Title: Condensed Matter - Lecture 10

Date: Nov 11, 2011 09:00 AM

URL: http://pirsa.org/11110032

Abstract:

Today Lecture 10

1) Vortices
2) Superfluidity

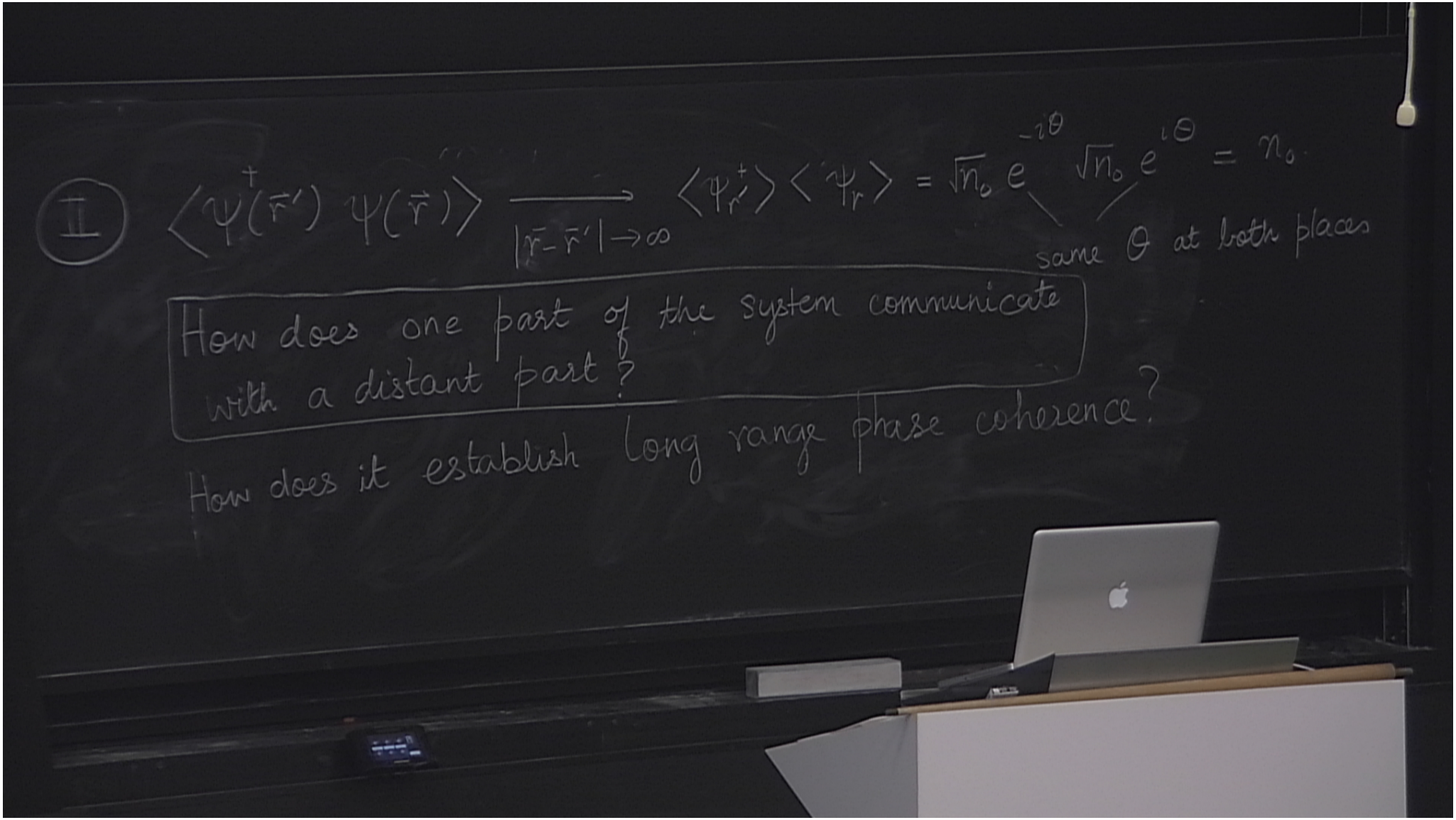
Food for thought...

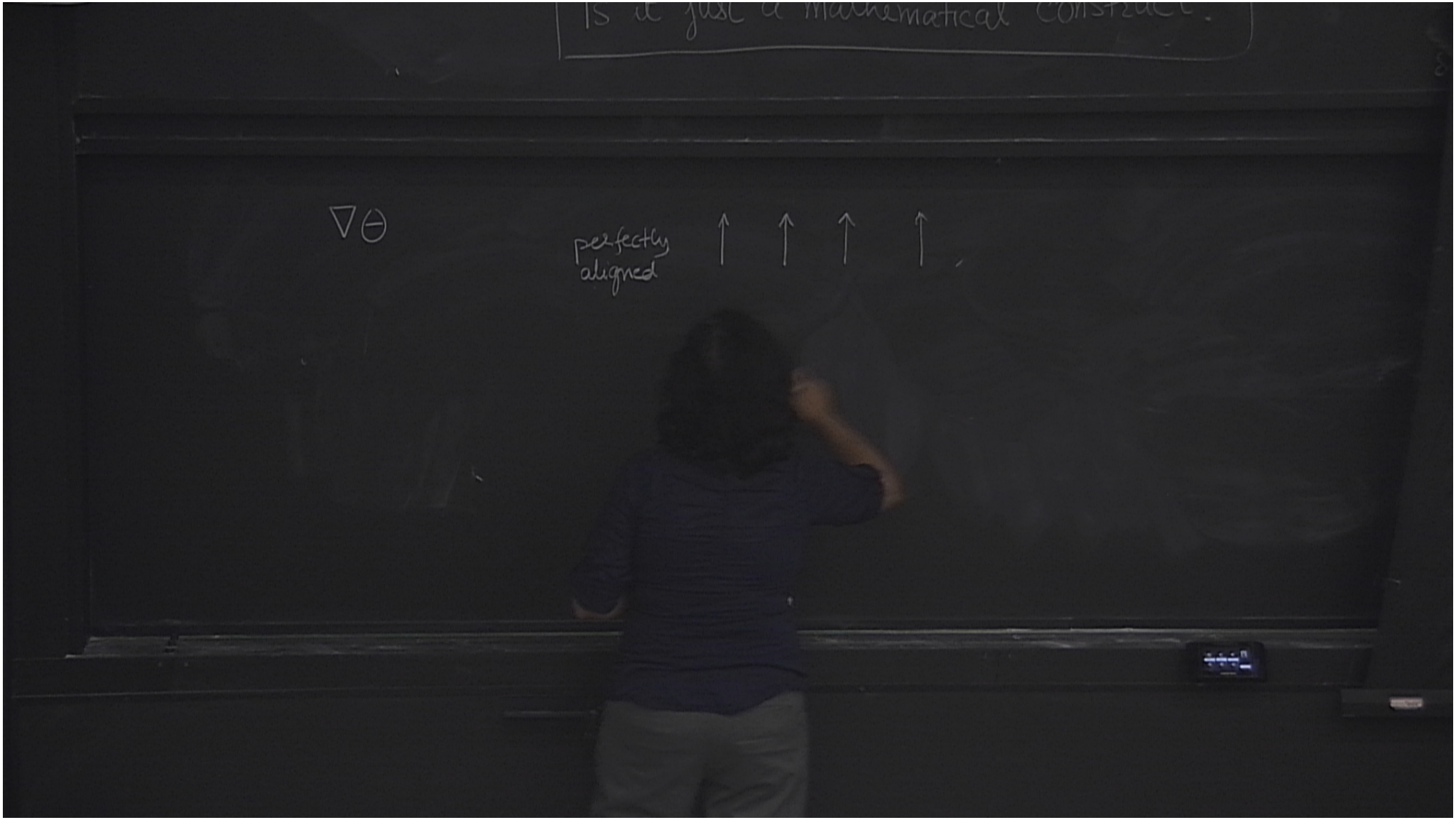
(I) order parameter for BEC

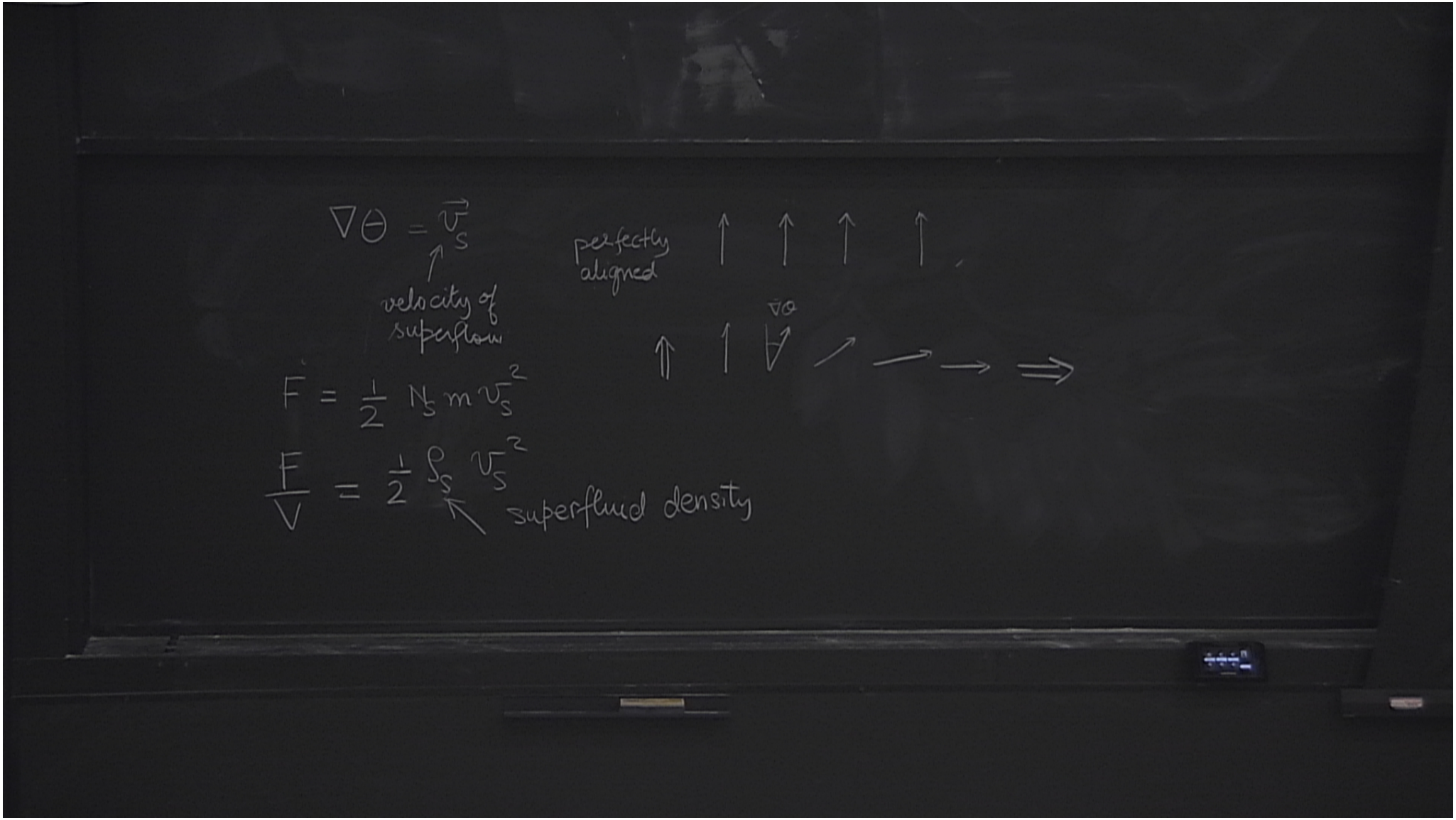
$\langle \psi(\vec{r}) \rangle = \sqrt{n_0}\, e^{i\theta}$ ← phase

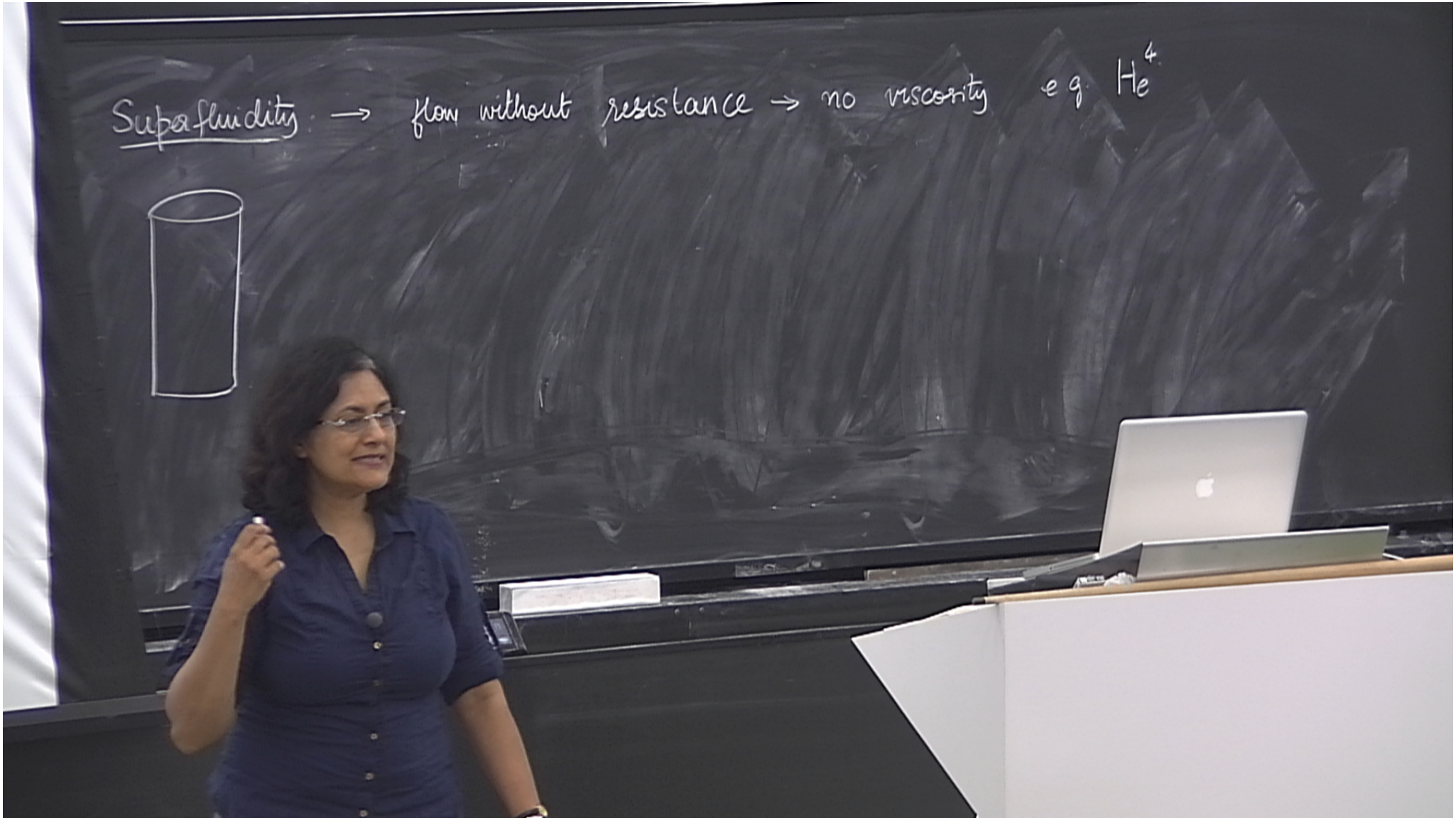
$\sqrt{n_0}$: amplitude ~ condensate density

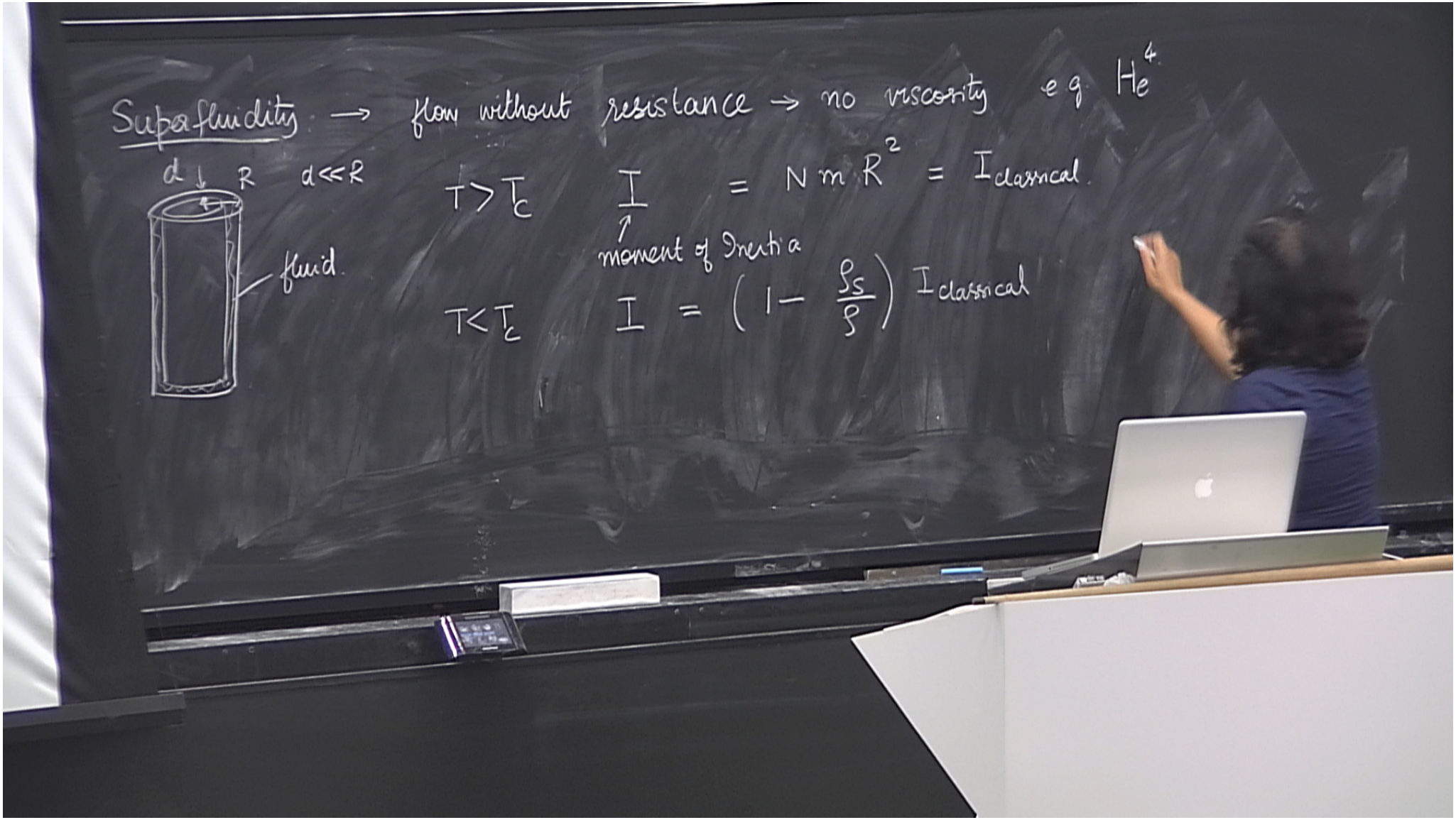
Does θ have a physical meaning?
Is it just a mathematical construct?

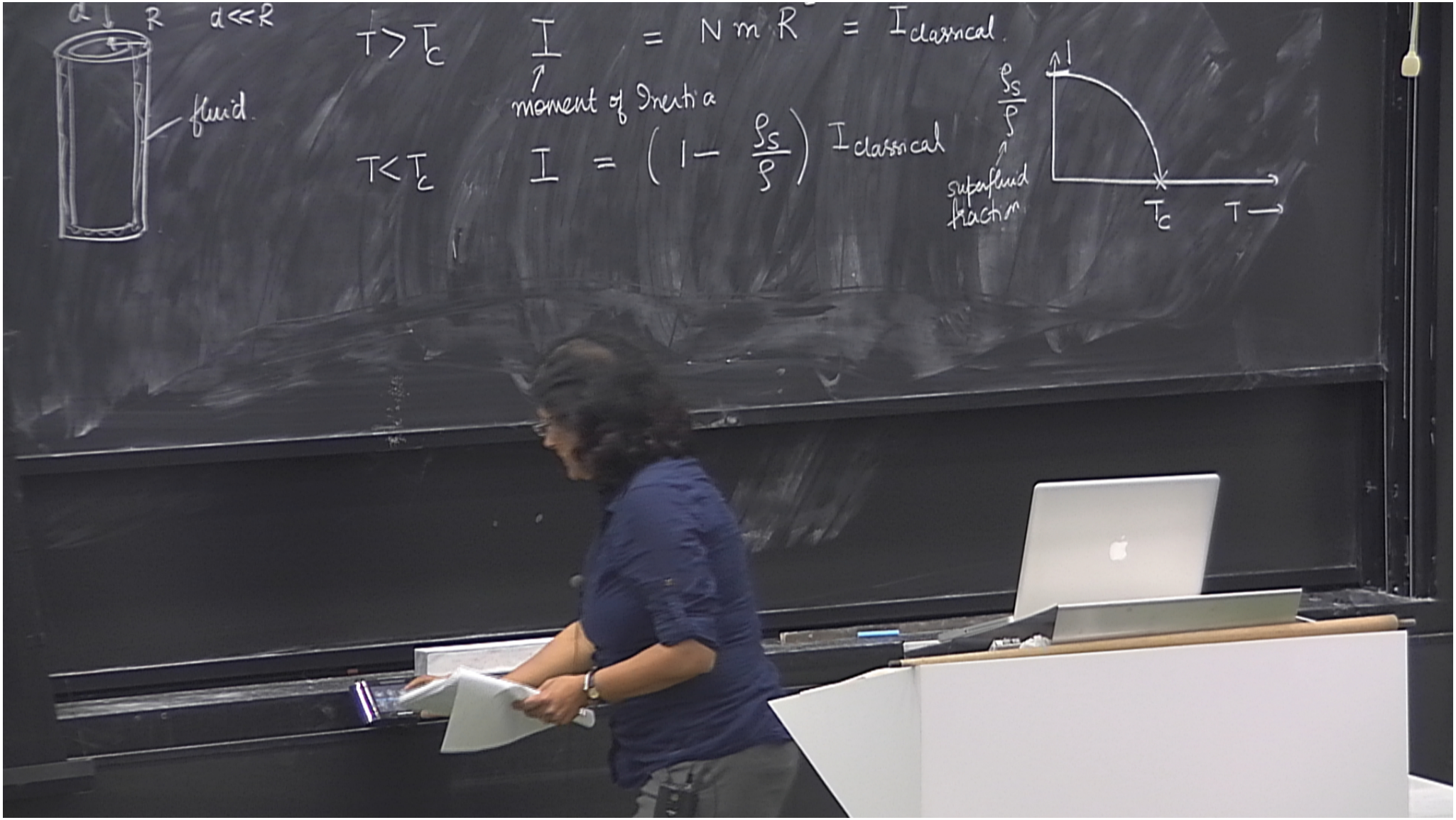

II
$\langle \psi^\dagger(\vec{r}') \psi(\vec{r}) \rangle \xrightarrow[|\vec{r}-\vec{r}'|\to\infty]{} \langle \psi^\dagger_{r'} \rangle \langle \psi_r \rangle = \sqrt{n_0}\, e^{-i\theta} \sqrt{n_0}\, e^{i\theta} = n_0$
same θ at both places
How does one part of the system communicate with a distant part?
How does it establish long range phase coherence?

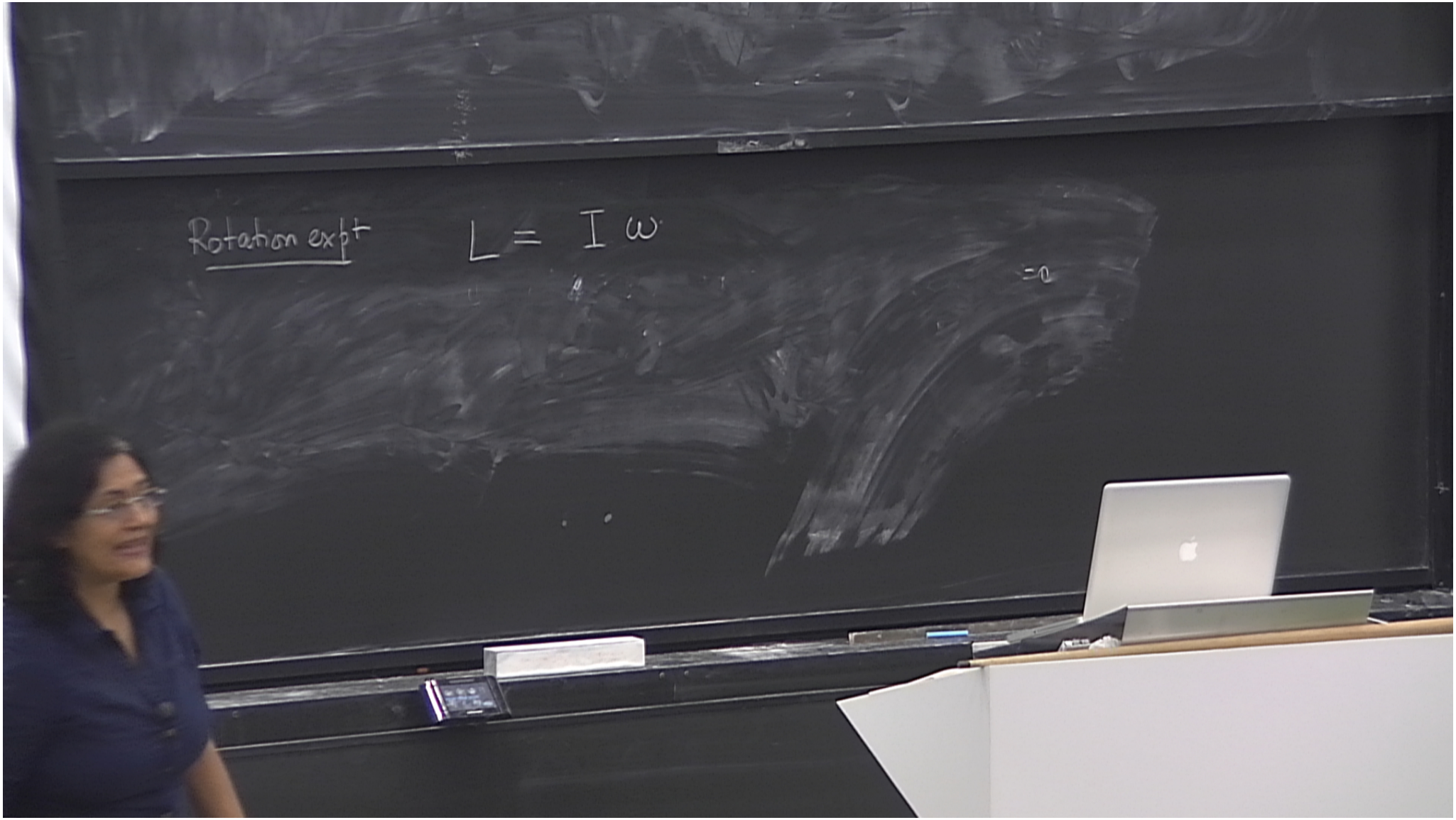

Is it just a mathematical construct.
$\nabla\Theta$
perfectly aligned

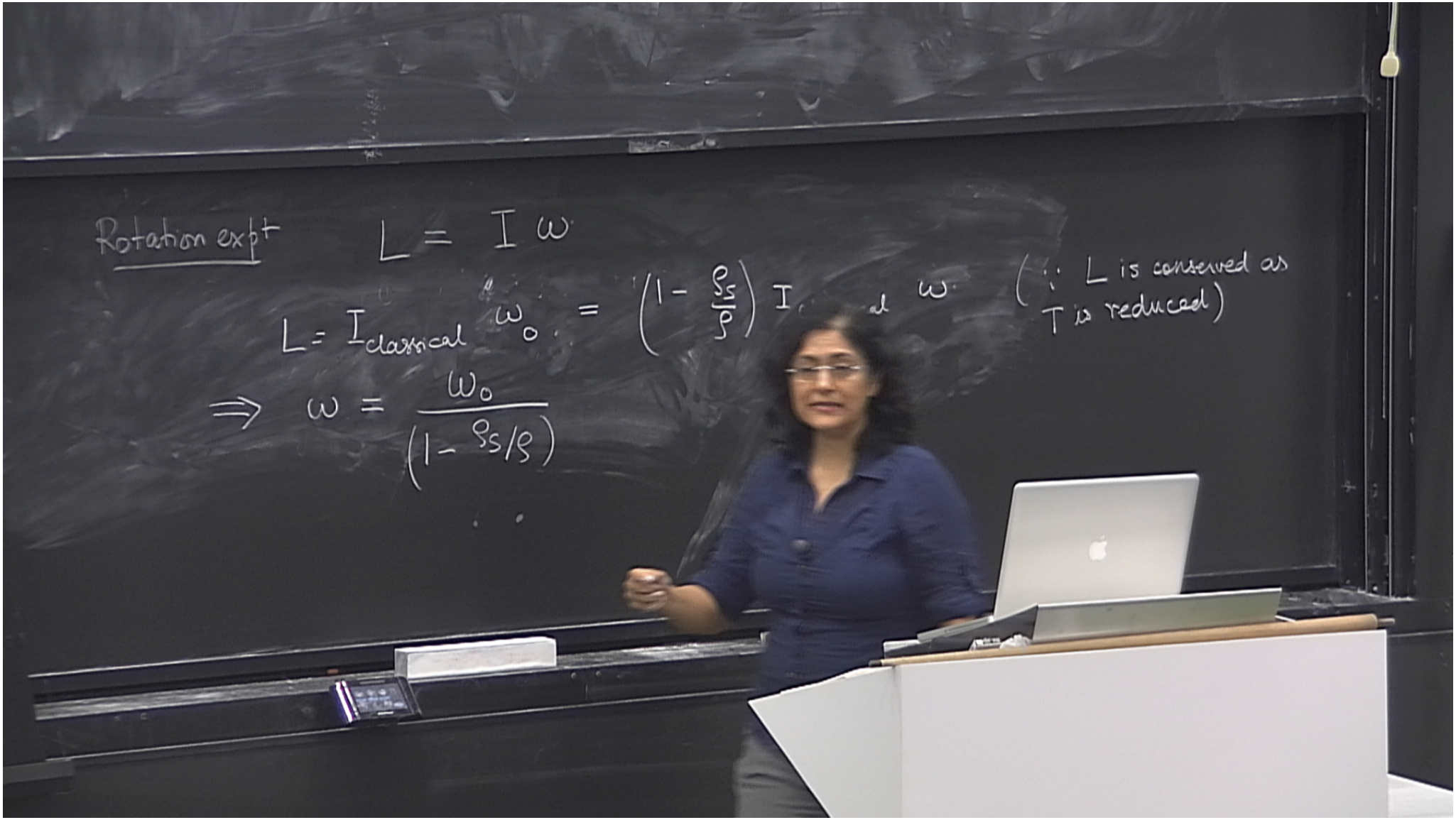

$\nabla\theta = \vec{v}_s$
velocity of superflow
perfectly aligned
v_c
$F = \frac{1}{2} N_s m v_s^2$
$\frac{F}{V} = \frac{1}{2} \rho_s v_s^2$
superfluid density

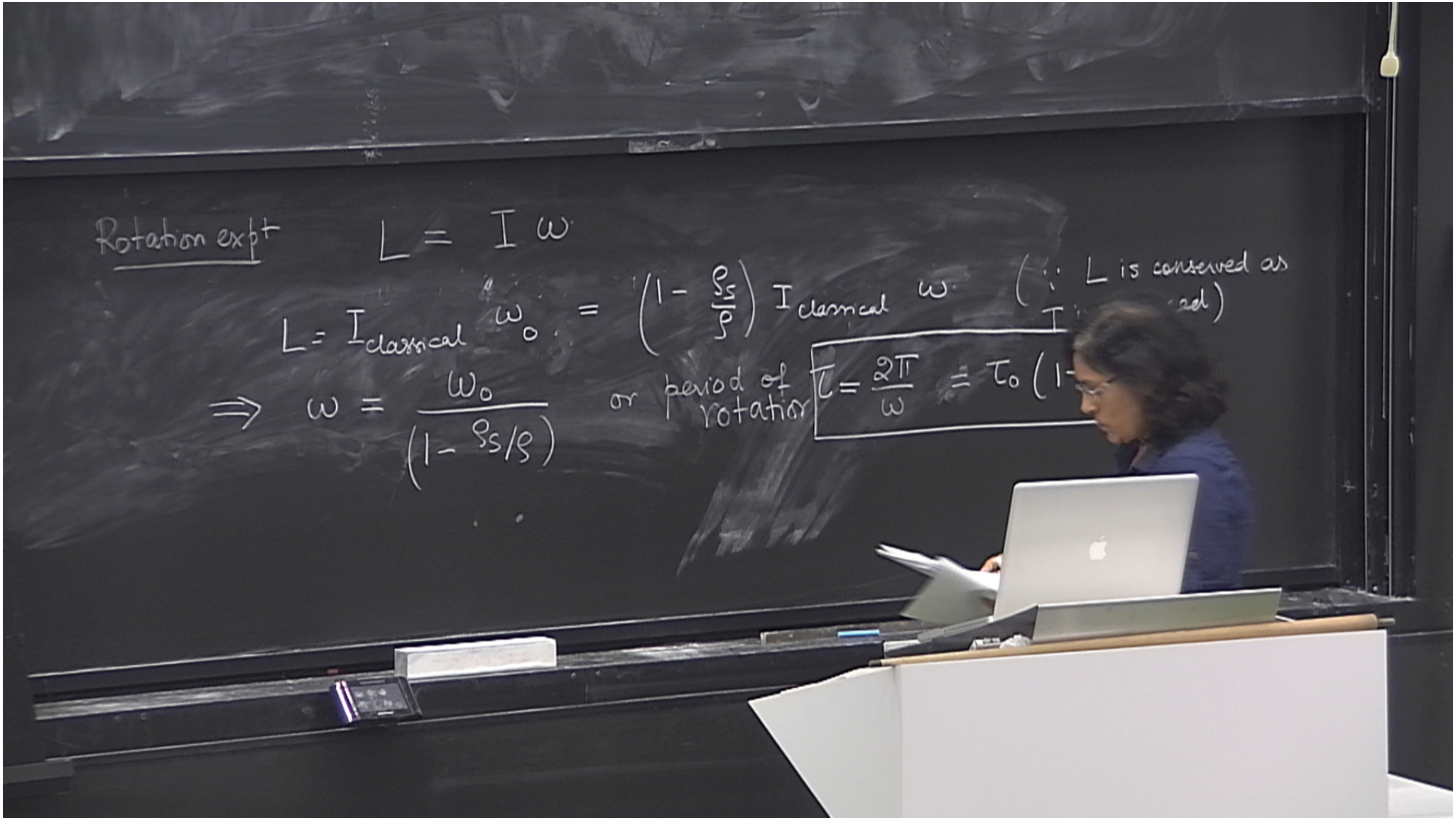

Superfluidity → flow without resistance → no viscosity e.g. He^4

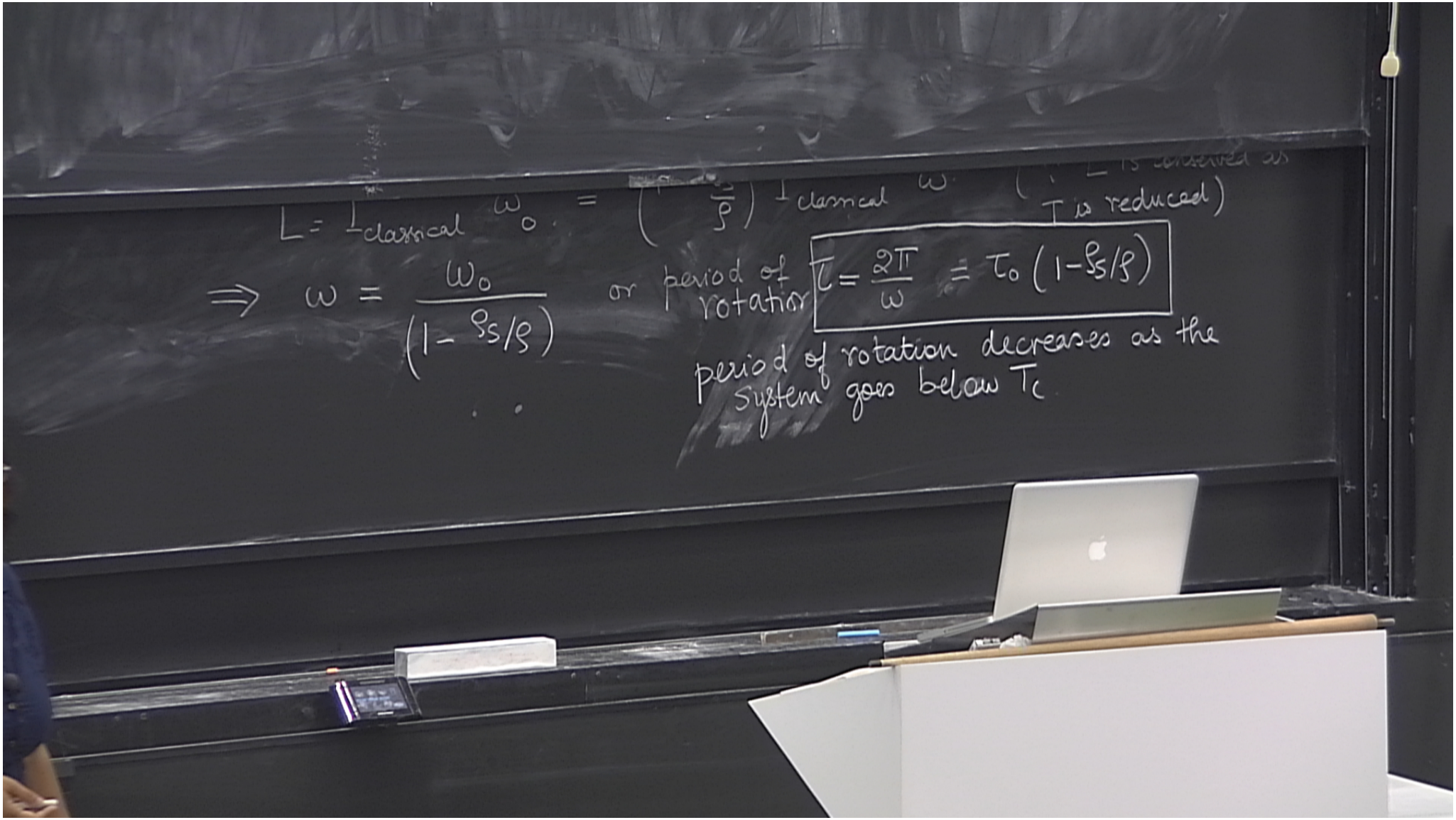

Supafluidity → flow without resistance → no viscosity e.g. He^4
d ↓ R d≪R
fluid
$T > T_c$ $I = N m R^2 = I_{classical}$
moment of Inertia
$T < T_c$ $I = \left(1 - \frac{\rho_s}{\rho}\right) I_{classical}$

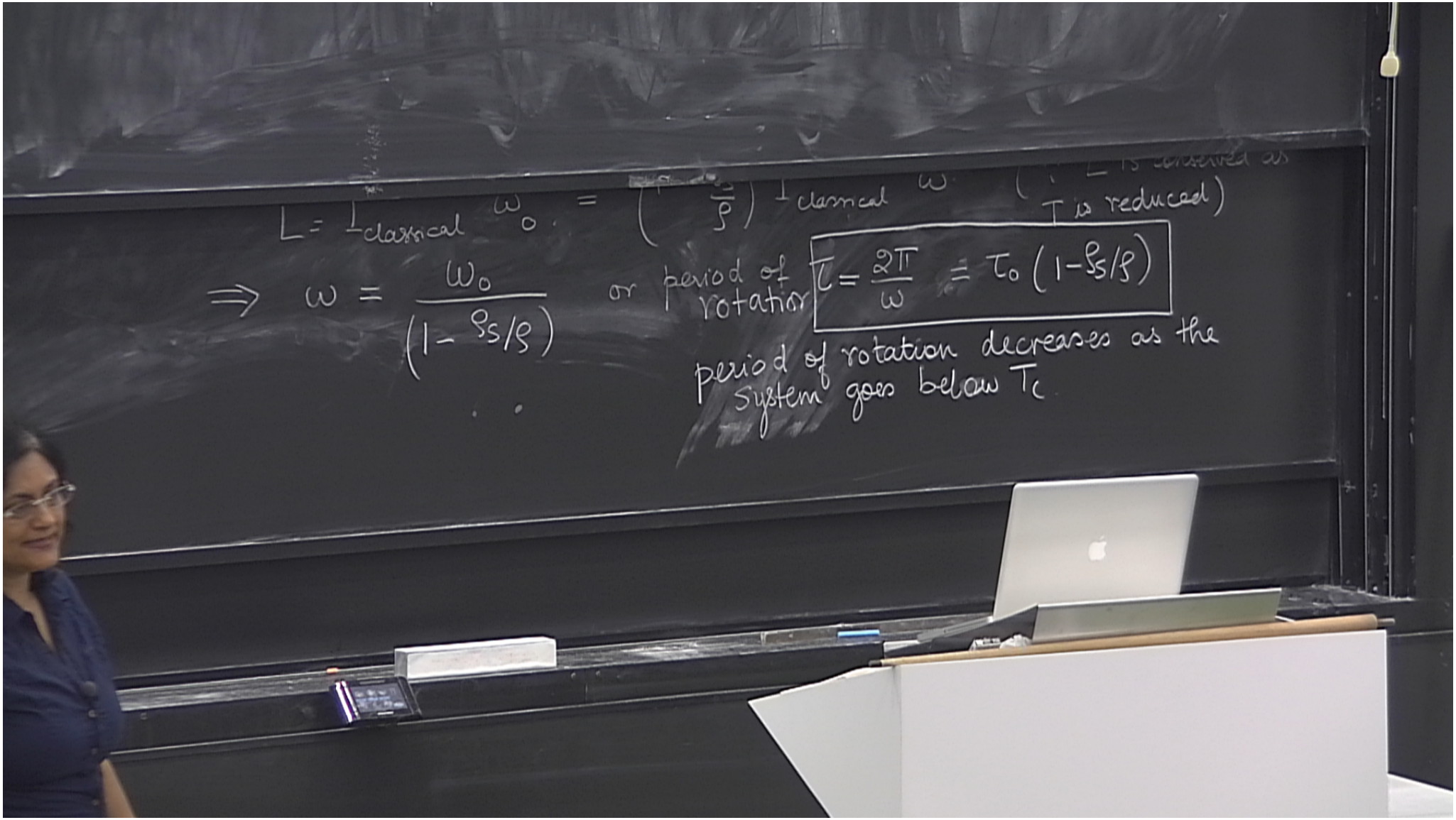

R
d≪R
fluid
T>Tc
I = N m R² = I classical
moment of Inertia
T<Tc
I = (1 − ρs/ρ) I classical
ρs/ρ
superfluid fraction
I
Tc
T

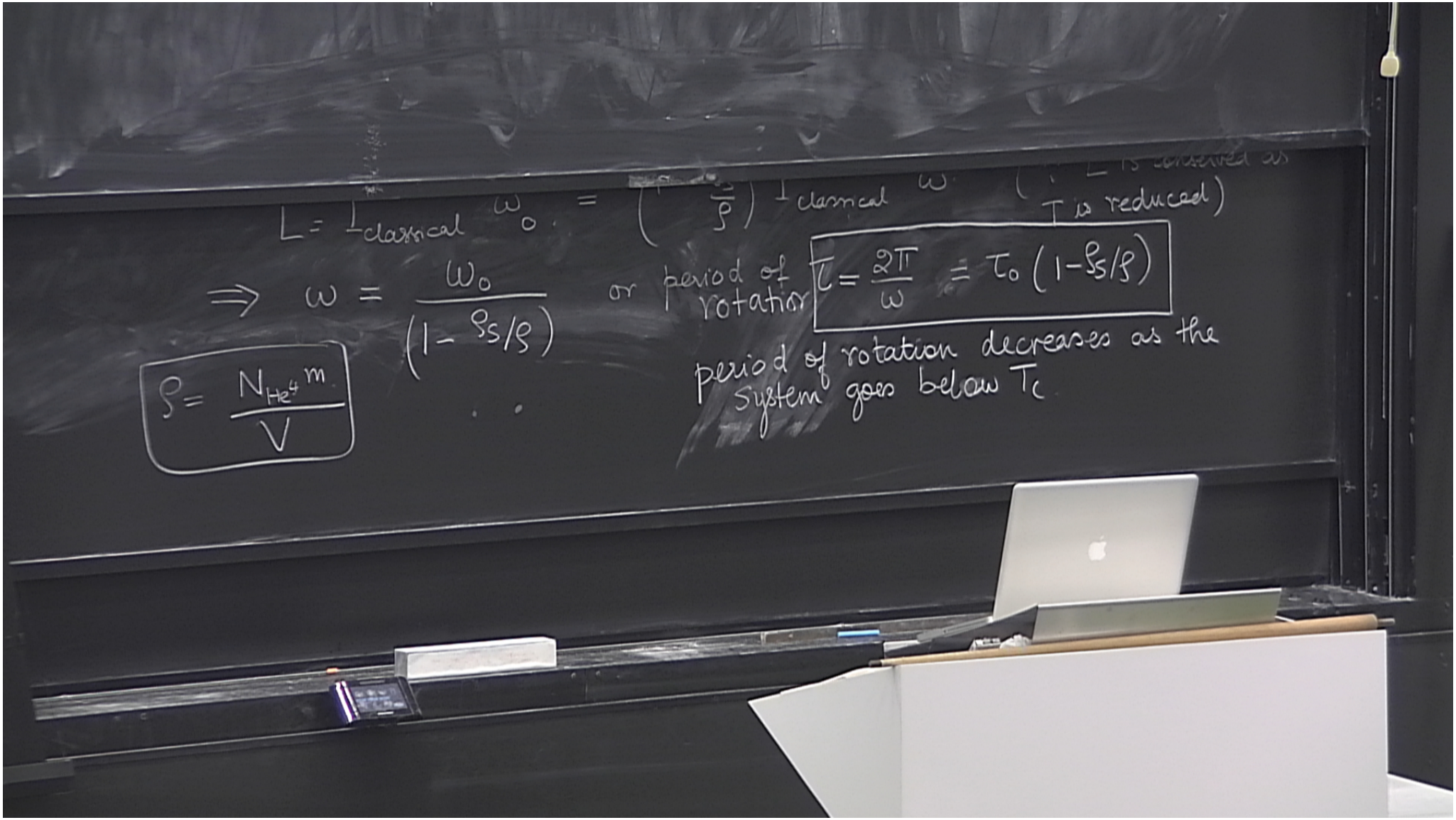

Rotation expt
L = I ω
=0


Rotation expt
$L = I\omega$
$L = I_{classical}\,\omega_0 = \left(1 - \frac{\rho_s}{\rho}\right) I \cdot \omega$ (∵ L is conserved as T is reduced)
$\Rightarrow \omega = \frac{\omega_0}{(1 - \rho_s/\rho)}$

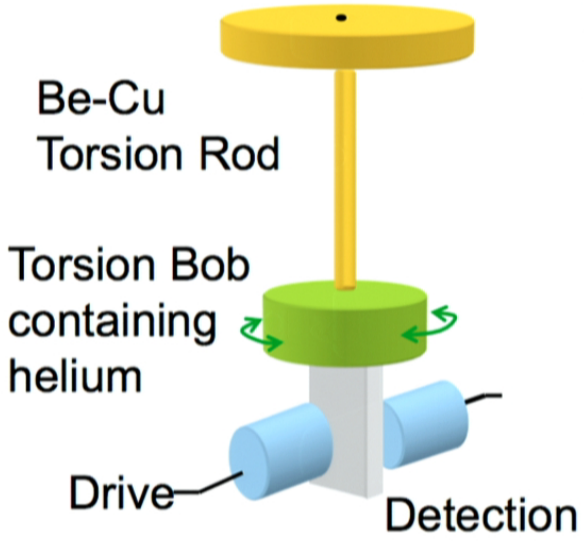

Rotation expt
$L = I\omega$
$L = I_{classical}\,\omega_0 = \left(1 - \frac{\rho_s}{\rho}\right) I_{classical}\,\omega$
$\Rightarrow \omega = \frac{\omega_0}{(1 - \rho_s/\rho)}$
or period of rotation
$\tau = \frac{2\pi}{\omega} = \tau_0 (1$


$L = I_{classical}\,\omega_0 = \left(\frac{2}{5}\right) I_{classical}\,\omega$
I is reduced
$\Rightarrow \omega = \frac{\omega_0}{(1-\rho_s/\rho)}$
or period of rotation $\tau = \frac{2\pi}{\omega} = \tau_0(1-\rho_s/\rho)$
period of rotation decreases as the system goes below T_c

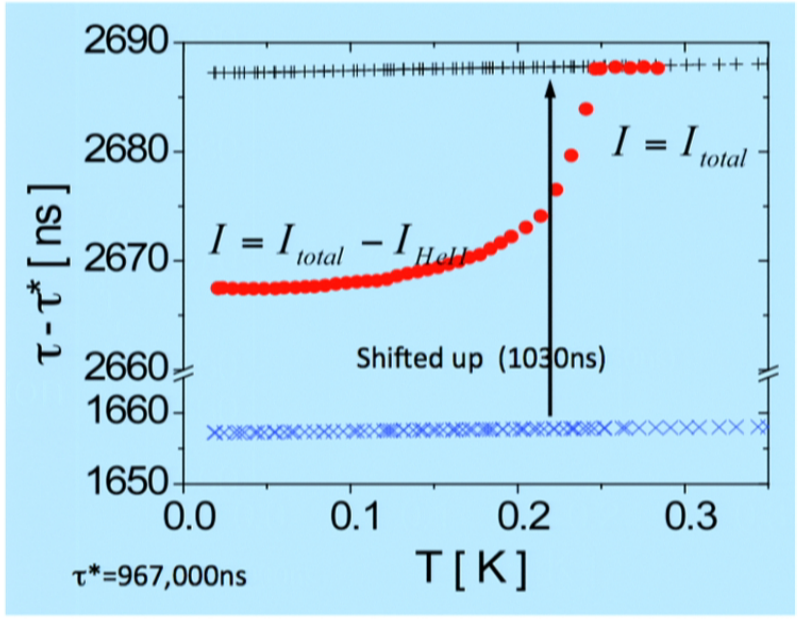

$L = I_{classical}\ \omega_0 = \left(\frac{2}{5}\right) I_{classical}\ \omega$
I is reduced)
$\Rightarrow \omega = \frac{\omega_0}{(1 - \rho_s/\rho)}$
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period of rotation decreases as the system goes below T_c


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I is reduced)
$\Rightarrow \omega = \frac{\omega_0}{(1-\rho_s/\rho)}$
or period of rotation $\tau = \frac{2\pi}{\omega} = \tau_0(1-\rho_s/\rho)$
$\rho = \frac{N_{He^4} m}{V}$
period of rotation decreases as the system goes below T_c

Torsional oscillator ideal for detection of superfluidity



$$I_{total} = I_{torsionbob} + I_{helium}$$

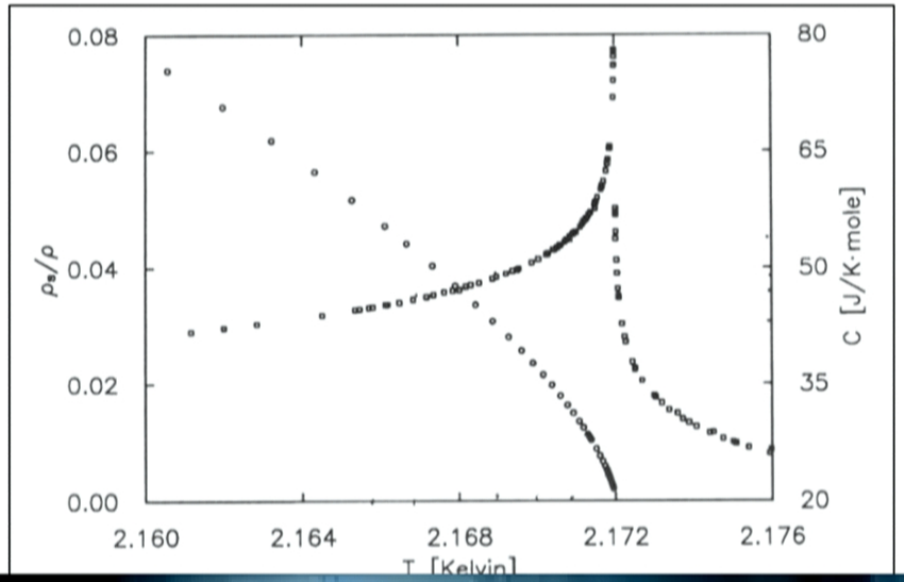


$$\tau_o = 2\pi\sqrt{\frac{I}{K}}$$

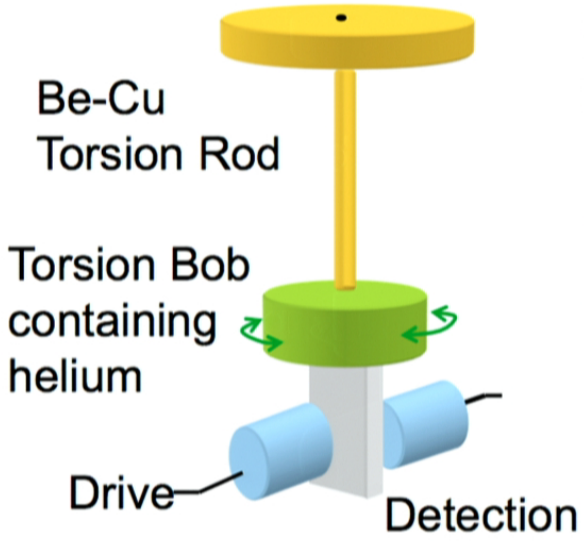
λ-transition in liquid ^{4}He

$$\rho = \rho_s + \rho_n , \quad \rho_S = \psi^* \psi , \quad \psi = |\psi| e^{i\varphi}$$

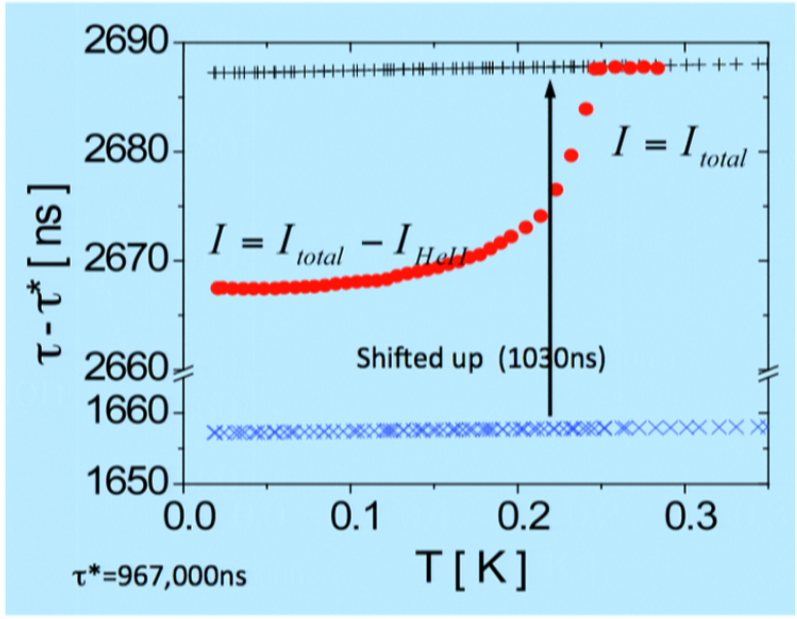
near transition $\frac{\rho_s}{\rho} = k_0 t^{\xi}$ with $\xi = 0.67$,$t=(T_\lambda - T)/T_\lambda$



Torsional oscillator ideal for detection of superfluidity



$$I_{total} = I_{torsionbob} + I_{helium}$$

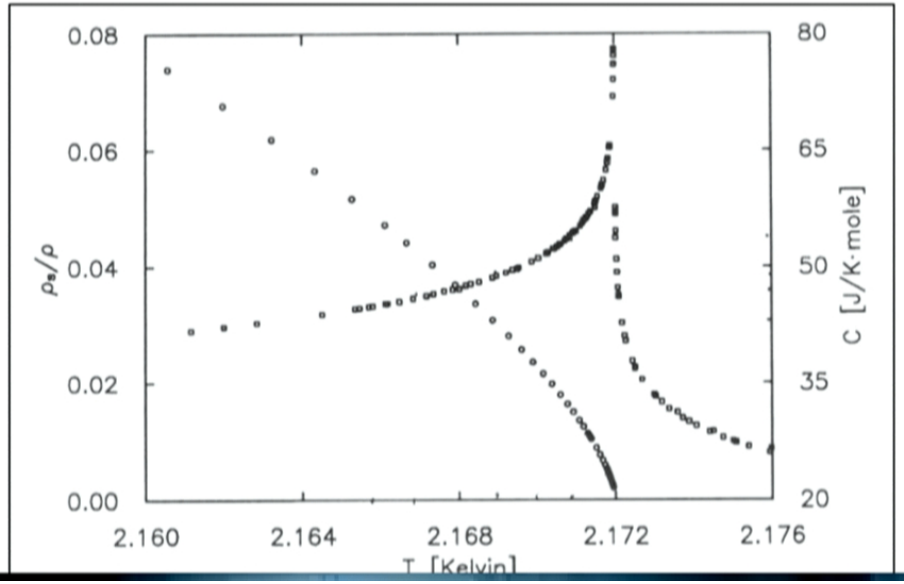


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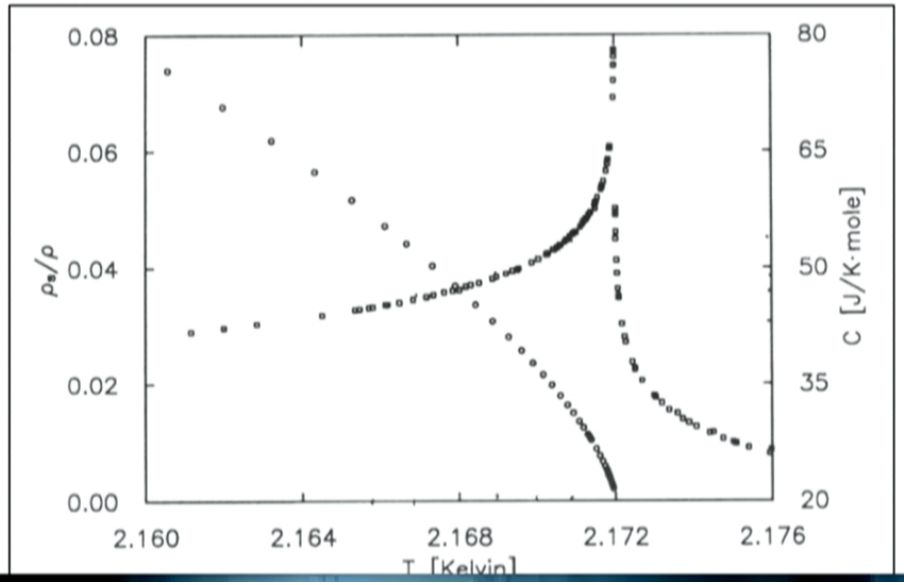
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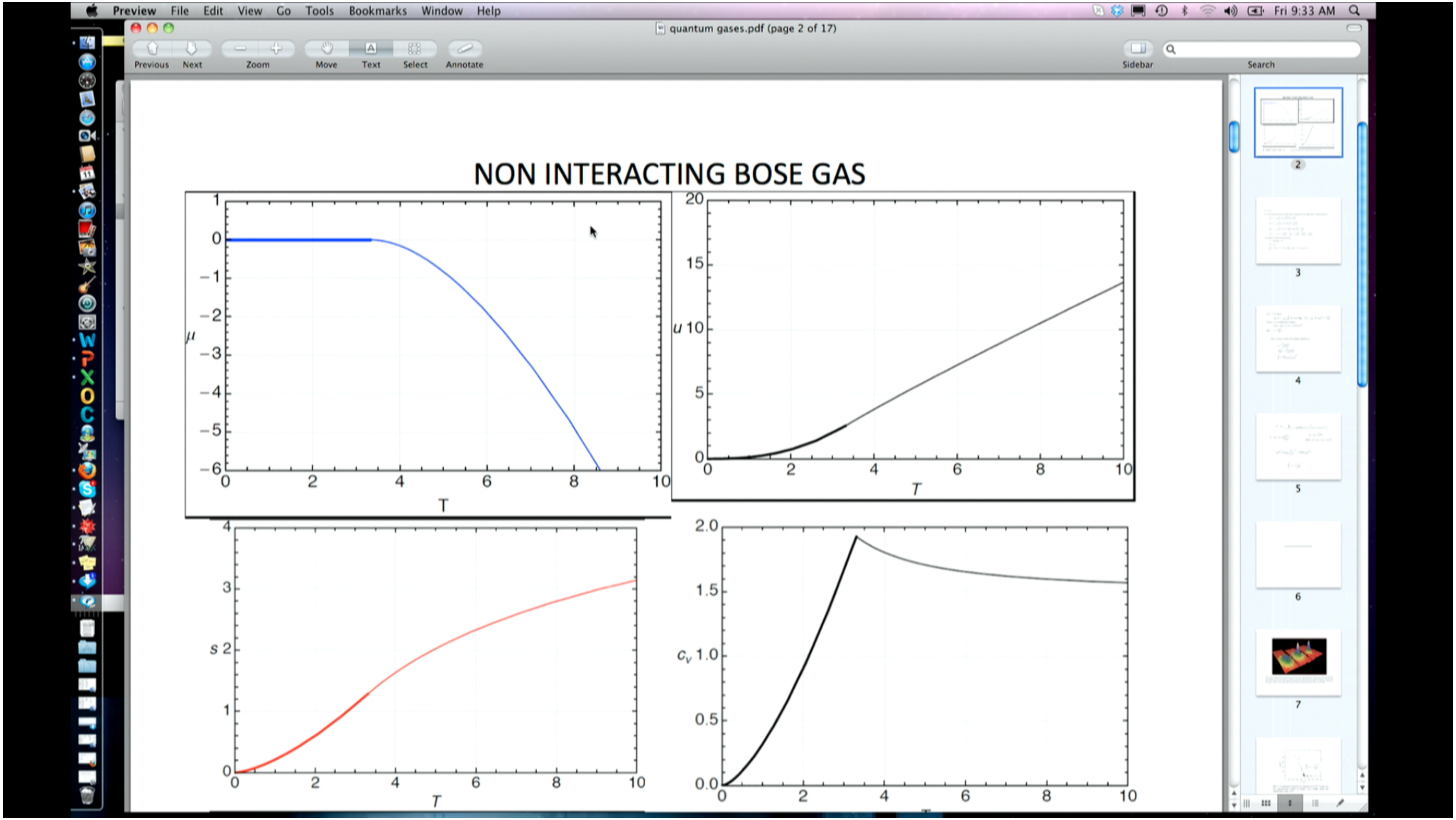


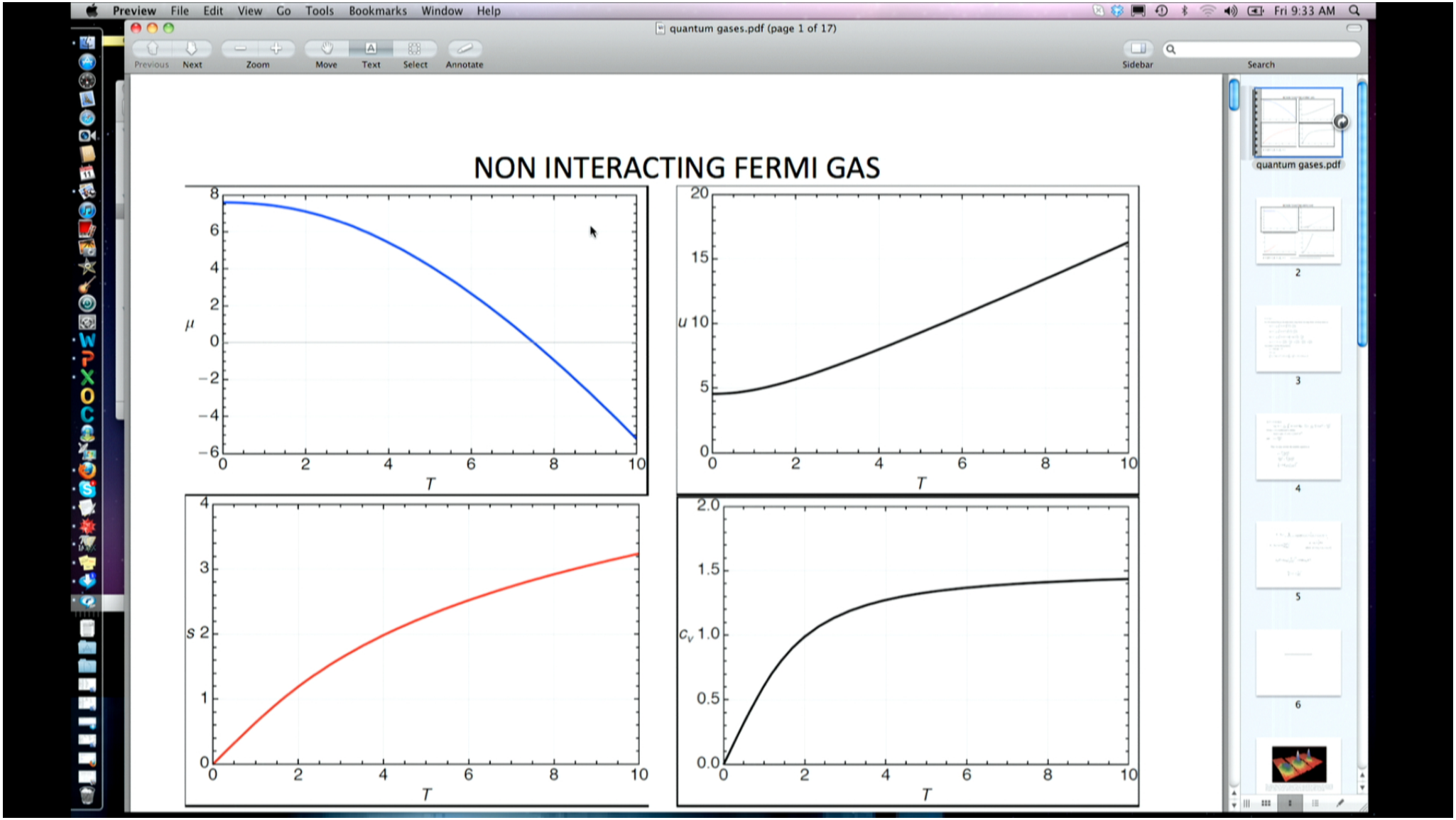
λ-transition in liquid ^{4}He

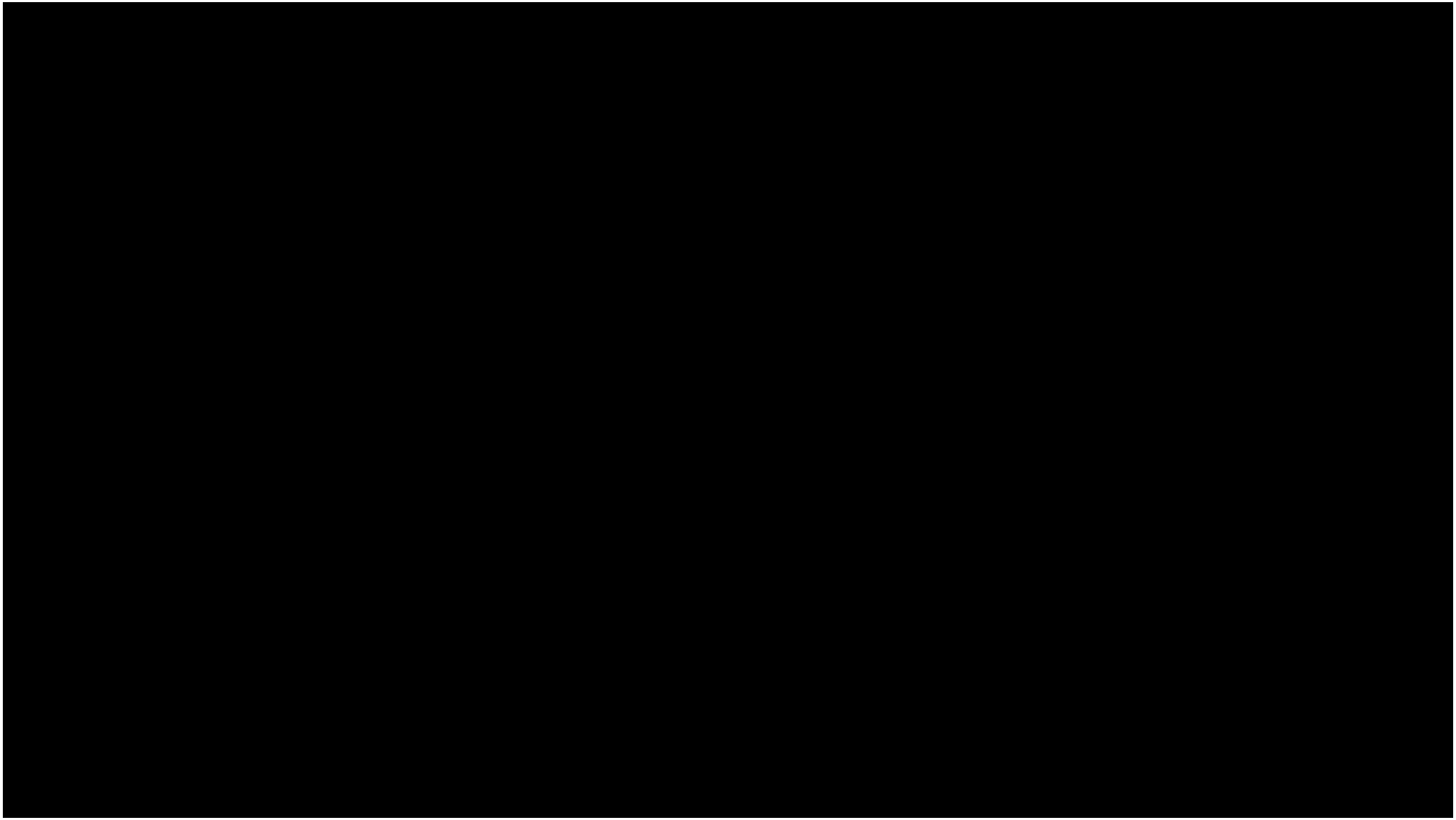
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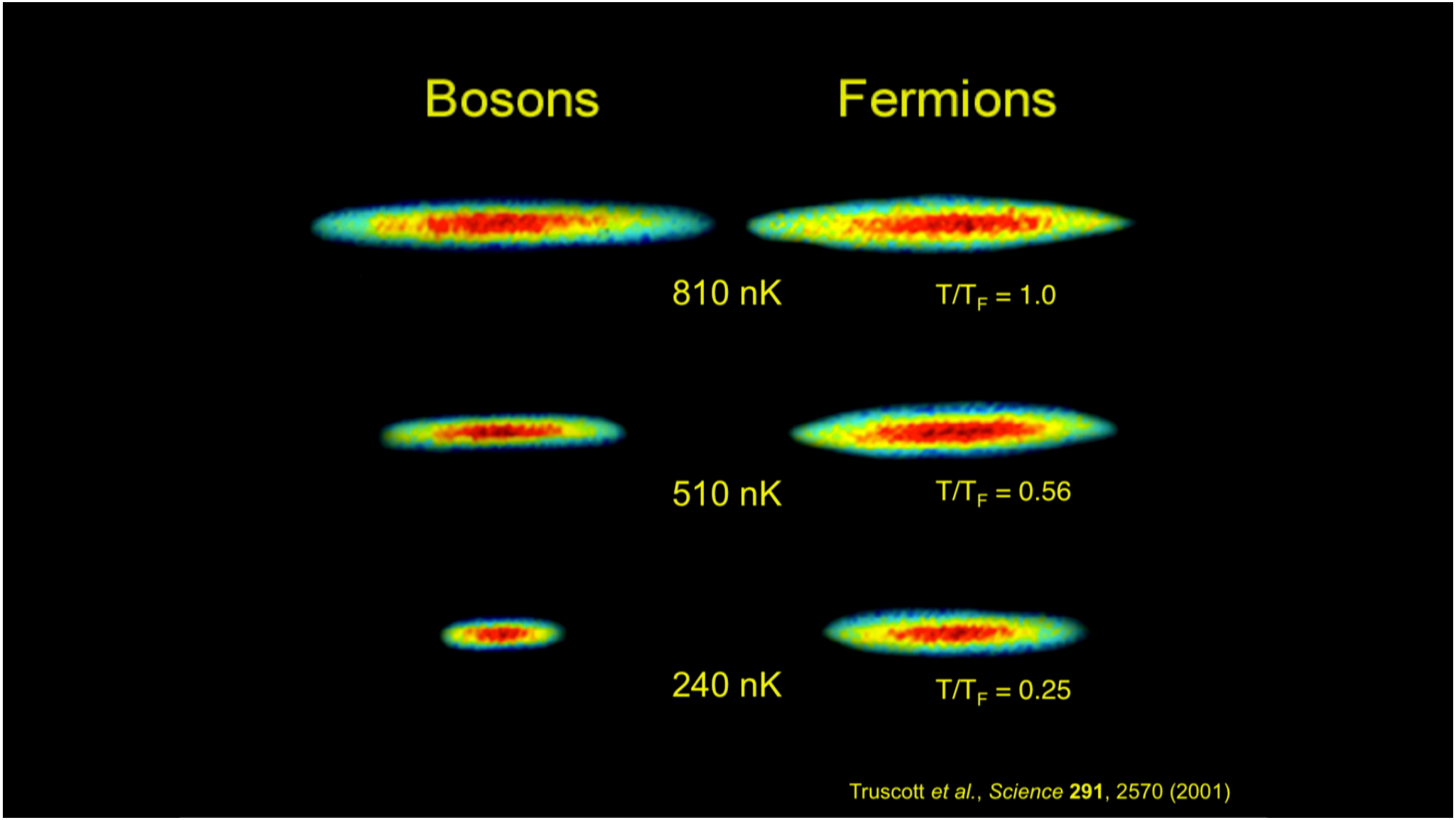
near transition $\frac{\rho_s}{\rho} = k_0 t^{\xi}$ with $\xi = 0.67$,$t=(T_\lambda - T)/T_\lambda$




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Bosons
Fermions
810 nK
T/T_F = 1.0
510 nK
T/T_F = 0.56
240 nK
T/T_F = 0.25
Truscott et al., Science 291, 2570 (2001)

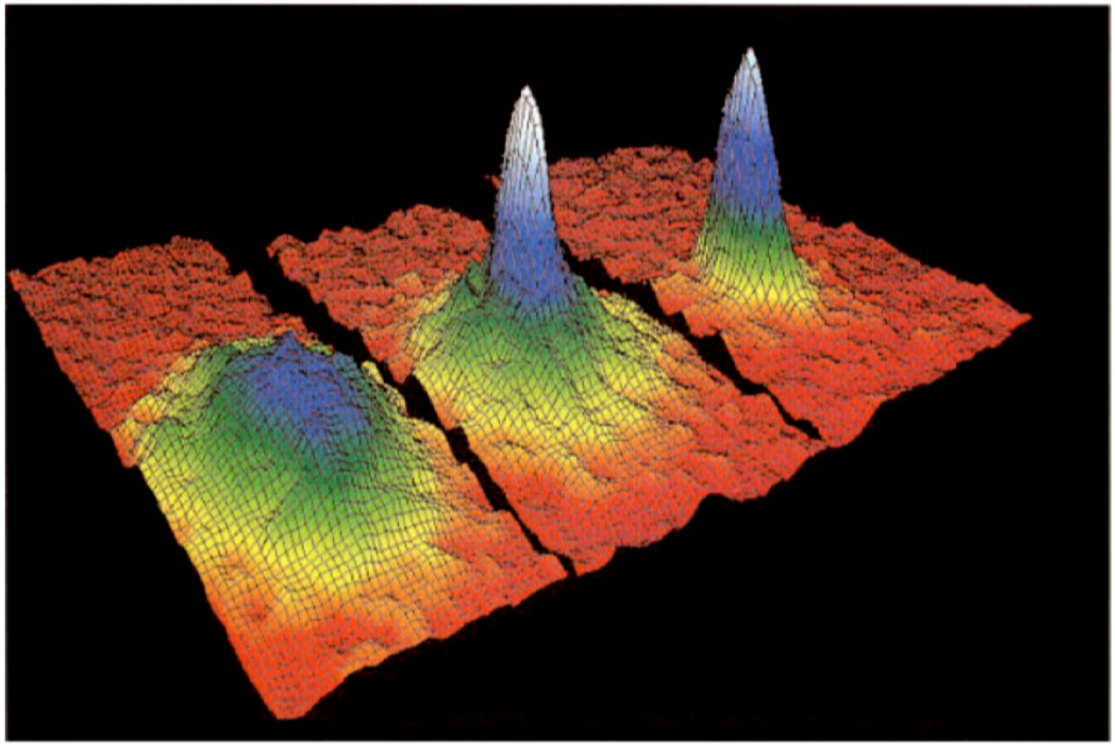
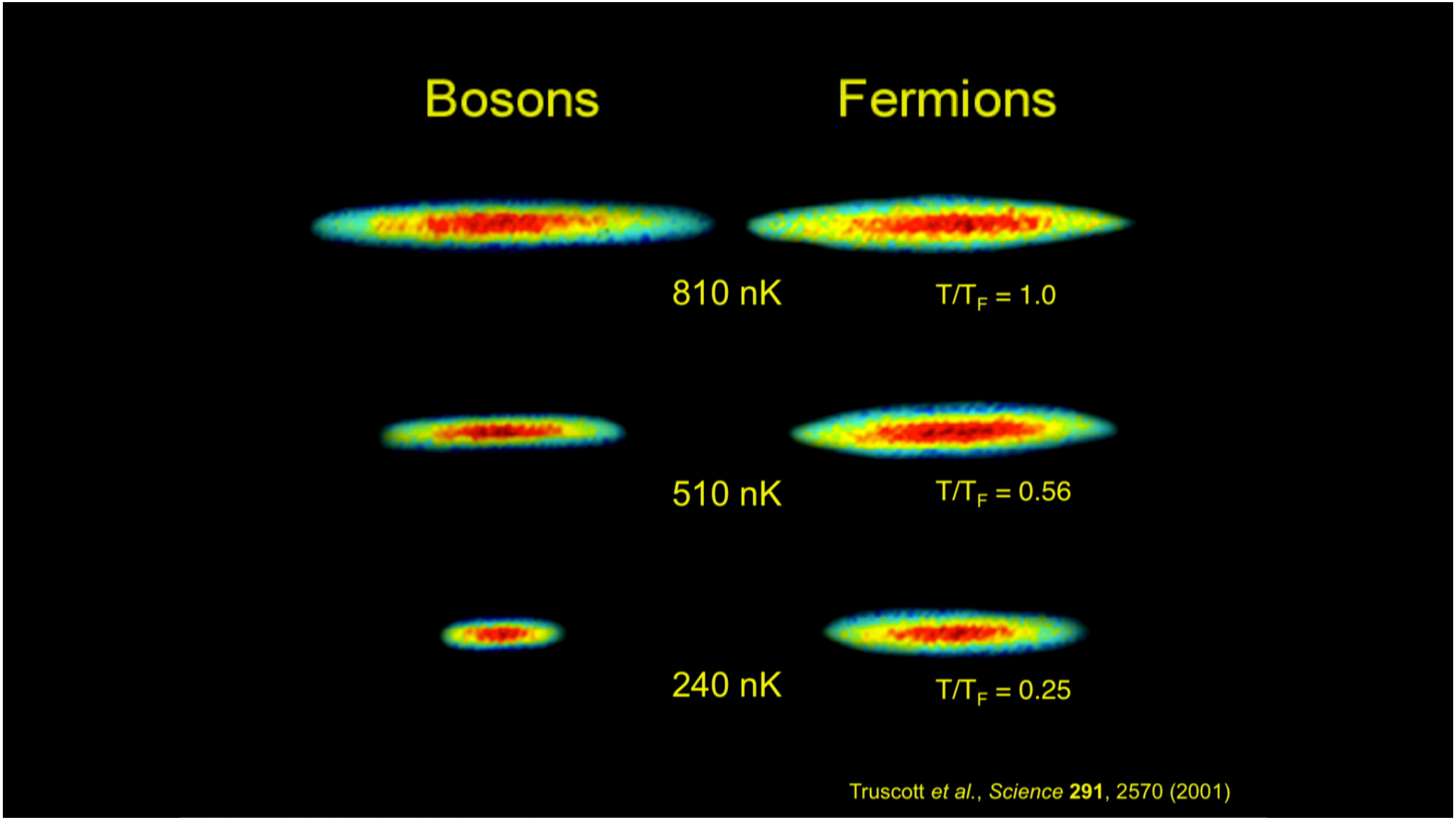
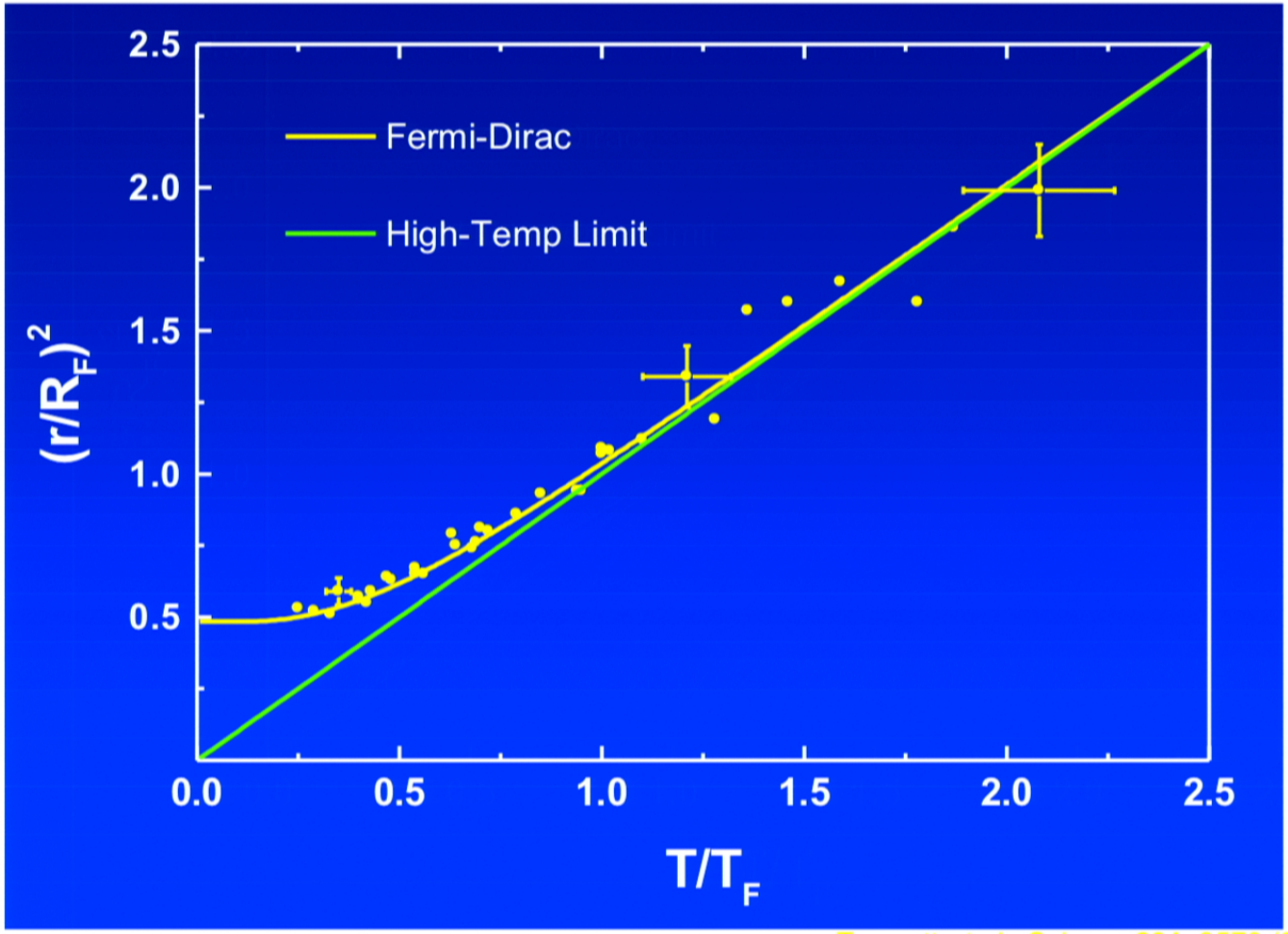


FIG. 1. (Color) Images of the velocity distribution of rubidium atoms in the experiment by Anderson *et al.* (1995), taken by means of the expansion method. The left frame corresponds to a gas at a temperature just above condensation; the center frame, just after the appearance of the condensate; the right frame, after further evaporation leaves a sample of nearly pure condensate. The field of view is 200 μm$\times$270 μm, and corresponds to the distance the atoms have moved in about 1/20 s. The color corresponds to the number of atoms at each velocity, with red being the fewest and white being the most. From Cornell (1996).

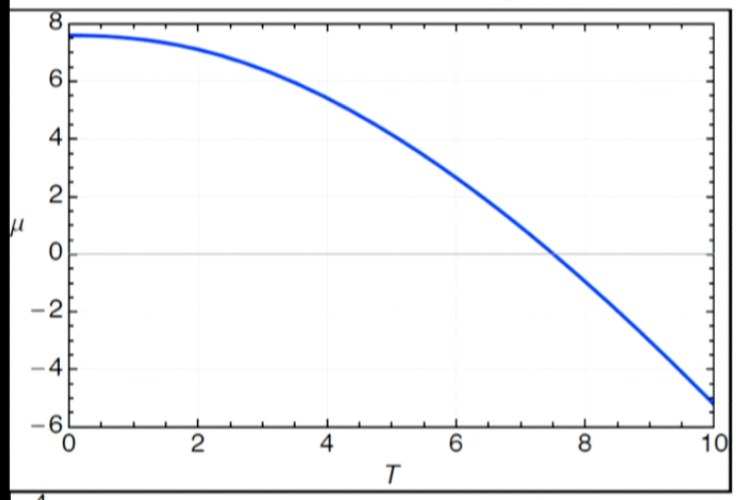

Bosons
Fermions
810 nK
T/T_F = 1.0
510 nK
T/T_F = 0.56
240 nK
T/T_F = 0.25
Truscott et al., Science 291, 2570 (2001)

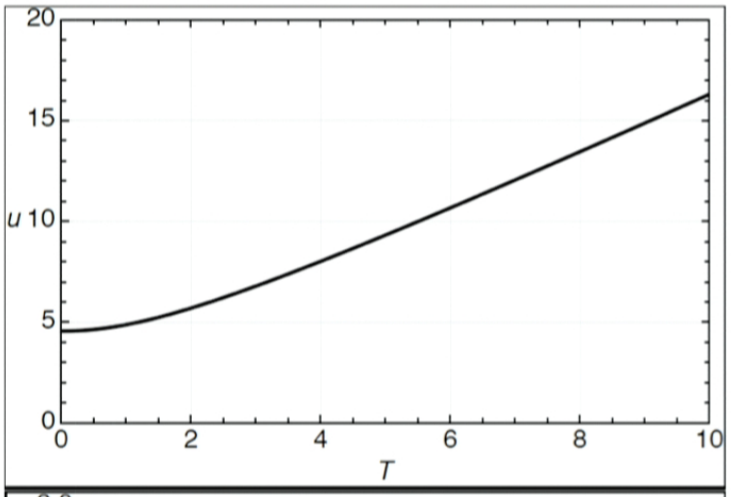
Fermi Pressure in Trapped Atoms

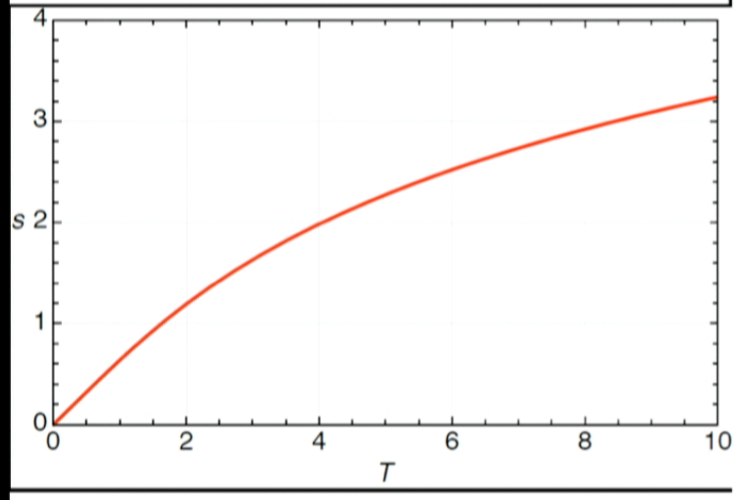


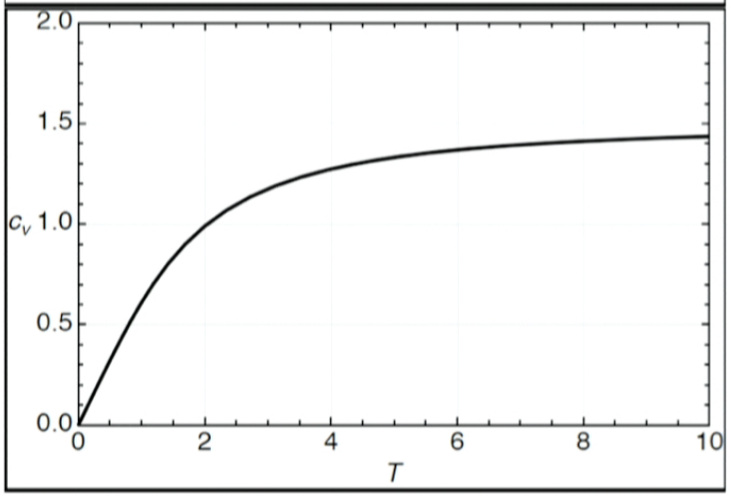
Truscott *et al.*, *Science* **291**, 2570 (2001)

NON INTERACTING FERMI GAS



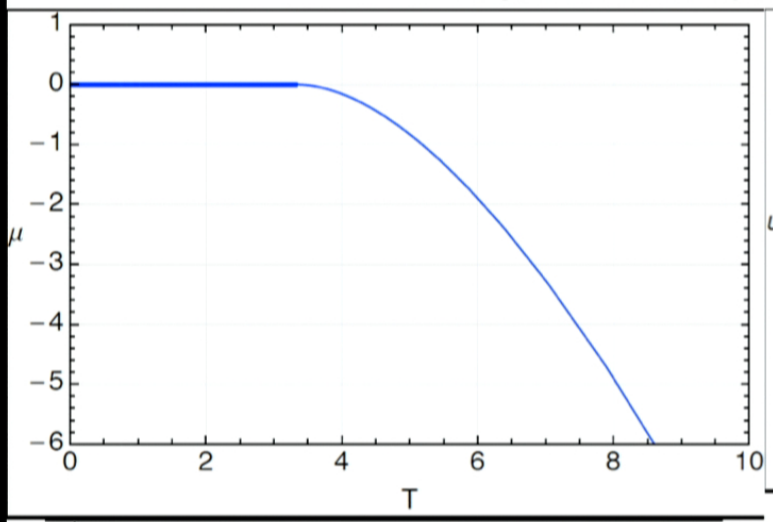


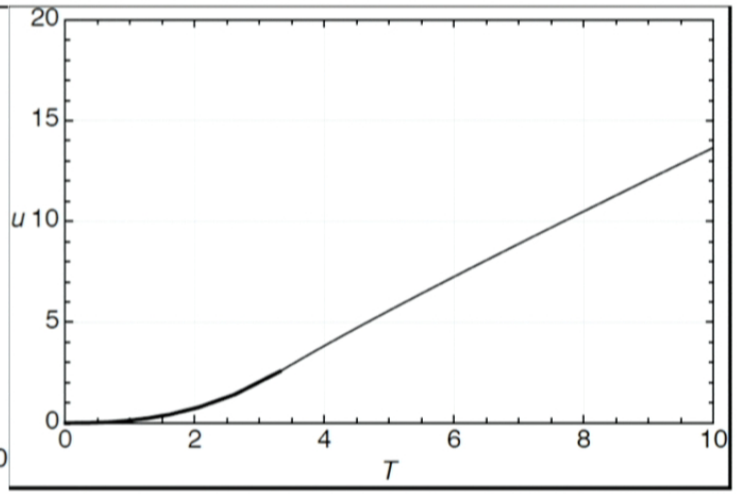


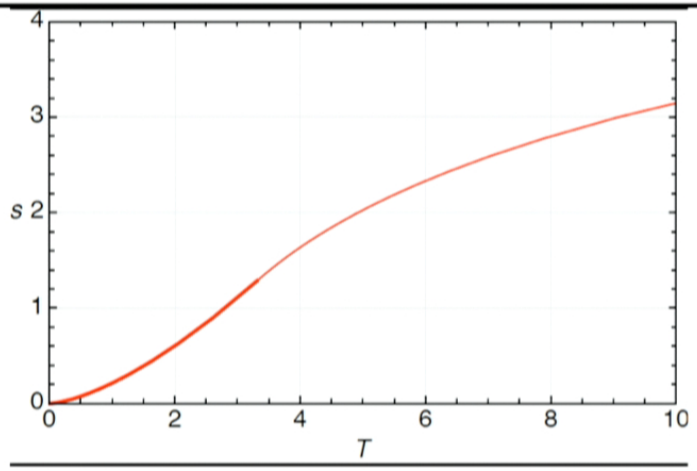


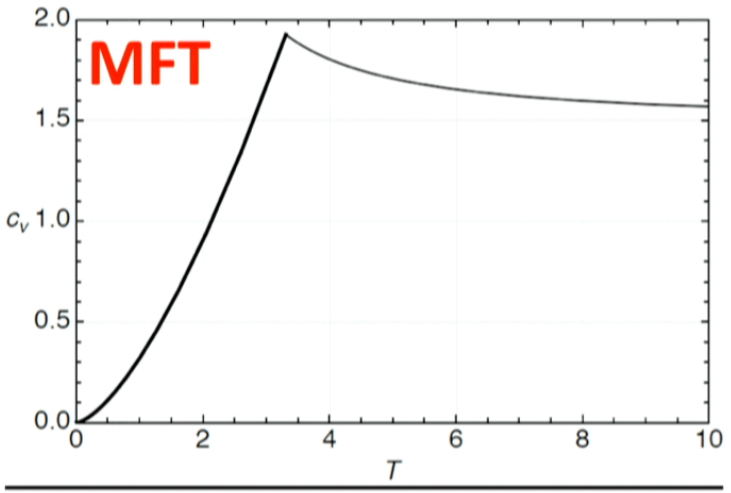
$$n = 1; \hbar = 1, m = 1, k_B = 1$$

NON INTERACTING BOSE GAS









$n = 1; \hbar = 1, m = 1, k_B = 1$

Thick lines are for behavior below T_bec

LAMBDA TRANSITION
XY MODEL PHASE FLUCTUATIONS

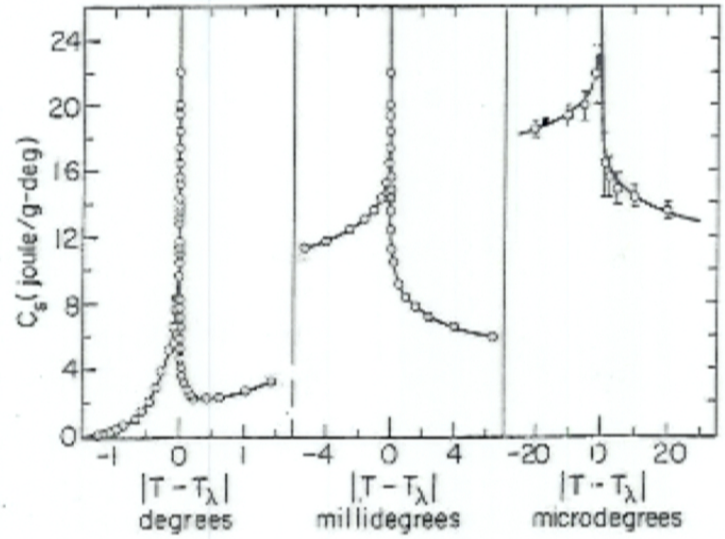
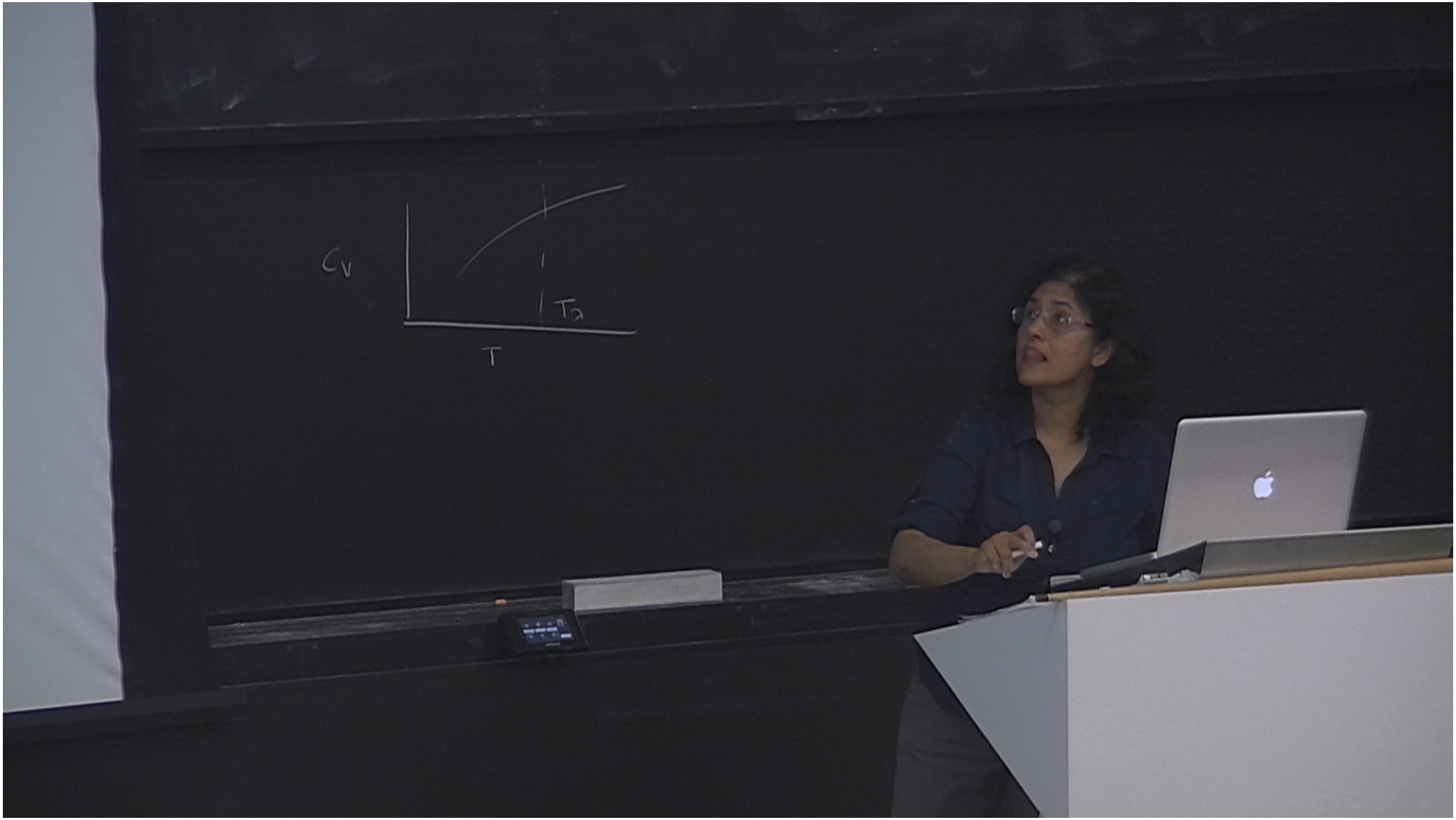


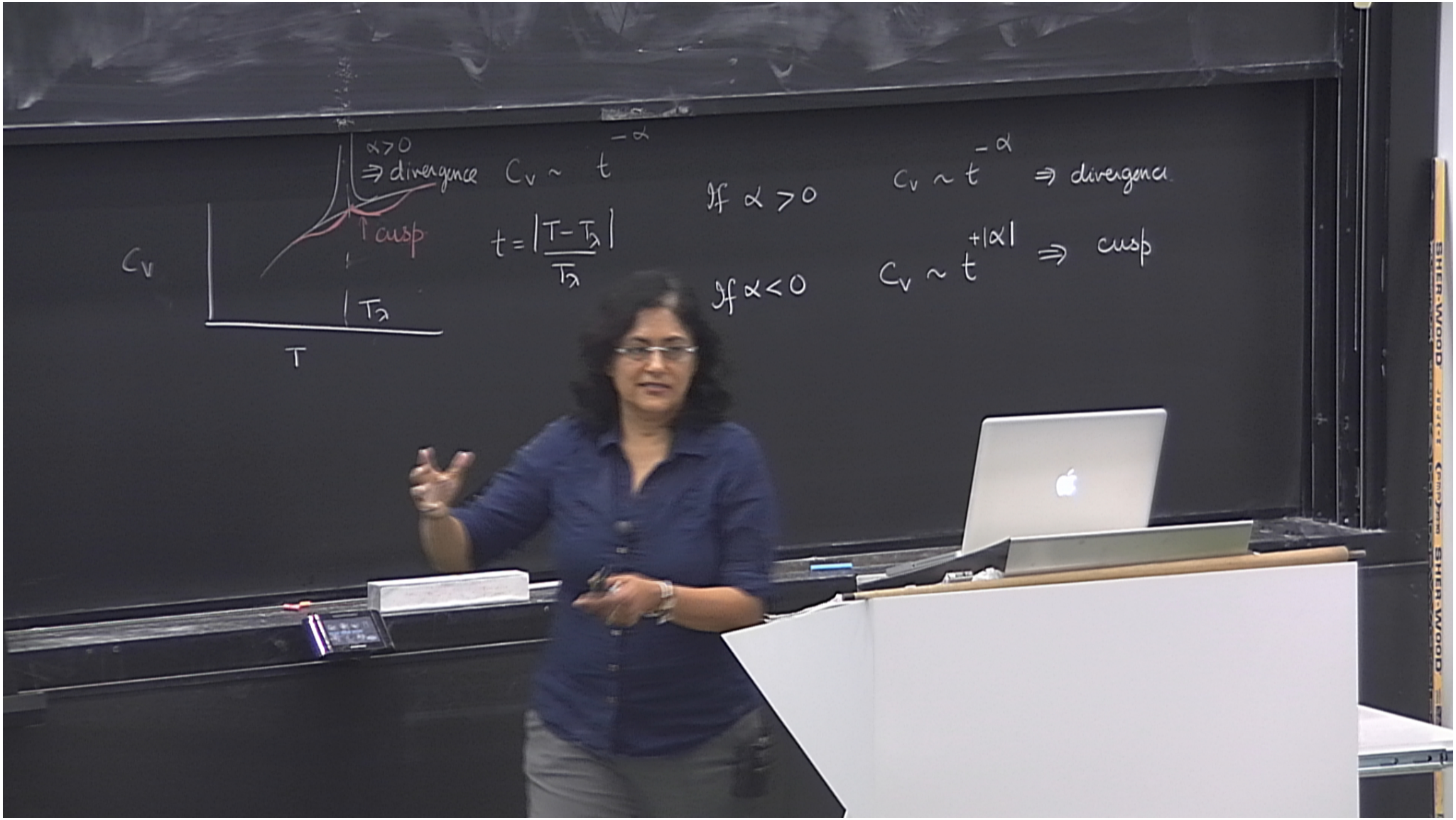
FIGURE 1.8. *Specific heat of liquid helium vs. $T - T_\lambda$ [from Buckingham and Fairbank (1961)].*

$C \sim A - B \log \left| \frac{T-T_\lambda}{T_\lambda} \right|$ (1960's)

Lipa + Chui, PRL 51, 2291 (1983)

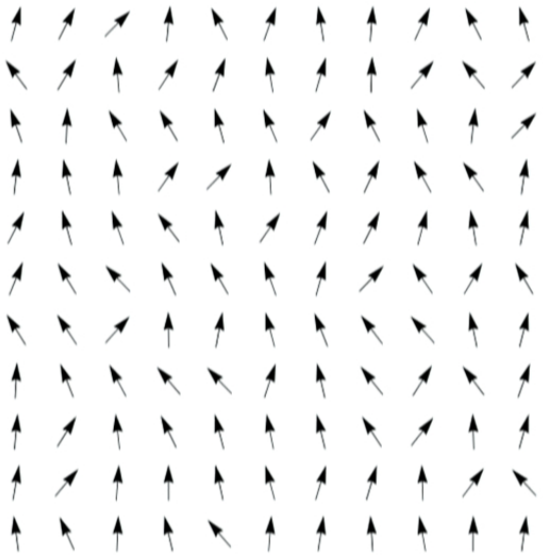
$C(T) \sim \left| \frac{T-T_\lambda}{T_\lambda} \right|^{-\alpha}$; $\alpha = -0.013 \pm 0.003$

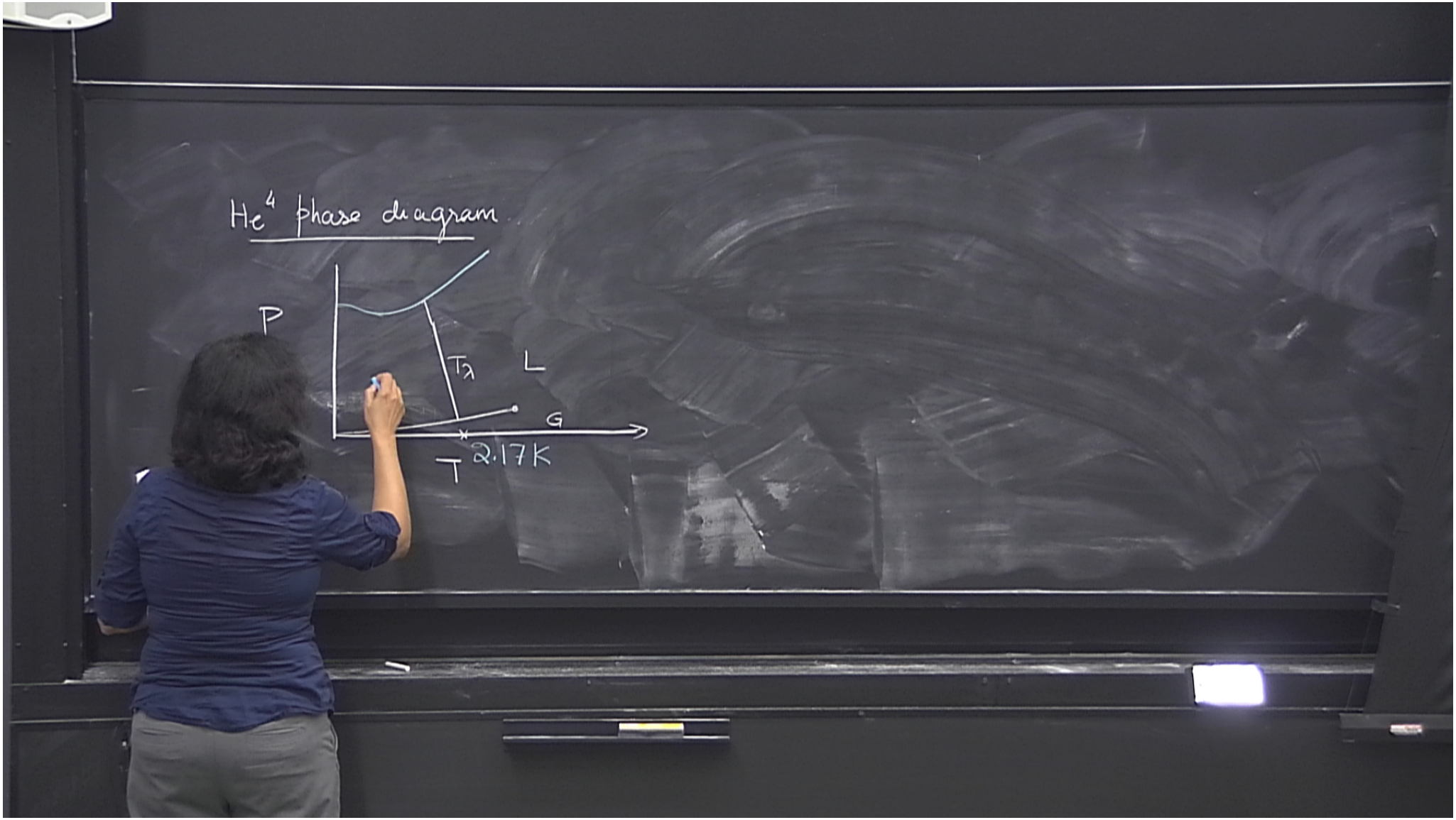

Cv
T
Ta

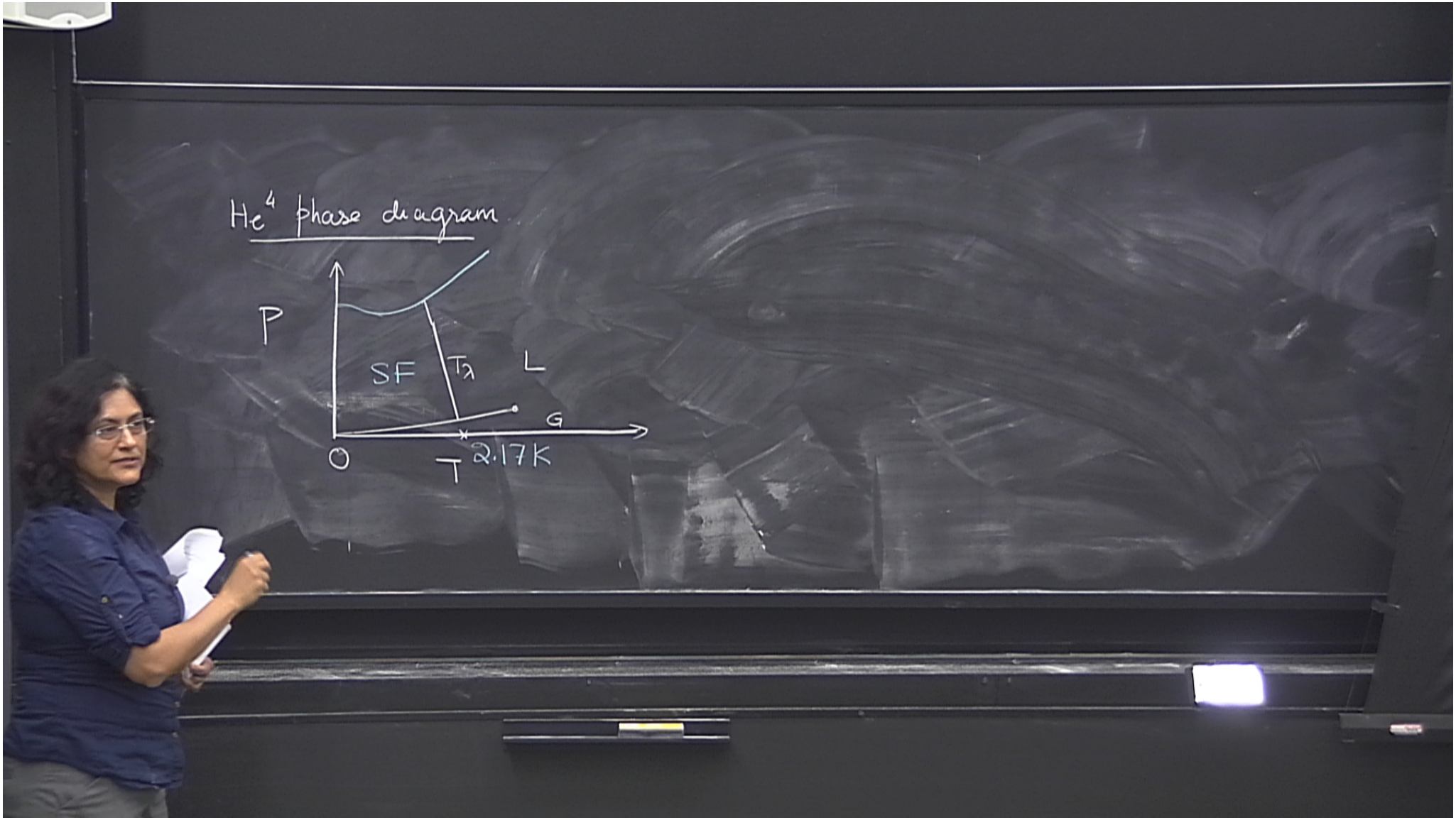

α>0
⇒ divergence
cusp
C_v
T_λ
T
$C_v \sim t^{-\alpha}$
$t = \left|\frac{T - T_\lambda}{T_\lambda}\right|$
If $\alpha > 0$
$C_v \sim t^{-\alpha}$ ⇒ divergence
If $\alpha < 0$
$C_v \sim t^{+|\alpha|}$ ⇒ cusp

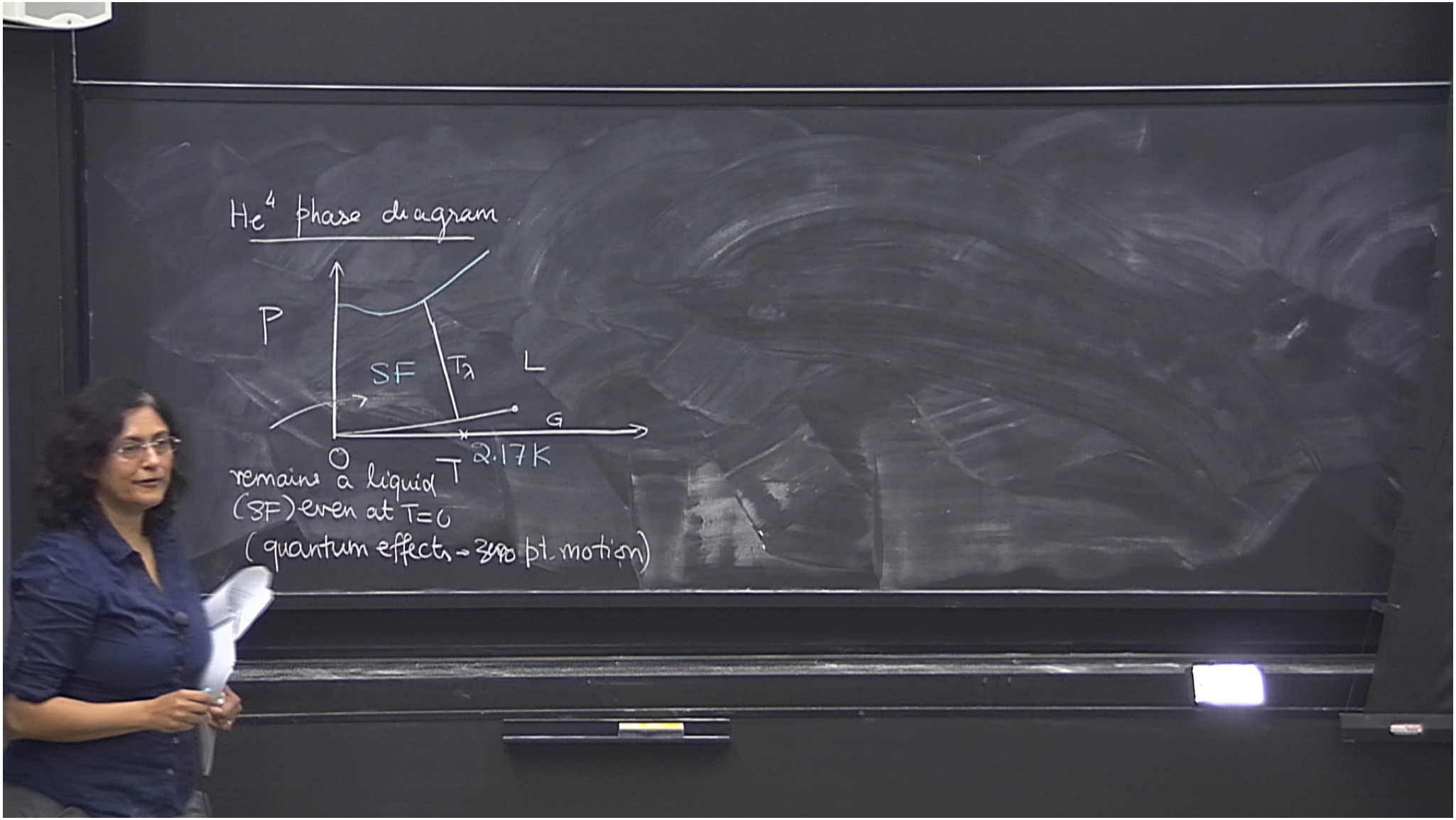
XY Hamiltonian and Vortices

$$H = -J \sum_{<i,j>} \boldsymbol{S}_i \cdot \boldsymbol{S}_j = -J \sum_{<i,j>} \mathrm{Cos}(\theta_i - \theta j)$$

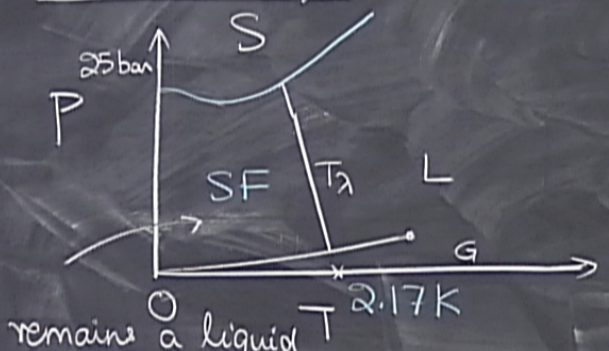



He^4 phase diagram
P
T_λ
L
G
T
2.17K

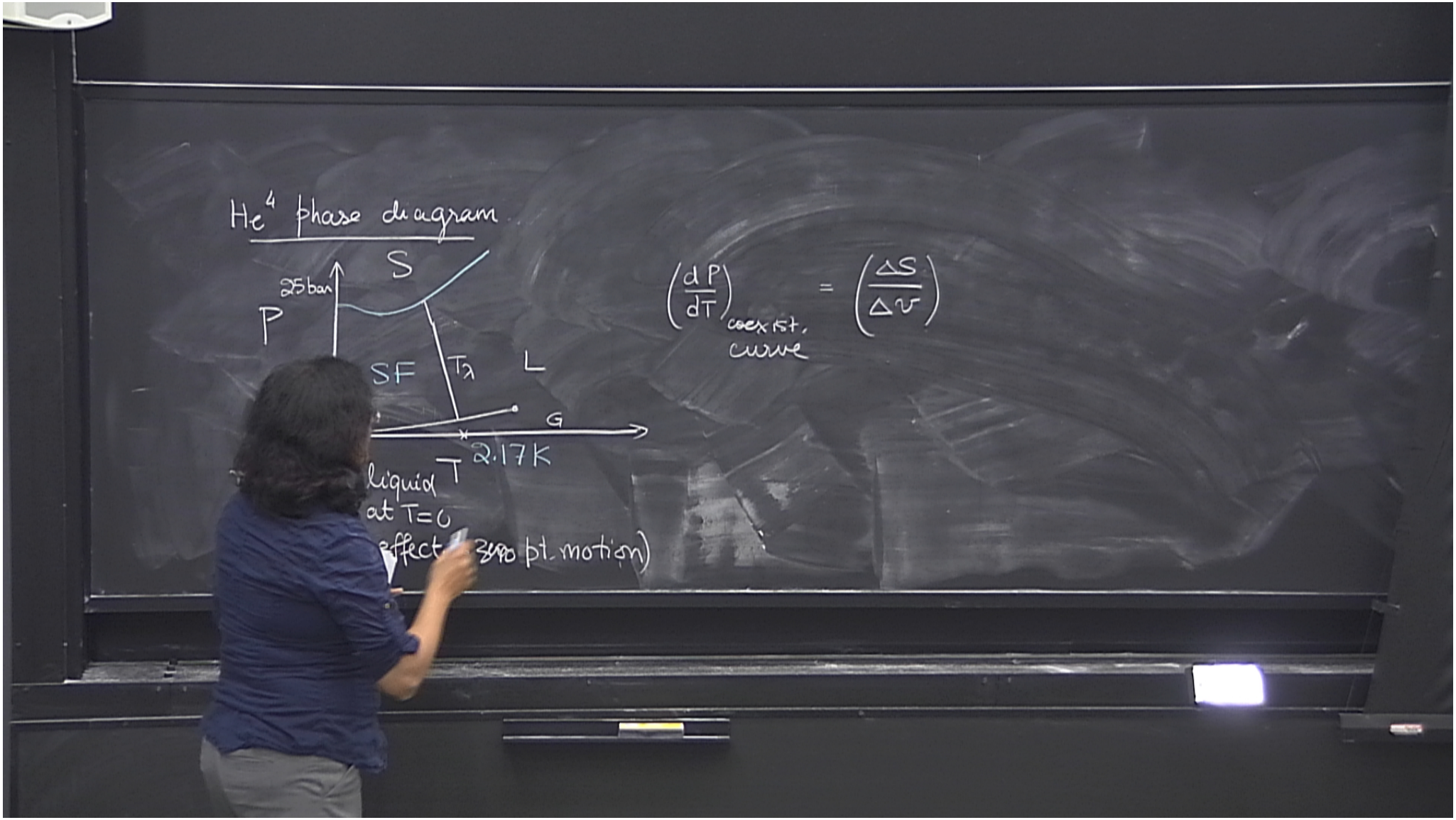

He^4 phase diagram
P
SF
T_λ
L
G
O
T
2.17K

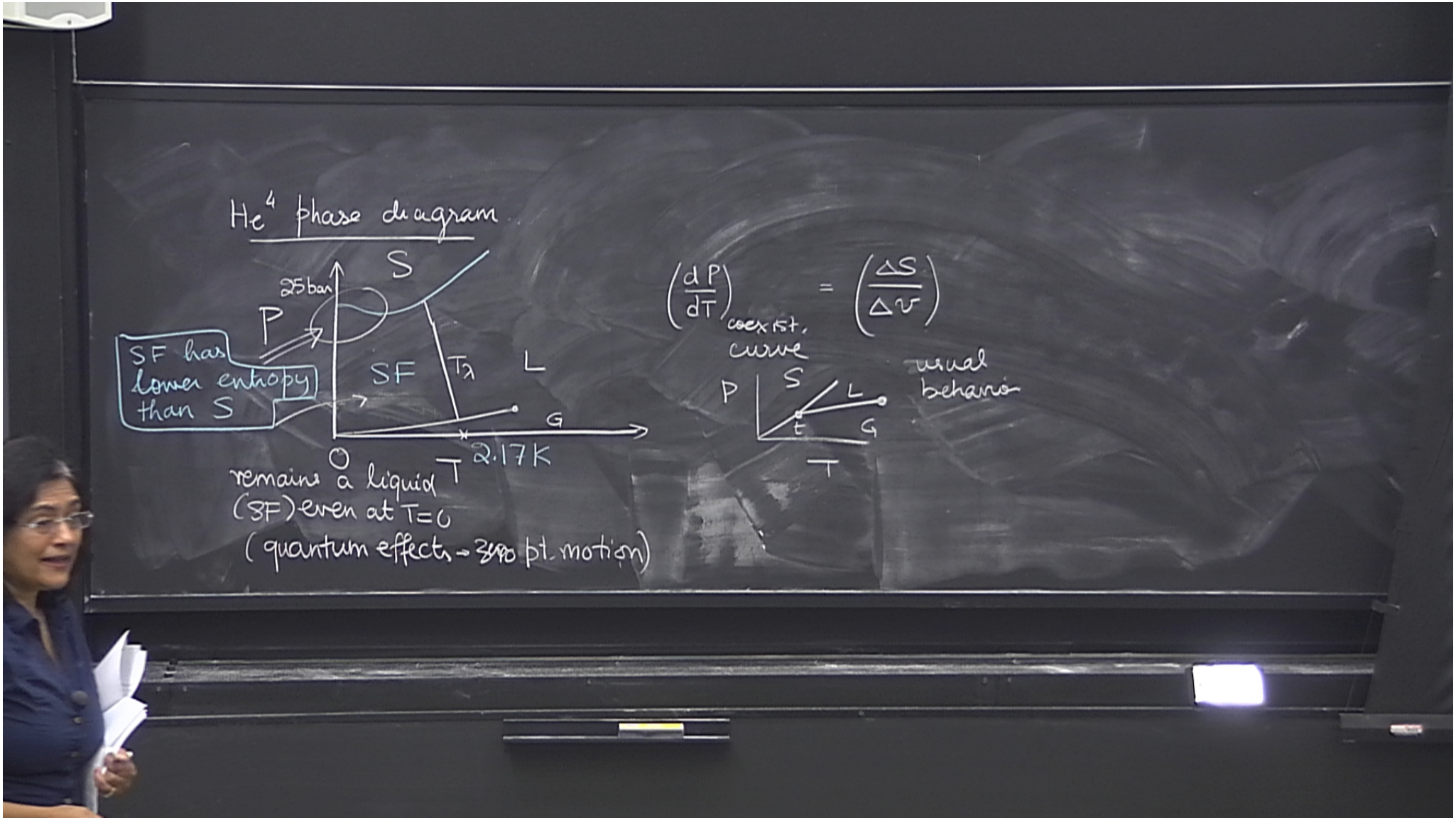

He^4 phase diagram
P
SF
T_λ
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2.17K
remains a liquid
(SF) even at T=0
(quantum effects - zero pt. motion)

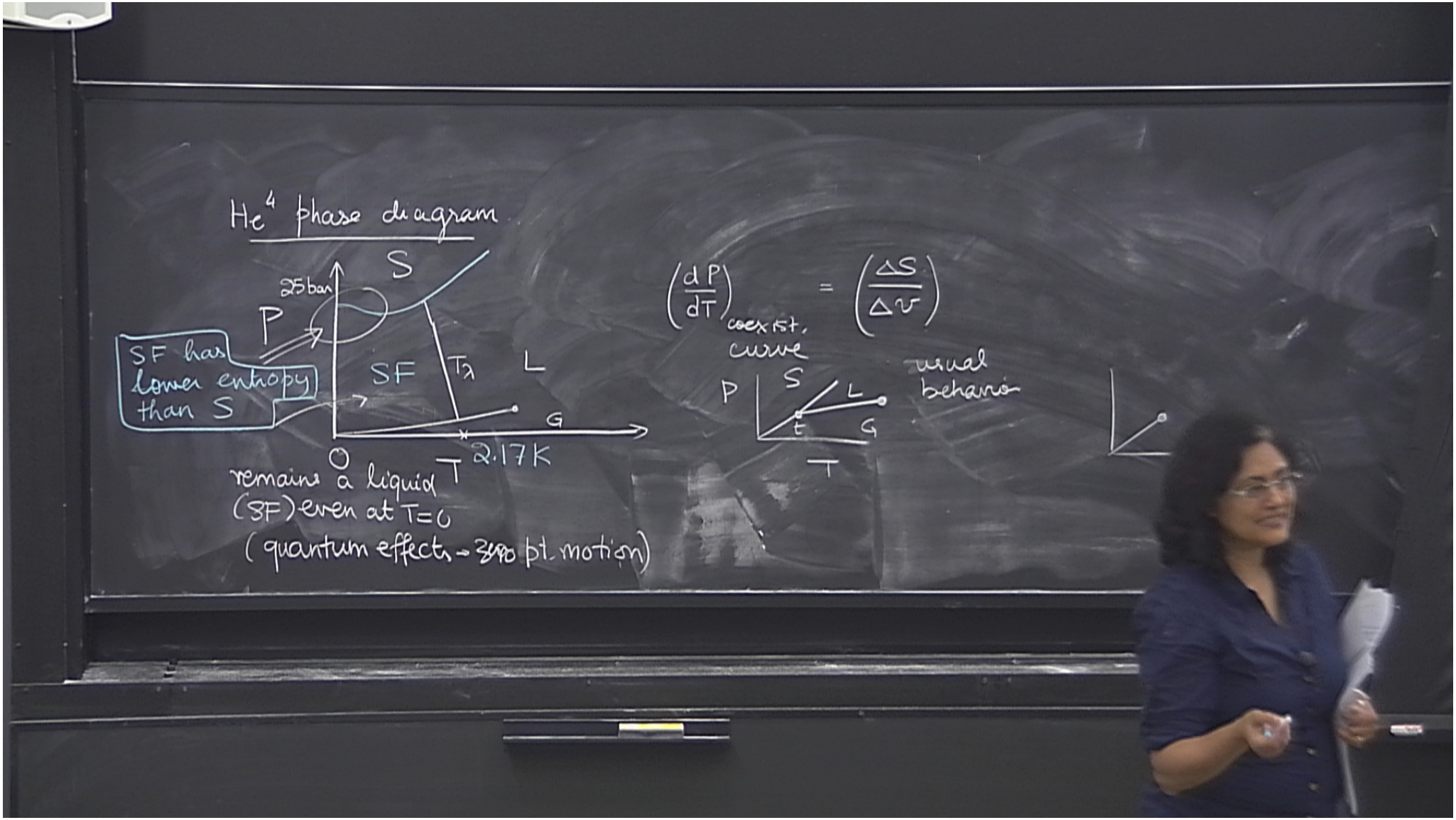
He^4 phase diagram

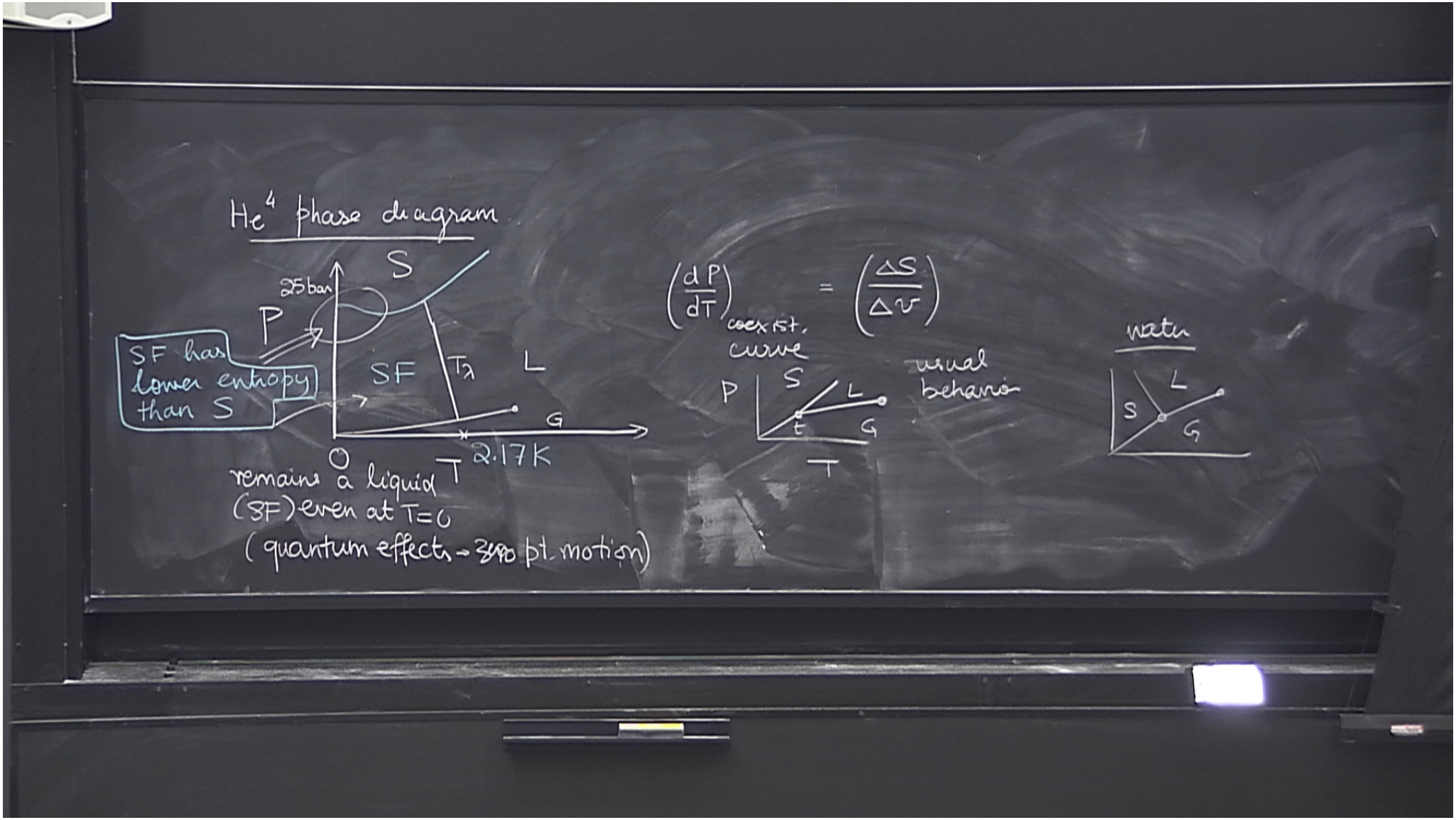


remains a liquid (SF) even at T=0
(quantum effects - zero pt. motion)


He^4 phase diagram
S
25 bar
P
SF
T_λ
L
G
2.17K
T
liquid
at T=0
$\left(\frac{dP}{dT}\right)_{\text{coexist. curve}} = \left(\frac{\Delta S}{\Delta v}\right)$

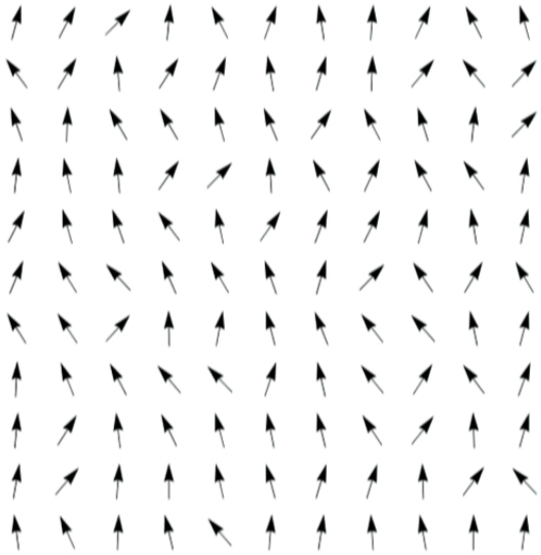

He^4 phase diagram
S
25 bar
P
SF has lower entropy than S
SF
T_λ
L
G
O
T
2.17K
remains a liquid (SF) even at T=0
(quantum effects - zero pt. motion)
$\left(\frac{dP}{dT}\right)_{coexist.\ curve} = \left(\frac{\Delta S}{\Delta v}\right)$
P
S
L
t
G
T
usual behavior

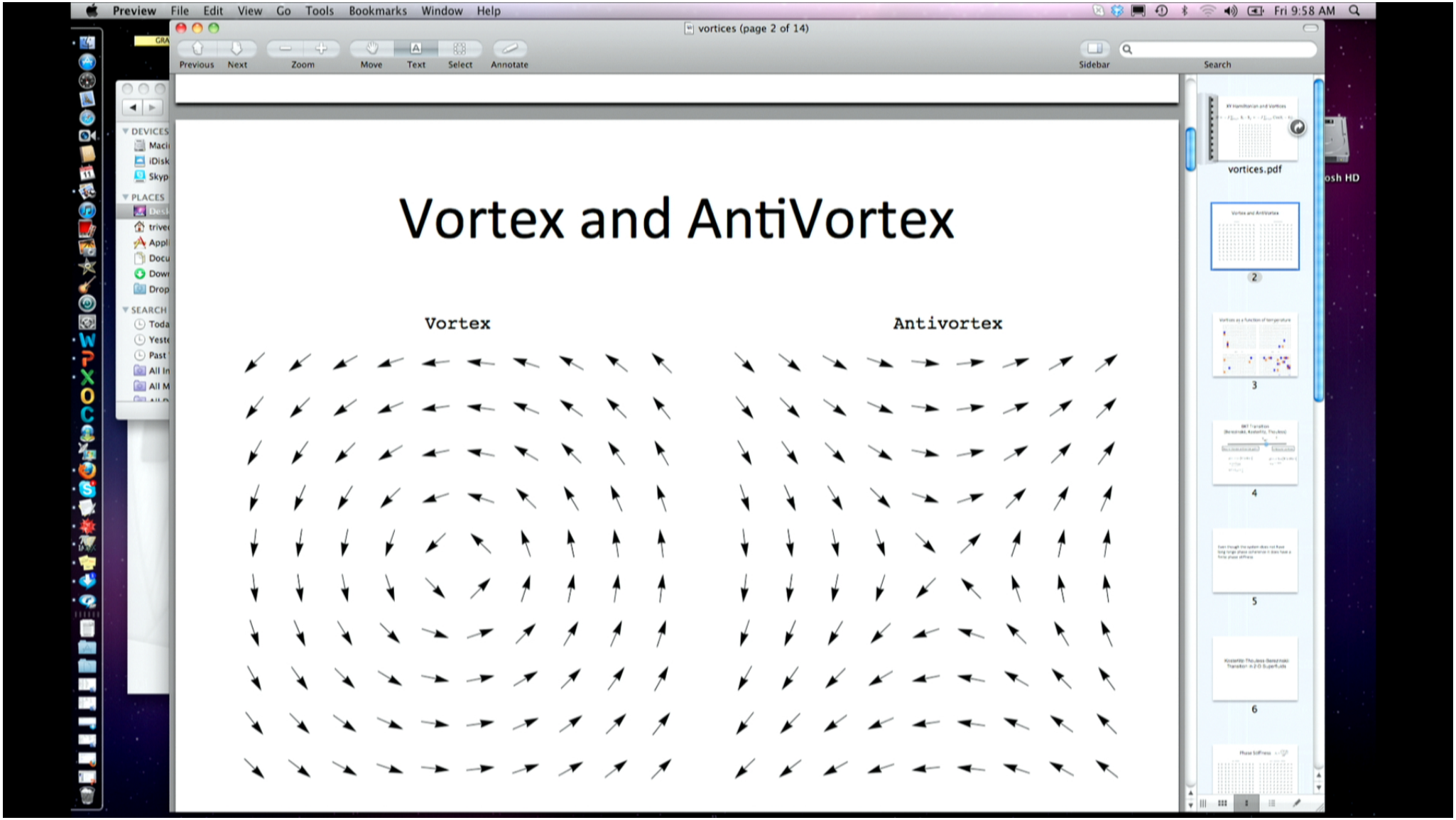

He^4 phase diagram
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T
usual behavior

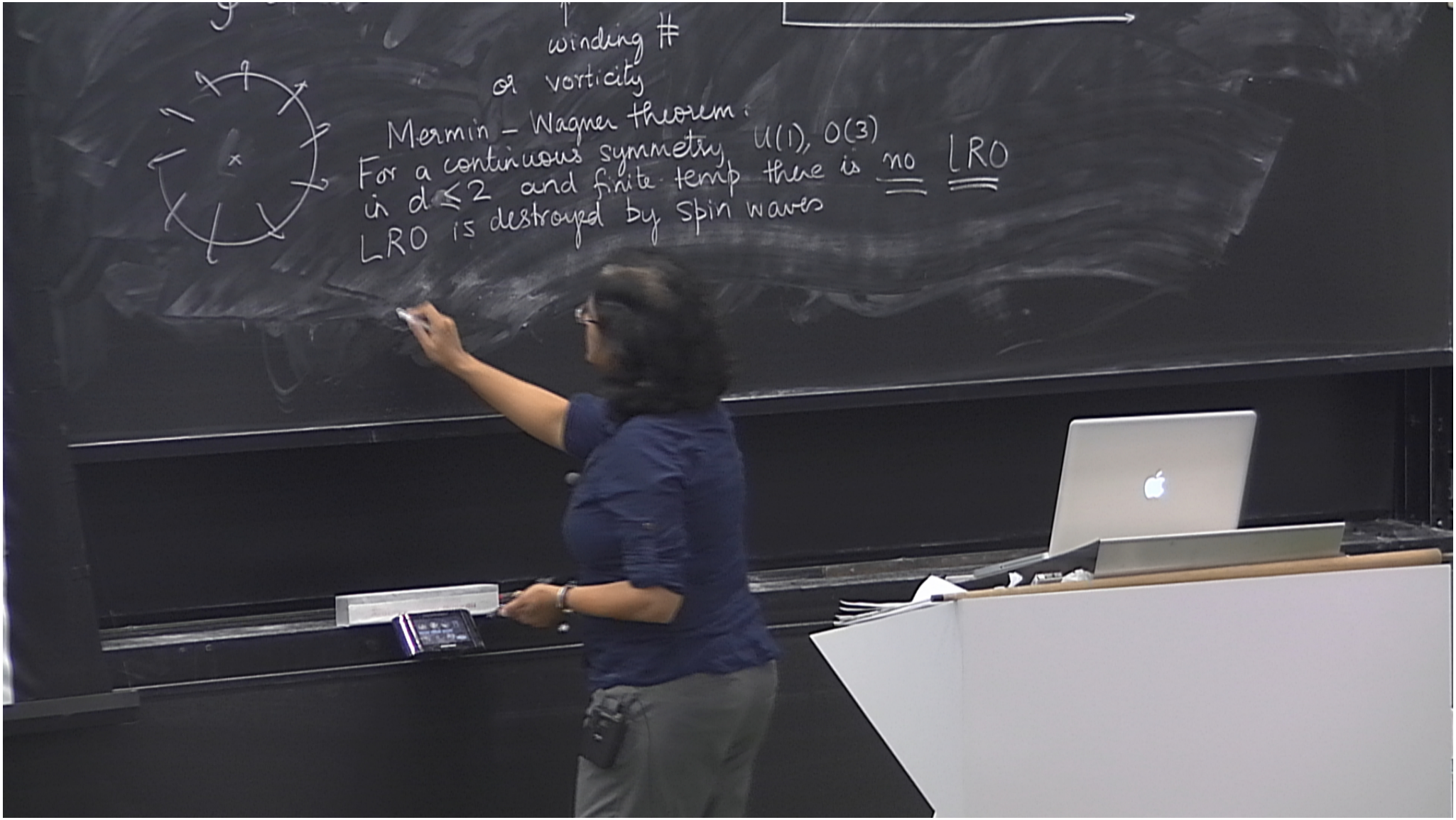

He^4 phase diagram
S
25 bar
P
SF has lower entropy than S
SF
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P
S
L
t
G
T
usual behavior
water
S
L
G

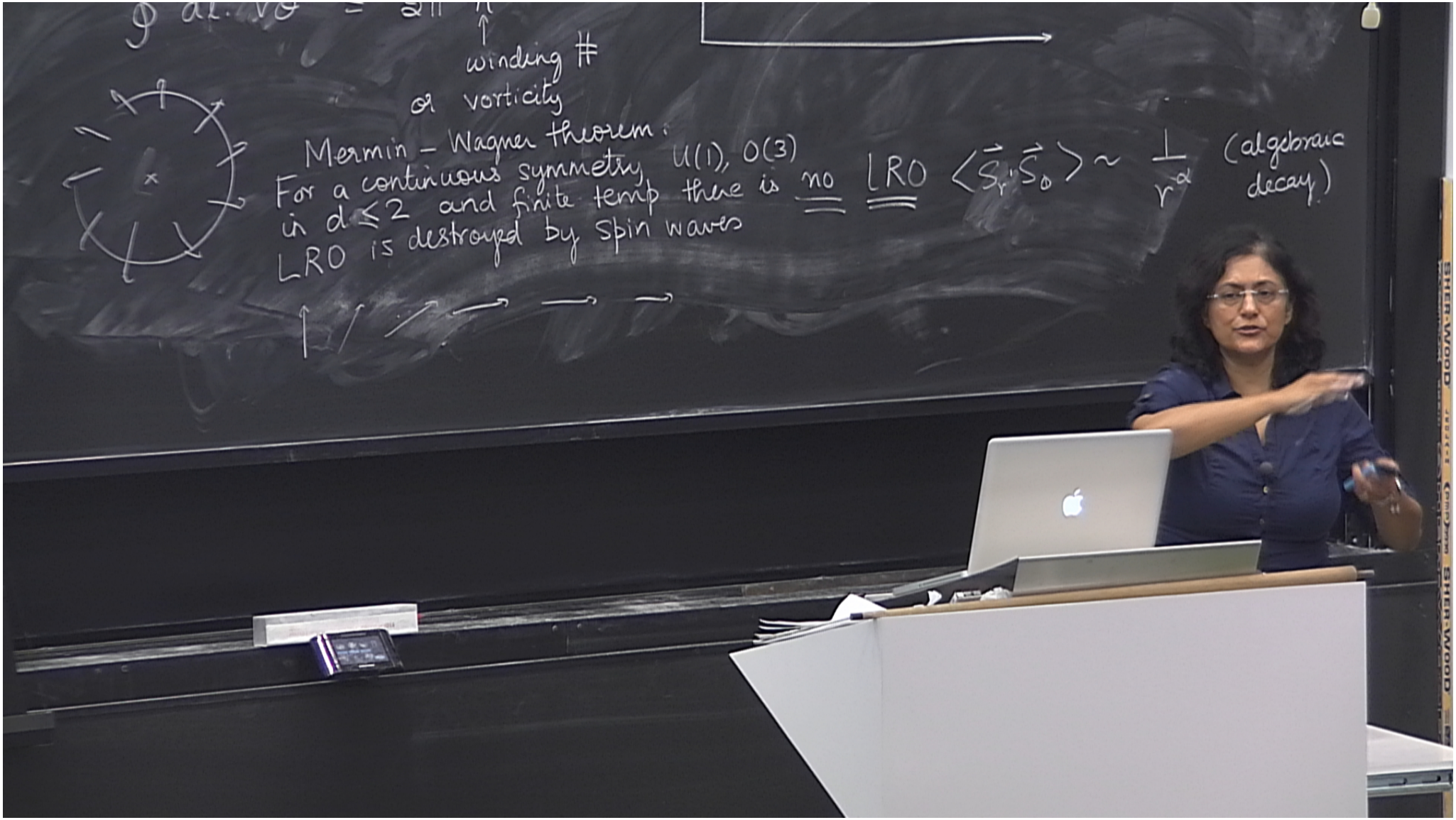
XY Hamiltonian and Vortices

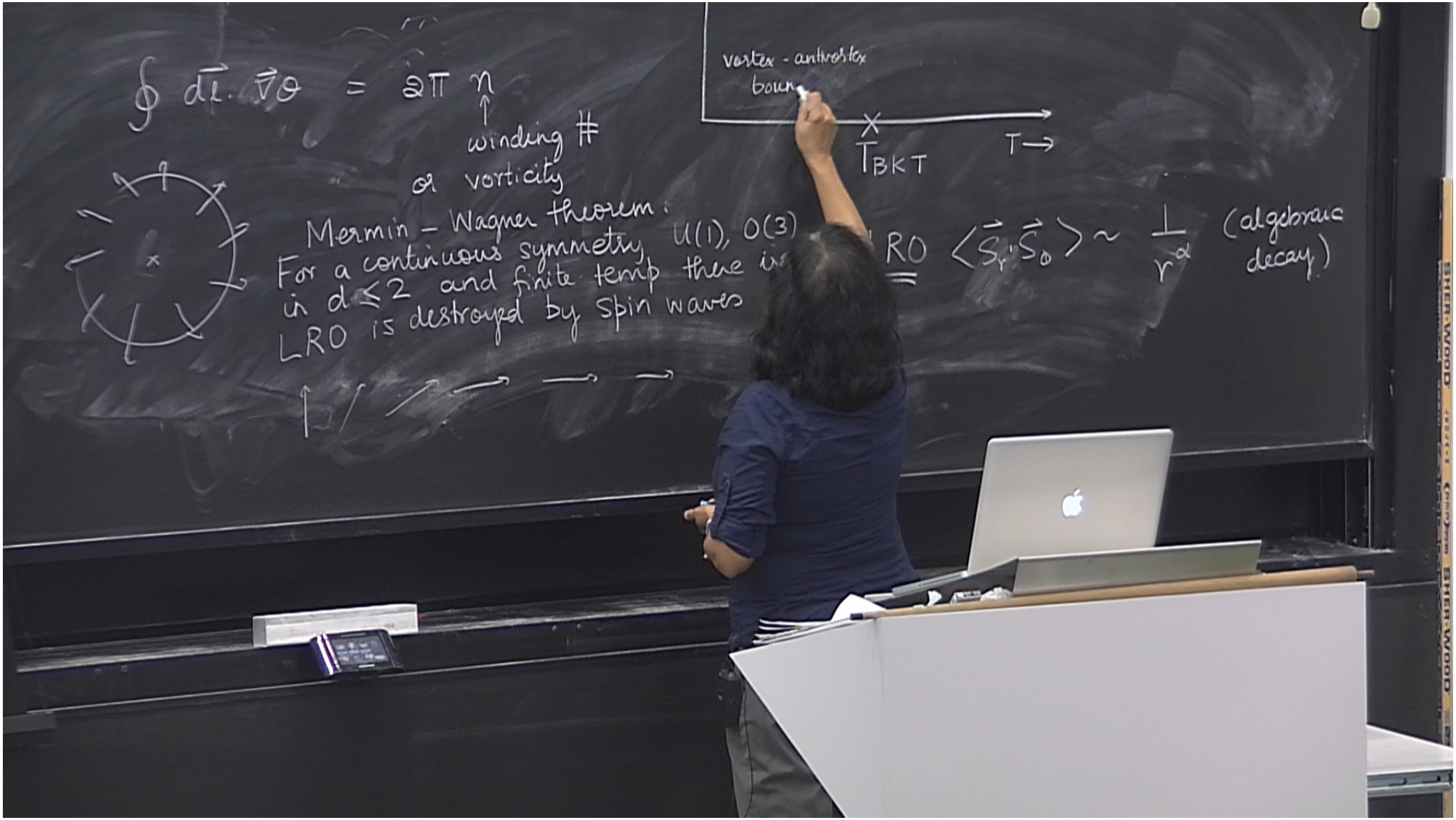
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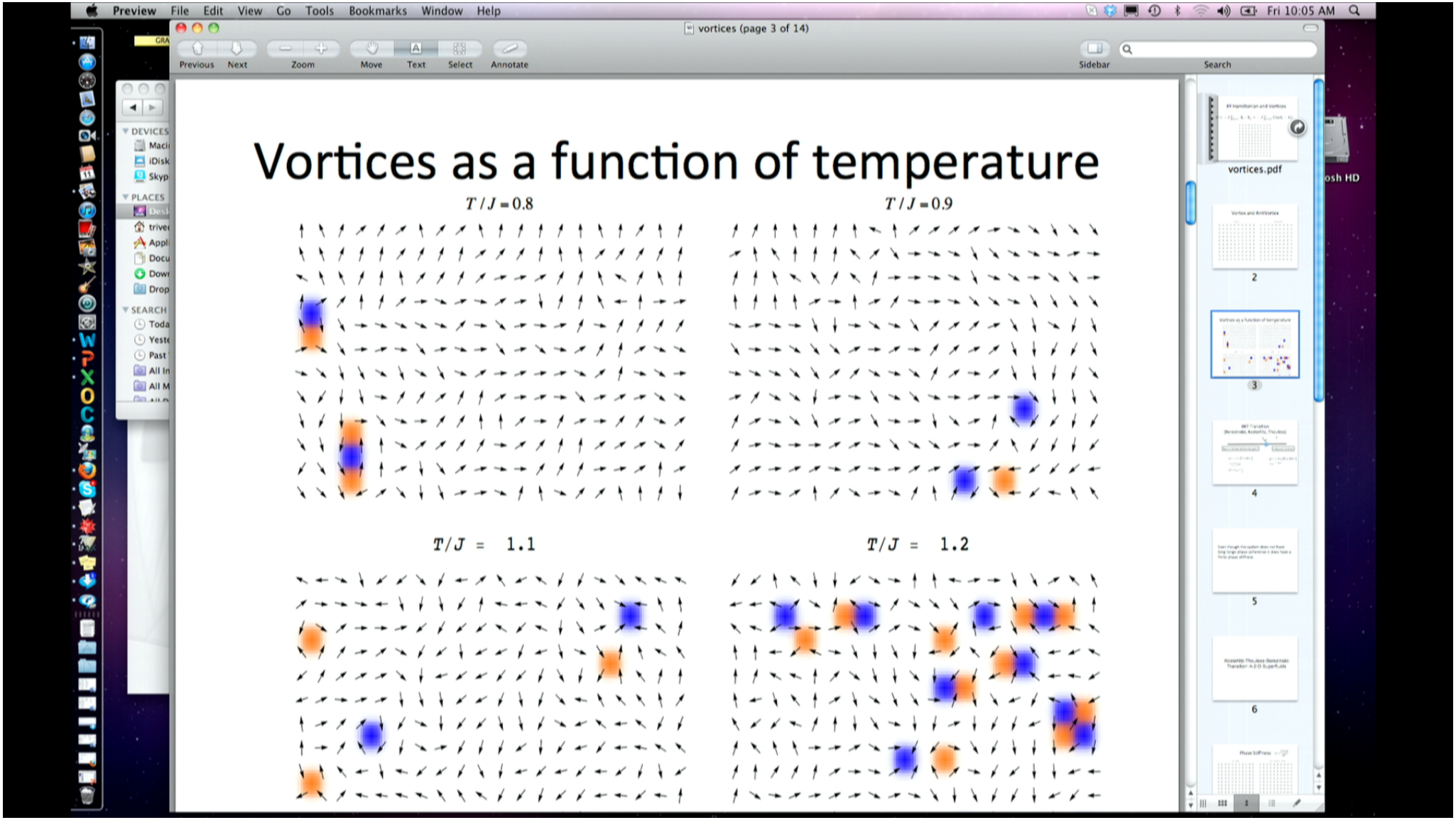


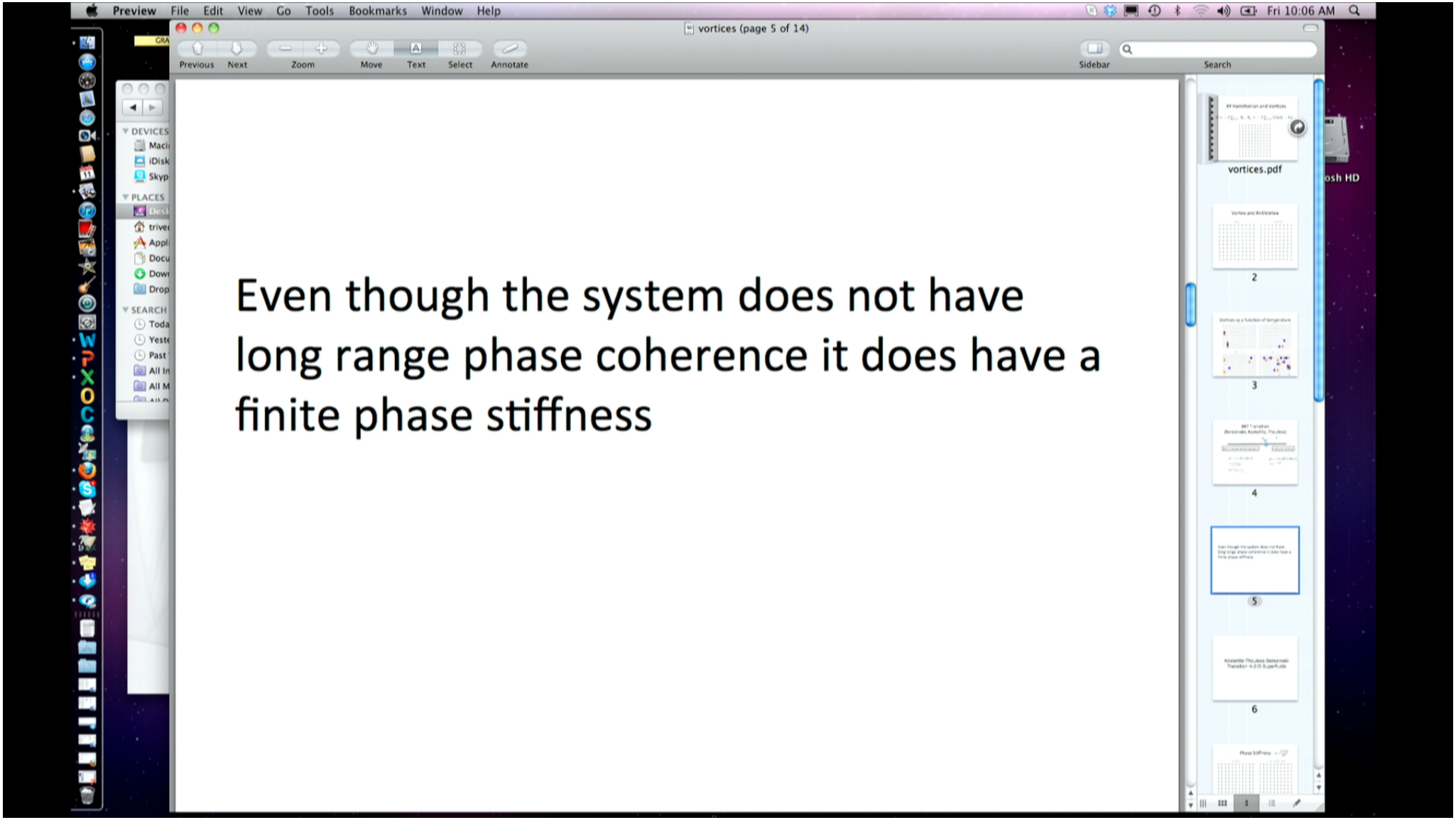

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Fri 9:58 AM
vortices (page 2 of 14)
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Vortex and AntiVortex
Vortex
Antivortex
vortices.pdf


winding #
or vorticity
Mermin – Wagner theorem:
For a continuous symmetry U(1), O(3)
in d ≤ 2 and finite temp there is no LRO
LRO is destroyed by spin waves


winding #
or vorticity
Mermin – Wagner theorem:
For a continuous symmetry U(1), O(3)
in d ≤ 2 and finite temp there is no LRO ⟨S_r · S_0⟩ ~ 1/r^α (algebraic decay)
LRO is destroyed by spin waves


$\oint d\vec{l} \cdot \vec{\nabla}\theta = 2\pi n$
winding #
or vorticity
Mermin – Wagner theorem:
For a continuous symmetry, U(1), O(3)
in $d \leq 2$ and finite temp there is
LRO is destroyed by spin waves
Vortex - antivortex
boun
T_{BKT}
T
RO $\langle \vec{S}_r \cdot \vec{S}_0 \rangle \sim \frac{1}{r^\alpha}$ (algebraic decay)


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Fri 10:05 AM
vortices (page 3 of 14)
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Vortices as a function of temperature
T / J = 0.8
T / J = 0.9
T/J = 1.1
T/J = 1.2
vortices.pdf

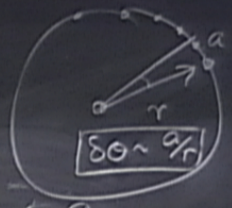

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vortices (page 5 of 14)
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Even though the system does not have long range phase coherence it does have a finite phase stiffness
vortices.pdf
2
3
4
5
6

remains a liquid
(SF) even at T=0
(quantum effects - zero pt. motion)

XY Model in d=2

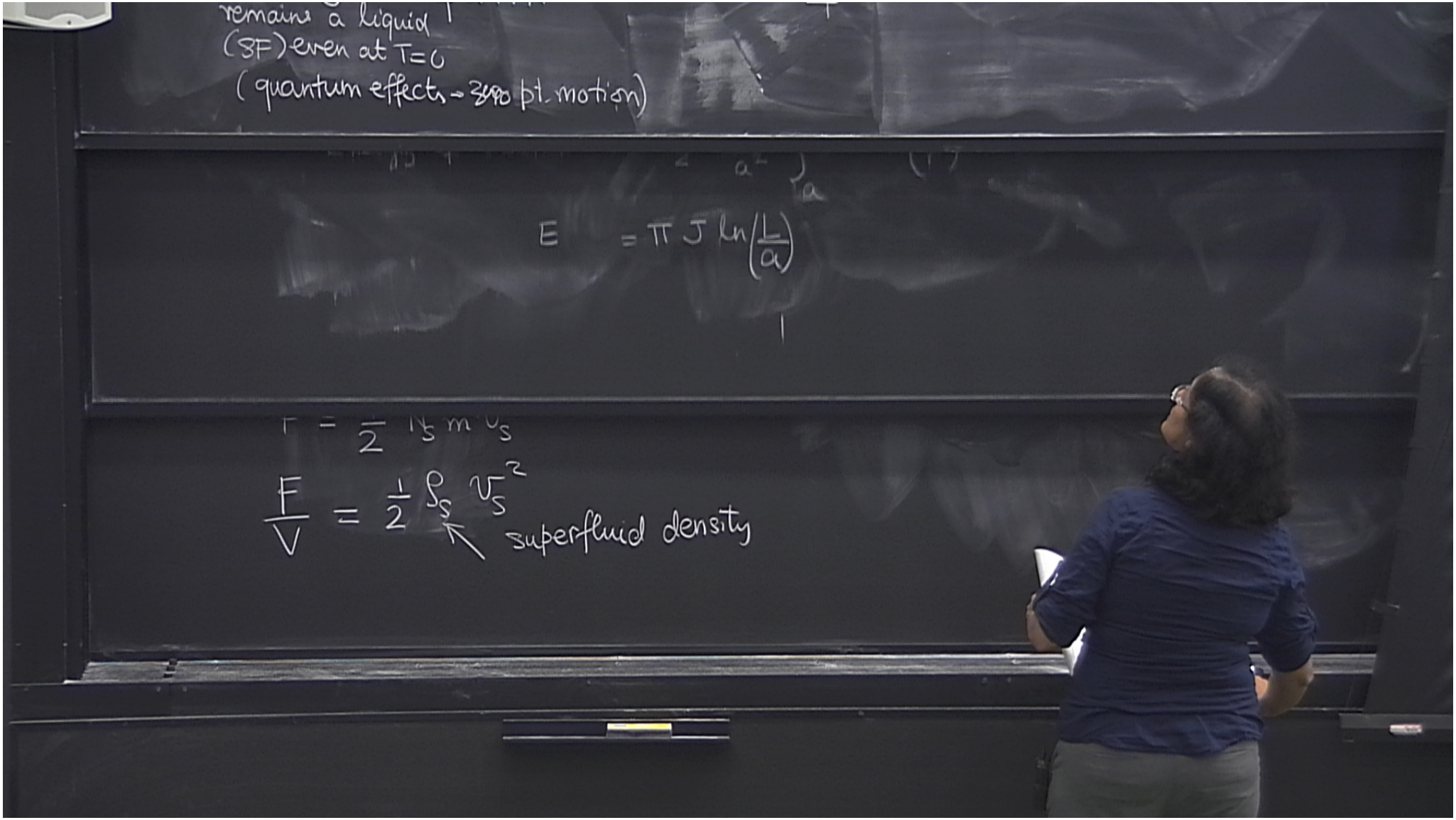
$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) \longrightarrow \frac{J}{2} \int \left(\frac{d^2\vec{r}}{a^2}\right) (\delta\theta)^2$$

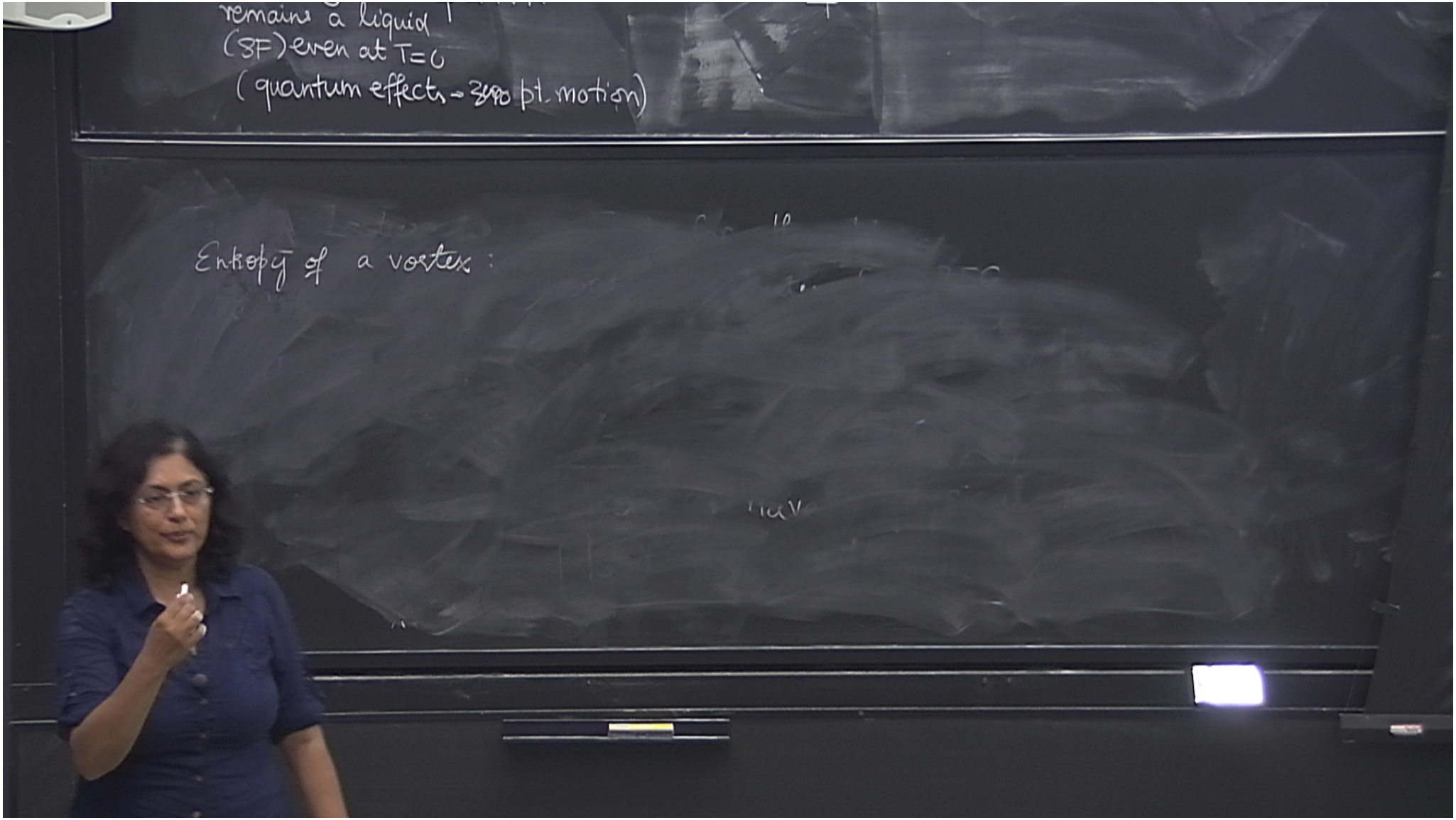
$J \sim \rho_s$



$\delta\theta \sim \frac{a}{r}$

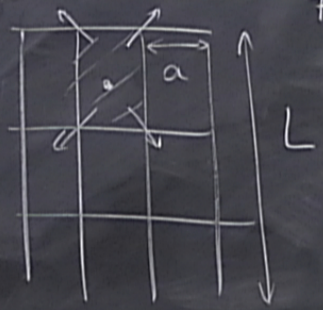
Energy of vortex $\sim \frac{J}{2} \frac{2\pi}{a^2}$ (a


remains a liquid
(SF) even at T=0
(quantum effects - Zero pt. motion)
$E = \pi J \ln\left(\frac{L}{a}\right)$
$\Gamma = \frac{1}{2} \rho_s m v_s$
$\frac{F}{V} = \frac{1}{2} \rho_s v_s^2$
superfluid density


remains a liquid
(SF) even at T=0
(quantum effects - Zero pt. motion)
Entropy of a vortex :

remains a liquid
(SF) even at T=0
(quantum effects - zero pt. motion)

Entropy of a vortex:

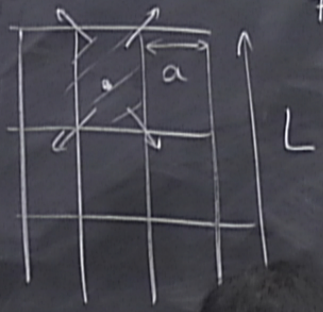


# of places where a vortex can be placed = $\frac{L^2}{a^2}$

$$S = k_B \ln\left(\frac{L}{a}\right)^2 = 2k_B \ln\left(\frac{L}{a}\right)$$

$$F = \left(\pi J - 2k_B T\right) \ln\left(\frac{L}{a}\right)$$

Entropy of a vortex:

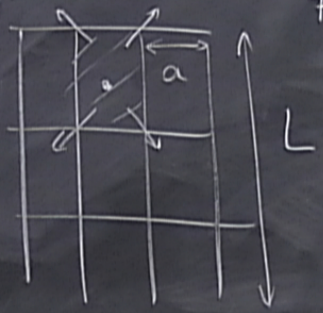


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$$F = E - TS = \left(\underbrace{\pi J}_{\text{dominates at low } T} - \underbrace{2k_B T}_{\text{dominates at high } T}\right) \ln\left(\frac{L}{a}\right)$$

Entropy of a vortex:



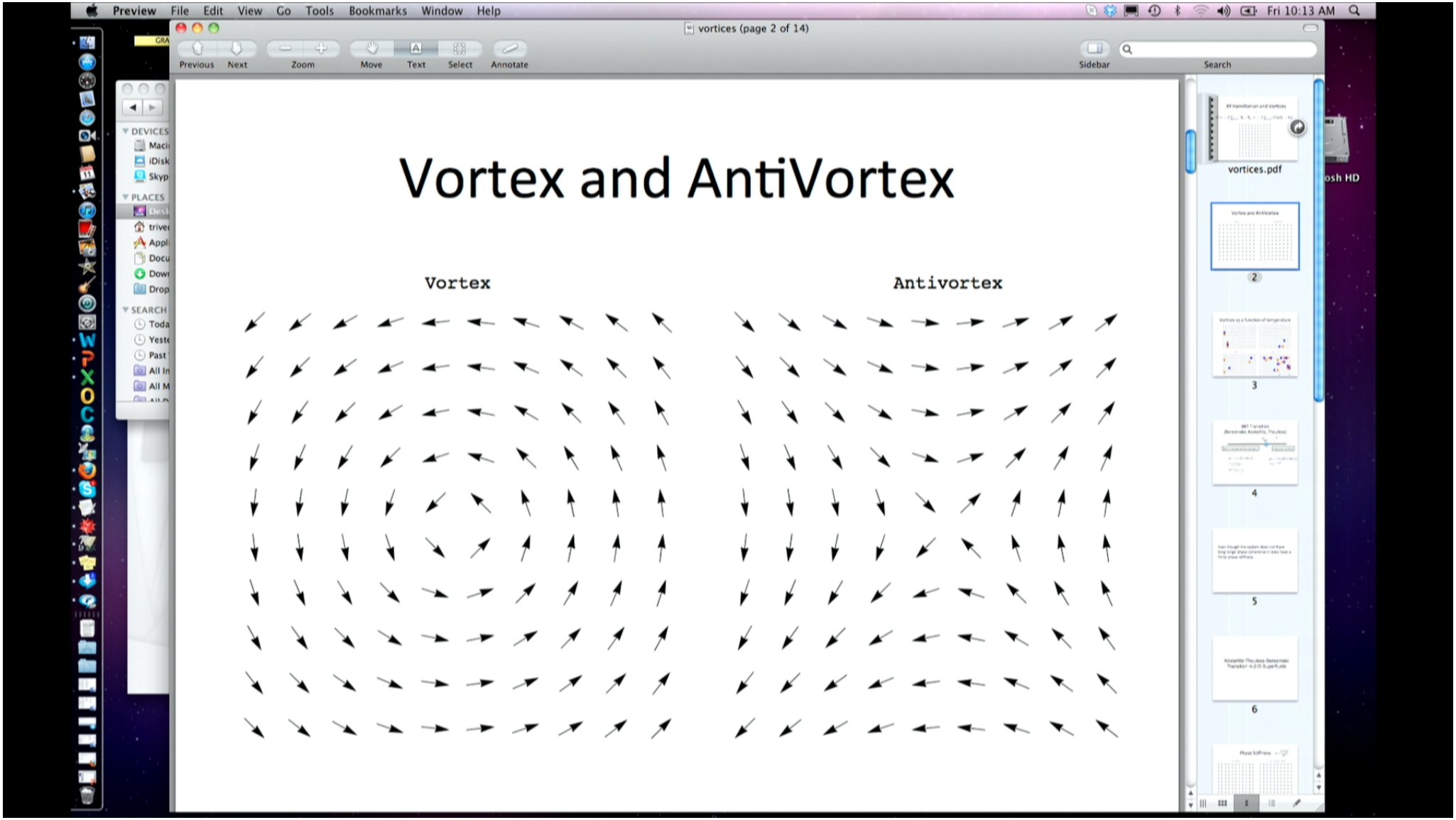
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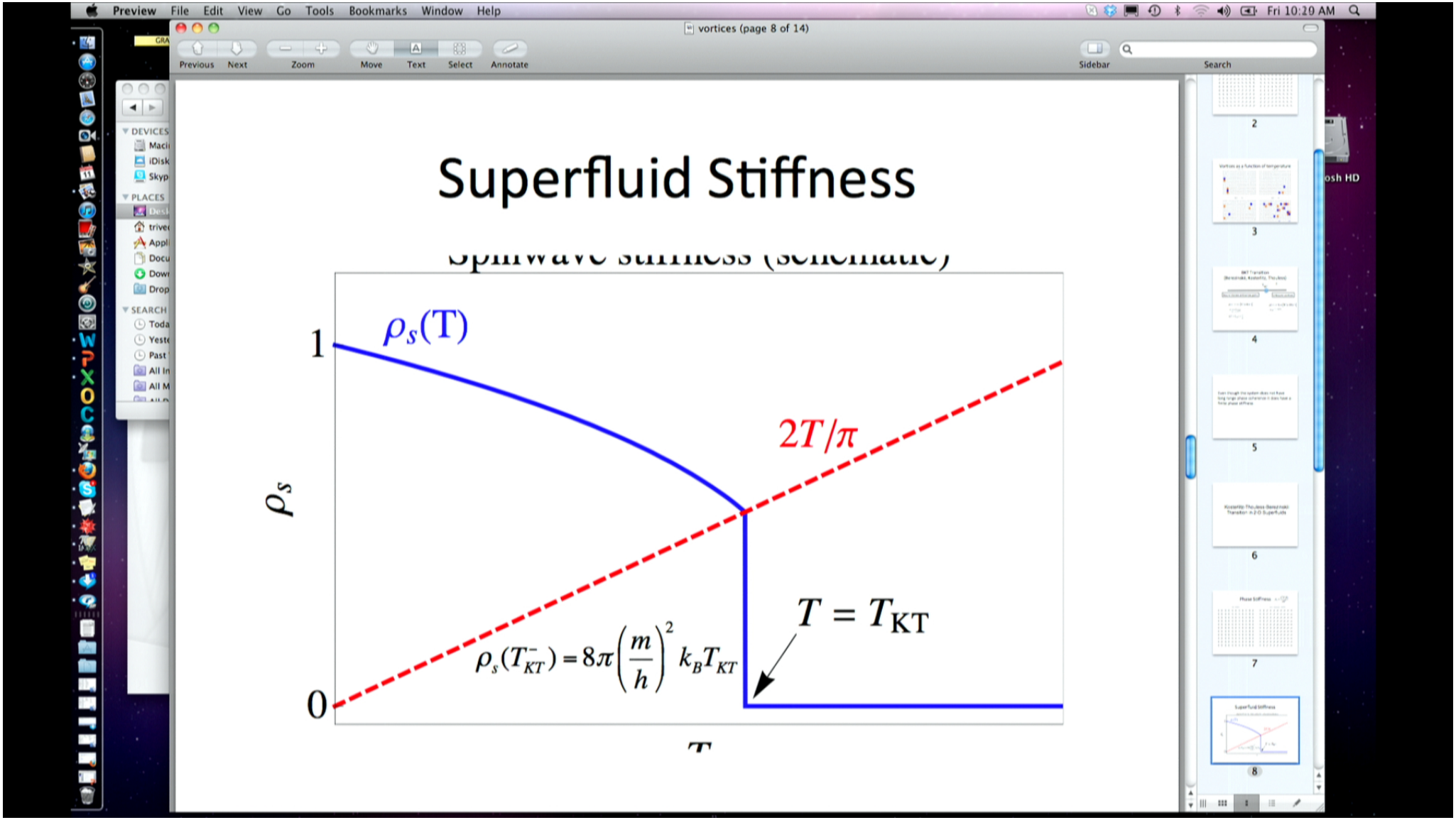
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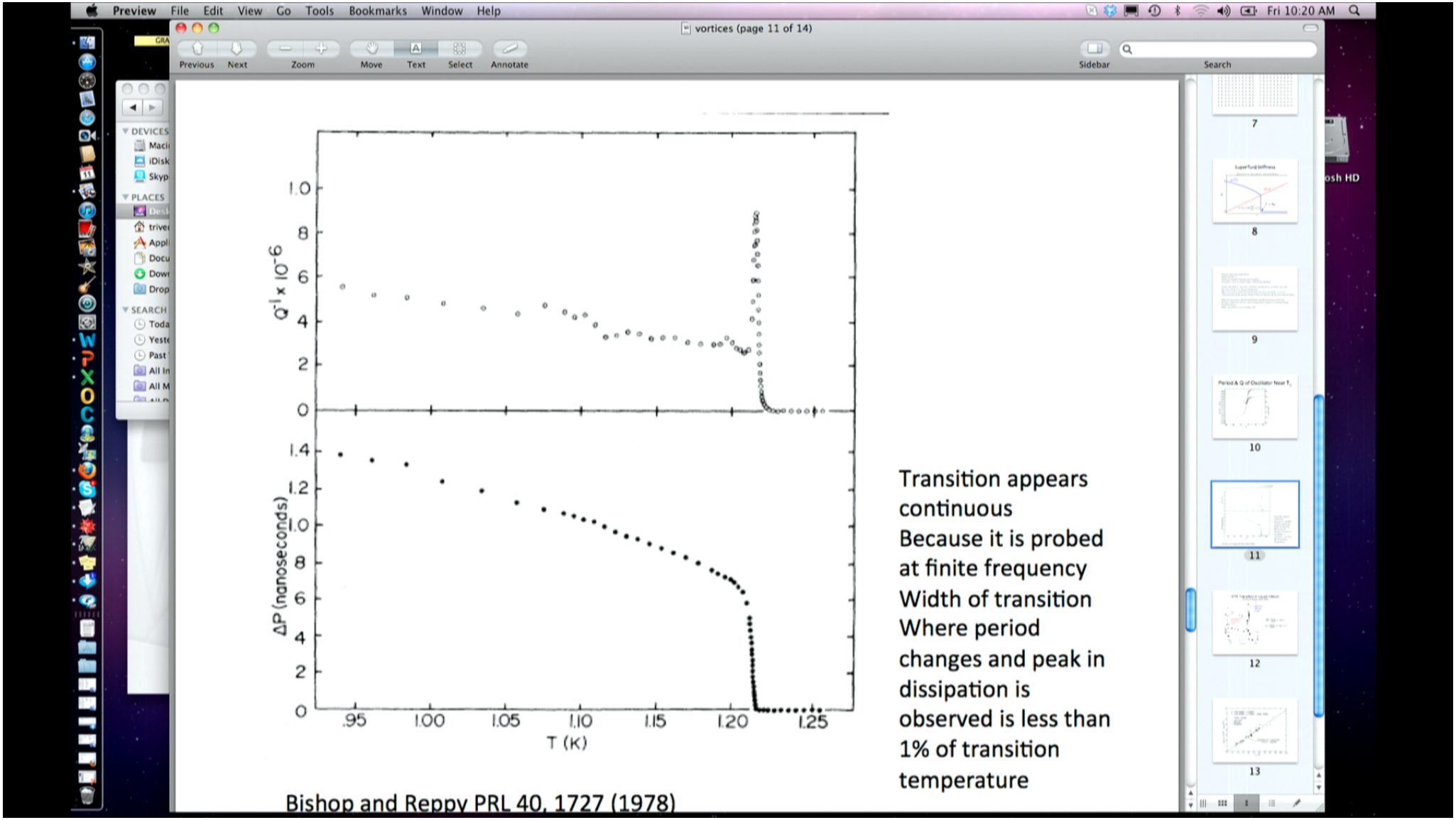
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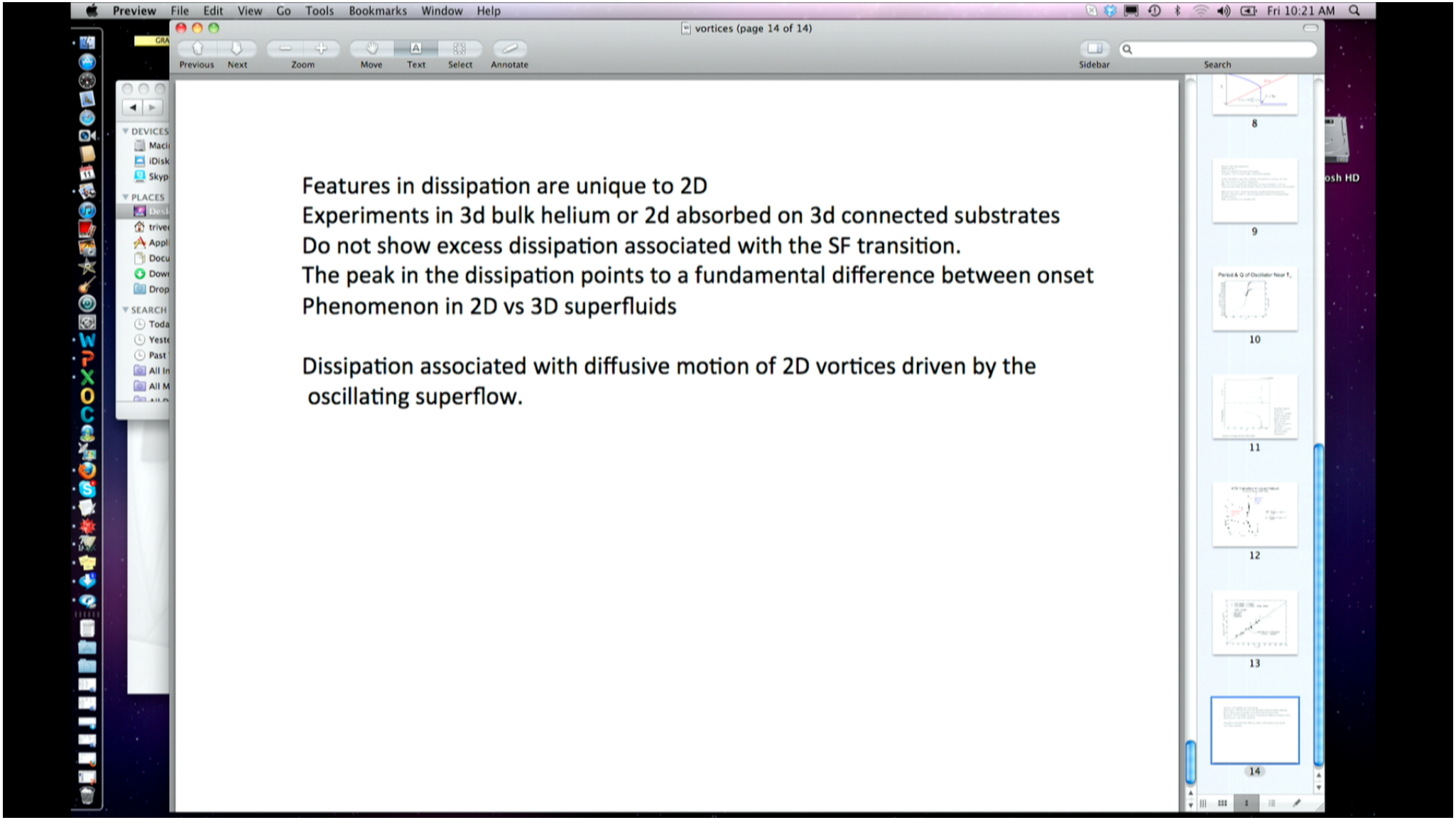
$$k_B T_c \simeq \pi J/2$$

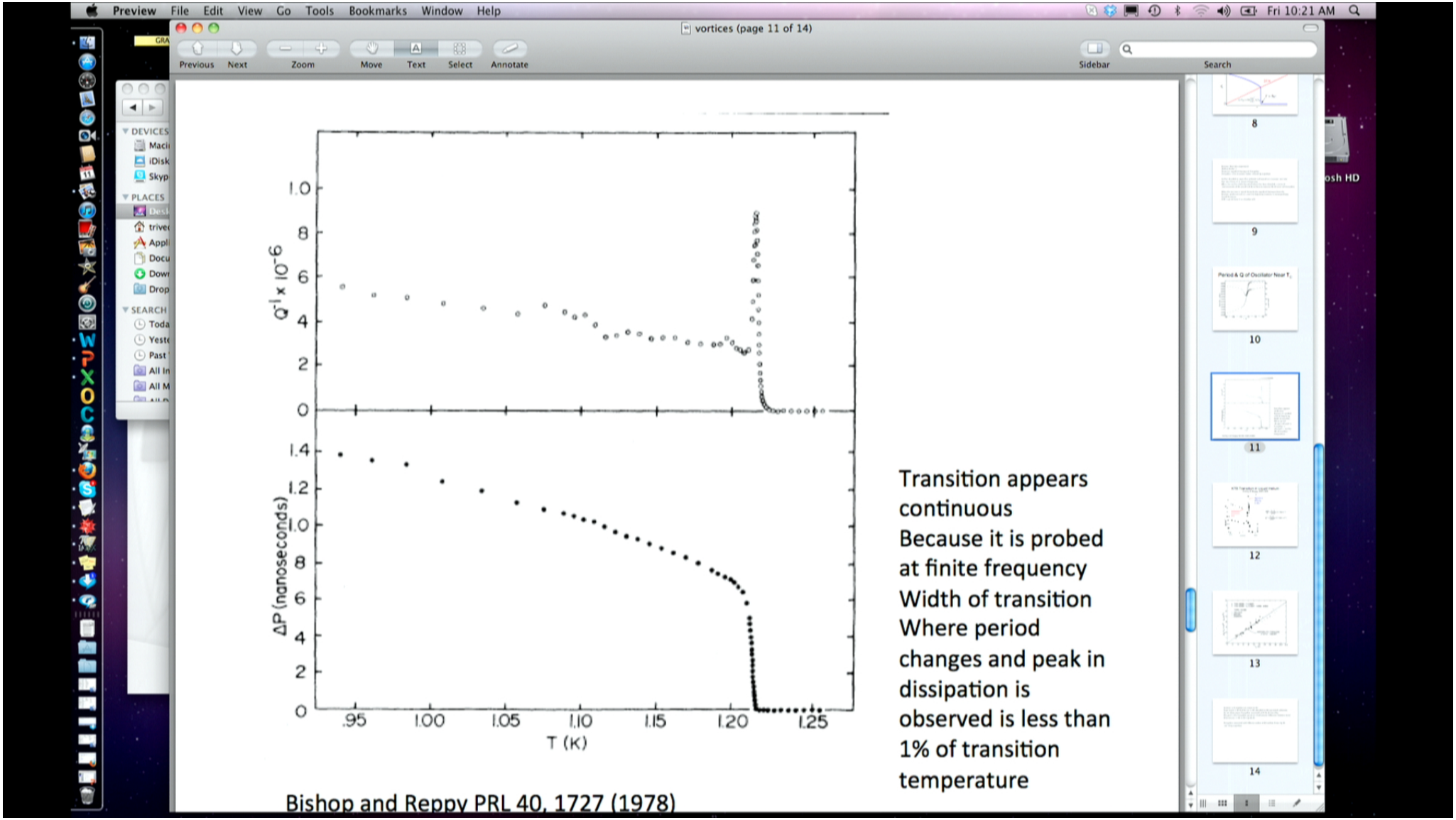
V — ↖ superfluid density

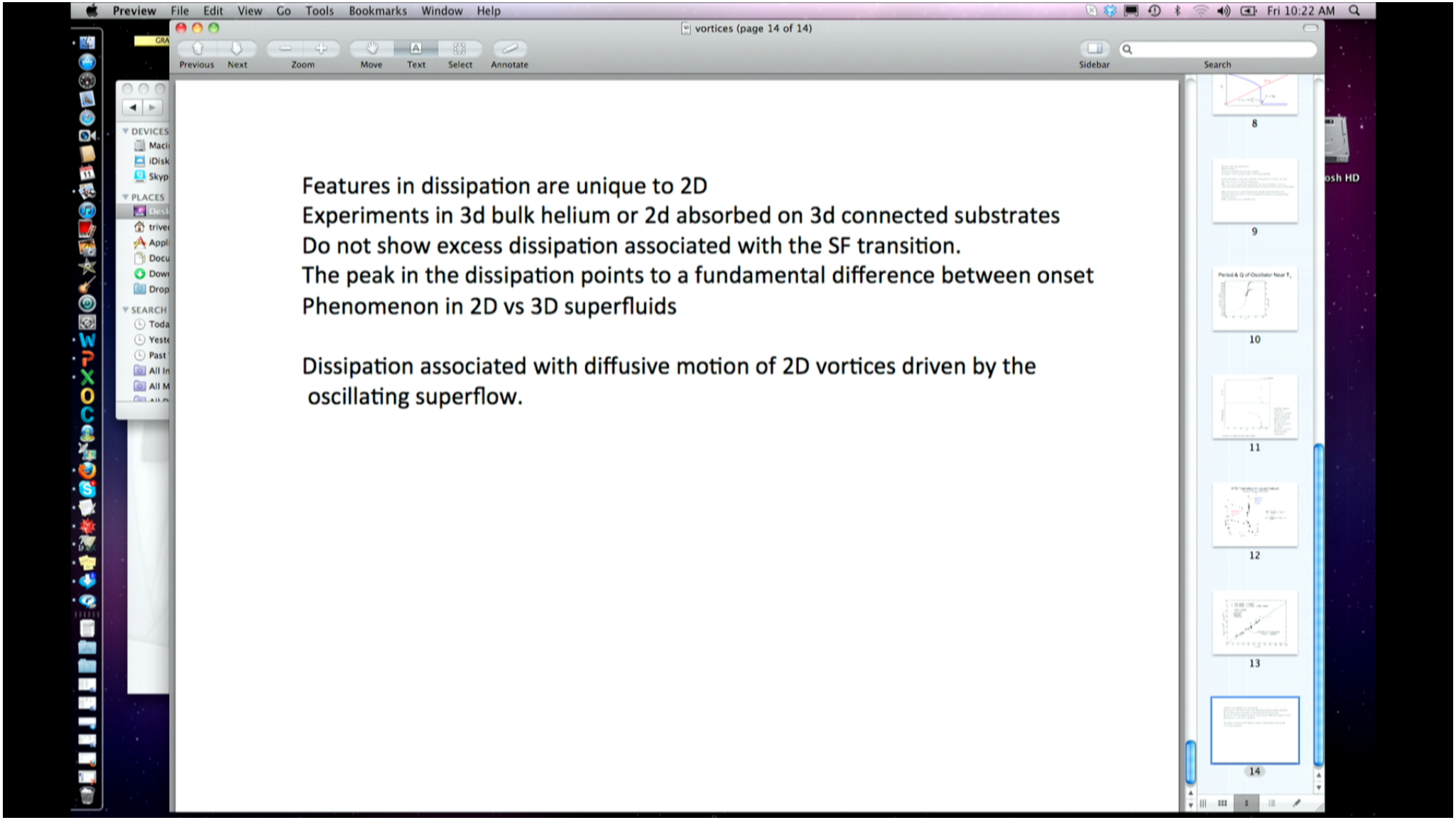

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Vortex and AntiVortex
Vortex
Antivortex
vortices.pdf

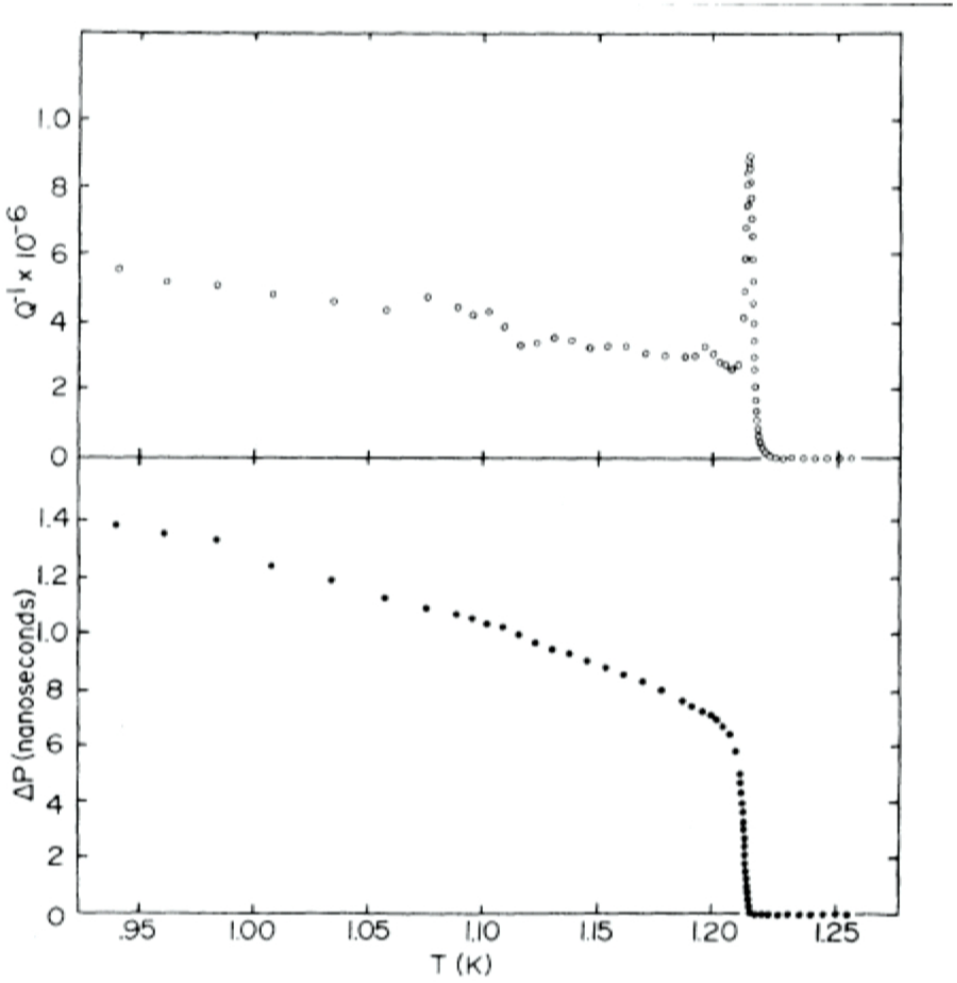

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Superfluid Stiffness
Spinwave stiffness (schematic)
$\rho_s(T)$
$2T/\pi$
ρ_s
1
0
$T = T_{KT}$
$\rho_s(T_{KT}^-) = 8\pi\left(\frac{m}{h}\right)^2 k_B T_{KT}$
T


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vortices (page 11 of 14)
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$Q^{-1} \times 10^{-6}$
ΔP (nanoseconds)
T (K)
.95 1.00 1.05 1.10 1.15 1.20 1.25
Transition appears continuous
Because it is probed at finite frequency
Width of transition
Where period changes and peak in dissipation is observed is less than 1% of transition temperature
Bishop and Reppy PRL 40, 1727 (1978)


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vortices (page 14 of 14)
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Features in dissipation are unique to 2D
Experiments in 3d bulk helium or 2d absorbed on 3d connected substrates
Do not show excess dissipation associated with the SF transition.
The peak in the dissipation points to a fundamental difference between onset
Phenomenon in 2D vs 3D superfluids
Dissipation associated with diffusive motion of 2D vortices driven by the
oscillating superflow.
8
9
10
11
12
13
14


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vortices (page 11 of 14)
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