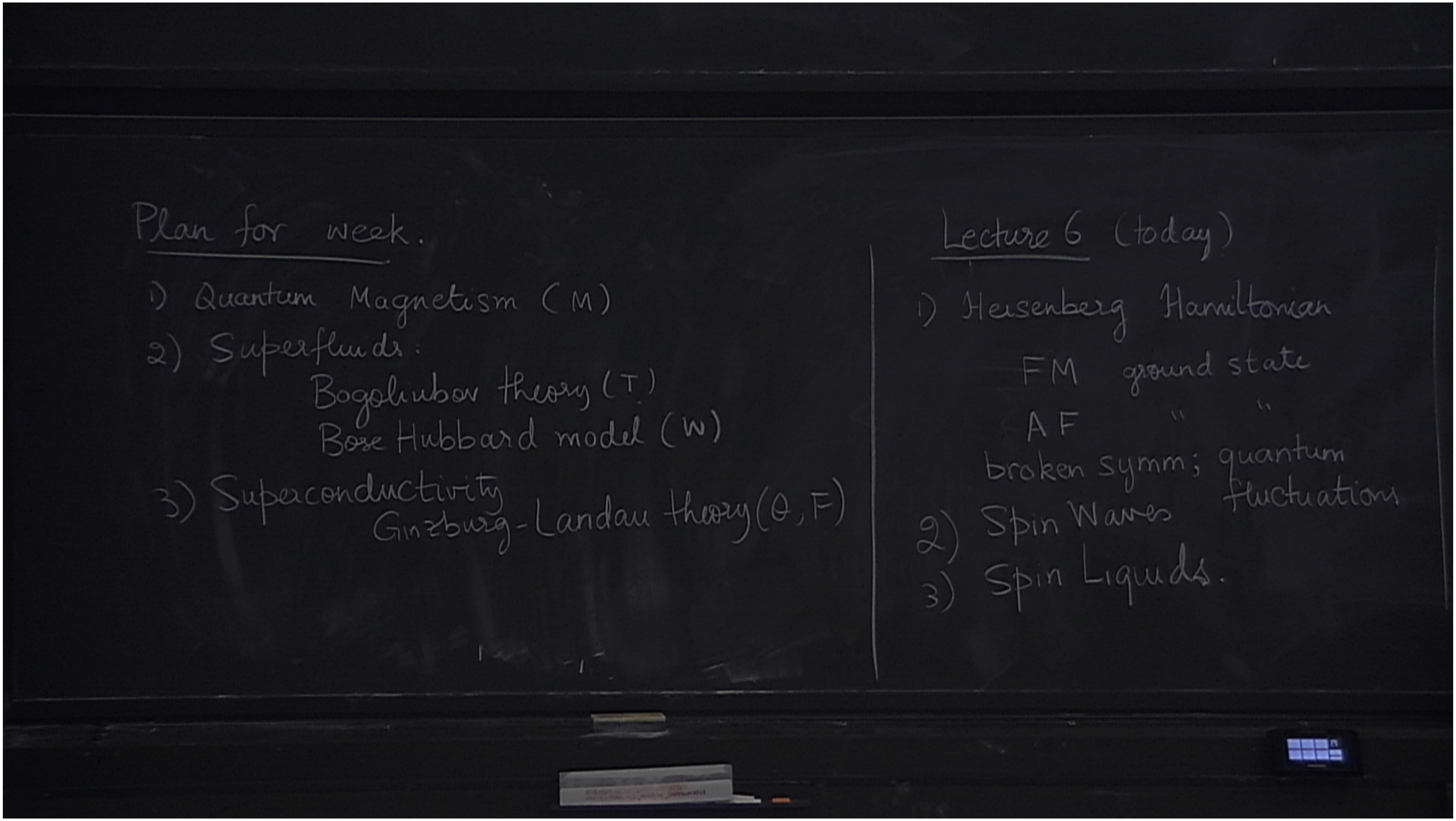
Title: Condensed Matter - Lecture 6

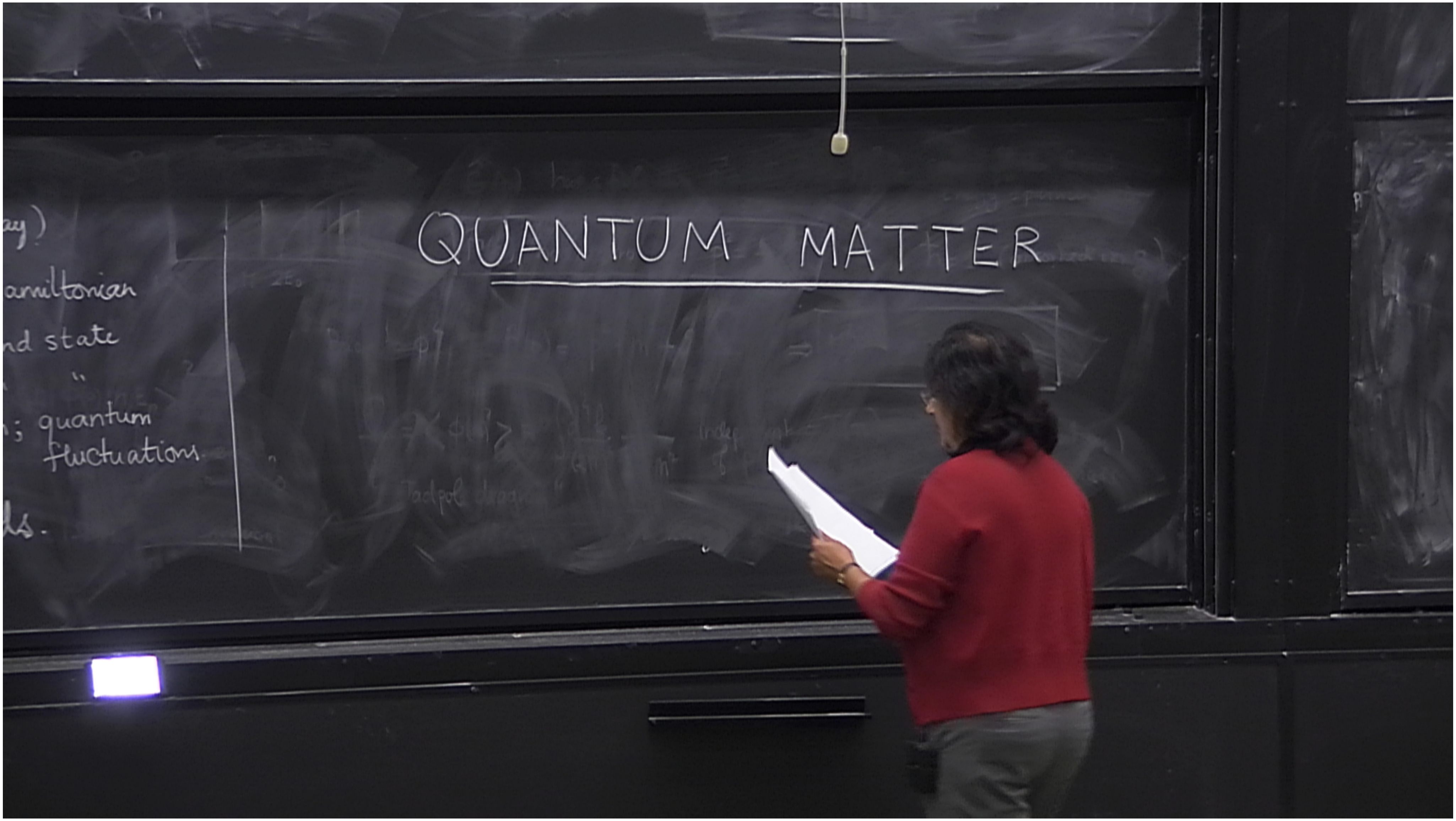
Date: Nov 07, 2011 10:30 AM

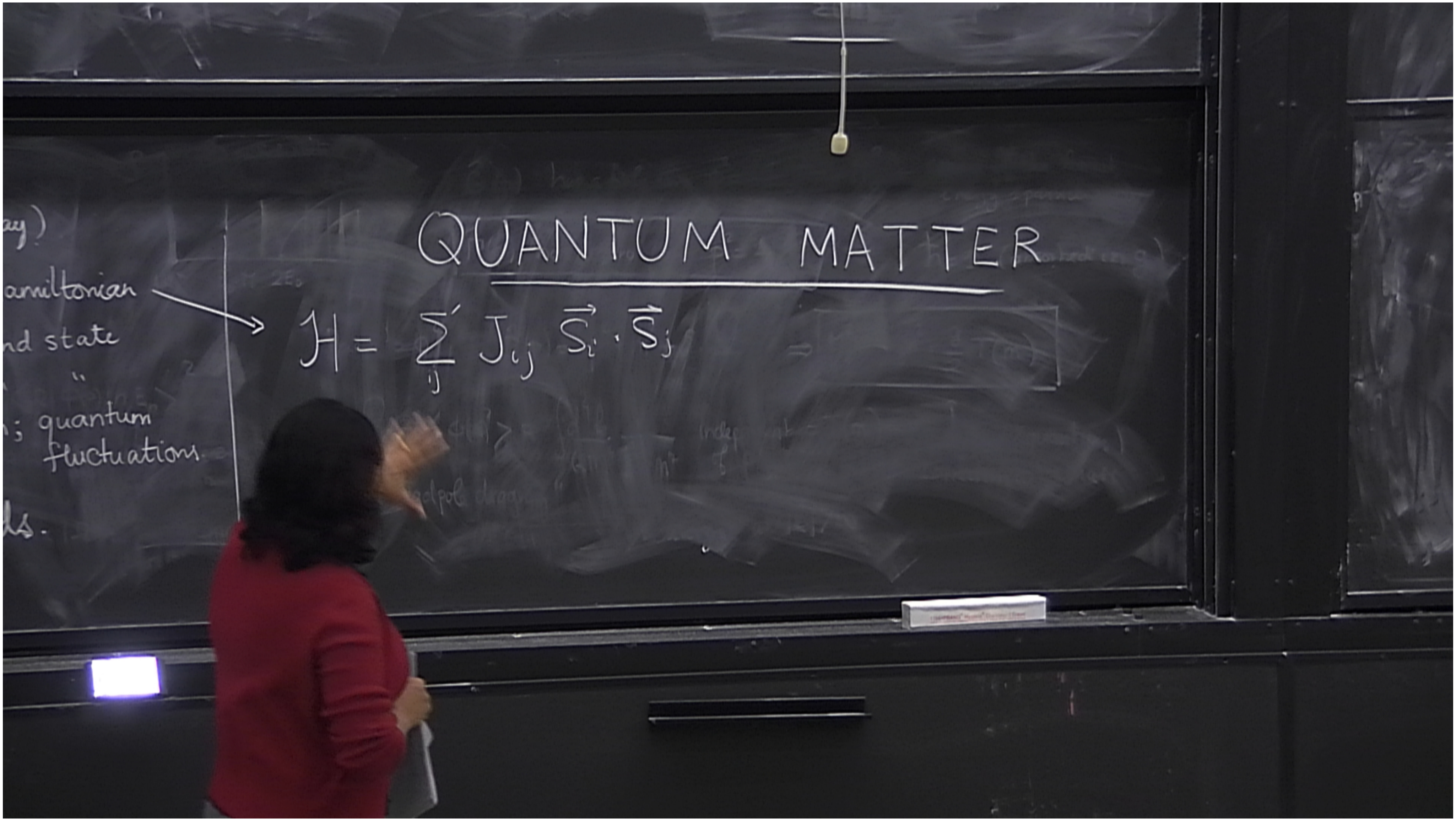
URL: http://pirsa.org/11110028

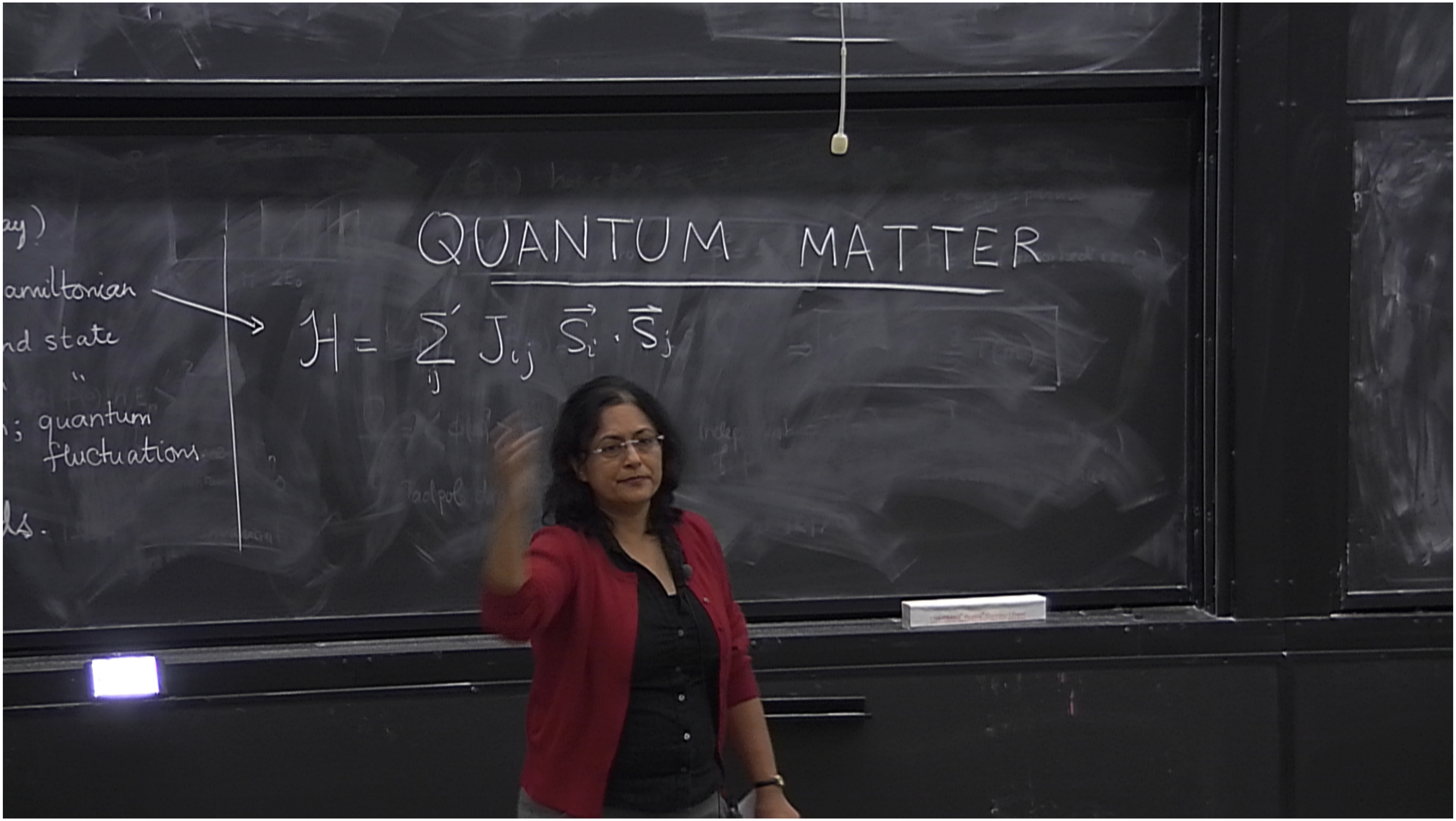
Abstract:


Plan for week.
1) Quantum Magnetism (M)
2) Superfluids:
Bogoliubov theory (T)
Bose Hubbard model (W)
3) Superconductivity
Ginzburg-Landau theory (Θ, F)
Lecture 6 (today)
1) Heisenberg Hamiltonian
FM ground state
AF " "
broken symm; quantum fluctuations
2) Spin Waves
3) Spin Liquids.


PreviousResearch
-MR-NT
DOC
Lecture2
DOCX
Mallika-research
DOCX
Fisher--
Conven...ulators


QUANTUM MATTER
quantum
fluctuations


QUANTUM MATTER
$\mathcal{H} = \sum_{ij}' J_{ij} \vec{S}_i \cdot \vec{S}_j$
amiltonian
nd state
; quantum fluctuations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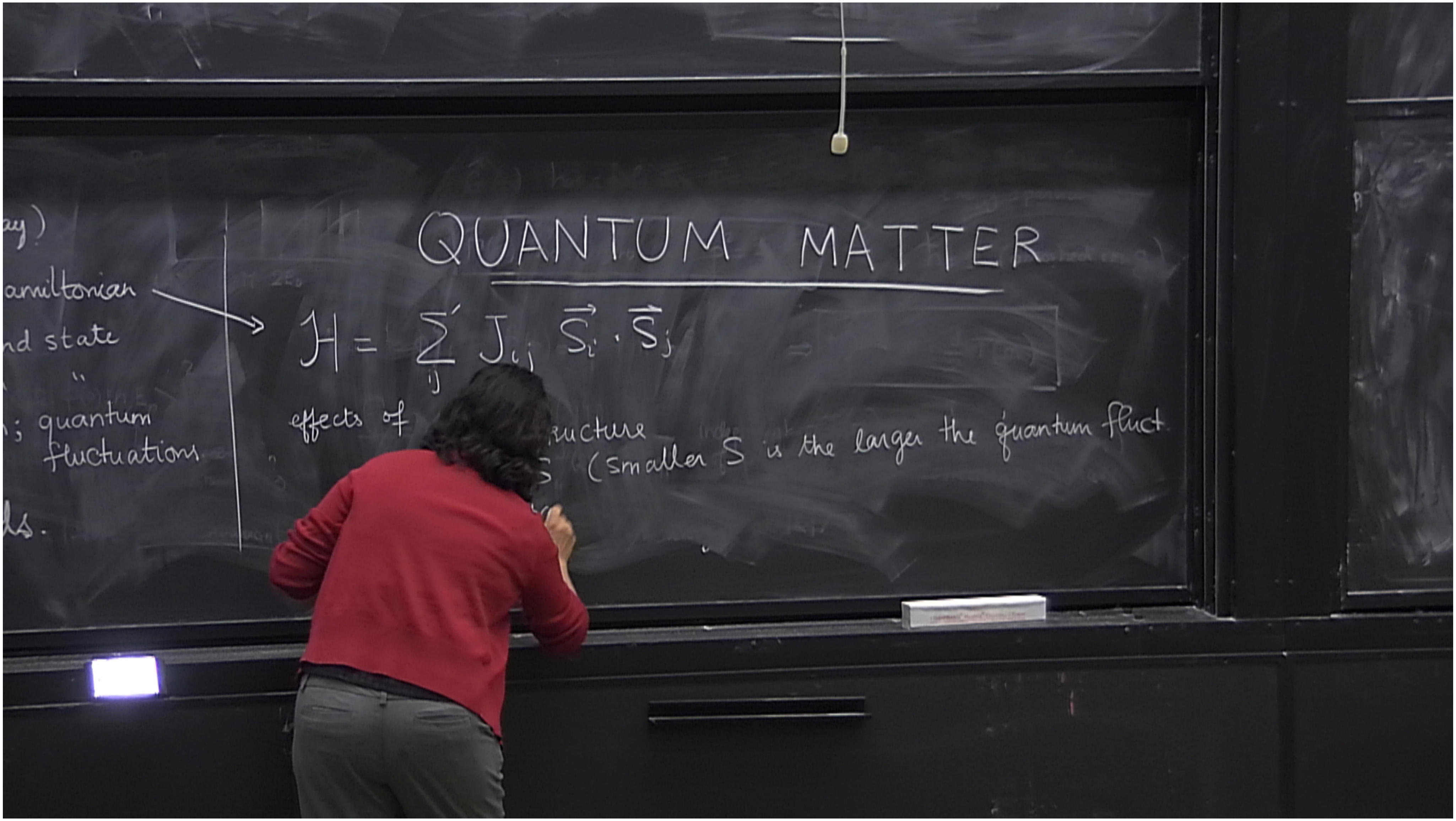
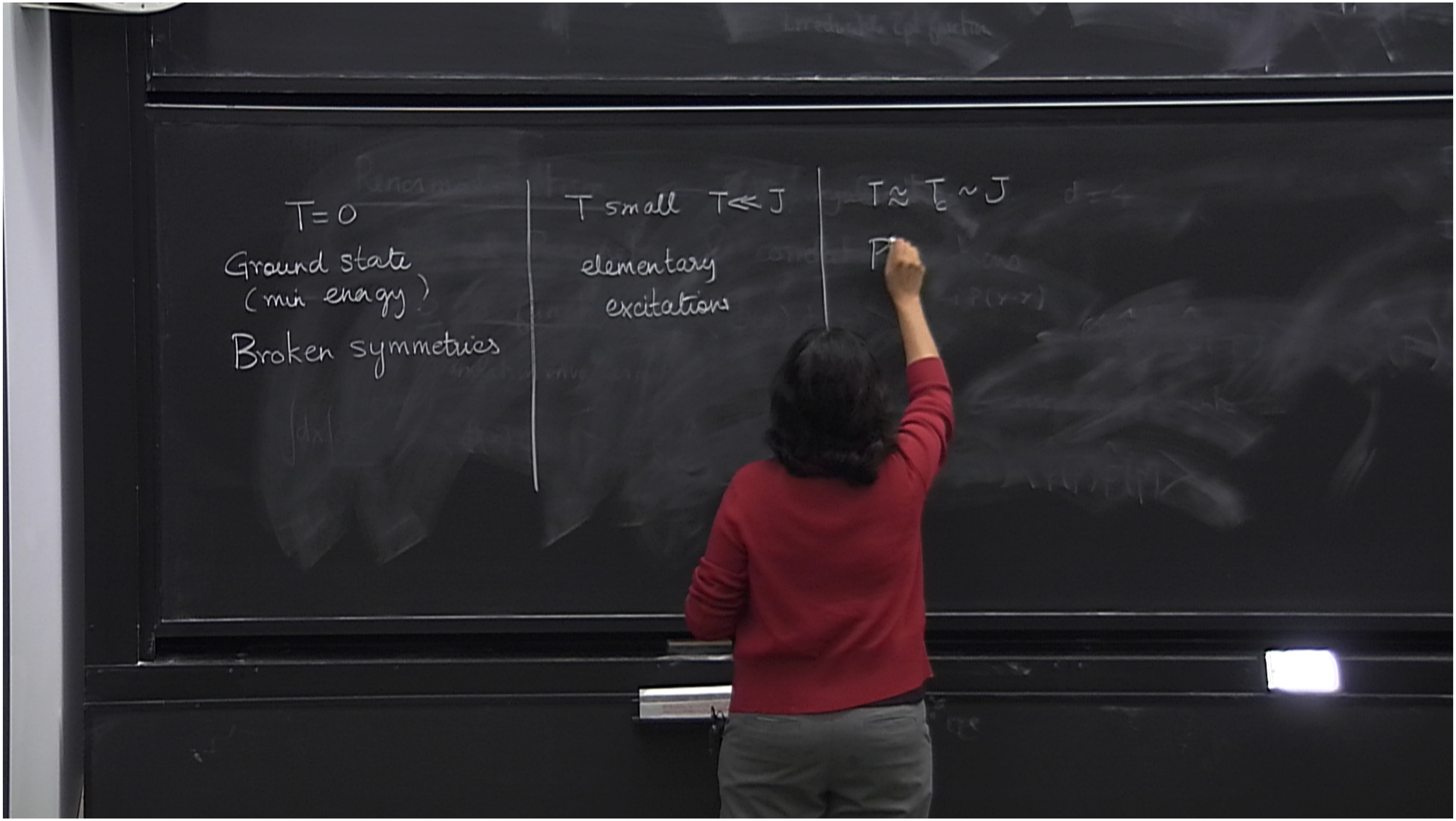
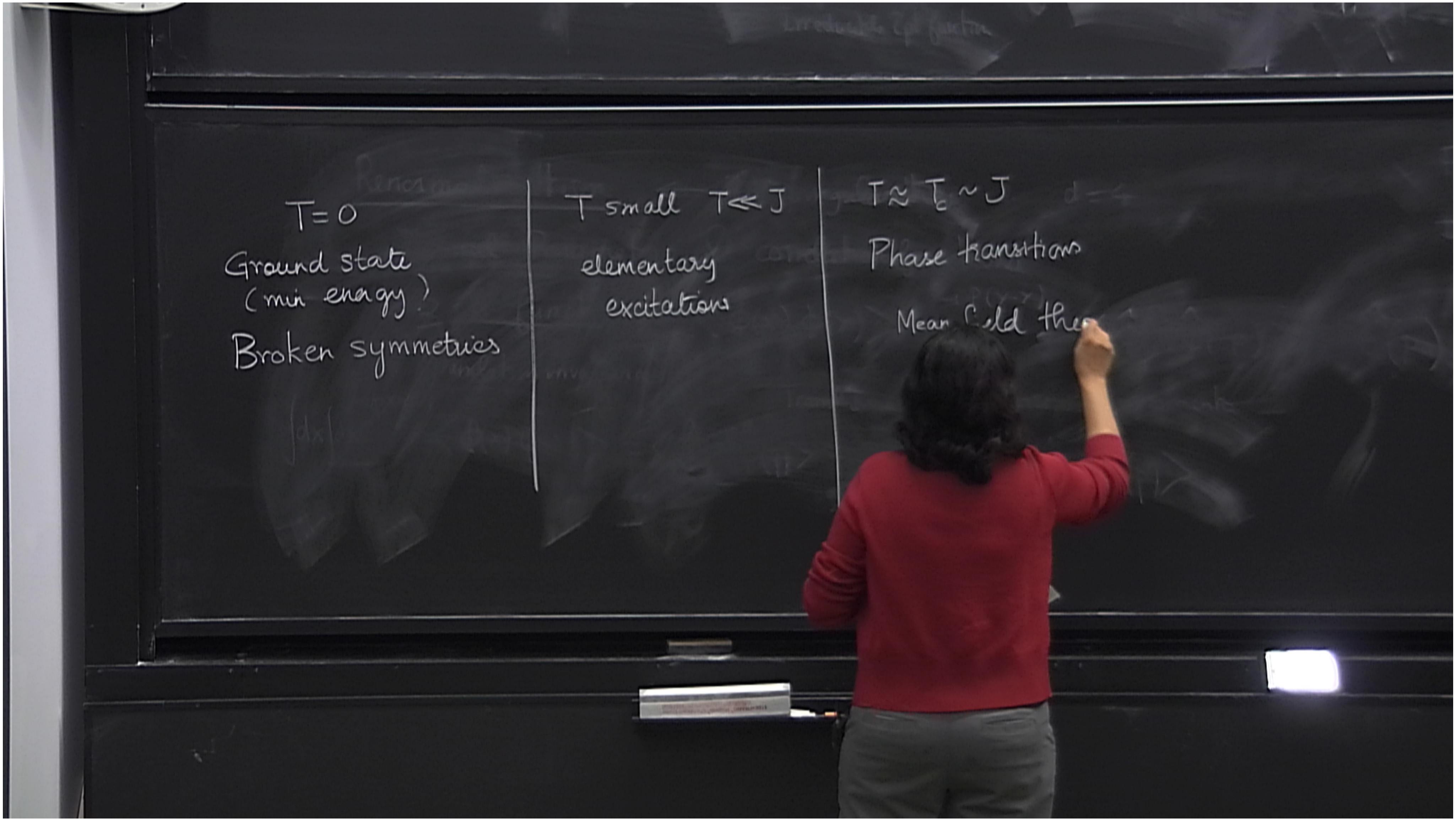

QUANTUM MATTER
$\mathcal{H} = \sum'_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$
effects of
ucture
(smaller S is the larger the quantum fluct.
amiltonian
nd state
; quantum
fluctuations


T=0
Ground state
(min energy)
Broken symmetries
T small T≪J
elementary
excitations
T≈ T_c ~ J


T=0
Ground state
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Broken symmetries
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excitations
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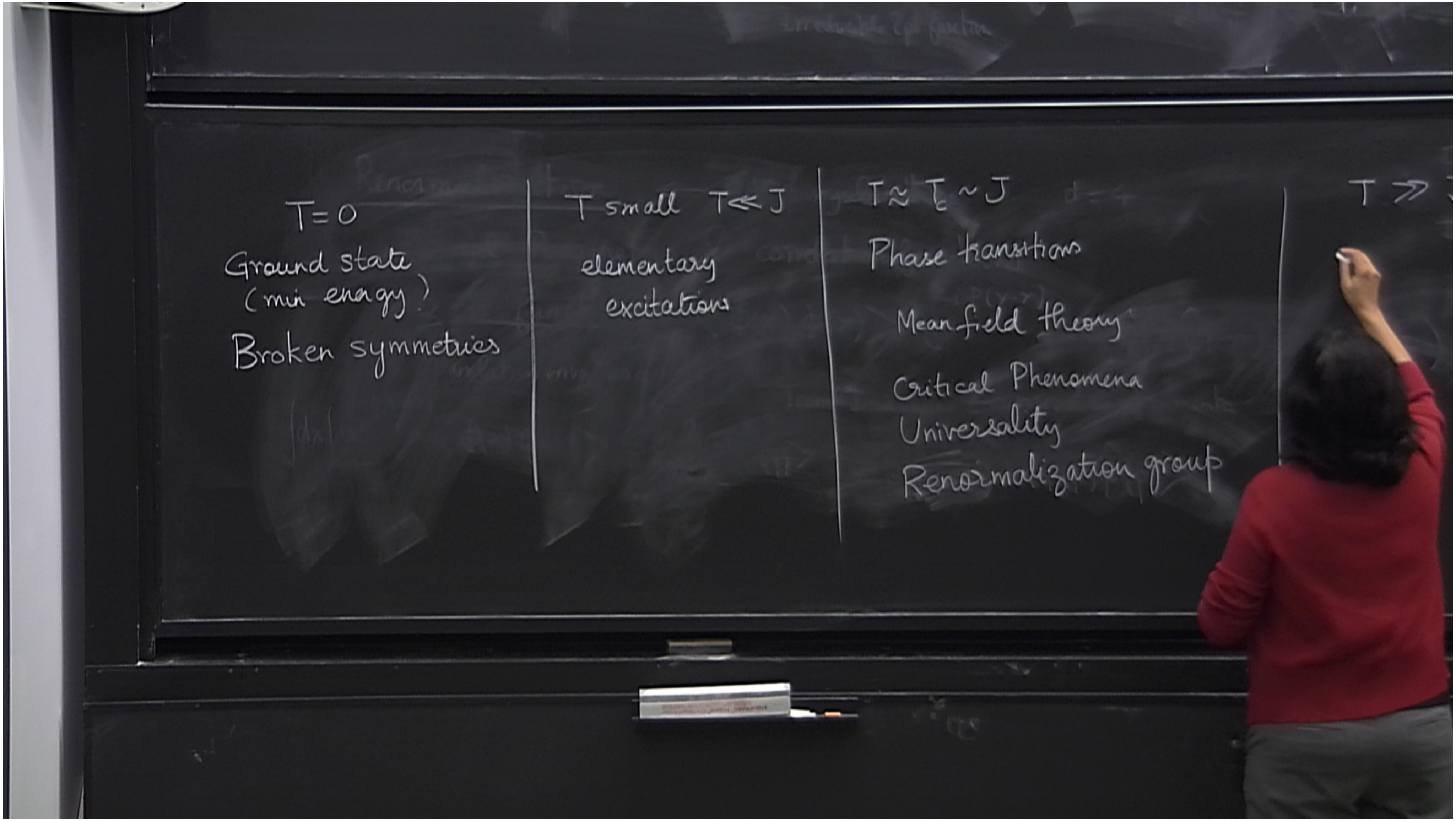
Phase transitions
Mean field the


T=0
Ground state
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Broken symmetries
T small T≪J
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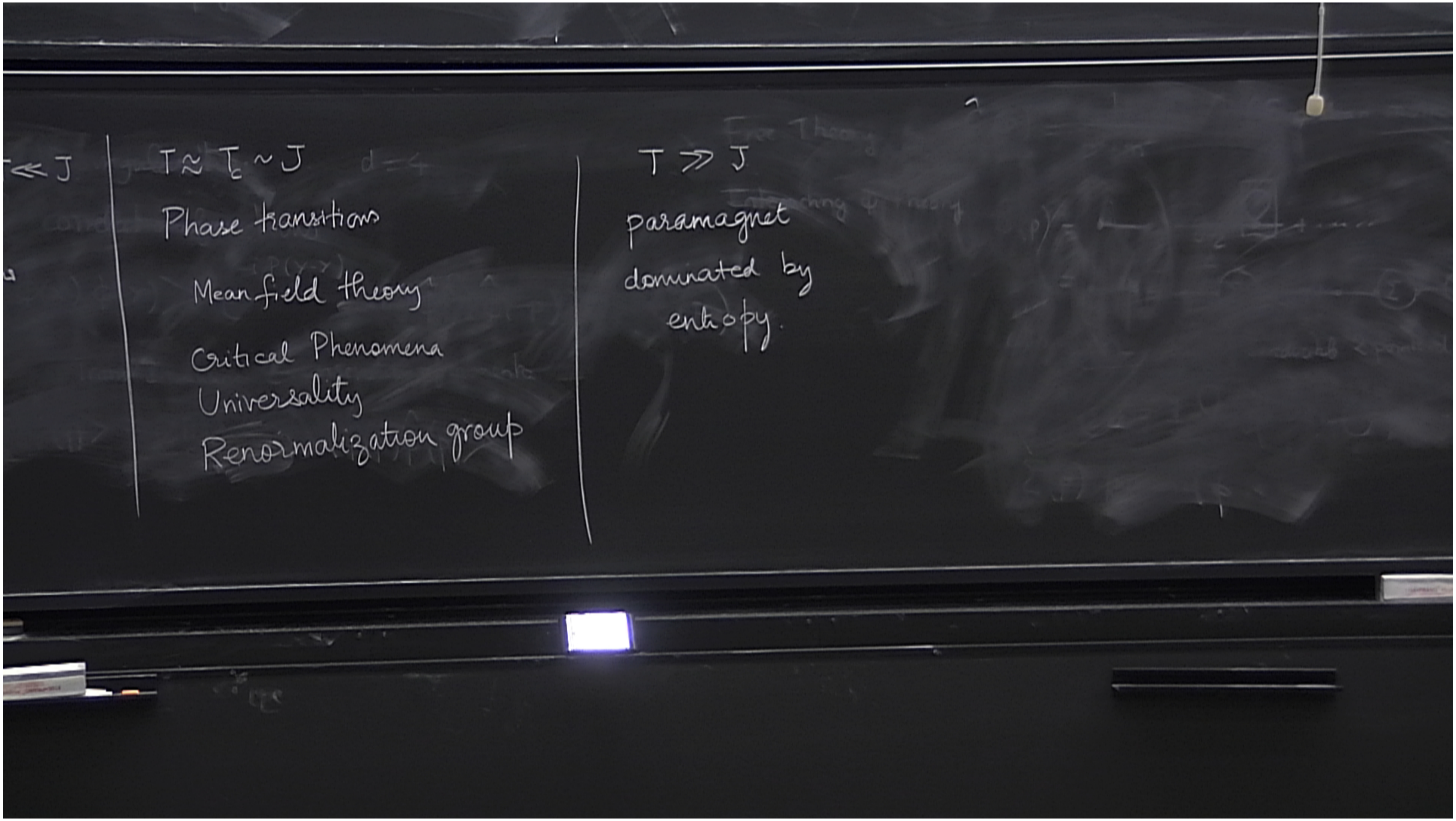
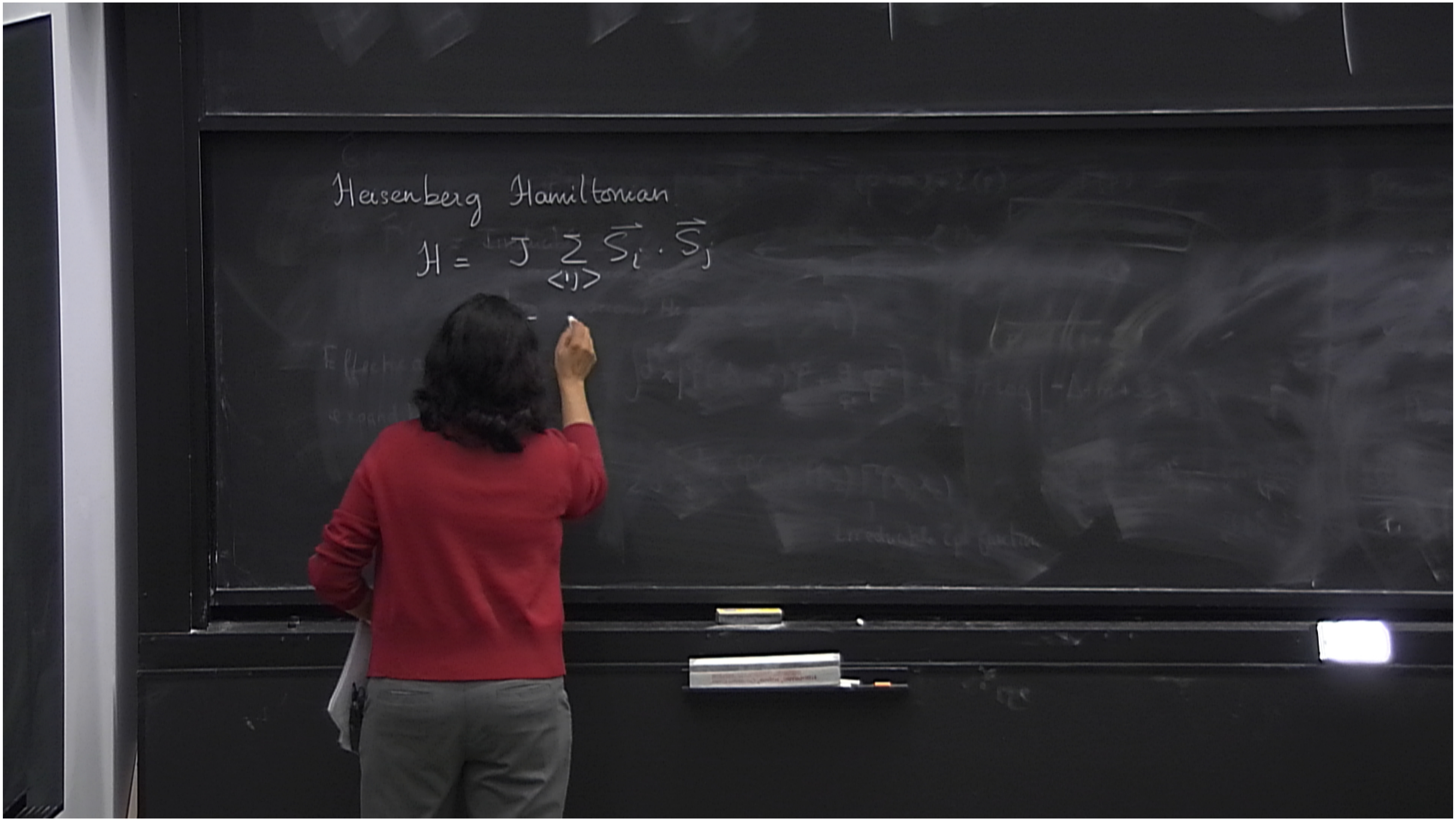
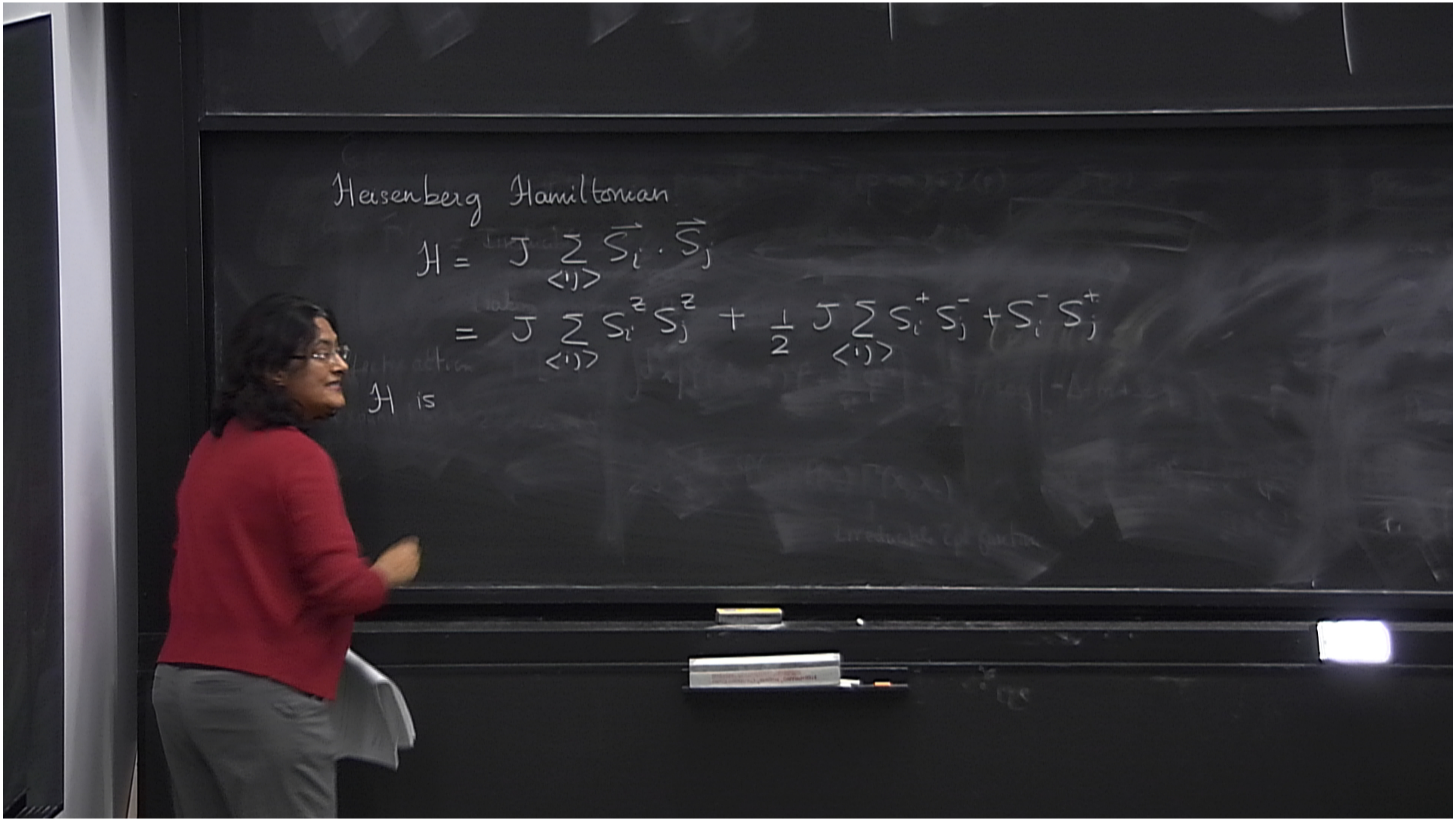
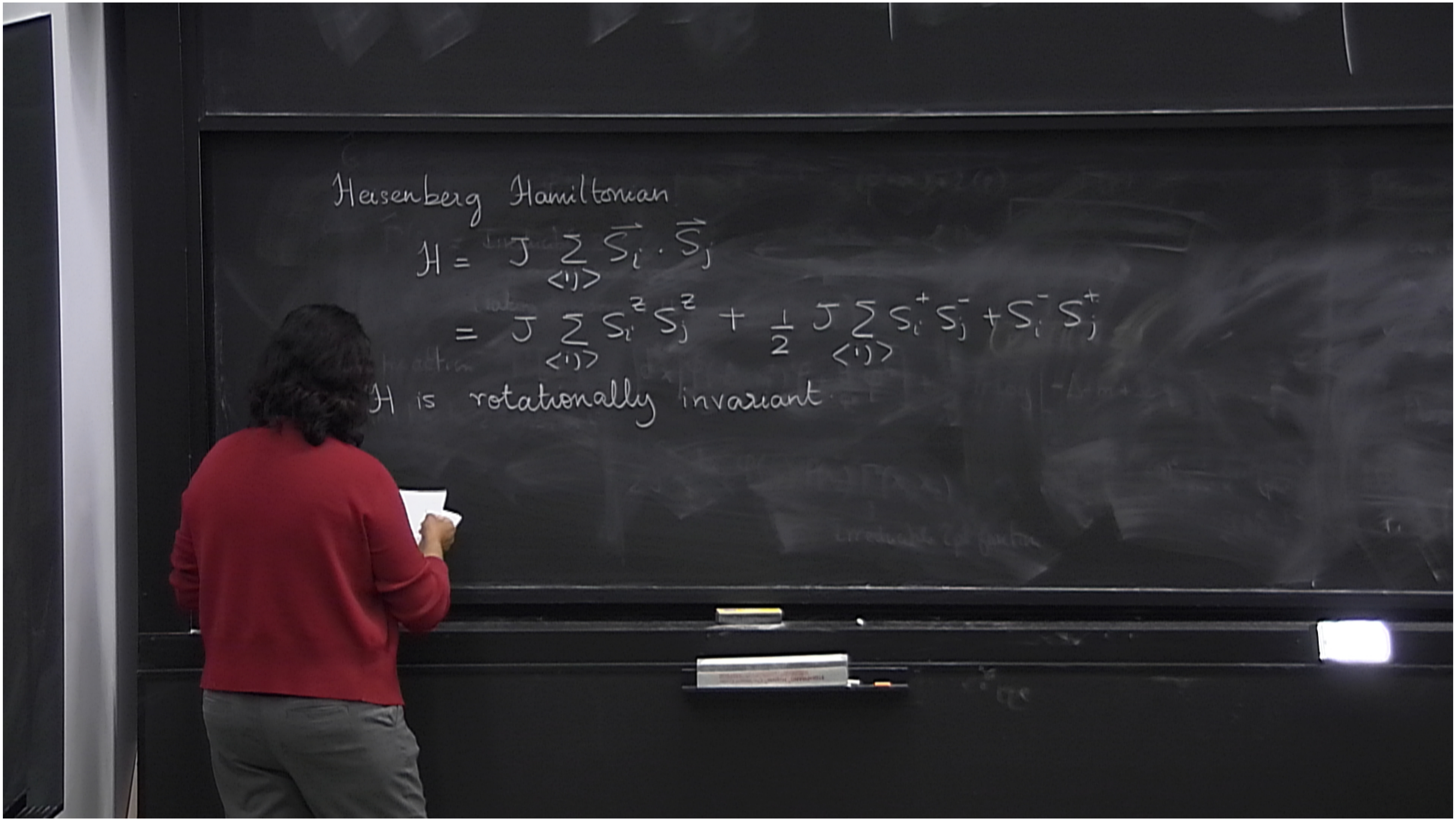
T≈ T_c ~ J
Phase transitions
Mean field theory
Critical Phenomena
Universality
Renormalization group
T ≫



T ≪ J
T ≈ Tc ~ J
Phase transitions
Mean field theory
Critical Phenomena
Universality
Renormalization group
T ≫ J
paramagnet
dominated by
entropy.


Heisenberg Hamiltonian
$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$


Heisenberg Hamiltonian
$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$
$= J \sum_{\langle ij \rangle} S_i^z S_j^z + \frac{1}{2} J \sum_{\langle ij \rangle} S_i^+ S_j^- + S_i^- S_j^+$
$\mathcal{H}$ is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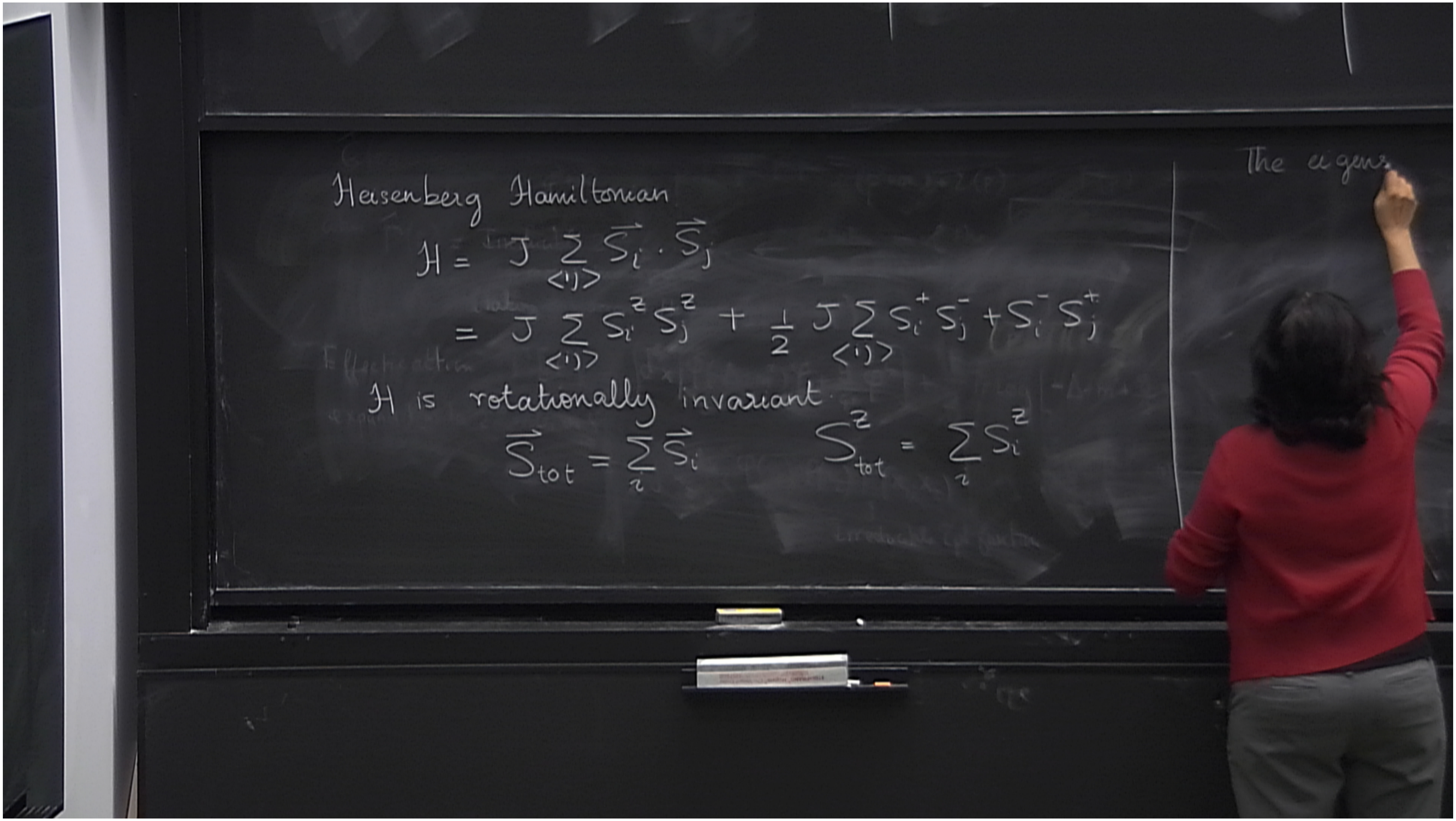
$\mathcal{H}$ is rotationally invariant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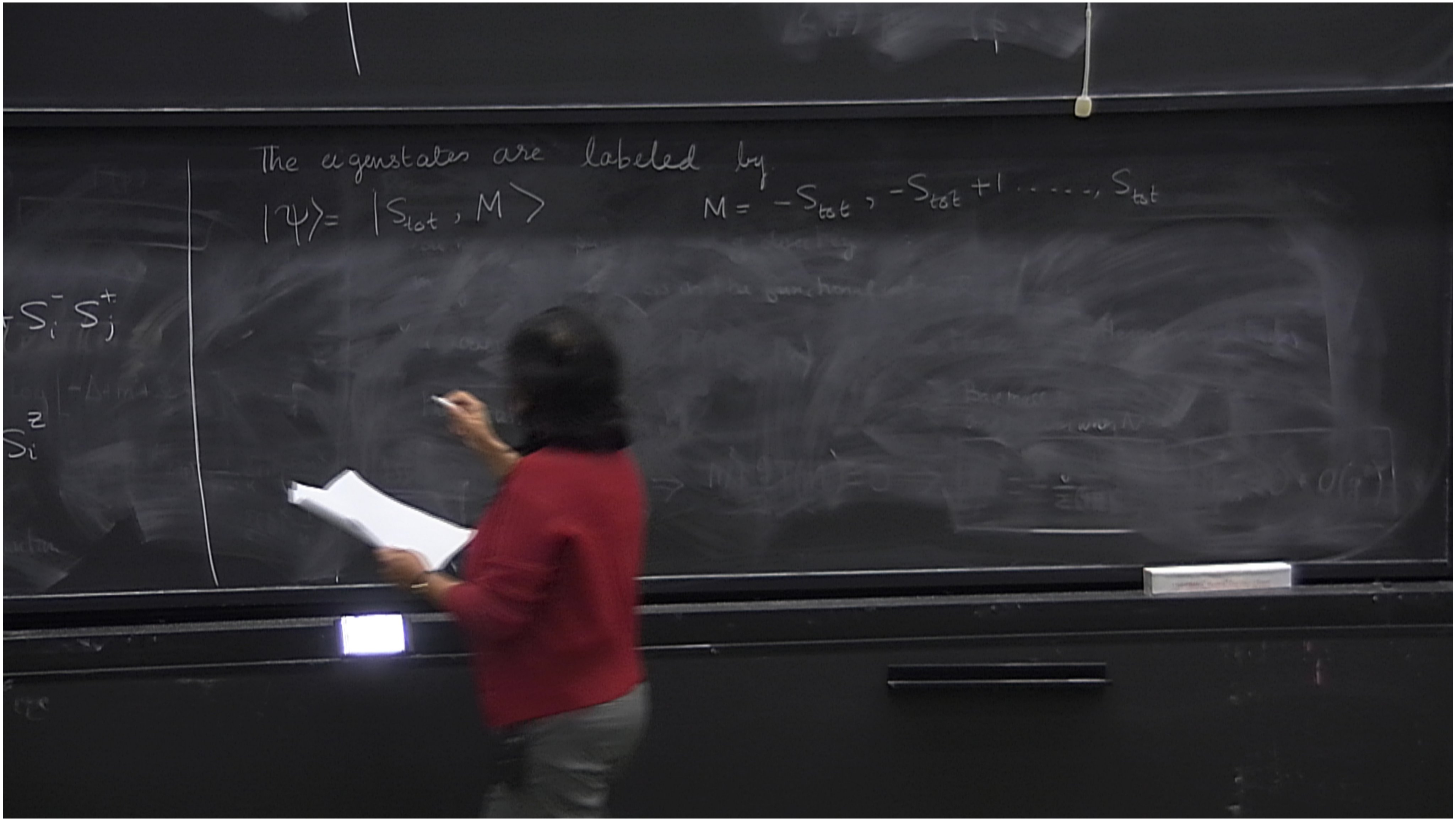
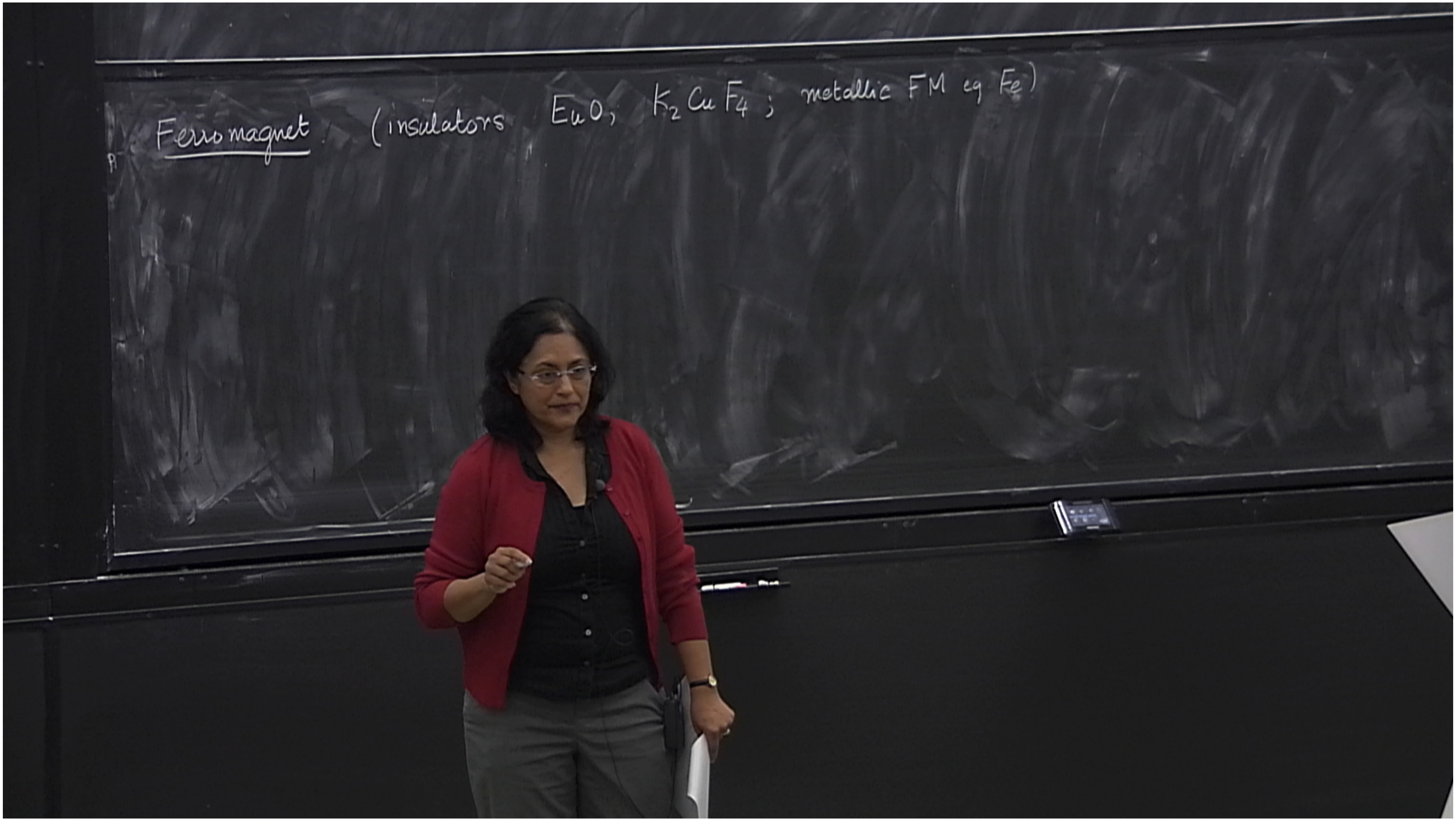
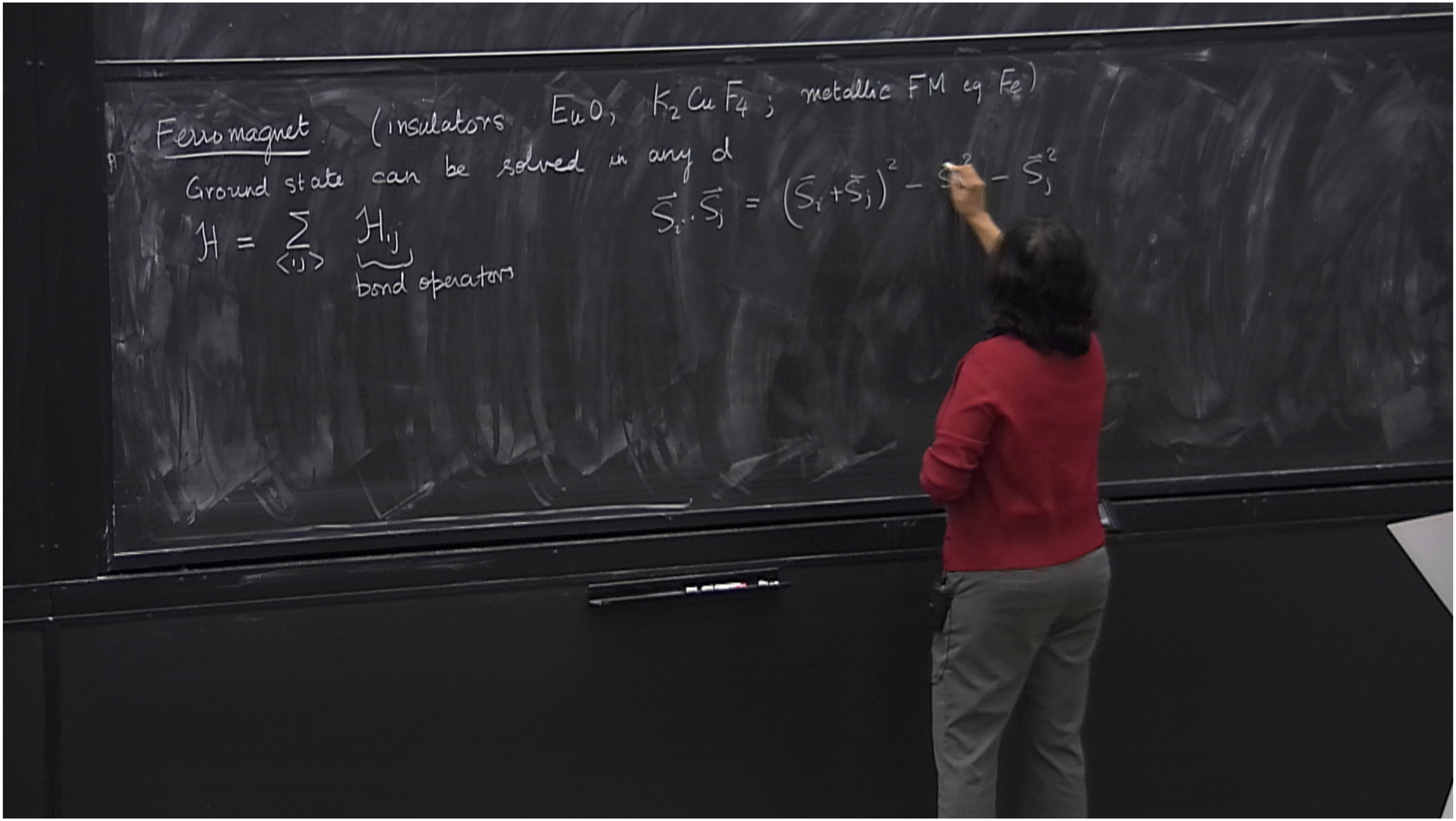
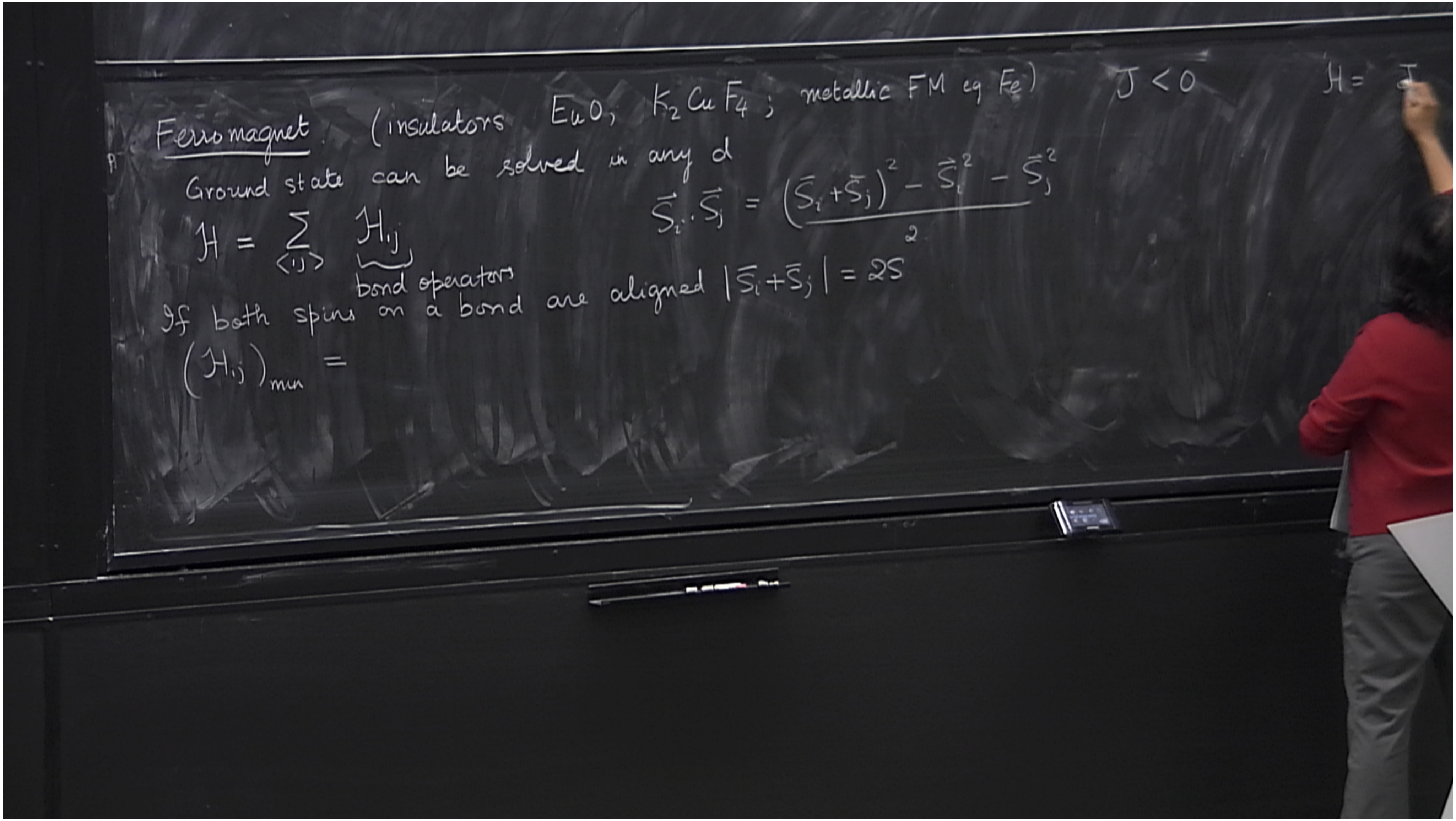
$\mathcal{H}$ is rotationally invariant
$\vec{S}_{tot} = \sum_i \vec{S}_i$
$S^z_{tot} = \sum_i S_i^z$
The eigens

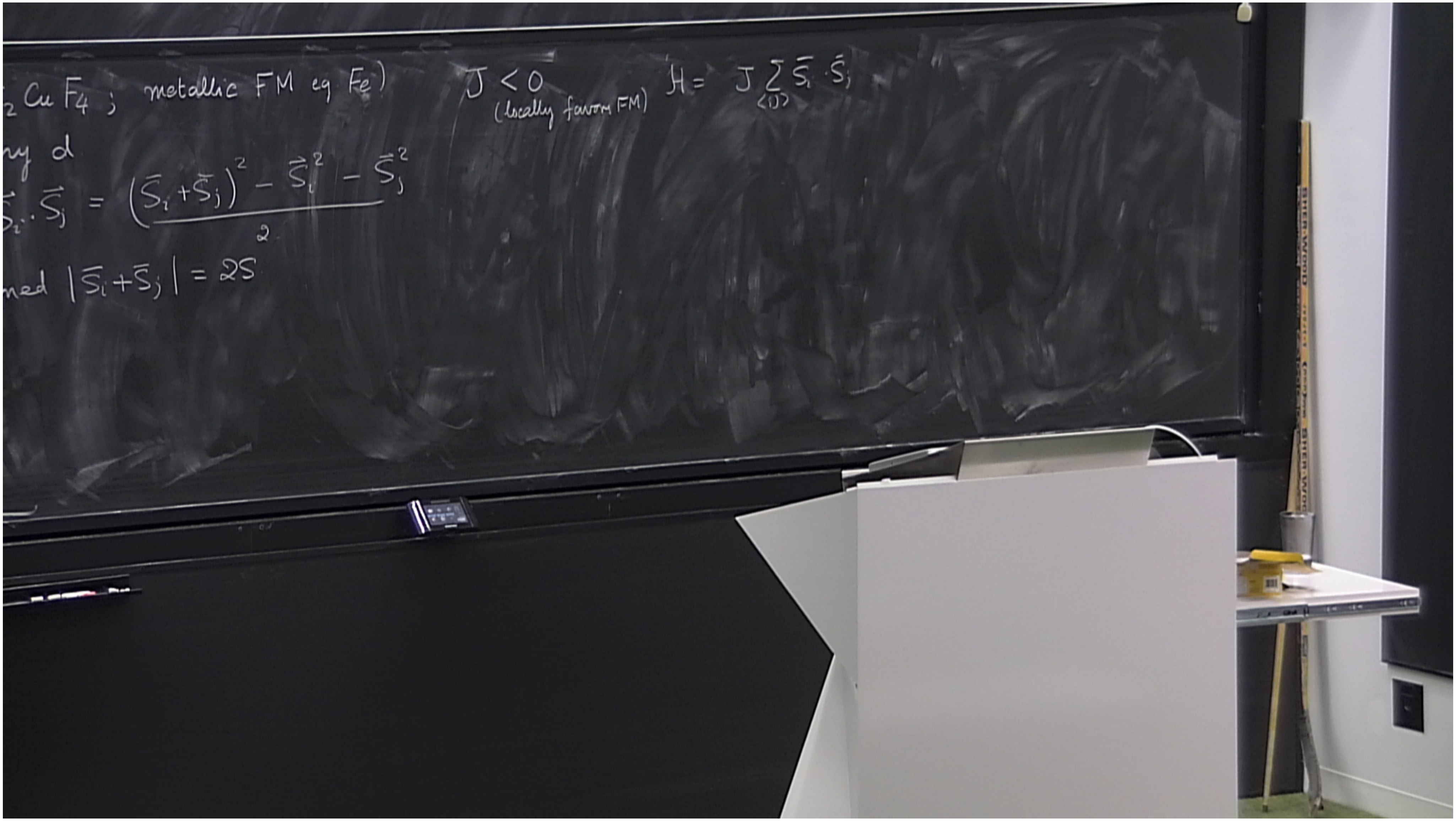

The eigenstates are labeled by
|ψ⟩ = |S_tot , M⟩
M = −S_tot , −S_tot +1 . . . . . , S_tot

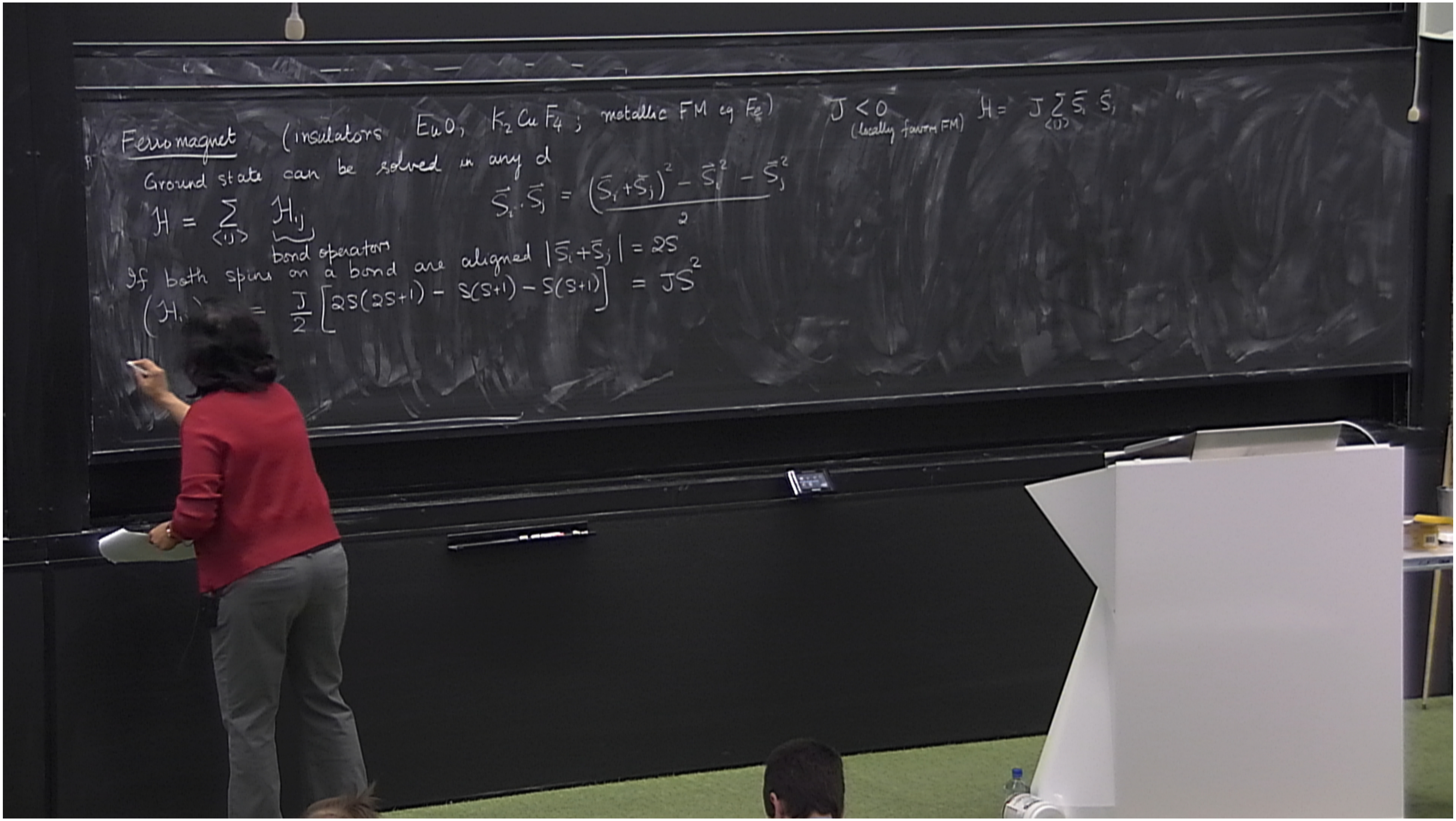

Ferromagnet (insulators


Ferromagnet (insulators EuO, K2 Cu F4 ; metallic FM eg Fe)


Ferromagnet (insulators EuO, K2CuF4 ; metallic FM eg Fe)
Ground state can be solved in any d
H = Σ_{⟨ij⟩} H_ij
bond operators
S_i · S_j = (S_i + S_j)^2 − S_i^2 − S_j^2


Ferromagnet (insulators EuO, K2CuF4 ; metallic FM eg Fe)
J < 0
H =
Ground state can be solved in any d
Si . Sj = ((Si + Sj)^2 − Si^2 − Sj^2) / 2
H = Σ<ij> Hij
bond operators
If both spins on a bond are aligned |Si + Sj| = 2S
(Hij)min =


Cu F4 ; metallic FM eg Fe)
J < 0
(locally favors FM)
H = J Σ S_i · S_j
⟨ij⟩
ny d
S_i · S_j = ((S_i + S_j)^2 − S_i^2 − S_j^2) / 2
ned |S_i + S_j| = 2S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H = J Σ<ij> Si Sj
Ground state can be solved in any d
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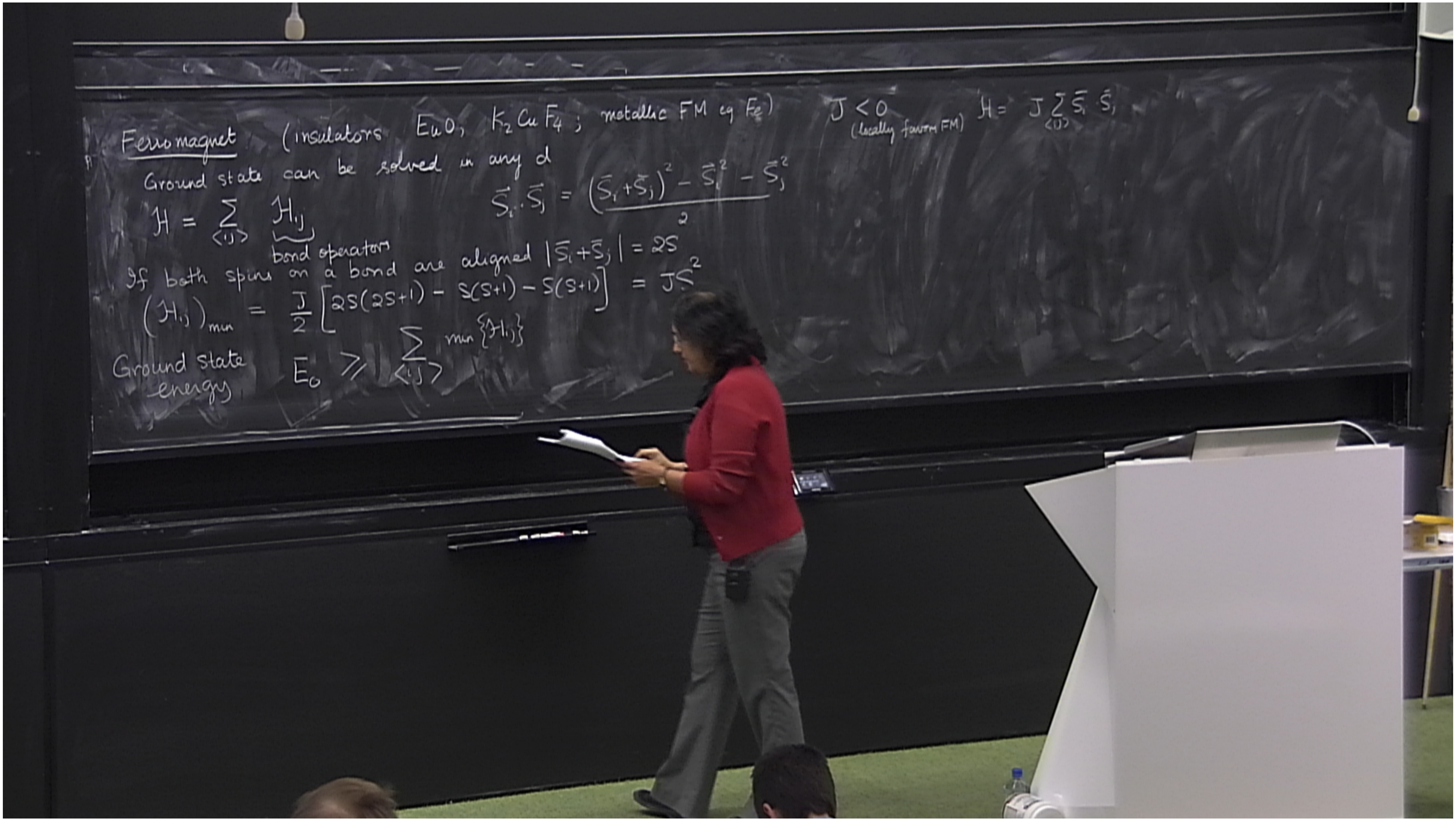
Si · Sj = ((Si + Sj)^2 − Si^2 − Sj^2) / 2
If both spins on a bond are aligned |Si + Sj| = 2S
J/2 [2S(2S+1) − S(S+1) − S(S+1)] = JS^2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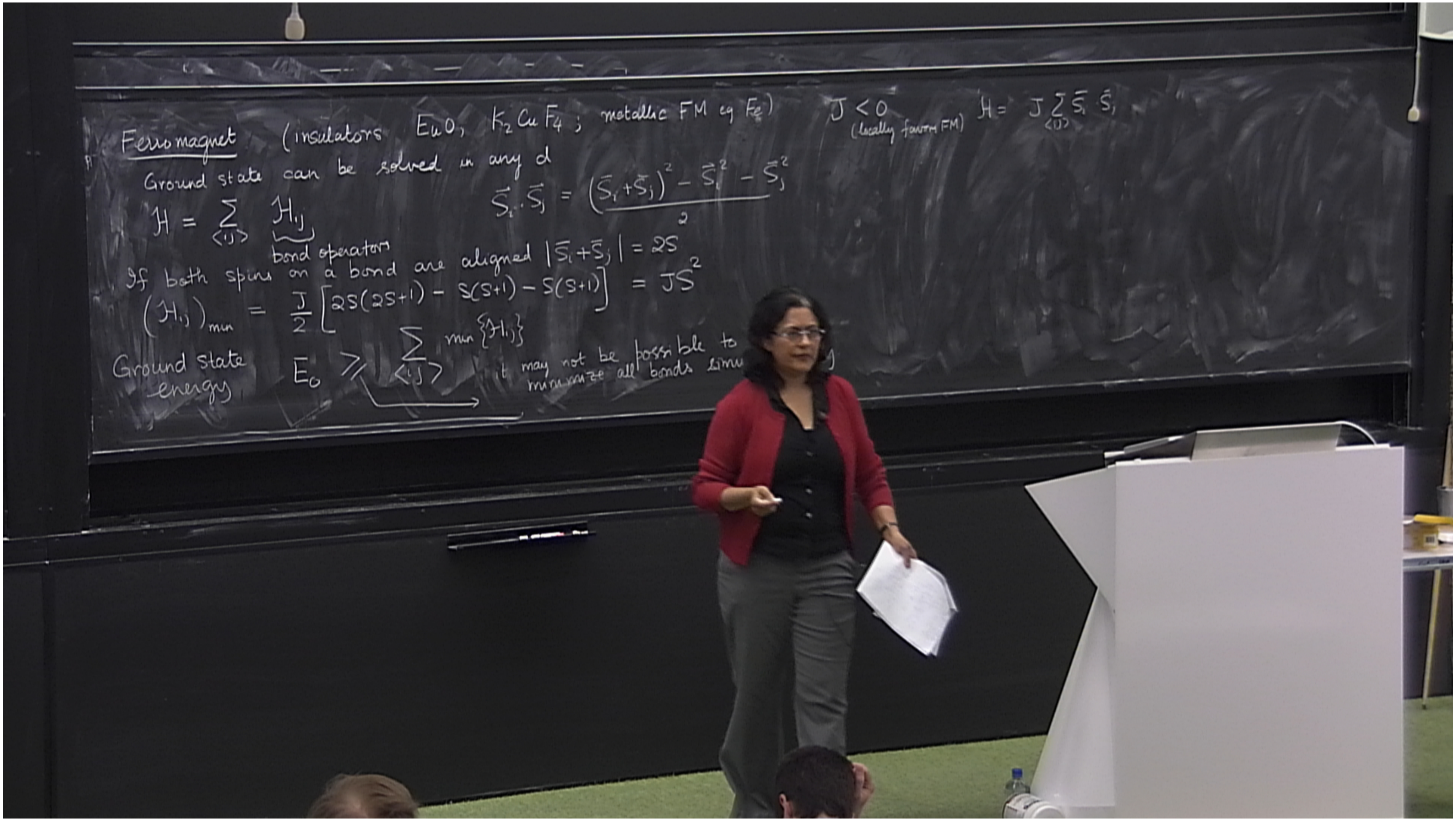
Ground state energy
E0 ≥ Σ<ij> min{Hij}


Ferromagnet (insulators EuO, K2CuF4 ; metallic FM eg Fe)
J < 0 (locally favors FM)
H = J Σ_⟨ij⟩ S_i · S_j
Ground state can be solved in any d
H = Σ_⟨ij⟩ H_ij
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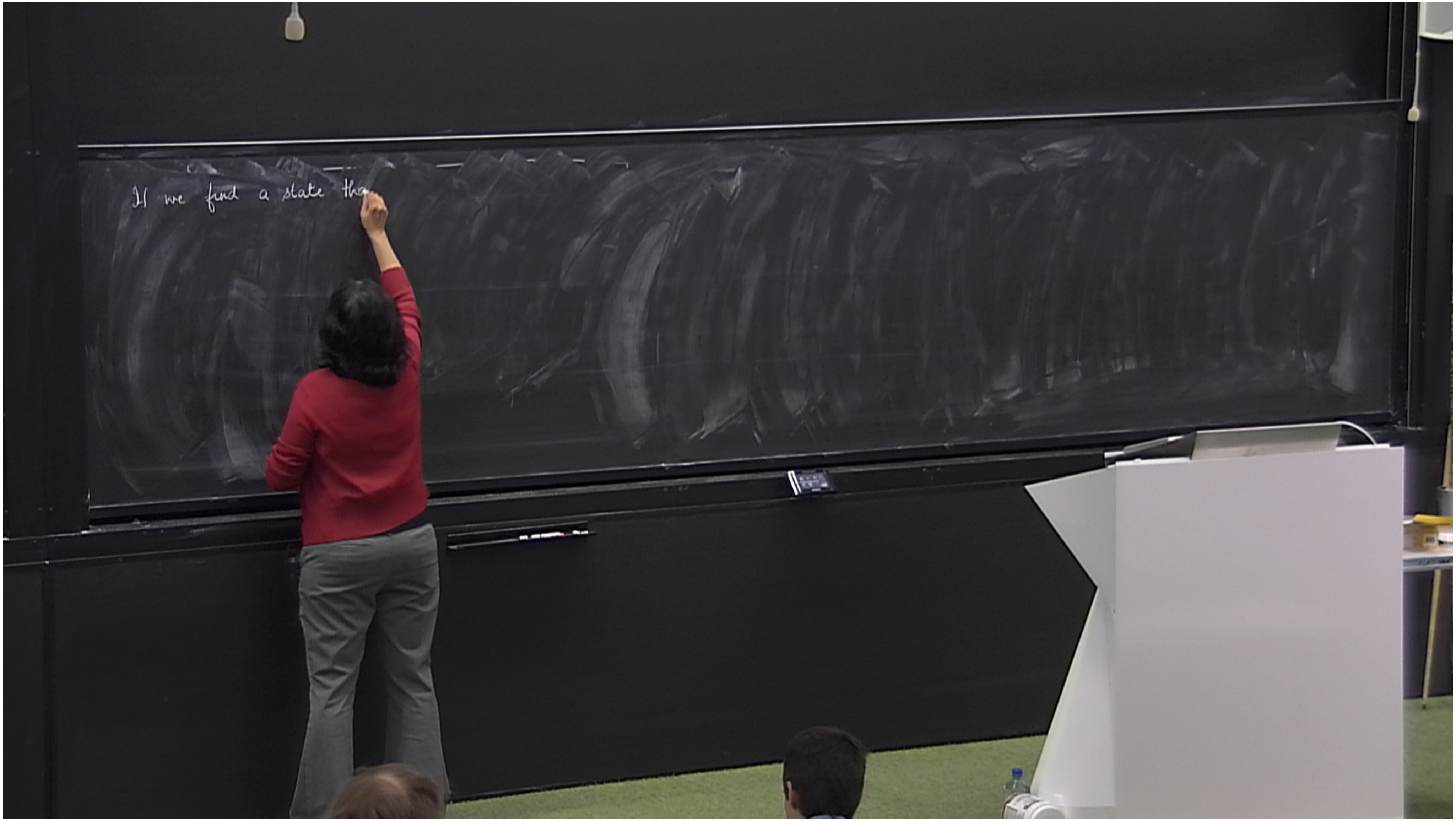
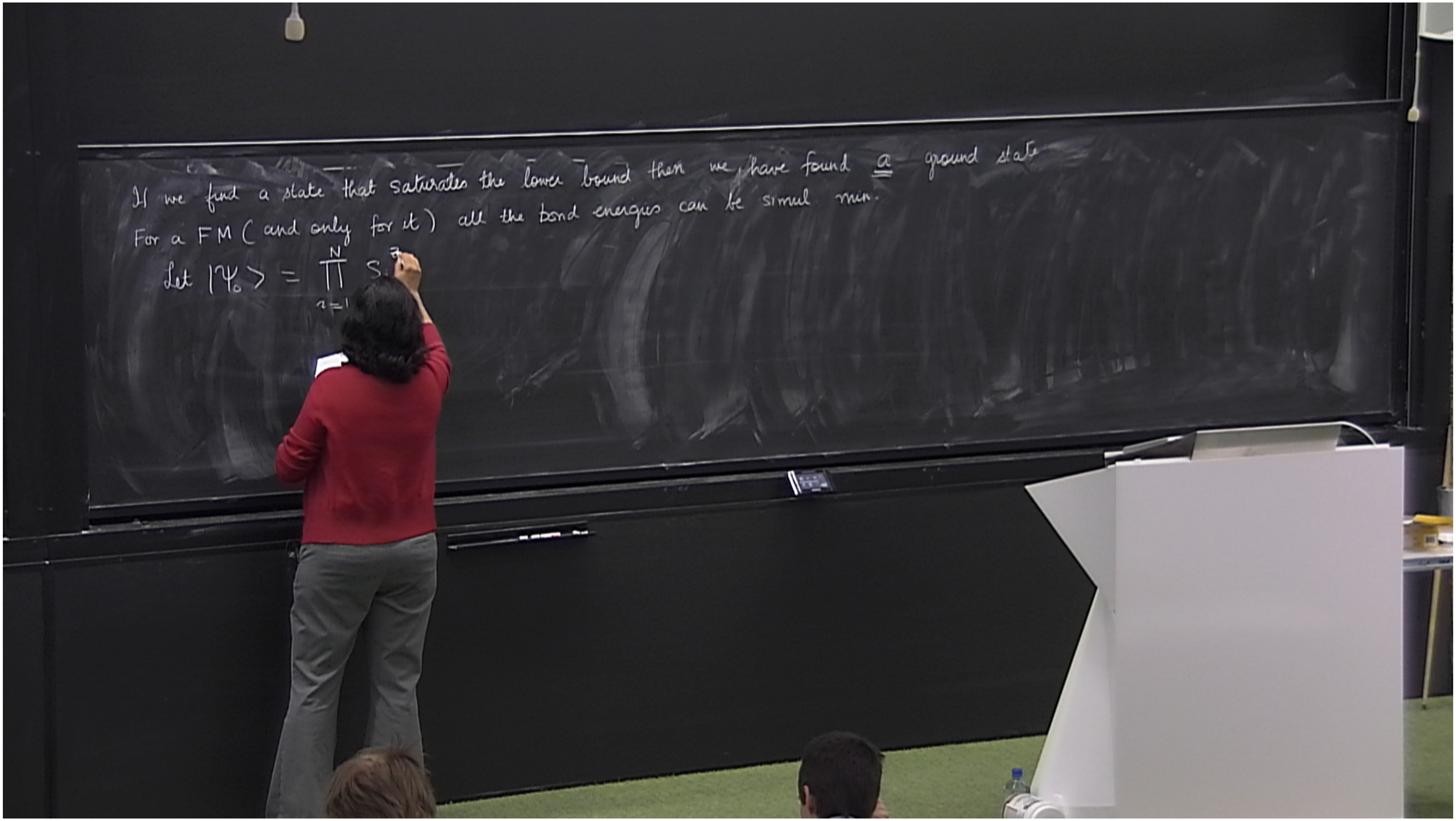
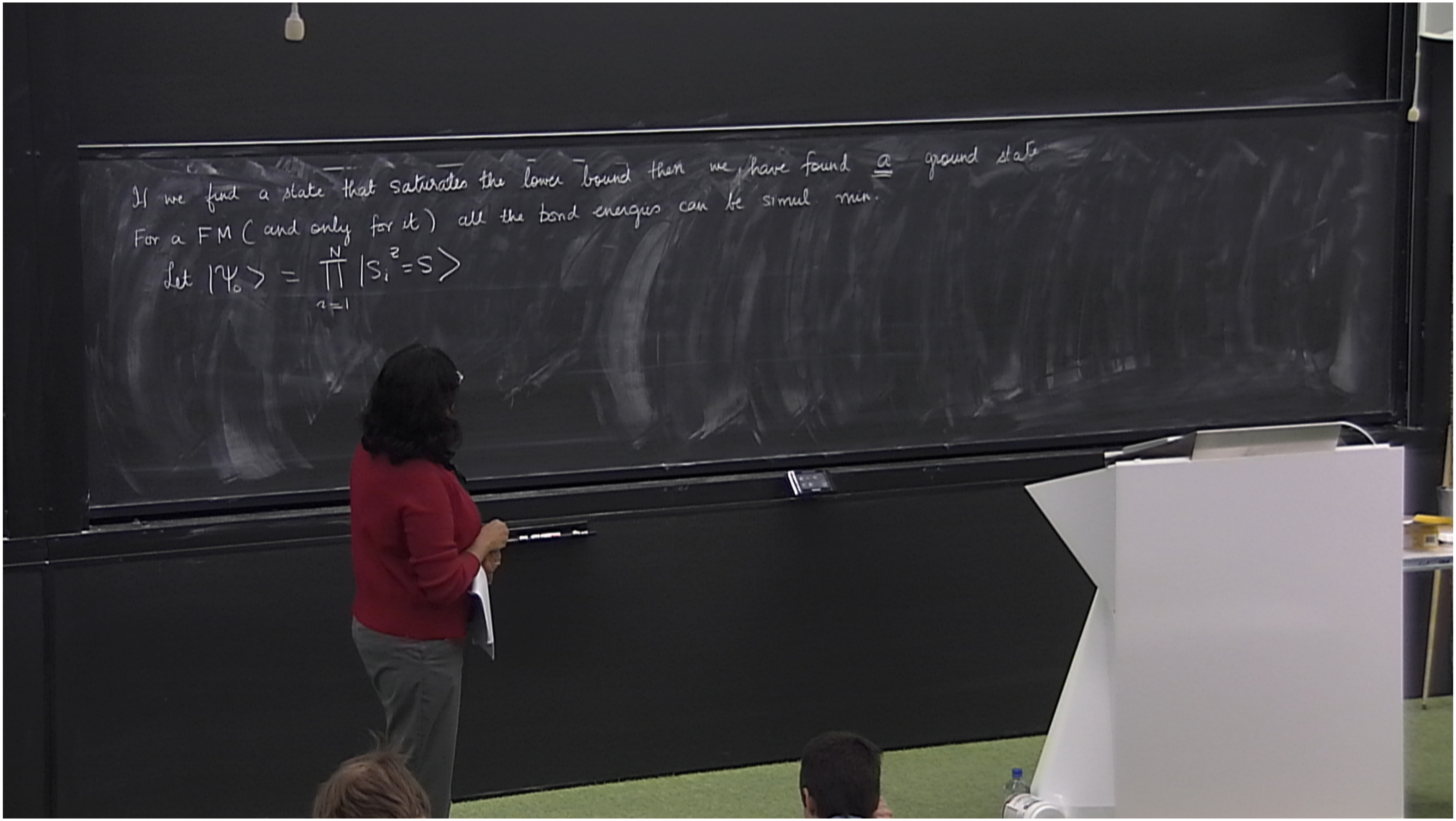
S_i · S_j = ((S_i + S_j)^2 − S_i^2 − S_j^2) / 2
If both spins on a bond are aligned |S_i + S_j| = 2S
(H_ij)_min = J/2 [2S(2S+1) − S(S+1) − S(S+1)] = JS^2
Ground state energy
E_0 ≥ Σ_⟨ij⟩ min {H_ij}
it may not be possible to minimize all bonds simu


If we find a state


If we find a state that saturates the lower bound then we have found a ground state
For a FM (and only for it) all the bond energies can be simul min.
Let |Ψ₀ > = ∏ S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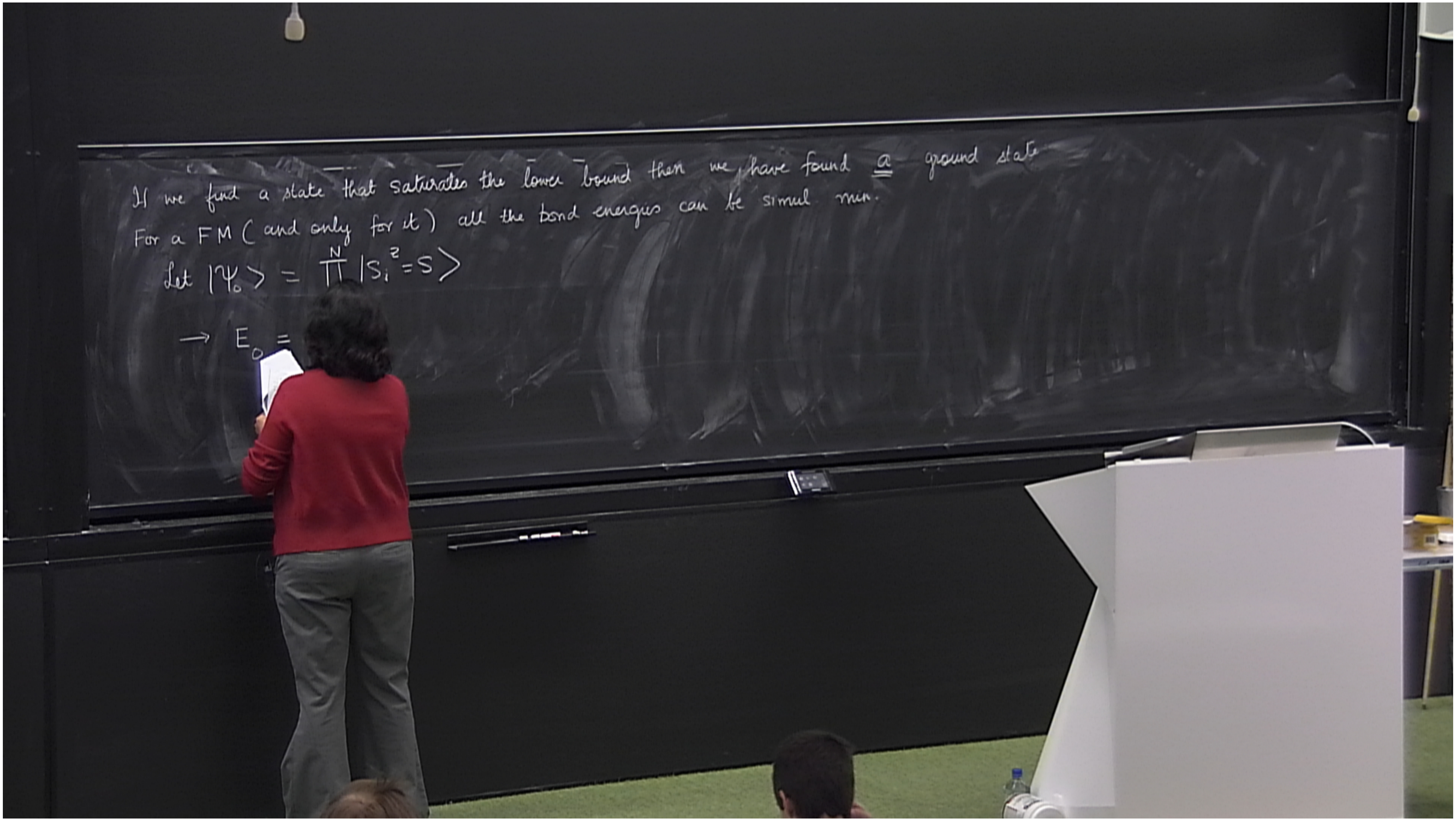
Let $|\Psi_0\rangle = \prod_{i=1}^{N} |S_i^z = S\rangle$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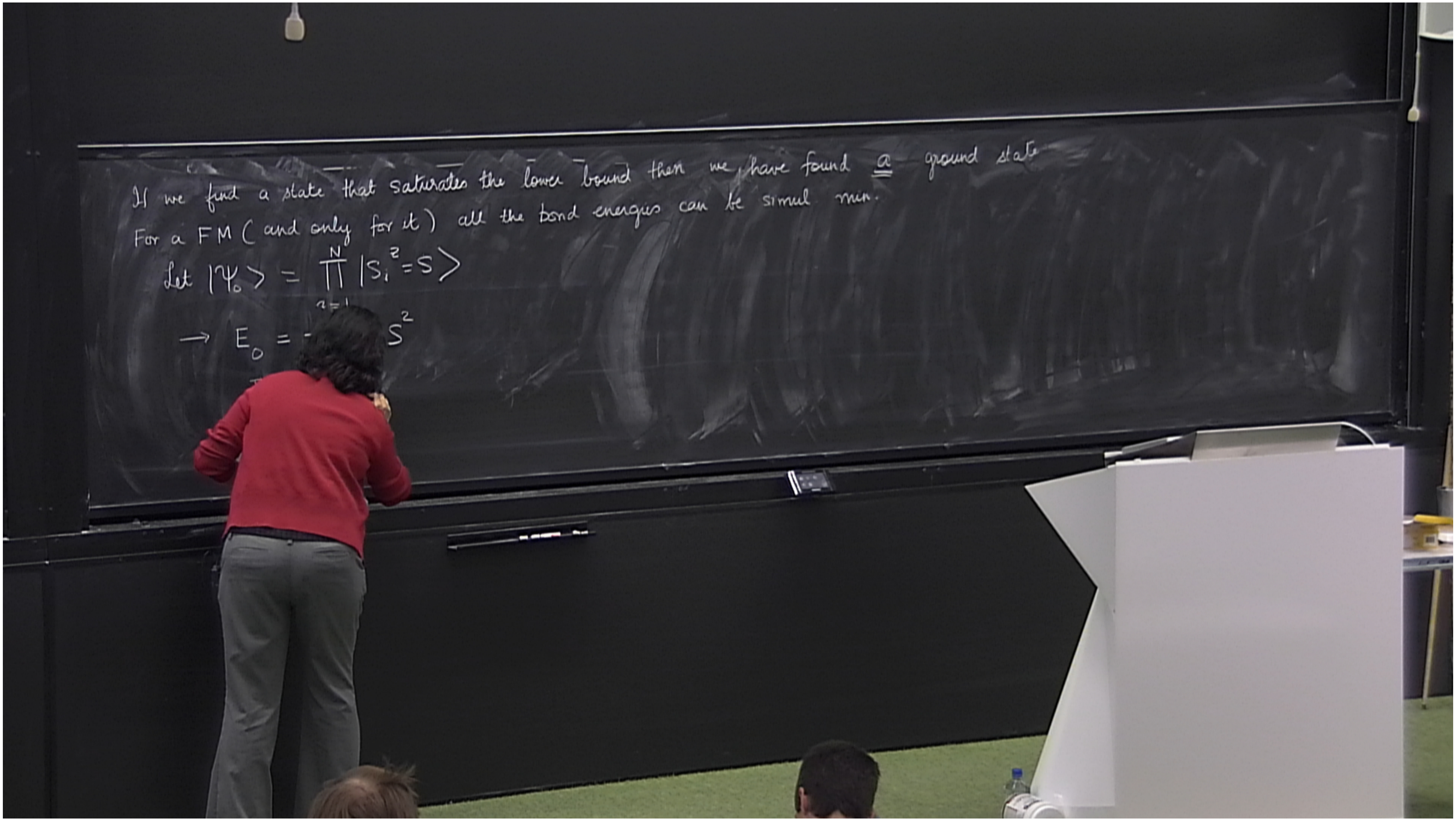
Let |Ψ₀ > = ∏^N |S_i^z = S >
→ E_0 =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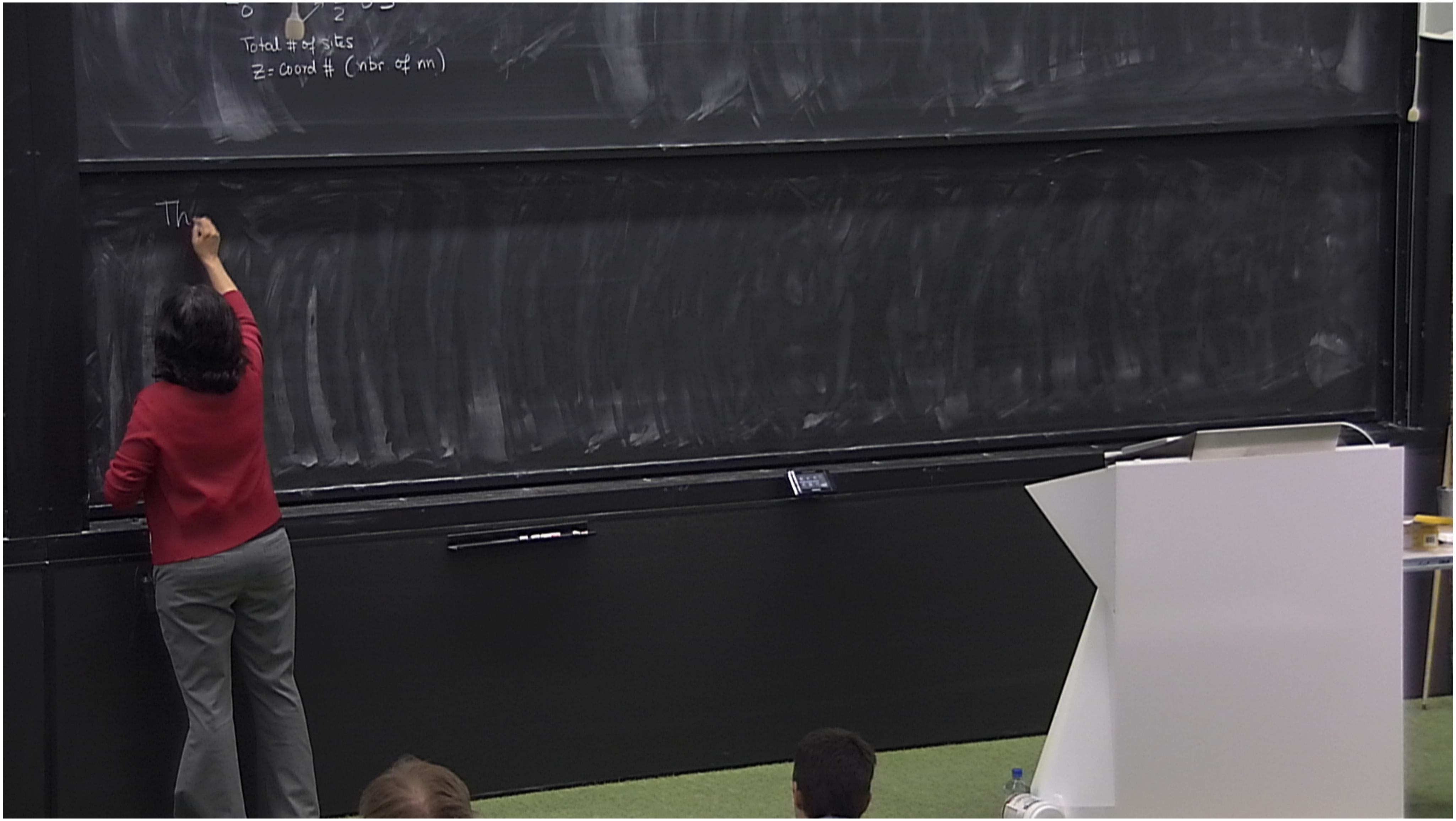
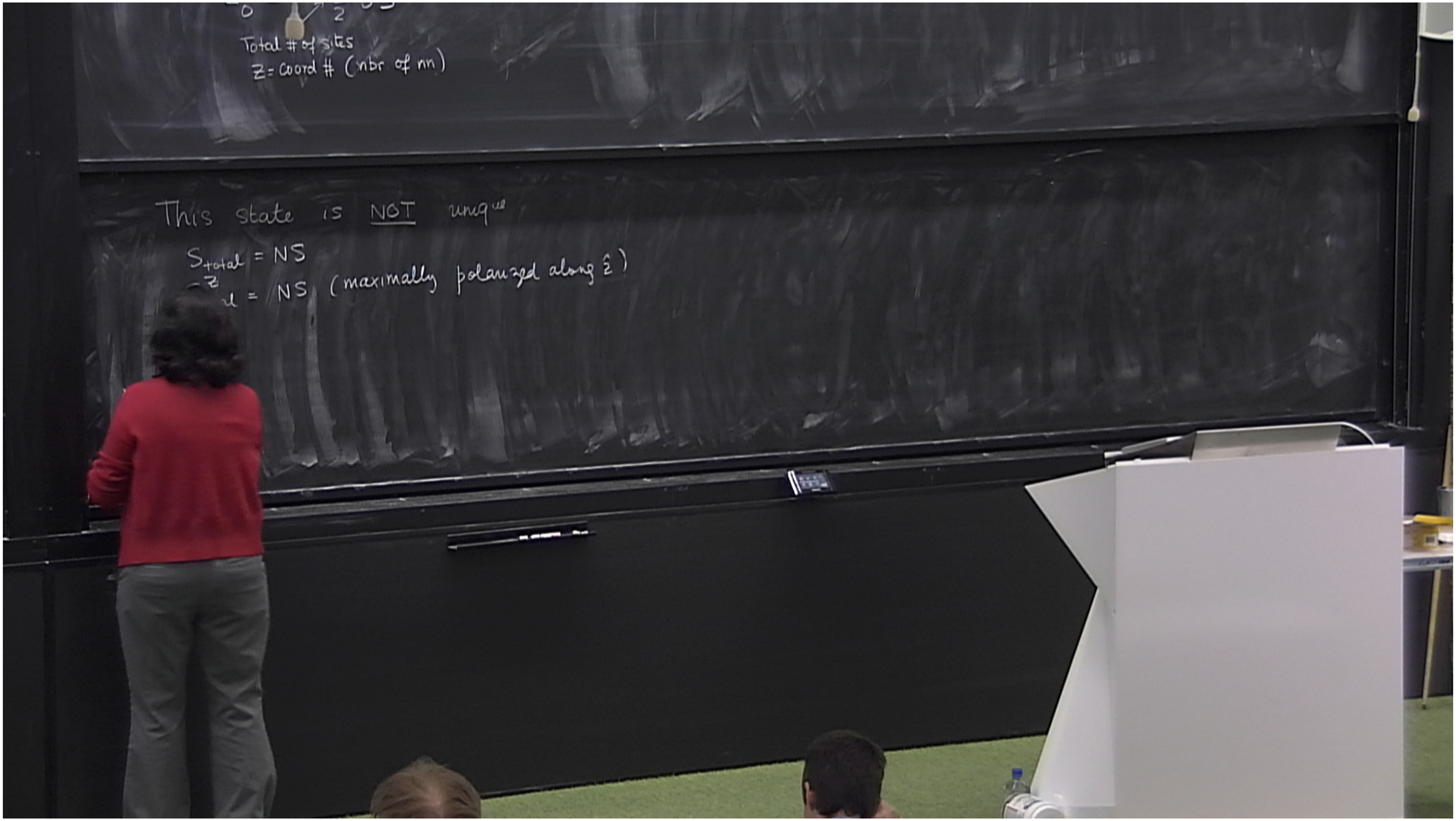
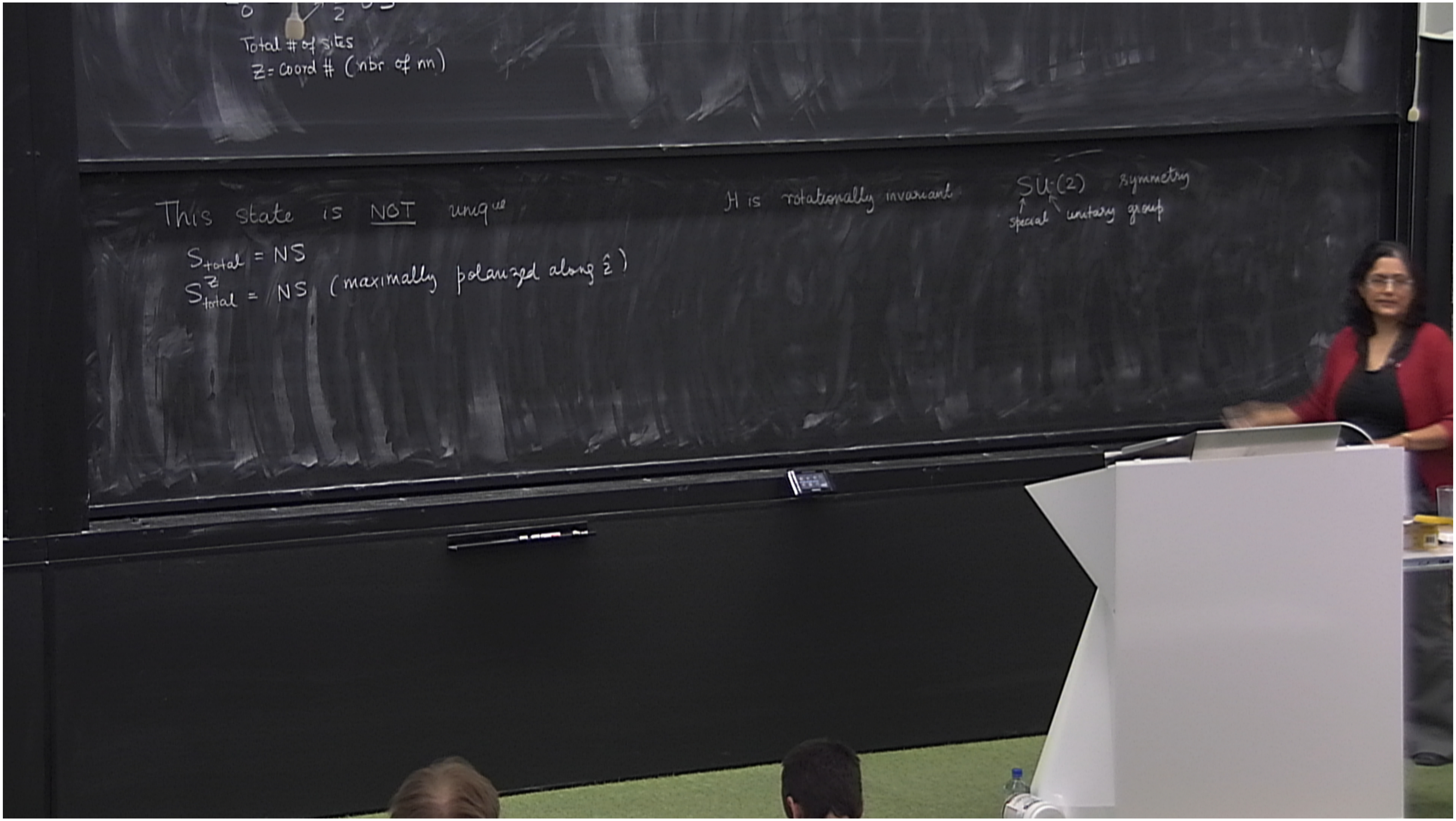
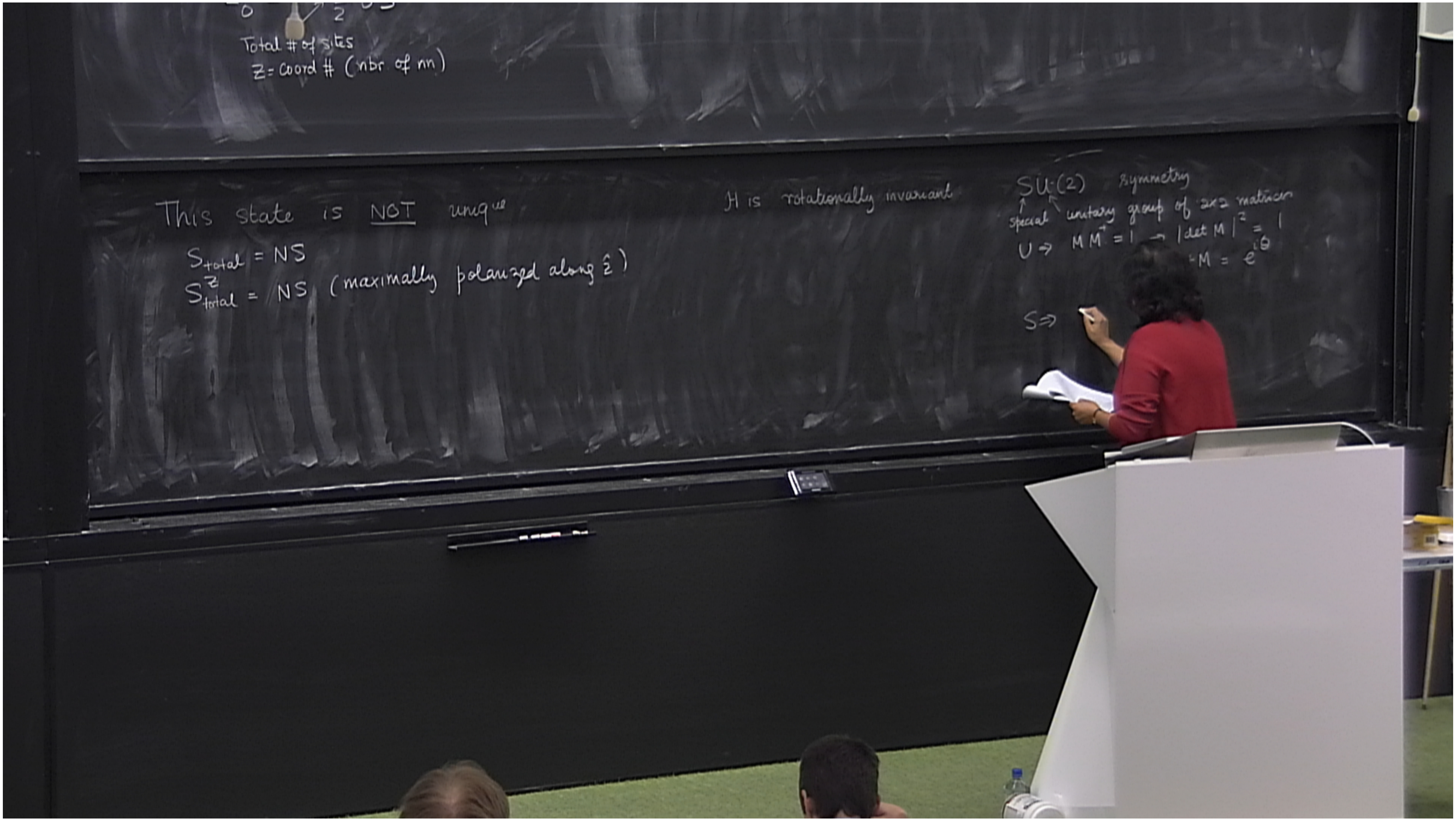
For a FM (and only for it) all the bond energies can be simul min.
Let |Ψ₀> = ∏ |Sᵢᶻ = S>
→ E₀ = S²


Total # of sites
z = coord # (nbr of nn)


Total # of sites
z = Coord # (nbr of nn)
This state is NOT unique
S_total = NS
= NS (maximally polarized along ẑ)


Total # of sites
z = Coord # (nbr of nn)
This state is NOT unique
H is rotationally invariant
SU(2) symmetry
special unitary group
S_total = NS
S^z_total = NS (maximally polarized along ẑ)


Total # of sites
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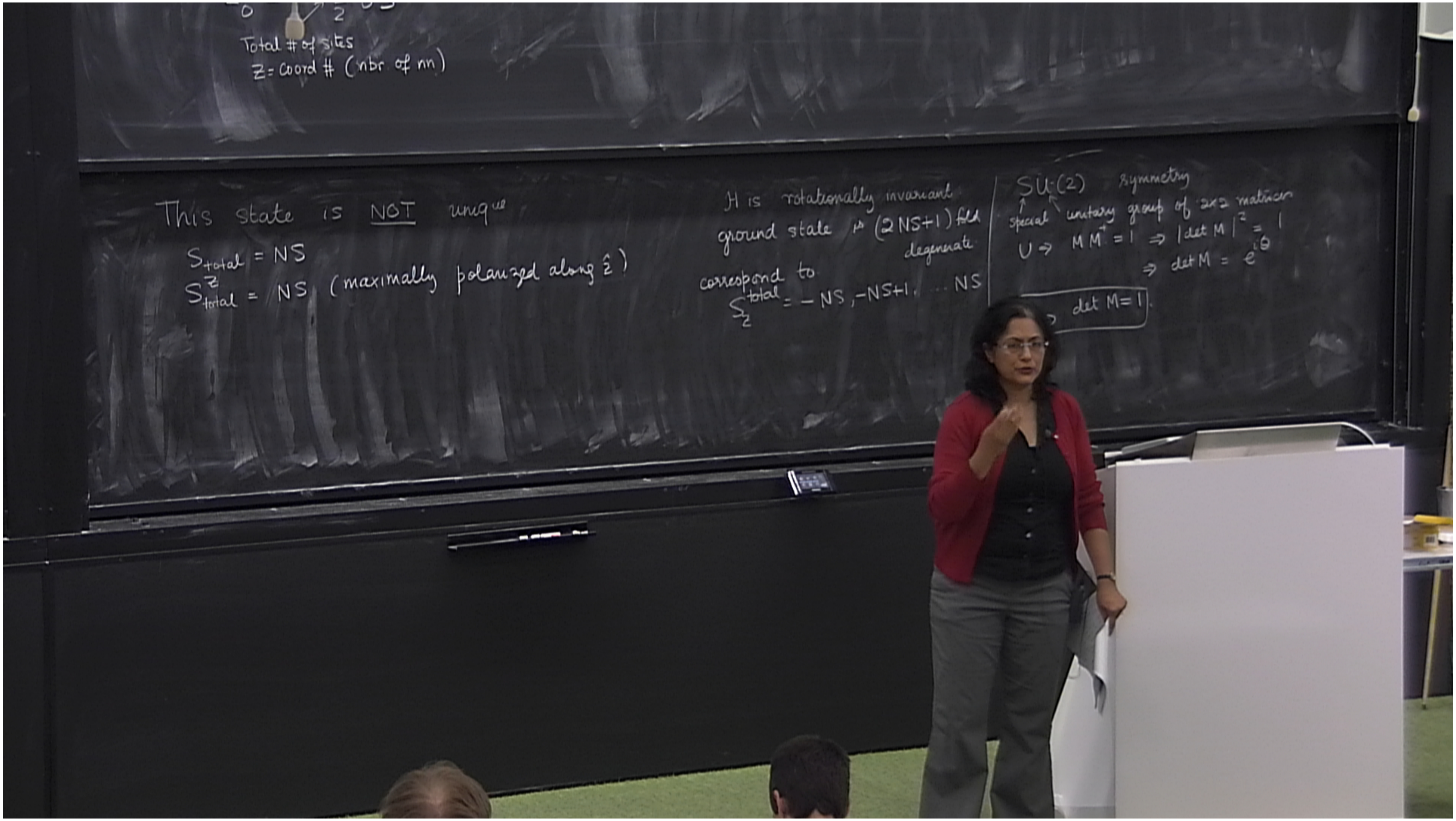
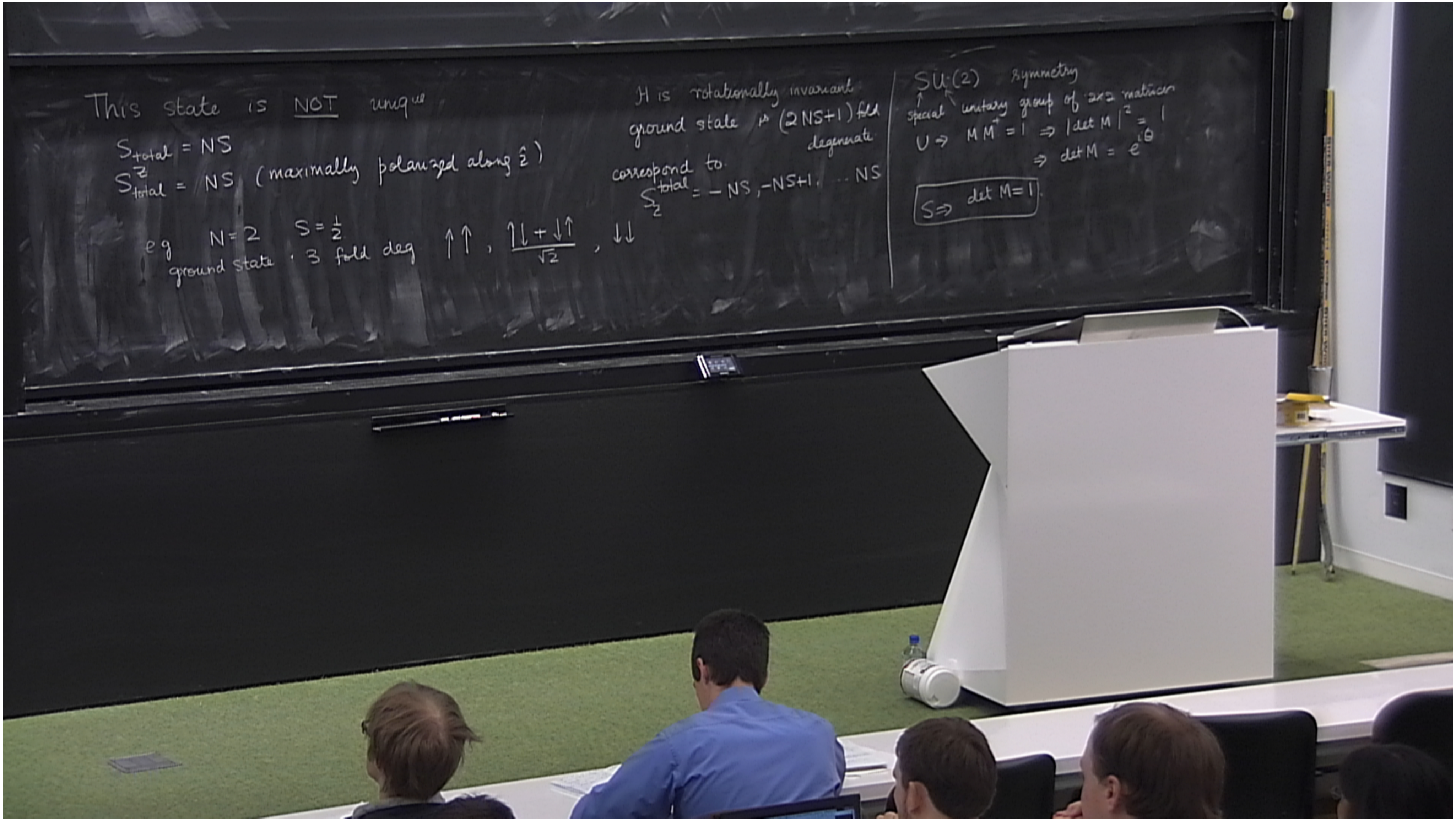
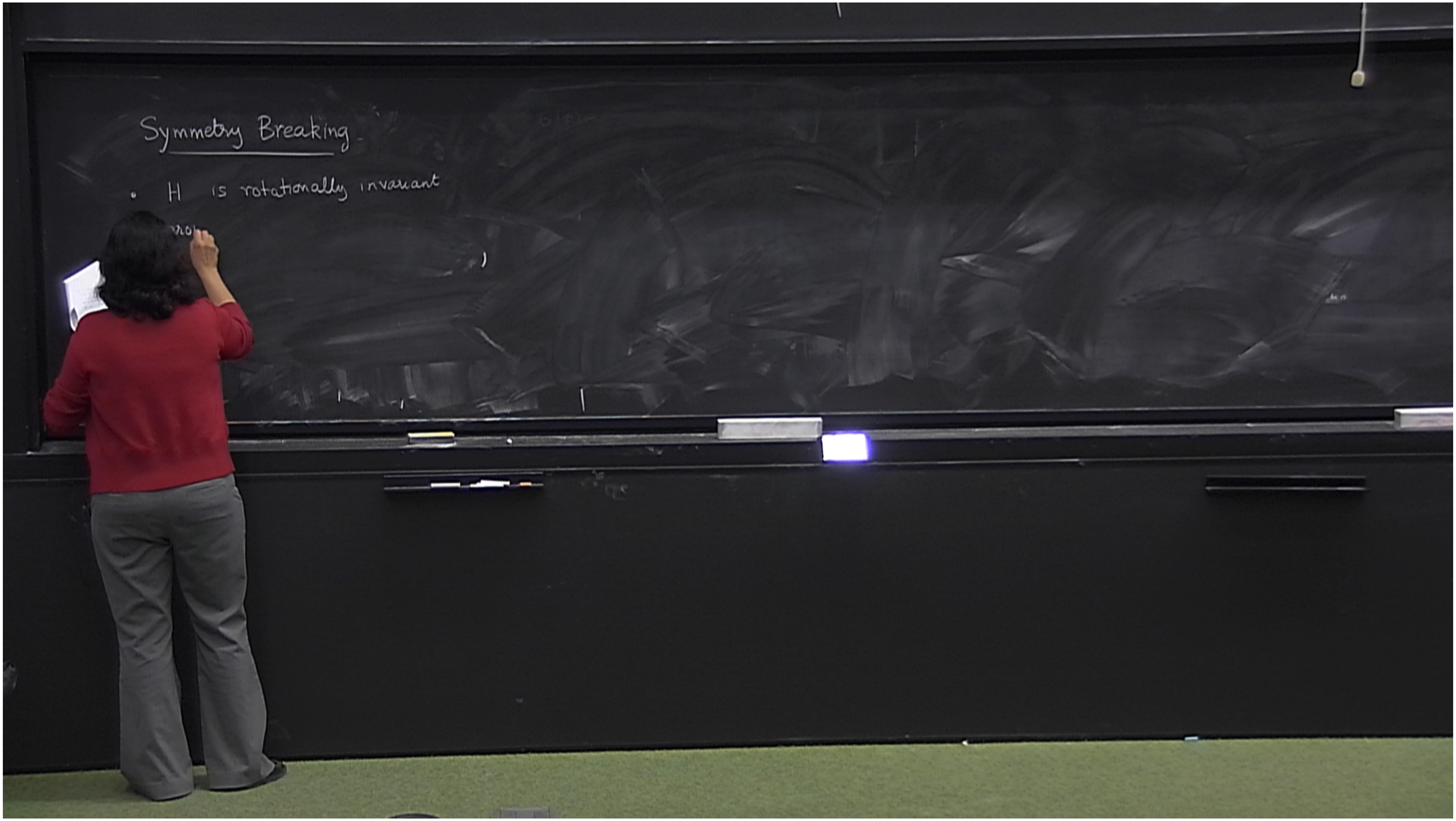
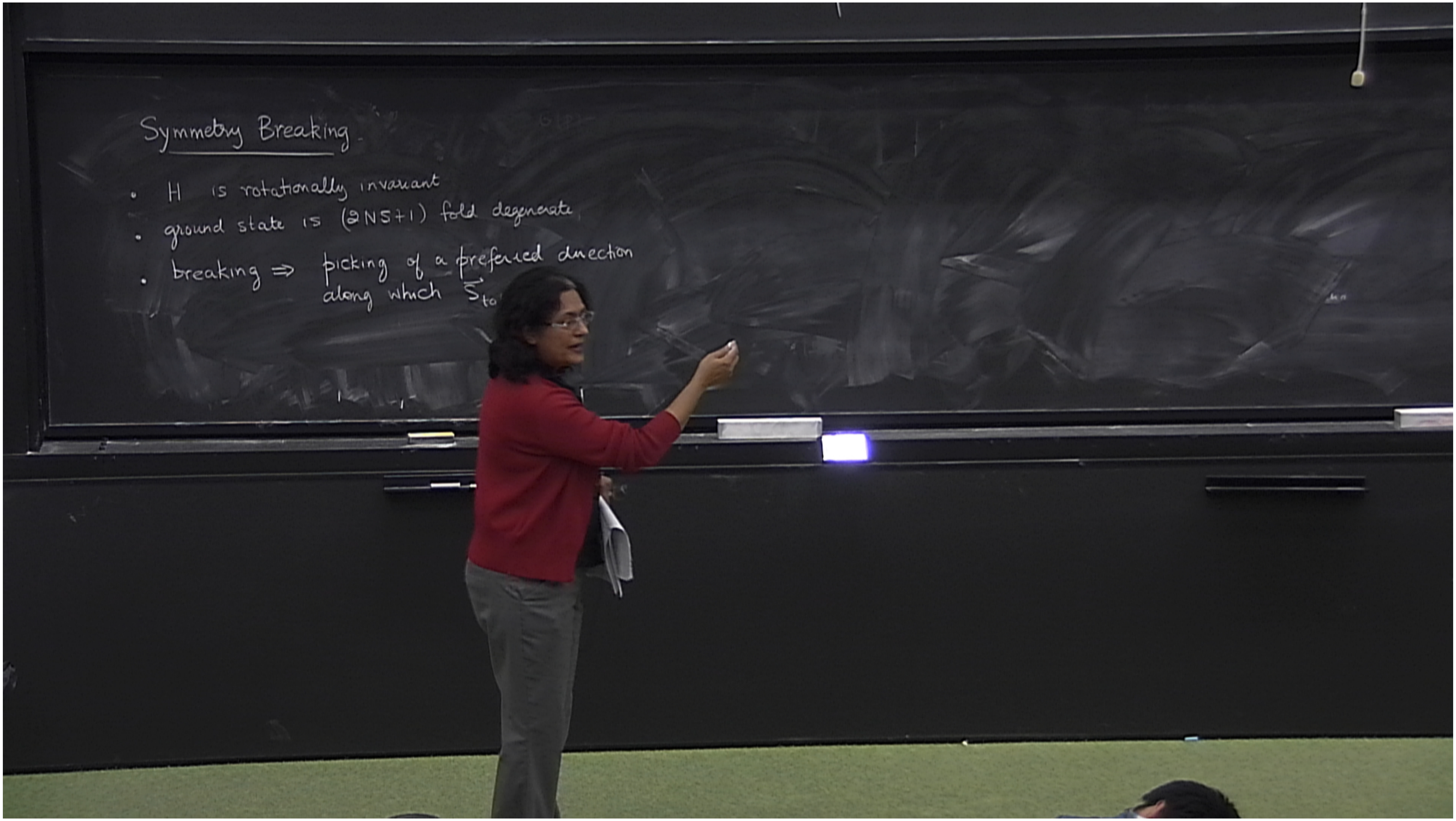
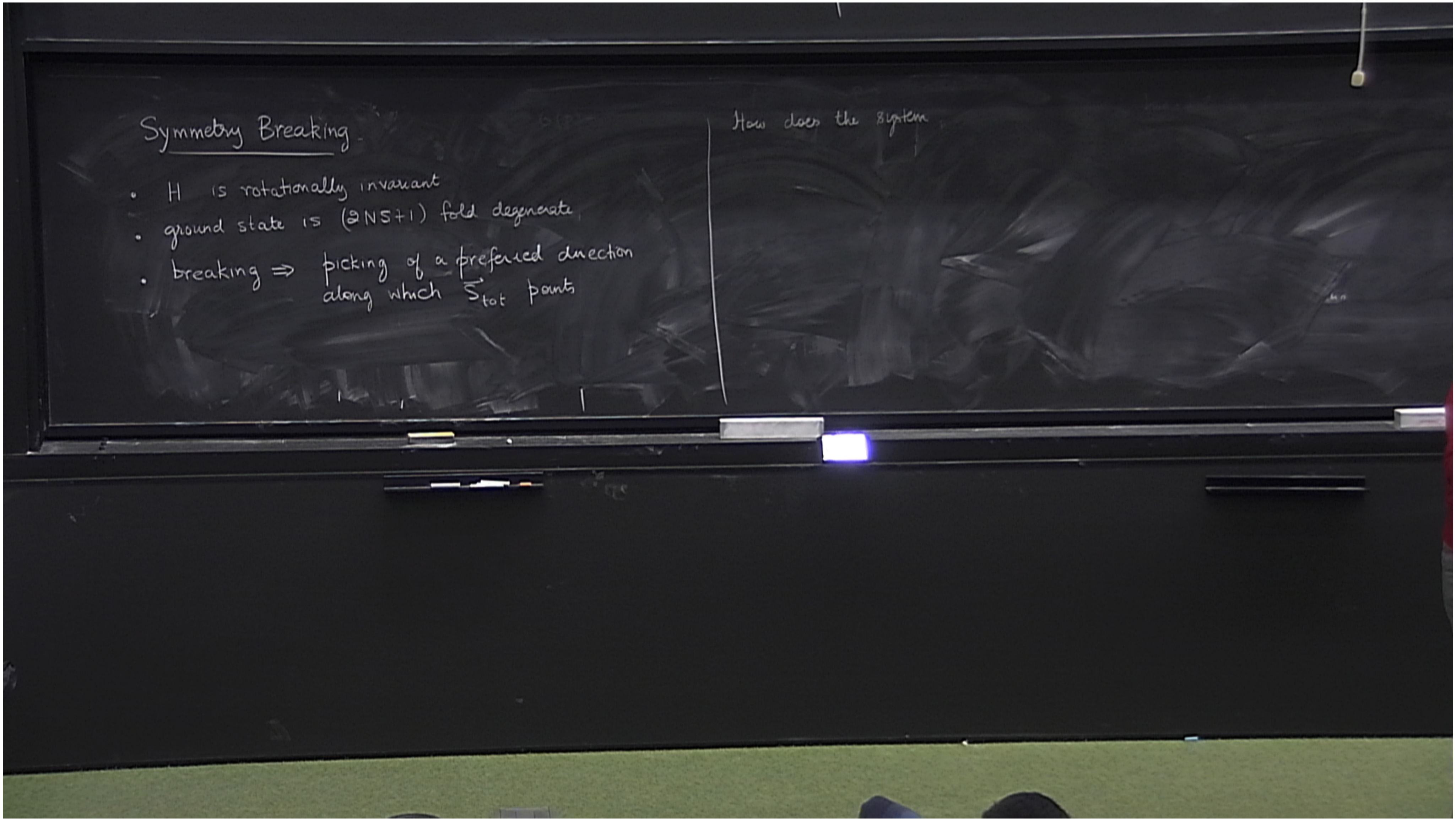
This state is NOT unique
S_total = NS
S^z_total = NS (maximally polarized along ẑ)
H is rotationally invariant
SU(2) symmetry
special unitary group of 2x2 matrices
U ⇒ M M† = 1 → |det M|² = 1
M = e^iθ
S ⇒

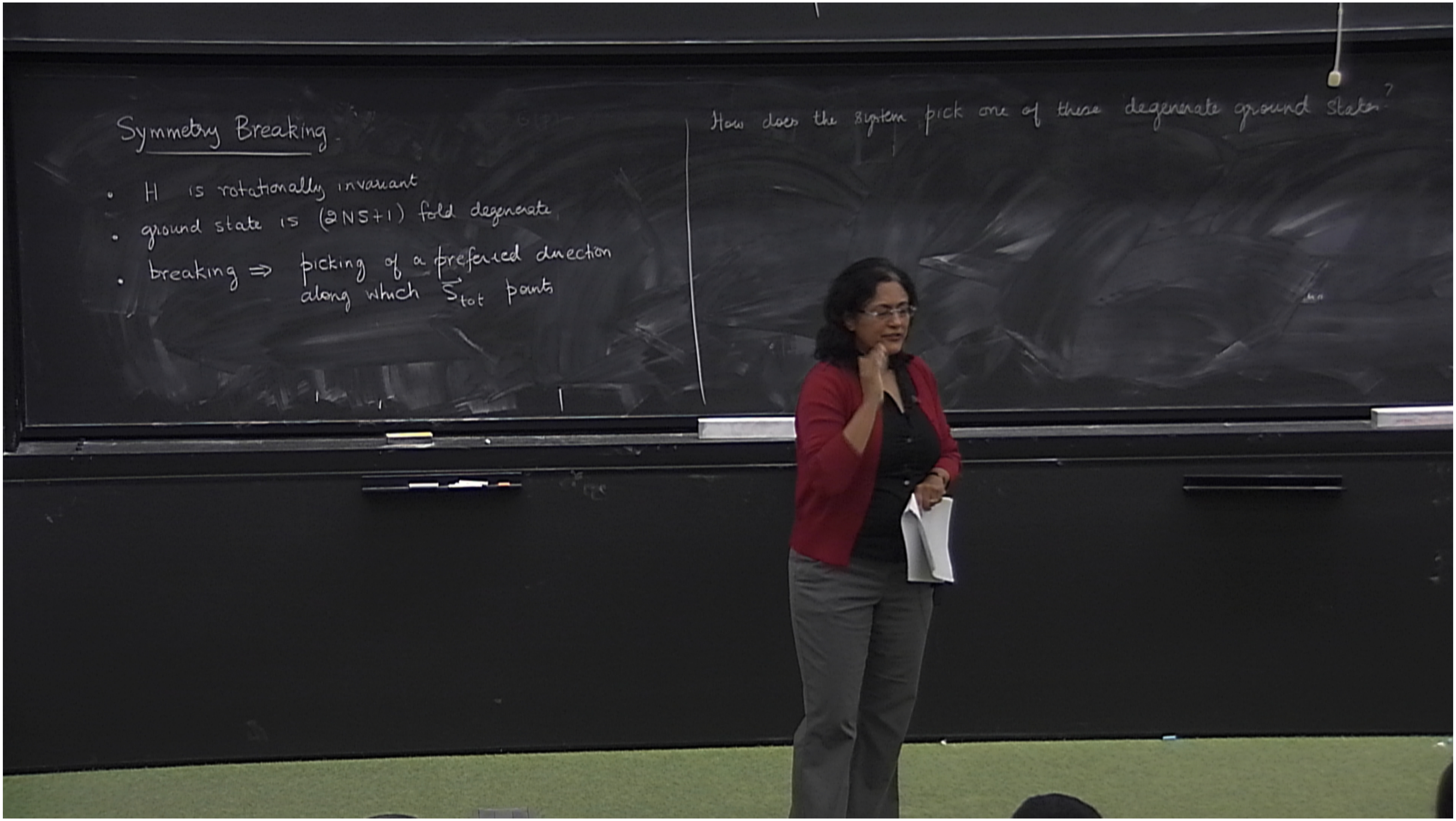

Total # of sites
z = coord # (nbr of nn)
This state is NOT unique
H is rotationally invariant
ground state is (2NS+1) fold degenerate
correspond to
SU(2) symmetry
special unitary group of 2x2 matrices
det M = 1


This state is NOT unique
S_total = NS
S^z_total = NS (maximally polarized along ẑ)
eg N = 2 S = 1/2
ground state : 3 fold deg ↑↑ , (↑↓ + ↓↑)/√2 , ↓↓
H is rotationally invariant
ground state is (2NS+1) fold degenerate
correspond to
S_z^total = −NS, −NS+1, … NS
SU(2) symmetry
special unitary group of 2×2 matrices
U ⇒ MM† = 1 ⇒ |det M|² = 1
⇒ det M = e^{iθ}
S ⇒ det M = 1


Symmetry Breaking
• H is rotationally invariant


Symmetry Breaking
• H is rotationally invariant
• ground state is (2NS+1) fold degenerate
• breaking ⇒ picking of a preferred direction
along which S


Symmetry Breaking
• H is rotationally invariant
• ground state is (2NS+1) fold degenerate
• breaking ⇒ picking of a preferred direction
along which S tot points
How does the system


Symmetry Breaking
• H is rotationally invariant
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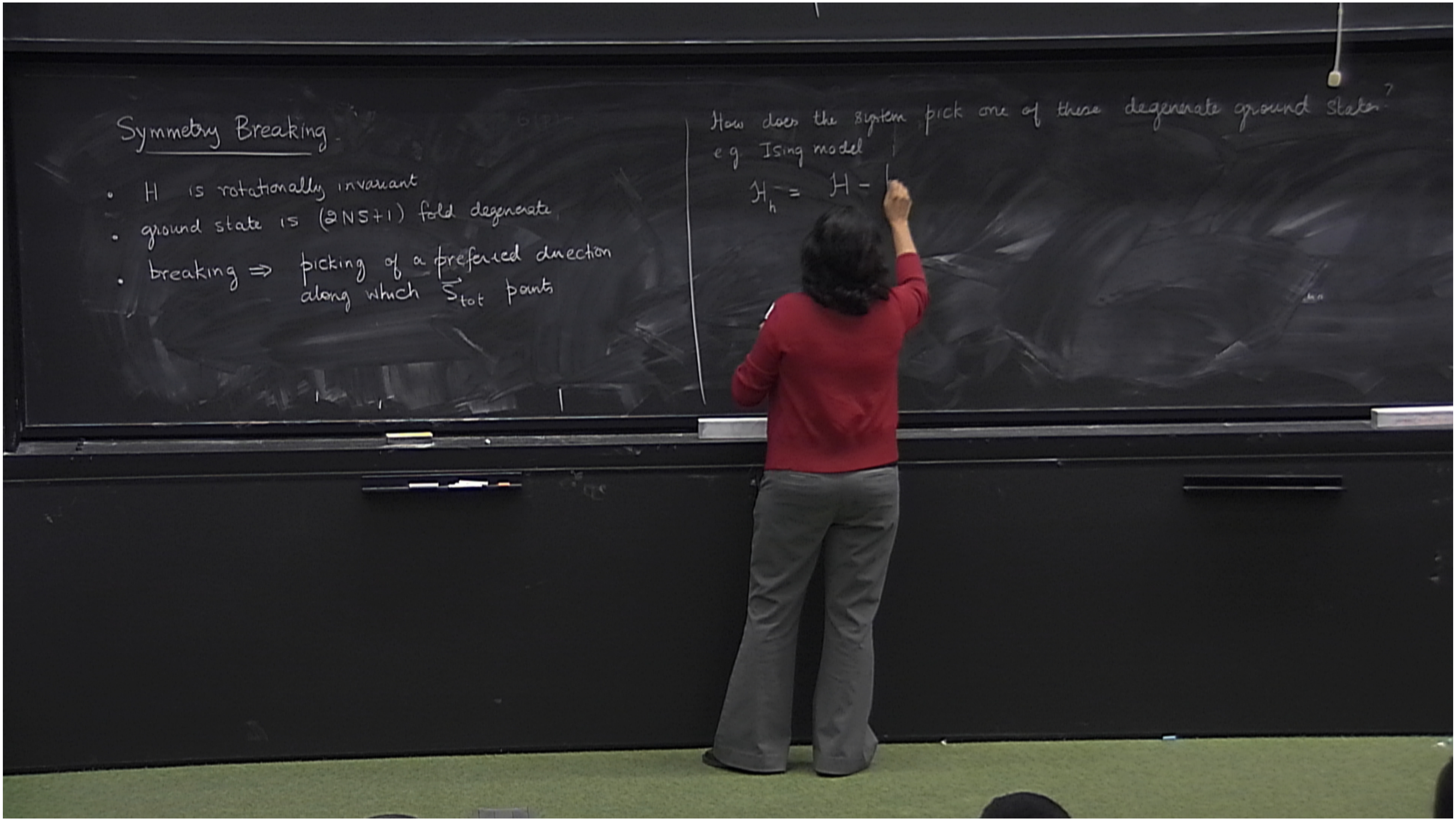
How does the system pick one of these degenerate ground states?


Symmetry Breaking
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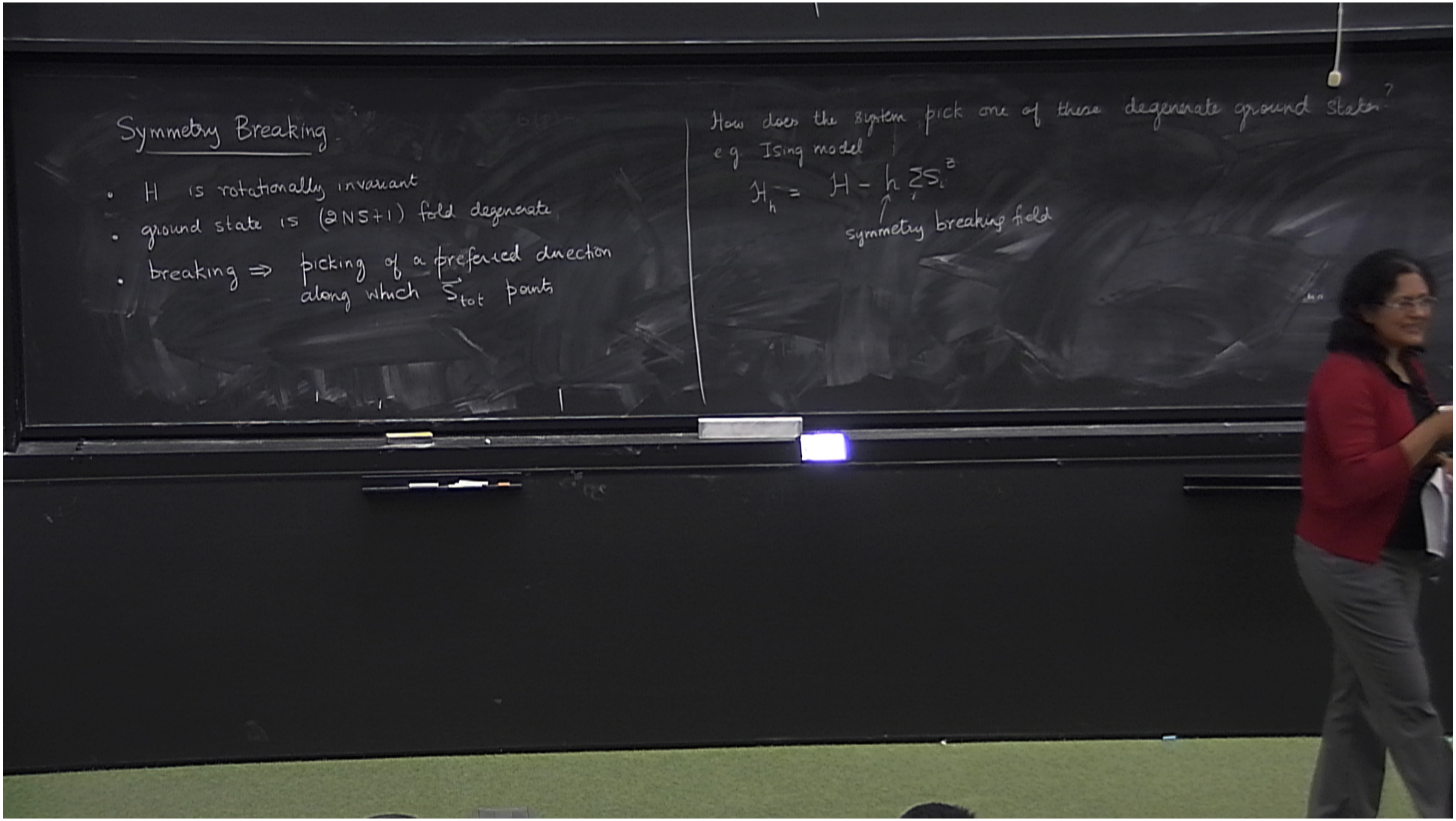
How does the system pick one of these degenerate ground states?
e.g. Ising model
$\mathcal{H}_h = \mathcal{H} -$


Symmetry Breaking
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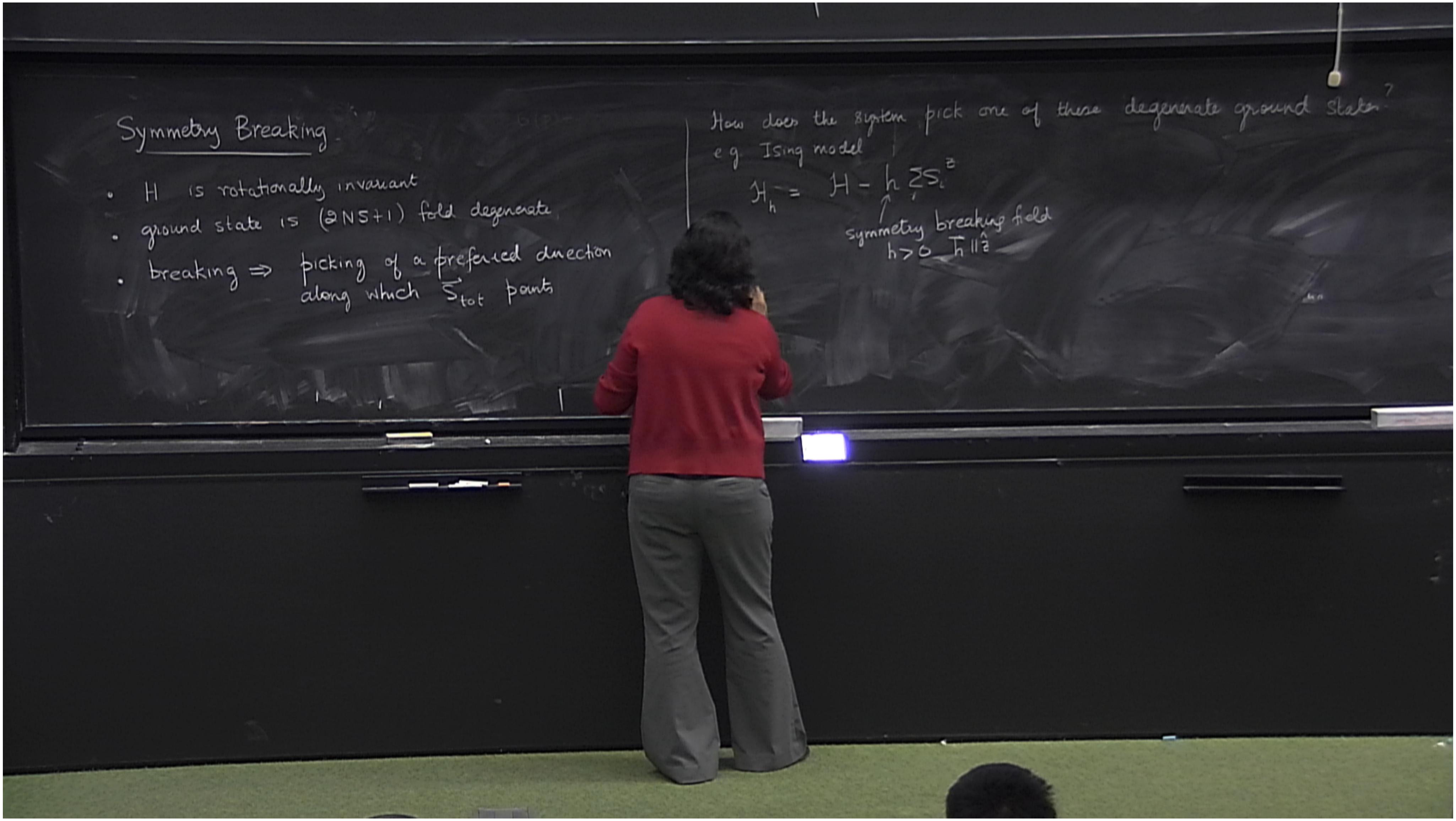
How does the system pick one of these degenerate ground states?
e.g. Ising model
$\mathcal{H}_h = H - h \sum_i S_i^z$
symmetry breaking field


Symmetry Breaking
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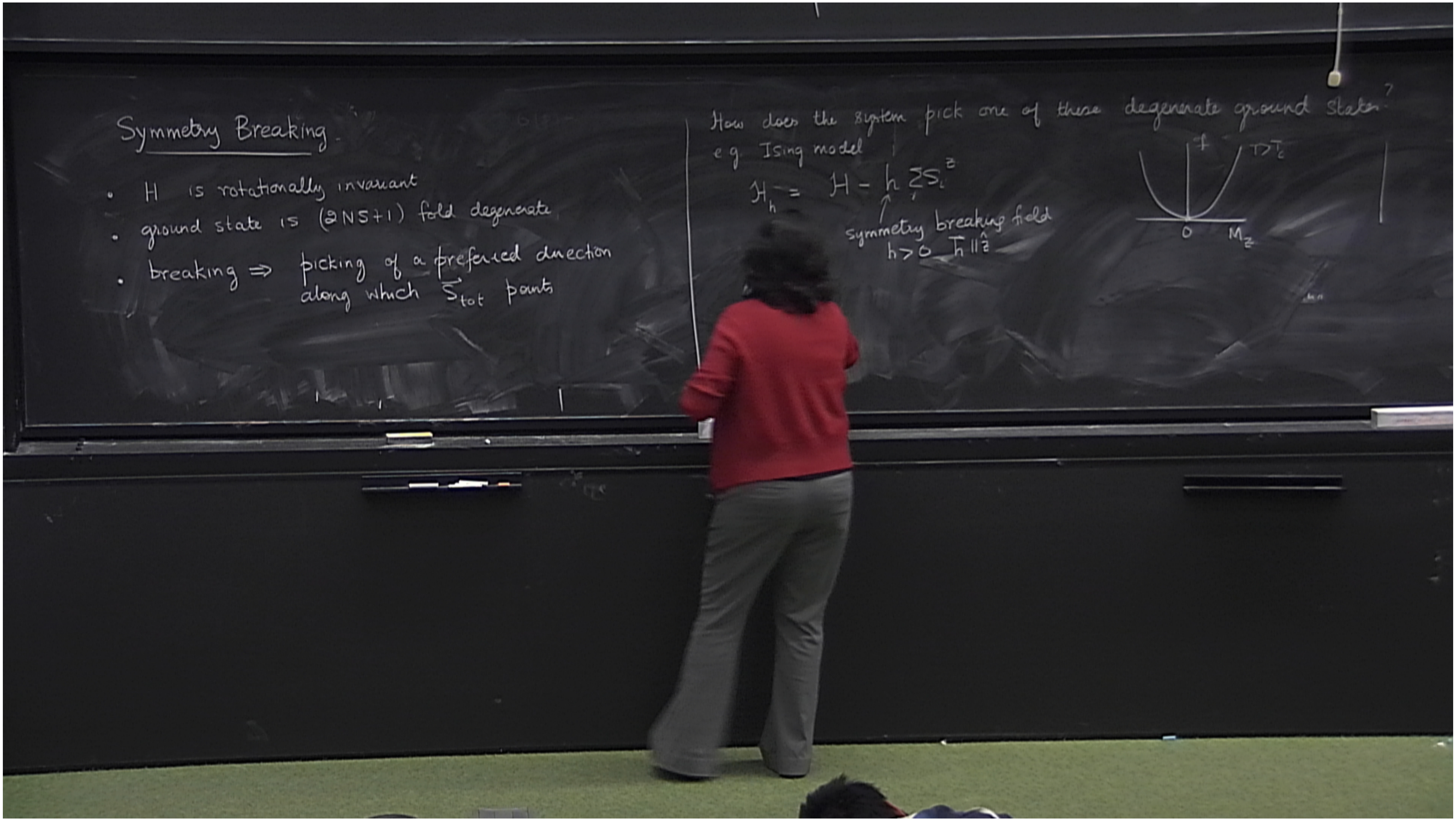
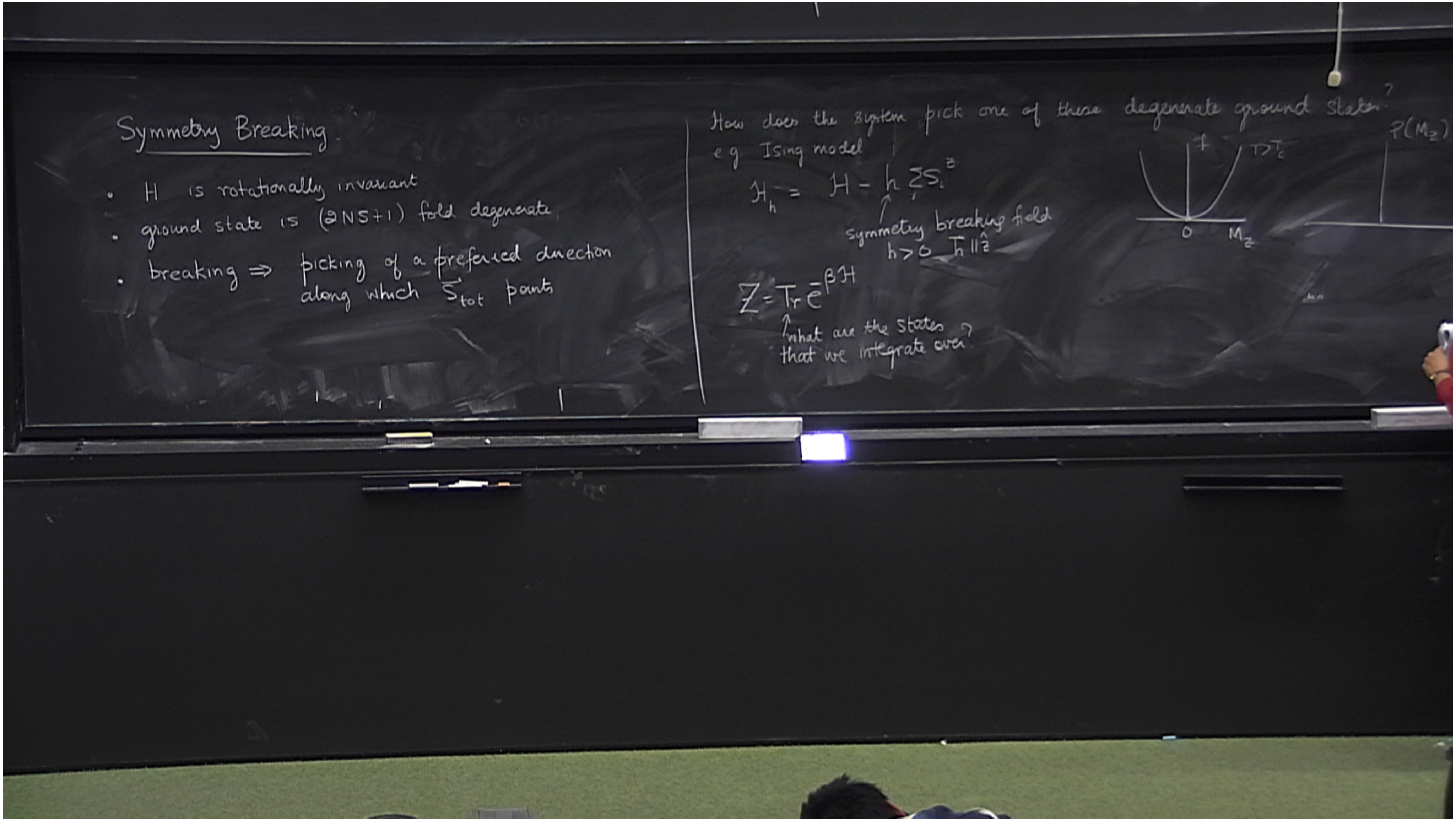
How does the system pick one of these degenerate ground states?
e.g. Ising model
$\mathcal{H}_h = \mathcal{H} - h \sum_i S_i^z$
symmetry breaking field
$h > 0$ $\vec{h} \parallel \hat{z}$


Symmetry Breaking
• H is rotationally invariant
• ground state is (2NS+1) fold degenerate
• breaking ⇒ picking of a preferred direction along which S_tot points
How does the system pick one of these degenerate ground states?
e.g. Ising model
H_h = H − h Σ_i S_i^z
symmetry breaking field
h>0 h ∥ ẑ
f
T>T_c
0
M_z


Symmetry Breaking
• H is rotationally invariant
• ground state is (2NS+1) fold degenerate
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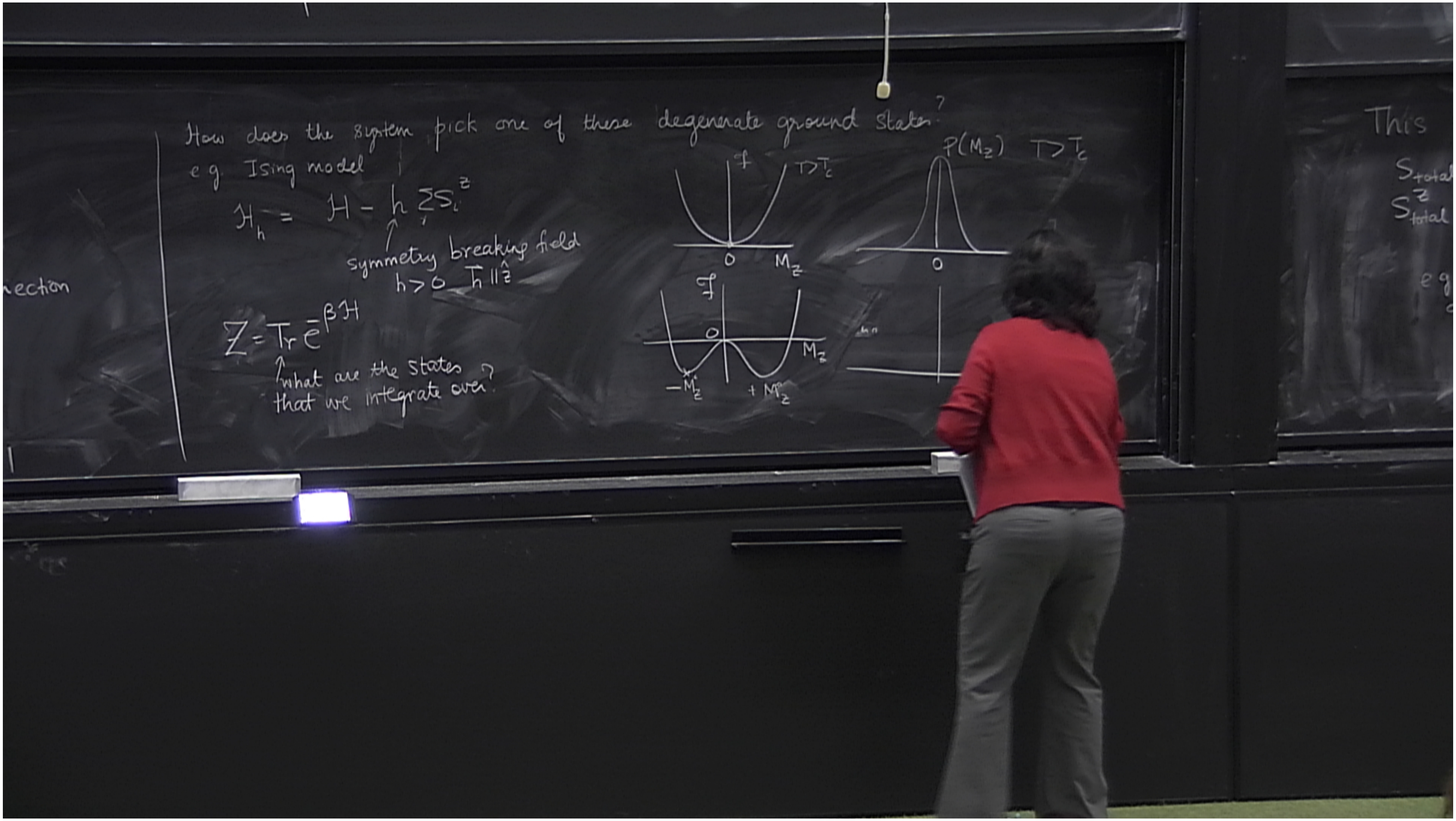
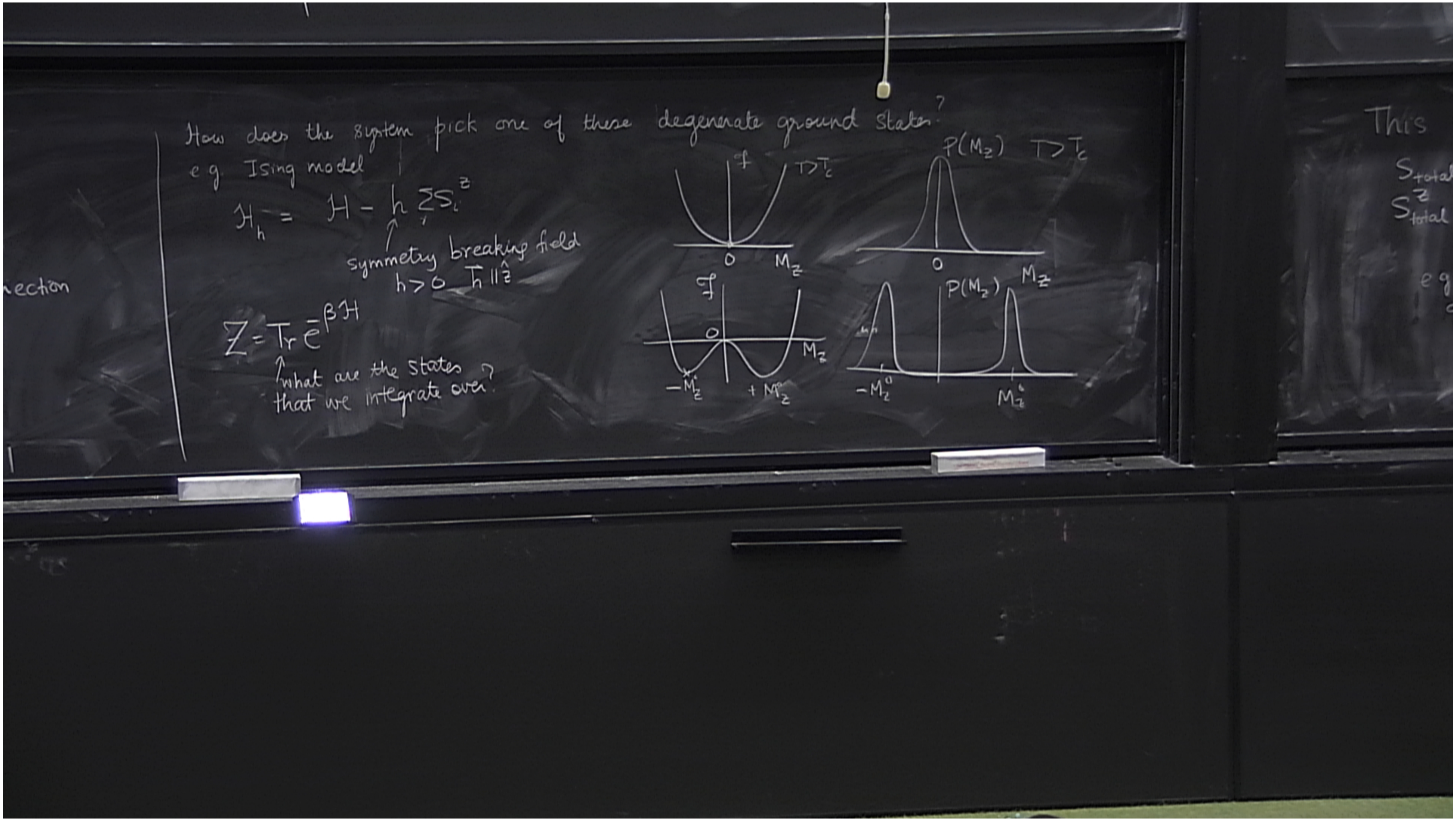
How does the system pick one of these degenerate ground states?
e.g. Ising model
H_h = H − h ΣS_i^z
symmetry breaking field
h>0 h ∥ ẑ
Z = Tr e^{−βH}
what are the states that we integrate over?
T>T_c
0
M_z
P(M_z)


How does the system pick one of these degenerate ground states?
e.g. Ising model
symmetry breaking field
what are the states that we integrate over?
T>Tc
P(Mz)
Mz
This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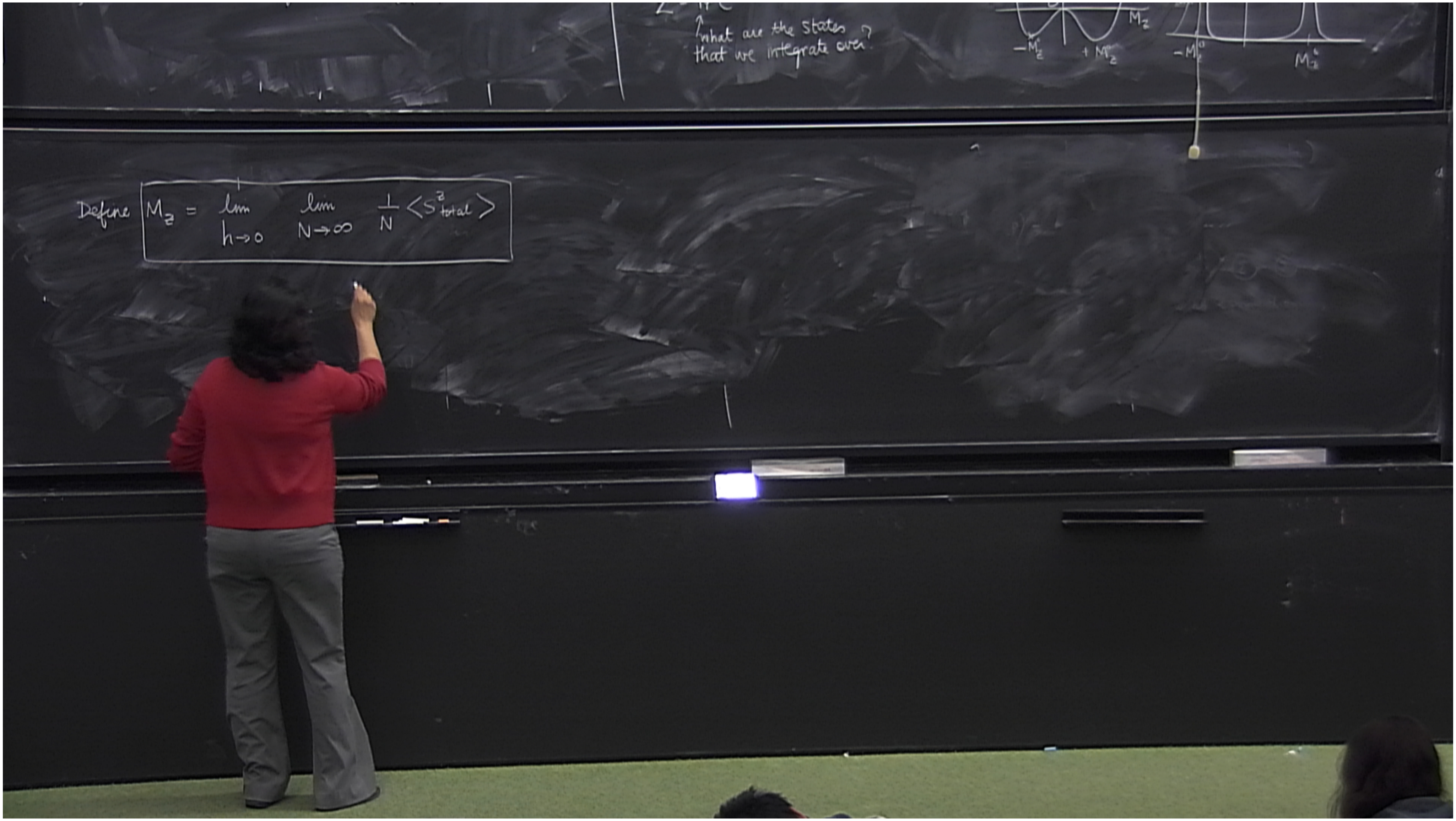
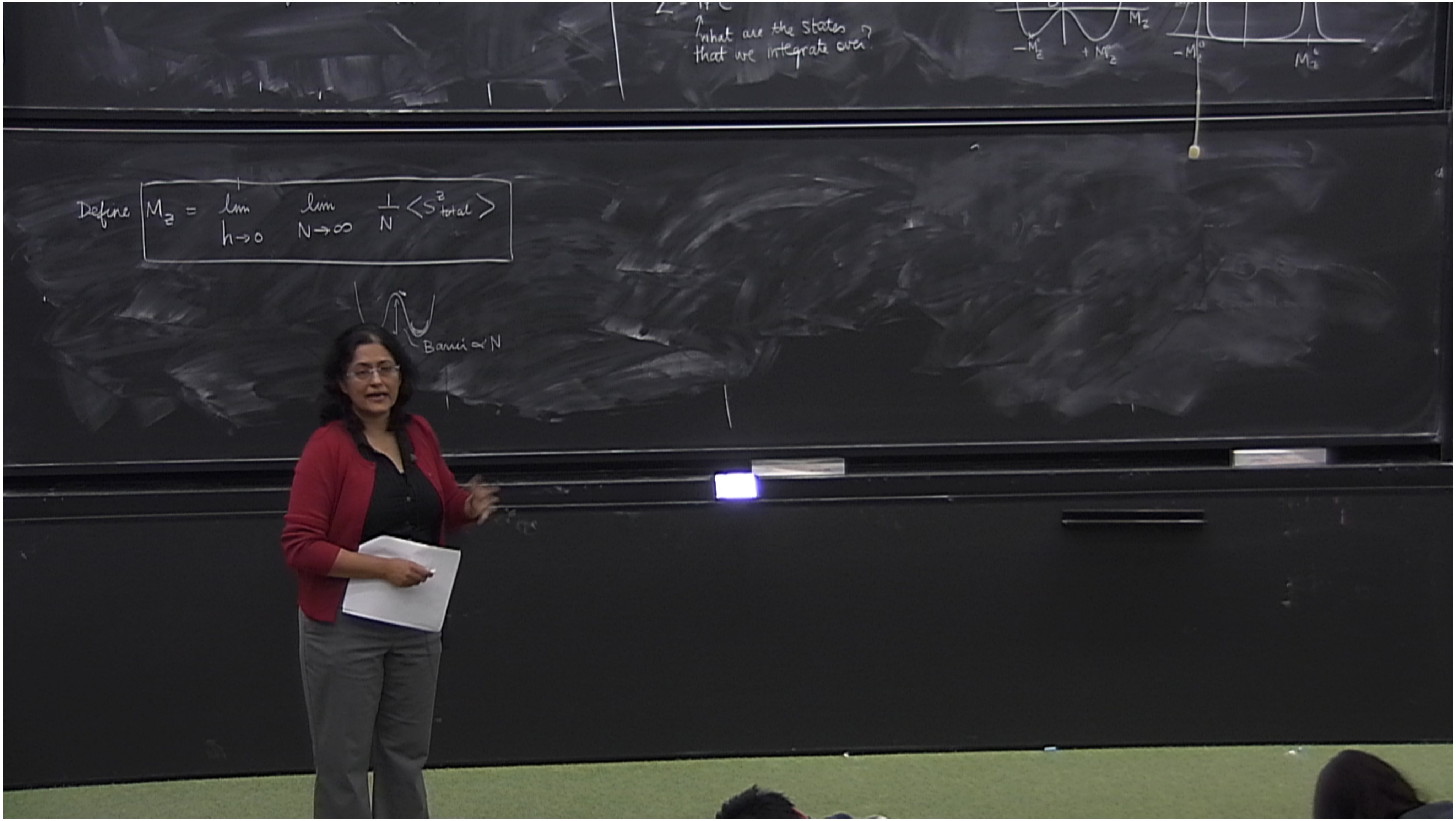
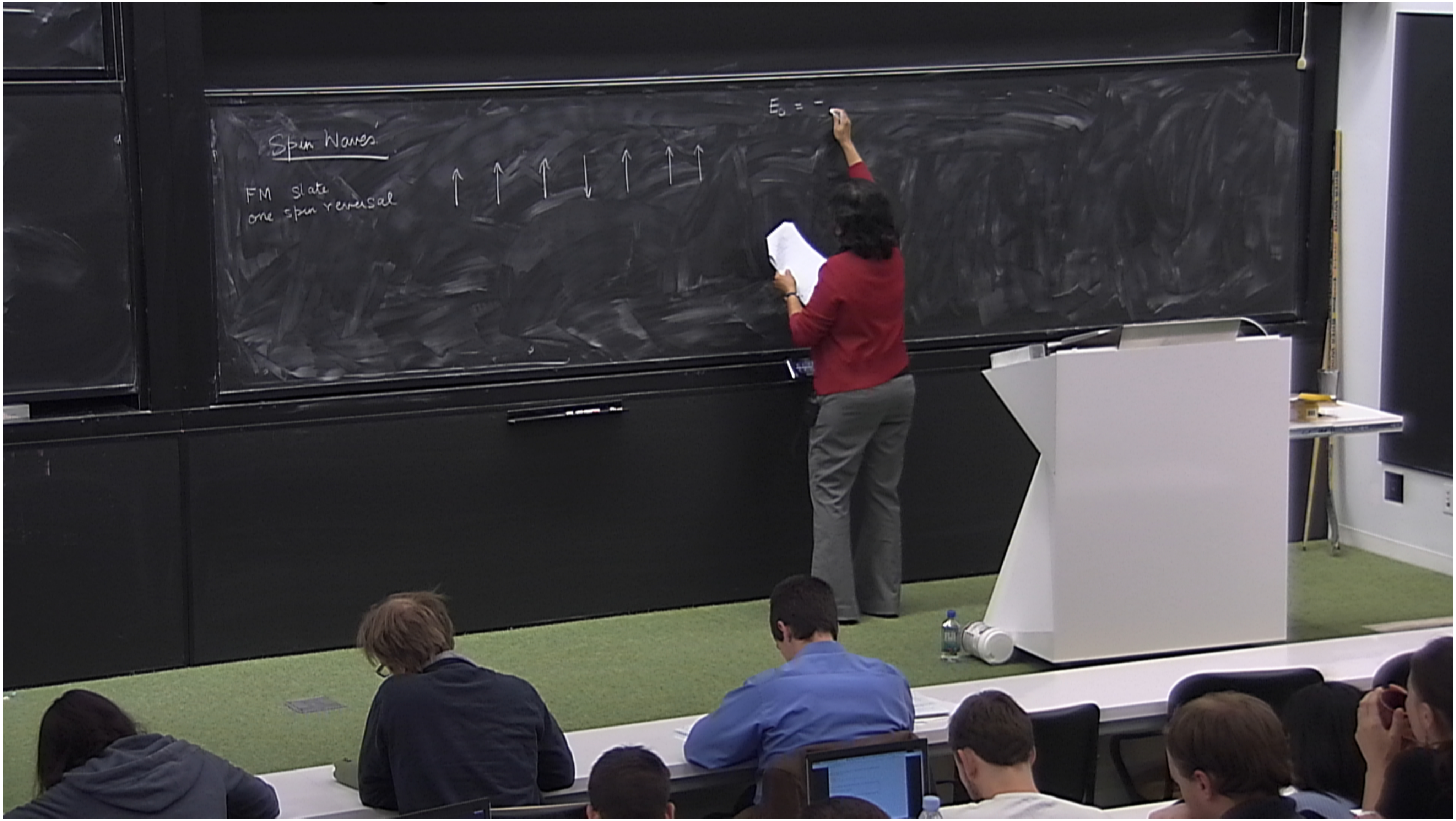
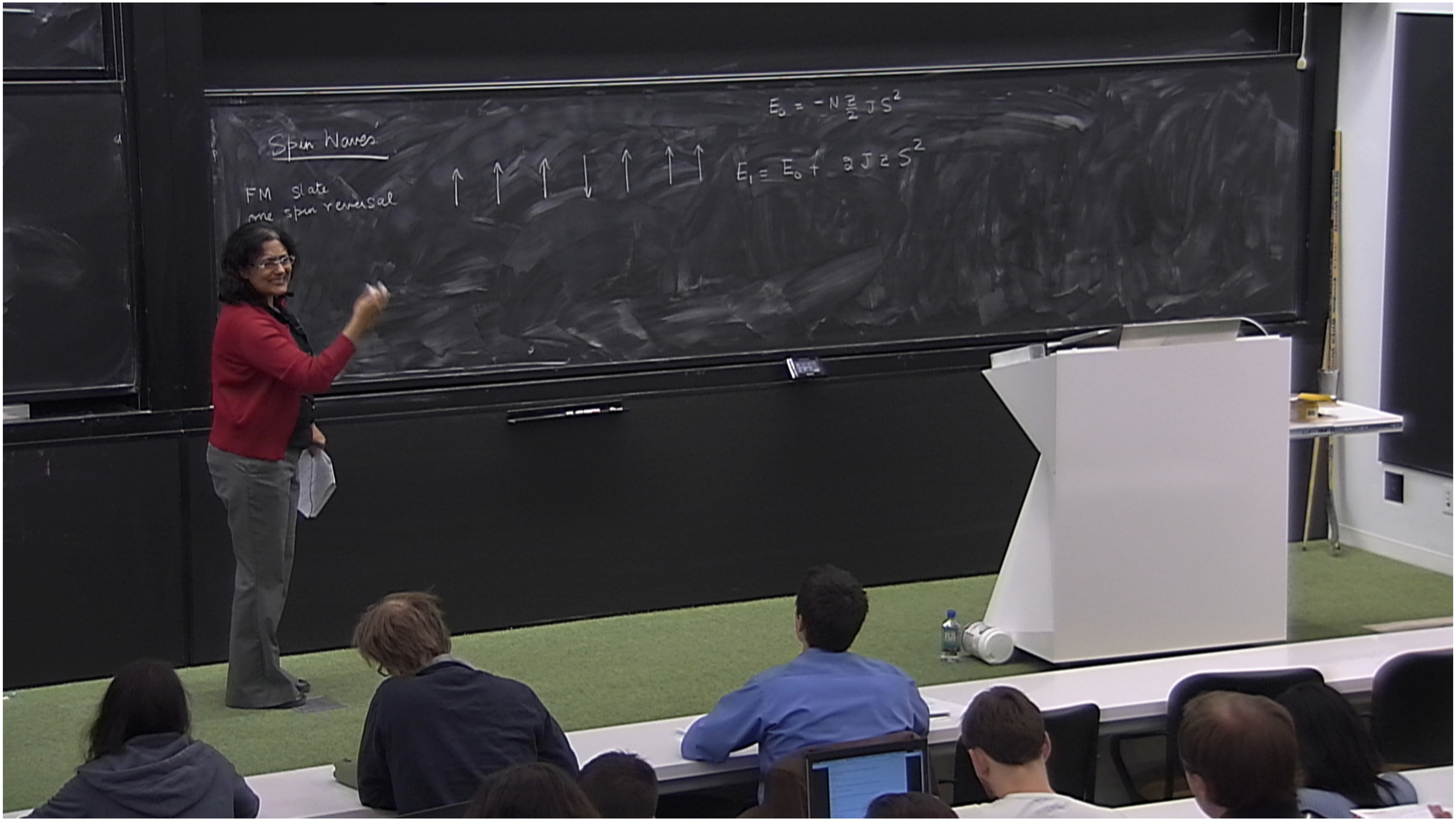
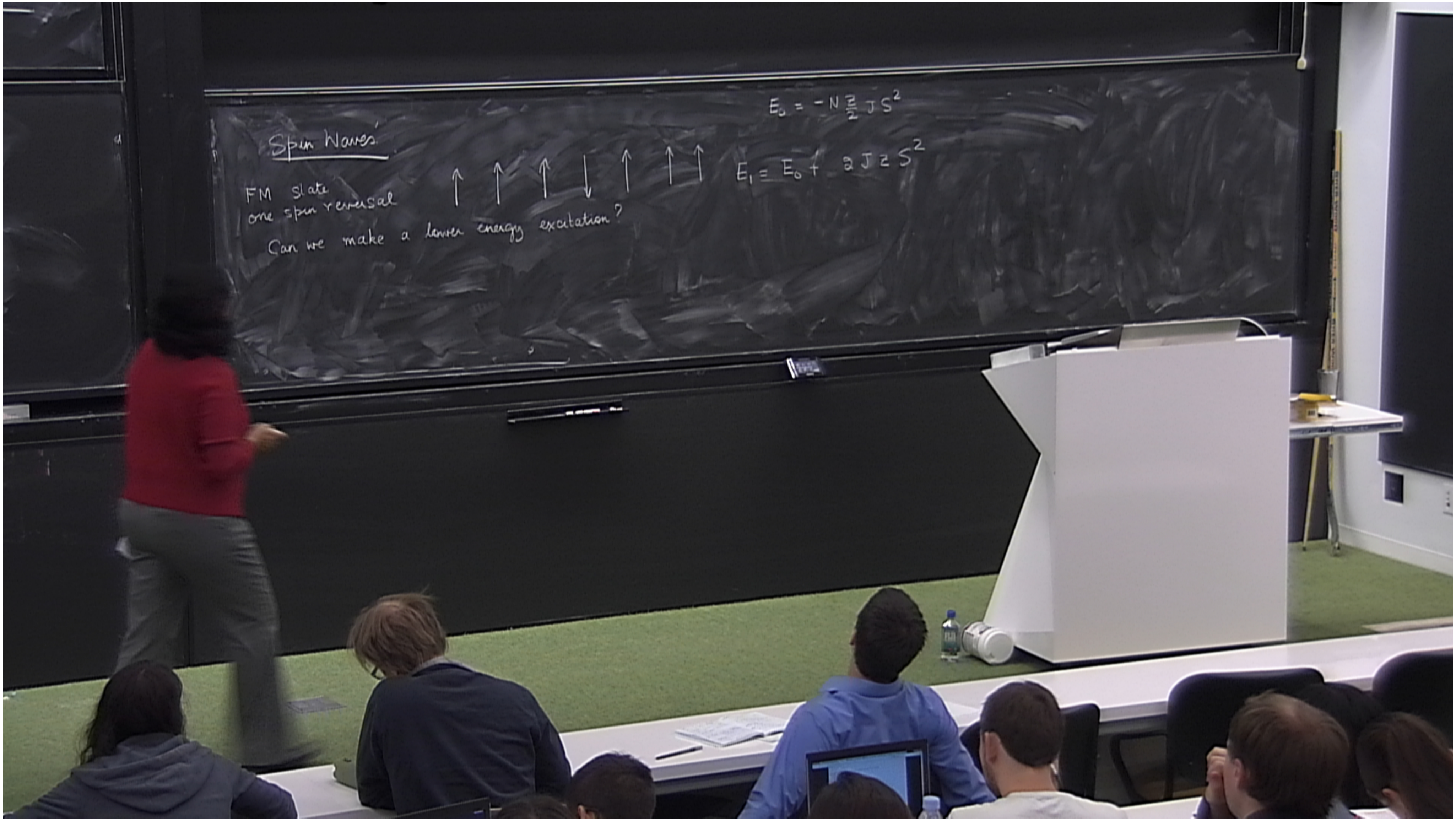
e.g. Ising model
$\mathcal{H}_h = \mathcal{H} - h \sum_i S_i^z$
symmetry breaking field
$h > 0$ $\vec{h} \parallel \hat{z}$
$Z = \mathrm{Tr}\, e^{-\beta \mathcal{H}}$
what are the states that we integrate over?
$T > T_c$
M_z
$P(M_z)$
$-M_z^0$
$+M_z^0$
M_z^0
This
S_{total}
S^z_{total}

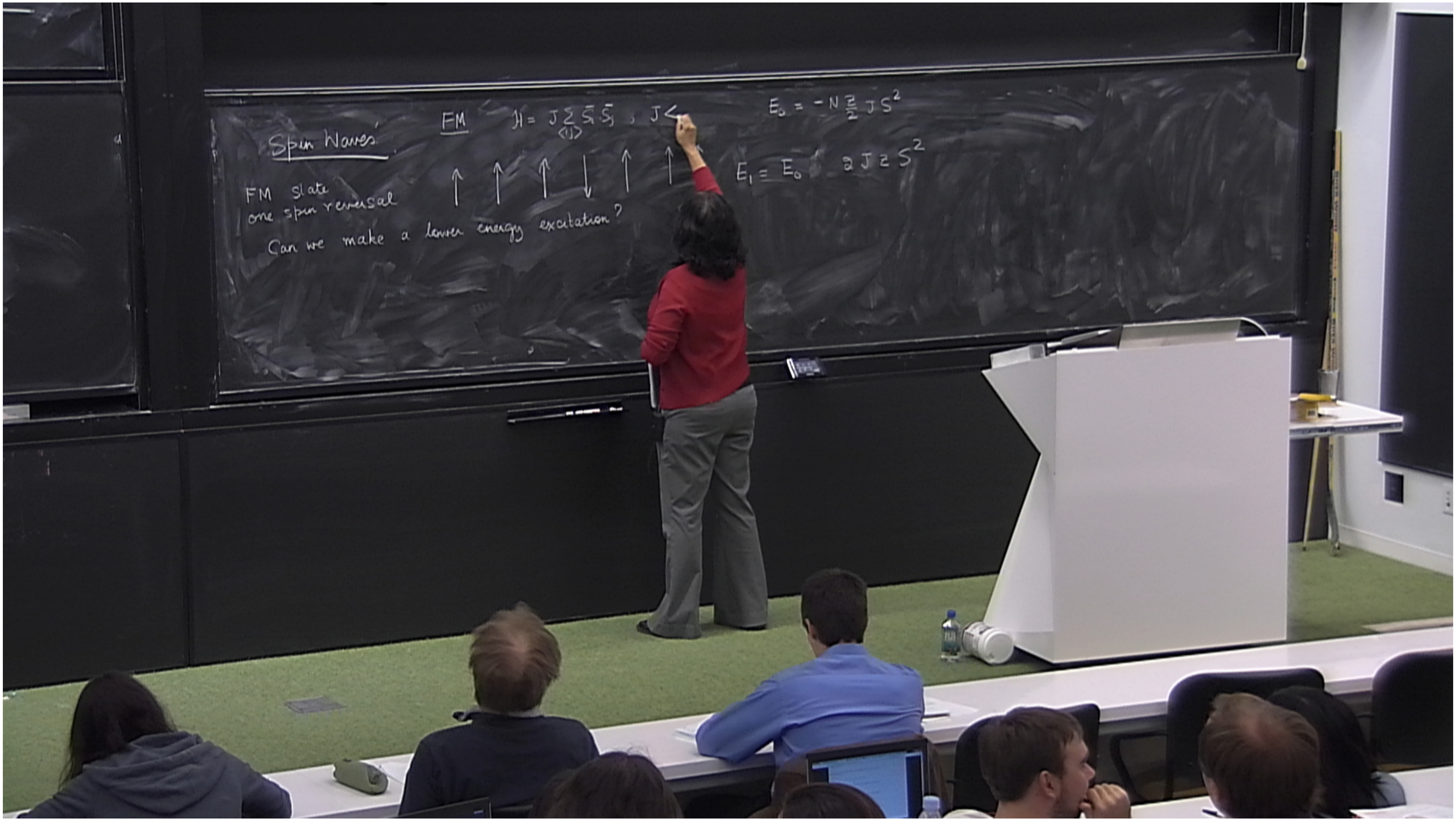

what are the states
that we integrate over?
Define $M_z = \lim_{h \to 0} \lim_{N \to \infty} \frac{1}{N} \langle S^z_{total} \rangle$

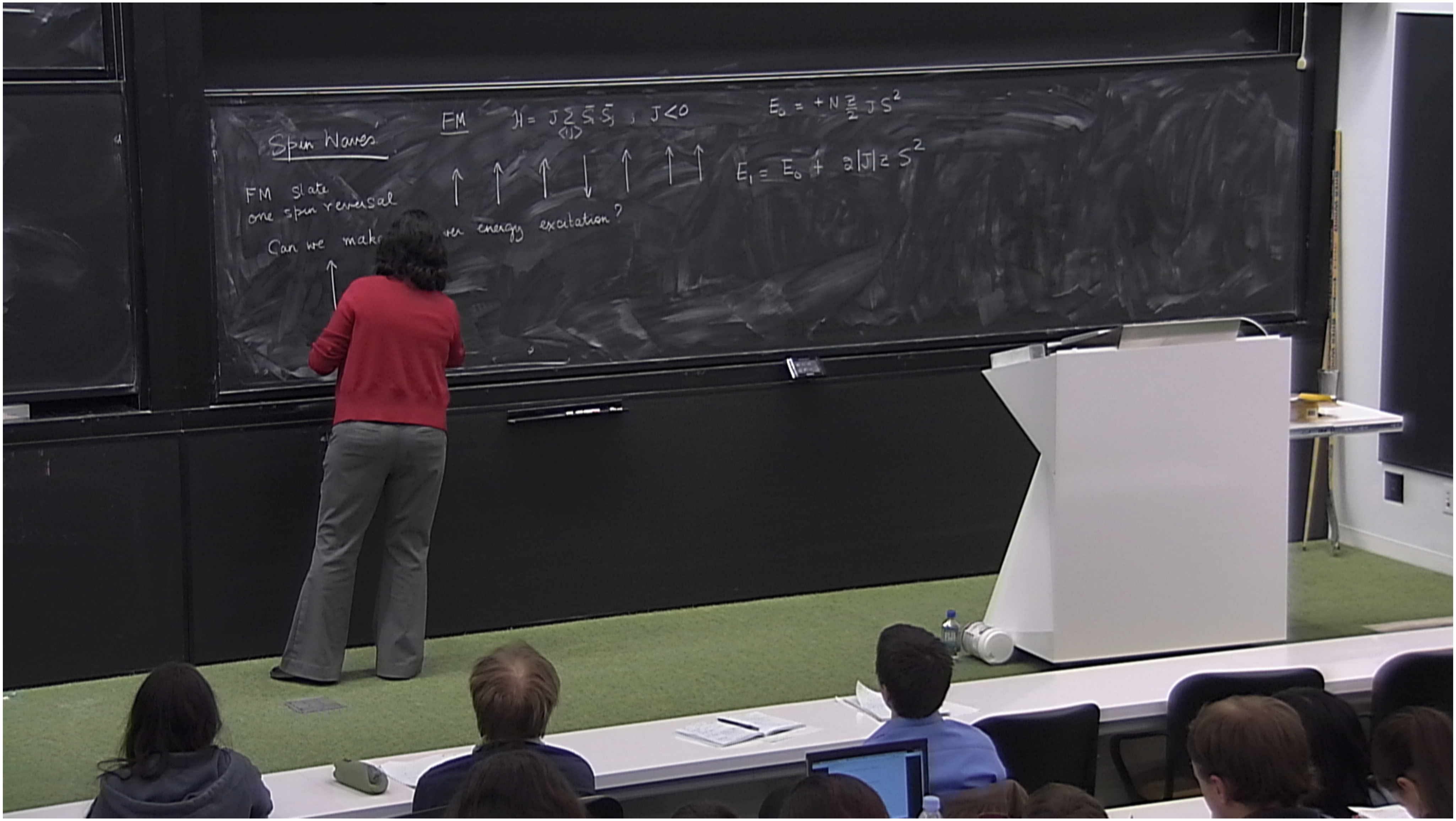

what are the states that we integrate over?
Define $M_z = \lim_{h\to 0} \lim_{N\to\infty} \frac{1}{N} \langle S^z_{total} \rangle$
Barrier ∝ N

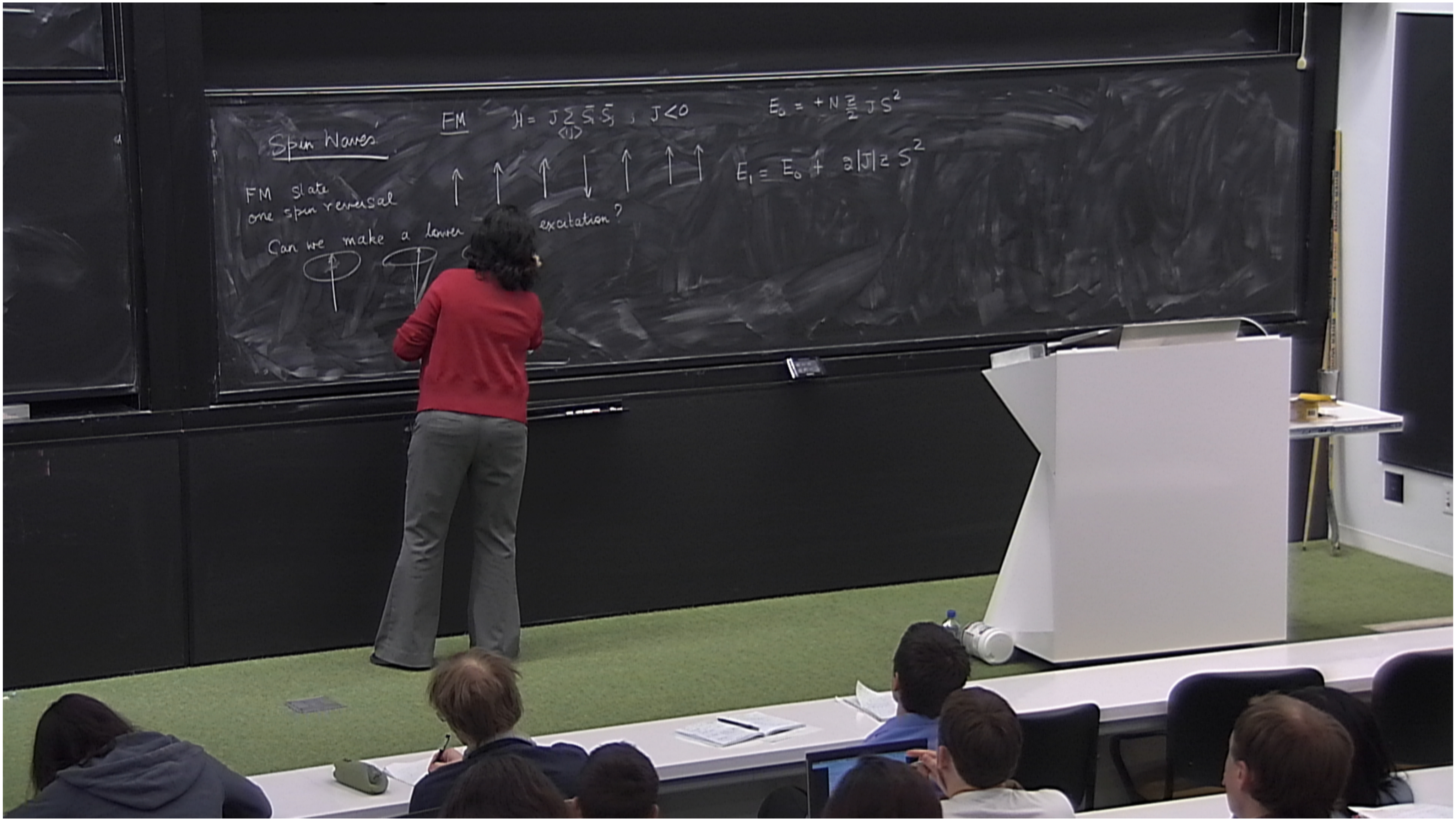

Spin Waves
FM state
one spin reversal

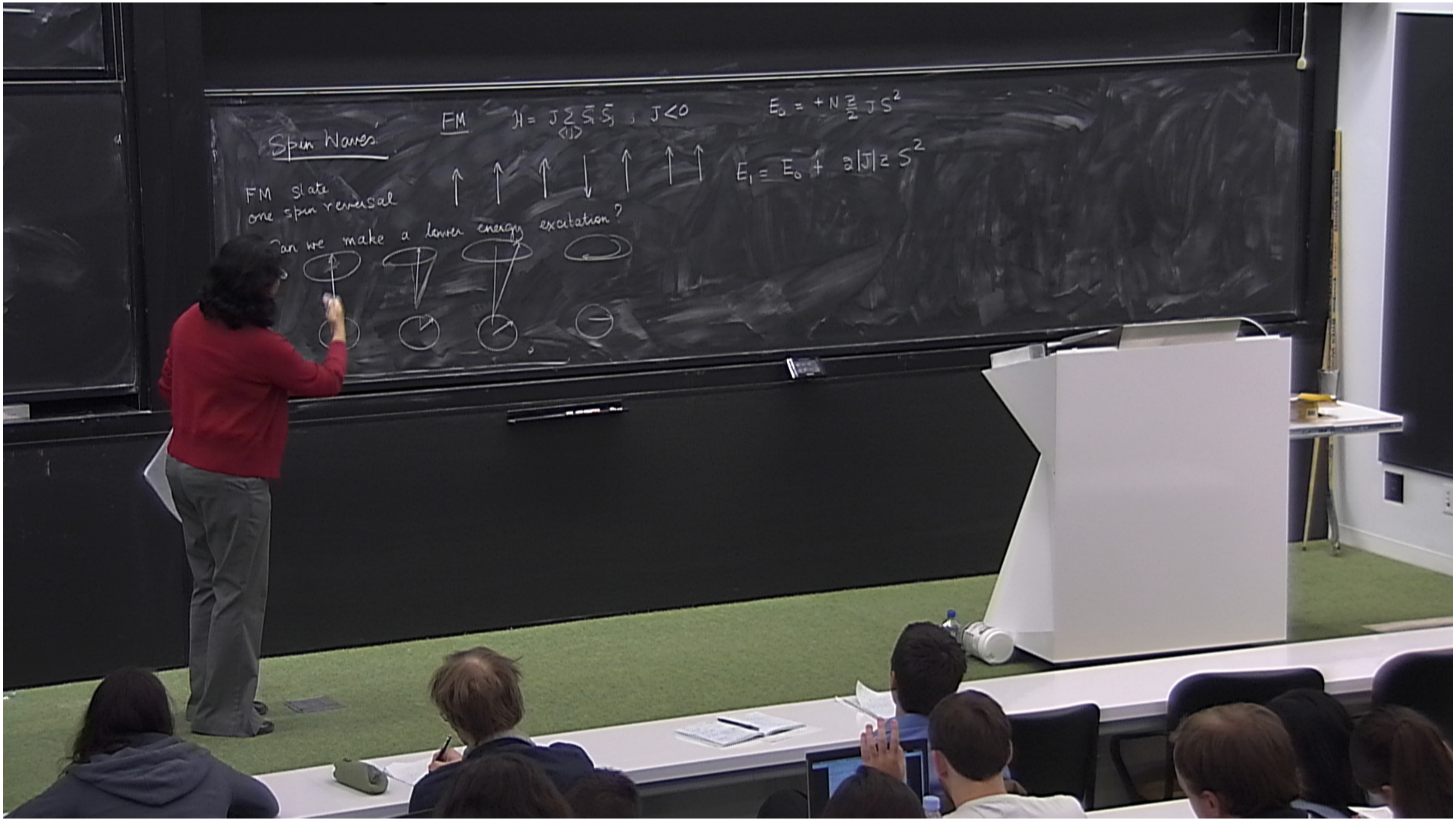

Spin Waves
FM state
one spin reversal
E₀ = − N z/2 J S²
E₁ = E₀ + 2 J z S²

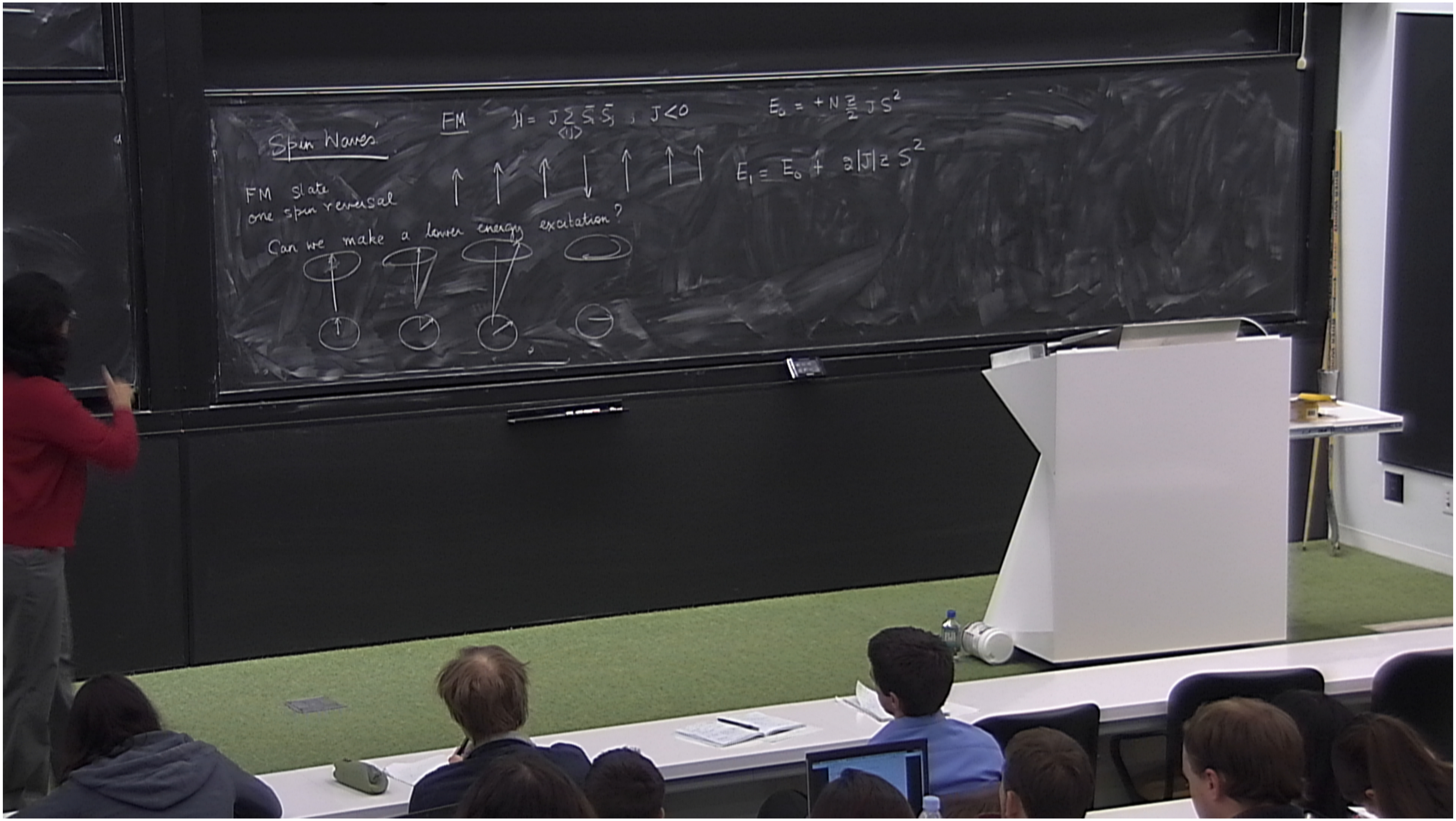

Spin Waves
FM state
one spin reversal
Can we make a lower energy excitation?
$E_0 = -N \frac{z}{2} J S^2$
$E_1 = E_0 + 2JzS^2$

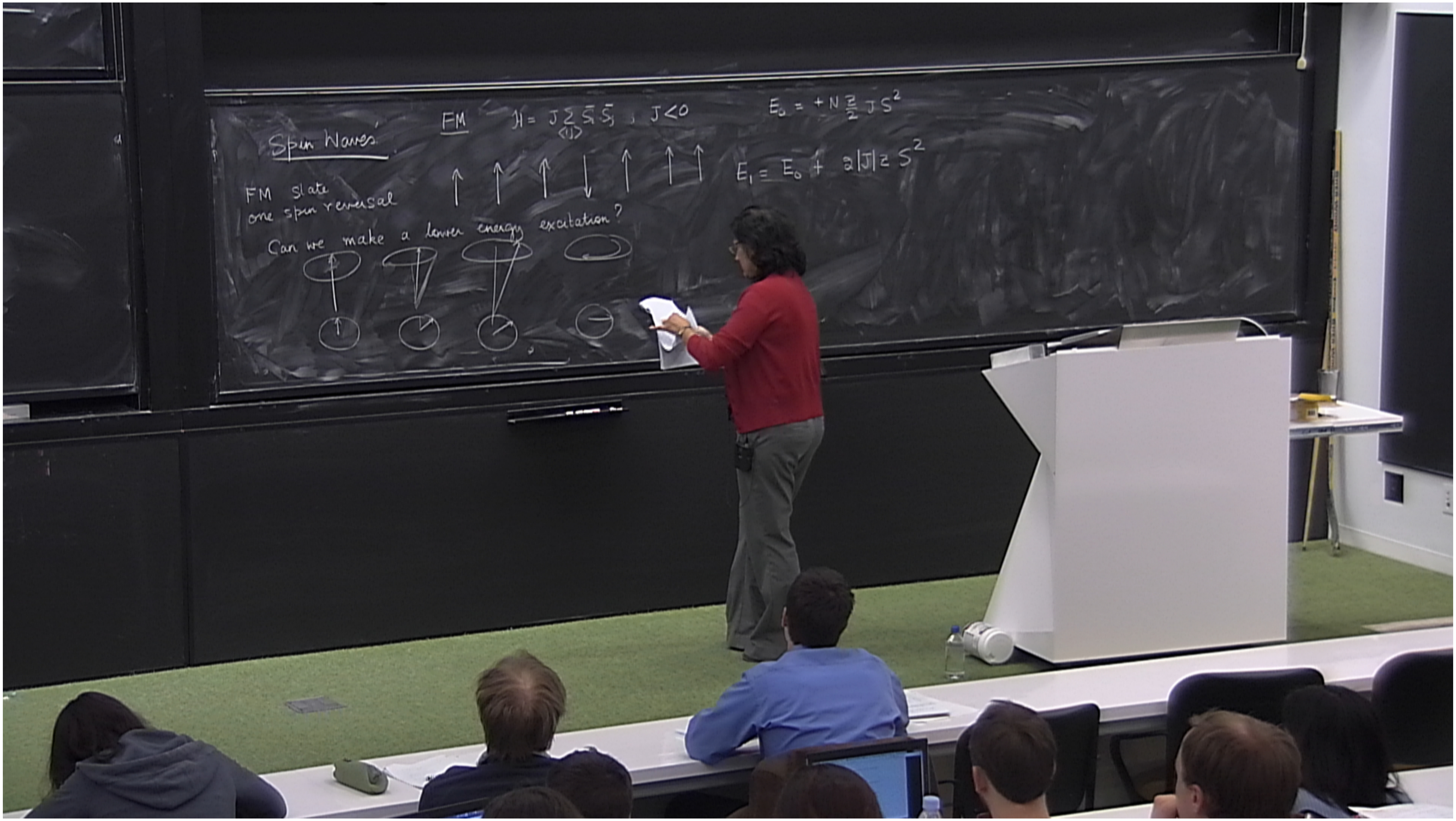

Spin Waves
FM
$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$, J <
$E_0 = -N \frac{z}{2} J S^2$
FM state
one spin reversal
$E_1 = E_0$ 2 J z S^2
Can we make a lower energy excitation ?

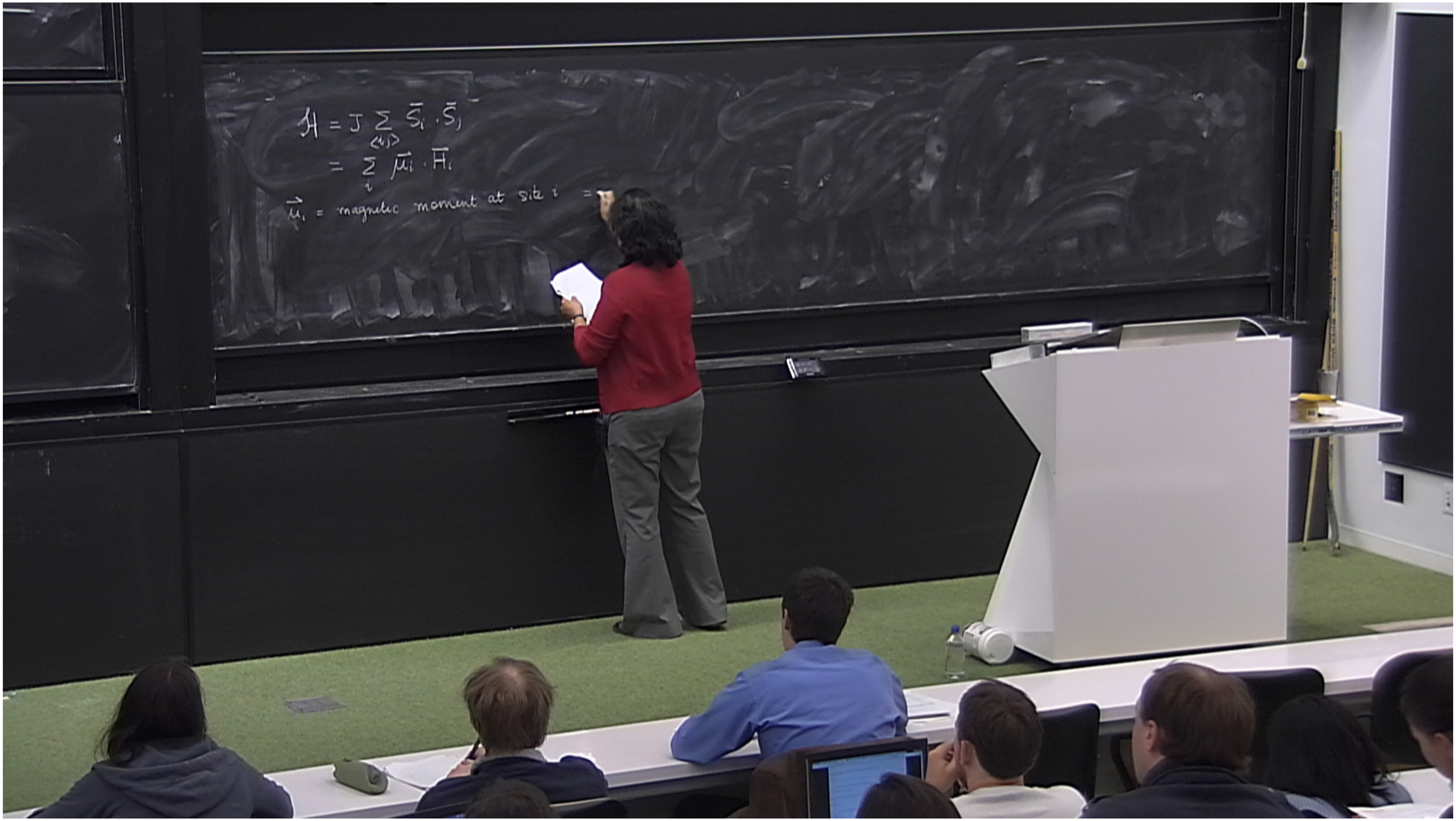

Spin Waves
FM
FM state
one spin reversal
Can we make
energy excitation?
E₁ = E₀ + 2|J|z S²
J<0

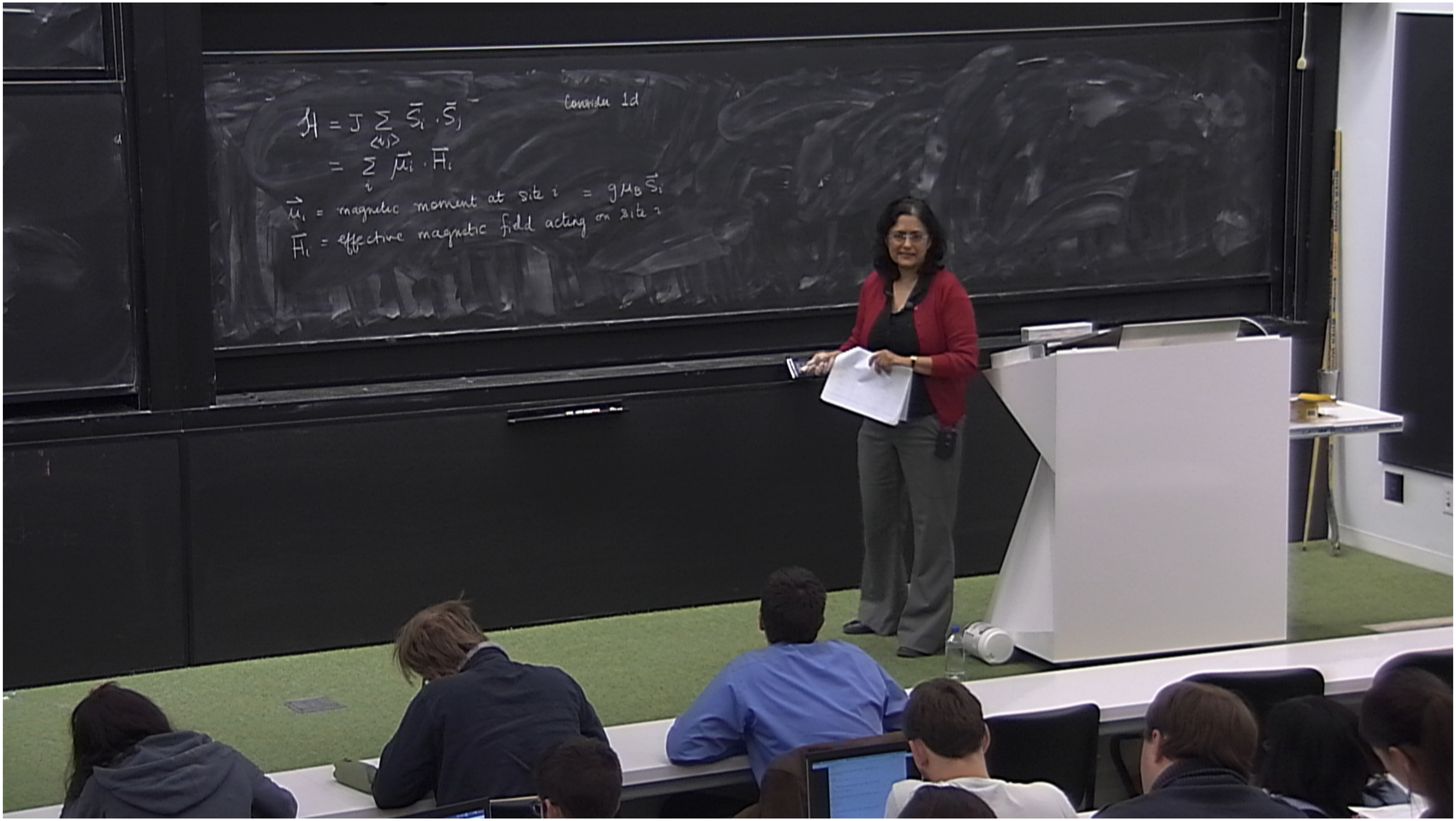

Spin Waves
FM
$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$, $J<0$
$E_0 = + N \frac{z}{2} J S^2$
FM state
one spin reversal
$E_1 = E_0 + 2|J| z S^2$
Can we make a lower
excitation?

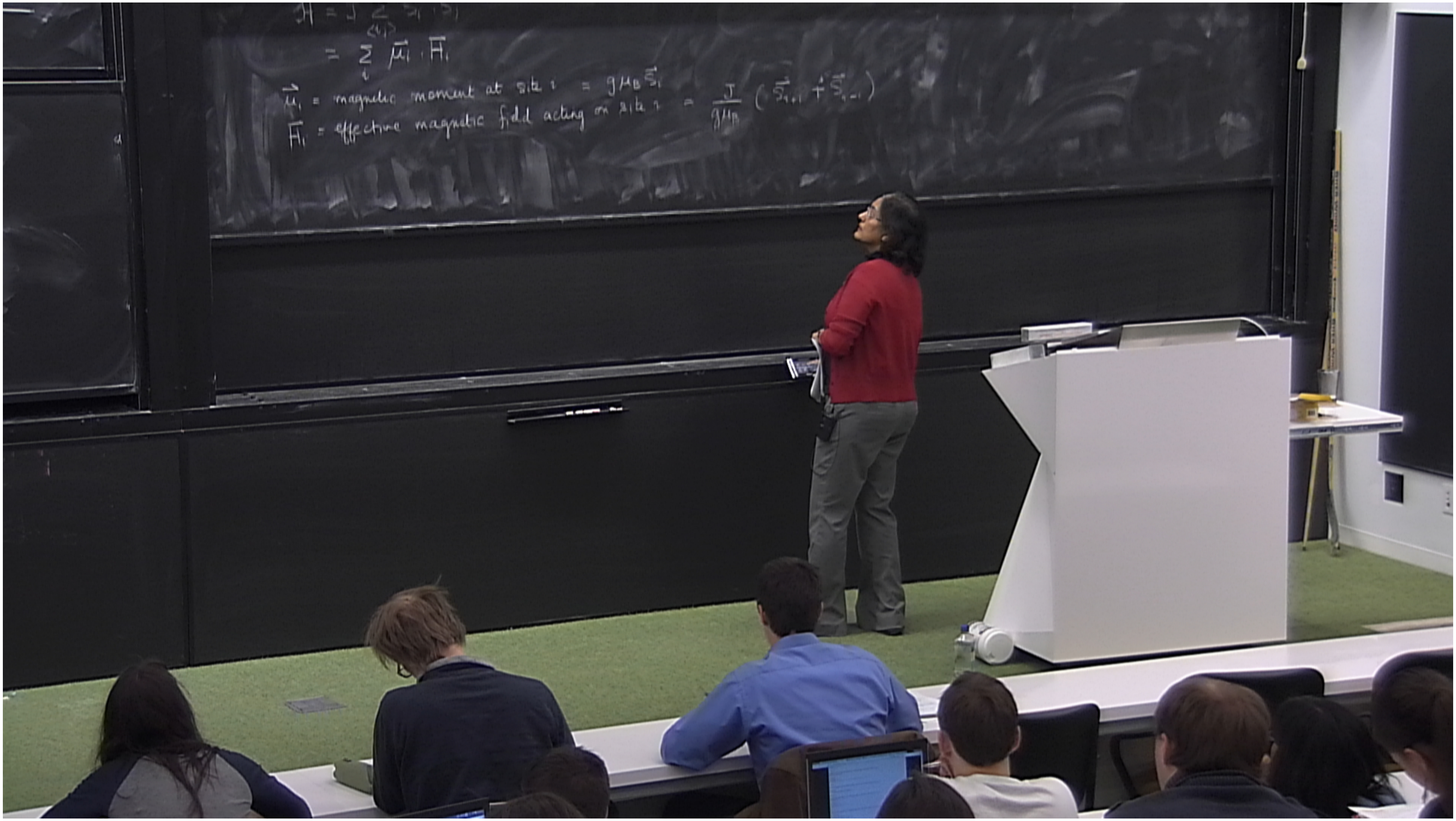

Spin Waves
FM
H = J Σ S_i S_j , J<0
E_0 = + N z/2 J S^2
FM state
one spin reversal
E_1 = E_0 + 2|J| z S^2
Can we make a lower energy excitation?

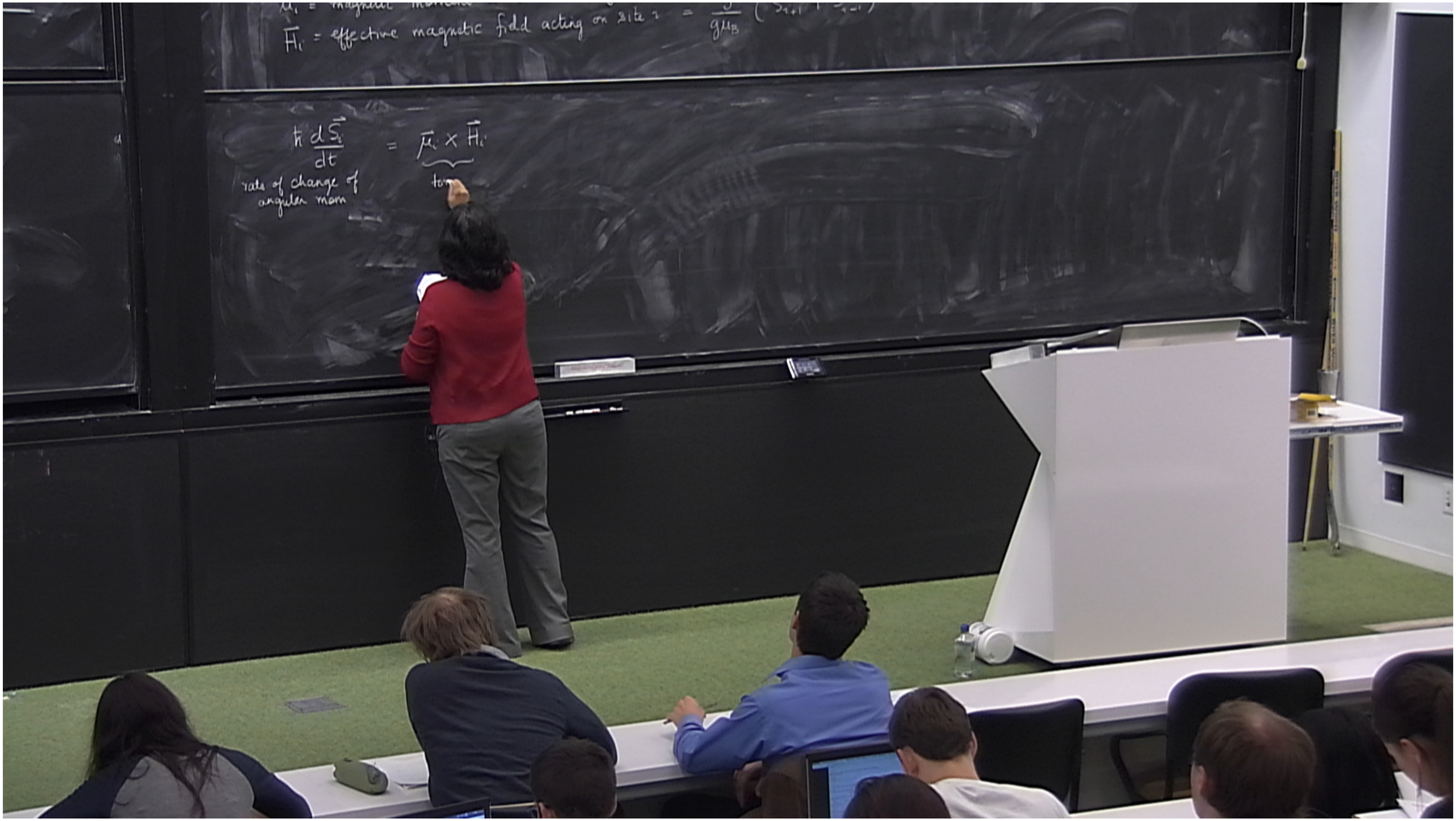

Spin Waves
FM
H = J Σ S_i · S_j , J < 0
E_0 = + N z/2 J S^2
FM state
one spin reversal
E_1 = E_0 + 2|J| z S^2
Can we make a lower energy excitation?

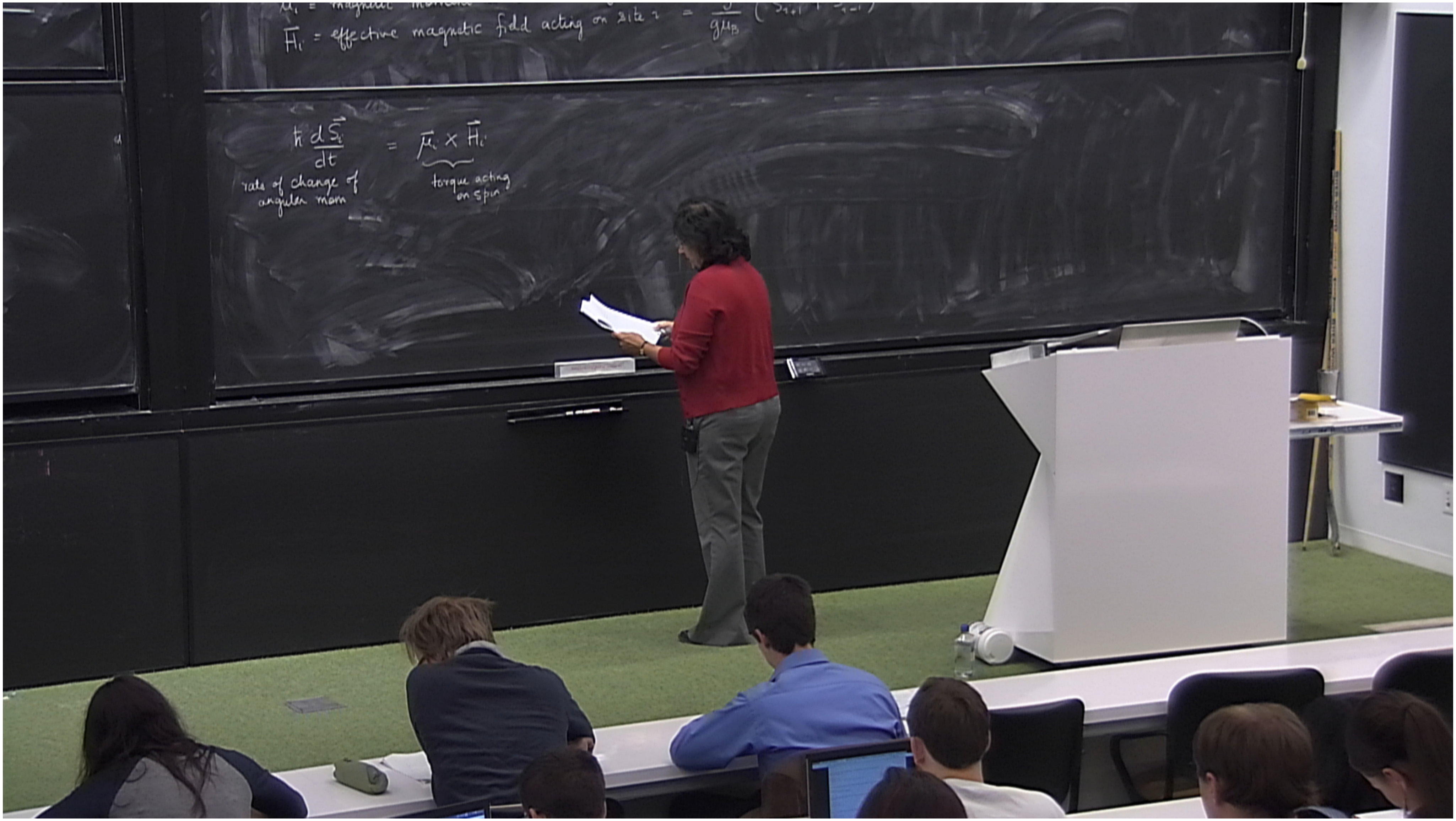

Spin Waves
FM
$\mathcal{H} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$, $J<0$
$E_0 = + N \frac{z}{2} J S^2$
$E_1 = E_0 + 2|J| z S^2$
FM state
one spin reversal
Can we make a lower energy excitation?


$\mathcal{H} = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$
$= \sum_i \vec{\mu}_i \cdot \vec{H}_i$
$\vec{\mu}_i$ = magnetic moment at site i =

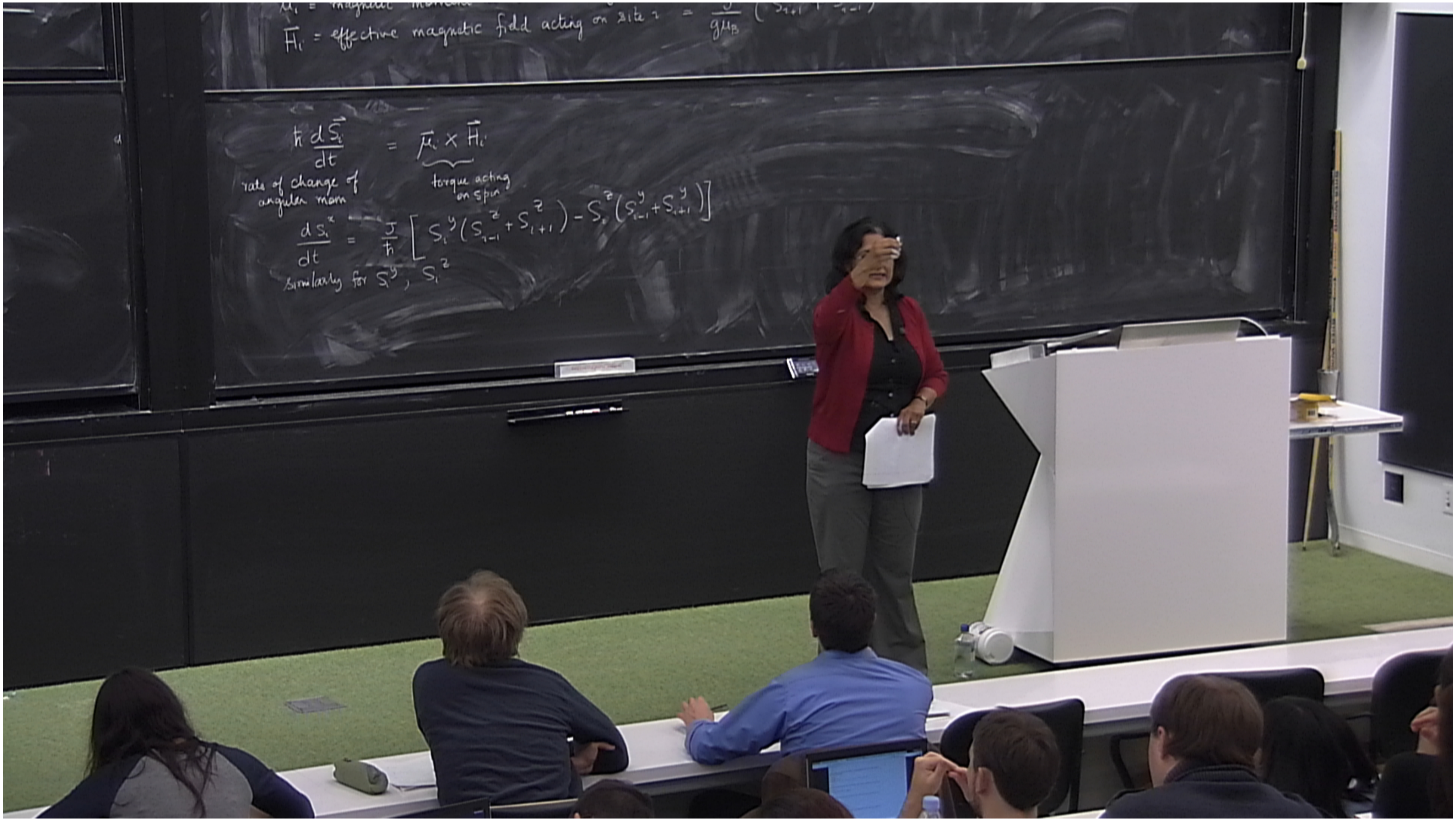

Consider 1d
H = J Σ⟨ij⟩ Si · Sj
= Σi μi · Hi
μi = magnetic moment at site i = g μB Si
Hi = effective magnetic field acting on site i

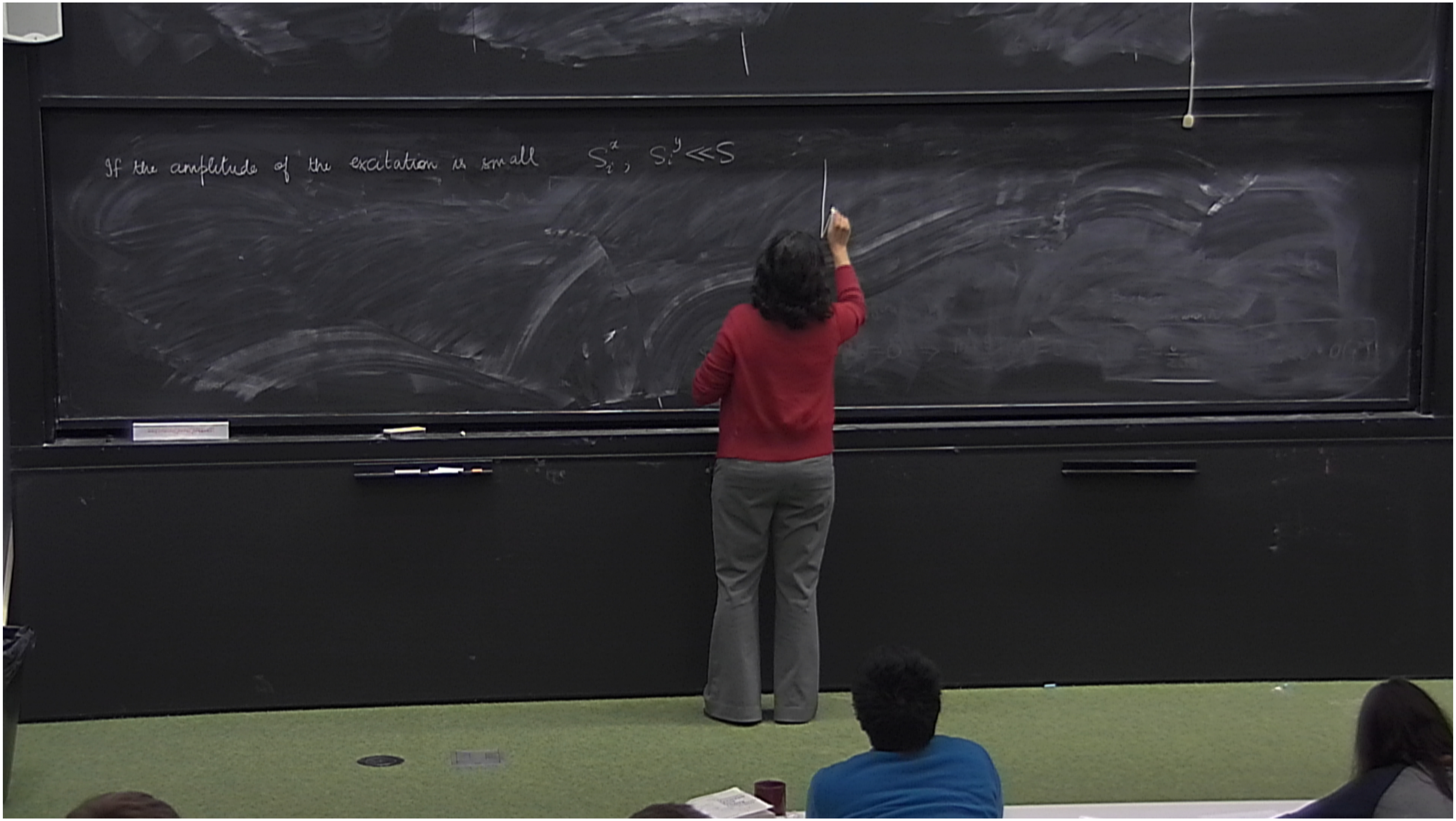

$= \sum_i \vec{\mu}_i \cdot \vec{H}_i$
$\vec{\mu}_i$ = magnetic moment at site i = $g\mu_B \vec{S}_i$
$\vec{H}_i$ = effective magnetic field acting on site i = $\frac{J}{g\mu_B}(\vec{S}_{i+1} + \vec{S}_{i-1})$

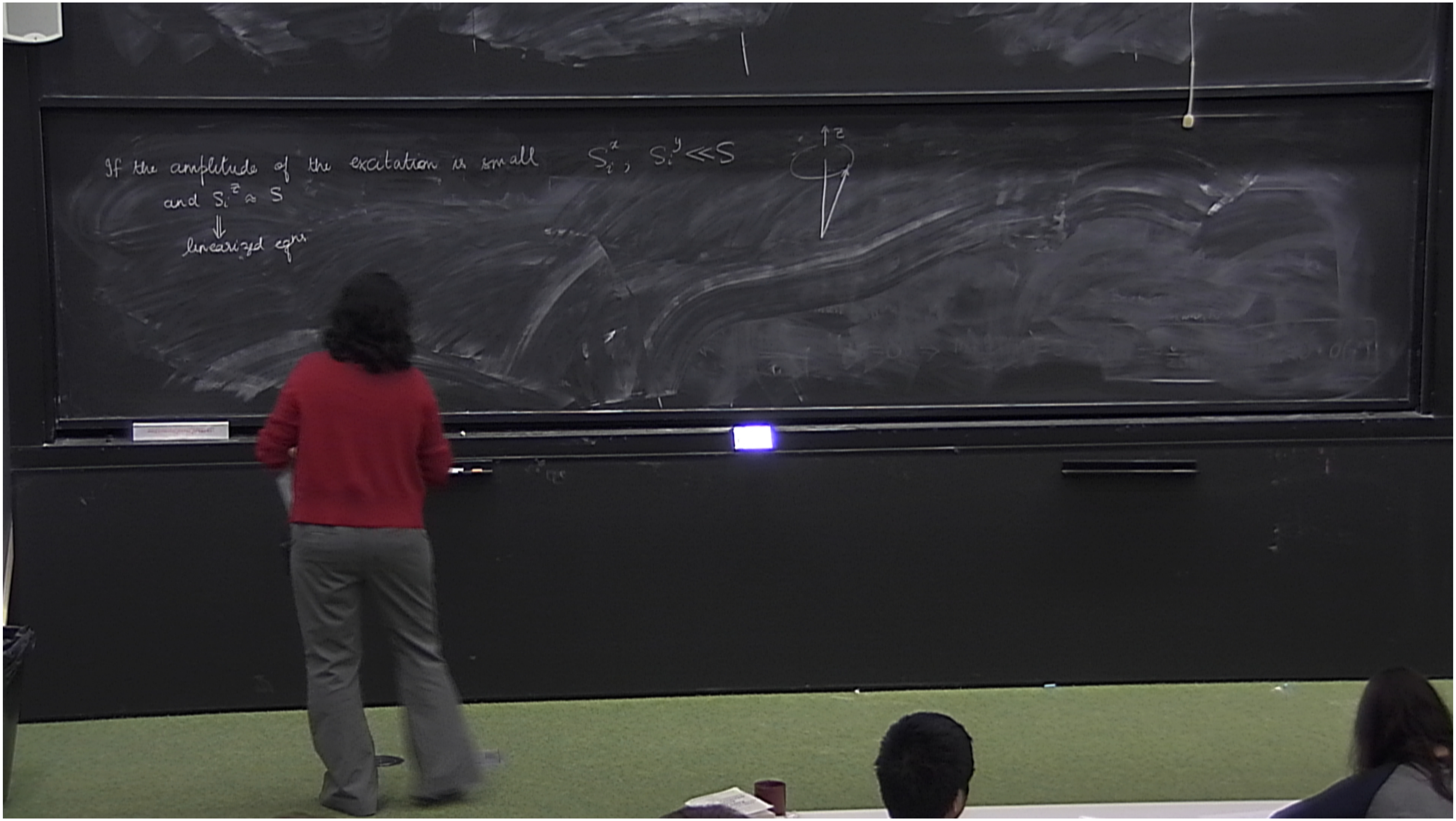

H_i = effective magnetic field acting on site i
ħ dS_i/dt = μ_i × H_i
rate of change of angular mom

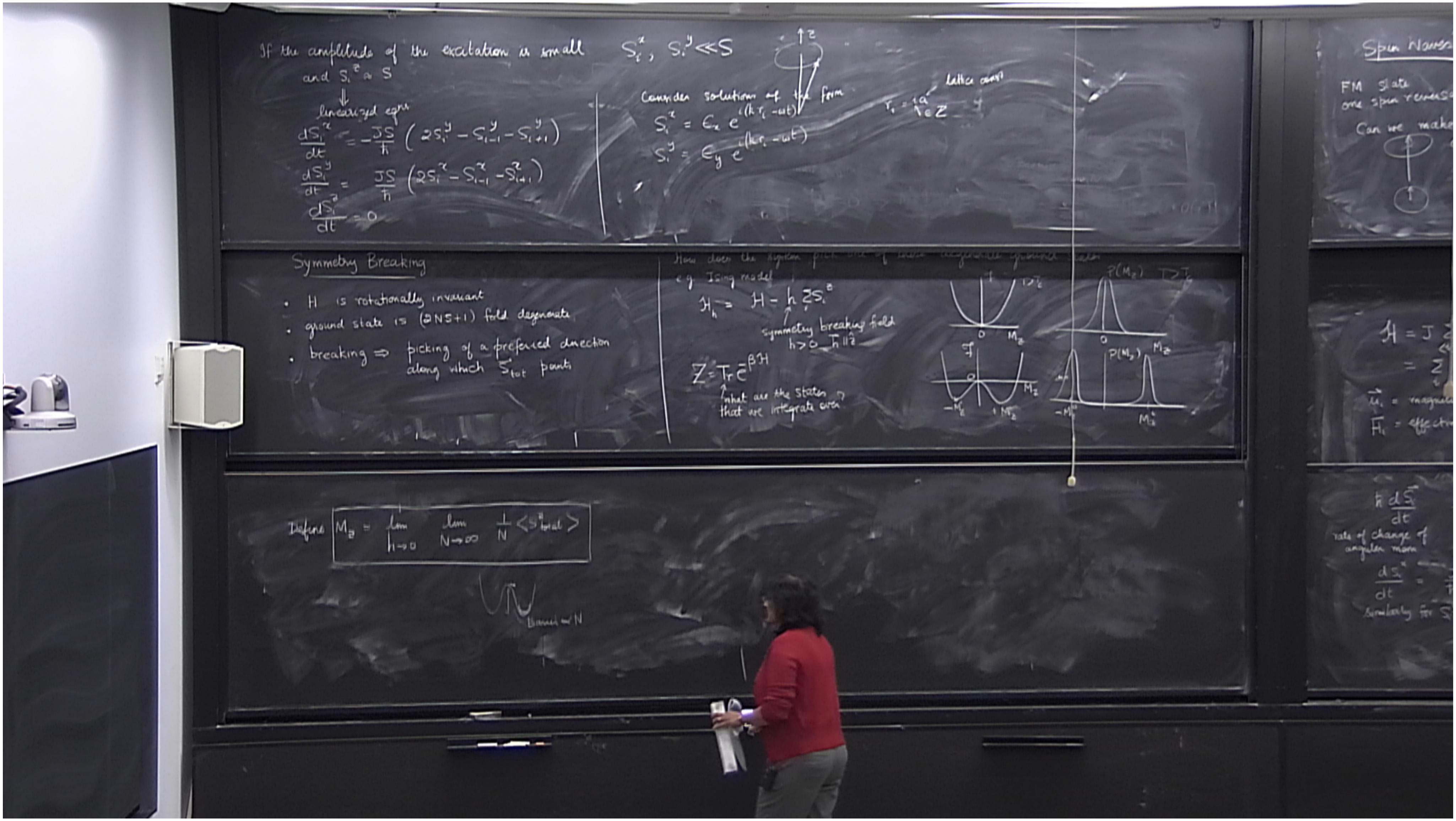


H_i = effective magnetic field acting on site i
ħ dS_i/dt = μ_i × H_i
rate of change of angular mom
torque acting on spin

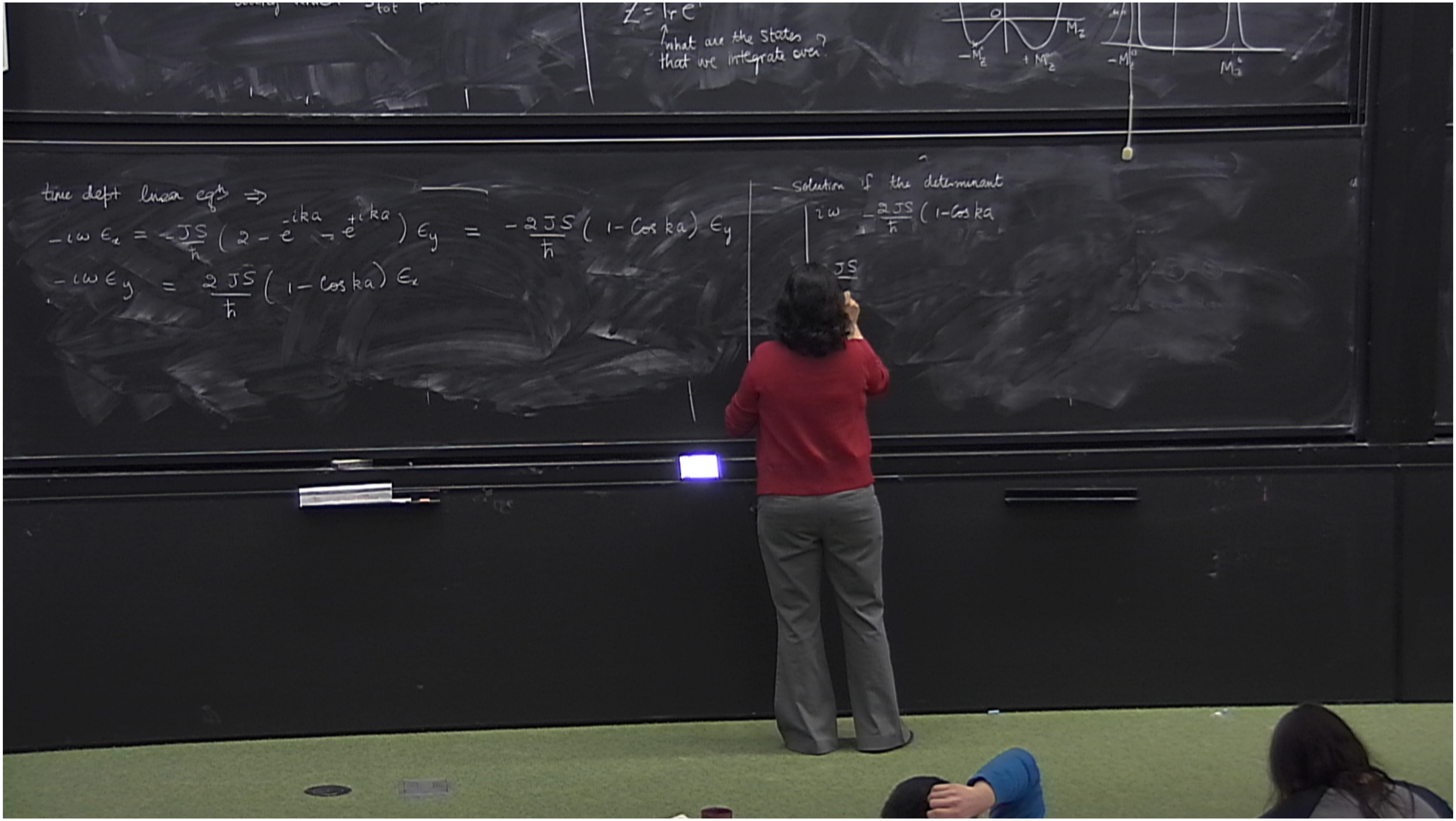

$\vec{H}_i$ = effective magnetic field acting on site i
$\hbar \frac{d\vec{S}_i}{dt} = \vec{\mu}_i \times \vec{H}_i$
rate of change of angular mom
torque acting on spin
$\frac{dS_i^x}{dt} = \frac{J}{\hbar}\left[S_i^y(S_{i-1}^z + S_{i+1}^z) - S_i^z(S_{i-1}^y + S_{i+1}^y)\right]$
similarly for S_i^y, S_i^z

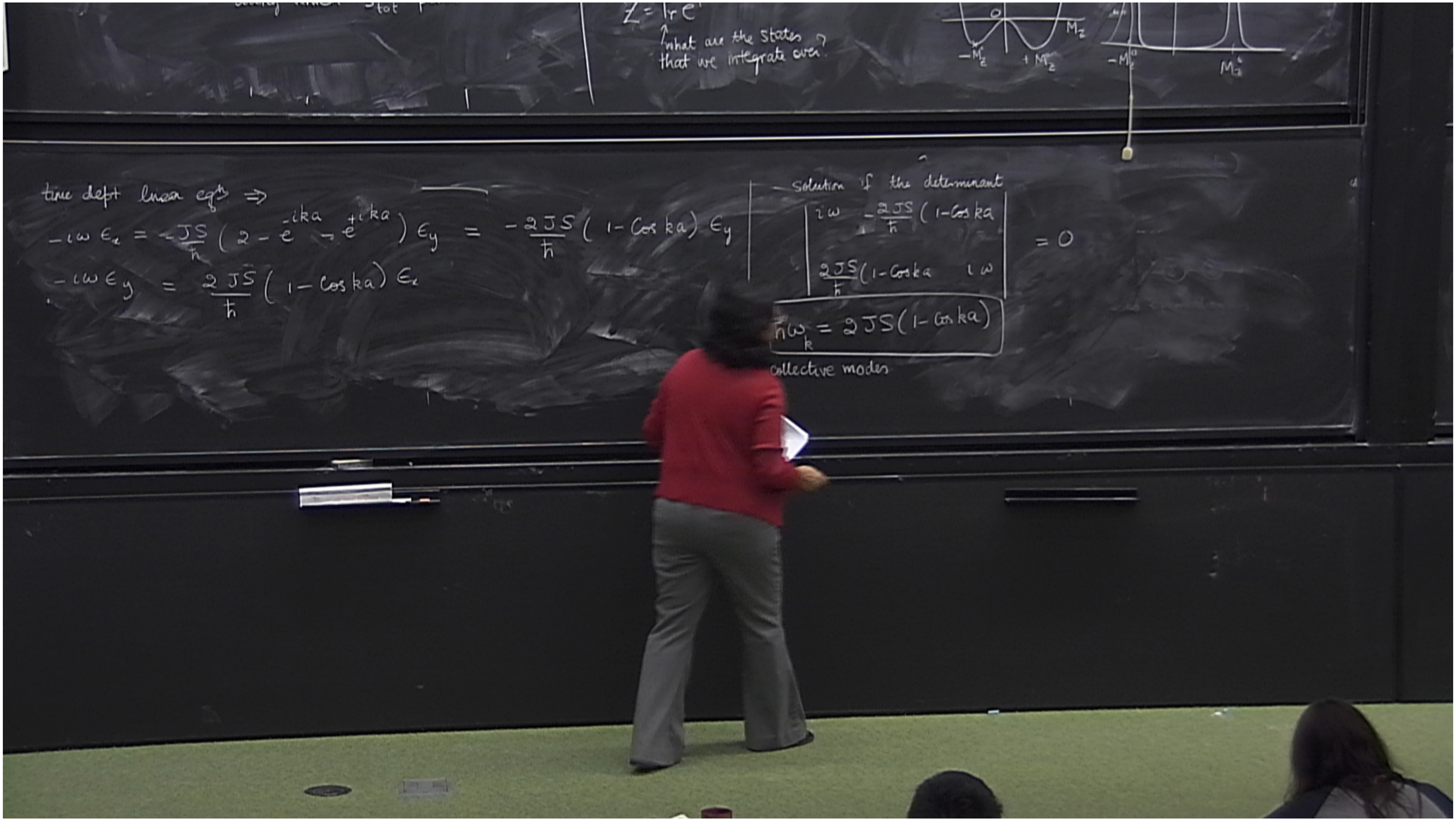

If the amplitude of the excitation is small $S_i^x, S_i^y \ll S$

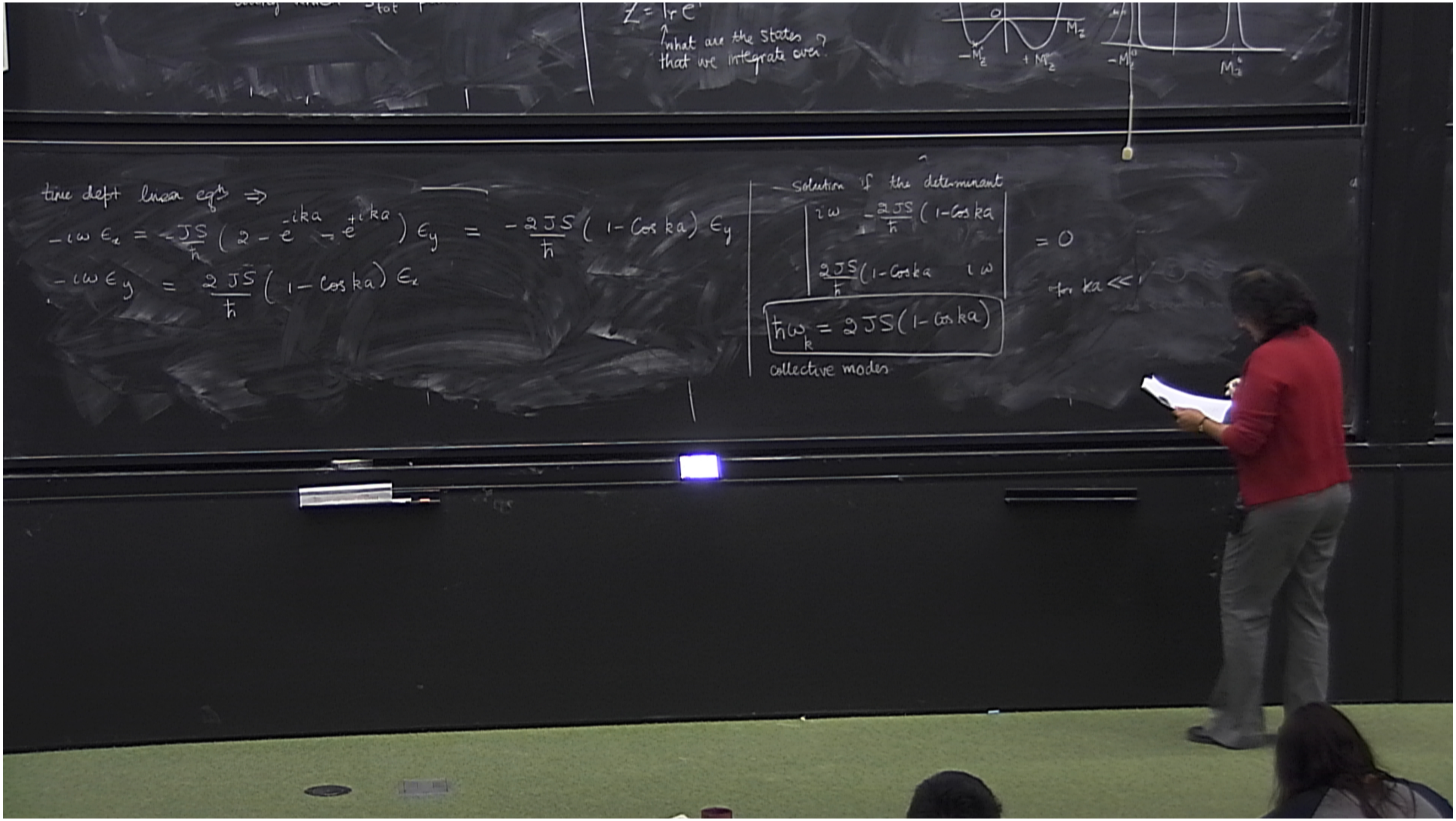

If the amplitude of the excitation is small $S_i^x, S_i^y \ll S$
and $S_i^z \approx S$
⇓
linearized eqns
z

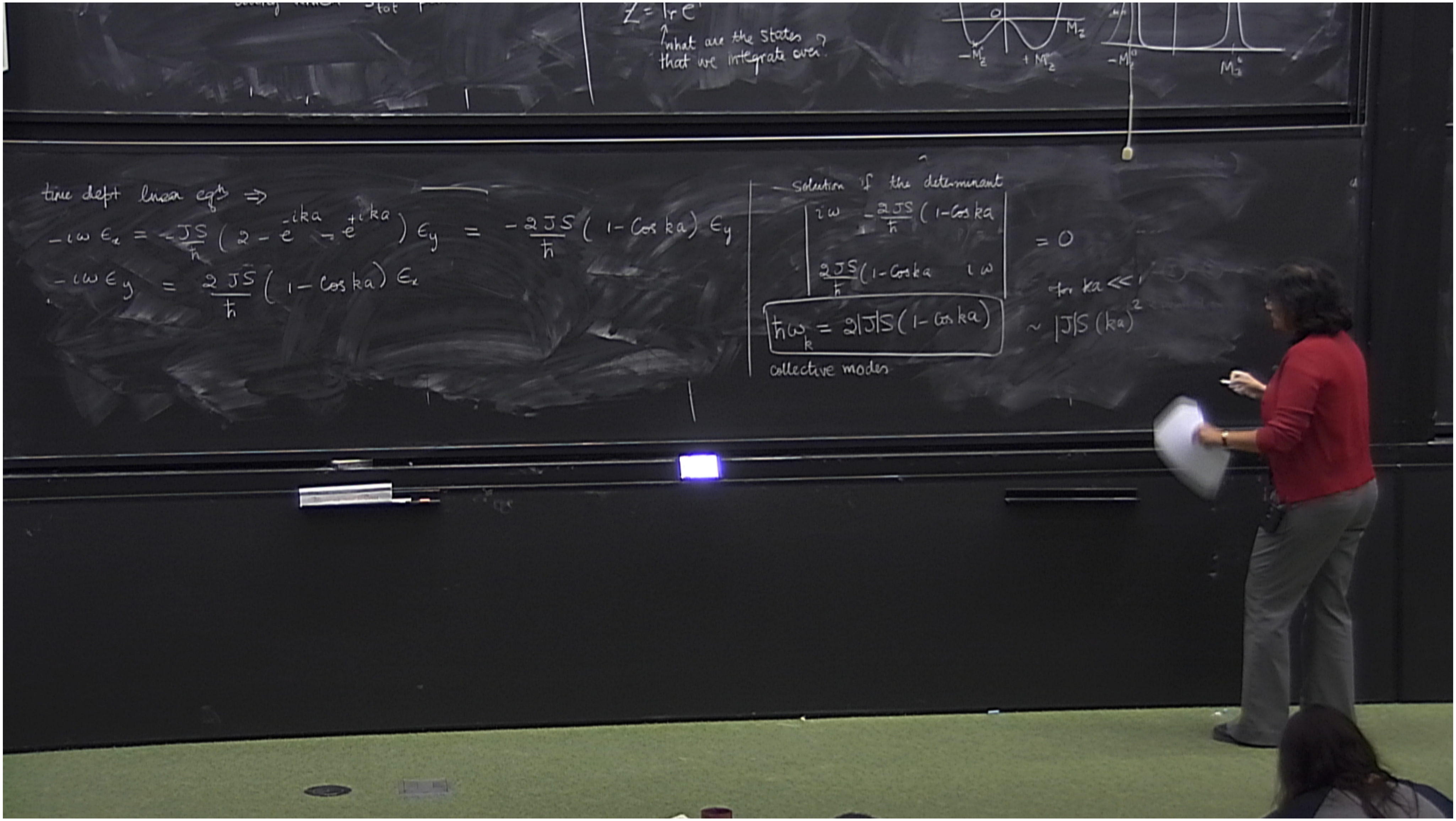


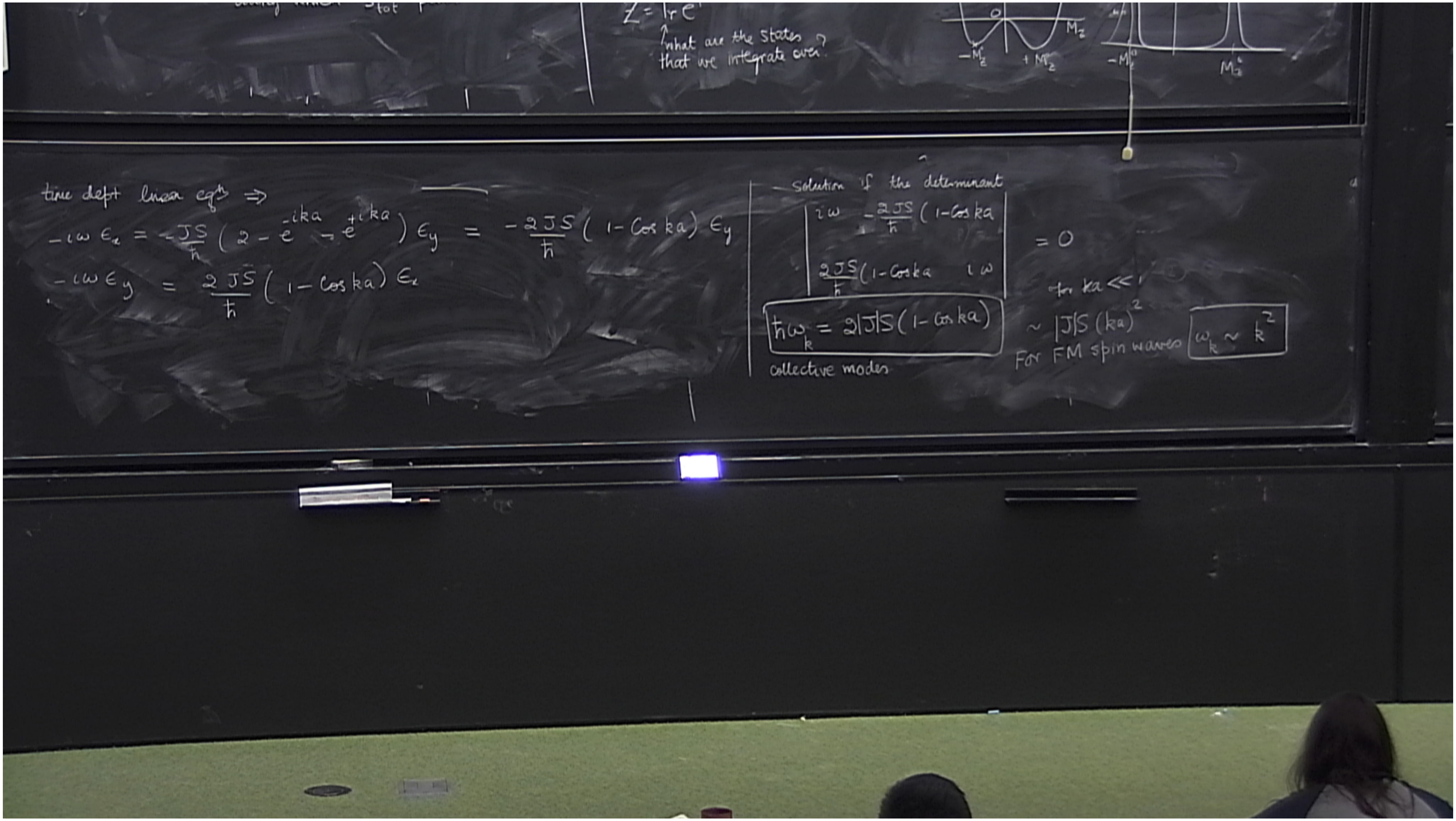
If the amplitude of the excitation is small
and S_i^z ≈ S
linearized eqn
dS_i^x/dt = −JS/ħ (2S_i^y − S_{i−1}^y − S_{i+1}^y)
dS_i^y/dt = JS/ħ (2S_i^x − S_{i−1}^x − S_{i+1}^x)
dS_i^z/dt = 0
S_i^x, S_i^y ≪ S
Consider solutions of the form
S_i^x = ε_x e^{i(k r_i − ωt)}
S_i^y = ε_y e^{i(k r_i − ωt)}
lattice const
Symmetry Breaking
H is rotationally invariant
ground state is (2NS+1) fold degenerate
breaking ⇒ picking of a preferred direction along which S_tot points
e.g. Ising model
H_h = H − h Σ S_i^z
Symmetry breaking field h > 0, h ∥ z
Z = Tr e^{−βH}
what are the states that we integrate over?
Define M_z = lim_{h→0} lim_{N→∞} 1/N ⟨S^z_total⟩
Spin Waves
FM state
Can we make
rate of change of angular mom
Similarly for

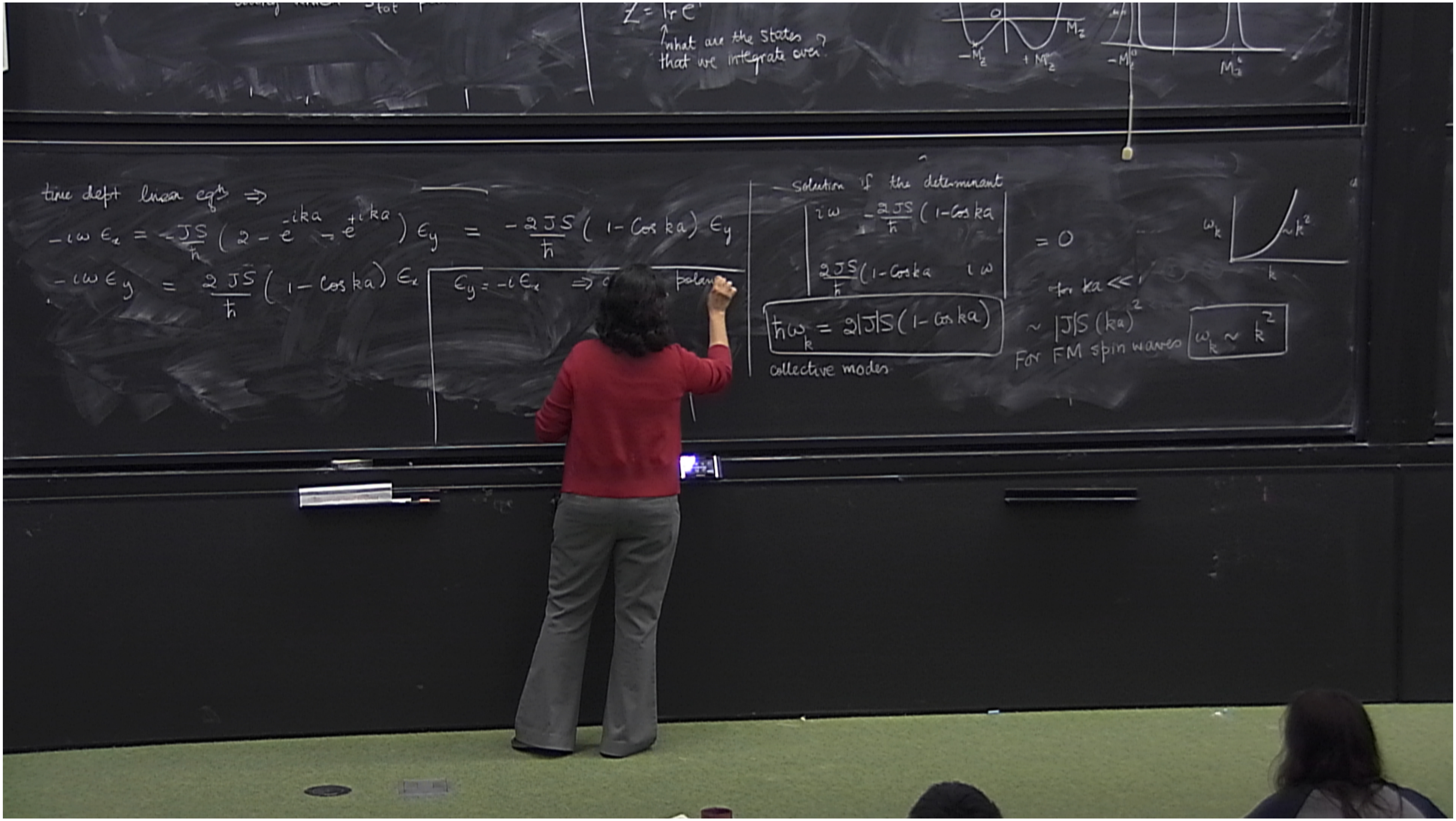

what are the states that we integrate over?
time dep't linear eq^n ⇒
-iω ε_x = -JS/ħ (2 - e^{-ika} - e^{+ika}) ε_y = -2JS/ħ (1 - Cos ka) ε_y
-iω ε_y = 2JS/ħ (1 - Cos ka) ε_x
Solution if the determinant
iω -2JS/ħ (1-Cos ka
JS

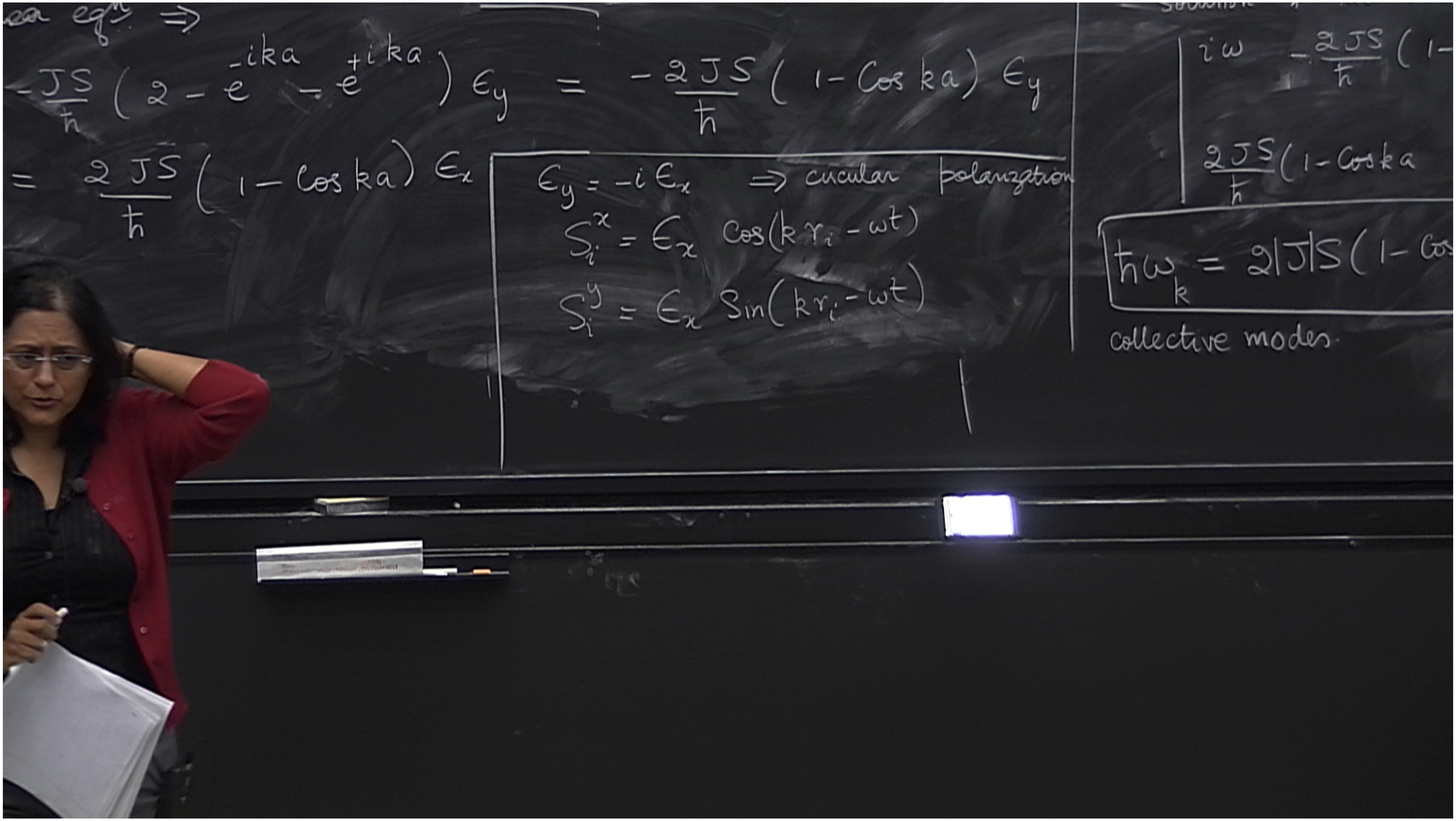

what are the states that we integrate over?
time dept linear eqⁿˢ ⇒
−iω ε_x = −JS/ħ (2 − e^{−ika} − e^{+ika}) ε_y = −2JS/ħ (1 − Cos ka) ε_y
−iω ε_y = 2JS/ħ (1 − Cos ka) ε_x
Solution if the determinant
iω −2JS/ħ (1−Cos ka
2JS/ħ (1−Cos ka iω
= 0
ħω_k = 2JS(1−Cos ka)
collective modes

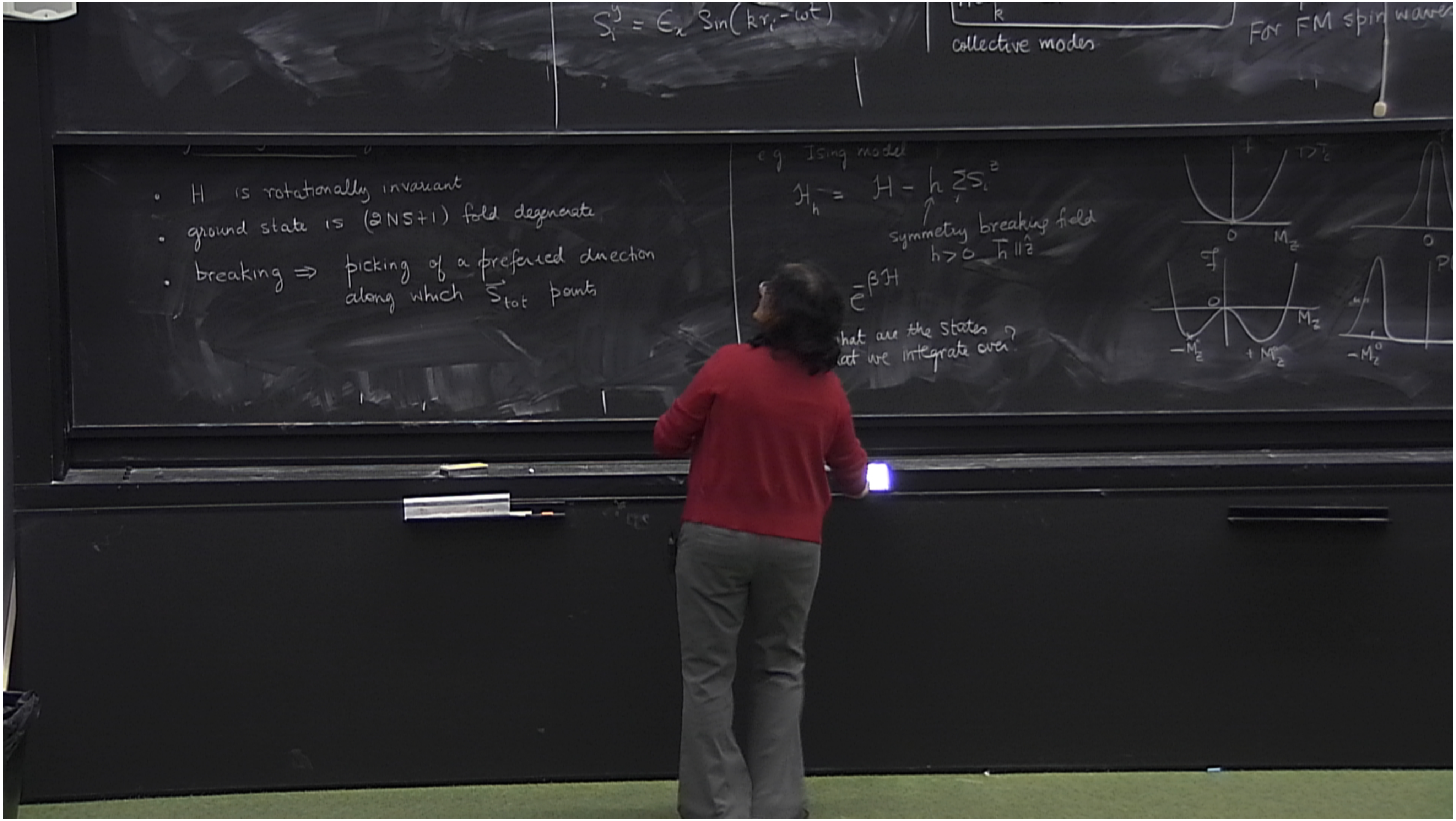

what are the states that we integrate over?
time dept linear eqs ⇒
$-i\omega \epsilon_x = -\frac{JS}{\hbar}(2 - e^{-ika} - e^{+ika})\epsilon_y = -\frac{2JS}{\hbar}(1-\cos ka)\epsilon_y$
$-i\omega \epsilon_y = \frac{2JS}{\hbar}(1-\cos ka)\epsilon_x$
Solution if the determinant
$\begin{vmatrix} i\omega & -\frac{2JS}{\hbar}(1-\cos ka) \\ \frac{2JS}{\hbar}(1-\cos ka) & i\omega \end{vmatrix} = 0$
$\hbar\omega_k = 2JS(1-\cos ka)$
collective modes
for $ka \ll 1$

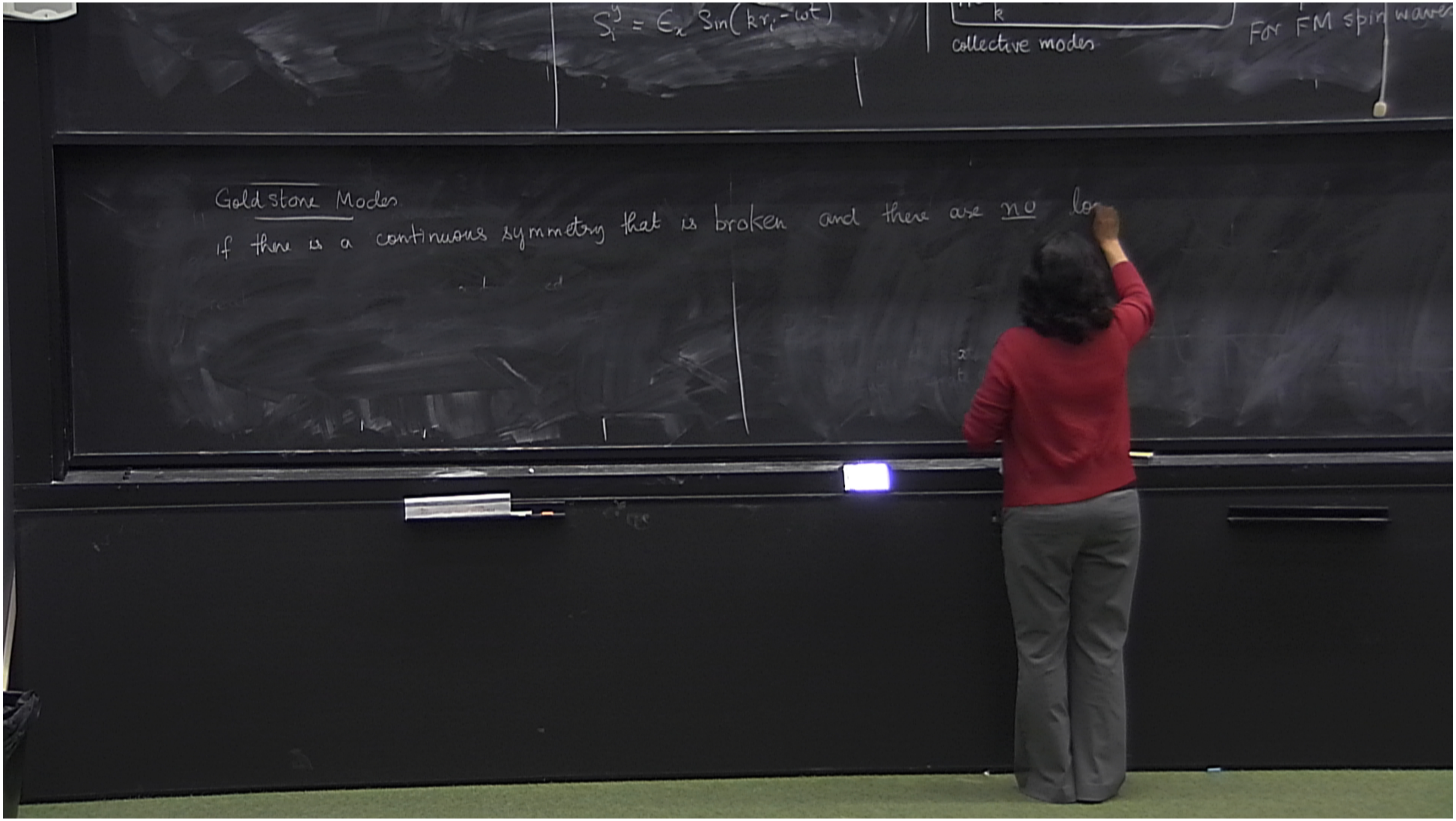

what are the States that we integrate over?
time dept linear eqns ⇒
−iω ε_x = −JS/ħ (2 − e^{−ika} − e^{+ika}) ε_y = −2JS/ħ (1 − Cos ka) ε_y
−iω ε_y = 2JS/ħ (1 − Cos ka) ε_x
Solution if the determinant
iω   −2JS/ħ (1 − Cos ka
2JS/ħ (1 − Cos ka   iω
= 0
ħω_k = 2|J|S (1 − Cos ka)
collective modes
for ka << 1
~ |J|S (ka)^2

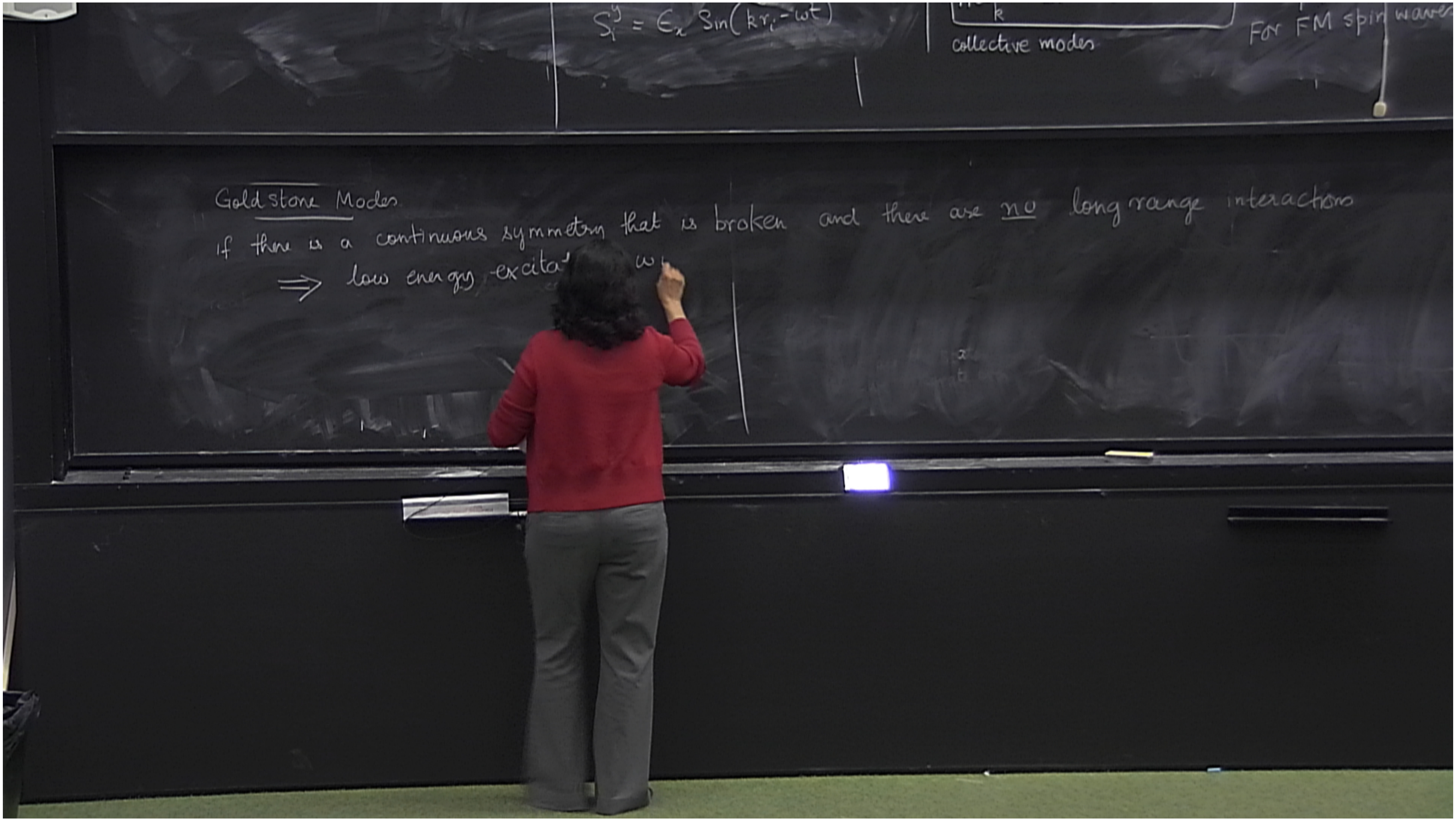

what are the states that we integrate over?
time dept linear eqns ⇒
−iω ε_x = −JS/ħ (2 − e^{−ika} − e^{+ika}) ε_y = −2JS/ħ (1 − Cos ka) ε_y
−iω ε_y = 2JS/ħ (1 − Cos ka) ε_x
Solution if the determinant
= 0
ħω_k = 2|J|S(1 − Cos ka)
collective modes
for ka ≪ 1
~ |J|S (ka)^2
For FM spin waves ω_k ~ k^2

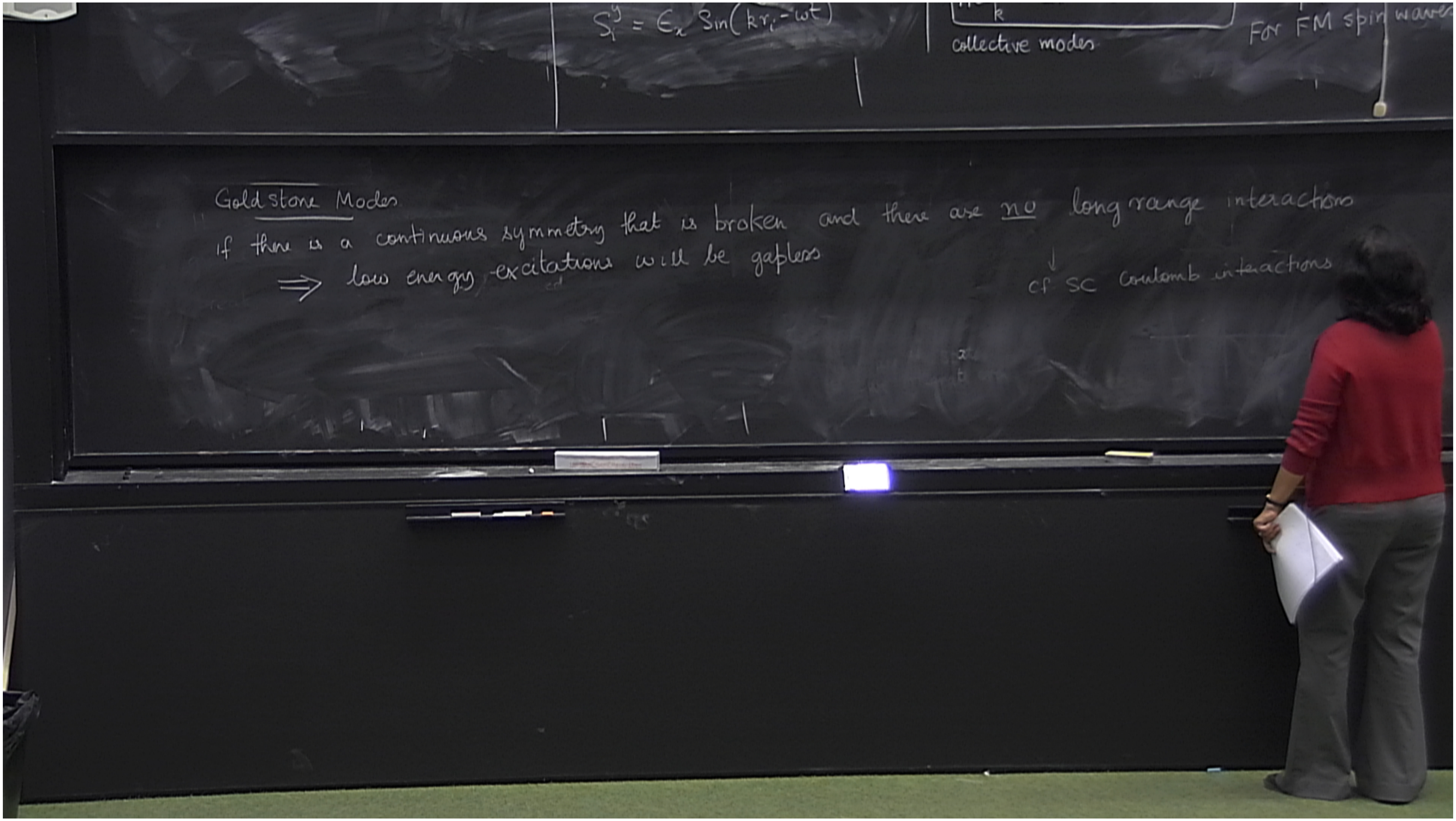

what are the states that we integrate over?
time dept linear eqns ⇒
$-i\omega \epsilon_x = -\frac{JS}{\hbar}(2 - e^{-ika} - e^{+ika})\epsilon_y = -\frac{2JS}{\hbar}(1-\cos ka)\epsilon_y$
$-i\omega \epsilon_y = \frac{2JS}{\hbar}(1-\cos ka)\epsilon_x$
$\epsilon_y = -i\epsilon_x$
Solution if the determinant
$= 0$
$\hbar\omega_k = 2|J|S(1-\cos ka)$
collective modes
for $ka \ll 1$
$\sim |J|S(ka)^2$
For FM spin waves
$\omega_k \sim k^2$

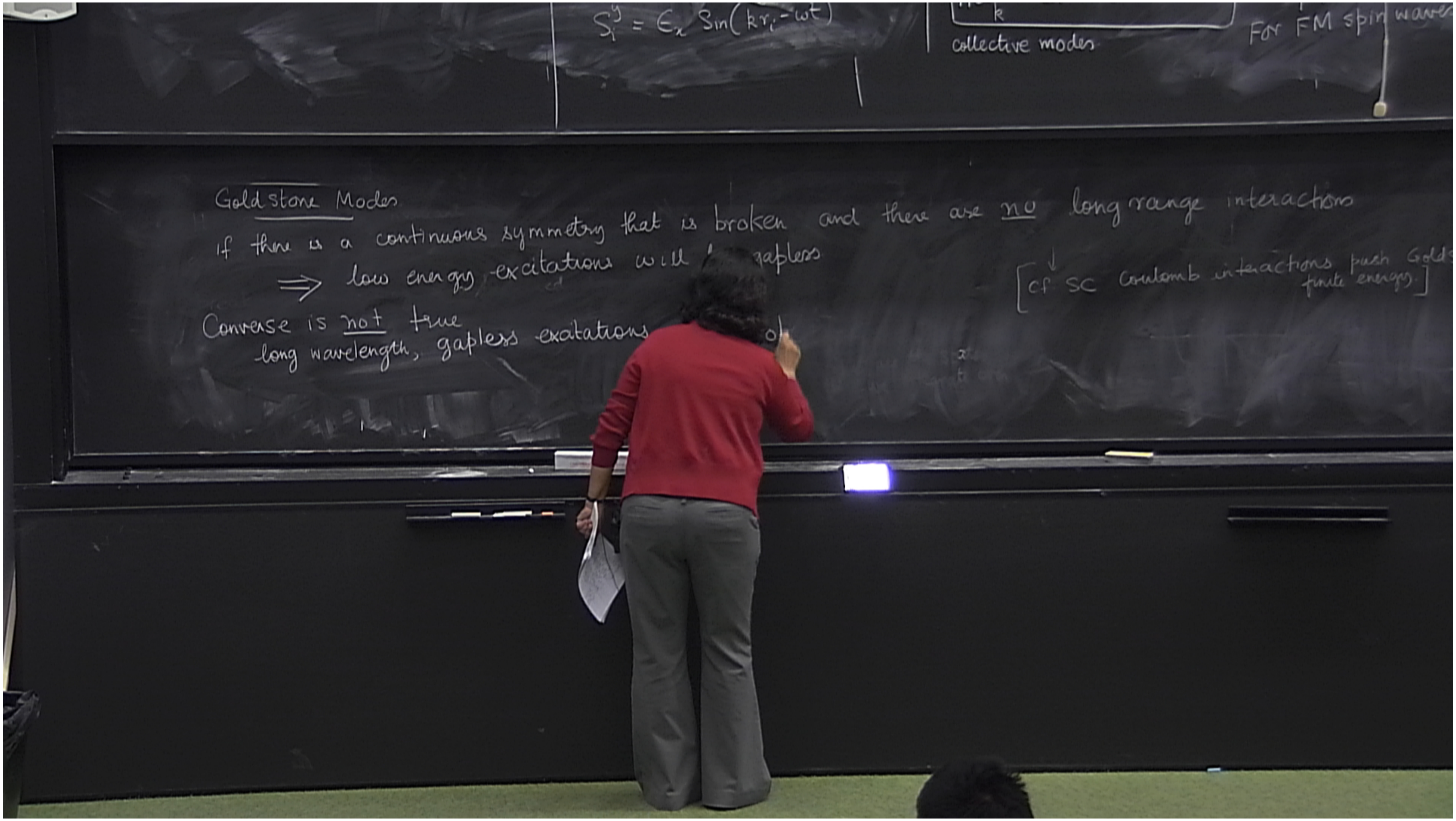

eqn ⇒
-JS/ħ (2 - e^-ika - e^+ika) εy = -2JS/ħ (1 - Cos ka) εy
= 2JS/ħ (1 - Cos ka) εx
εy = -i εx ⇒ circular polarization
Si^x = εx Cos(k ri - ωt)
Si^y = εx Sin(k ri - ωt)
iω -2JS/ħ (1-
2JS/ħ (1 - Cos ka
ħωk = 2|J|S (1- Cos
collective modes.


collective modes
For FM spin waves
H is rotationally invariant
ground state is (2NS+1) fold degenerate
breaking ⇒ picking of a preferred direction along which S_tot points
e.g. Ising model
symmetry breaking field
what are the states we integrate over?

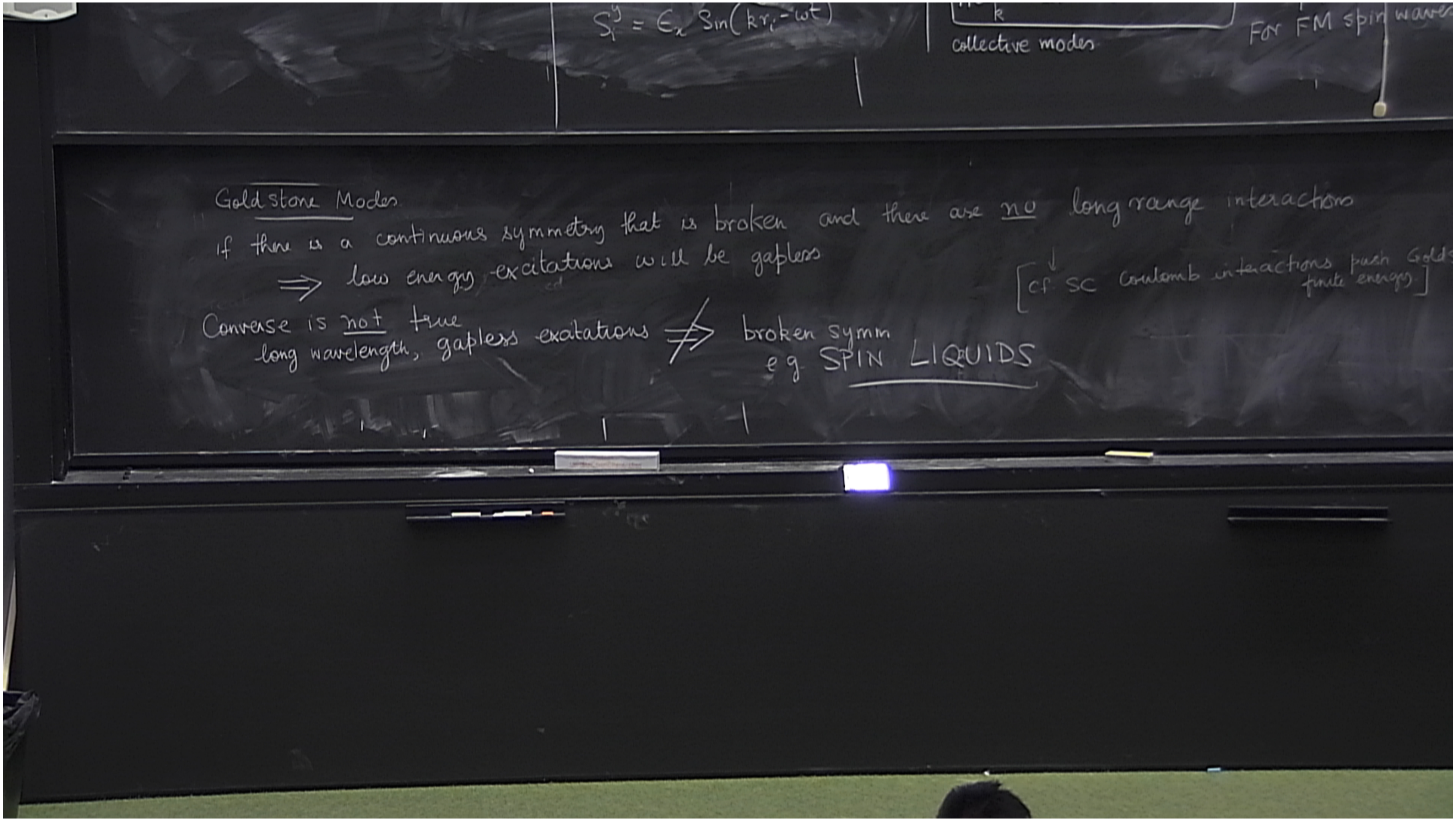

S_i^y = ε_x Sin(kr_i - ωt)
collective modes
For FM spin waves
Goldstone Modes
If there is a continuous symmetry that is broken and there are no lo

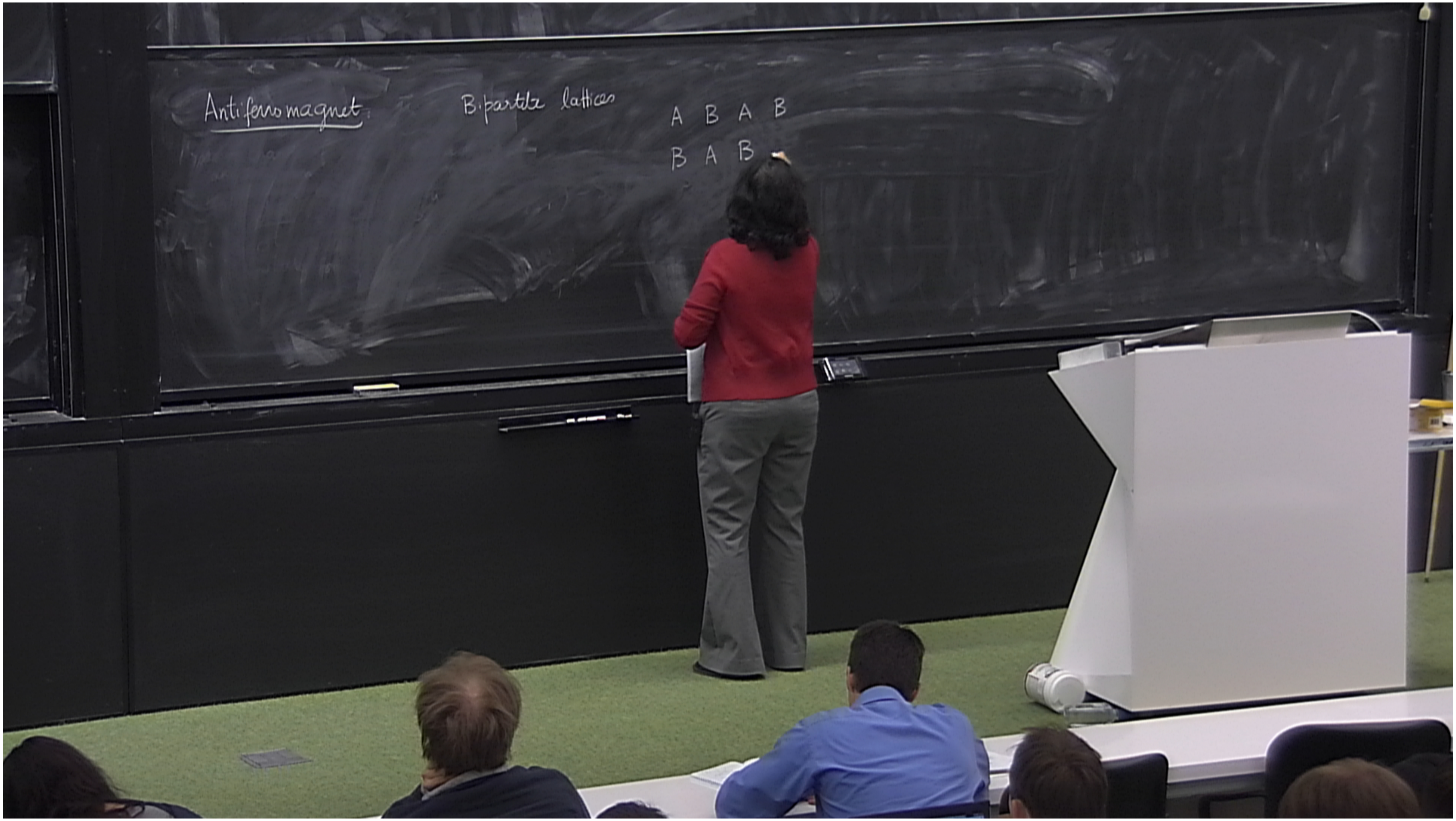

$S_i^y = \epsilon_x \sin(kr_i - \omega t)$
collective modes
For FM spin wave
Goldstone Modes
If there is a continuous symmetry that is broken and there are no long range interactions
⇒ low energy excita

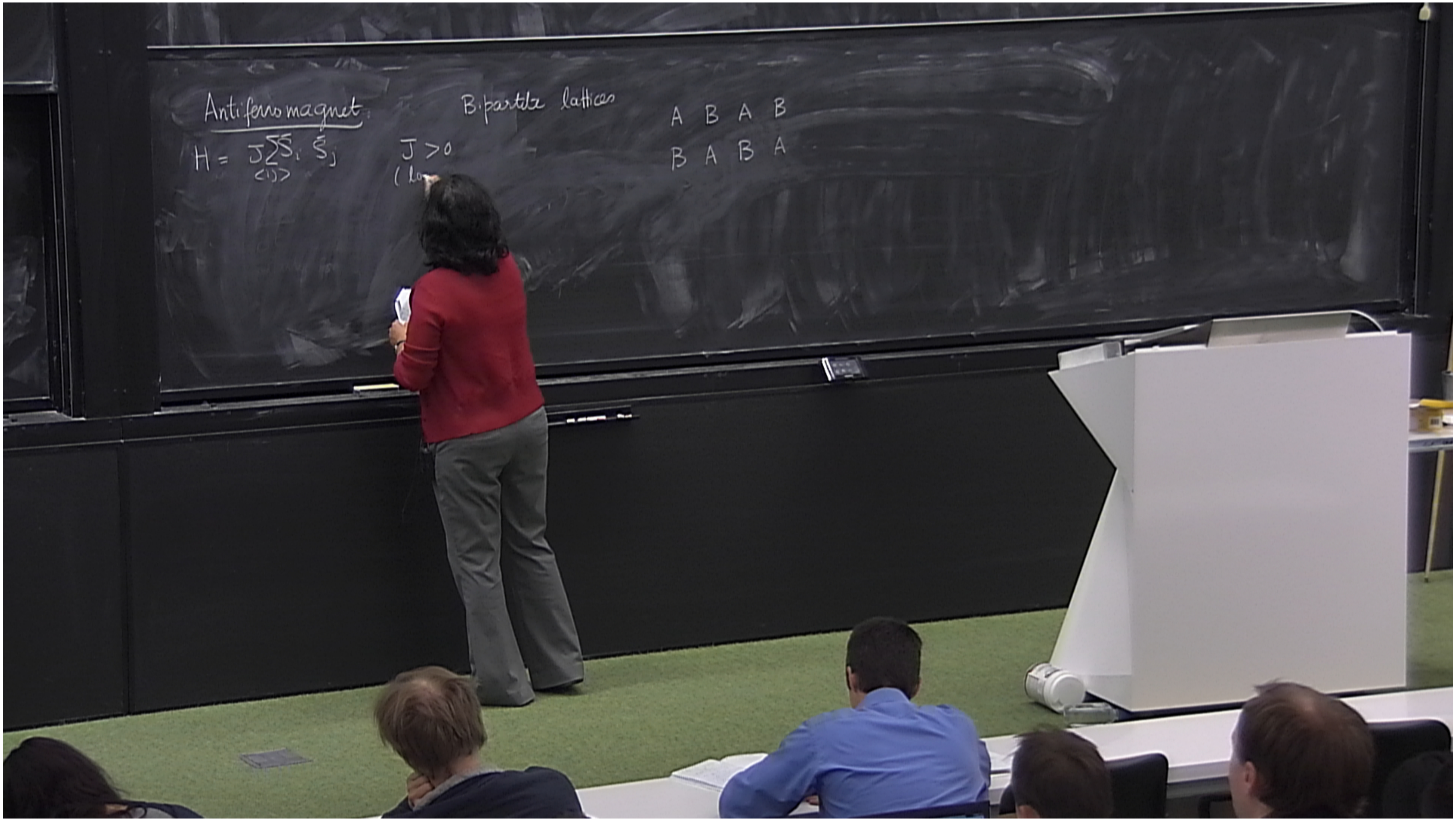

$S_i^y = \epsilon_x \sin(kr_i - \omega t)$
k
collective modes
For FM spin waves
Goldstone Modes
If there is a continuous symmetry that is broken and there are no long range interactions
⇒ low energy excitations will be gapless
cf SC Coulomb interactions

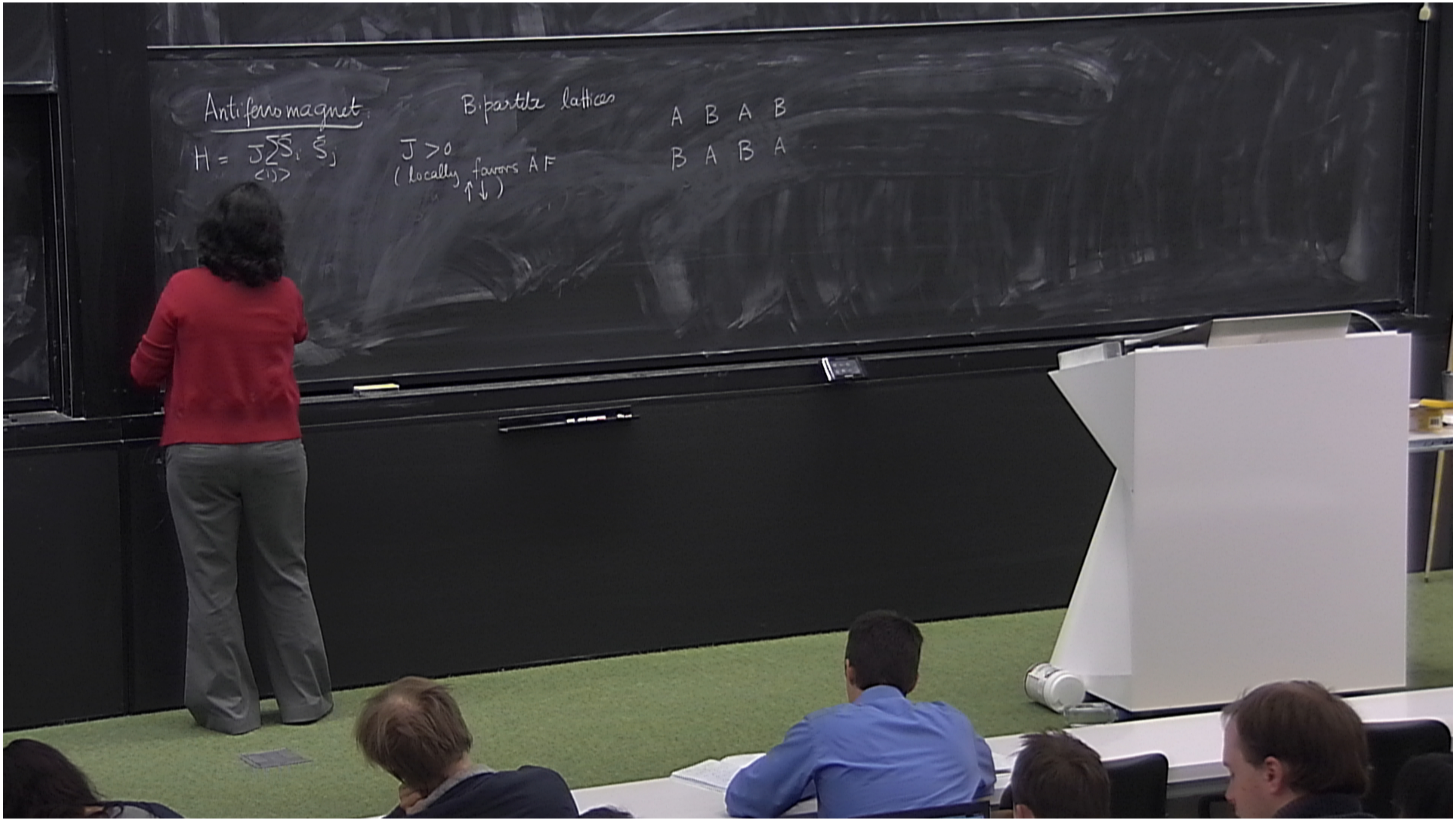


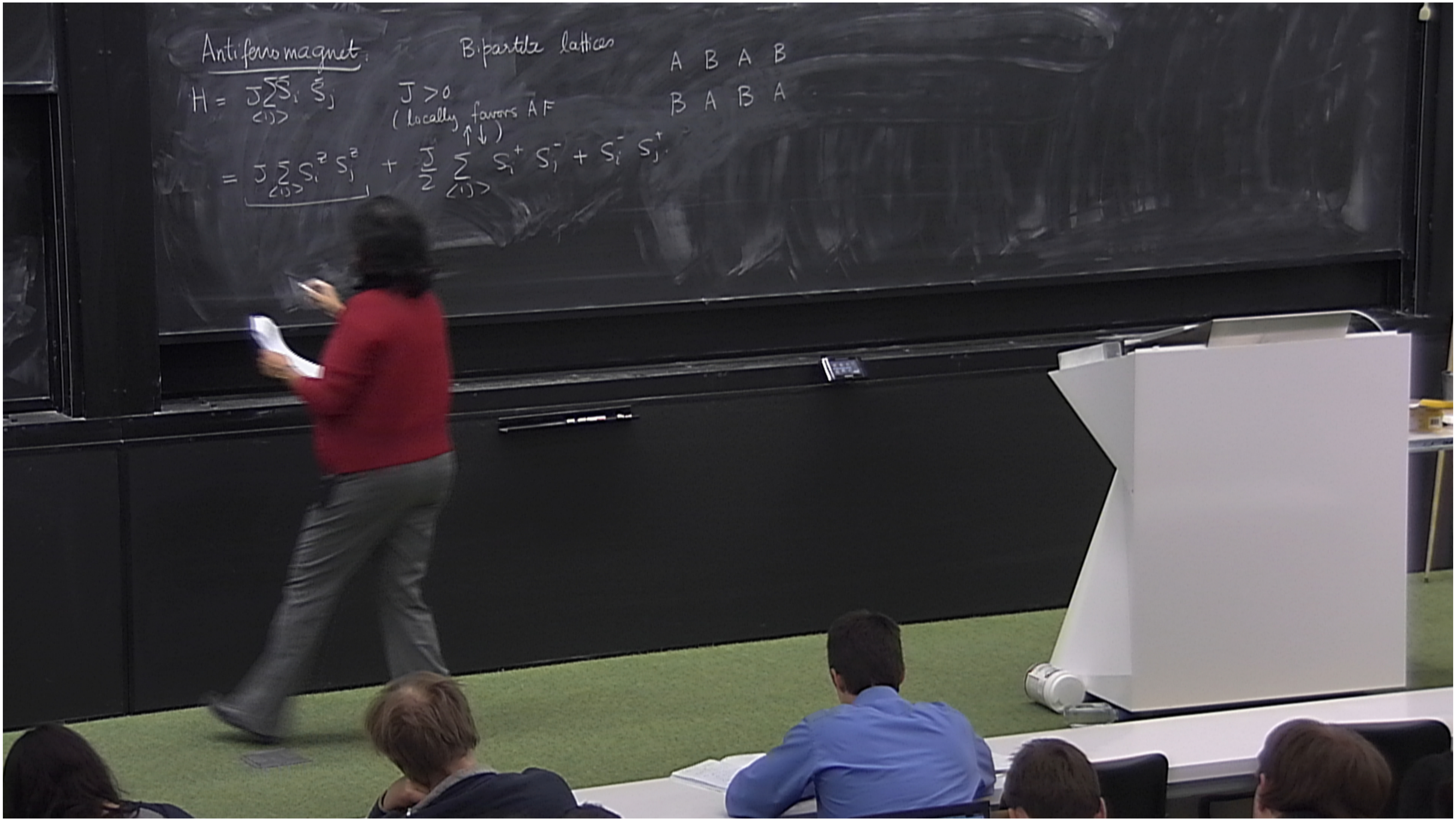
S_i^y = ε_x Sin(kr_i - ωt)
collective modes
For FM spin
Goldstone Modes
If there is a continuous symmetry that is broken and there are no long range interactions
⇒ low energy excitations will
gapless
cf SC Coulomb interactions push
finite energy.
Converse is not true
long wavelength, gapless excitations

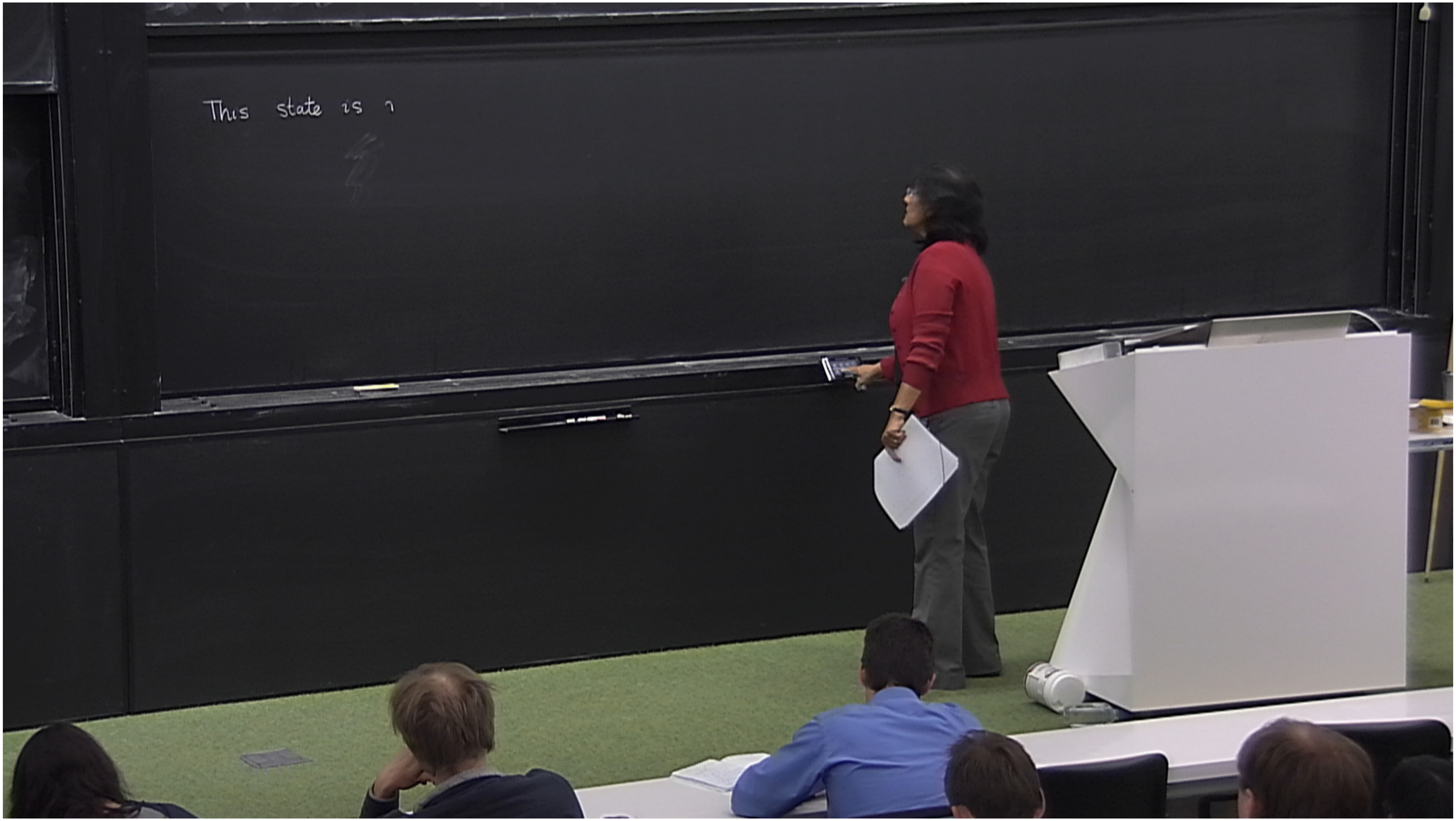

$S_i^y = \epsilon_x \sin(kr_i - \omega t)$
k
collective modes
For FM spin waves
Goldstone Modes
If there is a continuous symmetry that is broken and there are no long range interactions
⟹ low energy excitations will be gapless
[cf SC Coulomb interactions push Golds finite energies]
Converse is not true
long wavelength, gapless excitations ⇏ broken symm
e.g. SPIN LIQUIDS

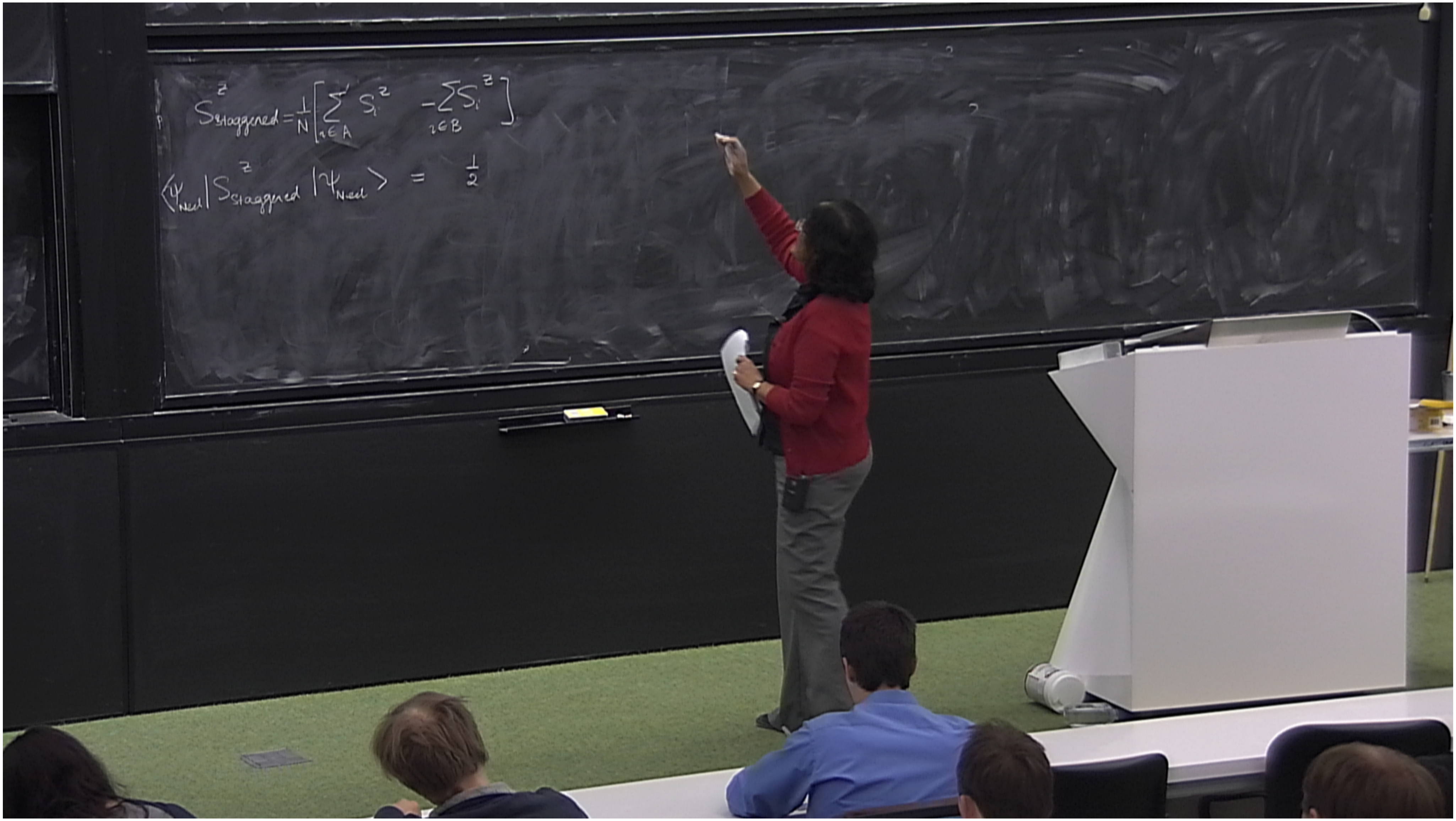

Antiferromagnet:
Bipartite lattices
A B A B
B A B

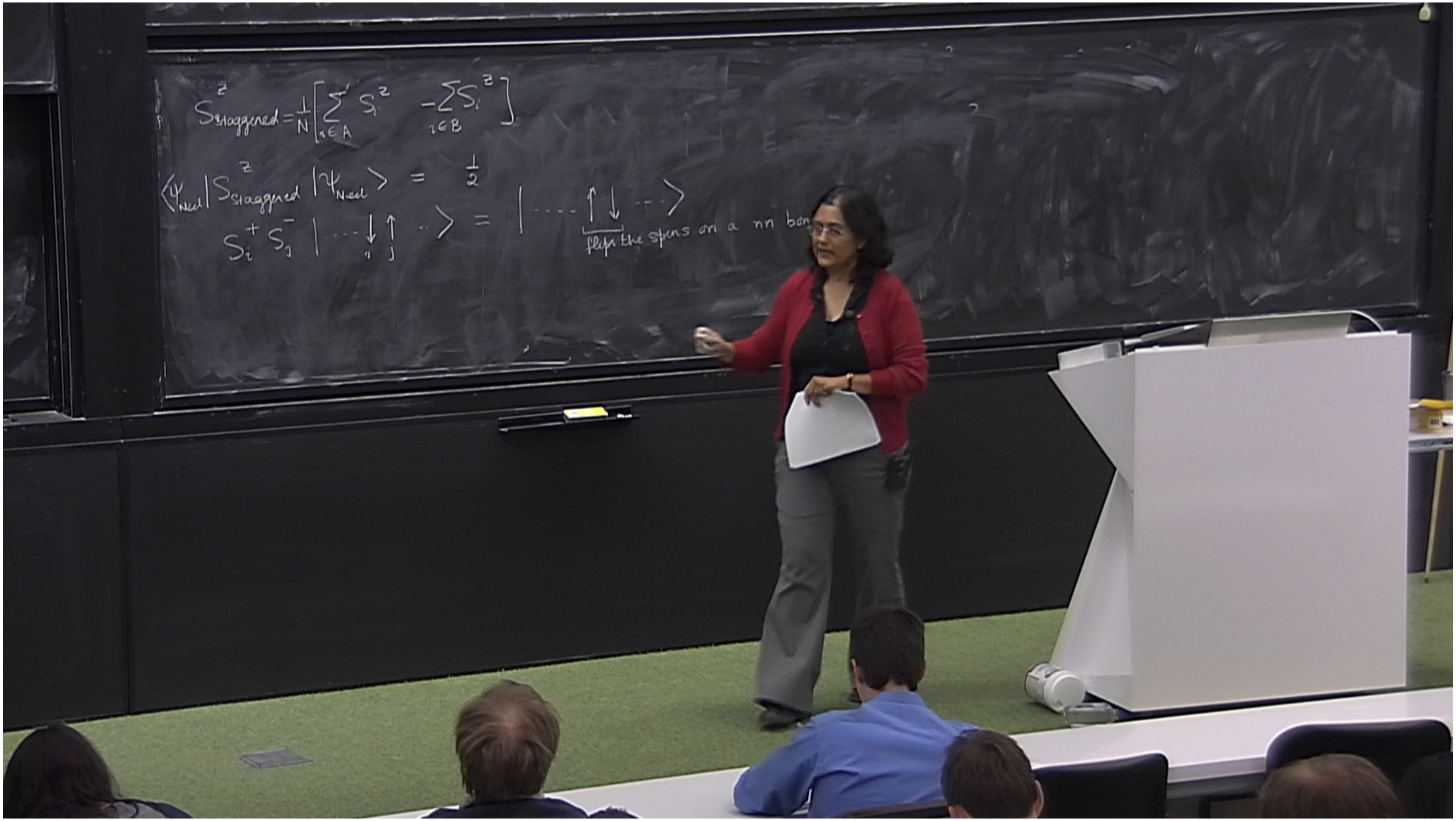

Antiferromagnet:
H = J∑S_i S_j
<ij>
J > 0
Bipartite lattices
A B A B
B A B A

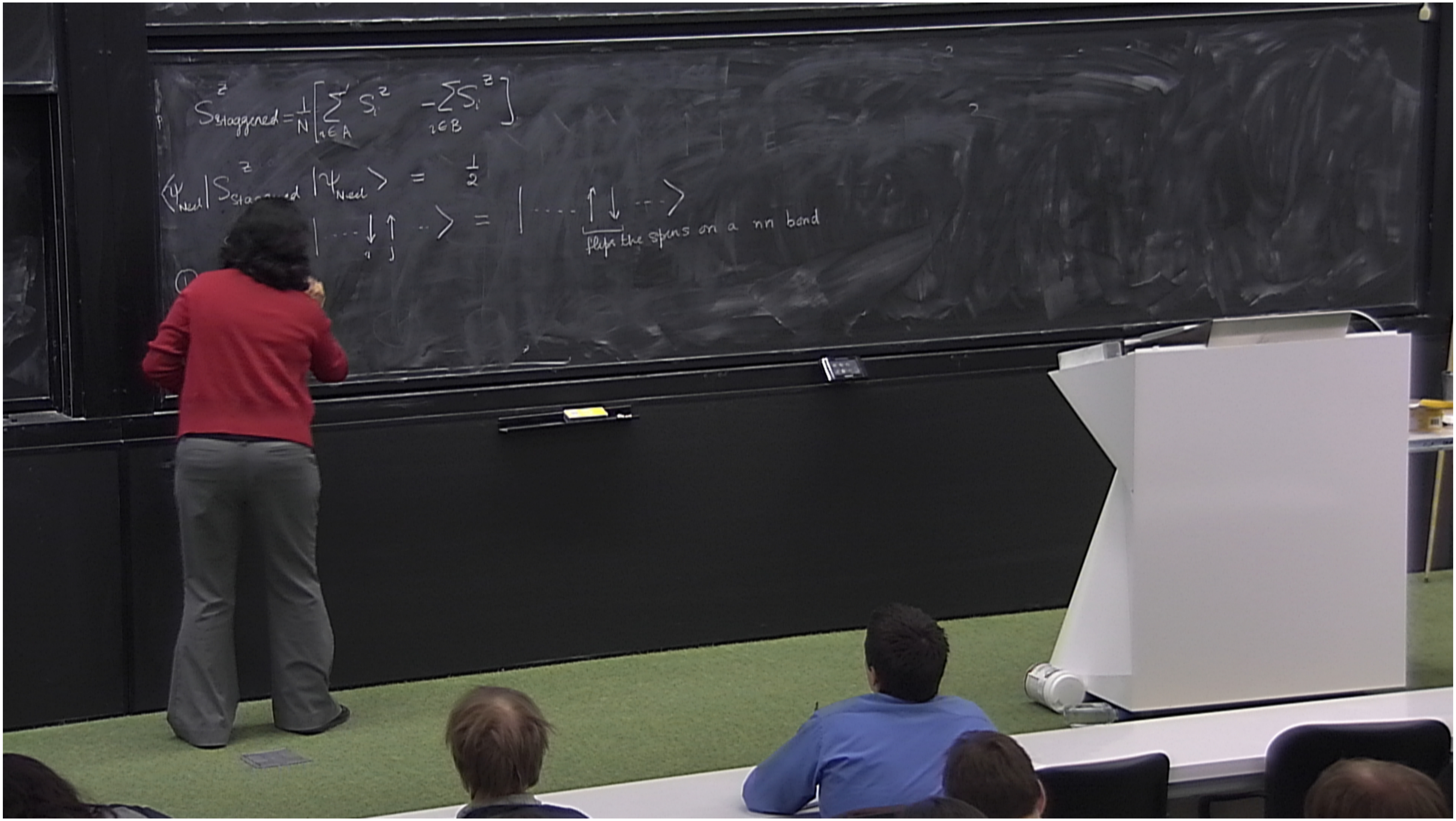

Antiferromagnet
H = J∑ S_i · S_j
<ij>
J > 0
(locally favors AF
↑↓)
Bipartite lattices
A B A B
B A B A

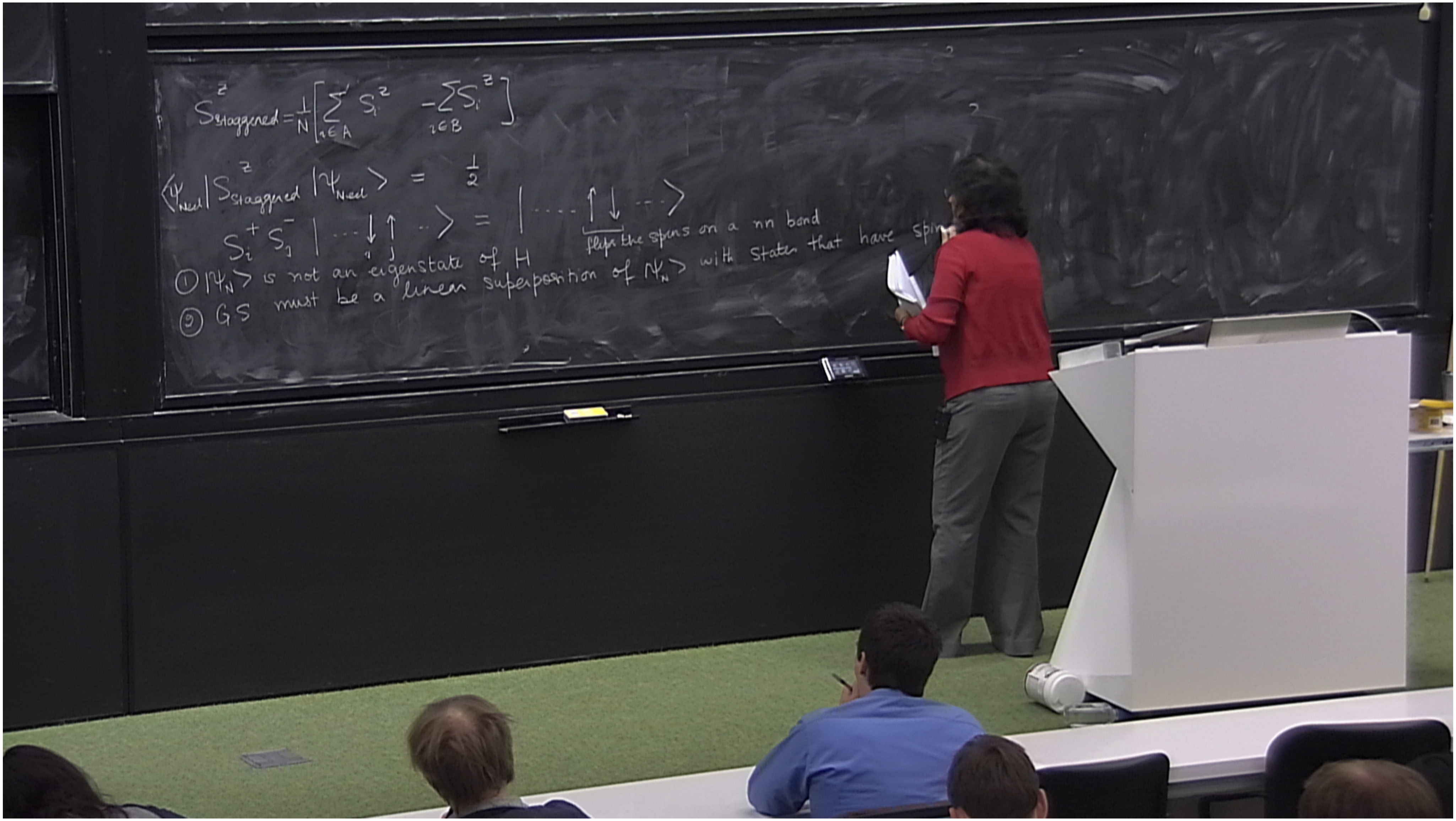

Antiferromagnet:
Bipartite lattices
A B A B
B A B A
H = J Σ<ij> S_i S_j
J > 0
(locally favors AF ↑↓)
= J Σ<ij> S_i^z S_j^z + J/2 Σ<ij> S_i^+ S_j^- + S_i^- S_j^+

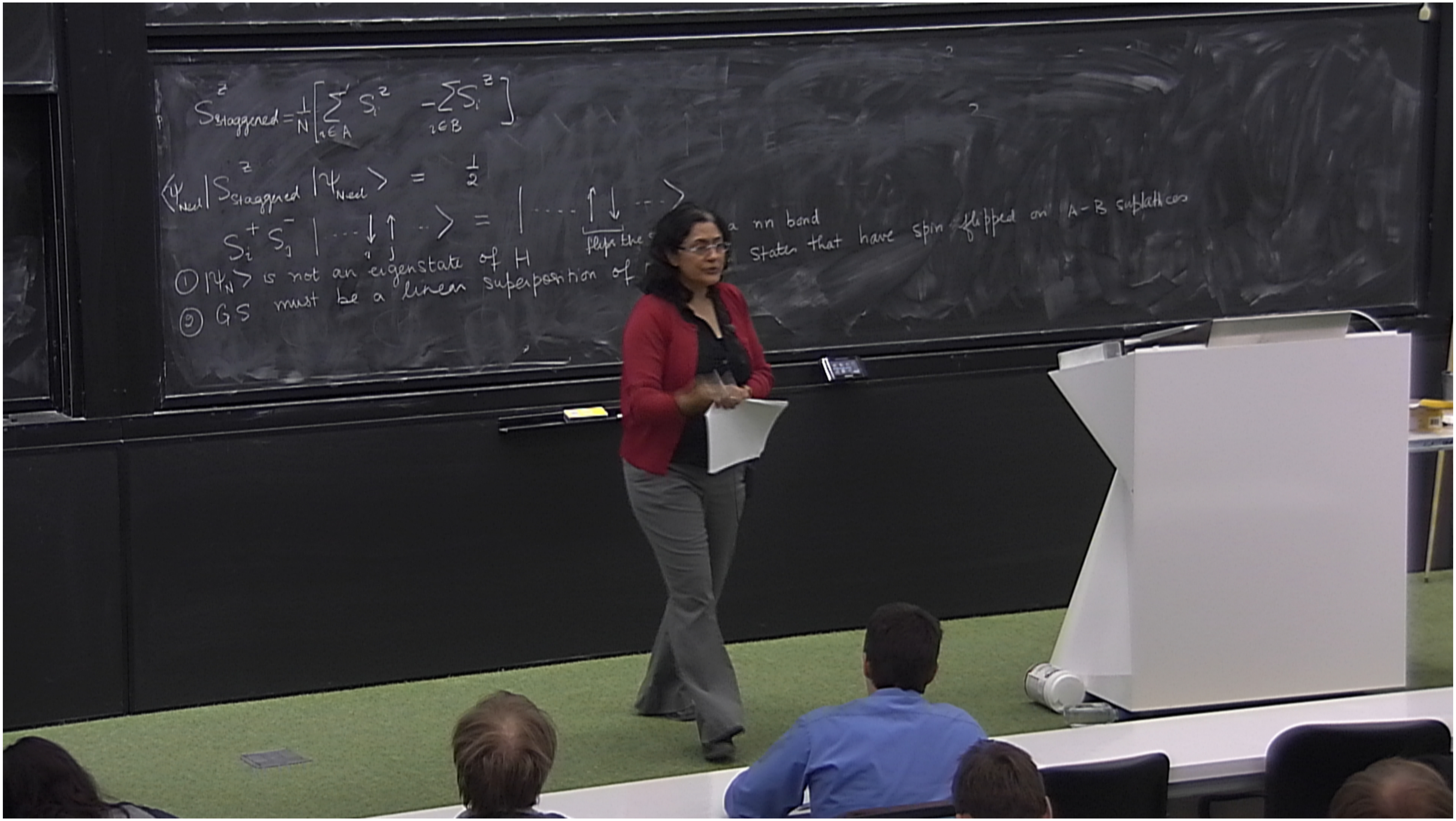

This state is ?


$S^z_{staggered} = \frac{1}{N}\left[\sum'_{i \in A} S_i^z - \sum_{i \in B} S_i^z\right]$
$\langle \psi_{Néel} | S^z_{staggered} | \psi_{Néel} \rangle = \frac{1}{2}$

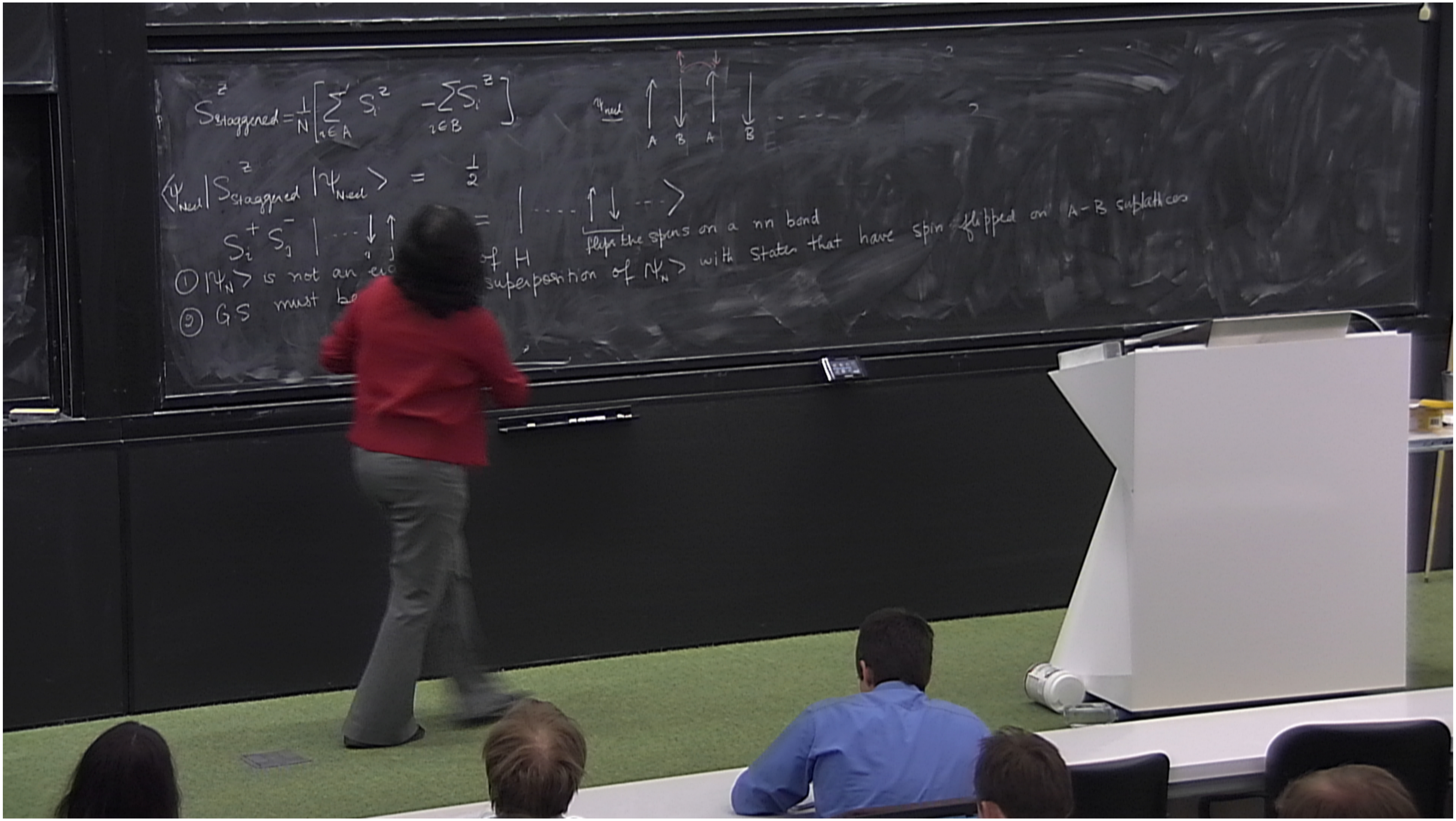

$S^z_{staggered} = \frac{1}{N}\left[\sum_{i\in A} S_i^z - \sum_{i\in B} S_i^z\right]$
$\langle\psi_{Neel}| S^z_{staggered} |\psi_{Neel}\rangle = \frac{1}{2}$
$S_i^+ S_j^- |\cdots\downarrow_i \uparrow_j \cdots\rangle = |\cdots\uparrow\downarrow\cdots\rangle$
flips the spins on a nn bon

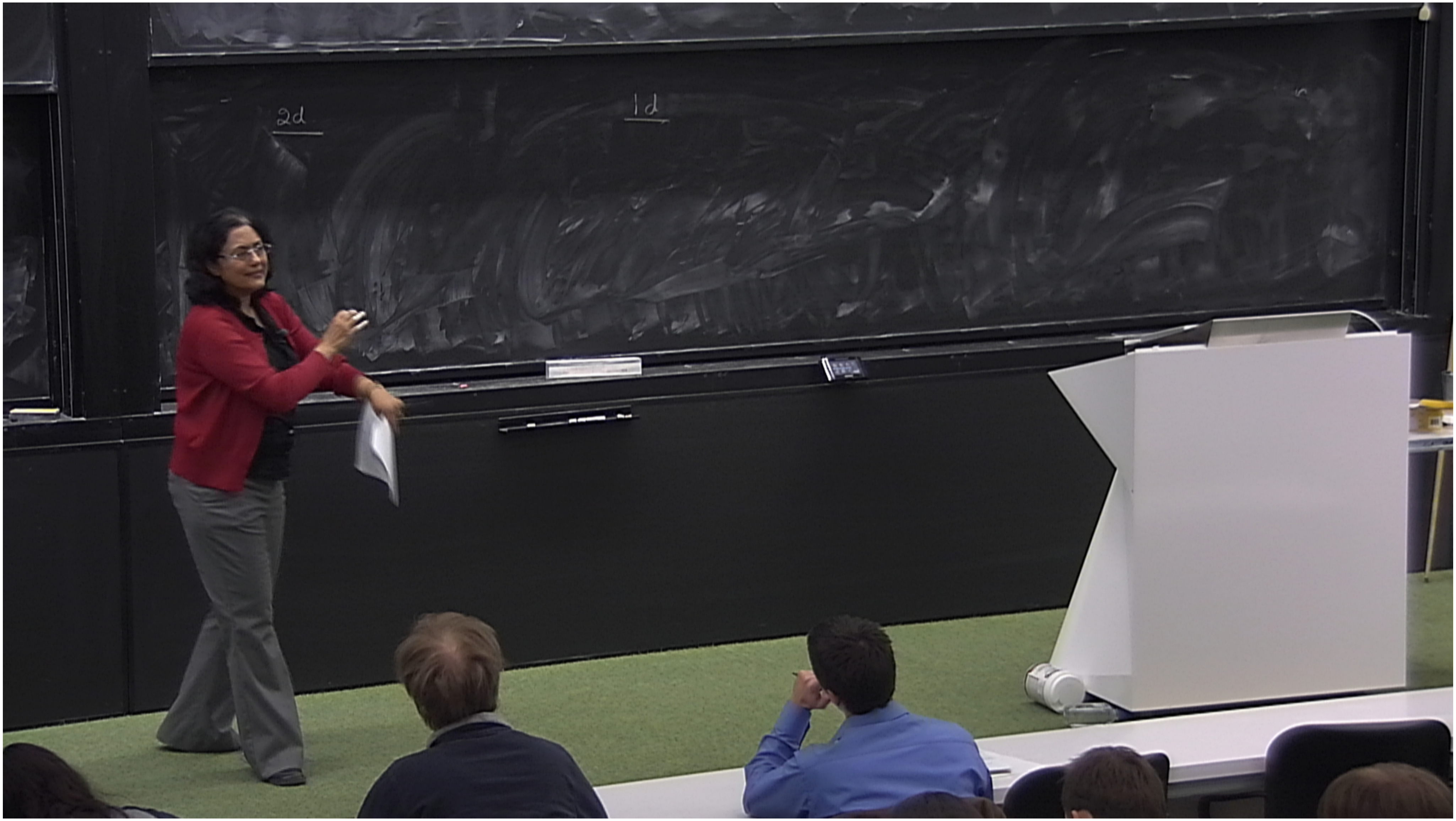

flips the spins on a nn bond

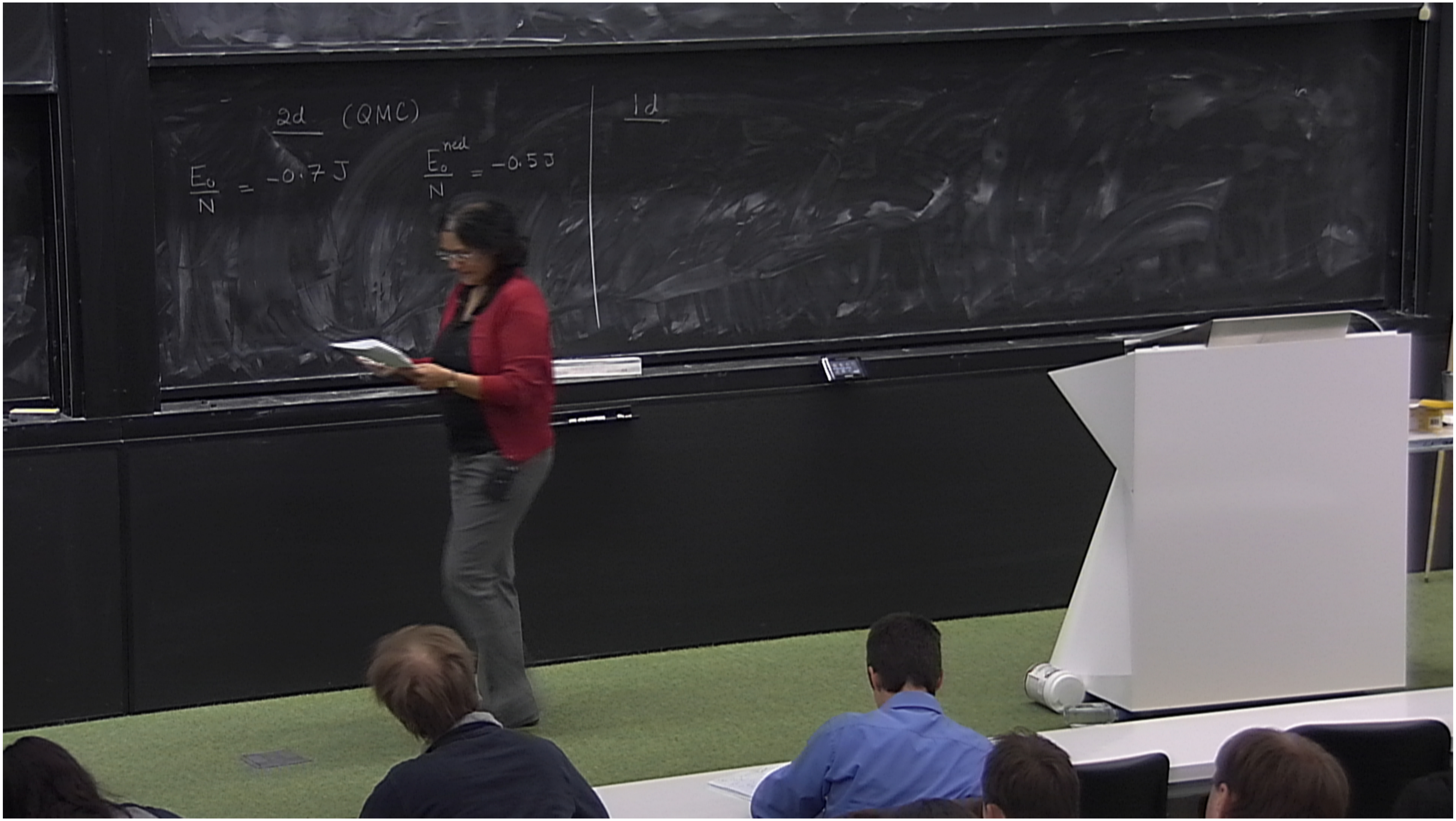

S^z staggered = 1/N [Σ' i∈A S_i^z − Σ i∈B S_i^z]
⟨Ψ Neel| S^z staggered |Ψ Neel⟩ = 1/2
S_i^+ S_j^- | ... ↓ ↑ .. ⟩ = | ..... ↑ ↓ ... ⟩
flips the spins on a nn bond
① |Ψ_N⟩ is not an eigenstate of H
② GS must be a linear superposition of |Ψ_N⟩ with states that have spin


$S^z_{staggered} = \frac{1}{N}\left[\sum_{i\in A} S_i^z - \sum_{i\in B} S_i^z\right]$
$\langle\Psi_{Neel}| S^z_{staggered} |\Psi_{Neel}\rangle = \frac{1}{2}$
$S_i^+ S_j^- | \cdots \downarrow \uparrow \cdots \rangle = | \cdots \uparrow \downarrow \cdots \rangle$
flips the
a nn bond
① $|\Psi_N\rangle$ is not an eigenstate of H
States that have spin flipped on A-B sublattices
② GS must be a linear superposition of


$S^z_{staggered} = \frac{1}{N}\left[\sum_{i\in A} S_i^z - \sum_{i\in B} S_i^z\right]$
ψ_{Neel} ↑ ↓ ↑ ↓ . . . .
A B A B
$\langle\psi_{Neel}| S^z_{staggered} |\psi_{Neel}\rangle = \frac{1}{2}$
a nn bond
① $|\psi_N\rangle$ is not an eigenstate of H
② GS must be a linear superposition of
states that have spin flipped on A-B sublattices

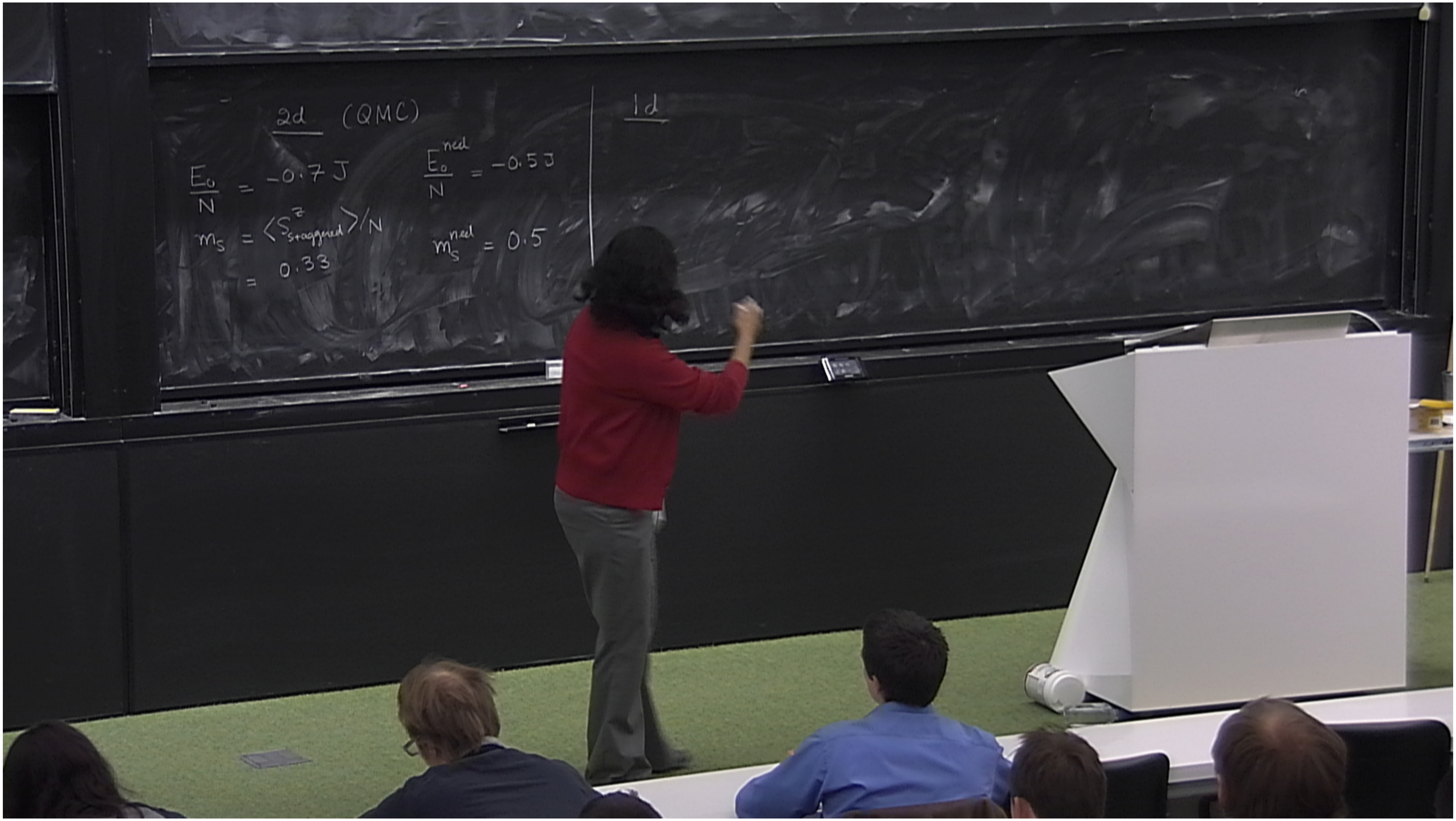

S^z_staggered = 1/N [Σ' S_i^z − Σ S_i^z]
i∈A
i∈B
ψ_Neel ↑ ↓ ↑ ↓ . . . .
A B A B
⟨ψ_Neel| S^z_staggered |ψ_Neel⟩ = 1/2
S_i^+ S_j^- | . . . ↓ ↑
= | . . . . ↑ ↓ . . . ⟩
flips the spins on a nn bond
of H
① |ψ_N⟩ is not an
superposition of |ψ_N⟩ with states that have spin flipped on A-B sublattices
② GS must

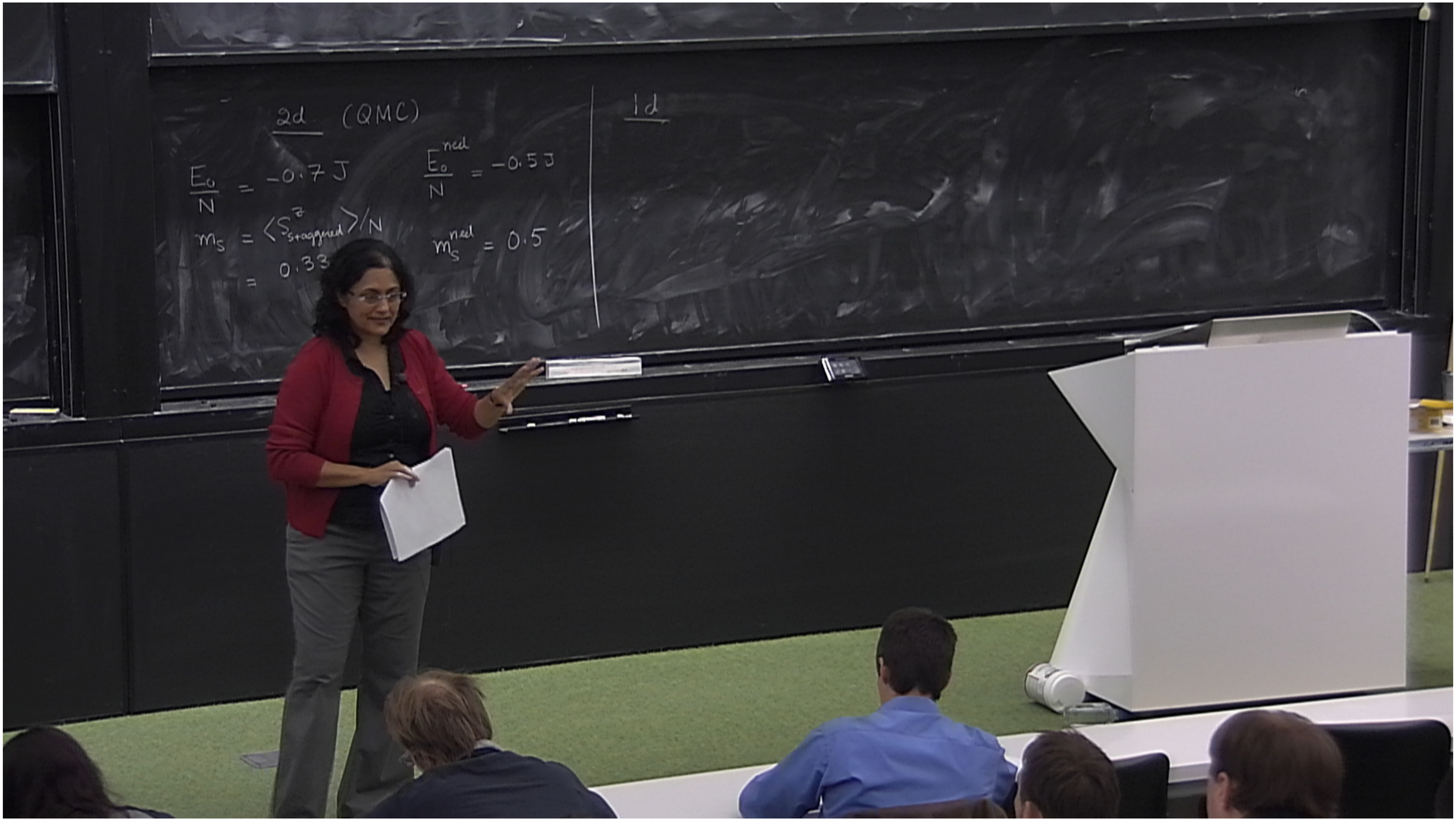

2d
1d



2d (QMC)
$\frac{E_0}{N} = -0.7J$
$\frac{E_0^{neel}}{N} = -0.5J$
1d


2d (QMC)
1d
E0/N = -0.7 J
E0^neel/N = -0.5 J
ms = ⟨S^z staggered⟩/N
= 0.33
ms^neel = 0.5


2d (QMC)
E0/N = -0.7 J
mS = ⟨S^z staggered⟩/N
= 0.33
E0^neel/N = -0.5 J
mS^neel = 0.5
1d


2d (QMC)
E0/N = -0.7 J
ms = ⟨S^z staggered⟩/N
= 0.33
E0^neel/N = -0.5 J
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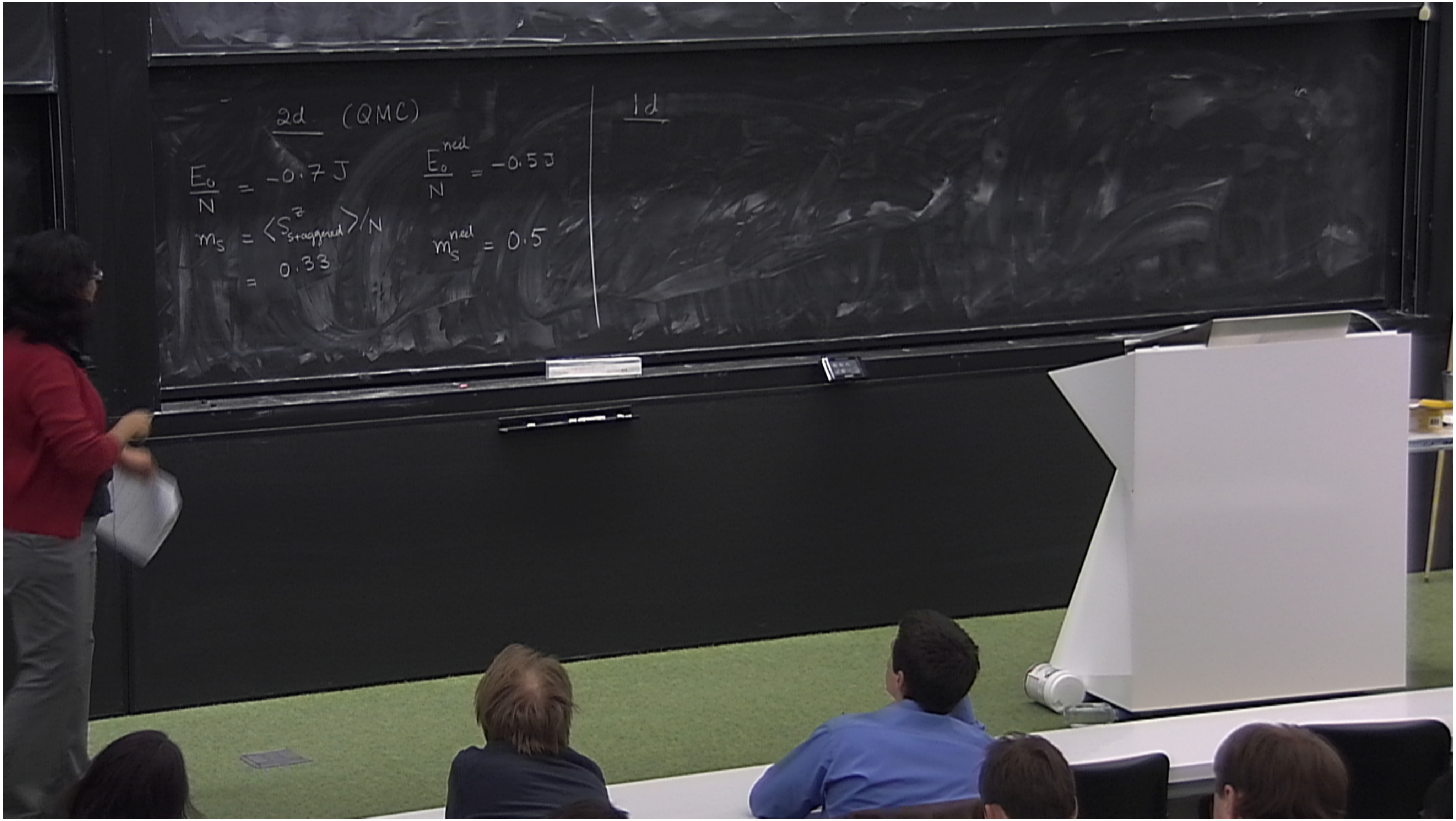
1d



2d (QMC)
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ms = ⟨S^z staggered⟩/N
= 0.33
E0^neel/N = -0.5 J
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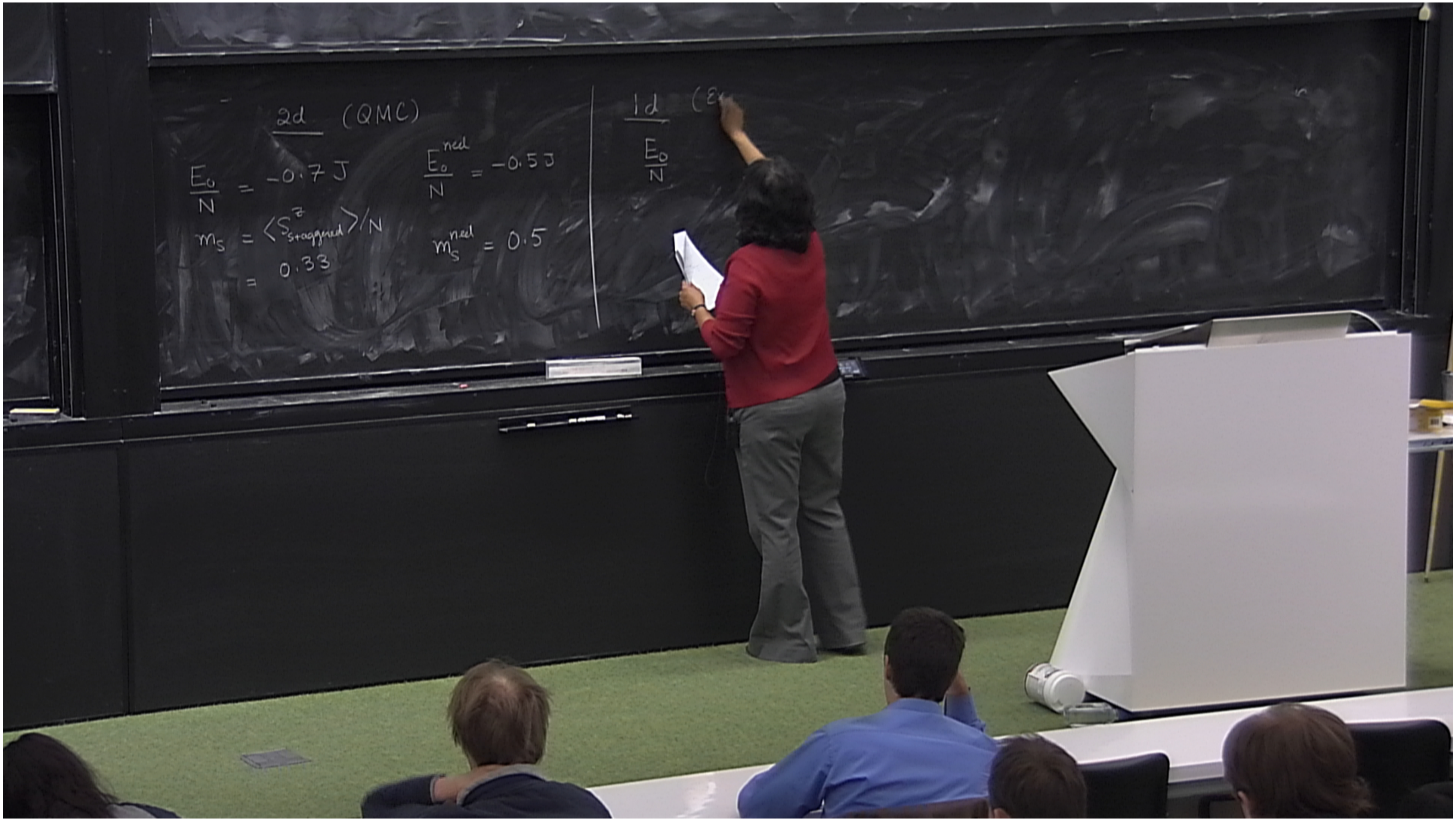
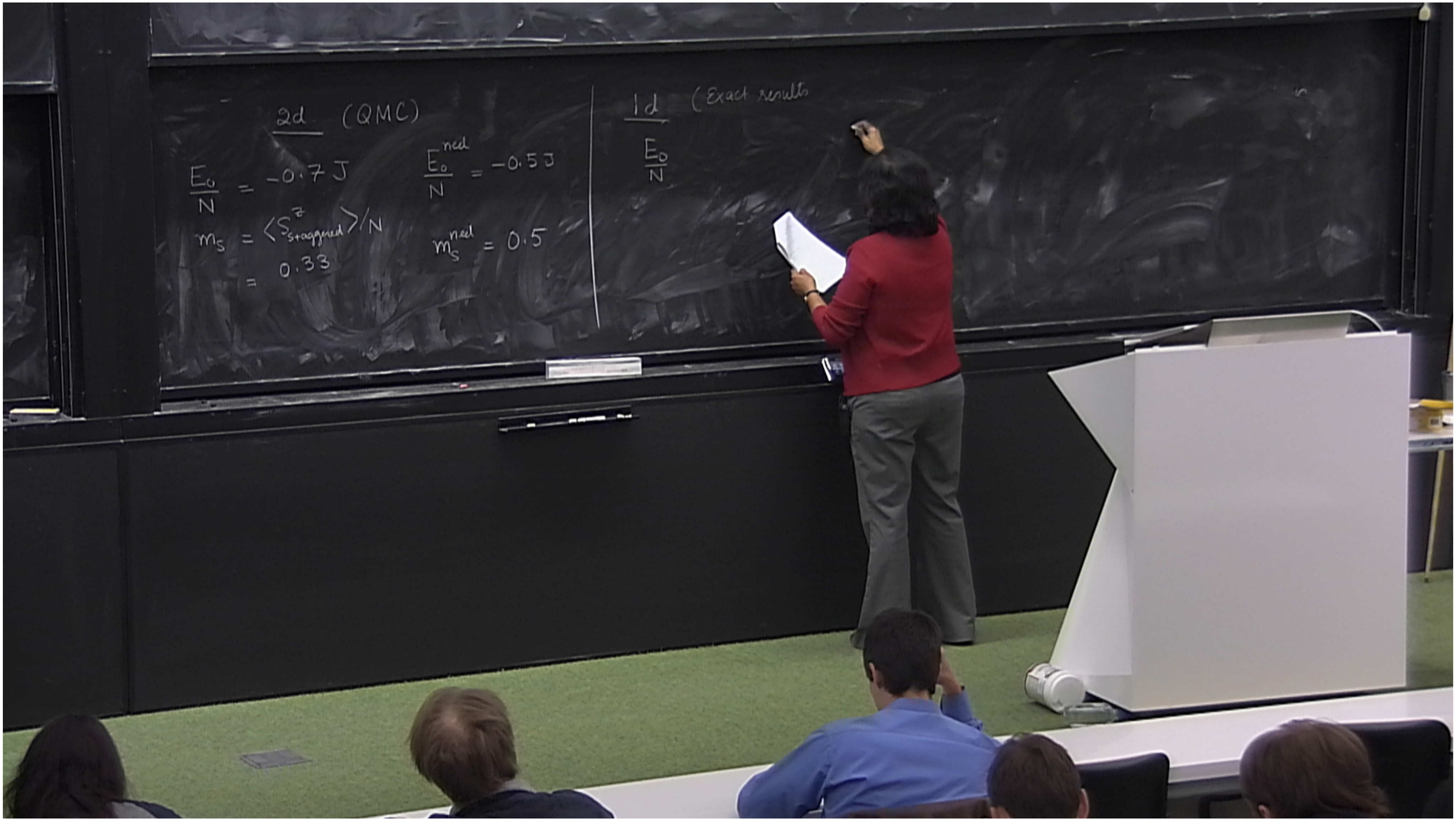
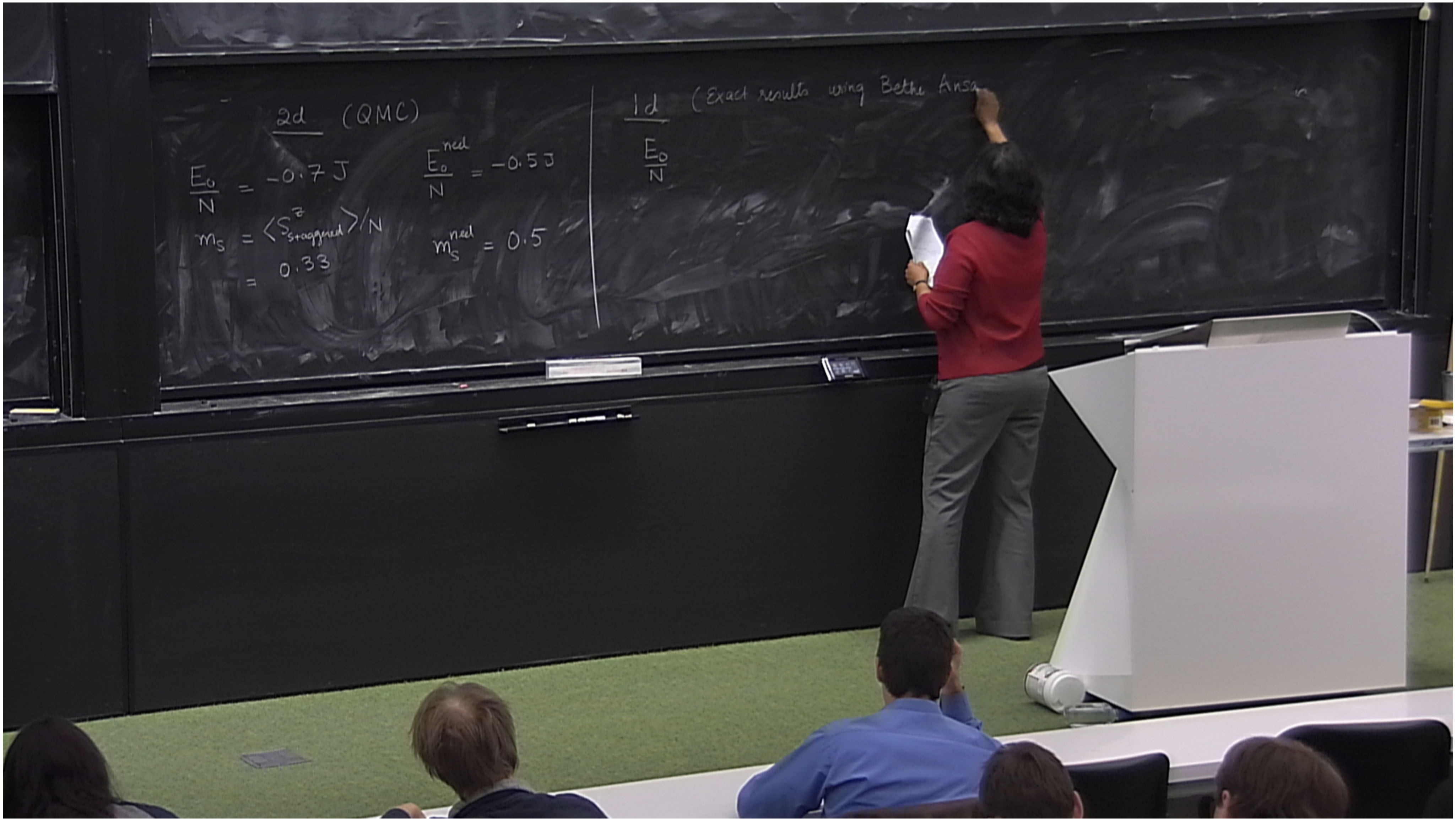
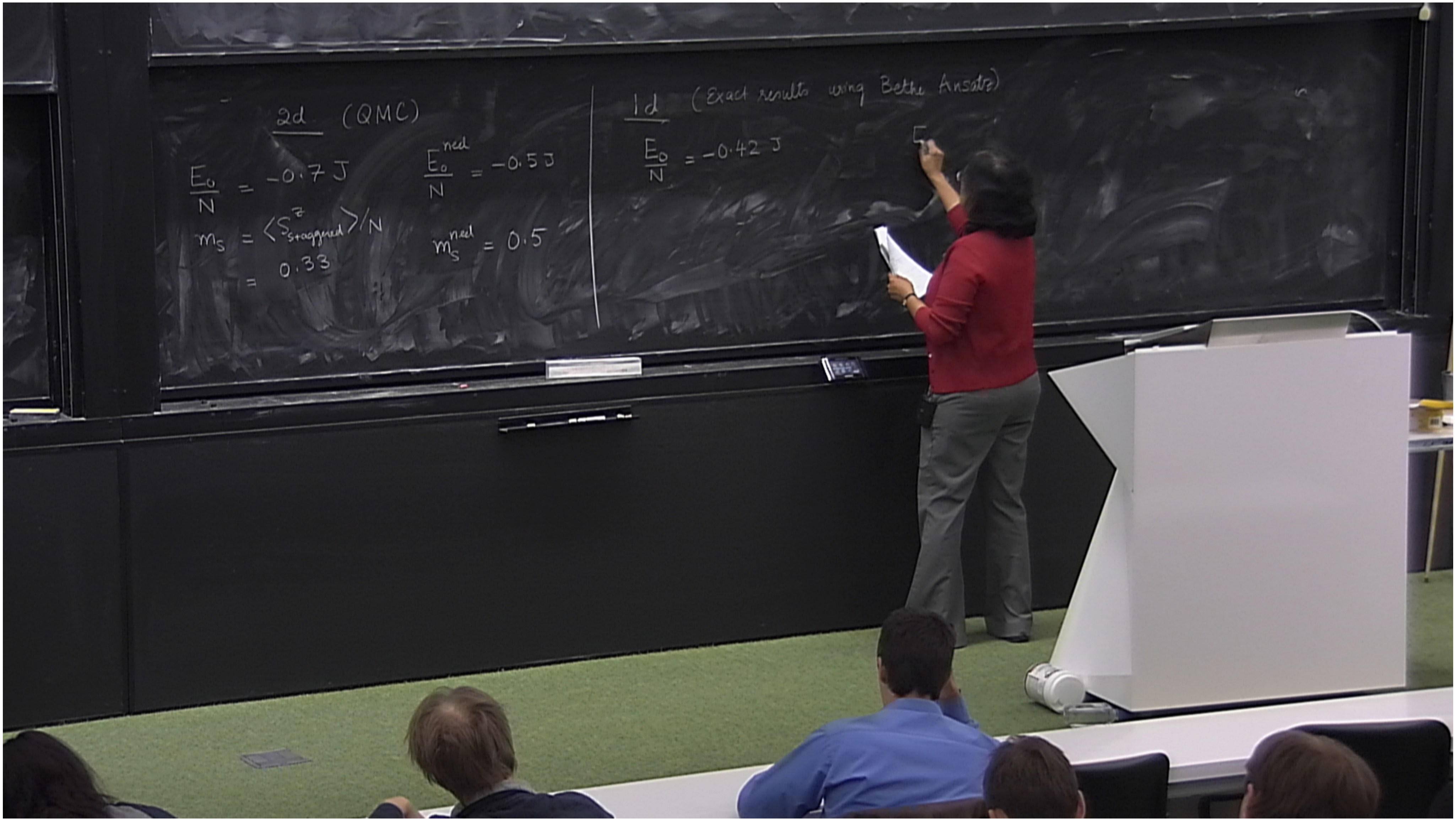
1d


2d (QMC)
1d
E0/N = -0.7 J
E0^neel/N = -0.5 J
m_s = ⟨S^z_staggered⟩/N
= 0.33
m_s^neel = 0.5

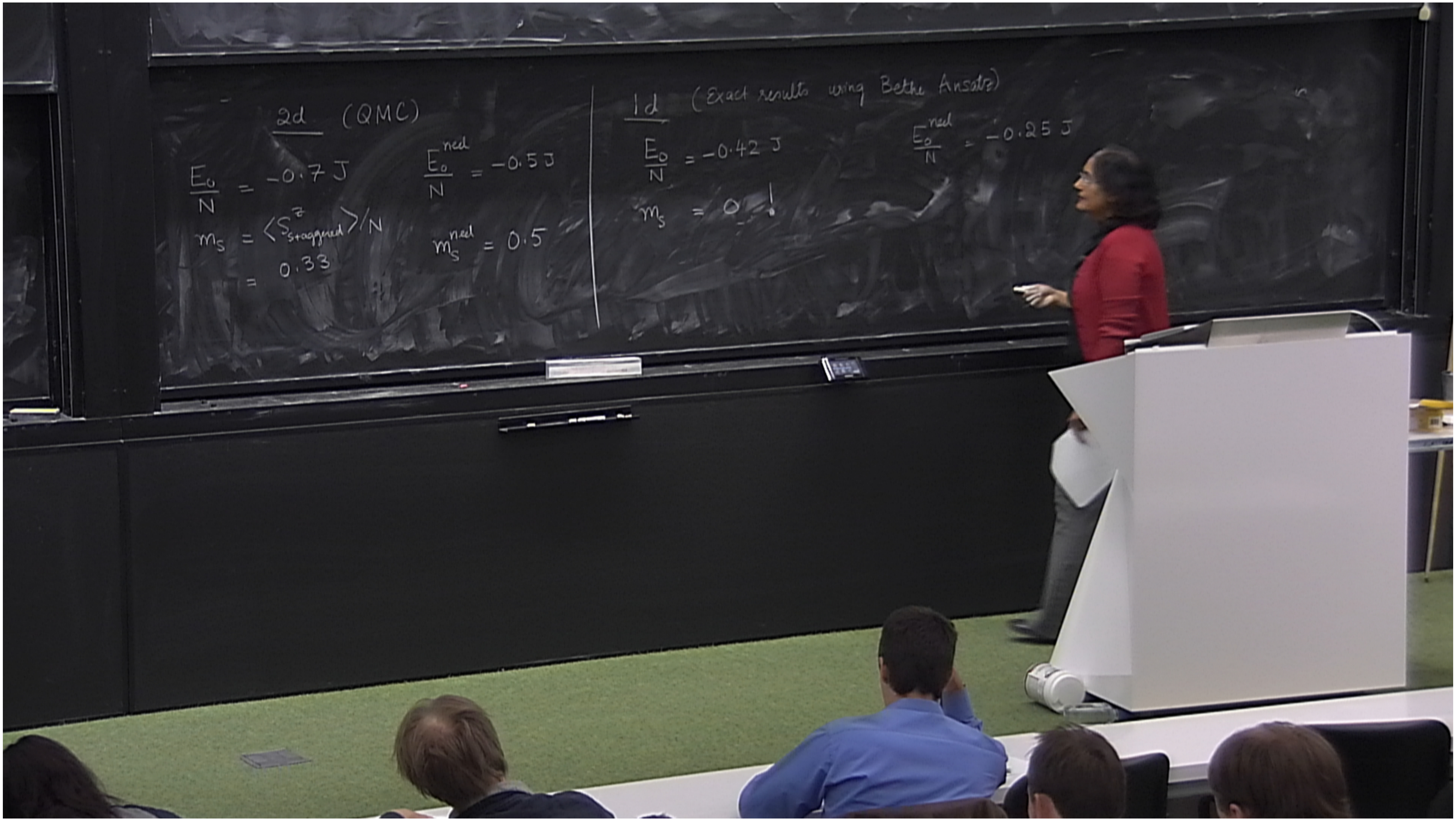

2d (QMC)
E_0/N = -0.7 J
m_s = ⟨S^z_staggered⟩/N
= 0.33
E_0^néel/N = -0.5 J
m_s^néel = 0.5
1d
E_0/N

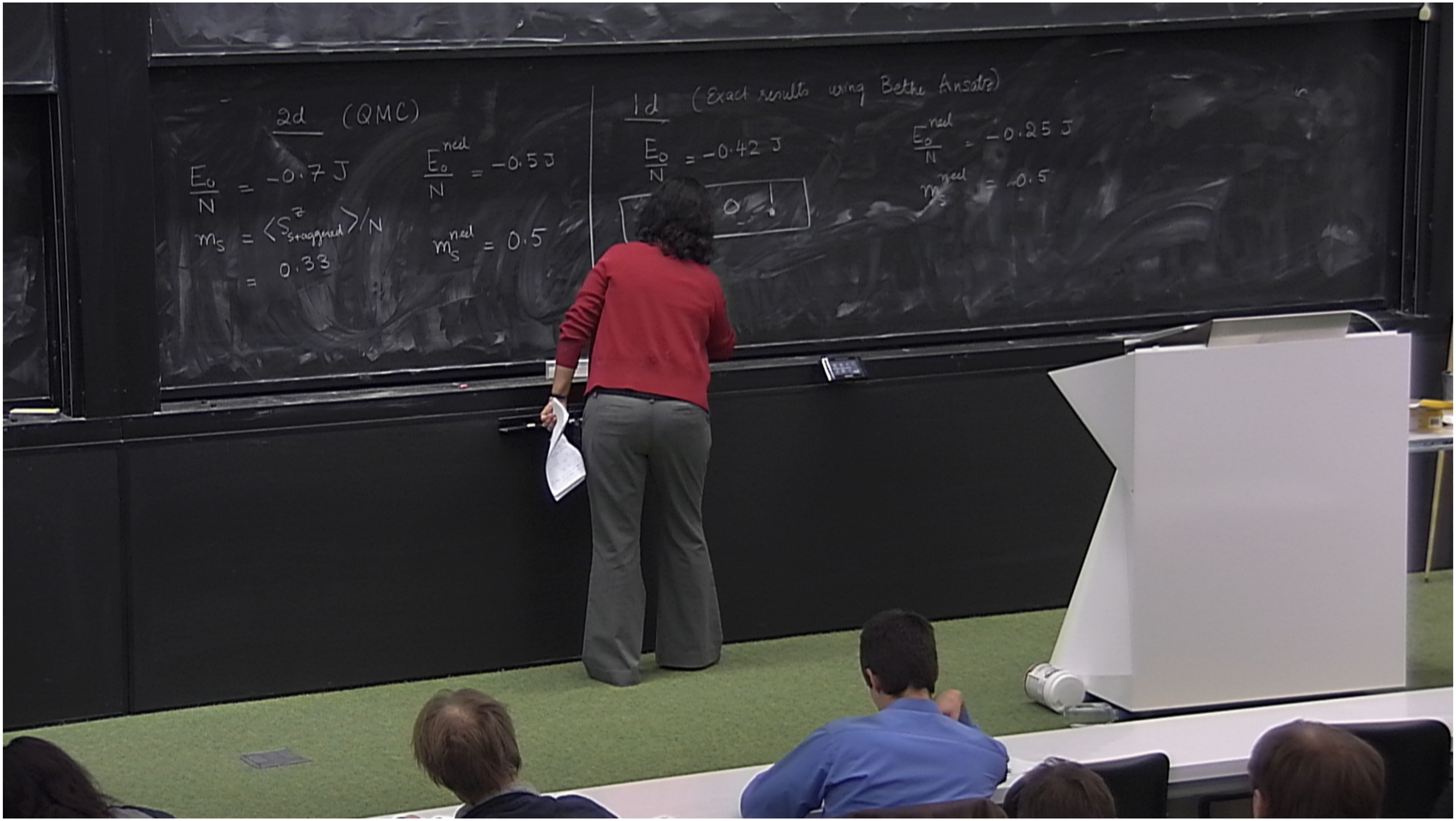

2d (QMC)
$\frac{E_0}{N} = -0.7J$
$\frac{E_0^{neel}}{N} = -0.5J$
$m_s = \langle S^z_{staggered} \rangle / N$
$= 0.33$
$m_s^{neel} = 0.5$
1d (Exact results
$\frac{E_0}{N}$

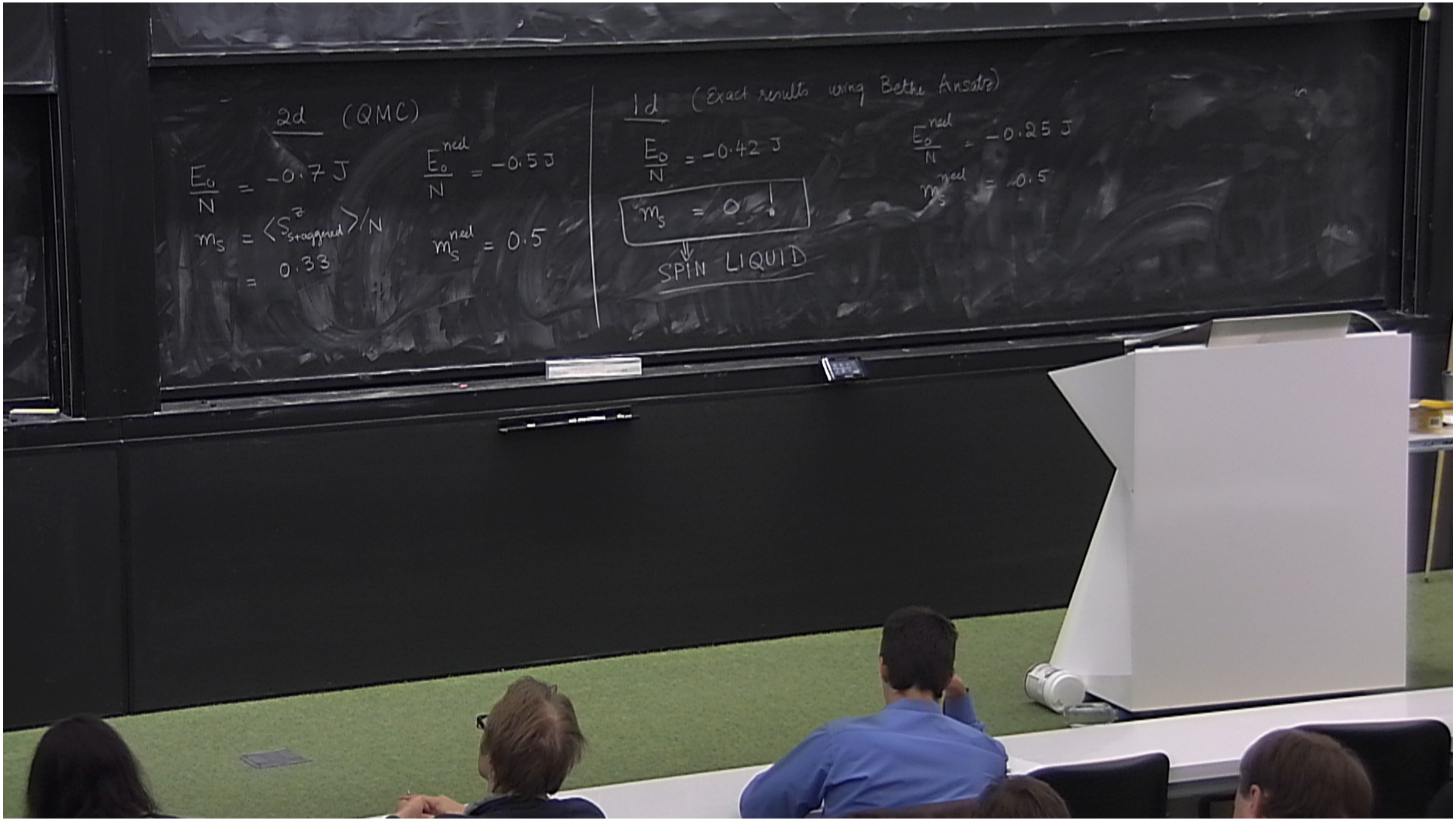

2d (QMC)
E_0/N = -0.7 J
E_0^{neel}/N = -0.5 J
m_s = ⟨S^z_staggered⟩/N
= 0.33
m_s^{neel} = 0.5
1d (Exact results using Bethe Ansa
E_0/N

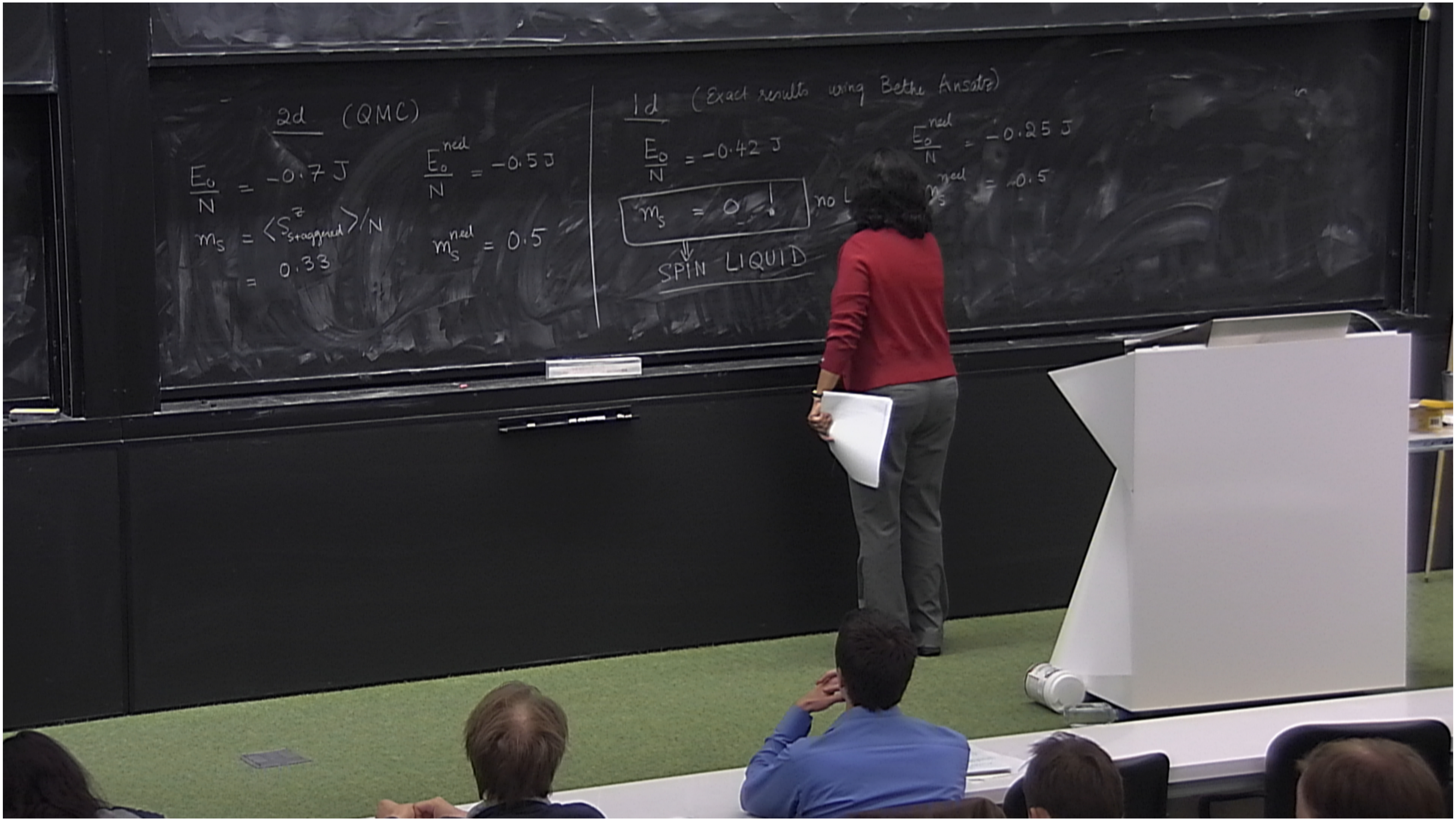

2d (QMC)
E0/N = -0.7 J
E0^neel/N = -0.5 J
ms = ⟨S^z staggered⟩/N = 0.33
ms^neel = 0.5
1d (Exact results using Bethe Ansatz)
E0/N = -0.42 J

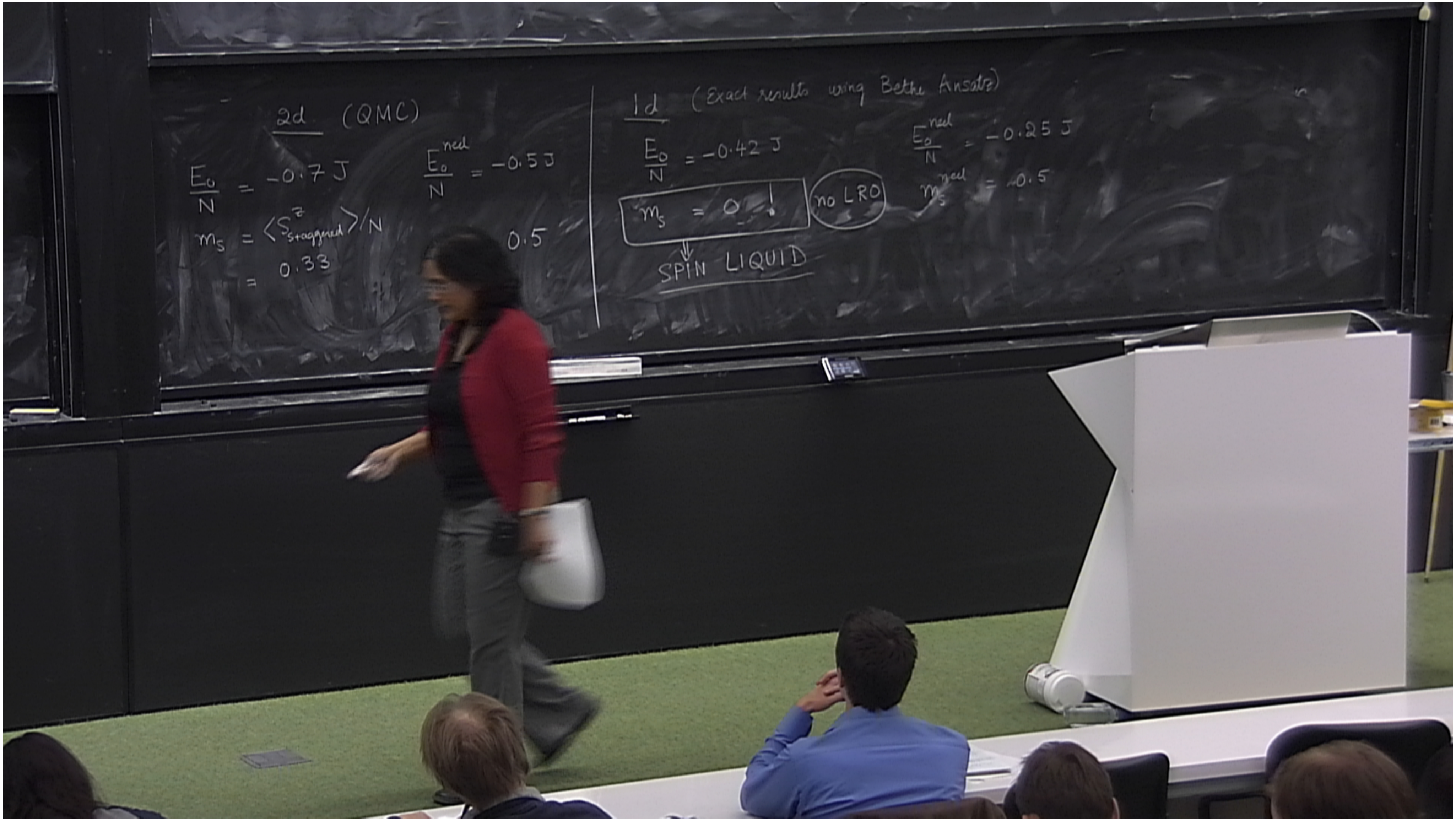

2d (QMC)
$\frac{E_0}{N} = -0.7 J$
$\frac{E_0^{neel}}{N} = -0.5 J$
$m_s = \langle S^z_{staggered} \rangle / N = 0.33$
$m_s^{neel} = 0.5$
1d (Exact results using Bethe Ansatz)
$\frac{E_0}{N} = -0.42 J$
$\frac{E_0^{neel}}{N} = -0.25 J$


2d (QMC)
$\frac{E_0}{N} = -0.7\,J$
$m_s = \langle S^z_{staggered} \rangle / N$
$= 0.33$
$\frac{E_0^{neel}}{N} = -0.5\,J$
$m_s^{neel} = 0.5$
1d (Exact results using Bethe Ansatz)
$\frac{E_0}{N} = -0.42\,J$
$m_s = 0\,!$
$\frac{E_0^{neel}}{N} = -0.25\,J$


2d (QMC)
E0/N = -0.7 J
E0^neel/N = -0.5 J
mS = ⟨S^z staggered⟩/N
= 0.33
mS^neel = 0.5
1d (Exact results using Bethe Ansatz)
E0/N = -0.42 J
0 !
E0^neel/N = -0.25 J
mS^neel = 0.5


2d (QMC)
E0/N = -0.7 J
Eo^neel/N = -0.5 J
ms = <S^z staggered>/N
= 0.33
ms^neel = 0.5
1d (Exact results using Bethe Ansatz)
E0/N = -0.42 J
Eo^neel/N = -0.25 J
ms = 0 !
ms^neel = 0.5
SPIN LIQUID


2d (QMC)
E0/N = -0.7 J
E0^neel/N = -0.5 J
ms = ⟨S^z_staggered⟩/N
= 0.33
ms^neel = 0.5
1d (Exact results using Bethe Ansatz)
E0/N = -0.42 J
E0^neel/N = -0.25 J
ms = 0 !
ms^neel = 0.5
SPIN LIQUID


2d (QMC)
E0/N = -0.7 J
E0^neel/N = -0.5 J
ms = ⟨S^z_staggered⟩/N
= 0.33
0.5
1d (Exact results using Bethe Ansatz)
E0/N = -0.42 J
E0^neel/N = -0.25 J
ms = 0 !
no LRO
ms^neel = 0.5
SPIN LIQUID