Title: Condensed Matter - Lecture 12

Date: Nov 15, 2011 10:30 AM

URL: http://pirsa.org/11110036

Abstract:

BCS theory:

$$\mathcal{H}_{MF} = E_{0S} + \sum_k E_k \left(\gamma^{+}_{k\uparrow} \gamma_{k\uparrow} + \gamma^{+}_{-k\downarrow} \gamma_{-k\downarrow} \right)$$

g.s defined by $\gamma_{k\sigma} |\Psi_{BCS}\rangle = 0 \quad \forall\ k, \sigma$

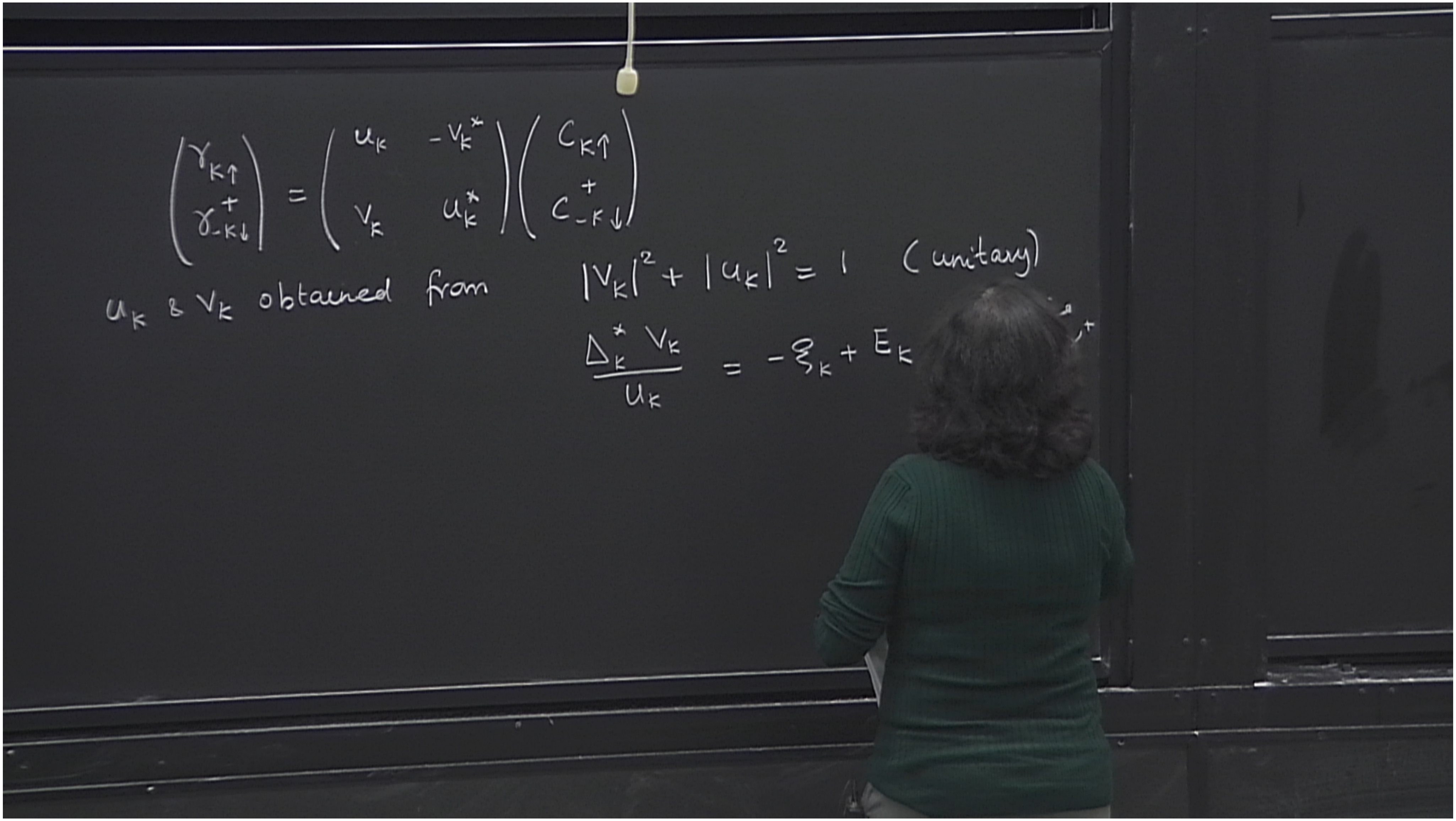
$$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$$

$$\xi_k = \epsilon_k - \mu$$

$$\gamma_{k\uparrow} + \gamma^{\dagger}_{-k\downarrow}\, \gamma_{-k\downarrow})$$

$$\forall \ k, \sigma$$

$$\begin{pmatrix} \gamma_{k\uparrow} \\ \gamma^{\dagger}_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} u_k & -v_k^* \\ v_k & u_k^* \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^{\dagger}_{-k\downarrow} \end{pmatrix}$$


$\begin{pmatrix} \gamma_{k\uparrow} \\ \gamma^{+}_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} u_k & -v_k^* \\ v_k & u_k^* \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^{+}_{-k\downarrow} \end{pmatrix}$
u_k & v_k obtained from
$|v_k|^2 + |u_k|^2 = 1$ (unitary)
$\frac{\Delta_k^* v_k}{u_k} = -\xi_k + E_k$

$+ \gamma^{+}_{-k\downarrow}\gamma_{-k\downarrow})$

E, σ

$$\begin{pmatrix} \gamma_{k\uparrow} \\ \gamma^{+}_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} u_k & -v_k^* \\ v_k & u_k^* \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^{+}_{-k\downarrow} \end{pmatrix}$$

u_k & v_k obtained from $|v_k|^2 + |u_k|^2 = 1$ (unitary)

$$\frac{\Delta_k^* v_k}{u_k} = -\xi_k + E_k \quad \left(\text{requires } \gamma\gamma, \gamma^{+}\gamma^{+} \text{ coeff} = 0\right)$$

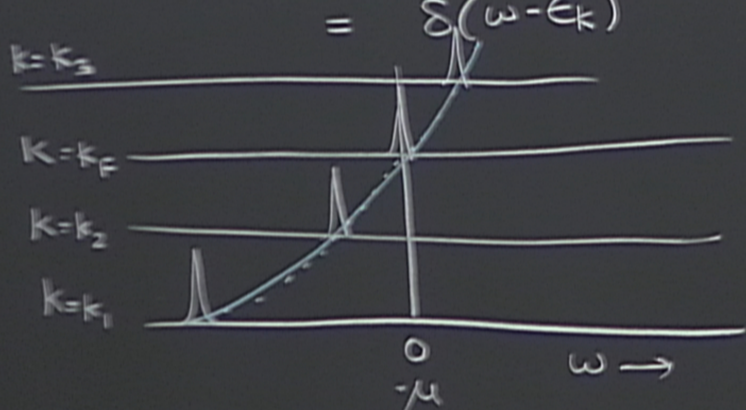
$$\boxed{\begin{aligned} |v_k|^2 &= \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) \\ |u_k|^2 &= \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) \\ 2u_k v_k &= \frac{\Delta_k}{E_k} \end{aligned}}$$

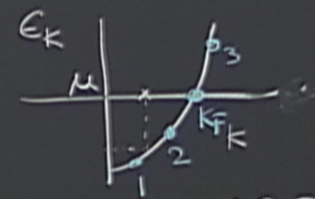
Spectral Function: understand particle-hole mixing.

non-interacting Fermi gas $(T=0)$

$$A_{\vec{k}}(\omega) = -\frac{1}{\pi} \operatorname{Im} G^R_k(\omega) = \text{prob to find an electron at } (\vec{k}, \omega)$$

$$= \delta(\omega - \epsilon_k)$$





$$\epsilon_k = \frac{\hbar^2 k^2}{2m}$$

ARPES

angle resolved photoemission spectroscopy

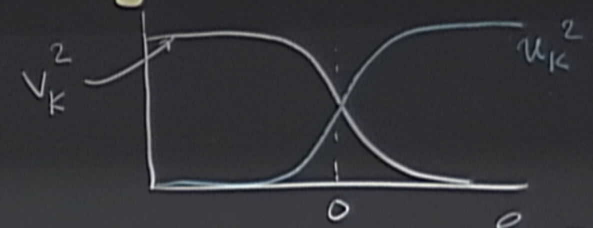
BCS Sc

BCS Superconductor $(T=0)$ $\quad A_k(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\, G_{11}^{R}(k,\omega)$ $\quad v_k$

$$A_k(\omega) = u_k\, \delta(\omega - E_k) + v_k^2\, \delta(\omega + E_k)$$

onductor $(T=0)$ $\quad A_k(\omega) = -\frac{1}{\pi} \operatorname{Im} G_{11}^R(k,\omega)$

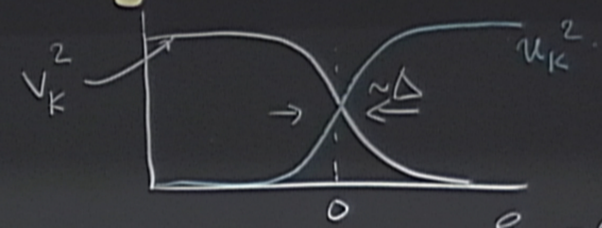
$u_k^2 \, \delta(\omega - E_k) + v_k^2 \, \delta(\omega + E_k)$



$\xi_k = \epsilon_k - \mu$

onductor $(T=0)$ $\quad A_k(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G^R_{11}(k,\omega)$

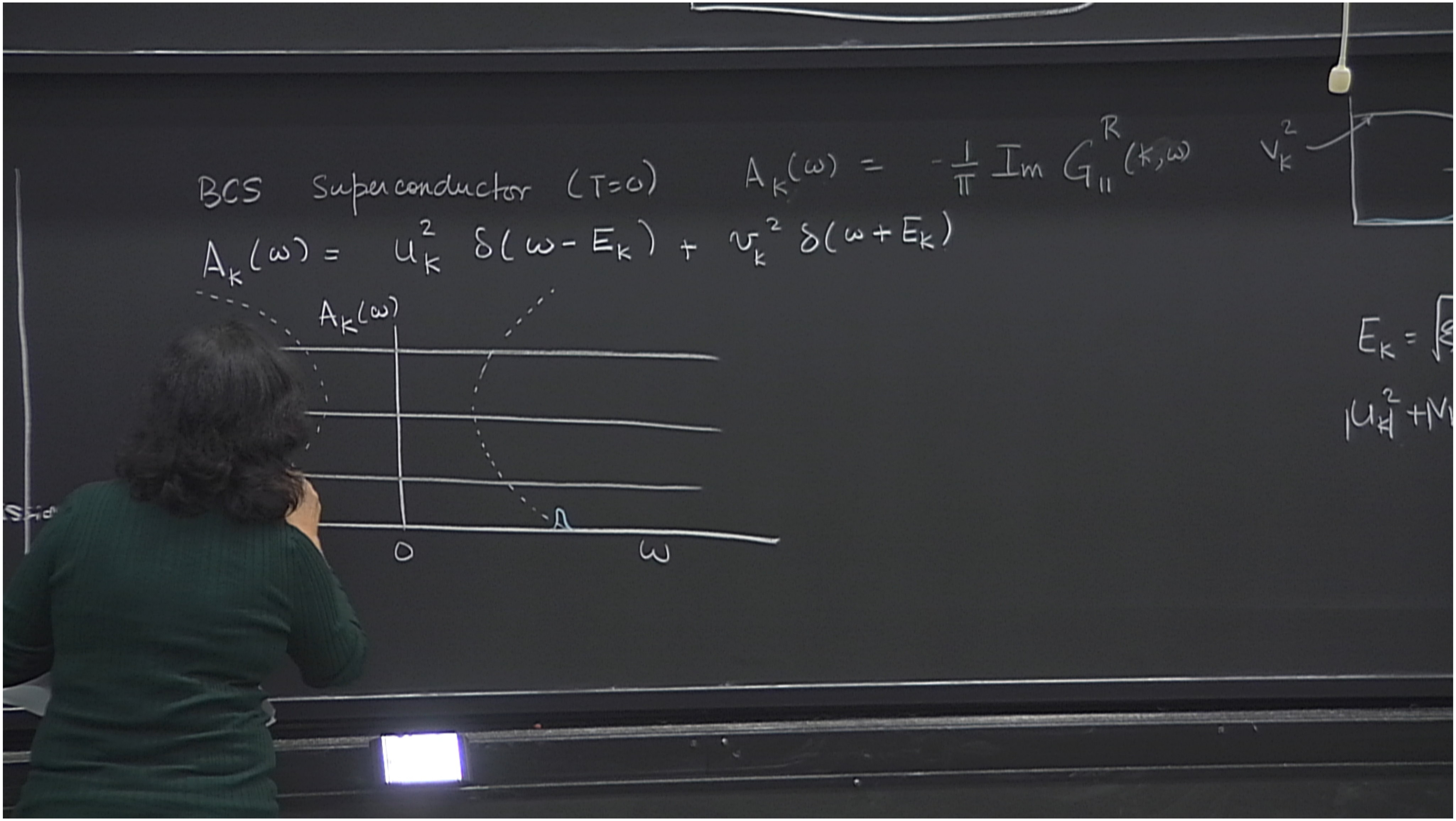
$u_k^2\,\delta(\omega - E_k) + v_k^2\,\delta(\omega + E_k)$



$\xi_k = \epsilon_k - \mu$

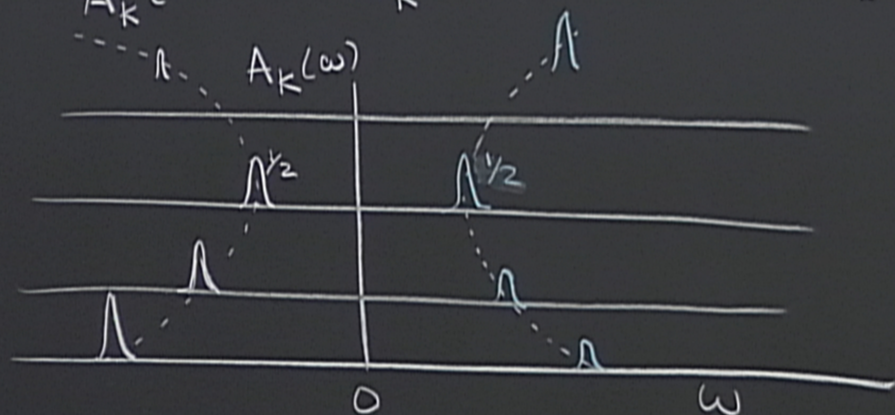
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$

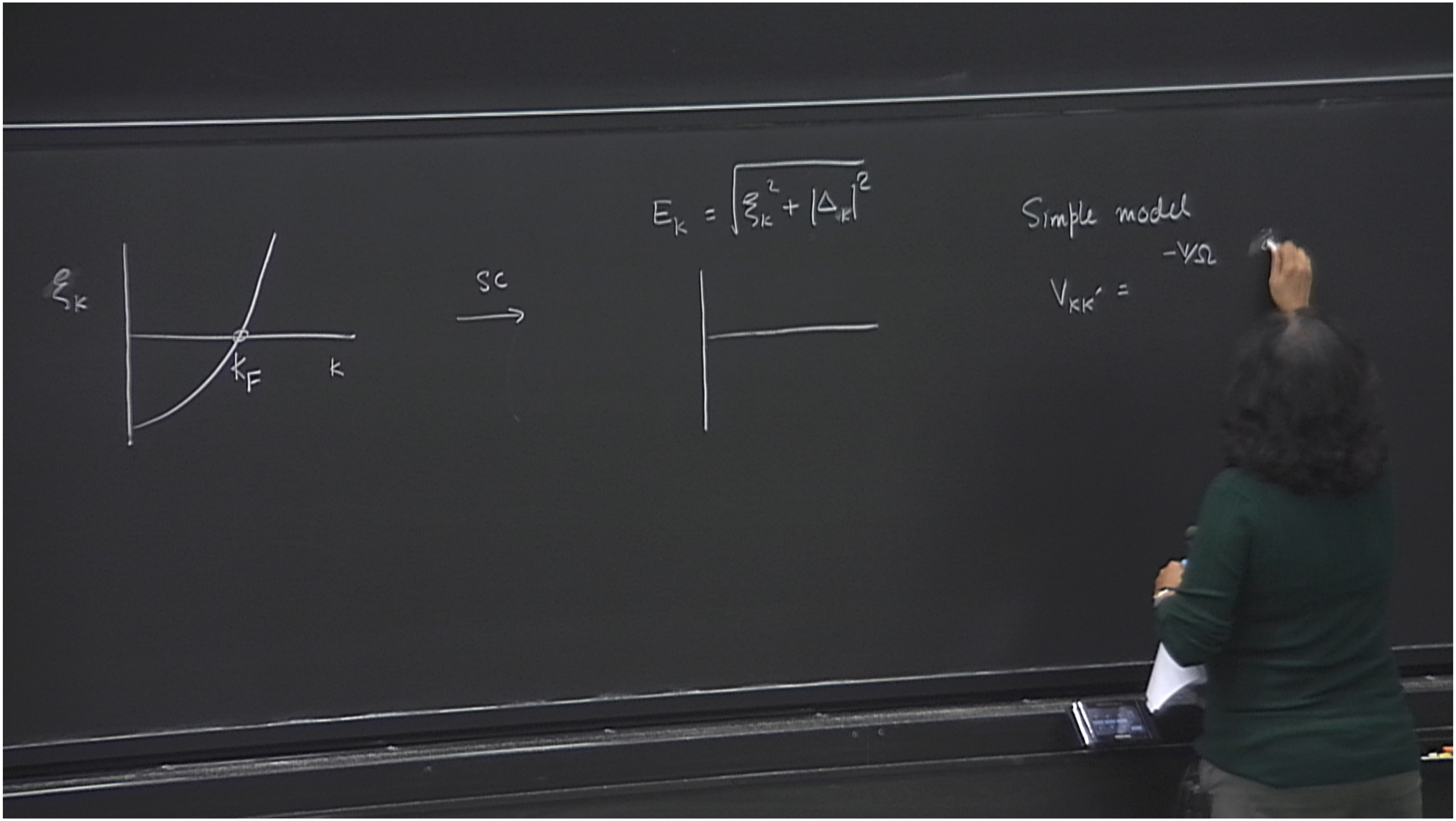
$|u_k|^2 + |v_k|^2 = 1$


BCS Superconductor (T=0)
$A_k(\omega) = -\frac{1}{\pi} \mathrm{Im}\, G^R_{11}(k,\omega)$
v_k^2
$A_k(\omega) = u_k^2 \, \delta(\omega - E_k) + v_k^2 \, \delta(\omega + E_k)$
$A_k(\omega)$
0
ω

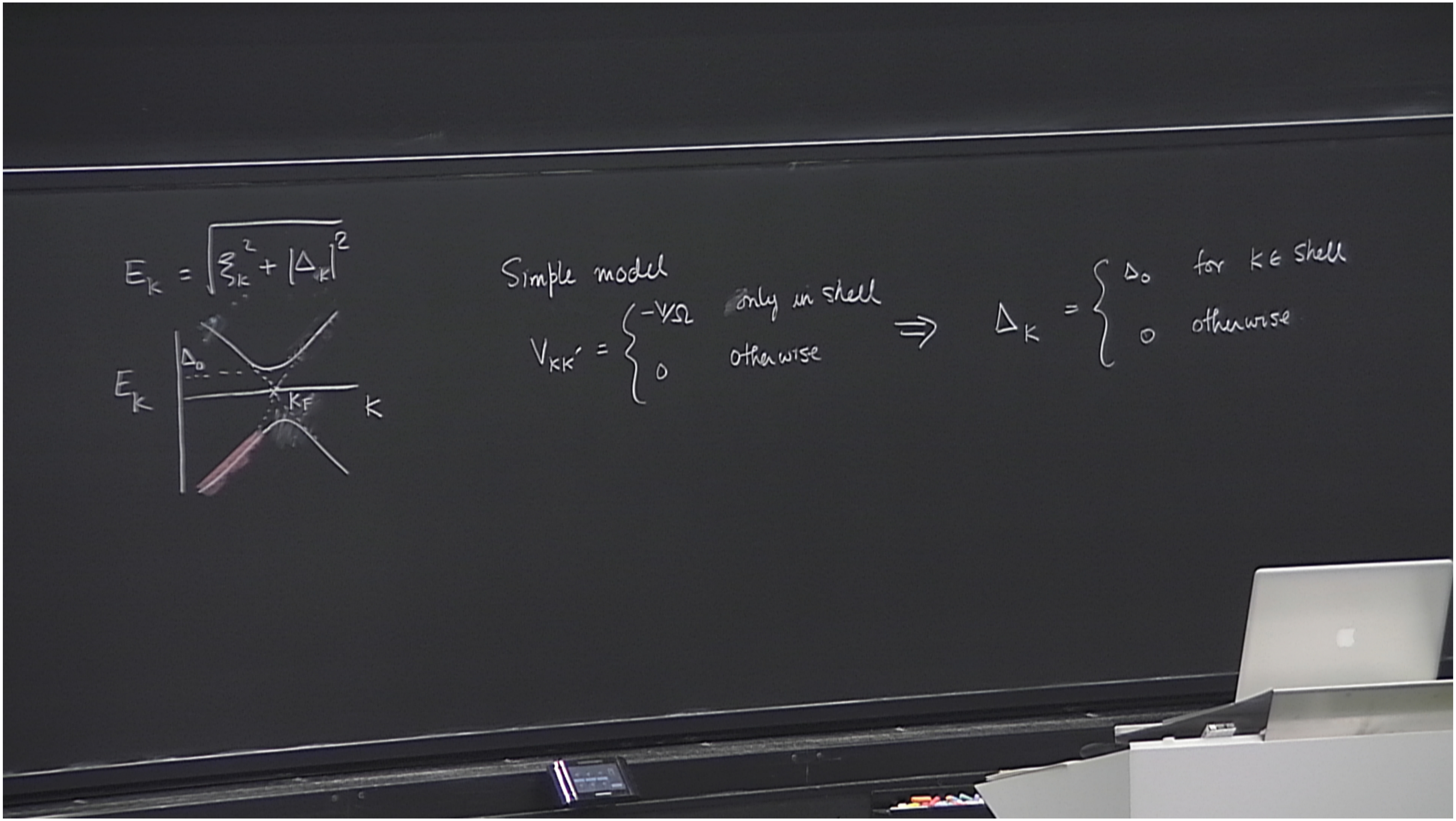
BCS Superconductor $(T=0)$ $\quad A_k(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G^R_{11}(k,\omega)$

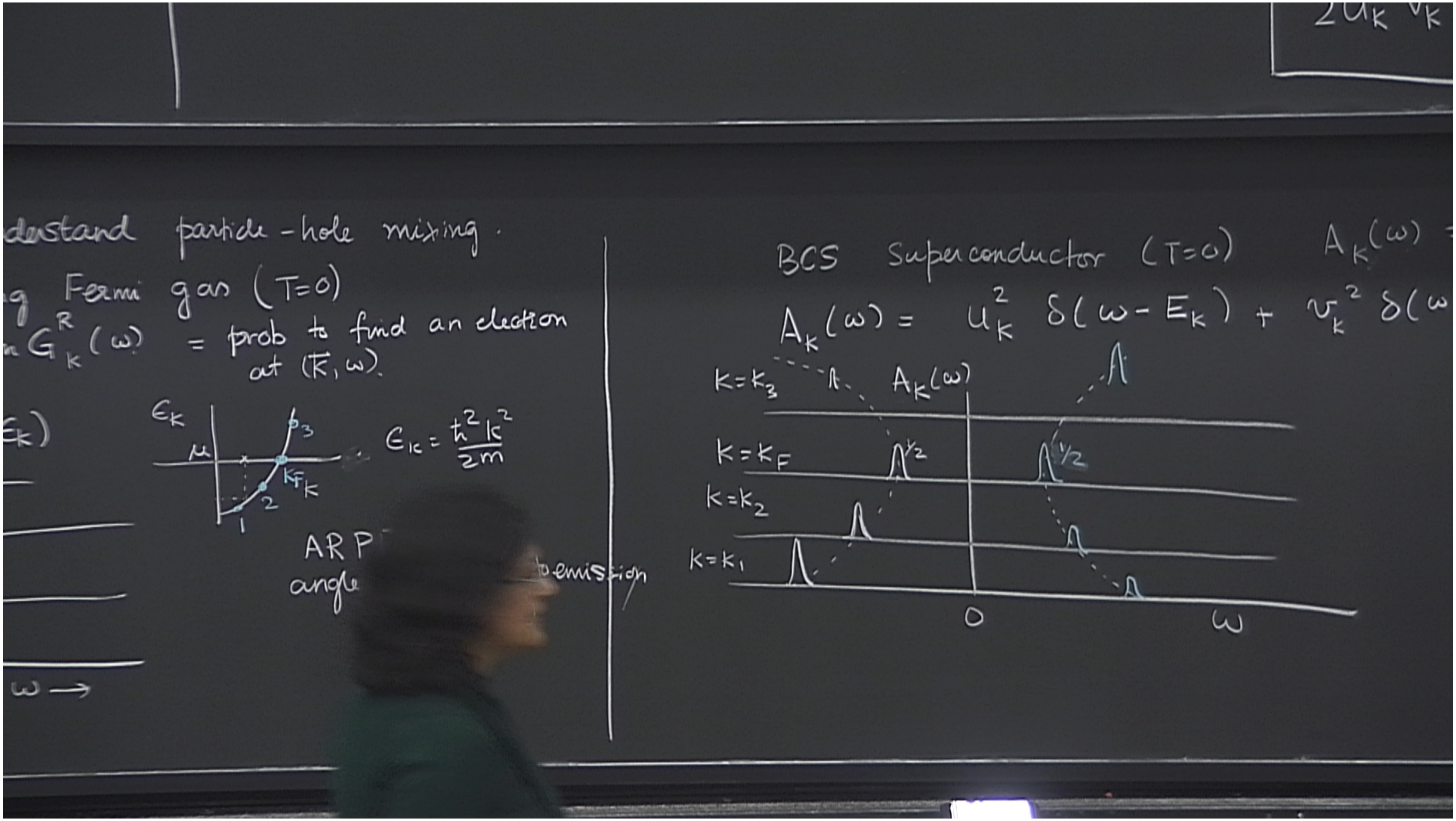
$$A_k(\omega) = u_k^2\,\delta(\omega - E_k) + v_k^2\,\delta(\omega + E_k)$$



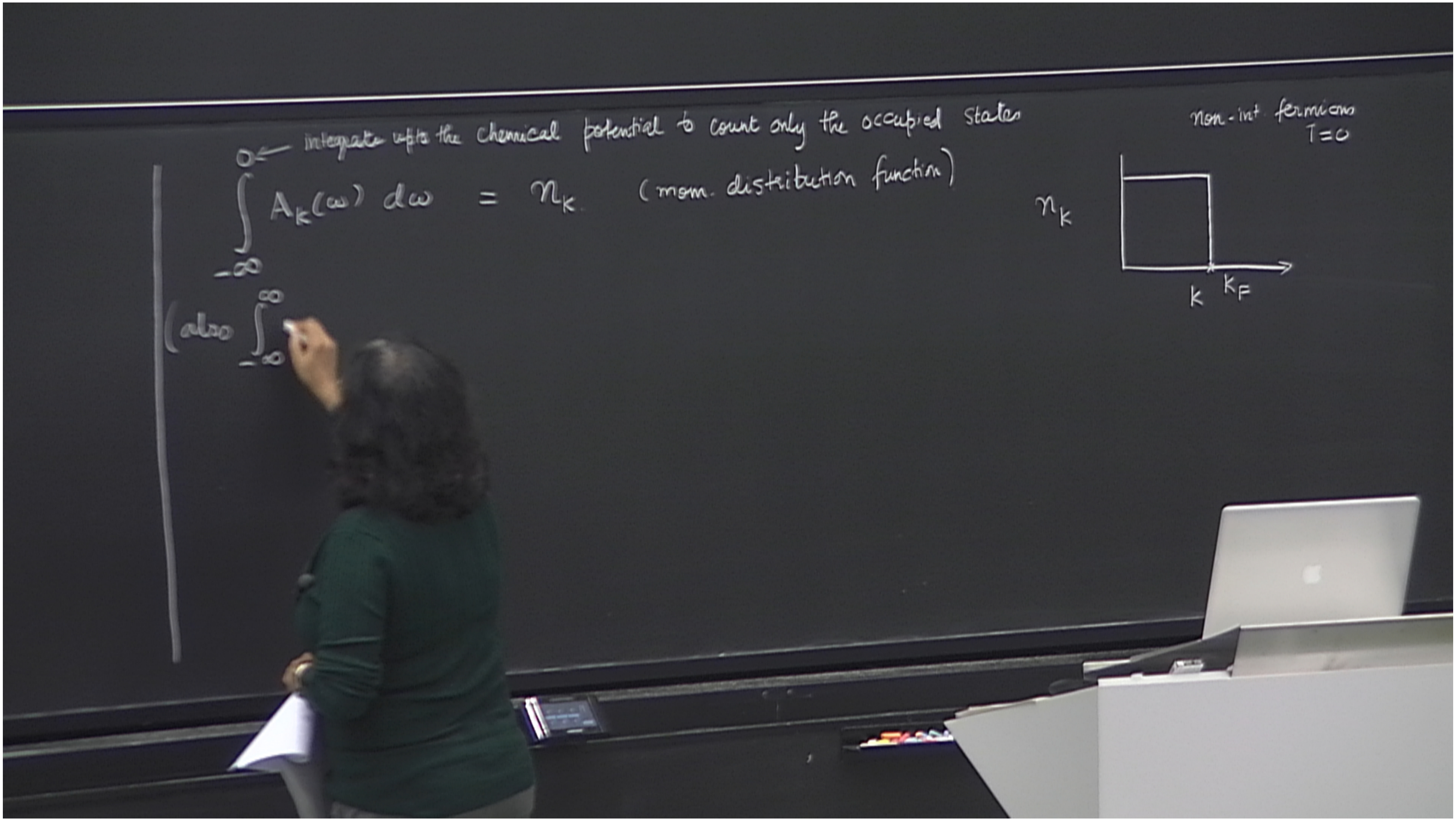


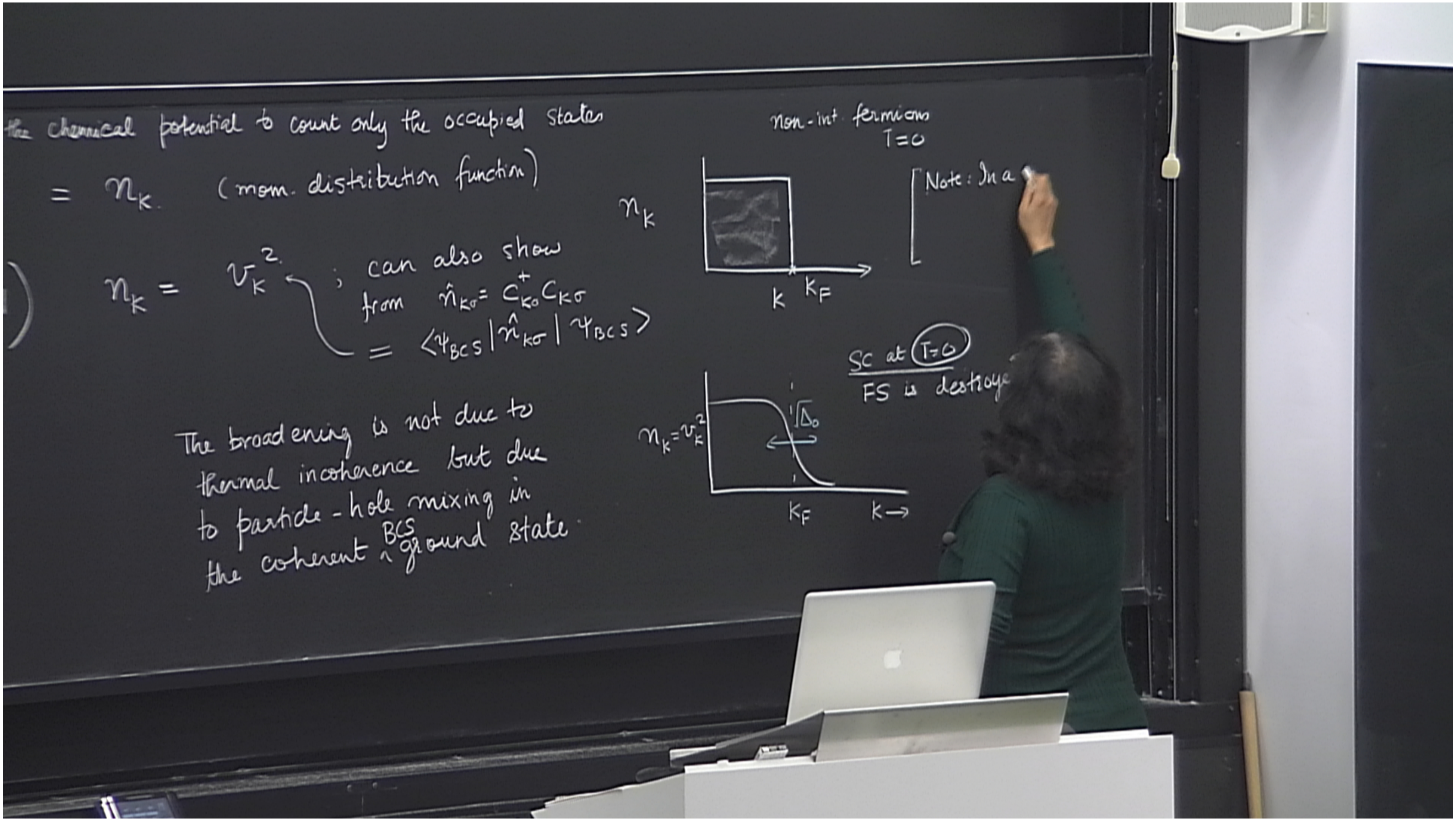
ξ_k
k_F
k
SC
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
Simple model
$V_{kk'} = -V/\Omega$

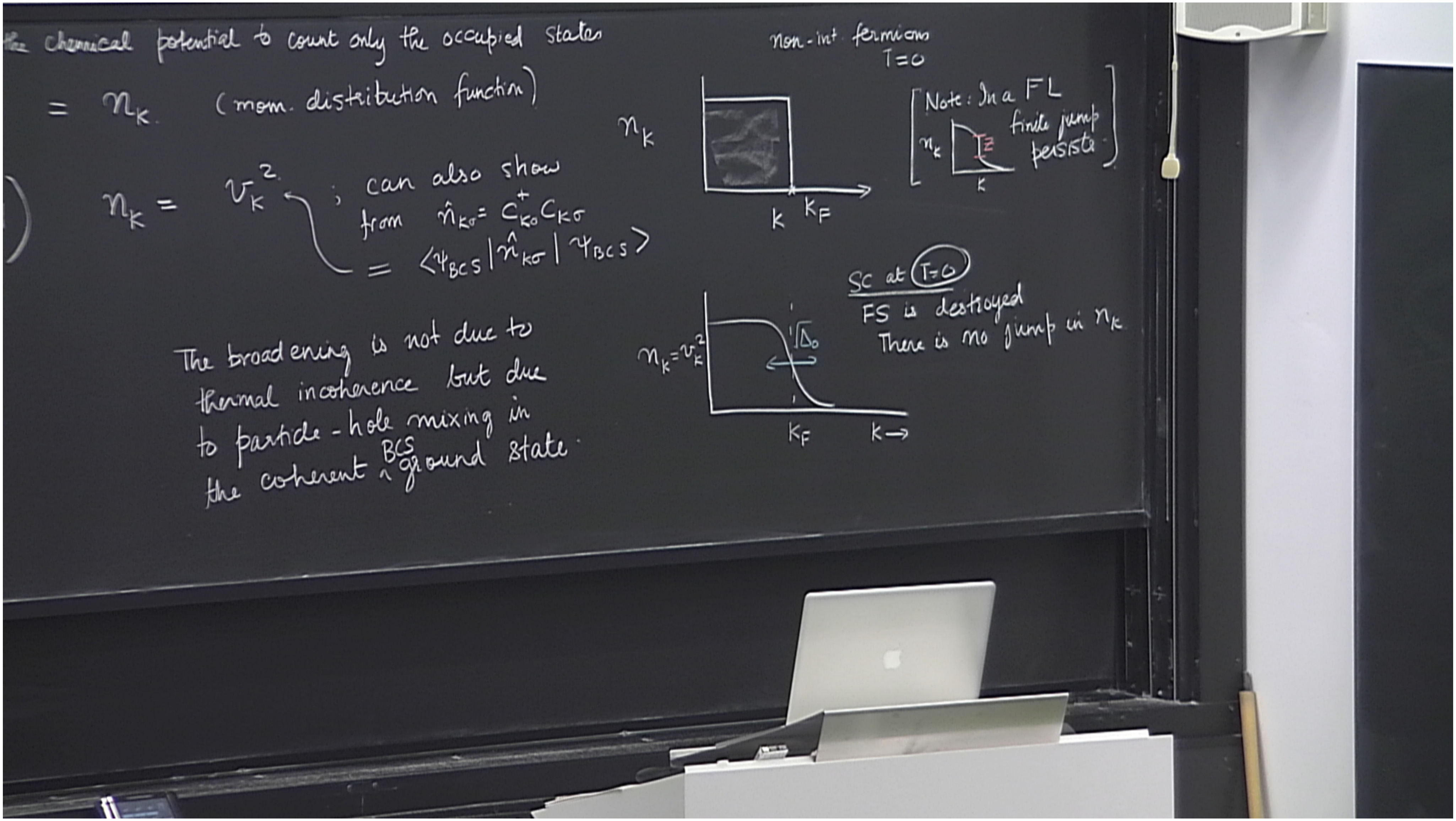

$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
E_k
Δ_0
k_F
k
Simple model
$V_{kk'} = \begin{cases} -V/\Omega & \text{only in shell} \\ 0 & \text{otherwise} \end{cases}$
$\Rightarrow \quad \Delta_k = \begin{cases} \Delta_0 & \text{for } k \in \text{shell} \\ 0 & \text{otherwise} \end{cases}$



...derstand particle-hole mixing.
...g Fermi gas (T=0)
$G^R_k(\omega)$ = prob to find an electron at $(\vec{k}, \omega)$.
ϵ_k
μ
k_F k
$\epsilon_k = \frac{\hbar^2 k^2}{2m}$
ARP...
angle...
...emission
$\omega \rightarrow$
BCS Superconductor (T=0)
$A_k(\omega) = u_k^2 \, \delta(\omega - E_k) + v_k^2 \, \delta(\omega$ ...
$A_k(\omega)$
$k=k_3$
$k=k_F$
$k=k_2$
$k=k_1$
1/2
0
ω


integrate upto the chemical potential to count only the occupied states
non-int fermions
T=0
(mom. distribution function)
$\int_{-\infty}^{0} A_k(\omega)\, d\omega = n_k$
(also $\int_{-\infty}^{\infty}$
n_k
k
k_F


the chemical potential to count only the occupied states
= n_k (mom. distribution function)
n_k = v_k^2 ; can also show from n̂_kσ = c†_kσ c_kσ = ⟨ψ_BCS | n̂_kσ | ψ_BCS⟩
The broadening is not due to thermal incoherence but due to particle - hole mixing in the coherent BCS ground state
non-int. fermions
T=0
n_k
k k_F
Note: In a
SC at T=0
FS is destroye
n_k=v_k^2
Δ_0
k_F
k→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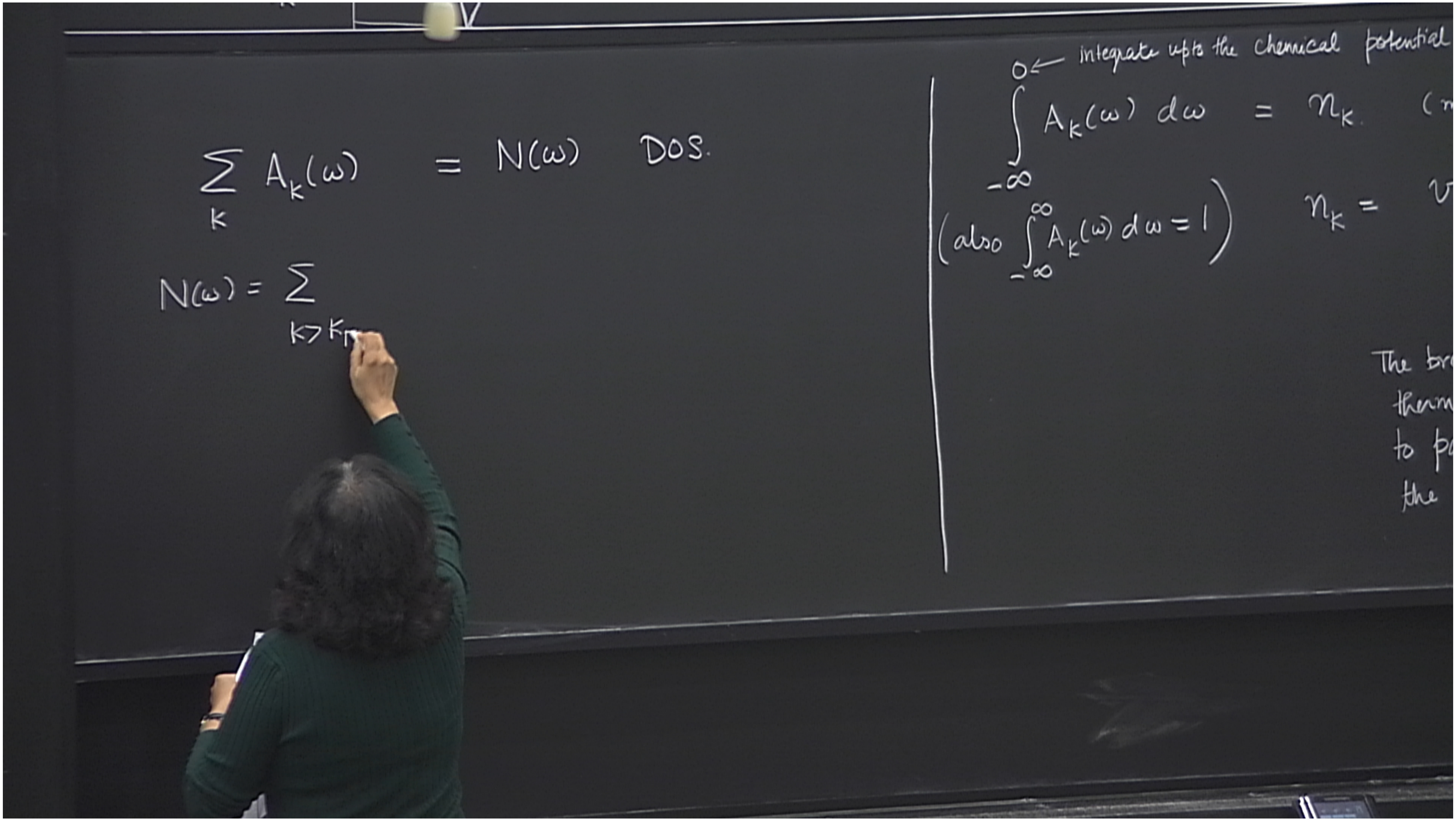
= ⟨ψ_BCS | n̂_kσ | ψ_BCS⟩
The broadening is not due to
thermal incoherence but due
to particle-hole mixing in
the coherent BCS ground state.
non-int. fermions
T=0
n_k
k
k_F
Note: In a FL
n_k
Z
k
finite jump
persists
SC at T=0
FS is destroyed
There is no jump in n_k
n_k = v_k^2
Δ_0
k_F
k


$\sum_k A_k(\omega) = N(\omega)$ DOS.
$N(\omega) = \sum_{k > k_F}$
$\int_{-\infty}^{0} A_k(\omega)\, d\omega = n_k$
integrate upto the chemical potential
(also $\int_{-\infty}^{\infty} A_k(\omega)\, d\omega = 1$)
$n_k =$

k_F k

$$\sum_k A_k(\omega) = N(\omega) \quad \text{DOS.}$$

$$N(\omega) = \sum_{k>k_F} \delta(\omega - E_k) = N(0)\int_0^\infty \underbrace{d\xi_k}\, \delta(\omega - E_k)$$

$$d\xi = \frac{\partial \xi}{\partial E} dE = \frac{E}{\xi} dE = \frac{E}{\sqrt{E^2-\Delta^2}} dE$$

$$= N(0)\frac{\omega}{\sqrt{\omega^2 - |\Delta_k|^2}} \; ; \; \omega > \Delta$$

$$= 0 \qquad \omega < \Delta$$

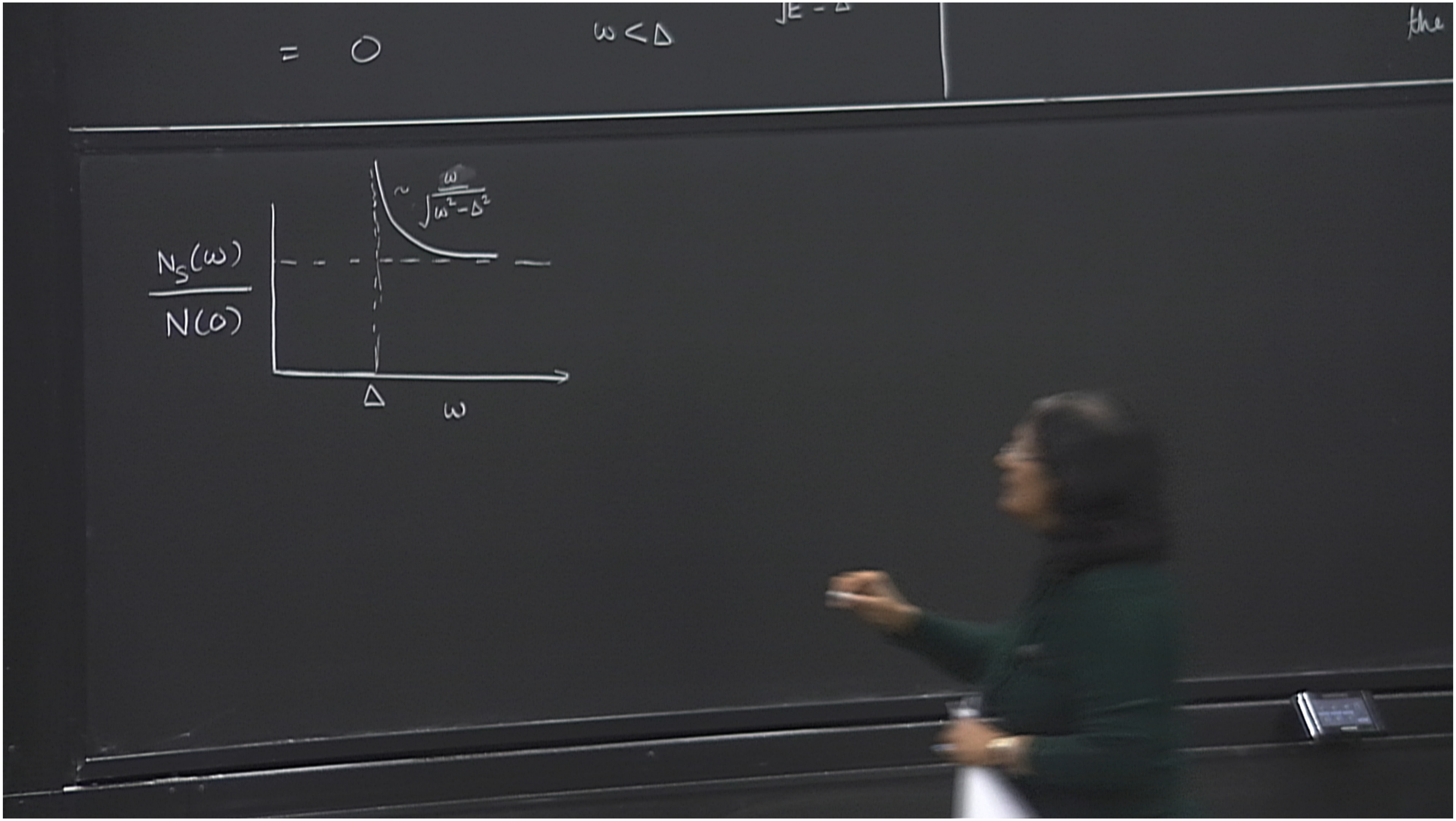
$$\int_{-\infty}^{0} A_k(\omega)\, d\omega = n_k \quad (m$$

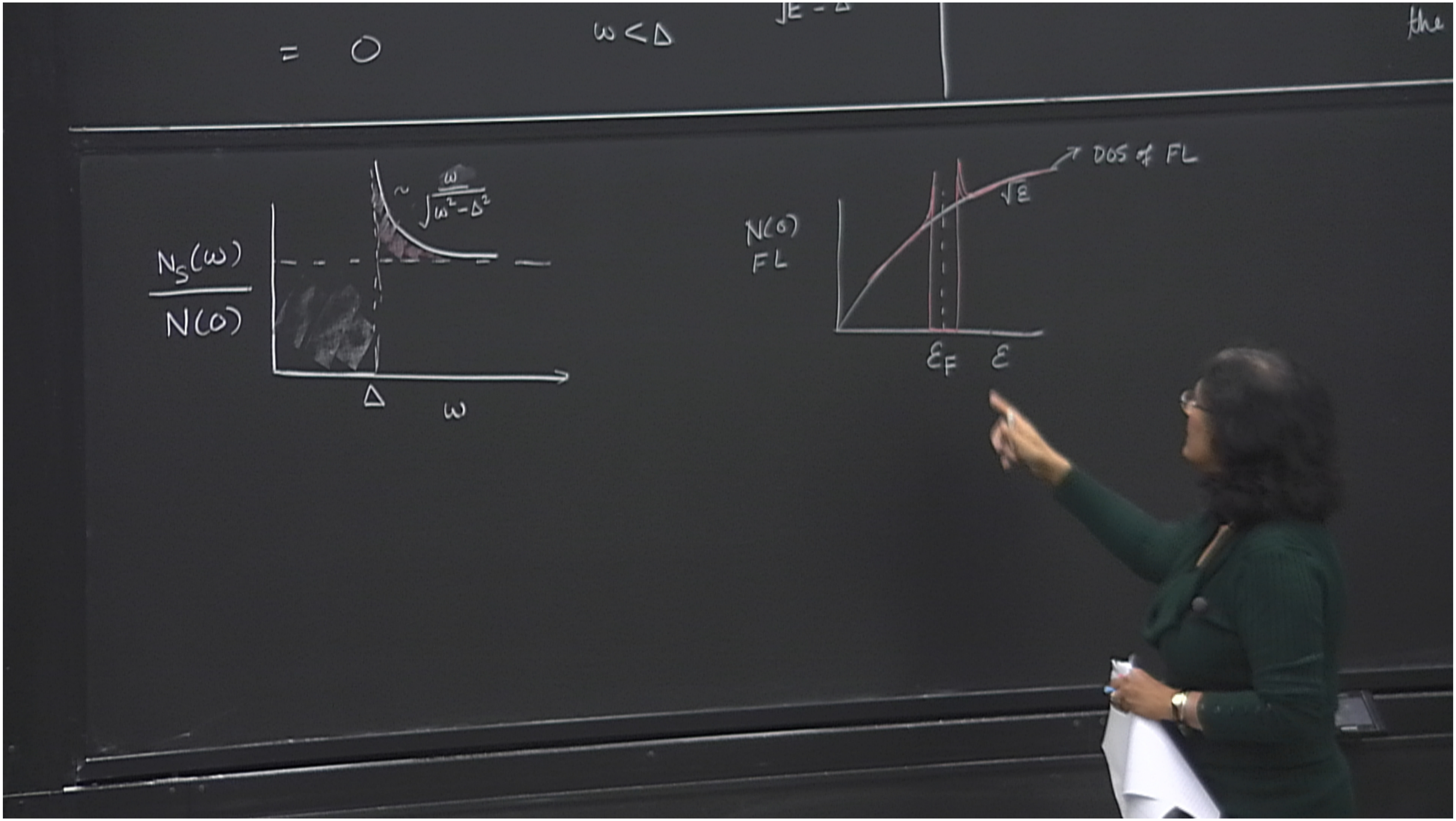
$0 \leftarrow$ integrate upto the chemical potential

$$\left(\text{also} \int_{-\infty}^{\infty} A_k(\omega)\, d\omega = 1\right)$$

$n_k = v$

The br
therm
to p
the


= 0
ω < Δ
the
N_s(ω) / N(0)
~ ω / √(ω² − Δ²)
Δ
ω


= 0
ω < Δ
the
N_S(ω) / N(0)
~ ω / √(ω² − Δ²)
Δ
ω
DOS of FL
√E
N(0) FL
ε_F
ε

PHASE TRANSITION AT Tc

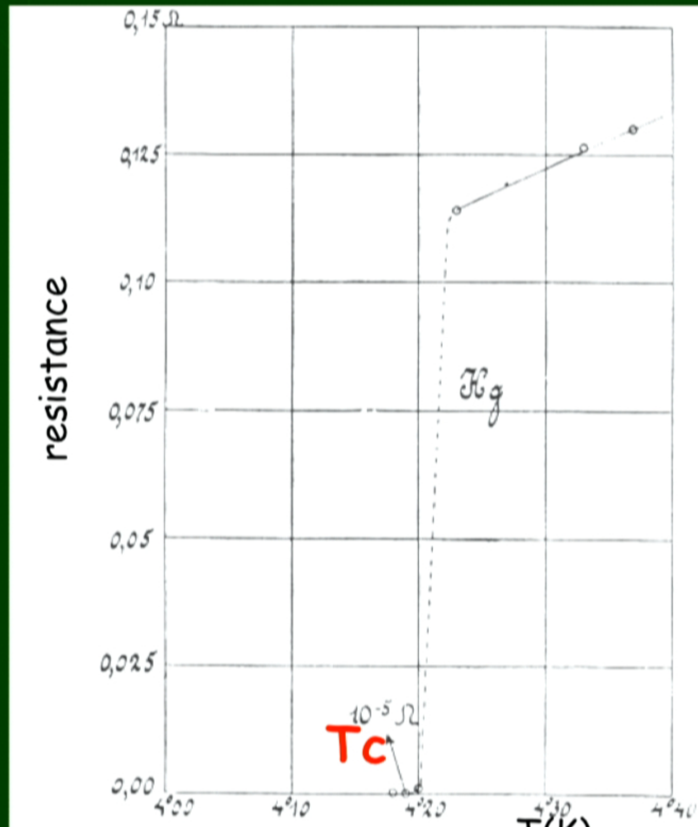


Figure 3.2 Resistance (vertical axis, in units of ohm) versus temperature (horizontal axis, in degrees Kelvin) for mercury (Hg) wires. (Courtesy of the Kamerlingh Onnes Laboratorium, Leiden. Copyright permission obtained from Elsevier Science Publishers.)

Lets do the numbers....

$$B = \frac{\mu_0(\#turns)I}{2\pi r} = \frac{4\pi\times10^{-7}(N/A^2)(10^5)(100A)}{2\pi(0.5m)}$$

$$B = 4N/(Am) = \boxed{4Tesla}$$

$$R = \frac{\rho L}{A} = \frac{(1micro\Omega - cm)(2\pi r)(\#turns)}{(\pi r_{wire}^2)}$$

$$R = \frac{(10^{-6}\Omega cm)(2\times0.5m)(10^5)}{(1mm)^2} = \frac{(10^{-6}\Omega cm)(2\times50cm)(10^5)}{(0.1cm)^2}$$

$$\boxed{R = 10^3\Omega = 1k\Omega}$$

$$P = I^2R = (100A)^2(10^3\Omega) = 10^7 Watts = \boxed{10MWatts}$$

$$Energy = P\times time = 10^7W\times1hr = 36\times10^9 Watts-\sec = 36\times10^9J$$

$$mass = density\times volume = (10g/cc)(10^5turns)(2\pi\times50cm)(\pi\times0.1^2cm^2) = 10^7g = 10^4Kg$$

$$Energy = (mass)(c_v)(\Delta T) = (10^7g)(0.4J/g^oK)(\Delta T) = 36\times10^9J$$

$$\boxed{\begin{array}{l}\Delta T \cong 10^4K \cong 10^4C \\ T_{melt} = 1000C\end{array}}$$

COIL HAS TO BE WATER COOLED TO PREVENT IT FROM MELTING!!!

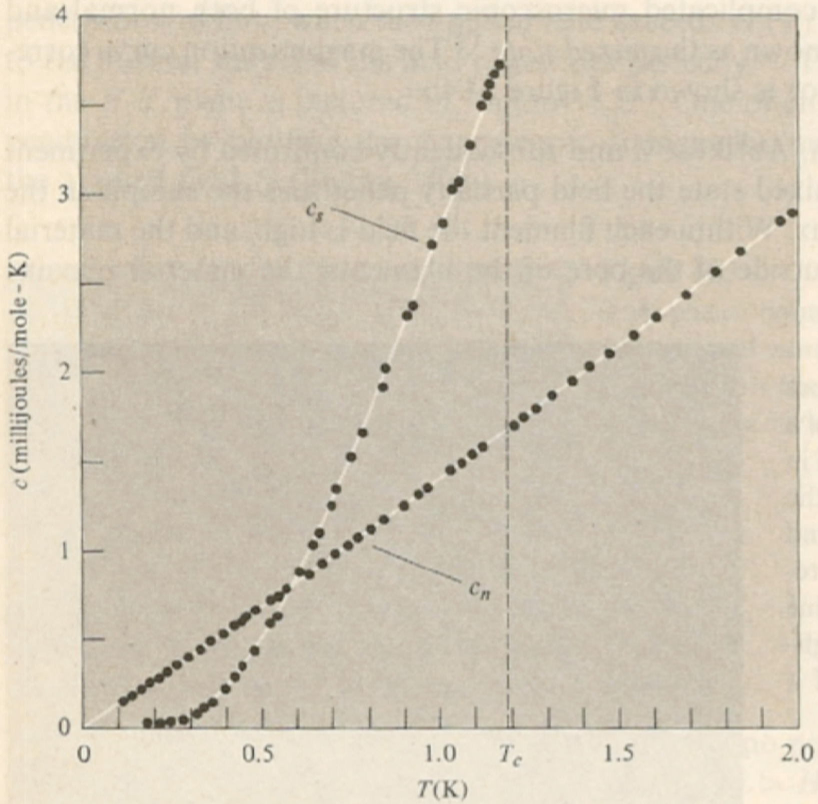
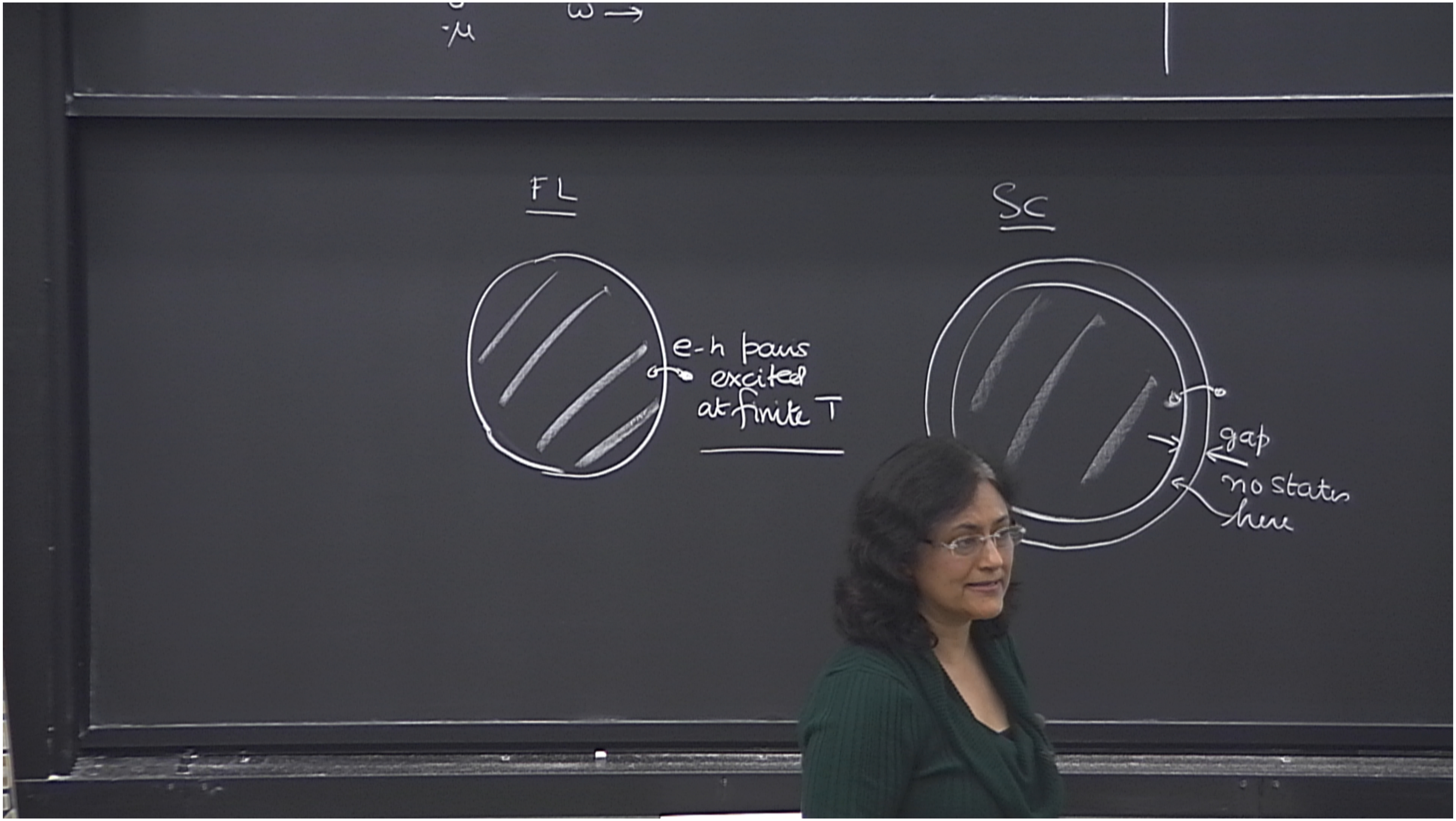


Figure 34.6
Low-temperature specific heat of normal and superconducting aluminum. The normal phase is produced below T_c by application of a weak (300-gauss) magnetic field, which destroys the superconducting ordering but has otherwise negligible effect on the specific heat. The Debye temperature is quite high in aluminum, so the specific heat is dominated by the electronic contribution throughout this temperature range (as can be seen from the fact that the normal-state curve is quite close to being linear). The discontinuity at T_c agrees well with the theoretical prediction (34.22) $[c_s - c_n]/c_n = 1.43$. Well below T_c, c_s drops far below c_n, suggesting the existence of an energy gap. (N. E. Phillips, *Phys. Rev.* **114**, 676 (1959).)



-μ
ω →
FL
e-h pairs
excited
at finite T
Sc
gap
no states
here

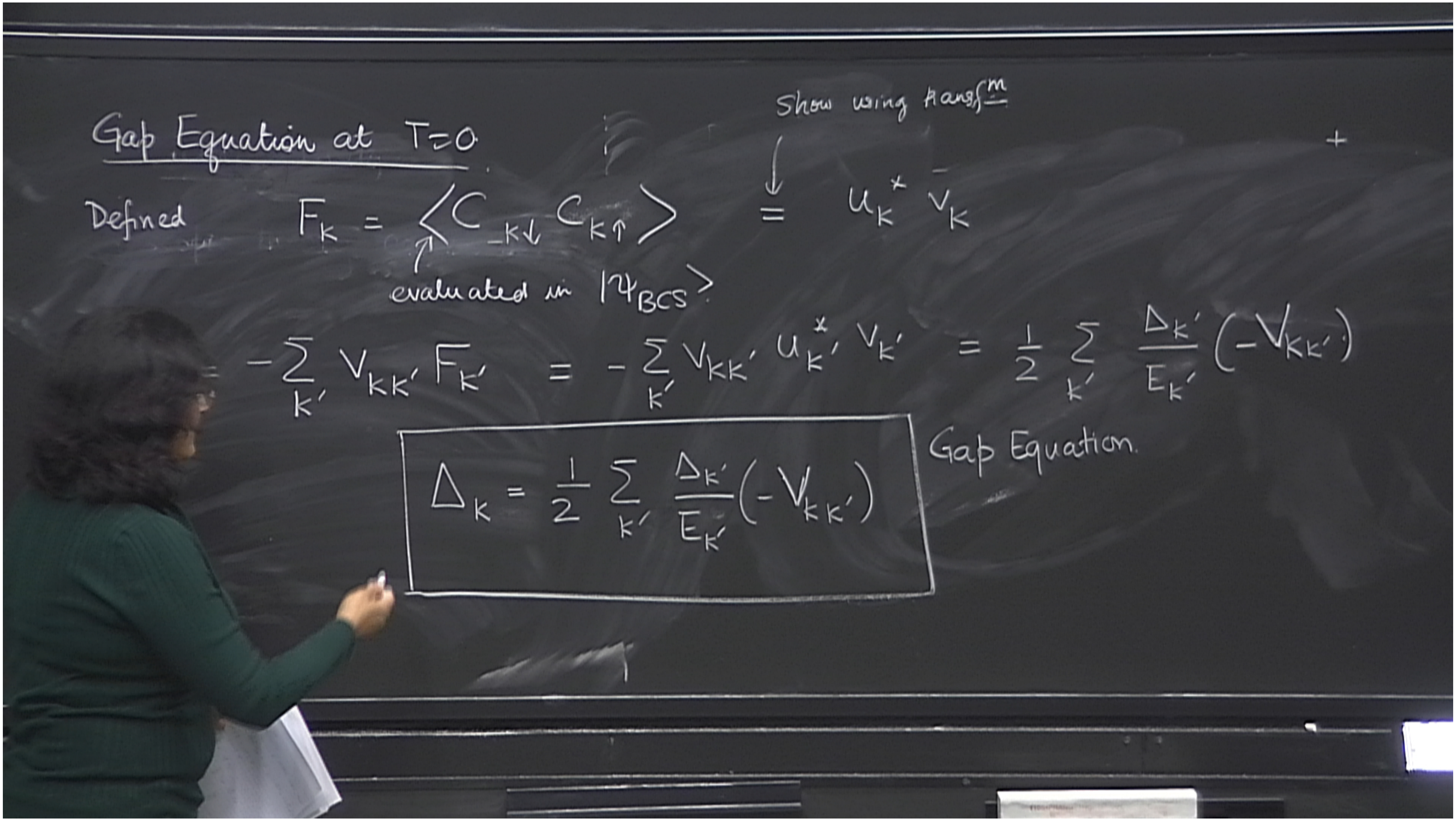
Gap Equation at $T=0$

Show using transf$^{\underline{m}}$

Defined $F_k = \langle C_{-k\downarrow} C_{k\uparrow} \rangle = u_k^* v_k$

evaluated in $|\Psi_{BCS}\rangle$

$$\Delta_k = -\sum_{k'} V_{kk'} F_{k'} = -\sum_{k'} V_{kk'} u_{k'}^* v_{k'}$$


Gap Equation at T=0
Show using transf$^{\underline{m}}$
Defined $F_k = \langle C_{-k\downarrow} C_{k\uparrow} \rangle = u_k^* v_k$
evaluated in $|\Psi_{BCS}\rangle$
$= -\sum_{k'} V_{kk'} F_{k'} = -\sum_{k'} V_{kk'} u_{k'}^* v_{k'} = \frac{1}{2} \sum_{k'} \frac{\Delta_{k'}}{E_{k'}} (-V_{kk'})$
$\boxed{\Delta_k = \frac{1}{2} \sum_{k'} \frac{\Delta_{k'}}{E_{k'}} (-V_{kk'})}$
Gap Equation.

$$\begin{pmatrix} \gamma_{k\uparrow} \\ \gamma^{+}_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} u_k & -v_k^* \\ v_k & u_k^* \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^{+}_{-k\downarrow} \end{pmatrix}$$

u_k & v_k obtained from $|v_k|^2 + |u_k|^2 = 1$ (unitary)

$$\frac{\Delta_k^* v_k}{u_k} = -\xi_k + E_k$$ (require $\gamma\gamma$, $\gamma^+\gamma^+$ coeff $= 0$)

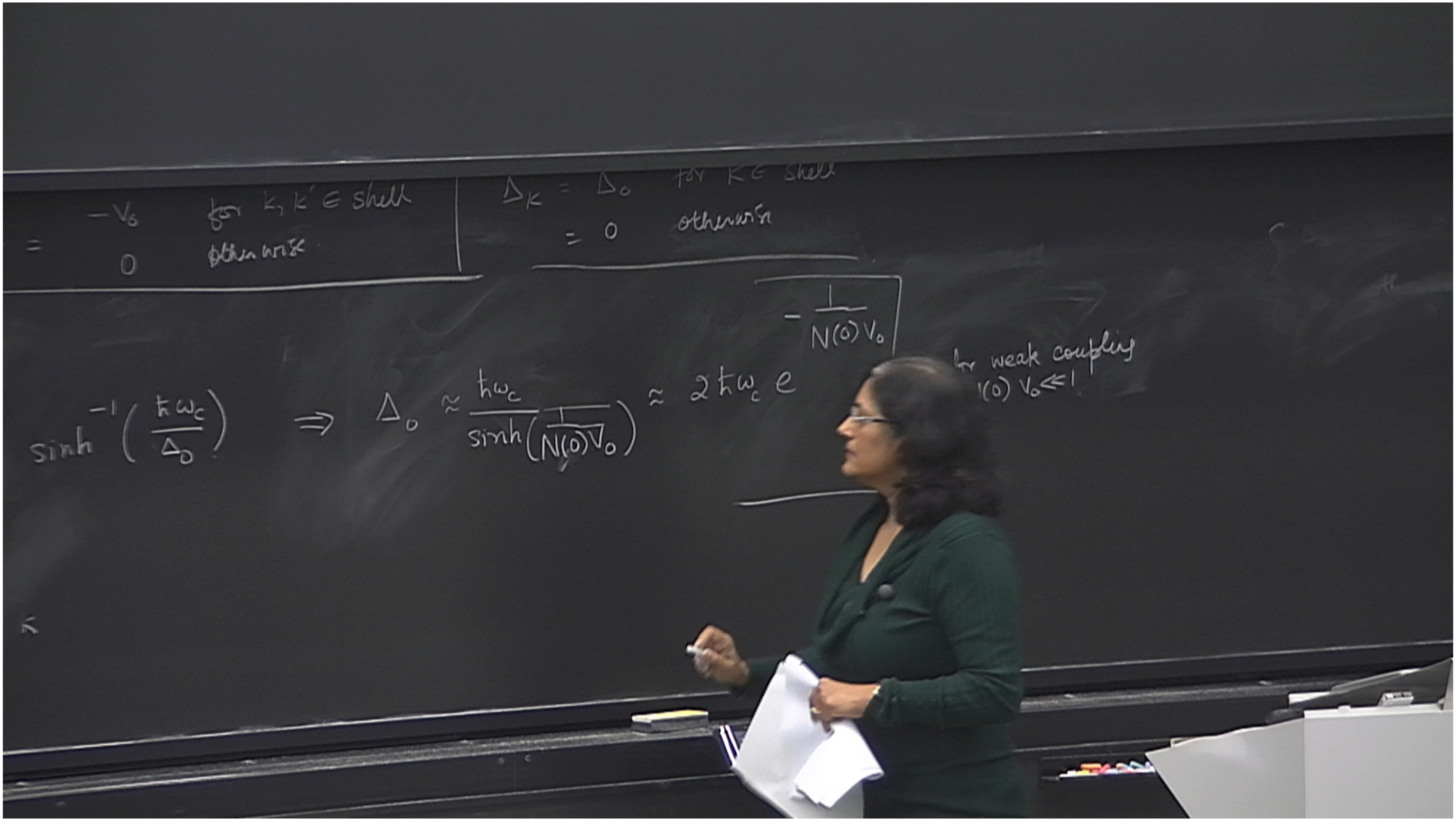
$$\boxed{\begin{aligned} |v_k|^2 &= \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) \\ |u_k|^2 &= \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) \\ 2u_k v_k &= \frac{\Delta_k}{E_k} \end{aligned}}$$

u_k can be chosen to be real

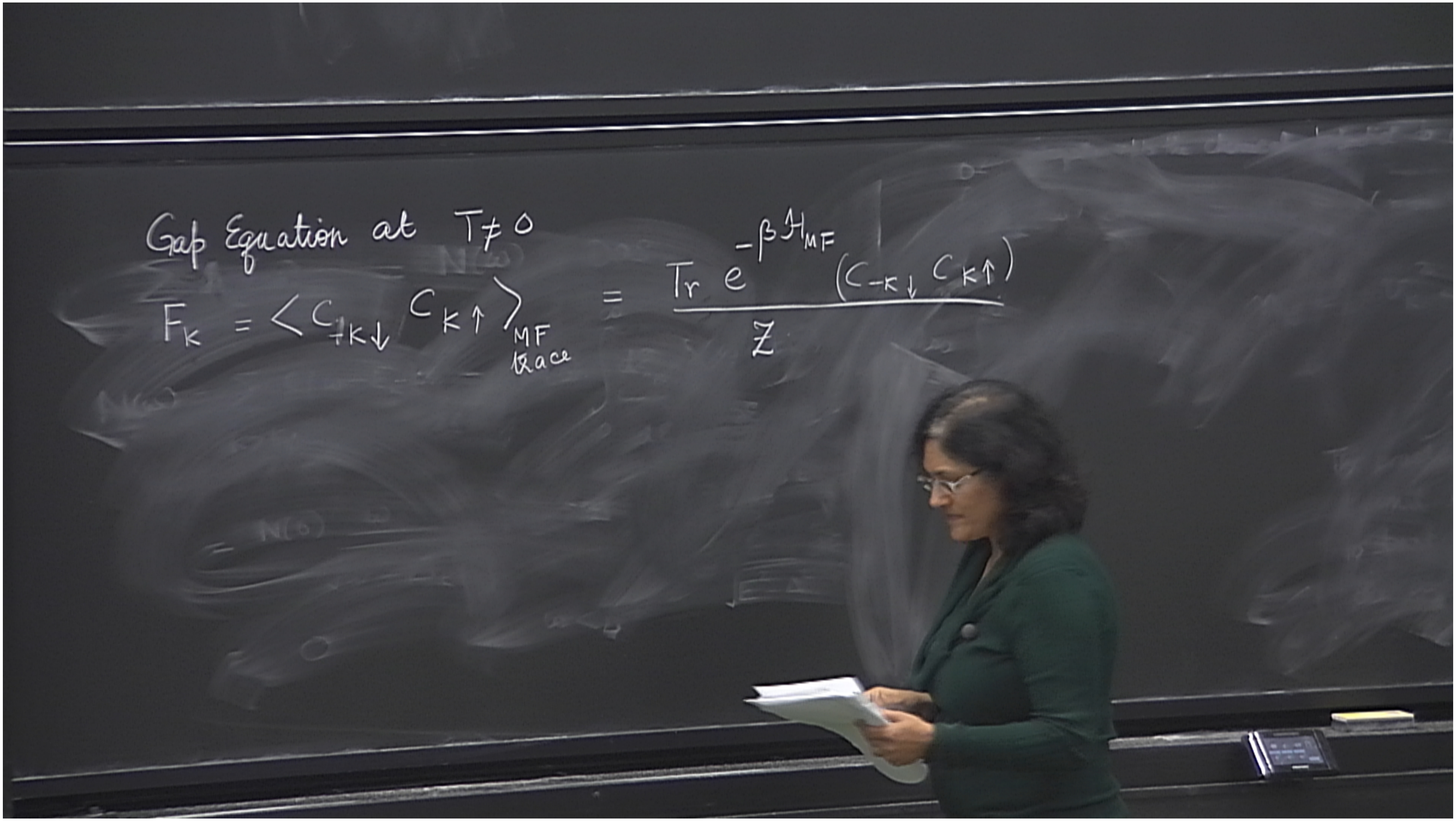
$v_k = |v_k| e^{i\theta}$

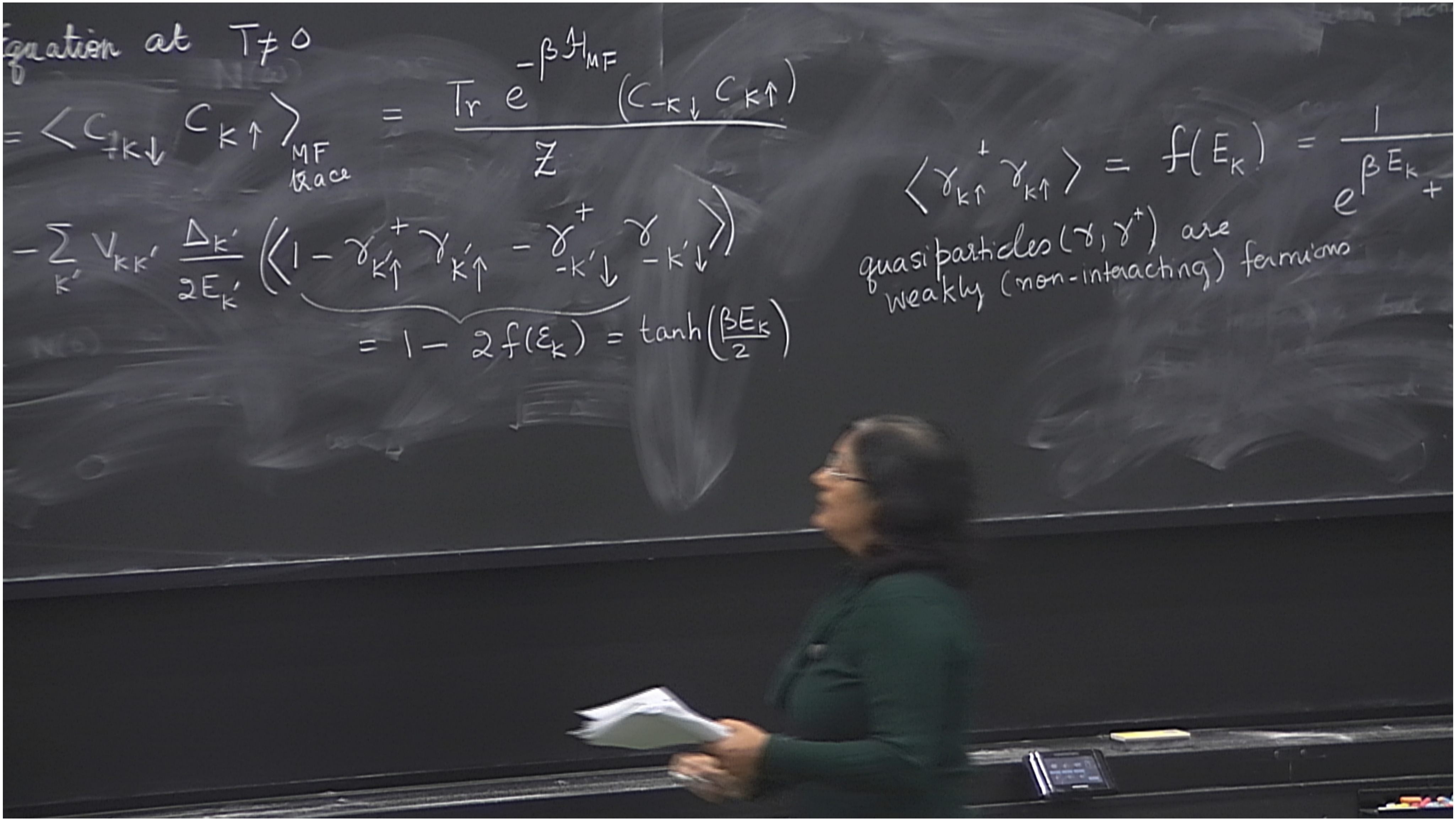
Δ_k must have the same phase $= |\Delta_k| e^{i\theta}$

(RHS) $-\xi_k + E_k$ is real



= −V0 for k, k' ∈ shell
0 otherwise
Δk = Δ0 for k ∈ shell
= 0 otherwise
for weak coupling


Gap Equation at $T \neq 0$
$F_k = \langle c_{-k\downarrow} c_{k\uparrow} \rangle_{\substack{MF \\ trace}} = \dfrac{Tr\, e^{-\beta H_{MF}} (c_{-k\downarrow} c_{k\uparrow})}{Z}$


quation at T ≠ 0
= ⟨c_{-k↓} c_{k↑}⟩_{MF trace} = Tr e^{-βH_MF} (c_{-k↓} c_{k↑}) / Z
-Σ_{k'} V_{kk'} Δ_{k'}/2E_{k'} ⟨⟨1 - γ⁺_{k'↑} γ_{k'↑} - γ⁺_{-k'↓} γ_{-k'↓}⟩⟩
= 1 - 2f(E_k) = tanh(βE_k/2)
⟨γ⁺_{k↑} γ_{k↑}⟩ = f(E_k) = 1/(e^{βE_k} +
quasiparticles (γ, γ⁺) are
weakly (non-interacting) fermions

Gap Equation at $T \neq 0$

$$F_k = \langle c_{-k\downarrow} c_{k\uparrow} \rangle_{\substack{MF \\ \text{trace}}} = \frac{\mathrm{Tr}\, e^{-\beta \mathcal{H}_{MF}} (c_{-k\downarrow} c_{k\uparrow})}{Z}$$

$$\langle \gamma^{+}_{k\uparrow} \gamma_{k\uparrow} \rangle = f($$

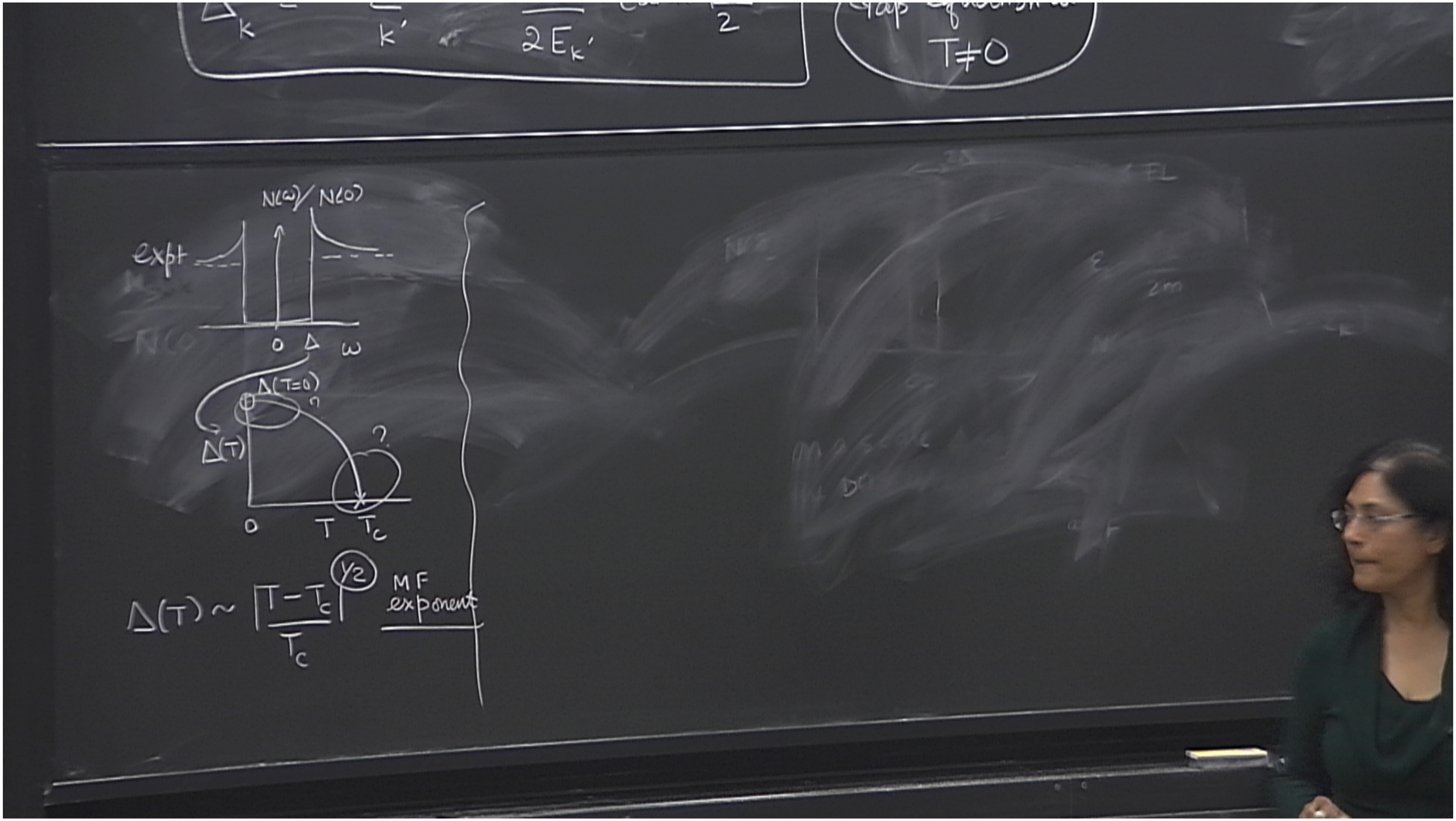
$$\Delta_k = -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \Big\langle 1 - \underbrace{\gamma^{+}_{k'\uparrow} \gamma_{k'\uparrow} - \gamma^{+}_{-k'\downarrow} \gamma_{-k'\downarrow}}_{} \Big\rangle$$

quasiparticles (γ, γ^{+}) a
weakly (non-interactin

$$= 1 - 2f(\mathcal{E}_k) = \tanh\left(\frac{\beta E_k}{2}\right)$$

$$\boxed{\Delta_k = -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \tanh \frac{\beta E_{k'}}{2}}$$

Gap equation at $T \neq 0$



T≠0
N(ω)/N(0)
expt
0 Δ ω
Δ(T=0)
Δ(T)
0 T T_c
Δ(T) ~ |(T − T_c)/T_c|^{1/2}
MF exponent

$\Delta_k = \sum_{k'} \quad \frac{\quad}{2E_{k'}} \quad \frac{\quad}{2}$

Gap equation at $T \neq 0$

$$\Delta(T) \sim \left|\frac{T-T_c}{T_c}\right|^{1/2}$$

MF exponent

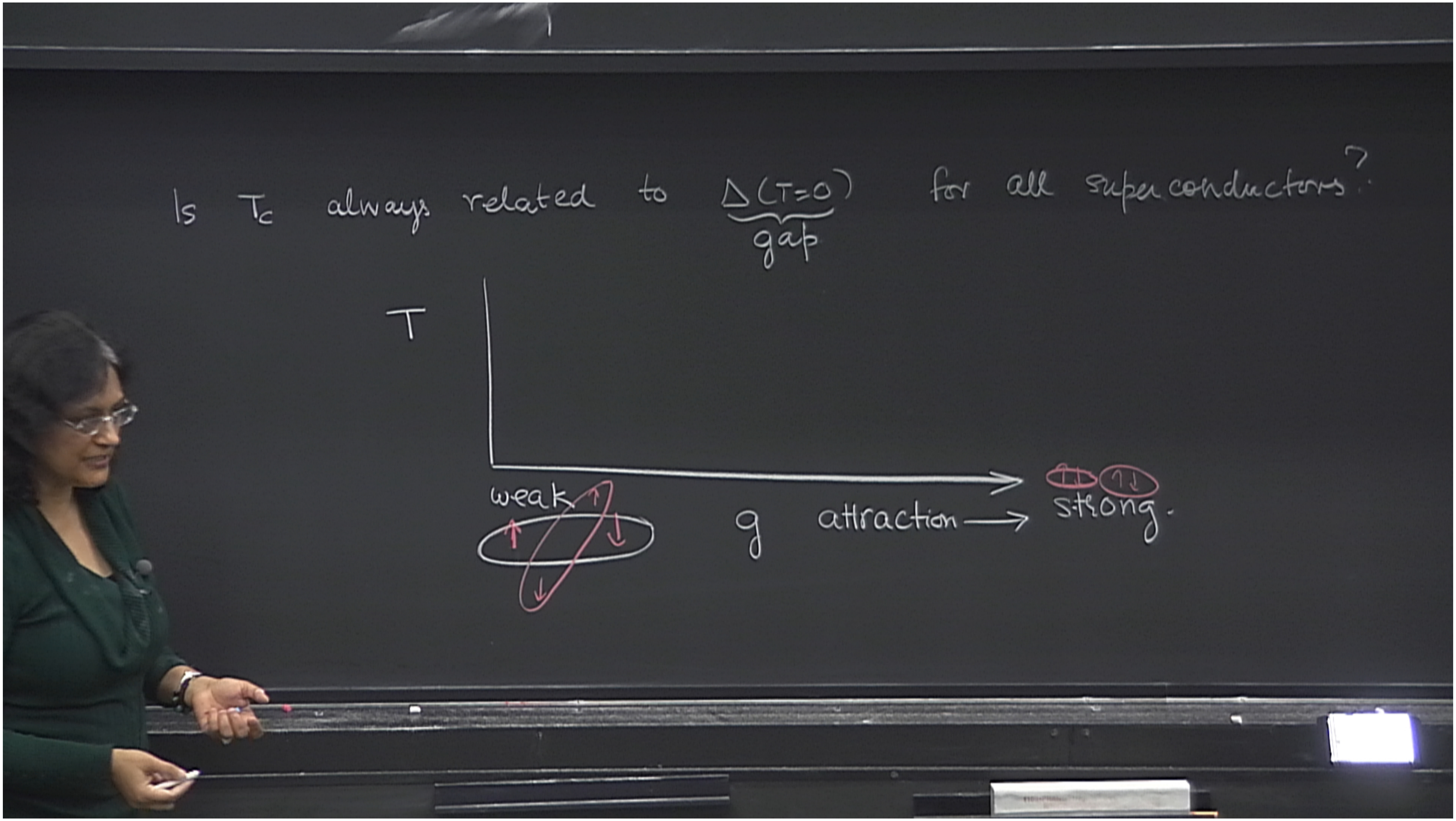
Determine T_c

as $T \to T_c^-$ $\quad \Delta(T) \to 0$

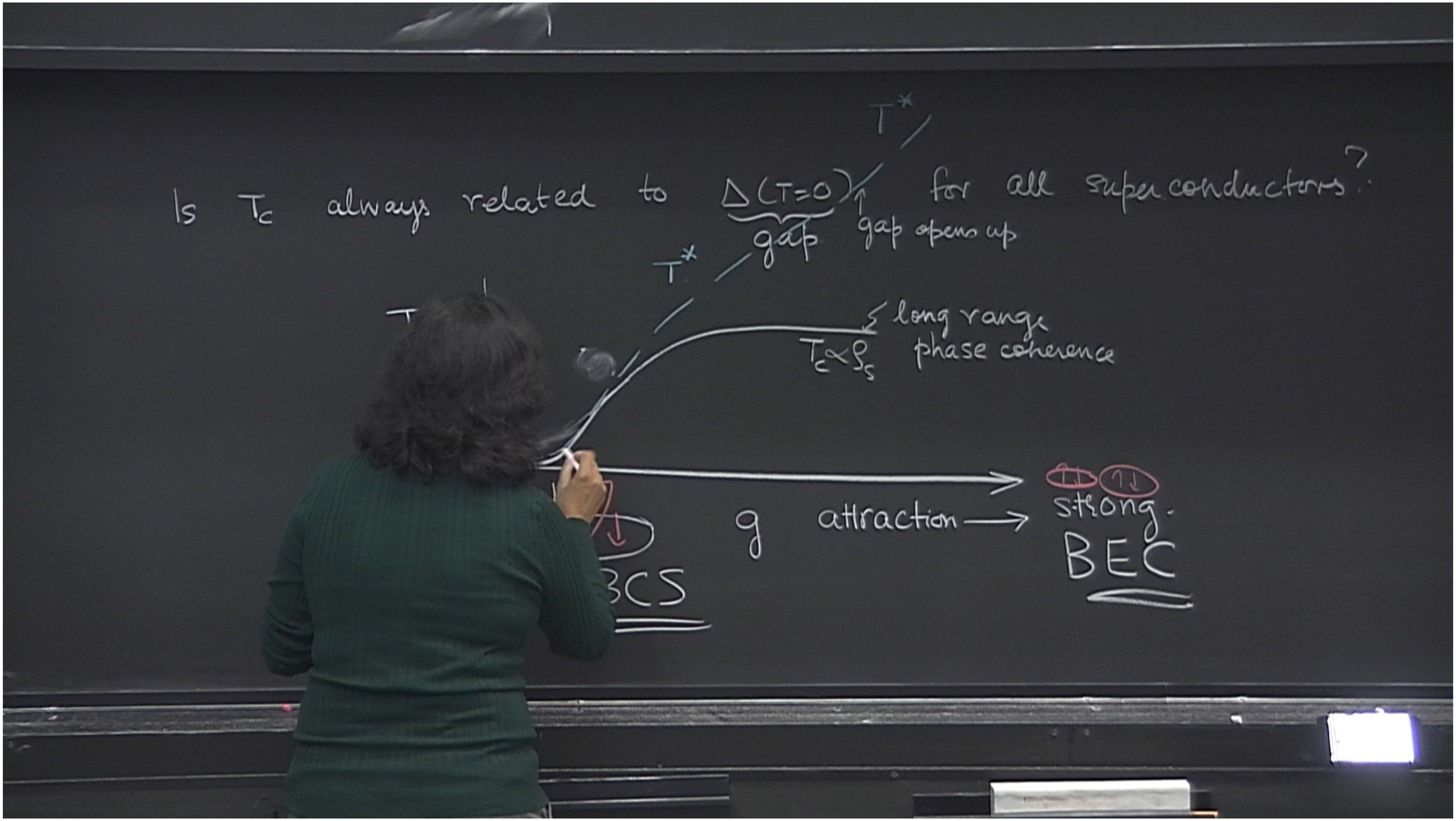
$$\frac{1}{N(0)V_0} = \int_0^{\omega_c} d\xi \, \frac{\tanh\left(\frac{\xi}{2T_c}\right)}{\xi}$$

$$T_c = \frac{2e^{\gamma}}{\pi}\,\omega_c \, e^{-\frac{1}{N(0)V_0}} \simeq 1.13\,\omega_c\, e^{-\frac{1}{N(0)V_0}}$$

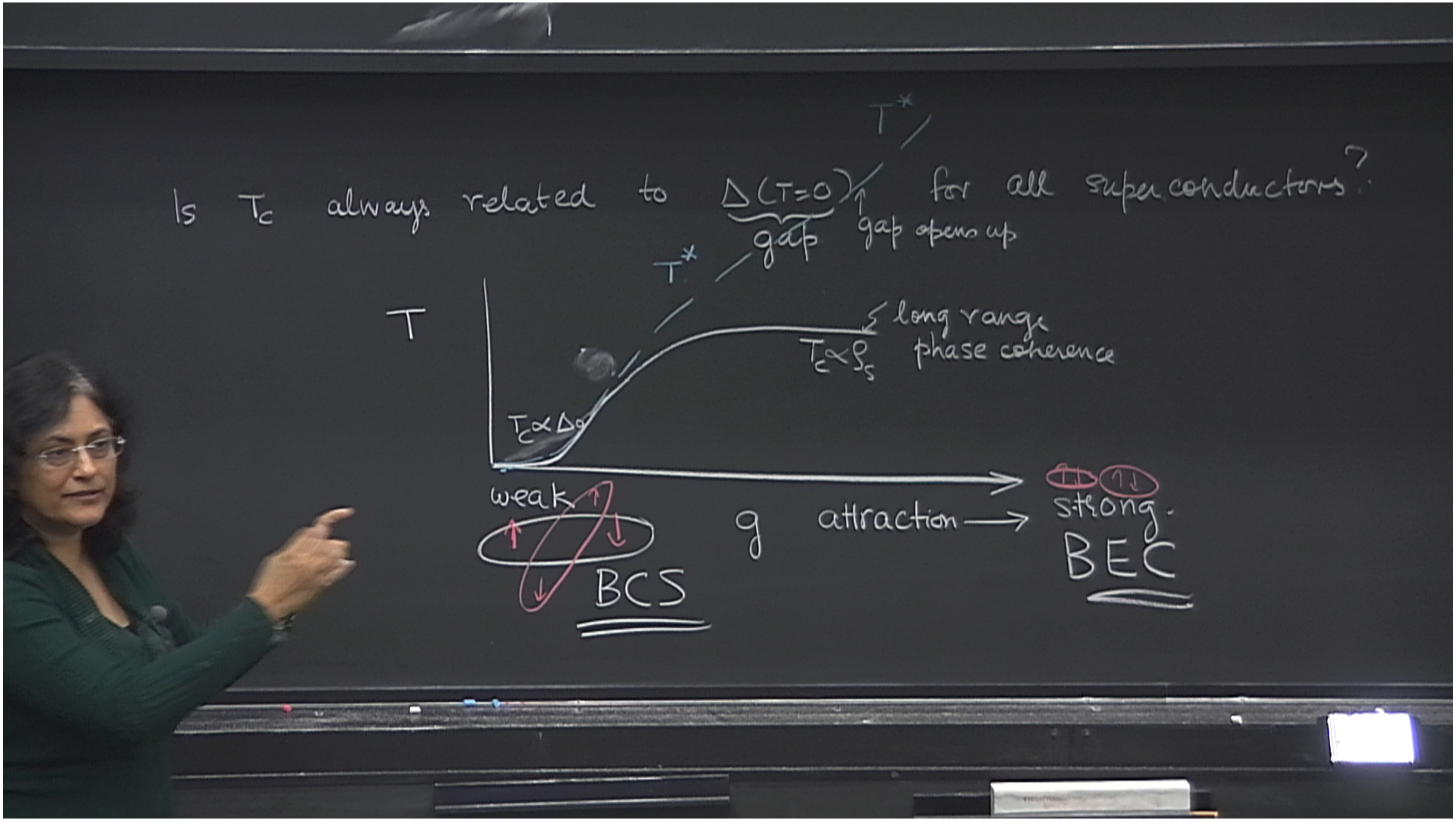
γ = Euler const $\simeq 0.5772$



Is Tc always related to Δ(T=0) for all superconductors?
gap
T
weak
g
attraction
strong.



Is T_c always related to $\Delta(T=0)$ for all superconductors?
gap
gap opens up
T^*
T^*
long range phase coherence
$T_c \propto \rho_s$
g attraction →
strong.
BEC



Is T_c always related to $\underbrace{\Delta(T=0)}_{\text{gap}}$ for all superconductors?
gap opens up
T^*
T^*
T
long range phase coherence
$T_c \propto \rho_s$
$T_c \propto \Delta_0$
weak
g
attraction →
strong.
BCS
BEC