Title: Mathematical Physics - Lecture 5

Date: Nov 25, 2011 09:00 AM

URL: http://pirsa.org/11110044

Abstract:

Some examples

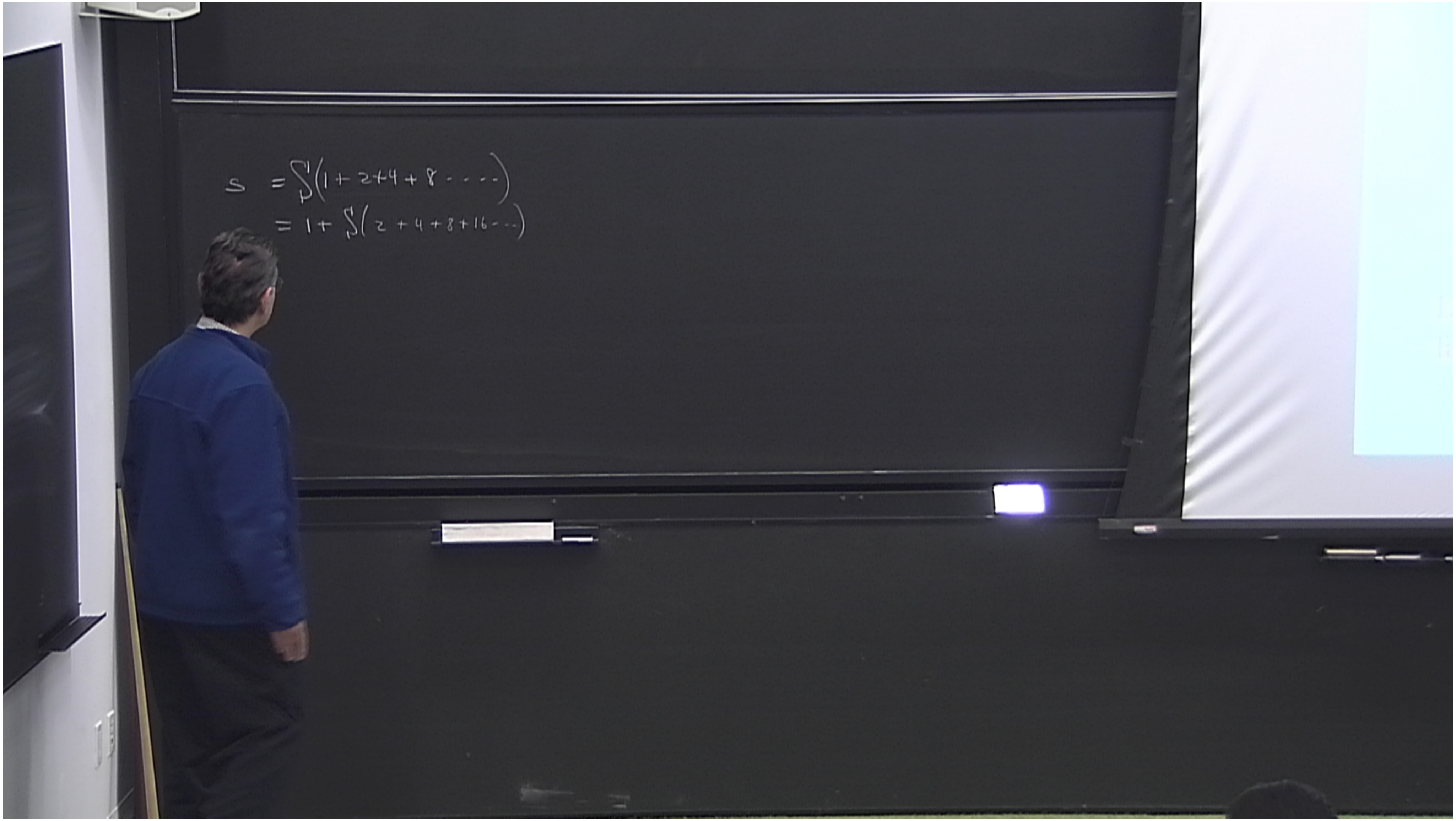
$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 + \ldots$

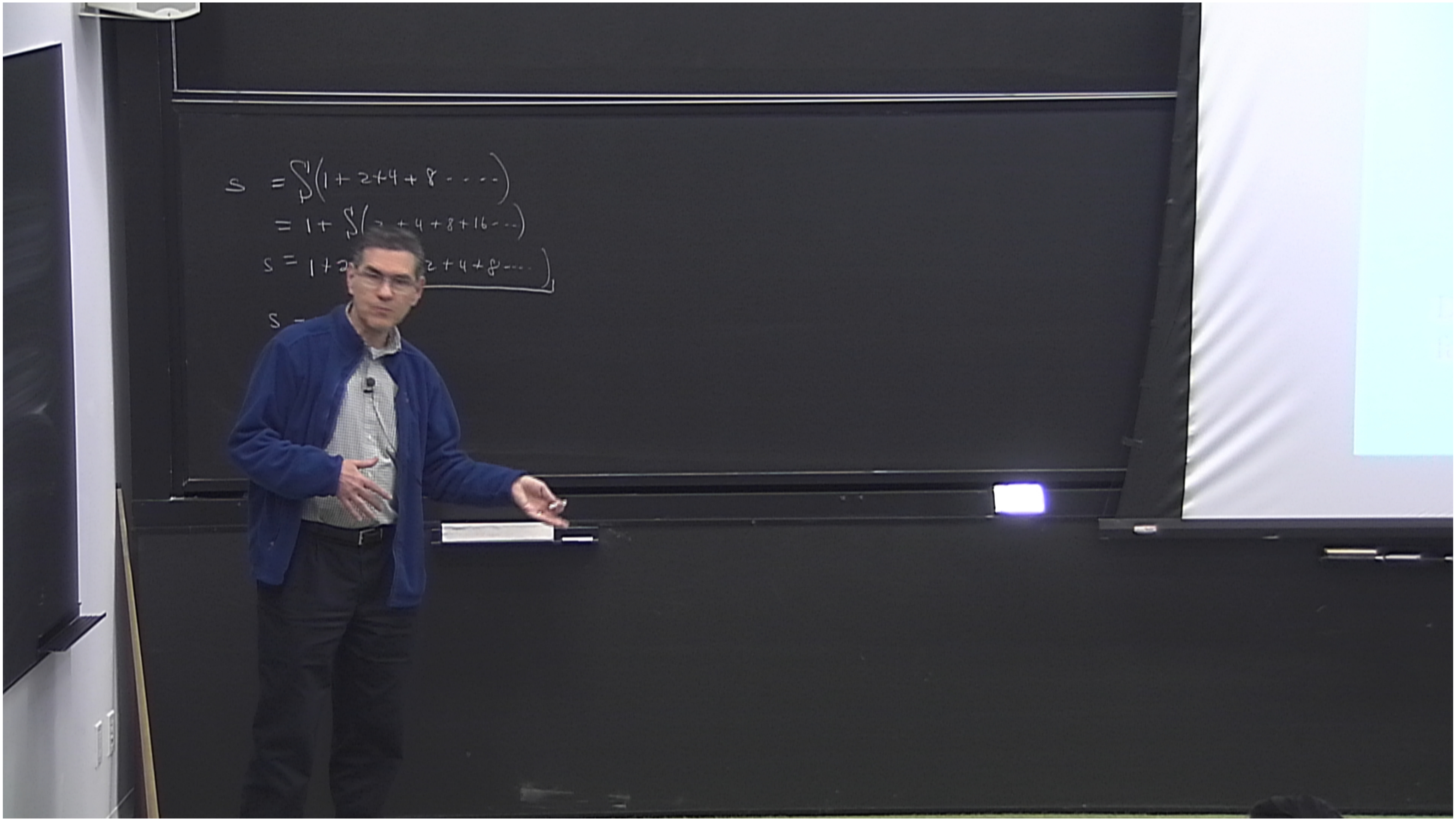
$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + \ldots$

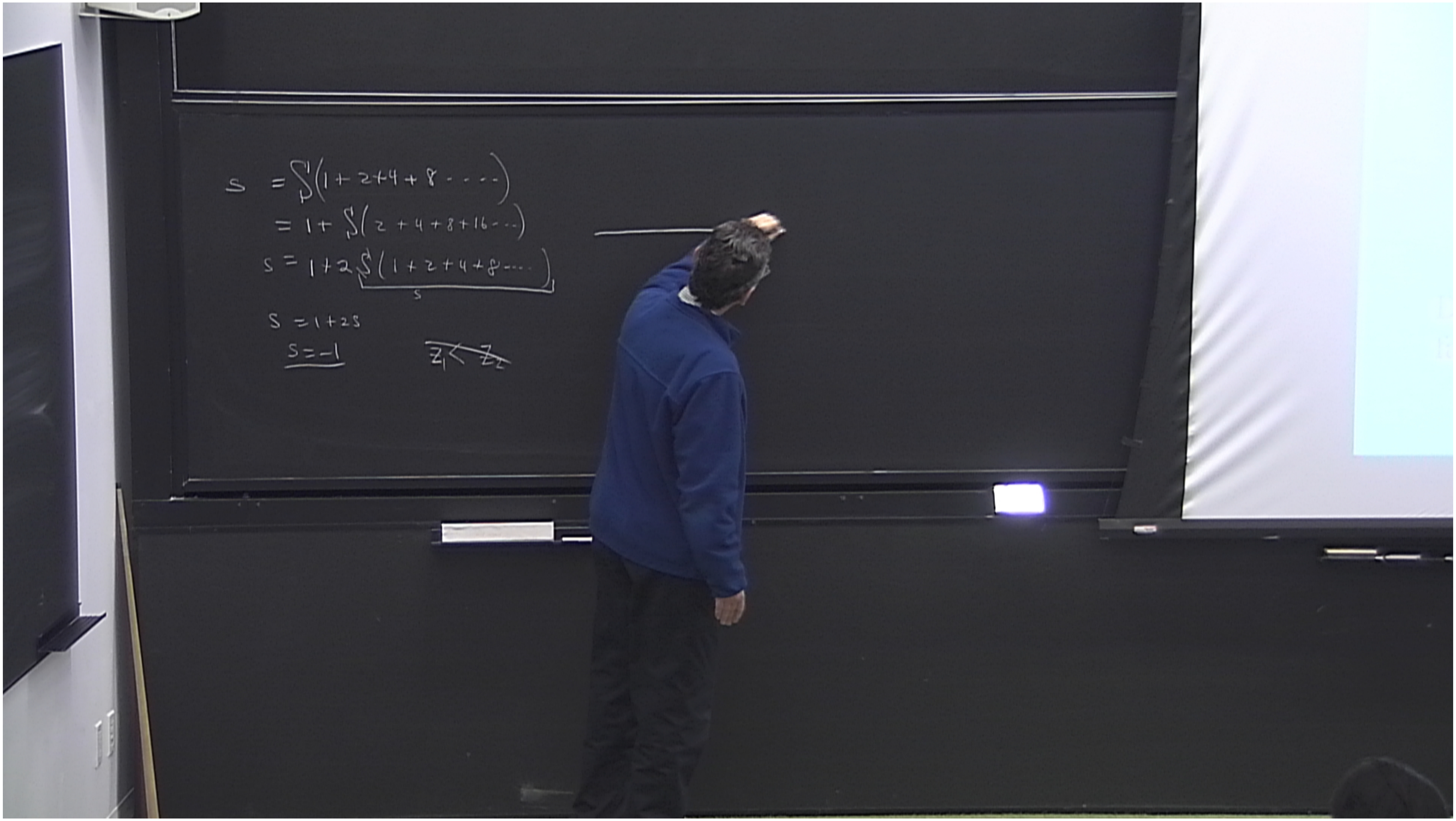
$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + \ldots$

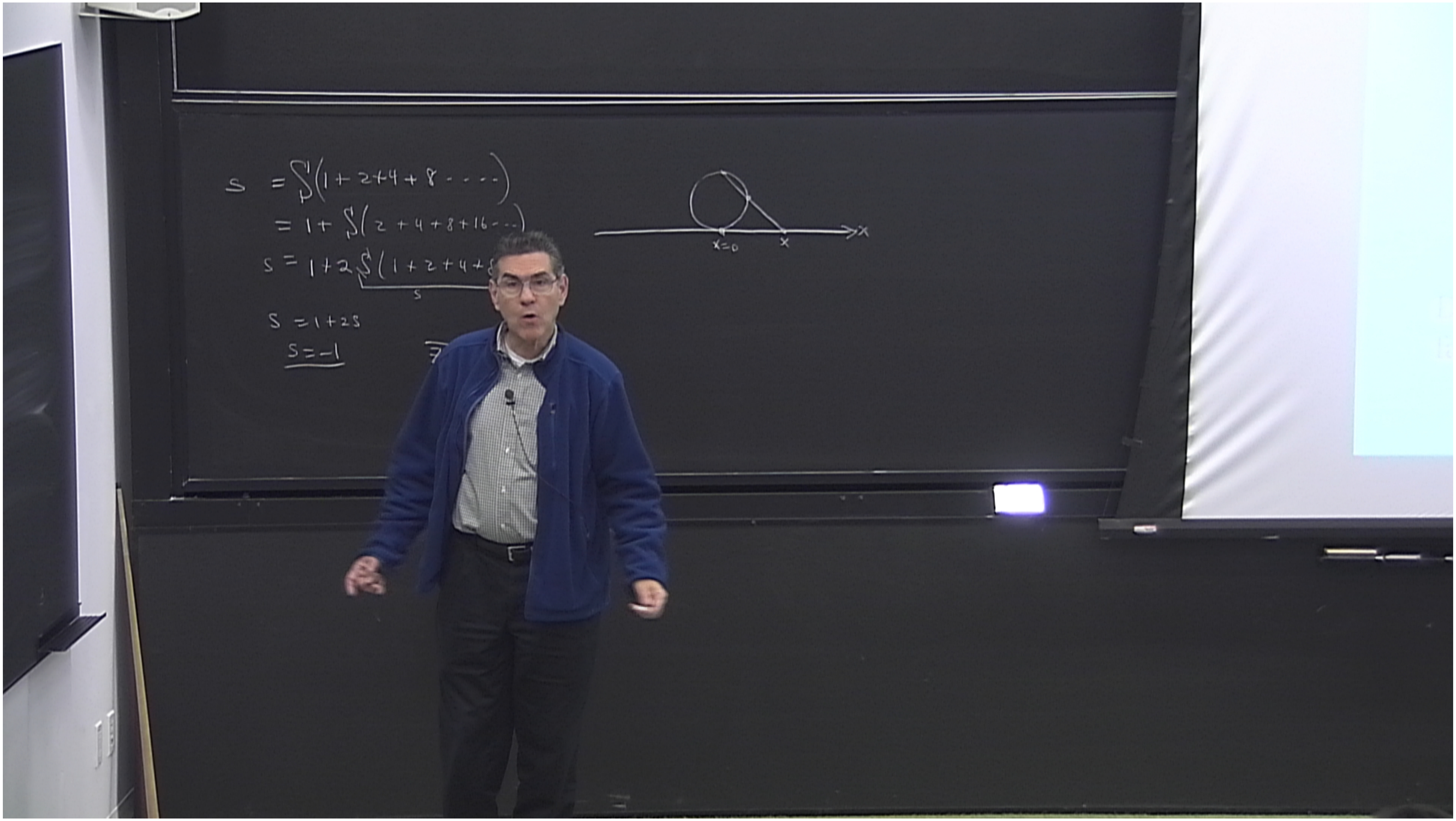
$1 + 0 - 1 + 1 + 0 - 1 + 1 + 0 - 1 + \ldots$

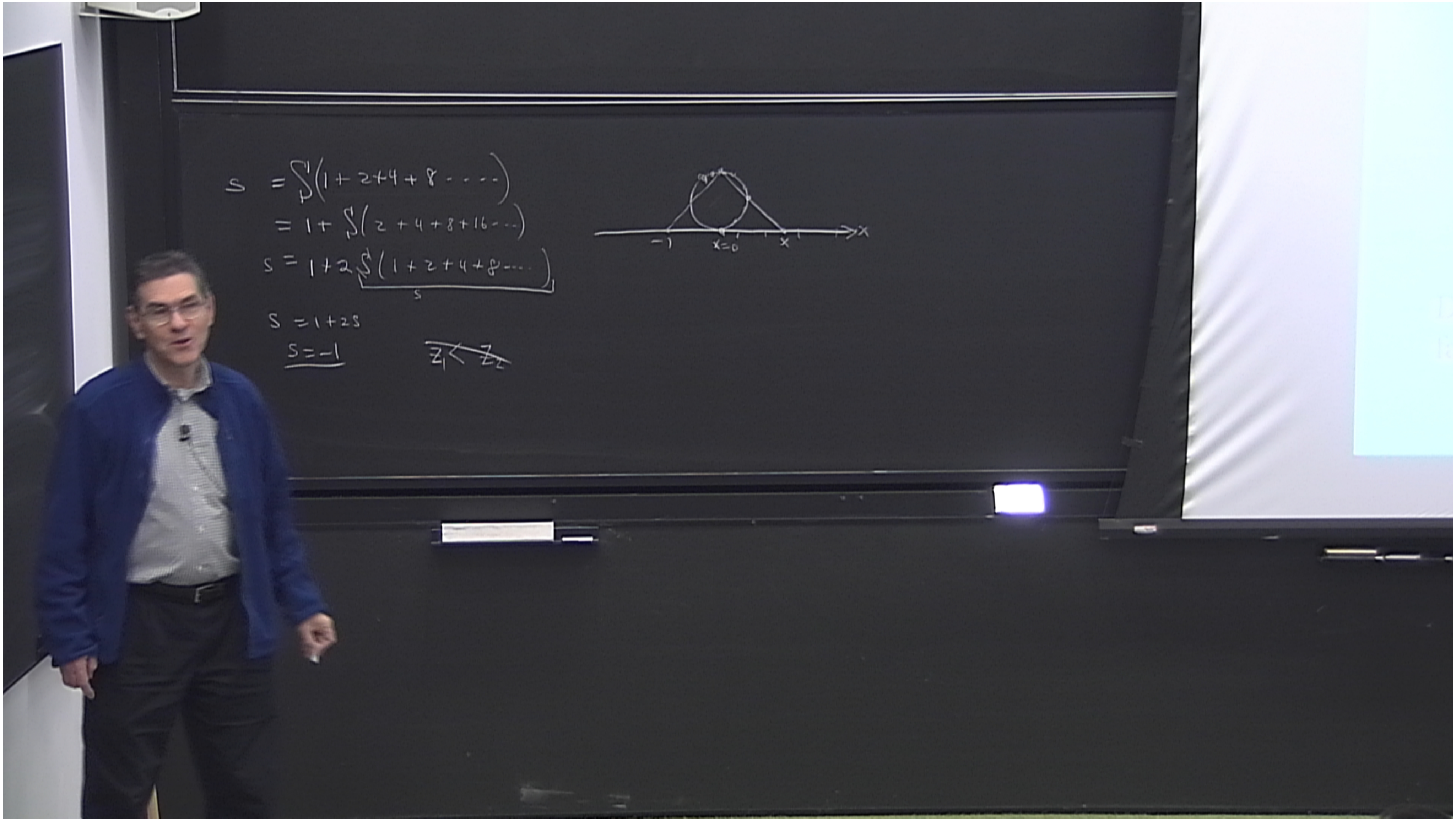
Divergent series are not bad! They are useful, and they converge faster than convergent series!

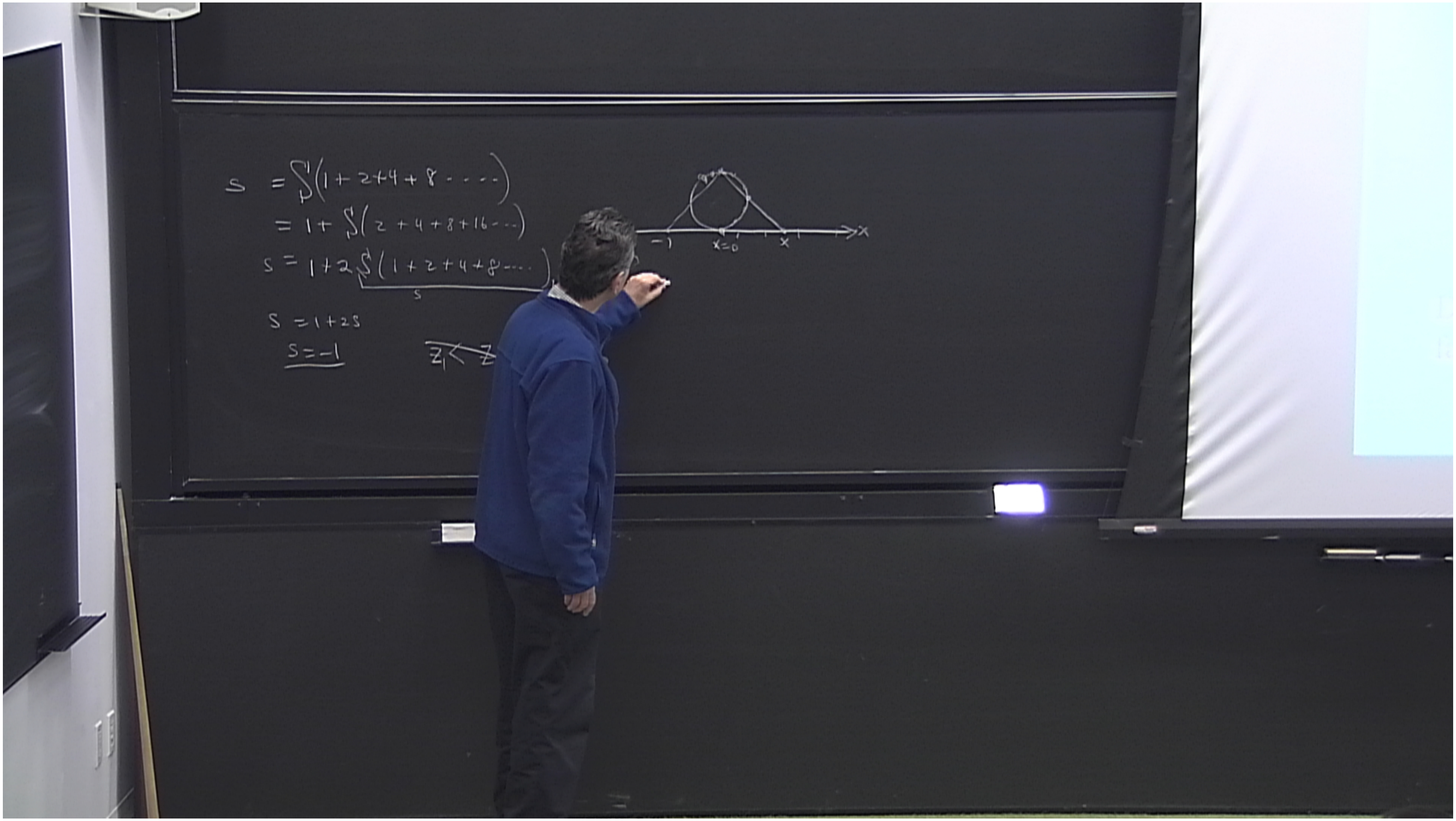

s = S(1+2+4+8 ....)
= 1+ S(2+4+8+16...)


s = S(1+2+4+8 - - - -)


s = S(1+2+4+8....)
= 1+ S(2+4+8+16...)
s = 1+2S(1+2+4+8....)
s
s = 1+2s
s = -1
Z1 < Z2


S = S(1+2+4+8....)
= 1+ S(2+4+8+16...)
S = 1+2S(1+2+4+
S = 1+2S
S = -1
x=0
x
x


s = ∫(1+2+4+8....)
= 1+∫(2+4+8+16...)
s = 1+2∫(1+2+4+8...)
s
S = 1+2S
S = -1
Z1 < Z2
-1
x=0
x
x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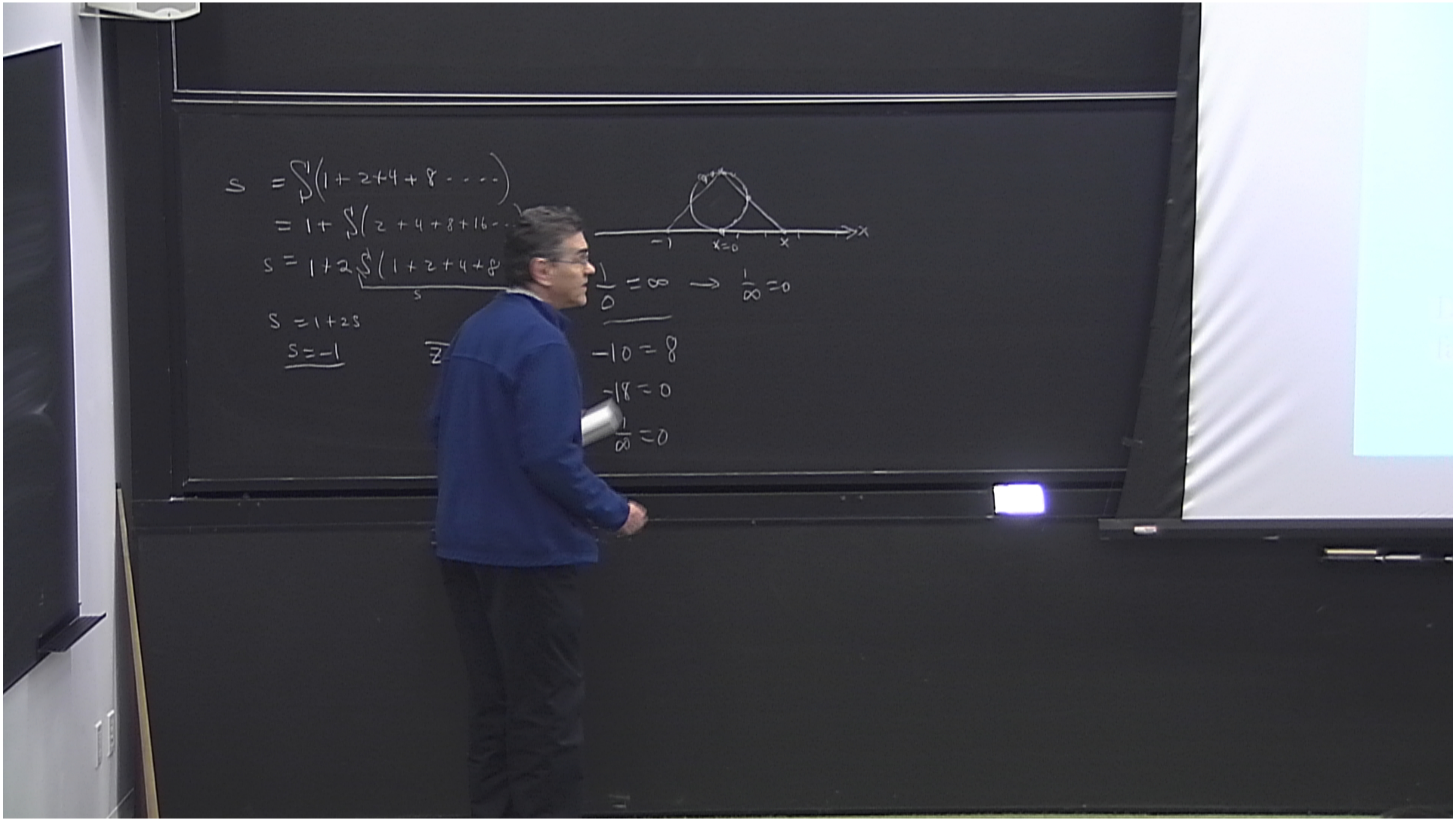
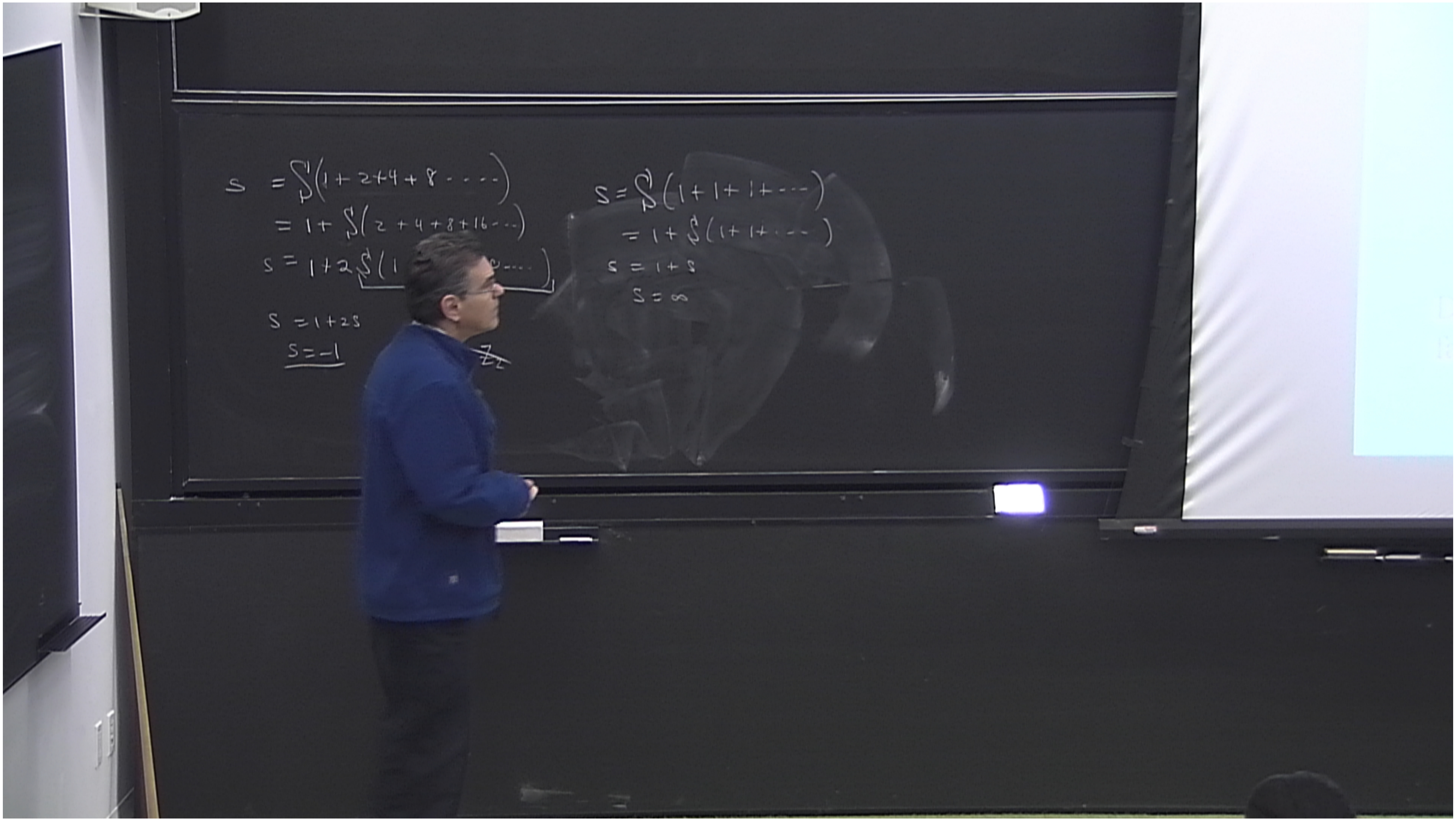
s = 1+2S(1+2+4+8- - - -)
s
s = 1+2s
s = -1
-1
x=0
x
x
Z1 < Z


S = 1+2S
S = -1
1/0 = ∞ → 1/∞ = 0
-10 = 8
-18 = 0
1/∞ = 0
-1
x=0
x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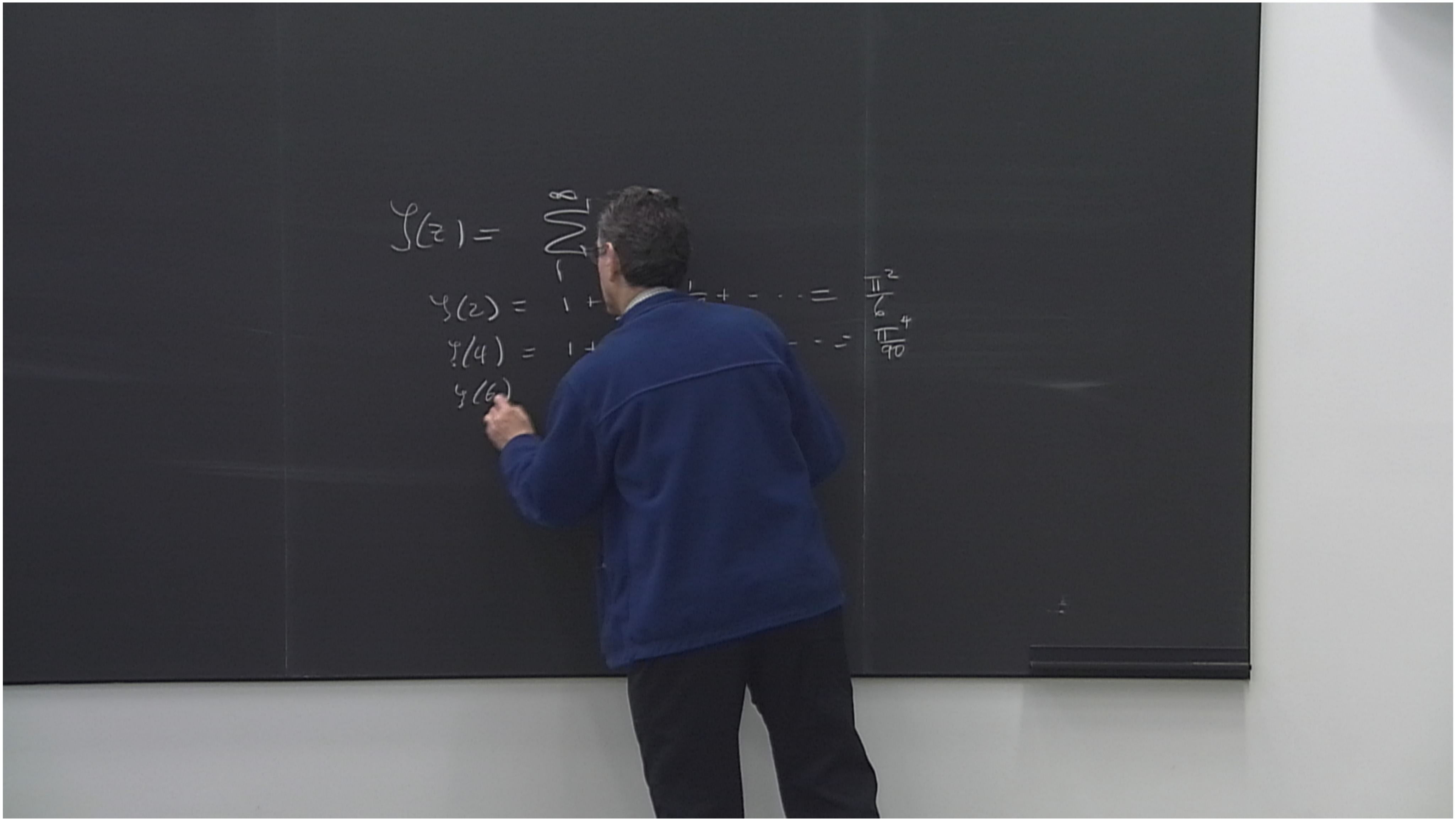
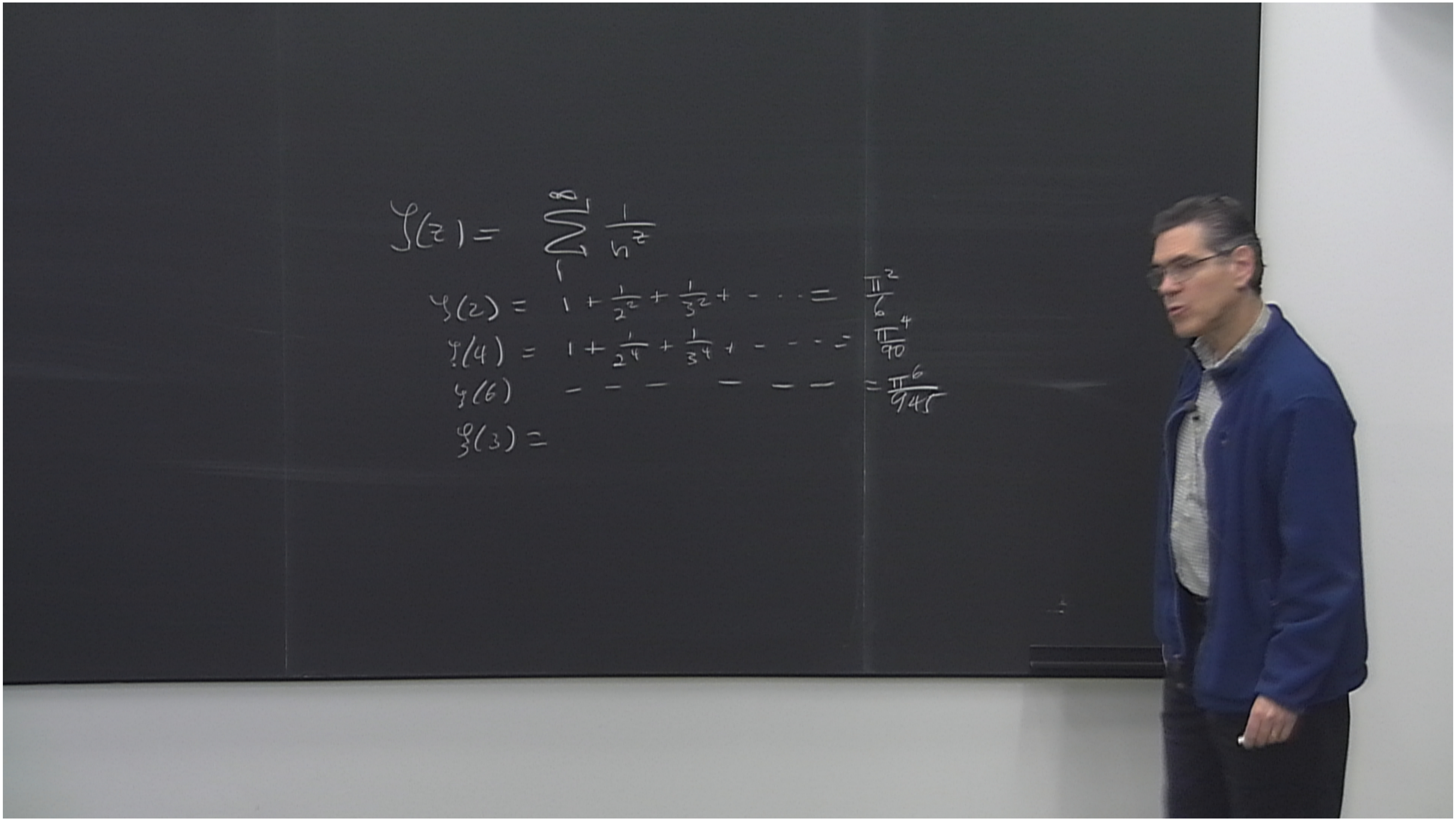
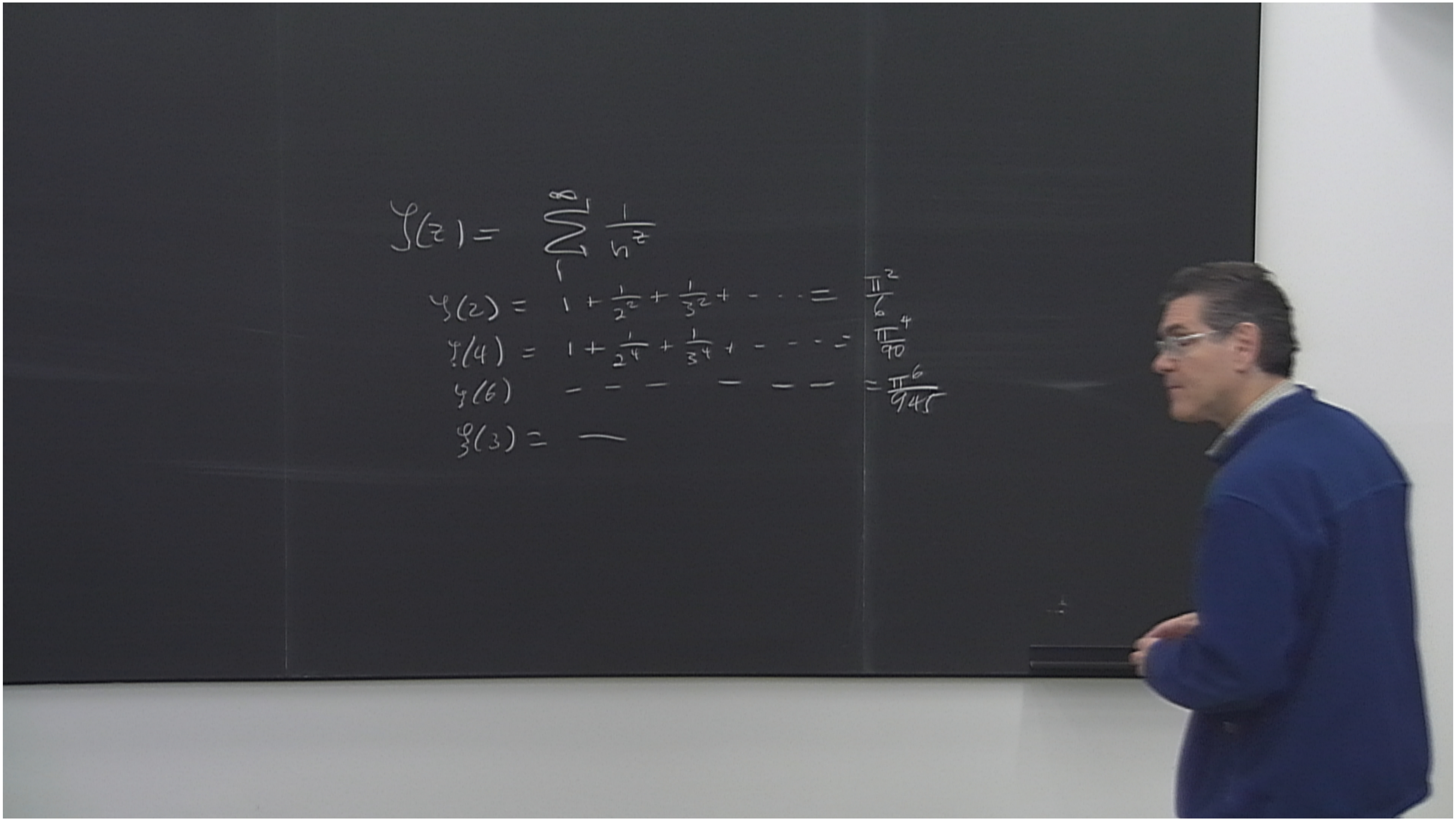
= 1+ S(2+4+8+16...)
s = 1+2S(1
s = 1+2s
s = -1
Z2
s = S(1+1+1+...)
= 1+S(1+1+...)
s = 1+s
s = ∞

Some divergent series REALLY sum up to infinity--- this is perfectly OK!

$$1 + \tfrac{1}{2} + 1/3 + \tfrac{1}{4} + 1/5 + \ldots = \zeta\,(1)$$


ζ(z) = ∑_1^∞
ζ(2) = 1 + … = π²/6
ζ(4) = 1 + … = π⁴/90
ζ(6)


$\zeta(z) = \sum_{1}^{\infty} \frac{1}{n^z}$
$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$
$\zeta(4) = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$
$\zeta(6) \quad \cdots = \frac{\pi^6}{945}$
$\zeta(3) =$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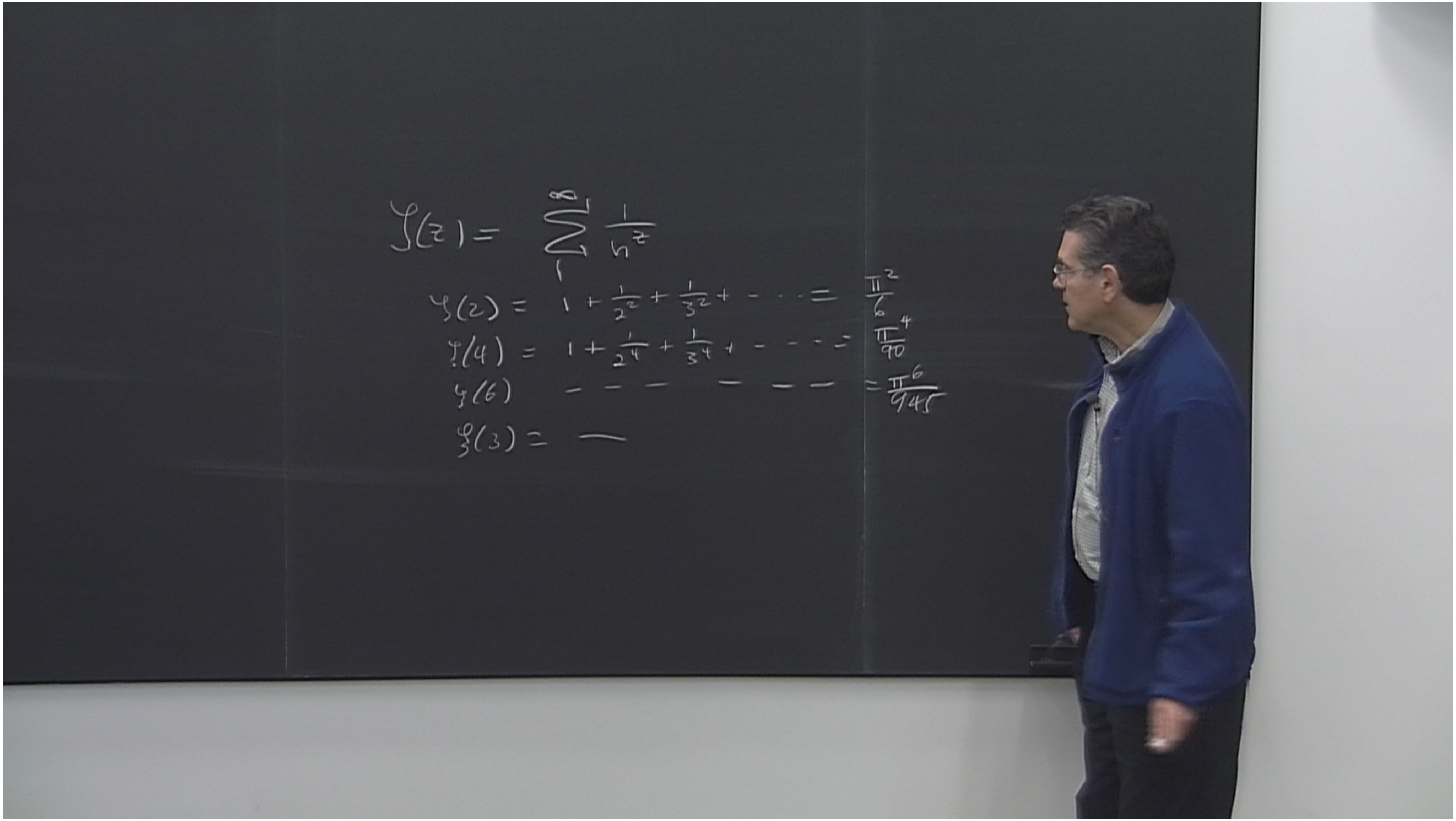
$\zeta(6) \cdots = \frac{\pi^6}{945}$
$\zeta(3) = —$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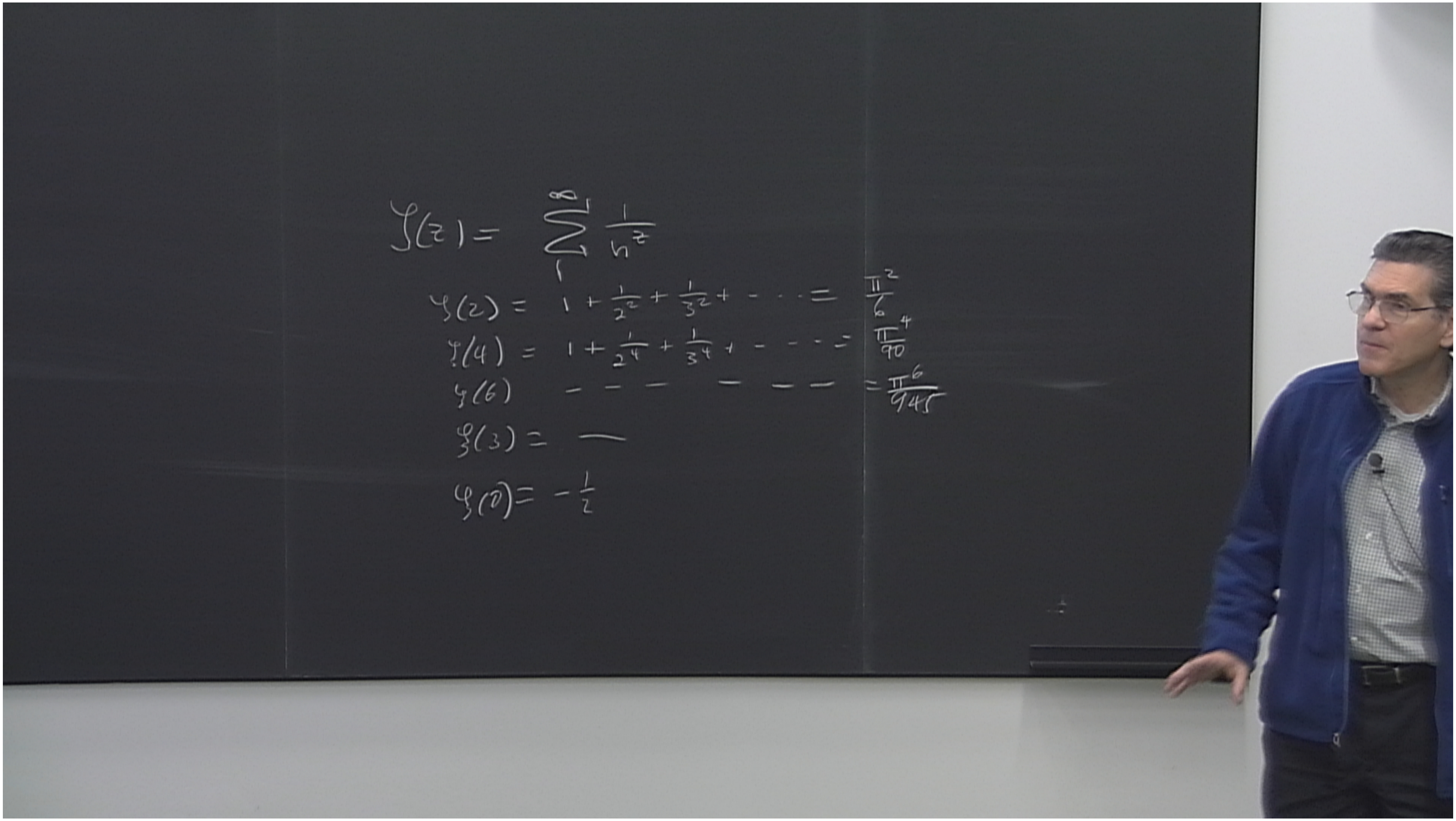
$\zeta(3) =$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$\zeta(6) \quad - - - \quad - - - = \frac{\pi^6}{945}$
$\zeta(3) = \text{—}$
$\zeta(0) = -\frac{1}{2}$

Techniques for summing series

- Note: addition is not infinitely commutative
- Euler summation
- Borel summation
- Generic summation methods
- Note: addition is not infinitely associative
- Zeta summation
- Continued functions

Zeta summation

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

$$\zeta(1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... = \infty$$

$$\zeta(0) = 1 + 1 + 1 + 1 + 1... = -\frac{1}{2}$$

$$\zeta(-1) = 1 + 2 + 3 + 4 + 5 + 6... = -\frac{1}{12}$$

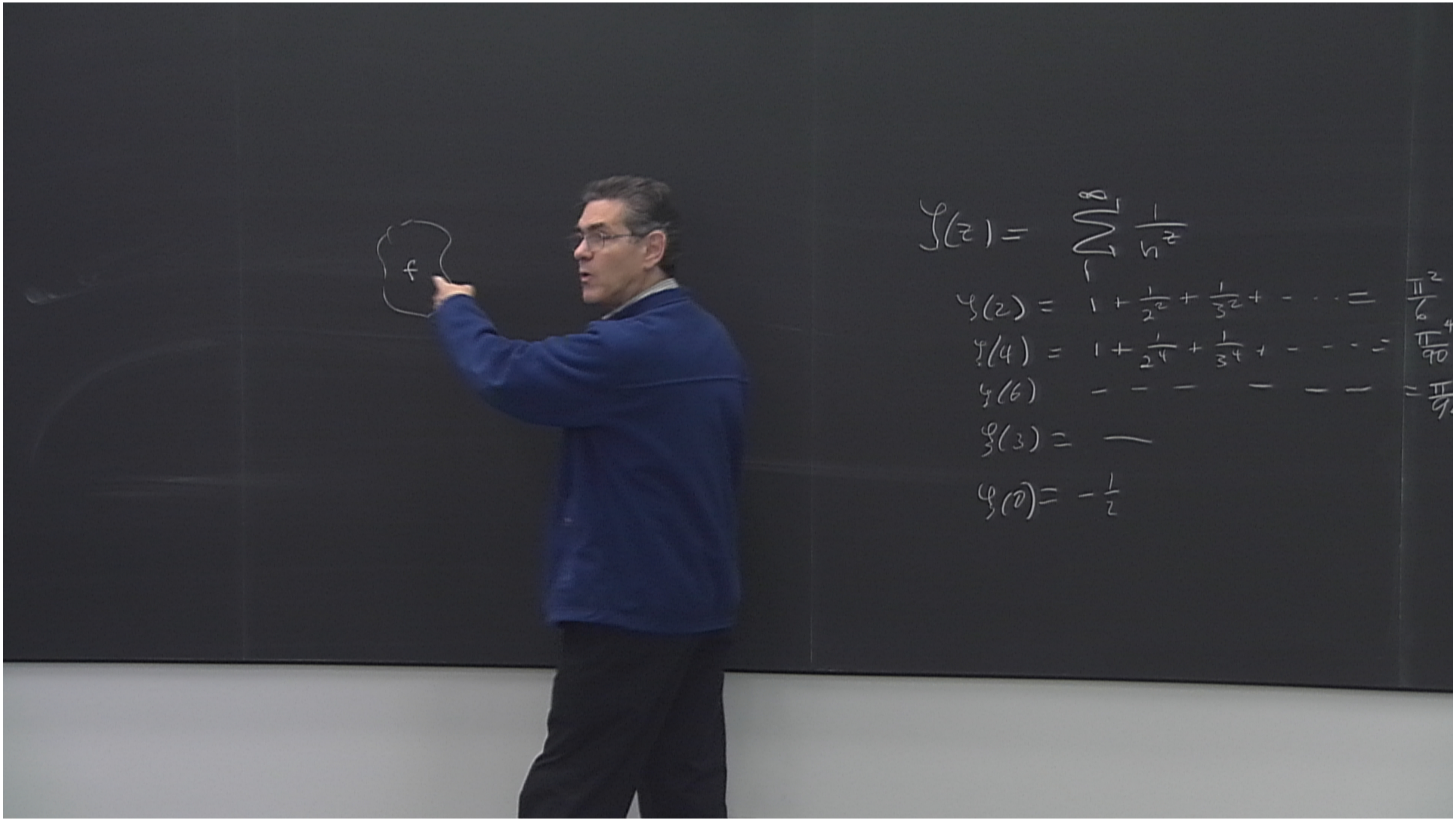
Zeta summation

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

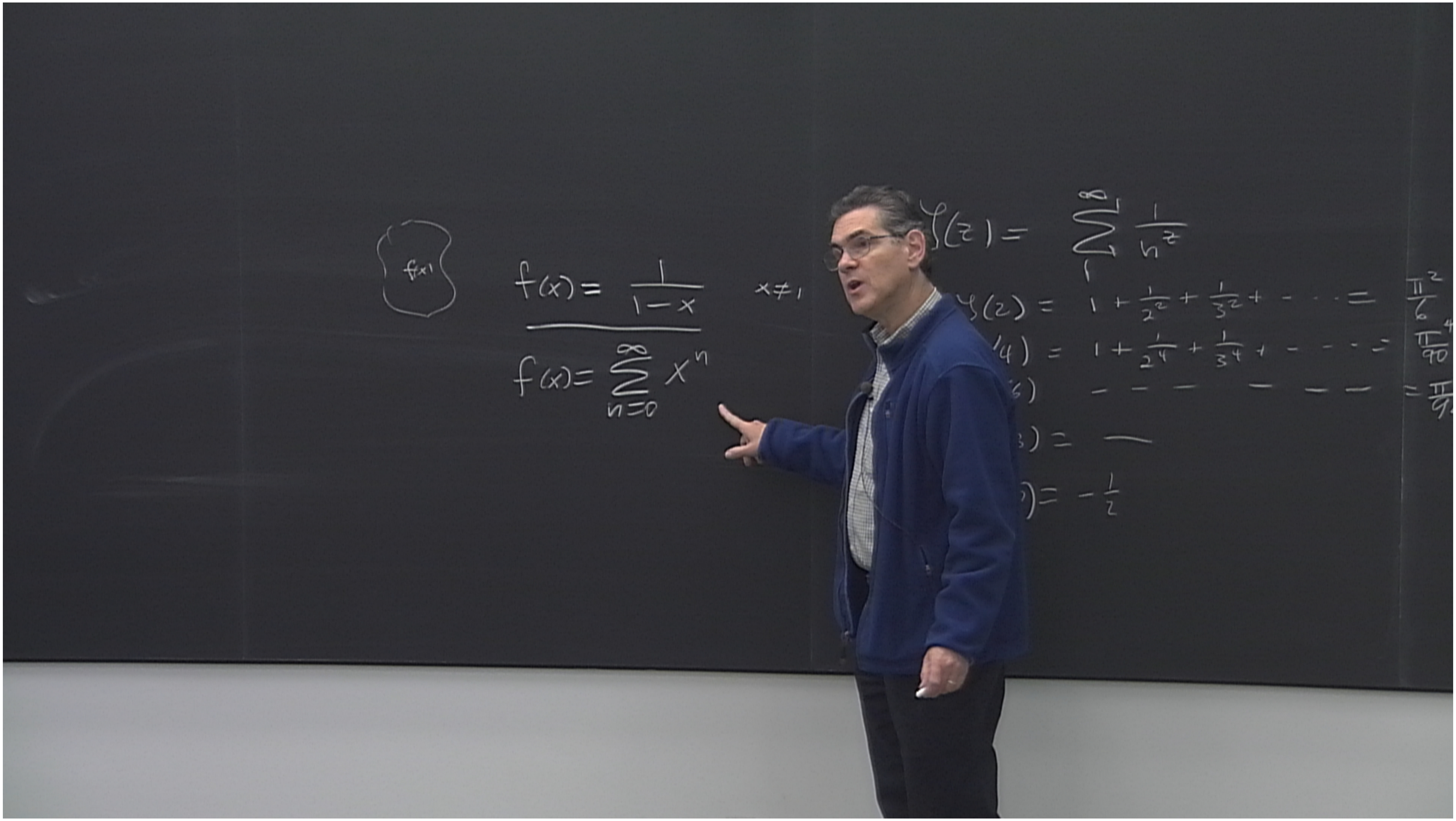
$$\zeta(1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots = \infty$$

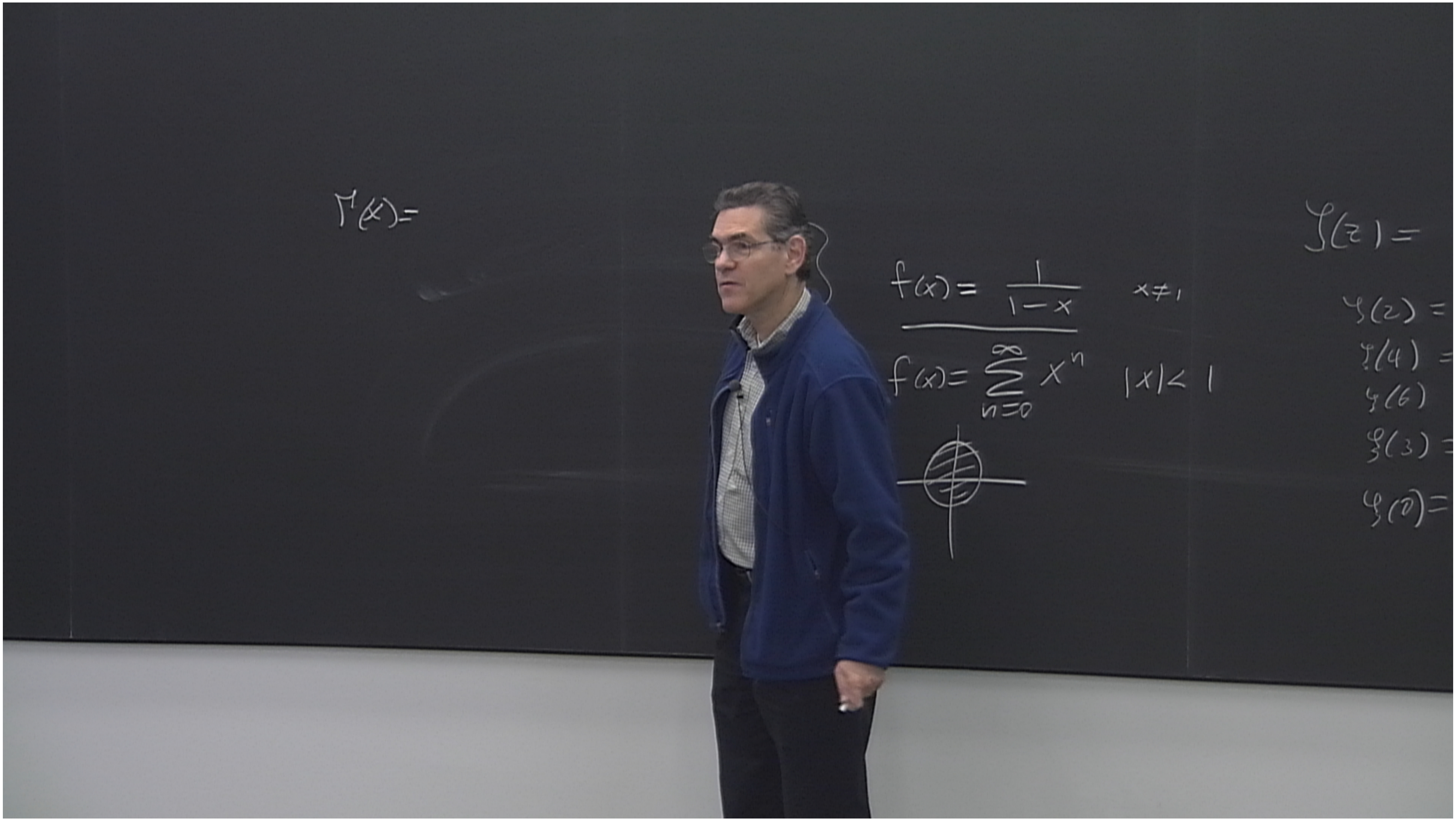
$$\zeta(0) = 1 + 1 + 1 + 1 + 1\ldots = -\frac{1}{2}$$

$$\zeta(-1) = 1 + 2 + 3 + 4 + 5 + 6\ldots = -\frac{1}{12}$$

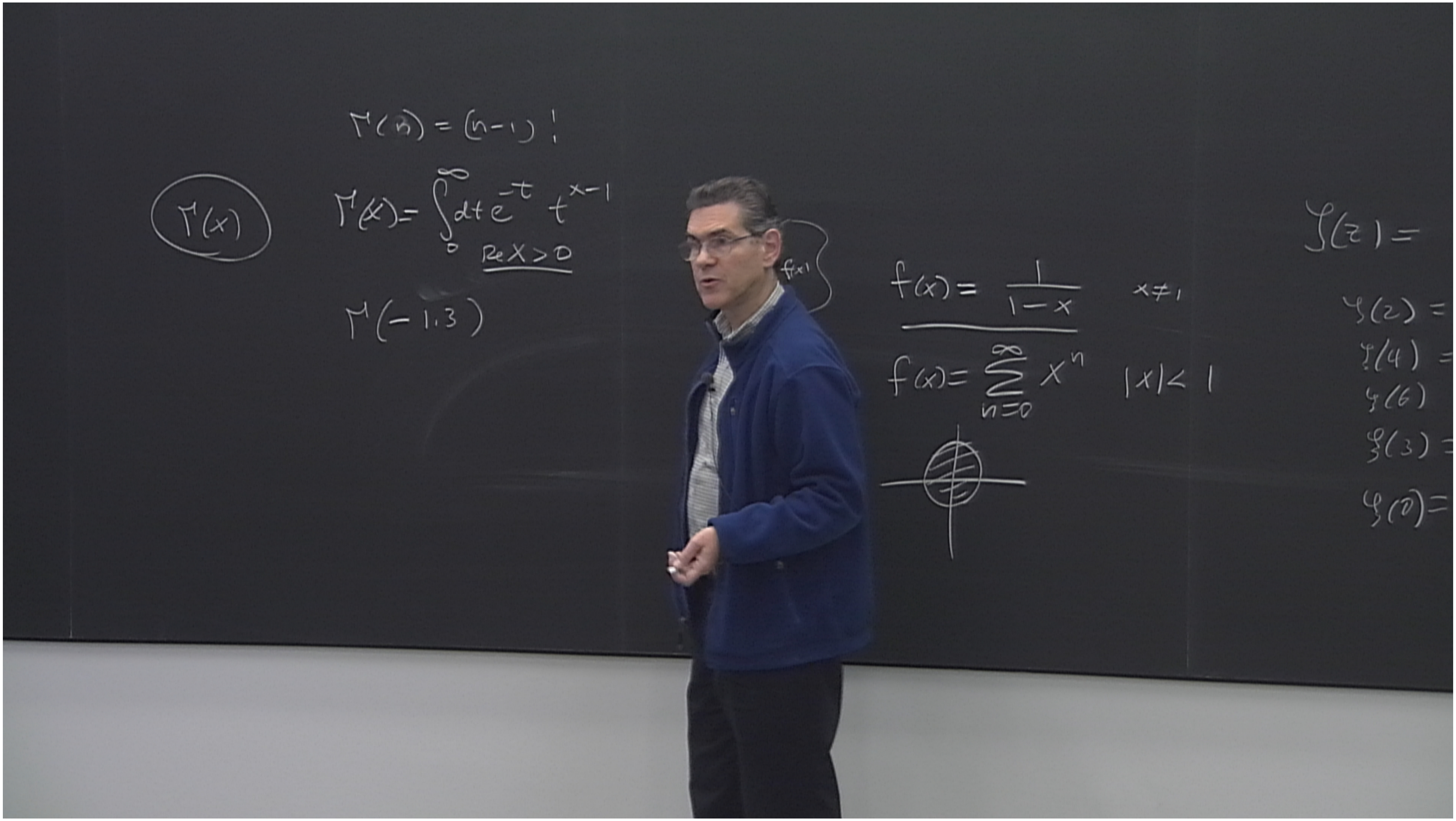


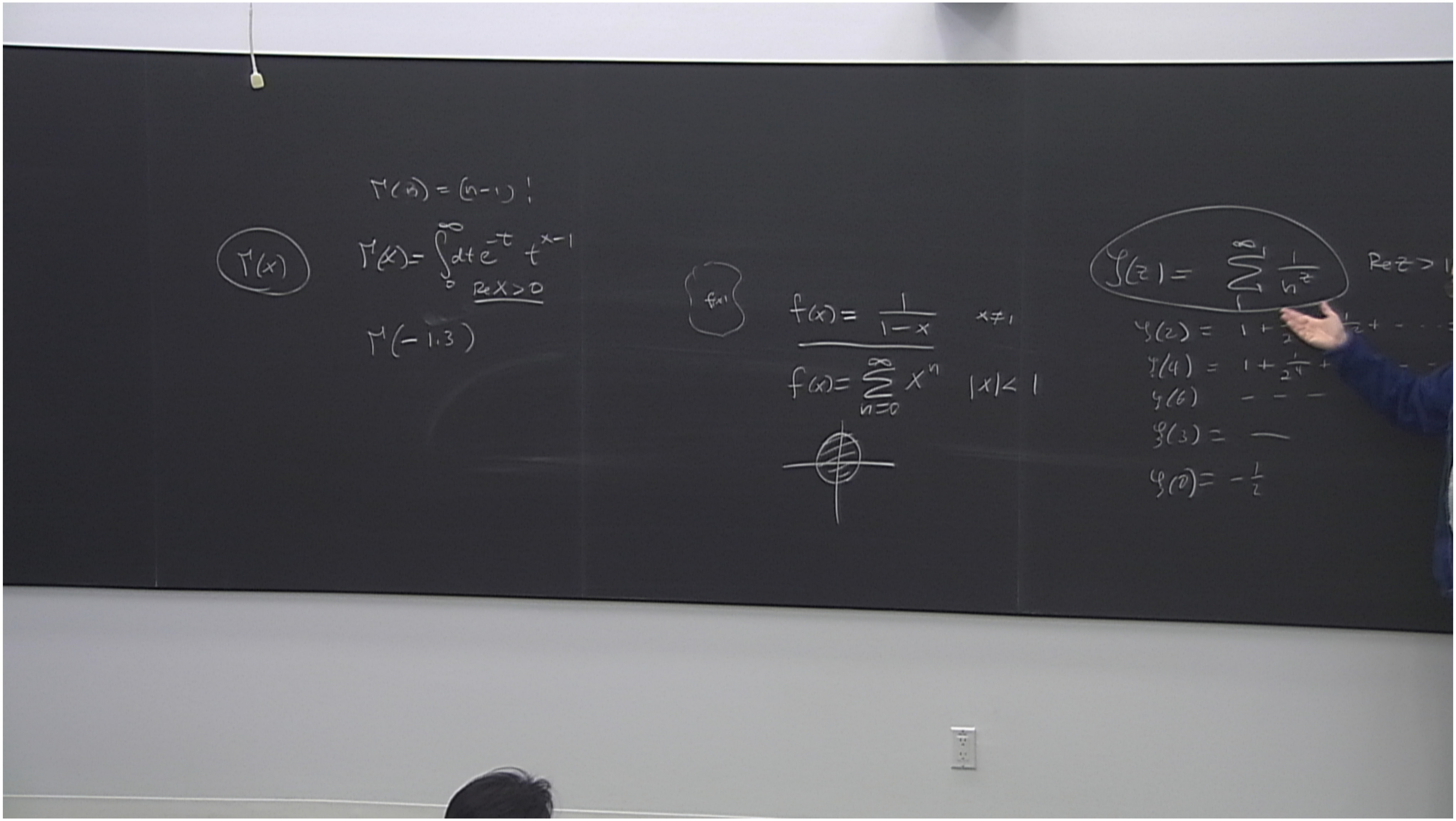
f
$\zeta(z) = \sum_{1}^{\infty} \frac{1}{n^z}$
$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$
$\zeta(4) = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$
$\zeta(6)$
$\zeta(3) =$
$\zeta(0) = -\frac{1}{2}$

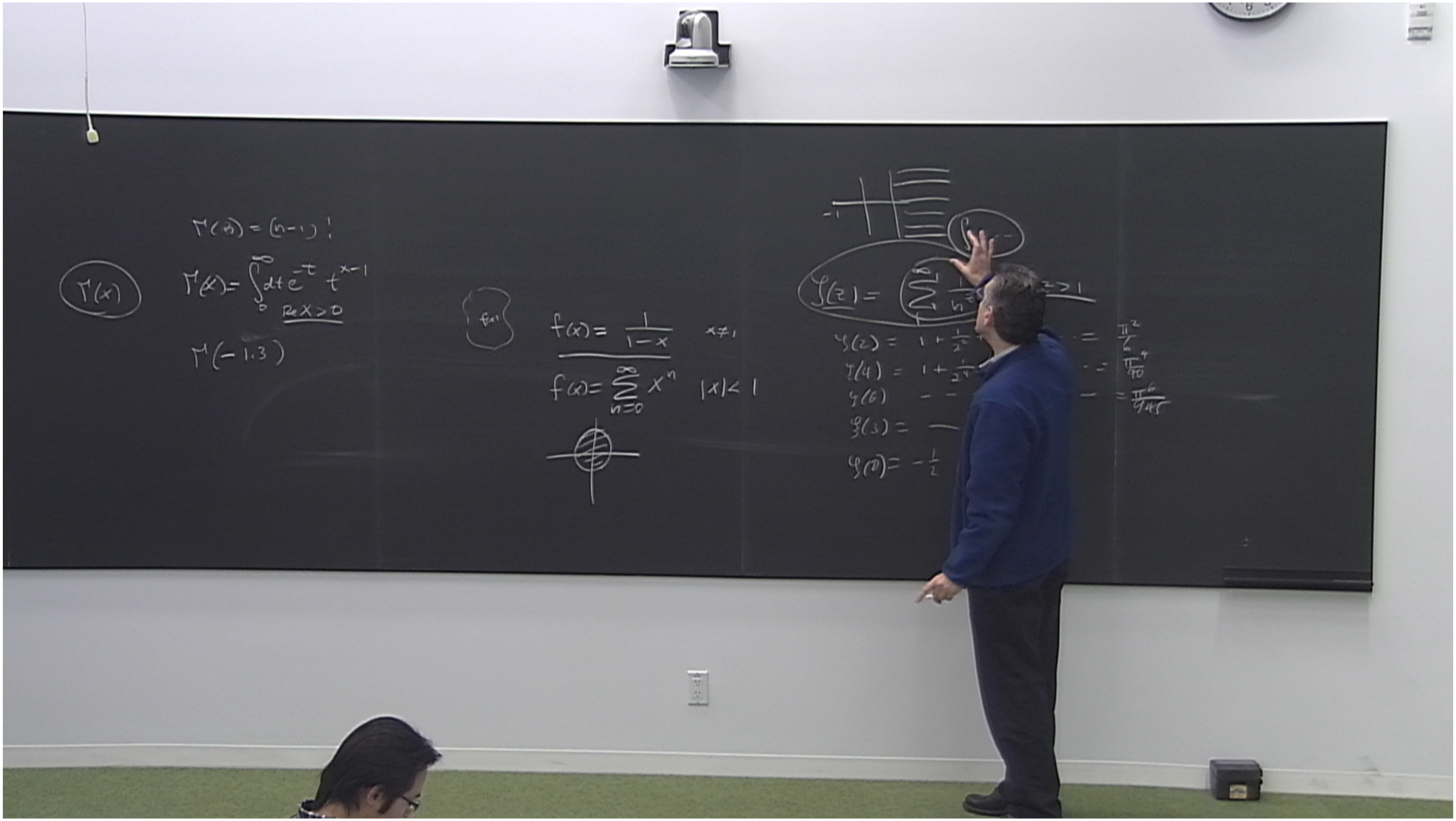

f(x) = 1/(1−x)
x ≠ 1
f(x) = Σ_{n=0}^∞ x^n
ζ(z) = Σ_1^∞ 1/n^z
ζ(2) = 1 + 1/2² + 1/3² + · · · = π²/6
ζ(4) = 1 + 1/2⁴ + 1/3⁴ + · · · = π⁴/90
)= −1/2



Γ(x) =
ζ(z) =
f(x) = 1/(1−x)   x ≠ 1
f(x) = Σ_{n=0}^∞ x^n |x| < 1
ζ(2) =
ζ(4) =
ζ(6)
ζ(3) =
ζ(0) =


Γ(n) = (n−1)!
Γ(x)
Γ(x) = ∫₀^∞ dt e^{−t} t^{x−1}
Re x > 0
Γ(−1.3)
f(x) = 1/(1−x)   x ≠ 1
f(x) = Σ_{n=0}^∞ x^n |x| < 1
ζ(z) =
ζ(2) =
ζ(4) =
ζ(6)
ζ(3) =
ζ(0) =


$\Gamma(n) = (n-1)!$
$\Gamma(x)$
$\Gamma(x) = \int_0^\infty dt\, e^{-t}\, t^{x-1}$
$\mathrm{Re}\, x > 0$
$\Gamma(-1.3)$
$f(x)$
$f(x) = \frac{1}{1-x}$ $x \neq 1$
$f(x) = \sum_{n=0}^{\infty} x^n$ $|x| < 1$
$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$ $\mathrm{Re}\, z > 1$
$\zeta(2) = 1 + \frac{1}{2^2} + \cdots$
$\zeta(4) = 1 + \frac{1}{2^4} + \cdots$
$\zeta(6)$ $---$
$\zeta(3) = —$
$\zeta(0) = -\frac{1}{2}$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Γ(x)
Γ(x) = ∫ dt e^{-t} t^{x-1}
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Γ(-1.3)
f(x)
f(x) = 1/(1-x)   x≠1
f(x) = Σ_{n=0}^∞ x^n |x| < 1
-1
ζ(z) =
ζ(2) = 1 + 1/2^2 = π^2/6
ζ(4) = 1 + 1/2^4 = π^4/90
ζ(6) = π^6/945
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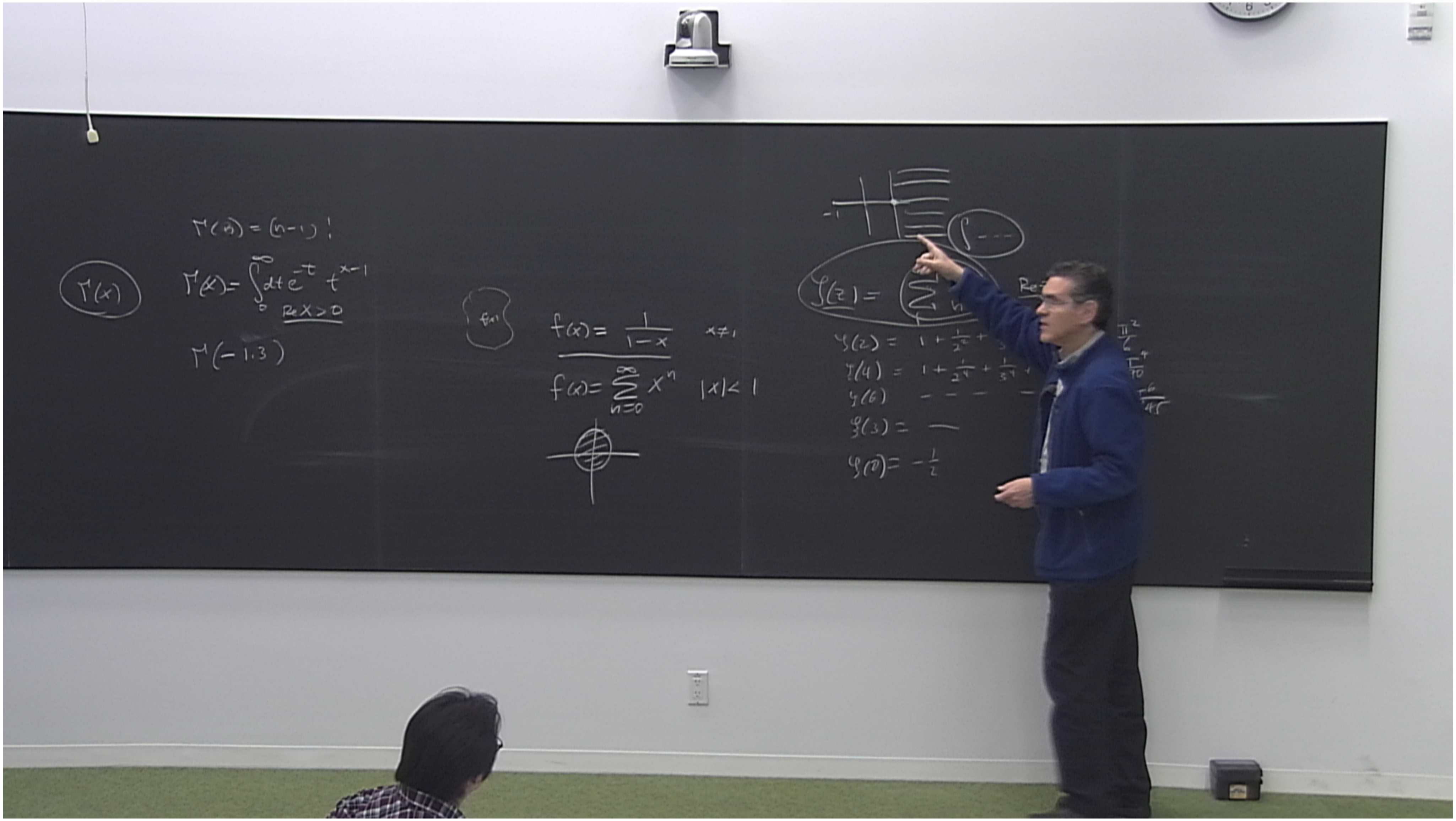
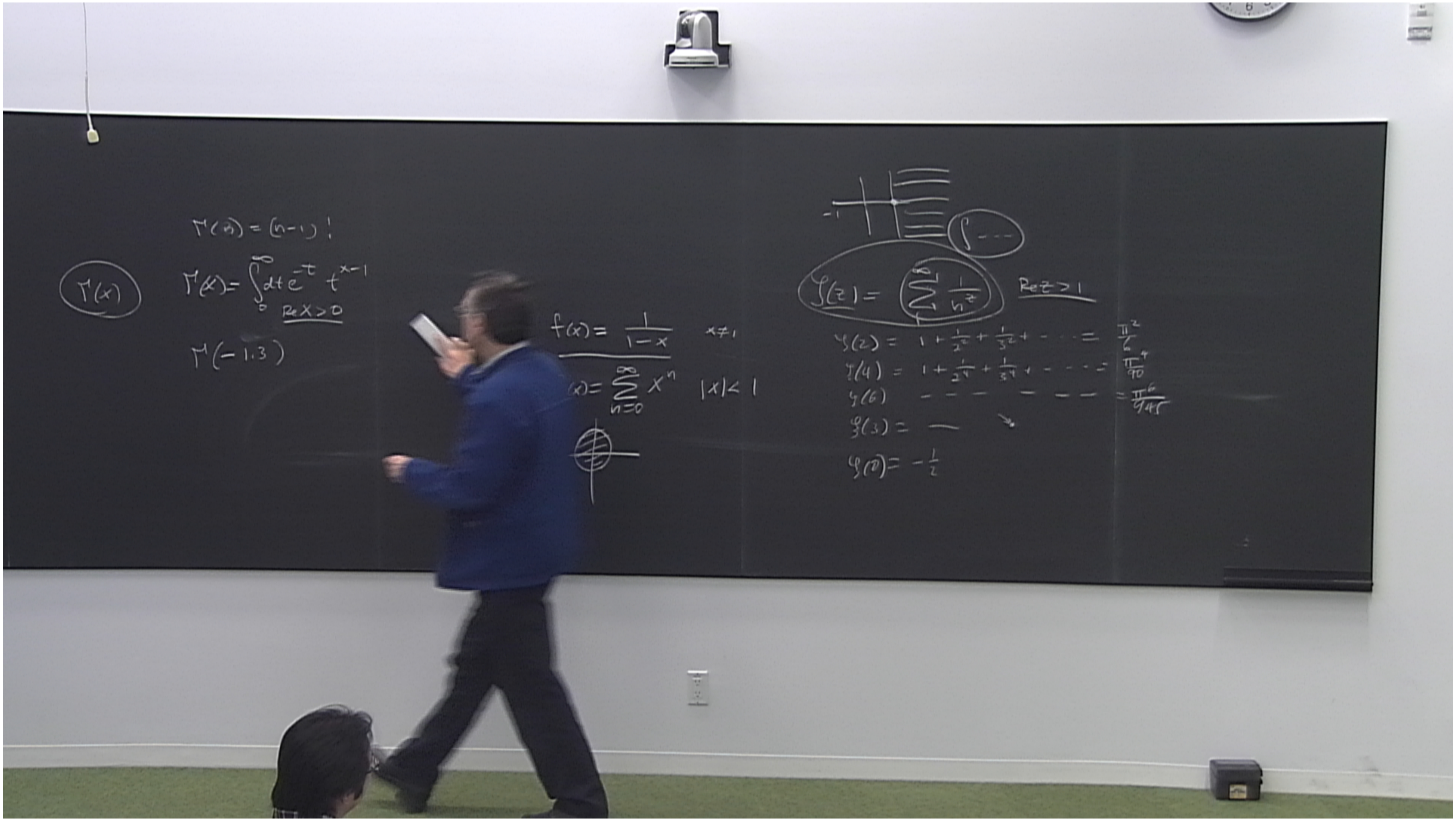
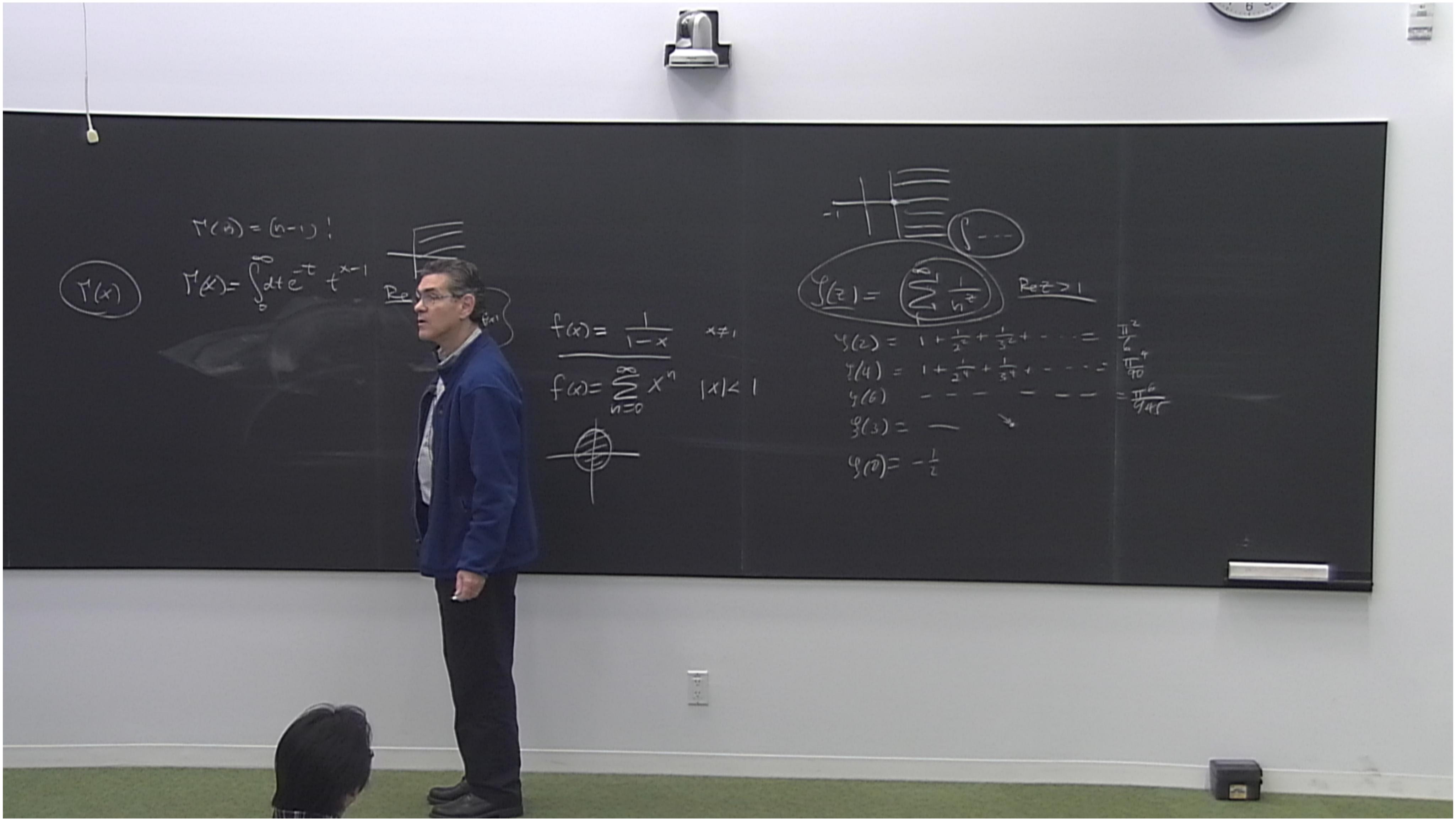

Γ(n) = (n−1)!
Γ(x)
Γ(x) = ∫ dt e^-t t^x−1
Re x > 0
Γ(−1.3)
f(x) = 1/(1−x)
x ≠ 1
f(x) = Σ x^n
n=0
|x| < 1
ζ(z) =
ζ(2) =
ζ(4) =
ζ(6)
ζ(3) =
ζ(0) = −1/2


Γ(n) = (n-1)!
Γ(x)
Γ(x) = ∫₀^∞ dt e^{-t} t^{x-1}
Re x > 0
Γ(-1.3)
f(x) = 1/(1-x)   x≠1
f(x) = Σ_{n=0}^∞ x^n |x| < 1
ζ(z) = Σ_{n=1}^∞ 1/n^z   Re z > 1
ζ(2) = 1 + 1/2² + 1/3² + ... = π²/6
ζ(4) = 1 + 1/2⁴ + 1/3⁴ + ... = π⁴/90
ζ(6) = ... = π⁶/945
ζ(3) =
ζ(0) = -1/2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Γ(x)
Γ(x) = ∫₀^∞ dt e^{-t} t^{x-1}
f(x) = 1/(1-x)   x≠1
f(x) = Σ_{n=0}^∞ x^n |x| < 1
ζ(z) = Σ 1/n^z
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ζ(2) = 1 + 1/2² + 1/3² + ... = π²/6
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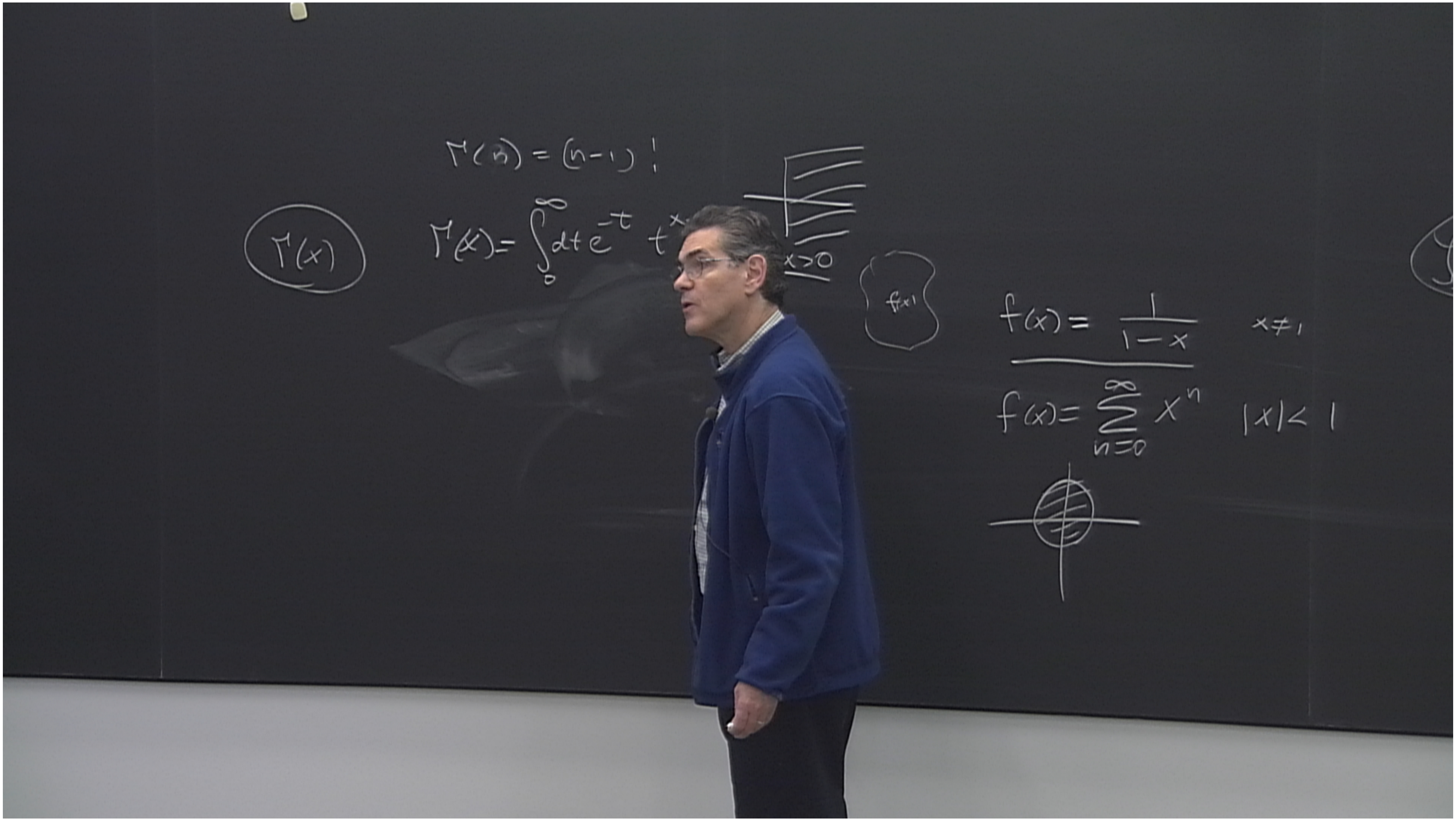
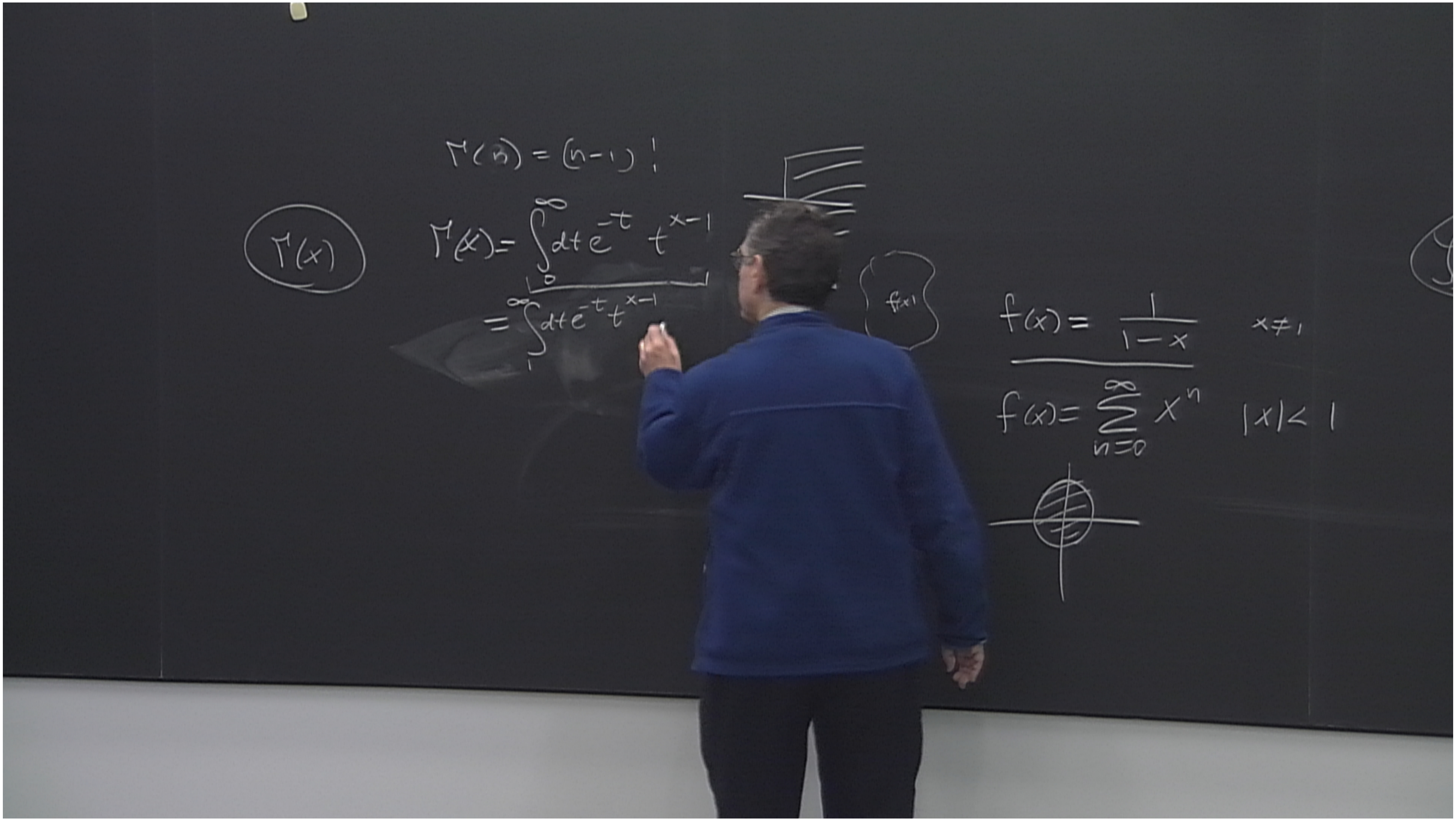
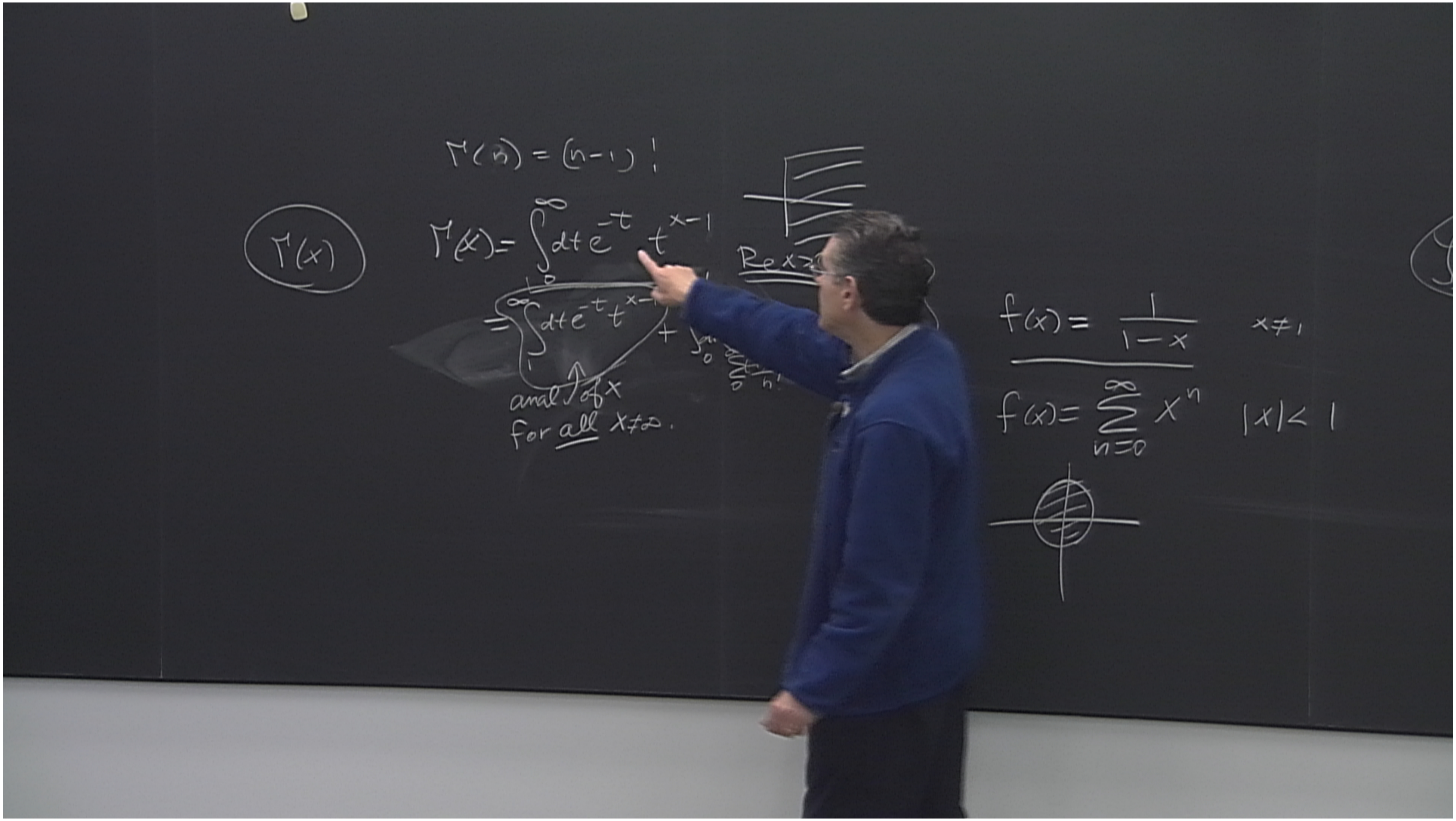
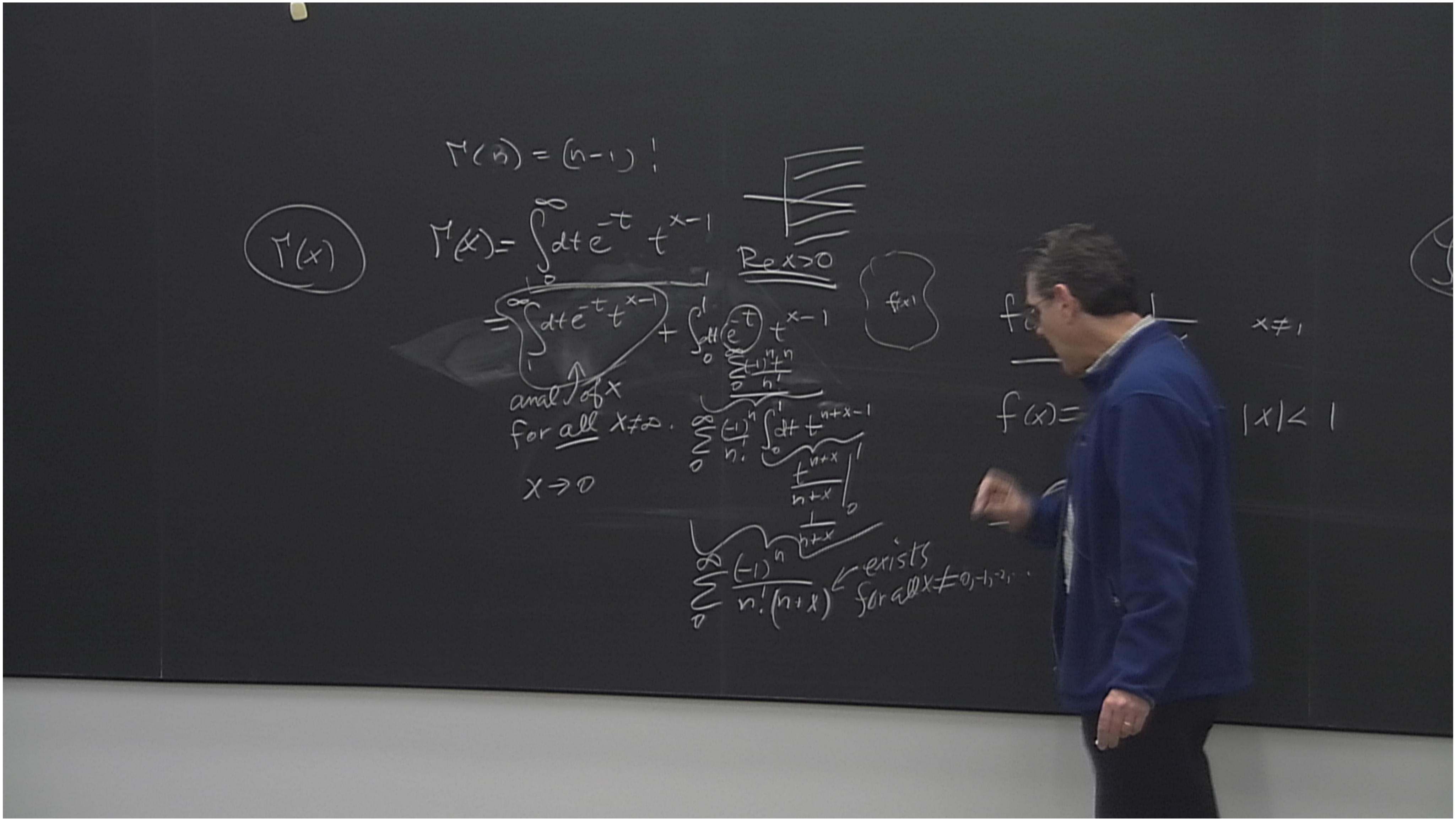


Γ(n) = (n-1)!
Γ(x)
Γ(x) = ∫₀^∞ dt e^-t t^x
x>0
f(x)
f(x) = 1/(1-x)   x≠1
f(x) = Σ_{n=0}^∞ x^n |x| < 1


$\Gamma(n) = (n-1)!$
$\Gamma(x)$
$\Gamma(x) = \int_0^\infty dt\, e^{-t}\, t^{x-1}$
$= \int_1^\infty dt\, e^{-t} t^{x-1}$
$f(x)$
$f(x) = \frac{1}{1-x}$ $x \neq 1$
$f(x) = \sum_{n=0}^\infty x^n$ $|x| < 1$


Γ(n) = (n-1)!
Γ(x)
Γ(x) = ∫ dt e^-t t^x-1
= ∫ dt e^-t t^x-1 +
anal of x
for all x≠∞.
f(x) = 1/(1-x)   x≠1
f(x) = Σ x^n |x|< 1
n=0

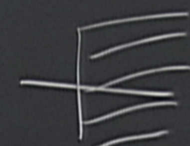


Γ(n) = (n-1)!
Γ(x)
Γ(x) = ∫₀^∞ dt e^{-t} t^{x-1}
Re x>0
f(x)
= ∫₁^∞ dt e^{-t} t^{x-1} + ∫₀^1 dt e^{-t} t^{x-1}
anal of x
for all x≠0.
x→0
exists
for all x≠0,-1,-2,...
x≠1
|x|< 1

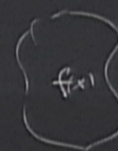
$\Gamma(n) = (n-1)!$

$\Gamma(x)$

$$\Gamma(x) = \int_0^\infty dt\, e^{-t}\, t^{x-1}$$



Re $x > 0$



$$= \int_1^\infty dt\, e^{-t}\, t^{x-1} + \int_0^1 dt\, e^{-t}\, t^{x-1}$$

anal. of x for all $x \neq \infty$.

$$e^{-t} = \sum_0^\infty \frac{(-1)^n t^n}{n!}$$

$$\sum_0^\infty \frac{(-1)^n}{n!} \underbrace{\int_0^1 dt\, t^{n+x-1}}_{\left.\frac{t^{n+x}}{n+x}\right|_0^1 = \frac{1}{n+x}}$$

$x \to 0$

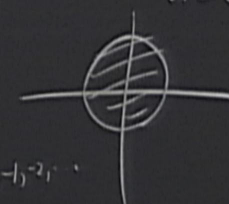
$\Gamma(x) \sim \frac{1}{x}$ as $x \to 0$.

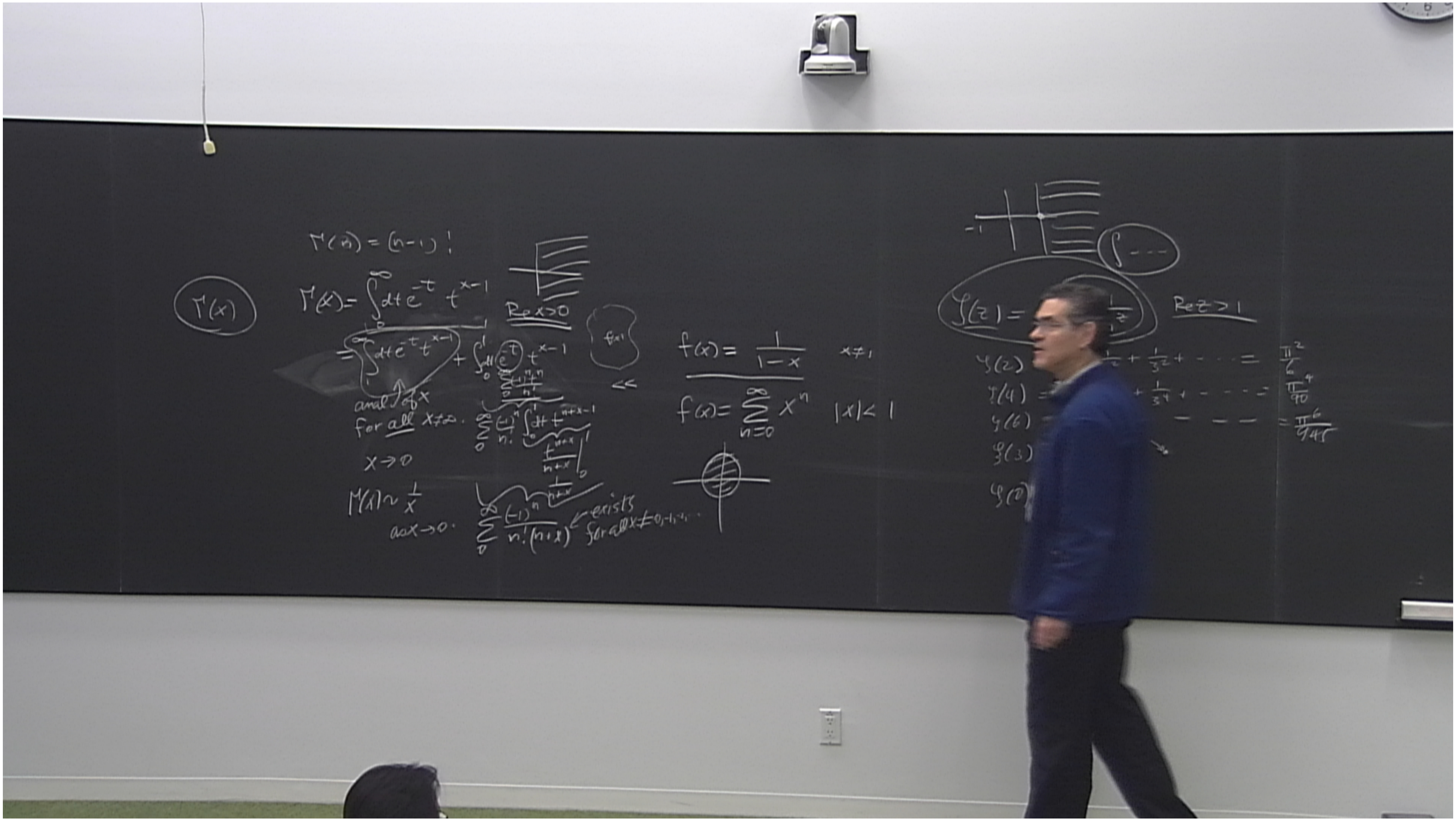
$$\sum_0^\infty \frac{(-1)^n}{n!\,(n+x)} \leftarrow \text{exists for all } x \neq 0, -1, -2, \ldots$$

$<<$

$f(x) = \frac{1}{1-x}$ $\quad x \neq 1$

$f(x) = \sum_{n=0}^\infty x^n$ $\quad |x| < 1$

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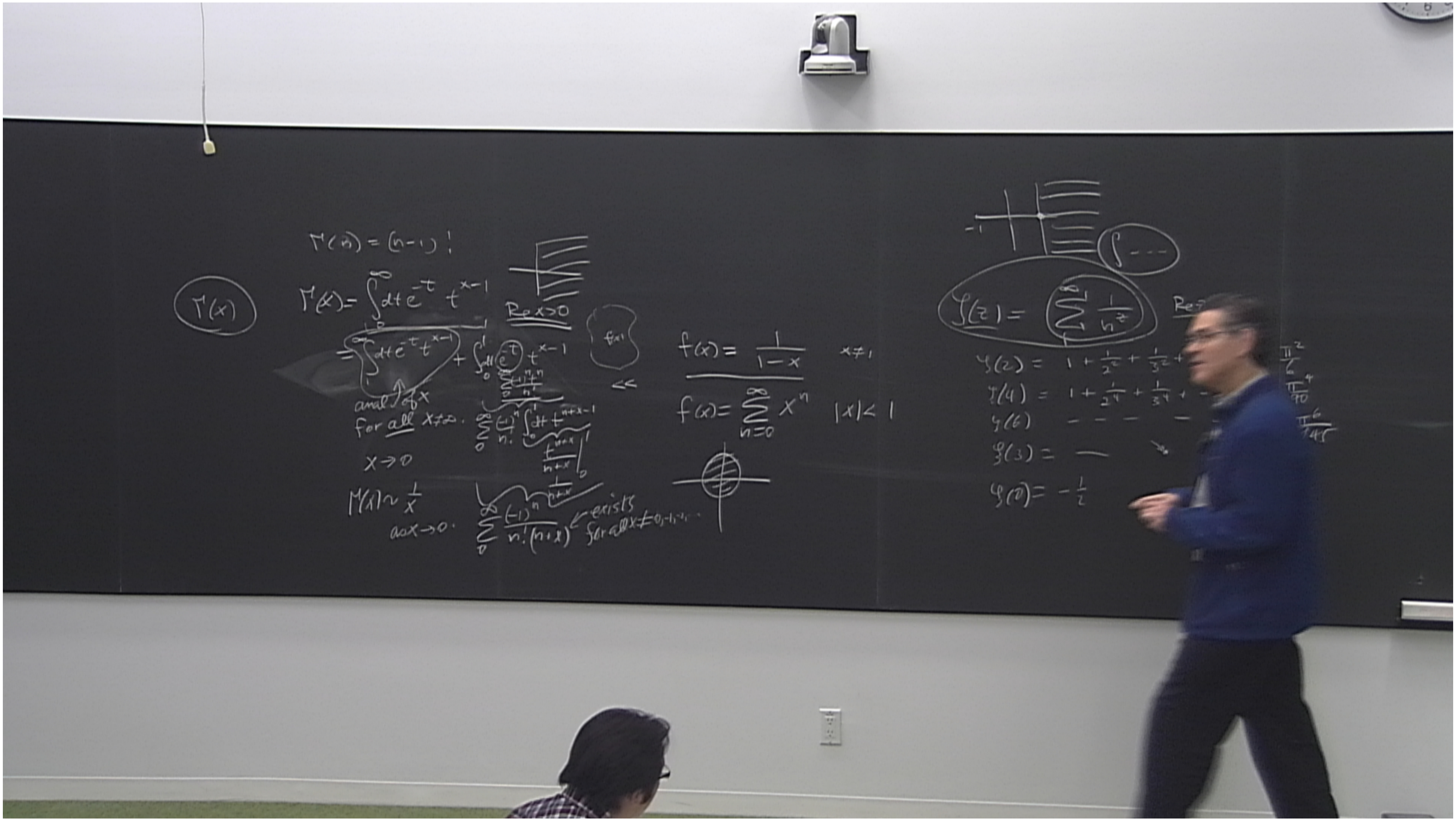
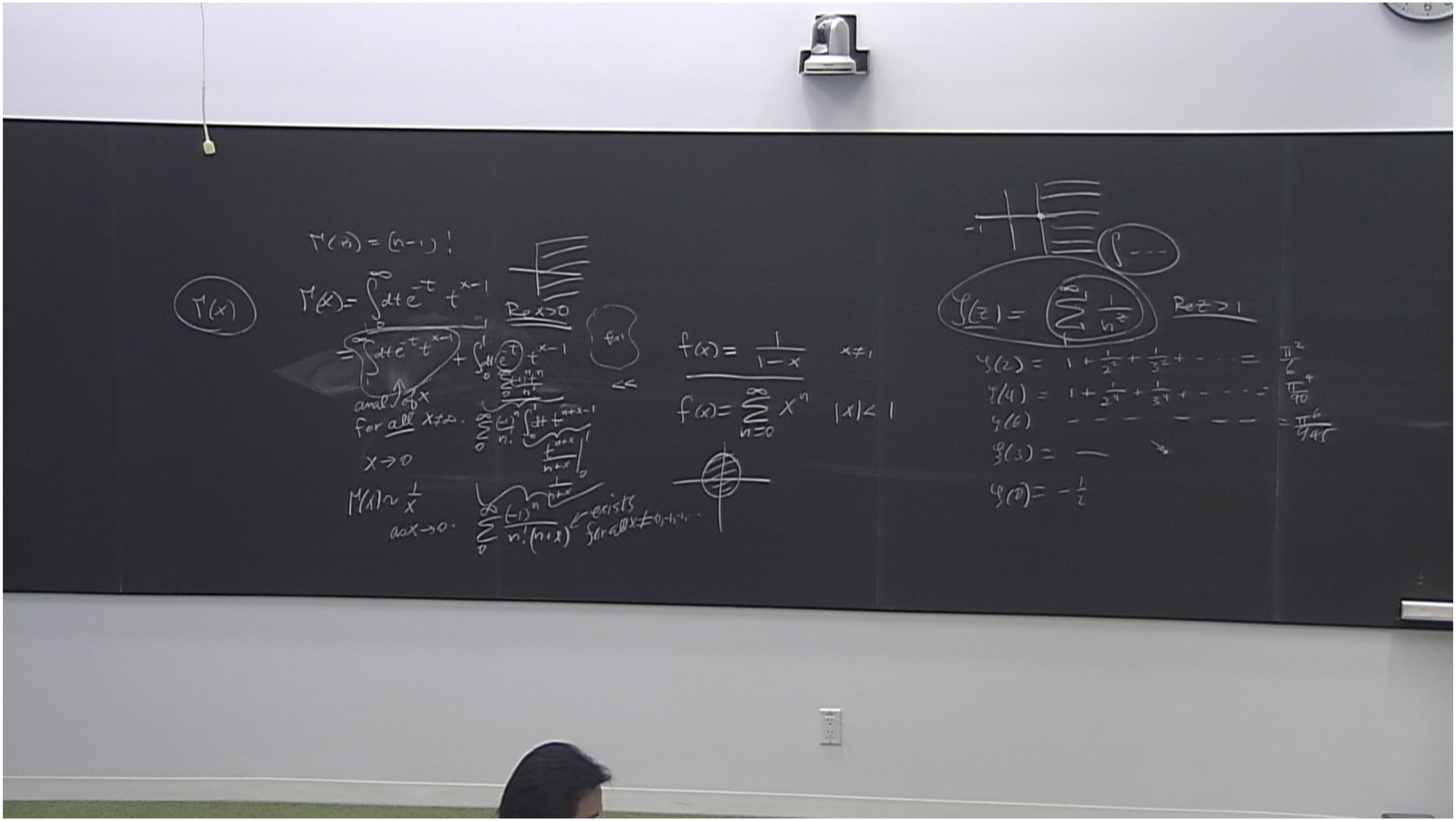
Re x>0
anal. of x
for all x≠0.
x→0
Γ(x) ~ 1/x
as x→0.
exists
for all x≠0,-1,-2,...
f(x) = 1/(1-x)
x≠1
f(x) = Σ_{n=0}^{∞} x^n
|x|< 1
ζ(z) =
Re z > 1
ζ(2)
ζ(4)
ζ(6)
ζ(3)
π²/6
π⁴/90
π⁶/945


Γ(n) = (n-1)!
Re x>0
f(x) = 1/(1-x)
x≠1
|x| < 1
exists
ζ(0) = -1/2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as x→0.
exists
for all x≠0,−1,−2…
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x≠1
f(x) = Σ x^n
n=0
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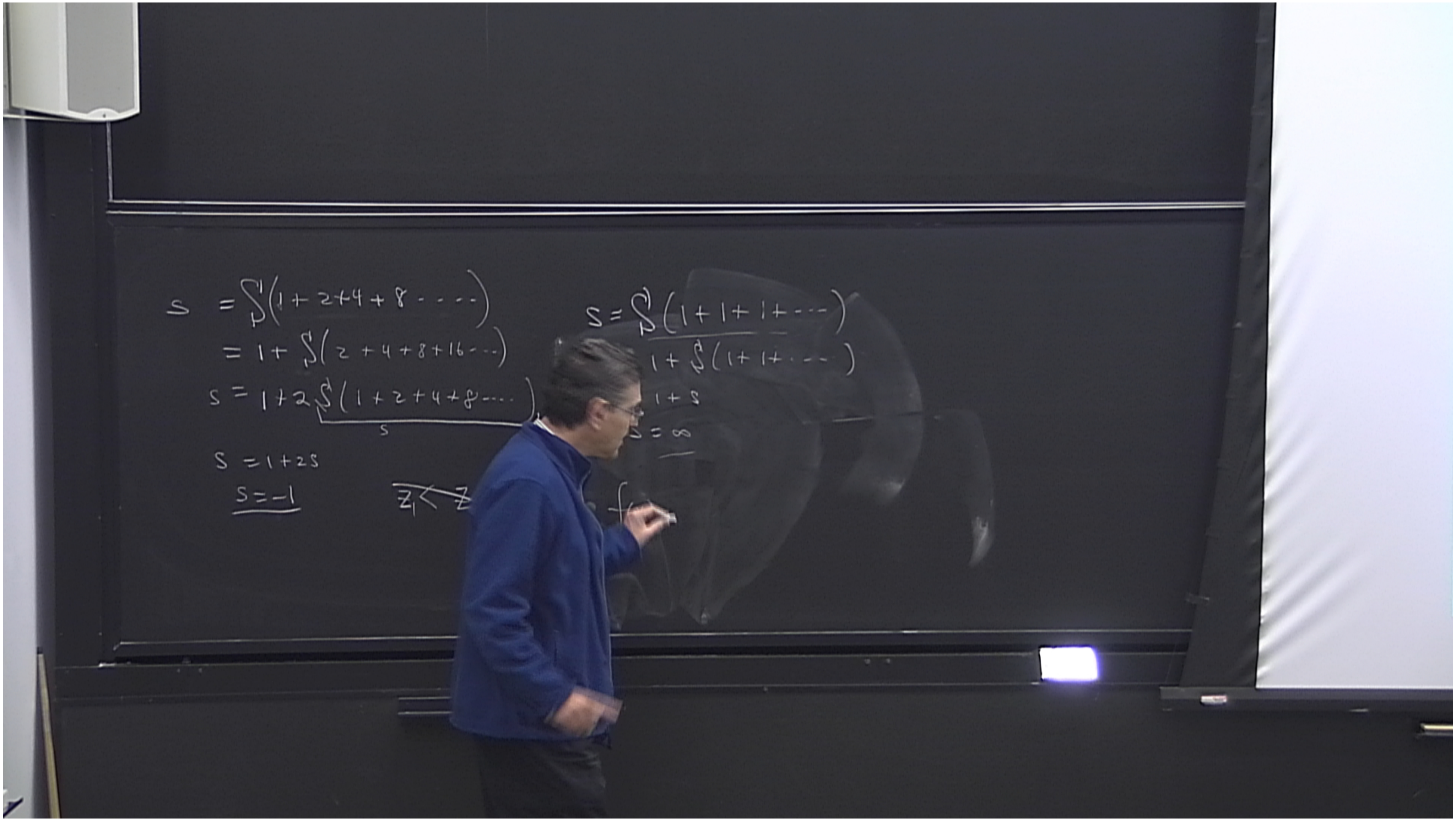
ζ(6) = π⁶/945
ζ(3) =
ζ(0) = −1/2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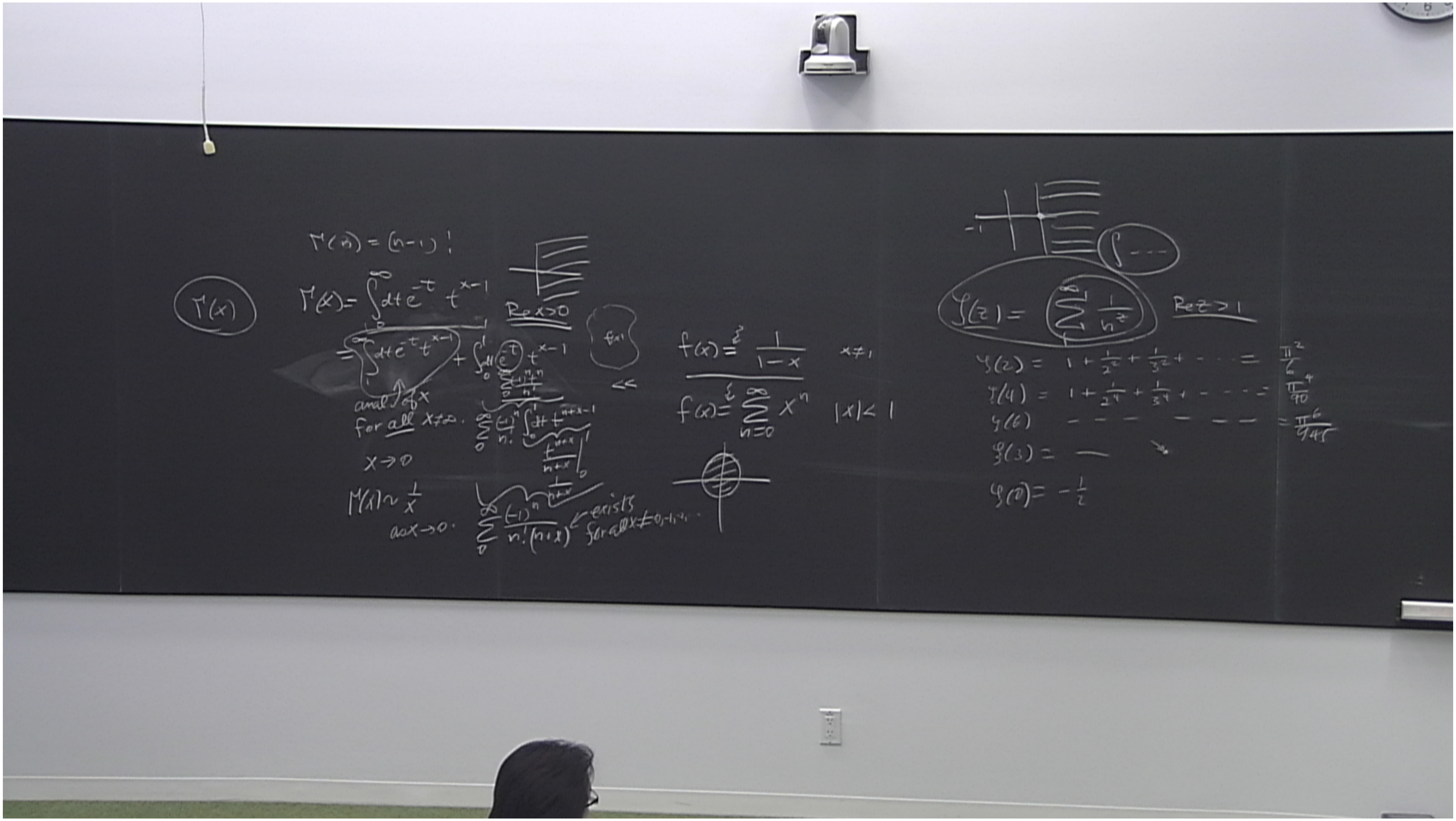
= 1+S(2+4+8+16...)
s = 1+2S(1+2+4+8....)
s
s = 1+2s
s = -1
s = S(1+1+1+...)
1+S(1+1+...)
1+s
s = ∞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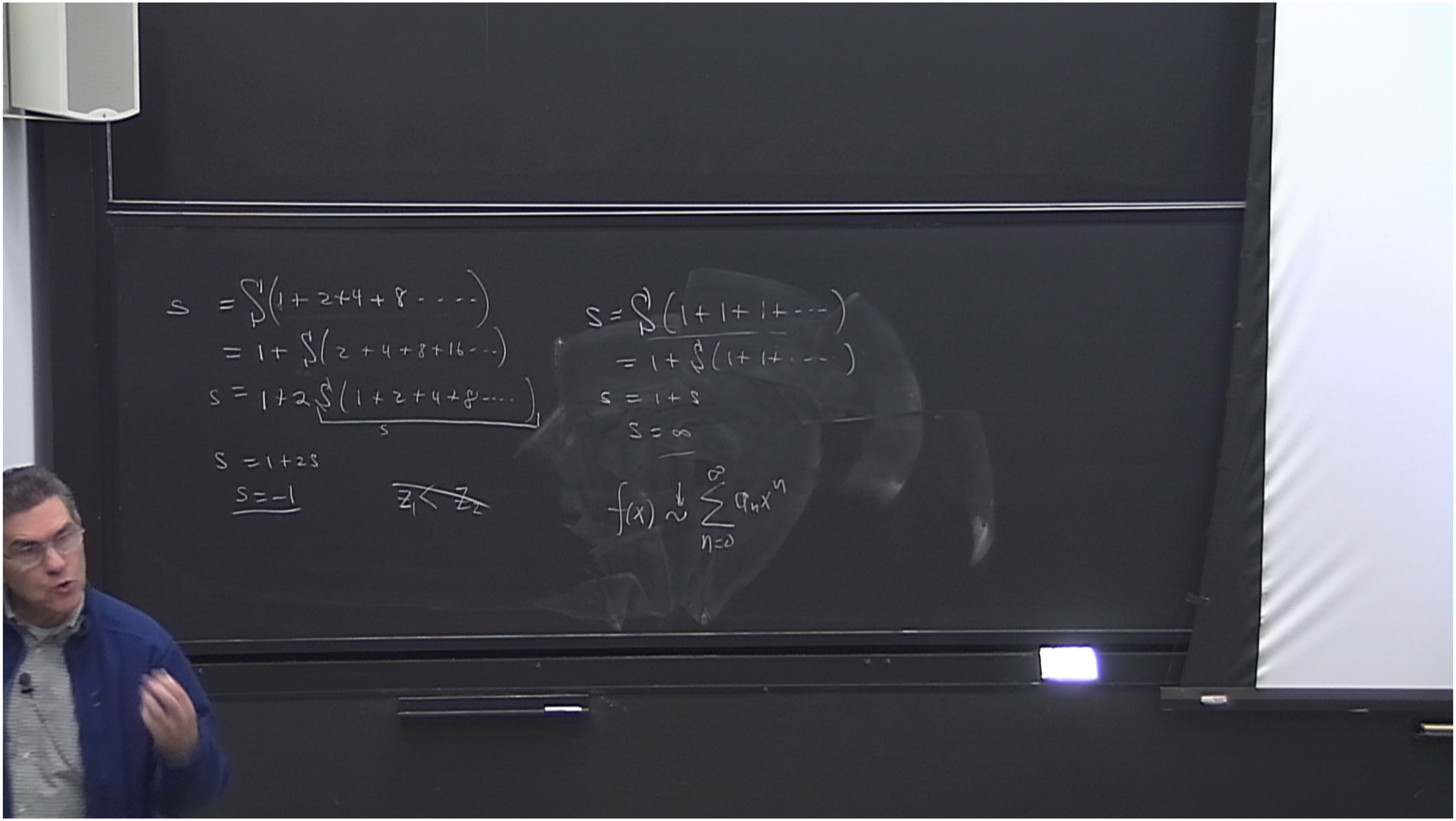
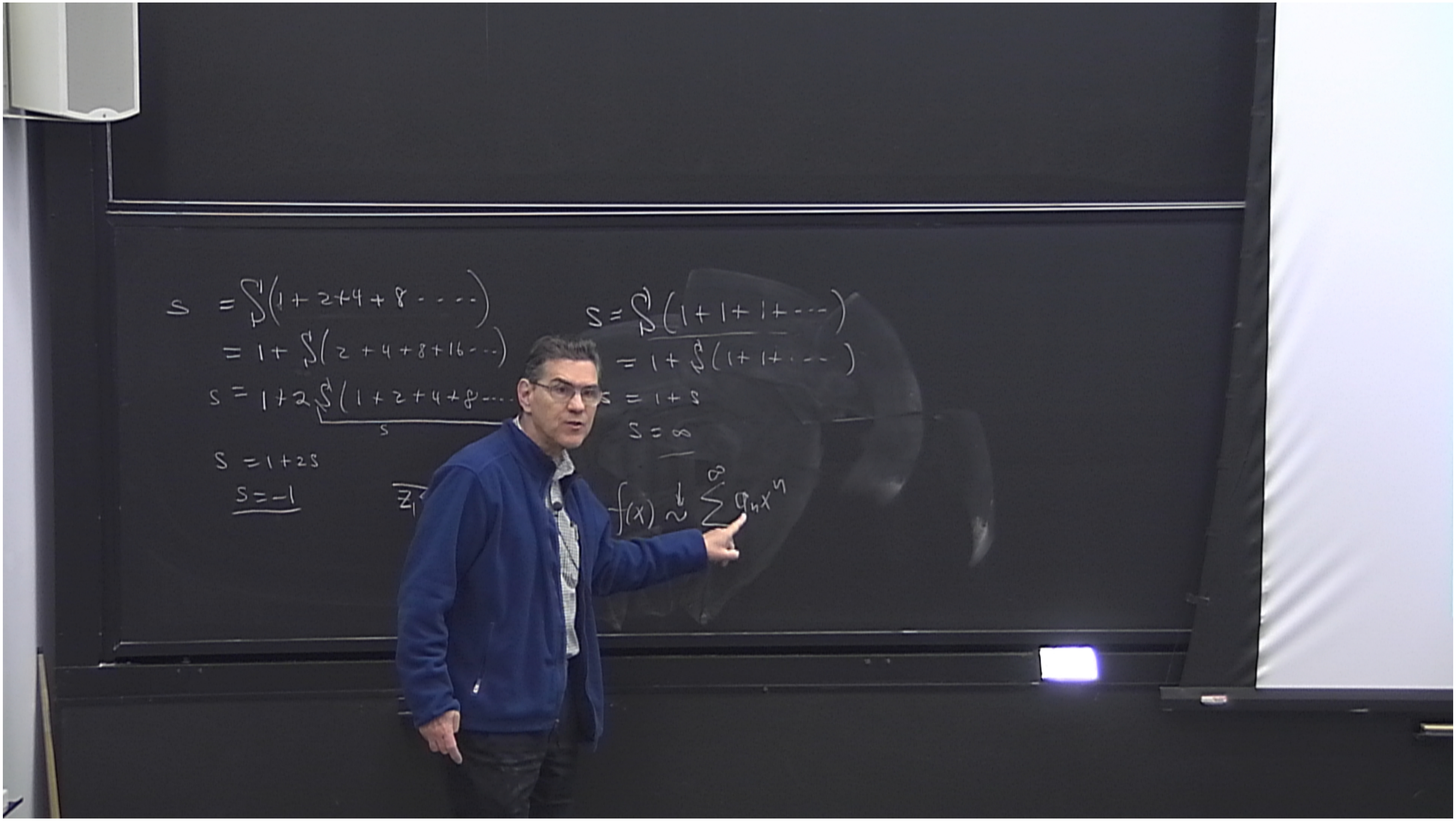
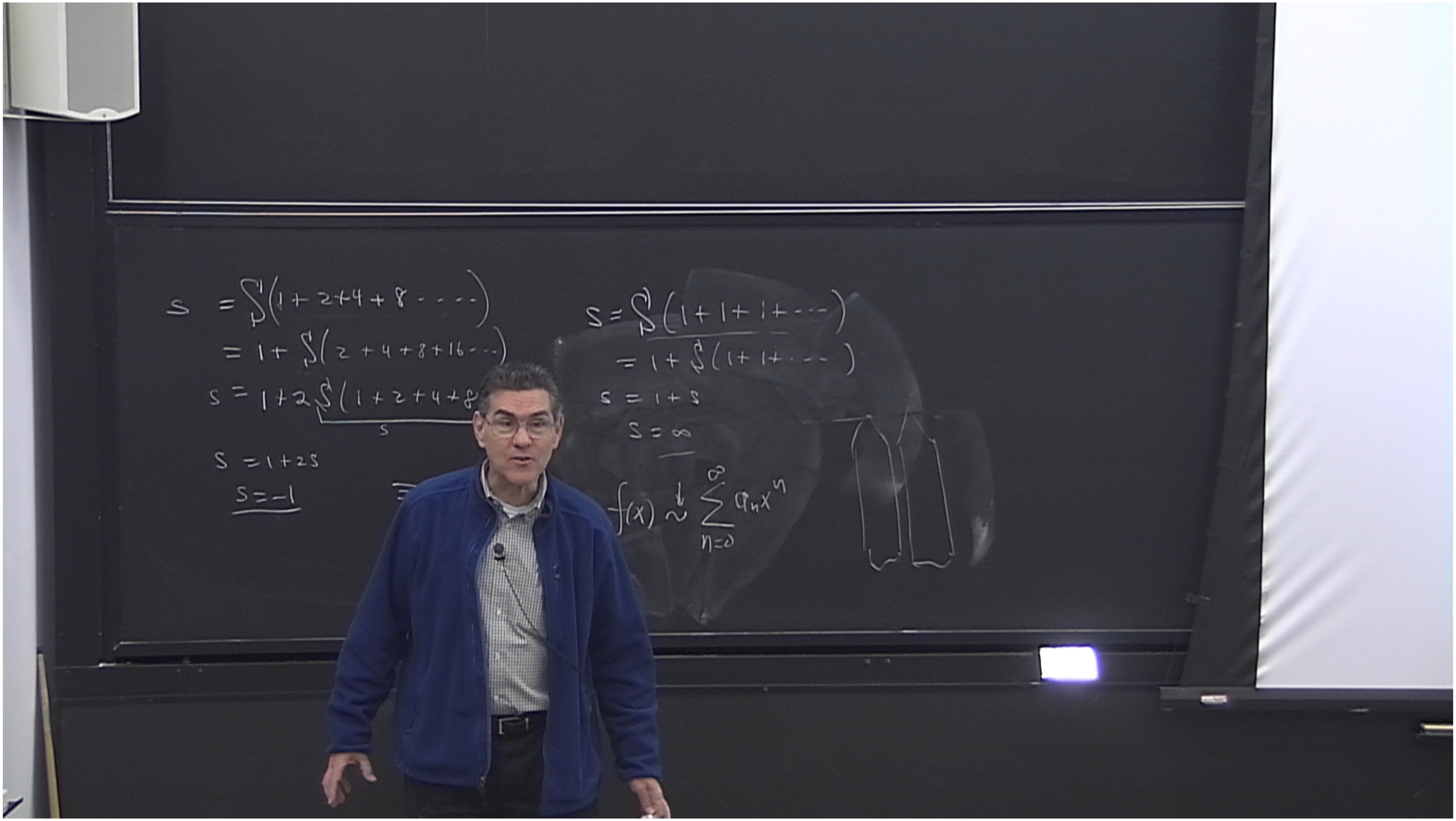
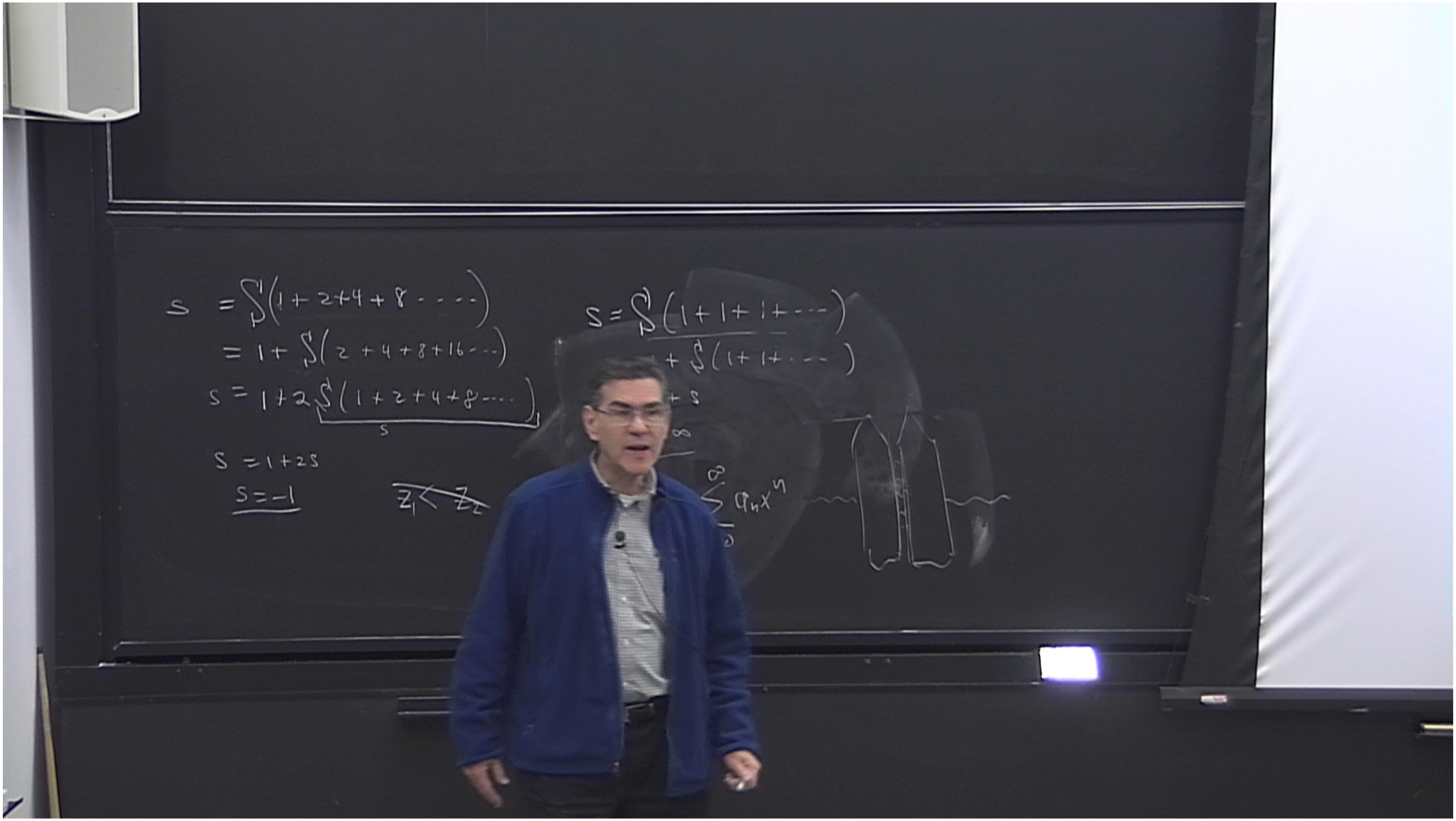
Γ(x)
Γ(x) = ∫ dt e^{-t} t^{x-1}
Re x > 0
anal. f of x
for all x ≠ 0.
x → 0
Γ(x) ~ 1/x
as x → 0.
exists
for all x ≠ 0, −1, −2, ...
f(x) = 1/(1−x)
x ≠ 1
f(x) = Σ_{n=0}^{∞} x^n
|x| < 1
ζ(z) = Σ_1^∞ 1/n^z
Re z > 1
ζ(2) = 1 + 1/2² + 1/3² + ... = π²/6
ζ(4) = 1 + 1/2⁴ + 1/3⁴ + ... = π⁴/90
ζ(6) = π⁶/945
ζ(3) =
ζ(0) = −1/2

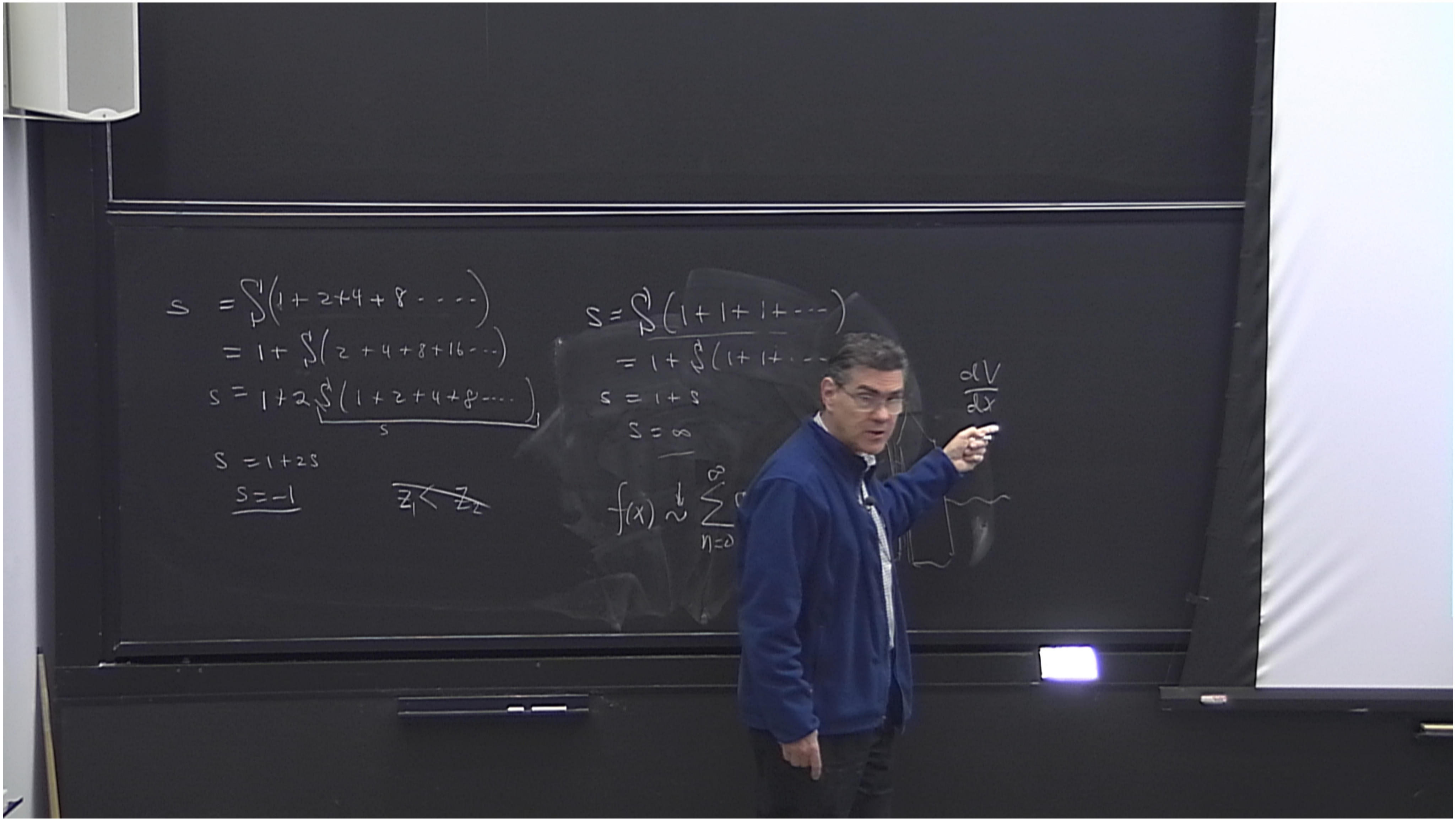

S = S(1+2+4+8....)
= 1+S(2+4+8+16...)
S = 1+2S(1+2+4+8....)
S
S = 1+2S
S = -1
Z1 < Z2
S = S(1+1+1+...)
= 1+S(1+1+...)
S = 1+S
S = ∞
f(x) ~ Σ n=0 ∞ anxn

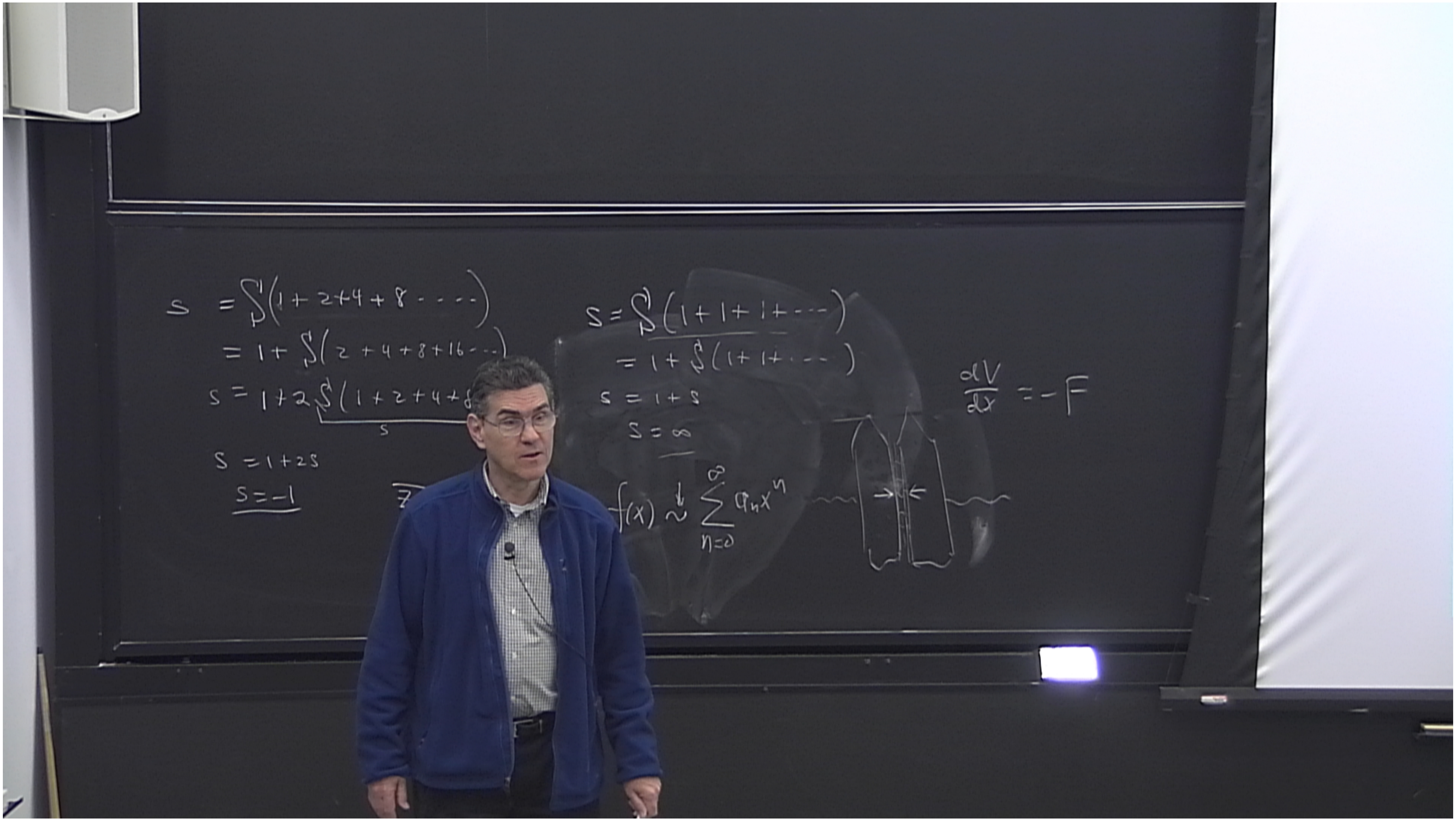


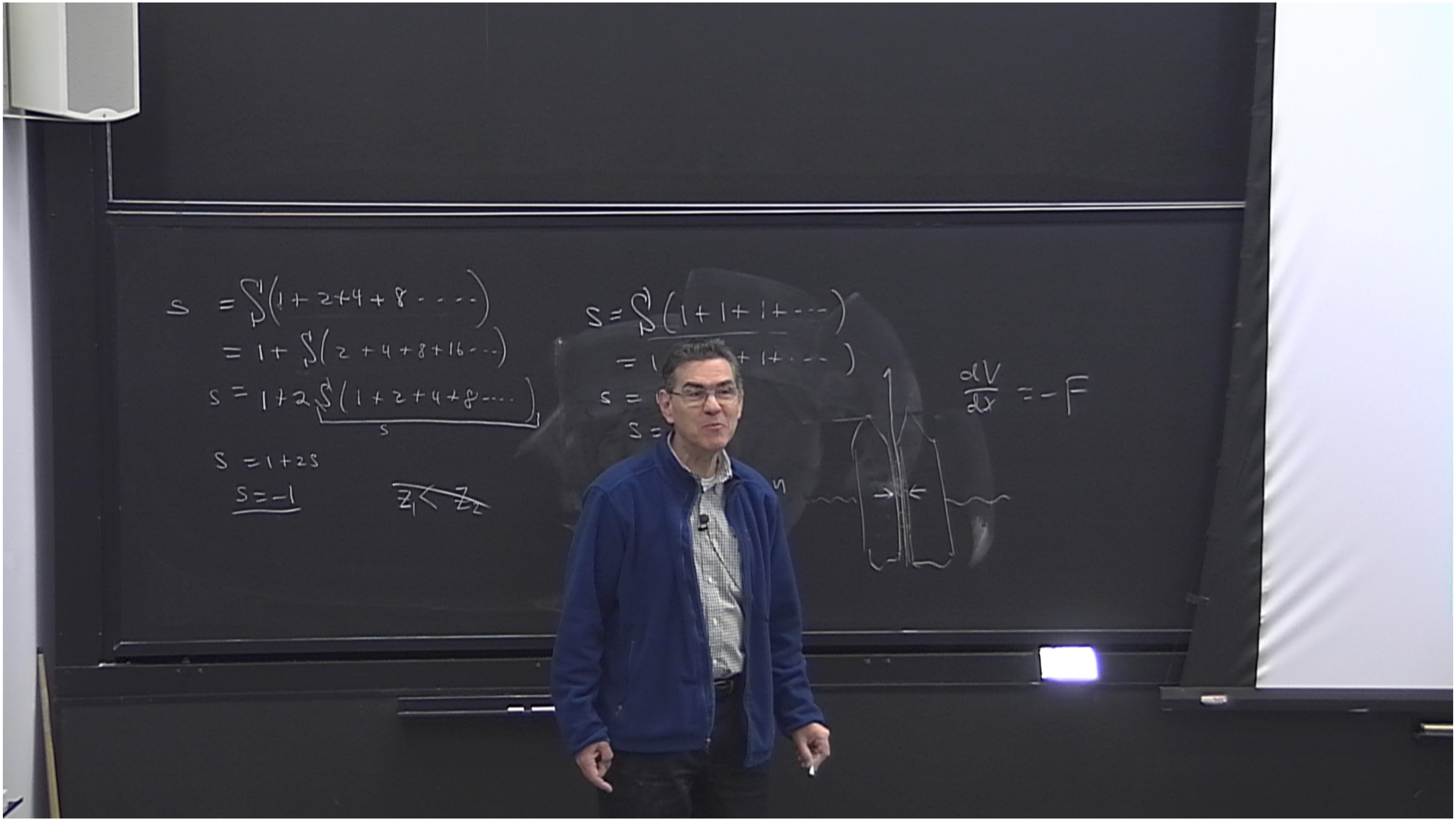
s = S(1+2+4+8....)
= 1+S(2+4+8+16...)
s = 1+2S(1+2+4+8....
s = 1+2s
s = -1
S = S(1+1+1+...)
= 1+S(1+1+...)
s = 1+s
S = ∞
f(x) ~ Σ a_n x^n


S = S(1+2+4+8....)
= 1+S(2+4+8+16...)
S = 1+2S(1+2+4+8
S = 1+2S
S = -1
S = S(1+1+1+...)
= 1+S(1+1+...)
S = 1+S
S = ∞
f(x) ~ Σ_{n=0}^∞ a_n x^n


s = S(1 + 2 + 4 + 8 ....)
= 1 + S(2 + 4 + 8 + 16 ...)
s = 1 + 2 S(1 + 2 + 4 + 8 ....)
s = 1 + 2s
s = -1
S = S(1 + 1 + 1 + ...)


S = S(1+2+4+8....)
= 1+ S(2+4+8+16...)
s = 1+2S(1+2+4+8....)
s
S = 1+2S
S = -1
S = S(1+1+1+...)
= 1+ S(1+1+...
S = 1+S
S = ∞
f(x) ~ ∑ n=0 ∞
dV/dx


dV/dx = −F



s = S(1+2+4+8 - - - -)
= 1+ S(2+4+8+16 - -)
s = 1+2S(1+2+4+8 - - - -)
s
s = 1+2s
s = -1
$dV/dx = -F$

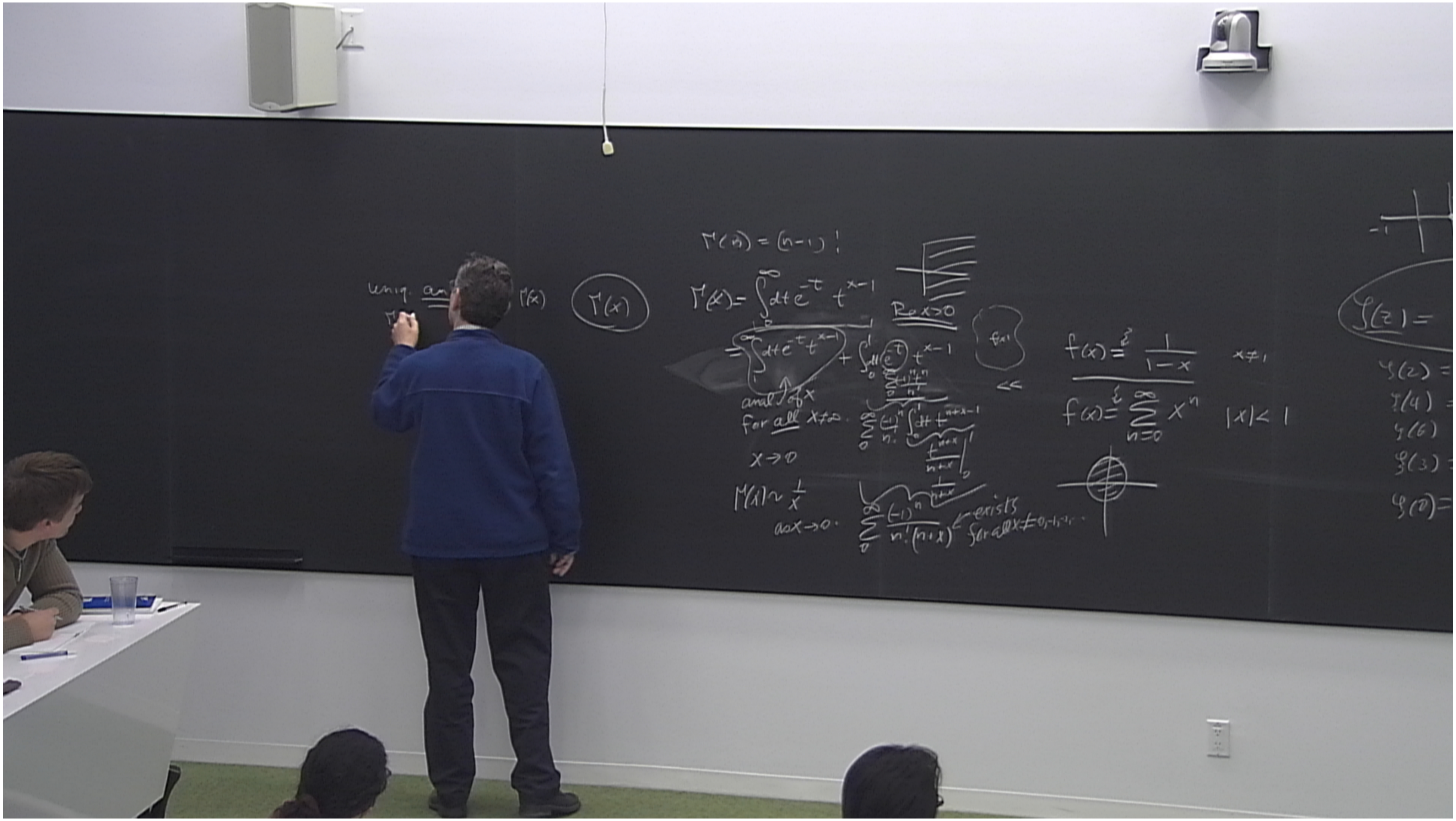
Zeta summation

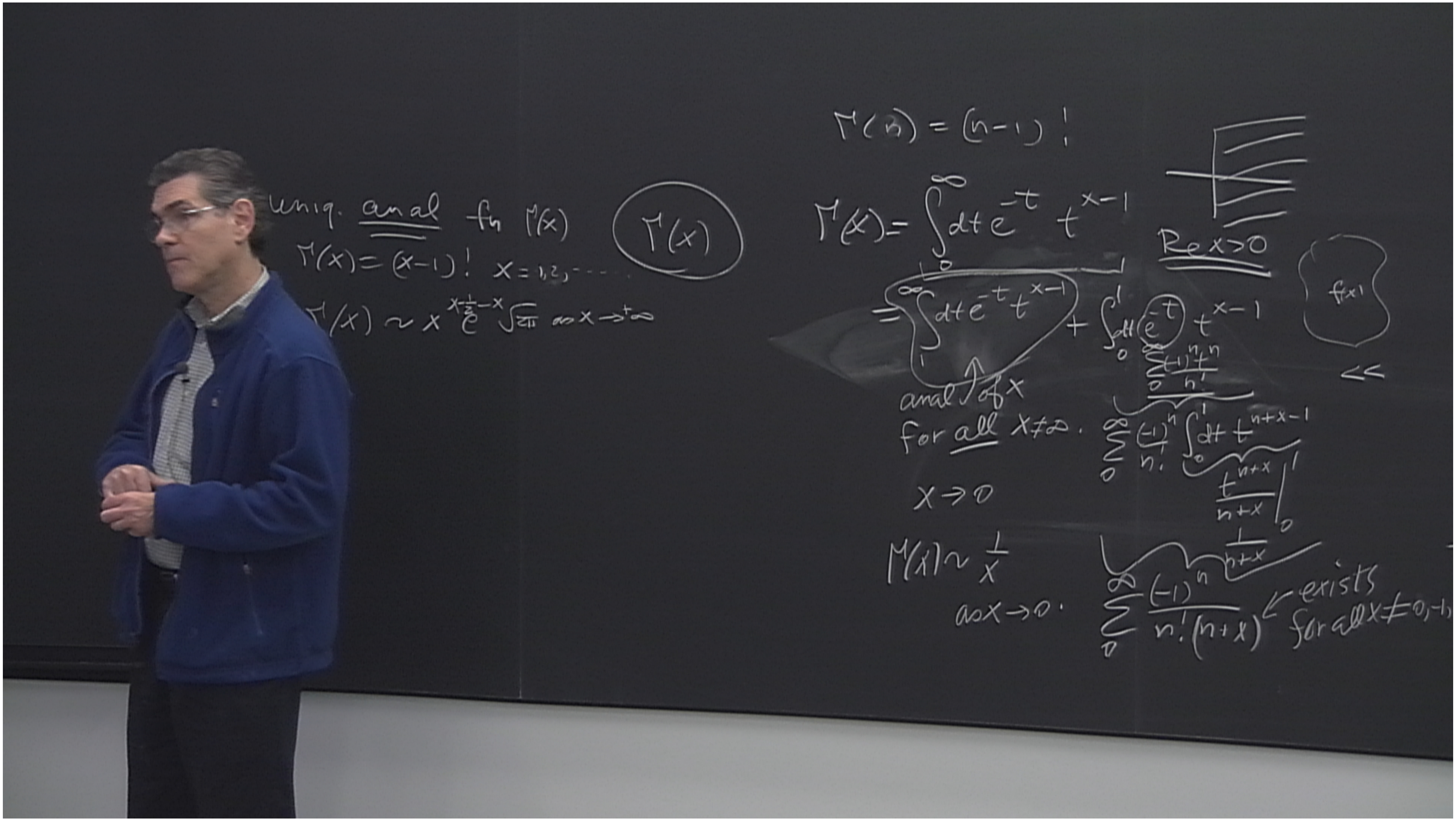
$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

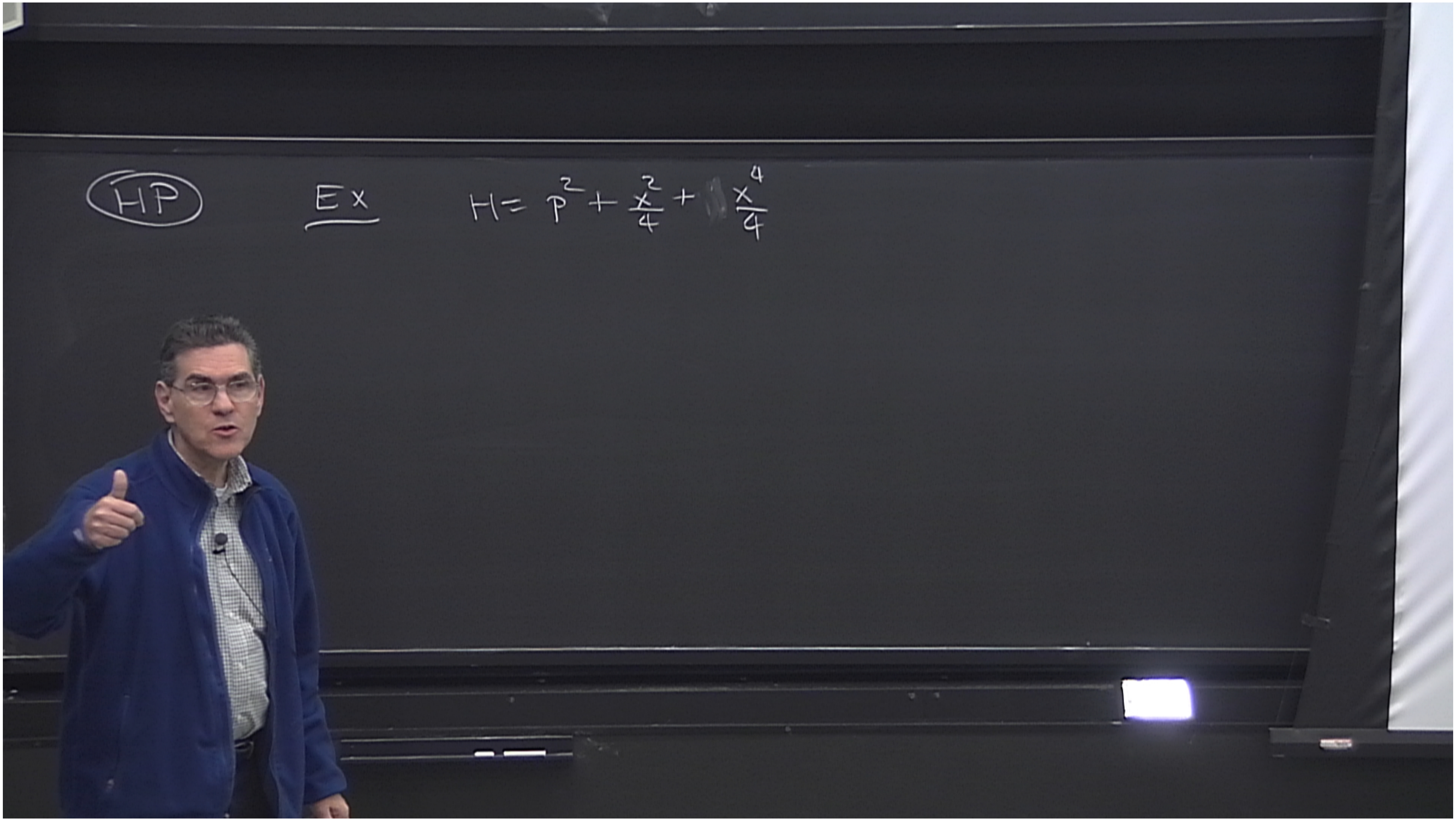
$$\zeta(1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots = \infty$$

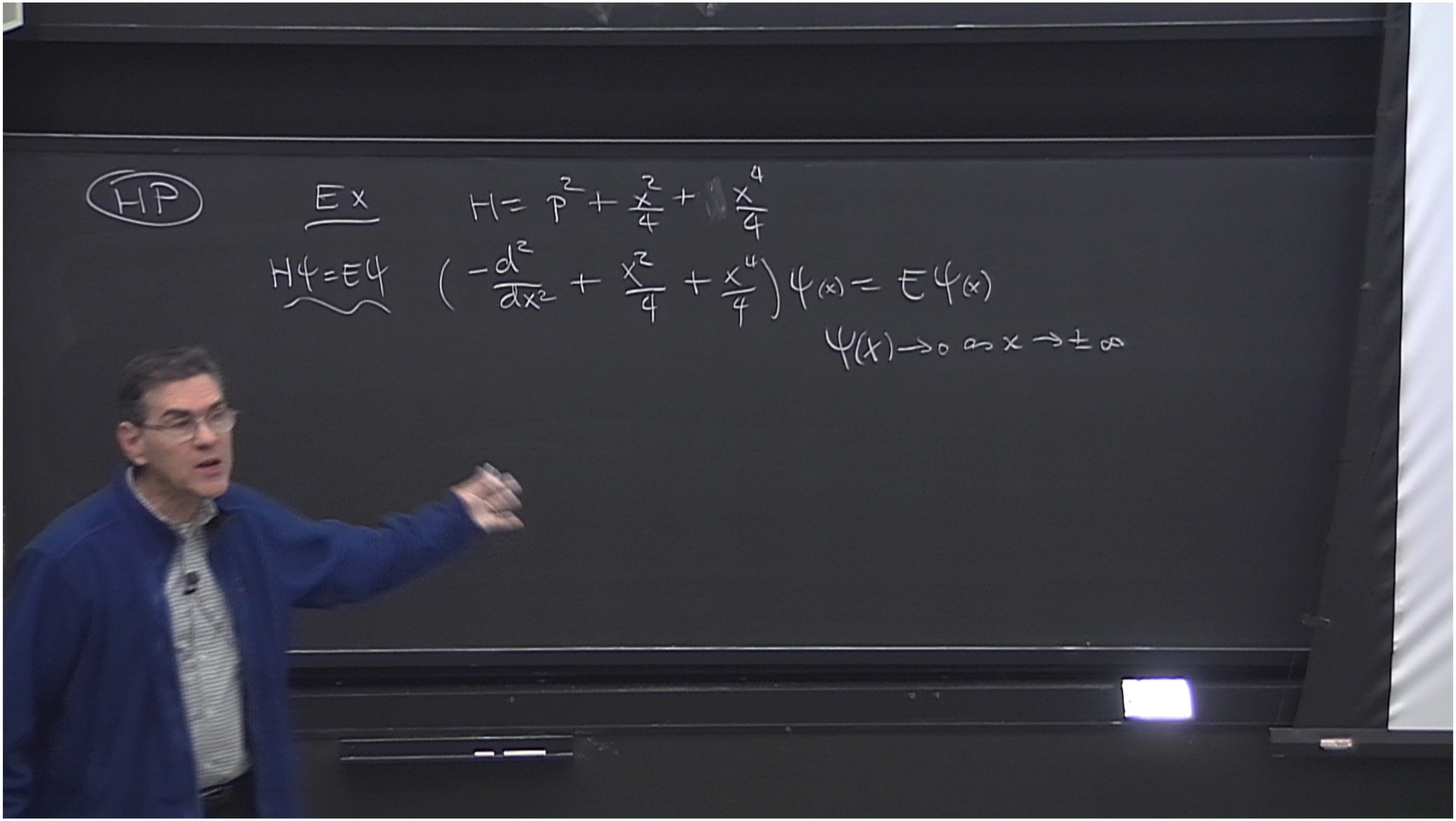
$$\zeta(0) = 1 + 1 + 1 + 1 + 1\ldots = -\frac{1}{2}$$

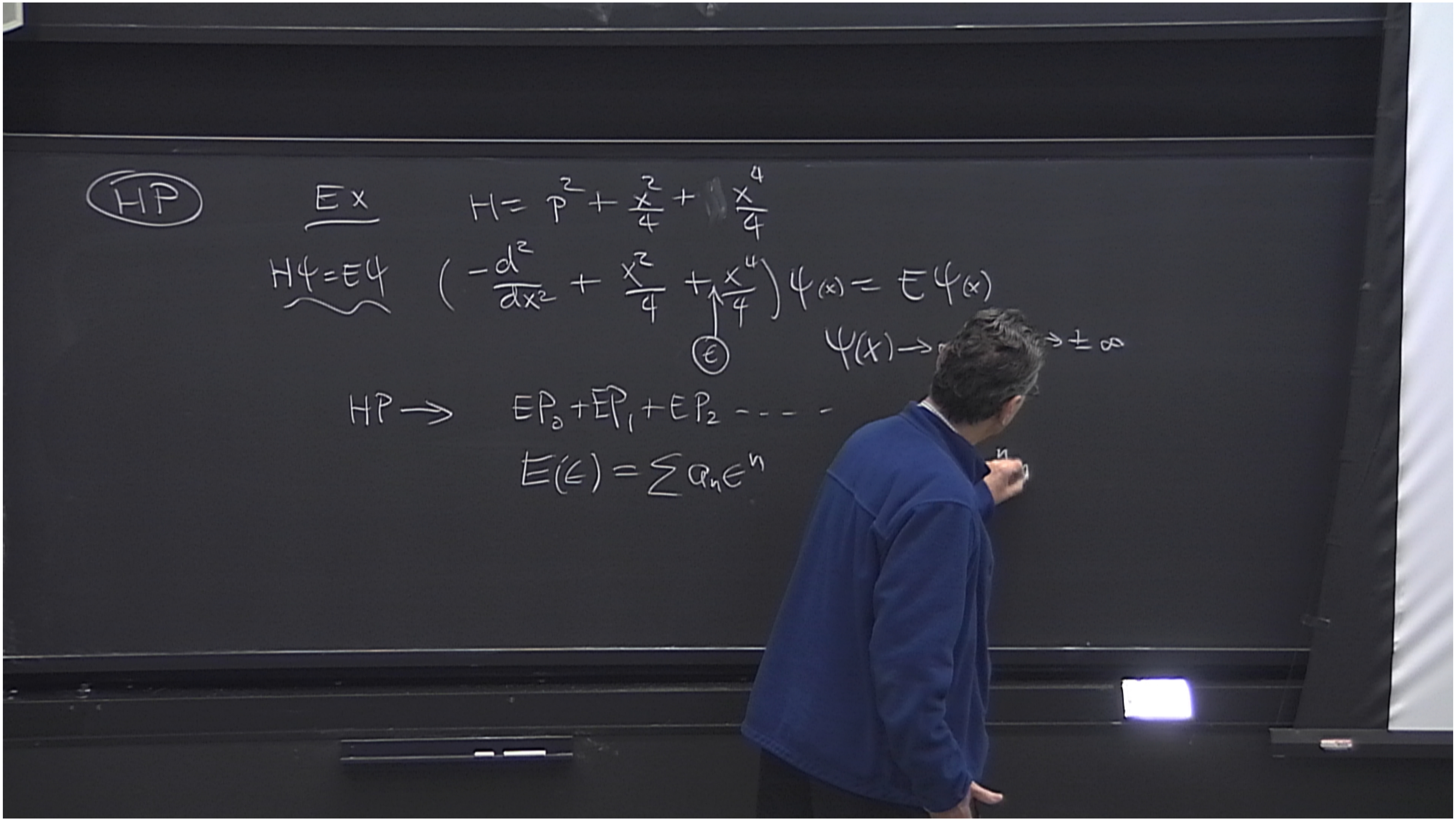
$$\zeta(-1) = 1 + 2 + 3 + 4 + 5 + 6\ldots = -\frac{1}{12}$$

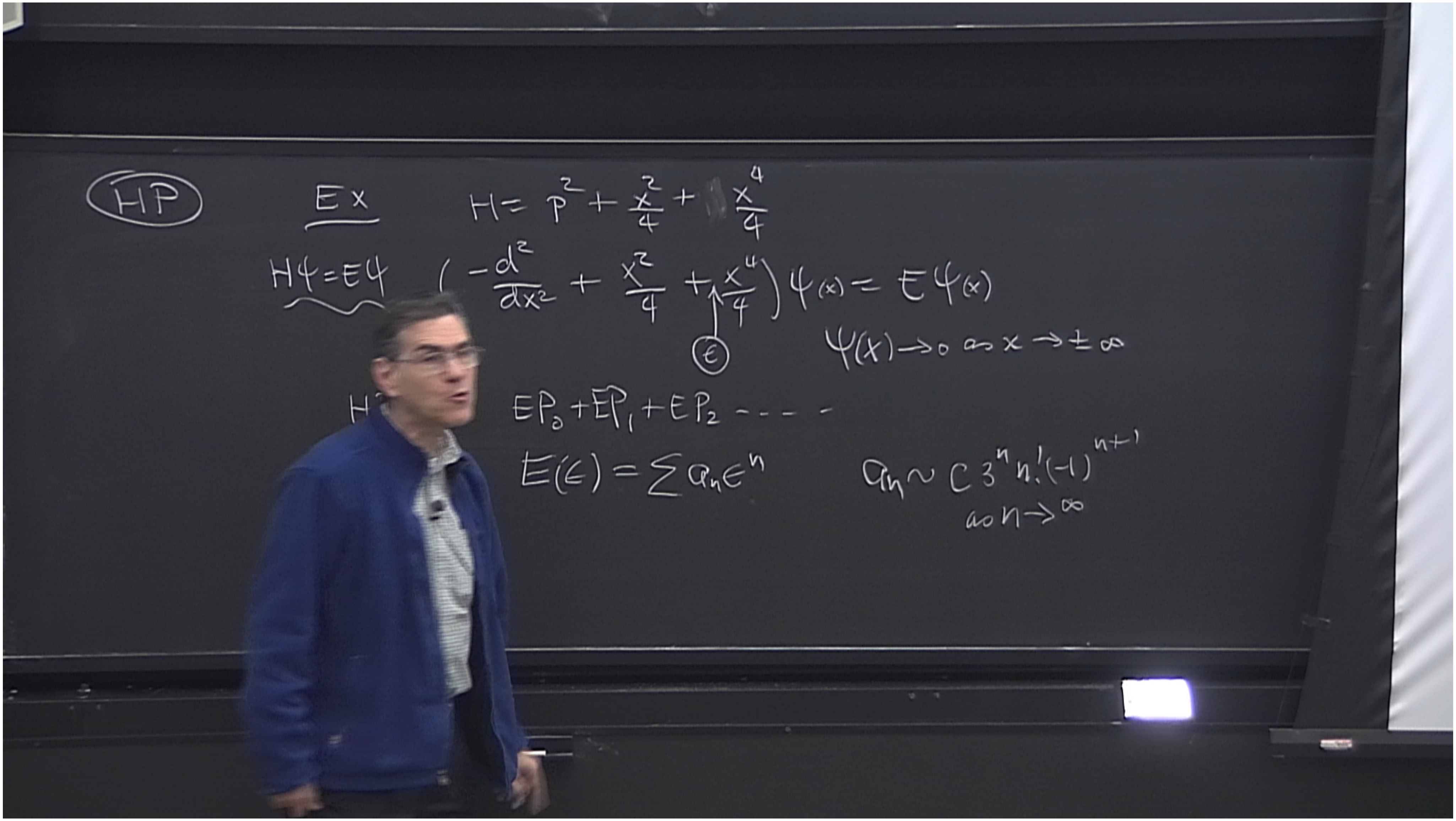


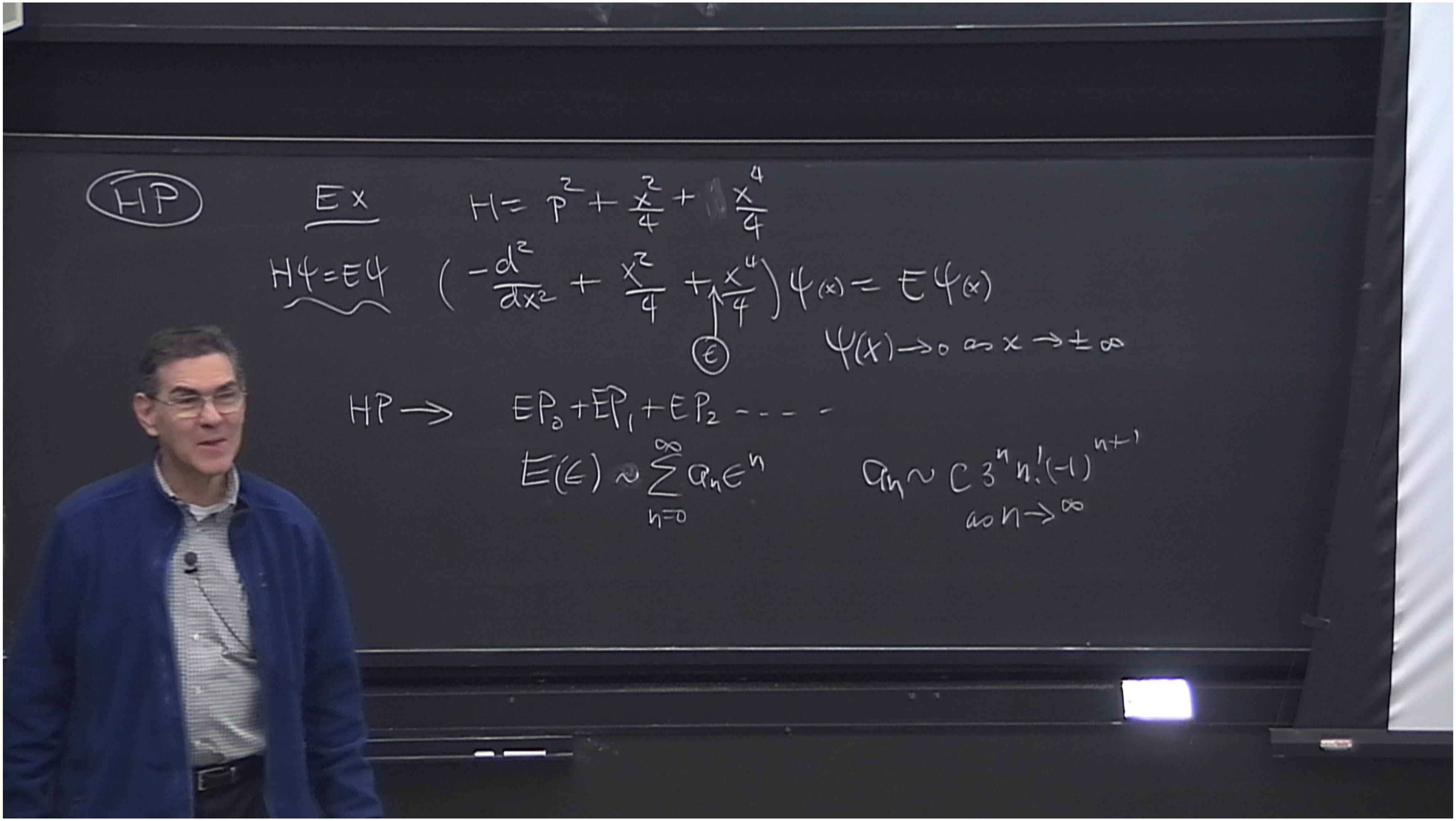

uniq. anal fn Γ(x)
Γ(x) = (x−1)! x = 1, 2, …
Γ(x) ~ x^(x−½) e^(−x) √2π as x → +∞
Γ(x)
Γ(n) = (n−1)!
Γ(x) = ∫₀^∞ dt e^(−t) t^(x−1)
Re x > 0
= ∫₁^∞ dt e^(−t) t^(x−1) + ∫₀¹ dt e^(−t) t^(x−1)
anal of x
for all x ≠ ∞.
x → 0
Γ(x) ~ 1/x
as x → 0.
Σ₀^∞ (−1)ⁿ tⁿ / n!
Σ₀^∞ (−1)ⁿ/n! ∫₀¹ dt t^(n+x−1)
t^(n+x)/(n+x) |₀¹
1/(n+x)
Σ₀^∞ (−1)ⁿ / (n!(n+x))
exists for all x ≠ 0, −1,
f(x)
<<

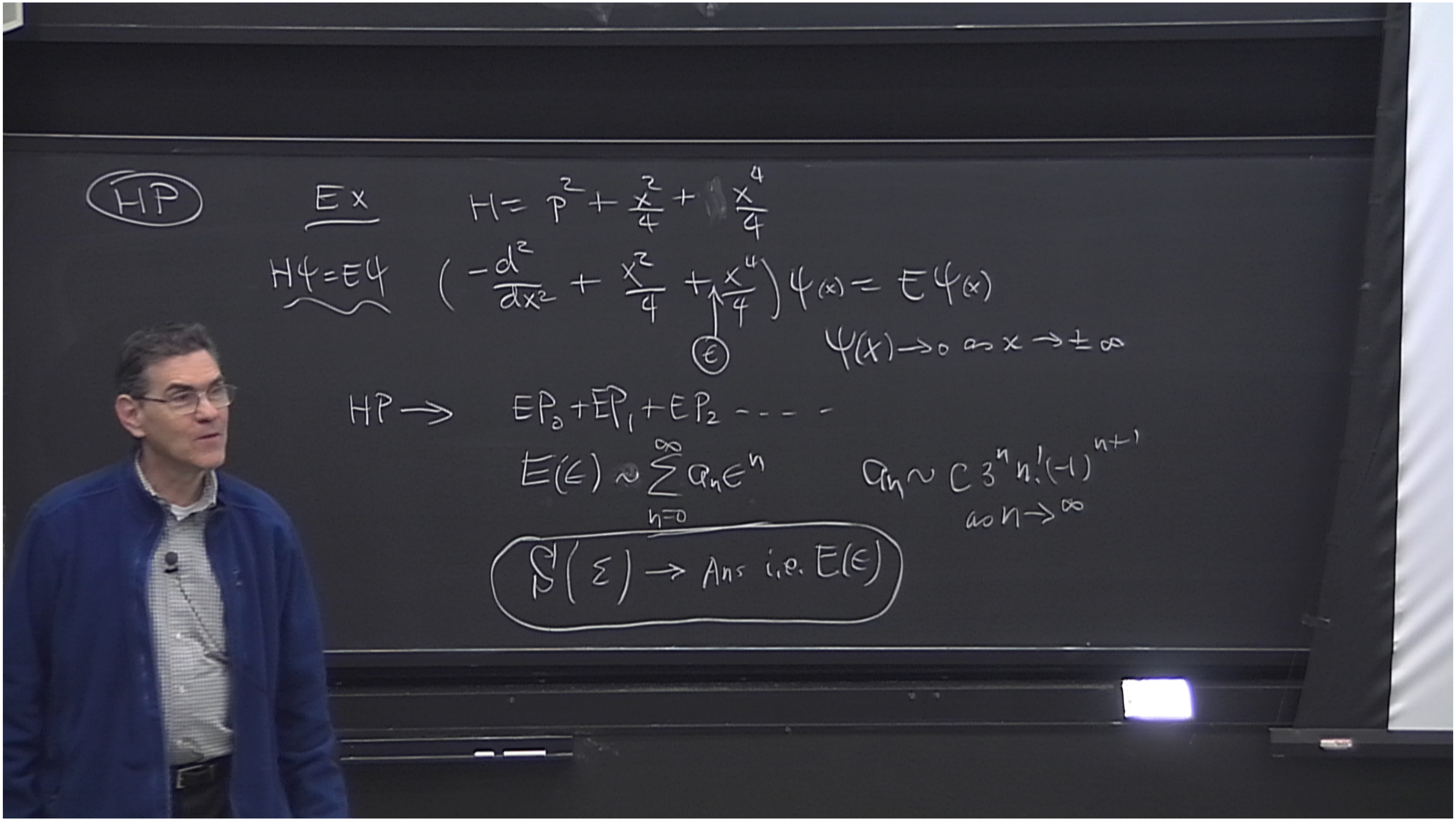

HP
Ex
$H = p^2 + \frac{x^2}{4} + \frac{x^4}{4}$

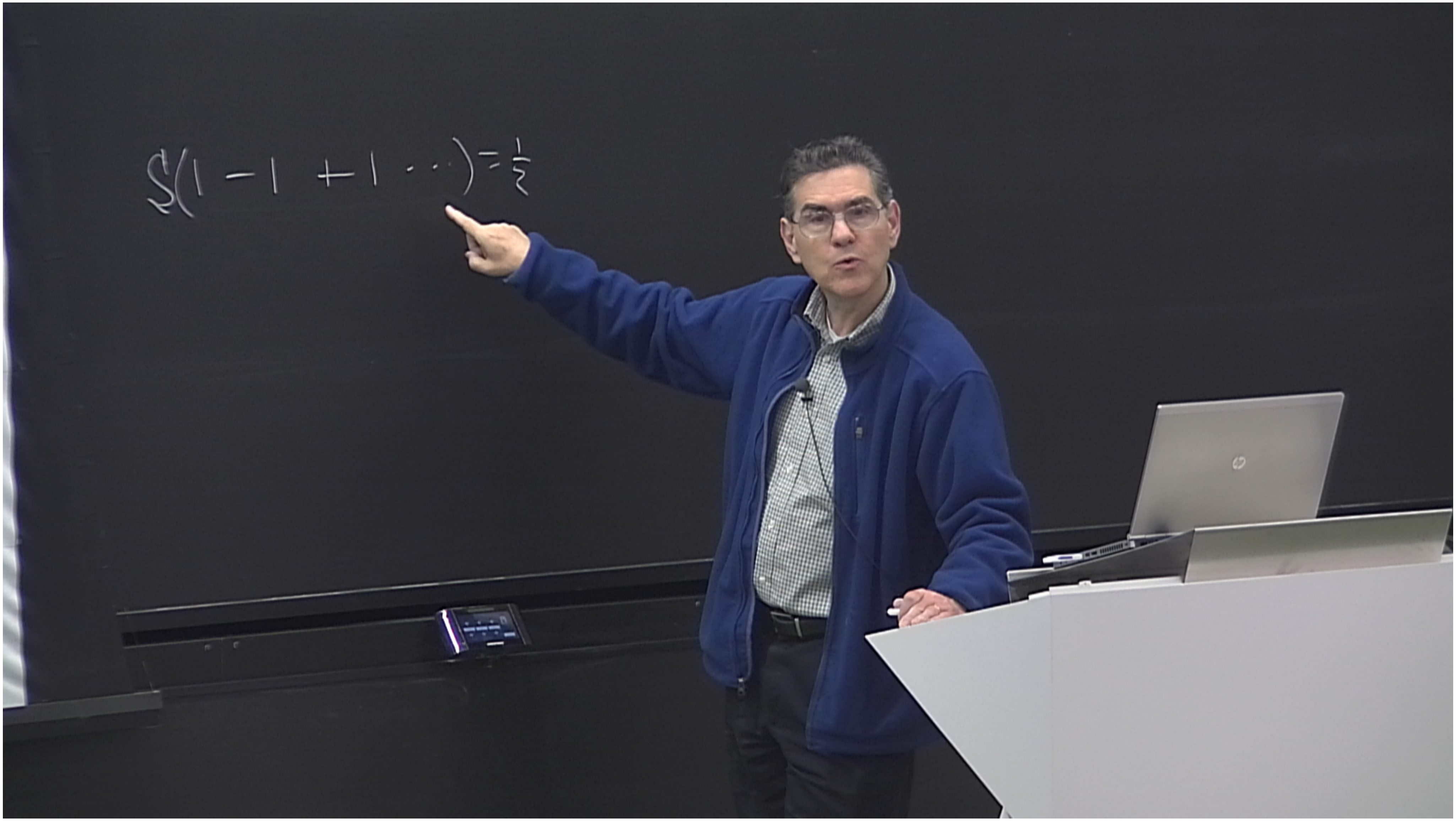

HP
Ex
$H = p^2 + \frac{x^2}{4} + \frac{x^4}{4}$
$H\psi = E\psi$
$\left(-\frac{d^2}{dx^2} + \frac{x^2}{4} + \frac{x^4}{4}\right)\psi(x) = E\psi(x)$
$\psi(x) \to 0$ as $x \to \pm\infty$


HP
Ex
$H = p^2 + \frac{x^2}{4} + \frac{x^4}{4}$
$H\psi = E\psi$
$\left(-\frac{d^2}{dx^2} + \frac{x^2}{4} + \epsilon\frac{x^4}{4}\right)\psi(x) = E\psi(x)$
$\psi(x) \to$
$\to \pm\infty$
$HP \to EP_0 + EP_1 + EP_2$ - - - -
$E(\epsilon) = \sum a_n \epsilon^n$


HP
Ex
$H = p^2 + \frac{x^2}{4} + \frac{x^4}{4}$
$H\psi = E\psi$
$\left(-\frac{d^2}{dx^2} + \frac{x^2}{4} + \frac{x^4}{4}\right)\psi(x) = E\psi(x)$
ϵ
$\psi(x) \to 0$ as $x \to \pm\infty$
$EP_0 + EP_1 + EP_2 - - - -$
$E(\epsilon) = \sum a_n \epsilon^n$
$a_n \sim C\, 3^n n! (-1)^{n+1}$
as $n \to \infty$

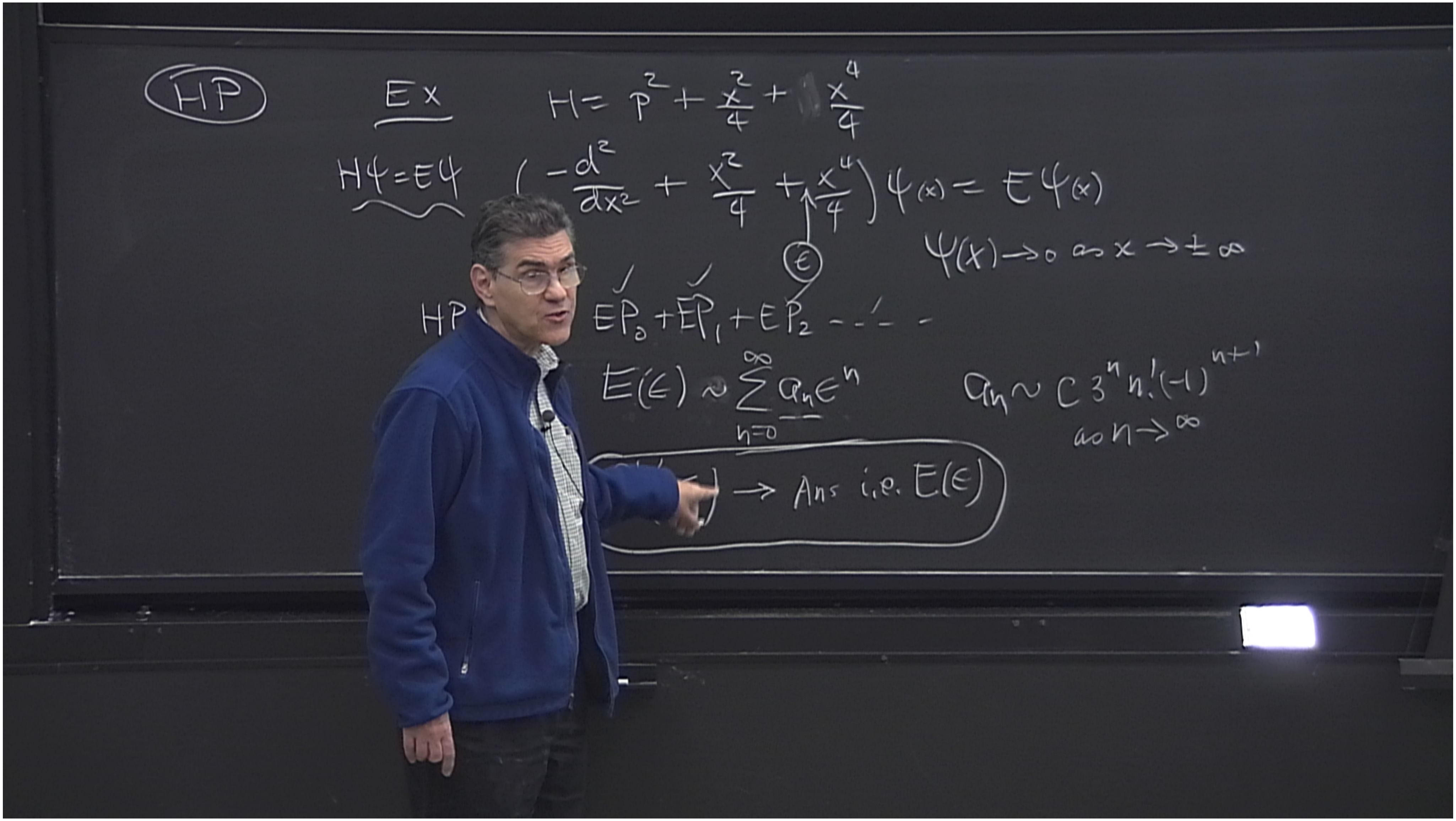

HP
Ex
$H = p^2 + \frac{x^2}{4} + \frac{x^4}{4}$
$H\psi = E\psi$
$\left(-\frac{d^2}{dx^2} + \frac{x^2}{4} + \frac{x^4}{4}\right)\psi(x) = E\psi(x)$
ϵ
$\psi(x) \to 0$ as $x \to \pm\infty$
$HP \to EP_0 + EP_1 + EP_2 \cdots$
$E(\epsilon) \sim \sum_{n=0}^{\infty} a_n \epsilon^n$
$a_n \sim C\, 3^n n! (-1)^{n+1}$
as $n \to \infty$

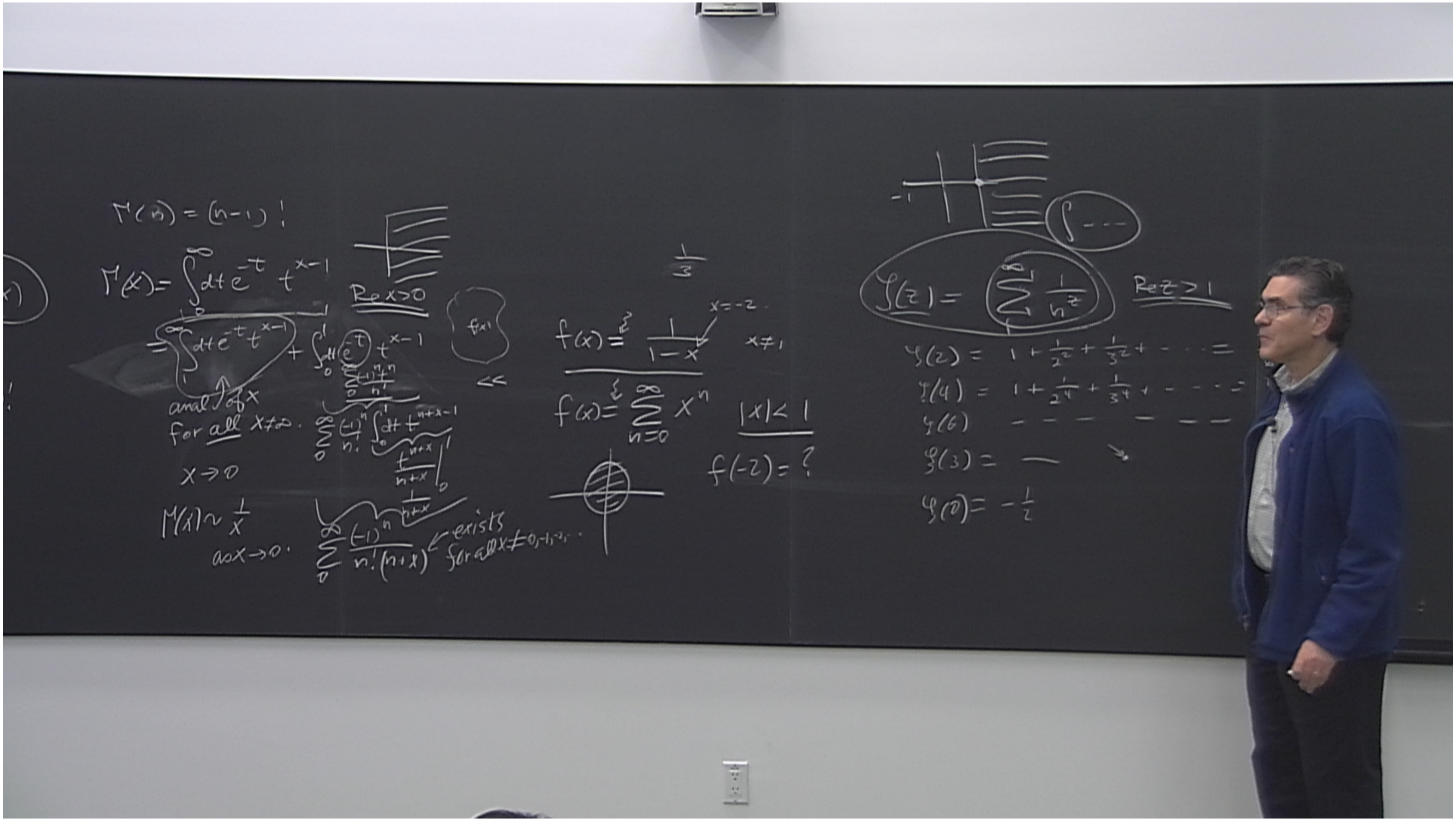

HP
Ex
H = p² + x²/4 + x⁴/4
Hψ = Eψ
(−d²/dx² + x²/4 + x⁴/4) ψ(x) = E ψ(x)
ε
ψ(x) → 0 as x → ±∞
HP → EP₀ + EP₁ + EP₂ - - - -
E(ε) ~ Σ_{n=0}^∞ aₙ εⁿ
aₙ ~ C 3ⁿ n! (−1)ⁿ⁺¹
as n → ∞
S(ε) → Ans i.e. E(ε)


S(1 − 1 + 1 ...) = 1/2

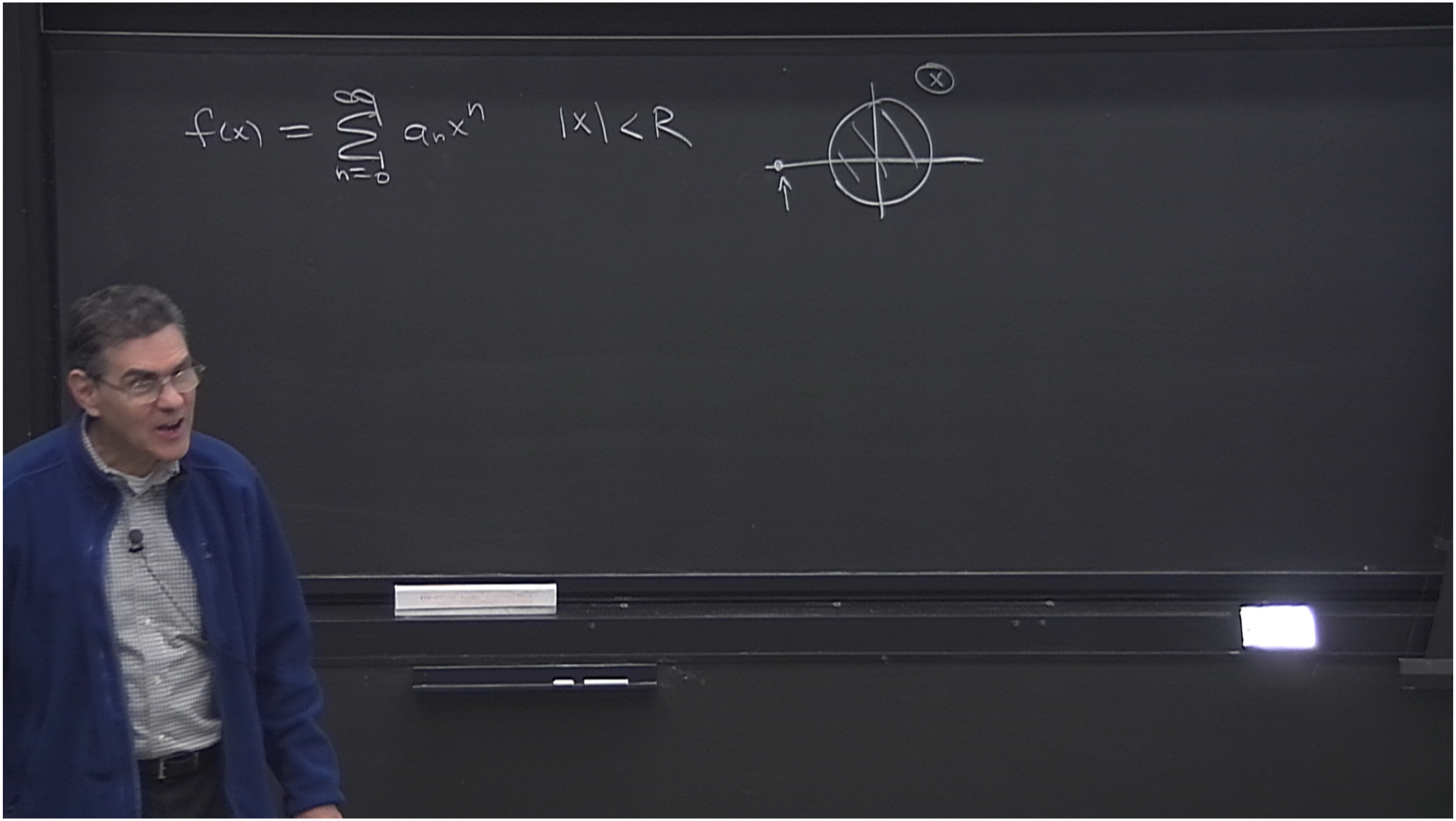
What if we don't know all the terms in the series??

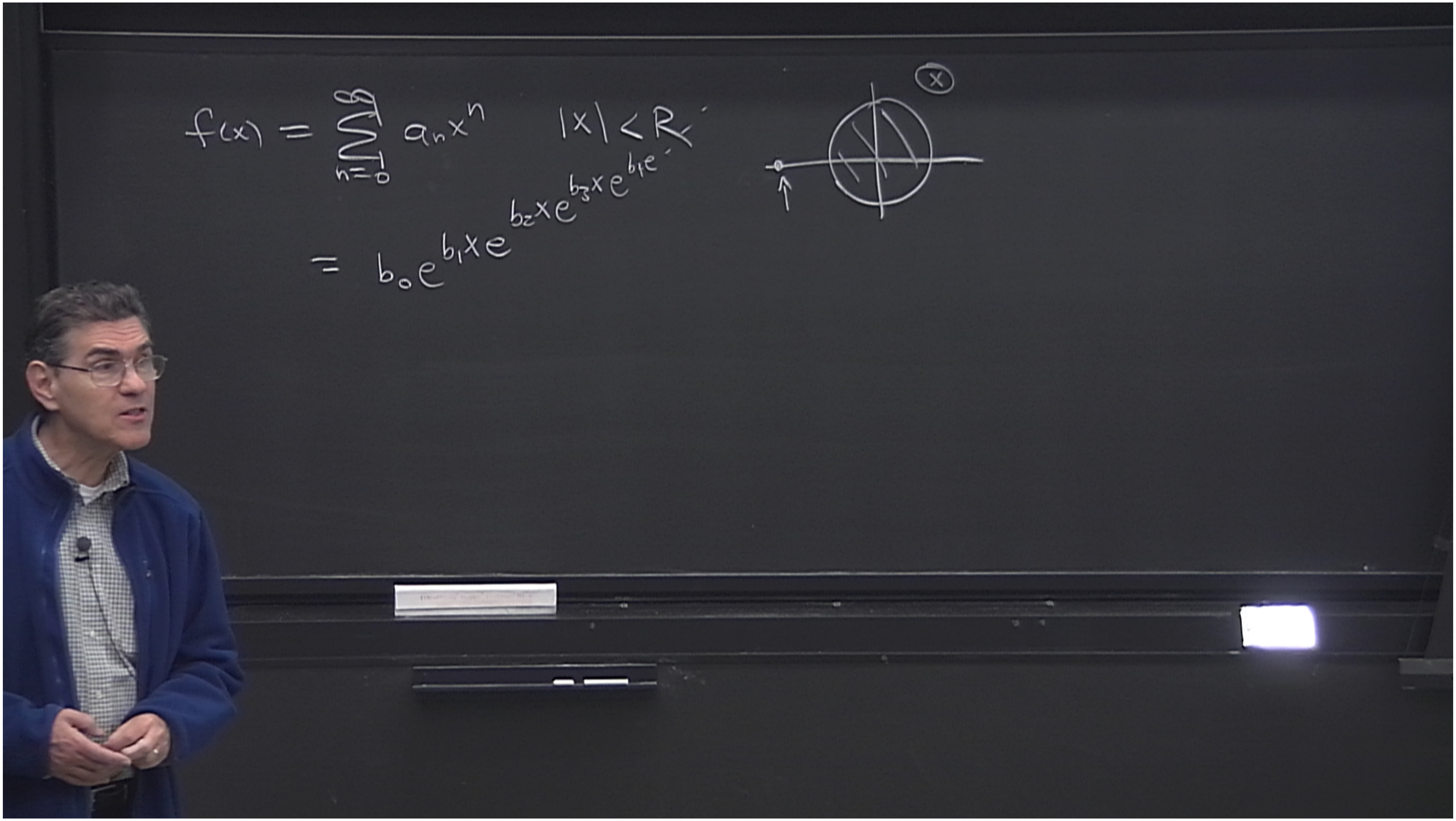
WE USE CONTINUED FUNCTIONS!

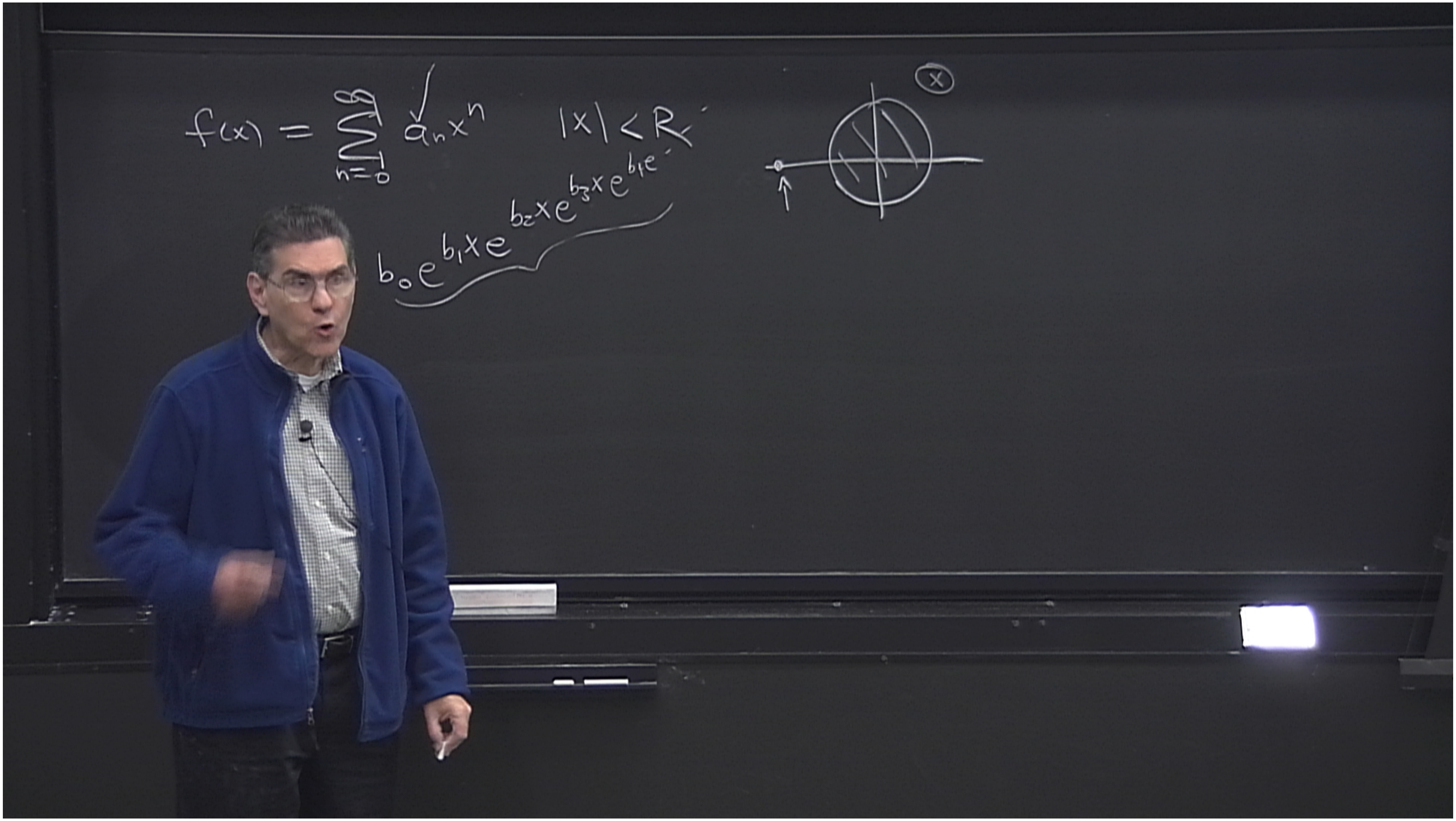

HP
Ex
$H = p^2 + \frac{x^2}{4} + \frac{x^4}{4}$
$H\psi = E\psi$
$\left(-\frac{d^2}{dx^2} + \frac{x^2}{4} + \epsilon\frac{x^4}{4}\right)\psi(x) = E\psi(x)$
$\psi(x) \to 0$ as $x \to \pm\infty$
HP
$EP_0 + EP_1 + EP_2 - - - -$
$E(\epsilon) \sim \sum_{n=0}^{\infty} a_n \epsilon^n$
$a_n \sim C\, 3^n n! (-1)^{n+1}$
as $n \to \infty$
→ Ans i.e. $E(\epsilon)$



Γ(n) = (n−1)!
Γ(x) = ∫ dt e^{−t} t^{x−1}
Re x > 0
anal. of x
for all x ≠ 0.
x → 0
Γ(x) ~ 1/x
as x → 0.
exists
for all x ≠ 0, −1, −2, ...
f(x) = 1/(1−x)
x = −2
x ≠ 1
f(x) = Σ_{n=0}^∞ x^n
|x| < 1
f(−2) = ?
ζ(z) = Σ 1/n^z
Re z > 1
ζ(2) = 1 + 1/2^2 + 1/3^2 + ...
ζ(4) = 1 + 1/2^4 + 1/3^4 + ...
ζ(6)
ζ(3) =
ζ(0) = −1/2

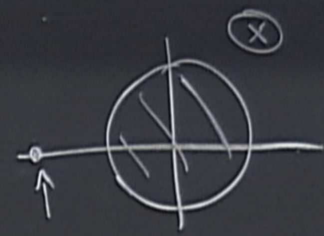

$f(x) = \sum_{n=0}^{\infty} a_n x^n$
$|x| < R$
x


$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad |x| < R$
$= b_0 e^{b_1 x e^{b_2 x e^{b_3 x e^{b_4 e}}}}$
$|x|$
x


$f(x) = \sum_{n=0}^{\infty} a_n x^n$
$|x| < R$
$b_0 e^{b_1 x e^{b_2 x e^{b_3 x e^{b_4 e}}}}$
x

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \qquad |x| < R$$

$$= b_0 e^{b_1 x e^{b_2 x e^{b_3 x e^{b_4 e^{\cdots}}}}}$$



$a_0 \to b_0$

$a_1 \to b_1$

$a_2 \to b_2$

Continued exponentials

$$a_0 e^{a_1 z e^{a_2 z \cdots}}$$

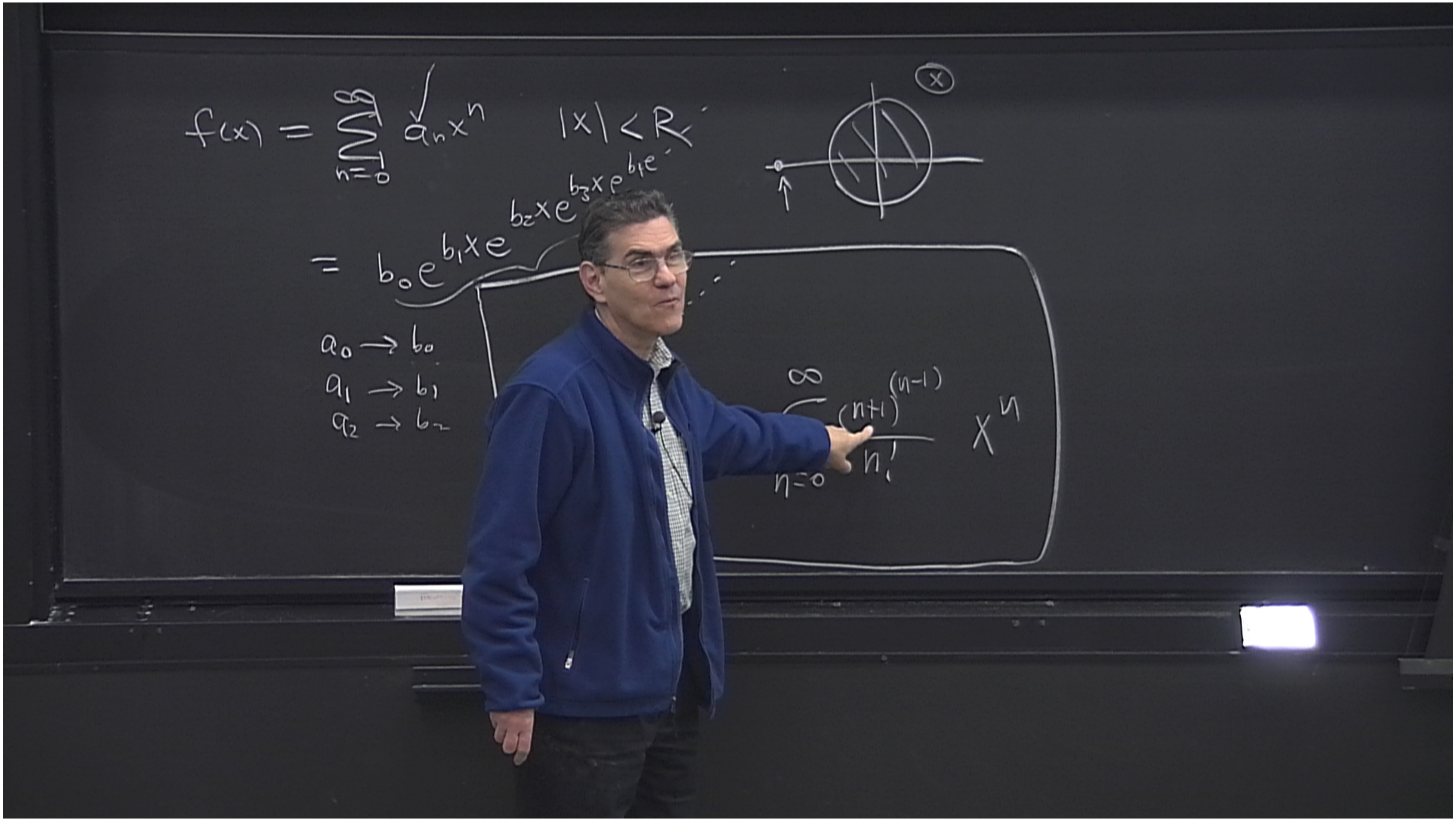
$$\sum_{n=0}^{\infty} c_n z^n$$

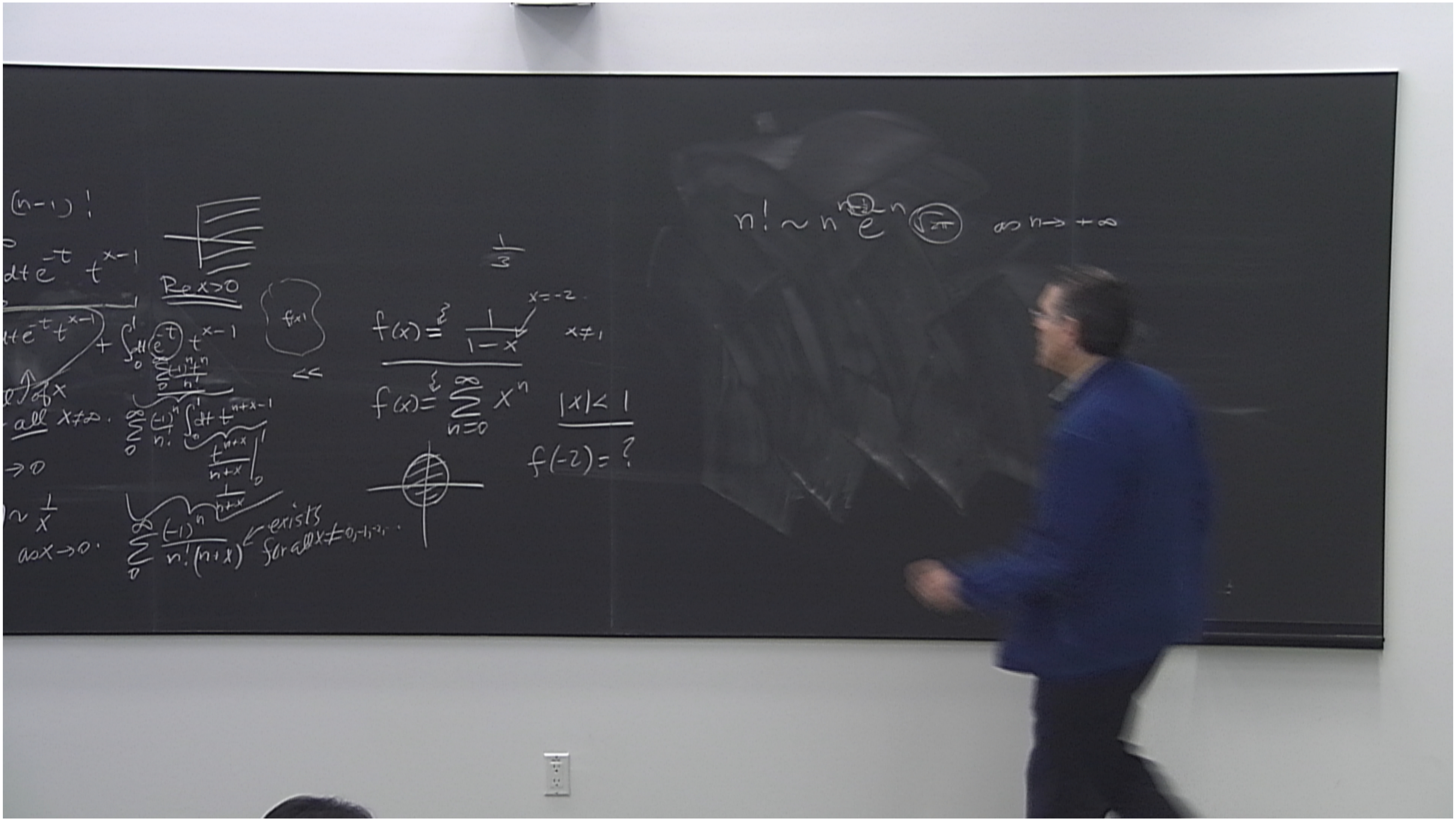
$$c_0 = a_0,$$

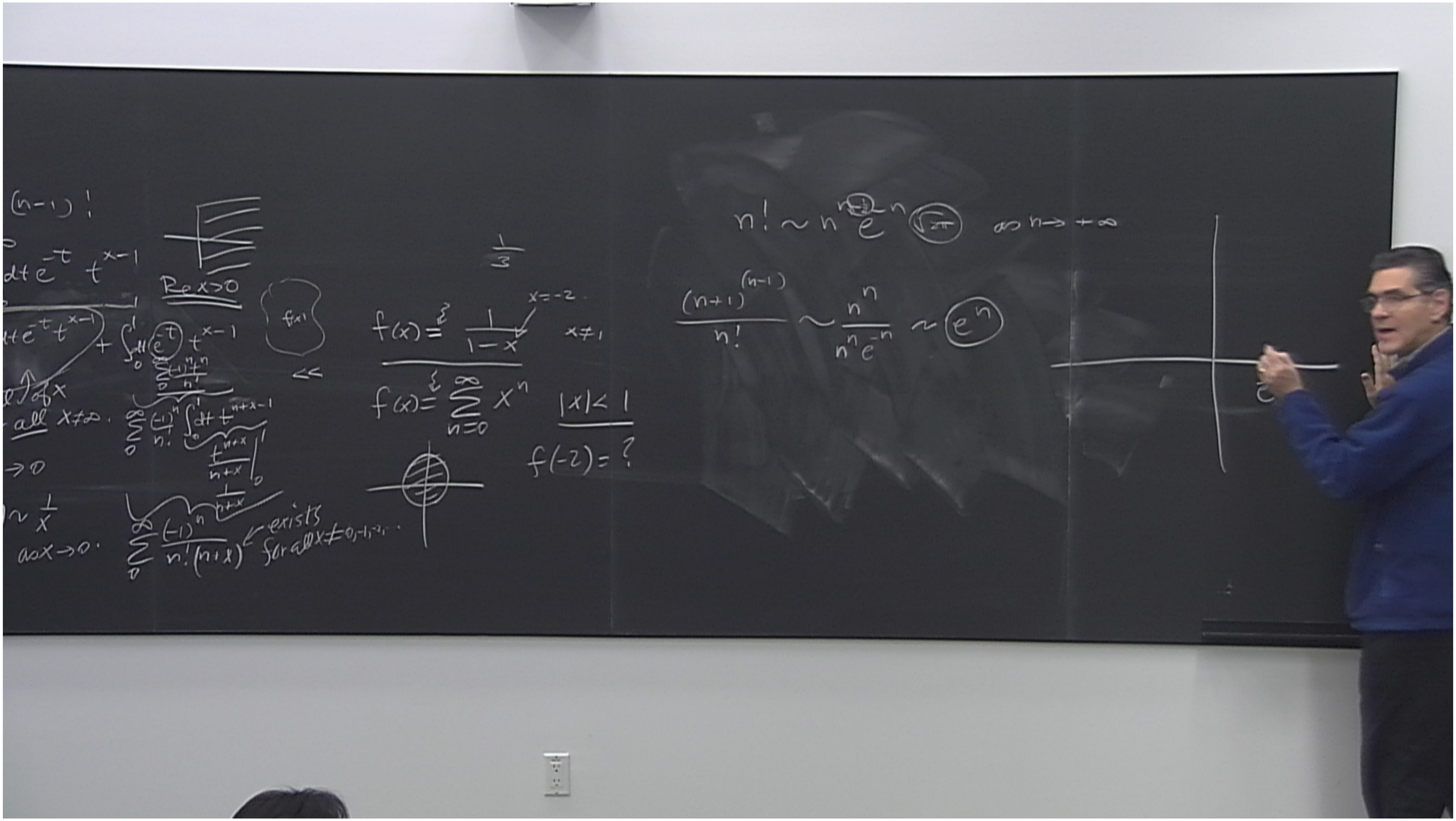
$$c_1 = a_1 a_0,$$

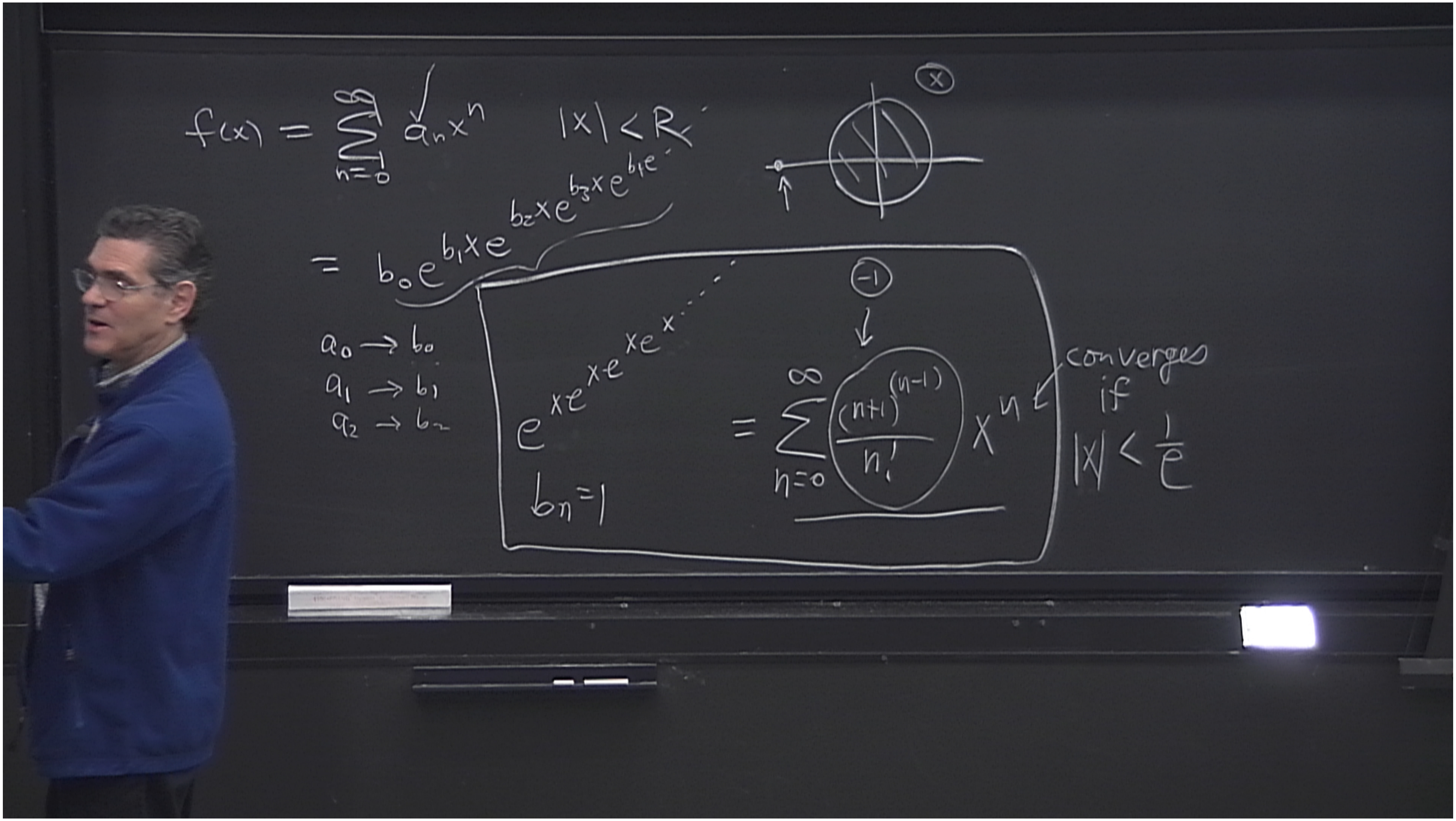
$$c_2 = a_0 a_1 a_2 + \frac{1}{2} a_0 a_1^2,$$

$$c_3 = a_0 a_1 a_2 a_3 + \frac{1}{2} a_0 a_1 a_2^2 + a_0 a_1^2 a_2 + \frac{1}{6} a_0 a_1^3$$

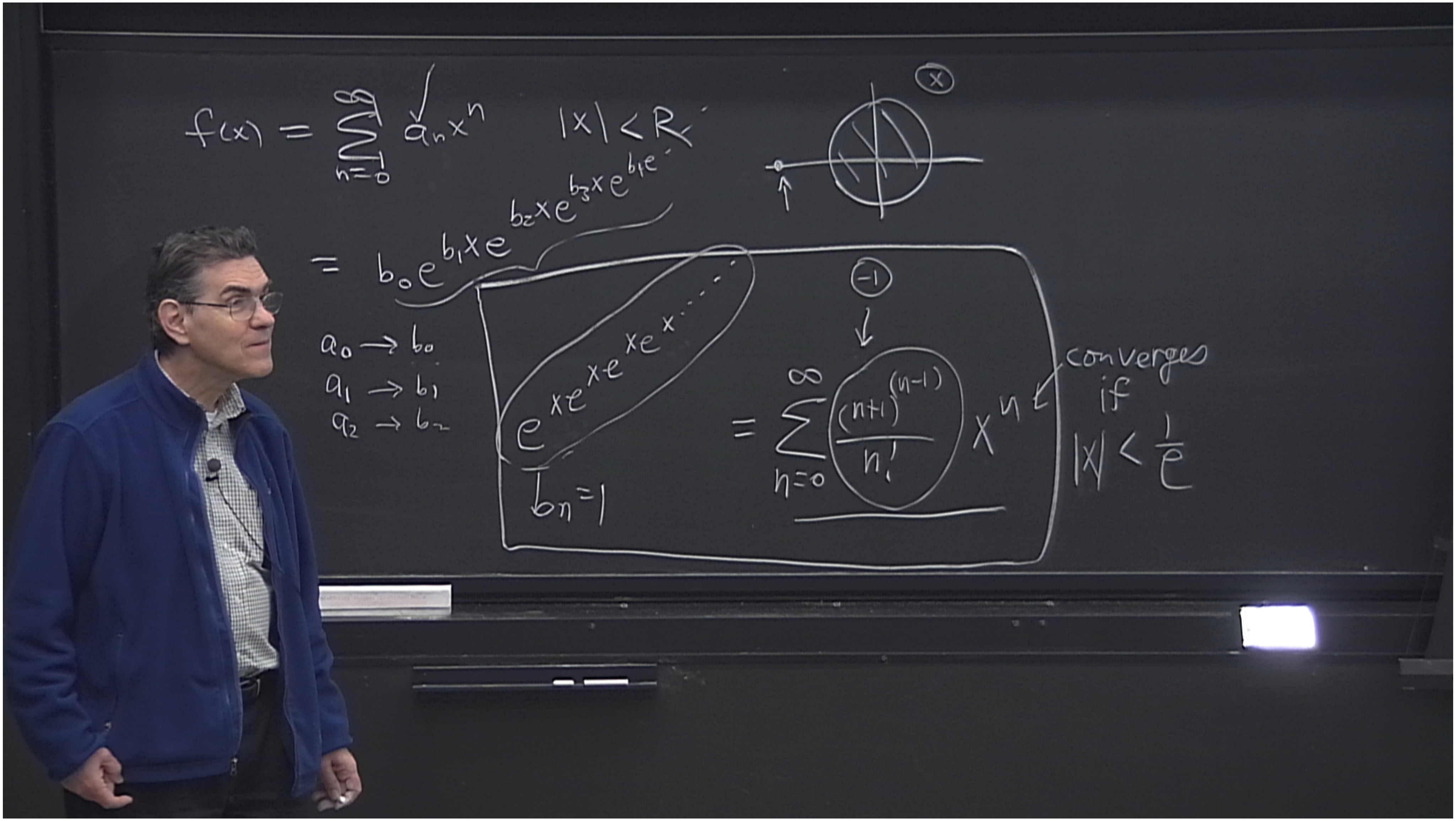

f(x) = Σ_{n=0}^{∞} a_n x^n
|x| < R
= b_0 e^{b_1 x e^{b_2 x e^{b_3 x e^{b_4 e}}}}
a_0 → b_0
a_1 → b_1
a_2 → b_2
Σ_{n=0}^{∞} (n+1)^{(n-1)} / n! x^n

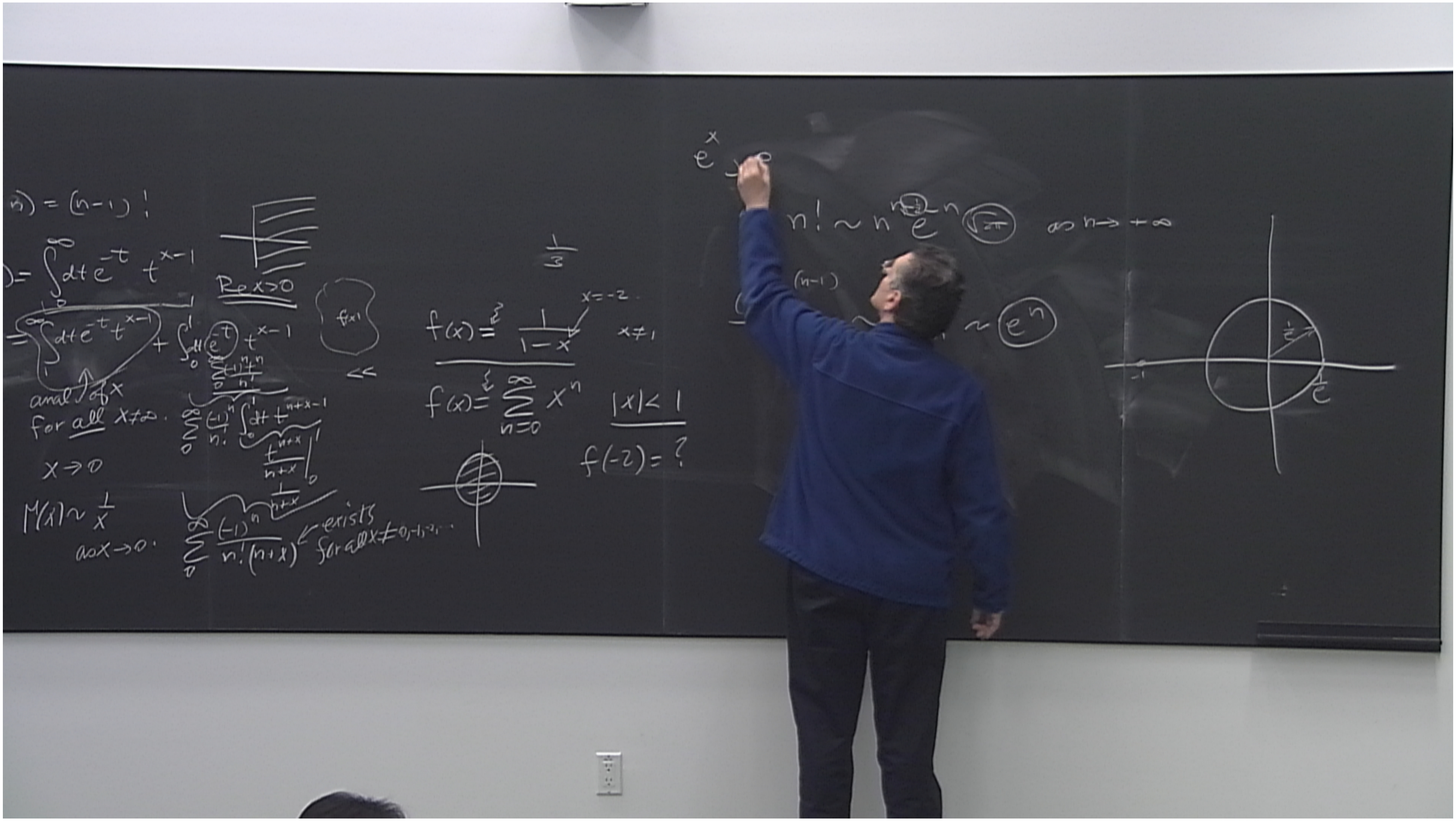

Re x>0
f(x) = 1/(1-x)
x = -2
x ≠ 1
f(x) = Σ_{n=0}^∞ x^n
|x| < 1
f(-2) = ?
exists for all x ≠ 0, -1, -2, ...
as x → 0
n! ~ n^{n+1/2} e^{-n} √2π as n → +∞

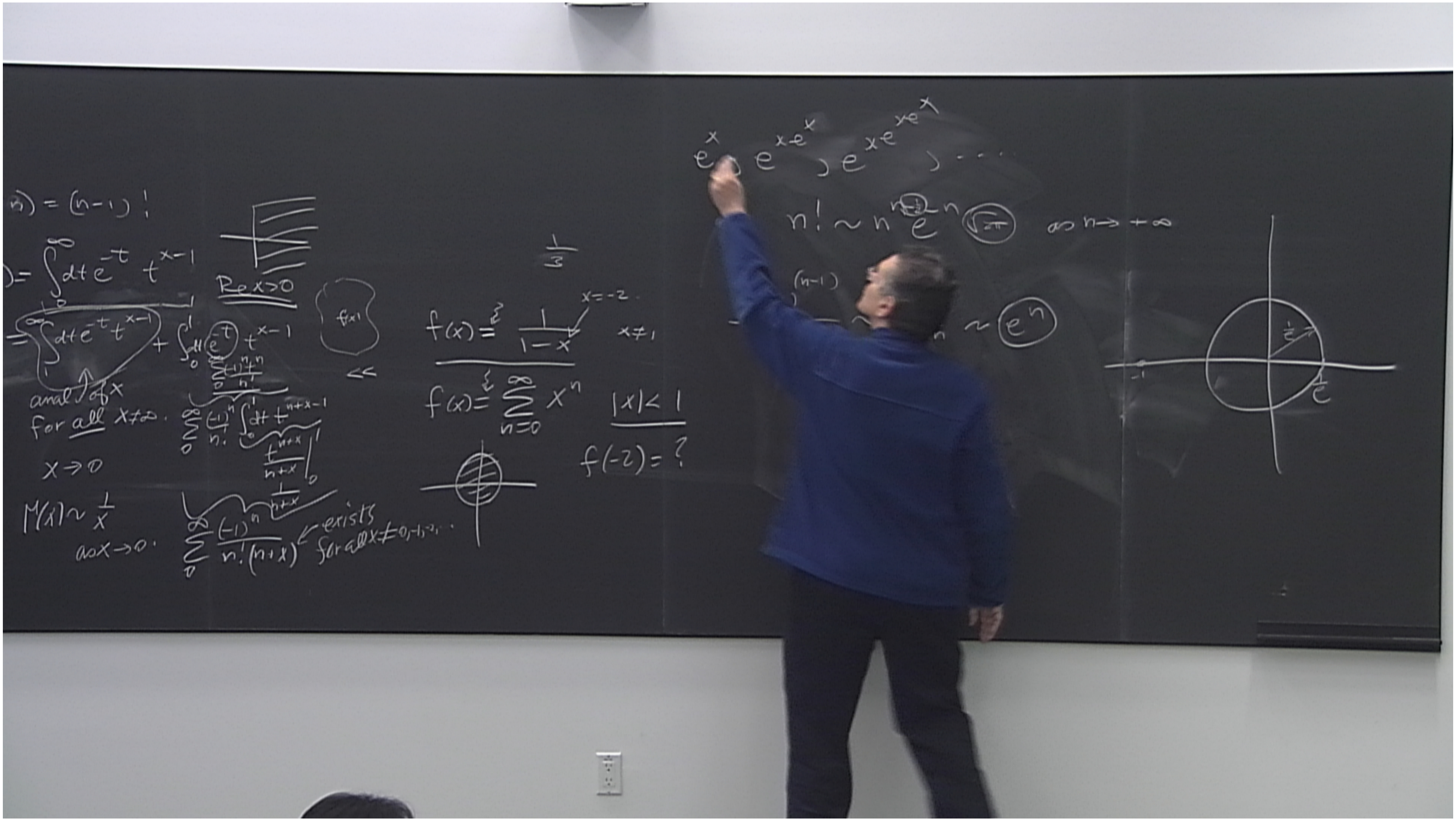

(n-1)!
Re x>0
f(x) = 1/(1-x)
x≠1
x=-2
f(x) = Σ_{n=0}^∞ x^n
|x|<1
f(-2) = ?
exists for all x≠0,-1,-2,...
as x→0
n! ~ n^{n+1/2} e^{-n} √2π as n→+∞
(n+1)^{(n-1)}/n! ~ n^n/(n^n e^{-n}) ~ e^n



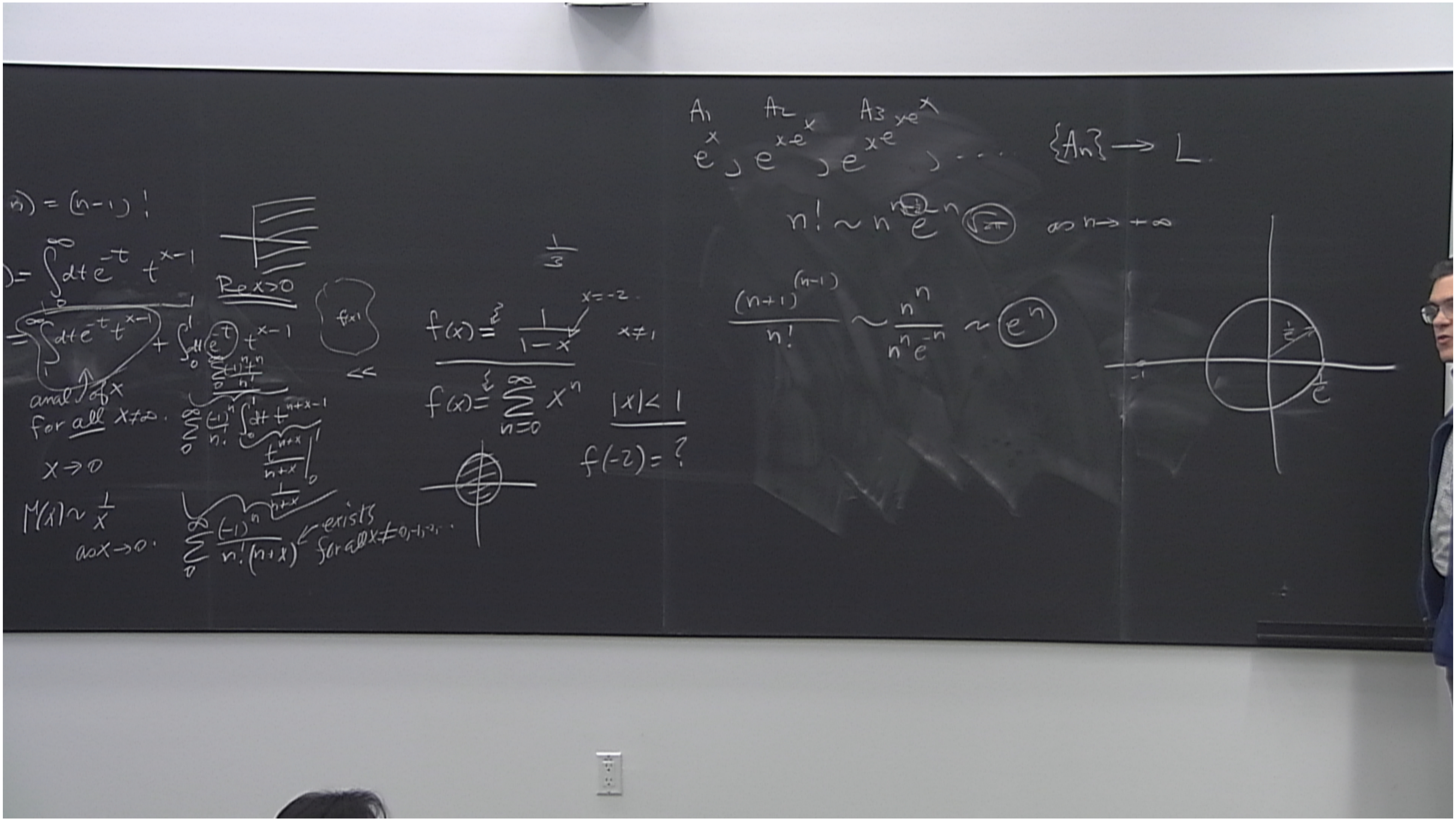
f(x) = ∑_{n=0}^∞ a_n x^n |x| < R
= b_0 e^{b_1 x e^{b_2 x e^{b_3 x e^{b_4 e ...}}}}
a_0 → b_0
a_1 → b_1
a_2 → b_2
e^{x e^{x e^{x e^{x ...}}}} = ∑_{n=0}^∞ (n+1)^{(n−1)} / n! x^n
b_n = 1
−1
x
converges if |x| < 1/e

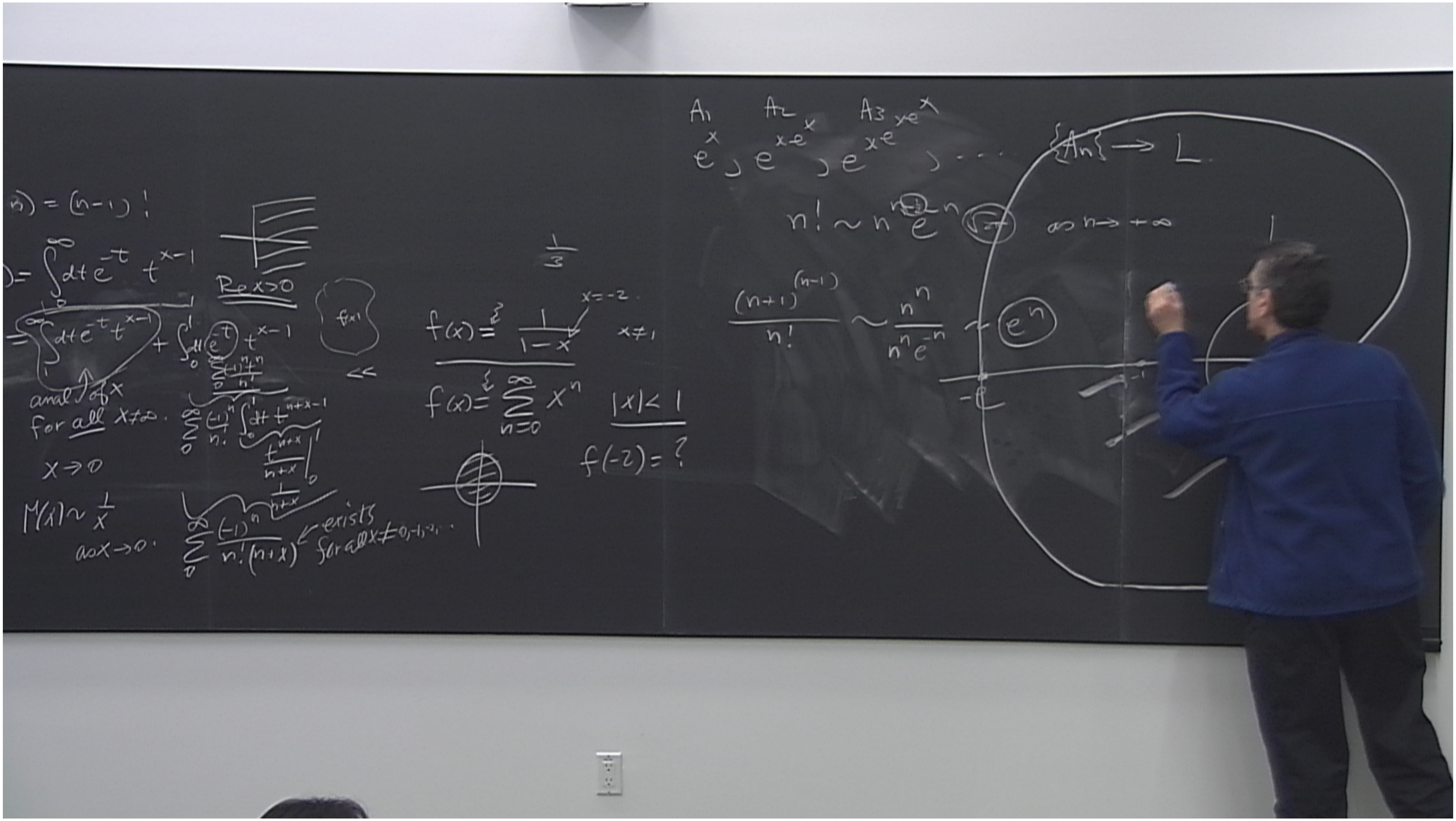

f(x) = ∑_{n=0}^{∞} a_n x^n |x| < R
= b_0 e^{b_1 x e^{b_2 x e^{b_3 x e^{b_4 e^{...}}}}}
a_0 → b_0
a_1 → b_1
a_2 → b_2
e^{x e^{x e^{x e^{x ...}}}}
b_n = 1
= ∑_{n=0}^{∞} ((n+1)^{(n-1)} / n!) x^n
-1
converges if |x| < 1/e
x


Re x > 0
f(x) = 1/(1 − x)
x = −2
x ≠ 1
|x| < 1
f(−2) = ?
exists for all x ≠ 0, −1, −2, ...
as n → +∞

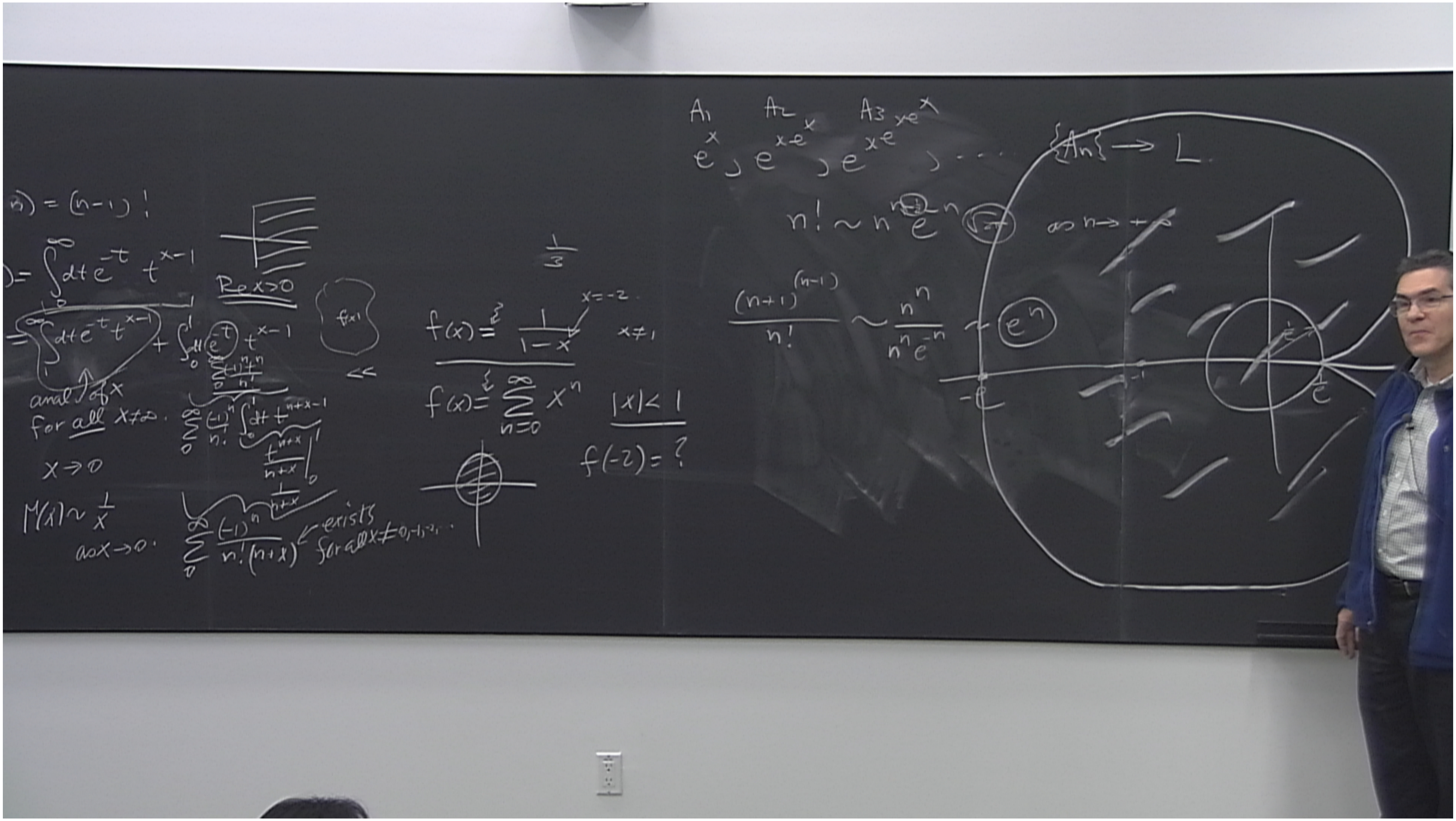


n! ~ n^(n-1/2) e^(-n) √2π as n → +∞
Re x > 0
f(x) = 1/(1−x)
x ≠ 1
x = −2
f(x) = Σ_{n=0}^∞ x^n
|x| < 1
f(−2) = ?
for all x ≠ 0
x → 0
as x → 0
exists for all x ≠ 0, −1, −2, ...


A1 A2 A3
{An} → L
n! ~ n^(n-1/2) e^(-n) √2π as n → +∞
Re x>0
f(x)= 1/(1-x) x≠1
x=-2
|x|< 1
f(-2)= ?
analytic of x
for all x≠0.
x→0
as x→0.
exists for all x≠0,-1,-2,...



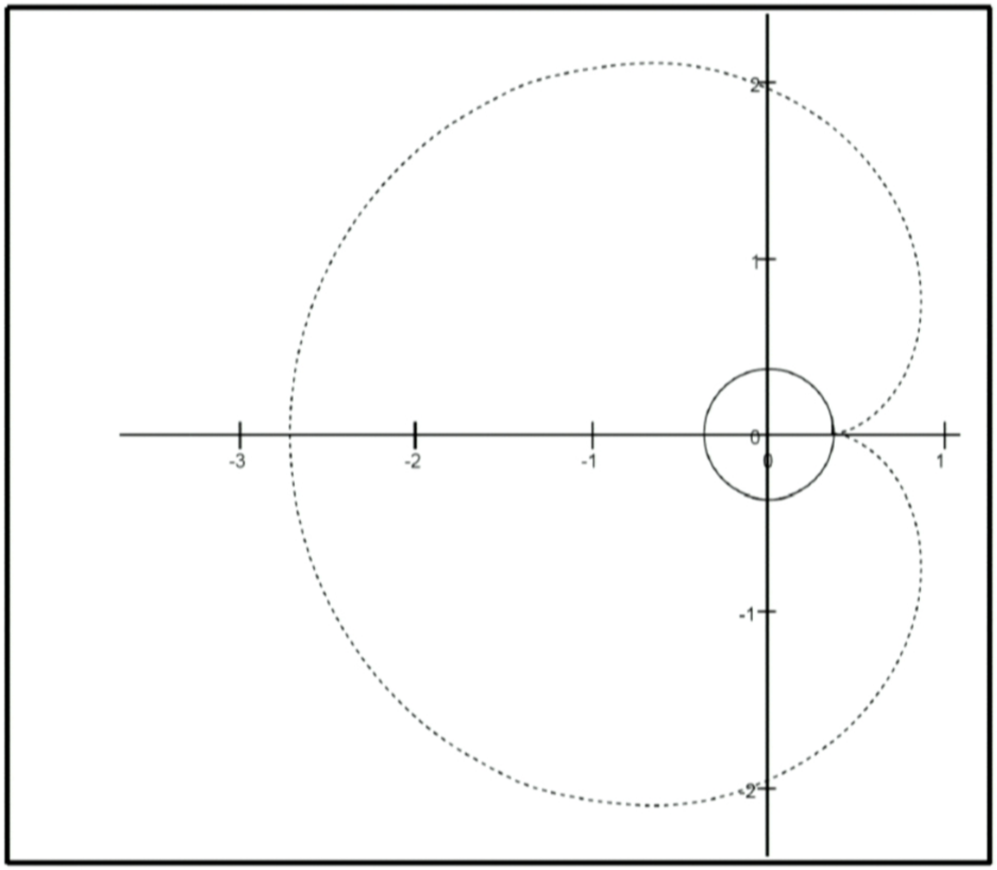
$\Gamma(n) = (n-1)!$
Re x>0
anal. of x
for all x≠0
x→0
$\Gamma(x) \sim \frac{1}{x}$
as x→0
exists for all x≠0,-1,-2,...
$f(x) = \frac{1}{1-x}$
x≠1
x=-2
$f(x) = \sum_{n=0}^{\infty} x^n$
|x|<1
f(-2) = ?
$A_1 \quad A_2 \quad A_3$
$\{A_n\} \to L$
$n! \sim n^{n+\frac{1}{2}} e^{-n} \sqrt{2\pi}$
as n→+∞
$\frac{(n+1)^{(n-1)}}{n!} \sim \frac{n^n}{n^n e^{-n}} \sim e^n$

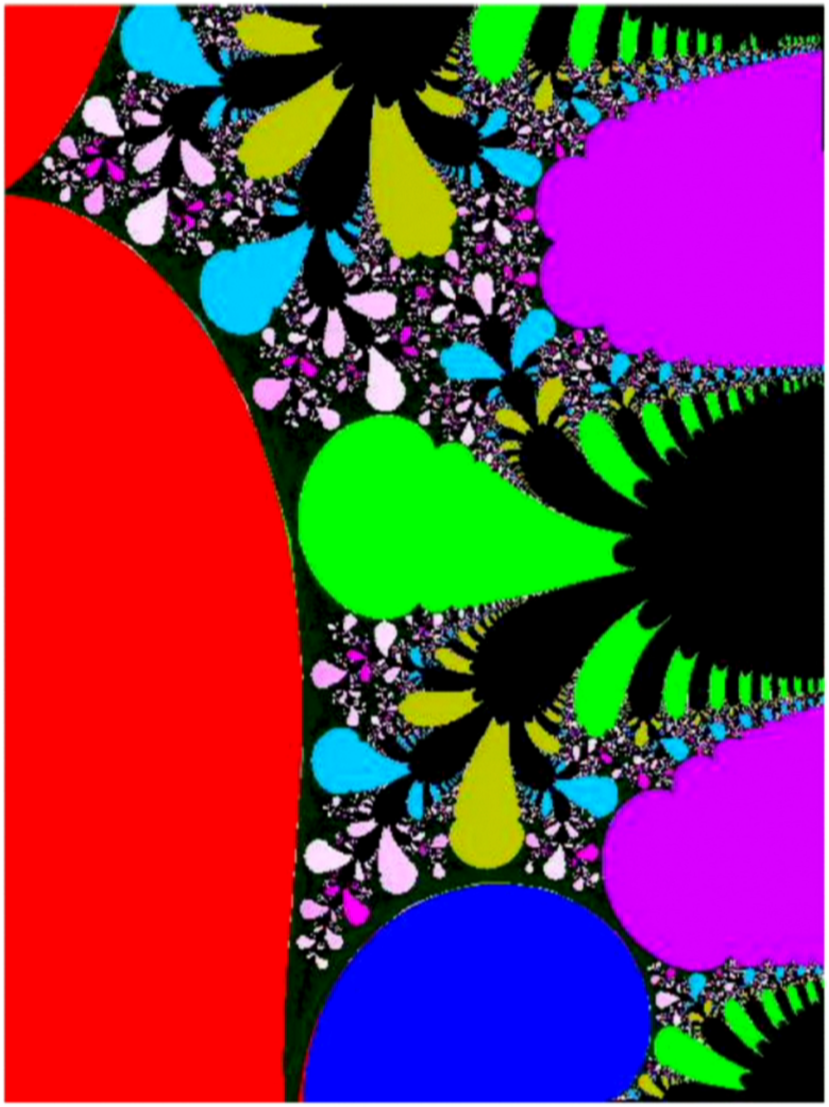

A1 A2 A3
{An} → L
n! ~ n^n e^-n √2πn as n → +∞
Re x>0
f(x)
f(x) = 1/(1−x)
x = −2
x ≠ 1
f(x) = Σ_{n=0}^∞ x^n
|x| < 1
f(−2) = ?
for all x ≠ 0
x → 0
Γ(x) ~ 1/x as x → 0
exists for all x ≠ 0, −1, −2, ...
−e
e

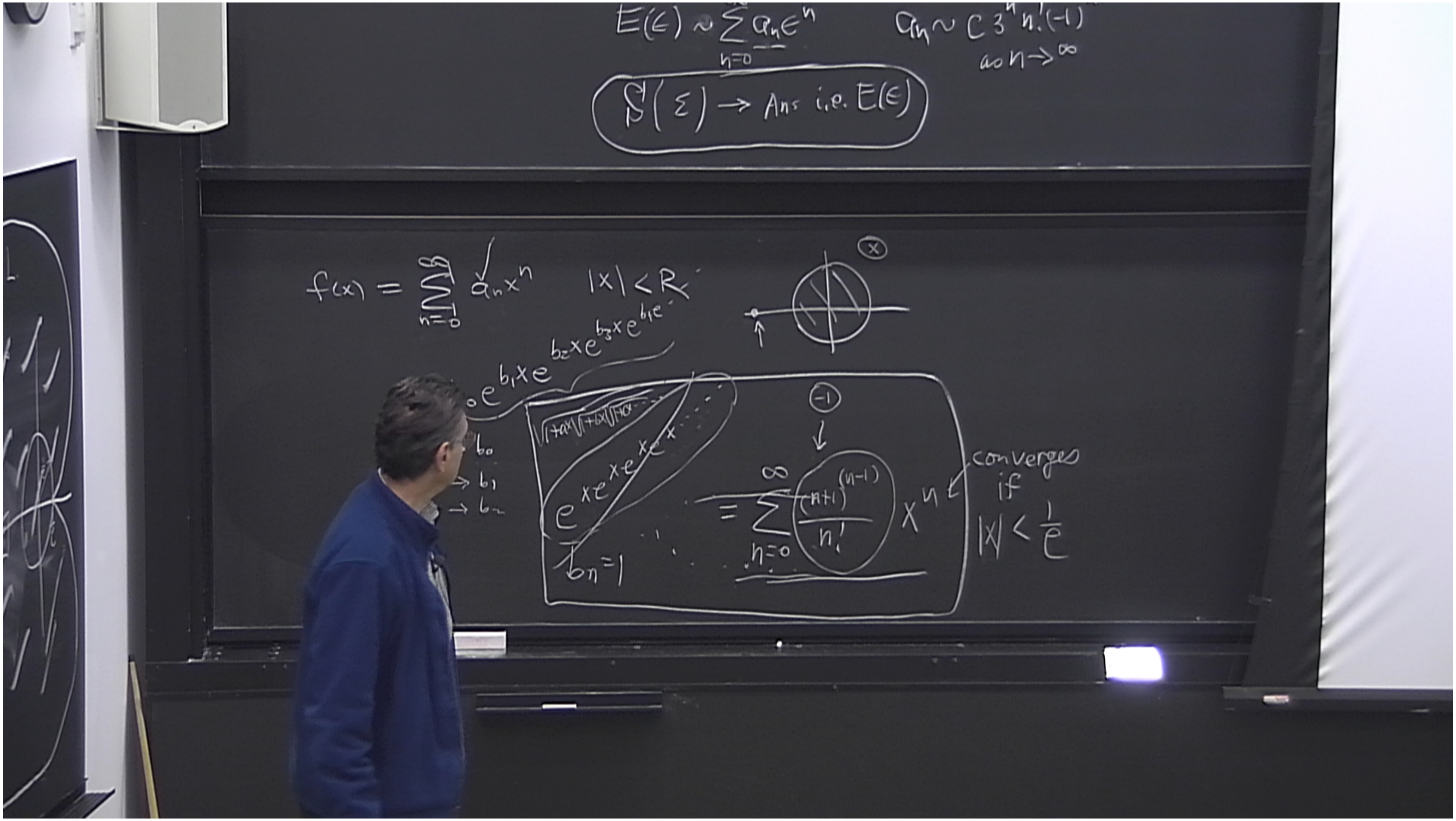
Example

$$e^{ze^{ze^{ze^{z\cdots}}}} = \sum_{n=0}^{\infty} \frac{(n+1)^{n-1}}{n!} z^n$$

Region of convergence






E(ε) ~ Σ a_n ε^n
n=0
a_n ~ C 3^n n! (-1)^n
as n → ∞
S(ε) → Ans i.e. E(ε)
f(x) = Σ a_n x^n
n=0
|x| < R
e^x e^x e^x e^x
b_n = 1
converges
if
|x| < 1/e