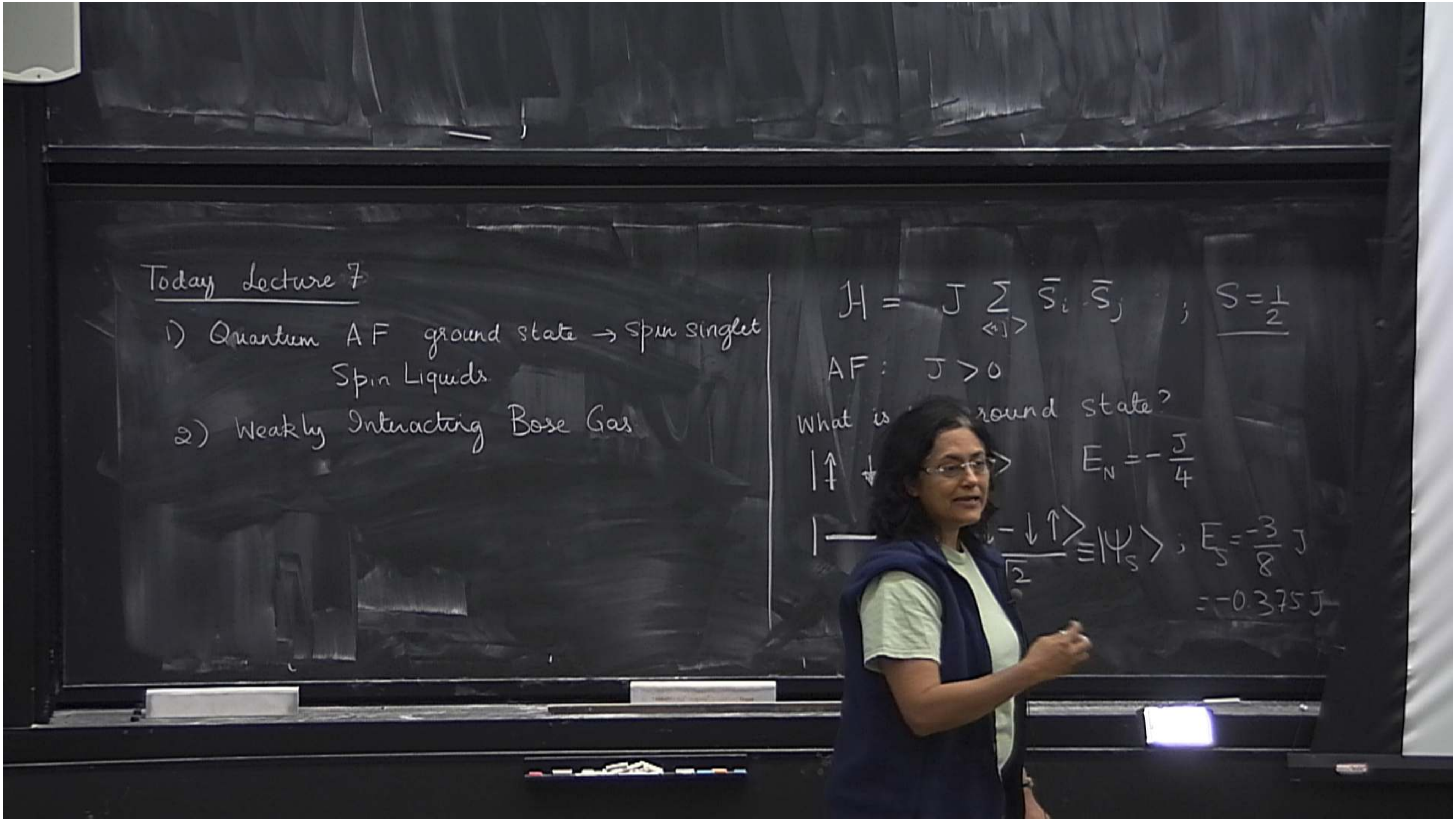
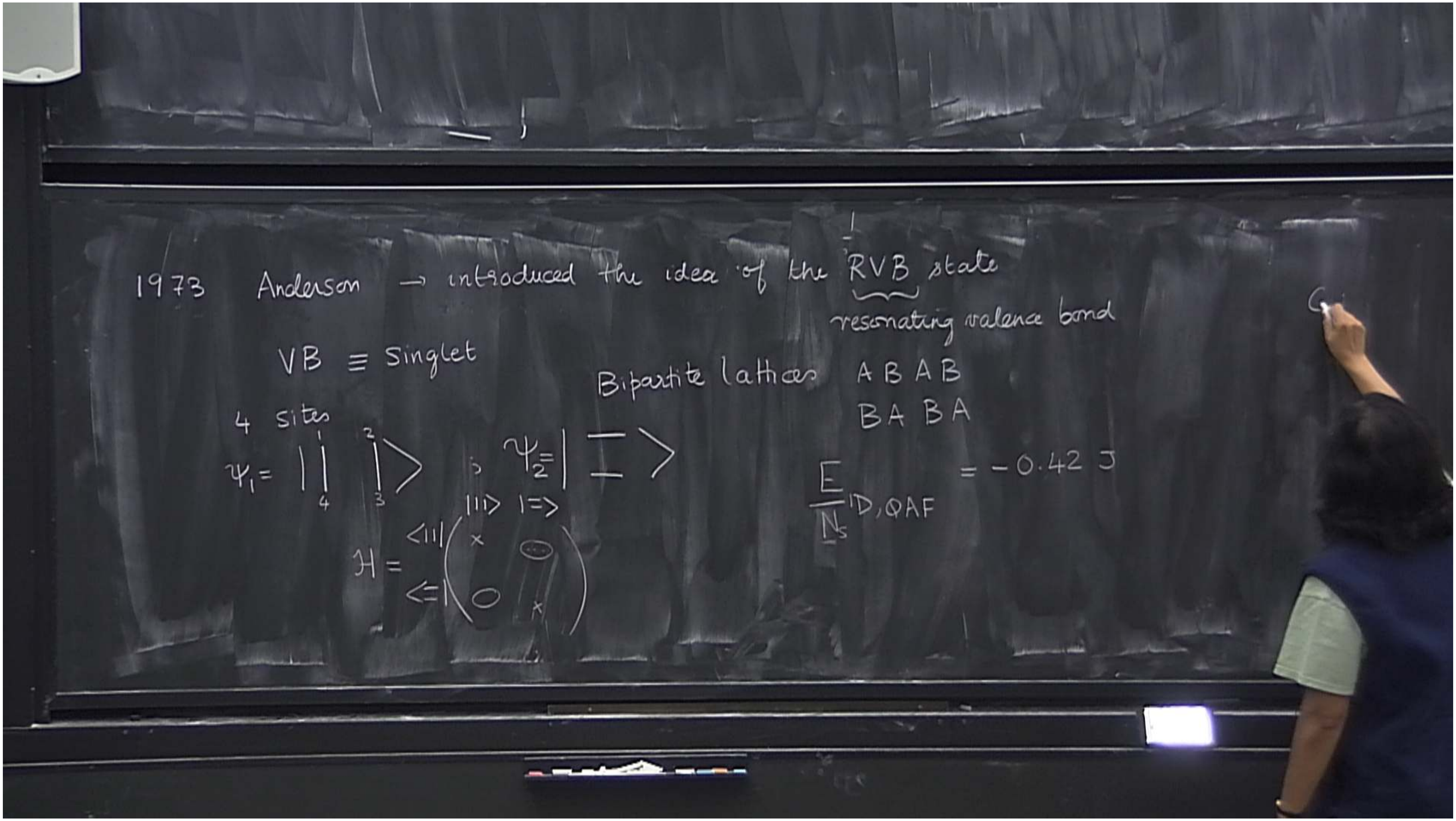
Title: Condensed Matter - Lecture 7

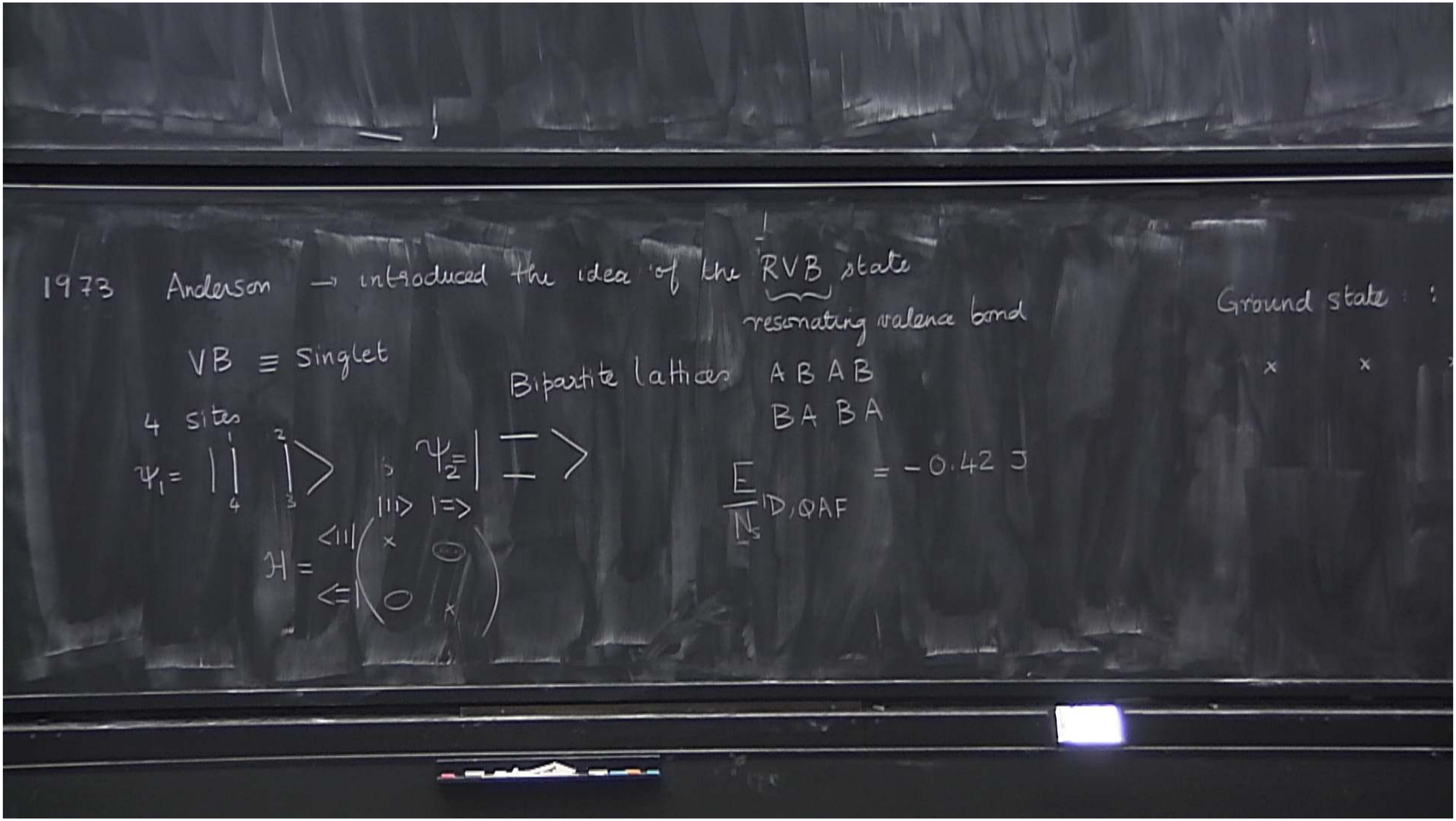
Date: Nov 08, 2011 10:30 AM

URL: http://pirsa.org/11110029

Abstract:


Today Lecture 7
1) Quantum AF ground state → spin singlet
Spin Liquids
2) Weakly Interacting Bose Gas
AF : J > 0
E_N = -J/4
≡|Ψ_s⟩ ; E_s = -3/8 J
= -0.375 J


1973 Anderson → introduced the idea of the RVB state
resonating valence bond
VB ≡ Singlet
Bipartite lattices
A B A B
B A B A
4 sites
ψ1 =
ψ2 =
|||> |=>
H =
E/Ns 1D, QAF = −0.42 J


1973 Anderson → introduced the idea of the RVB state
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Ground state :
VB ≡ Singlet
Bipartite lattices ABAB
BABA
4 sites
ψ1 = | ||> , ψ2 = | = >
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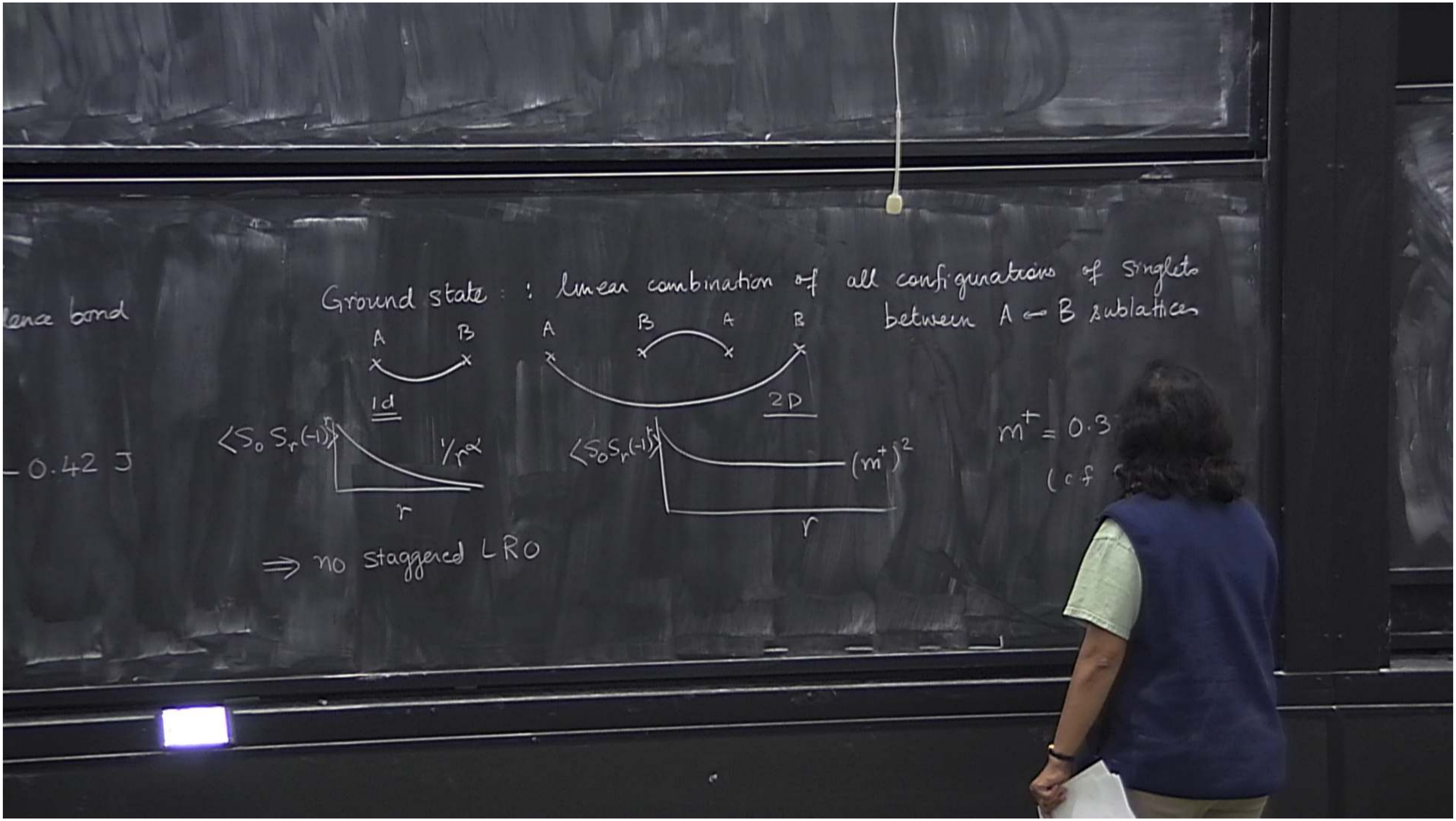
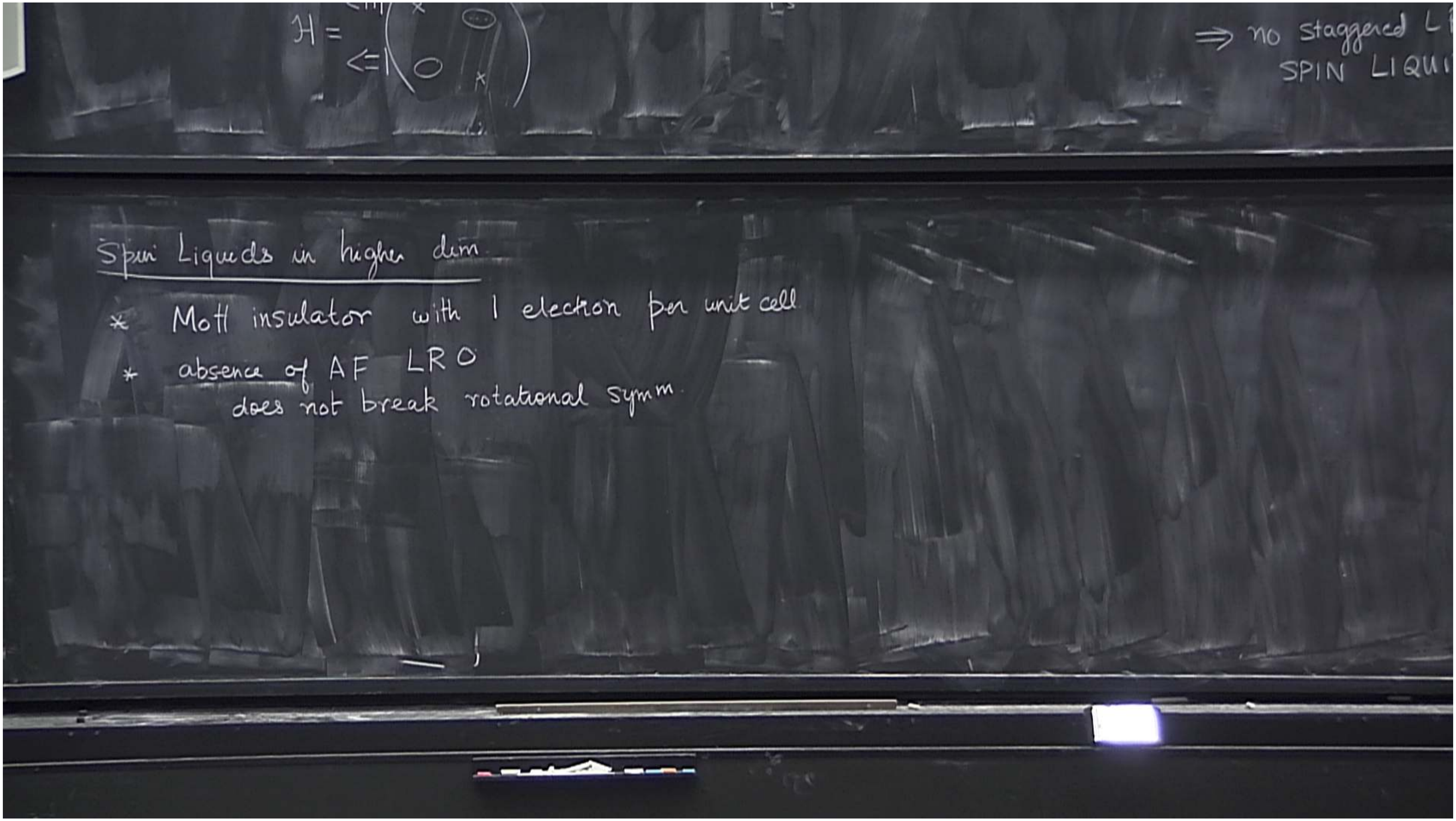
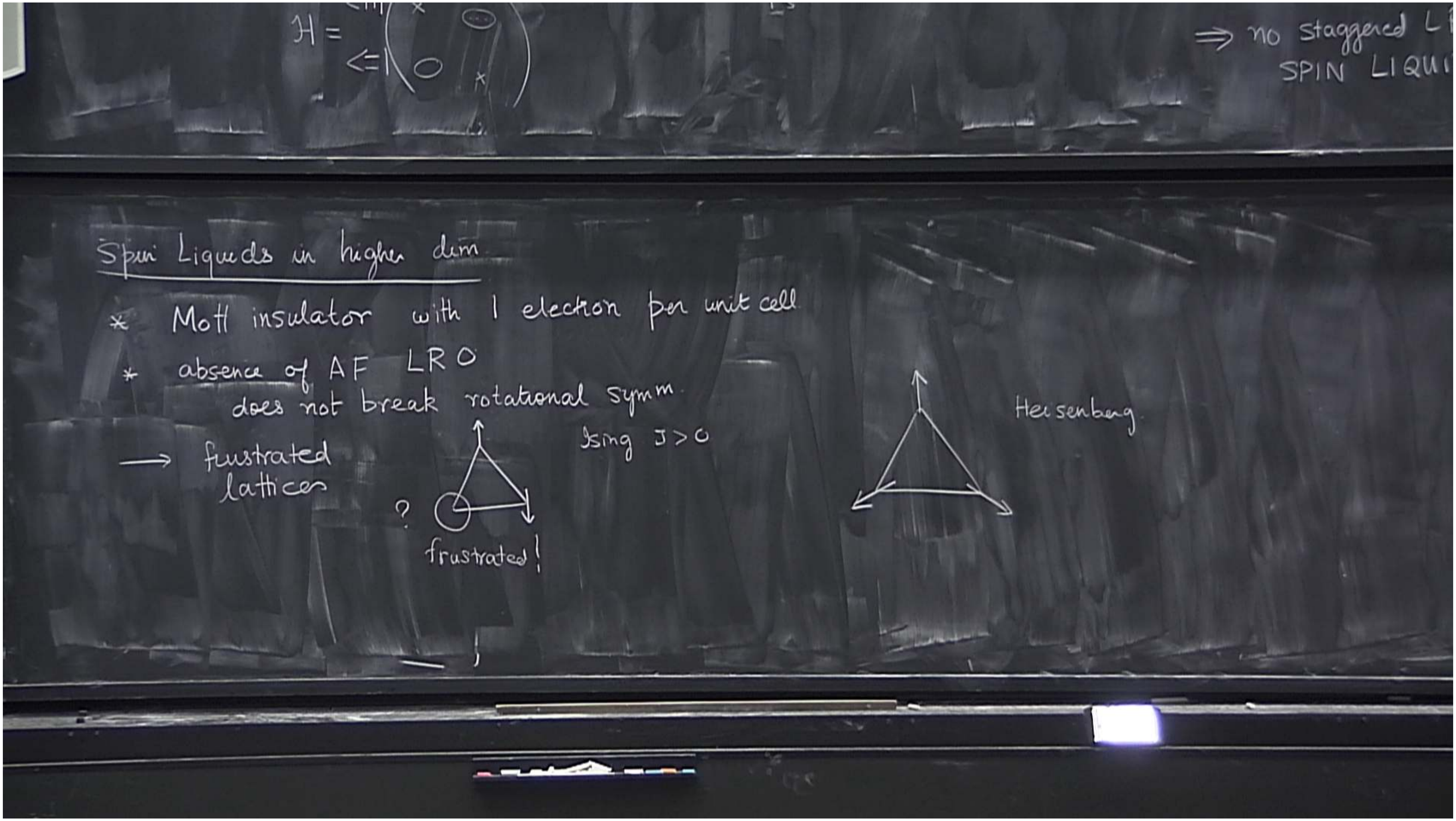
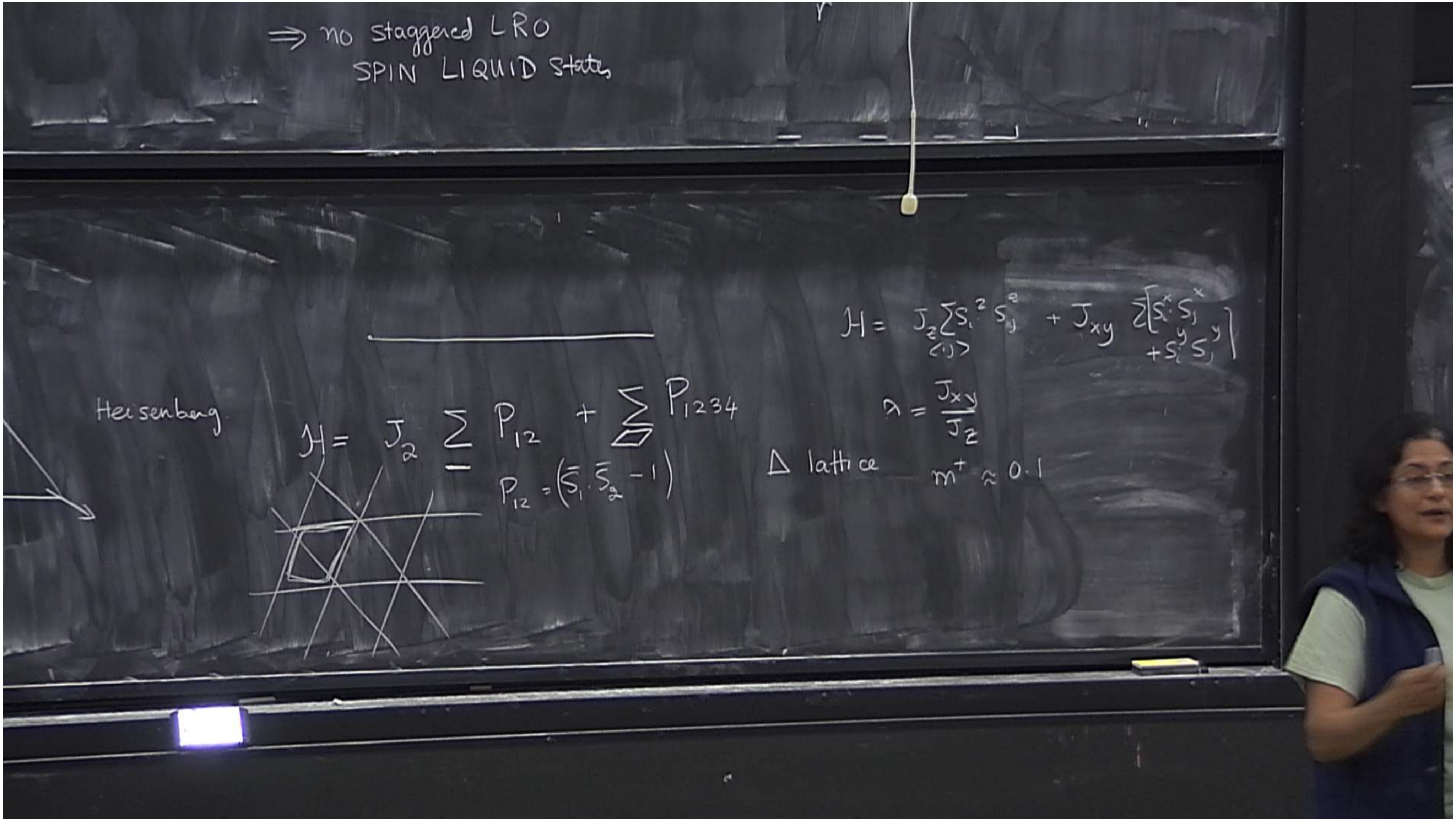
H =
<||| <=|
E/Ns 1D, QAF = −0.42 J

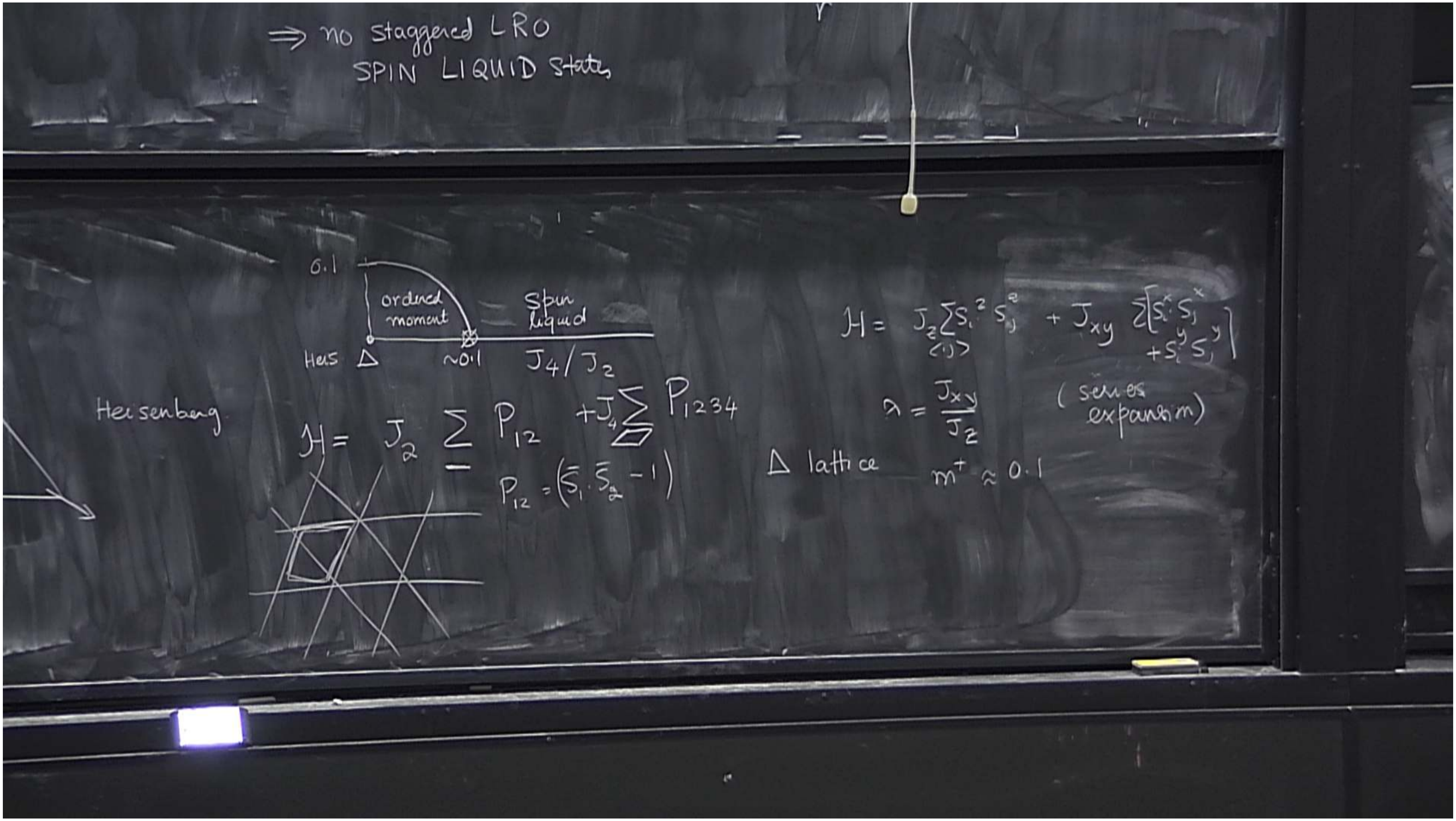

Ground state : linear combination of all configurations of singlets between A – B sublattices
lence bond
A B
A B A B
1d
2D
⟨S0 Sr (-1)^r⟩
1/r^α
⟨S0 Sr (-1)^r⟩
(m+)^2
r
r
– 0.42 J
m+ = 0.3
(cf
⇒ no staggered LRO

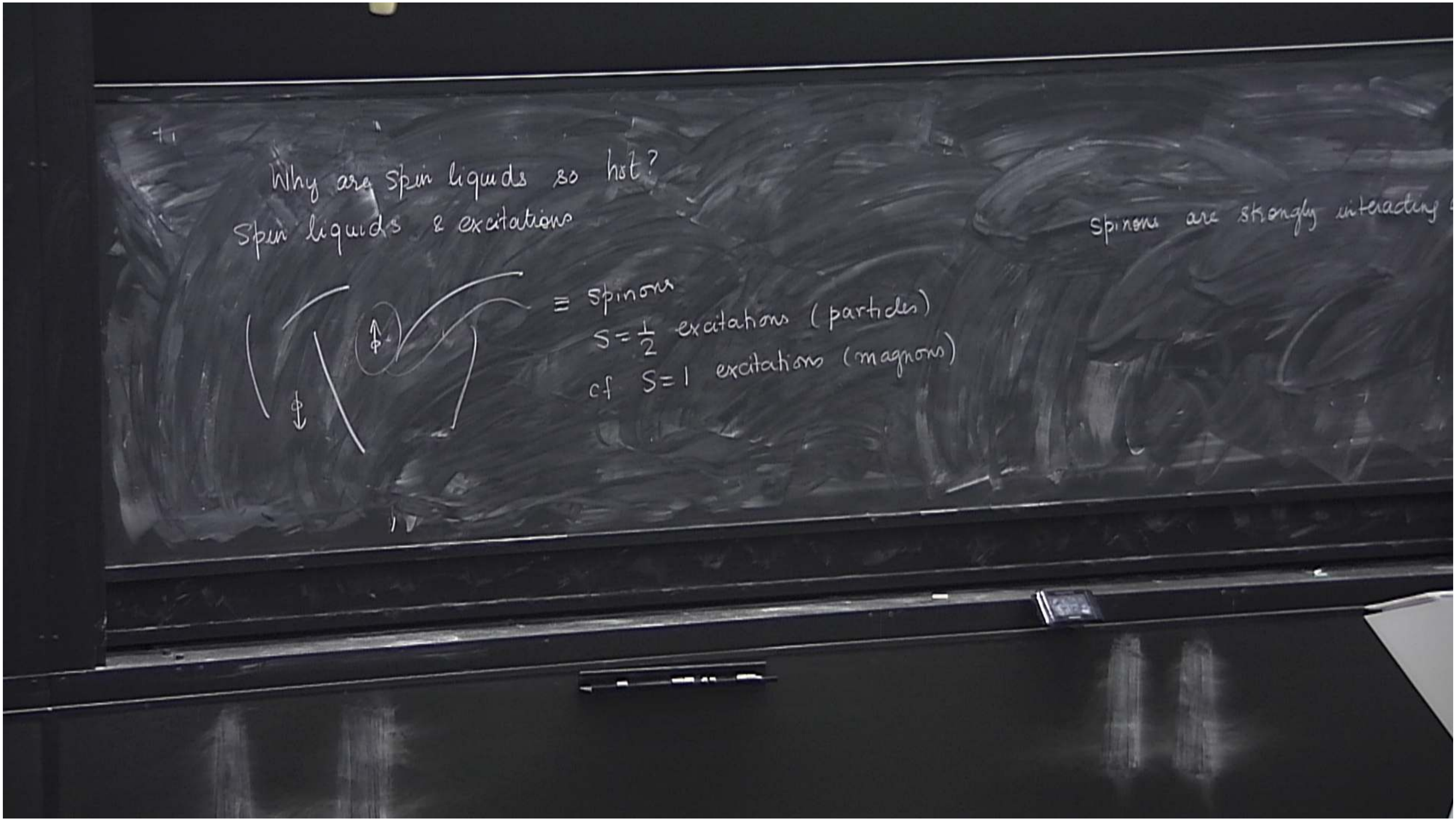

⇒ no staggered
SPIN
Spin Liquids in higher dim
* Mott insulator with 1 electron per unit cell
* absence of AF LRO
does not break rotational symm

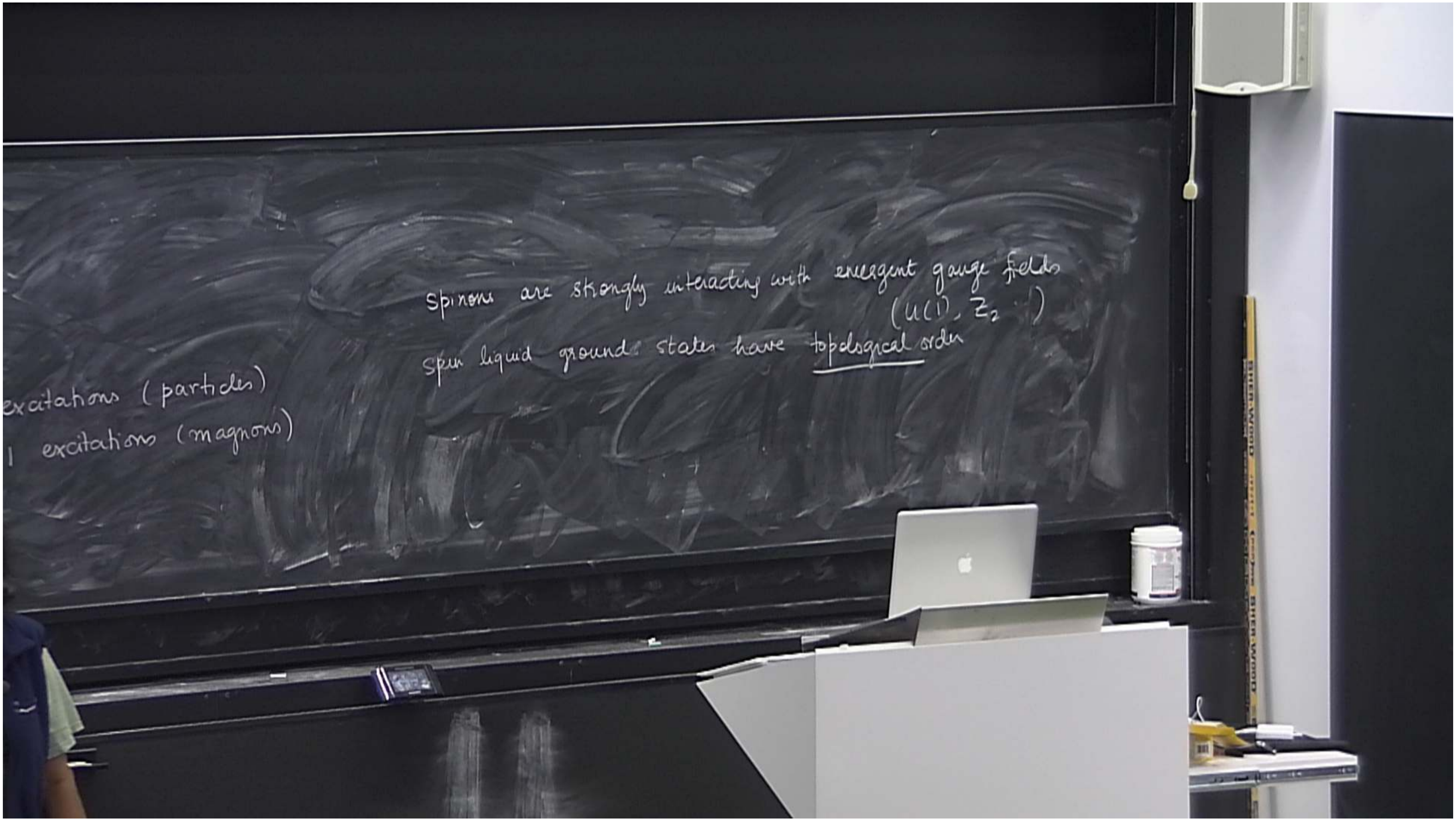


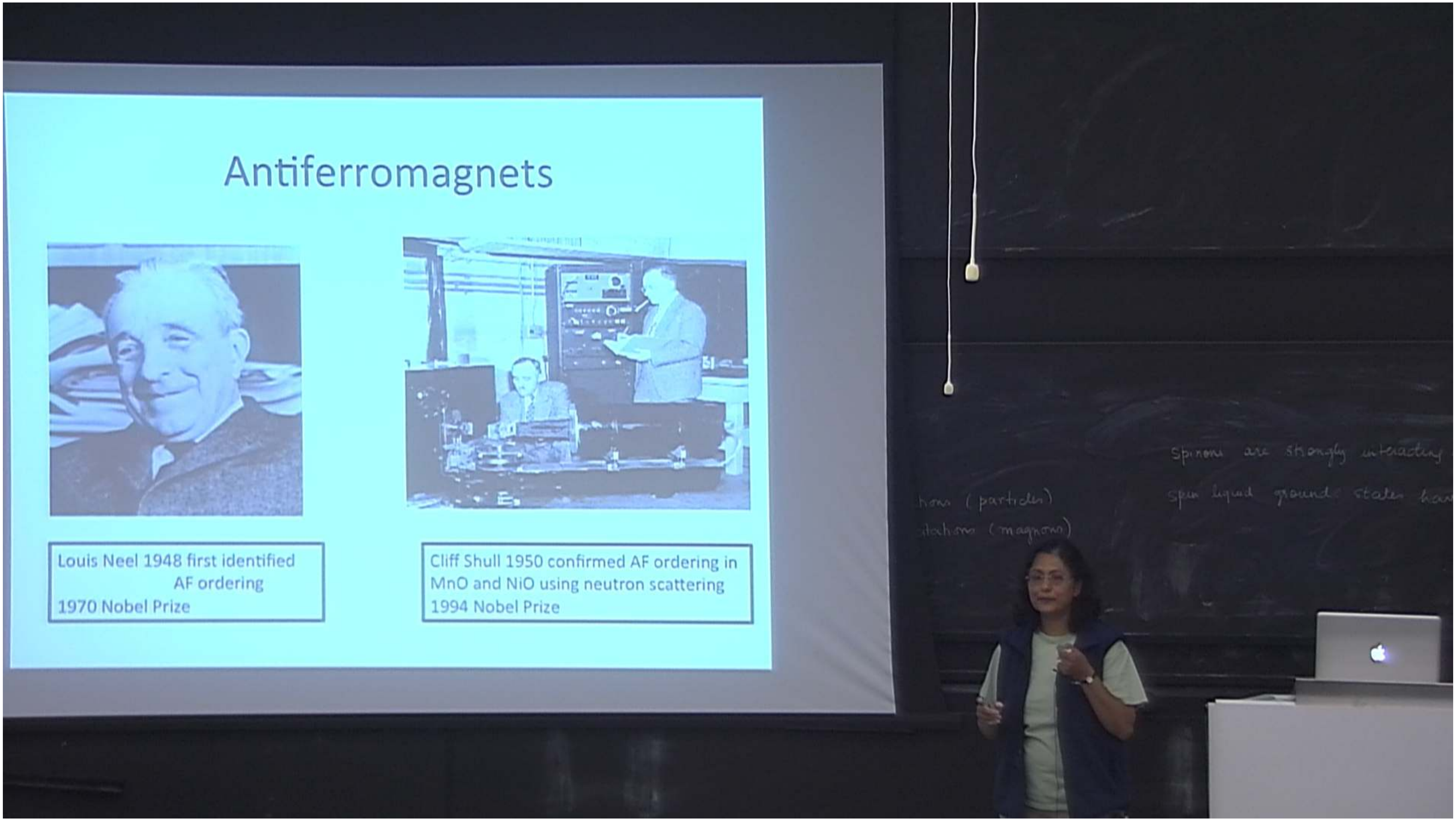
Spin Liquids in higher dim
* Mott insulator with 1 electron per unit cell
* absence of AF LRO
does not break rotational symm
→ frustrated lattices
Ising J > 0
?
frustrated!
Heisenberg

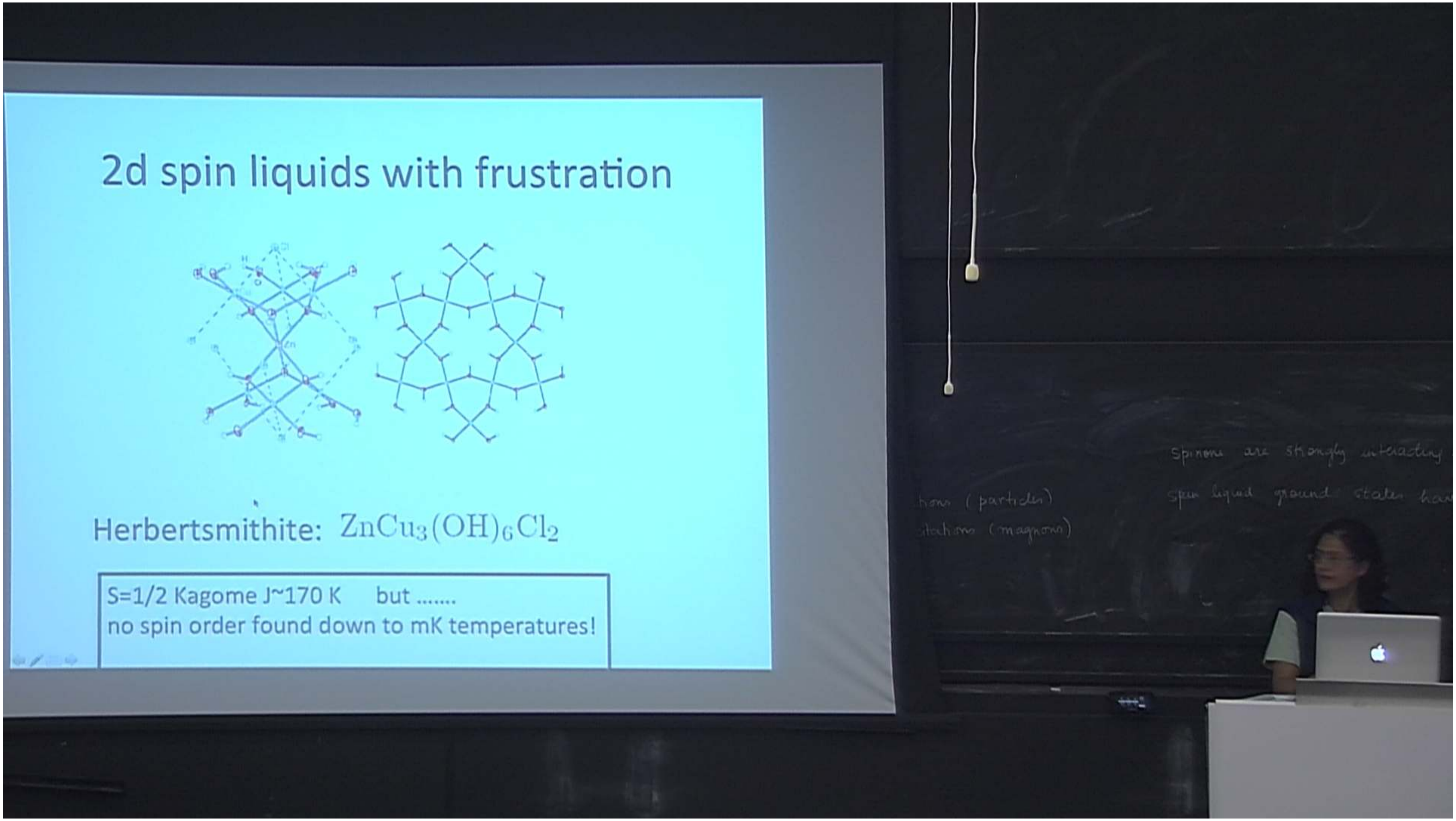

⇒ no staggered LRO
SPIN LIQUID states
Heisenberg
$\mathcal{H} = J_2 \sum P_{12} + \sum P_{1234}$
$P_{12} = (\bar{S}_1 \cdot \bar{S}_2 - 1)$
Δ lattice
$\mathcal{H} = J_z \sum_{\langle ij \rangle} S_i^z S_j^z + J_{xy} \sum [S_i^x S_j^x + S_i^y S_j^y]$
$\alpha = \frac{J_{xy}}{J_z}$
$m^+ \approx 0.1$

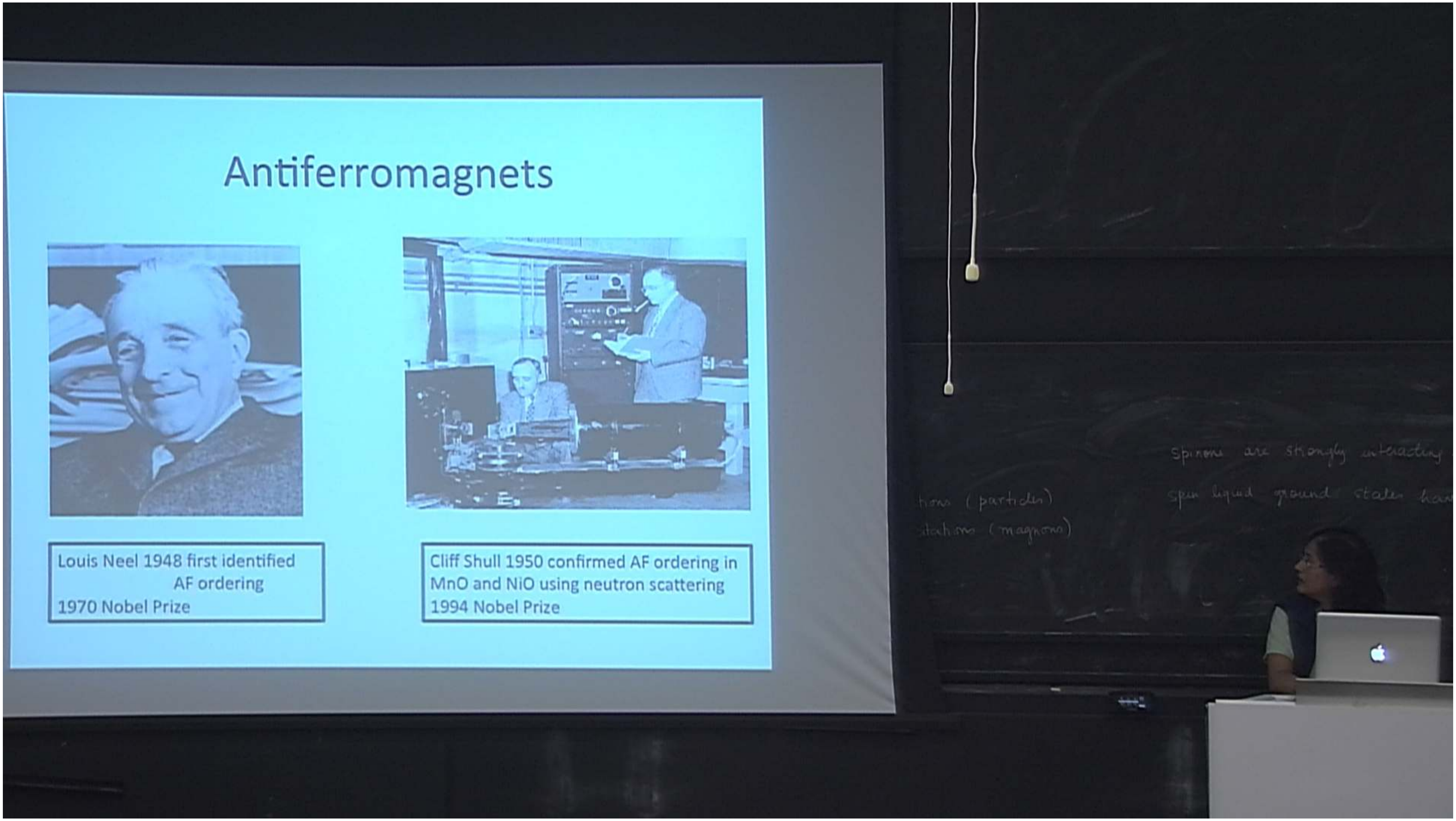

⇒ no staggered LRO
SPIN LIQUID states
0.1
ordered moment
spin liquid
Heis Δ
~0.1
J4 / J2
Heisenberg
H = J2 Σ P12 + J4 Σ P1234
P12 = (S1 · S2 − 1)
Δ lattice
H = Jz Σ Si^z Sj^z + Jxy Σ [Si^x Sj^x + Si^y Sj^y]
α = Jxy / Jz
(series expansion)
m+ ≈ 0.1


Why are spin liquids so hot?
Spin liquids & excitations
≡ spinons
S = 1/2 excitations (particles)
cf S = 1 excitations (magnons)
spinons are strongly interacting


spinons are strongly interacting with emergent gauge fields
(U(1), Z2 ...)
spin liquid ground states have topological order
excitations (particles)
excitations (magnons)


Antiferromagnets
Louis Neel 1948 first identified AF ordering
1970 Nobel Prize
Cliff Shull 1950 confirmed AF ordering in MnO and NiO using neutron scattering
1994 Nobel Prize


2d spin liquids with frustration
Zn
Herbertsmithite: $ZnCu_3(OH)_6Cl_2$
S=1/2 Kagome J~170 K but ....
no spin order found down to mK temperatures!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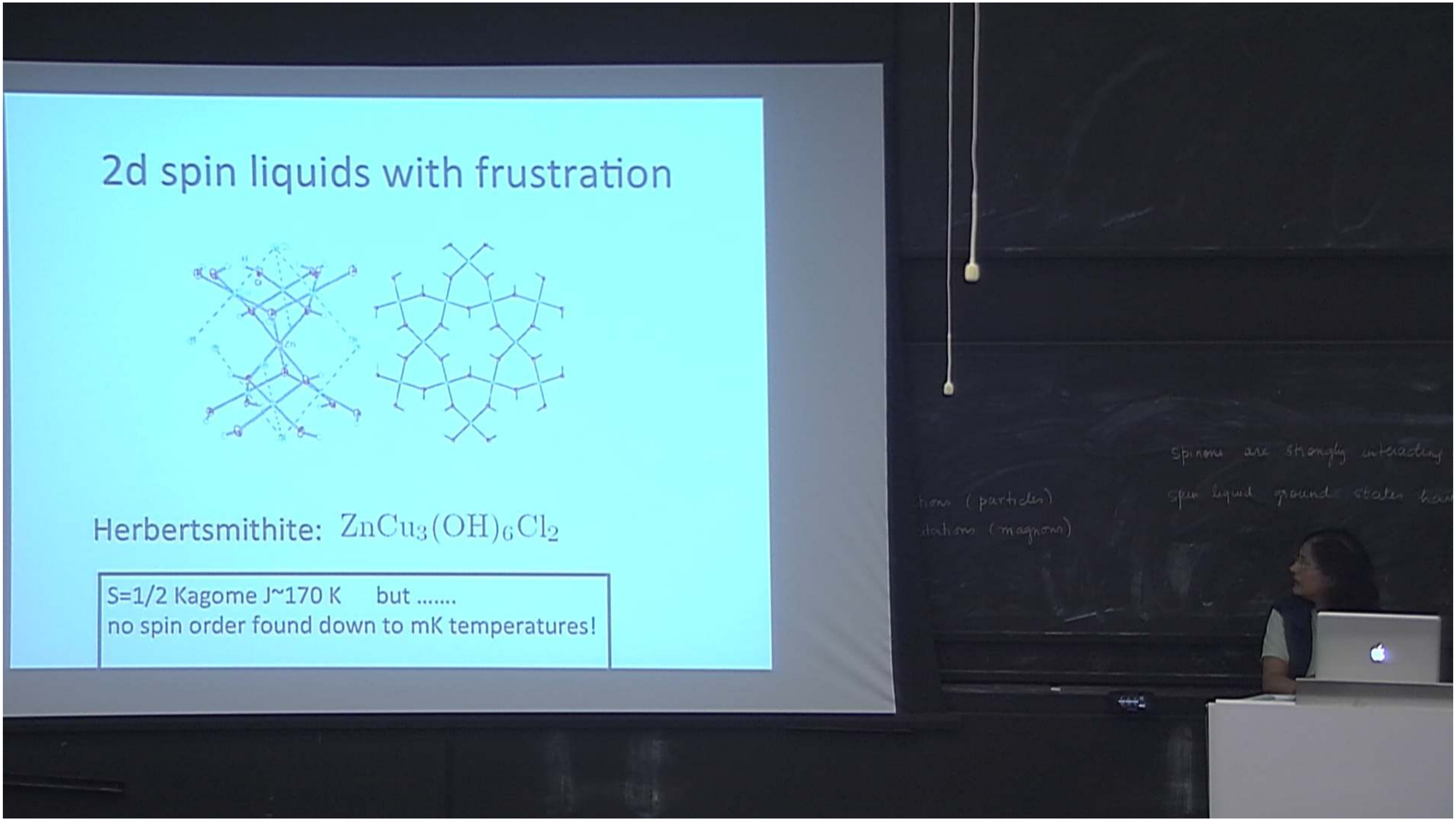
2d spin liquids with frustration
Herbertsmithite: ZnCu3(OH)6Cl2
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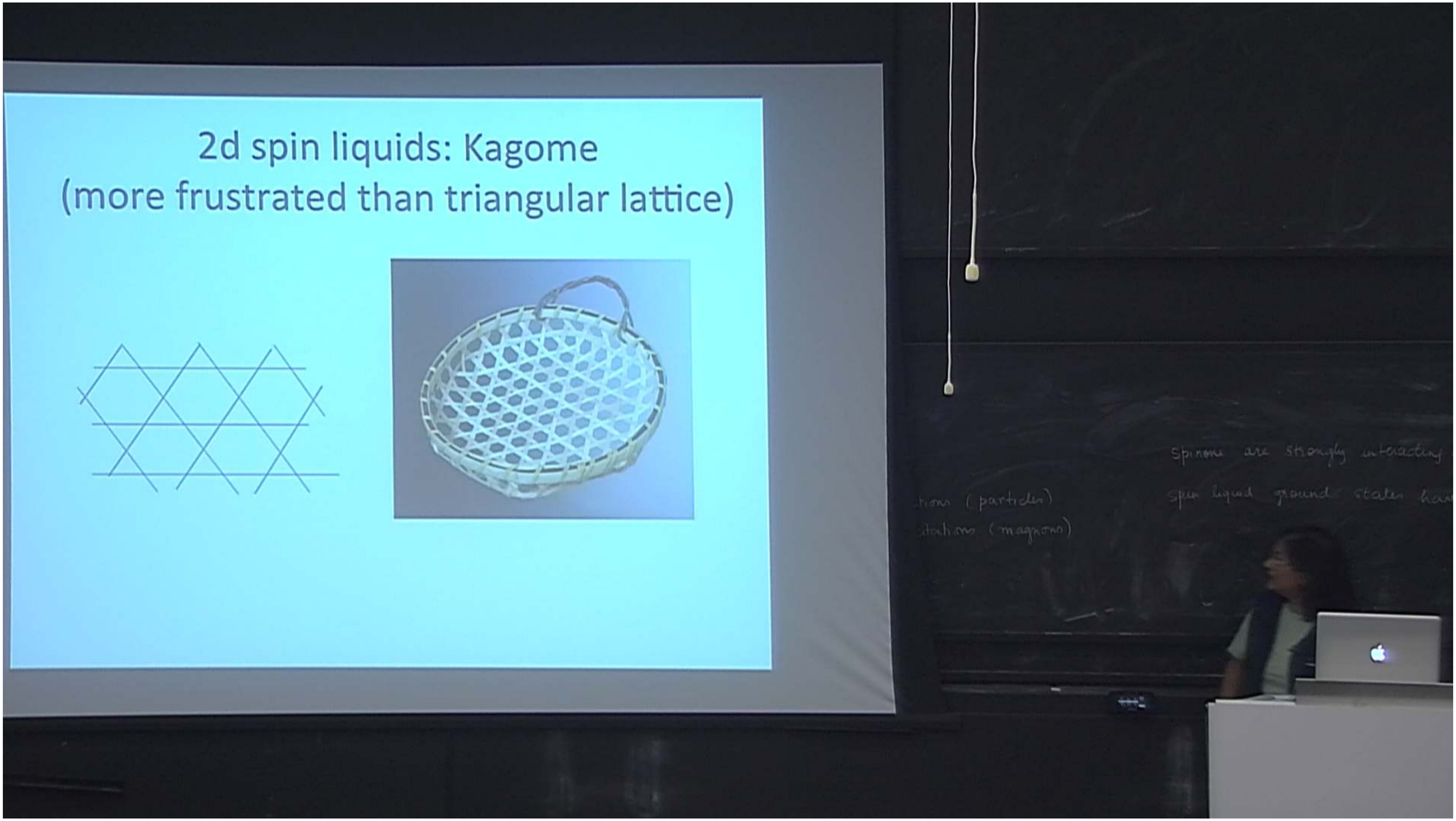
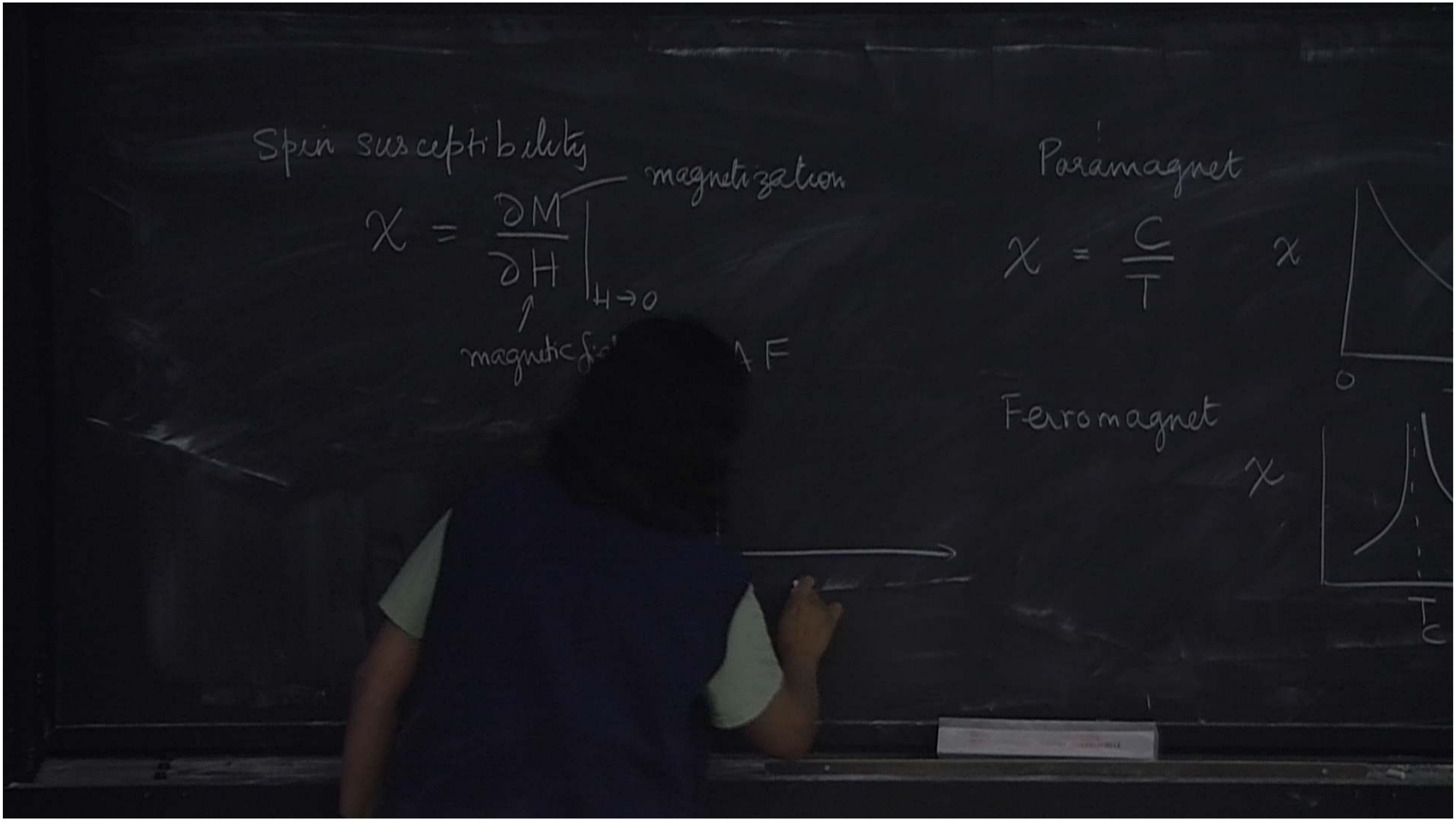
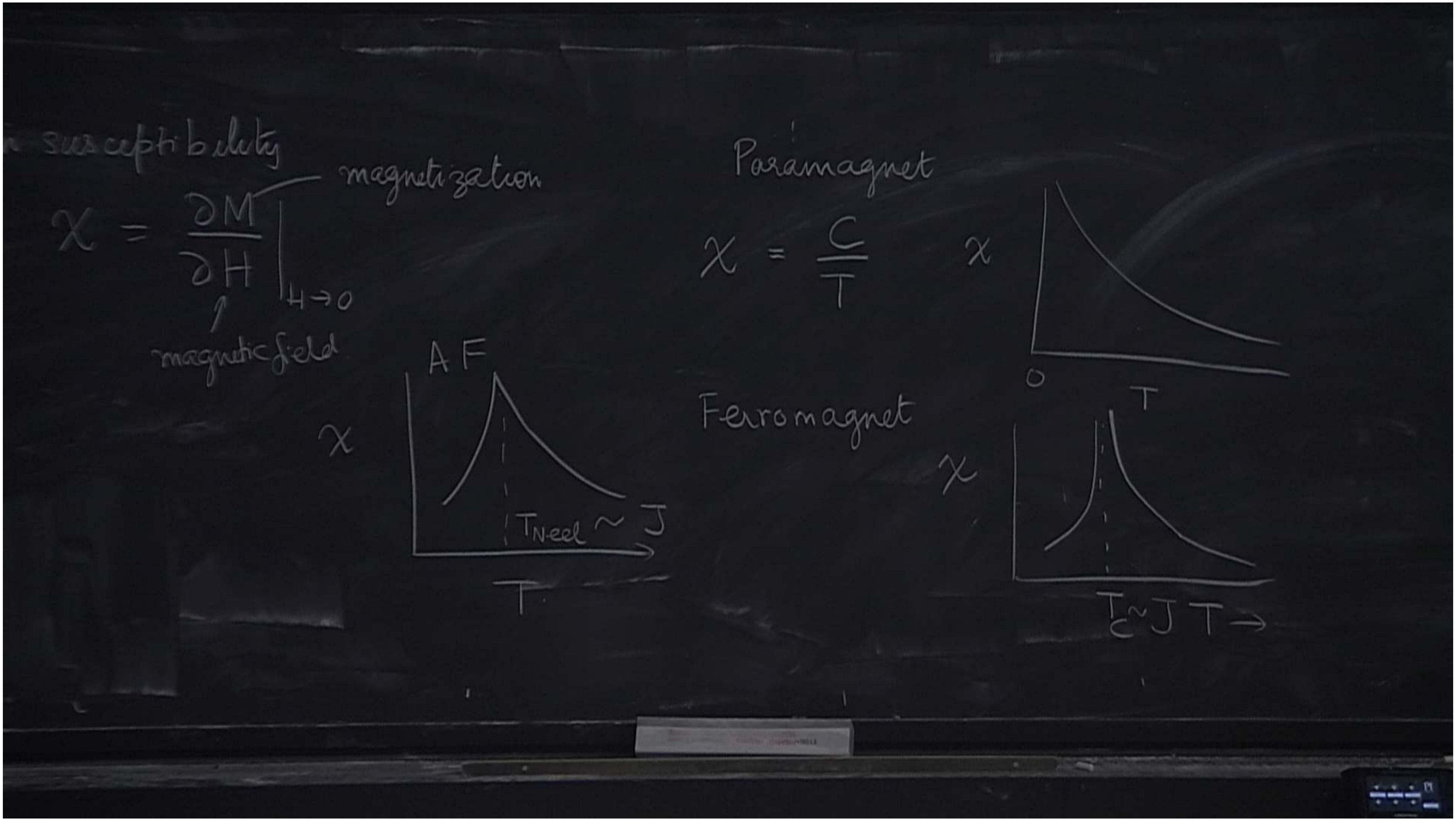
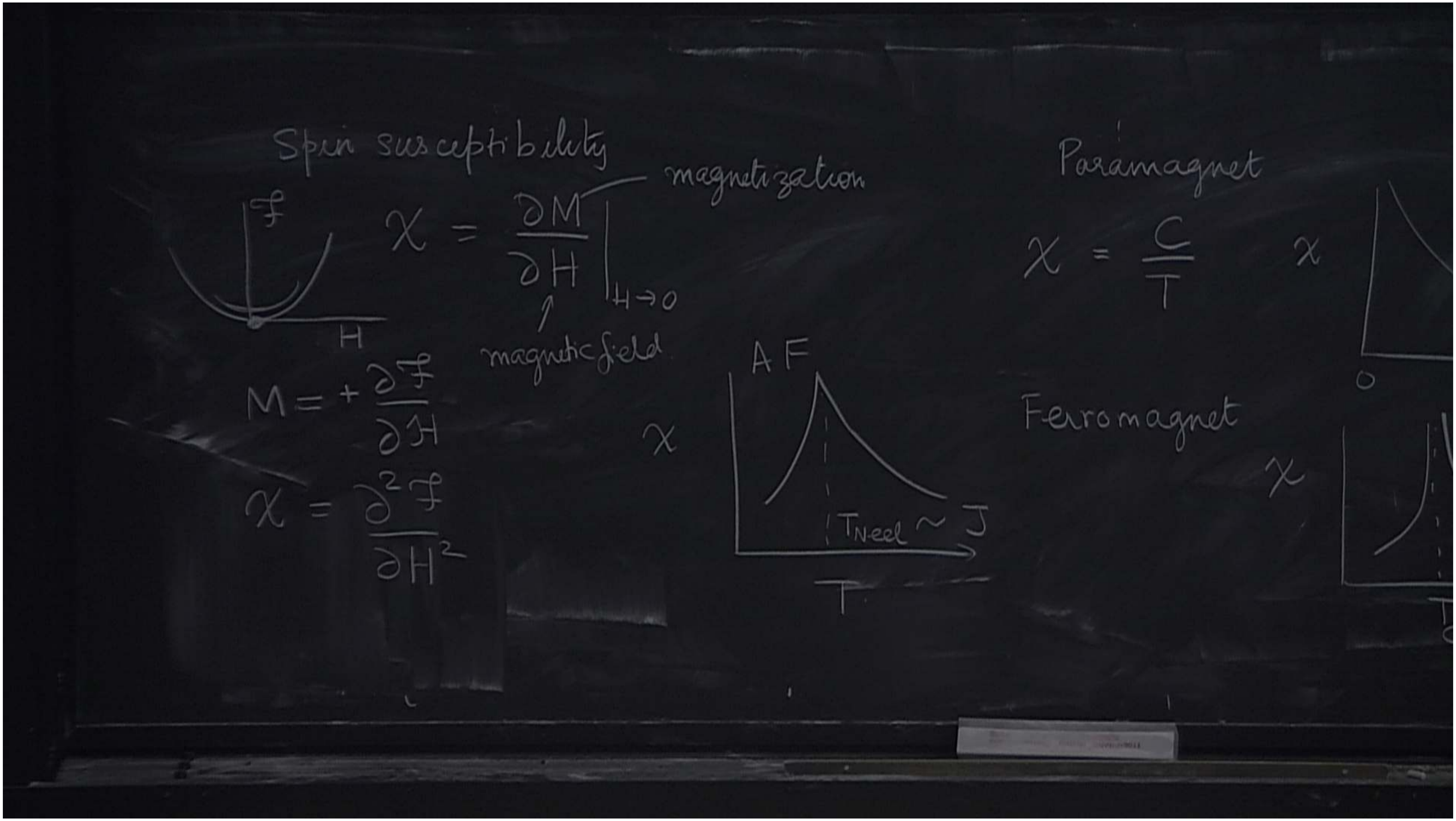

2d spin liquids: Kagome
(more frustrated than triangular lattice)

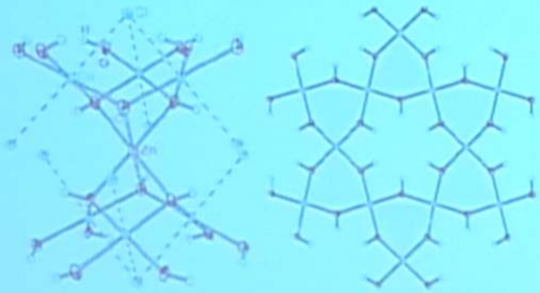

Spin susceptibility
χ = ∂M/∂H |H→0
magnetization
magnetic
Paramagnet
χ = C/T
χ
0
Ferromagnet
χ
Tc



susceptibility
magnetization
χ = ∂M/∂H |H→0
magnetic field
AF
χ
TNeel ~ J
T
Paramagnet
χ = C/T
χ
0
T
Ferromagnet
χ
Tc ~ J T →

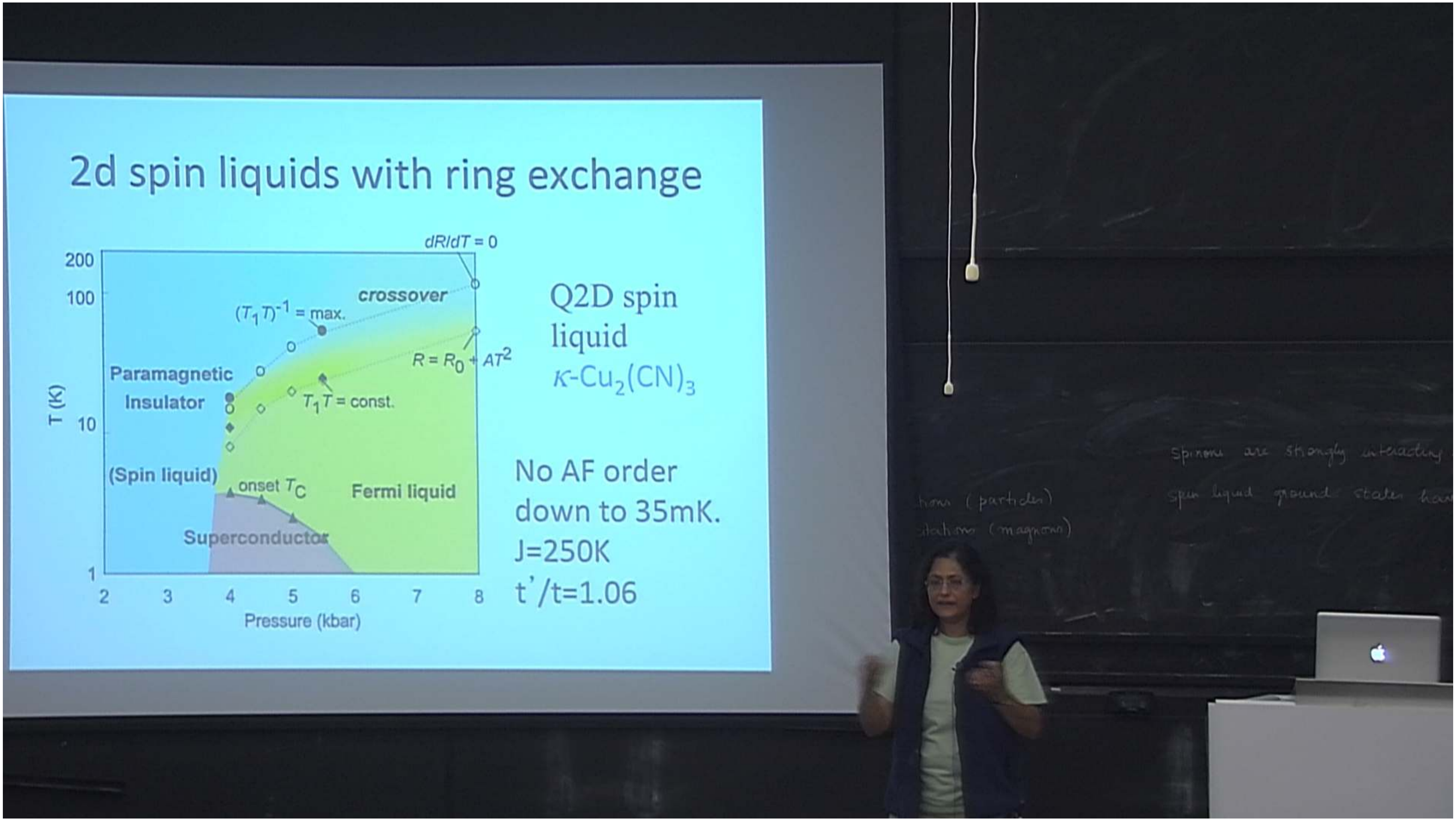

Spin susceptibility
magnetization
χ = ∂M/∂H |H→0
magnetic field
H
M = + ∂F/∂H
χ = ∂²F/∂H²
AF
χ
TNeel ~ J
T
Paramagnet
χ = C/T
χ
0
Ferromagnet
χ

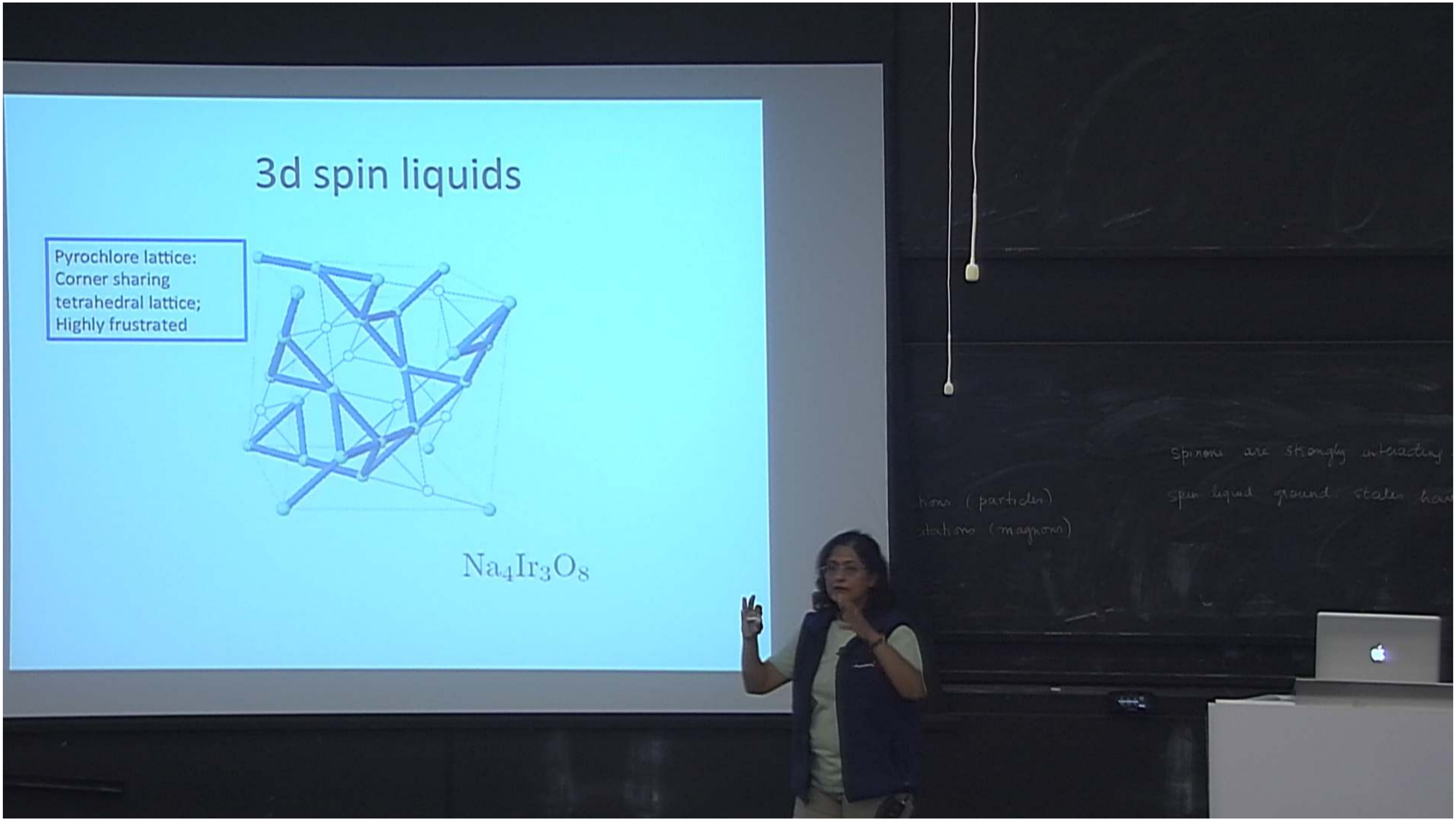
2d spin liquids with frustration

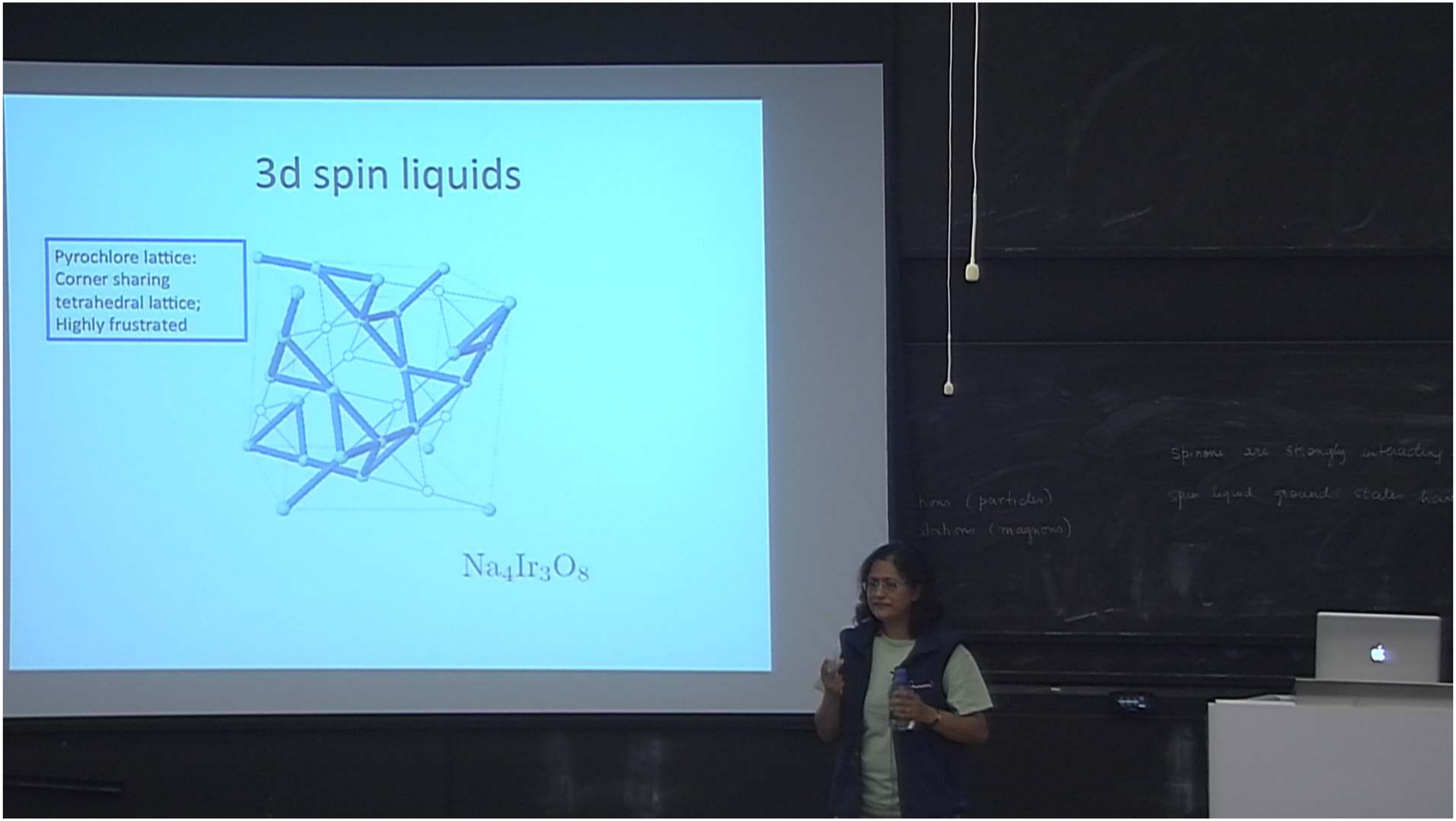


Herbertsmithite: $ZnCu_3(OH)_6Cl_2$

S=1/2 Kagome J~170 K   but ....
no spin order found down to mK temperatures!


2d spin liquids with ring exchange
dR/dT = 0
200
100
10
1
T (K)
crossover
(T1T)-1 = max.
R = R0 + AT2
Paramagnetic
Insulator
T1T = const.
(Spin liquid)
onset TC
Fermi liquid
Superconductor
2 3 4 5 6 7 8
Pressure (kbar)
Q2D spin liquid
κ-Cu2(CN)3
No AF order down to 35mK.
J=250K
t'/t=1.06


3d spin liquids
Pyrochlore lattice:
Corner sharing
tetrahedral lattice;
Highly frustrated
$Na_4Ir_3O_8$


3d spin liquids
Pyrochlore lattice:
Corner sharing
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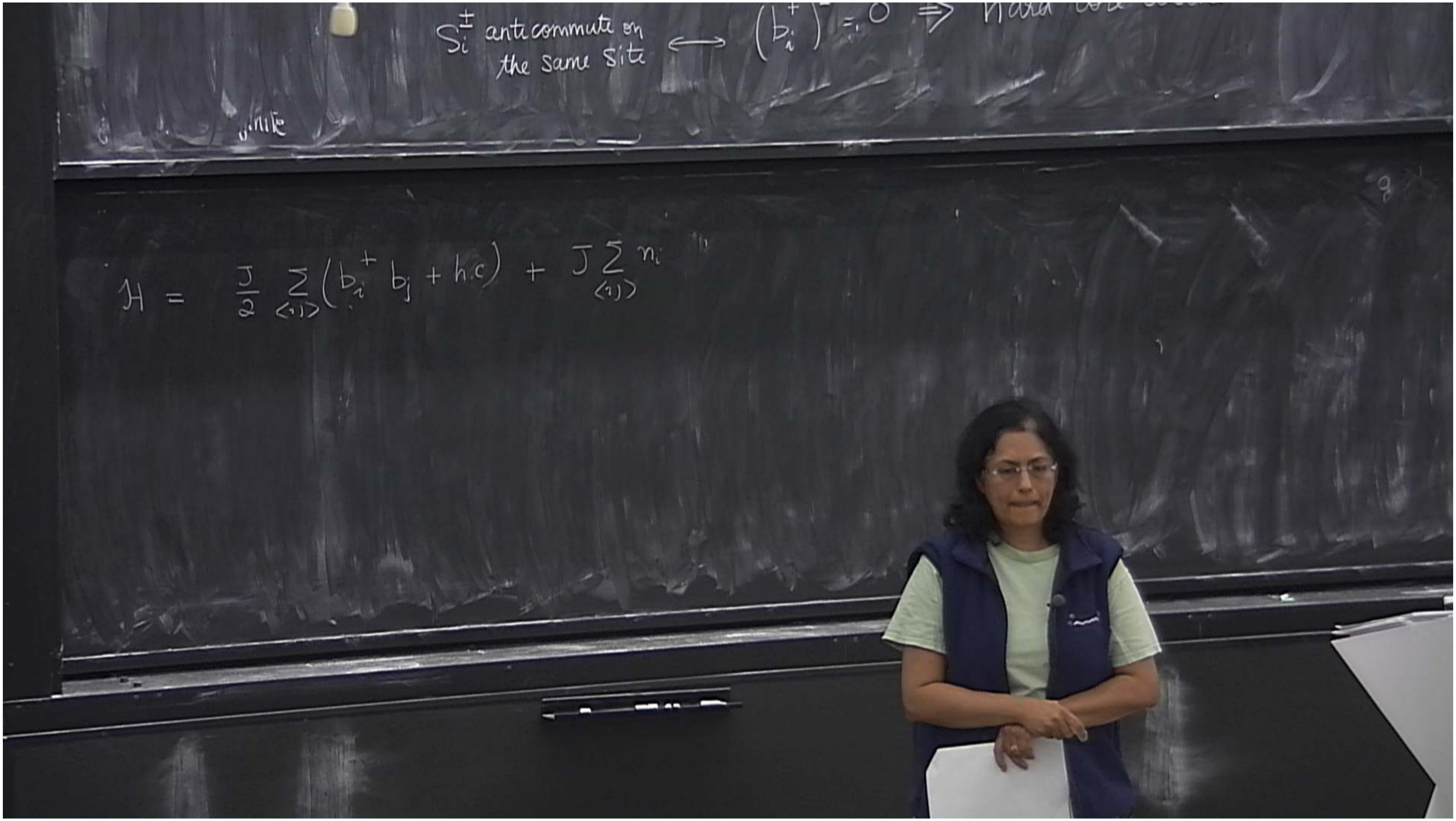
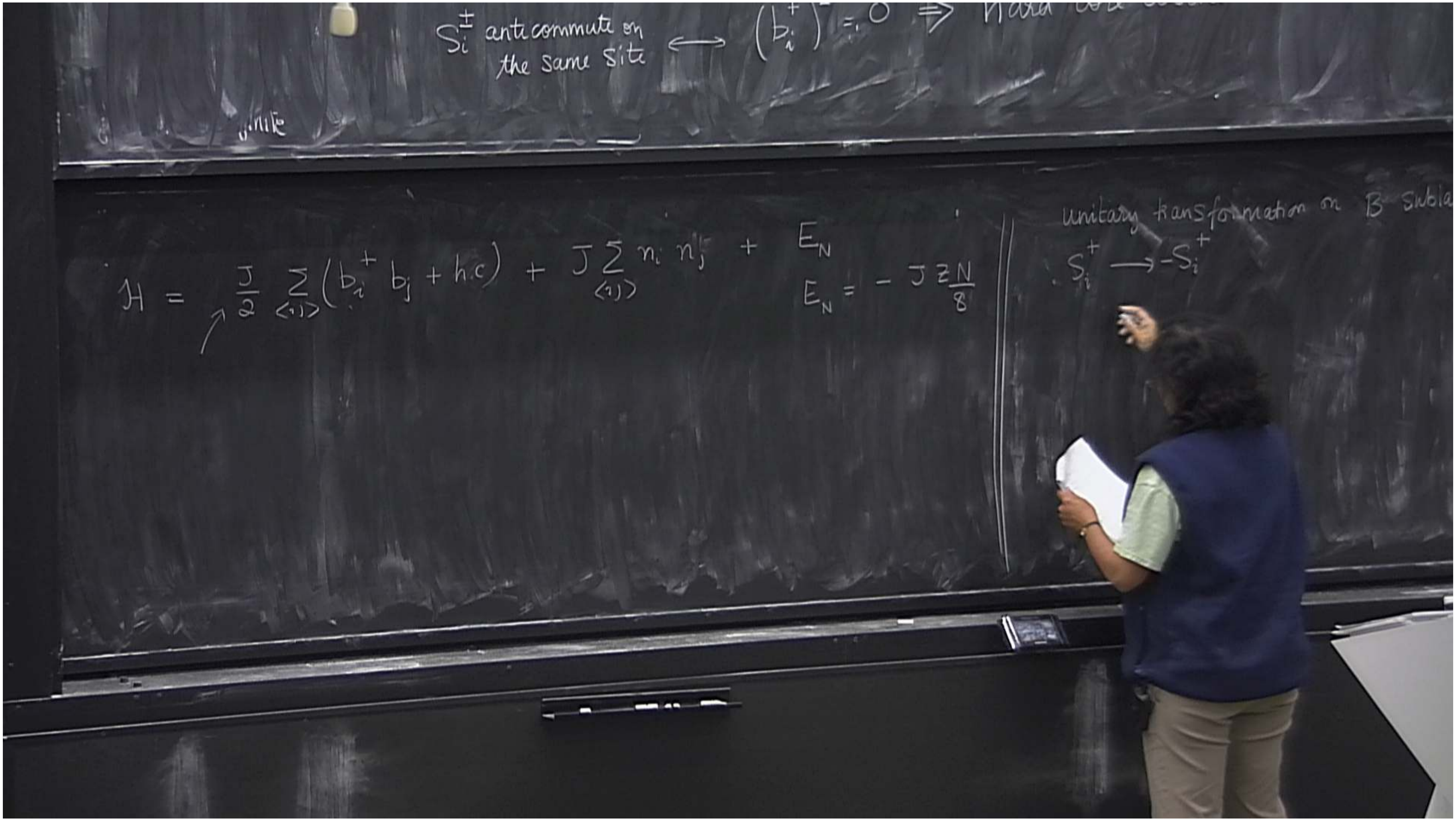
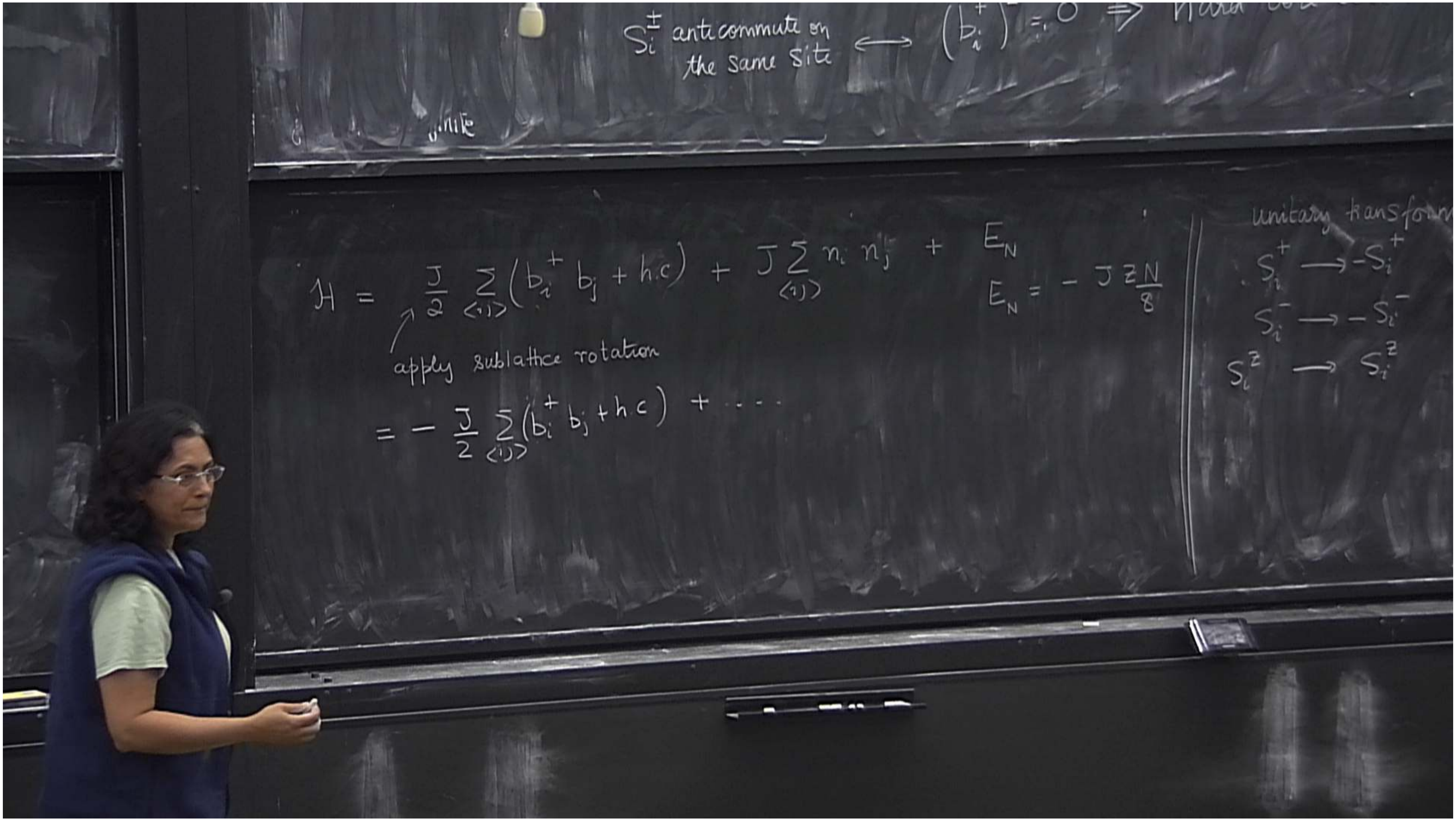
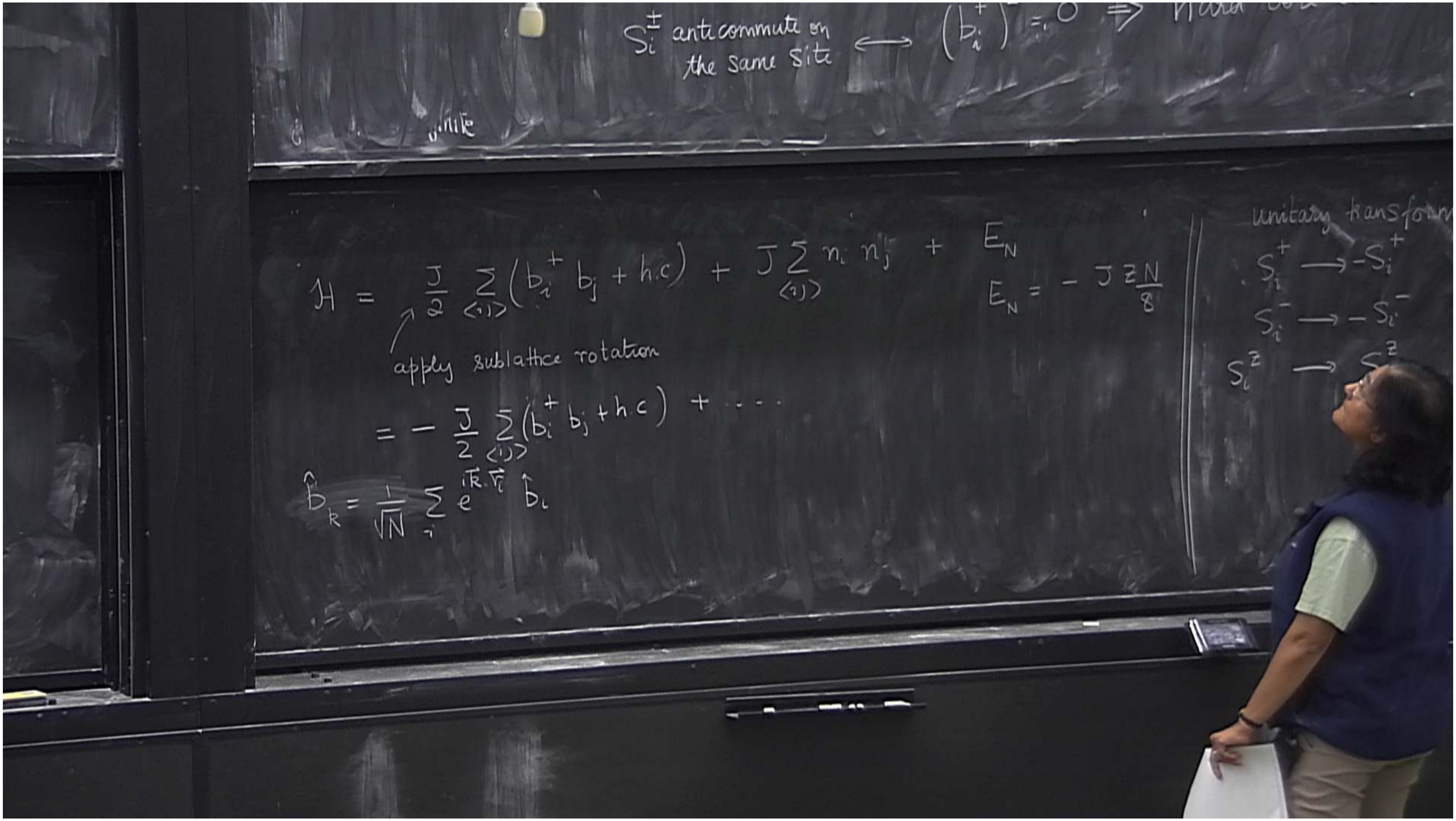
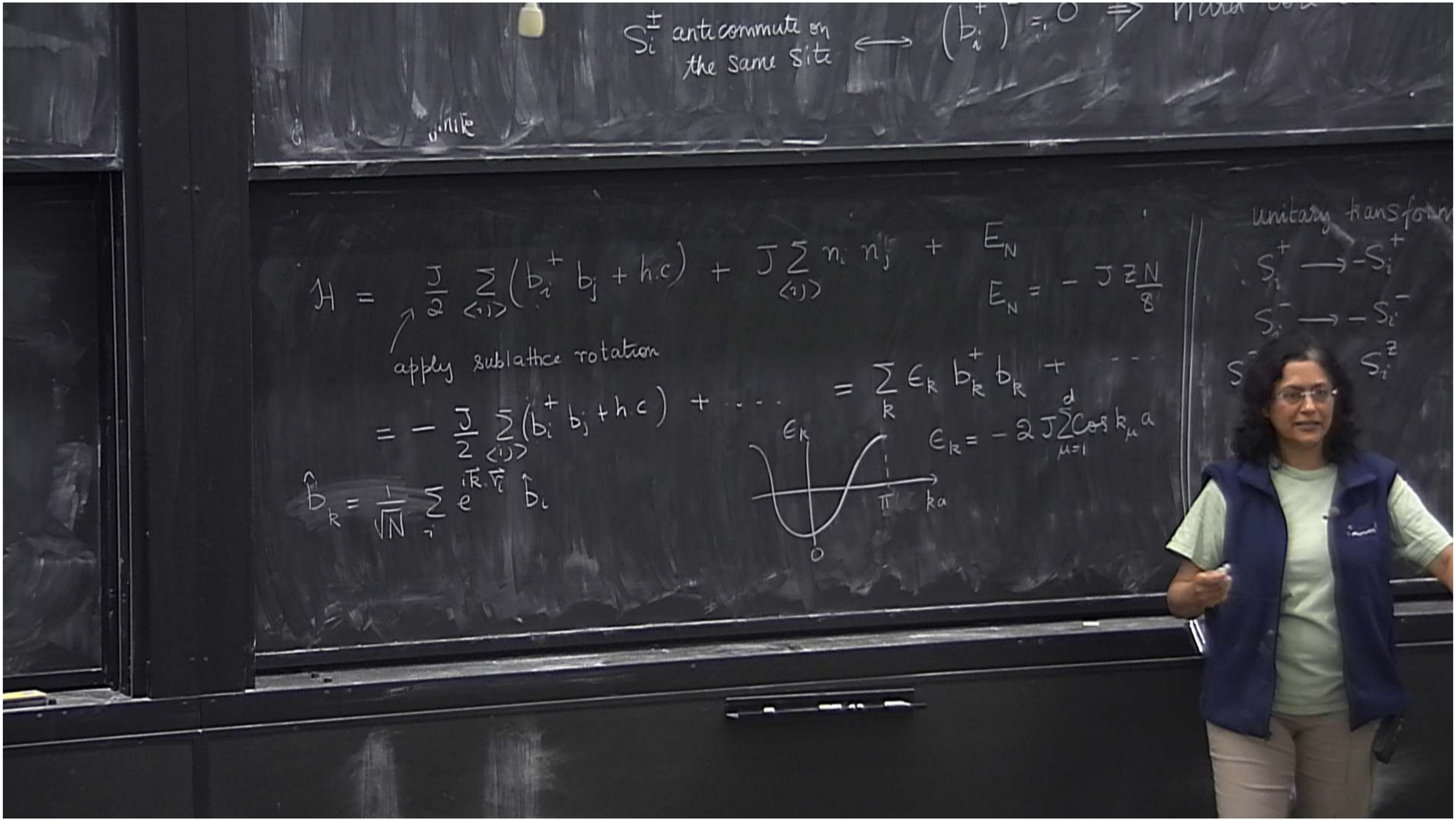

S_i^± anticommute on the same site ⟷ (b_i^+)^2 = 0 ⇒
H = J/2 Σ_<ij> (b_i^+ b_j + h.c) + J Σ_<ij> n_i


S_i^± anticommute on the same site ⟷ (b_i^+)^2 = 0
H = J/2 Σ_⟨ij⟩ (b_i^+ b_j + h.c) + J Σ_⟨ij⟩ n_i n_j + E_N
E_N = − J z N/8
Unitary transformation on B
S_i^+ ⟶ −S_i^+

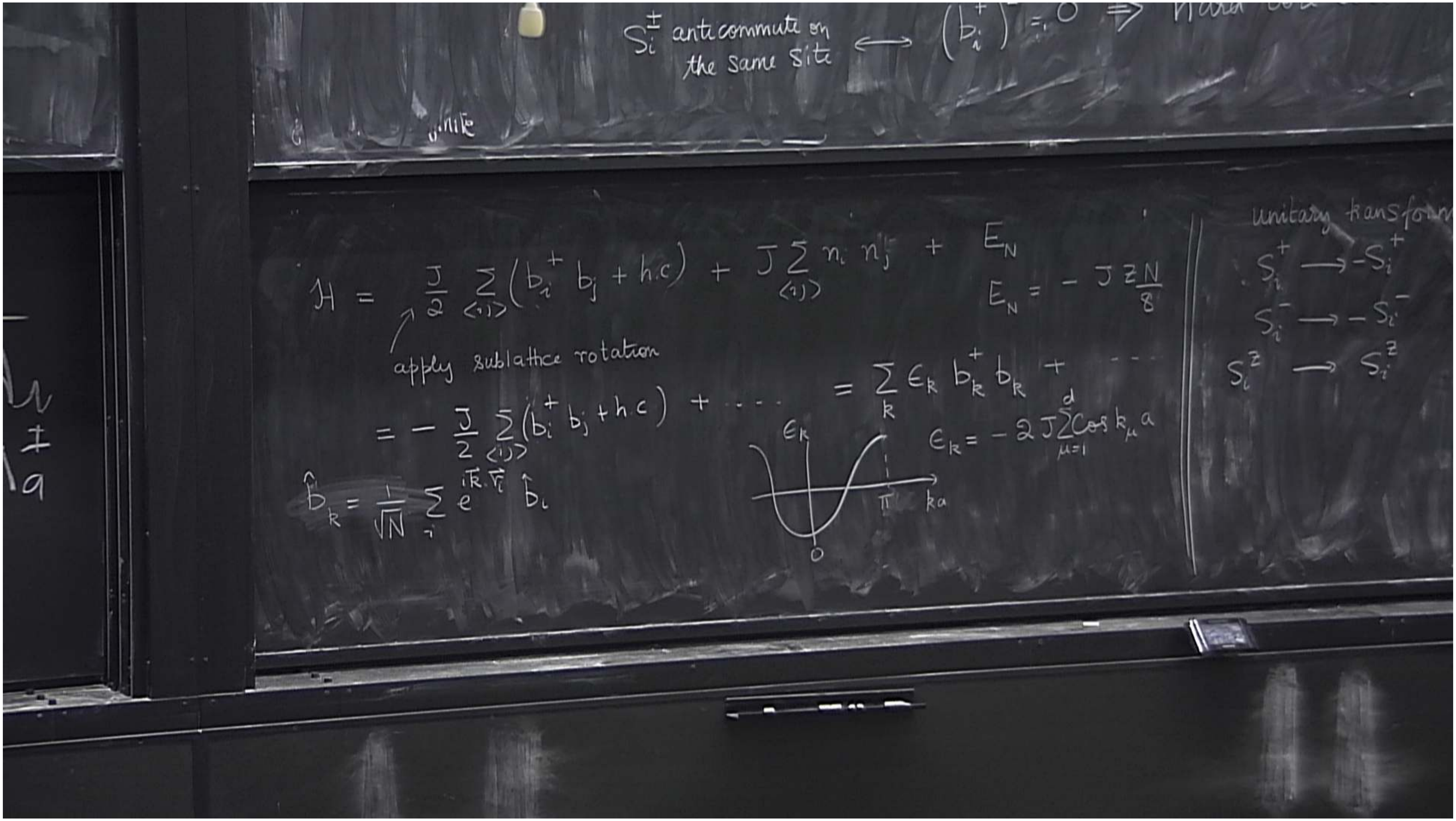

S_i^± anticommute on the same site ⟷ (b_i^+)^2 = 0
H = J/2 Σ_⟨ij⟩ (b_i^+ b_j + h.c) + J Σ_⟨ij⟩ n_i n_j + E_N
E_N = − J z N/8
apply sublattice rotation
= − J/2 Σ_⟨ij⟩ (b_i^+ b_j + h c) + ...
unitary transform
S_i^+ ⟶ −S_i^+
S_i^− ⟶ − S_i^−
S_i^z ⟶ S_i^z

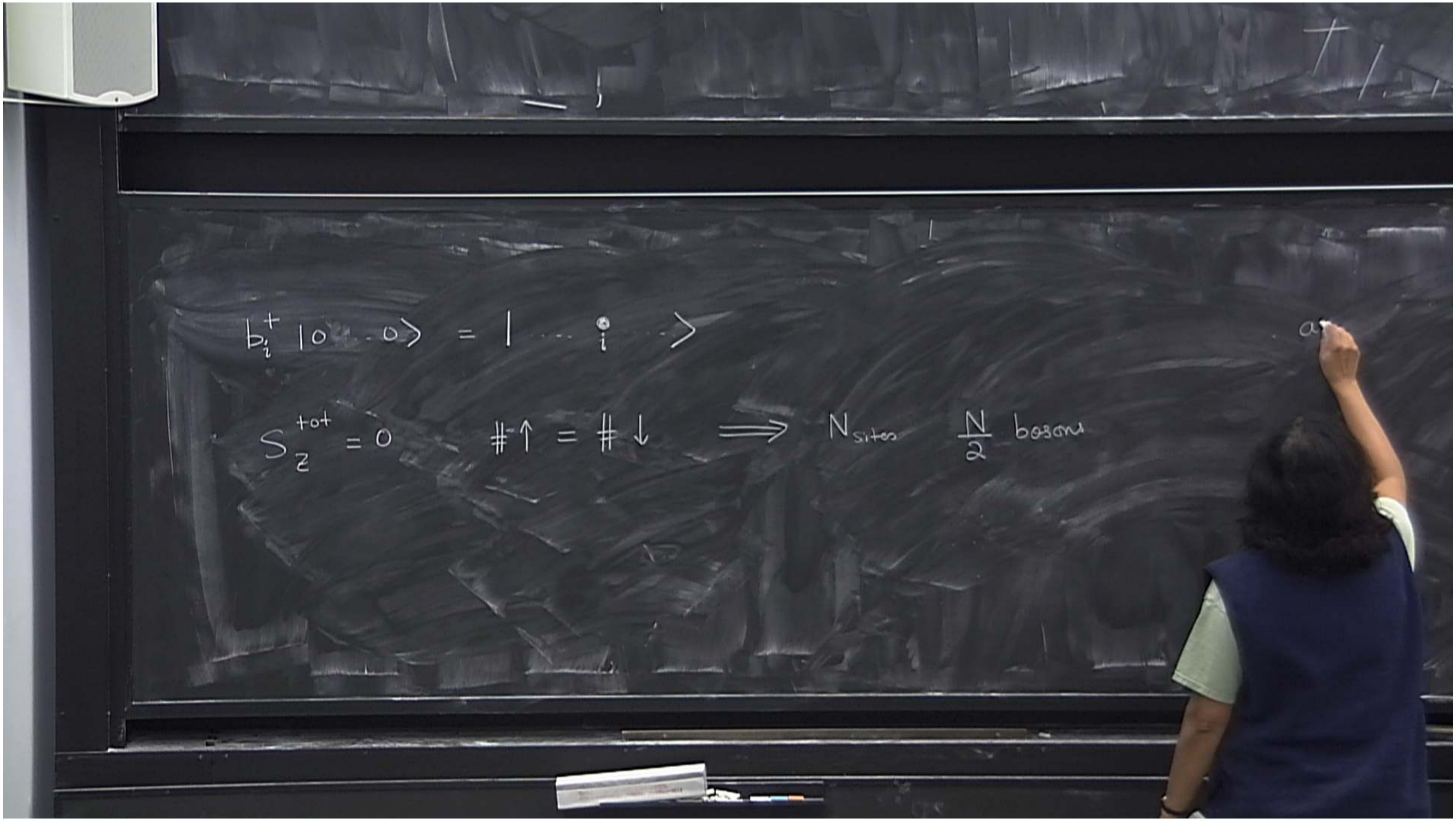


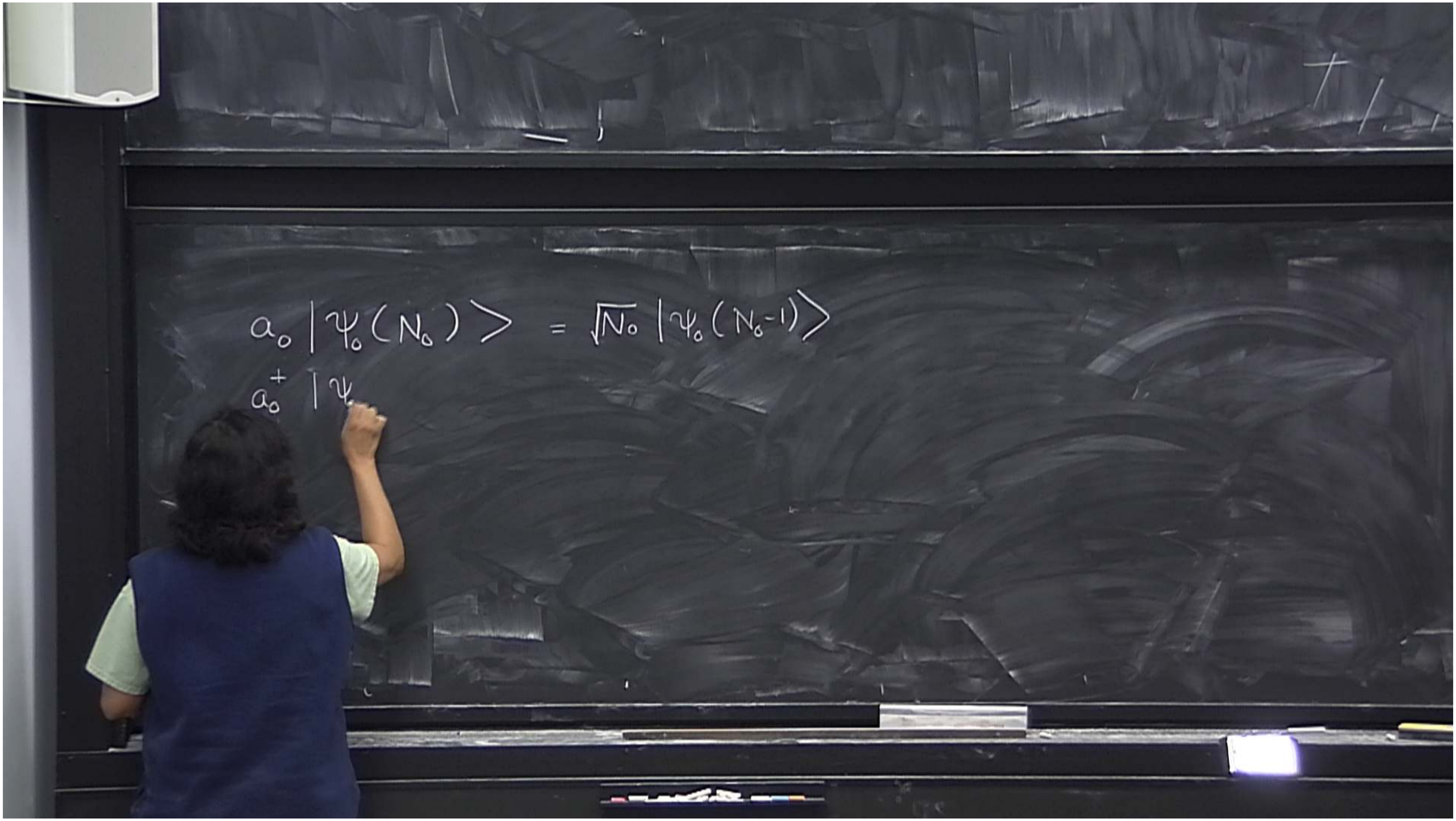
S_i^± anticommute on the same site ⟷ (b_i^+)^2 = 0 ⟹
H = J/2 Σ_⟨ij⟩ (b_i^+ b_j + h.c) + J Σ_⟨ij⟩ n_i n_j + E_N
E_N = − J z N/8
apply sublattice rotation
= − J/2 Σ_⟨ij⟩ (b_i^+ b_j + h c) + ...
b_k = 1/√N Σ_i e^{ik·r_i} b_i
unitary transform
S_i^+ ⟶ −S_i^+
S_i^- ⟶ − S_i^-
S_i^z ⟶

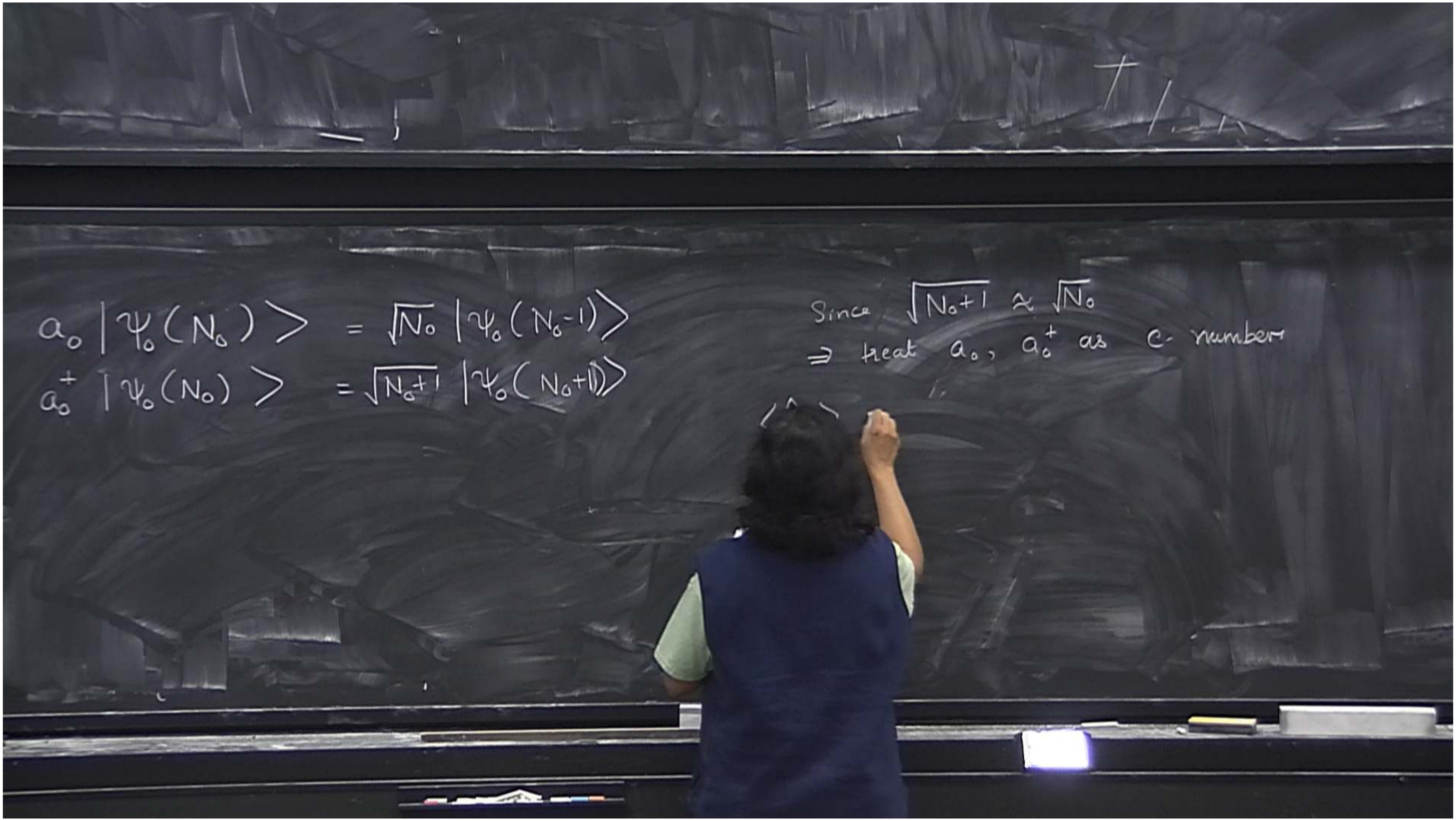


$S_i^\pm$ anticommute on the same site $\longleftrightarrow$ $(b_i^+)^2 = 0$
$\mathcal{H} = \frac{J}{2}\sum_{\langle ij\rangle}(b_i^+ b_j + h.c) + J\sum_{\langle ij\rangle} n_i n_j + E_N$
$E_N = -JzN/8$
apply sublattice rotation
$= -\frac{J}{2}\sum_{\langle ij\rangle}(b_i^+ b_j + h.c) + \cdots = \sum_k \epsilon_k b_k^+ b_k + \cdots$
$\epsilon_k = -2J\sum_{\mu=1}^{d}\cos k_\mu a$
$\hat{b}_k = \frac{1}{\sqrt{N}}\sum_i e^{i\vec{k}\cdot\vec{r}_i}\hat{b}_i$
ϵ_k
0
π
ka
unitary transform
$S_i^+ \longrightarrow -S_i^+$
$S_i^- \longrightarrow -S_i^-$
S_i^z

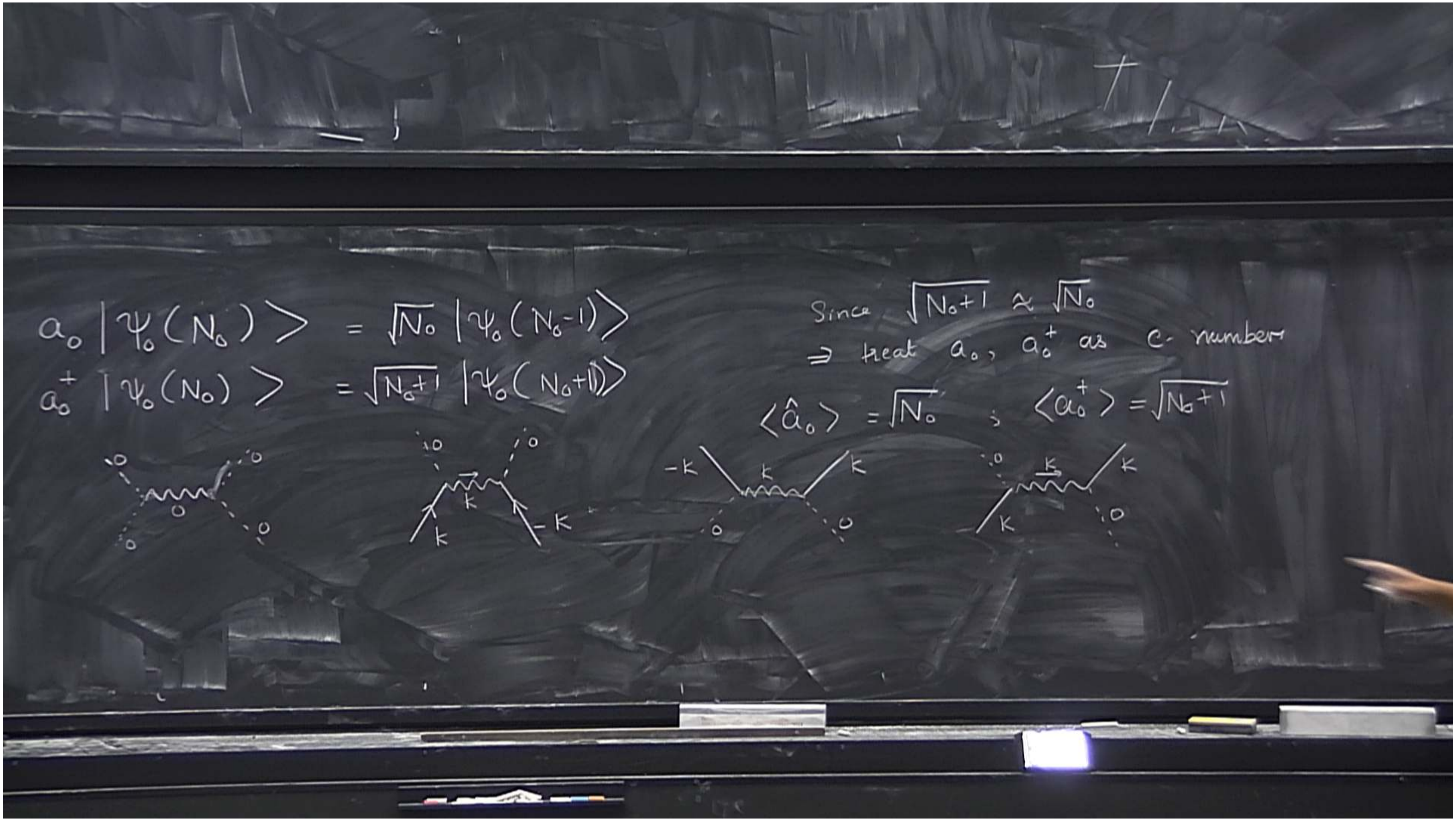

$S_i^\pm$ anticommute on the same site $\longleftrightarrow$ $(b_i^+)^2 = 0$ $\Rightarrow$
$\mathcal{H} = \frac{J}{2}\sum_{\langle ij\rangle}(b_i^+ b_j + h.c) + J\sum_{\langle ij\rangle} n_i n_j + E_N$
$E_N = -JZ\frac{N}{8}$
apply sublattice rotation
$= -\frac{J}{2}\sum_{\langle ij\rangle}(b_i^+ b_j + h.c) + \cdots = \sum_k \epsilon_k b_k^+ b_k + \cdots$
$\hat{b}_k = \frac{1}{\sqrt{N}}\sum_i e^{i\vec{k}\cdot\vec{r}_i}\hat{b}_i$
$\epsilon_k = -2J\sum_{\mu=1}^{d}\cos k_\mu a$
ϵ_k
ka
π
0
unitary transform
$S_i^+ \longrightarrow -S_i^+$
$S_i^- \longrightarrow -S_i^-$
$S_i^z \longrightarrow S_i^z$

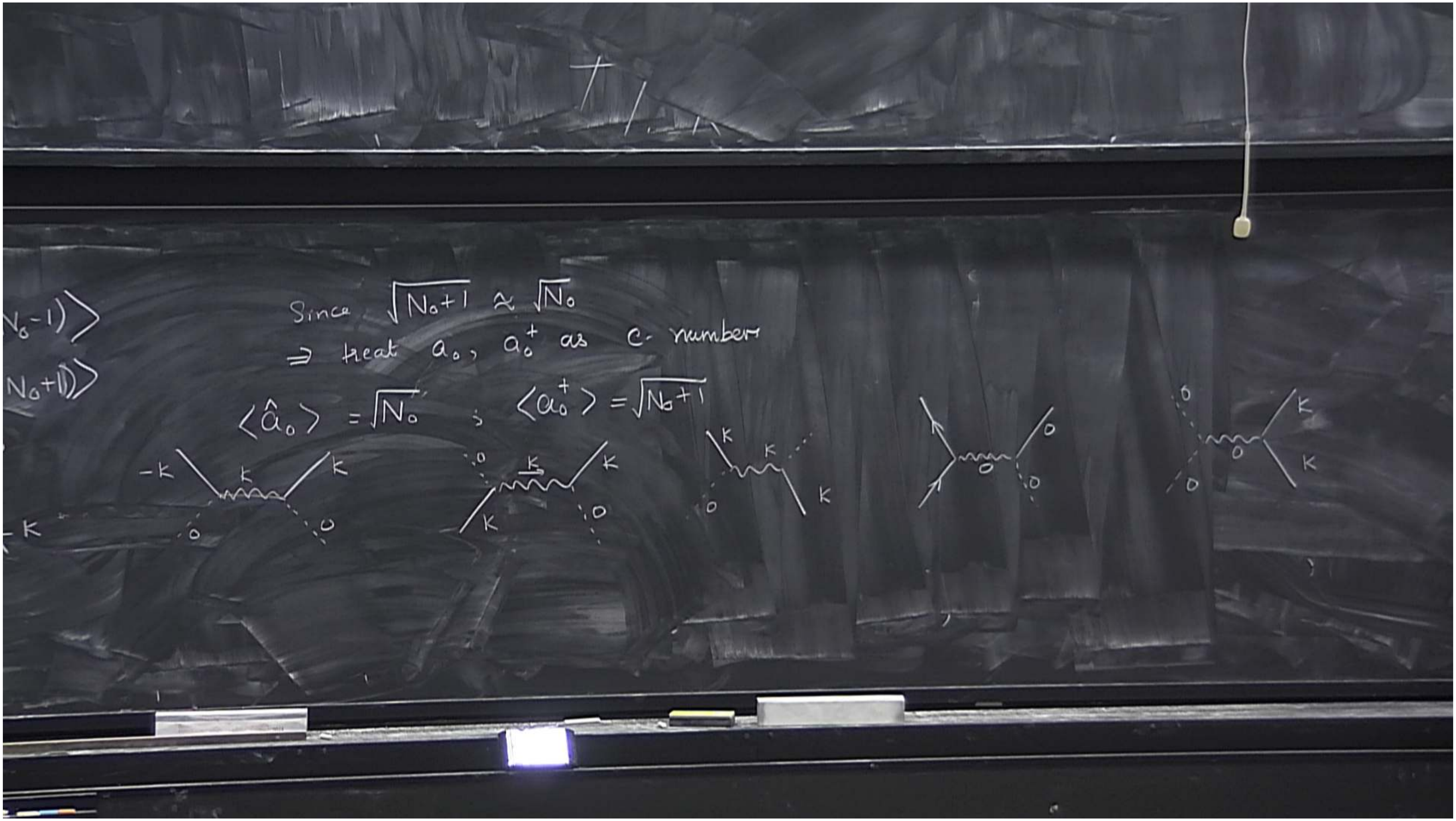

$b_i^+ |0 \cdots 0\rangle = |\cdots \underset{i}{\bullet} \cdots\rangle$
$S_z^{tot} = 0 \quad \#\uparrow = \#\downarrow \Longrightarrow N_{sites} \quad \frac{N}{2}$ bosons

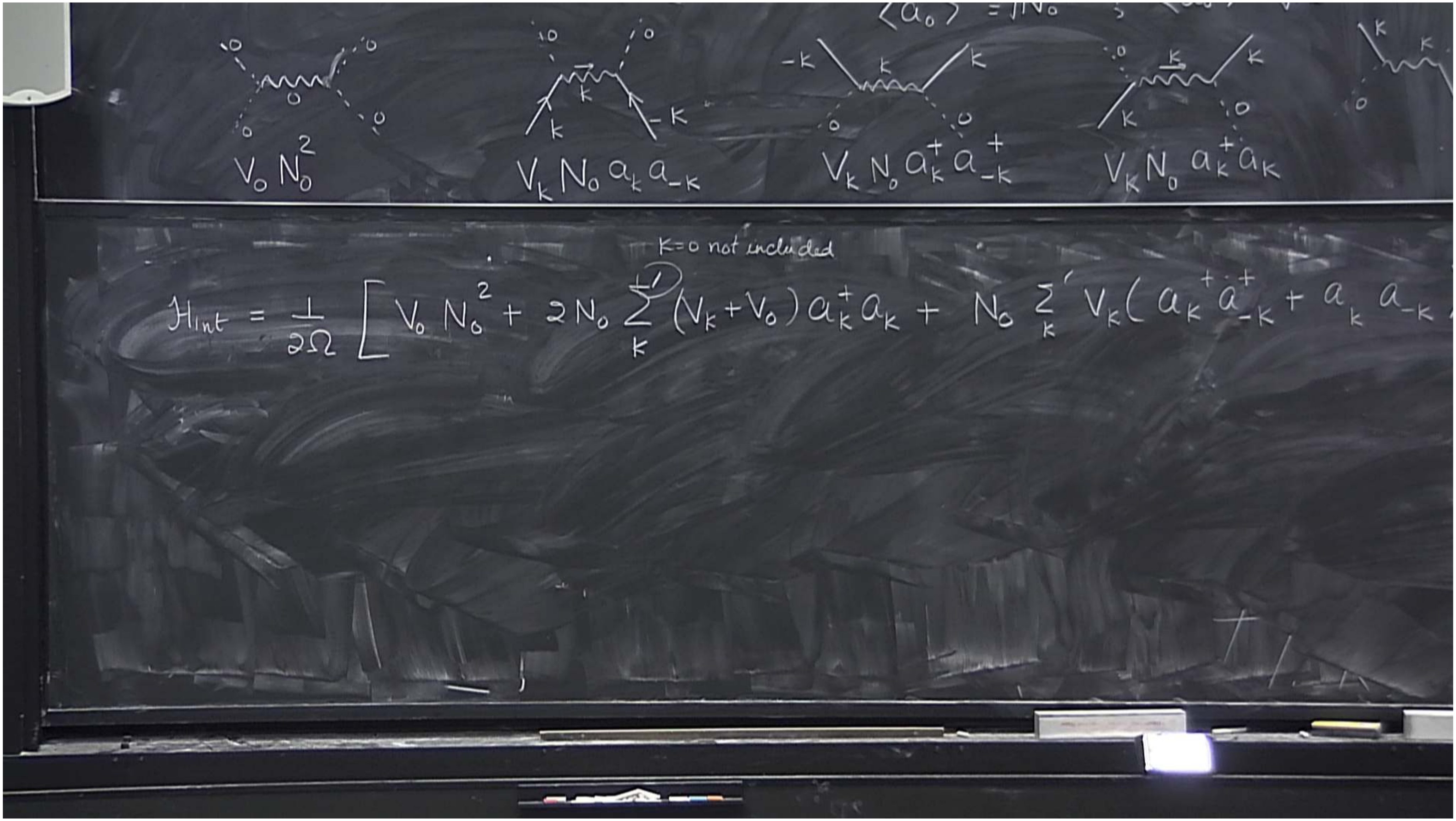

$a_0 |\psi_0(N_0)\rangle = \sqrt{N_0}\, |\psi_0(N_0-1)\rangle$
$a_0^{+} |\psi$

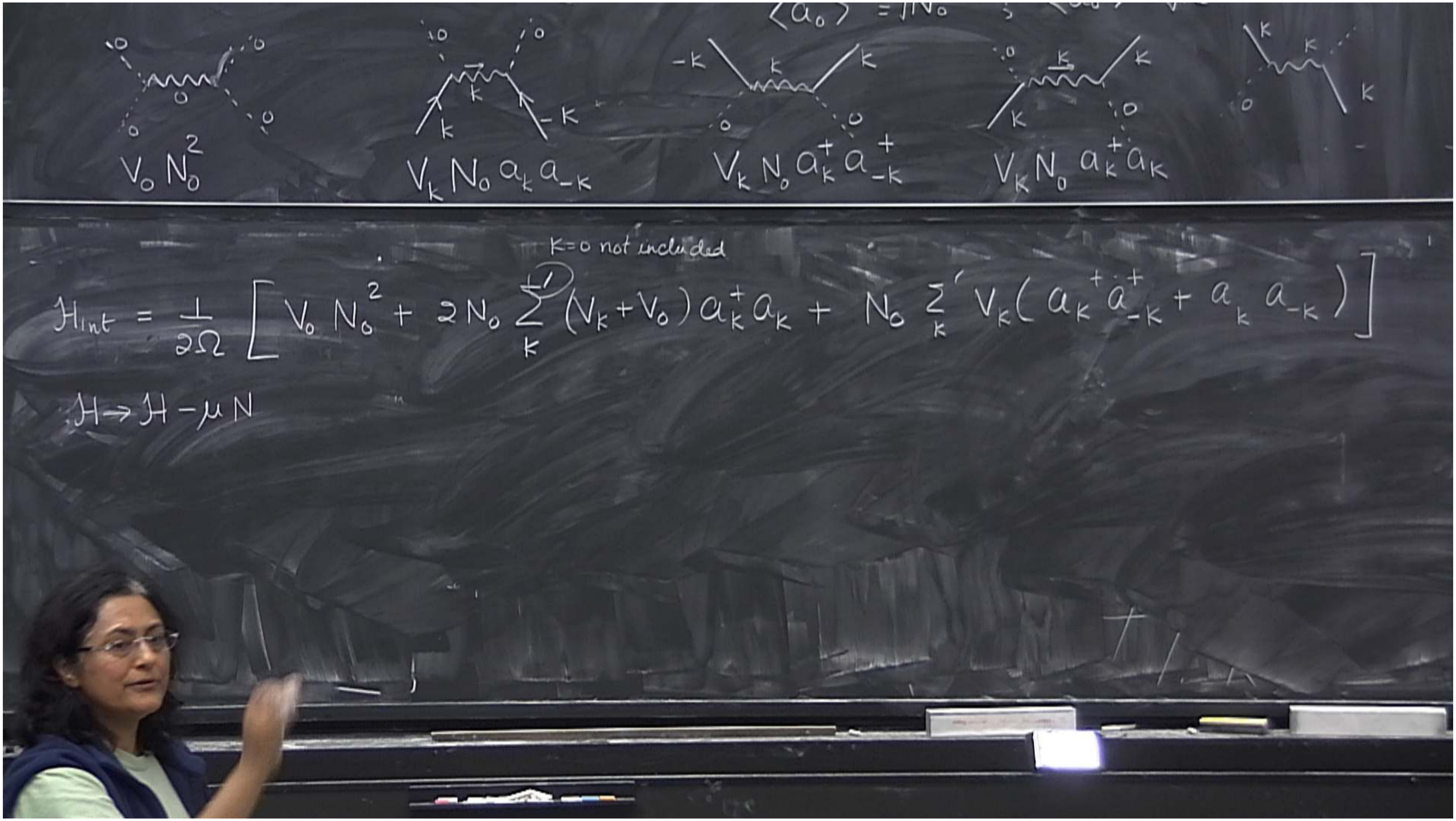


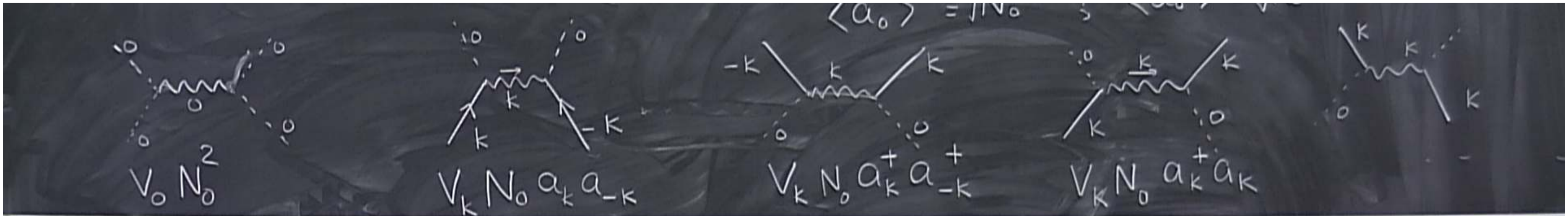
$a_0 |\psi_0(N_0)\rangle = \sqrt{N_0}\, |\psi_0(N_0-1)\rangle$
$a_0^+ |\psi_0(N_0)\rangle = \sqrt{N_0+1}\, |\psi_0(N_0+1)\rangle$
Since $\sqrt{N_0+1} \approx \sqrt{N_0}$
$\Rightarrow$ treat a_0, a_0^+ as c-number


a₀ |ψ₀(N₀)⟩ = √N₀ |ψ₀(N₀−1)⟩
a₀⁺ |ψ₀(N₀)⟩ = √(N₀+1) |ψ₀(N₀+1)⟩
Since √(N₀+1) ≈ √N₀
⇒ treat a₀, a₀⁺ as c-number
⟨â₀⟩ = √N₀ ; ⟨a₀⁺⟩ = √(N₀+1)


Since $\sqrt{N_0+1} \approx \sqrt{N_0}$
$\Rightarrow$ treat a_0, a_0^+ as c-number
$\langle \hat{a}_0 \rangle = \sqrt{N_0}$; $\langle a_0^+ \rangle = \sqrt{N_0+1}$


$\langle a_0 \rangle = \sqrt{N_0}$
$V_0 N_0^2$
$V_k N_0 a_k a_{-k}$
$V_k N_0 a_k^+ a_{-k}^+$
$V_k N_0 a_k^+ a_k$
k=0 not included
$H_{int} = \frac{1}{2\Omega} \Big[V_0 N_0^2 + 2 N_0 \sum_k' (V_k + V_0) a_k^+ a_k + N_0 \sum_k' V_k (a_k^+ a_{-k}^+ + a_k a_{-k}$


$V_0 N_0^2$
$V_k N_0 a_k a_{-k}$
$V_k N_0 a_k^+ a_{-k}^+$
$V_k N_0 a_k^+ a_k$
k=0 not included
$\mathcal{H}_{int} = \frac{1}{2\Omega}\left[V_0 N_0^2 + 2N_0 \sum_k{}' (V_k + V_0) a_k^+ a_k + N_0 \sum_k{}' V_k (a_k^+ a_{-k}^+ + a_k a_{-k}) \right]$
$\mathcal{H} \to \mathcal{H} - \mu N$



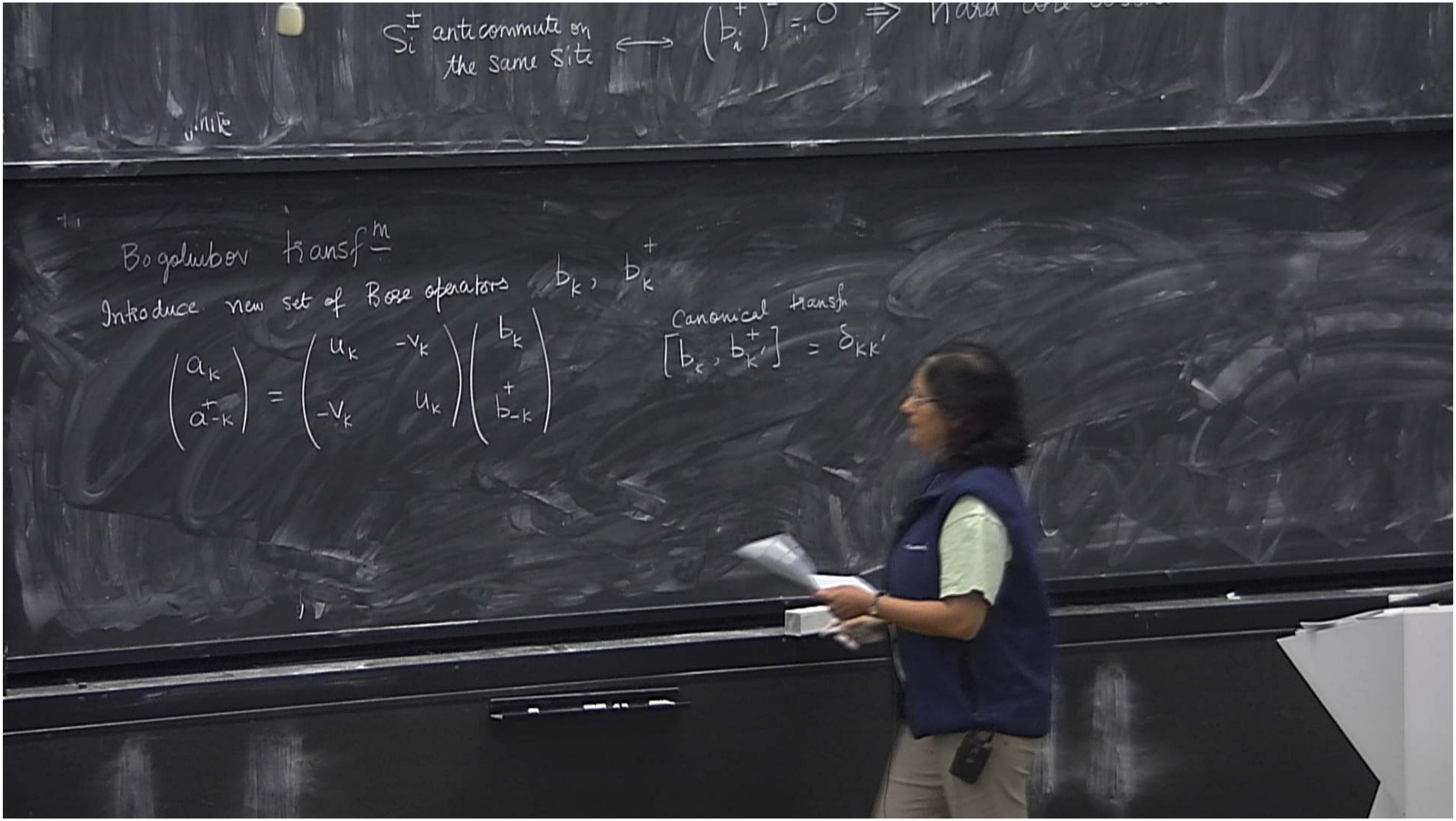
k=0 not included

$$\mathcal{H}_{int} = \frac{1}{2\Omega}\left[V_0 N_0^2 + 2N_0 \sum_k{}' (V_k + V_0) a_k^+ a_k + N_0 \sum_k{}' V_k (a_k^+ a_{-k}^+ + a_k a_{-k}) \right]$$

$$\mathcal{H} \to \mathcal{H} - \mu N$$

$$\mathcal{H} = \frac{1}{2}\Omega n^2 V_0 + \frac{1}{2}\sum_k{}' (\epsilon_k^0 + n V_k)(a_k^+ a_k + a_{-k}^+ a_{-k}) + \frac{1}{2} n \sum_k{}' V_k (a_k^+ a_{-k}^+ + a_k a_{-k})$$

$$n = \frac{N}{\Omega}$$


$S_i^{\pm}$ anticommute on the same site $\longleftrightarrow$ $(b_i^+)^2 = 0$ $\Rightarrow$
Bogolubov transf$^{\underline{m}}$
Introduce new set of Bose operators b_k, b_k^+
$\begin{pmatrix} a_k \\ a^+_{-k} \end{pmatrix} = \begin{pmatrix} u_k & -v_k \\ -v_k & u_k \end{pmatrix} \begin{pmatrix} b_k \\ b^+_{-k} \end{pmatrix}$
Canonical transf^n
$[b_k, b^+_{k'}] = \delta_{kk'}$

$$\begin{pmatrix} b^+_{-k} \end{pmatrix} = \begin{pmatrix} & \\ v_k & u_k \end{pmatrix} \begin{pmatrix} a^+_{-k} \end{pmatrix}$$

Substitute the Bog. transf^m in H

$$H = E_0 + \sum_k{}' E_k \, b_k^+ b_k$$

$$E_k = k\left[\frac{1}{2m}\left(\frac{k^2}{2m} + 2nV_k\right)\right]^{1/2}$$

Bare dispersion $\epsilon_k \sim k^2$

as $k \to 0$ $\quad E_k = ck \Rightarrow$ sound modes