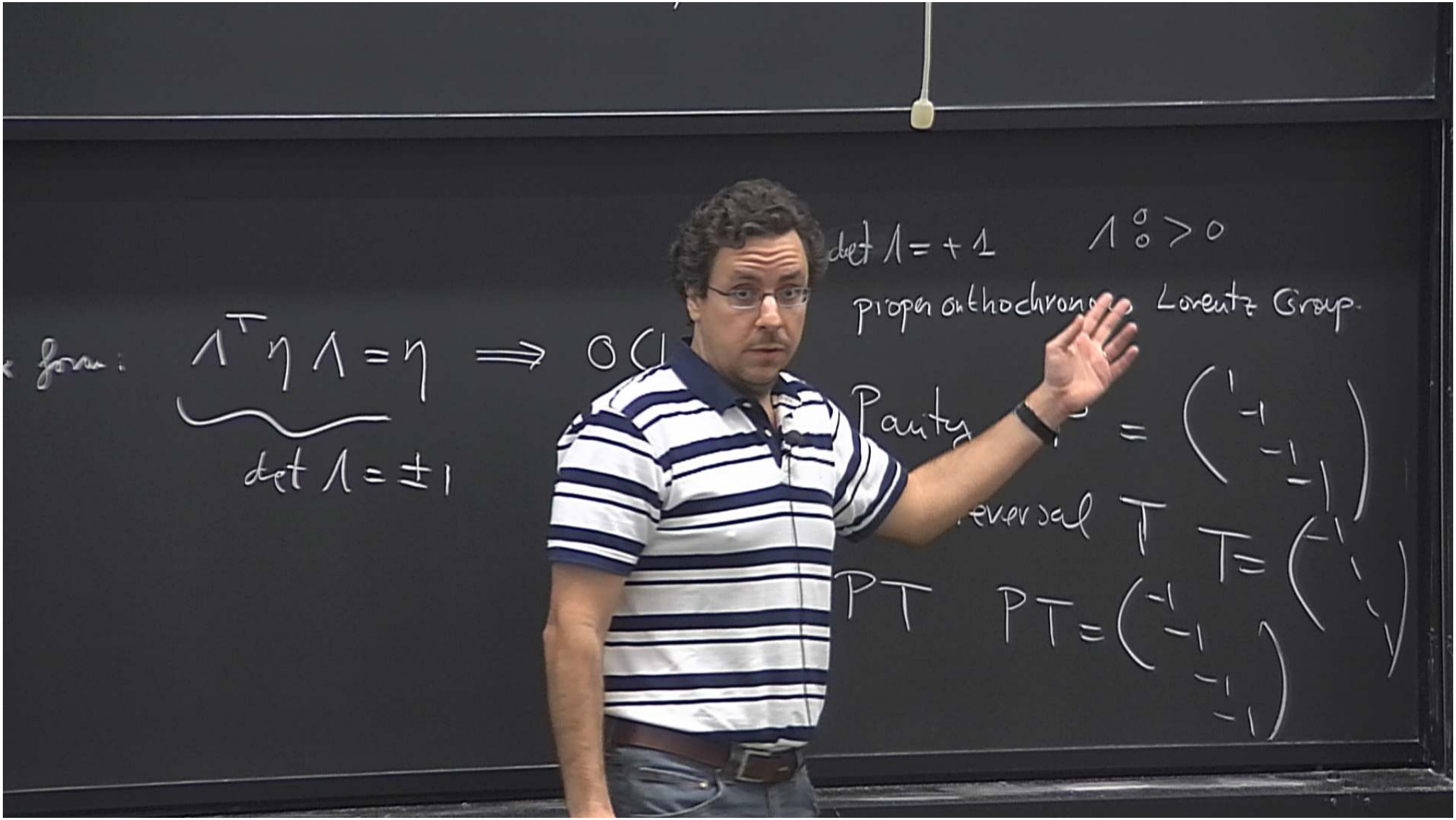
Title: Conformal Field Theory - Lecture 3

Date: Nov 23, 2011 10:30 AM

URL: http://pirsa.org/11110052

Abstract:

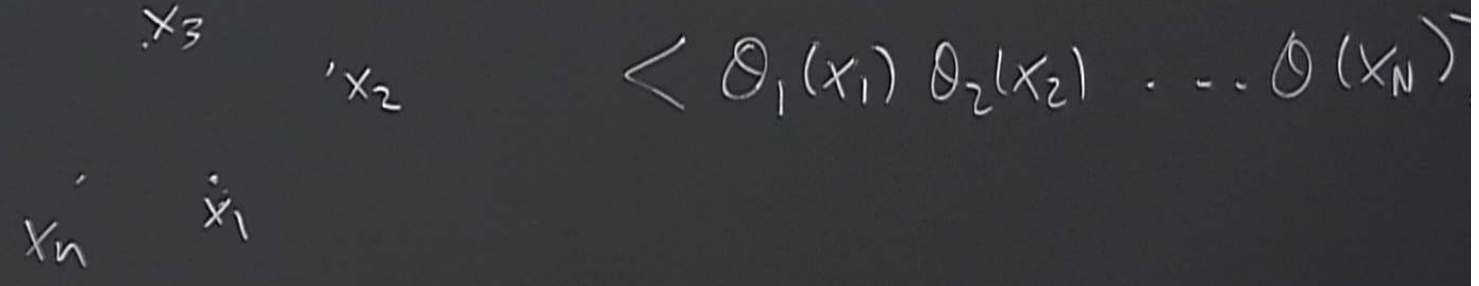

$\Lambda^T \eta \Lambda = \eta \Longrightarrow O(1$
$\det \Lambda = \pm 1$
$\det \Lambda = +1$
$\Lambda^0_0 > 0$
proper orthochron
Lorentz Group
Parity
reversal
$T = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$
PT
$PT = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

- Ultimate Goal: extract physics from CFT.

① Constraints that Conformal imposes on observables

② Determine exactly the observables

- correlation function of local operators



- Ultimate Goal: extract physics from CFT.

  ① Constraints that Conformal imposes on observables

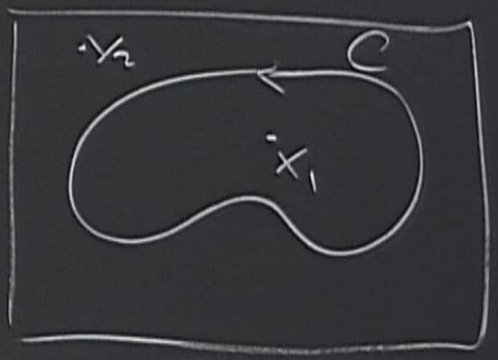
  ② Determine exactly the observables

- correlation function of local operators

x_3 x_2 $\quad \langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \cdots \mathcal{O}(x_N) \rangle$

x_n x_1

- correlations of non-local operators



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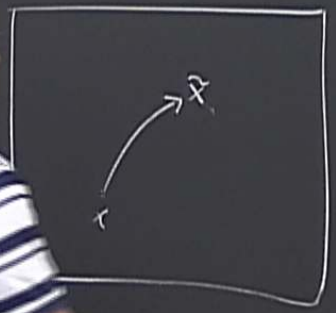
- Conformal Symmetries: Let's first review Poincaré transformations.

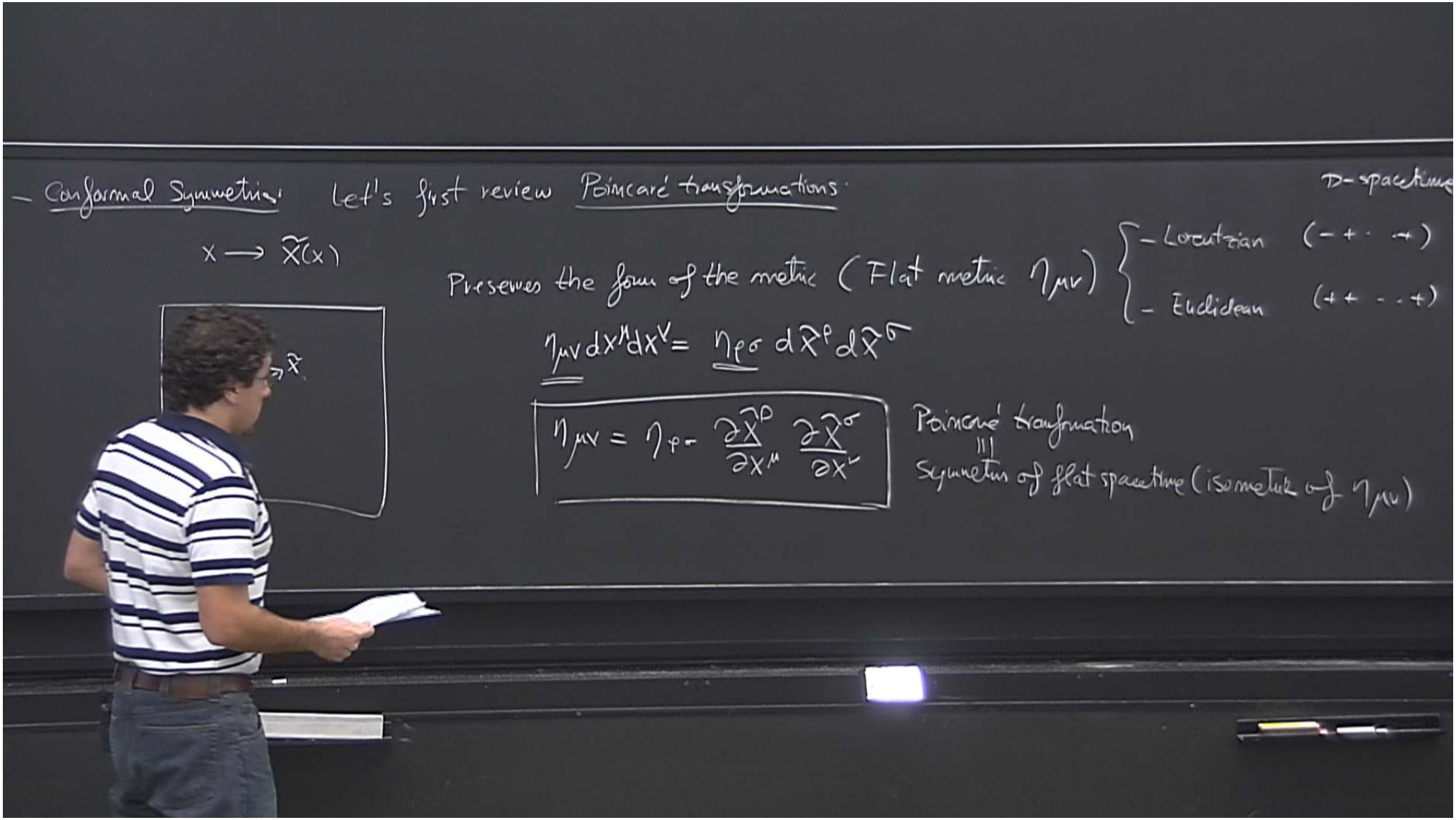
D-spacetime

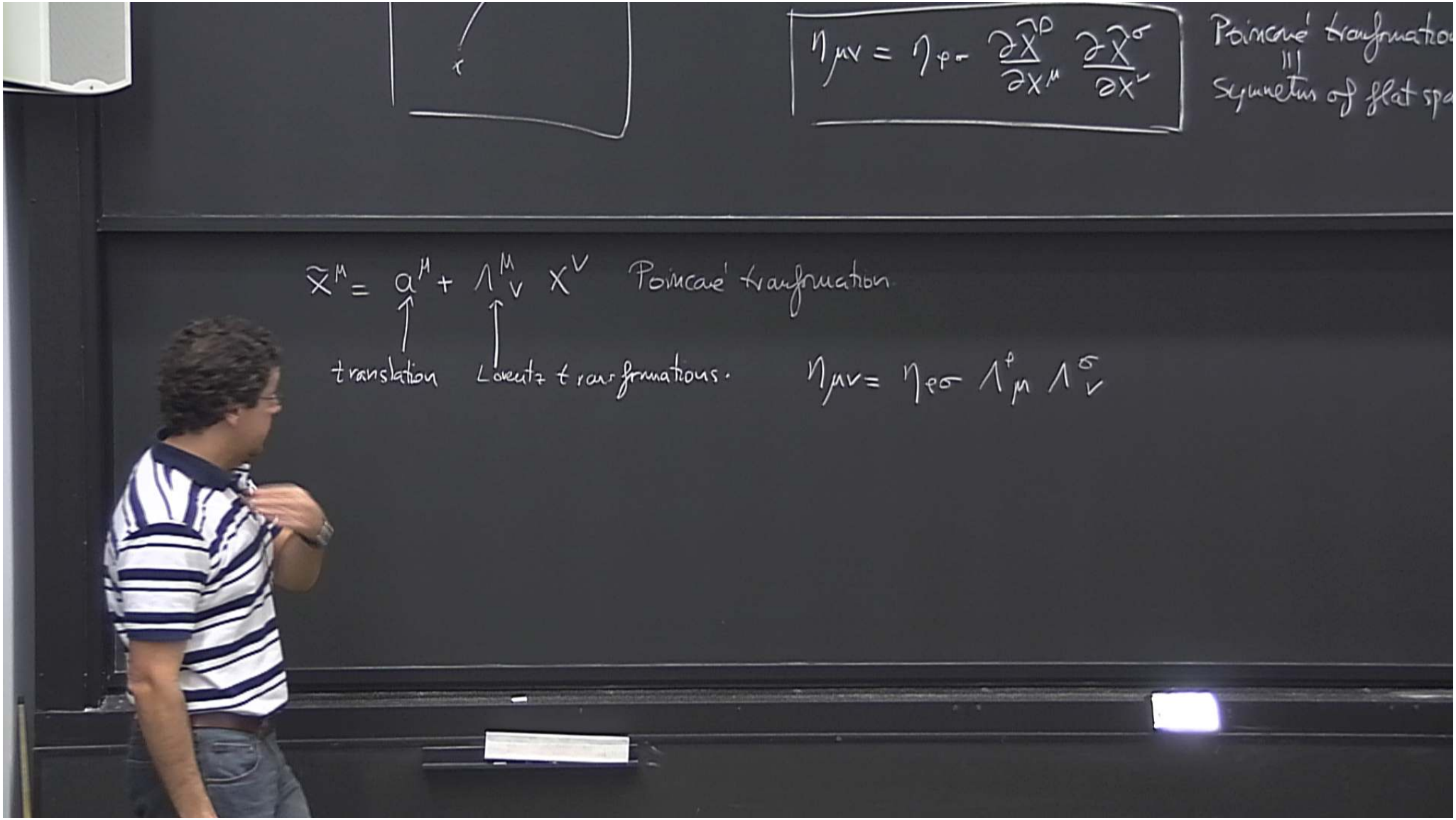
$x \longrightarrow \tilde{x}(x)$

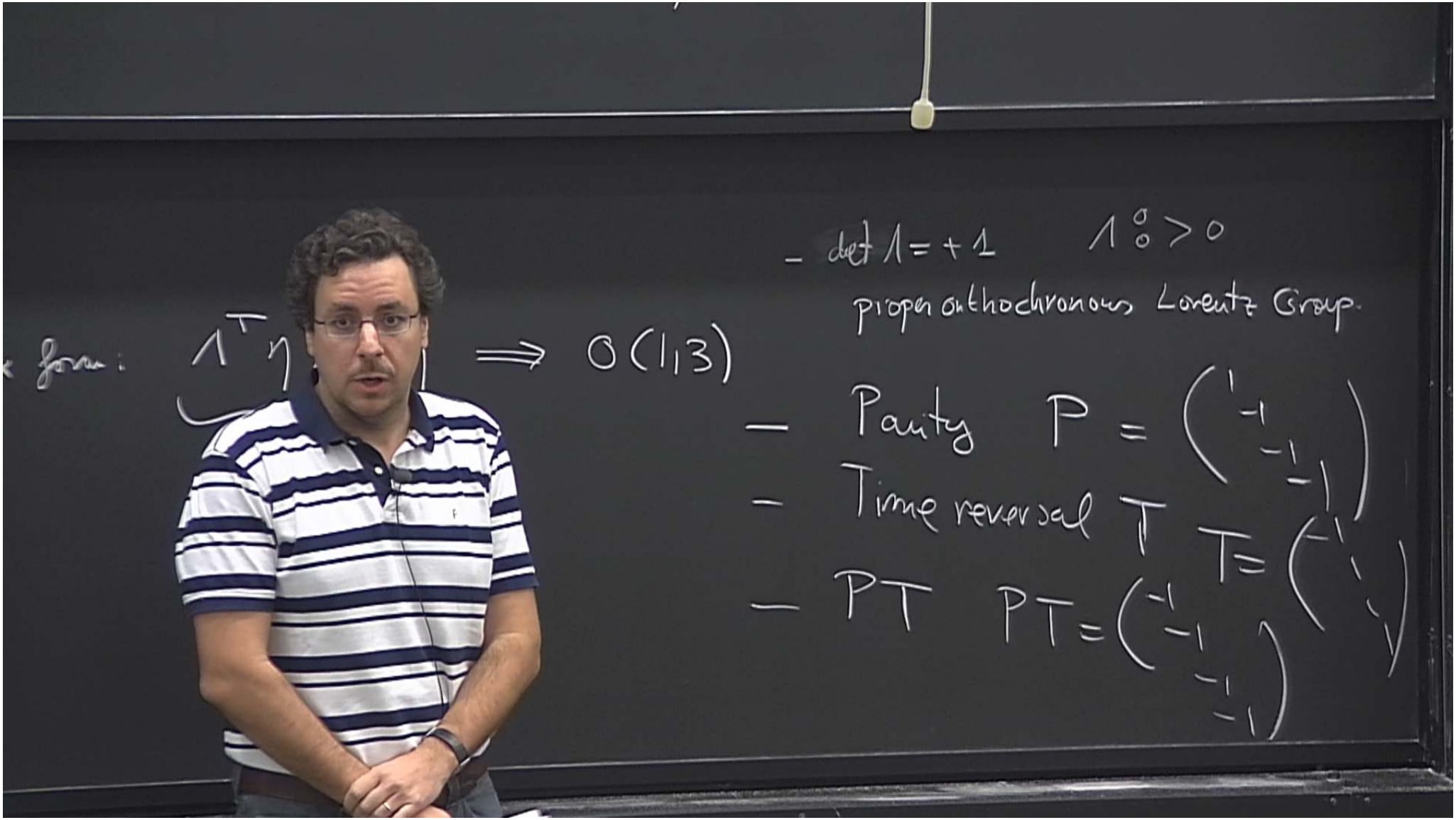


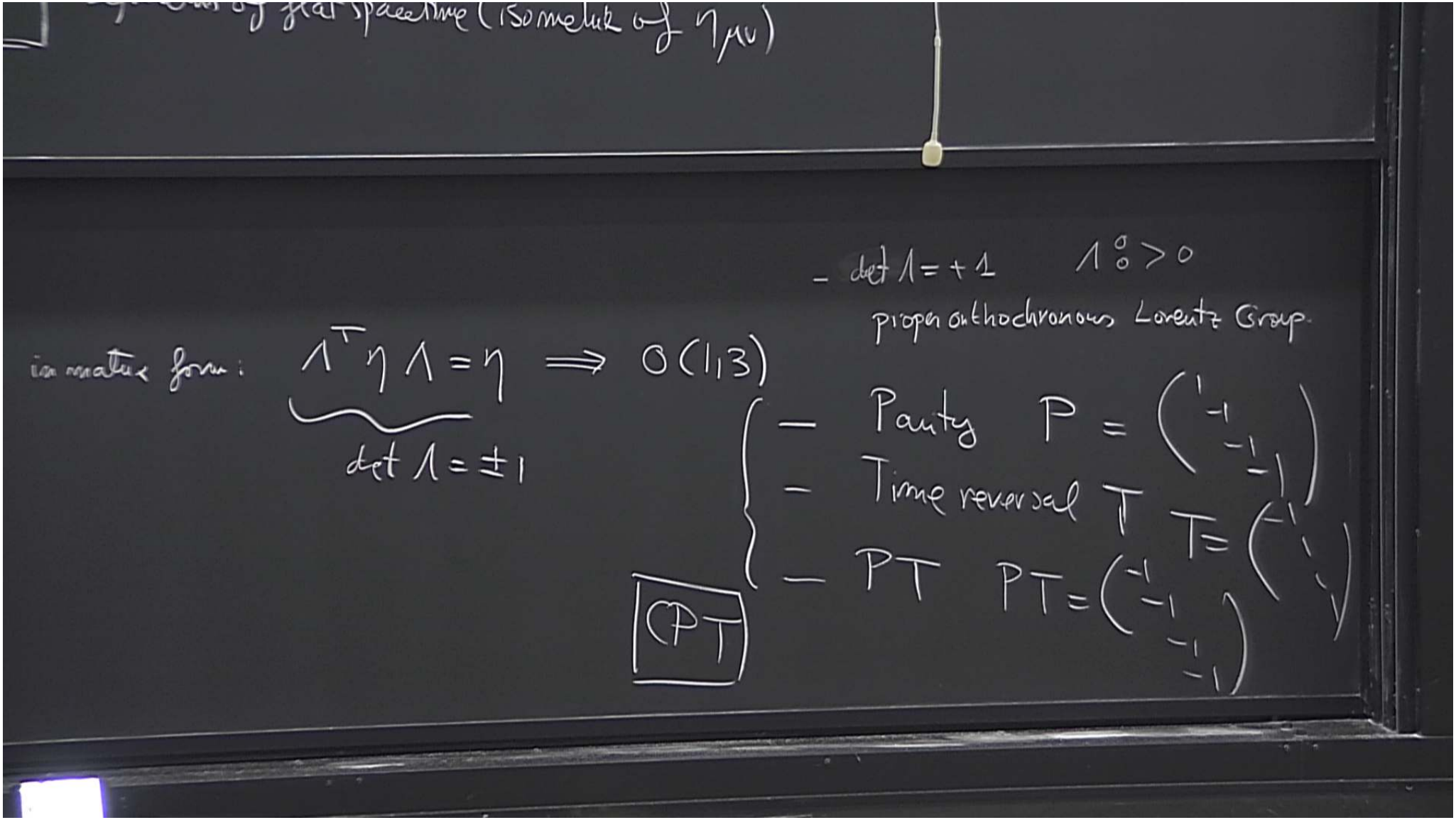
Preserves the form of the metric (Flat metric $\eta_{\mu\nu}$)
- Lorentzian $(- + \cdots +)$
- Euclidean $(+ + \cdots +)$

$$\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma$$


- Conformal Symmetries: Let's first review Poincaré transformations.
D-spacetime
$x \longrightarrow \tilde{x}(x)$
Preserves the form of the metric (Flat metric $\eta_{\mu\nu}$)
- Lorentzian $(- + \cdots +)$
- Euclidean $(+ + \cdots +)$
$\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma$
$\eta_{\mu\nu} = \eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu}$
Poincaré transformation
=
Symmetries of flat spacetime (isometries of $\eta_{\mu\nu}$)


$\eta_{\mu\nu} = \eta_{\rho\sigma} \frac{\partial x'^{\rho}}{\partial x^{\mu}} \frac{\partial x'^{\sigma}}{\partial x^{\nu}}$
Poincaré transformation
$\tilde{x}^{\mu} = a^{\mu} + \Lambda^{\mu}_{\ \nu} x^{\nu}$ Poincaré transformation
translation
Lorentz transformations.
$\eta_{\mu\nu} = \eta_{\rho\sigma} \Lambda^{\rho}_{\mu} \Lambda^{\sigma}_{\nu}$


$\Lambda^T \eta \Lambda = \eta \Longrightarrow O(1,3)$
- $\det \Lambda = +1$ $\Lambda^0_0 > 0$
proper orthochronous Lorentz Group
- Parity $P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
- Time reversal T $T = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$
- PT $PT = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$


of flat spacetime (isometric of $\eta_{\mu\nu}$)
- $\det \Lambda = +1$ $\Lambda^0_0 > 0$
proper orthochronous Lorentz Group.
in matrix form: $\Lambda^T \eta \Lambda = \eta \Longrightarrow O(1,3)$
$\det \Lambda = \pm 1$
- Parity $P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
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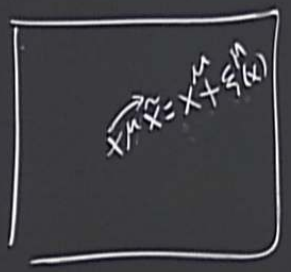
CPT

Transformations

↑ Infinitesimal



$\xi = \xi^\mu \partial_\mu$ vector field that generates the transformation.

$\eta_{\mu\nu} = \eta_{\rho\sigma} (\delta^\rho_\mu + \partial_\mu \xi^\rho)(\delta^\sigma_\nu + \partial_\nu \xi^\sigma)$

$O(\xi^0)$: $\eta_{\mu\nu} = \eta_{\mu\nu}$

Transformations

↑ Infinitesimal

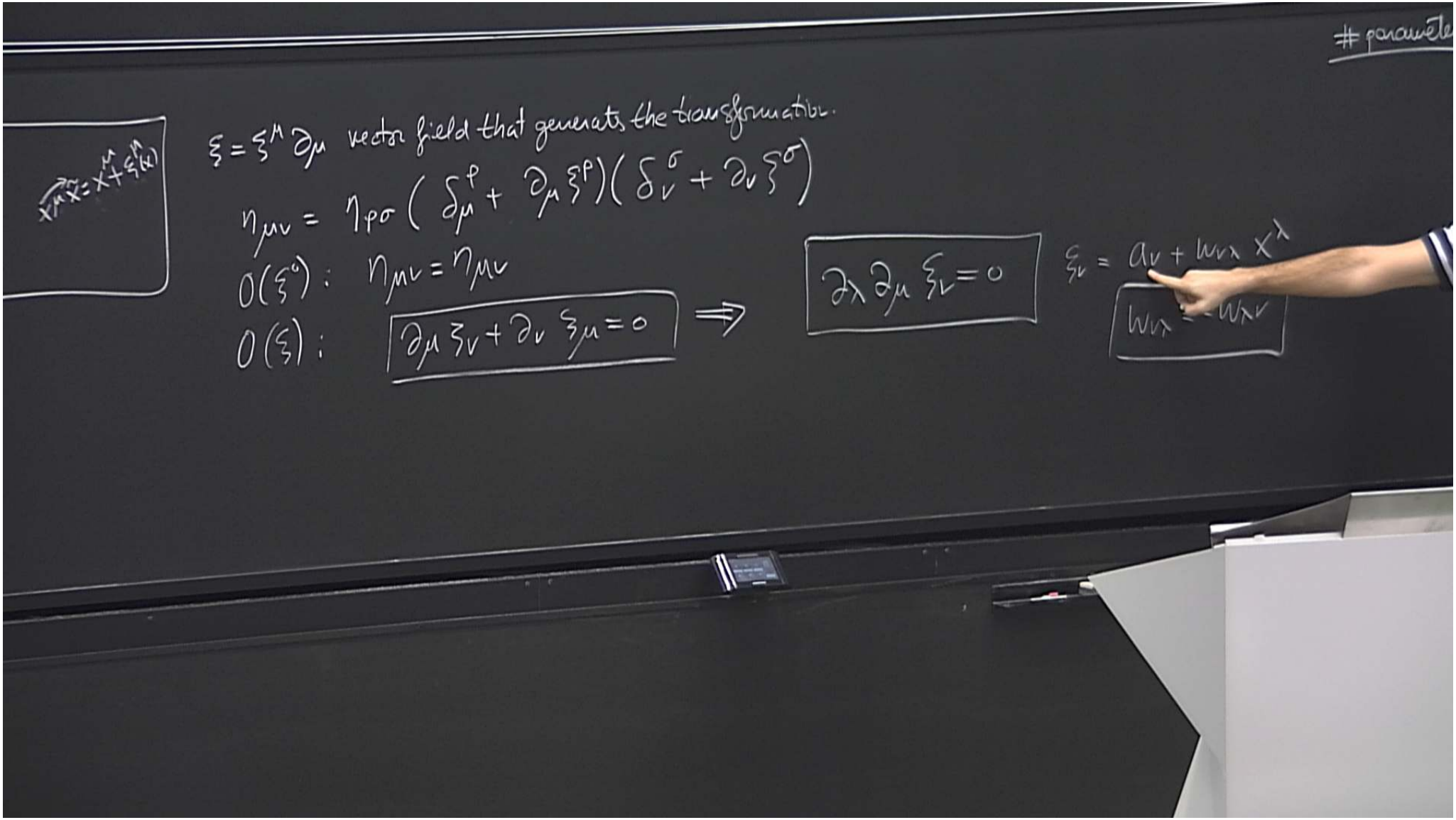
$x'^\mu = x^\mu + \xi^\mu(x)$

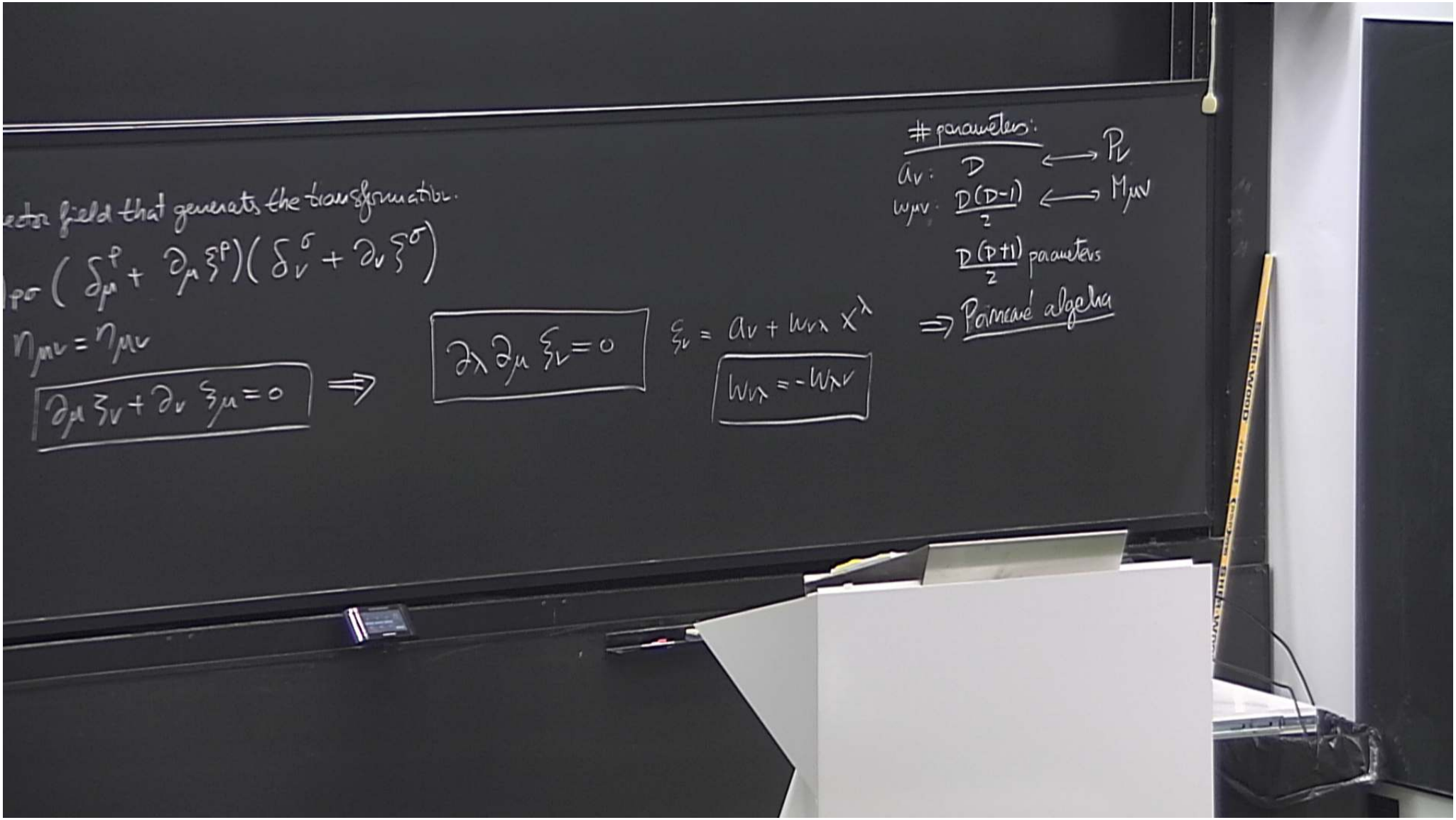
$\xi = \xi^\mu \partial_\mu$ vector field that generates the transformation.

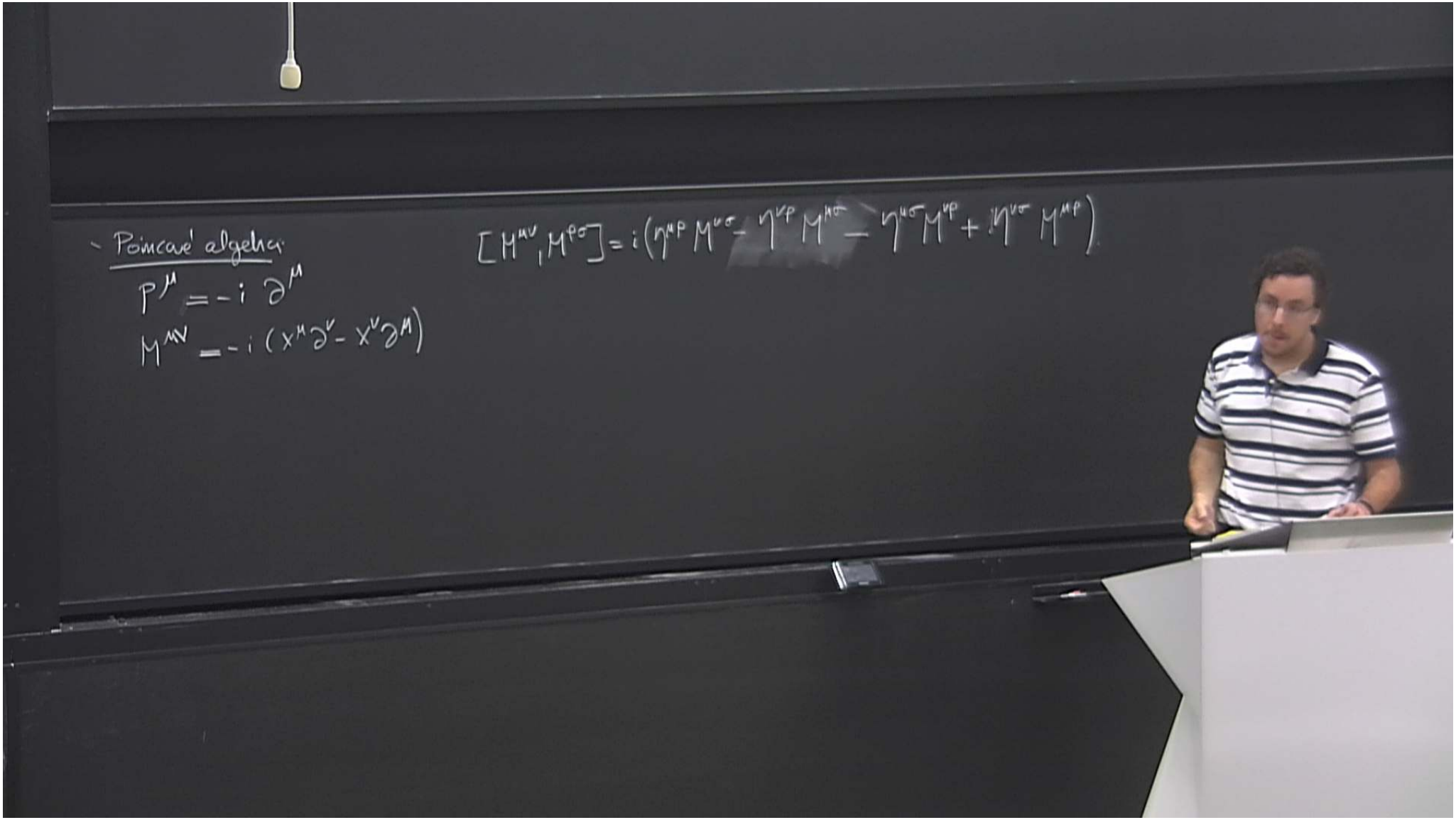
$\eta_{\mu\nu} = \eta_{\rho\sigma}\left(\delta^\rho_\mu + \partial_\mu \xi^\rho\right)\left(\delta^\sigma_\nu + \partial_\nu \xi^\sigma\right)$

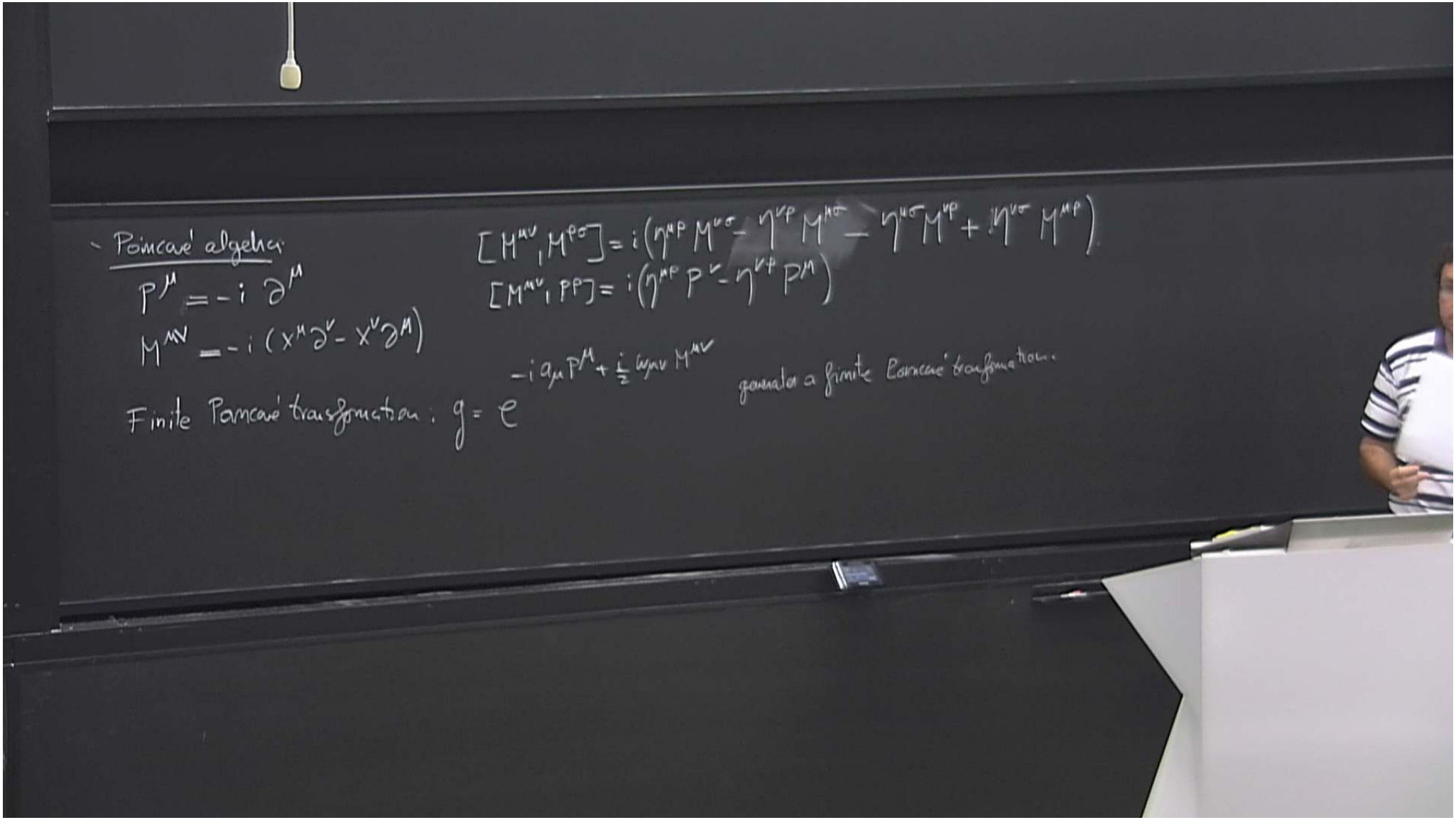
$O(\xi^0)$: $\eta_{\mu\nu} = \eta_{\mu\nu}$

$O(\xi)$: $\boxed{\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0}$ →


#parametr
$x'^\mu = x^\mu + \xi^\mu(x)$
$\xi = \xi^\mu \partial_\mu$ vector field that generates the transformation.
$\eta_{\mu\nu} = \eta_{\rho\sigma}(\delta^\rho_\mu + \partial_\mu \xi^\rho)(\delta^\sigma_\nu + \partial_\nu \xi^\sigma)$
$O(\xi^0)$: $\eta_{\mu\nu} = \eta_{\mu\nu}$
$O(\xi)$: $\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0$
$\Rightarrow$
$\partial_\lambda \partial_\mu \xi_\nu = 0$
$\xi_\nu = a_\nu + w_{\nu\lambda} x^\lambda$
$w_{\nu\lambda} = -w_{\lambda\nu}$


ector field that generates the transformation.
$\partial_\mu \left(\delta^\rho_\mu + \partial_\mu \xi^\rho \right)\left(\delta^\sigma_\nu + \partial_\nu \xi^\sigma \right)$
$\eta_{\mu\nu} = \eta_{\mu\nu}$
$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0$
$\Rightarrow$
$\partial_\lambda \partial_\mu \xi_\nu = 0$
$\xi_\nu = a_\nu + w_{\nu\lambda} x^\lambda$
$w_{\nu\lambda} = -w_{\lambda\nu}$
parameters:
a_ν: D $\longleftrightarrow$ P_ν
$w_{\mu\nu}$: $\frac{D(D-1)}{2}$ $\longleftrightarrow$ $M_{\mu\nu}$
$\frac{D(D+1)}{2}$ parameters
$\Rightarrow$ Poincaré algebra


- Poincaré algebra:
$P^\mu = -i\,\partial^\mu$
$M^{\mu\nu} = -i\,(x^\mu \partial^\nu - x^\nu \partial^\mu)$
$[M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\mu\rho} M^{\nu\sigma} - \eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\sigma} M^{\nu\rho} + \eta^{\nu\sigma} M^{\mu\rho})$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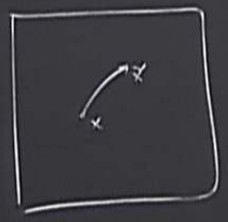
$[M^{\mu\nu}, P^\rho] = i(\eta^{\mu\rho} P^\nu - \eta^{\nu\rho} P^\mu)$
Finite Poincaré transformation: $g = e^{-i a_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}}$
generator a finite Poincaré transformation.

$M^{\mu\nu} = -i(x^\mu \partial - x^\nu \partial^\mu)$

Finite Poincaré transformation: $g = e^{-i a_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}}$ generates a finite Poincaré transformation.

Conformal Transformations of Flat spacetime.

$x \longrightarrow \tilde{x}(x)$



$\Omega^2(x)\, \eta_{\mu\nu}\, dx^\mu dx^\nu = \eta_{\rho\sigma}\, d\tilde{x}^\rho d\tilde{x}^\sigma$

($\Omega^2(x)$: scale factor.)

$$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = \Omega^2(x)\, \eta_{\mu\nu}$$

the solutions to these equations are the conformal transformations.

$M^{\mu\nu} = -i(x^\mu \partial - x^\nu \partial^\mu)$

Finite Poincaré transformation: $g = e^{-i a_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}}$ generates a finite Poincaré transformation.

Conformal Transformations of Flat spacetime.

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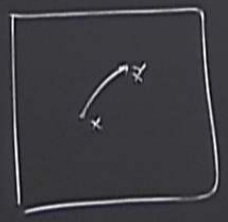

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$\Omega^2(x)$: scale factor.

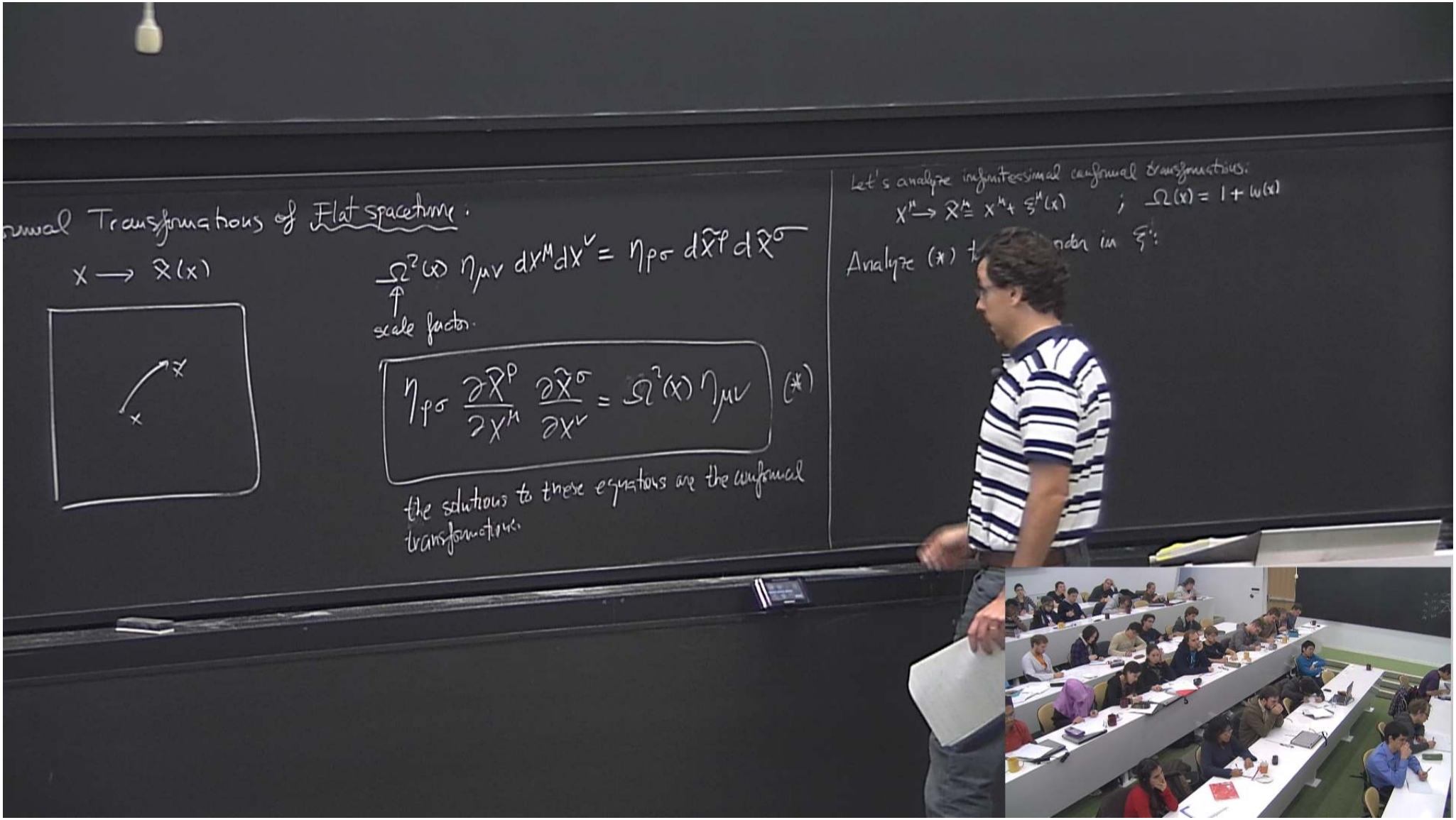
$$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = \Omega^2(x)\, \eta_{\mu\nu} \quad (*)$$

the solutions to these equations are the conformal transformations.

Let's analyze infinitesimal conformal transformations.

$x^\mu \longrightarrow \tilde{x}^\mu = x^\mu + \xi^\mu(x)$

Analyze $(*)$ to linear order in ξ^μ:


nmal Transformations of Flat spacetime.
$x \longrightarrow \hat{x}(x)$
$\Omega^2(x)\,\eta_{\mu\nu}\,dx^\mu dx^\nu = \eta_{\rho\sigma}\,d\hat{x}^\rho\,d\hat{x}^\sigma$
scale factor.
$\eta_{\rho\sigma}\,\frac{\partial \hat{x}^\rho}{\partial x^\mu}\,\frac{\partial \hat{x}^\sigma}{\partial x^\nu} = \Omega^2(x)\,\eta_{\mu\nu}$ (*)
the solutions to these equations are the conformal transformations.
Let's analyze infinitessimal conformal transformations.
$x^\mu \longrightarrow \hat{x}^\mu = x^\mu + \xi^\mu(x)$; $\Omega(x) = 1 + \omega(x)$
Analyze (*) t
der in ξ:

$-\frac{1}{4}\partial_\mu\Gamma^\mu + \frac{1}{2}\omega_{\mu\nu}\Gamma$

uction: $g = e$

generates a finite Poincaré transformation.

tions of Flat spacetime:

$$\Omega^2(x)\,\eta_{\mu\nu}\,dx^\mu dx^\nu = \eta_{\rho\sigma}\,d\tilde{x}^\rho\,d\tilde{x}^\sigma$$

$\Omega^2(x)$: scale factor.

$$\eta_{\rho\sigma}\frac{\partial \tilde{x}^\rho}{\partial x^\mu}\frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = \Omega^2(x)\,\eta_{\mu\nu} \qquad (*)$$

the solutions to these equations are the conformal transformations.

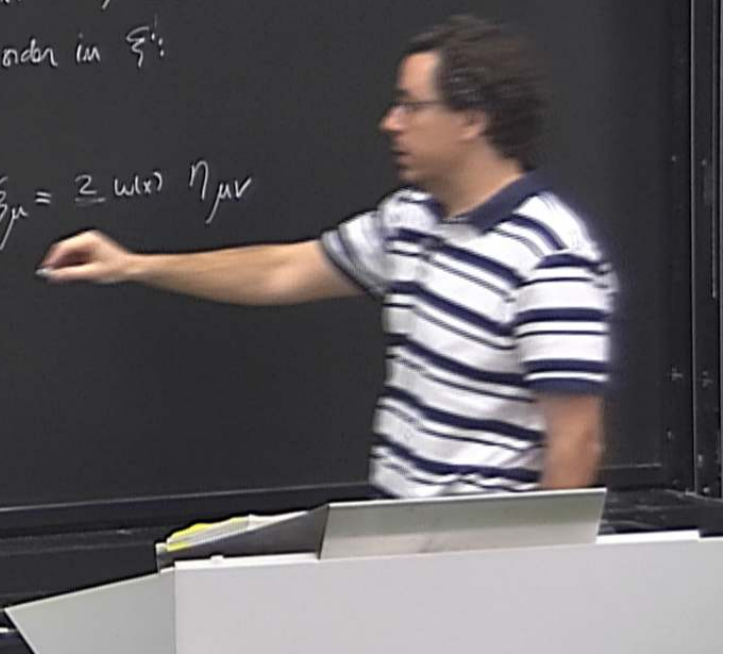
Let's analyze infinitesimal conformal transformations.

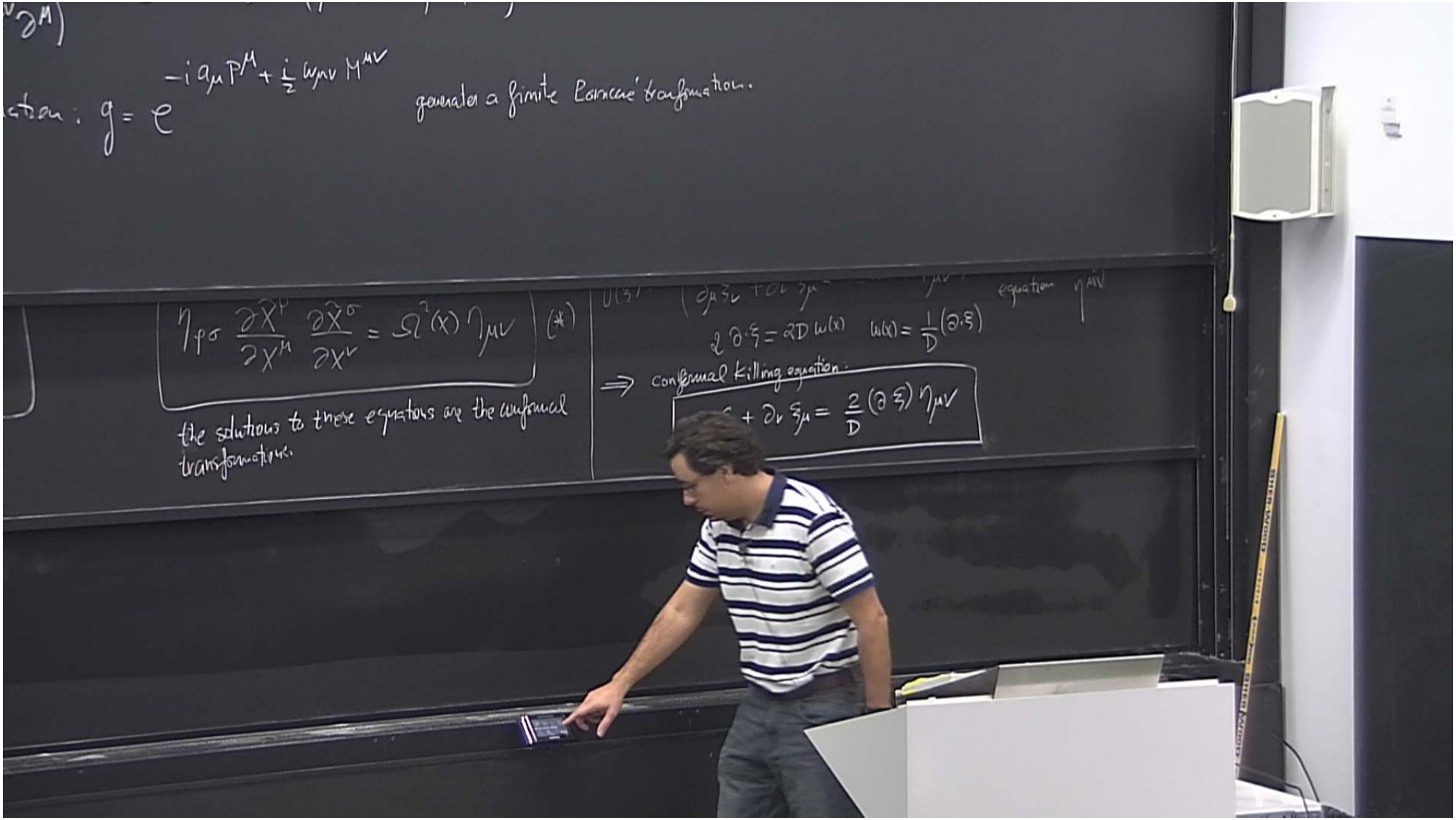
$x^\mu \to \tilde{x}^\mu = x^\mu + \xi^\mu(x)$; $\Omega(x) = 1 + \omega(x)$

Analyze $(*)$ to linear order in ξ:

$O(\xi^0)$ $\eta_{\mu\nu} = \eta_{\mu\nu}$

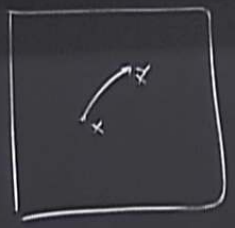
$O(\xi)$: $\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 2\,\omega(x)\,\eta_{\mu\nu}$




∂_M)
$-i a_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}$
generates a finite Poincaré transformation.
$\eta_{\rho\sigma} \frac{\partial \tilde{X}^\rho}{\partial X^\mu} \frac{\partial \tilde{X}^\sigma}{\partial X^\nu} = \Omega^2(x) \eta_{\mu\nu}$ (*)
the solutions to these equations are the conformal transformations.
$\Rightarrow$ conformal killing equation.
$\partial_\nu \xi_\mu = \frac{2}{D}(\partial \cdot \xi) \eta_{\mu\nu}$

$M^{\mu\nu} = -i(x^\mu \partial^\nu - x^\nu \partial^\mu)$

Finite Poincaré transformation: $g = e^{-i a_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}}$ generates a finite Poincaré transformation.



scale factor

$$\eta_{\rho\sigma} \frac{\partial x'^\rho}{\partial x^\mu} \frac{\partial x'^\sigma}{\partial x^\nu} = \Omega^2(x)\, \eta_{\mu\nu} \qquad (*)$$

the solutions to these equations are the conformal transformations.

Analyze (*) to linear order in ξ:

$O(\xi^0)$: $\eta_{\mu\nu} = \eta_{\mu\nu}$

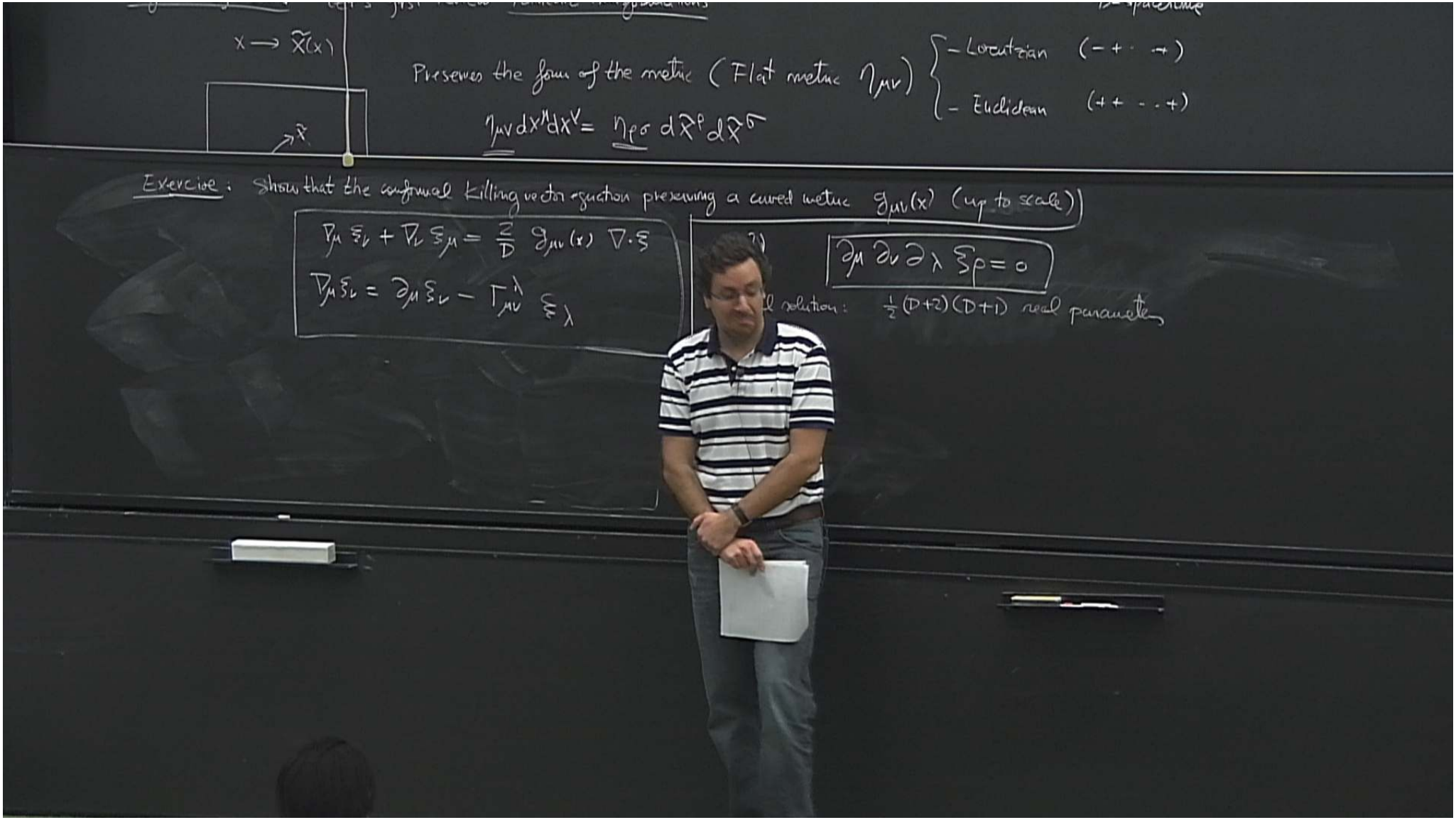
$O(\xi)$: $\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 2\omega(x)\, \eta_{\mu\nu}$ take the trace of this equation $\eta^{\mu\nu}$

$2\,\partial\cdot\xi = 2D\,\omega(x) \qquad \omega(x) = \frac{1}{D}(\partial\cdot\xi)$

$\Rightarrow$ conformal killing equation

$$(**) \quad \partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D}(\partial\cdot\xi)\, \eta_{\mu\nu}$$

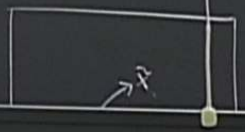
Minkowski space


$x \longrightarrow \tilde{x}(x)$
Preserves the form of the metric (Flat metric $\eta_{\mu\nu}$)
- Lorentzian $(- + \cdots +)$
- Euclidean $(+ + \cdots +)$
$\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma$
Exercise: Show that the conformal Killing vector equation preserving a curved metric $g_{\mu\nu}(x)$ (up to scale)
$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \frac{2}{D} g_{\mu\nu}(x) \nabla \cdot \xi$
$\nabla_\mu \xi_\nu = \partial_\mu \xi_\nu - \Gamma^\lambda_{\mu\nu} \xi_\lambda$
$\partial_\mu \partial_\nu \partial_\lambda \xi_\rho = 0$
solution: $\frac{1}{2}(D+2)(D+1)$ real parameters

$x \longrightarrow \tilde{x}(x)$

Preserves the form of the metric (Flat metric $\eta_{\mu\nu}$) $\begin{cases} - \text{Lorentzian} & (- + \cdots +) \\ - \text{Euclidean} & (+ + \cdots +) \end{cases}$

$\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma$



Exercise: Show that the conformal Killing vector equation preserving a curved metric $g_{\mu\nu}(x)$ (up to scale)

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \frac{2}{D} g_{\mu\nu}(x) \nabla \cdot \xi$$

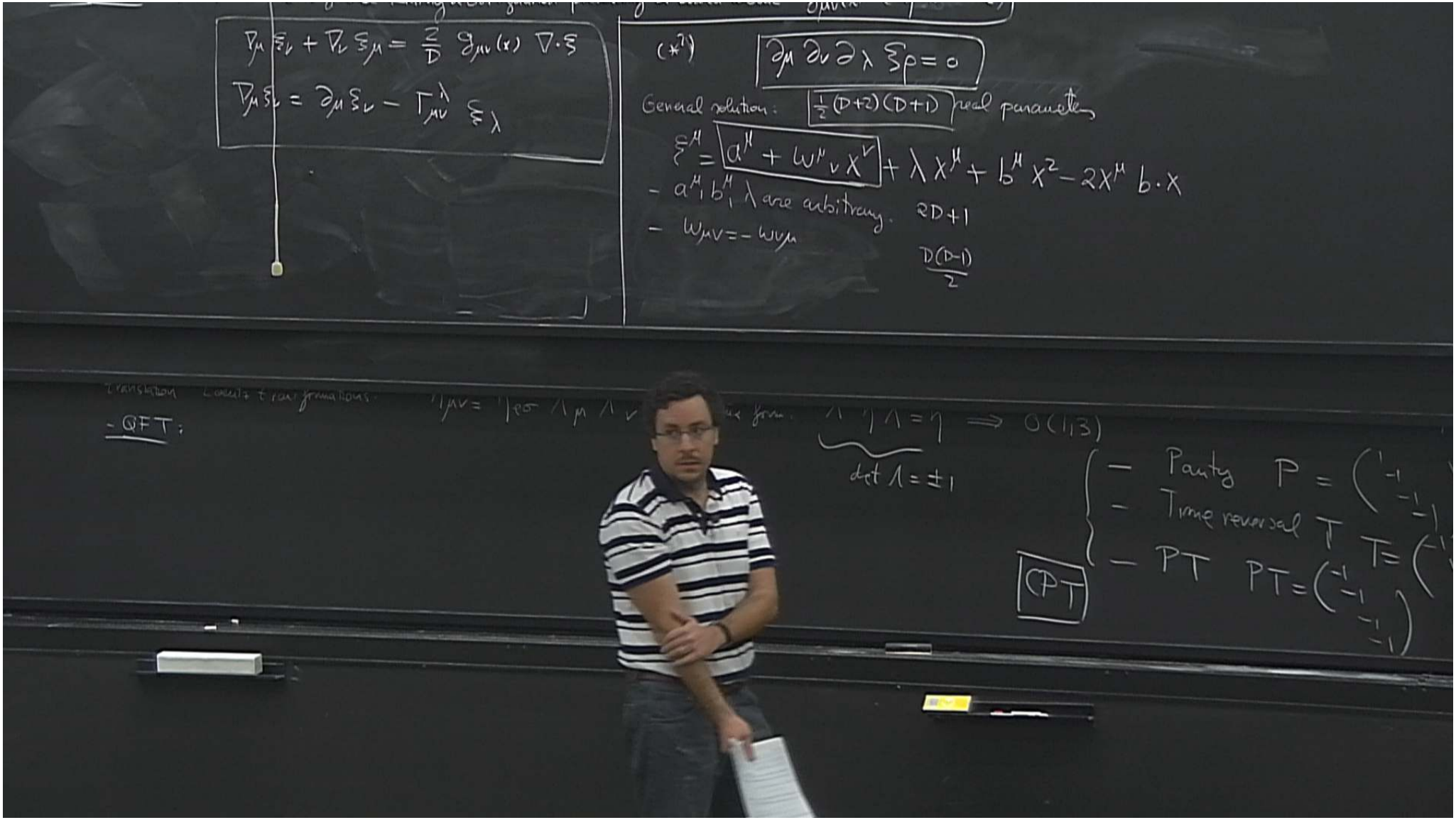
$$\nabla_\mu \xi_\nu = \partial_\mu \xi_\nu - \Gamma^\lambda_{\mu\nu} \xi_\lambda$$

(x^2) $\quad \partial_\mu \partial_\nu \partial_\lambda \xi_\rho = 0$

General solution: $\frac{1}{2}(D+2)(D+1)$ real parameters

$$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu b \cdot x$$

- a^μ, b^μ, λ are arbitrary.
- $\omega_{\mu\nu} = -\omega_{\nu\mu}$


$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D} \eta_{\mu\nu}(x)\, \partial\cdot\xi$
$\nabla_\mu \xi_\nu = \partial_\mu \xi_\nu - \Gamma^\lambda_{\mu\nu}\, \xi_\lambda$
$(*^2)$ $\partial_\mu \partial_\nu \partial_\lambda \xi_\rho = 0$
General solution: $\frac{1}{2}(D+2)(D+1)$ real parameters
$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu\, b\cdot x$
- a^μ, b^μ, λ are arbitrary. $2D+1$
- $\omega_{\mu\nu} = -\omega_{\nu\mu}$ $\frac{D(D-1)}{2}$
- QFT:
$\det \Lambda = \pm 1$
- Parity $P = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
- Time reversal T $T = \begin{pmatrix} -1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \end{pmatrix}$
- PT $PT = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
CPT

$a^\mu \longrightarrow P^\mu : x^\mu \longrightarrow x^\mu + a^\mu$

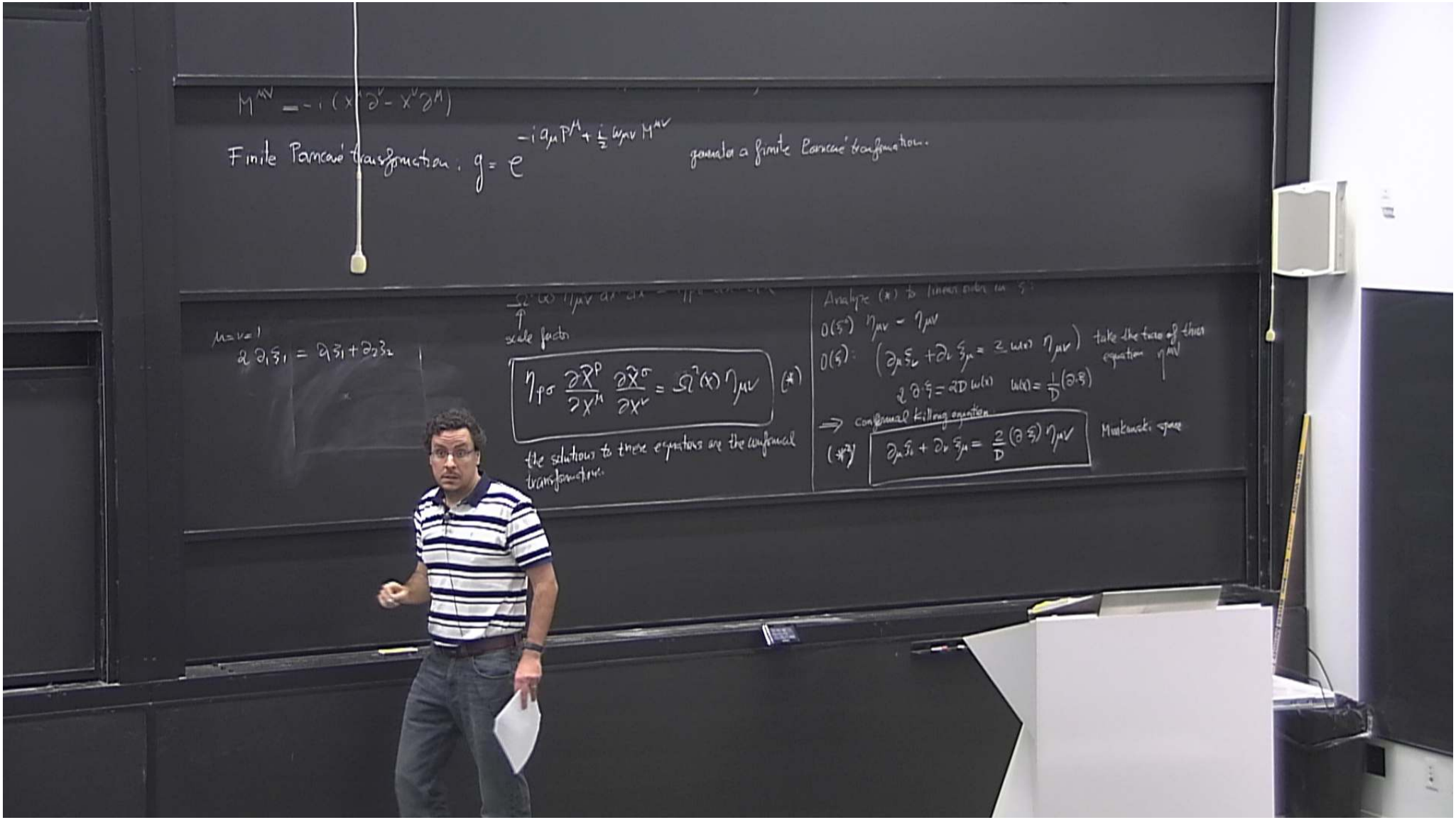
$\omega^\mu{}_\nu \longrightarrow M^{\mu\nu} : x^\mu \longrightarrow x^\mu + \omega^\mu{}_\nu x^\nu$

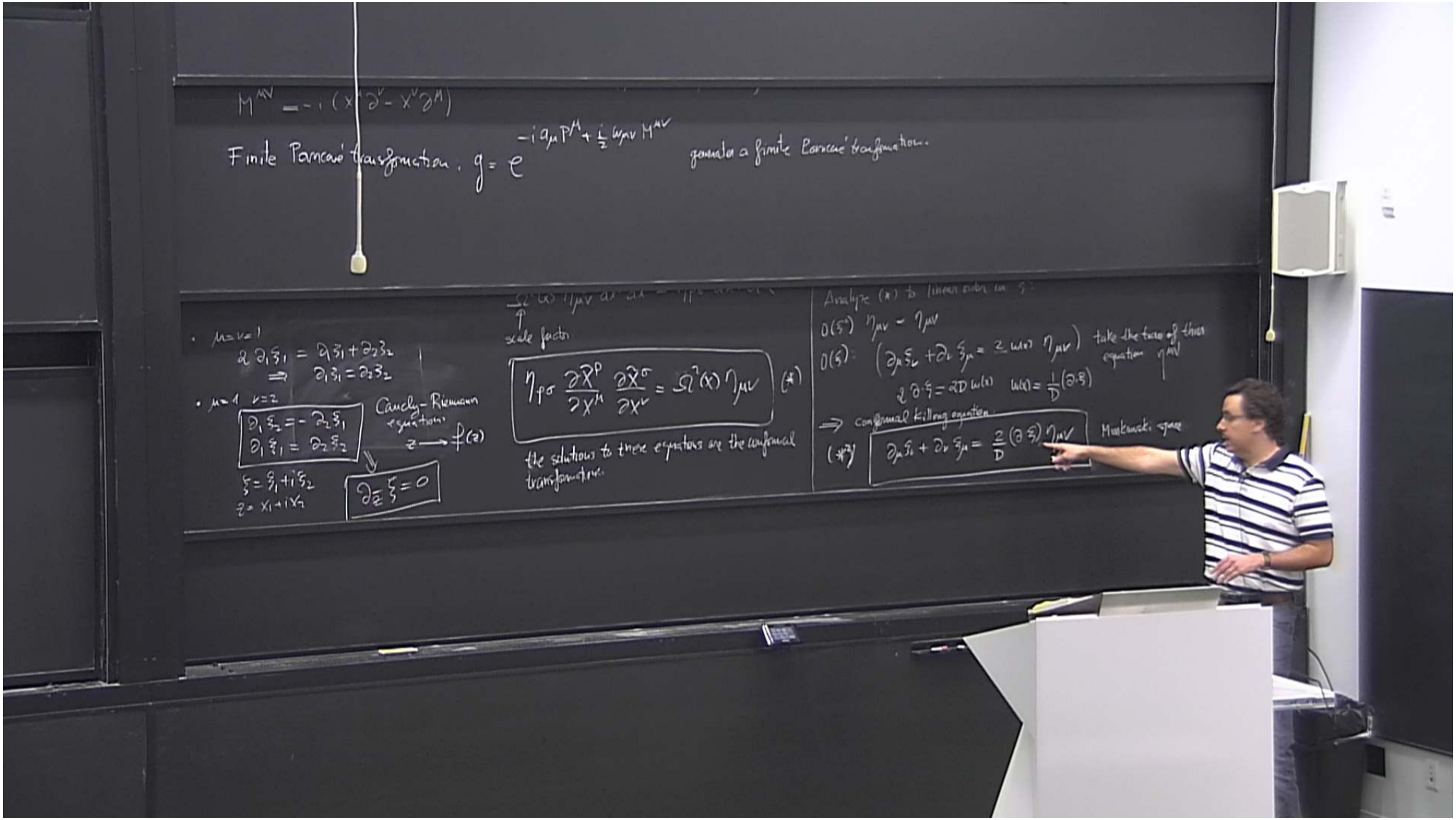
$\lambda \longrightarrow D : x^\mu \longrightarrow x^\mu + \lambda x^\mu = (1+\lambda) x^\mu$

scale or dilatation generator

$b^\mu \longrightarrow K^\mu : x^\mu \longrightarrow x^\mu + b^\mu x^2 - 2 x^\mu b \cdot x$

special conformal transformation


$M^{\mu\nu} = -i(x^\mu\partial^\nu - x^\nu\partial^\mu)$
Finite Poincaré transformation: $g = e^{-i a_\mu P^\mu + \frac{1}{2}\omega_{\mu\nu} M^{\mu\nu}}$ generates a finite Poincaré transformation.
$\mu=\nu=1$
$2\,\partial_1\xi_1 = 2\partial_1\xi_1 + 2\partial_2\xi_2$
scale factor
$\eta_{\rho\sigma}\frac{\partial\tilde{x}^\rho}{\partial x^\mu}\frac{\partial\tilde{x}^\sigma}{\partial x^\nu} = \Omega^2(x)\eta_{\mu\nu}$ (*)
the solutions to these equations are the conformal transformations.
Analyze (*) to linear order in ξ:
$O(\xi^0)$ $\eta_{\mu\nu} = \eta_{\mu\nu}$
$O(\xi)$: $(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu = 2\omega(x)\eta_{\mu\nu})$ take the trace of this equation $\eta^{\mu\nu}$
$2\,\partial\cdot\xi = 2D\,\omega(x)$ $\omega(x) = \frac{1}{D}(\partial\cdot\xi)$
$\Rightarrow$ conformal killing equation
(**) $\partial_\mu\xi_\nu + \partial_\nu\xi_\mu = \frac{2}{D}(\partial\cdot\xi)\eta_{\mu\nu}$
Minkowski space


$M^{\mu\nu} = -i(x^\mu\partial^\nu - x^\nu\partial^\mu)$
Finite Poincaré transformation. $g = e^{-ia_\mu P^\mu + \frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}}$ generates a finite Poincaré transformation.
$\mu=\nu=1$
$2\,\partial_1\xi_1 = \partial_1\xi_1 + \partial_2\xi_2$
$\Rightarrow \partial_1\xi_1 = \partial_2\xi_2$
$\mu=1\ \ \nu=2$
$\partial_1\xi_2 = -\partial_2\xi_1$
$\partial_1\xi_1 = \partial_2\xi_2$
Cauchy-Riemann equations
$z \longrightarrow f(z)$
$\xi = \xi_1 + i\xi_2$
$z = x_1 + ix_2$
$\partial_{\bar z}\xi = 0$
scale factor
$\eta_{\rho\sigma}\frac{\partial X^\rho}{\partial x^\mu}\frac{\partial X^\sigma}{\partial x^\nu} = \Omega^2(x)\eta_{\mu\nu}$ (*)
the solutions to these equations are the conformal transformations.
Analyze (*) to linear order in ξ:
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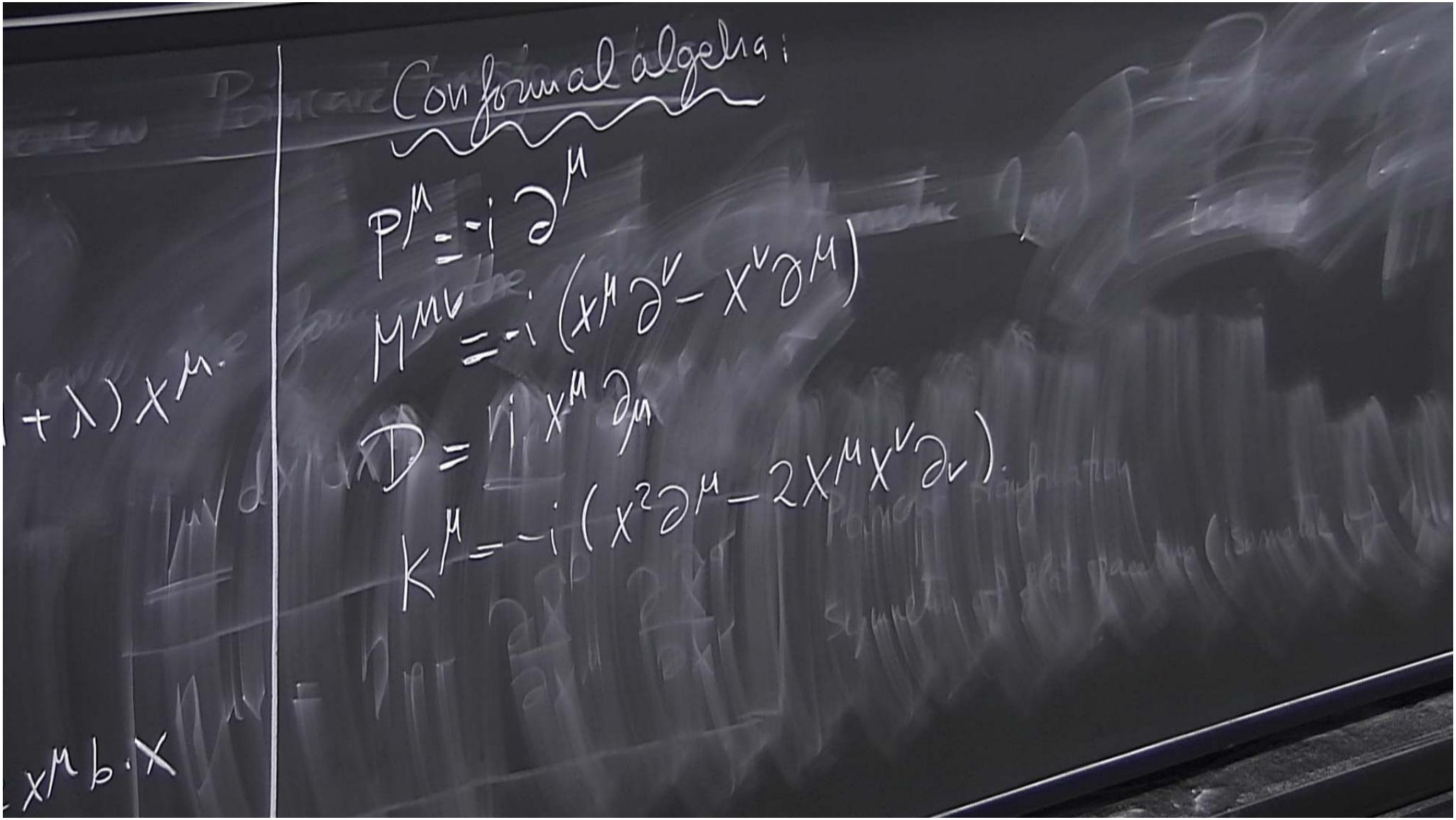
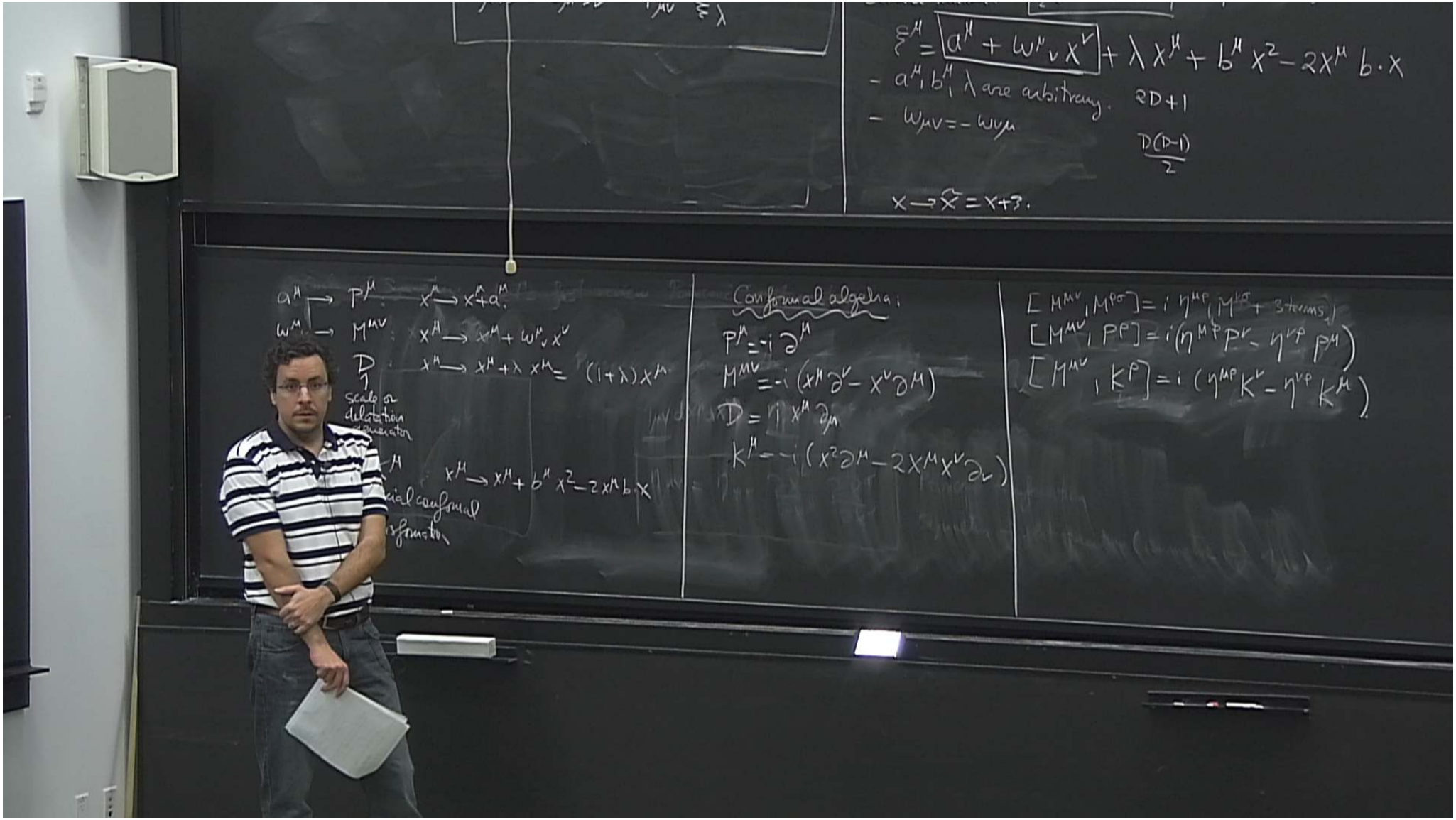
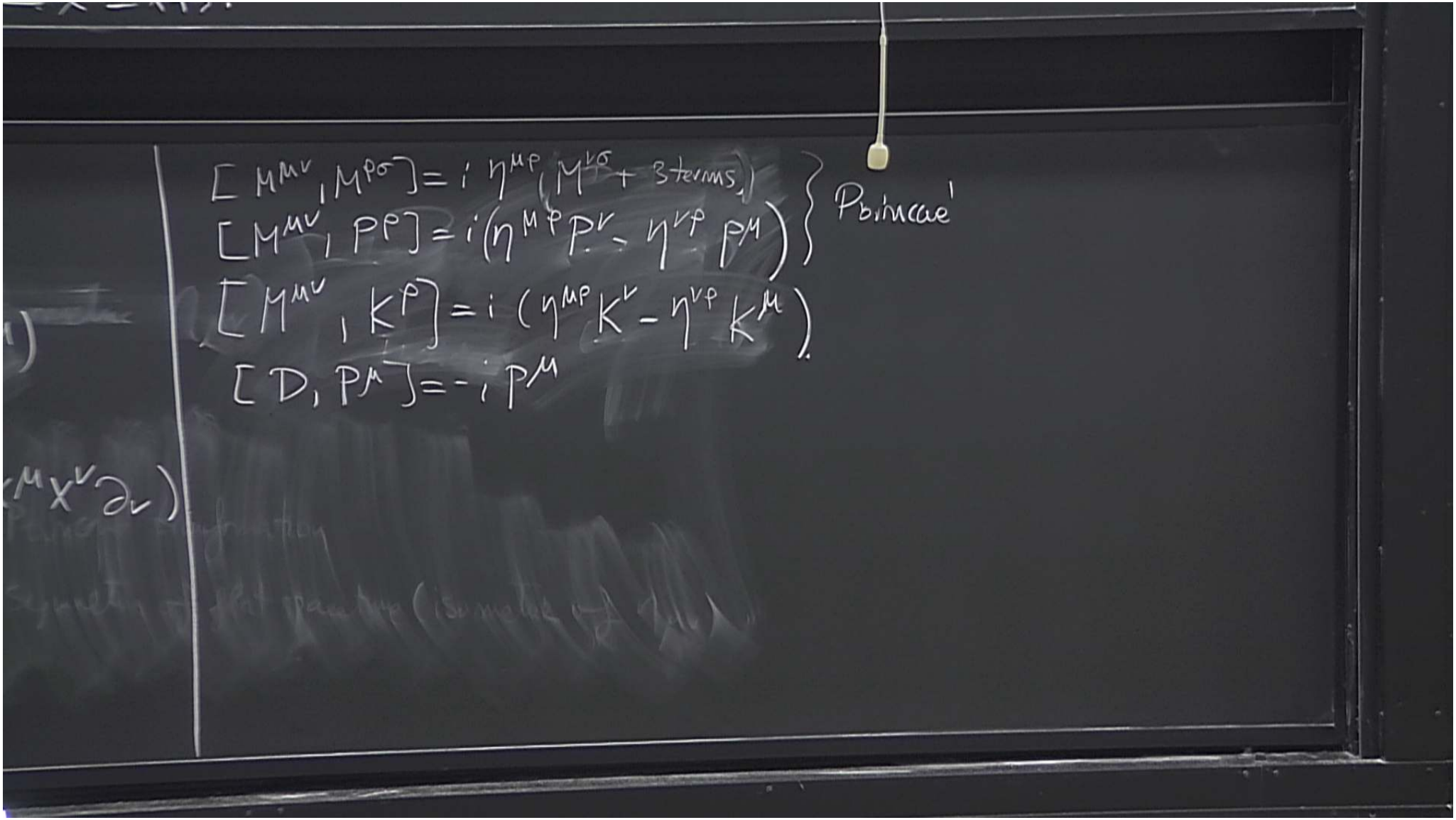
Minkowski space


Conformal algebra:
$P^\mu = -i\partial^\mu$
$M^{\mu\nu} = -i(x^\mu\partial^\nu - x^\nu\partial^\mu)$
$D = -i x^\mu \partial_\mu$
$K^\mu = -i(x^2\partial^\mu - 2x^\mu x^\nu \partial_\nu)$


$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu b\cdot x$
- a^μ, b^μ, λ are arbitrary. $2D+1$
- $\omega_{\mu\nu} = -\omega_{\nu\mu}$ $\frac{D(D-1)}{2}$
$x \to \tilde{x} = x + a$.
$a^\mu \to P^\mu$: $x^\mu \to x^\mu + a^\mu$
$\omega^\mu{}_\nu \to M^{\mu\nu}$: $x^\mu \to x^\mu + \omega^\mu{}_\nu x^\nu$
D : $x^\mu \to x^\mu + \lambda x^\mu = (1+\lambda)x^\mu$
scale or dilatation
b^μ : $x^\mu \to x^\mu + b^\mu x^2 - 2x^\mu b\cdot x$
Conformal algebra:
$P^\mu = -i\partial^\mu$
$M^{\mu\nu} = -i(x^\mu\partial^\nu - x^\nu\partial^\mu)$
$D = -i x^\mu \partial_\mu$
$K^\mu = -i(x^2\partial^\mu - 2x^\mu x^\nu \partial_\nu)$
$[M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\mu\rho}M^{\nu\sigma} + 3\text{ terms})$
$[M^{\mu\nu}, P^\rho] = i(\eta^{\mu\rho}P^\nu - \eta^{\nu\rho}P^\mu)$
$[M^{\mu\nu}, K^\rho] = i(\eta^{\mu\rho}K^\nu - \eta^{\nu\rho}K^\mu)$


$[M^{\mu\nu}, M^{\rho\sigma}] = i\,\eta^{\mu\rho}(M^{\nu\sigma} + 3\text{ terms})$
$[M^{\mu\nu}, P^{\rho}] = i(\eta^{\mu\rho}P^{\nu} - \eta^{\nu\rho}P^{\mu})$
$[M^{\mu\nu}, K^{\rho}] = i(\eta^{\mu\rho}K^{\nu} - \eta^{\nu\rho}K^{\mu})$
Poincaré
$[D, P^{\mu}] = -iP^{\mu}$

$\omega_{\mu\nu} = -\omega_{\nu\mu}$

$\frac{D(D-1)}{2}$

$x \to \tilde{x} = x + a.$

algebra:

$[M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\mu\rho} M^{\nu\sigma} + 3\text{ terms})$

$[M^{\mu\nu}, P^\rho] = i(\eta^{\mu\rho} P^\nu - \eta^{\nu\rho} P^\mu)$ } Poincaré

$[M^{\mu\nu}, K^\rho] = i(\eta^{\mu\rho} K^\nu - \eta^{\nu\rho} K^\mu)$

$[D, P^\mu] = -i P^\mu \qquad [D, K^\mu] = i K^\mu$ ⇐ P^μ, K^ν are raising and lowering operators for D

$[P^\mu, K^\nu] =$

$\partial^\nu - x^\nu \partial^\mu)$

∂_μ

$x^2 \partial^\mu - 2x^\mu x^\nu \partial_\nu)$

- $\mu = \nu = 1$

$2\,\partial_1 \xi_1 = \partial_1 \xi_1$

$\Rightarrow \partial_1 \xi_1 =$

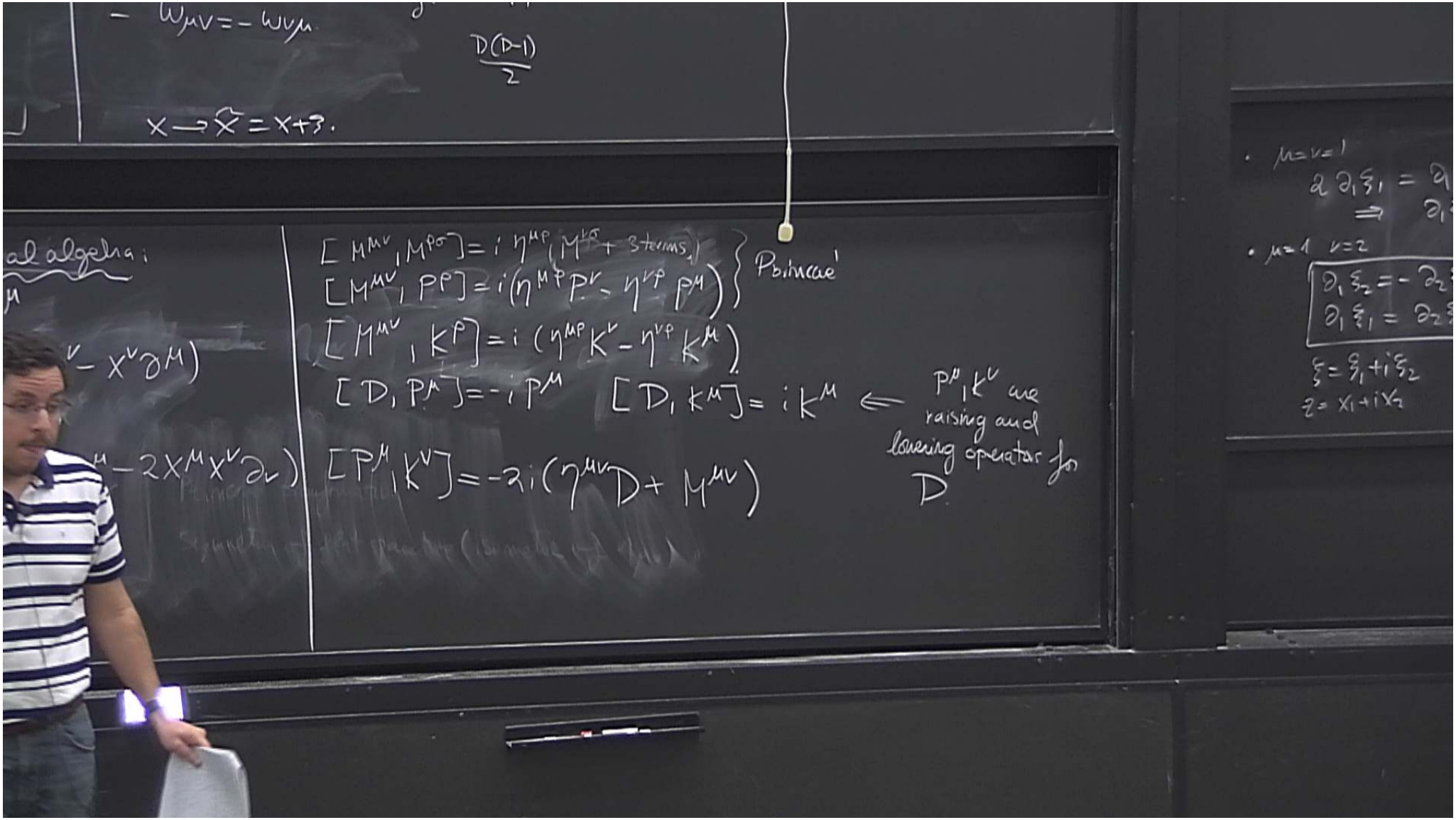
- $\mu = 1 \quad \nu = 2$

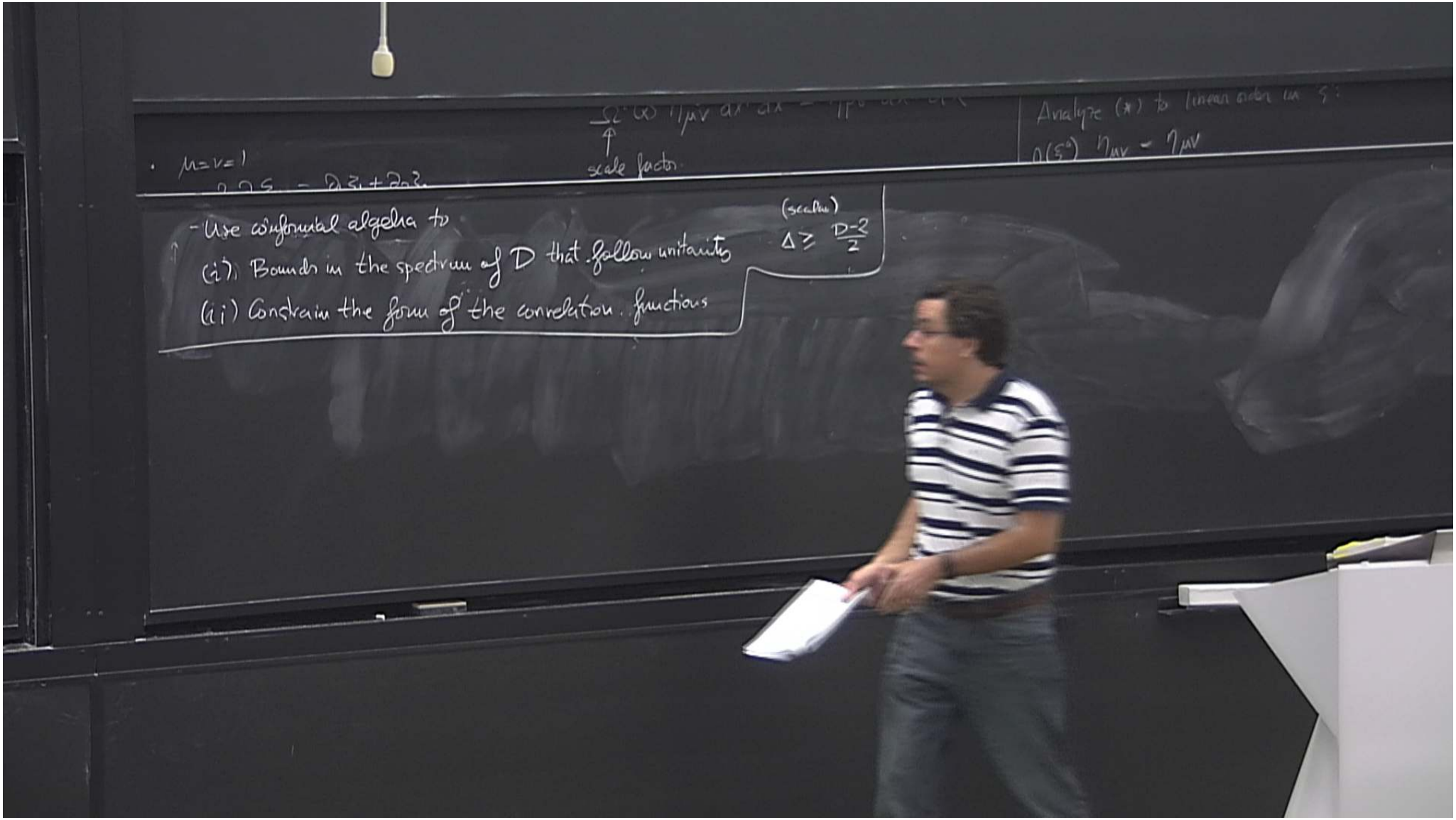
$\partial_1 \xi_2 = -\partial_2 \xi_1$

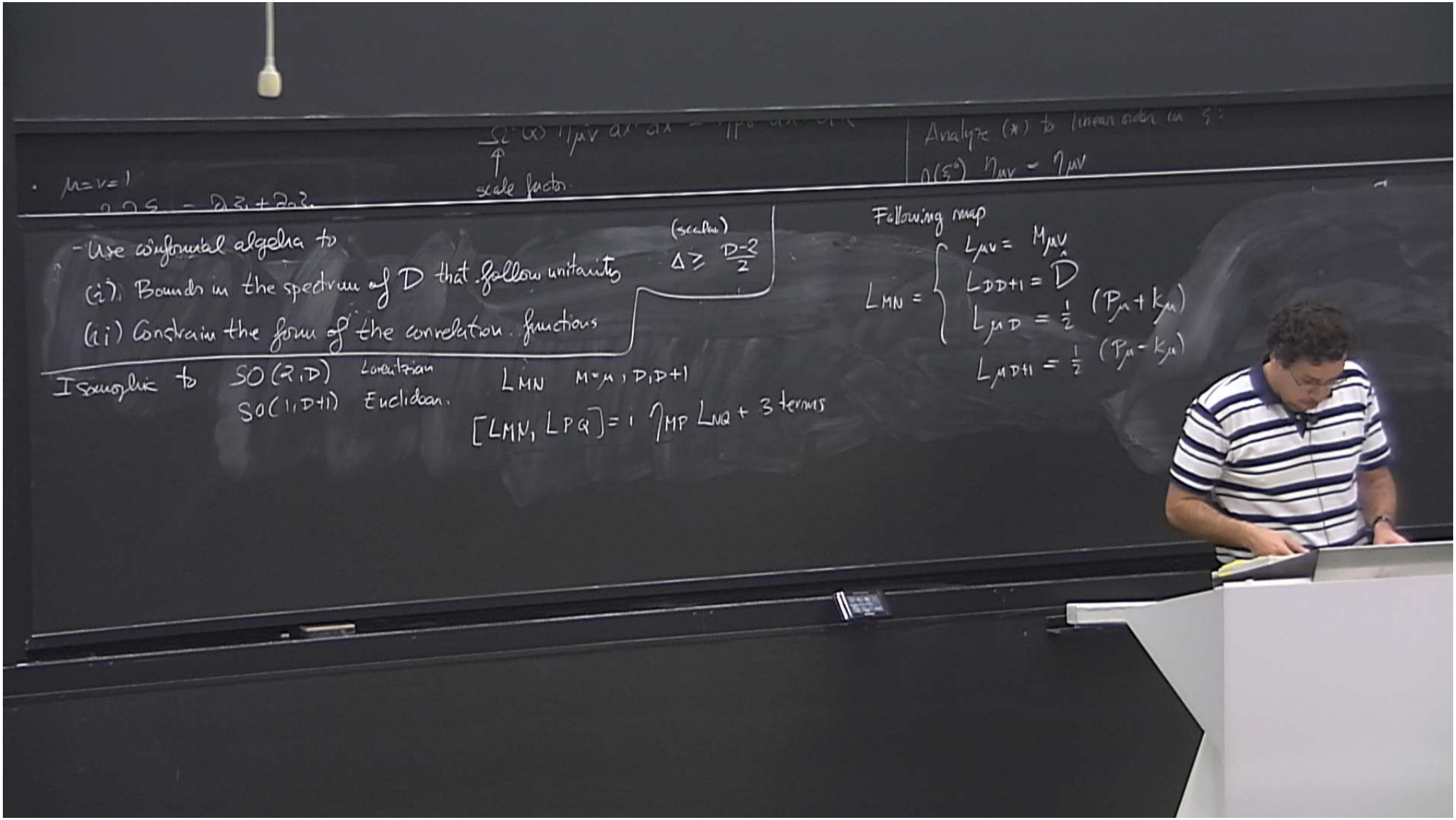
$\partial_1 \xi_1 = \partial_2 \xi_2$

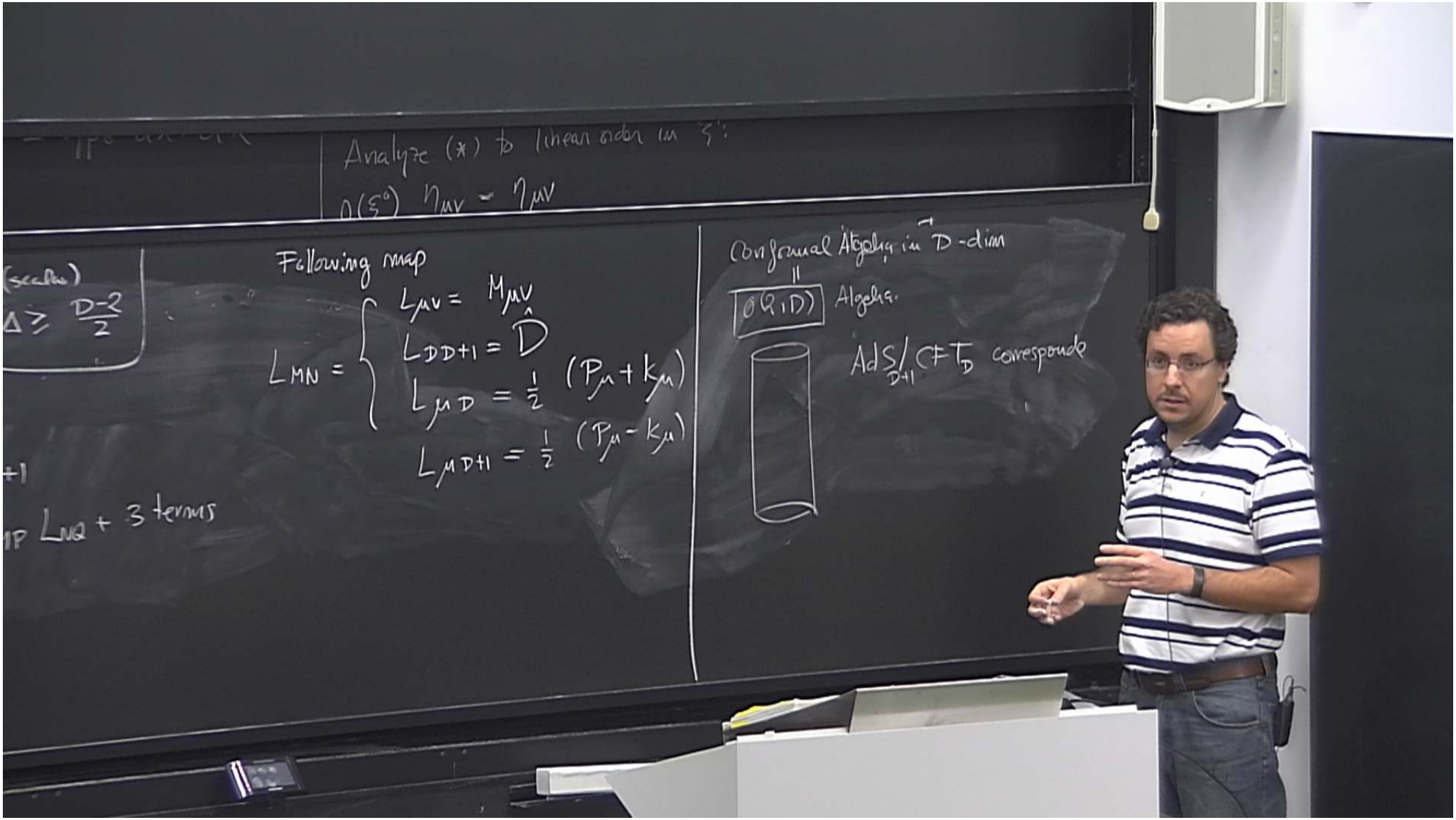
$\xi = \xi_1 + i\xi_2$

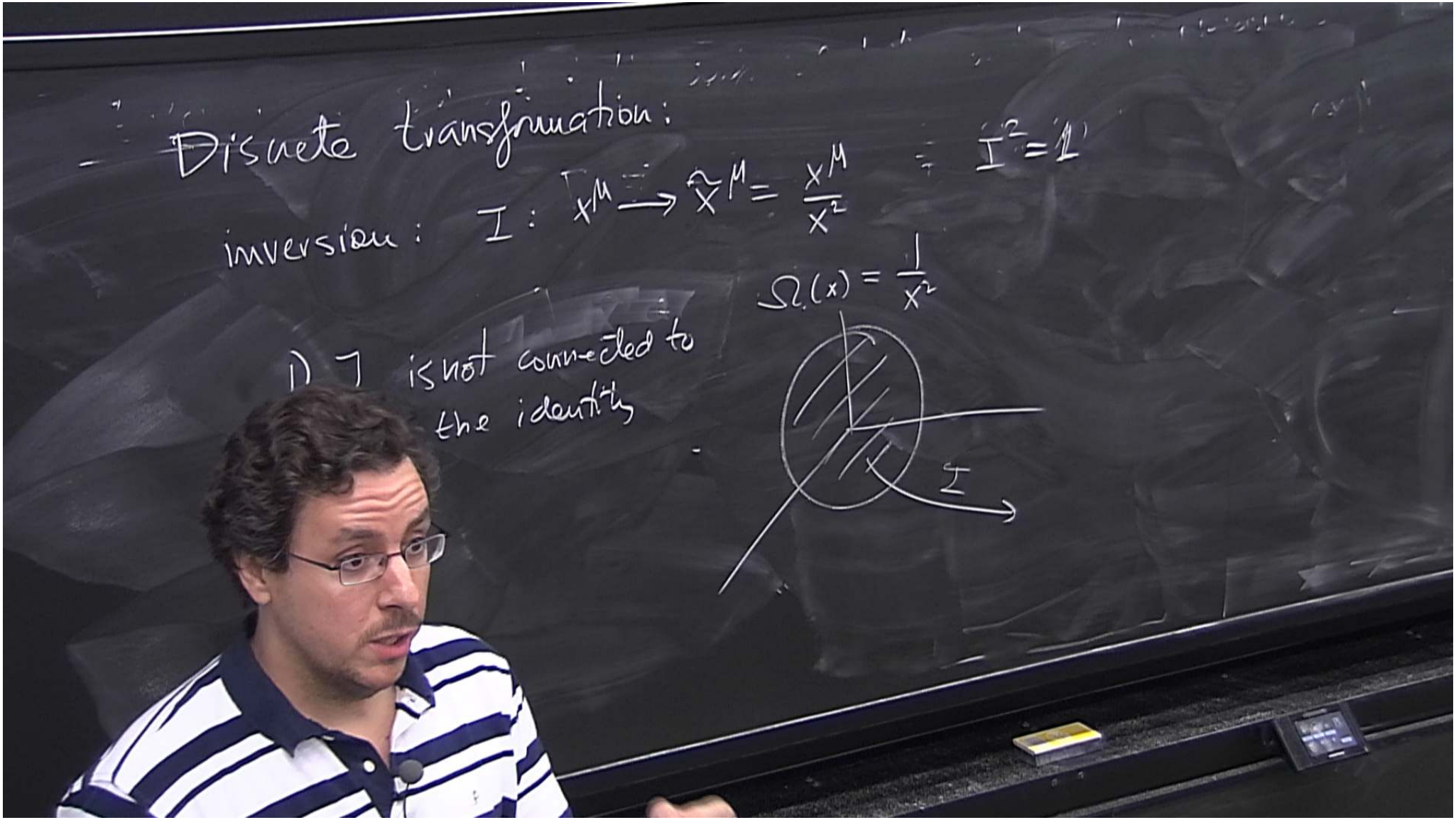
$z = x_1 + i x_2$

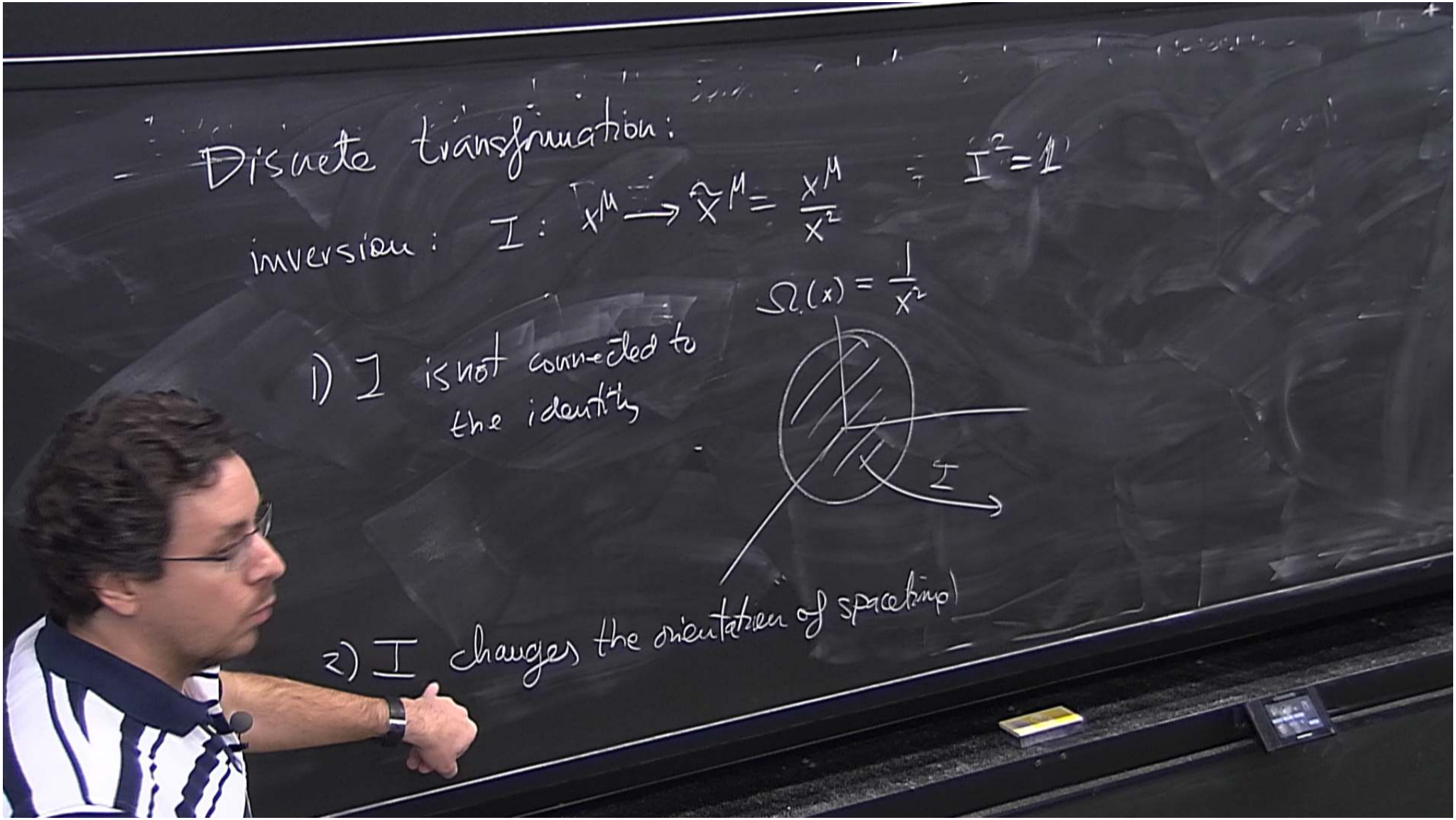

$\omega_{\mu\nu} = -\omega_{\nu\mu}$
$\frac{D(D-1)}{2}$
$x \to \tilde{x} = x + \xi$.
$[M^{\mu\nu}, M^{\rho\sigma}] = i\,\eta^{\mu\rho}(M^{\nu\sigma} + 3\text{ terms})$
$[M^{\mu\nu}, P^\rho] = i(\eta^{\mu\rho}P^\nu - \eta^{\nu\rho}P^\mu)$
Poincaré
$[M^{\mu\nu}, K^\rho] = i(\eta^{\mu\rho}K^\nu - \eta^{\nu\rho}K^\mu)$
$[D, P^\mu] = -iP^\mu$
$[D, K^\mu] = iK^\mu$
P^μ, K^ν are raising and lowering operators for D
$[P^\mu, K^\nu] = -2i(\eta^{\mu\nu}D + M^{\mu\nu})$
$\mu = \nu = 1$
$\mu = 1 \quad \nu = 2$
$\partial_1\xi_2 = -\partial_2$
$\partial_1\xi_1 = \partial_2$
$\xi = \xi_1 + i\xi_2$
$z = x_1 + ix_2$


scale factor
Analyze (*) to linear order in ε:
- Use conformal algebra to
(i) Bounds in the spectrum of D that follow unitarity
(ii) Constrain the form of the correlation functions
(scalar)
Δ ≥ (D-2)/2


scale factor
Analyze (*) to linear order in ξ:
Following map
- Use conformal algebra to
(i) Bounds in the spectrum of D that follow unitarity
(ii) Constrain the form of the correlation functions
(scalar)
Δ ≥ (D-2)/2
Isomorphic to SO(2,D) Lorentzian
SO(1,D+1) Euclidean.
L_MN M=μ, D, D+1
[L_MN, L_PQ] = η_MP L_NQ + 3 terms


Analyze (*) to linear order in ξ:
$O(\xi^0)$ $\eta_{\mu\nu} = \eta_{\mu\nu}$
Following map
$L_{MN} = \begin{cases} L_{\mu\nu} = M_{\mu\nu} \\ L_{DD+1} = \hat{D} \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$
Conformal Algebra in D-dim
$O(2,D)$ Algebra.
AdS_{D+1}/CFT_D corespondo
$\Delta \geq \frac{D-2}{2}$
$+1$
$P\, L_{\nu Q} + 3$ terms


Discrete transformation:
inversion: $I: x^\mu \to \tilde{x}^\mu = \frac{x^\mu}{x^2}$ $I^2 = 1$
$\Omega(x) = \frac{1}{x^2}$
I) I is not connected to the identity
I


Discrete transformation:
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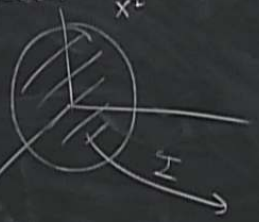
1) I is not connected to the identity
I
2) I changes the orientation of spacetime

Discrete transformation:

inversion: $I: x^\mu \to \tilde{x}^\mu = \frac{x^\mu}{x^2}$ $\quad I^2 = 1$

1) I is not connected to the identity

$\Omega(x) = \frac{1}{x^2}$



2) I changes the orientation of spacetime

Conformal Transformations by combining

- Poincaré transformations ⊕
- Inversion

} $O(2,D)$ group of transformations

↑ connected component

$- I P_\mu I \quad x^\mu \to \frac{x^\mu + a^\mu x^2}{1 + a^2 x^2 + 2a\cdot x} \simeq x^\mu + a^\mu x^2 - 2x^\mu\, a\cdot x \Longleftarrow$ spe tra