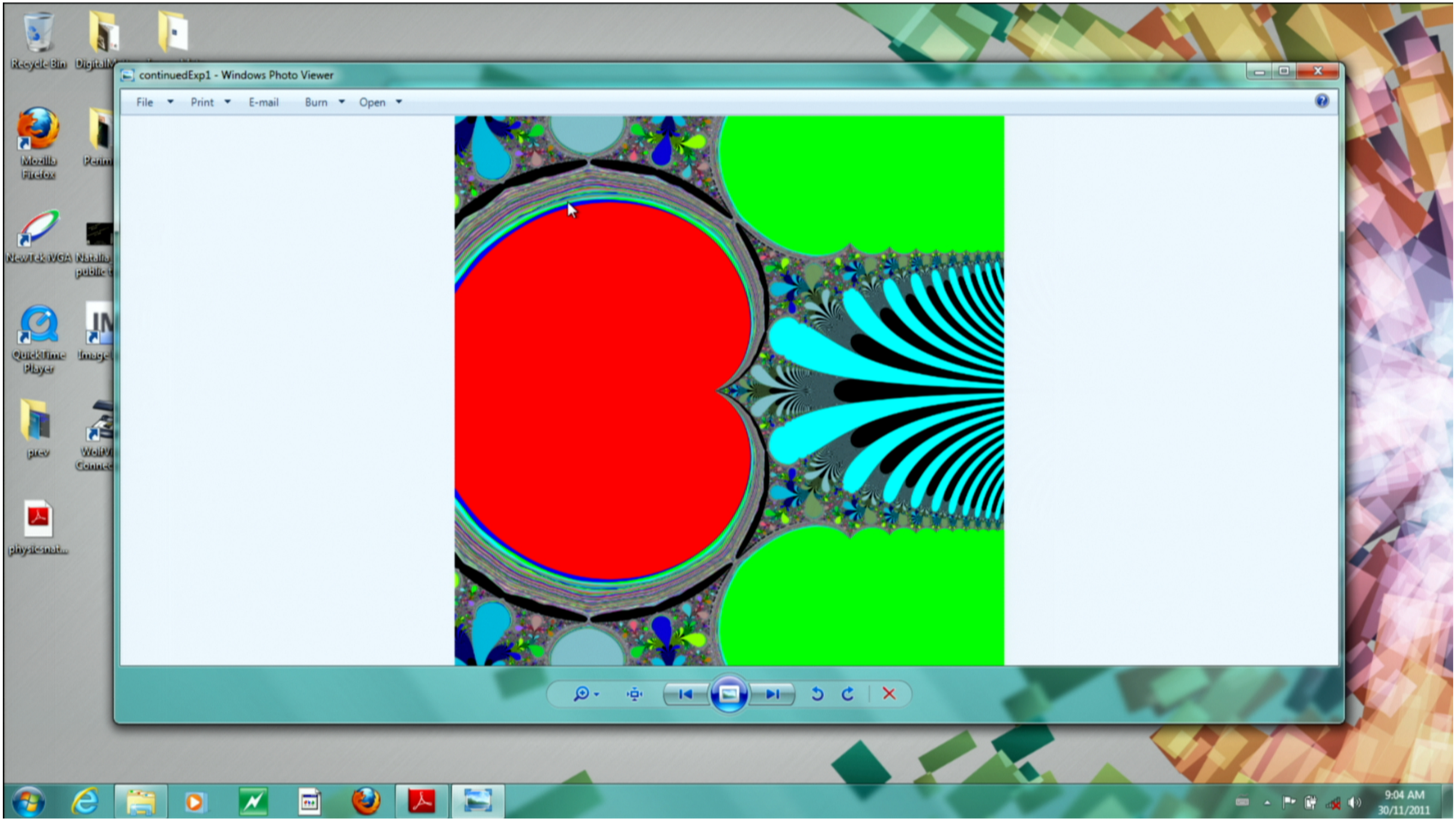
Title: Mathematical Physics - Lecture 8

Date: Nov 30, 2011 09:00 AM

URL: http://pirsa.org/11110049

Abstract: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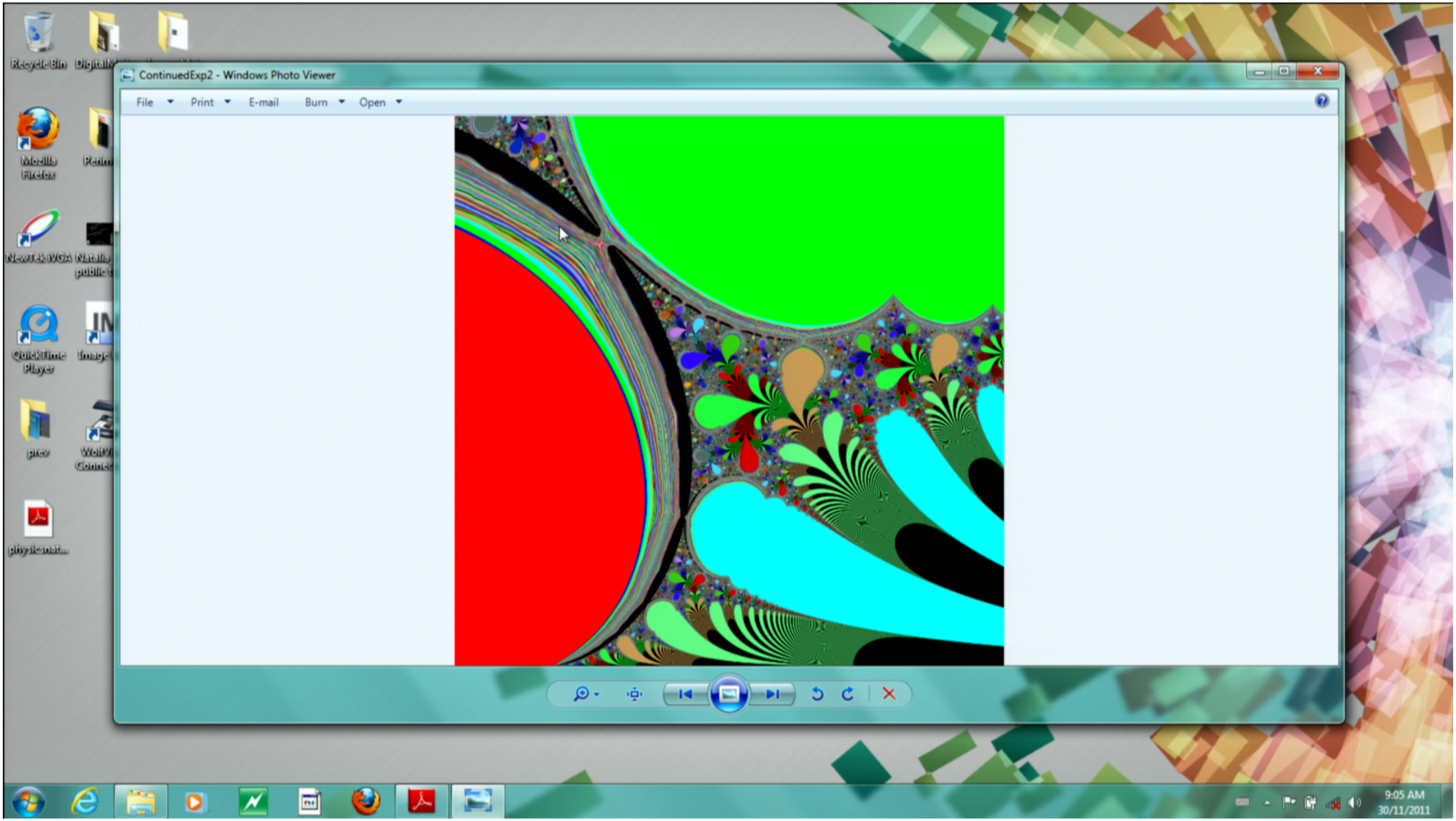
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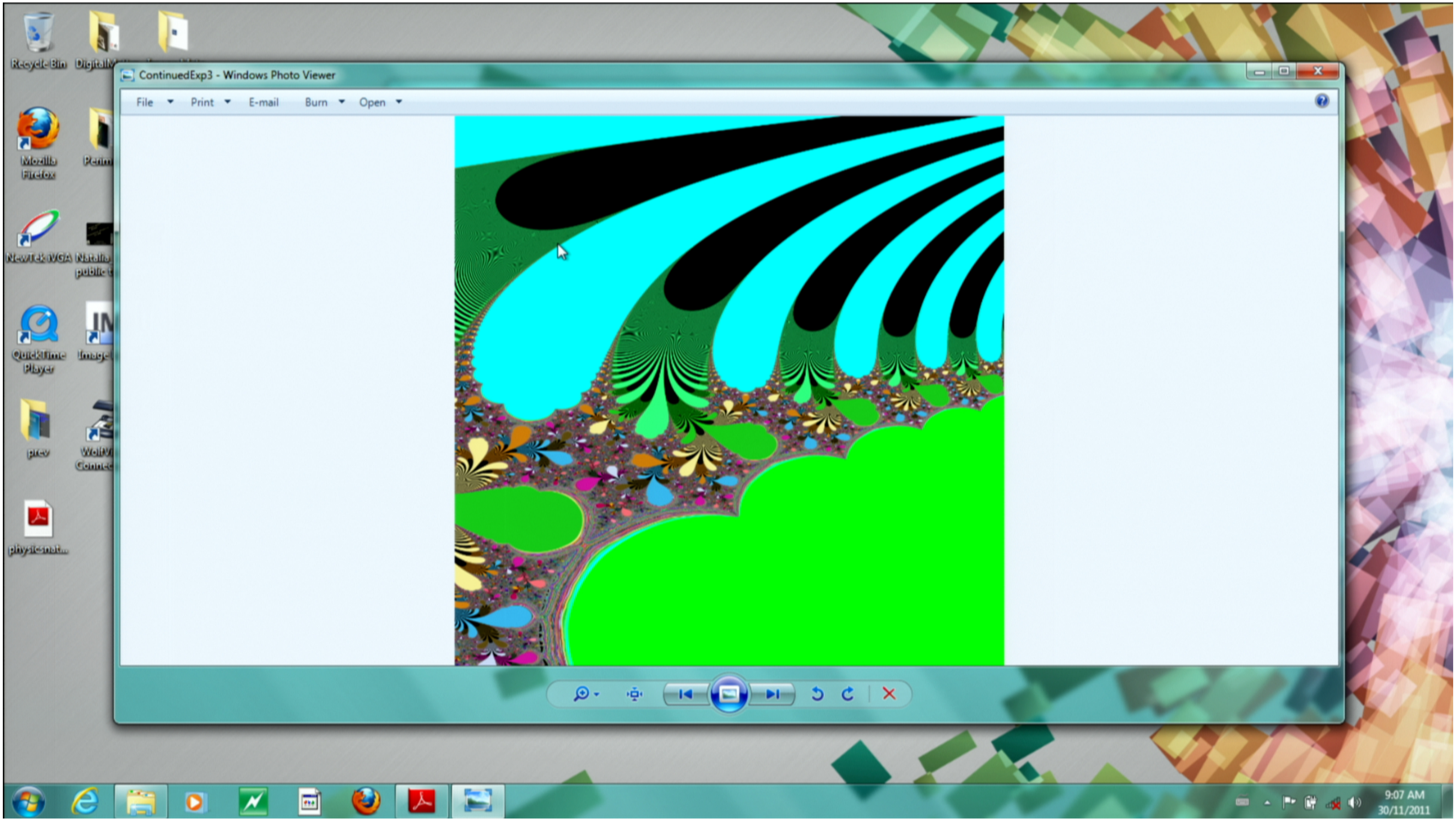
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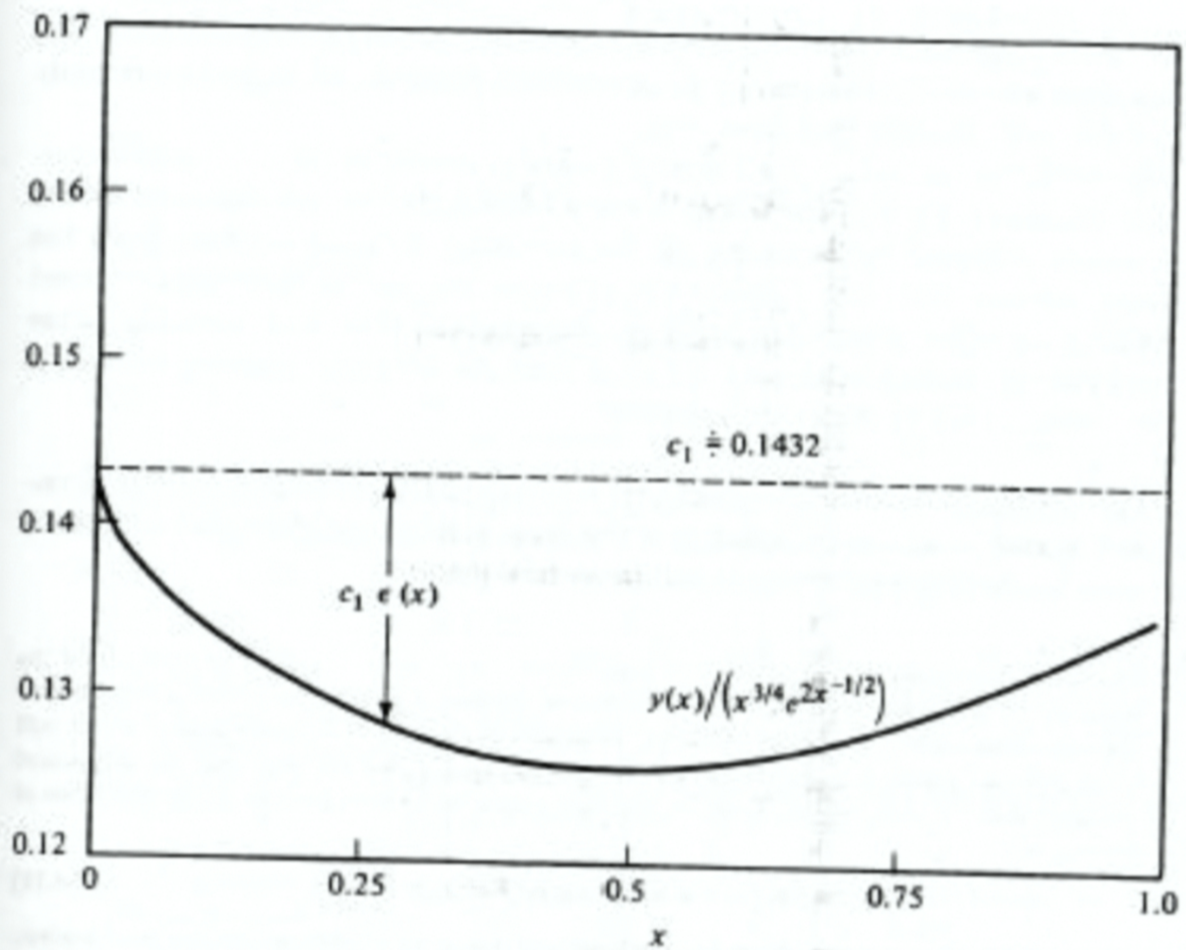
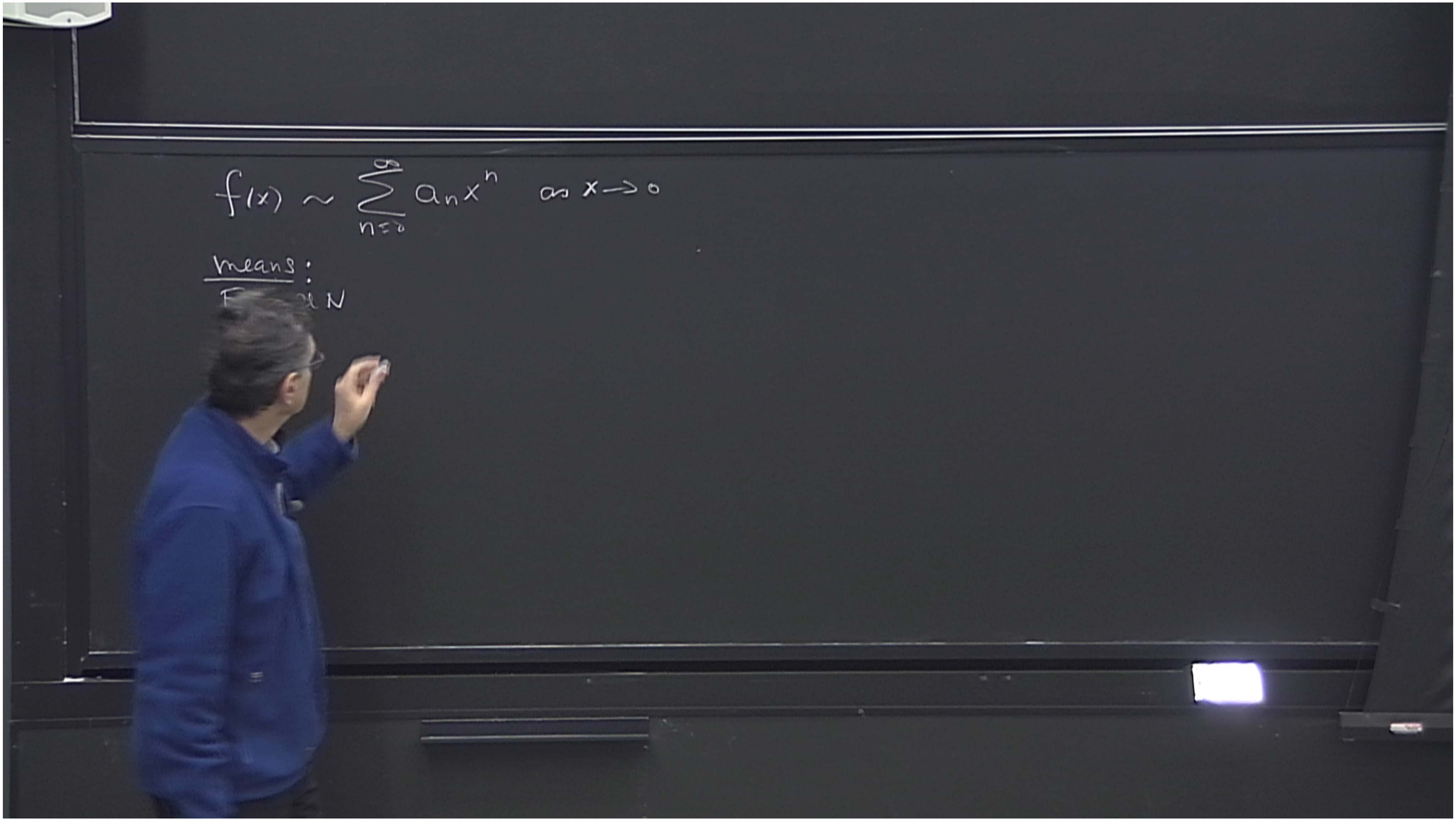
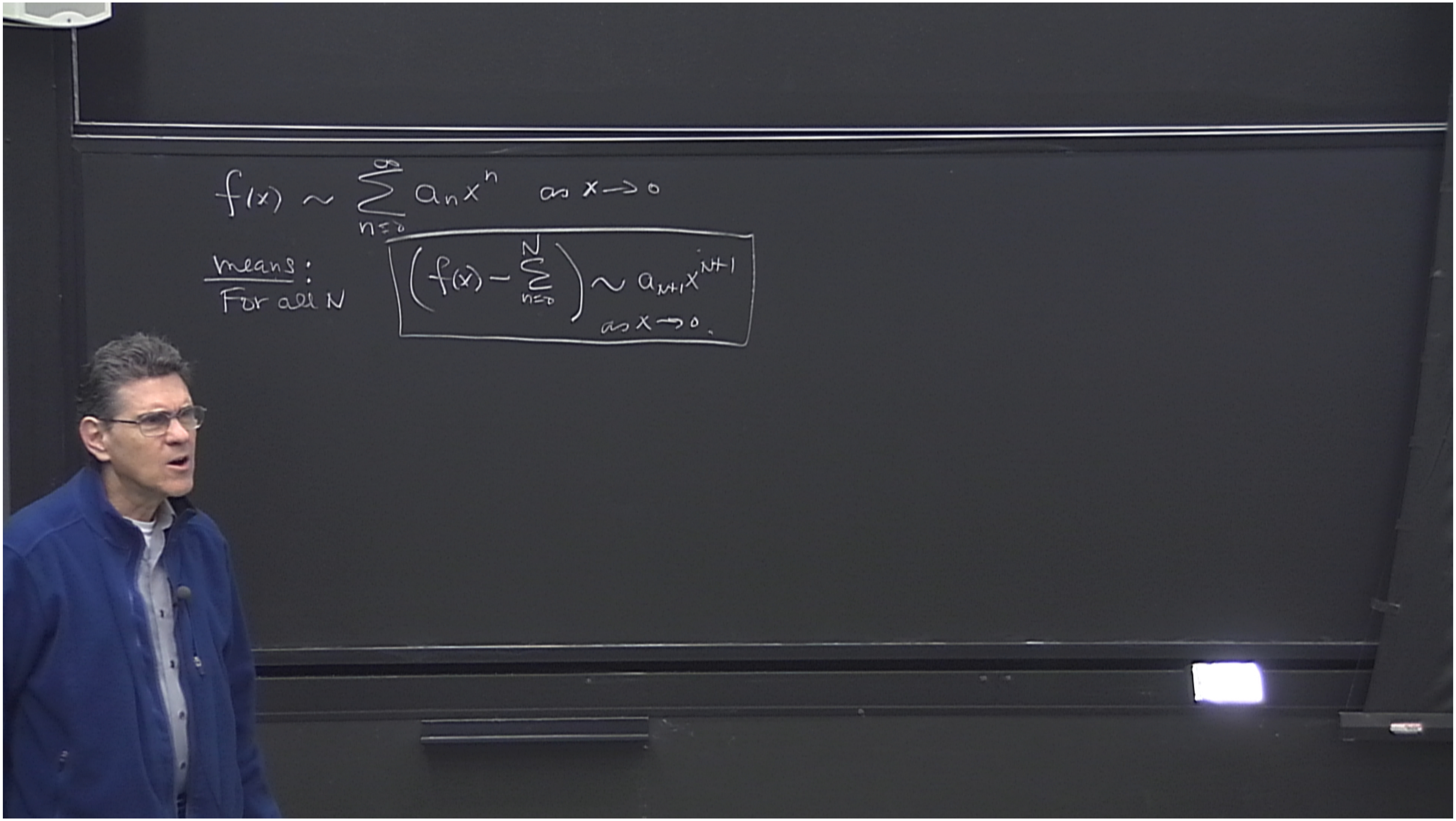
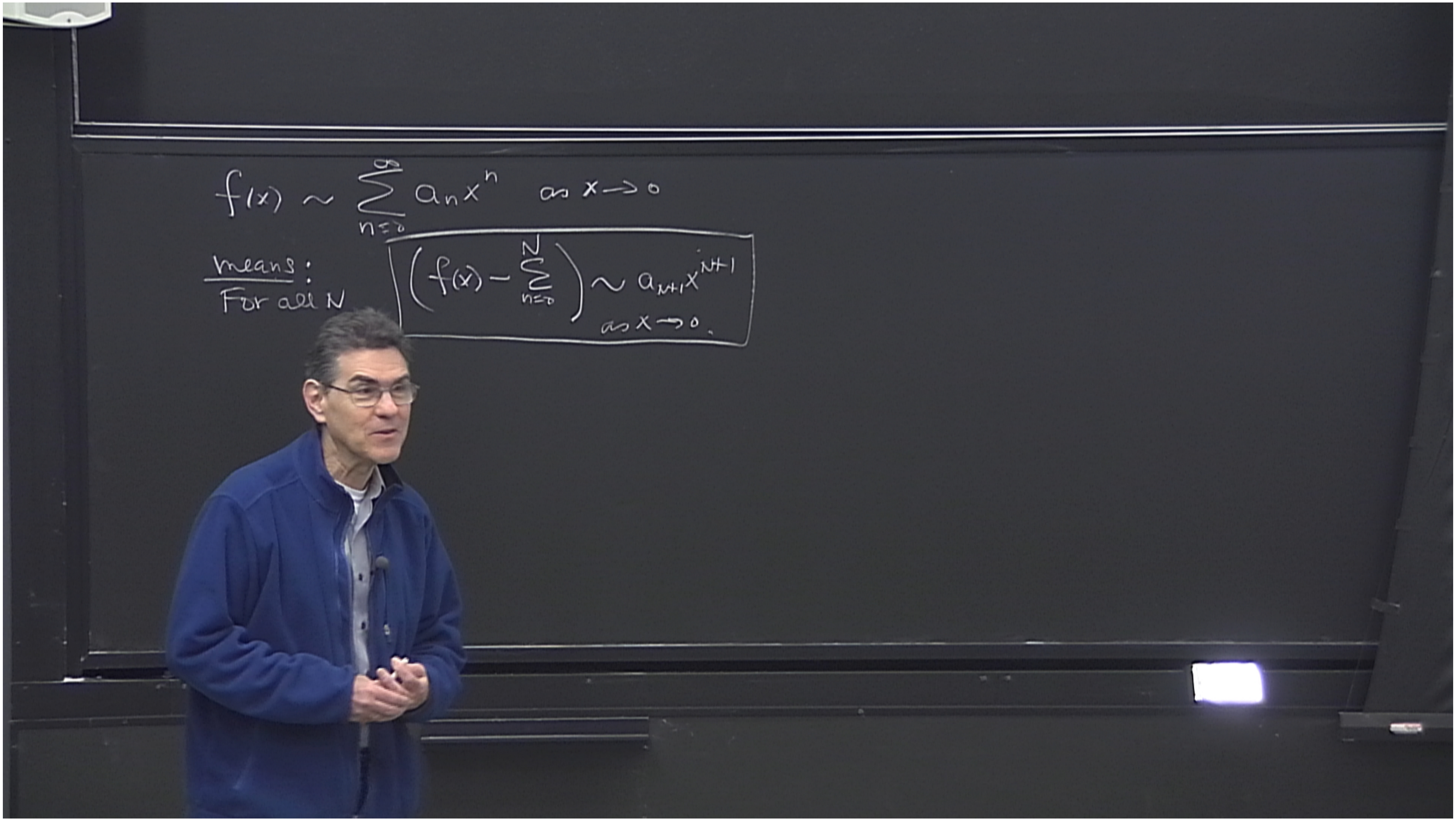


Figure 3.2 A plot of the ratio of $y(x)$, the numerical solution of (3.4.1) with $y(1) = 1$ and $y'(1) = 0$, to $x^{3/4}\exp(2x^{-1/2})$ for $0 \le x \le 1$. Asymptotic analysis [see (3.4.17)] shows that this ratio approaches a constant c_1. For these initial conditions, $c_1 \doteq 0.1432$. The difference between the plotted ratio and c_1 is $c_1\epsilon(x)$.


$f(x) \sim \sum_{n=0}^{\infty} a_n x^n$ as $x \to 0$
means:


$f(x) \sim \sum_{n=0}^{\infty} a_n x^n$ as $x \to 0$
means:
For all N
$\boxed{\left(f(x) - \sum_{n=0}^{N}\right) \sim a_{N+1} x^{N+1} \text{ as } x \to 0.}$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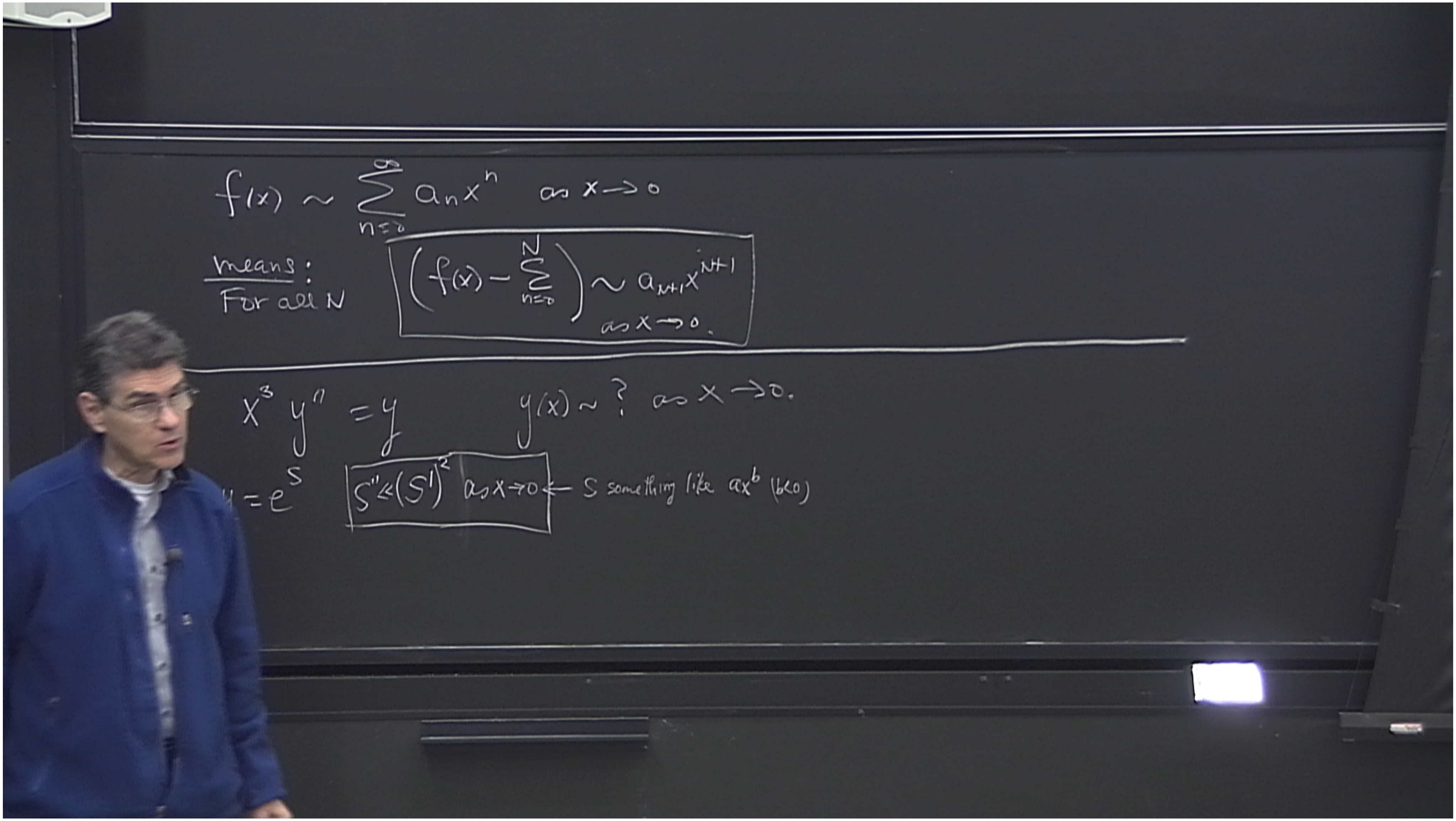
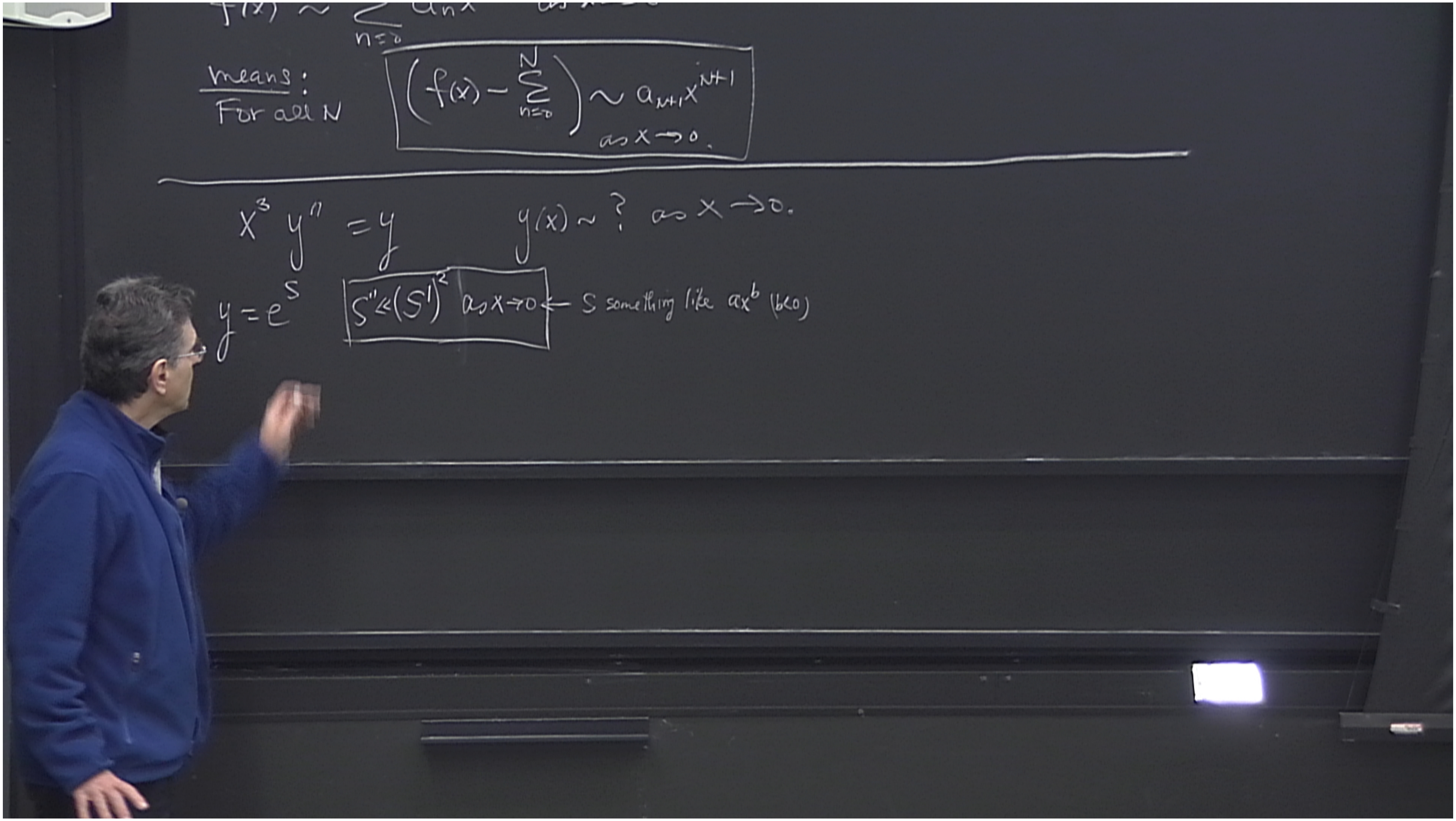
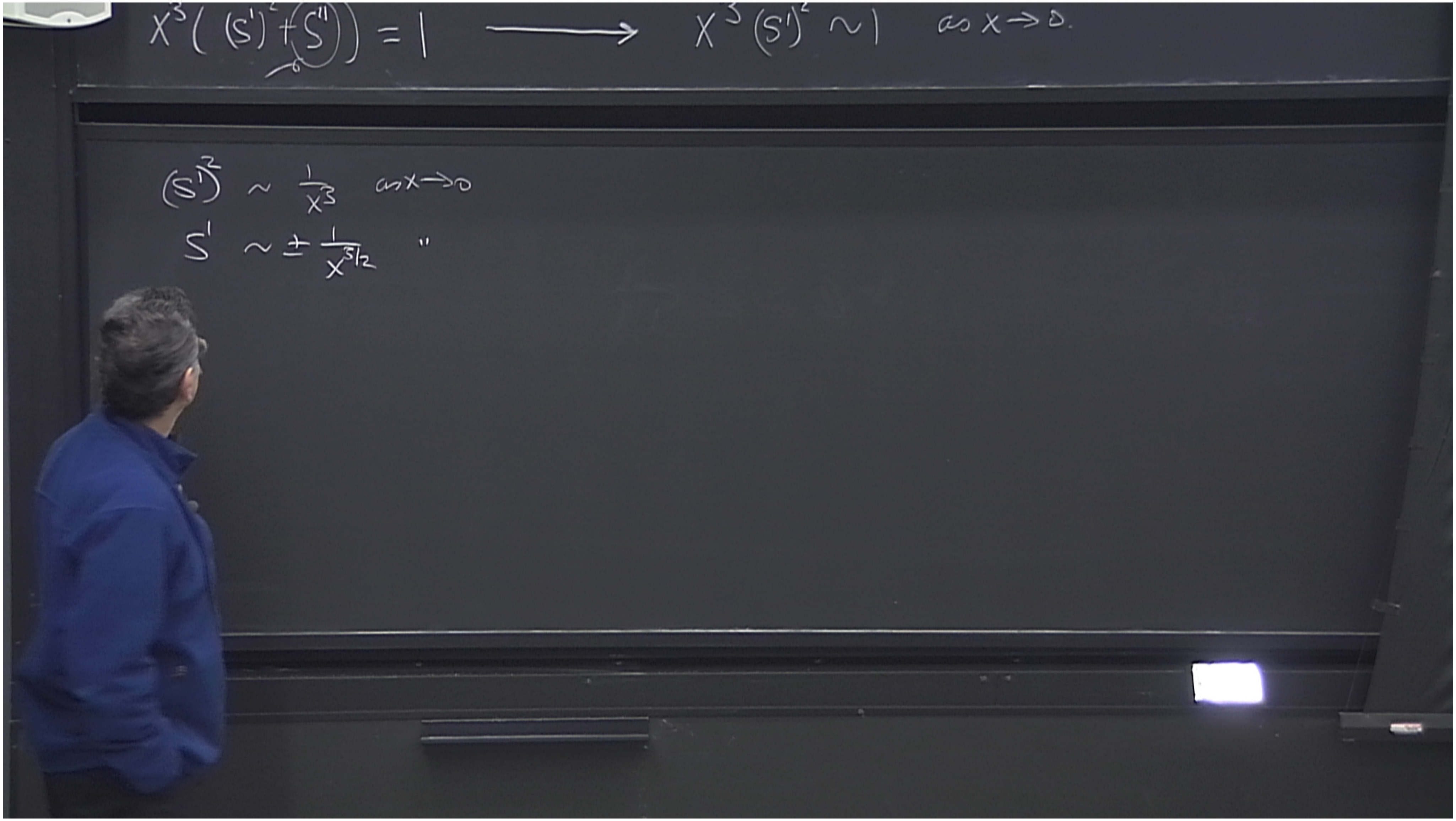
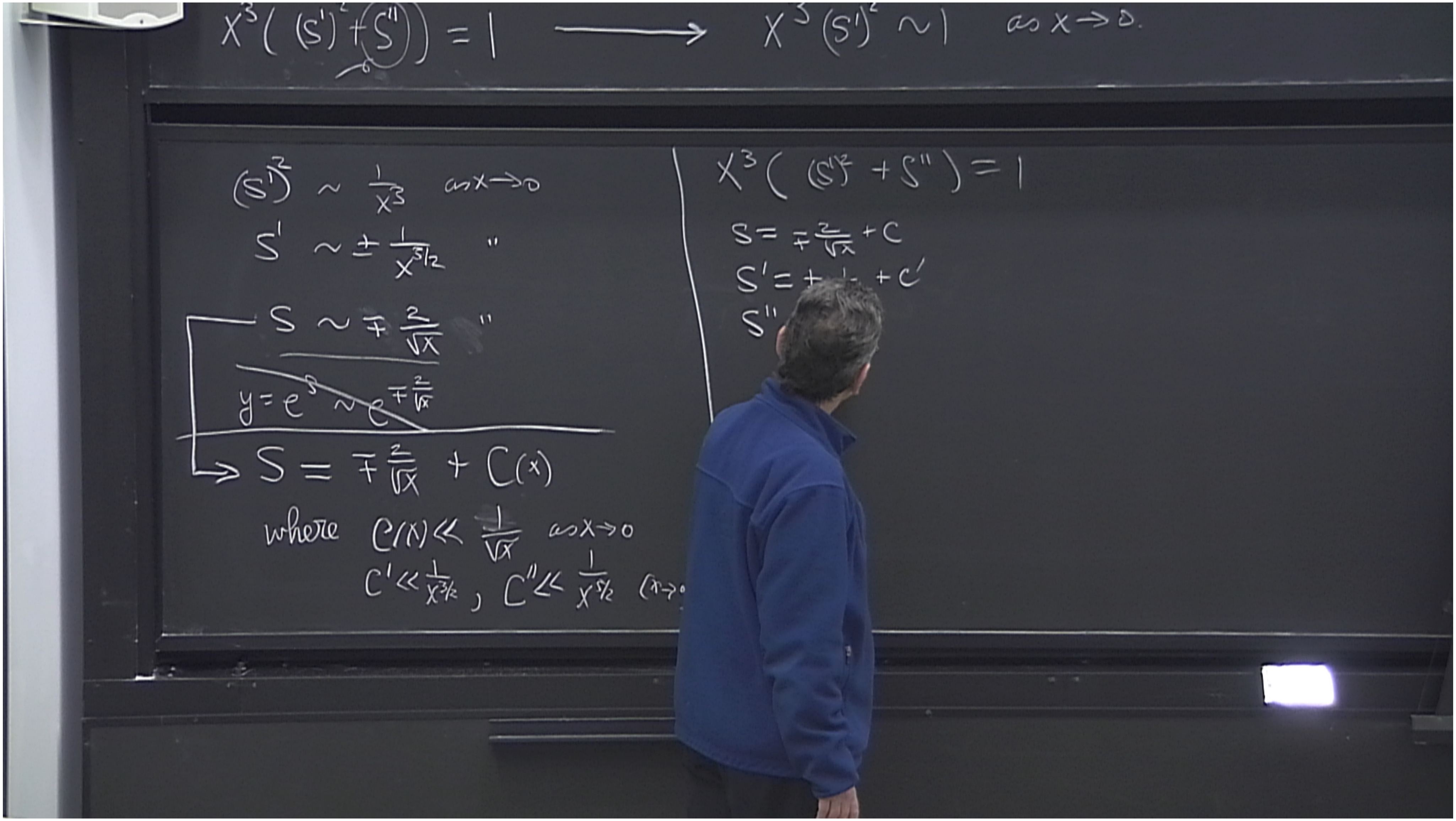

$f(x) \sim \sum_{n=0}^{\infty} a_n x^n$ as $x \to 0$
means:
For all N
$\boxed{\left(f(x) - \sum_{n=0}^{N}\right) \sim a_{N+1} x^{N+1} \text{ as } x \to 0.}$
$x^3 y'' = y$
$y(x) \sim ?$ as $x \to 0.$
$y = e^S$
$\boxed{S'' \ll (S')^2 \text{ as } x \to 0}$ ← S something like ax^b $(b<0)$


means: For all N
$\left(f(x) - \sum_{n=0}^{N}\right) \sim a_{N+1}x^{N+1}$ as $x \to 0$.
$x^3 y'' = y$
$y(x) \sim ?$ as $x \to 0$.
$y = e^S$
$S'' \ll (S')^2$ as $x \to 0$
S something like ax^b $(b<0)$


$x^3((S')^2 + S'') = 1 \longrightarrow x^3 (S')^2 \sim 1$ as $x \to 0$.
$(S')^2 \sim \frac{1}{x^3}$ as $x \to 0$
$S' \sim \pm \frac{1}{x^{3/2}}$ "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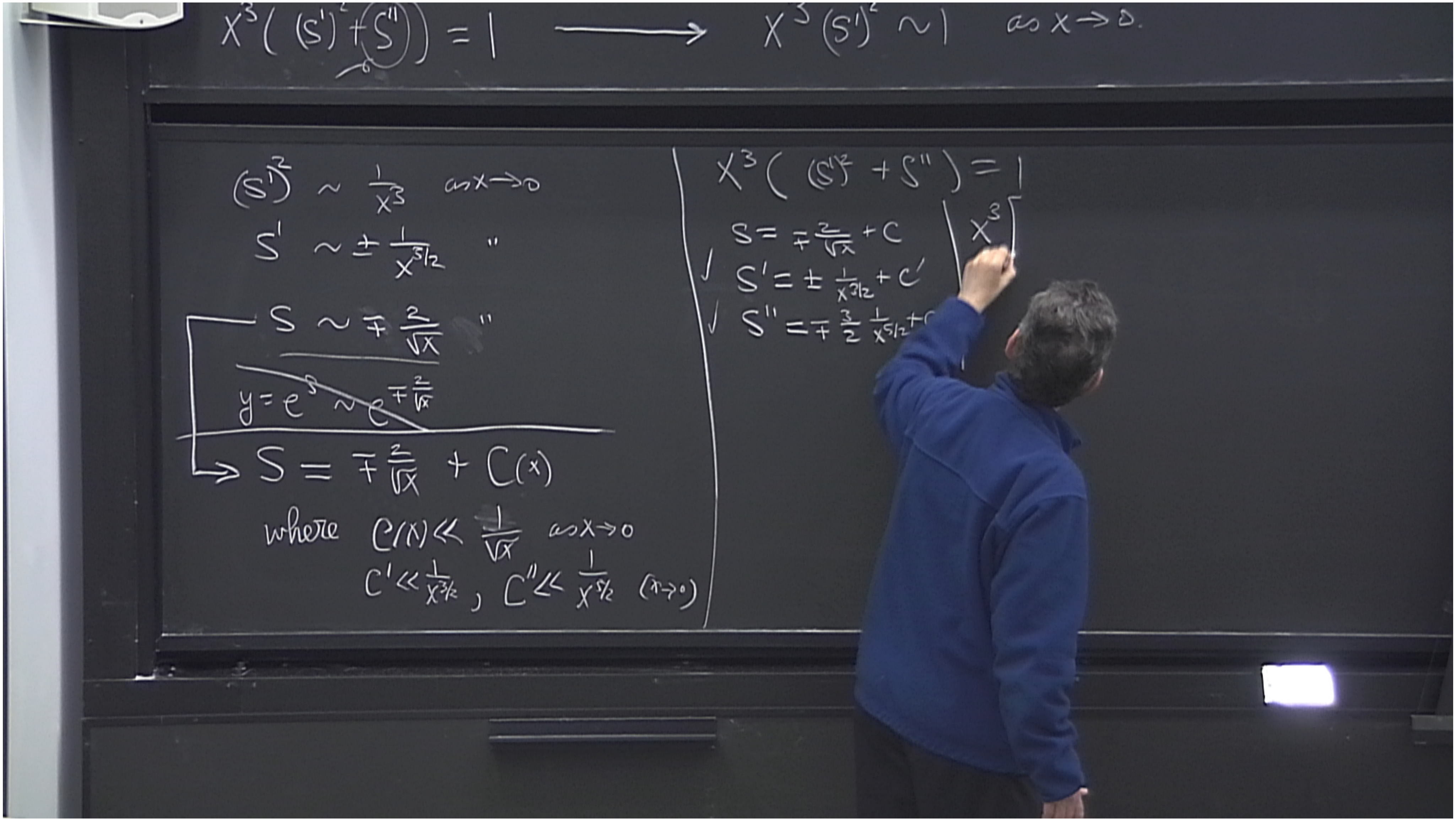
$(S')^2 \sim \frac{1}{x^3}$ as $x \to 0$
$S' \sim \pm \frac{1}{x^{3/2}}$ "
$S \sim \mp \frac{2}{\sqrt{x}}$ "
$y = e^S \sim e^{\mp \frac{2}{\sqrt{x}}}$
$S = \mp \frac{2}{\sqrt{x}} + C(x)$
where $C(x) \ll \frac{1}{\sqrt{x}}$ as $x \to 0$
$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0$
$x^3((S')^2 + S'') = 1$
$S = \mp \frac{2}{\sqrt{x}} + C$
$S' = \pm$
$+ C'$
S''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$S \sim \mp \frac{2}{\sqrt{x}}$ "
$\cancel{y = e^S \sim e^{\mp \frac{2}{\sqrt{x}}}}$
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where $C(x) \ll \frac{1}{\sqrt{x}}$ as $x \to 0$
$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0)$
$x^3((S')^2 + S'') = 1$
$S = \mp \frac{2}{\sqrt{x}} + C$
$S' = \pm \frac{1}{x^{3/2}} + C'$
$S'' = \mp \frac{3}{2} \frac{1}{x^{5/2}} + C''$
$x^3[$

$$x^3\left((S')^2 + S''\right) = 1 \longrightarrow x^3 (S')^2 \sim 1 \quad \text{as } x \to 0.$$

$(S')^2 \sim \frac{1}{x^3}$ as $x \to 0$

$S' \sim \pm \frac{1}{x^{3/2}}$ "

$S \sim \mp \frac{2}{\sqrt{x}}$ "

$\cancel{y = e^S \sim e^{\mp \frac{2}{\sqrt{x}}}}$

$S = \mp \frac{2}{\sqrt{x}} + C(x)$

where $C(x) \ll \frac{1}{\sqrt{x}}$ as $x \to 0$

$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0)$

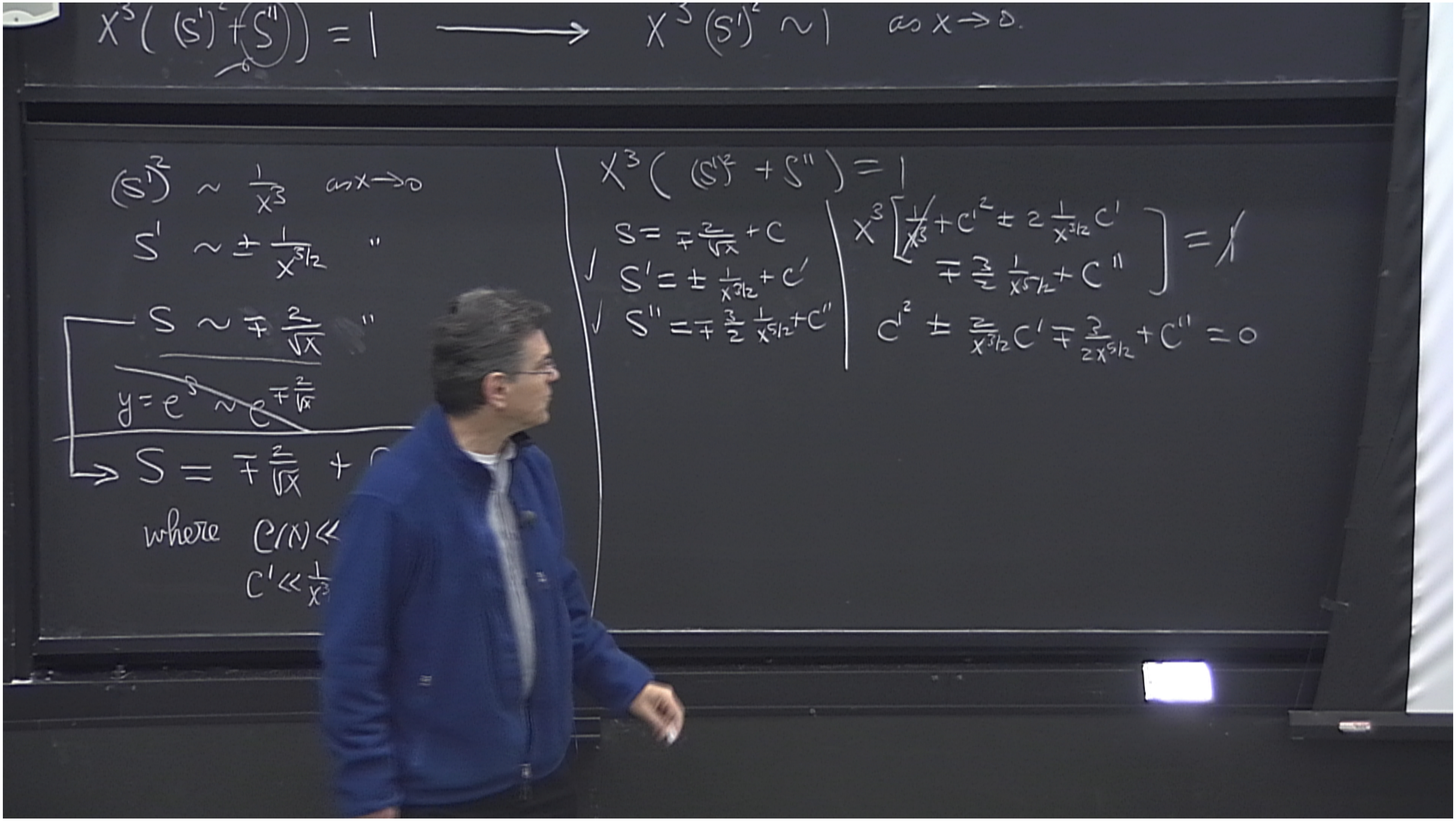
$x^3\left((S')^2 + S''\right) = 1$

$S = \mp \frac{2}{\sqrt{x}} + C$

✓ $S' = \pm \frac{1}{x^{3/2}} + C'$

✓ $S'' = \mp \frac{3}{2} \frac{1}{x^{5/2}} + C''$

$$x^3\left[\frac{1}{x^3} + C'^2 \pm 2\frac{1}{x^{3/2}}C' \mp \frac{3}{2}\frac{1}{x^{5/2}} + C''\right] = 1$$


$x^3((S')^2 + S'') = 1 \longrightarrow x^3 (S')^2 \sim 1$ as $x \to 0$.
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$S \sim \mp \frac{2}{\sqrt{x}}$ "
$y = e^S \sim e^{\mp \frac{2}{\sqrt{x}}}$
$S = \mp \frac{2}{\sqrt{x}} + C$
where $C(x) \ll$
$C' \ll \frac{1}{x^3}$
$x^3((S')^2 + S'') = 1$
$S = \mp \frac{2}{\sqrt{x}} + C$
$S' = \pm \frac{1}{x^{3/2}} + C'$
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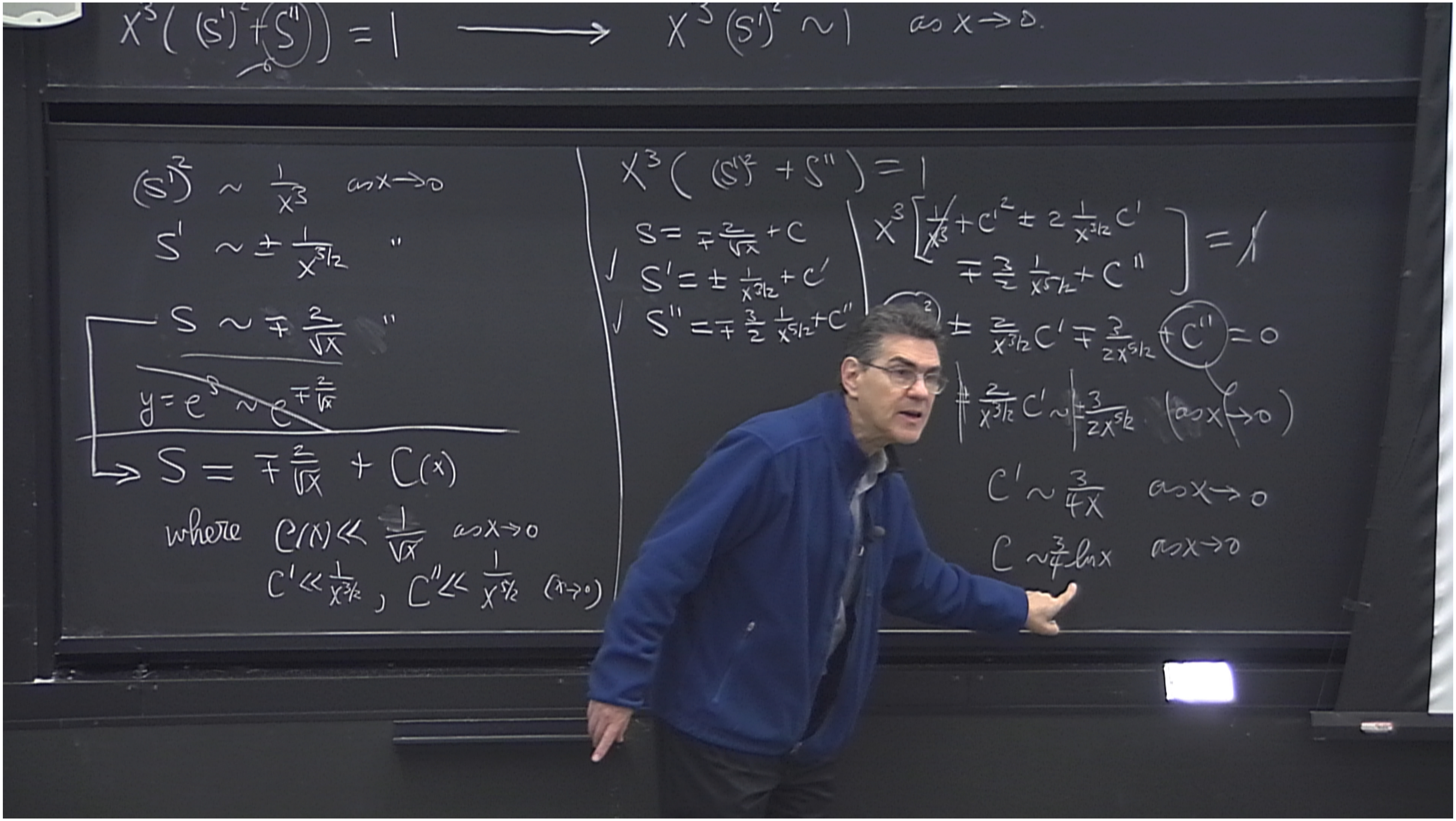
$x^3 \left[\frac{1}{x^3} + C'^2 \pm 2 \frac{1}{x^{3/2}} C' \mp \frac{3}{2} \frac{1}{x^{5/2}} + C'' \right] = 1$
$C'^2 \pm \frac{2}{x^{3/2}} C' \mp \frac{3}{2x^{5/2}} + C'' = 0$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$S = \mp \frac{2}{\sqrt{x}} + C(x)$
where $C(x) \ll \frac{1}{\sqrt{x}}$ as $x \to 0$
$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0)$
$x^3((S')^2 + S'') = 1$
$S = \mp \frac{2}{\sqrt{x}} + C$
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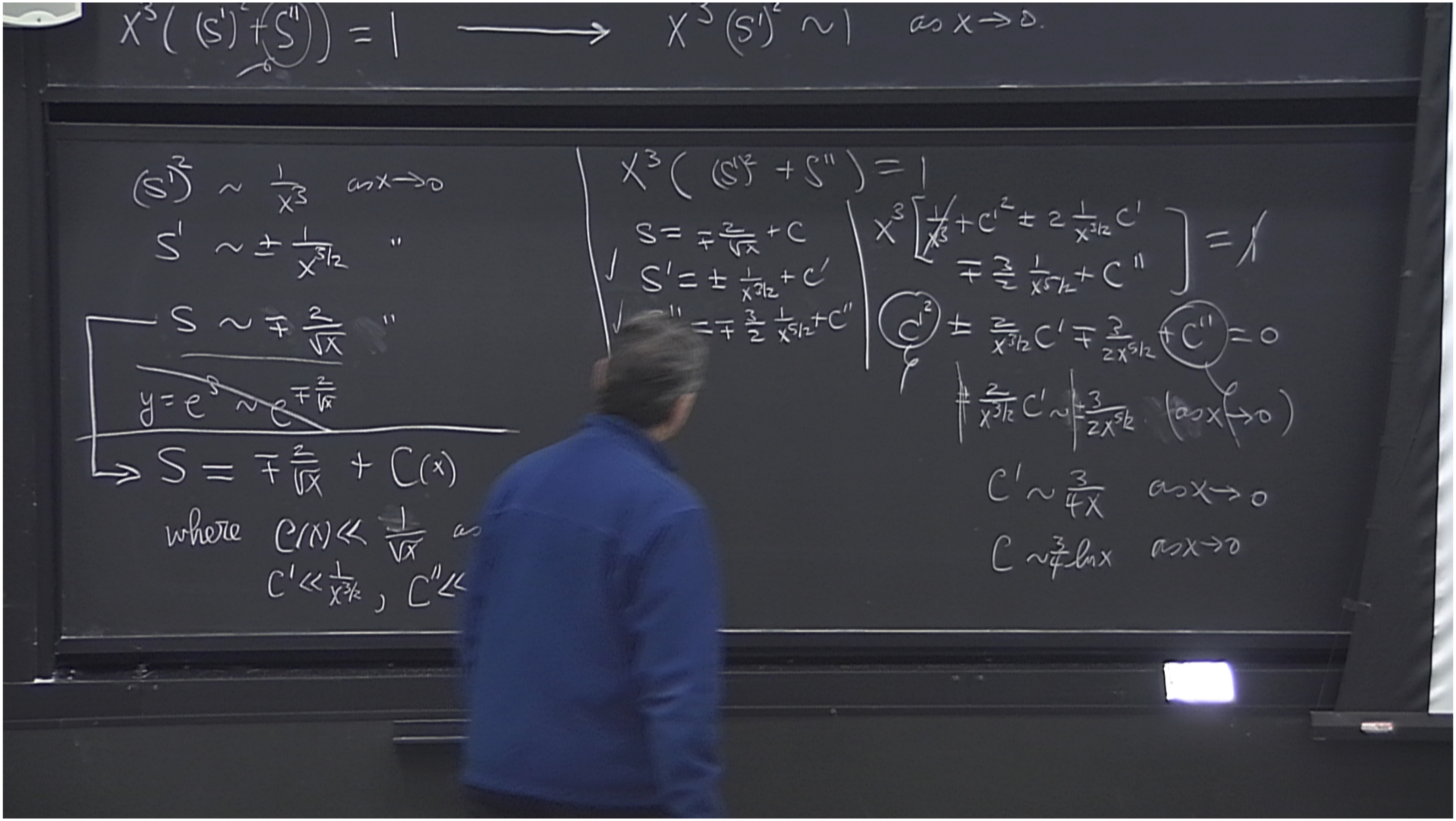
$x^3 \left[\frac{1}{x^3} + C'^2 \pm 2 \frac{1}{x^{3/2}} C' \mp \frac{3}{2} \frac{1}{x^{5/2}} + C'' \right] = 1$
$\pm \frac{2}{x^{3/2}} C' \mp \frac{3}{2x^{5/2}} + C'' = 0$
$\frac{2}{x^{3/2}} C' \sim \frac{3}{2x^{5/2}}$ (as $x \to 0$)
$C' \sim \frac{3}{4x}$ as $x \to 0$
$C \sim \frac{3}{4} \ln x$ as $x \to 0$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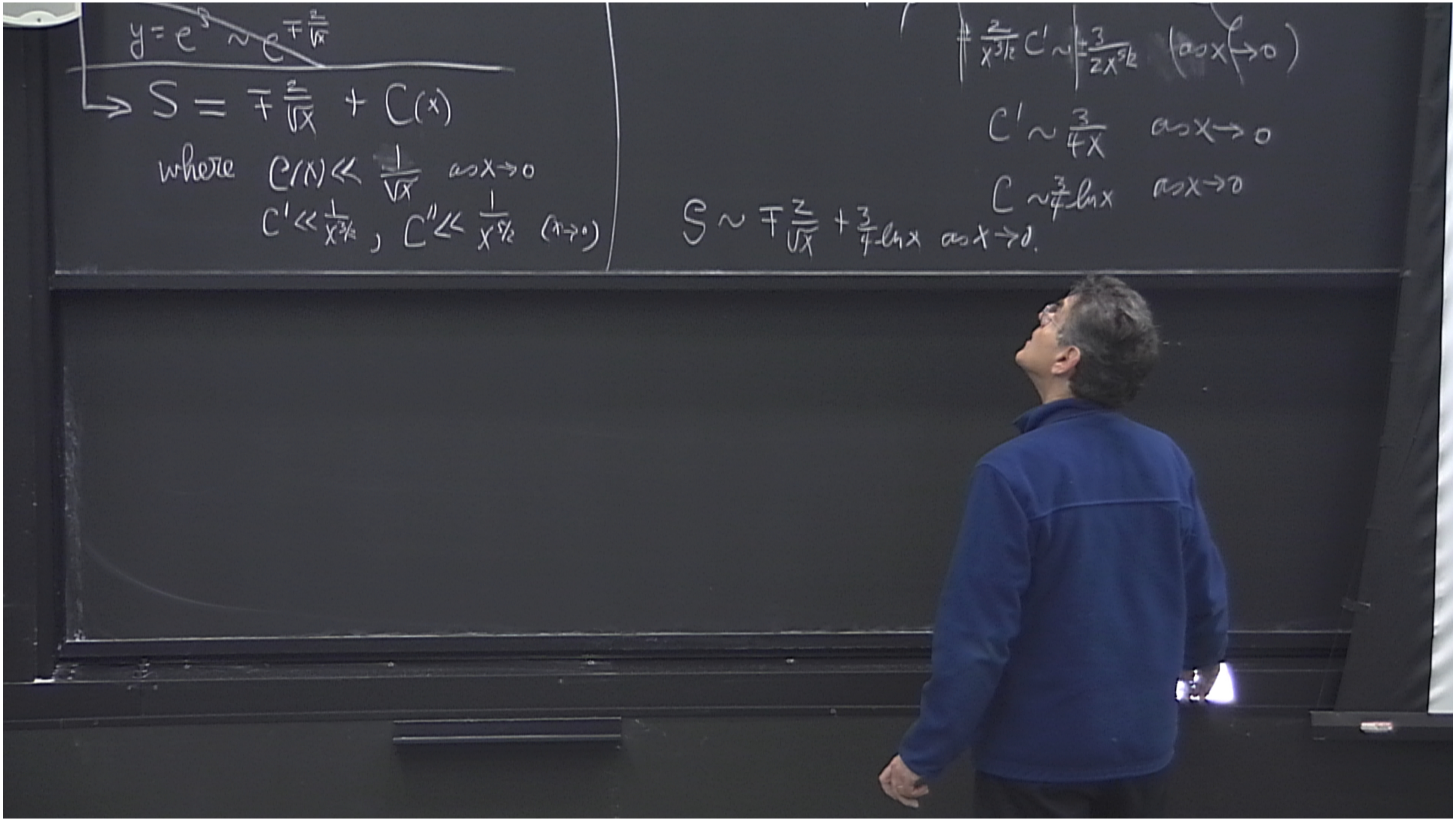
$y=e^S \sim e^{\mp\frac{2}{\sqrt{x}}}$
$S = \mp\frac{2}{\sqrt{x}} + C(x)$
where $C(x) \ll \frac{1}{\sqrt{x}}$ as
$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll$
$x^3((S')^2+S'')=1$
$S=\mp\frac{2}{\sqrt{x}}+C$
$S'=\pm\frac{1}{x^{3/2}}+C'$
$S''=\mp\frac{3}{2}\frac{1}{x^{5/2}}+C''$
$x^3\left[\frac{1}{x^3}+C'^2\pm 2\frac{1}{x^{3/2}}C' \mp\frac{3}{2}\frac{1}{x^{5/2}}+C''\right]=1$
$C'^2 \pm \frac{2}{x^{3/2}}C' \mp \frac{3}{2x^{5/2}} + C'' = 0$
$\pm\frac{2}{x^{3/2}}C' \sim \pm\frac{3}{2x^{5/2}}$ (as $x\to 0$)
$C' \sim \frac{3}{4x}$ as $x\to 0$
$C \sim \frac{3}{4}\ln x$ as $x\to 0$


$y = e^S \sim e^{\mp \frac{2}{\sqrt{x}}}$
$S = \mp \frac{2}{\sqrt{x}} + C(x)$
where $C(x) \ll \frac{1}{\sqrt{x}}$ as $x \to 0$
$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0)$
$\pm \frac{2}{x^{3/2}} C' \sim \pm \frac{3}{2x^{5/2}}$ (as $x \to 0$)
$C' \sim \frac{3}{4x}$ as $x \to 0$
$C \sim \frac{3}{4} \ln x$ as $x \to 0$
$S \sim \mp \frac{2}{\sqrt{x}} + \frac{3}{4} \ln x$ as $x \to 0$.

$$\hookrightarrow S = \mp \frac{2}{\sqrt{x}} + C(x)$$

where $C(x) \ll \frac{1}{\sqrt{x}}$ as $x \to 0$

$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0)$

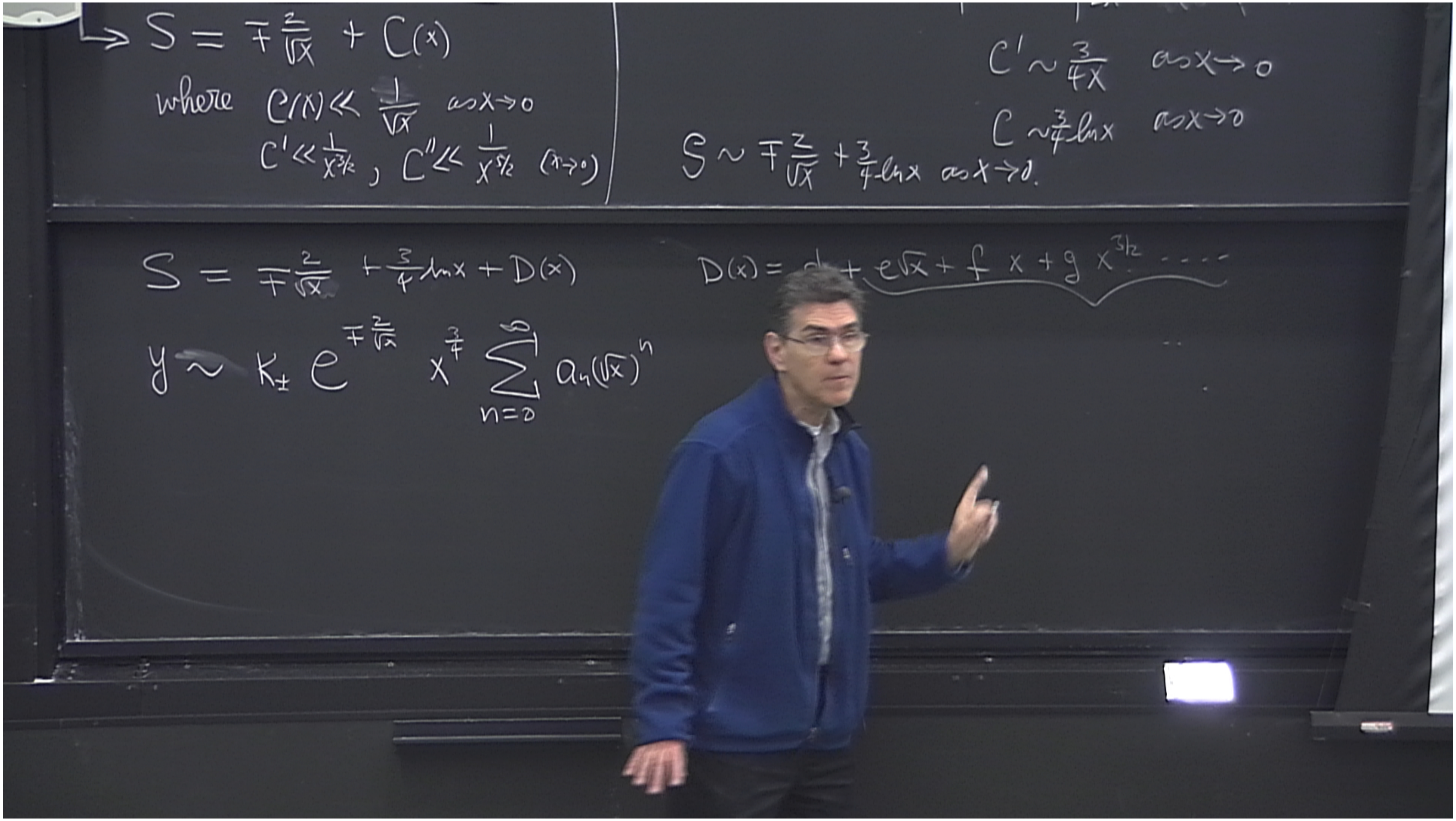
$C' \sim \frac{3}{4x}$ as $x \to 0$

$C \sim \frac{3}{4}\ln x$ as $x \to 0$

$S \sim \mp \frac{2}{\sqrt{x}} + \frac{3}{4}\ln x$ as $x \to 0$.

$$S = \mp \frac{2}{\sqrt{x}} + \frac{3}{4}\ln x + D(x)$$

$$D(x) = d + e\sqrt{x} + f\,x + g\,x^{3/2} \dots$$


$\hookrightarrow S = \mp \frac{2}{\sqrt{x}} + C(x)$
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$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0)$
$C' \sim \frac{3}{4x}$ as $x \to 0$
$C \sim \frac{3}{4} \ln x$ as $x \to 0$
$S \sim \mp \frac{2}{\sqrt{x}} + \frac{3}{4} \ln x$ as $x \to 0$.
$S = \mp \frac{2}{\sqrt{x}} + \frac{3}{4} \ln x + D(x)$
$D(x) = d + e\sqrt{x} + f\,x + g\,x^{3/2} \cdots$
$y \sim K_{\pm}\, e^{\mp \frac{2}{\sqrt{x}}}\, x^{\frac{3}{4}} \sum_{n=0}^{\infty} a_n (\sqrt{x})^n$

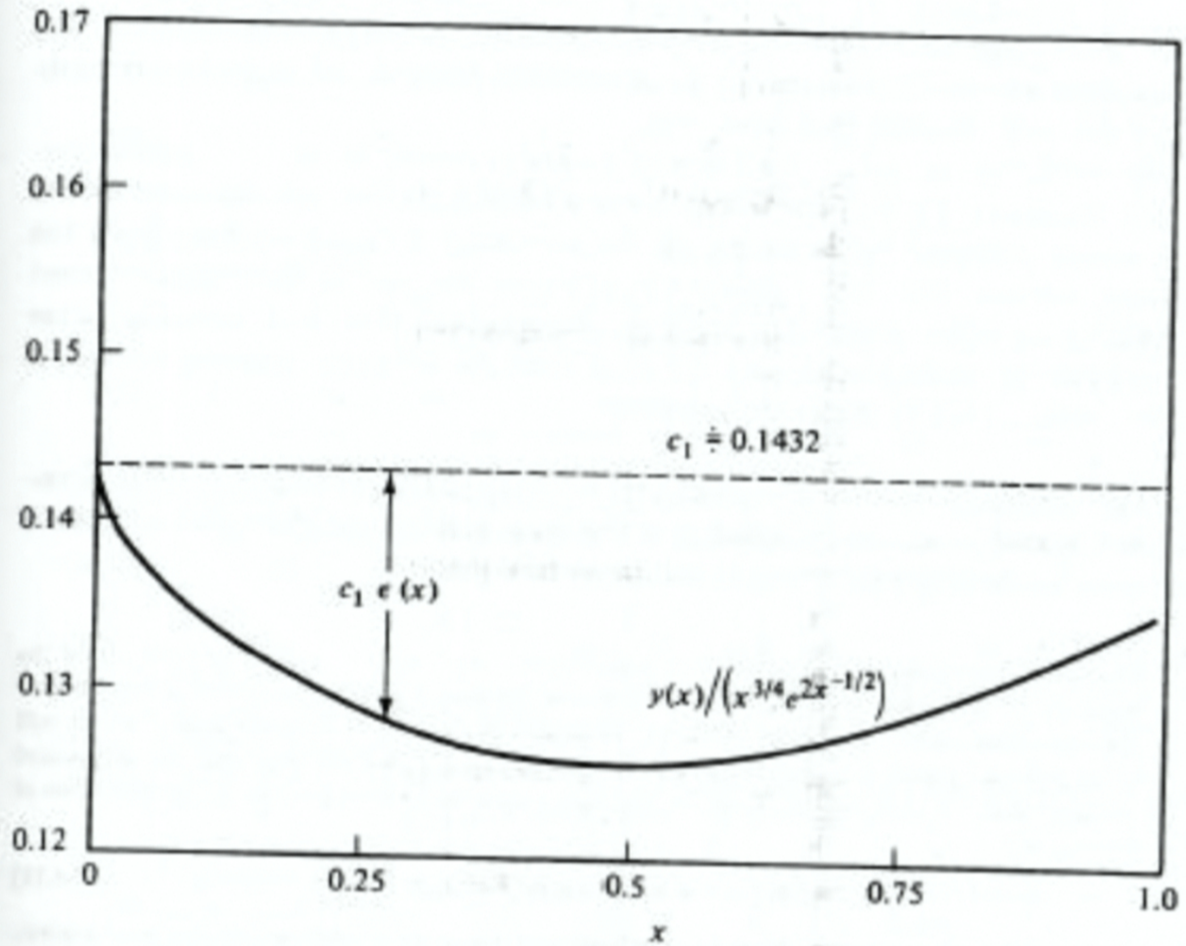
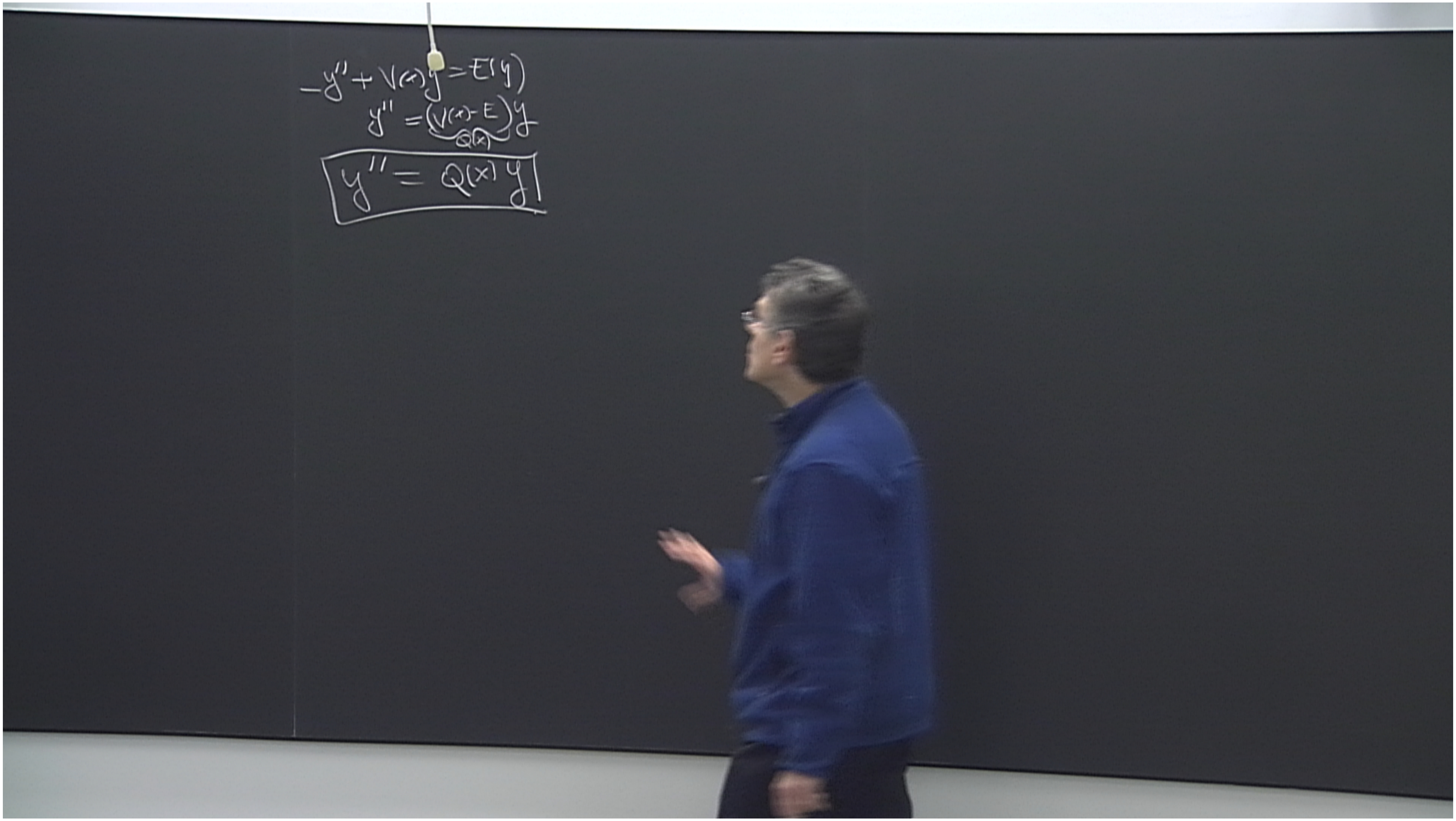
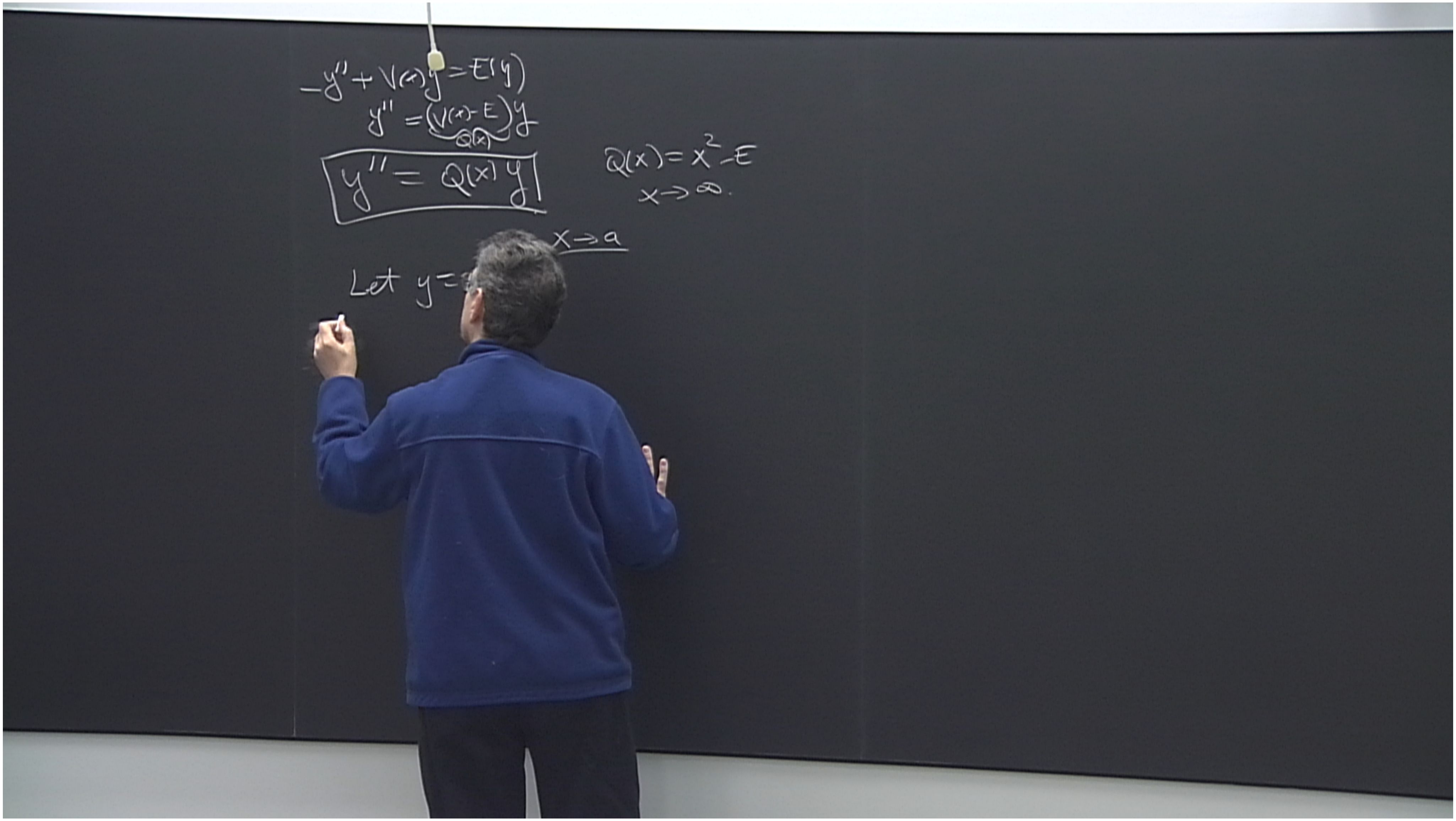
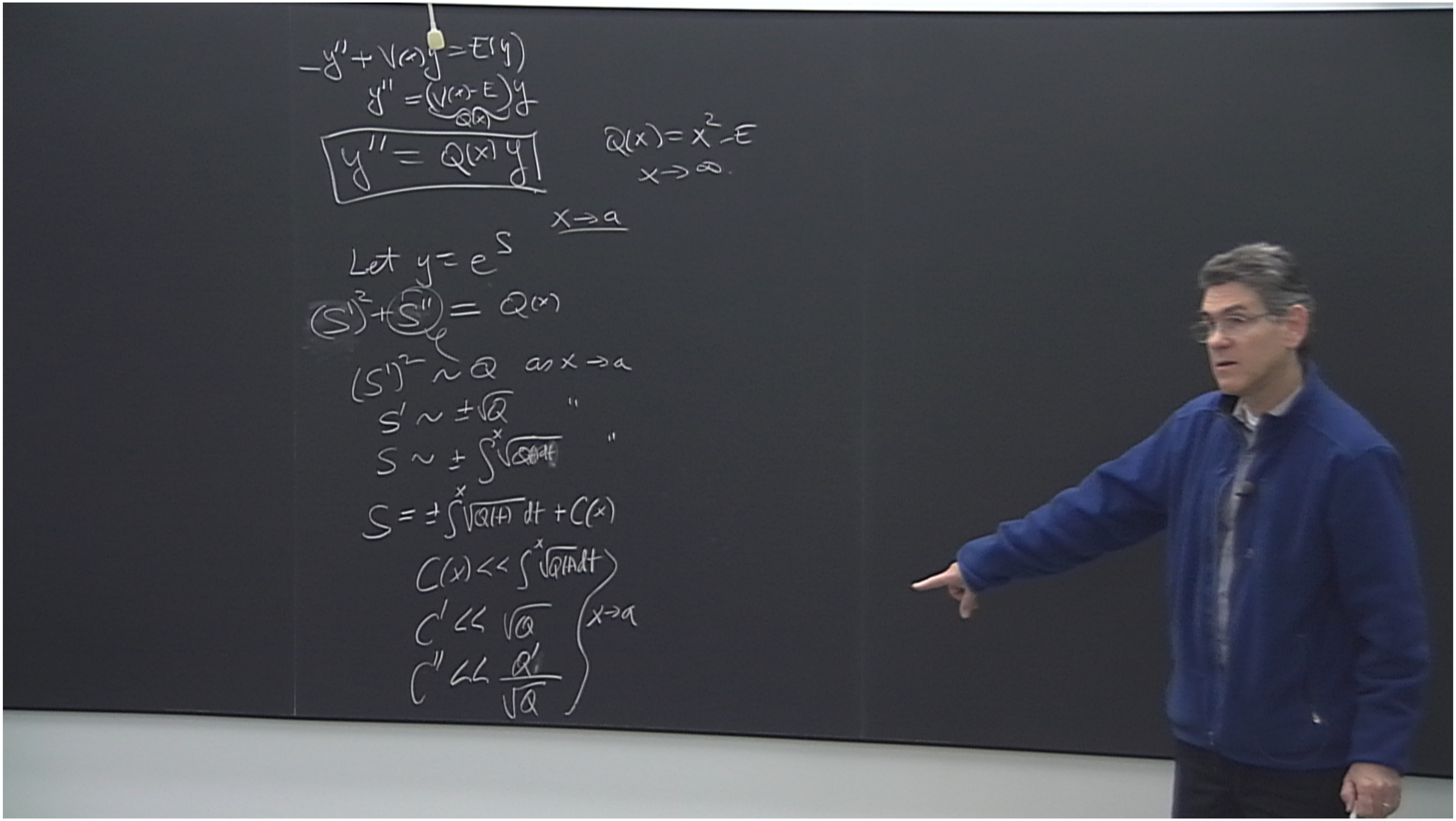
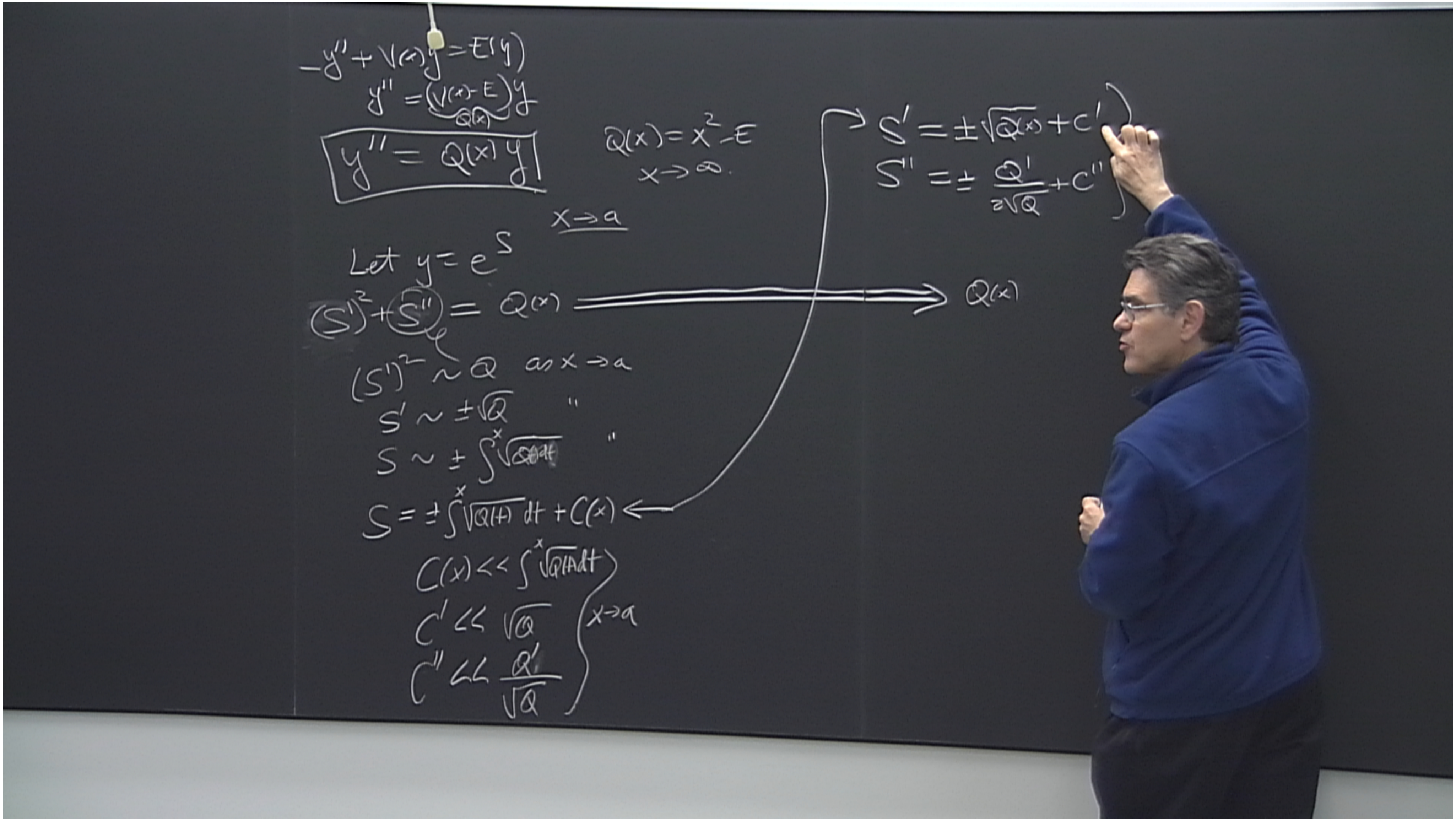


Figure 3.2 A plot of the ratio of $y(x)$, the numerical solution of (3.4.1) with $y(1) = 1$ and $y'(1) = 0$, to $x^{3/4}\exp(2x^{-1/2})$ for $0 \le x \le 1$. Asymptotic analysis [see (3.4.17)] shows that this ratio approaches a constant c_1. For these initial conditions, $c_1 \doteq 0.1432$. The difference between the plotted ratio and c_1 is $c_1\epsilon(x)$.


$-y'' + V(x)y = E(y)$
$y'' = \underbrace{(V(x) - E)}_{Q(x)} y$
$\boxed{y'' = Q(x)\, y}$


-y'' + V(x)y = E(y)
y'' = (V(x) - E)y
Q(x)
y'' = Q(x) y
Q(x) = x^2 - E
x → ∞.
x → a
Let y =


$-y'' + V(x)y = E(y)$
$y'' = \underbrace{(V(x) - E)}_{Q(x)} y$
$\boxed{y'' = Q(x)y}$
$Q(x) = x^2 - E$
$x \to \infty$
$x \to a$
Let $y = e^S$
$(S')^2 + S'' = Q(x)$
$(S')^2 \sim Q$ as $x \to a$
$S' \sim \pm\sqrt{Q}$ "
$S \sim \pm \int^x \sqrt{Q(t)}dt$ "
$S = \pm \int^x \sqrt{Q(t)}\,dt + C(x)$
$C(x) << \int^x \sqrt{Q(t)}dt$
$C' << \sqrt{Q}$
$C'' << \frac{Q'}{\sqrt{Q}}$
$x \to a$



-y'' + V(x)y = E(y)
y'' = (V(x) - E)y
Q(x)
y'' = Q(x) y
Q(x) = x^2 - E
x → ∞
x → a
Let y = e^S
(S')^2 + S'' = Q(x)
Q(x)
(S')^2 ~ Q as x → a
S' ~ ±√Q "
S ~ ± ∫^x √Q(t)dt "
S = ± ∫^x √Q(t) dt + C(x)
C(x) << ∫^x √Q(t)dt
C' << √Q
C'' << Q'/√Q
x → a
S' = ±√Q(x) + C'
S'' = ± Q'/(2√Q) + C''

$$-y'' + V(x)y = E\,y$$

$$y'' = \underbrace{(V(x) - E)}_{Q(x)}\,y$$

$$\boxed{y'' = Q(x)\,y}$$

$$Q(x) = x^2 - E$$

$$x \to \infty.$$

$\underline{x \to a}$

Let $y = e^S$

$$(S')^2 + S'' = Q(x) \Longrightarrow Q(x) + c'^2 \pm 2c'\sqrt{Q} \pm \frac{Q'}{2\sqrt{Q}} + c'' = Q$$

$$(S')^2 \sim Q \quad \text{as } x \to a$$

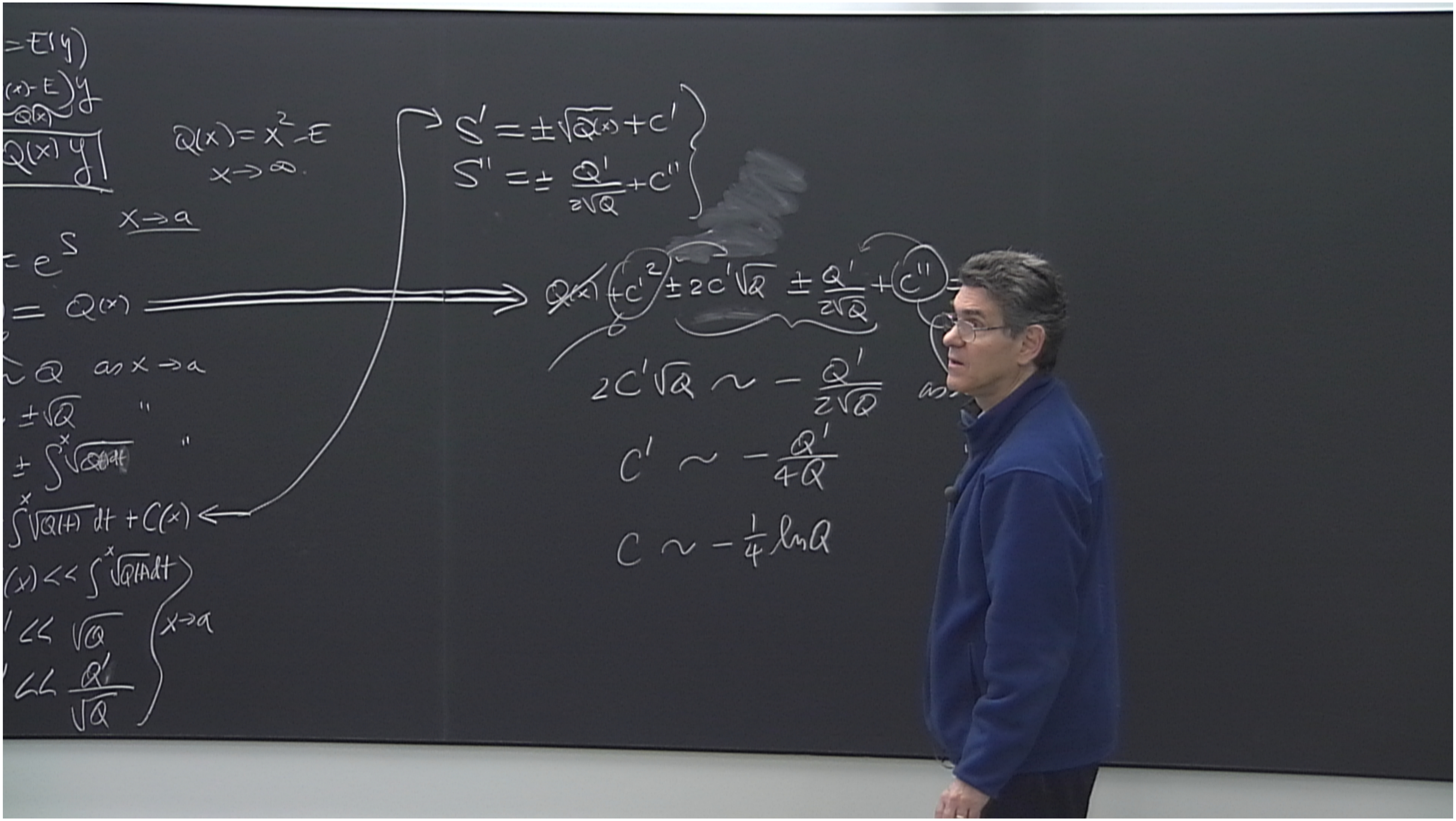
$$S' \sim \pm\sqrt{Q} \quad \text{"}$$

$$S \sim \pm\int^x \sqrt{Q(t)}\,dt \quad \text{"}$$

$$S = \pm\int^x \sqrt{Q(t)}\,dt + C(x)$$

$$\left.\begin{aligned} C(x) &<< \int^x \sqrt{Q(t)}\,dt \\ C' &<< \sqrt{Q} \\ C'' &<< \frac{Q'}{\sqrt{Q}} \end{aligned}\right\} x \to a$$

$$\left.\begin{aligned} S' &= \pm\sqrt{Q(x)} + c' \\ S'' &= \pm\frac{Q'}{2\sqrt{Q}} + c'' \end{aligned}\right\}$$


Q(x) = x² − E
x → ∞
x → a
S' = ± √Q(x) + C'
S'' = ± Q' / 2√Q + C''
2C'√Q ~ − Q' / 2√Q
C' ~ − Q' / 4Q
C ~ − ¼ ln Q

$$S' = \pm\sqrt{Q(x)} + C'$$
$$S'' = \pm \frac{Q'}{2\sqrt{Q}} + C''$$

$$\Rightarrow \cancel{Q(x)} + C'^2 \pm 2C'\sqrt{Q} \pm \frac{Q'}{2\sqrt{Q}} + C'' = \cancel{0}$$

$$2C'\sqrt{Q} \sim -\frac{Q'}{2\sqrt{Q}} \quad \text{as } x \to a$$

$$C' \sim -\frac{Q'}{4Q} \quad ''$$

$$C \sim -\frac{1}{4}\ln Q$$

$$S = \pm \int^x dt\,\sqrt{Q(t)} - \frac{1}{4}\ln Q$$

$$y = e^S$$

$$y \sim K_\pm \frac{e^{\int^x dt\,\sqrt{Q(t)}}}{Q^{\frac{1}{4}}(x)} \quad \text{as } x \to 0$$

$$\left.\begin{aligned} S' &= \pm\sqrt{Q(x)} + C' \\ S'' &= \pm\frac{Q'}{2\sqrt{Q}} + C'' \end{aligned}\right\}$$

$$\Longrightarrow \cancel{Q(x)} + C'^2 \pm 2C'\sqrt{Q} \pm \frac{Q'}{2\sqrt{Q}} + C'' = \cancel{Q}$$

$$2C'\sqrt{Q} \sim -\frac{Q'}{2\sqrt{Q}} \quad \text{as } x \to a$$

$$C' \sim -\frac{Q'}{4Q} \quad \text{"}$$

$$C \sim -\frac{1}{4}\ln Q$$

$$S = \pm\int^x dt\,\sqrt{Q(t)} - \frac{1}{4}\ln Q$$

$$y = e^S$$

$$\boxed{y \sim K_\pm \frac{e^{\pm\int^x dt\,\sqrt{Q(t)}}}{Q^{\frac{1}{4}}(x)} \quad \text{as } x \to a}$$

$$S = \pm \int^x dt \sqrt{Q(t)} - \frac{1}{4}\ln Q \quad \underline{+\text{const}} \quad + \left(\frac{\to 0}{\text{as } x \to a}\right)$$

$$\underbrace{\sqrt{Q} \pm \frac{Q'}{2\sqrt{Q}}} + C'' = \cancel{0}$$

$$\sim -\frac{Q'}{2\sqrt{Q}} \quad \text{as } x \to a$$

$$-\frac{Q'}{4Q} \quad ''$$

$$-\frac{1}{4}\ln Q$$

$$y = e^S$$

$$\boxed{y \sim K_\pm \frac{e^{\pm\int^x dt\sqrt{Q(t)}}}{Q^{\frac{1}{4}}(x)} \quad \text{as } x \to a}$$

$$-y'' + V(x)y = E\,y$$
$$y'' = \underbrace{(V(x) - E)}_{Q(x)}\,y$$
$$\boxed{y'' = Q(x)\,y}$$

$Q(x) = x^2 - E$
$x \to \infty$

$x \to a$

$\sim x^2 + \sin(x^{25})$ as $x \to \infty$
$\sim 2x + 25x^{24}\cos(x^{25})$

Let $y = e^S$

$$(S')^2 + S'' = Q(x)$$
$$(S')^2 \sim Q \quad \text{as } x \to a$$
$$S' \sim \pm\sqrt{Q} \quad ''$$
$$S \sim \pm\int^x \sqrt{Q(t)}\,dt \quad ''$$
$$S = \pm\int^x \sqrt{Q(t)}\,dt + C(x)$$

$$\left.\begin{aligned} C(x) &\ll \int^x \sqrt{Q(t)}\,dt \\ C' &\ll \sqrt{Q} \\ C'' &\ll \frac{Q'}{\sqrt{Q}} \end{aligned}\right\} x \to a$$

$$\left.\begin{aligned} S' &= \pm\sqrt{Q(x)} + C' \\ S'' &= \pm\frac{Q'}{2\sqrt{Q}} + C'' \end{aligned}\right\}$$

$$\cancel{Q(x)} + C'^2 \pm 2C'\sqrt{Q} \pm \frac{Q'}{2\sqrt{Q}} + \ldots$$

$$2C'\sqrt{Q} \sim -\frac{Q'}{2\sqrt{Q}}$$
$$C' \sim -\frac{Q'}{4Q}$$
$$C \sim -\frac{1}{4}\ln Q$$

$$\hookrightarrow S = \mp \frac{2}{\sqrt{x}} + C(x)$$

where $C(x) \ll \frac{1}{\sqrt{x}}$ as $x \to 0$

$C' \ll \frac{1}{x^{3/2}}$, $C'' \ll \frac{1}{x^{5/2}}$ $(x \to 0)$

$C' \sim \frac{3}{4x}$ as $x \to 0$

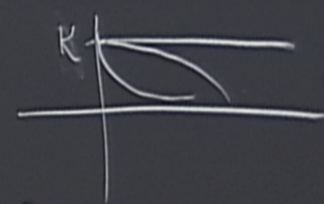
$C \sim \frac{3}{4}\ln x$ as $x \to 0$

$$S \sim \mp \frac{2}{\sqrt{x}} + \frac{3}{4}\ln x \quad \text{as } x \to 0.$$

$$S = \mp \frac{2}{\sqrt{x}} + \frac{3}{4}\ln x + D(x)$$

$$D(x) = d + \underbrace{e\sqrt{x} + f\,x + g\,x^{3/2} \cdots}$$

$$\sim K_\pm e^{\mp \frac{2}{\sqrt{x}}} x^{\frac{3}{4}} \sum_{n=0}^{\infty} a_n (\sqrt{x})^n \quad \text{as } x \to 0$$



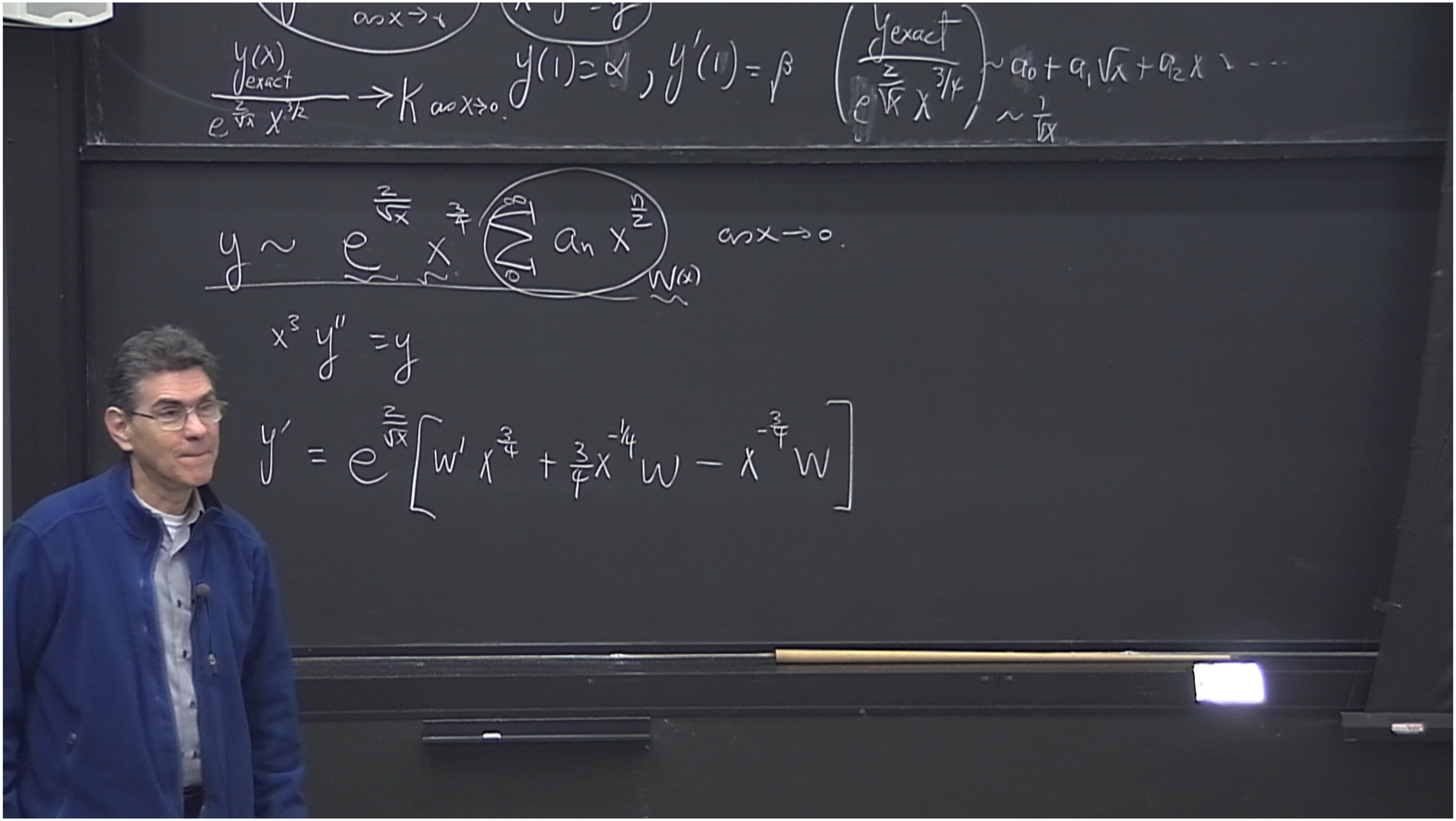
$$y(x) \sim K e^{\frac{2}{\sqrt{x}}} x^{3/4} \quad \text{as } x \to 0$$

$$x^3 y'' = y$$

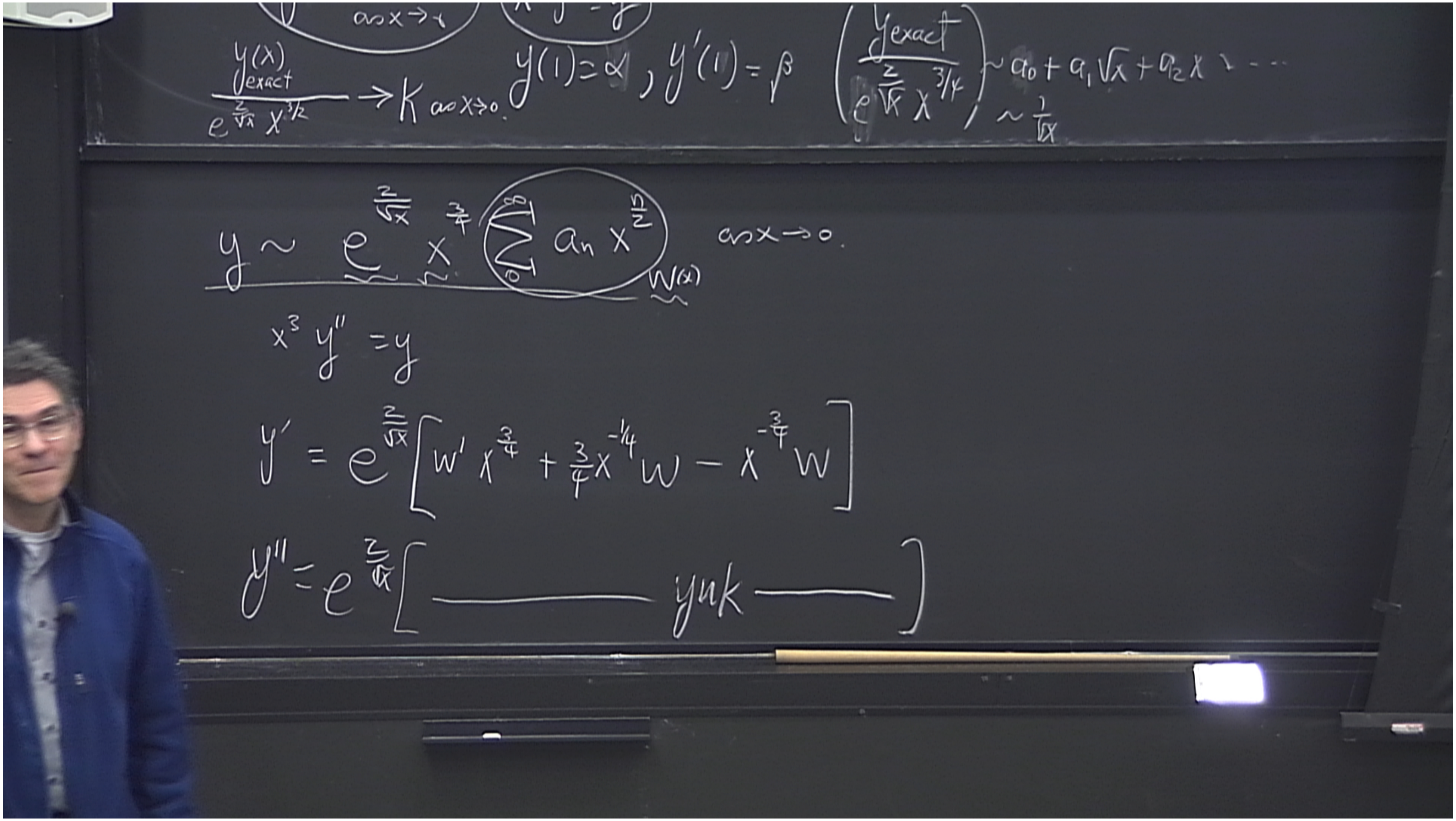
$$\frac{y_{exact}(x)}{e^{\frac{2}{\sqrt{x}}} x^{3/2}} \to K \quad \text{as } x \to 0.$$

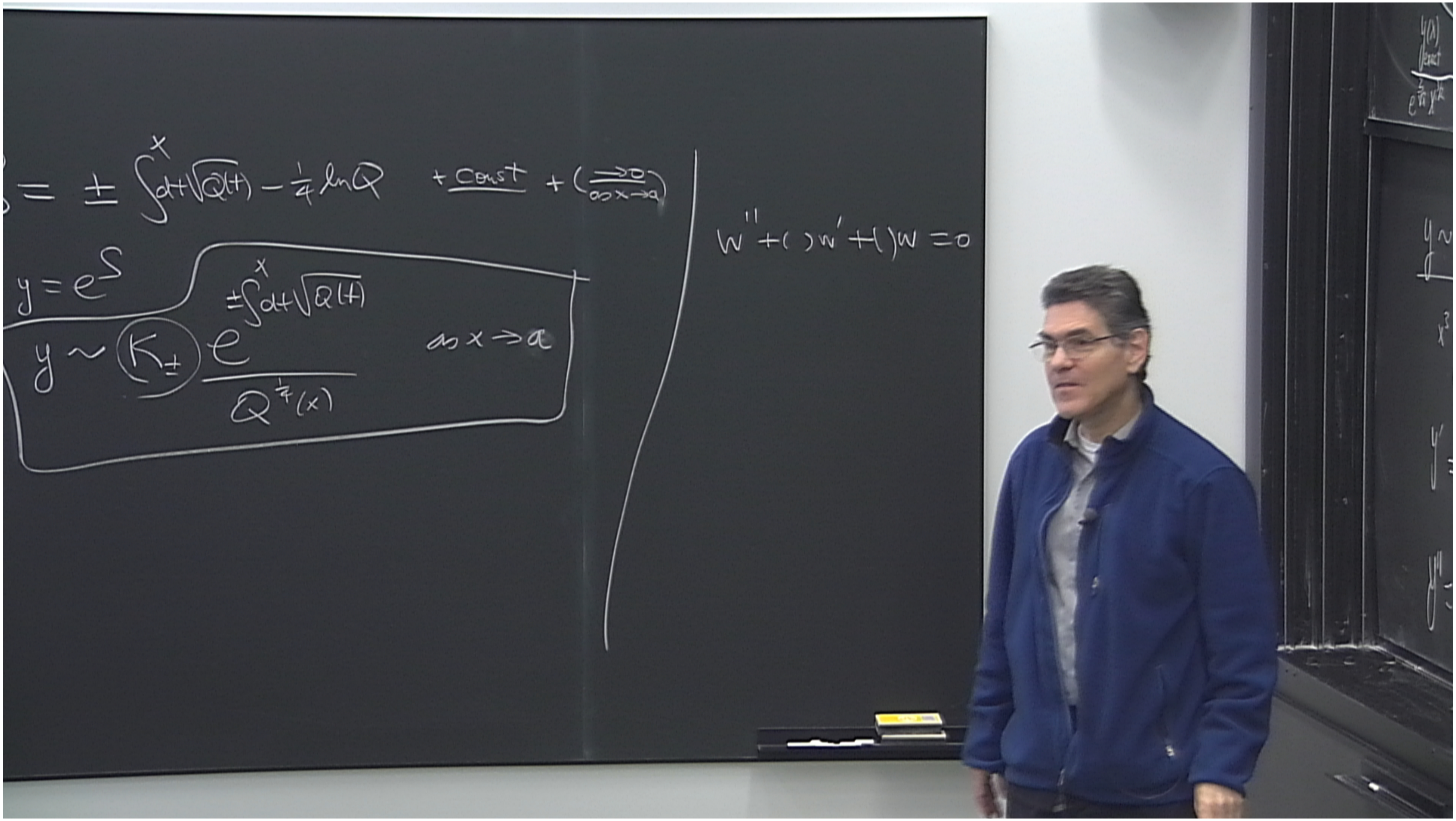
$y(1) = \alpha$, $y'(1) = \beta$

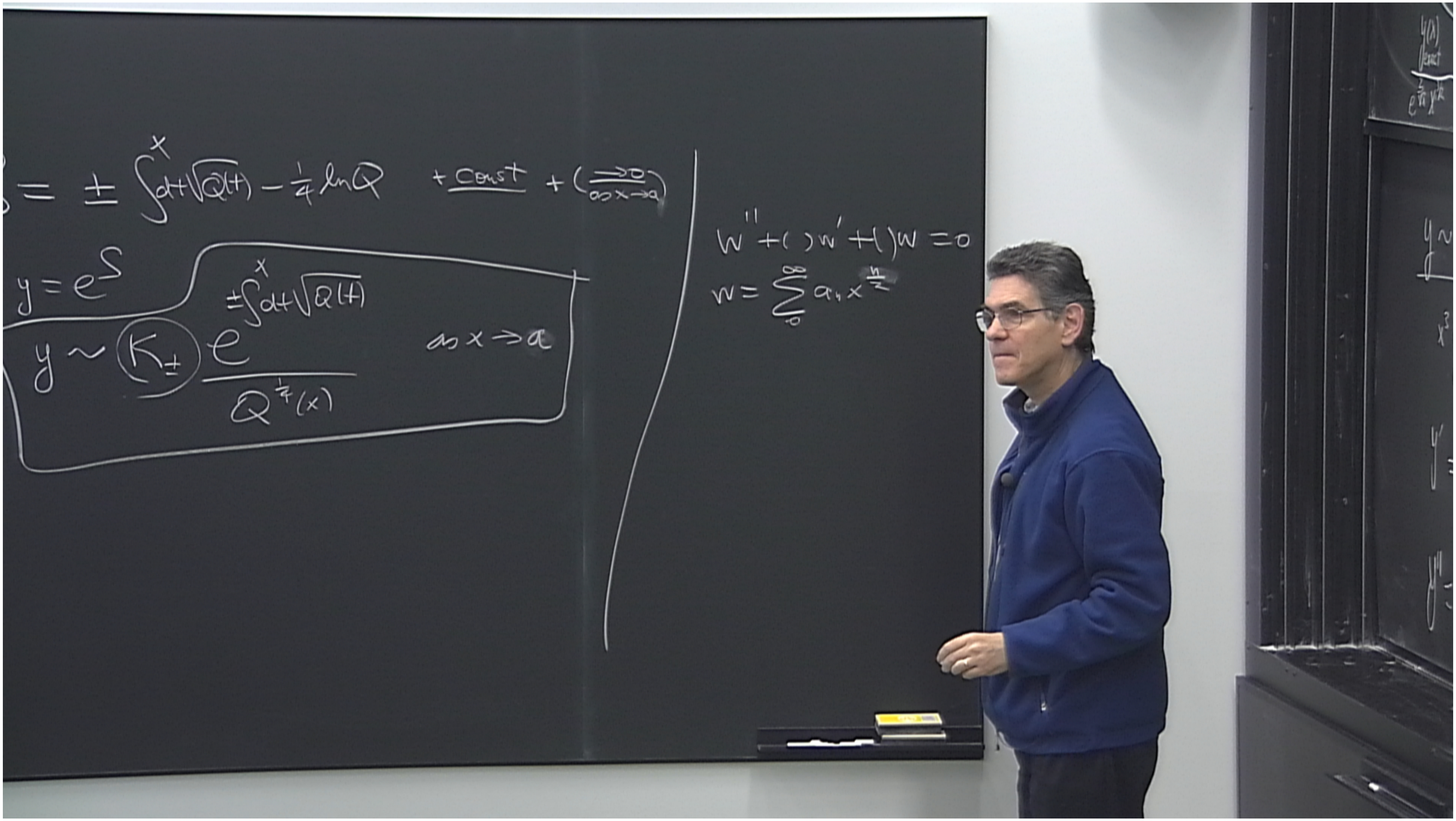
$$\left(\frac{y_{exact}}{e^{\frac{2}{\sqrt{x}}} x^{3/4}}\right) \sim a_0 + a_1\sqrt{x} + a_2 x + \cdots$$

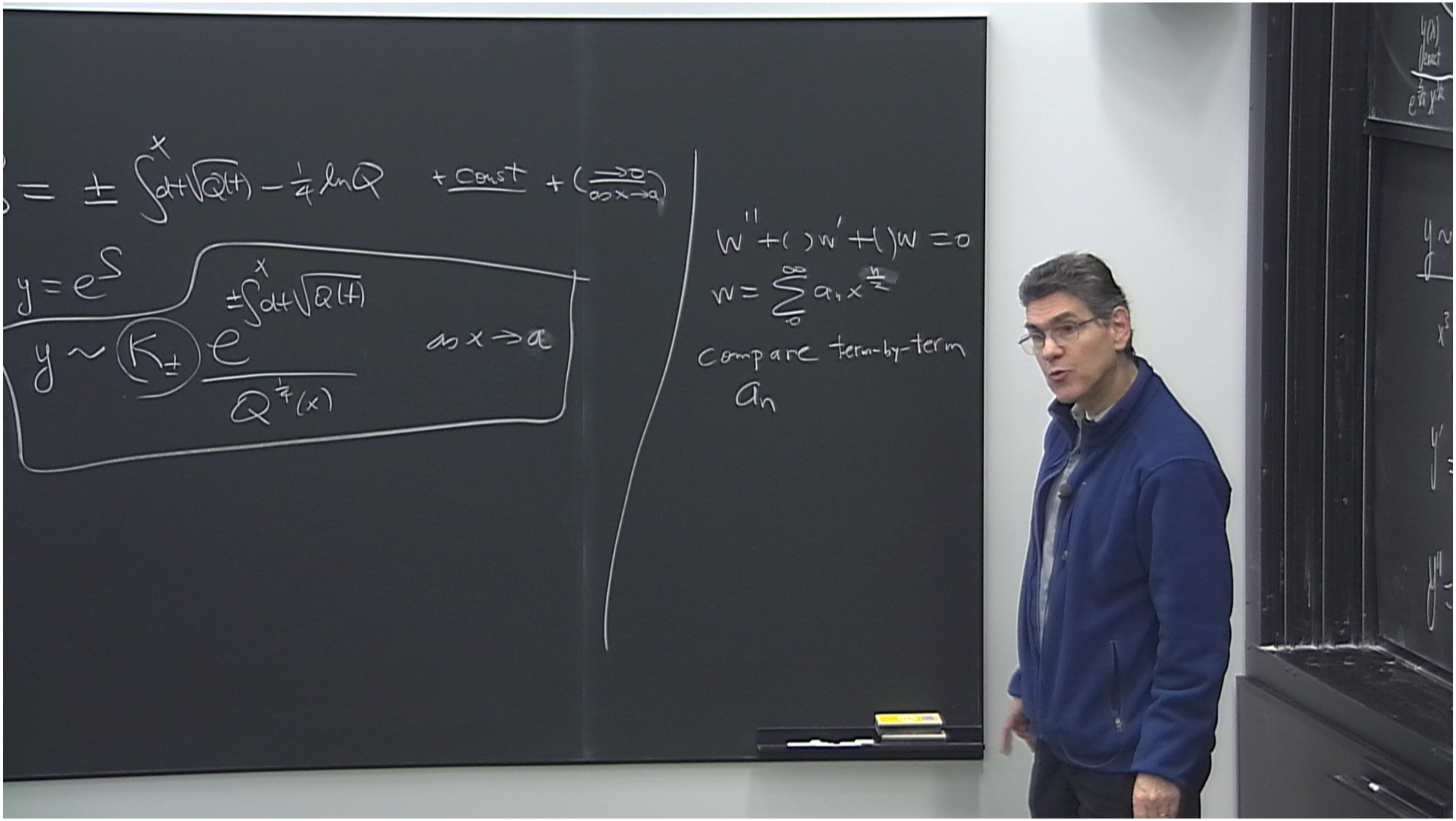


as $x \to \infty$
$\frac{y(x)_{exact}}{e^{\frac{2}{\sqrt{x}}} x^{3/2}} \to K$ as $x \to 0$.
$y(1) = \alpha$, $y'(1) = \beta$
$\left(\frac{y_{exact}}{e^{\frac{2}{\sqrt{x}}} x^{3/4}} \right) \sim a_0 + a_1 \sqrt{x} + a_2 x + \ldots$
$\sim \frac{1}{\sqrt{x}}$
$y \sim e^{\frac{2}{\sqrt{x}}} x^{\frac{3}{4}} \sum_{0}^{\infty} a_n x^{\frac{n}{2}}$ as $x \to 0$.
$W(x)$
$x^3 y'' = y$
$y' = e^{\frac{2}{\sqrt{x}}} \left[W' x^{\frac{3}{4}} + \frac{3}{4} x^{-1/4} W - x^{-\frac{3}{4}} W \right]$


as $x \to \infty$
$\frac{y(x)_{exact}}{e^{\frac{2}{\sqrt{x}}} x^{3/2}} \to K$ as $x \to 0$.
$y(1) = \alpha$, $y'(1) = \beta$
$\left(\frac{y_{exact}}{e^{\frac{2}{\sqrt{x}}} x^{3/4}} \right) \sim a_0 + a_1 \sqrt{x} + a_2 x + \dots$
$\sim \frac{1}{\sqrt{x}}$
$y \sim e^{\frac{2}{\sqrt{x}}} x^{\frac{3}{4}} \sum_{0}^{\infty} a_n x^{\frac{n}{2}}$ as $x \to 0$.
$W(x)$
$x^3 y'' = y$
$y' = e^{\frac{2}{\sqrt{x}}} \left[w' x^{\frac{3}{4}} + \frac{3}{4} x^{-1/4} w - x^{-\frac{3}{4}} w \right]$
$y'' = e^{\frac{2}{\sqrt{x}}} \left[\text{———— yuk ————} \right]$


$= \pm \int^x dt \sqrt{Q(t)} - \frac{1}{4} \ln Q + \text{const} + (\to 0 \text{ as } x \to a)$
$y = e^S$
$y \sim K_\pm \dfrac{e^{\pm \int^x dt \sqrt{Q(t)}}}{Q^{\frac{1}{4}}(x)}$ as $x \to a$
$w'' + (\ )w' + (\ )w = 0$


$= \pm \int^x dt\sqrt{Q(t)} - \frac{1}{4}\ln Q + \underline{\text{const}} + \left(\frac{\to 0}{\text{as } x \to a}\right)$
$y = e^S$
$y \sim K_\pm \frac{e^{\pm\int^x dt\sqrt{Q(t)}}}{Q^{\frac{1}{4}}(x)}$ as $x \to a$
$w'' + (\ )w' + (\ )w = 0$
$w = \sum_0^\infty a_n x^{\frac{n}{2}}$


$= \pm \int^x dt \sqrt{Q(t)} - \frac{1}{4} \ln Q + \text{const} + (\to 0 \text{ as } x \to a)$
$y = e^S$
$y \sim K_\pm \frac{e^{\pm \int^x dt \sqrt{Q(t)}}}{Q^{\frac{1}{4}}(x)}$ as $x \to a$
$w'' + (\ )w' + (\ )w = 0$
$w = \sum_0^\infty a_n x^{\frac{n}{2}}$
compare term-by-term
a_n

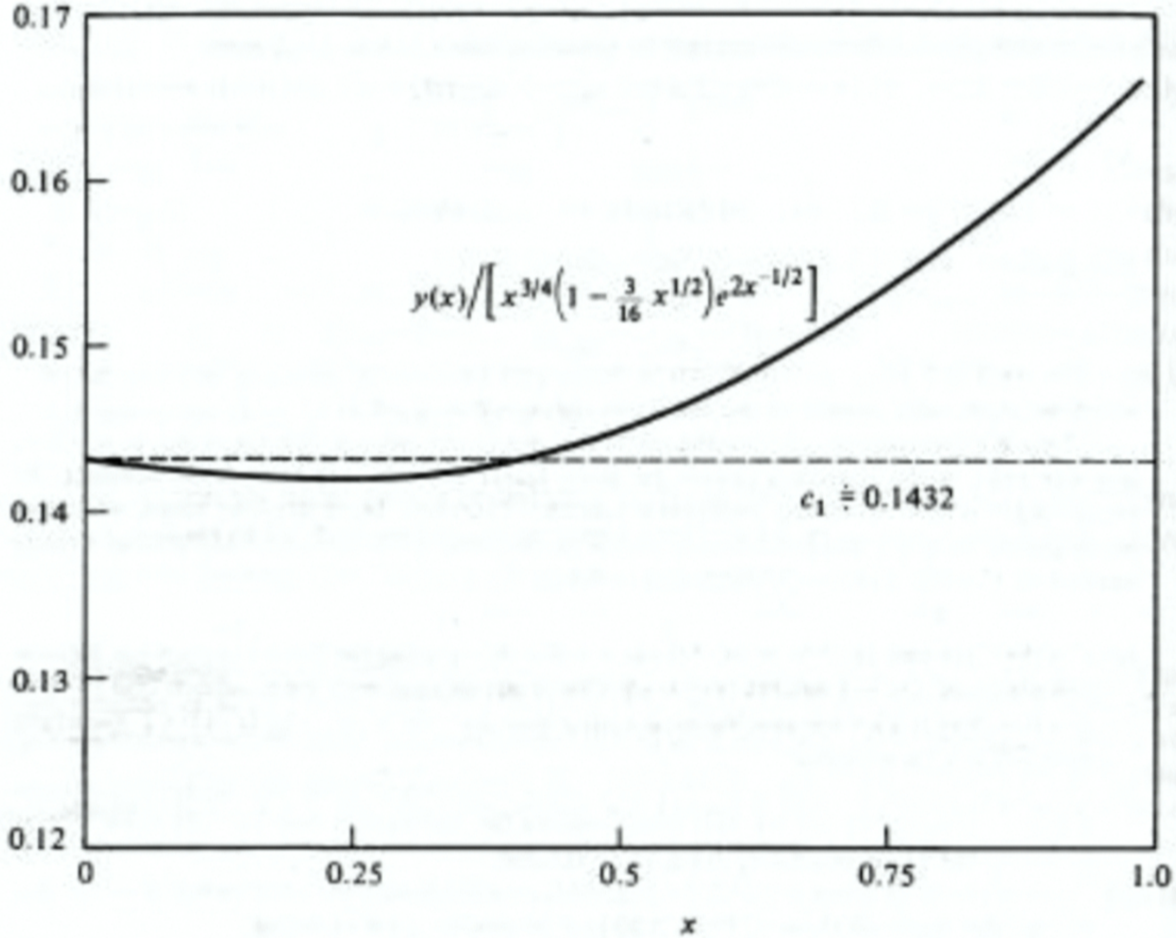
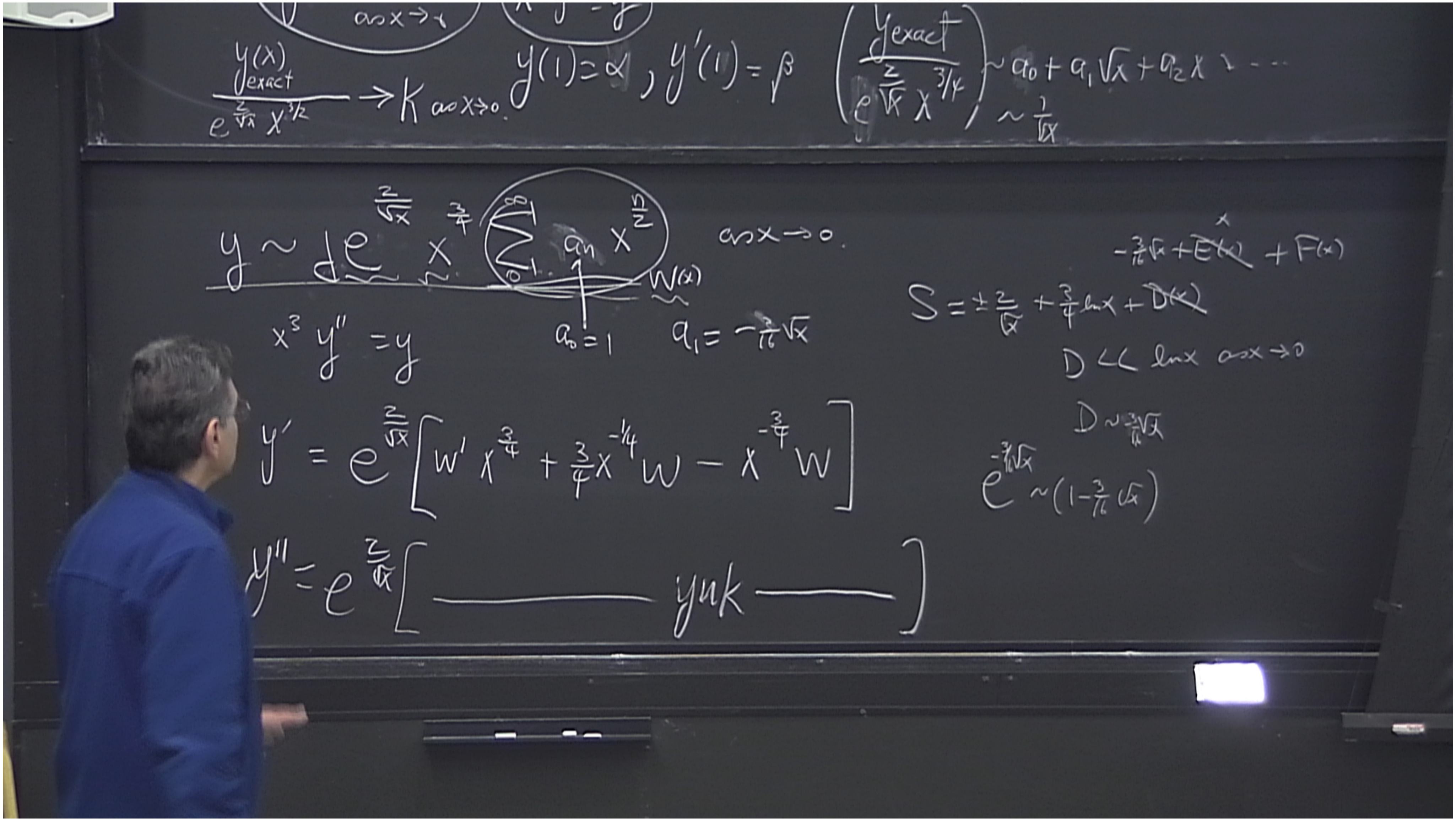
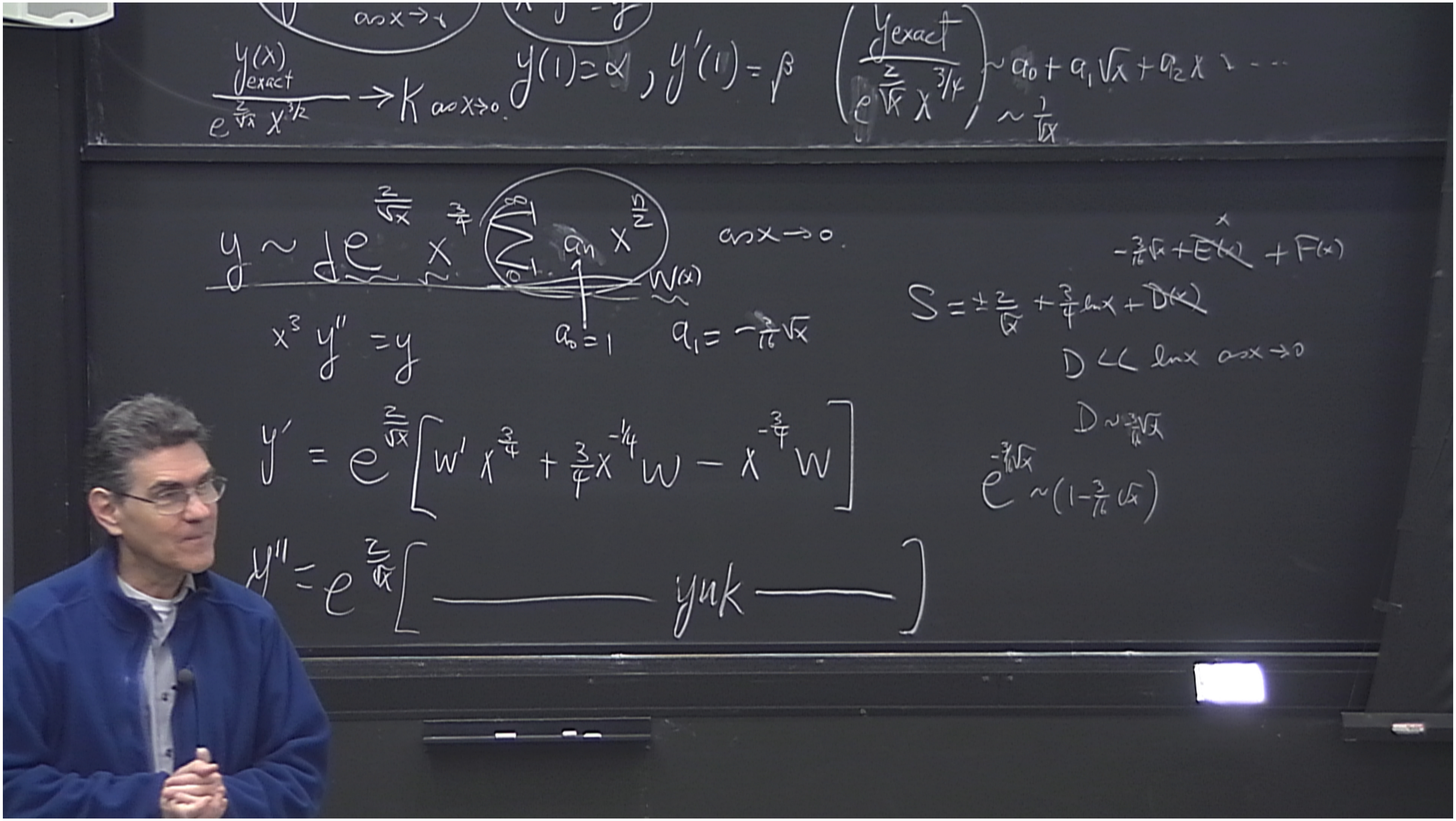


Figure 3.3 A plot of the ratio $y(x)$, the exact solution to (3.4.1) with $y(1)=1$ and $y'(1)=0$, to $x^{3/4}(1-\frac{3}{16}x^{1/2})\exp(2x^{-1/2})$ for $0 \le x \le 1$. A comparison of the results plotted in this figure with those of Fig. 3.2 shows that including the term $-\frac{3}{16}x^{1/2}$ in $w(x)$ improves the estimate of the numerical solution $y(x)$ as $x \to 0+$.


as x → 1
y(x) / (y_exact e^{2/√x} x^{3/2}) → K as x → 0
y(1) = α , y'(1) = β
(y_exact / (e^{2/√x} x^{3/4})) ~ a_0 + a_1 √x + a_2 x + ...
~ 1/√x
y ~ d e^{2/√x} x^{3/4} Σ_0^∞ a_n x^{n/2} as x → 0
W(x)
x^3 y'' = y
a_0 = 1
a_1 = -3/16 √x
-3/16 √x + E(x) + F(x)
S = ± 2/√x + 3/4 ln x + D(x)
D << ln x as x → 0
D ~ -3/16 √x
e^{-3/16 √x} ~ (1 - 3/16 √x)
y' = e^{2/√x} [w' x^{3/4} + 3/4 x^{-1/4} w - x^{-3/4} w]
y'' = e^{2/√x} [yuk]



as x → ∞
y(x) exact / e^(2/√x) x^(3/2) → K as x→0.
y(1) = α , y'(1) = β
(y exact / e^(2/√x) x^(3/4)) ~ a0 + a1 √x + a2 x + ...
~ 1/√x
y ~ d e^(2/√x) x^(3/4) Σ_{0}^{∞} an x^(n/2) as x → 0.
W(x)
x^3 y'' = y
a0 = 1
a1 = −3/16 √x
−3/16 √x + E(x) + F(x)
S = ± 2/√x + 3/4 ln x + D(x)
D << ln x as x → 0
D ~ −3/16 √x
e^(−3/16 √x) ~ (1 − 3/16 √x)
y' = e^(2/√x) [w' x^(3/4) + 3/4 x^(−1/4) w − x^(−3/4) w]
y'' = e^(2/√x) [yuk]