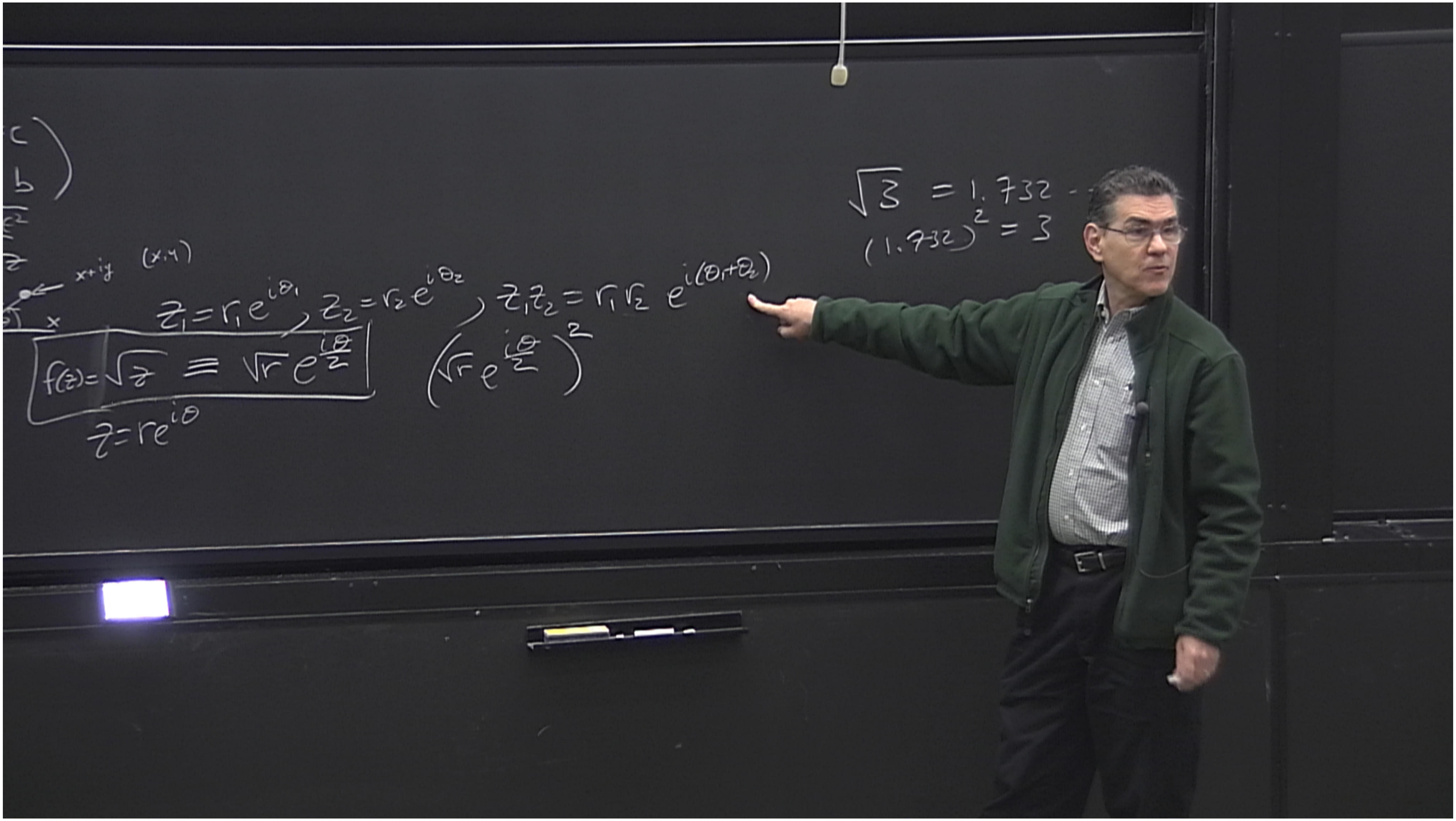
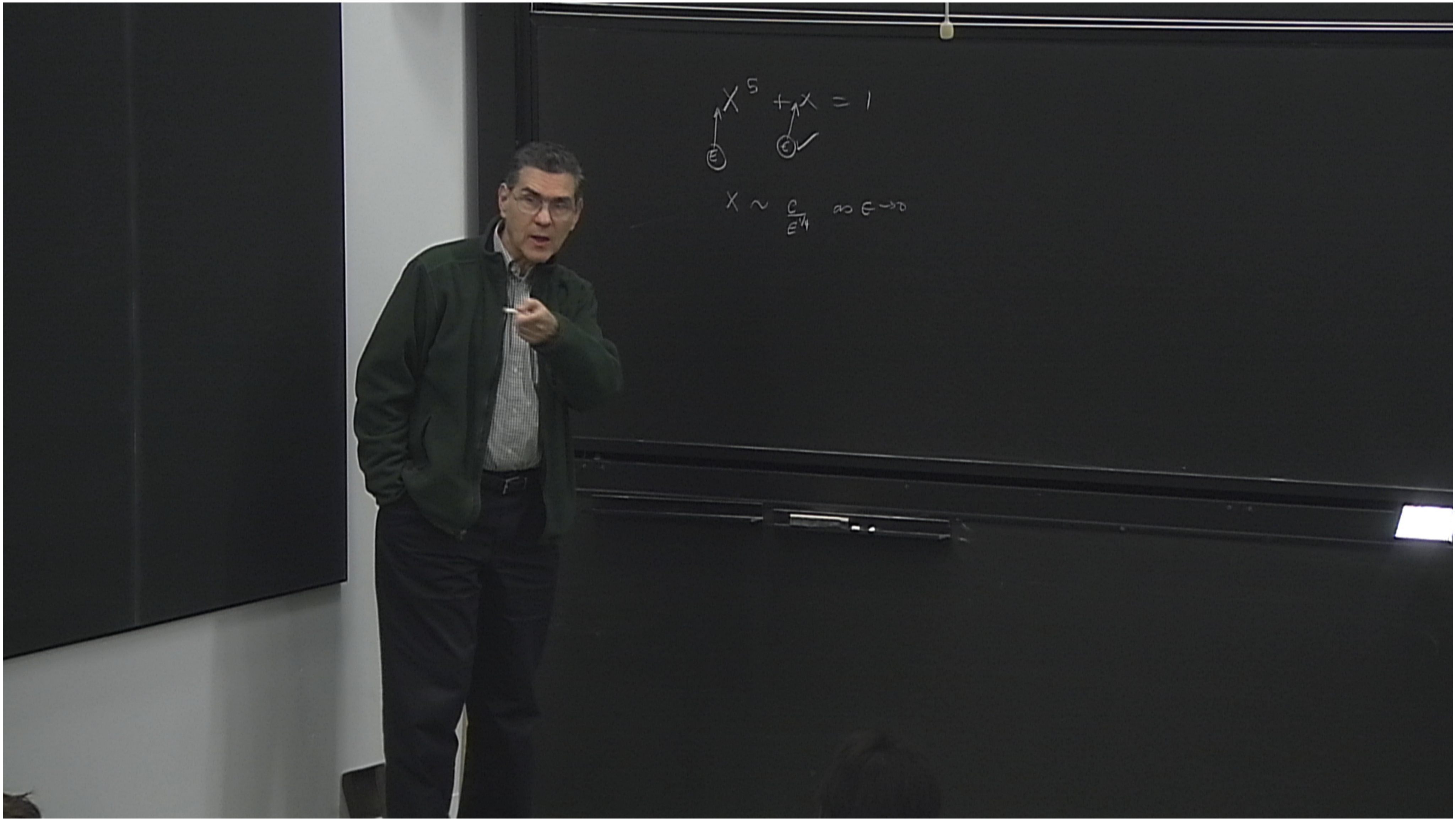
Title: Mathematical Physics - Lecture 3

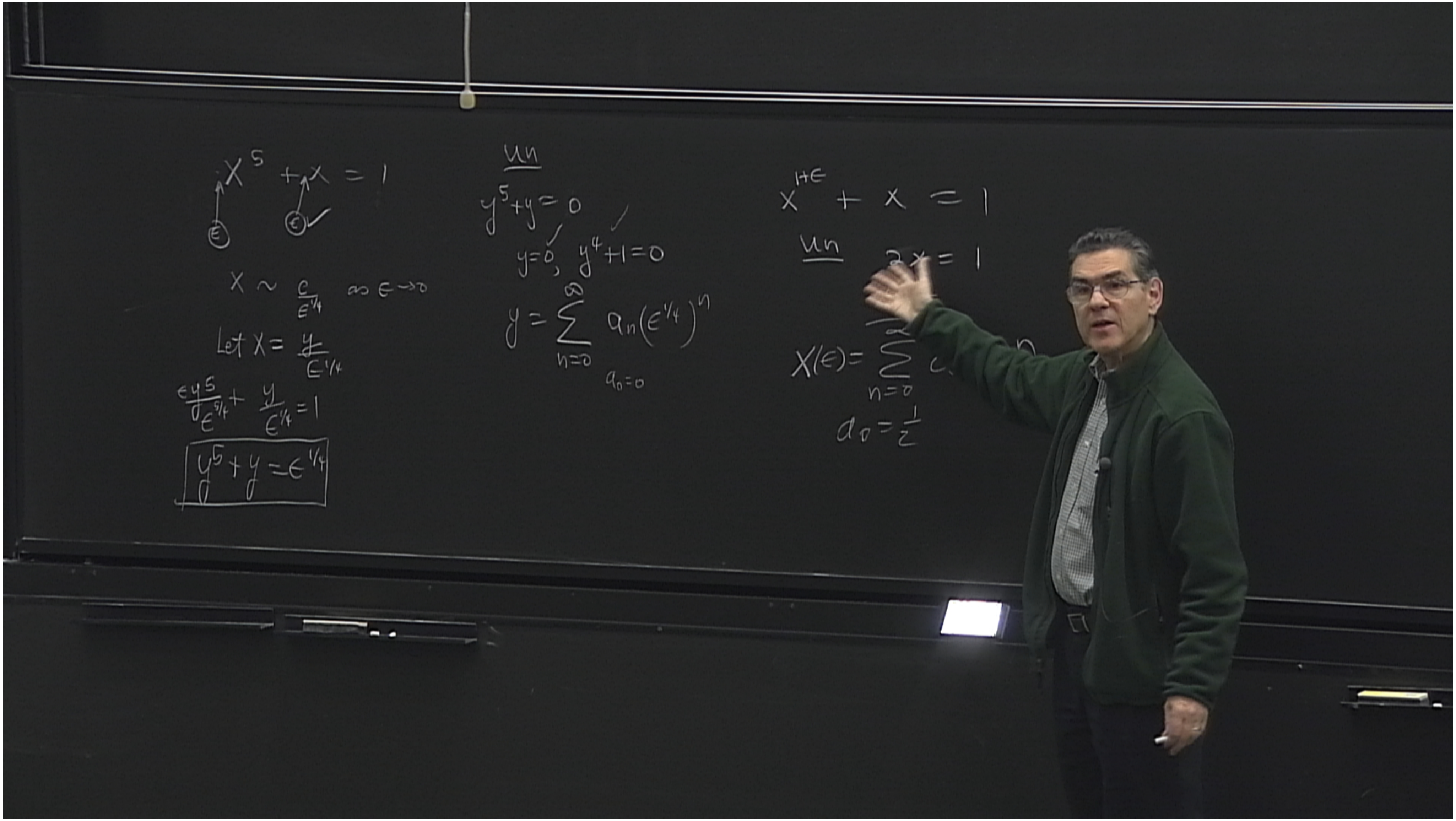
Date: Nov 23, 2011 09:00 AM

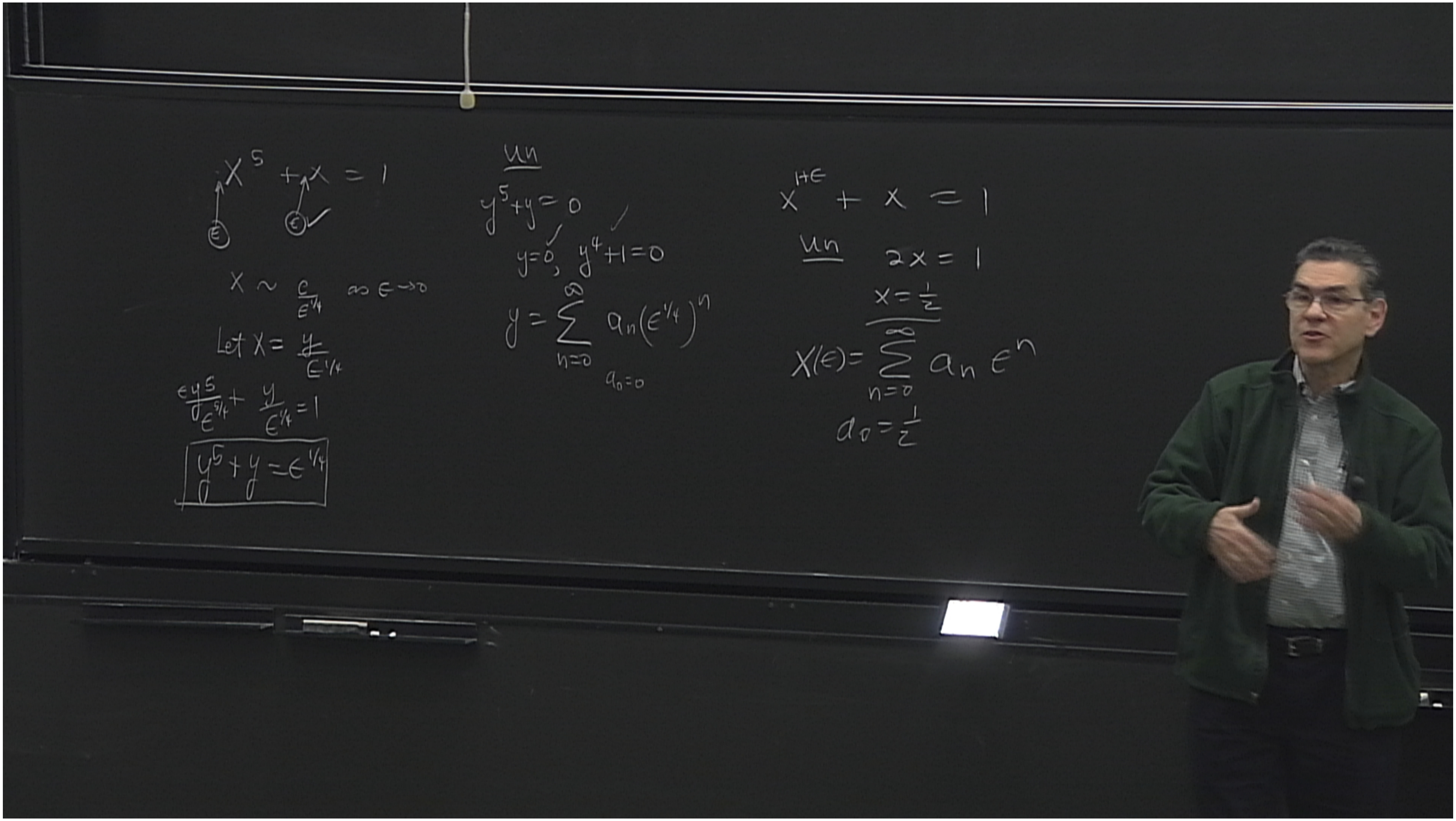
URL: http://pirsa.org/11110042

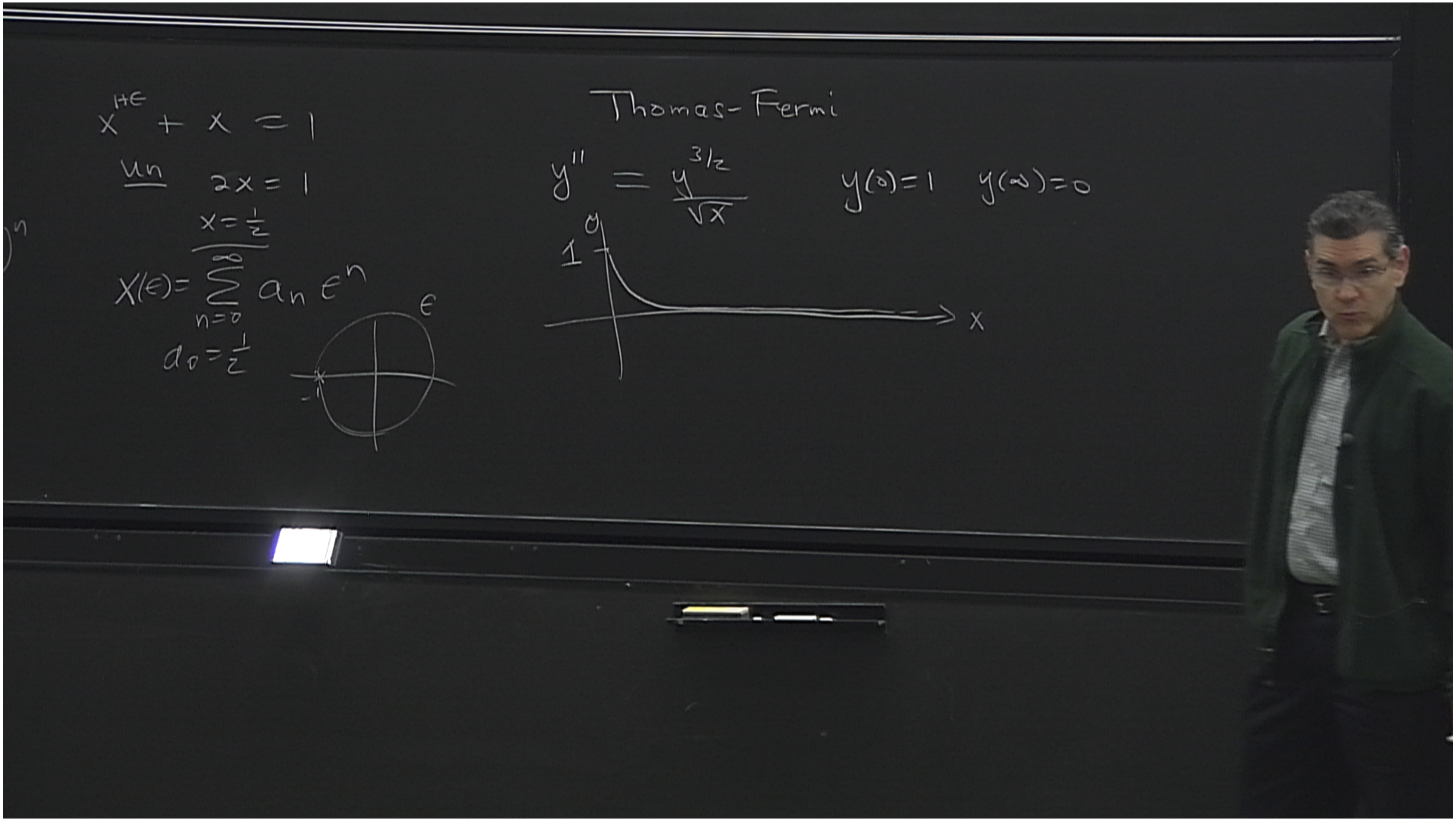
Abstract:

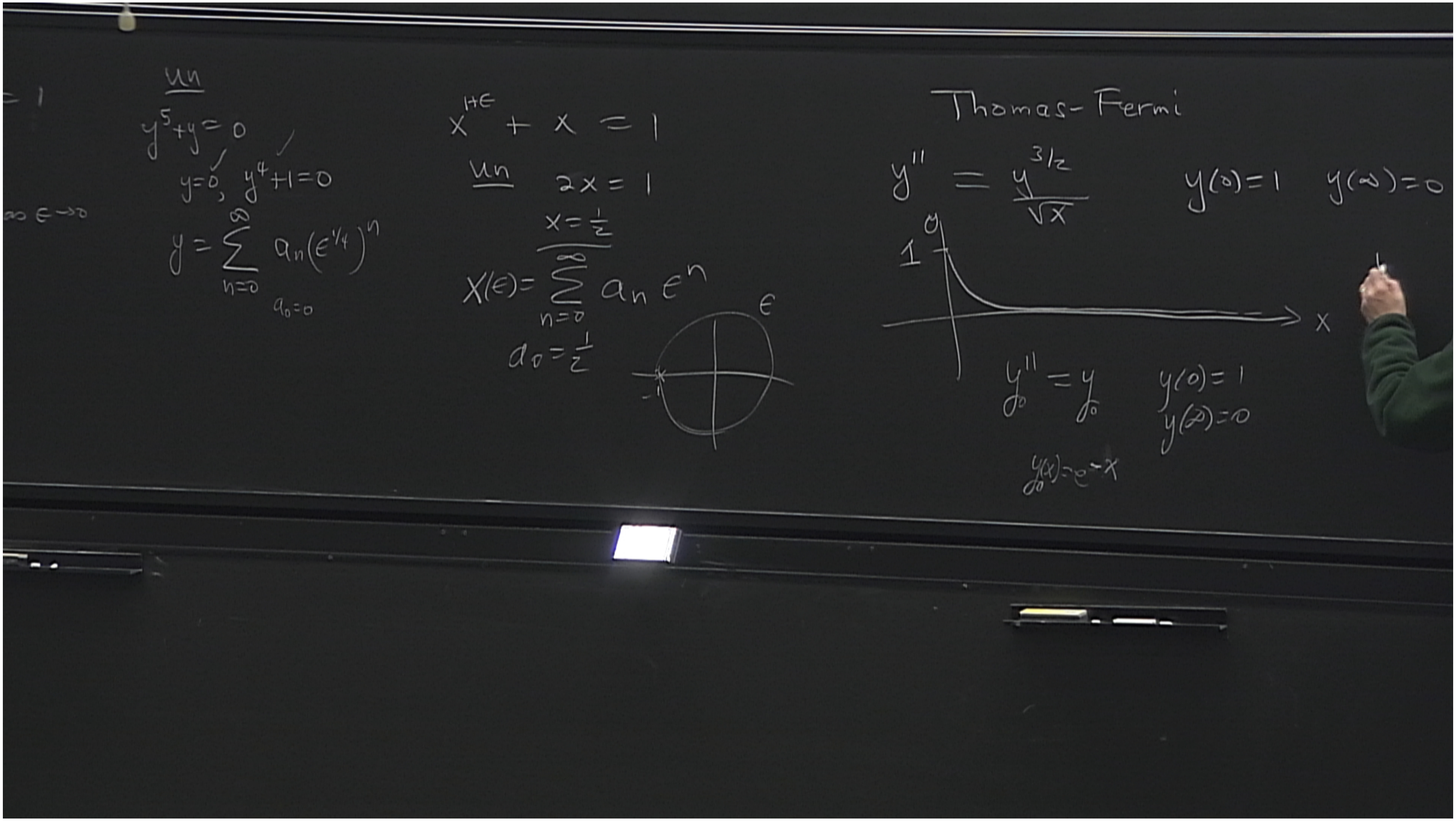

x+iy (x,y)
z1=r1e^iθ1, z2=r2e^iθ2, z1z2=r1r2 e^i(θ1+θ2)
f(z)=√z ≡ √r e^(iθ/2)
(√r e^(iθ/2))^2
z=re^iθ
√3 = 1.732 ...
(1.732)^2 = 3


X^5 + X = 1
ε ε
X ~ c/ε^{1/4} as ε → 0


X^5 + x = 1
x ~ c/ε^(1/4) as ε→0
Let x = y/ε^(1/4)
y^5 + y = ε^(1/4)
un
y^5 + y = 0
y = 0, y^4 + 1 = 0
a_0 = 0
x^(1+ε) + x = 1
un
2x = 1
n=0
a_0 = 1/2

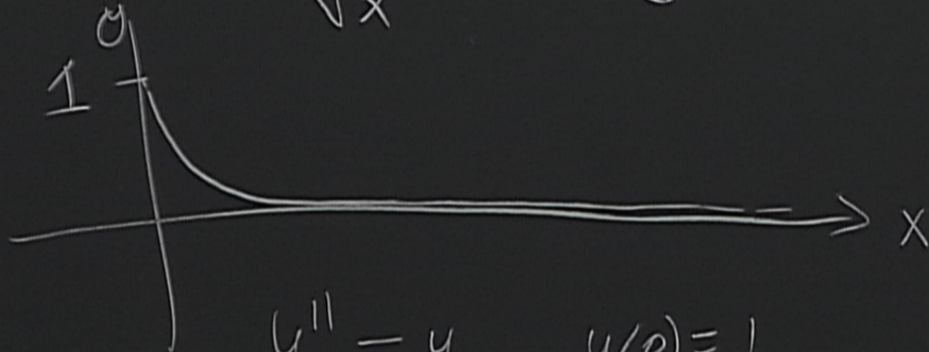

$x^5 + \epsilon x = 1$
$x \sim \frac{c}{\epsilon^{1/4}}$ as $\epsilon \to 0$
Let $x = \frac{y}{\epsilon^{1/4}}$
$\frac{\epsilon y^5}{\epsilon^{5/4}} + \frac{y}{\epsilon^{1/4}} = 1$
$y^5 + y = \epsilon^{1/4}$
un
$y^5 + y = 0$
$y = 0, \; y^4 + 1 = 0$
$y = \sum_{n=0}^{\infty} a_n (\epsilon^{1/4})^n$
$a_0 = 0$
$x^{1+\epsilon} + x = 1$
un
$2x = 1$
$x = \frac{1}{2}$
$x(\epsilon) = \sum_{n=0}^{\infty} a_n \epsilon^n$
$a_0 = \frac{1}{2}$


x^(1+ε) + x = 1
un
2x = 1
x = 1/2
X(ε) = Σ_(n=0)^∞ a_n ε^n
a_0 = 1/2
ε
Thomas-Fermi
y'' = y^(3/2)/√x
y(0)=1
y(∞)=0
1
x


un
y^5 + y = 0
y = 0, y^4 + 1 = 0
y = Σ_{n=0}^∞ a_n (ε^{1/4})^n
a_0 = 0
x^{1+ε} + x = 1
un 2x = 1
x = 1/2
X(ε) = Σ_{n=0}^∞ a_n ε^n
a_0 = 1/2
ε
Thomas-Fermi
y'' = y^{3/2} / √x
y(0) = 1
y(∞) = 0
y'' = y
y(0) = 1
y(∞) = 0
y_0(x) = e^{-x}

Thomas-Fermi

$$y'' = \frac{y^{3/2}}{\sqrt{x}} \qquad y(0)=1 \qquad y(\infty)=0$$



$$y_0'' = y_0 \qquad y(0)=1 \quad y(\infty)=0$$

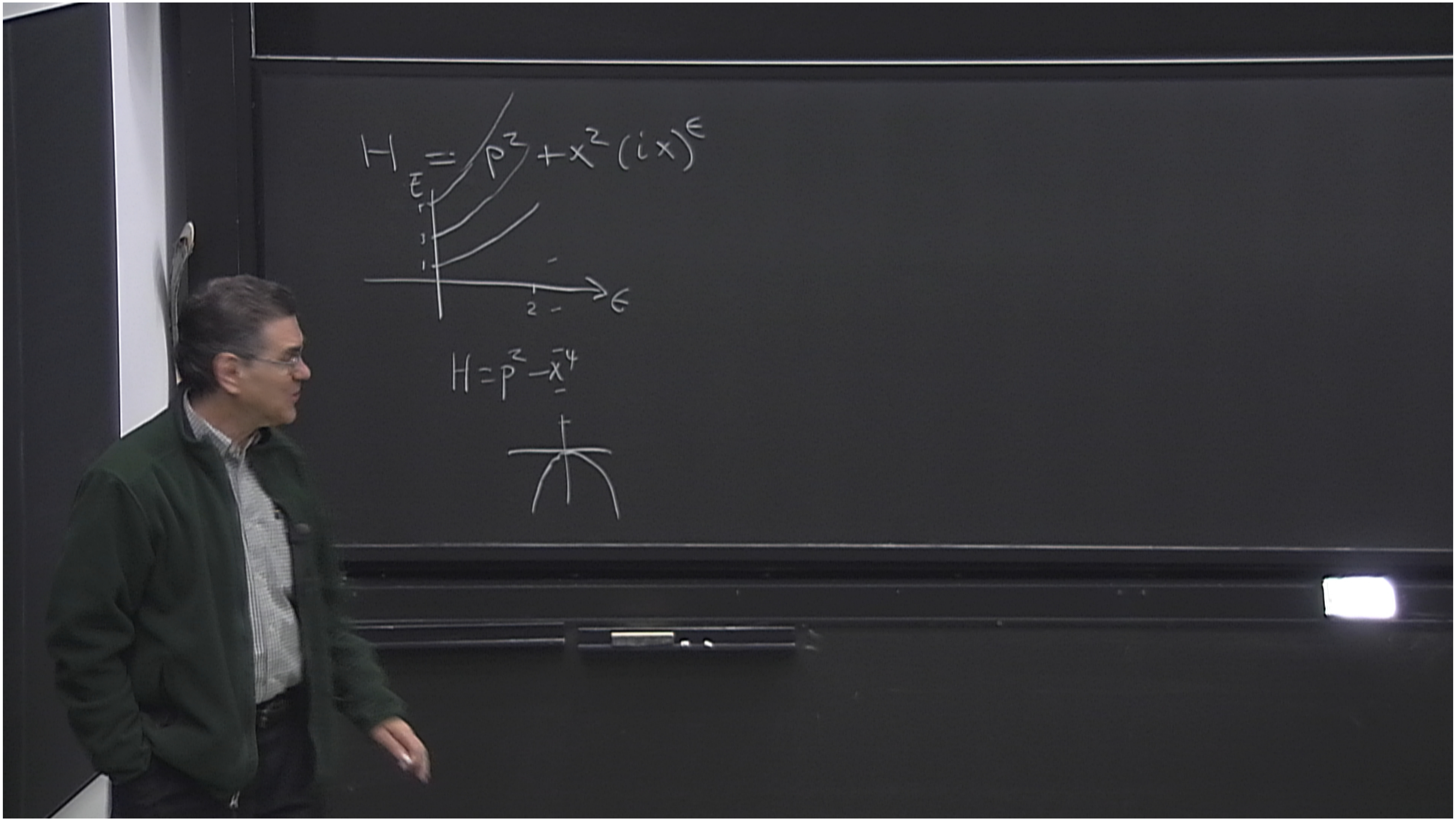
$$y_0(x)=e^{-x}$$

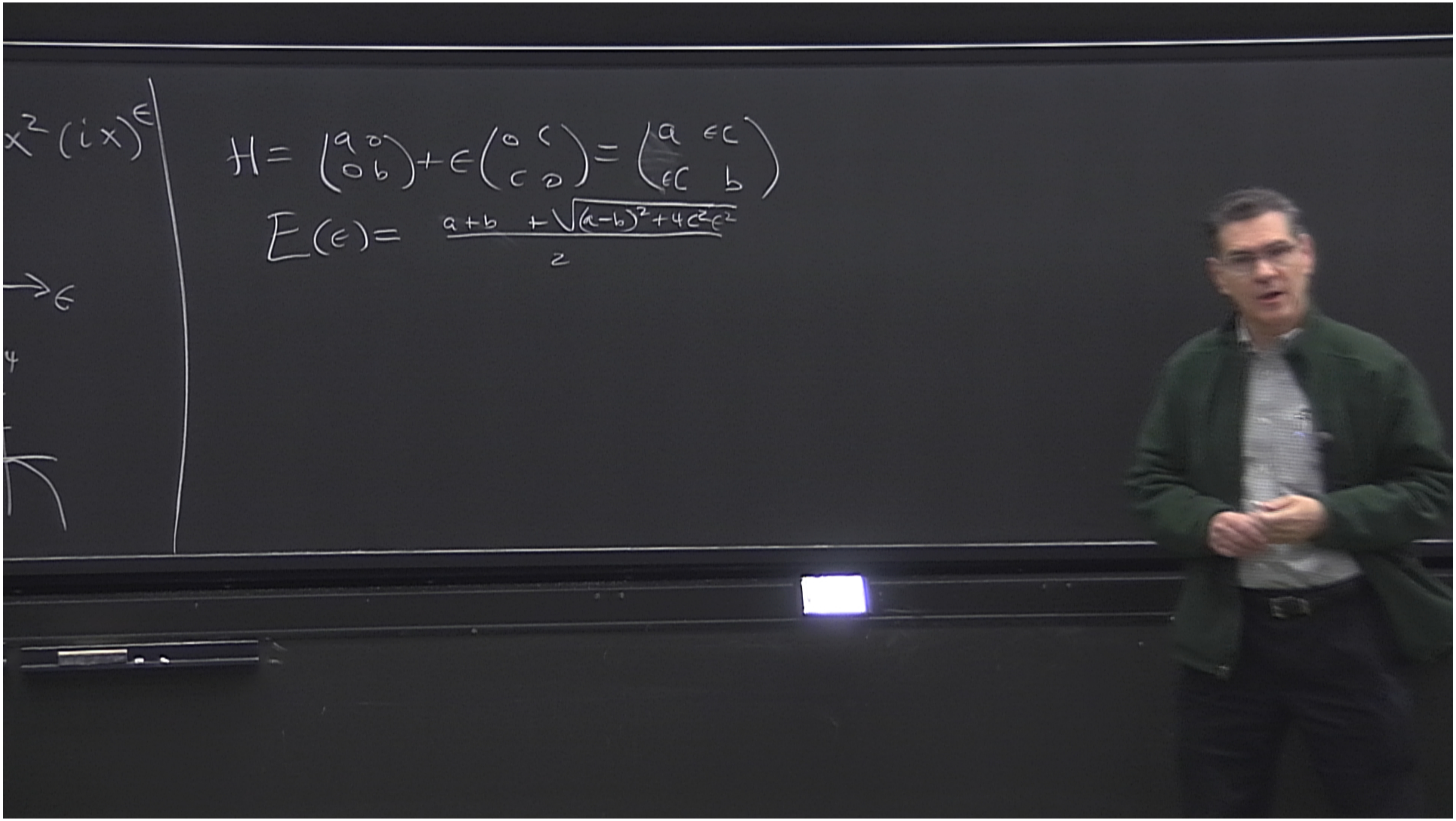
$$y'' = y\left(\frac{y}{x}\right)^{\epsilon}$$

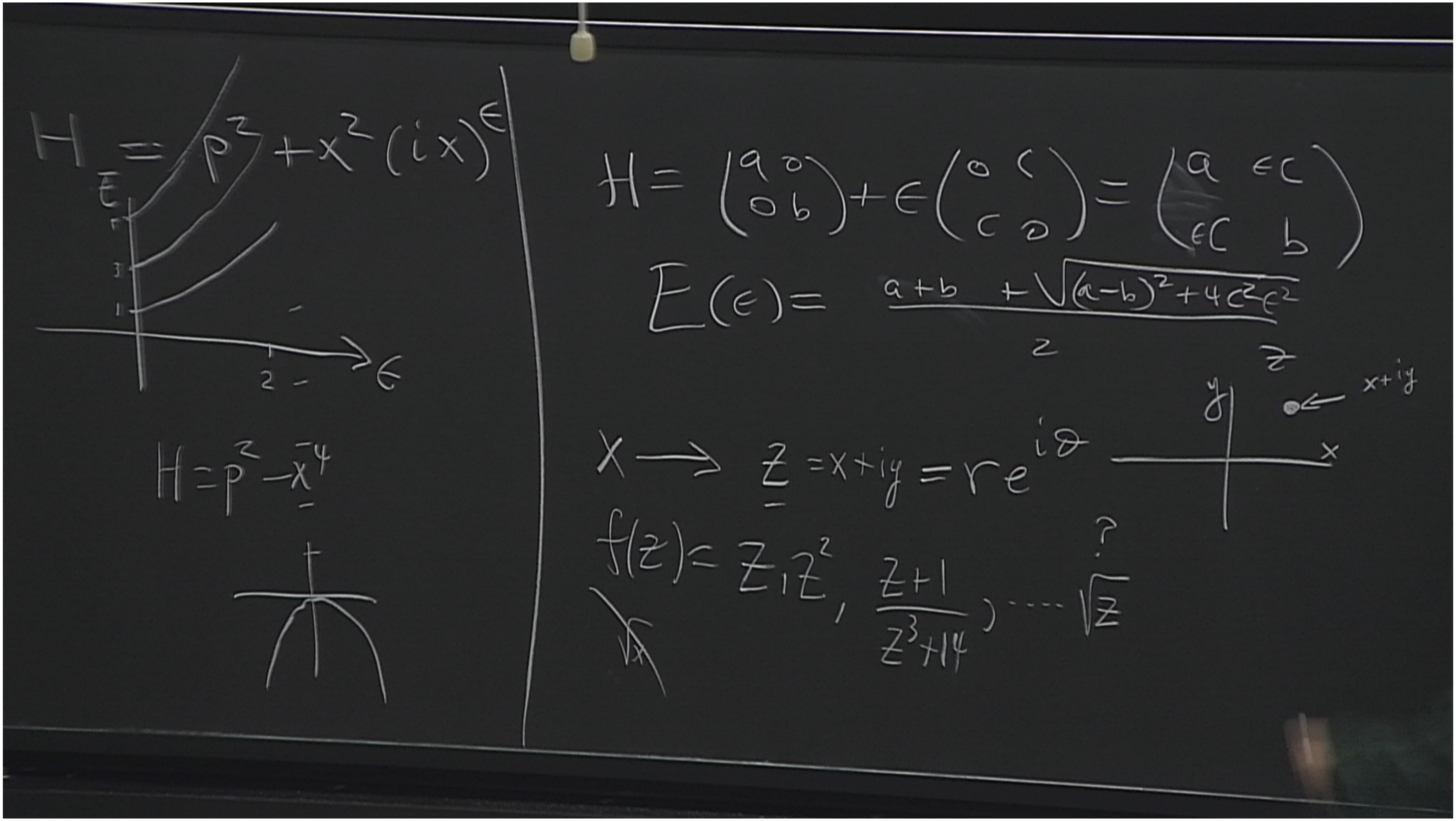
$$y=\sum_{0}^{\infty} y_n(x)\,\epsilon^n$$

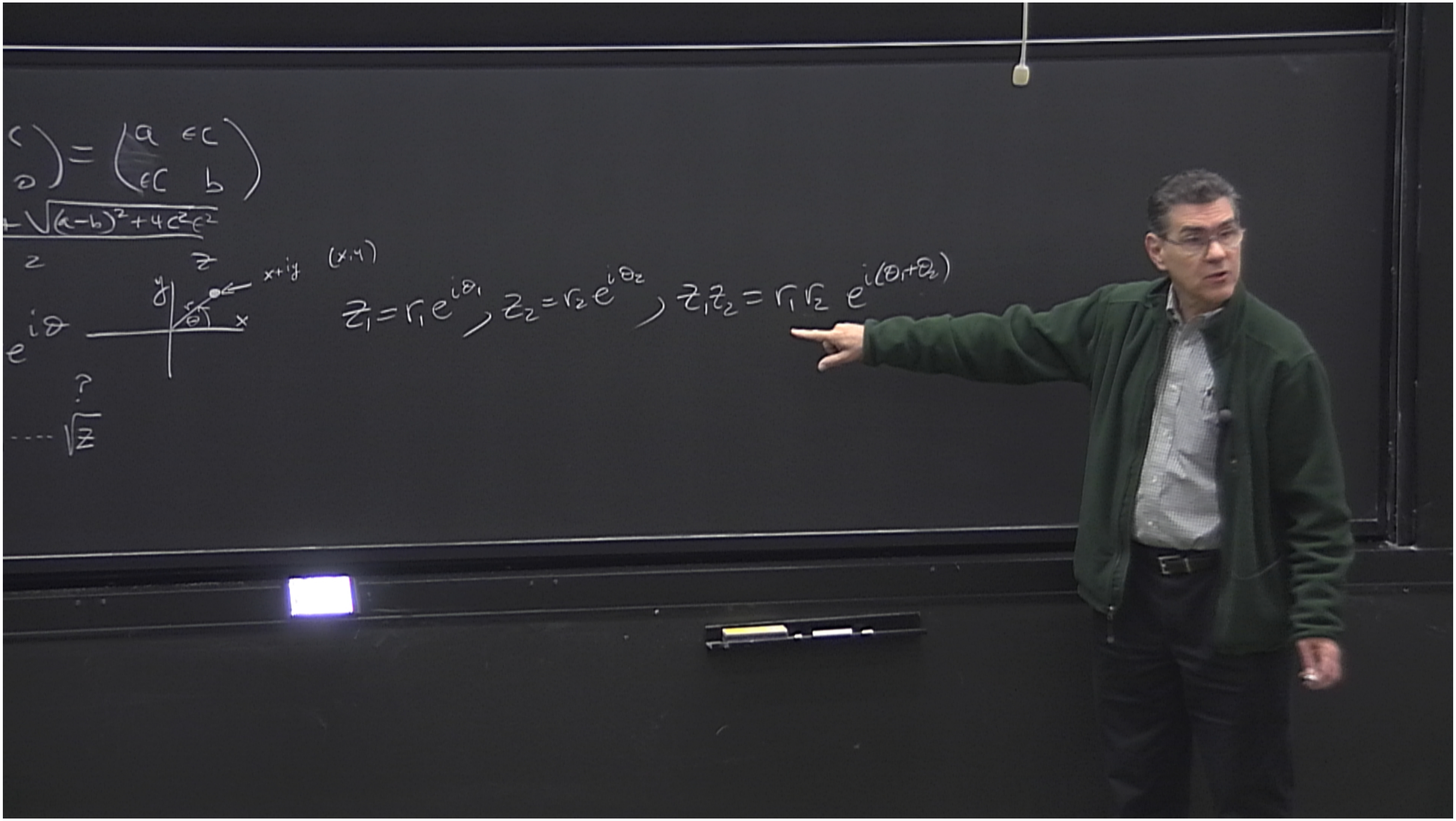
$$y_0=e^{-x}$$

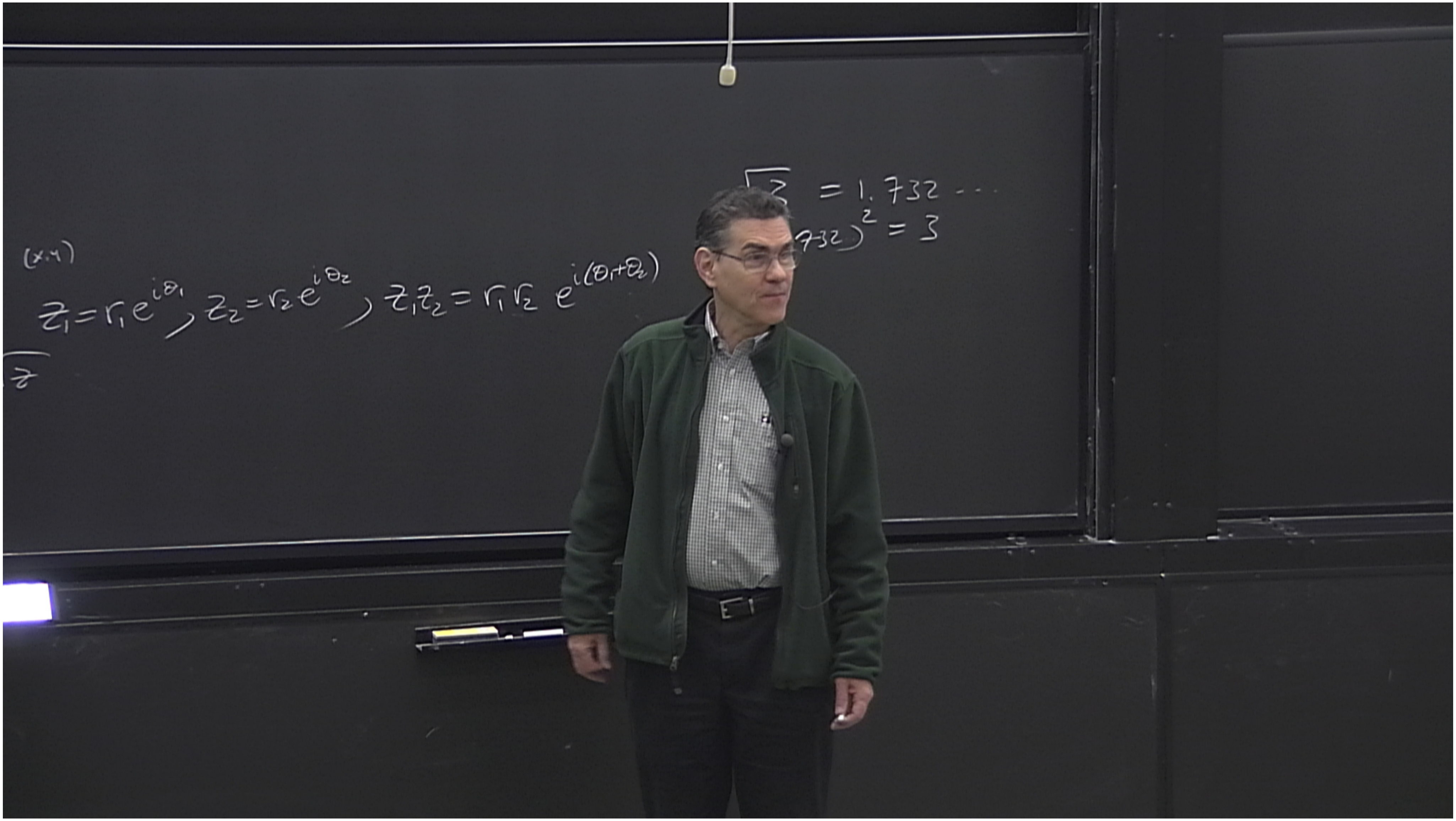
KdV $u_t + u^{\epsilon} u_x + u_{xxx} = 0$

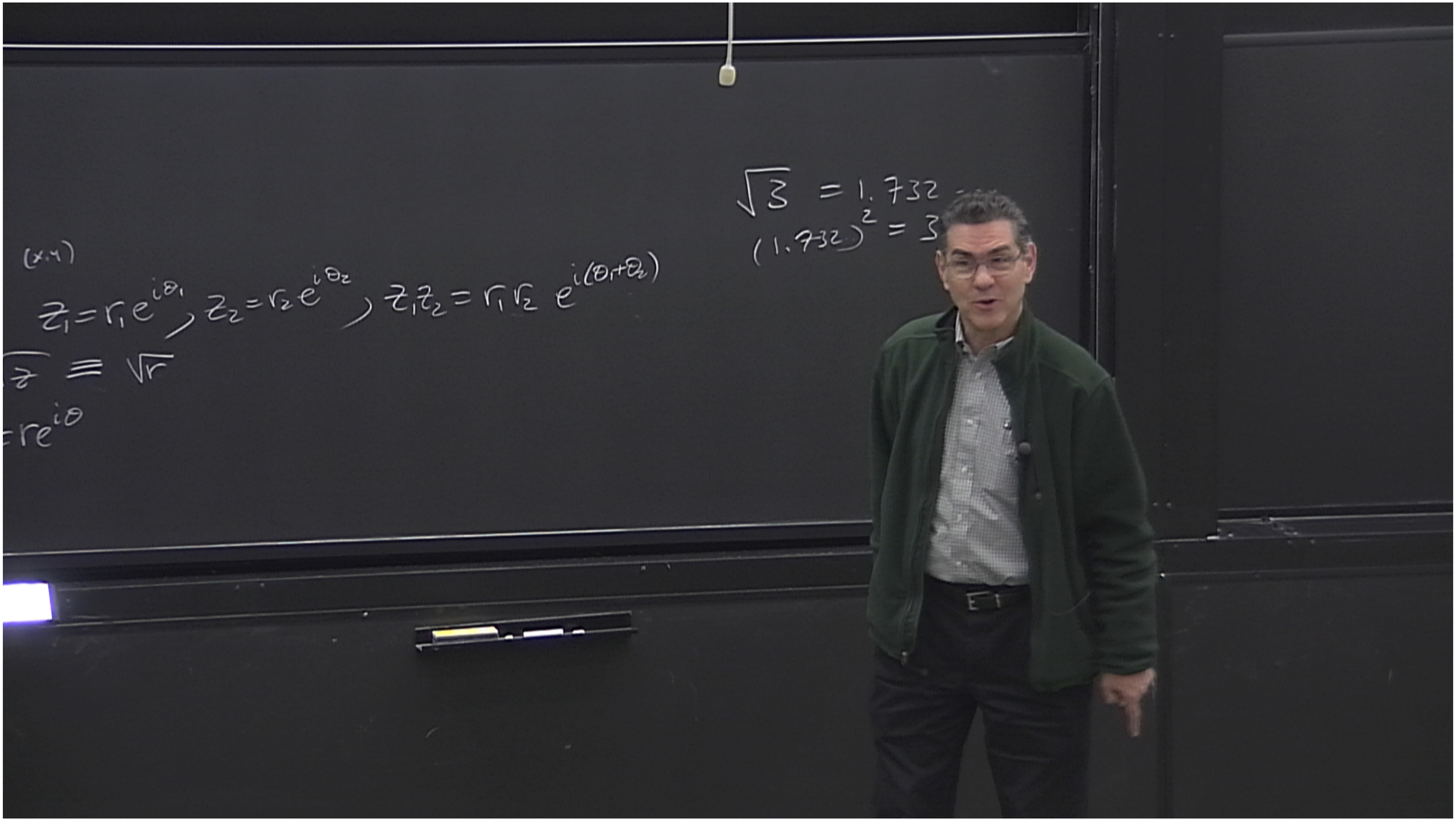

H = p² + x²(ix)^ε
E
ε
H = p² − x⁴

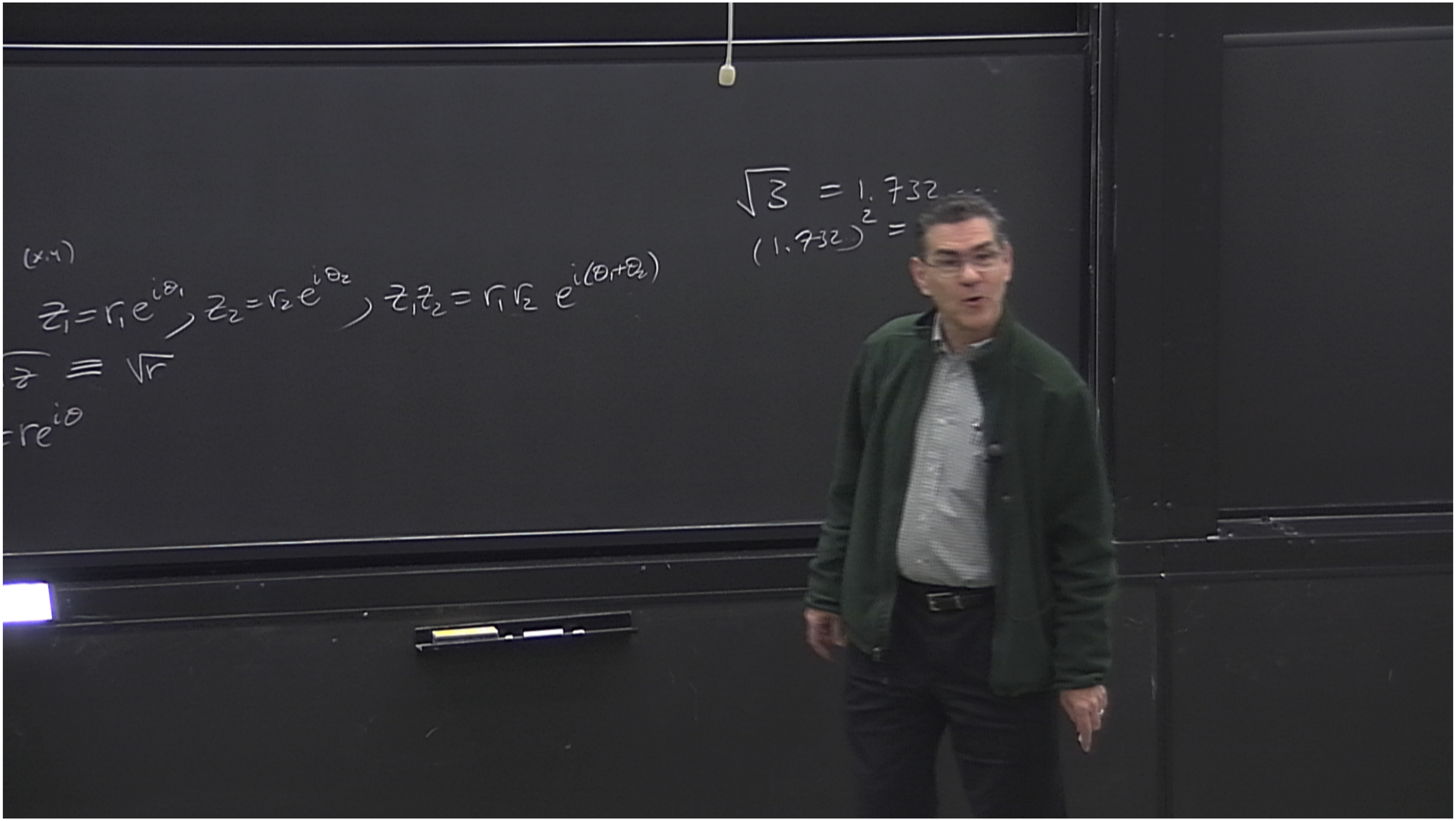

$x^2 (ix)^\epsilon$
$H = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + \epsilon \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} = \begin{pmatrix} a & \epsilon c \\ \epsilon c & b \end{pmatrix}$
$E(\epsilon) = \frac{a+b + \sqrt{(a-b)^2 + 4c^2\epsilon^2}}{2}$
$\rightarrow \epsilon$

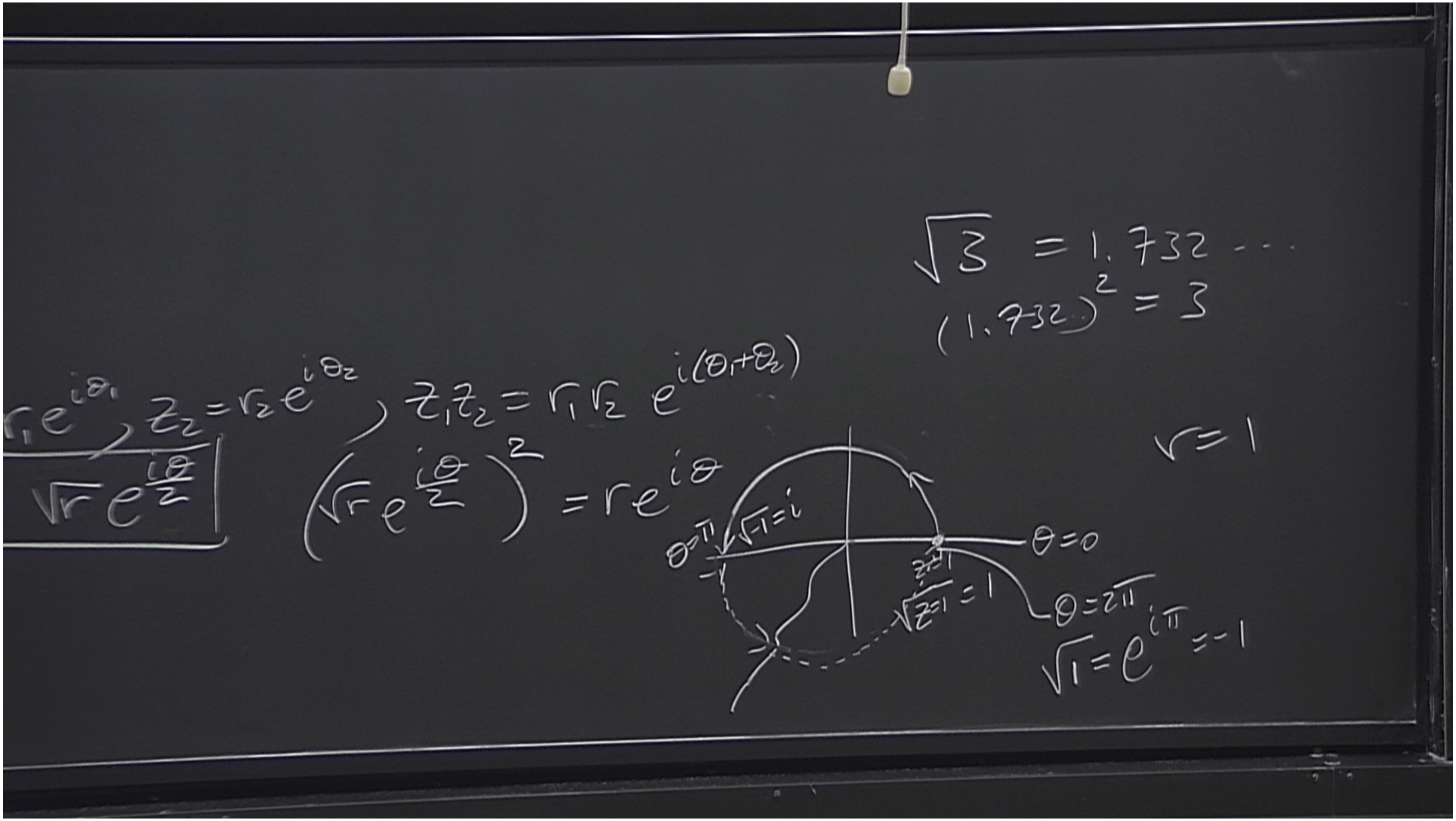

H_ε = p^2 + x^2 (ix)^ε
E
ε
H = p^2 − x^4
H = (a 0 / 0 b) + ε (0 c / c 0) = (a εc / εc b)
E(ε) = (a+b ± √((a−b)^2 + 4c^2 ε^2)) / 2
z
x + iy
y
x
x → z = x+iy = re^{iθ}
f(z) = z, z^2, (z+1)/(z^3+14), ... √z ?
√x


z1=r1 e^iθ1, z2=r2 e^iθ2, z1z2=r1r2 e^i(θ1+θ2)
x+iy (x,y)

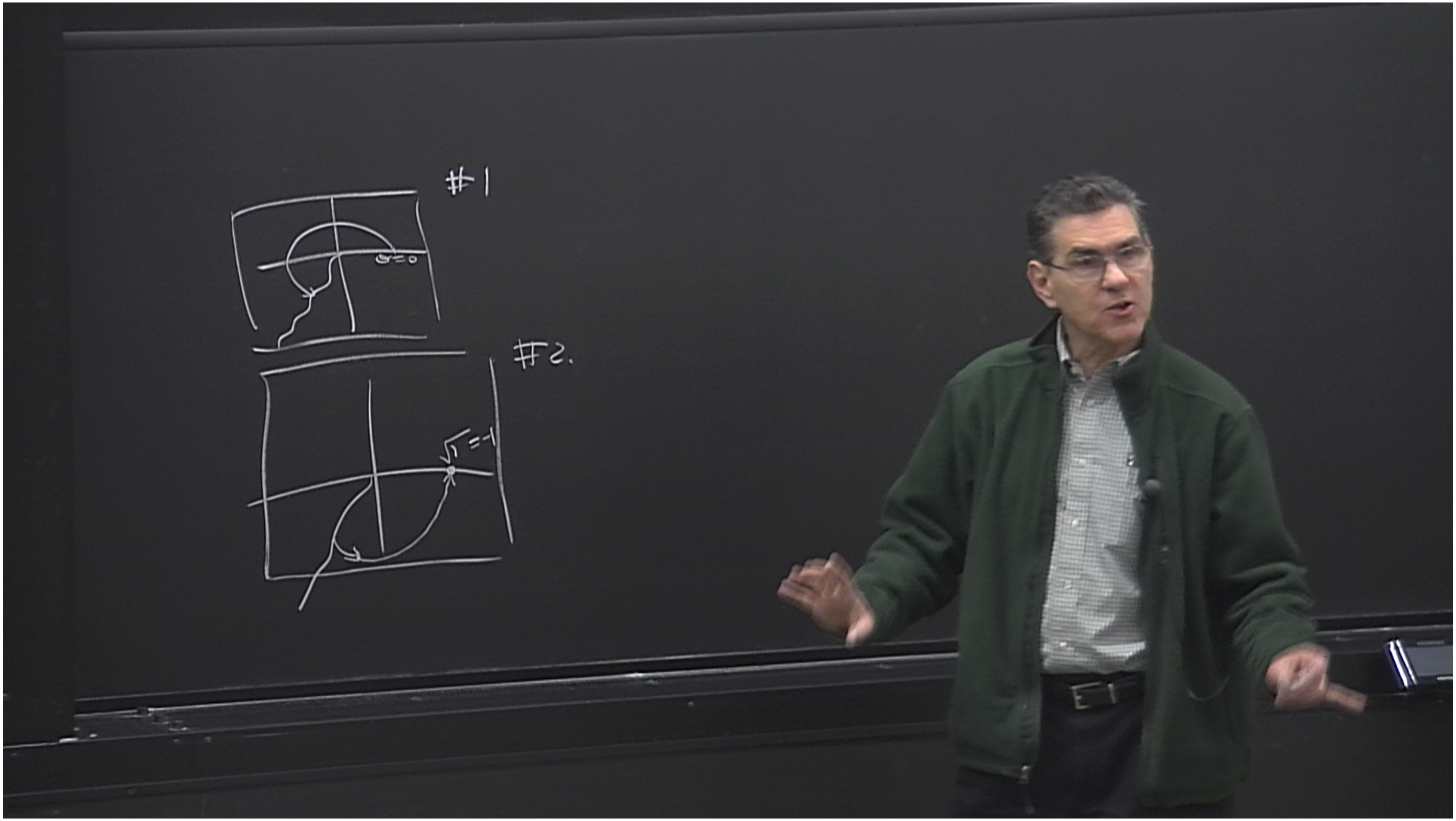

(x,y)
z1=r1e^iθ1, z2=r2e^iθ2, z1z2=r1r2 e^i(θ1+θ2)
= 1.732 ...
= 3

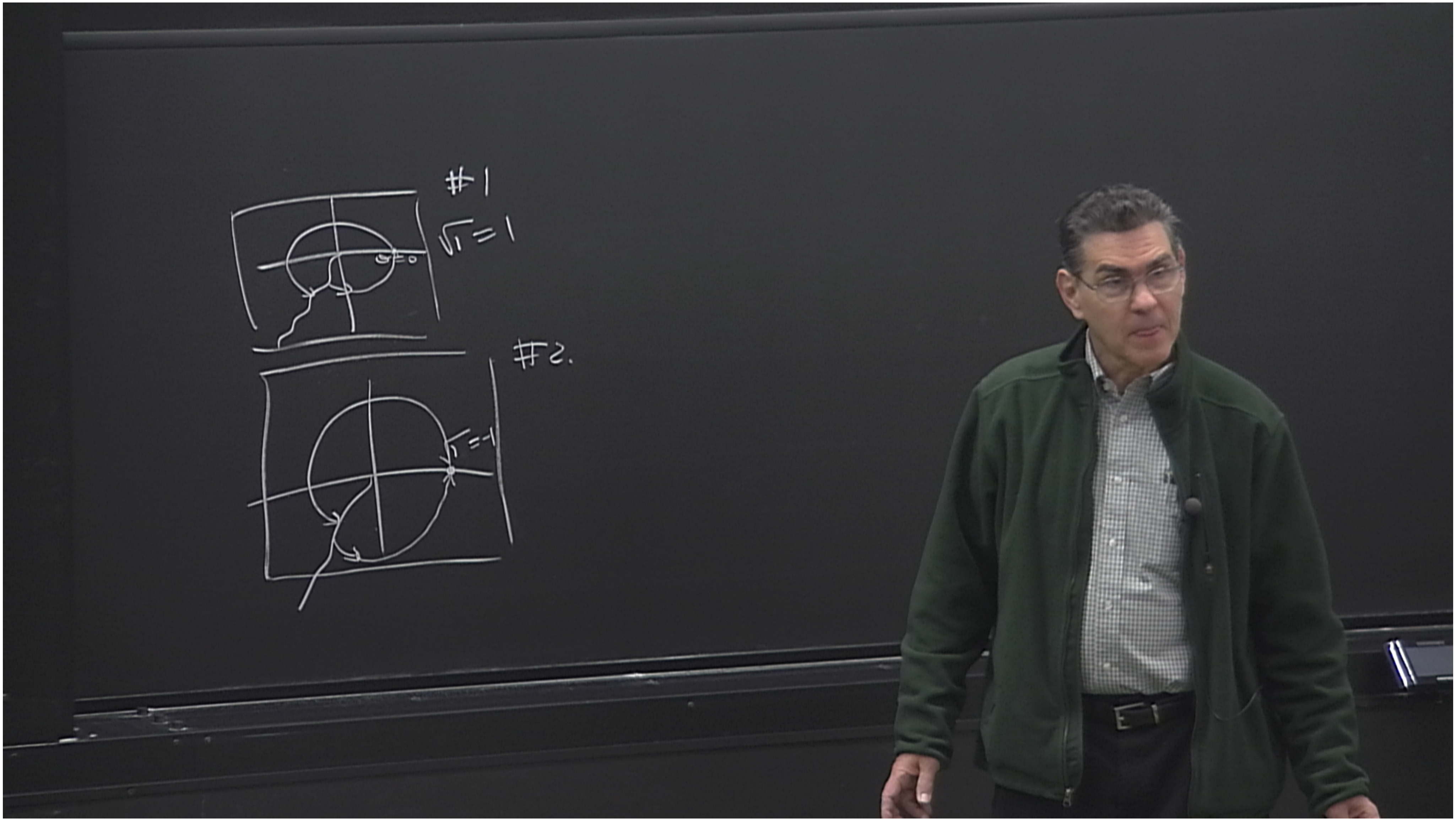

(x,y)
$z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$, $z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$
$\sqrt{z} \equiv \sqrt{r}$
$= re^{i\theta}$
$\sqrt{3} = 1.732$
$(1.732)^2 = 3$

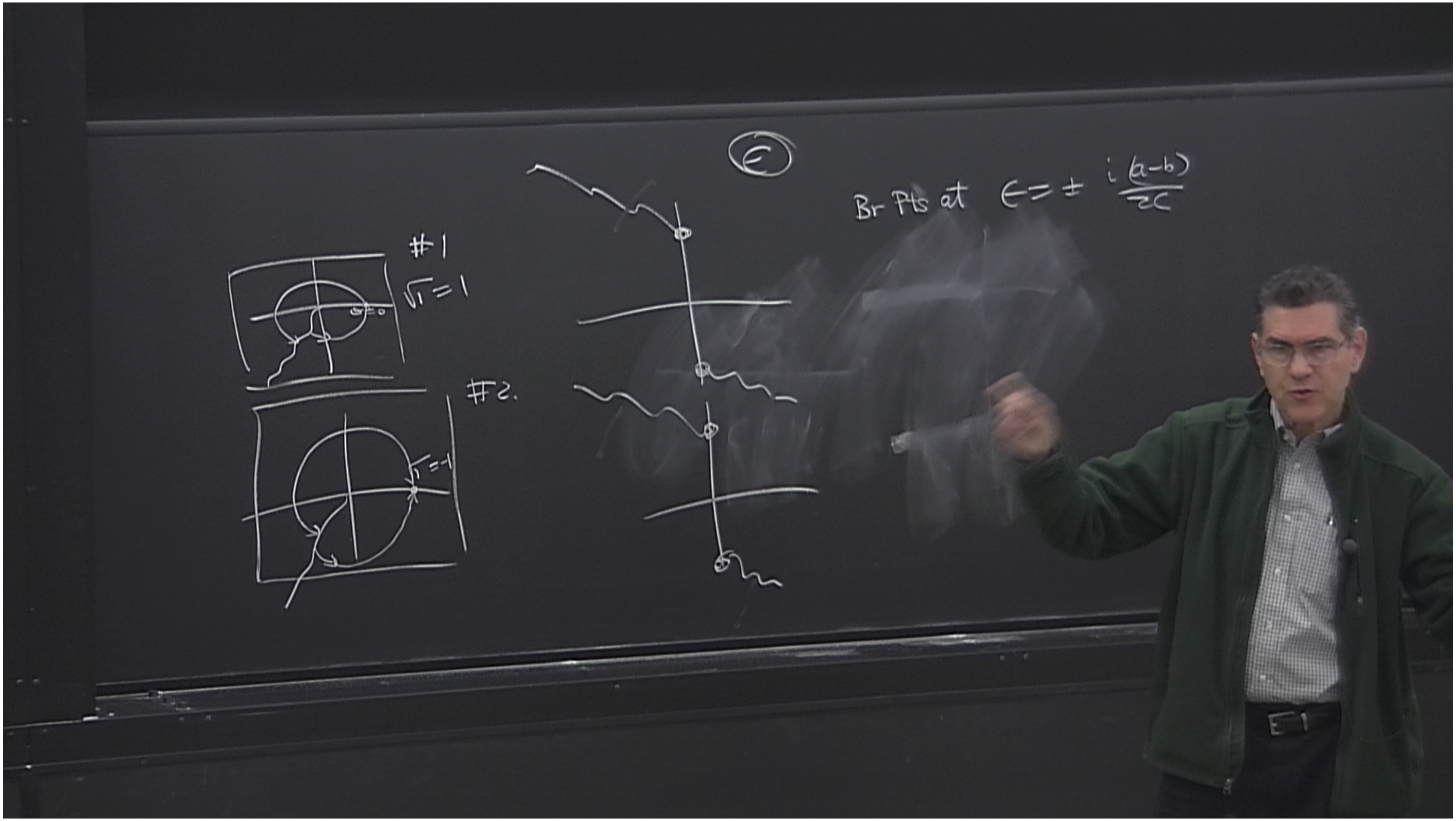

(x,y)
z1 = r1 e^iθ1, z2 = r2 e^iθ2, z1z2 = r1r2 e^i(θ1+θ2)
√3 = 1.732 ...
(1.732)^2 =



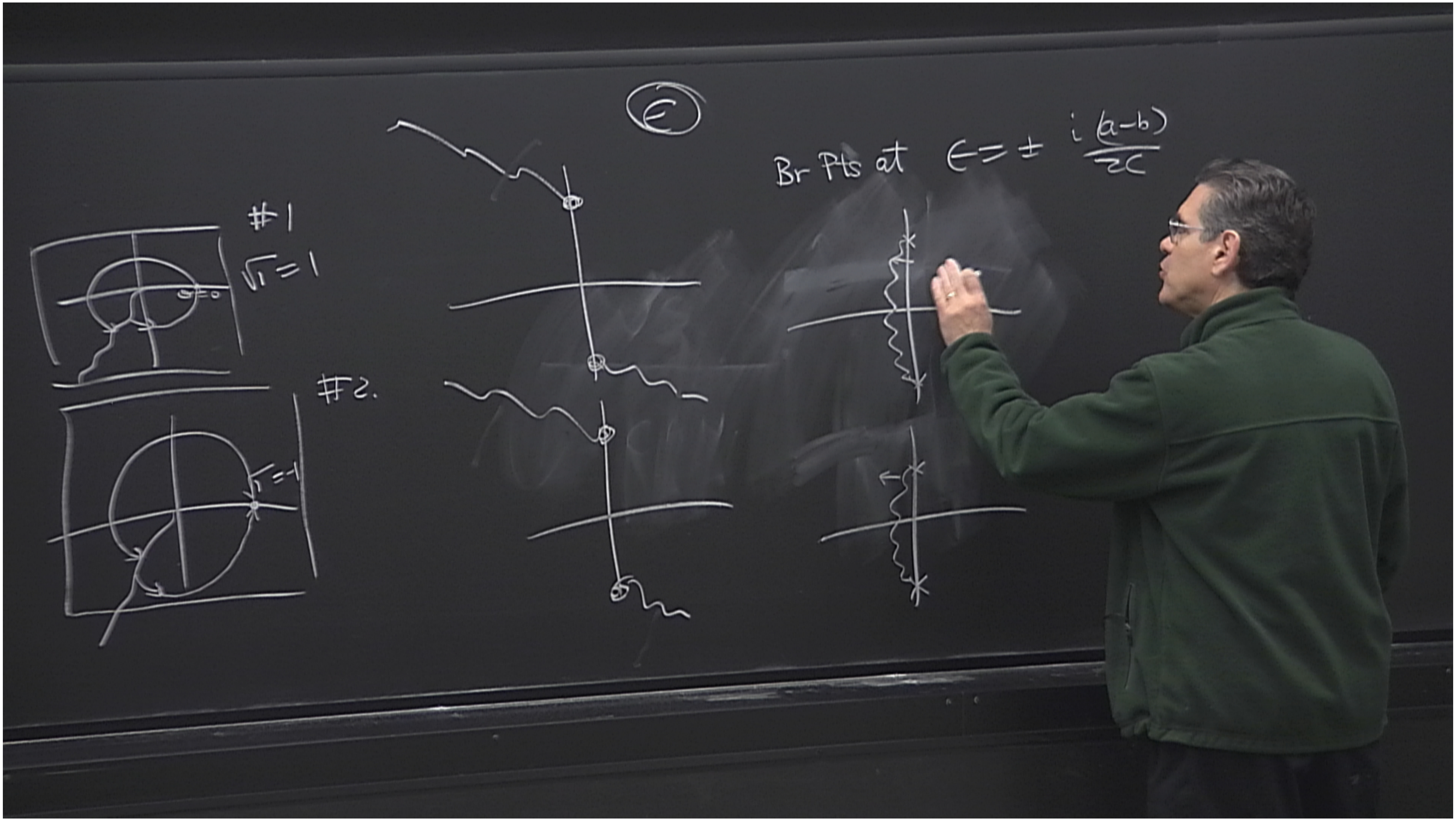
√3 = 1.732 ...
(1.732...)² = 3
r₁e^{iθ₁}, z₂ = r₂e^{iθ₂}, z₁z₂ = r₁r₂ e^{i(θ₁+θ₂)}
√r e^{iθ/2}
(√r e^{iθ/2})² = re^{iθ}
r = 1
θ = 0
θ = 2π
√-1 = i
√1 = e^{iπ} = -1

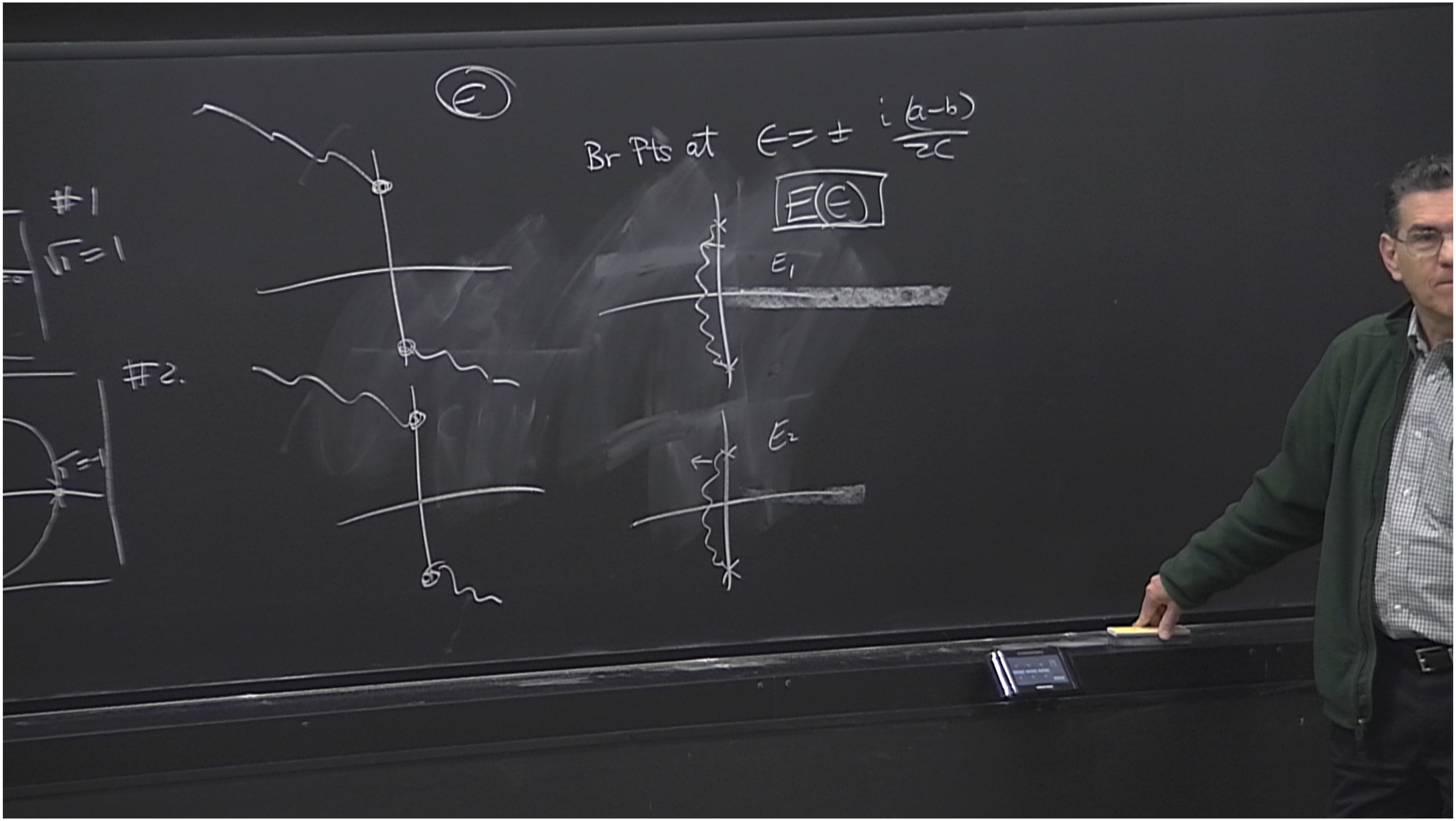

#1
#2.

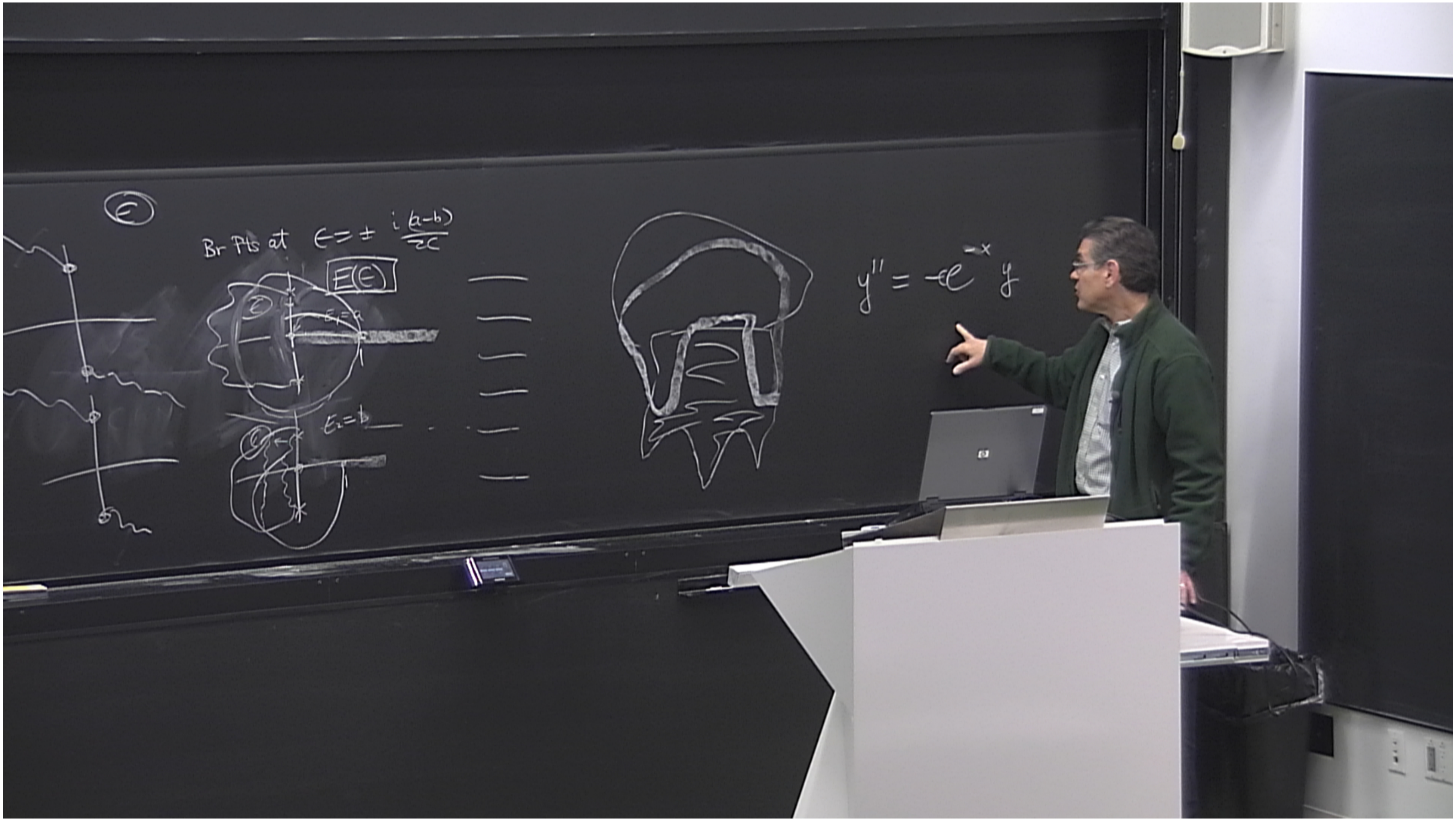

#1
#2.

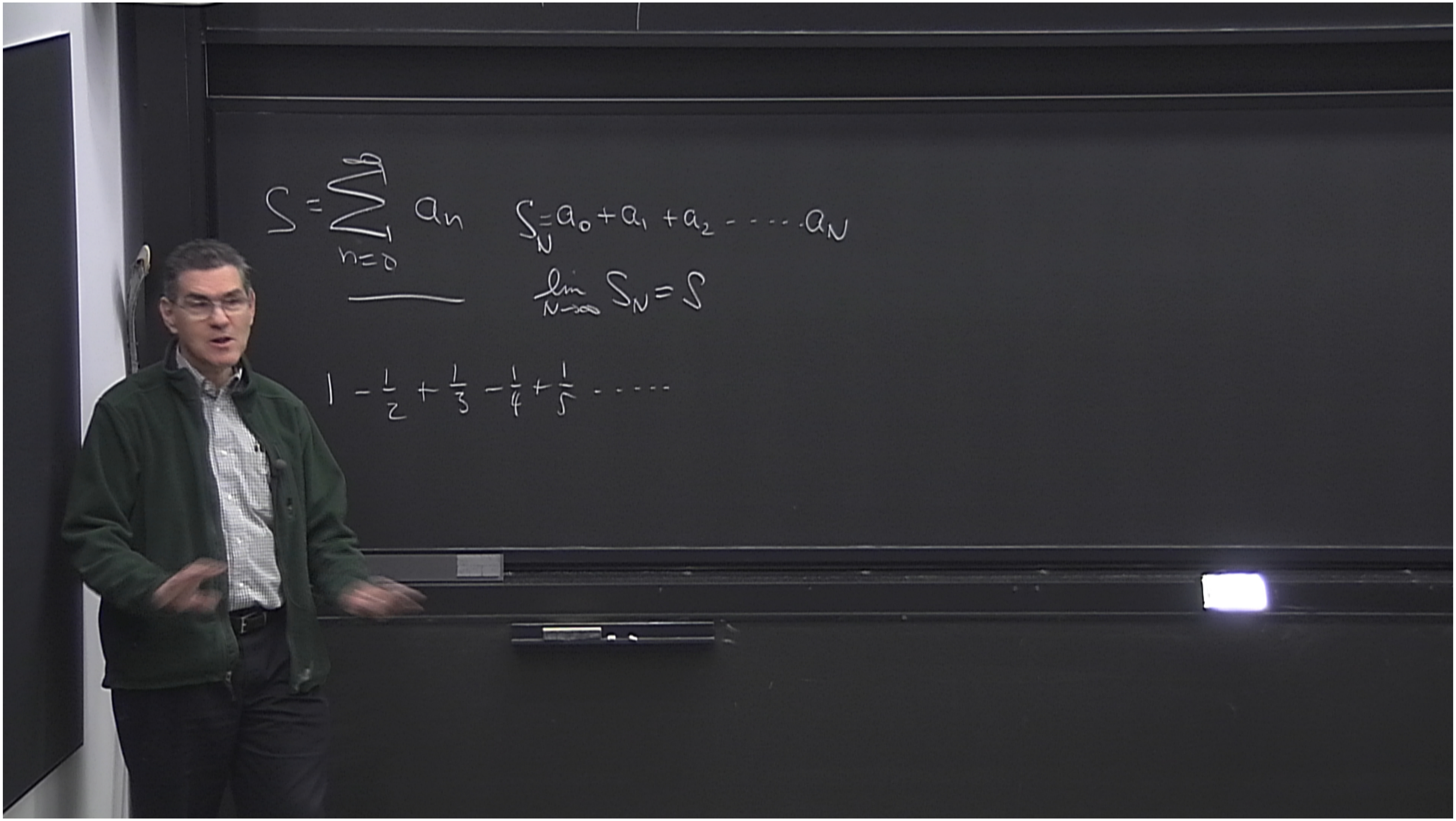


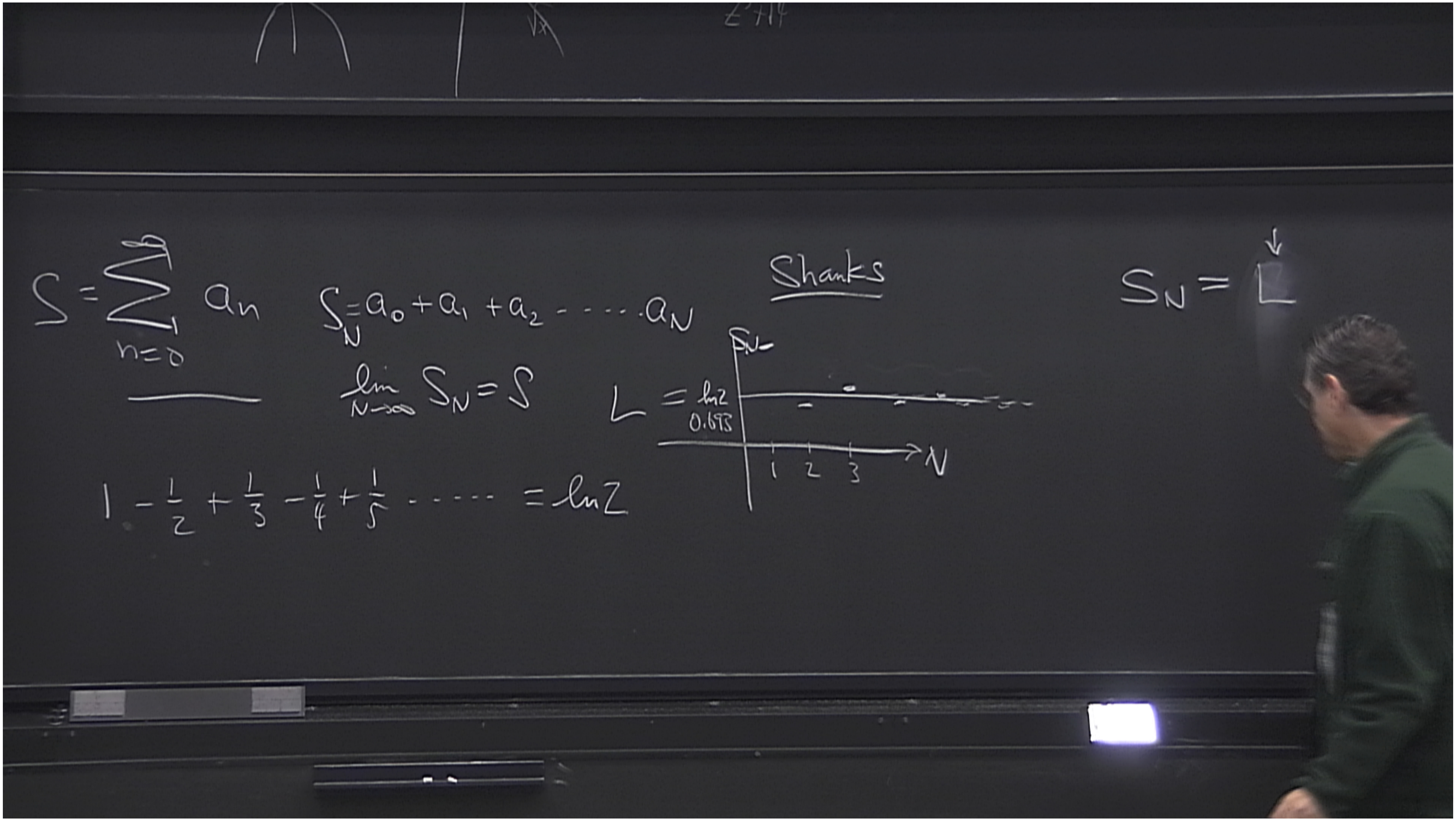
Br Pts at $\epsilon = \pm \frac{i(a-b)}{2c}$
#1
$\sqrt{1} = 1$
#2.

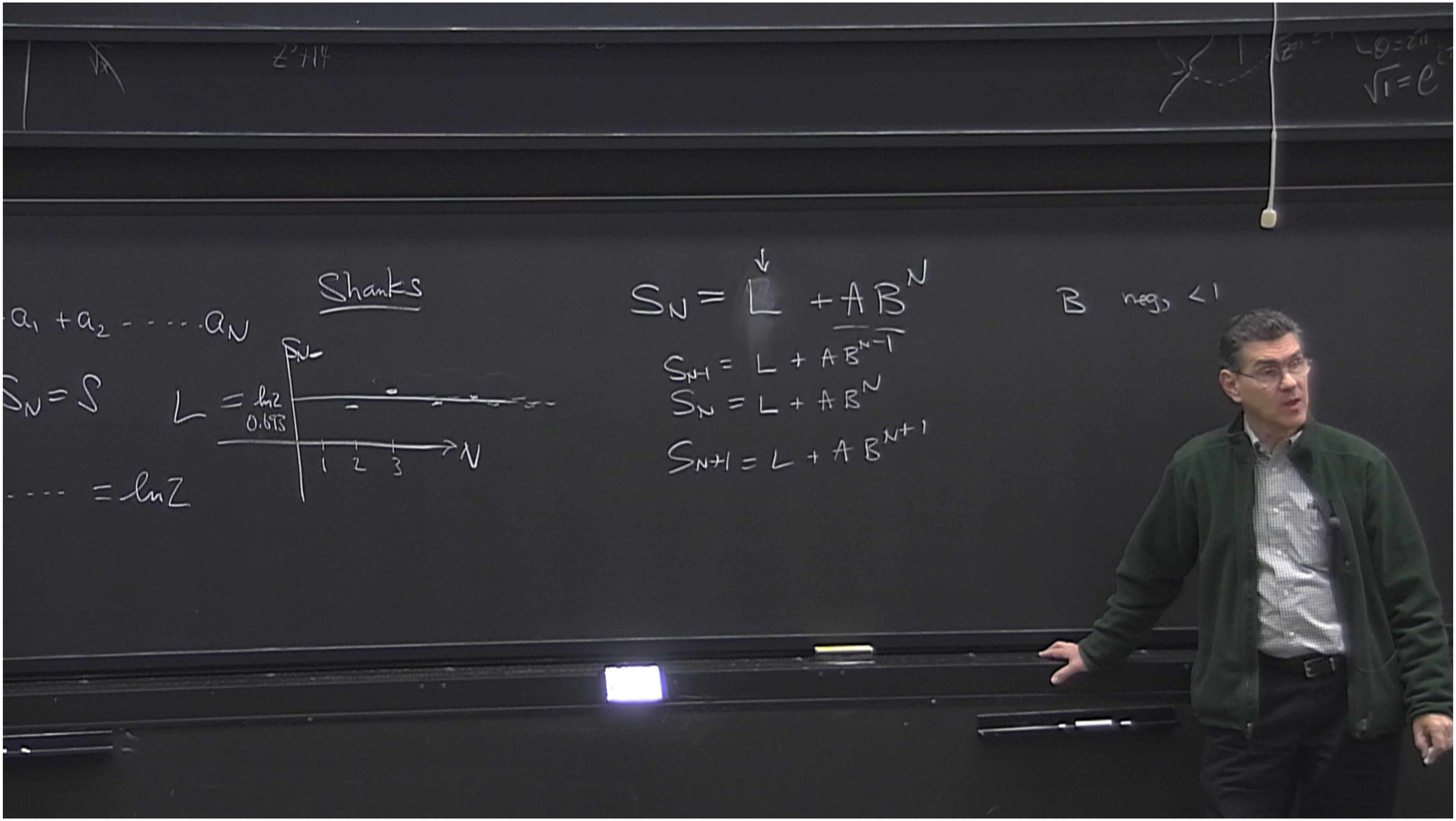

ϵ
Br Pts at ϵ = ± i(a−b)/2c
#1
√1 = 1
#2.
√1 = −1

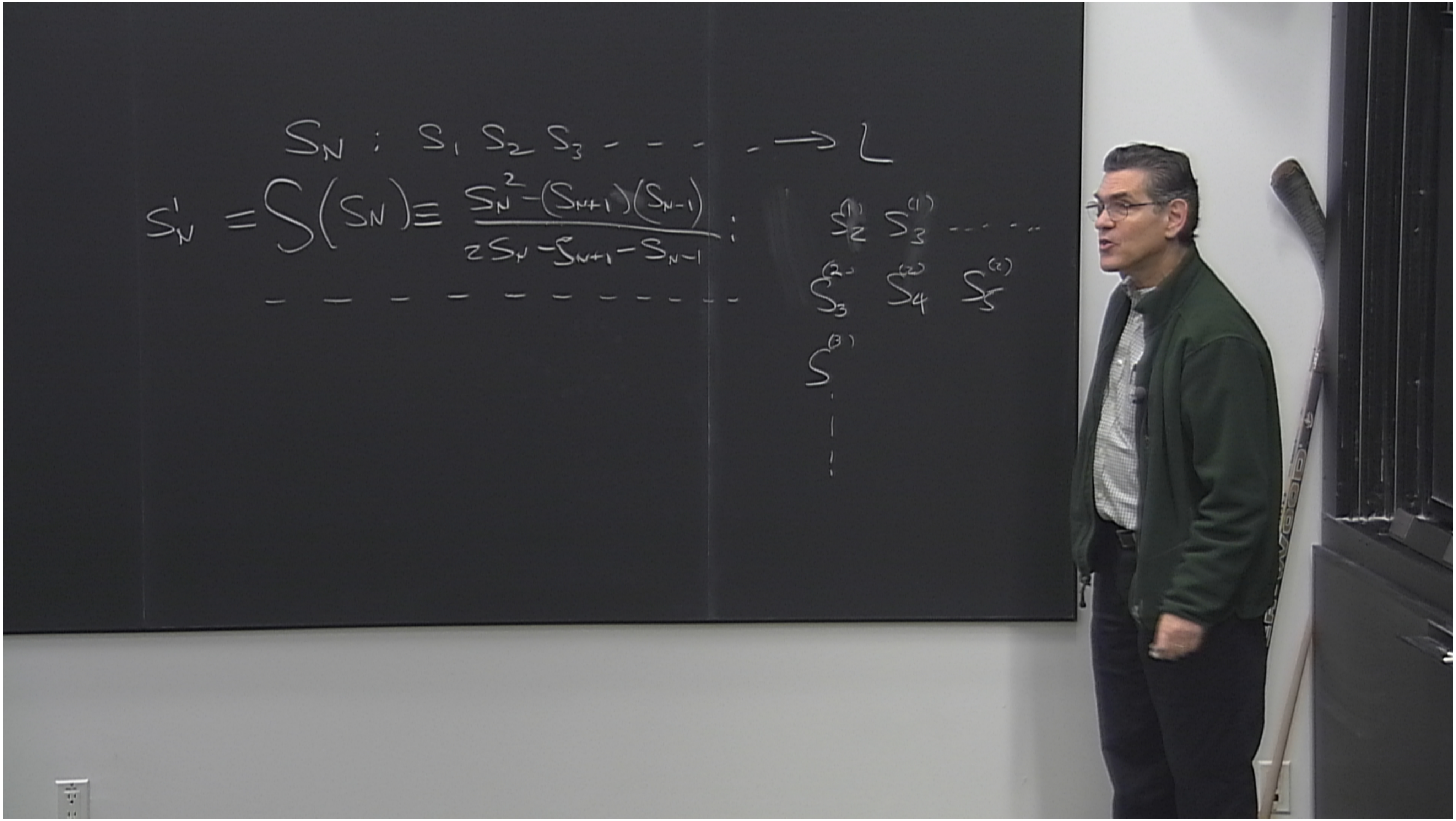

ε
Br Pts at ε = ± i(a-b)/2c
E(ε)
E1
E2
#1
√1 = 1
#2.


Br Pts at ϵ = ± i(a−b)/2c
E(ϵ)
ϵ₁=a
ϵ₂=b
y'' = −ϵe^{−x} y


S = Σ a_n, n=0 to ∞
S_N = a_0 + a_1 + a_2 - - - - a_N
lim S_N = S, N→∞
1 - 1/2 + 1/3 - 1/4 + 1/5 - - - - -


S = Σ a_n (n=0 to ∞)
S_N = a_0 + a_1 + a_2 ..... a_N
lim S_N = S (N→∞)
1 - 1/2 + 1/3 - 1/4 + 1/5 ..... = ln2
Shanks
S_N
L = ln2
0.693
1 2 3
N
S_N = L


Shanks
S_N = L + A B^N
B neg, < 1
S_{N-1} = L + A B^{N-1}
S_N = L + A B^N
S_{N+1} = L + A B^{N+1}
S_N = S
L = ln2
0.693
= ln2

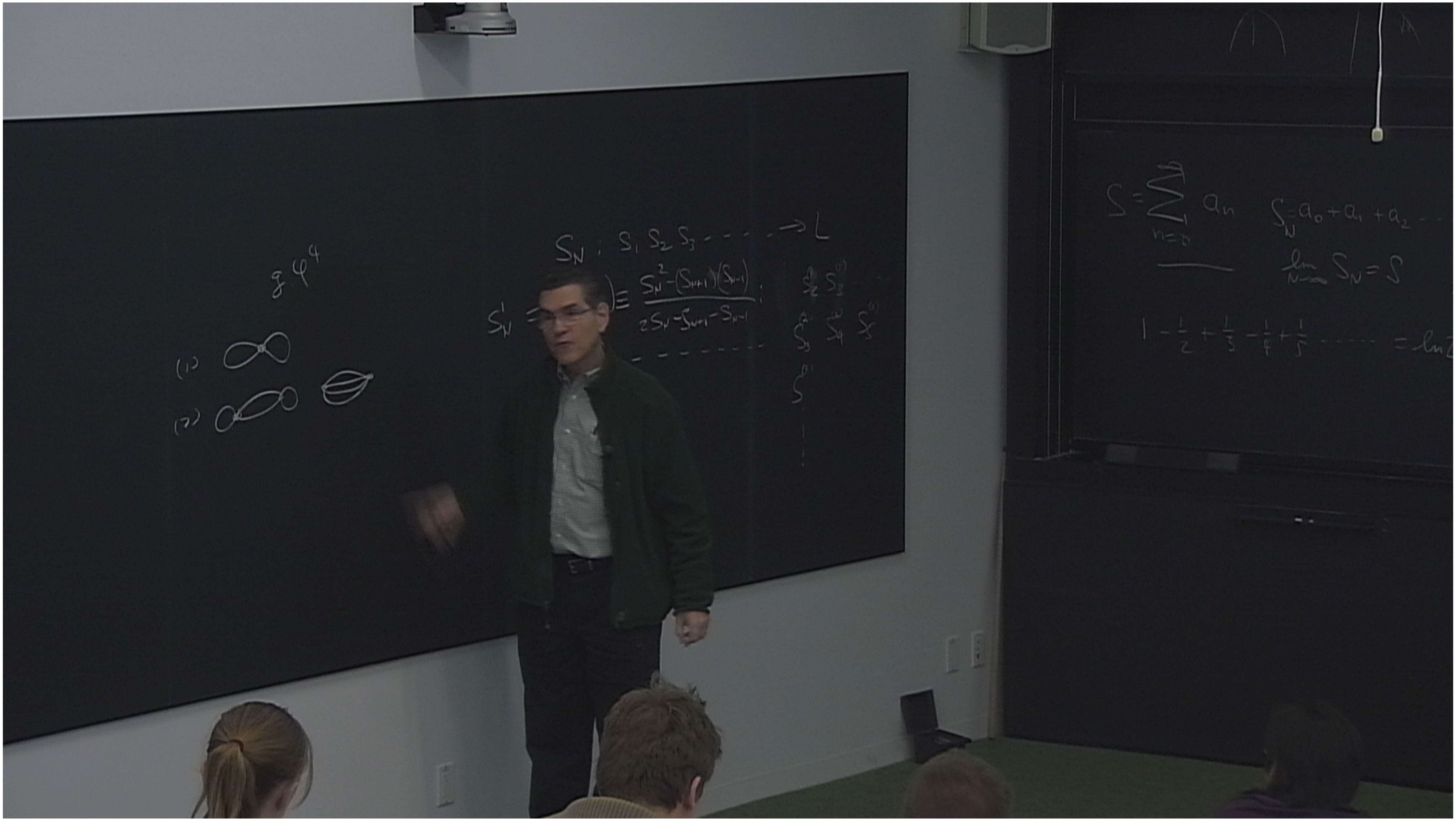

$S_N : S_1\ S_2\ S_3 \cdots \to L$
$S'_N = S(S_N) \equiv \frac{S_N^2 - (S_{N+1})(S_{N-1})}{2S_N - S_{N+1} - S_{N-1}}$;
$S_2^{(1)}\ S_3^{(1)} \cdots$
$S_3^{(2)}\ S_4^{(2)}\ S_5^{(2)}$
$S^{(3)}$

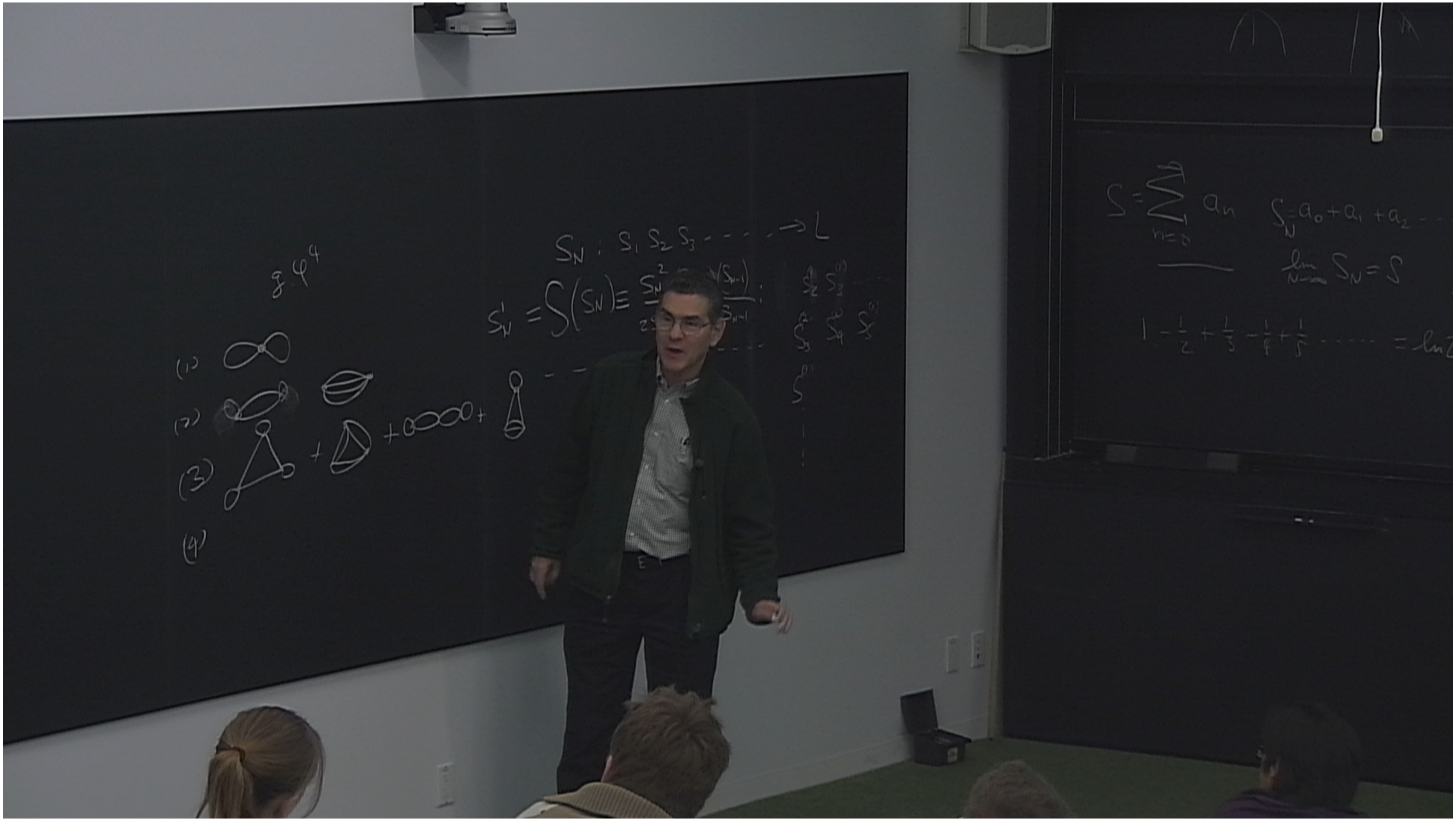
Shanks transform of $\log 2 = 1 - 1/2 + 1/3 - 1/4 \ldots = 0.693\ 147\ 2\ldots$

n	A	S(A)	S(S(A))	S(S(S(A)))
1	**1.000 000 0**			
2	**0.500 000 0**	**0.700 000 0**		
3	**0.833 333 3**	**0.690 476 2**	**0.693 277 3**	
4	**0.583 333 3**	**0.694 444 4**	**0.693 105 8**	**0.693 148 9**
5	**0.783 333 3**	**0.692 424 2**	**0.693 163 3**	**0.693 146 7**
6	**0.616 666 7**	**0.693 589 7**	**0.693 139 9**	
7	**0.759 523 8**	**0.692 857 1**		
8	**0.634 523 8**			

Shanks transform of 1-1/√2+1/√3-1/√4…=0.604 898 6…

n	A	S(A)	S(S(A))	S(S(S(A)))
1	**1.000 000 0**			
2	**0.292 893 2**	**0.610 730 5**		
3	**0.870 243 5**	**0.602 294 3**	**0.605 015 6**	
4	**0.370 243 5**	**0.606 311 5**	**0.604 858 5**	**0.604 900 3**
5	**0.817 457 1**	**0.604 035 3**	**0.604 915 5**	
6	**0.409 208 8**	**0.605 470 4**		
7	**0.787 173 3**			


$S_N : S_1\ S_2\ S_3 \cdots \to L$
$g\,\varphi^4$
$\lim_{N\to\infty} S_N = S$
$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \cdots = \ln 2$


g φ⁴
S_N : S_1 S_2 S_3 - - - - - → L
lim N→∞ S_N = S
1 - 1/2 + 1/3 - 1/4 + 1/5 - - - - - = ln2