

Title: Mathematical Physics - Lecture 2

Date: Nov 22, 2011 09:00 AM

URL: <http://pirsa.org/11110041>

Abstract:

EASY $y' + a(x)y = b(x)$

$$y'' + a(x)y' + b(x)y = 0$$

$$y = U(z) z(x)$$

$$U''z + Uz'' + \cancel{2U'z'} + aU'z + \cancel{aUz'} + bUz = 0$$

$$2U' + aU = 0$$

$$\frac{U'}{U} + \frac{a}{z} = 0$$

$$\ln U + \int \frac{a(s) ds}{z} = 0$$

$$U = e^{-\int \frac{a(s) ds}{z}}$$

$$y'' + Q(x)y = 0$$

$$y'' + a(x)y' + b(x)y = 0$$

$$y'' + Q(x)y = 0$$

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$$y = U(x) z(x)$$

$$U''z + U z'' + 2U'z' + aU'z + aUz'$$

$$+ bUz = 0$$

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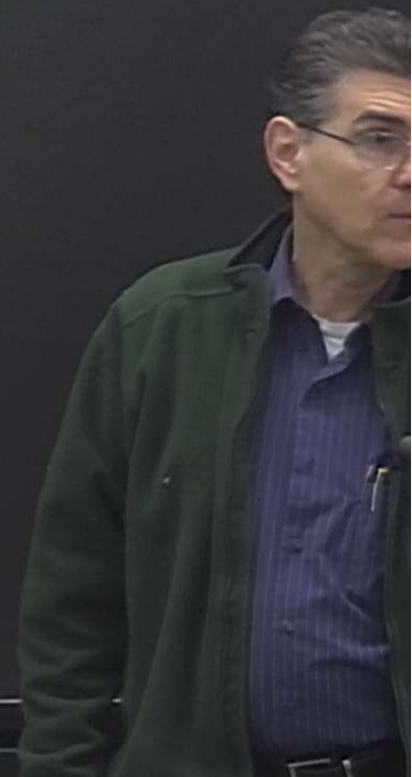
$$\frac{U'}{U} + \frac{a}{2} = 0$$

$$\ln U + \int \frac{a(s)}{2} ds = 0 \quad U = e^{-\int \frac{a(s)}{2} ds}$$

$$y'' + Q(x)y = 0$$

$$\frac{d}{dx} = D$$

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$$(D^2 + a(x)D + b(x))y(x) = 0$$
$$(D + A(x))(D + B(x))y(x) = 0$$



$$y'' - y = 0$$
$$(D^2 - 1)y = 0$$

$$\frac{d}{dx} = D$$
$$(D^2 + a(x)D + b(x))y(x) = 0$$
$$(D + A(x))(D + B(x))y(x) = 0$$



$$y = 0$$

$$y'' - y = 0$$

$$(D^2 - 1)y = 0$$

$$\left\{ (D-1)(D+1)y = 0 \right.$$

$$\left. (D - \tanh x)(D + \tanh x)y = 0 \right.$$

$$\frac{d}{dx} = D$$

$$(D^2 + a(x)D + b(x))y(x) = 0$$

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$$\left\{ \begin{array}{l} (D-1)(D+1)y = 0 \\ (D - \tanh x)(D + \tanh x)y = 0 \end{array} \right.$$

$$\frac{d}{dx} = D$$

$$\left(D^2 + a(x)D + b(x) \right) y(x) = 0$$

$$\rightarrow \underbrace{(D + A(x))(D + B(x))}_{W(x)} y(x) = 0$$

$$W' + A(x)W = 0$$

$W \checkmark$

$$y' + By = W$$

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$$y'' + Q(x)y = 0$$

$$y' - y = 0$$

$$(D^2 - 1)y = 0$$

$$(D-1)(D+1)y = 0$$

$$(D+1)y = 0$$

$$\frac{d}{dx} = D$$

$$\left[\begin{array}{l} D^2 + a(x)D \\ \rightarrow (D+A(x))(D+B(x)) \end{array} \right]$$

$$W' + A(x)W = 0$$

$$W = \int$$

$$y' +$$

$$y'' - y = 0$$

$$(x^2 - 1)y = 0$$

$$(D+1)y = 0$$

$$\cosh(x)(D + \tanh(x))y = 0$$

$$\frac{d}{dx} = D$$

$$\left(D^2 + a(x)D + b(x) \right) y(x) = 0$$

$$\left(\underbrace{D + A(x)}_{\text{mim}} \right) \underbrace{\left(D + B(x) \right)}_{W(x)} y(x) = 0$$

$$W' + A(x)W = 0$$

$$W \checkmark$$

$$y' + By = W$$

$$\begin{aligned}
 & +b(x) \Big) y(x) = 0 \\
 \underbrace{(D + B(x))}_{w(x)} y(x) = 0 & \implies (D^2 + AD + AB + BD) y = 0 \\
 & \underbrace{\hspace{10em}}_{D^2 + A}
 \end{aligned}$$

$v = 0$
 $-By = W$

$$+b(x) y(x) = 0$$

$$\underbrace{(D + B(x)) y(x) = 0}_{w(x)} \implies \begin{aligned} &(D^2 + AD + AB + B' + BD) y = 0 \\ &(D^2 + (A+B)D + AB + B') y = 0 \end{aligned}$$

$$N=0$$

$$A+B = a(x) \quad AB + B' = b$$

$$-By = W$$

$$y(x) = 0$$

$$y(x) = 0 \implies (D^2 + AD + AB + B' + BD)y = 0$$
$$(D^2 + (A+B)D + AB + B')y = 0$$

$$A+B = a(x)$$
$$A = a - B$$
$$AB + B' = b$$
$$aB - B^2 + B' = b$$

$$y(x) = 0$$

$$y(x) = 0 \implies (D^2 + AD + AB + B' + BD)y = 0$$
$$(D^2 + (A+B)D + AB + B')y = 0$$

$$A+B = a(x)$$

$$A = a - B$$

$$AB + B' = b$$

$$aB - B^2 + B' = b$$

$$y' = \alpha y^2 + \beta y + \gamma$$

$$y = Q \frac{w'}{w}$$

$$Q' \frac{w'}{w} + Q \frac{w''}{w} - Q \frac{w'^2}{w^2} = \alpha Q^2$$

$$y' = \alpha y^2 + \beta y + \gamma$$

$$y = Q \frac{w'}{w}$$

$$Q' \frac{w'}{w} + Q \frac{w''}{w} - Q \frac{w'^2}{w^2} = \alpha Q^2 \frac{w'^2}{w^2} + \beta Q \frac{w'}{w} + \gamma$$

$$y' = \alpha y^2 + \beta y + \gamma$$

$$y = Q \frac{w'}{w}$$

$$Q' \frac{w'}{w} + Q \frac{w''}{w} - Q \frac{w' w''}{w^2} = \alpha \frac{Q^2 w'^2}{w^2} + \beta Q \frac{w'}{w} + \gamma$$

$$-Q = \alpha Q^2$$

$$Q = -\frac{1}{\alpha}$$

$$y' = \alpha y^2 + \beta y + \gamma$$

$$y = Q \frac{w'}{w}$$

$$Q' \frac{w'}{w} + Q \frac{w''}{w} - \cancel{Q \frac{w'^2}{w^2}} = \alpha \cancel{Q^2 \frac{w'^2}{w^2}} + \beta Q \frac{w'}{w} + \gamma$$

$$-Q = \alpha Q^2$$

$$Q = -\frac{1}{\alpha}$$

$$Q' w' + Q w'' = \beta Q w' + \gamma w$$

$$B' = 1B^2 - aB + b$$

Riccati

$$B = -\frac{w'}{w}$$

$$\frac{w'}{w} + \frac{\cancel{w'^2}}{\cancel{w^2}} = \frac{\cancel{w'^2}}{\cancel{w^2}} + a\frac{w'}{w} + b$$

$$Q' \frac{w'}{w} +$$

HIP 1

$$y'' + Q(x)y = 0$$

$$y(0) = \alpha \quad y'(0) = \beta$$

Unpert: $y_0'' = 0$

$$y_0(x) = \alpha + \beta x$$

$$y(x) = \sum_{n=0}^{\infty} a_n(x) \epsilon^n$$

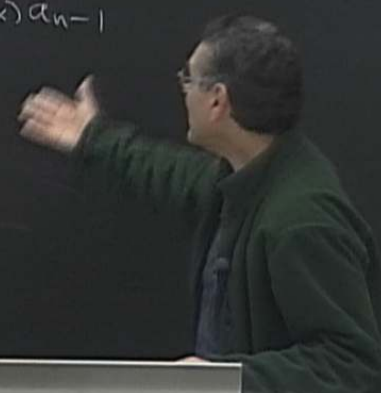
$$a_0(x) = \alpha + \beta x$$

$$\sum_{n=0}^{\infty} a_n''(x) \epsilon^n + \sum_{n=0}^{\infty} \sum_{m=n-1}^{\infty} Q(x) a_m(x) \epsilon^{m+1} = 0$$

$$\sum_{n=1}^{\infty} Q(x) a_{n-1}(x) \epsilon^n = 0$$

$$\epsilon^0 \quad a_0'' = 0 \quad a_0 = \alpha + \beta x$$

$$\epsilon^n \quad n \geq 0 \quad a_n'' = -Q(x) a_{n-1}$$



HP 1

$$y'' + Q(x)y = 0$$

$$y(0) = \alpha \quad y'(0) = \beta$$

Unpert: $y_0'' = 0$

$$y_0(x) = \alpha + \beta x$$

$$y(x) = \sum_{n=0}^{\infty} a_n(x) \epsilon^n$$

$$a_0(x) = \alpha + \beta x$$

$$\sum_{n=0}^{\infty} a_n''(x) \epsilon^n + \sum_{n=0}^{\infty} \underbrace{Q(x) a_n(x)}_{n \rightarrow n-1} \epsilon^n = 0$$

$$\sum_{n=1}^{\infty} Q(x) a_{n-1}(x) \epsilon^n = 0$$

$$\epsilon^0 \quad a_0'' = 0 \quad a_0 = \alpha + \beta x$$

$$\begin{matrix} a_n(0) = 0 \\ a_n'(0) = 0 \\ n > 0 \end{matrix}$$

$$\epsilon^n \quad n > 0$$

$$a_n'' = -Q(x) a_{n-1}$$

$$a_n' = - \int ds Q(s) a_{n-1}(s)$$

$$\left| \int_a^b f(x) dx \right| \leq (b-a)M, \quad f(x) \in [K, M]$$

$$a_n(x) = (-1)^n \iint_Q \iint_Q \iint_Q \iint_Q \dots \iint_Q (Q)^{\alpha+ps}$$

$\text{Max}|Q| = M$ $\text{Max}|x+px| = m$

$$|a_n(x)| \leq M^n m \underbrace{\int_0^x ds \int_0^{z^n} dt \int_0^t du \int_0^u dv \dots}_{\frac{1}{(2n)!} x^{2n}}$$

$$|a_n(x)| \leq M^n m x^{2n} \frac{1}{(2n)!}$$

$$\sum \frac{K^n}{(2n)!} e^n$$

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$$\frac{U'}{U} + \frac{a}{2} = 0$$

$$\ln U + \int \frac{a}{2} ds = 0 \quad \Rightarrow \quad U = e^{-\int \frac{a}{2} ds}$$

$$\left(-\frac{d^2}{dx^2} + V(x) \right) \psi(x) = E \psi(x)$$

H.O. $V(x) = \frac{x^2}{4}$ $E_n = n + \frac{1}{2}$

$$\left(-\frac{d^2}{dx^2} + \frac{x^2}{4} + \frac{x^4}{4} \right) \psi = E \psi$$

$$F = -V' = -\frac{x}{2} - x^3$$

$$E(\epsilon) = \sum_{n=0}^{\infty} a_n \psi_n$$

$$\psi(x) = E \psi(x)$$

$$= n + \frac{1}{2}$$

$$= E_0 \psi \leftarrow$$

$$E(\psi) = \sum_{n=0}^{\infty} a_n E^n$$

$$\psi(x) = \sum_{n=0}^{\infty} \psi_n(x) E^n$$

Ground state

$$\begin{cases} a_0 = \frac{1}{2} \\ \psi_0 = e^{-x^2/4} \end{cases}$$

$$E_{\text{ground state}} = \frac{1}{2} + \frac{3}{4}e - \frac{21}{8}e^2 + \frac{333}{16}e^3 + \dots$$

(as $n \rightarrow \infty$) $a_n \sim (-1)^{n+1} \frac{\sqrt{6}}{11^{3/2}} 3^n \sqrt{n + \frac{1}{2}}$

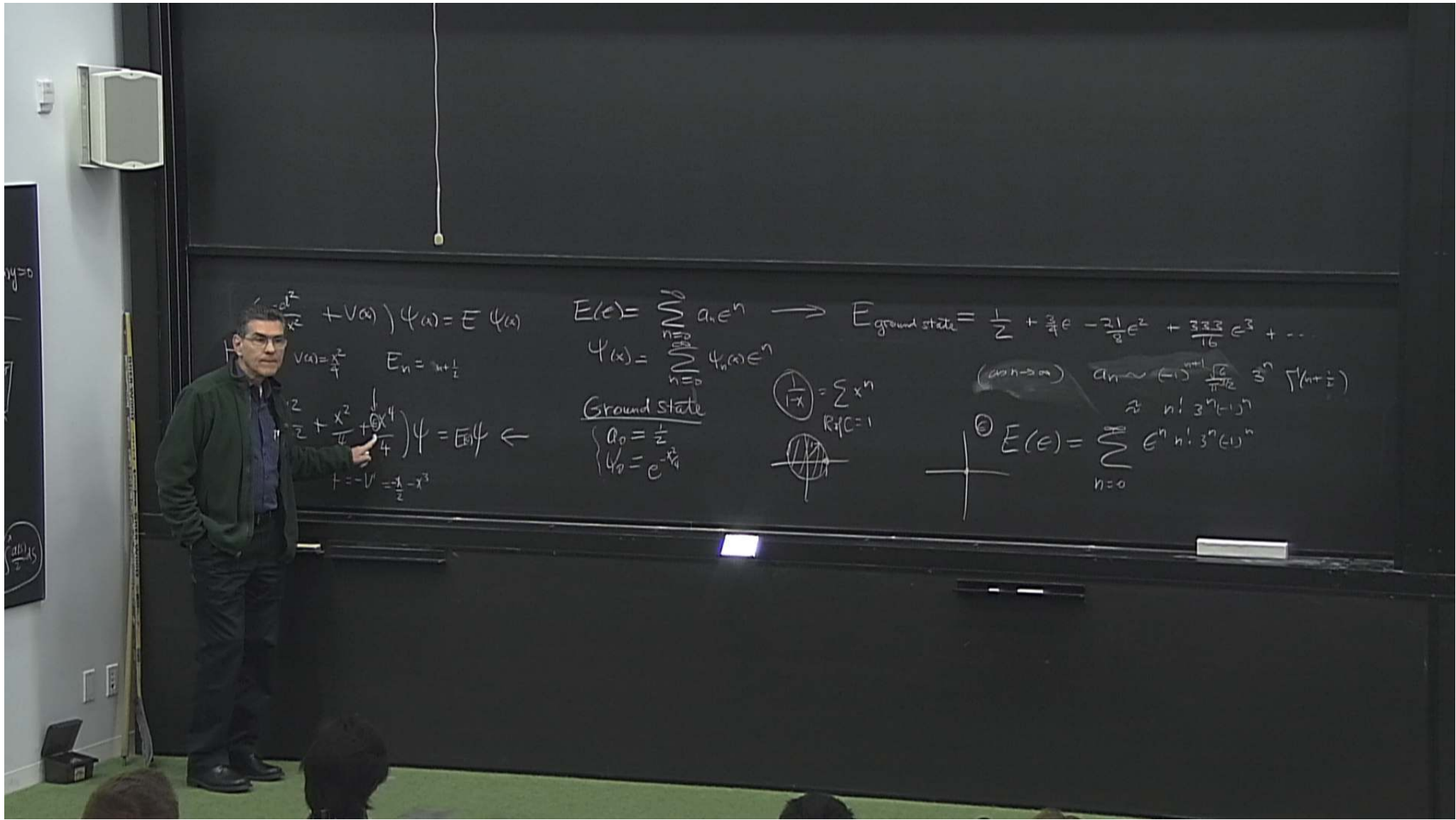
$$E_{\text{ground state}} = \frac{1}{2} + \frac{3}{4}e - \frac{21}{8}e^2 + \frac{333}{16}e^3 + \dots$$

(as $n \rightarrow \infty$)

$$a_n \sim (-1)^{n+1} \frac{\sqrt{6}}{\Gamma(3/2)} 3^n \Gamma(n + \frac{1}{2})$$

$$\approx n! 3^n (-1)^n$$

$$E(e) = \sum_{n=0}^{\infty} e^n n! 3^n (-1)^n$$



$$\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right) \psi(x) = E \psi(x)$$

$$V(x) = \frac{1}{2} k x^2 \quad E_n = \hbar \omega \left(n + \frac{1}{2} \right)$$

$$\left(\frac{1}{2} + \frac{x^2}{4} + \frac{\hbar^2}{4} \frac{d^2}{dx^2} \right) \psi = E \psi \leftarrow$$

$$V = -V = -\frac{1}{2} x^2$$

$$E(\epsilon) = \sum_{n=0}^{\infty} a_n \epsilon^n \rightarrow E_{\text{ground state}} = \frac{1}{2} + \frac{3}{4} \epsilon - \frac{21}{8} \epsilon^2 + \frac{333}{16} \epsilon^3 + \dots$$

$$\psi(x) = \sum_{n=0}^{\infty} \psi_n(x) \epsilon^n$$

Ground state

$$\begin{cases} a_0 = \frac{1}{2} \\ \psi_0 = e^{-x^2/4} \end{cases}$$

$$\frac{1}{1-x} = \sum x^n$$

R: |C| = 1

(as $n \rightarrow \infty$)

$$a_n \sim (-1)^{n+1} \frac{\sqrt{6}}{\pi^{3/2}} 3^n \Gamma\left(n + \frac{1}{2}\right)$$

$$\sim n! 3^n (-1)^n$$

$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n n! 3^n (-1)^n$$

$$H = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + \epsilon \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$H = \begin{pmatrix} a & \epsilon c \\ \epsilon c & b \end{pmatrix}$$

$$\det \begin{pmatrix} a - E & \epsilon c \\ \epsilon c & b - E \end{pmatrix} = 0$$
$$E^2 - (a+b)E + ab - \epsilon^2 c^2 = 0$$

$$H = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + e \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} a & ec \\ ec & b \end{pmatrix}$$

$$\det \begin{pmatrix} a-E & ec \\ ec & b-E \end{pmatrix} = 0$$

$$E^2 - (a+b)E + ab - e^2c^2 = 0$$

$$E_{\pm} = \frac{a+b \pm \sqrt{(a+b)^2 - 4ab + 4e^2c^2}}{2}$$

$$H = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + \epsilon \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & a \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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$\sum a_n$

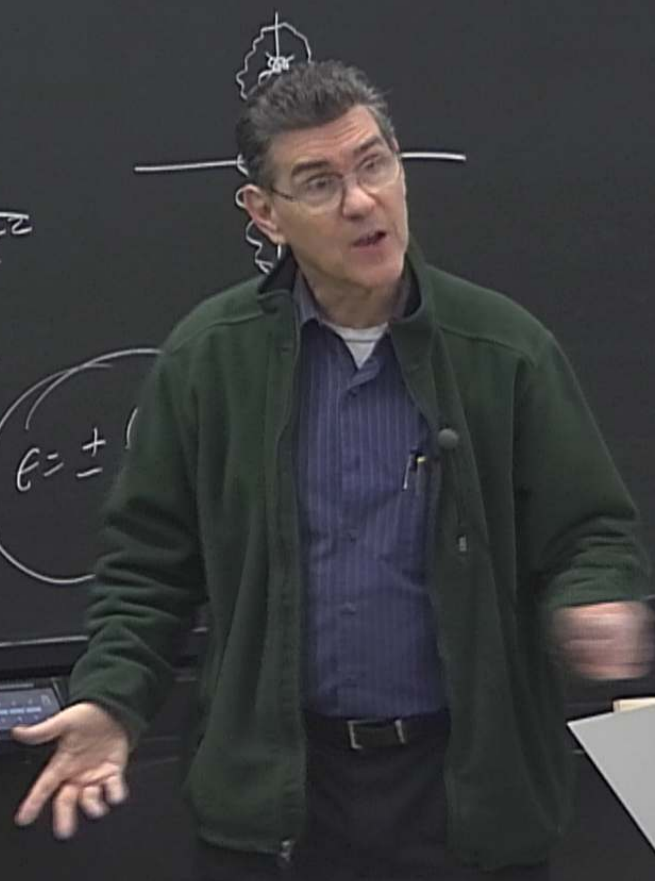
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$$E(\epsilon) = \sum a_n \epsilon^n$$



$$\det \begin{pmatrix} a-E & eC \\ eC & b-E \end{pmatrix} = 0$$

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$$= \frac{a+b \pm \sqrt{(a-b)^2 + 4e^2C^2}}{2}$$

$$E = \pm \frac{(a-b)}{2C}$$

$$E(\epsilon) = \sum a_n \epsilon^n \quad R_1 C = R$$

