

Title: Condensed Matter - Lecture 7

Date: Nov 08, 2011 10:30 AM

URL: <http://pirsa.org/11110029>

Abstract:

Today Lecture 7

- 1) Quantum AF ground state \rightarrow spin singlet
Spin Liquids
- 2) Weakly Interacting Bose Gas

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \quad ; \quad S = \frac{1}{2}$$

AF: $J > 0$

What is ground state?

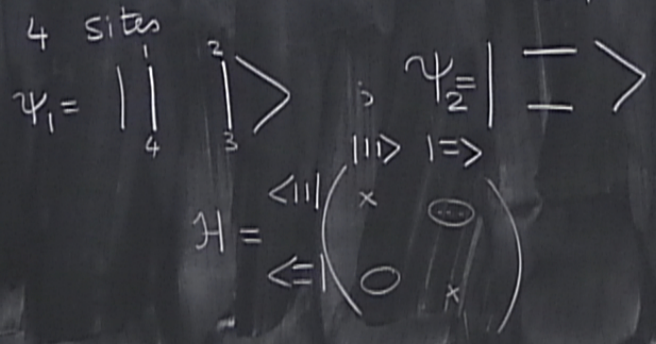
$$|\uparrow \downarrow\rangle \quad E_N = -\frac{J}{4}$$

$$|\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\rangle = |\psi_s\rangle \quad ; \quad E_s = -\frac{3}{8}J = -0.375J$$

1973 Anderson → introduced the idea of the RVB state
 resonating valence bond

VB ≡ Singlet

Bipartite lattice A B A B
 B A B A



$\frac{E}{N_s} = -0.42 J$

$\frac{E}{N_s} = D, \text{QAF}$

1973 Anderson → introduced the idea of the RVB state
 resonating valence bond

VB \equiv Singlet

4 sites
 $\psi_1 = \begin{vmatrix} | & | \\ | & | \\ | & | \\ | & | \end{vmatrix}$

$\psi_2 = \begin{vmatrix} | & | \\ | & | \\ | & | \\ | & | \end{vmatrix}$

$$H = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

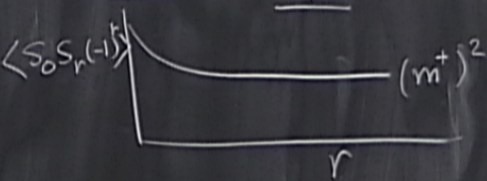
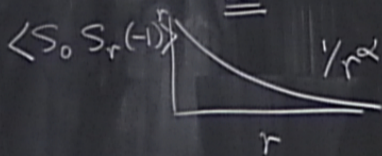
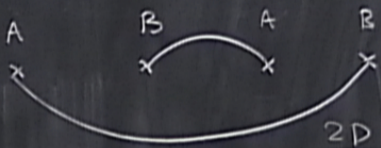
Bipartite lattices
 A B A B
 B A B A

$$\frac{E}{N_s} D, QAF = -0.42 J$$

Ground state :
 x x

lence bond

Ground state : linear combination of all configurations of singlets between A-B sublattices

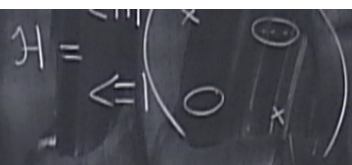


$m^+ = 0.3$

(of ...)

\Rightarrow no staggered LRO

- 0.42 J



\Rightarrow no staggered L
SPIN LIQUID

Spin Liquids in higher dim

- * Mott insulator with 1 electron per unit cell
- * absence of AF LRO
does not break rotational symm.

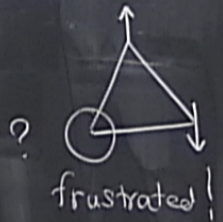
$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

\Rightarrow no staggered L
SPIN LIQUID

Spin Liquids in higher dim

- * Mott insulator with 1 electron per unit cell.
- * absence of AF LRO
does not break rotational symm.

\rightarrow frustrated lattices



using $J > 0$



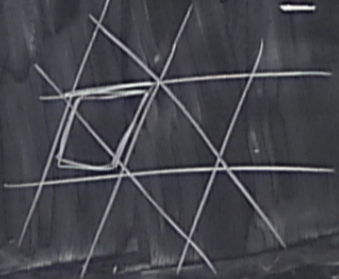
Heisenberg



⇒ no staggered LRO
SPIN LIQUID state

Heisenberg

$$H = J_2 \sum_{\langle ij \rangle} P_{ij} + \sum_{\langle ijk \rangle} P_{ijk}$$

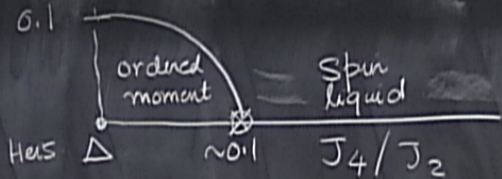
$$P_{ij} = (\vec{S}_i \cdot \vec{S}_j - 1)$$


$$H = J_z \sum_{\langle ij \rangle} S_i^z S_j^z + J_{xy} \sum_{\langle ij \rangle} [S_i^x S_j^x + S_i^y S_j^y]$$

$$\lambda = \frac{J_{xy}}{J_z}$$

Δ lattice $m^+ \approx 0.1$

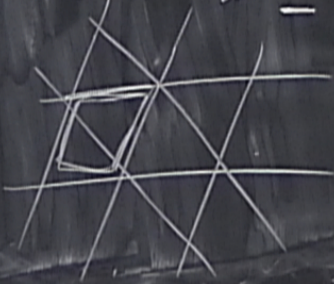
⇒ no staggered LRO
SPIN LIQUID state



Heisenberg

$$\mathcal{H} = J_2 \sum_{\langle ij \rangle} P_{ij} + J_4 \sum_{\langle ijkl \rangle} P_{ijkl}$$

$$P_{ij} = (\vec{S}_i \cdot \vec{S}_j - 1)$$

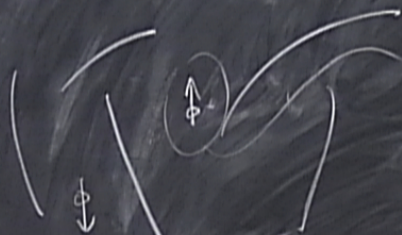


$$\mathcal{H} = J_z \sum_{\langle ij \rangle} S_i^z S_j^z + J_{xy} \sum_{\langle ij \rangle} [S_i^x S_j^x + S_i^y S_j^y]$$

$$\lambda = \frac{J_{xy}}{J_z} \quad (\text{series expansion})$$

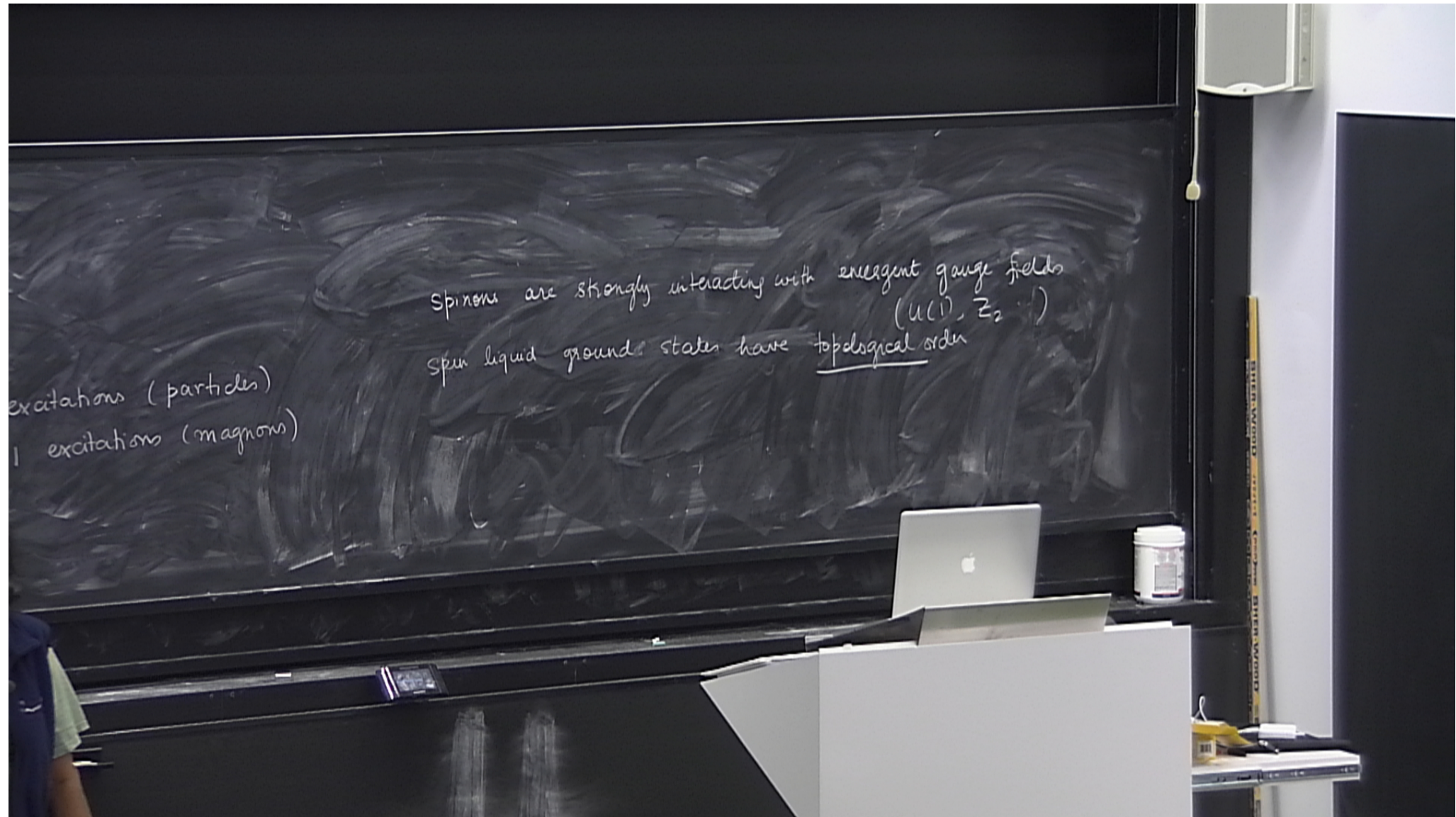
Δ lattice $m^+ \approx 0.1$

Why are spin liquids so hot?
Spin liquids & excitations



\equiv spinons
 $S = \frac{1}{2}$ excitations (particles)
of $S = 1$ excitations (magnons)

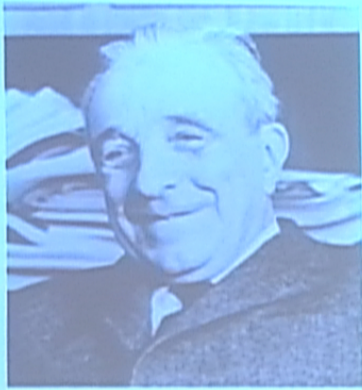
Spinons are strongly interacting



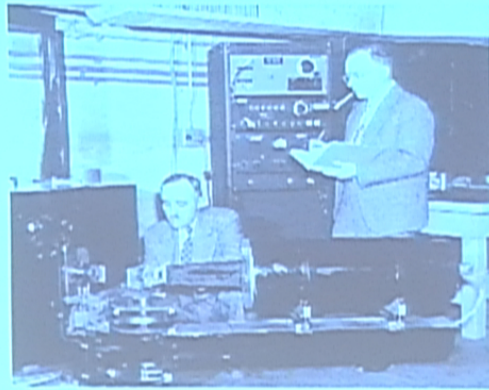
excitations (partides)
excitations (magnons)

Spinons are strongly interacting with emergent gauge fields
($U(1)$, Z_2)
spin liquid ground states have topological order

Antiferromagnets



Louis Néel 1948 first identified
AF ordering
1970 Nobel Prize

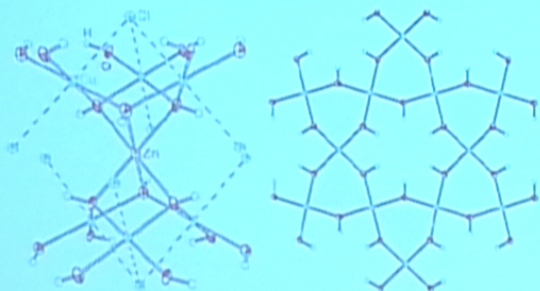


Cliff Shull 1950 confirmed AF ordering in
MnO and NiO using neutron scattering
1994 Nobel Prize

how (particles)
dynam (magnon)

Spinons are strongly interacting
spin liquid ground states have

2d spin liquids with frustration



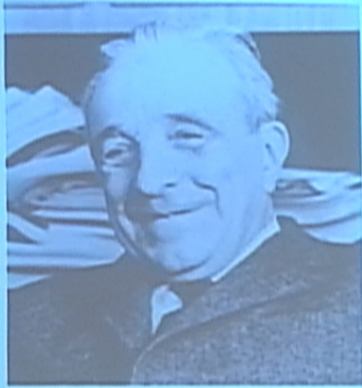
Herbertsmithite: $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

$S=1/2$ Kagome $J \sim 170$ K but
no spin order found down to mK temperatures!

how (particles)
atoms (magnons)

Spinons are strongly interacting
spin liquid ground states have

Antiferromagnets



Louis Neel 1948 first identified
AF ordering
1970 Nobel Prize

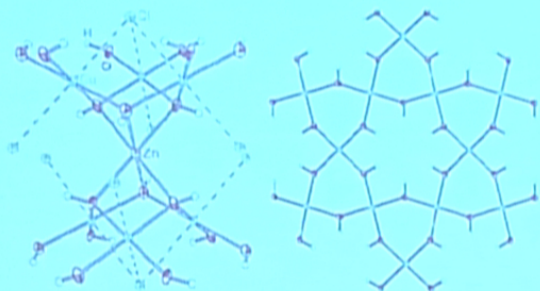


Cliff Shull 1950 confirmed AF ordering in
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ions (particles)
excitations (magnons)

Spinons are strongly interacting
spin liquid ground states have

2d spin liquids with frustration



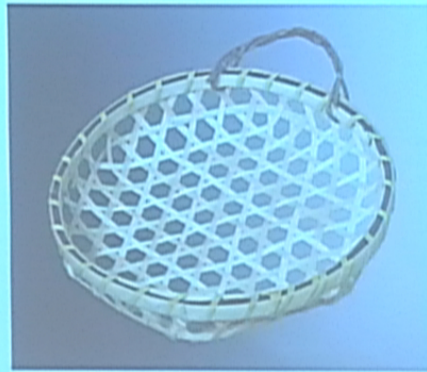
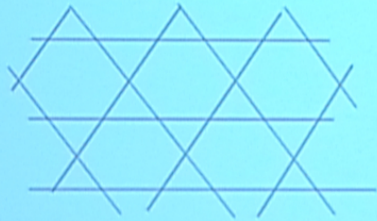
Herbertsmithite: $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

$S=1/2$ Kagome $J \sim 170$ K but
no spin order found down to mK temperatures!

how (particles)
dations (magnons)

Spinons are strongly interacting
spin liquid ground states have

2d spin liquids: Kagome (more frustrated than triangular lattice)



ions (particles)
excitations (magnons)

Spinons are strongly interacting
spin liquid ground states have

Spin susceptibility

$$\chi = \frac{\partial M}{\partial H} \Big|_{H \rightarrow 0}$$

magnetization

magnetic field

Paramagnet

$$\chi = \frac{C}{T} \quad \chi$$

Ferromagnet

χ

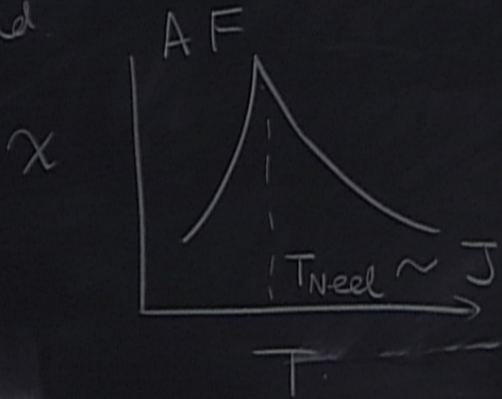


susceptibility

magnetization

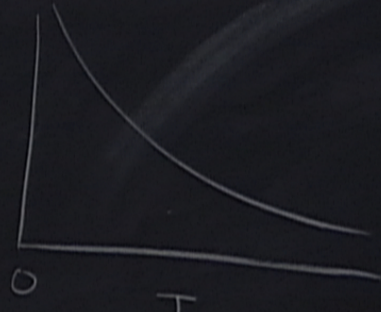
$$\chi = \left. \frac{\partial M}{\partial H} \right|_{H \rightarrow 0}$$

↑
magnetic field

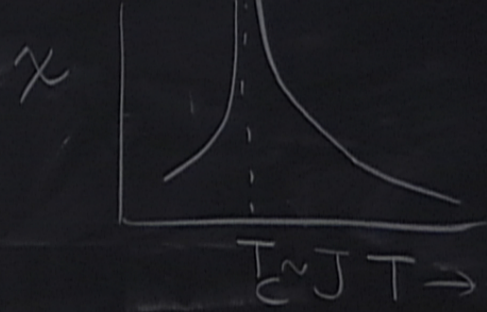


Paramagnet

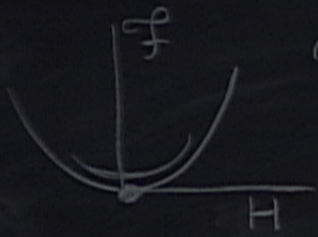
$$\chi = \frac{C}{T}$$



Ferromagnet



Spin susceptibility



$$\chi = \left. \frac{\partial M}{\partial H} \right|_{H \rightarrow 0}$$

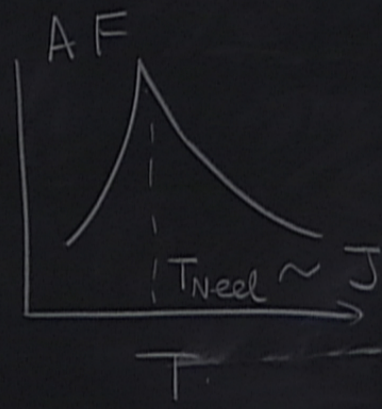
↑
magnetic field

magnetization

$$M = + \frac{\partial \phi}{\partial H}$$

$$\chi = \frac{\partial^2 \phi}{\partial H^2}$$

χ



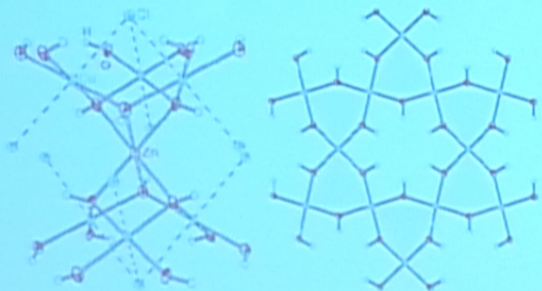
Paramagnet

$$\chi = \frac{C}{T} \quad \chi$$

Ferromagnet

χ

2d spin liquids with frustration



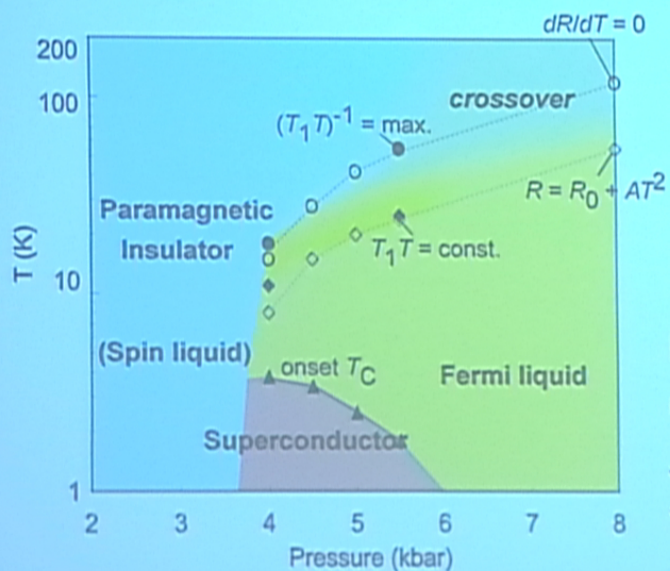
Herbertsmithite: $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

$S=1/2$ Kagome $J \sim 170$ K but
no spin order found down to mK temperatures!

Spinons are strongly interacting
non (particles)
dimer (mag)

Spin liquid ground state has

2d spin liquids with ring exchange



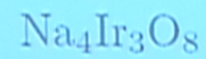
Q2D spin liquid
 $\text{K-Cu}_2(\text{CN})_3$

No AF order
 down to 35mK.
 $J=250\text{K}$
 $t'/t=1.06$

Spinons are strongly interacting
 holons (particles)
 magnons (magnons)
 spin liquid ground states have

3d spin liquids

Pyrochlore lattice:
Corner sharing
tetrahedral lattice;
Highly frustrated

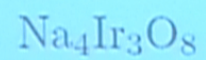


ions (particles)
excitations (magnons)

Spinons are strongly interacting
spin liquid ground states have

3d spin liquids

Pyrochlore lattice:
Corner sharing
tetrahedral lattice;
Highly frustrated

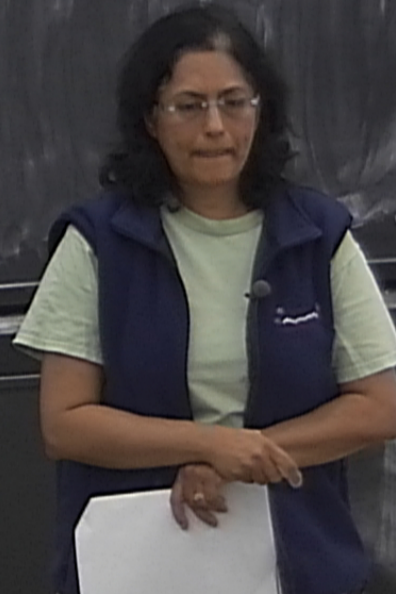


hons (particles)
dashons (magnons)

Spinons are strongly interacting
spin liquid ground states have

S_i^\pm anticommute on the same site $\iff (b_i^\pm)^2 = 0 \implies$ Maximal occupancy

$$\mathcal{H} = \frac{J}{2} \sum_{\langle ij \rangle} (b_i^\dagger b_j + \text{h.c.}) + J \sum_{\langle ij \rangle} n_i n_j$$



S_i^\pm anticommute on the same site $\iff (b_i^\pm)^2 = 0 \implies$ Maximal occupancy

$$H = \frac{J}{2} \sum_{\langle i,j \rangle} (b_i^\dagger b_j + \text{h.c.}) + J \sum_{\langle i,j \rangle} n_i n_j + E_N$$
$$E_N = -JzN/8$$

Unitary transformation on B-sublattice

$$S_i^\dagger \rightarrow -S_i^\dagger$$

S_i^\pm anticommute on the same site $\iff (b_i^\pm)^2 = 0 \implies$ Majorana fermions

$$\begin{aligned}
 \mathcal{H} &= \frac{J}{2} \sum_{\langle ij \rangle} (b_i^+ b_j + h.c.) + J \sum_{\langle ij \rangle} n_i n_j + E_N \\
 &\quad \text{apply sublattice rotation} \\
 &= -\frac{J}{2} \sum_{\langle ij \rangle} (b_i^+ b_j + h.c.) + \dots
 \end{aligned}$$

$$E_N = -\frac{JZN}{8}$$

Unitary transform

$$\begin{aligned}
 S_i^+ &\rightarrow -S_i^+ \\
 S_i^- &\rightarrow -S_i^- \\
 S_i^z &\rightarrow S_i^z
 \end{aligned}$$

S_i^\pm anticommute on the same site $\iff (b_i^\pm)^2 = 0 \implies$ Majorana fermions

$$\mathcal{H} = \frac{J}{2} \sum_{\langle ij \rangle} (b_i^+ b_j + h.c.) + J \sum_{\langle ij \rangle} n_i n_j + E_N$$

↑
apply sublattice rotation

$$= -\frac{J}{2} \sum_{\langle ij \rangle} (b_i^+ b_j + h.c.) + \dots$$

$$\hat{b}_k = \frac{1}{\sqrt{N}} \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \hat{b}_i$$

$$E_N = -J \frac{ZN}{8}$$

Unitary transform

$$S_i^+ \rightarrow -S_i^+$$

$$S_i^- \rightarrow -S_i^-$$

$$S_i^z \rightarrow S_i^z$$

S_i^\pm anticommute on the same site $\iff (b_i^\pm)^2 = 0 \implies$ Pauli exclusion

$$H = \frac{J}{2} \sum_{\langle i,j \rangle} (b_i^+ b_j + h.c.) + J \sum_{\langle i,j \rangle} n_i n_j + E_N$$

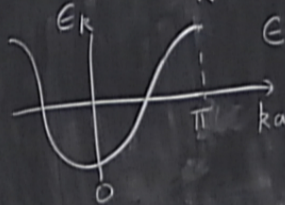
$$E_N = -JZN \frac{8}{8}$$

apply sublattice rotation

$$= -\frac{J}{2} \sum_{\langle i,j \rangle} (b_i^+ b_j + h.c.) + \dots = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} b_{\mathbf{k}}^+ b_{\mathbf{k}} + \dots$$

$$\epsilon_{\mathbf{k}} = -2J \sum_{\mu=1}^d \cos k_{\mu} a$$

$$\hat{b}_{\mathbf{k}} = \frac{1}{\sqrt{N}} \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \hat{b}_i$$

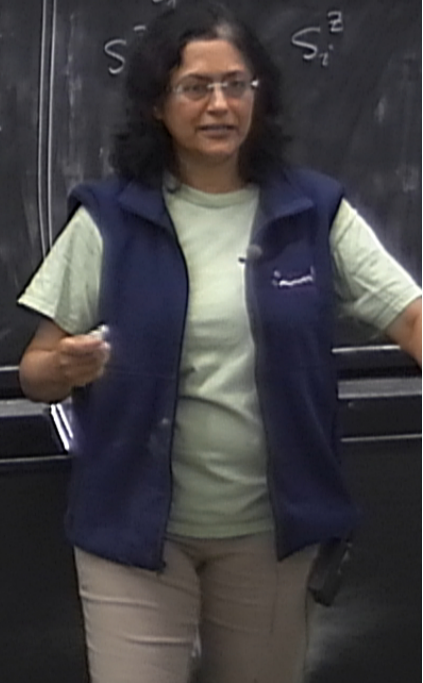


Unitary transform

$$S_i^+ \rightarrow -S_i^+$$

$$S_i^- \rightarrow -S_i^-$$

$$S_i^z \rightarrow S_i^z$$



S_i^\pm anticommute on the same site $\iff (b_i^\pm) = 0 \implies$ Majorana fermions

$$H = \frac{J}{2} \sum_{\langle ij \rangle} (b_i^+ b_j + h.c.) + J \sum_{\langle ij \rangle} n_i n_j + E_N$$

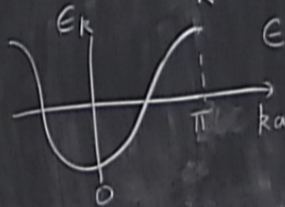
$$E_N = -JZ \frac{N}{8}$$

apply sublattice rotation

$$= -\frac{J}{2} \sum_{\langle ij \rangle} (b_i^+ b_j + h.c.) + \dots = \sum_k \epsilon_k b_k^+ b_k + \dots$$

$$\epsilon_k = -2J \sum_{\mu=1}^d \cos k_\mu a$$

$$\hat{b}_k = \frac{1}{\sqrt{N}} \sum_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \hat{b}_i$$



Unitary transform

$$S_i^+ \rightarrow -S_i^+$$

$$S_i^- \rightarrow -S_i^-$$

$$S_i^z \rightarrow S_i^z$$

$$b_i^\dagger |0 \dots 0\rangle = | \dots i \dots \rangle$$

$$S_z^{\text{tot}} = 0 \quad \# \uparrow = \# \downarrow \quad \implies \quad N_{\text{sites}} \quad \frac{N}{2} \text{ bosons}$$

a

$$a_0 |\psi_0(N_0)\rangle = \sqrt{N_0} |\psi_0(N_0-1)\rangle$$
$$a_0^+ |\psi$$

$$a_0 |\psi_0(N_0)\rangle = \sqrt{N_0} |\psi_0(N_0-1)\rangle$$
$$a_0^+ |\psi_0(N_0)\rangle = \sqrt{N_0+1} |\psi_0(N_0+1)\rangle$$

Since $\sqrt{N_0+1} \approx \sqrt{N_0}$
 \Rightarrow treat a_0, a_0^+ as c-number

$$a_0 |\psi_0(N_0)\rangle = \sqrt{N_0} |\psi_0(N_0-1)\rangle$$

$$a_0^+ |\psi_0(N_0)\rangle = \sqrt{N_0+1} |\psi_0(N_0+1)\rangle$$

Since $\sqrt{N_0+1} \approx \sqrt{N_0}$
 \Rightarrow treat a_0, a_0^+ as c-number

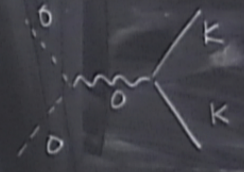
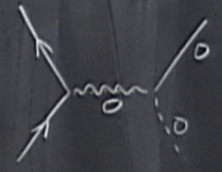
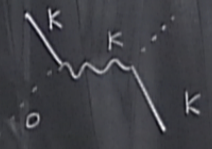
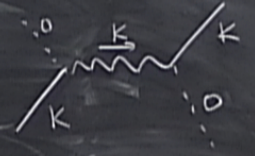
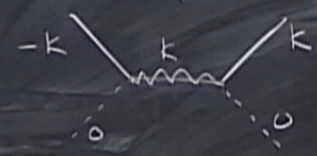
$$\langle \hat{a}_0 \rangle = \sqrt{N_0} \quad ; \quad \langle a_0^+ \rangle = \sqrt{N_0+1}$$

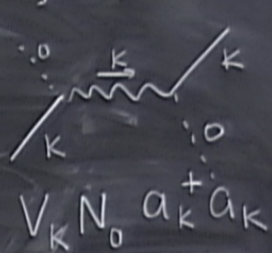
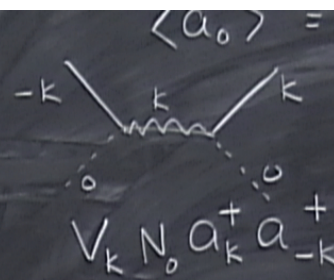
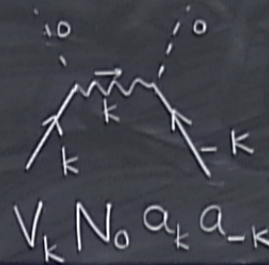
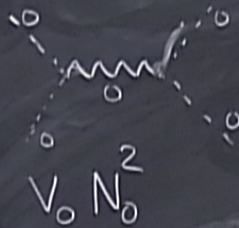


$|N_0 - 1\rangle$
 $|N_0 + 1\rangle$

Since $\sqrt{N_0 + 1} \approx \sqrt{N_0}$
 \Rightarrow treat a_0, a_0^+ as c-numbers

$\langle \hat{a}_0 \rangle = \sqrt{N_0}$; $\langle a_0^+ \rangle = \sqrt{N_0 + 1}$

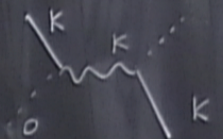
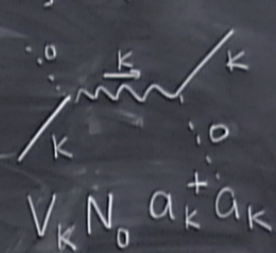
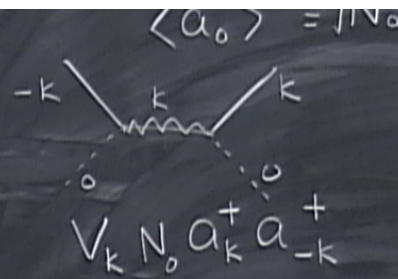
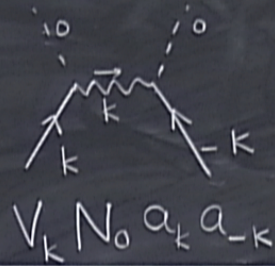
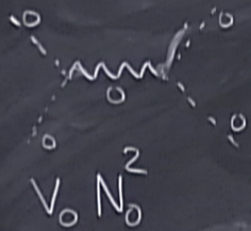




$$\langle a_0 \rangle = \sqrt{N_0}$$

k=0 not included

$$H_{int} = \frac{1}{2\Omega} \left[V_0 N_0^2 + 2N_0 \sum_k (V_k + V_0) a_k^+ a_k + N_0 \sum_k' V_k (a_k^+ a_{-k}^+ + a_k a_{-k}) \right]$$

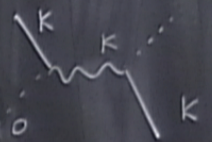
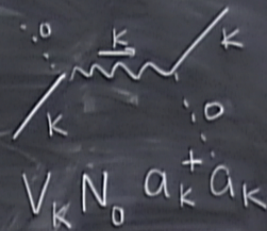
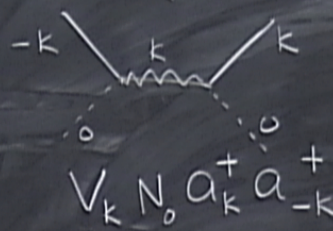
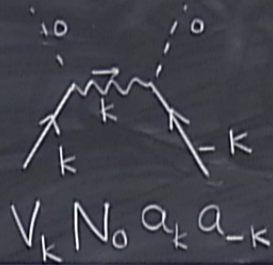
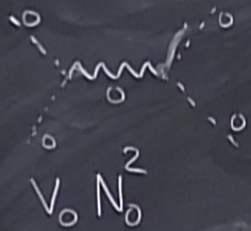


$k=0$ not included

$$H_{int} = \frac{1}{2\Omega} \left[V_0 N_0^2 + 2N_0 \sum_k (V_k + V_0) a_k^+ a_k + N_0 \sum_k' V_k (a_k^+ a_{-k}^+ + a_k a_{-k}) \right]$$

$$H \rightarrow H - \mu N$$





$k=0$ not included

$$H_{int} = \frac{1}{2\Omega} \left[V_0 N_0^2 + 2N_0 \sum_k (V_k + V_0) a_k^+ a_k + N_0 \sum_k' V_k (a_k^+ a_{-k}^+ + a_k a_{-k}) \right]$$

$$H \rightarrow H - \mu N$$

$$H = \frac{1}{2} \Omega n^2 V_0 + \frac{1}{2} \sum_k' (\epsilon_k^0 + n V_k) (a_k^+ a_k + a_{-k}^+ a_{-k}) + \frac{1}{2} n \sum_k' V_k (a_k^+ a_{-k}^+ + a_k a_{-k})$$

$$n = \frac{N}{\Omega}$$

S_i^\pm anticommute on the same site $\iff (b_i^\pm)^2 = 0 \implies$ Majorana fermions

Bogoliubov transfⁿ

Introduce new set of Bose operators b_k, b_k^+

$$\begin{pmatrix} a_k \\ a_{-k}^+ \end{pmatrix} = \begin{pmatrix} u_k & -v_k \\ -v_k & u_k \end{pmatrix} \begin{pmatrix} b_k \\ b_{-k}^+ \end{pmatrix}$$

Canonical transfⁿ

$$[b_k, b_{k'}^+] = \delta_{kk'}$$

$$\begin{pmatrix} b_k^+ \\ b_{-k} \end{pmatrix} = \begin{pmatrix} v_k & u_k \end{pmatrix} \begin{pmatrix} a_k^+ \\ a_{-k} \end{pmatrix}$$

Substitute the Bog. transf.^m in H

$$H = E_0 + \sum_k' E_k b_k^+ b_k$$

$$E_k = k \left[\frac{1}{2m} \left(\frac{k^2}{2m} + 2n v_k \right)^{1/2} \right]$$

as $k \rightarrow 0$ $E_k = ck \Rightarrow$ sound modes

Free dispersion $E_k \sim k^2$