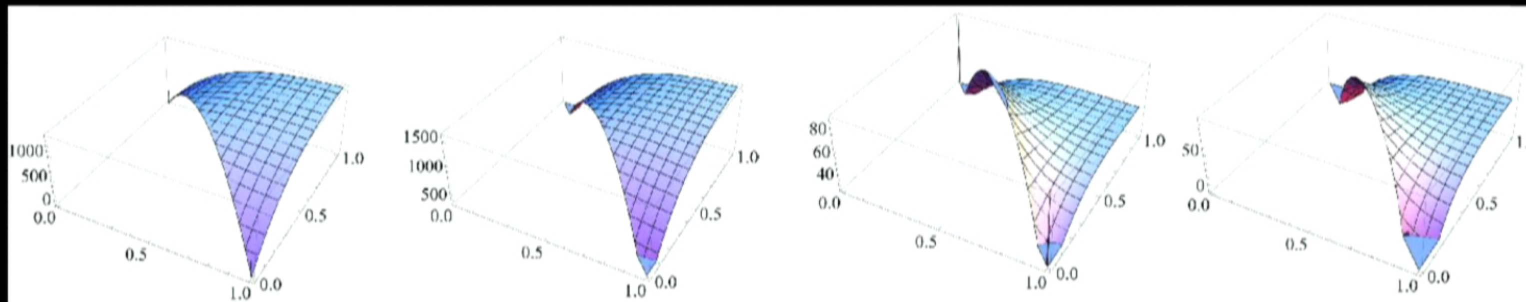
Fast-roll phenomenology

$$f_{NL}^{\frac{\lambda}{\Sigma}\gg 1} \sim 3\frac{1+\epsilon}{n_s-2}\frac{\lambda}{\Sigma}\cos\left(\alpha_2\frac{\pi}{2}\right)\Gamma(3+\alpha_2)$$

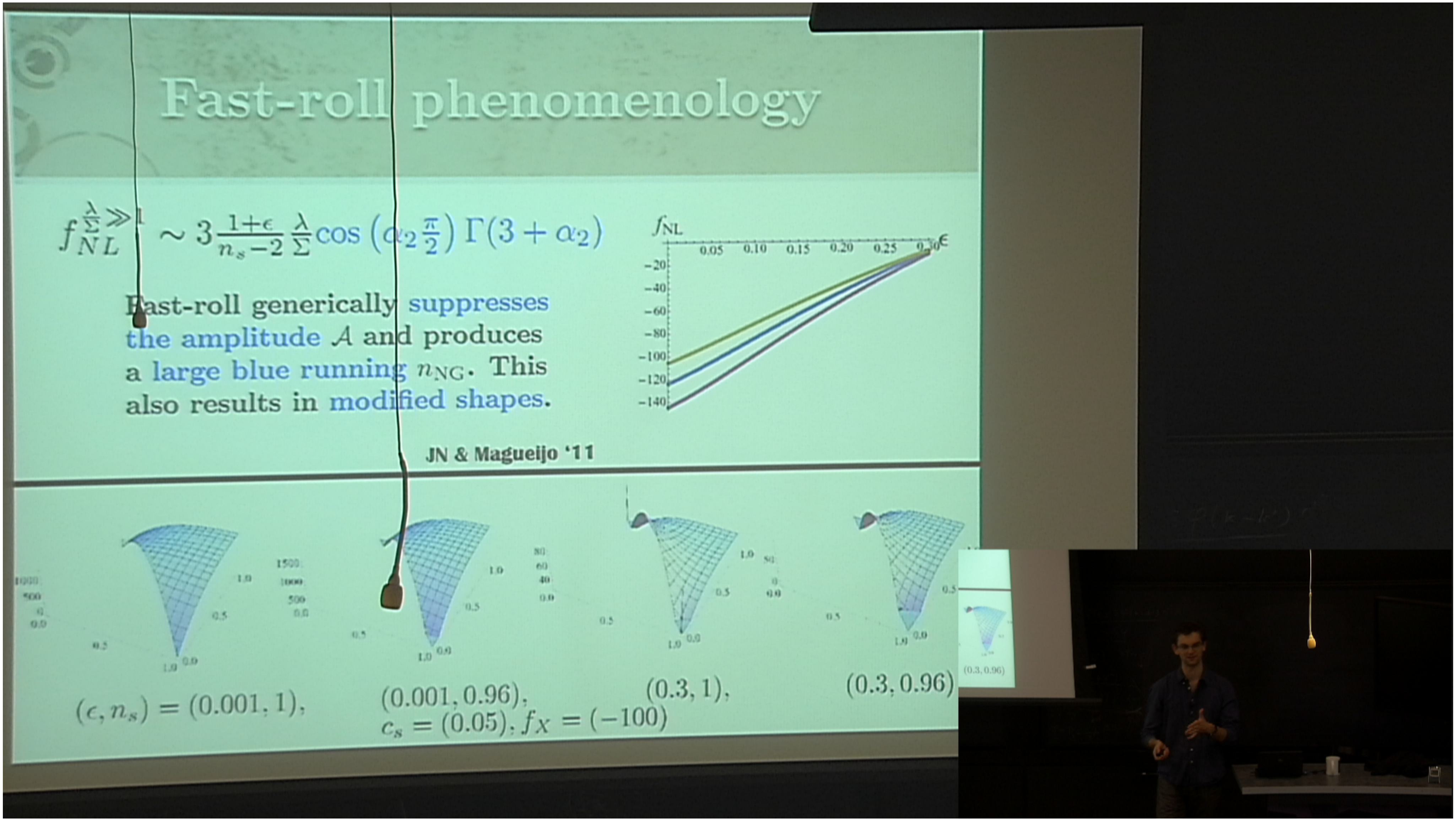


Fast-roll generically suppresses the amplitude $\mathcal{A}$ and produces a large blue running $n_{\rm NG}$. This also results in modified shapes.

JN & Magueijo '11



$(\epsilon, n_s) = (0.001, 1)$, $(0.001, 0.96)$, $(0.3, 1)$, $(0.3, 0.96)$
$c_s = (0.05)$, $f_X = (-100)$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$f_{\rm NL}$
ϵ
0.05 0.10 0.15 0.20 0.25 0.30
-20 -40 -60 -80 -100 -120 -140
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(0.001, 0.96),
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(0.3, 1),
(0.3, 0.96)

The tensor-to-scalar ratio r

Tensor modes: $$r \approx 2^{2\mu-3} \frac{(1-\varepsilon)^{2\mu-1}}{(\epsilon_s+\varepsilon-1)^2} \left| \frac{\Gamma(\mu)}{\Gamma\left(\frac{3}{2}\right)} \right|^2 16 c_s(k_\zeta) \varepsilon \left(\frac{H_h}{H_\zeta} \right)^2$$

$\varepsilon c_s < 0.023$ at 2σ confidence

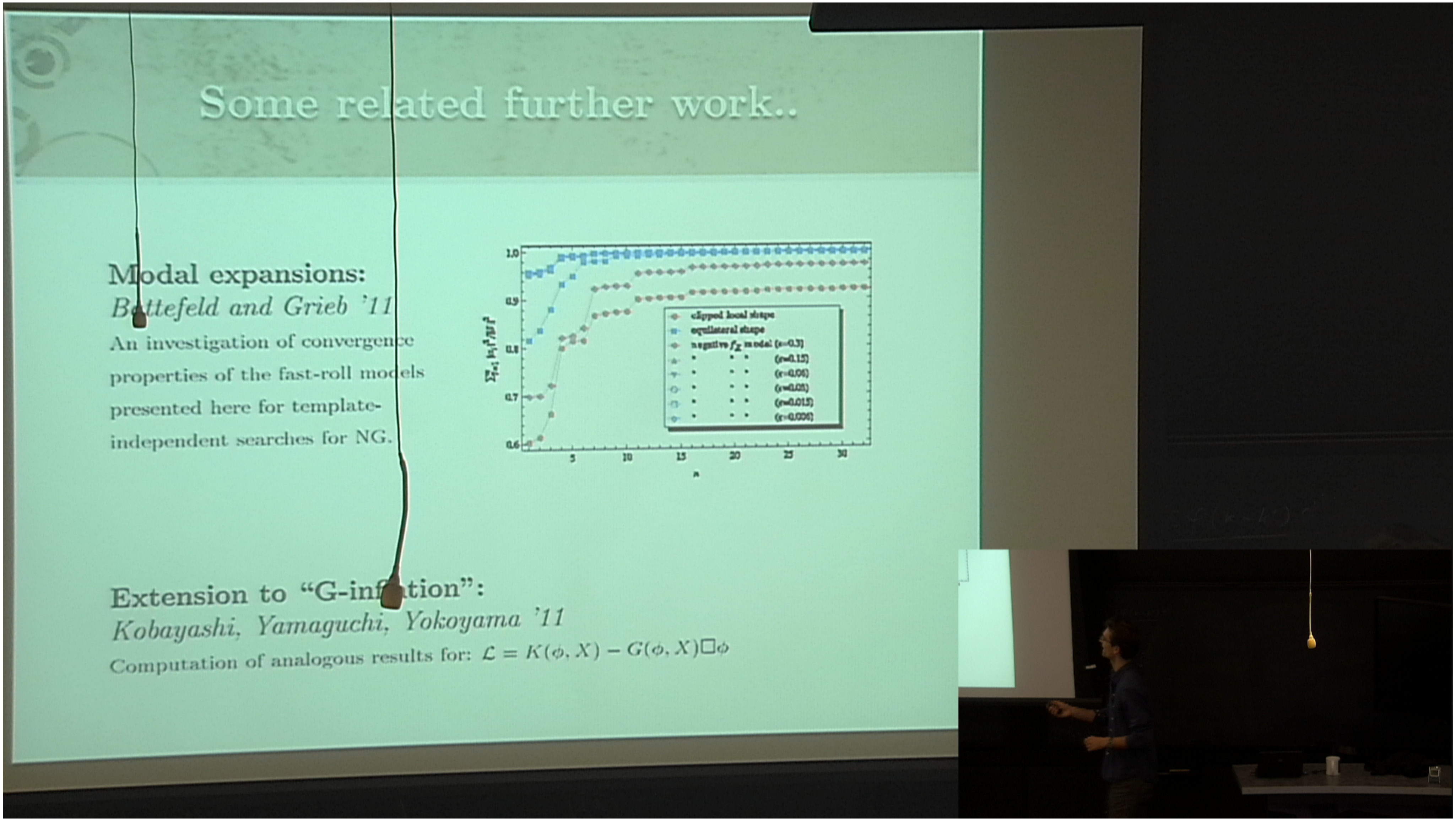
Agarwal & Bean '09, Lorenz et al. '08, Lidsey & Huston '07, Stewart & Lyth '93

$$\varepsilon \lesssim 0.68 \, ^*$$

$^*\bar{c}_s \gtrsim 0.034$

* does not take into account fast-roll suppression & assumes DBI-type action

$$n_s = 0.973 \pm 0.028, \quad r < 0.24 \qquad \text{(WMAP7, } 2\sigma \text{ confidence)}$$


Some related further work..
Modal expansions:
Battefeld and Grieb '11
An investigation of convergence properties of the fast-roll models presented here for template-independent searches for NG.
Extension to "G-inflation":
Kobayashi, Yamaguchi, Yokoyama '11
Computation of analogous results for: $\mathcal{L} = K(\phi, X) - G(\phi, X)\Box\phi$

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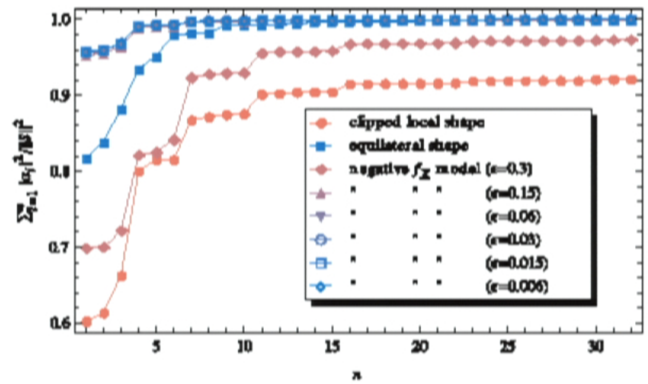

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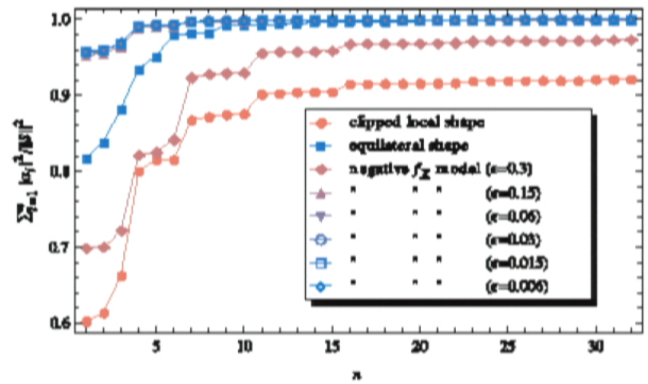

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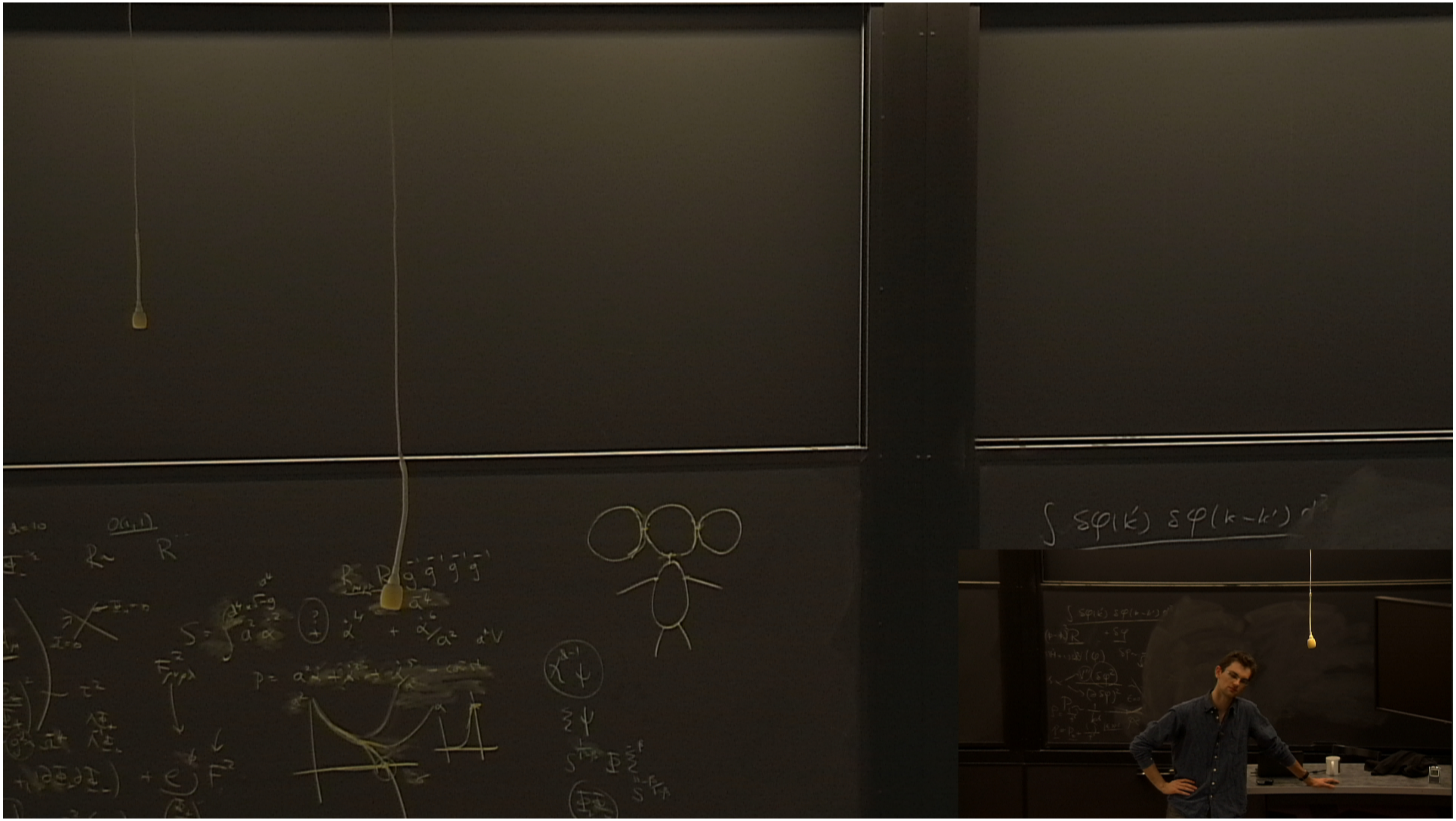
Summary

- **Slow-roll violations observationally allowed**
  Bounds compatible with $\epsilon \lesssim 0.3$
- **A variety of observational signatures:**
  Suppressed f_{NL}, modified shape-space, large running n_{NG}, large tensor-to-scalar ratio r

Thank you!

JN, in progress
JN & Magueijo, (PRD 83, 103511), arXiv:1102.0275
Magueijo, JN, Piazza, (PRD 82 043521), arXiv:1006.3216


∫ δφ(k') δφ(k − k') d³