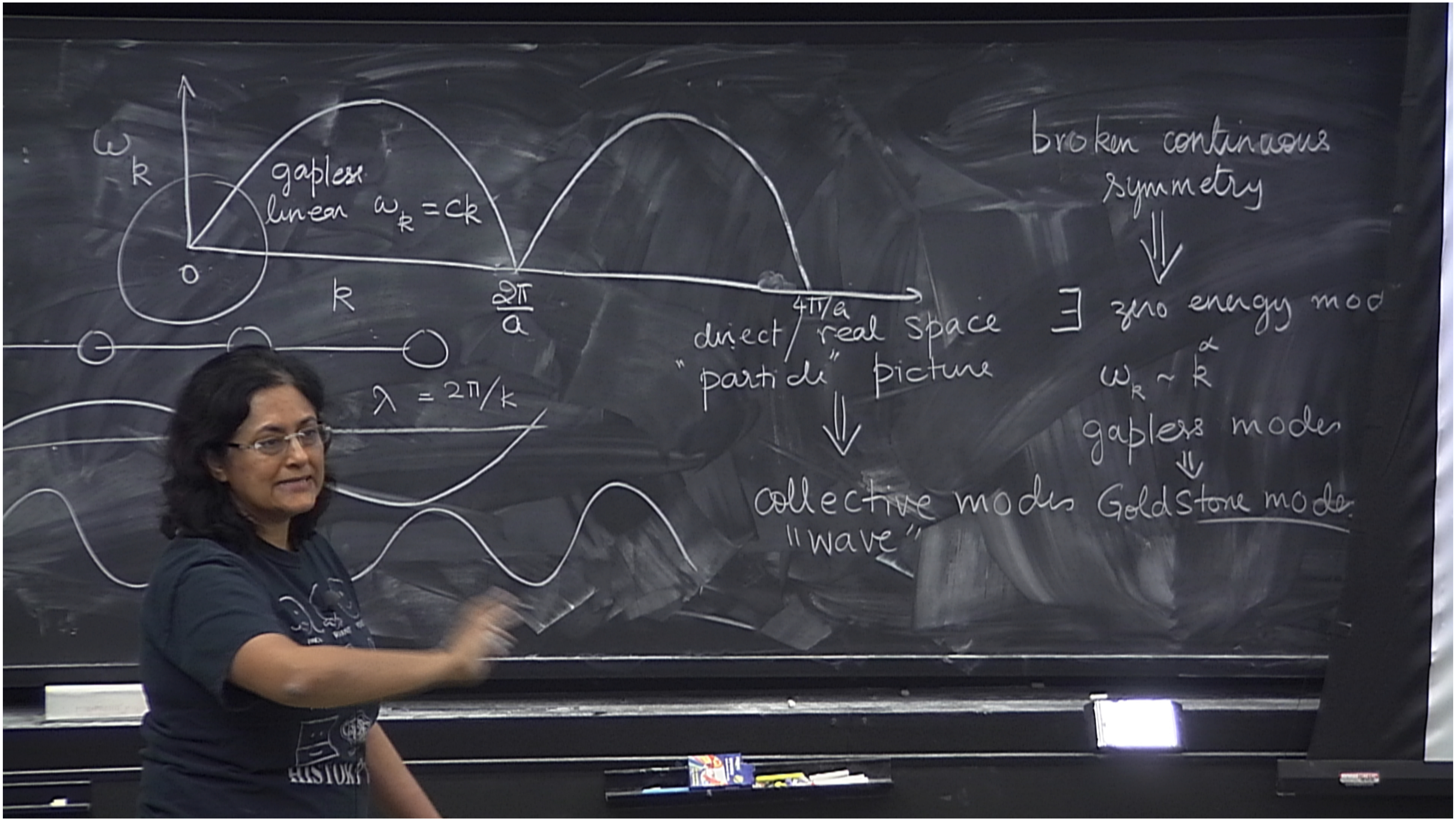
Title: Condensed Matter - Lecture 3

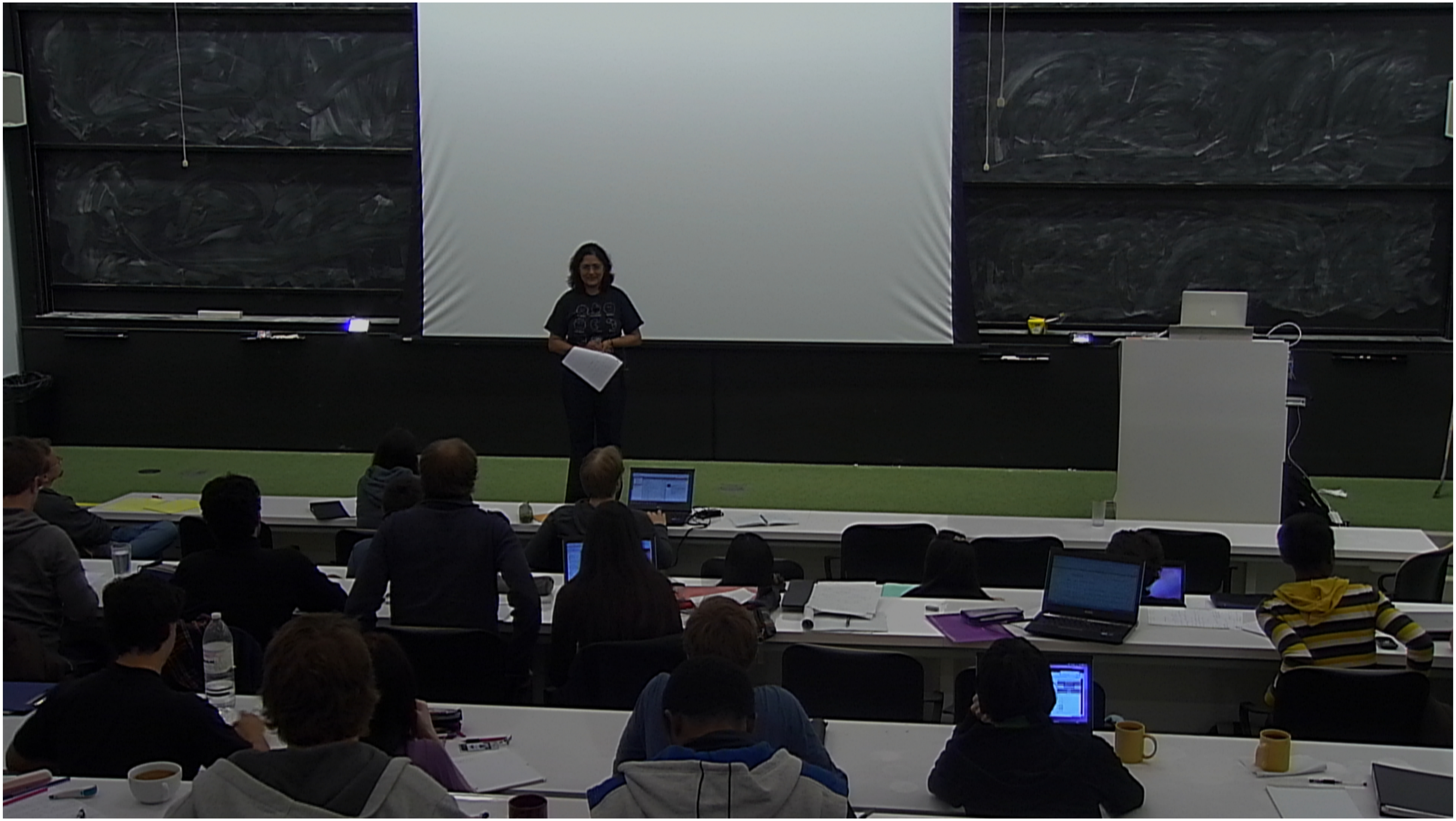
Date: Nov 02, 2011 10:30 AM

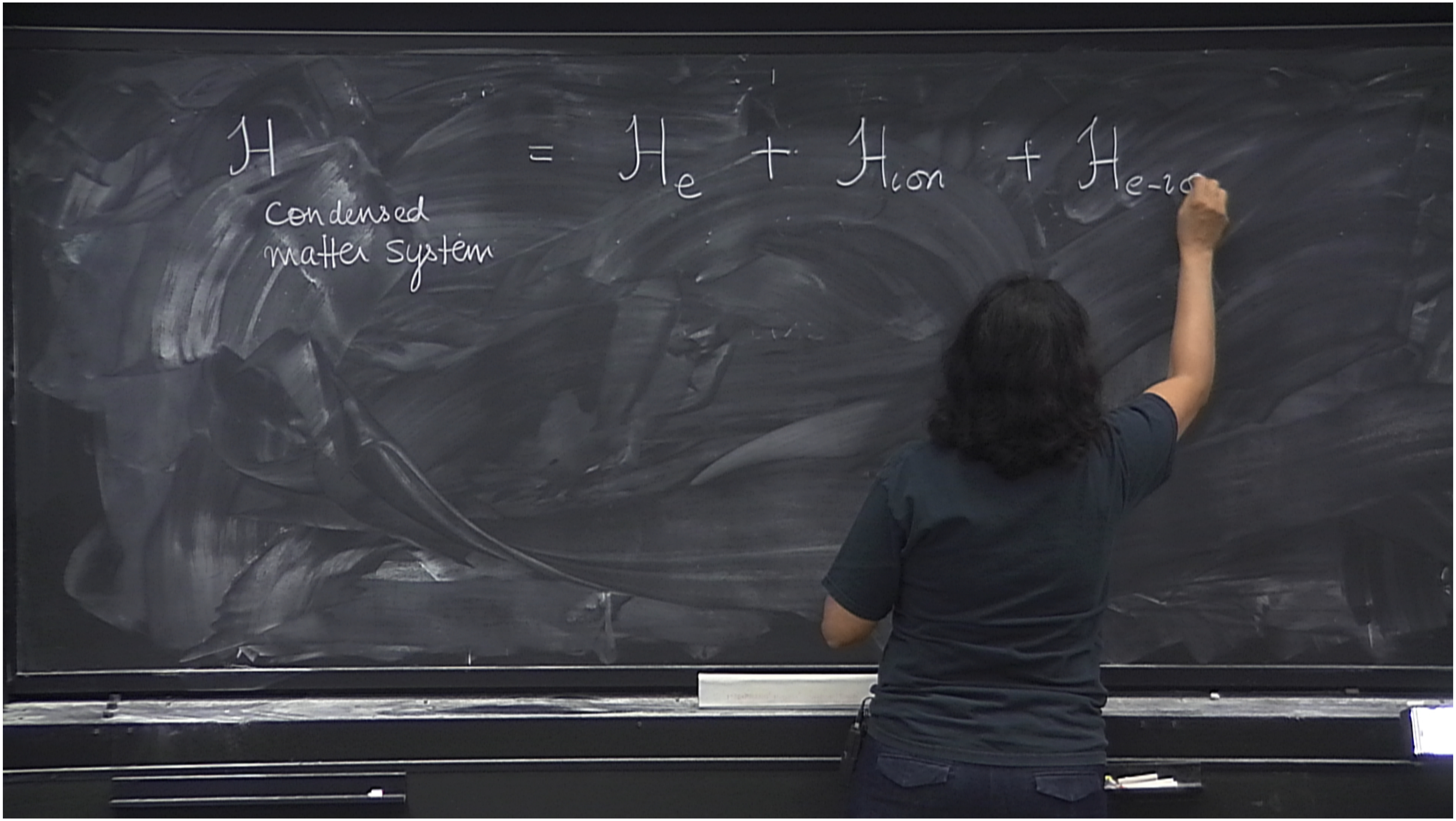
URL: http://pirsa.org/11110023

Abstract:

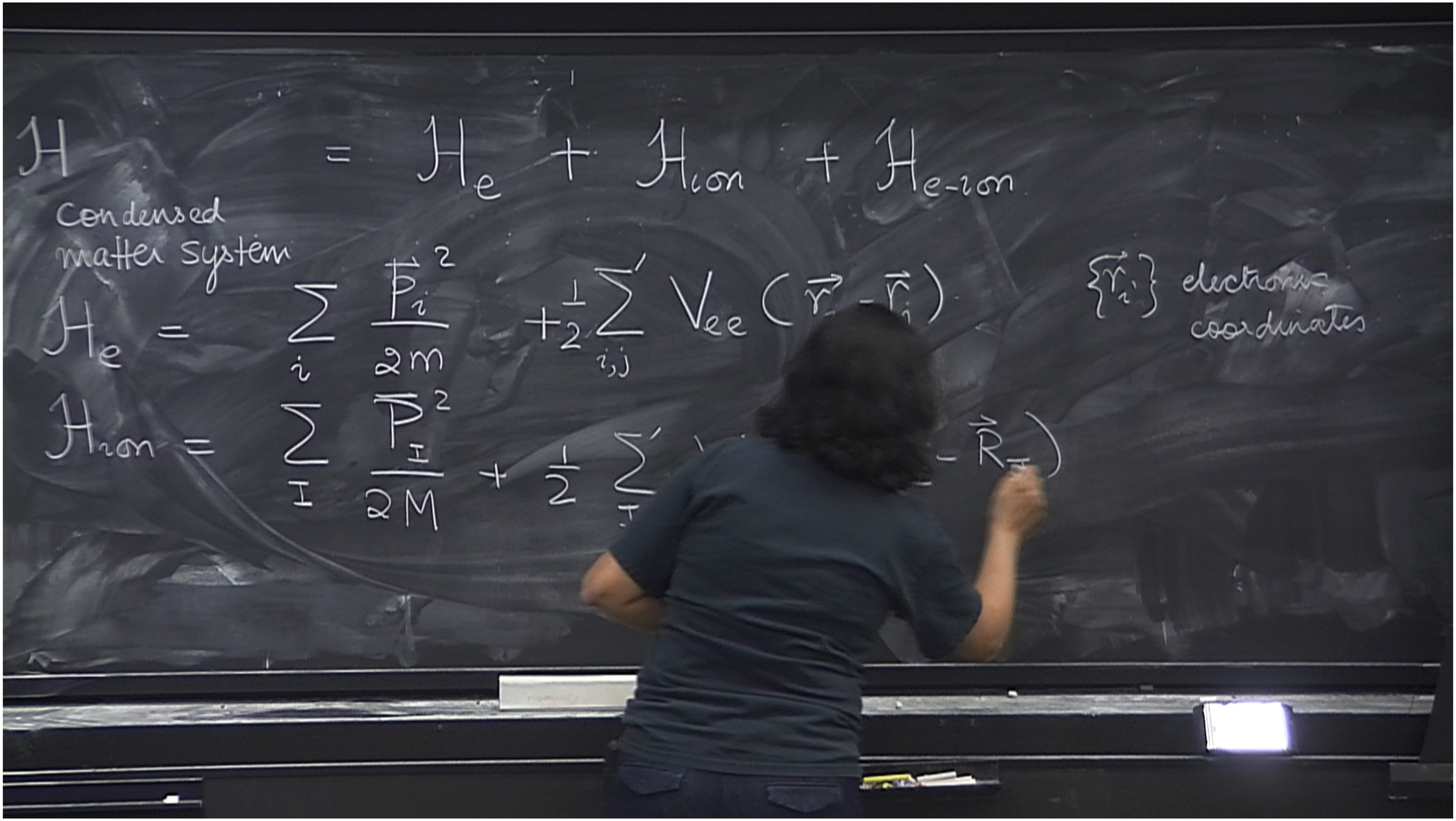


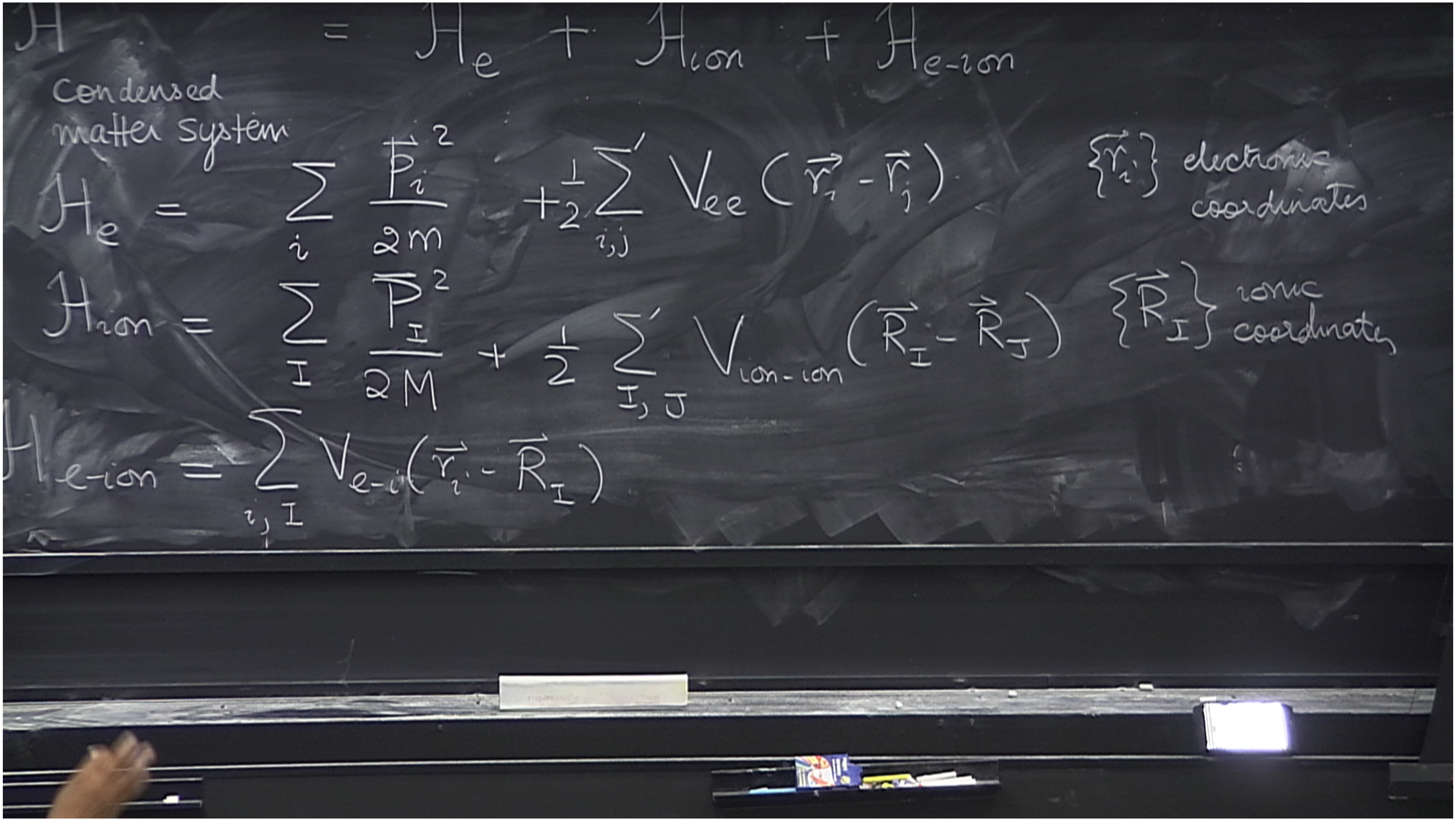
ω_k
k
gapless
linear ω_k = ck
0
k
2π/a
4π/a
λ = 2π/k
direct / real space
"particle" picture
collective modes
"wave"
broken continuous
symmetry
∃ zero energy mod
ω_k ~ k^α
gapless modes
Goldstone modes

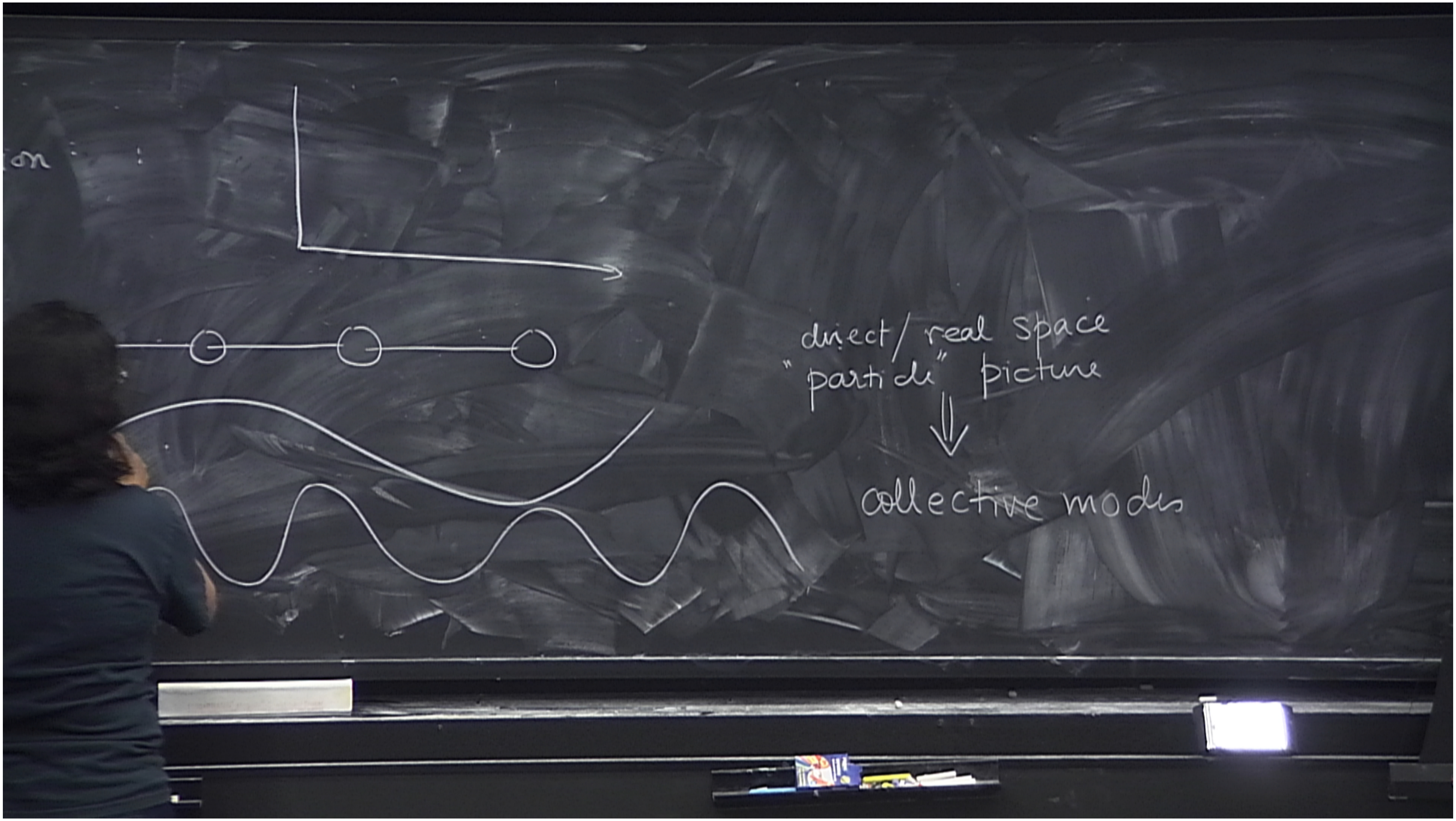


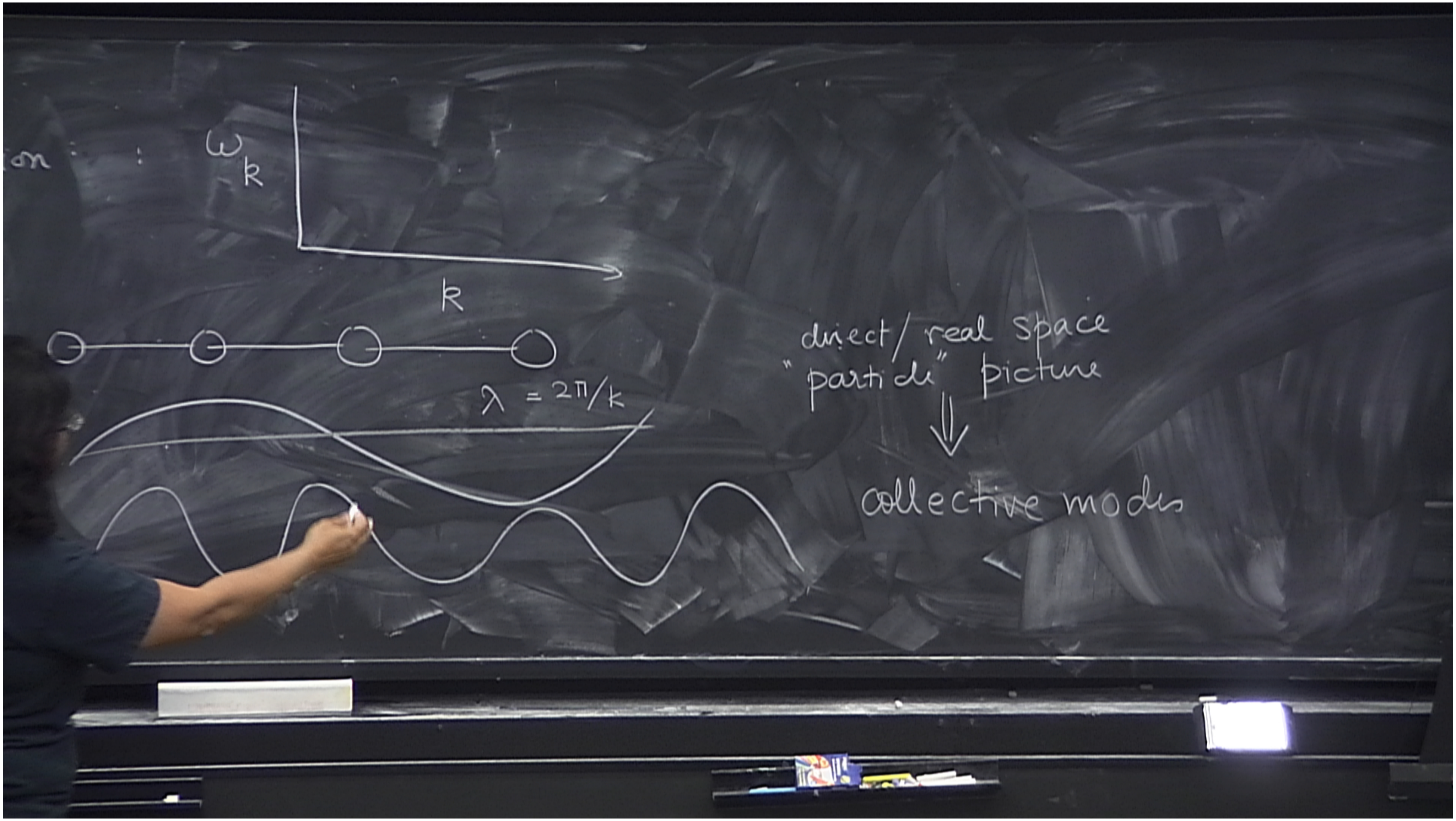


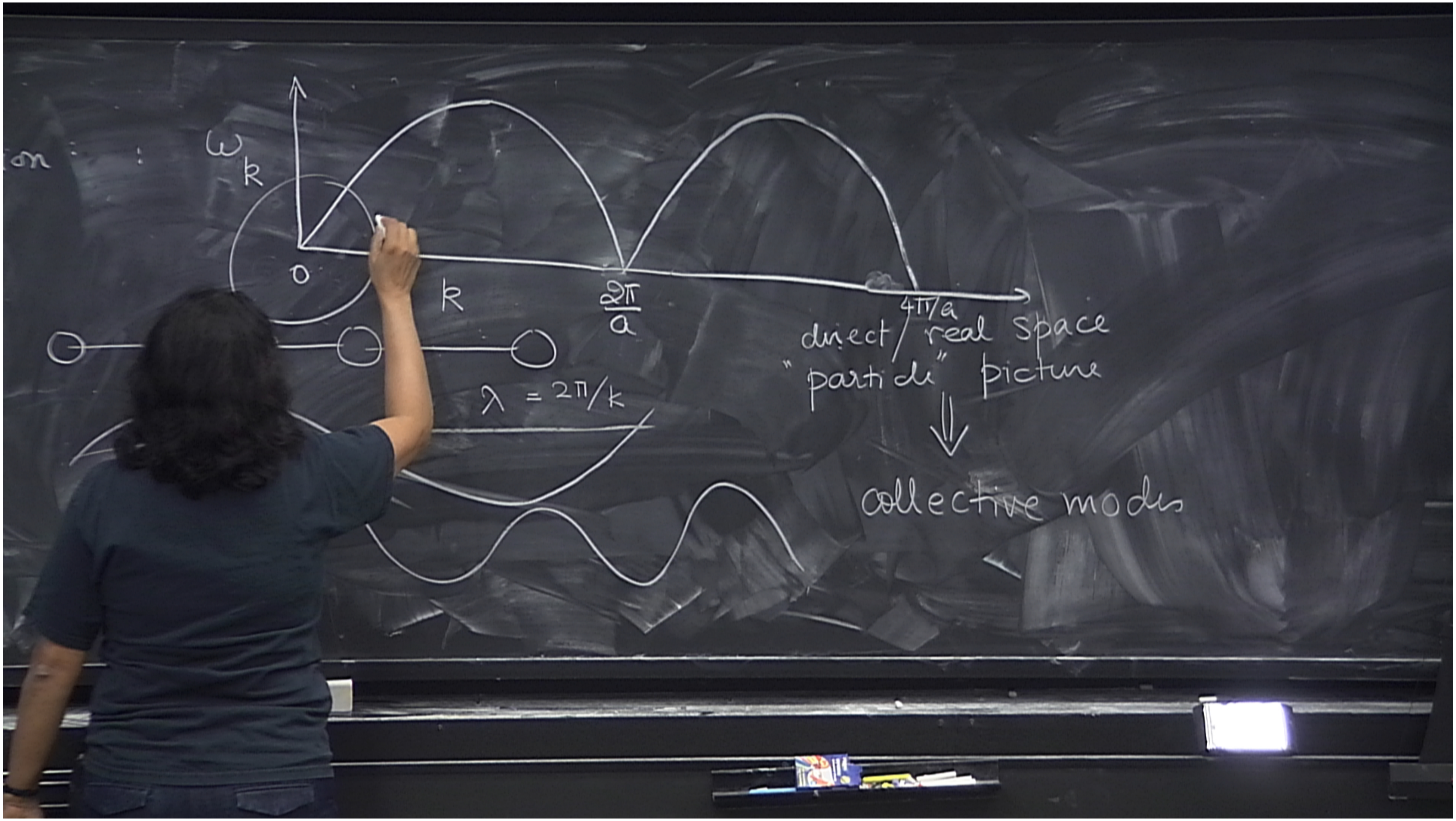
$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_{ion} + \mathcal{H}_{e\text{-}io}$
Condensed
matter system


$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_{ion} + \mathcal{H}_{e\text{-}ion}$
condensed matter system
$\mathcal{H}_e = \sum_i \frac{\vec{P}_i^2}{2m} + \frac{1}{2}\sum_{i,j}' V_{ee}(\vec{r}$
$\{\vec{r}_i\}$ electrons coordinates
$\mathcal{H}_{ion} = \sum_I \frac{\vec{P}_I^2}{2M} + \frac{1}{2}\sum'$
$- \vec{R}_J)$

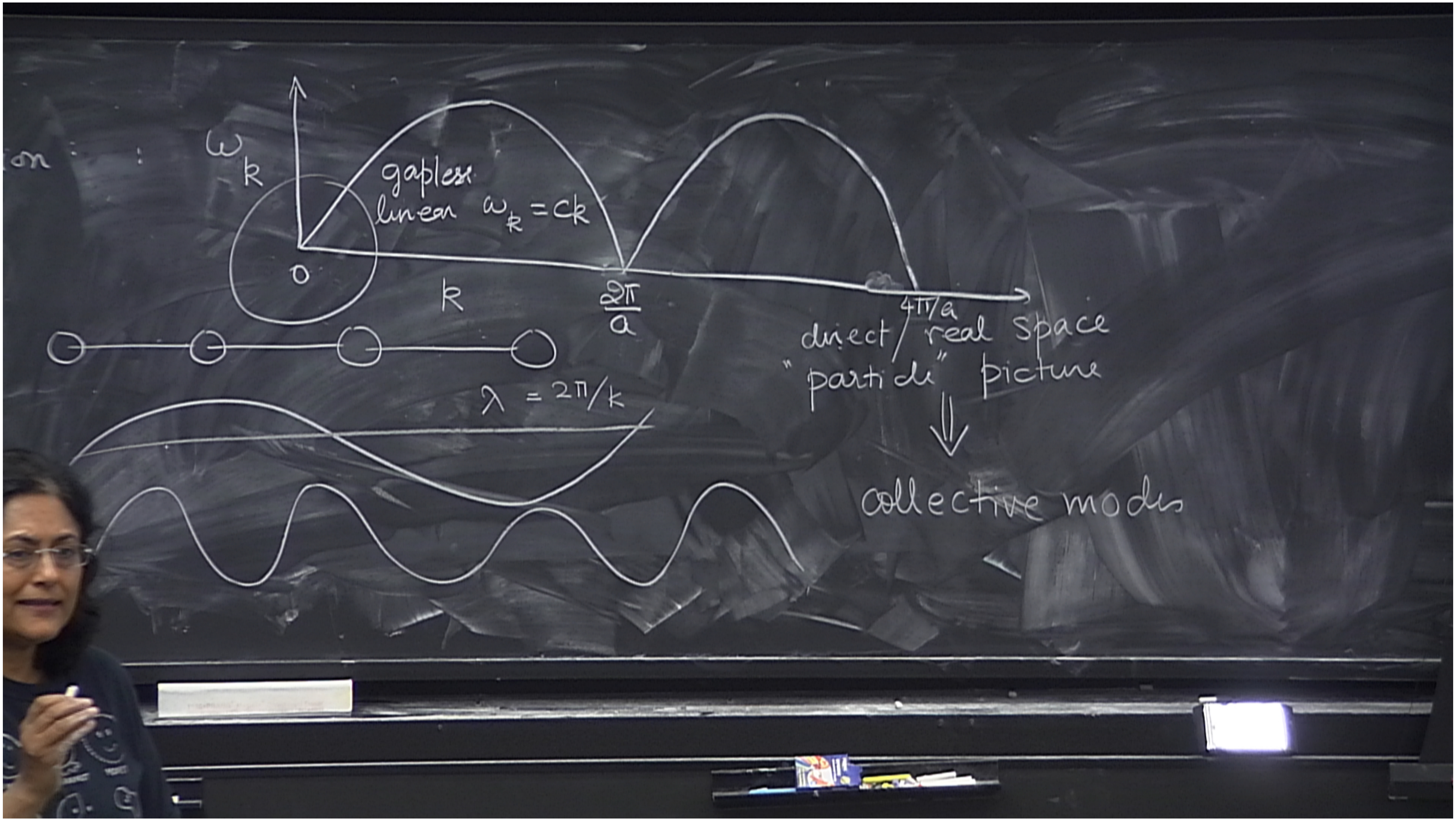

$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_{ion} + \mathcal{H}_{e-ion}$
Condensed matter system
$\mathcal{H}_e = \sum_i \frac{\vec{P}_i^2}{2m} + \frac{1}{2}\sum_{i,j}' V_{ee}(\vec{r}_i - \vec{r}_j)$
$\{\vec{r}_i\}$ electronic coordinates
$\mathcal{H}_{ion} = \sum_I \frac{\vec{P}_I^2}{2M} + \frac{1}{2}\sum_{I,J}' V_{ion-ion}(\vec{R}_I - \vec{R}_J)$
$\{\vec{R}_I\}$ ionic coordinates
$\mathcal{H}_{e-ion} = \sum_{i,I} V_{e-i}(\vec{r}_i - \vec{R}_I)$


direct/ real space
"particle" picture
collective modes

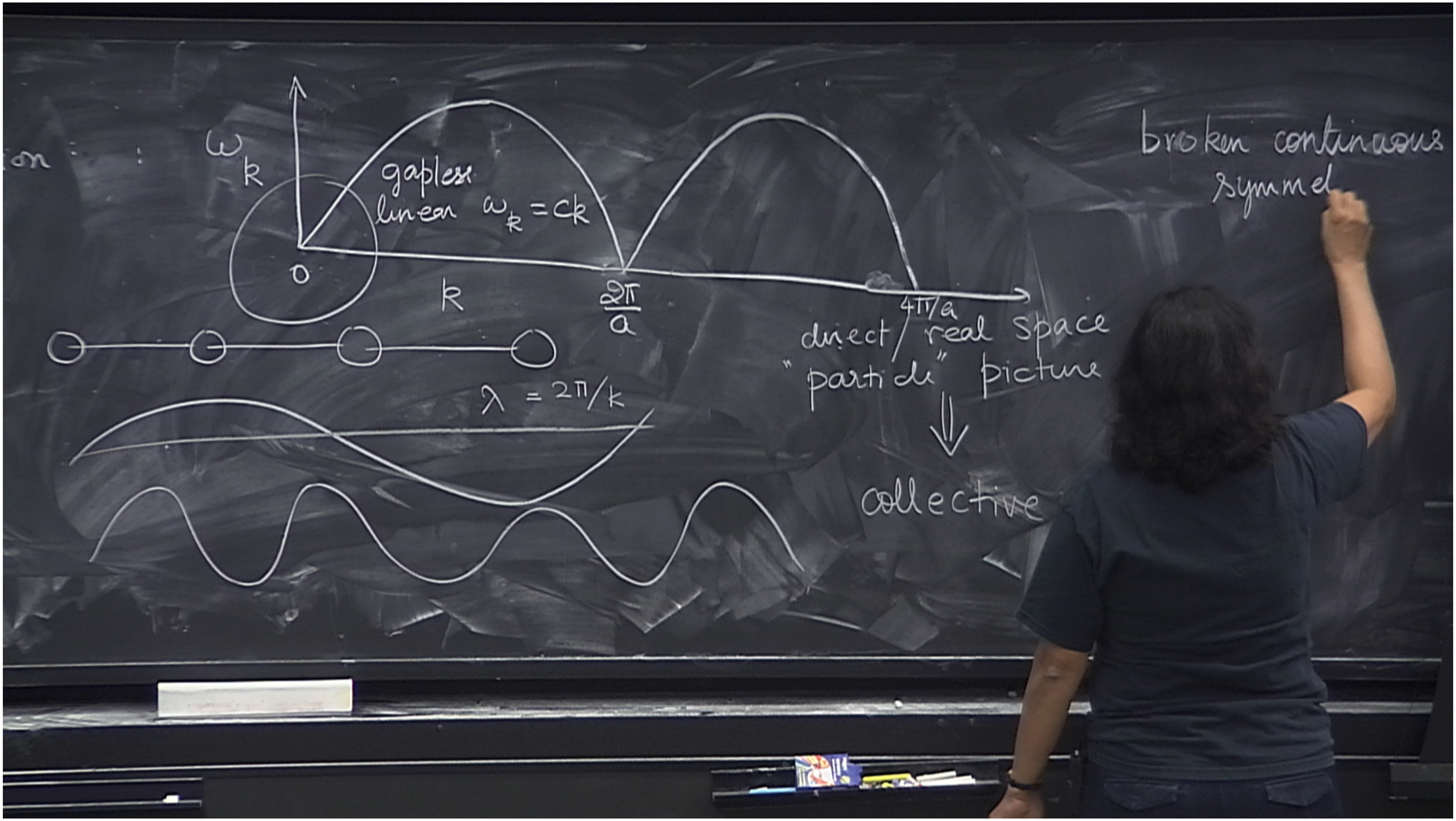

ion : ω_k
k
$\lambda = 2\pi/k$
direct / real space
"particle" picture
collective modes



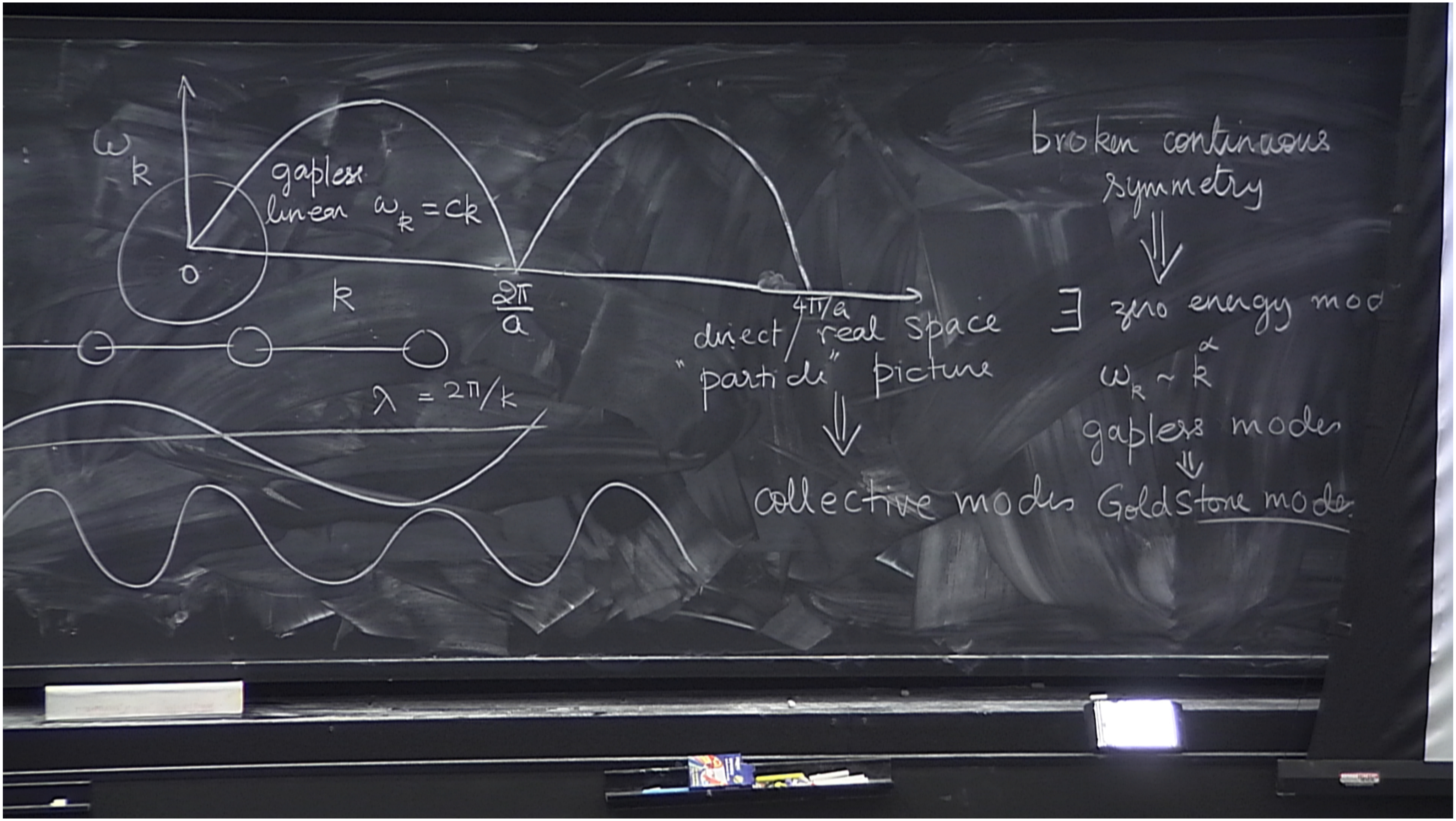
ωk
0
k
2π/a
4π/a
λ = 2π/k
direct / real space
"particle" picture
collective modes

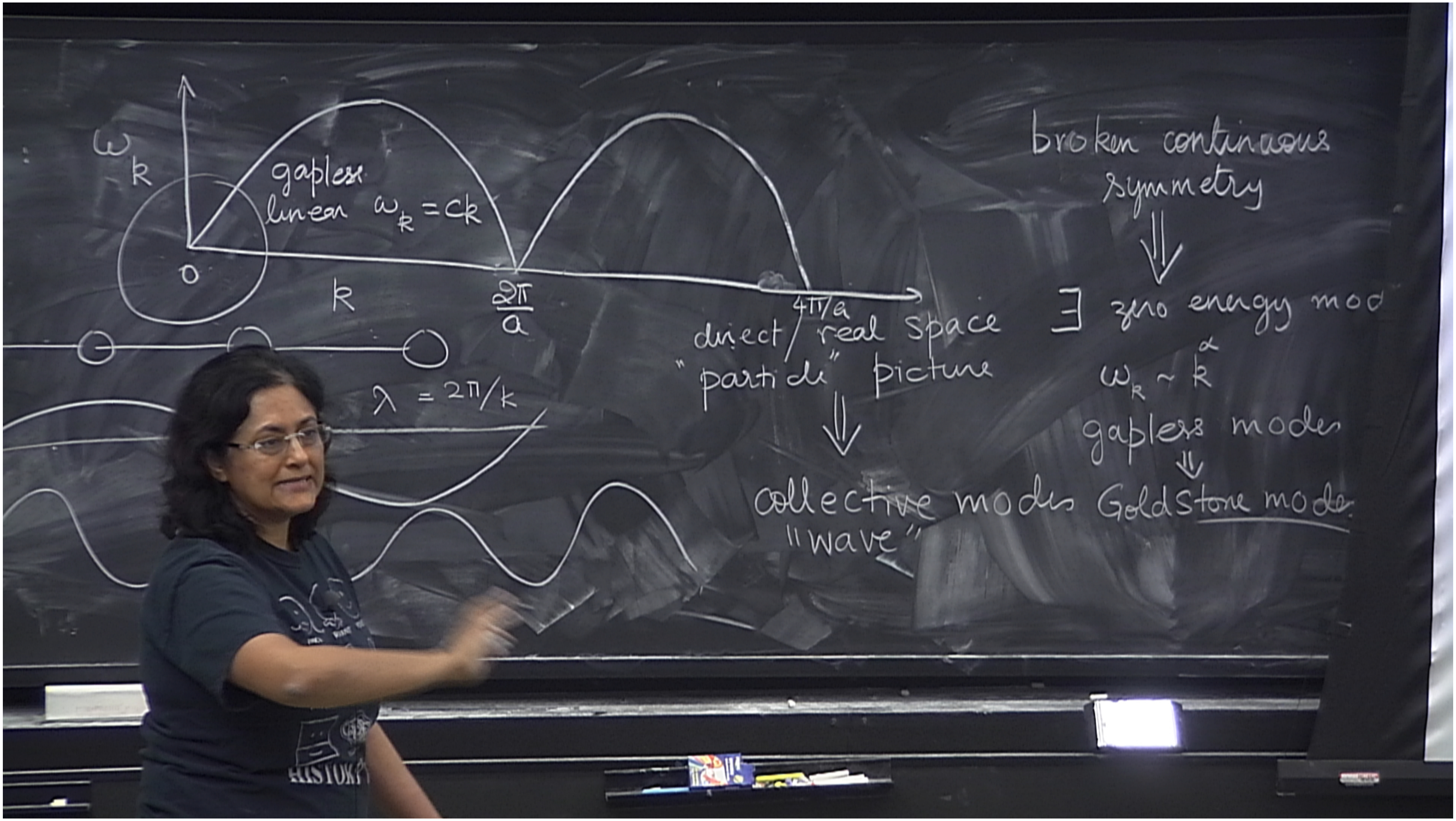


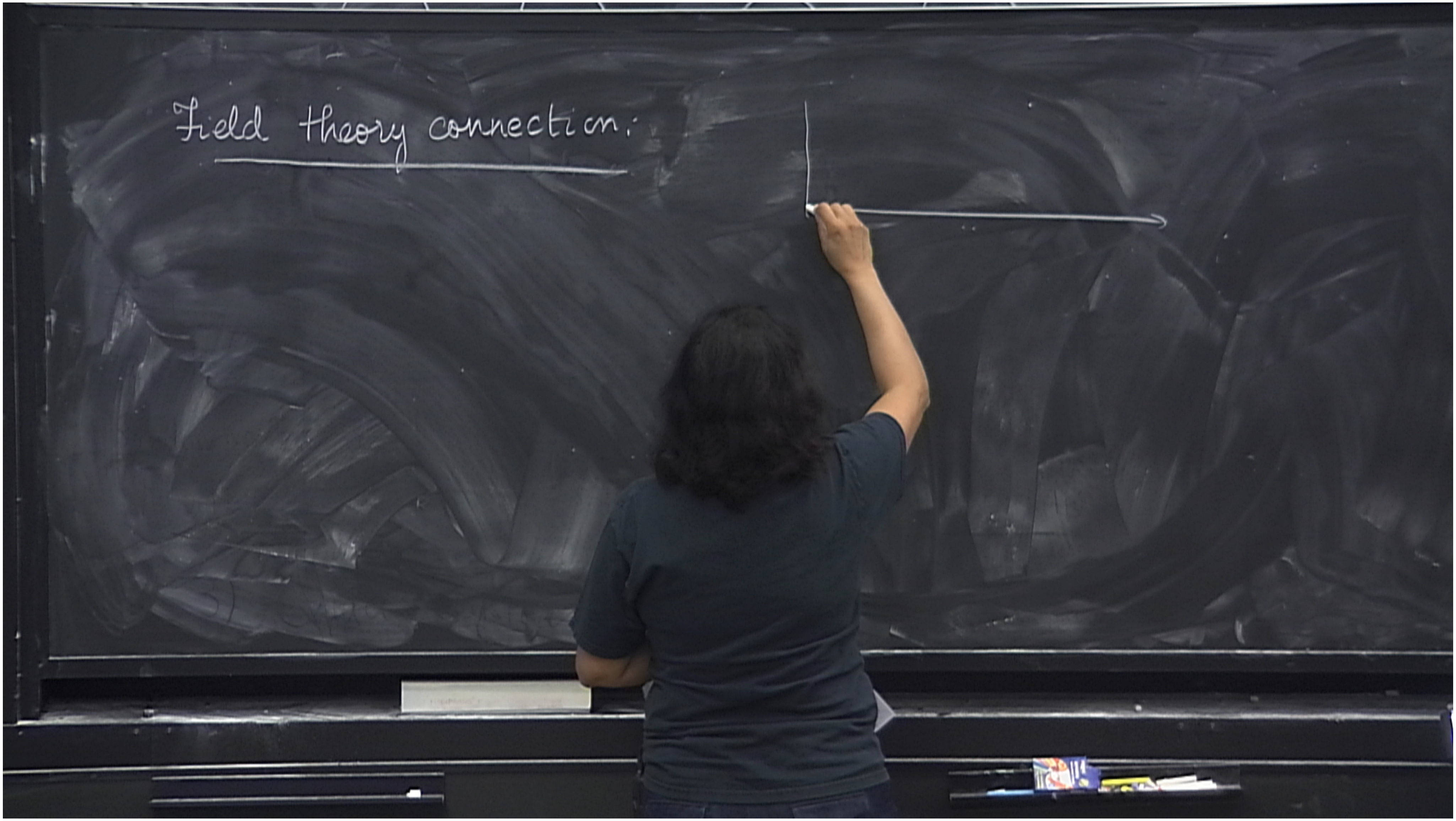
ion :
ω_k
gapless
linear $\omega_k = ck$
0
k
$\frac{2\pi}{a}$
$4\pi/a$
$\lambda = 2\pi/k$
direct / real space
"particle" picture
collective modes

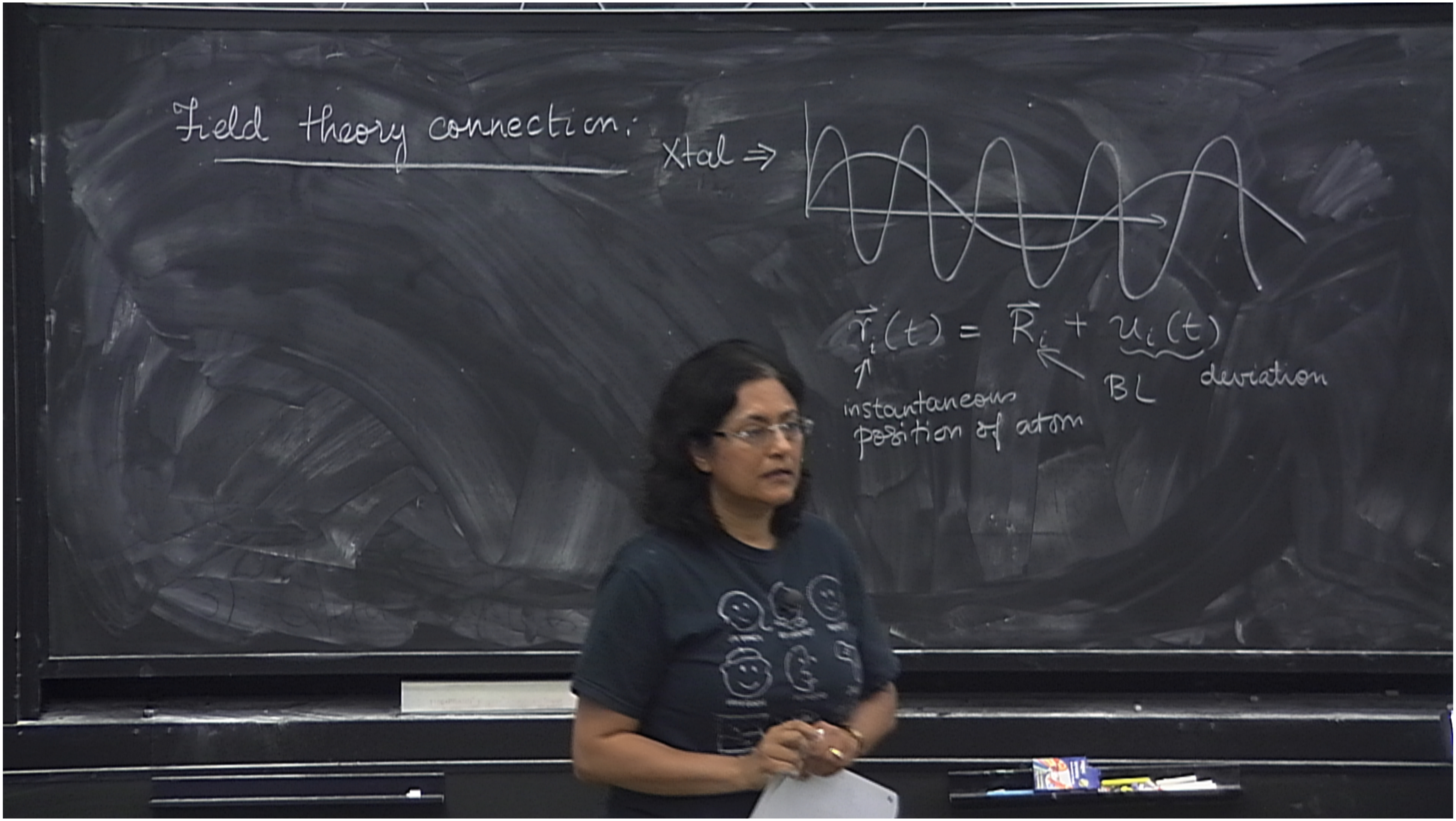


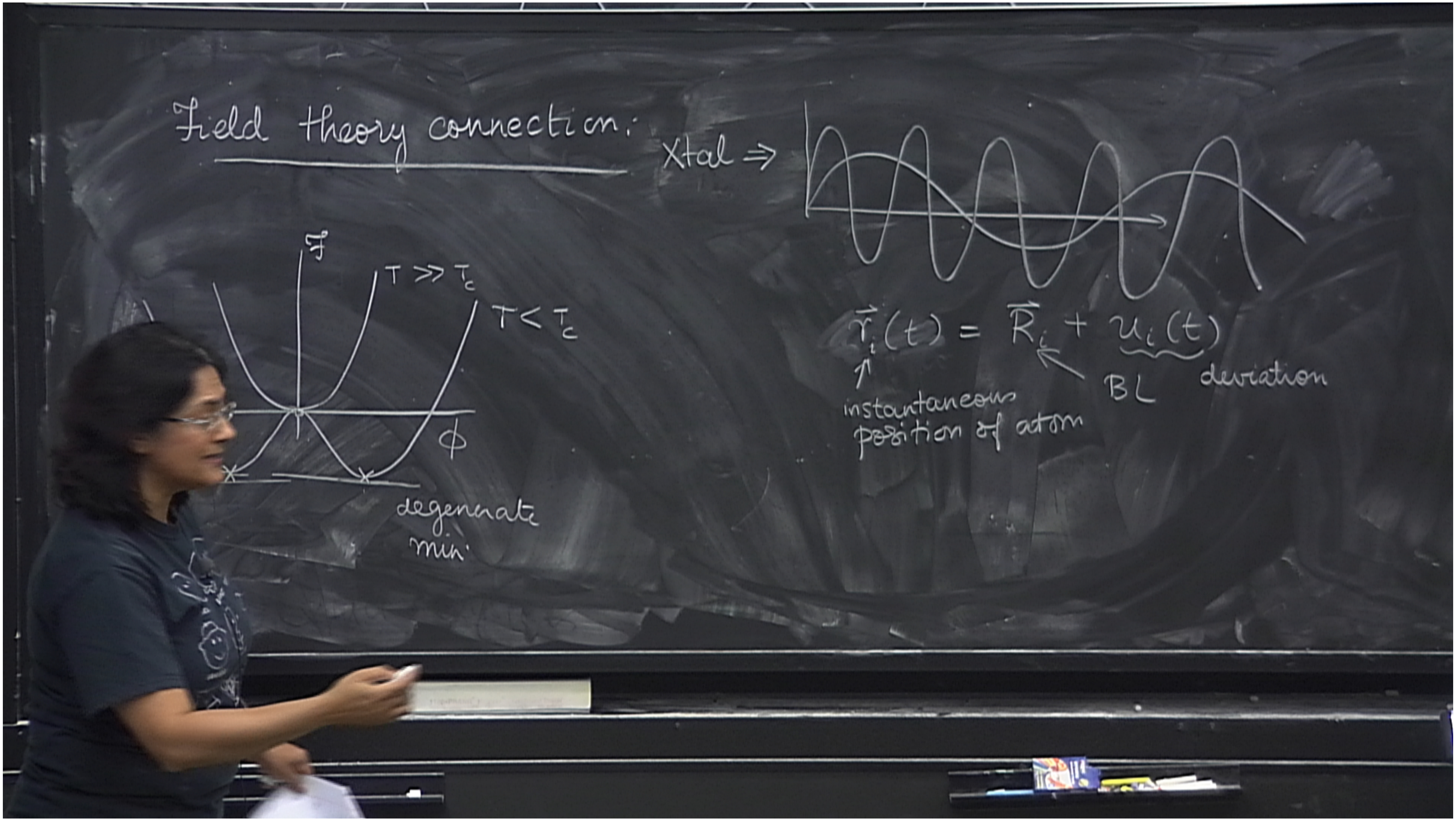
gapless
linear $\omega_k = ck$
ω_k
0
k
$2\pi/a$
$4\pi/a$
$\lambda = 2\pi/k$
direct/real space
"particle" picture
collective
broken continuous
symmet

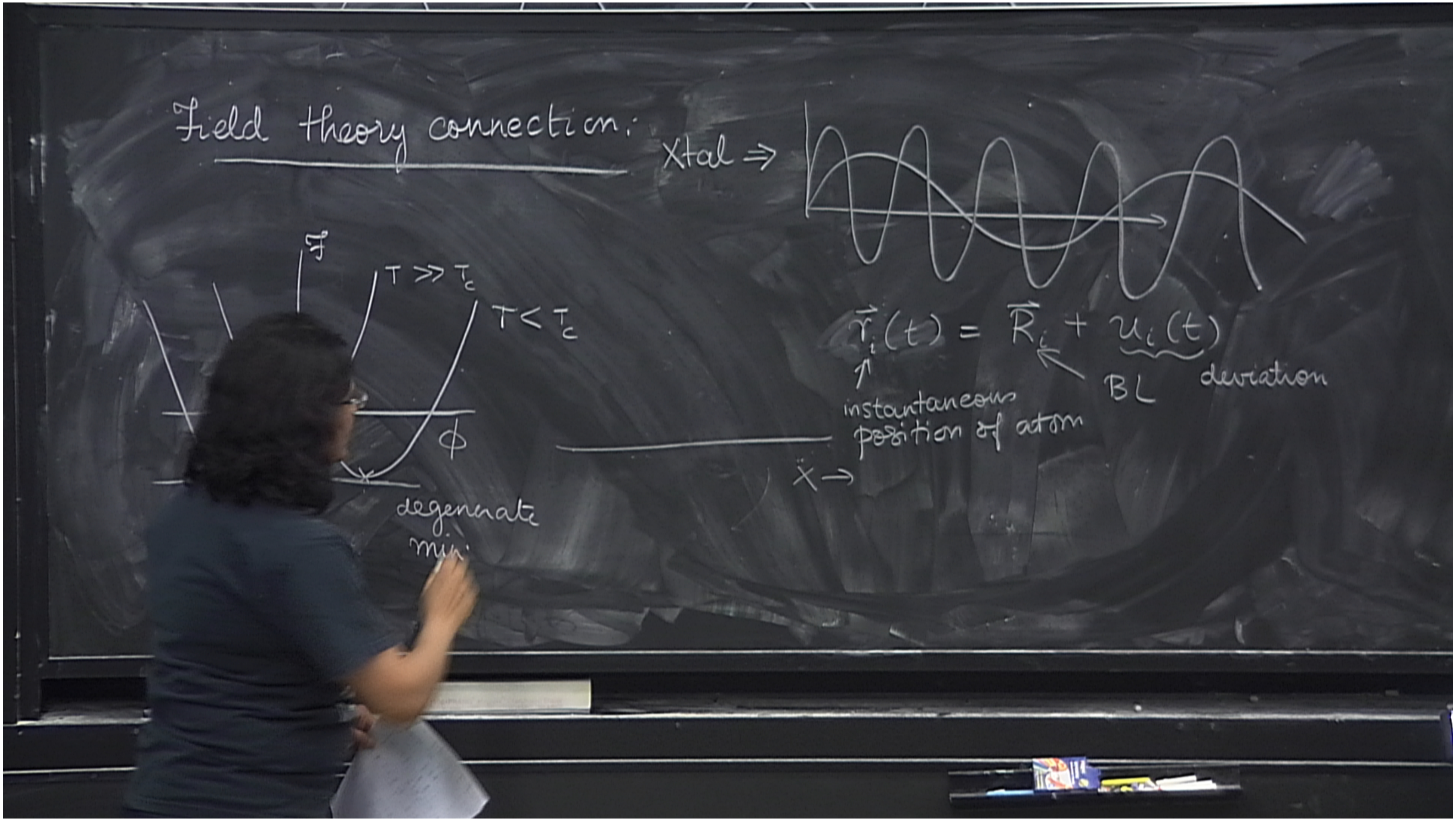

ω_k
gapless
linear ω_k = ck
0
k
2π/a
4π/a
λ = 2π/k
direct / real space
"particle" picture
collective modes
broken continuous
symmetry
∃ zero energy mod
ω_k ~ k^α
gapless modes
Goldstone mode

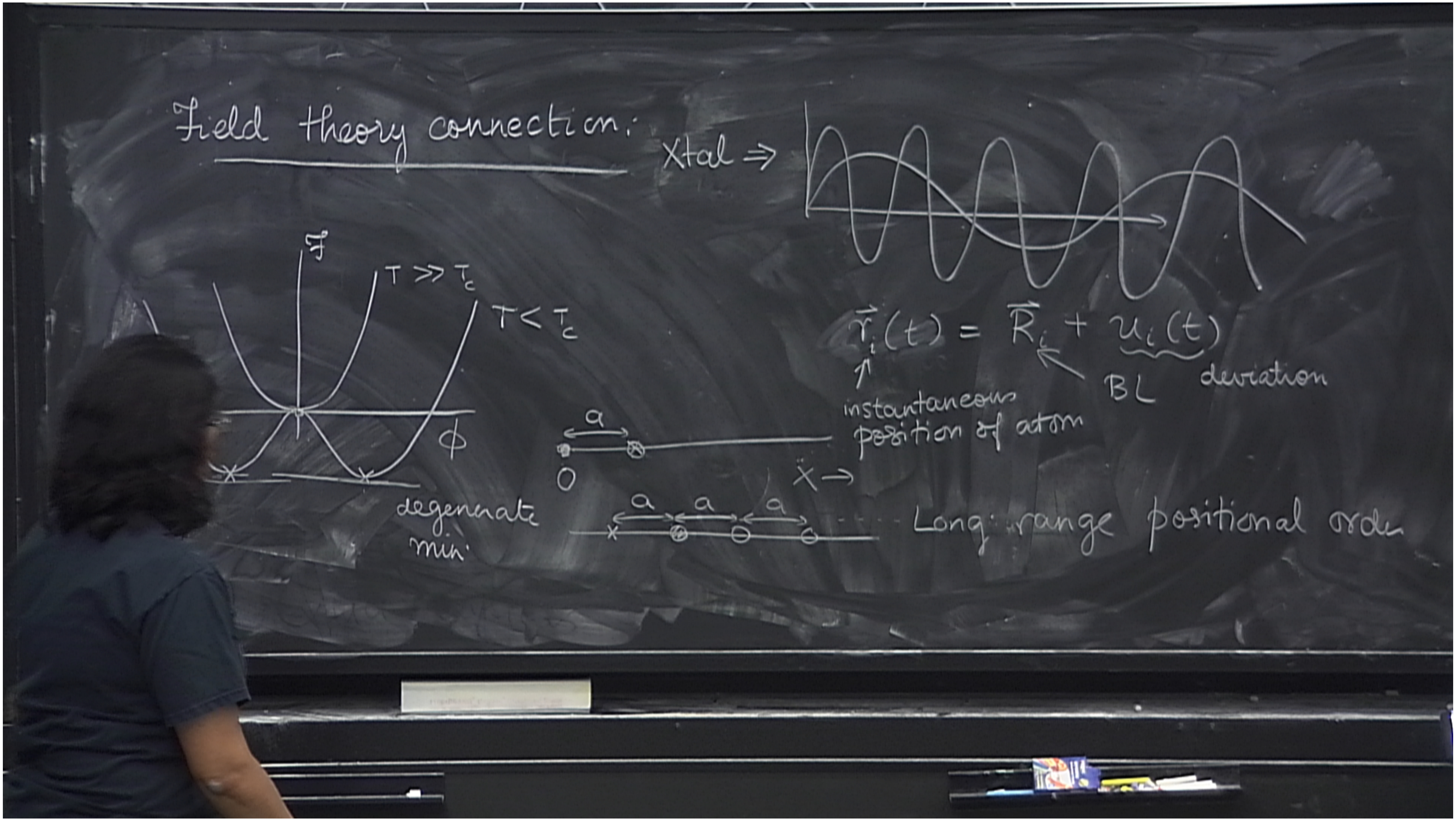

ω_k
gapless
linear $\omega_k = ck$
0
k
$2\pi/a$
$4\pi/a$
$\lambda = 2\pi/k$
direct / real space
"particle" picture
collective modes
"wave"
broken continuous symmetry
∃ zero energy mod
$\omega_k \sim k^\alpha$
gapless modes
Goldstone modes

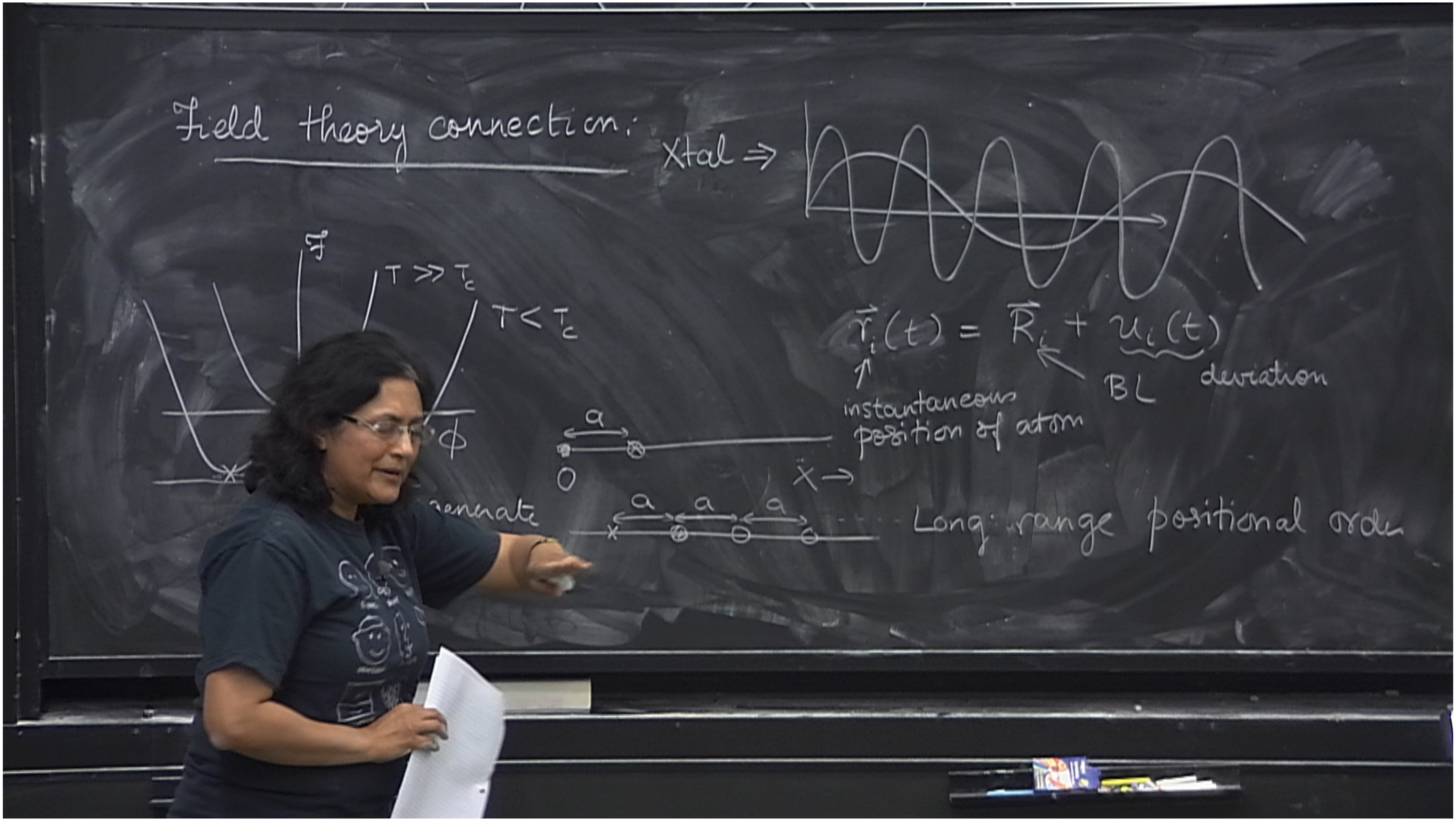

Field theory connection:

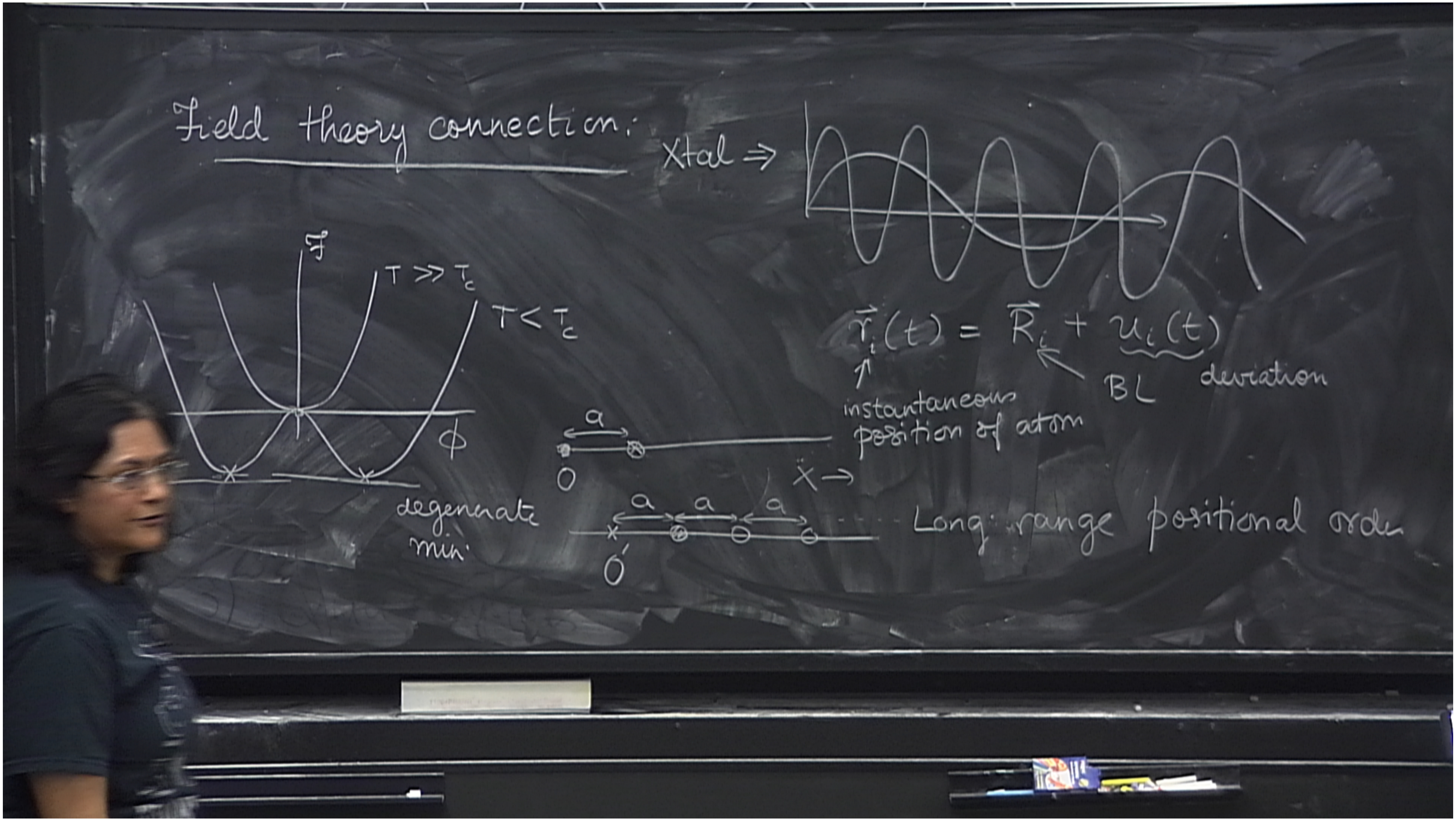

Field theory connection:
Xtal ⇒
$\vec{r}_i(t) = \vec{R}_i + u_i(t)$
instantaneous position of atom
BL
deviation


Field theory connection:
Xtal ⇒
T >> Tc
T < Tc
ϕ
degenerate min
ri(t) = Ri + ui(t)
instantaneous position of atom
BL
deviation

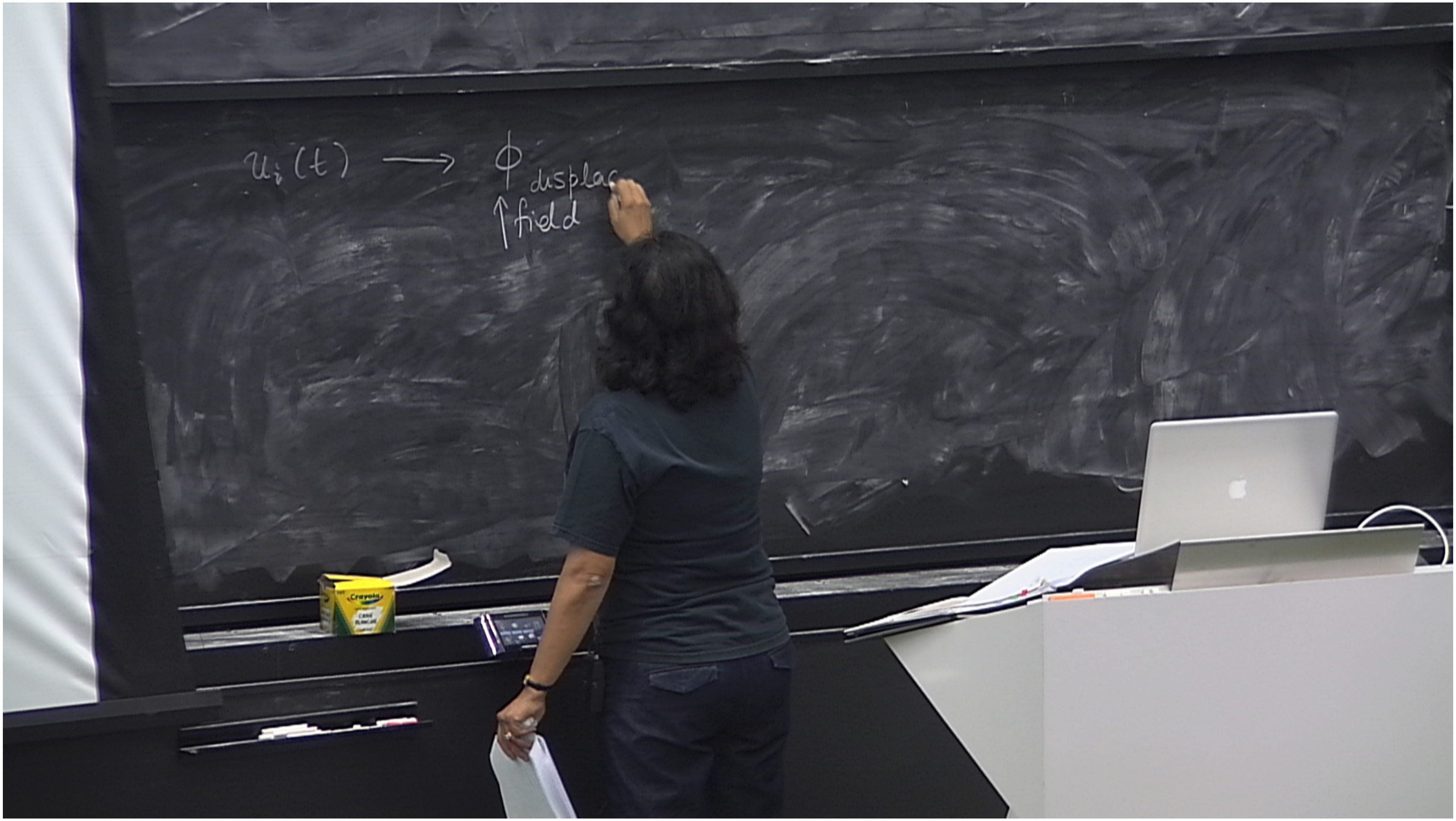

Field theory connection:
Xtal ⇒
$\mathcal{F}$
T >> T_c
T < T_c
ϕ
degenerate
min
$\vec{r}_i(t) = \vec{R}_i + u_i(t)$
instantaneous position of atom
BL
deviation
$\ddot{X}$ →


Field theory connection:
Xtal ⇒
T >> Tc
T < Tc
ϕ
degenerate min
a
O
X →
r_i(t) = R_i + u_i(t)
instantaneous position of atom
BL
deviation
Long range positional order

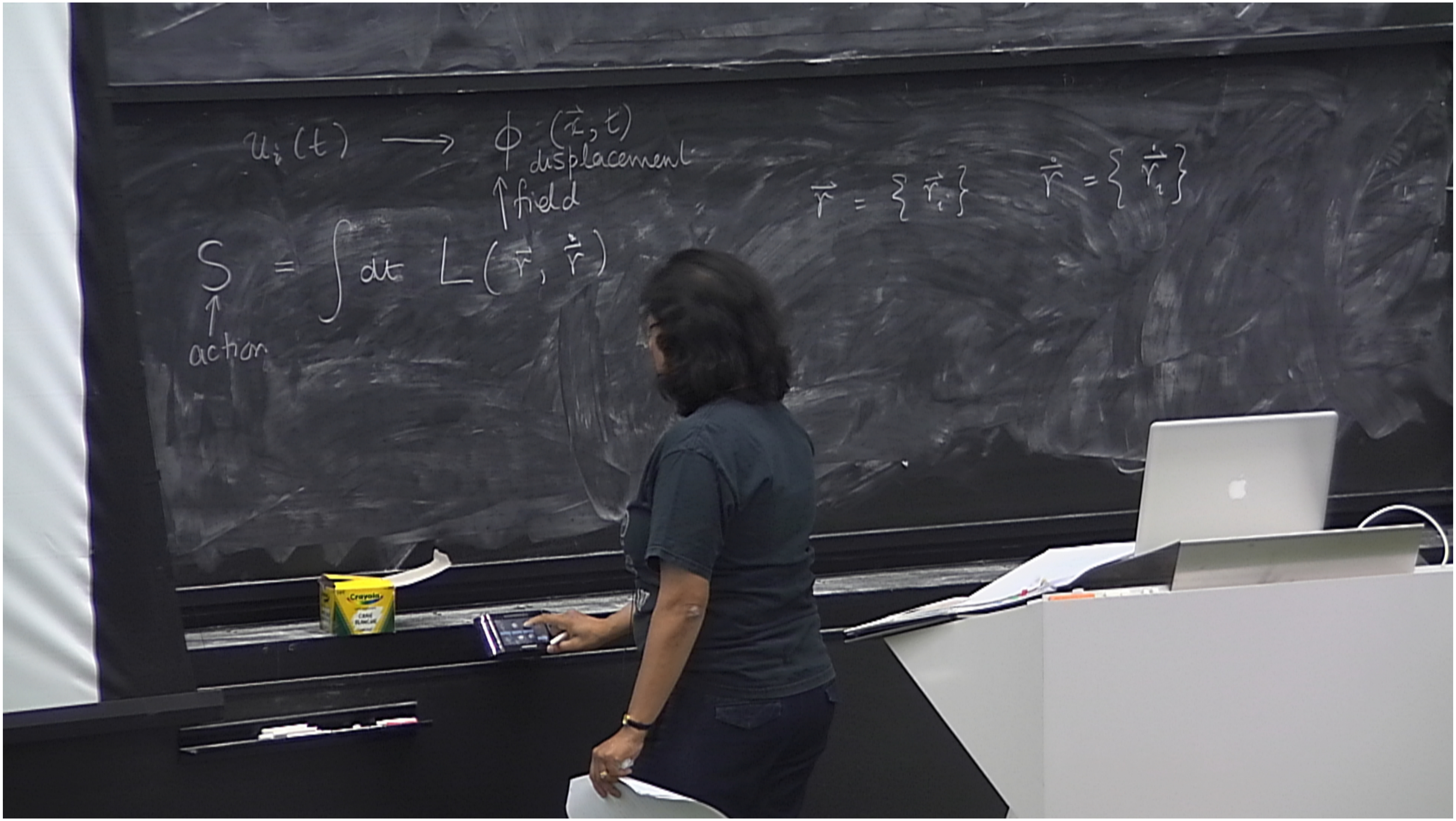

Field theory connection:
Xtal ⇒
T >> Tc
T < Tc
φ
a
0
x →
a a a
Long range positional order
r_i(t) = R_i + u_i(t)
instantaneous position of atom
BL
deviation

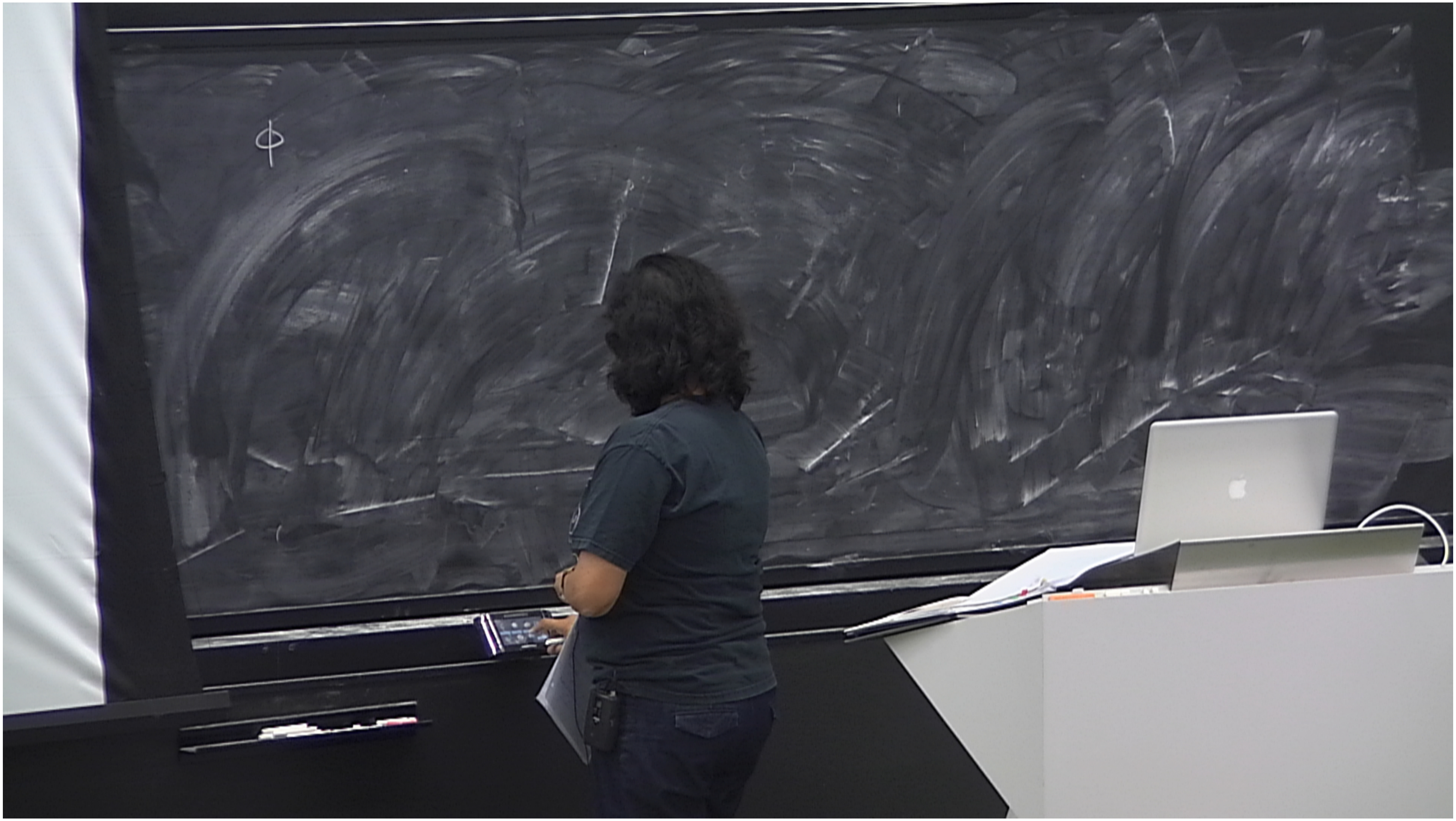


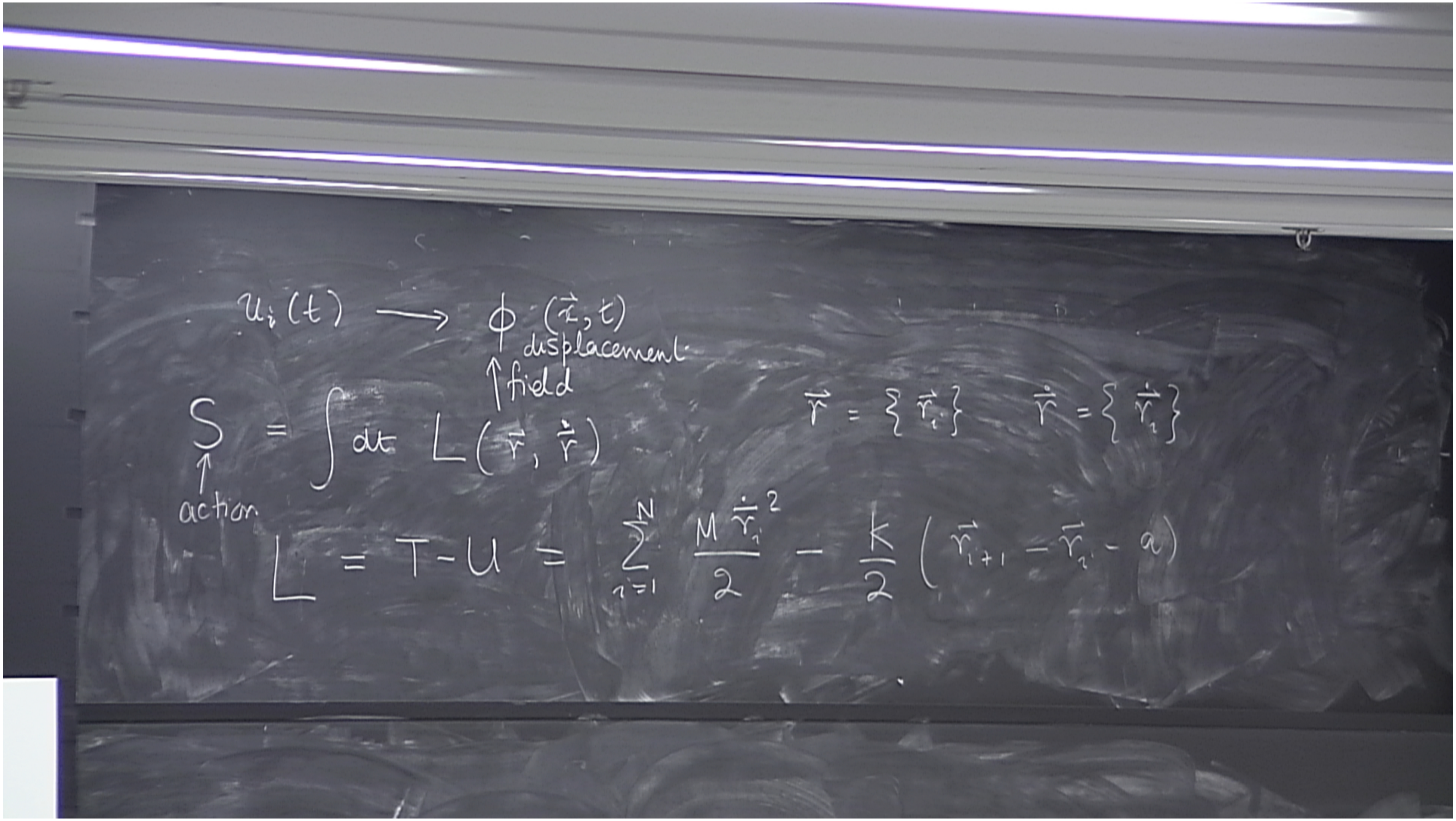
Field theory connection:
Xtal ⇒
$\mathcal{F}$
T >> Tc
T < Tc
φ
degenerate min
a
O
X →
O'
Long range positional order
$\vec{r}_i(t) = \vec{R}_i + u_i(t)$
instantaneous position of atom
BL
deviation



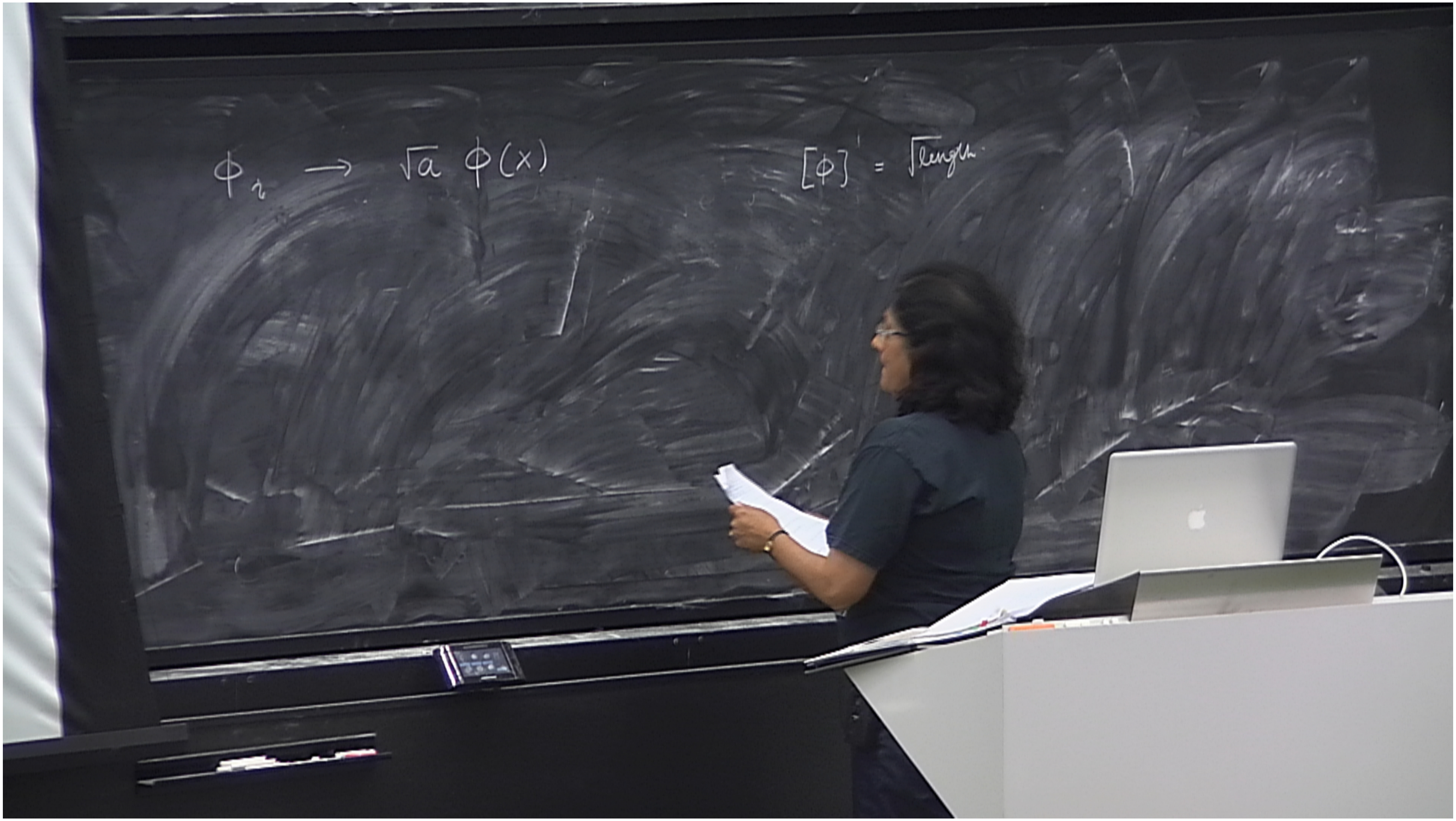
$u_i(t) \longrightarrow \phi$ displac
↑ field

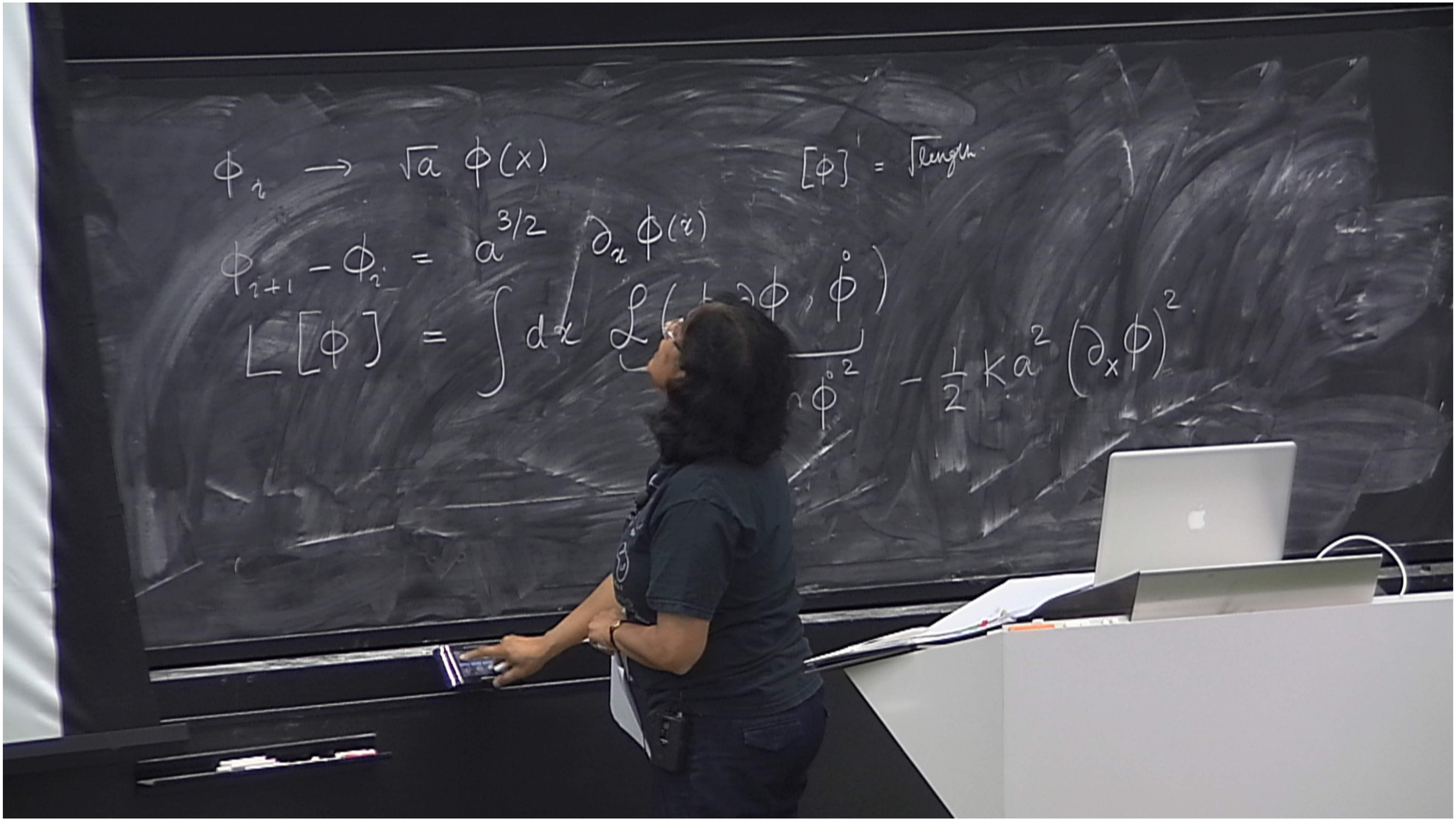

$u_i(t) \longrightarrow \phi(\vec{x}, t)$
displacement
field
$S = \int dt\, L(\vec{r}, \dot{\vec{r}})$
action
$\vec{r} = \{\vec{r}_i\}$
$\dot{\vec{r}} = \{\dot{\vec{r}}_i\}$
Crayola

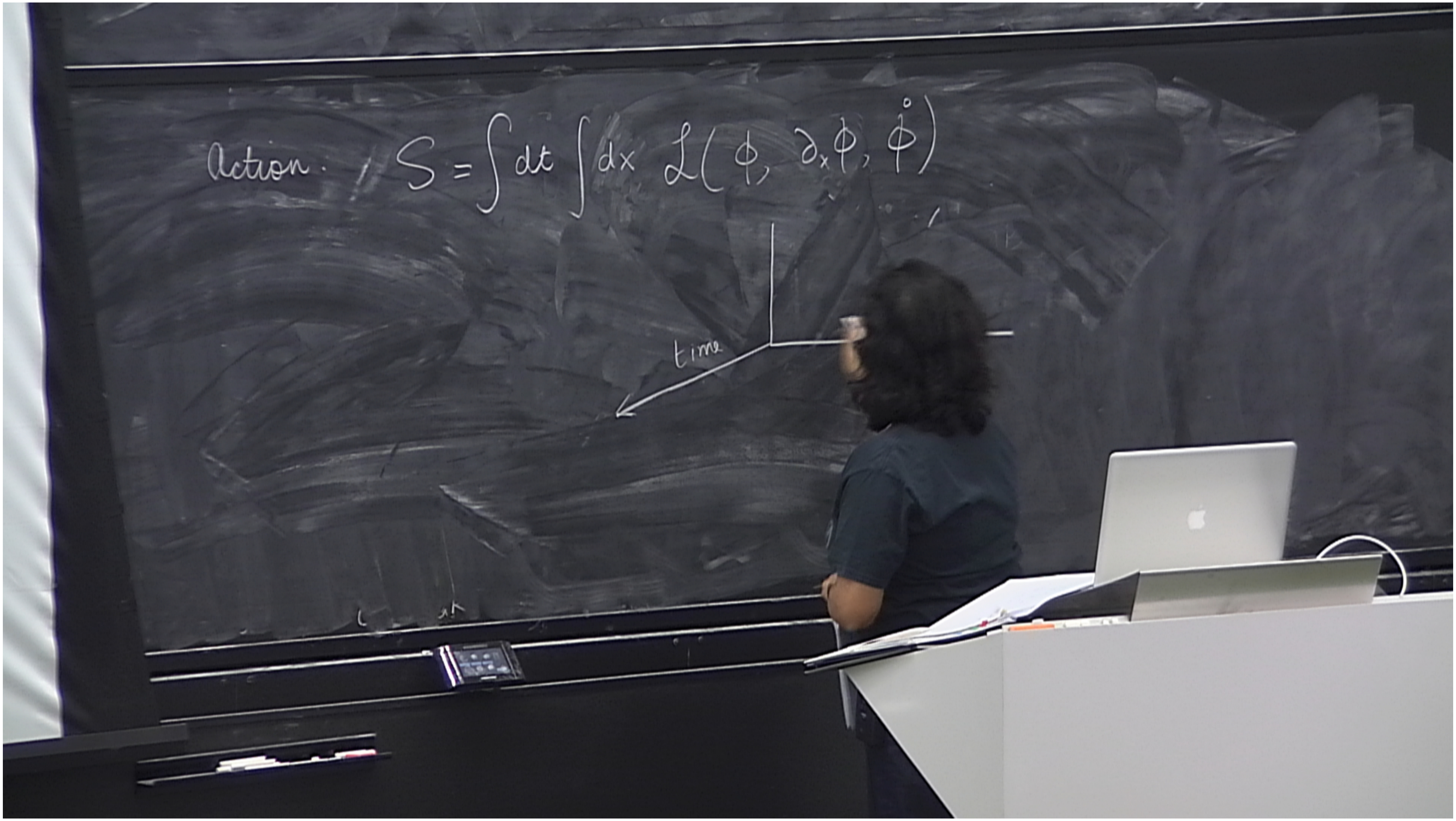

ϕ

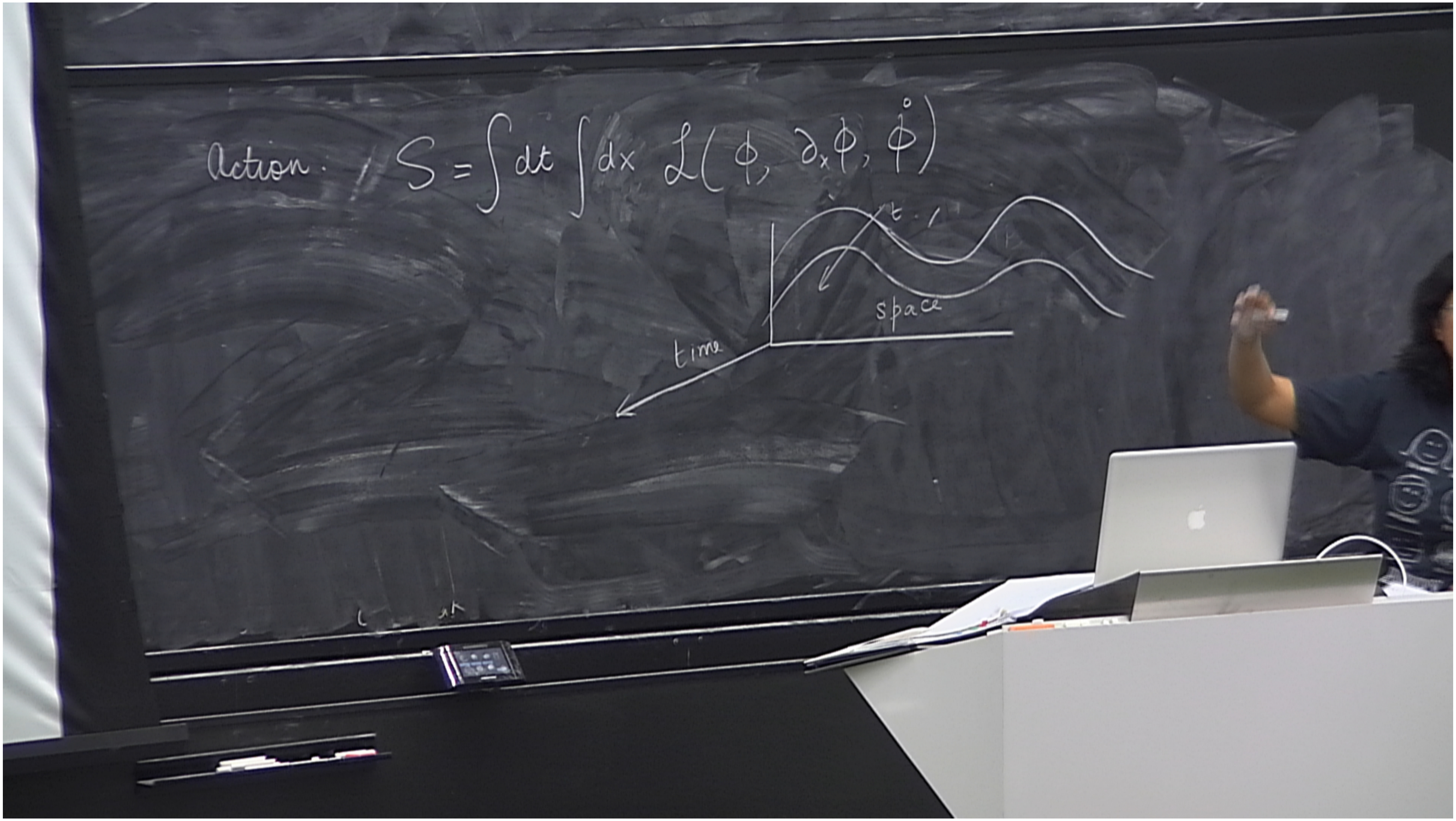


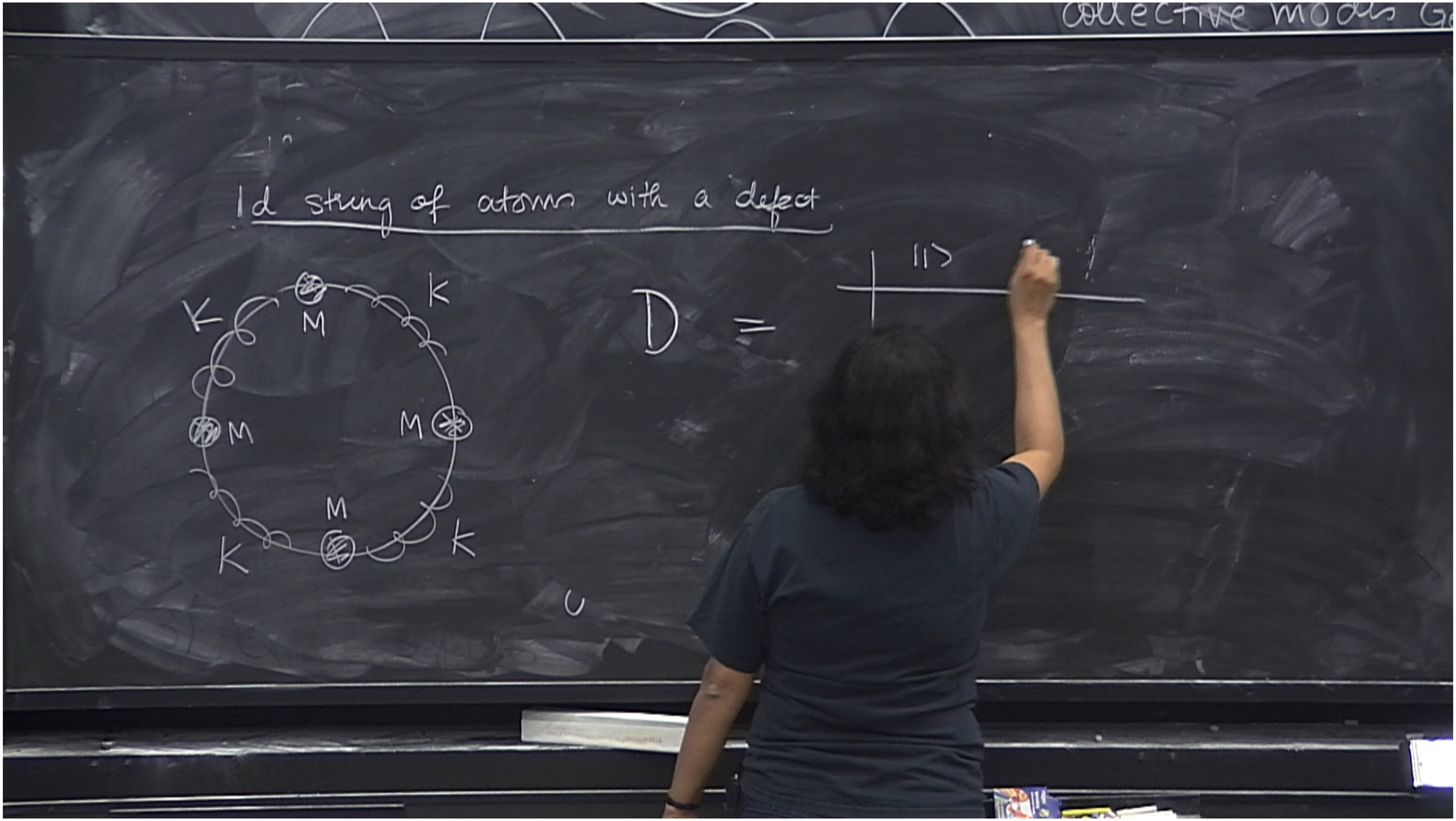
$u_i(t) \longrightarrow \phi(\vec{x}, t)$
displacement
field
$S = \int dt\, L(\vec{r}, \dot{\vec{r}})$
action
$\vec{r} = \{\vec{r}_i\}$ $\dot{\vec{r}} = \{\dot{\vec{r}}_i\}$
$L = T - U = \sum_{i=1}^{N} \frac{M \dot{\vec{r}}_i^{\,2}}{2} - \frac{k}{2}(\vec{r}_{i+1} - \vec{r}_i - a)$

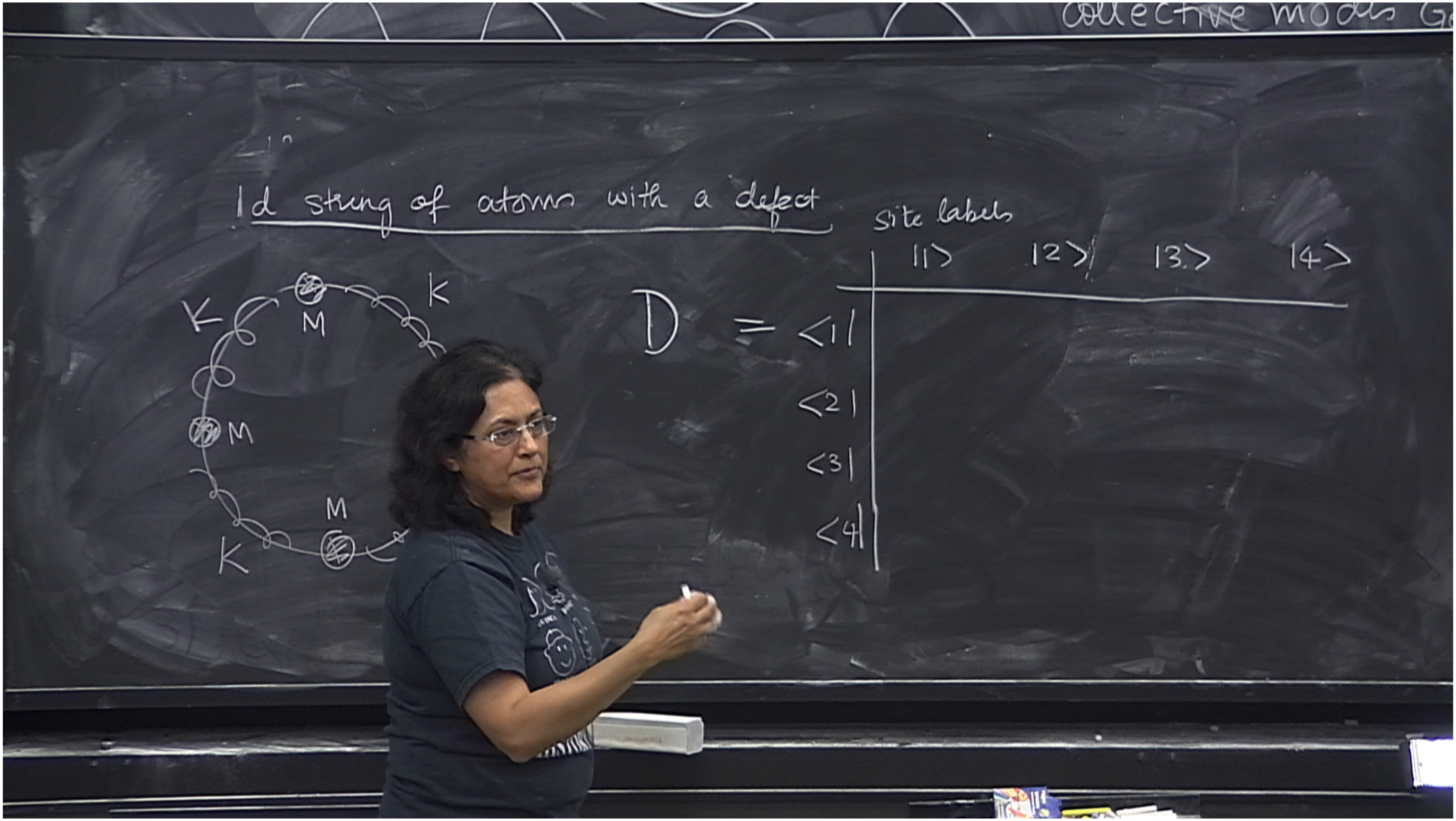

$\phi_i \longrightarrow \sqrt{a}\,\phi(x)$
$[\phi] = \sqrt{\text{length}}$

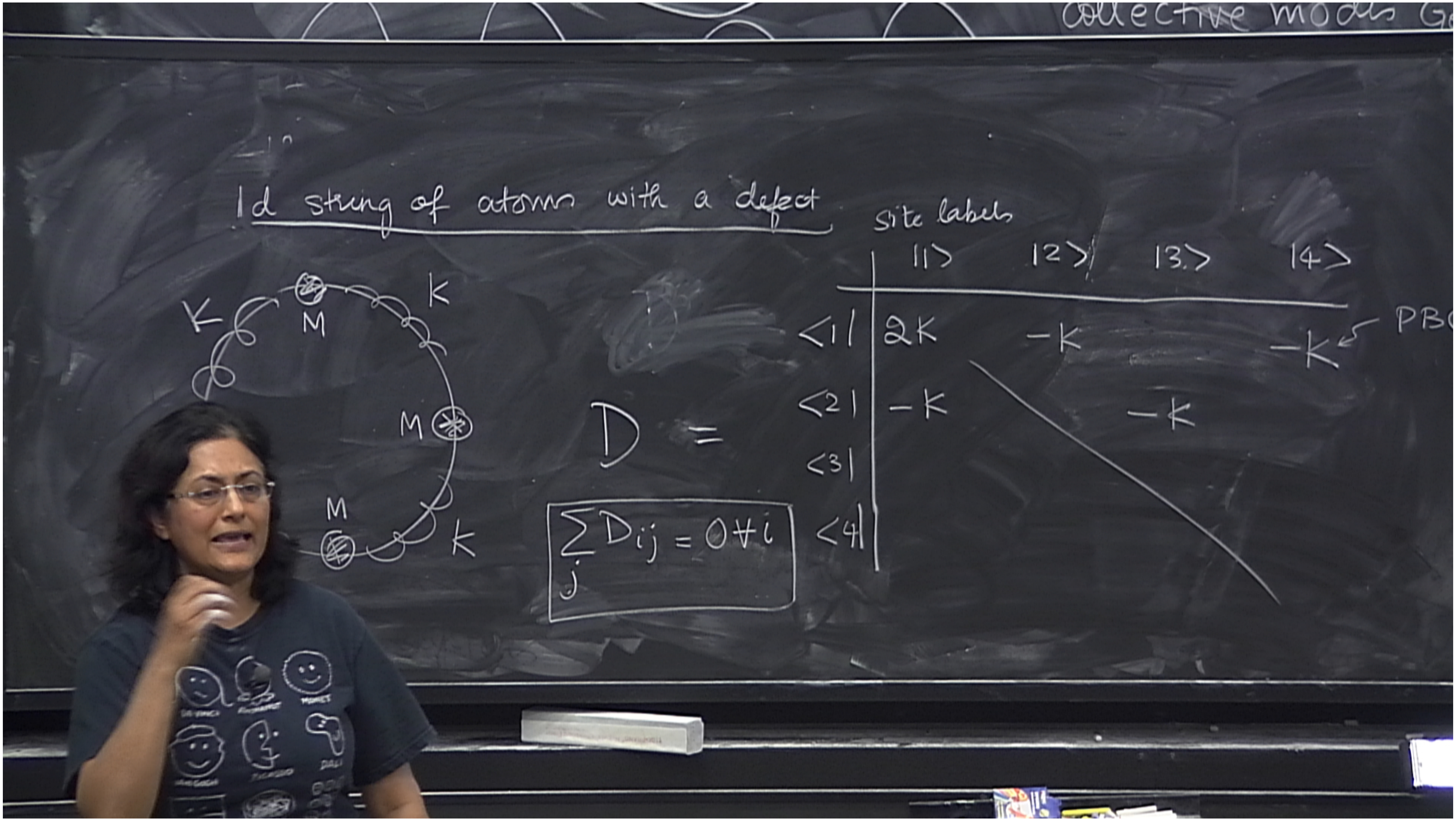

φ_i → √a φ(x)
[φ] = √length
φ_{i+1} − φ_i = a^{3/2} ∂_x φ(i)
L[φ] = ∫ dx
− 1/2 K a^2 (∂_x φ)^2


Action. $S = \int dt \int dx \, \mathcal{L}(\phi, \partial_x \phi, \dot{\phi})$
time

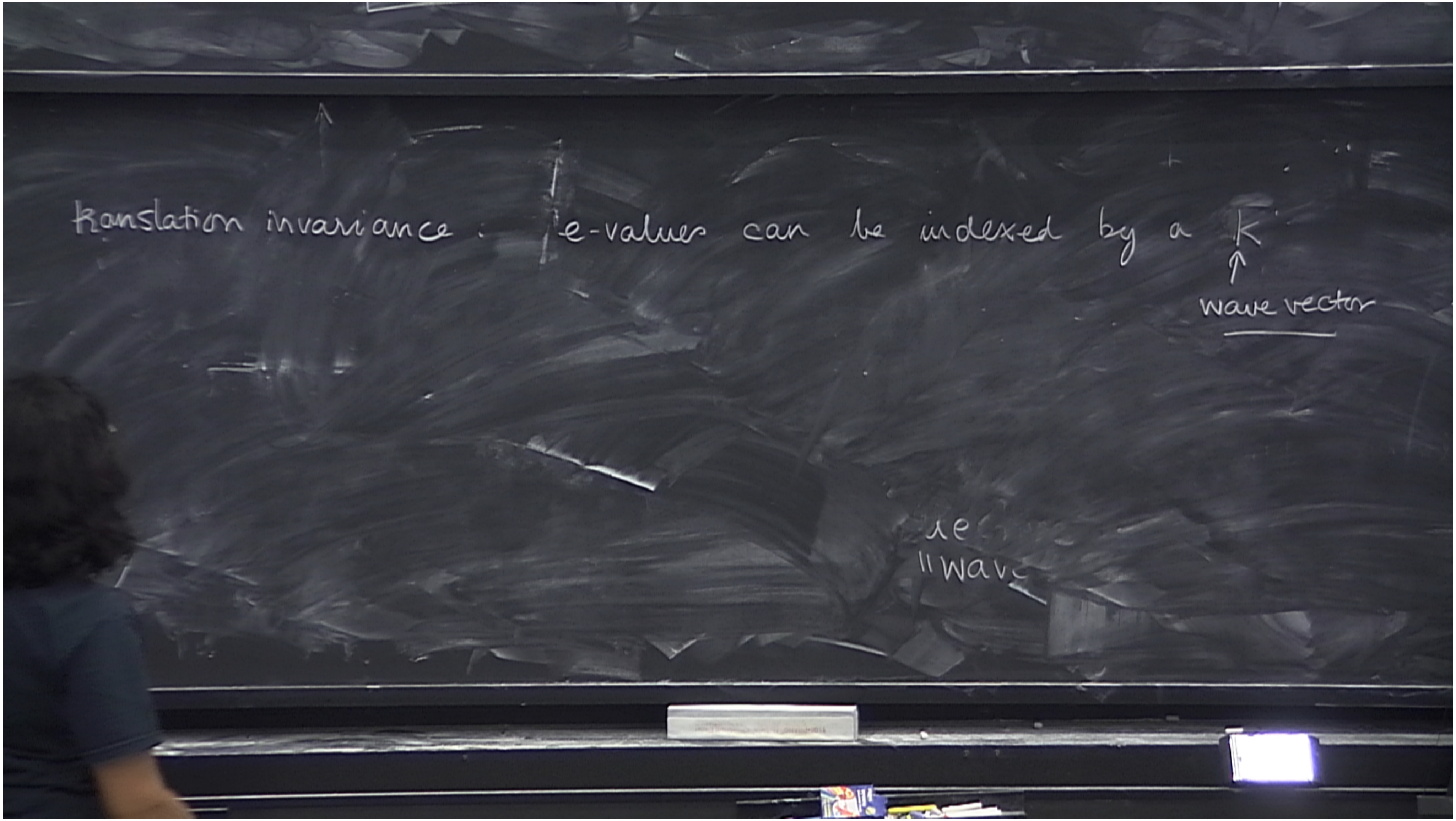

Action: $S = \int dt \int dx \, \mathcal{L}(\phi, \partial_x \phi, \dot{\phi})$
time
space

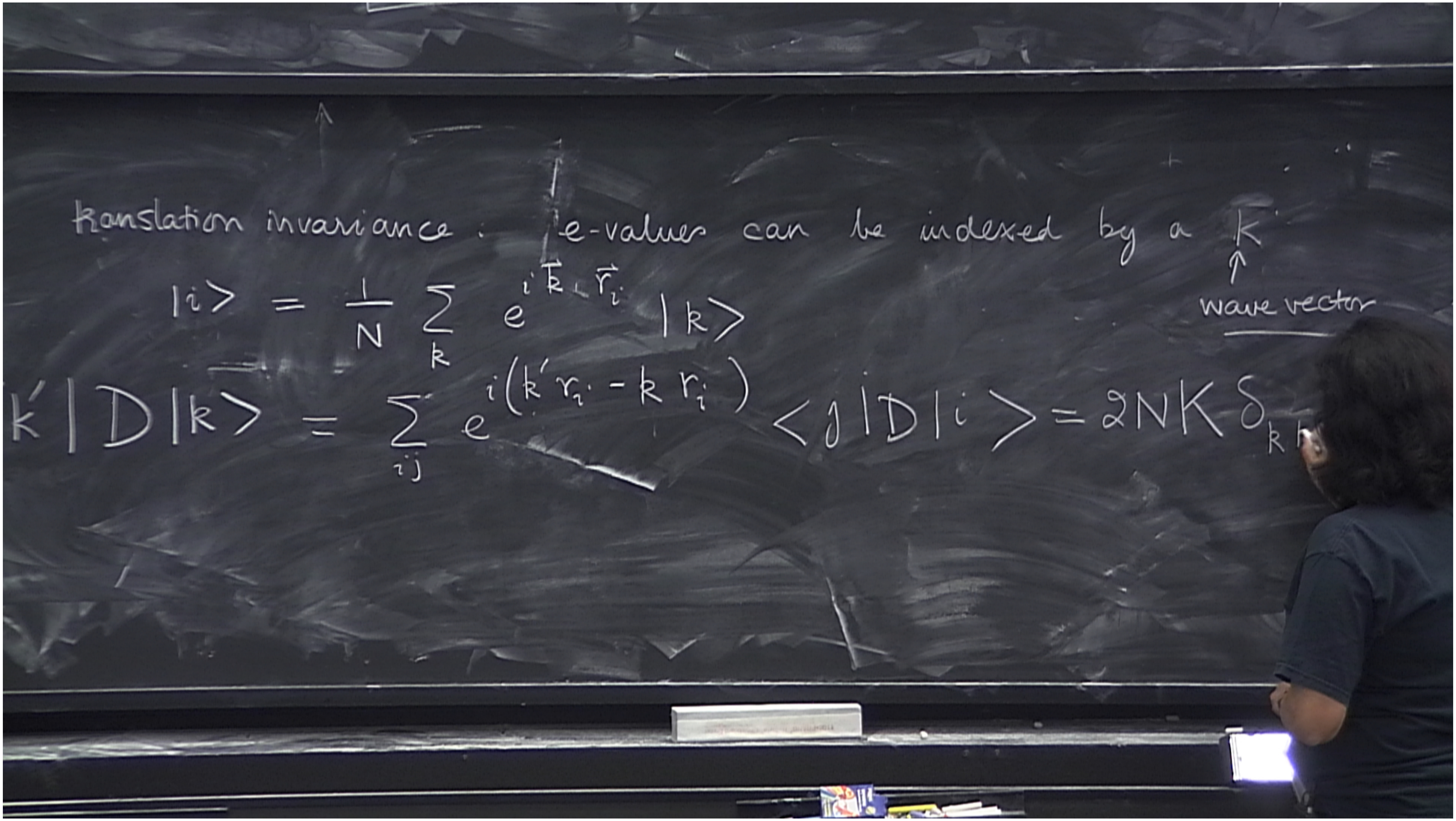

collective modes
1d string of atoms with a defect
K
M
K
M
M
M
K
K
D =
|1>


1d string of atoms with a defect
site labels
|1> |2> |3> |4>
D = <1|
<2|
<3|
<4|
K
M
K
M
M
K

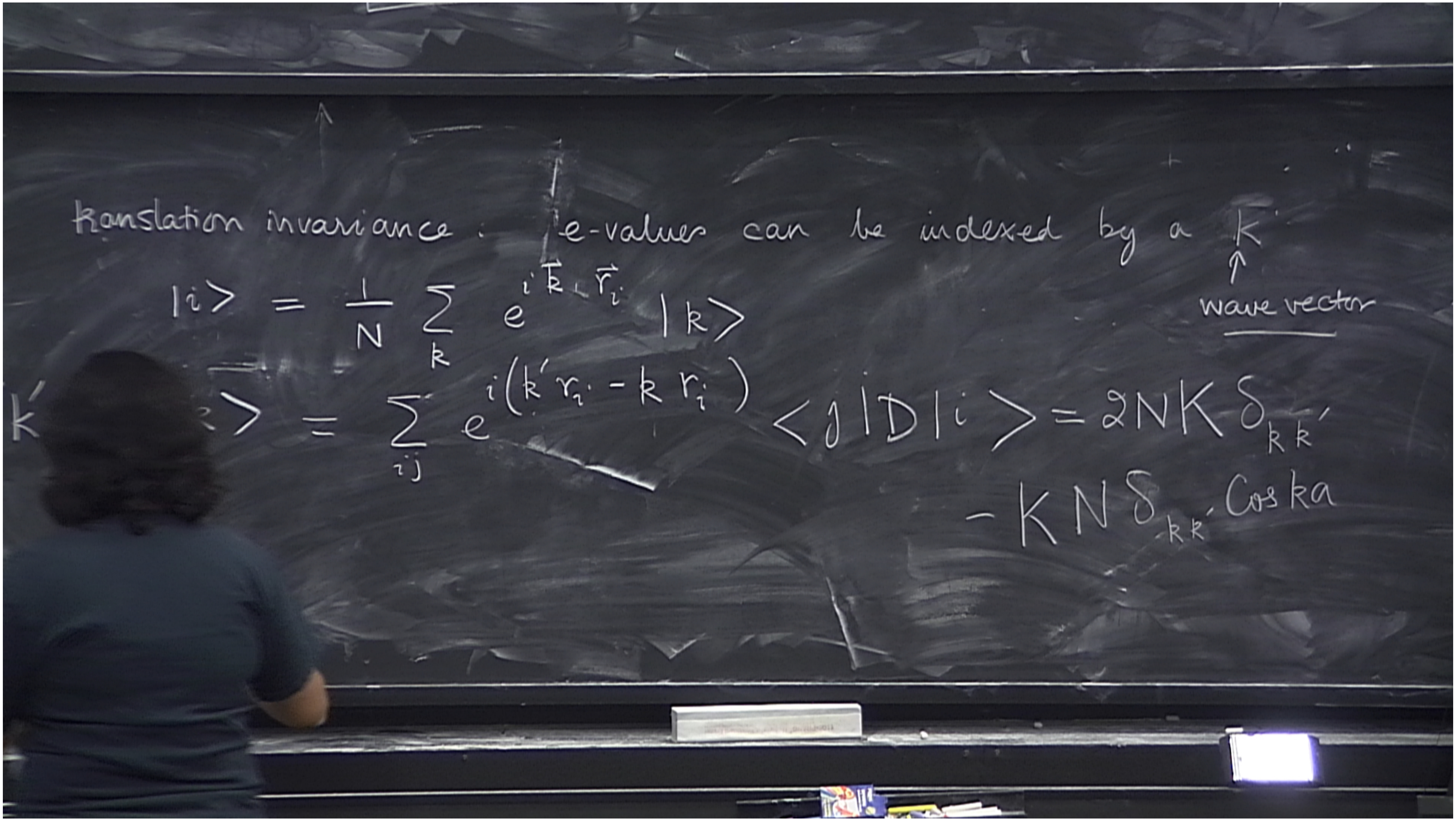


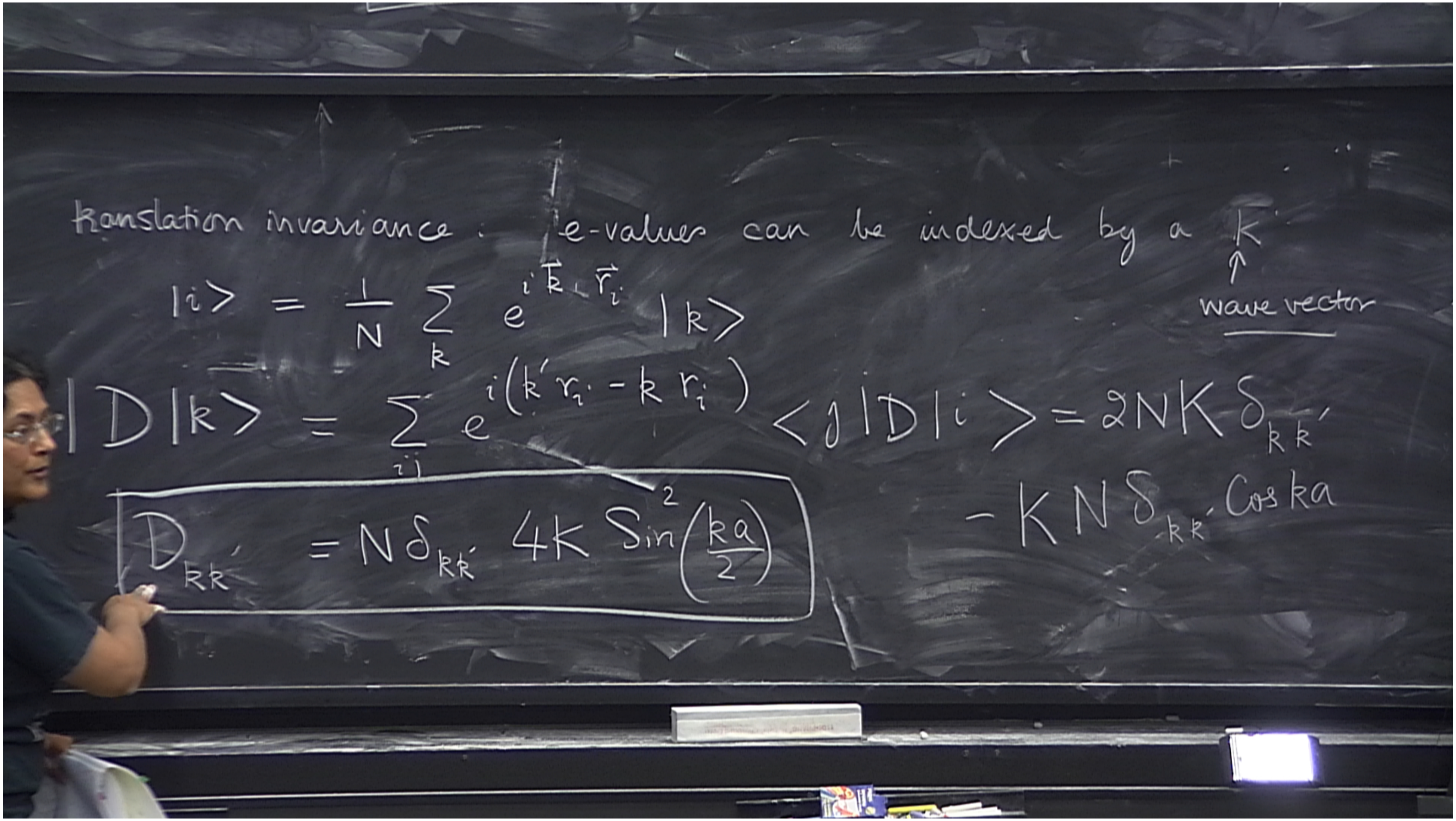
1d string of atoms with a defect
site labels
|1> |2> |3> |4>
<1| 2K −K −K
<2| −K −K
<3|
<4|
D =
∑_j D_ij = 0 ∀ i
K
M

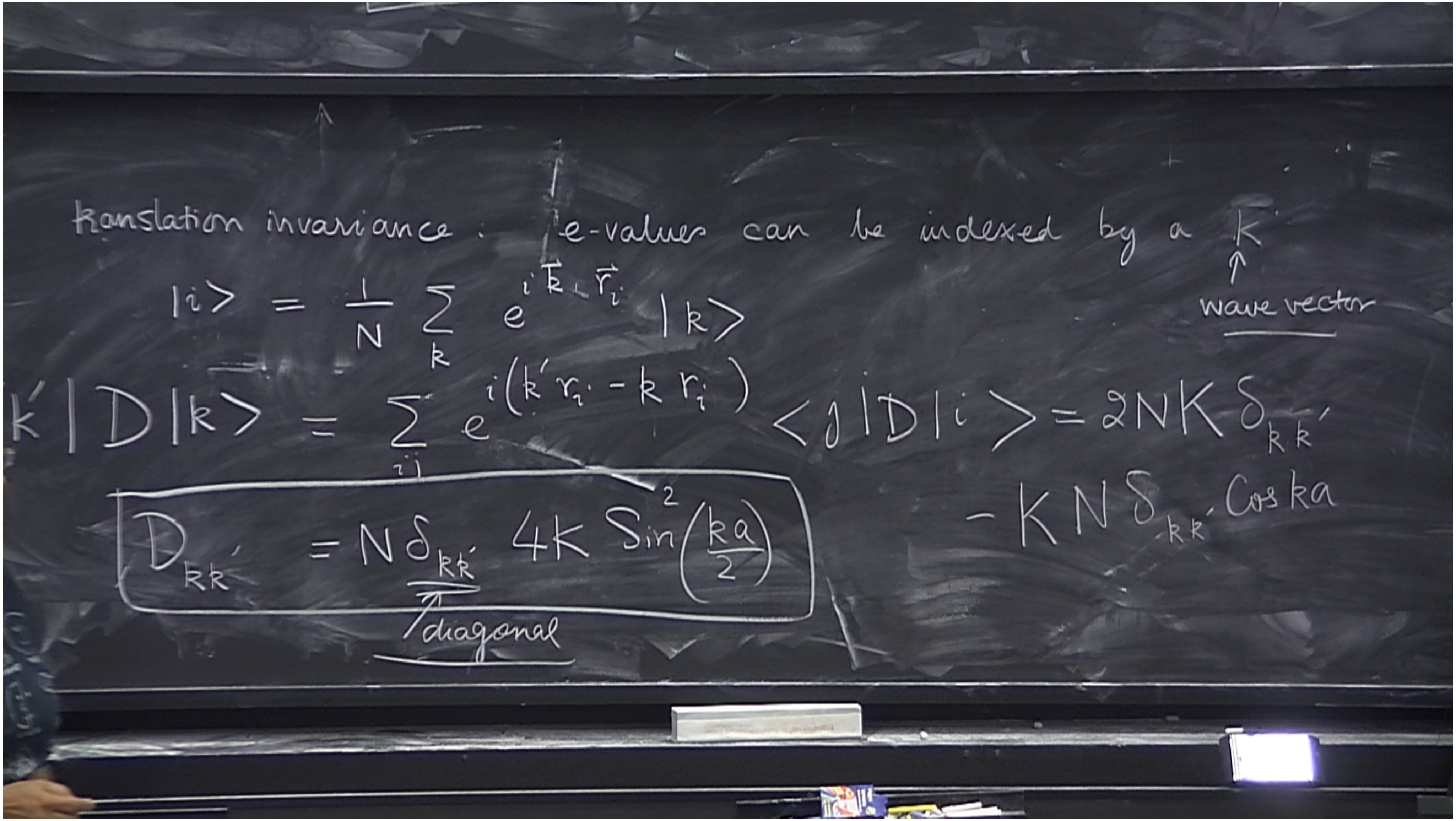

translation invariance : e-values can be indexed by a k
wave vector

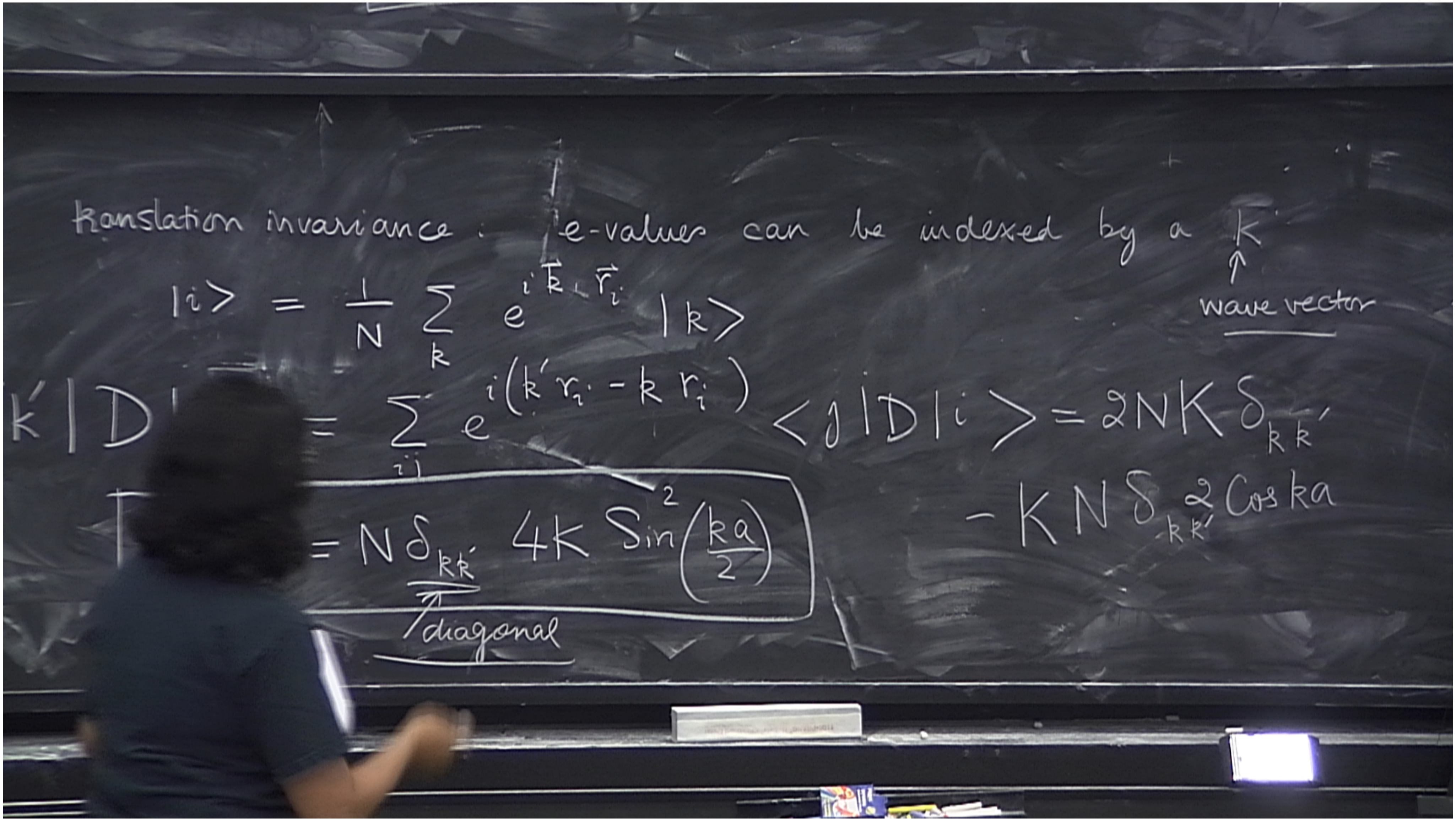


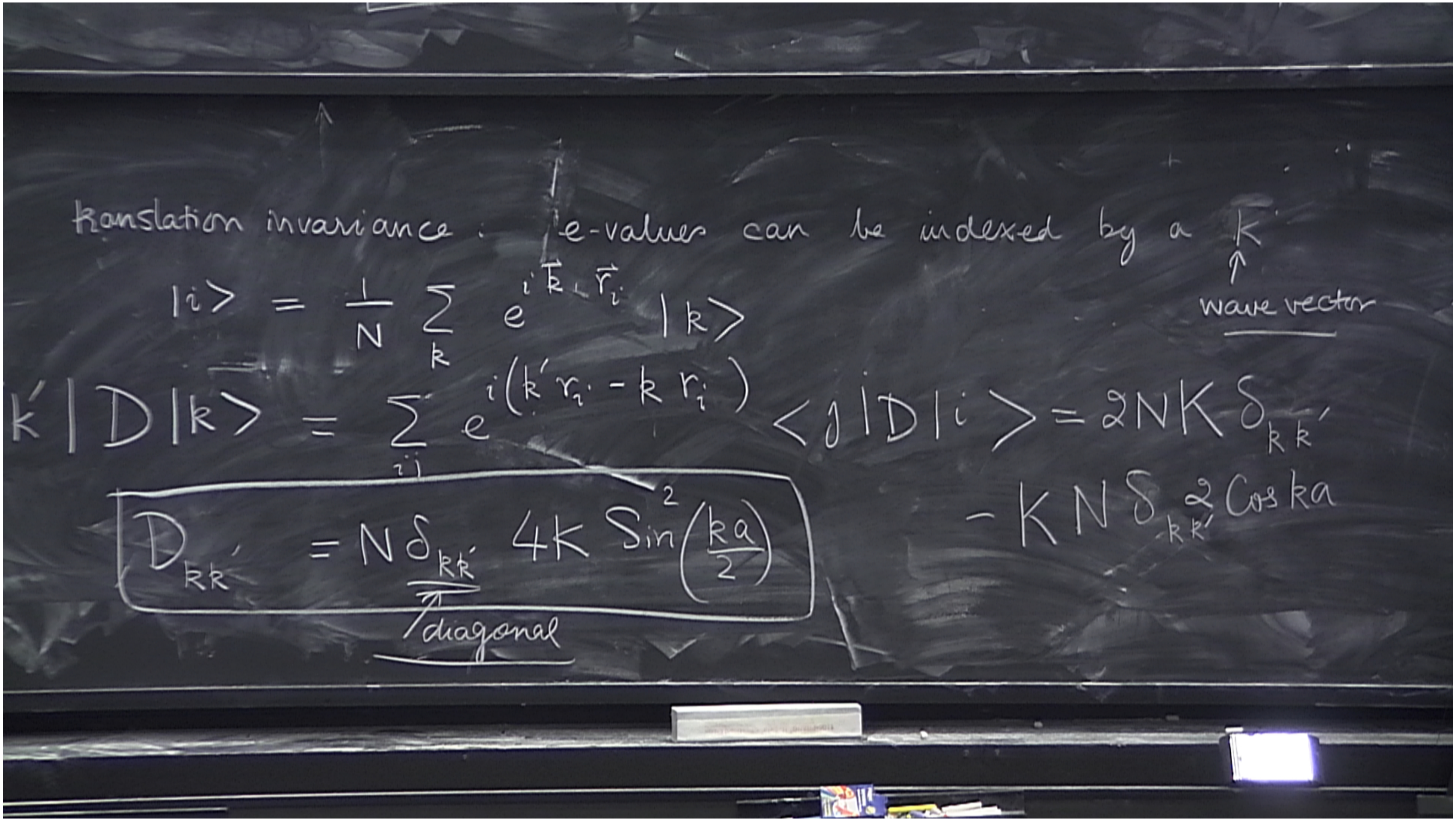
translation invariance : e-values can be indexed by a k
wave vector
|i> = 1/N ∑_k e^{i k·r_i} |k>
k'|D|k> = ∑_{ij} e^{i(k' r_j - k r_i)}
<j|D|i> = 2NK δ_{k k'}

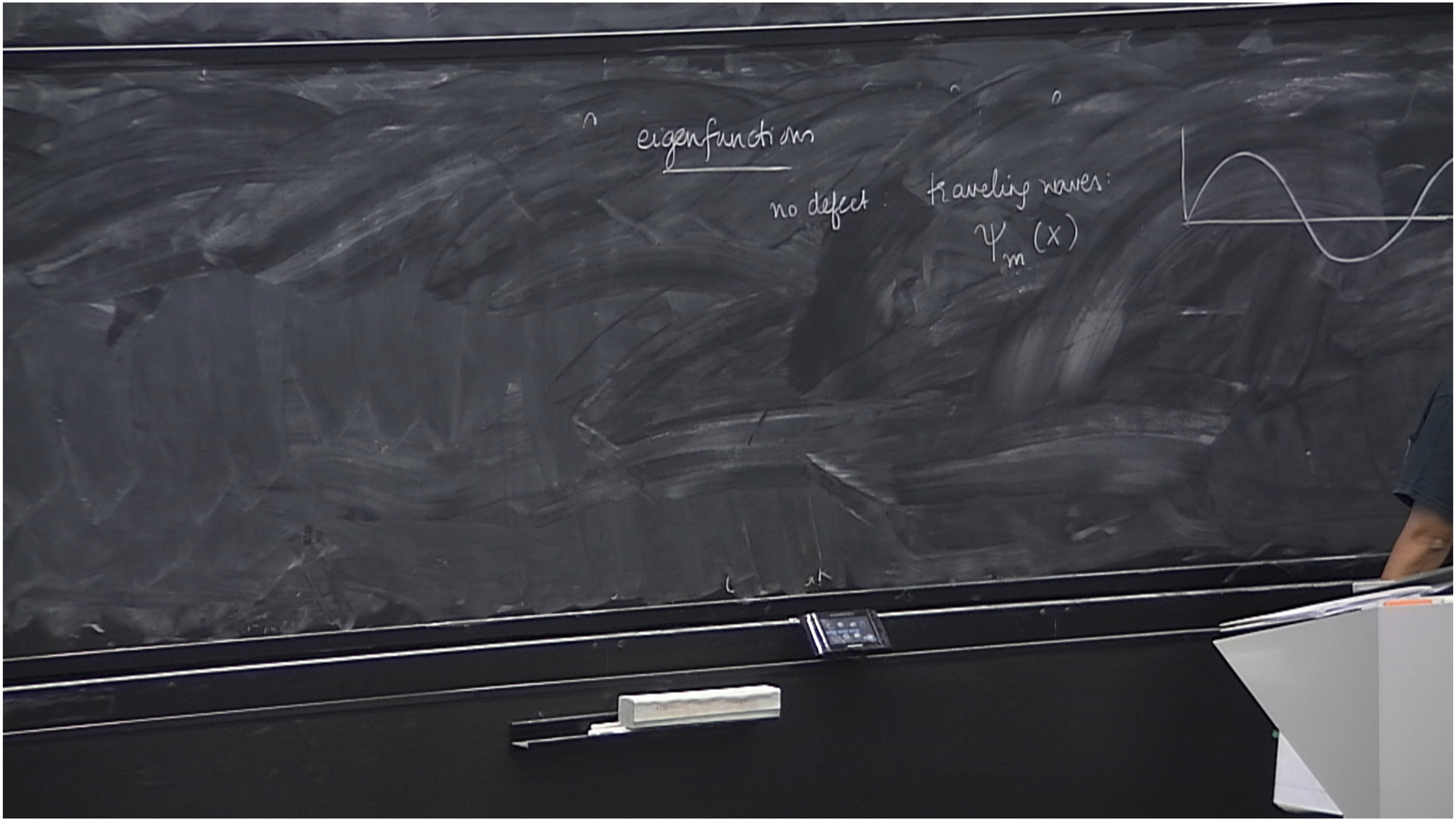

translation invariance: e-values can be indexed by a k
wave vector
|i> = 1/N Σ_k e^{i k·r_i} |k>
> = Σ_ij e^{i(k' r_i - k r_i)}
<j|D|i> = 2NK δ_kk'
- KN δ_kk' Cos ka

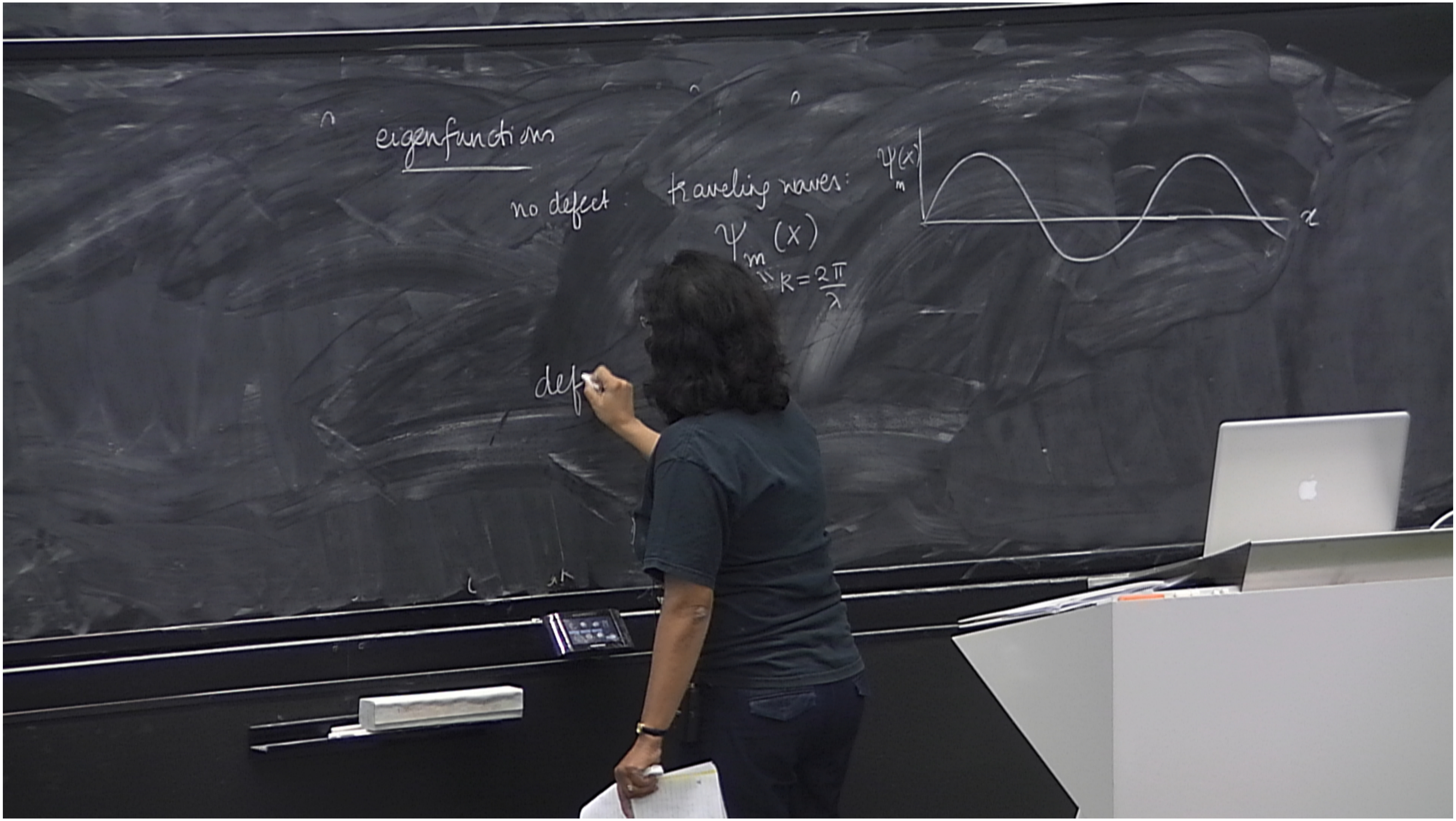

translation invariance : e-values can be indexed by a k
wave vector
|i> = 1/N Σ_k e^{i k·r_i} |k>
|D|k> = Σ_{ij} e^{i(k' r_i - k r_i)}
<j|D|i> = 2NK δ_{kk'}
- KN δ_{kk'} Cos ka
D_{kk'} = N δ_{kk'} 4K Sin^2(ka/2)

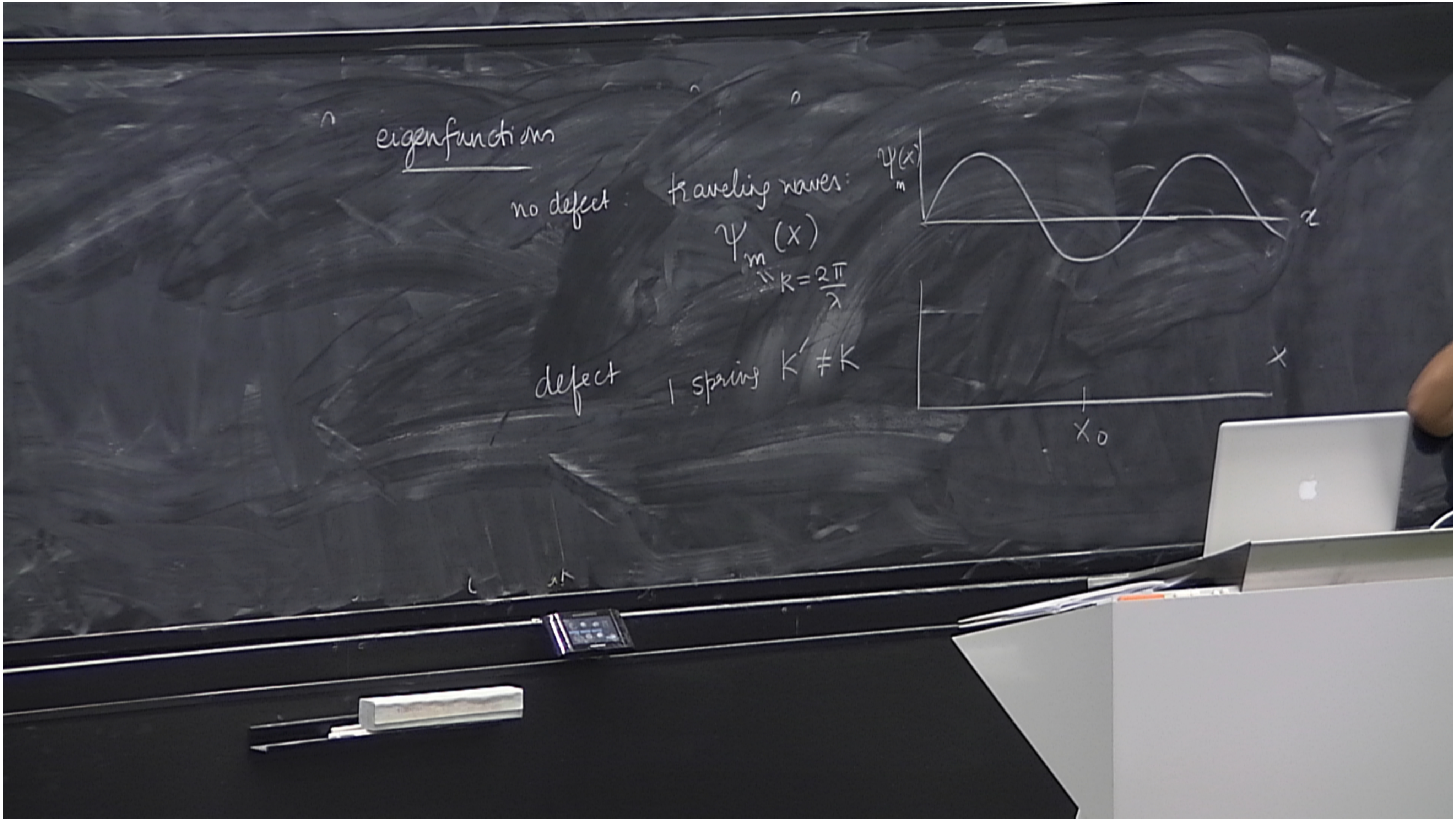

translation invariance: e-values can be indexed by a k
wave vector
$|i\rangle = \frac{1}{N} \sum_k e^{i\vec{k}\cdot\vec{r}_i} |k\rangle$
$\langle k'|D|k\rangle = \sum_{ij} e^{i(k' r_j - k r_i)} \langle j|D|i\rangle = 2NK\delta_{kk'}$
$- KN\delta_{kk'} \cos ka$
$D_{kk'} = N\delta_{kk'} \, 4K \sin^2\left(\frac{ka}{2}\right)$
diagonal

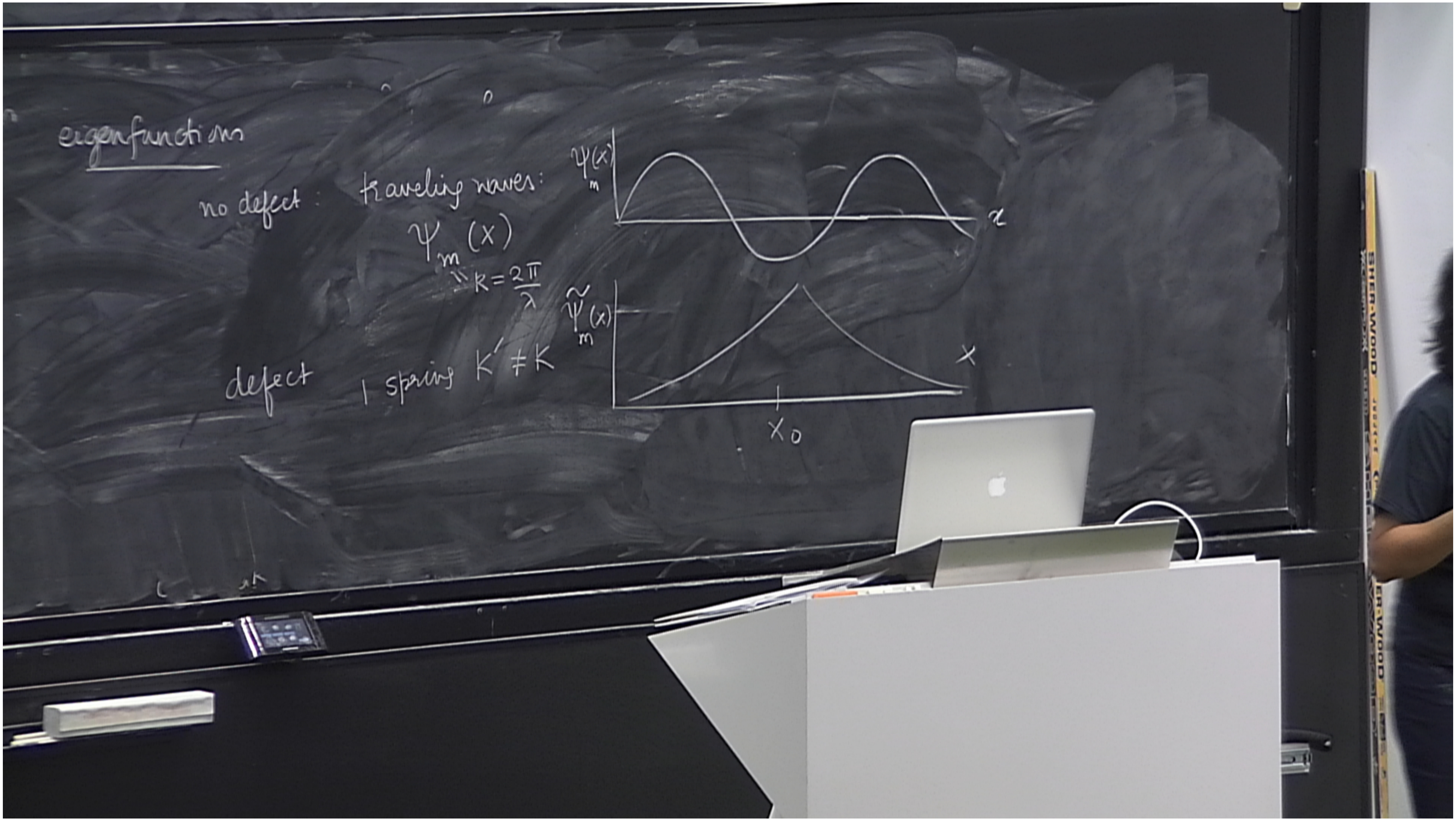

translation invariance : e-values can be indexed by a k
wave vector
|i> = 1/N ∑_k e^{i k·r_i} |k>
<j|D|i> = 2NK δ_{kk'}
−KN δ_{kk'} 2 Cos ka
= N δ_{kk'} 4K Sin^2(ka/2)
diagonal

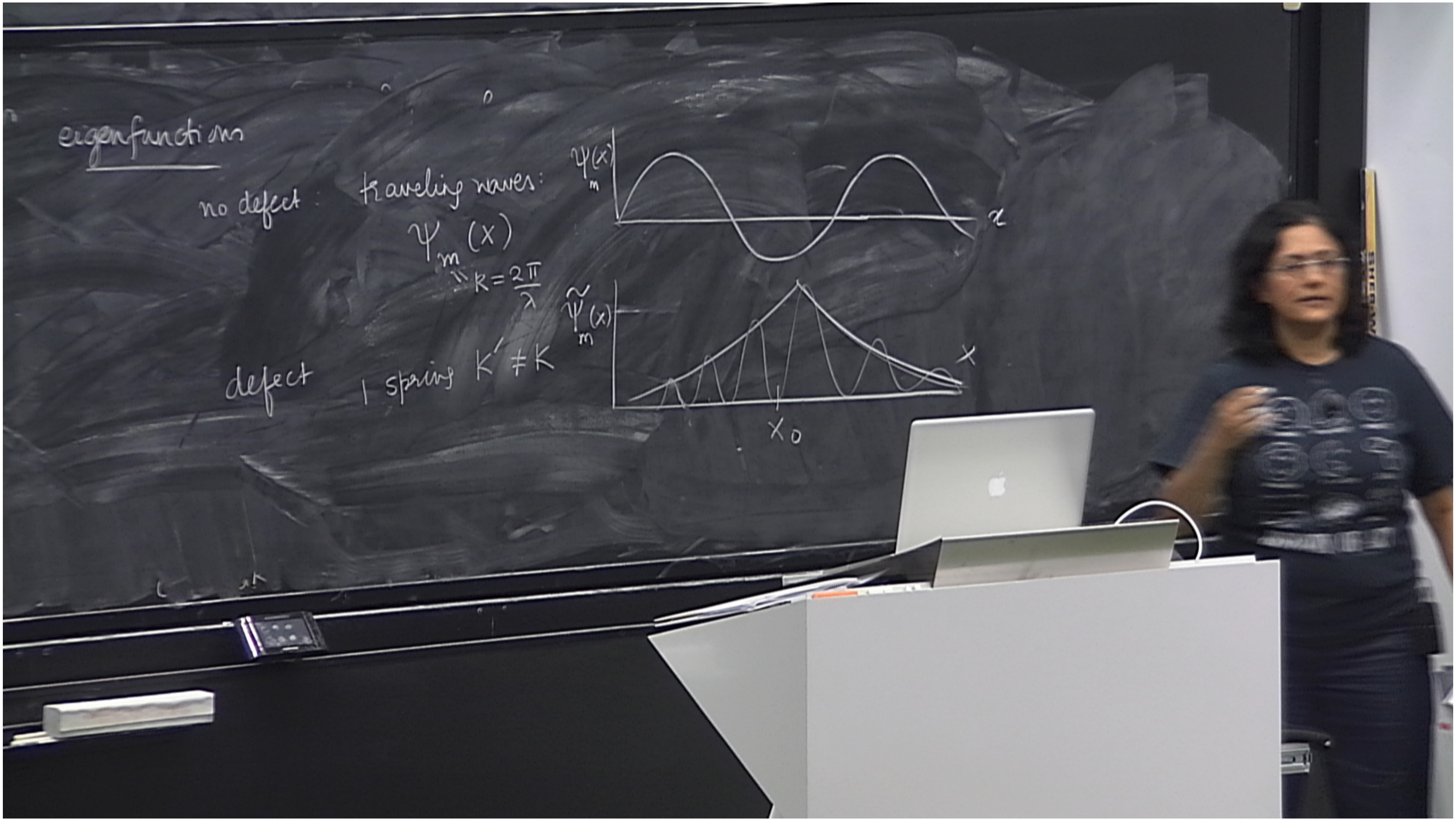

translation invariance: e-values can be indexed by a k
wave vector
|i> = 1/N Σ_k e^{i k·r_i} |k>
<k'|D|k> = Σ_{ij} e^{i(k' r_i - k r_i)}
<j|D|i> = 2NK δ_{kk'}
- KN δ_{kk'} 2 Cos ka
D_{kk'} = N δ_{kk'} 4K Sin^2(ka/2)
diagonal

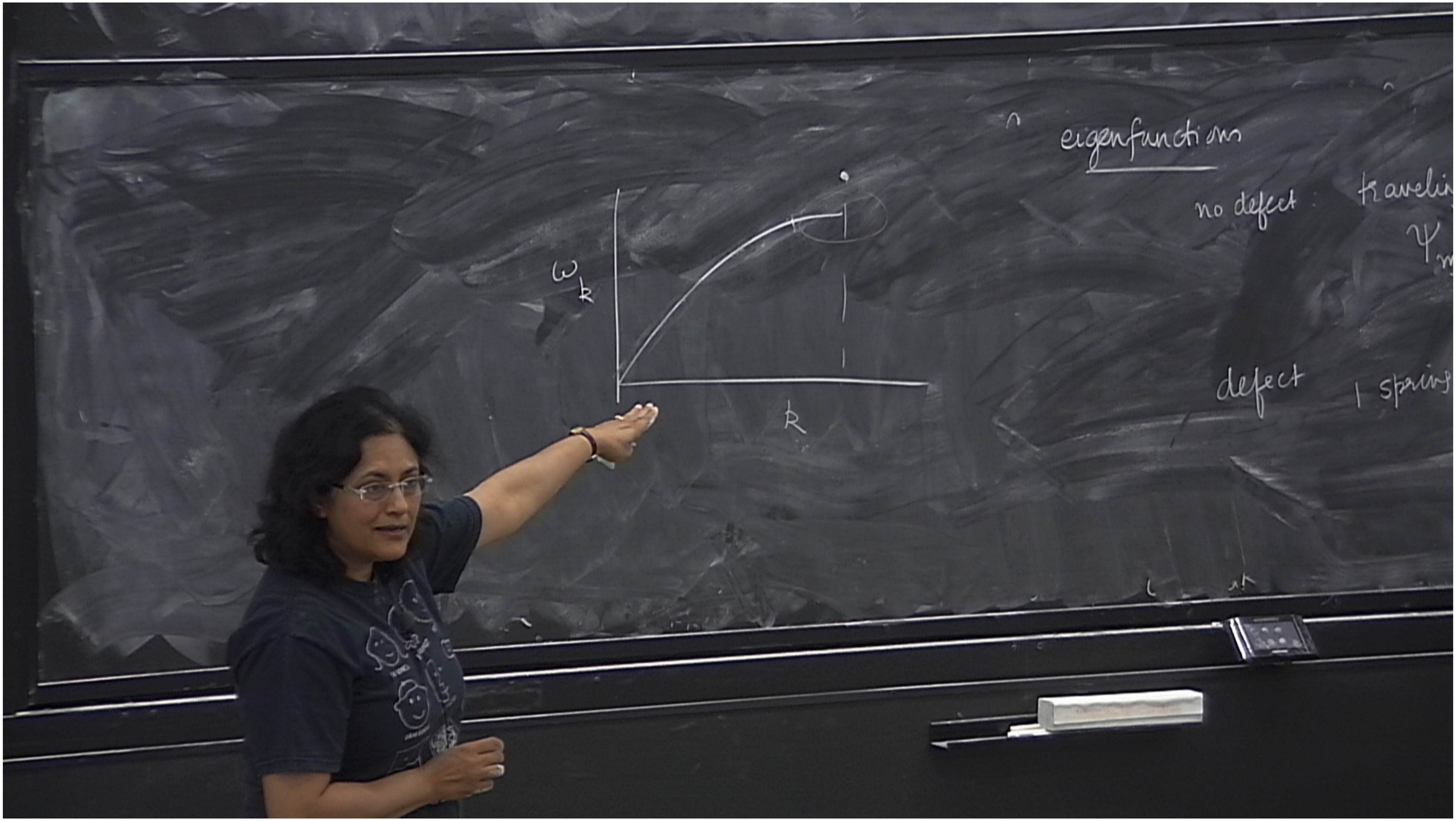

eigenfunctions
no defect :
traveling waves:
ψm (x)


eigenfunctions
no defect
traveling waves:
$\psi_m(x)$
$k = \frac{2\pi}{\lambda}$
$\psi_m(x)$
x
def

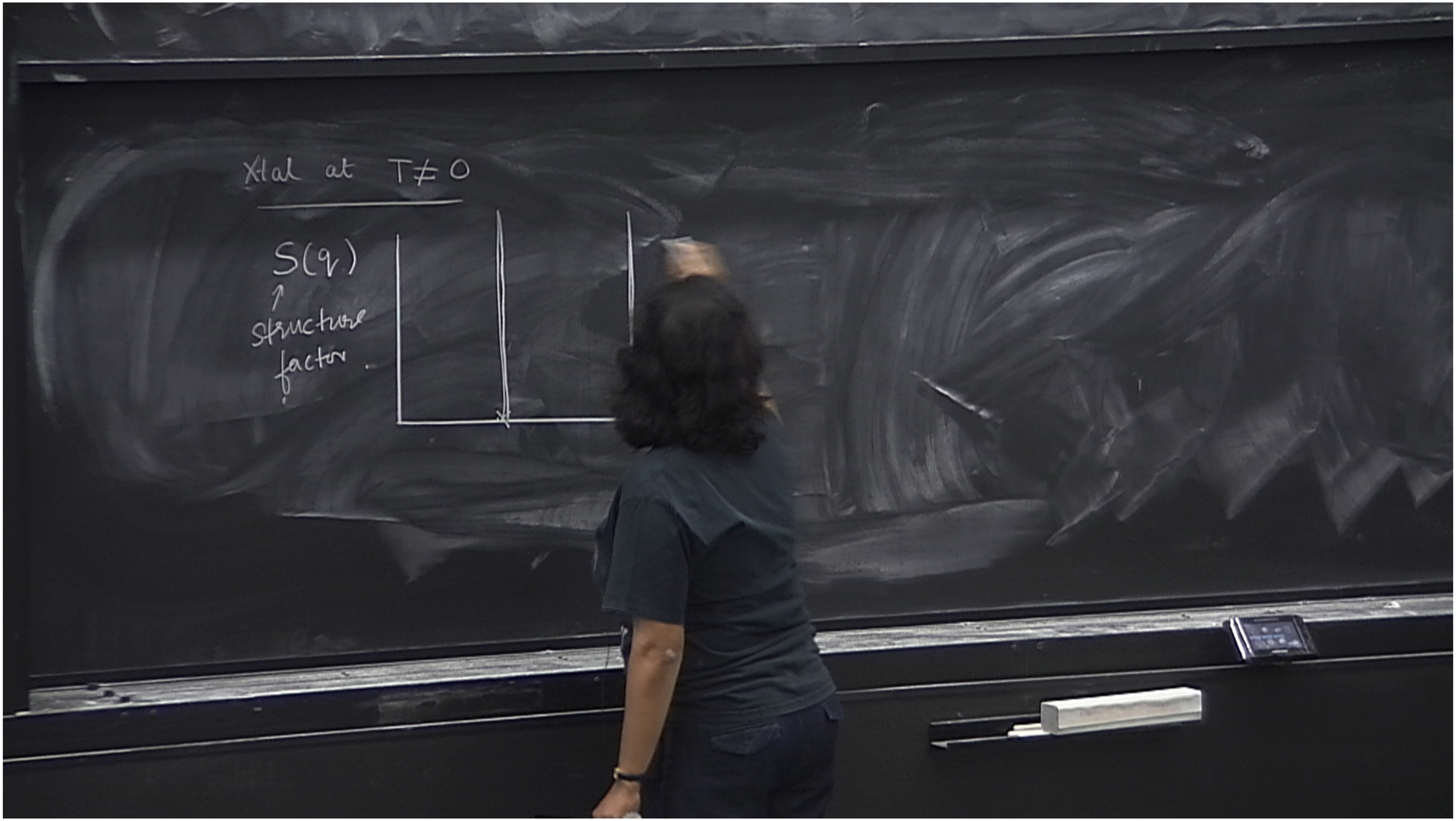

eigenfunctions
no defect : traveling waves:
ψm (x)
k = 2π/λ
defect
1 spring K' ≠ K
ψm(x)
x
x
X0

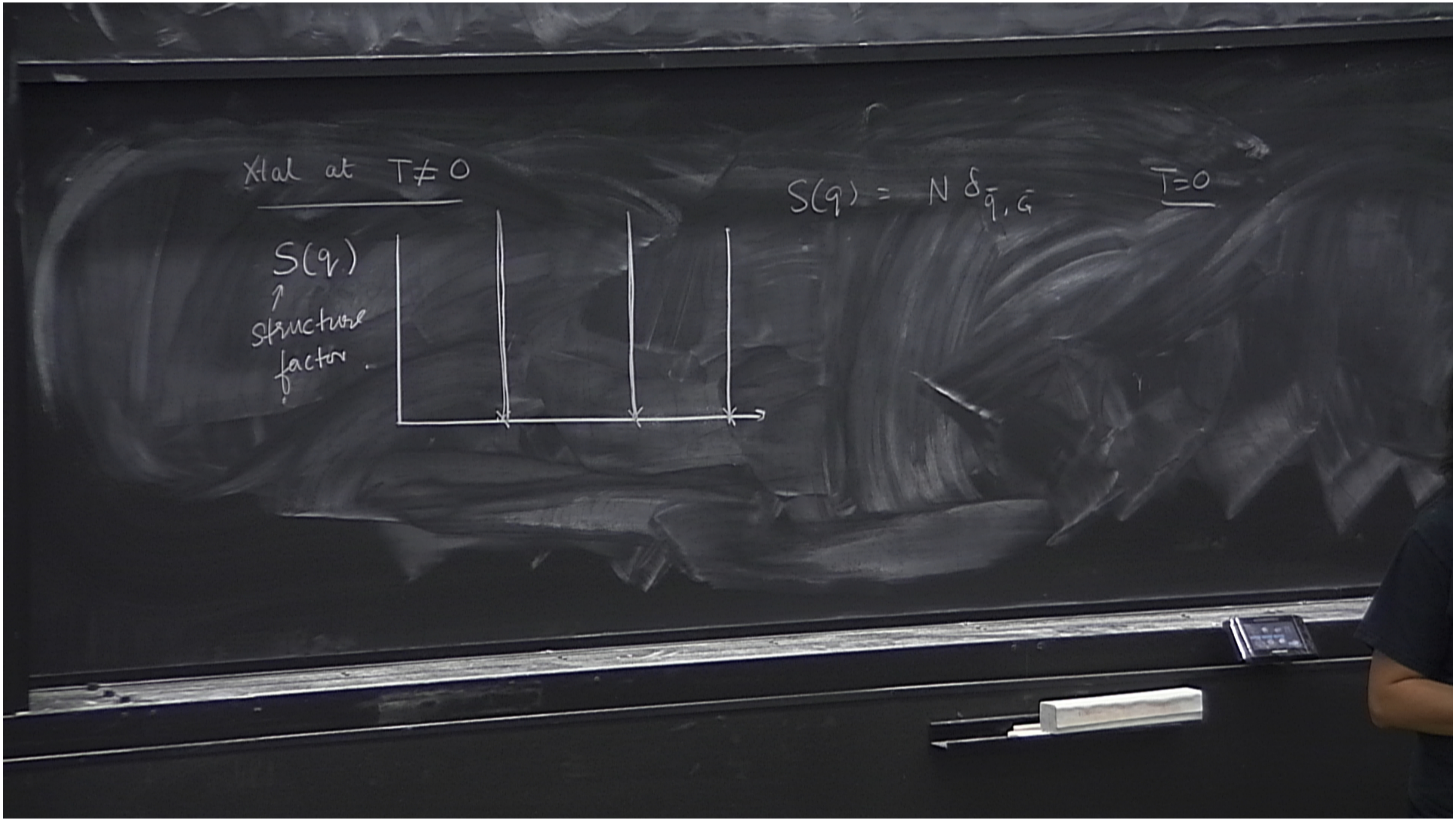

eigenfunctions
no defect : traveling waves:
$\psi_m(x)$
$k = \frac{2\pi}{\lambda}$
defect 1 spring $K' \neq K$
$\psi_m(x)$
$\tilde{\psi}_m(x)$
x
x_0

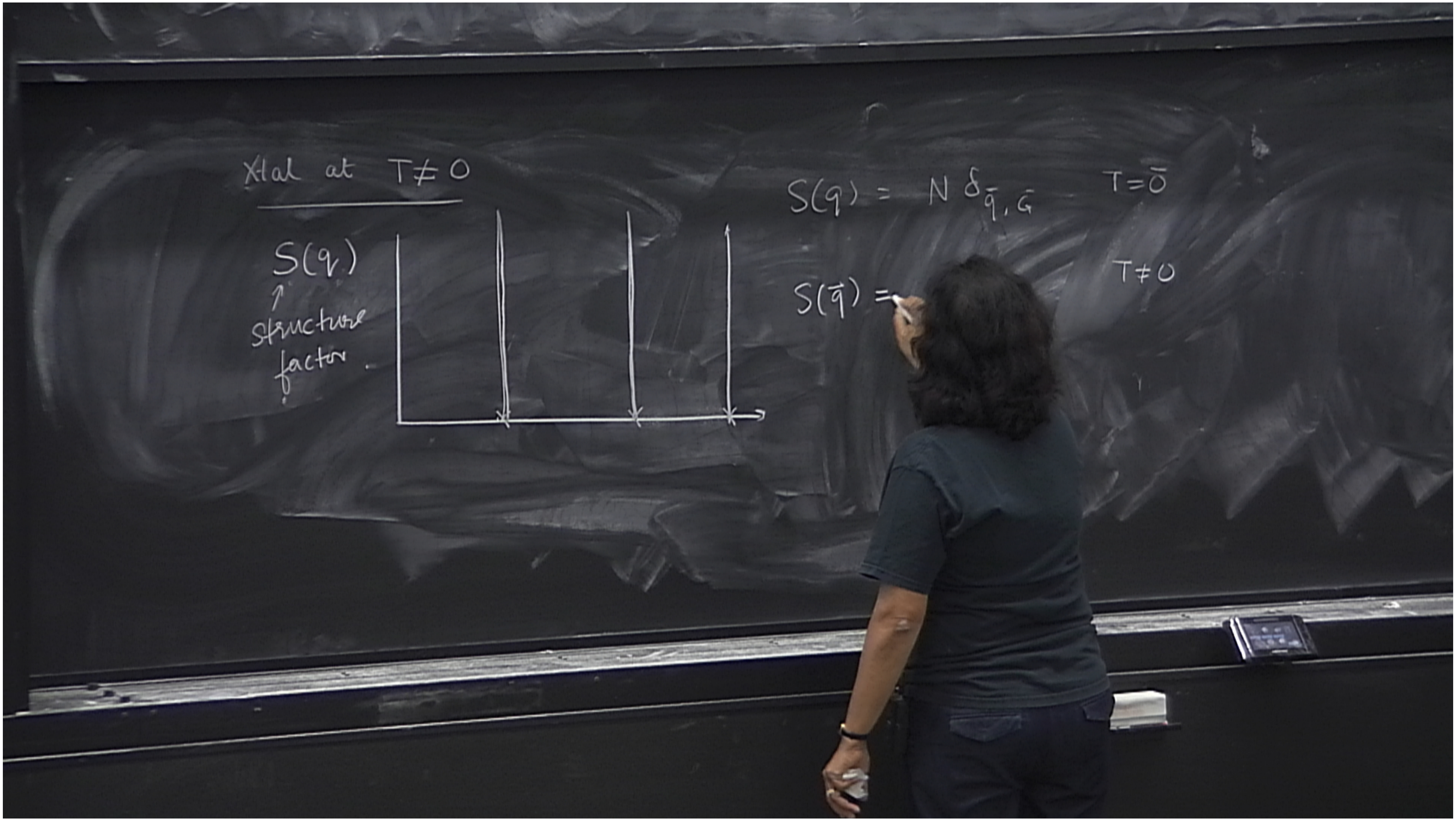

eigenfunctions
no defect
traveling waves:
ψm (x)
k = 2π/λ
defect
1 spring K′ ≠ K
ψm(x)
x
ψ̃m(x)
x
x0

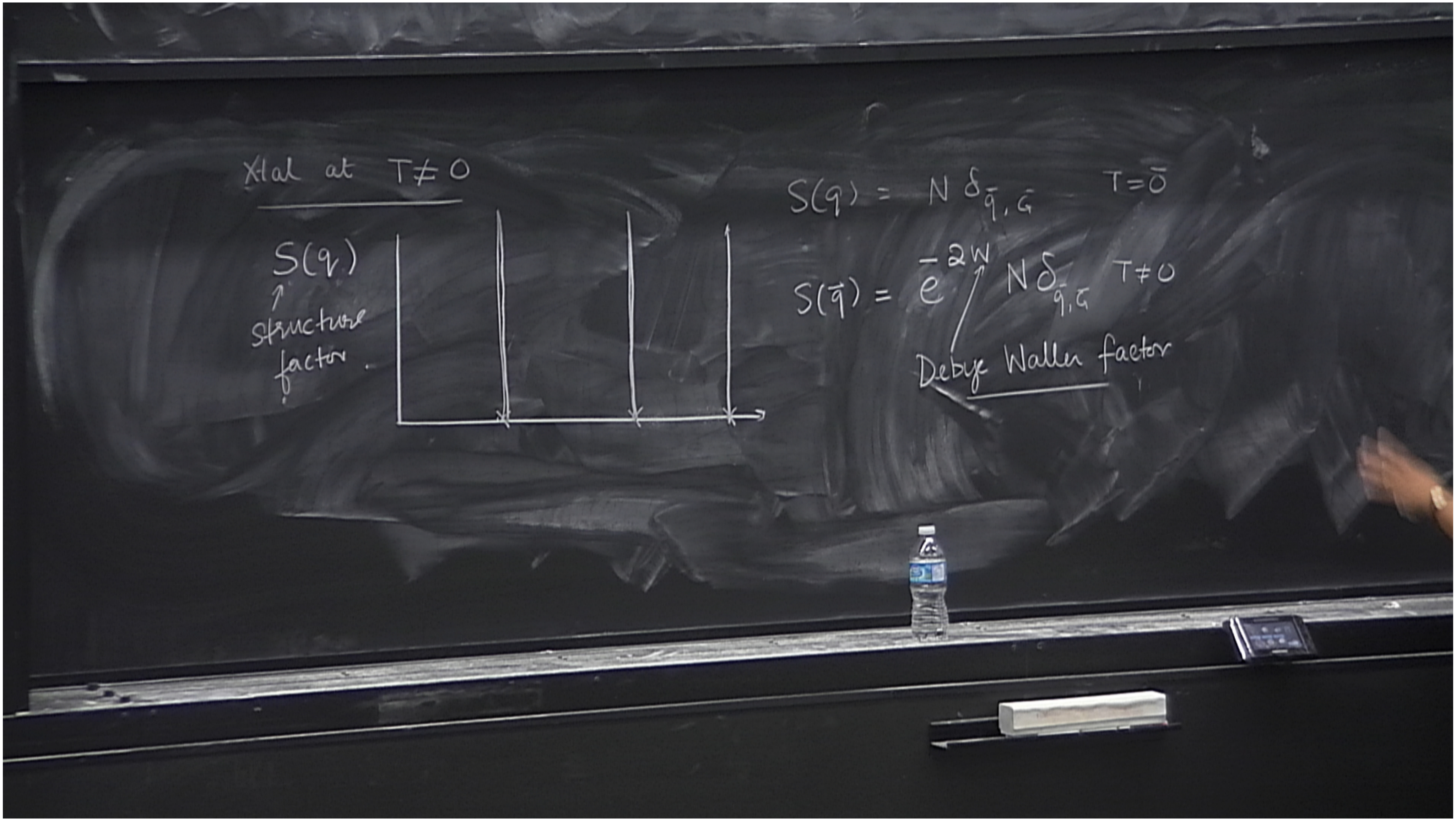


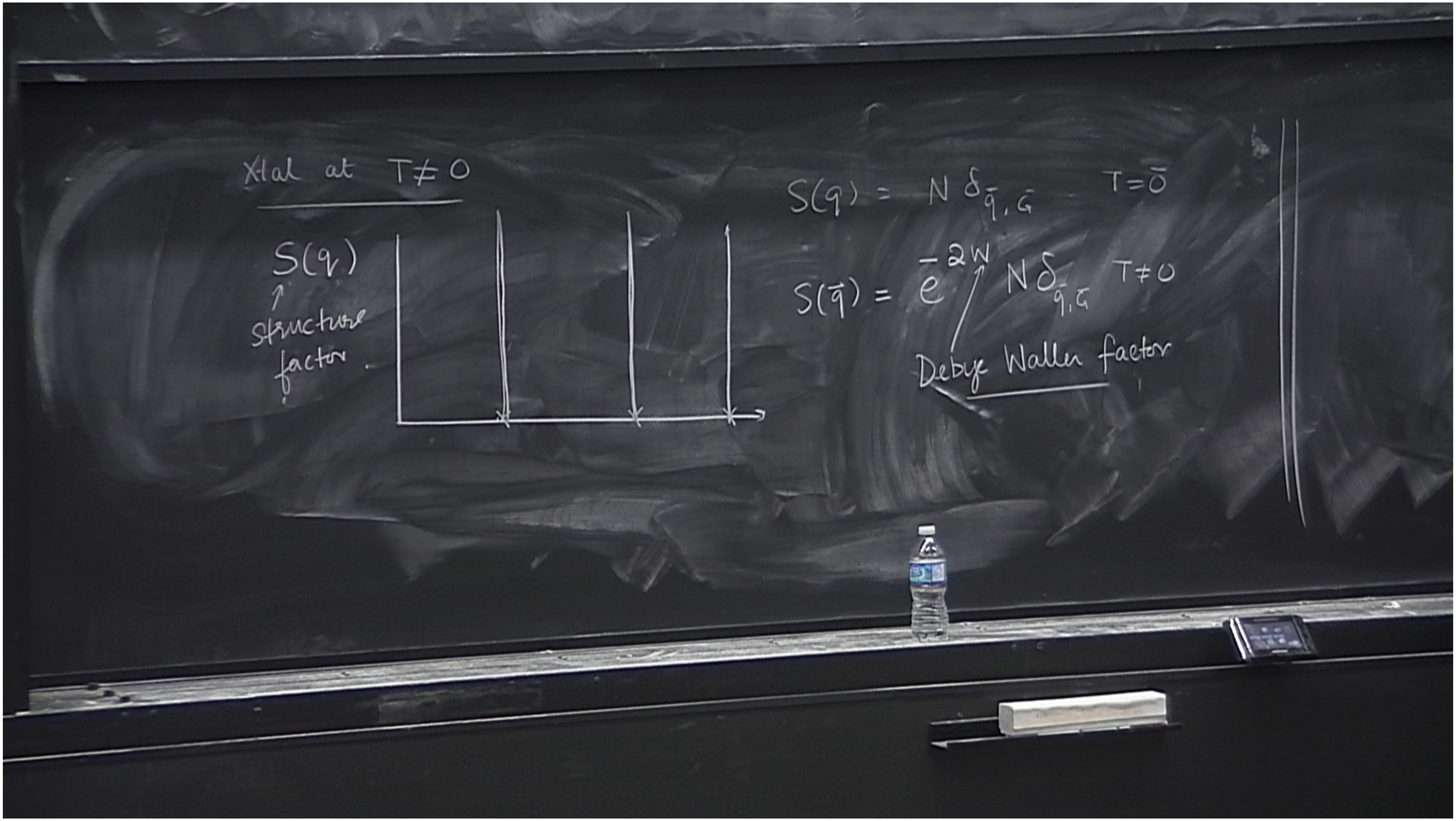
eigenfunctions
no defect
defect
ω_k
k

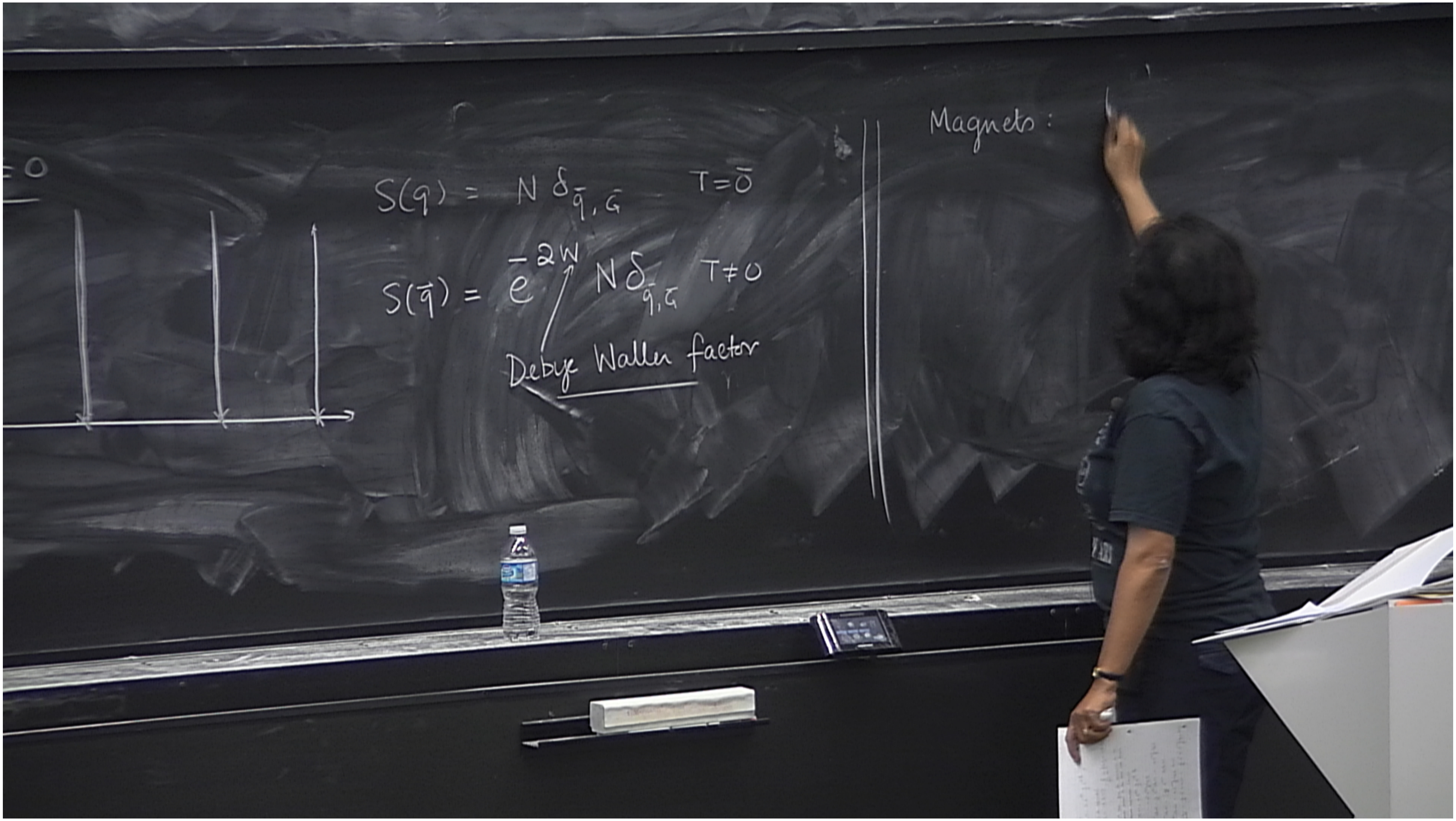

Xtal at T≠0
S(q)
structure
factor

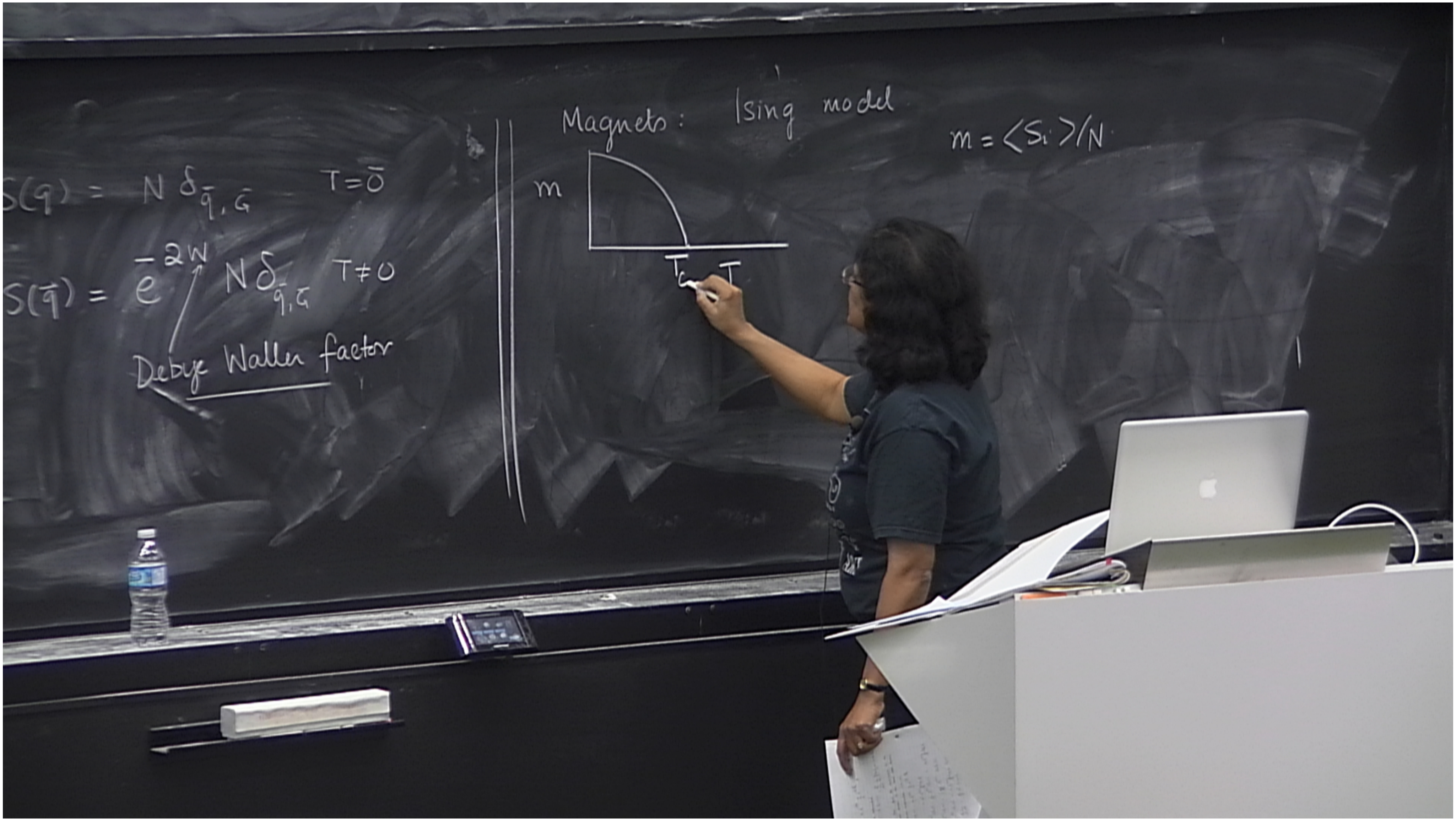

Xtal at T≠0
S(q)
structure factor
S(q) = N δ q, G
T=0


Xtal at T≠0
S(q)
structure factor
S(q) = N δq,G
T=0
T≠0
S(q) =

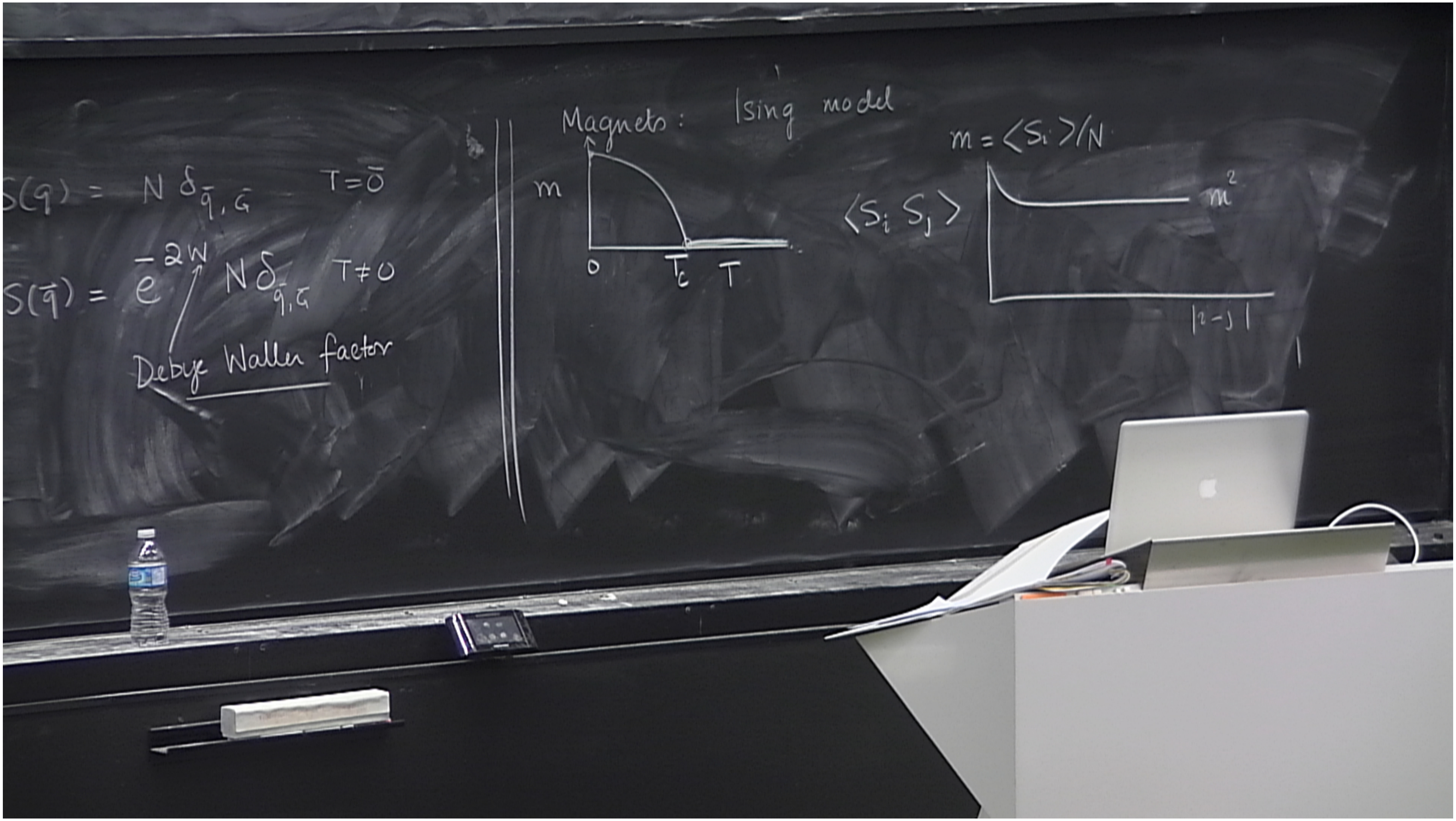

Xtal at T≠0
S(q)
structure factor
S(q) = N δ q, G   T=0
S(q) = e^-2W N δ q, G   T≠0
Debye Waller factor


Xtal at T≠0
S(q)
structure factor
S(q) = N δ q,G T=0
S(q) = e -2W N δ q,G T≠0
Debye Waller factor

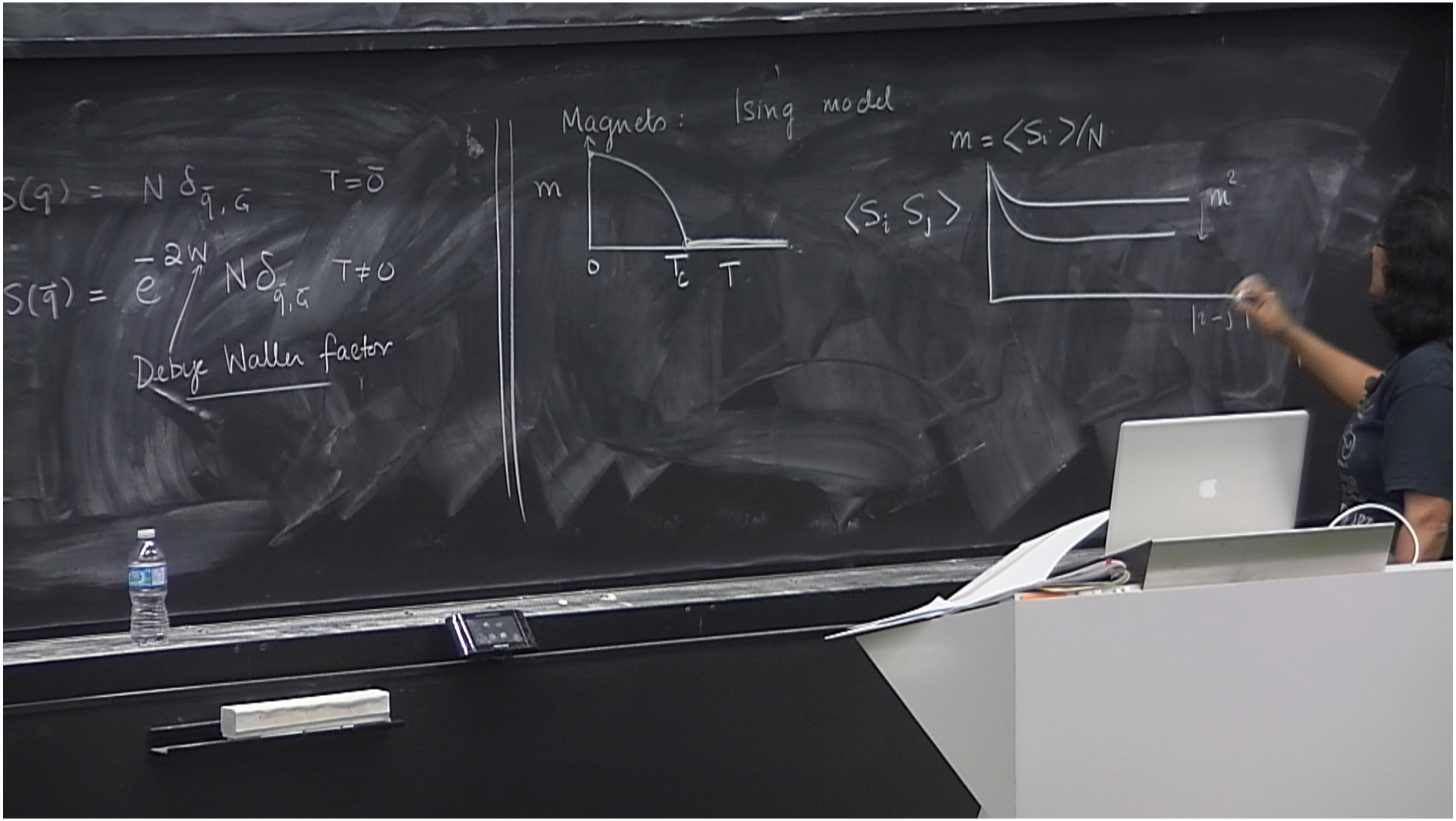

Magnets :
S(q) = N δ q, G T=0
S(q) = e -2W N δ q, G T≠0
Debye Waller factor

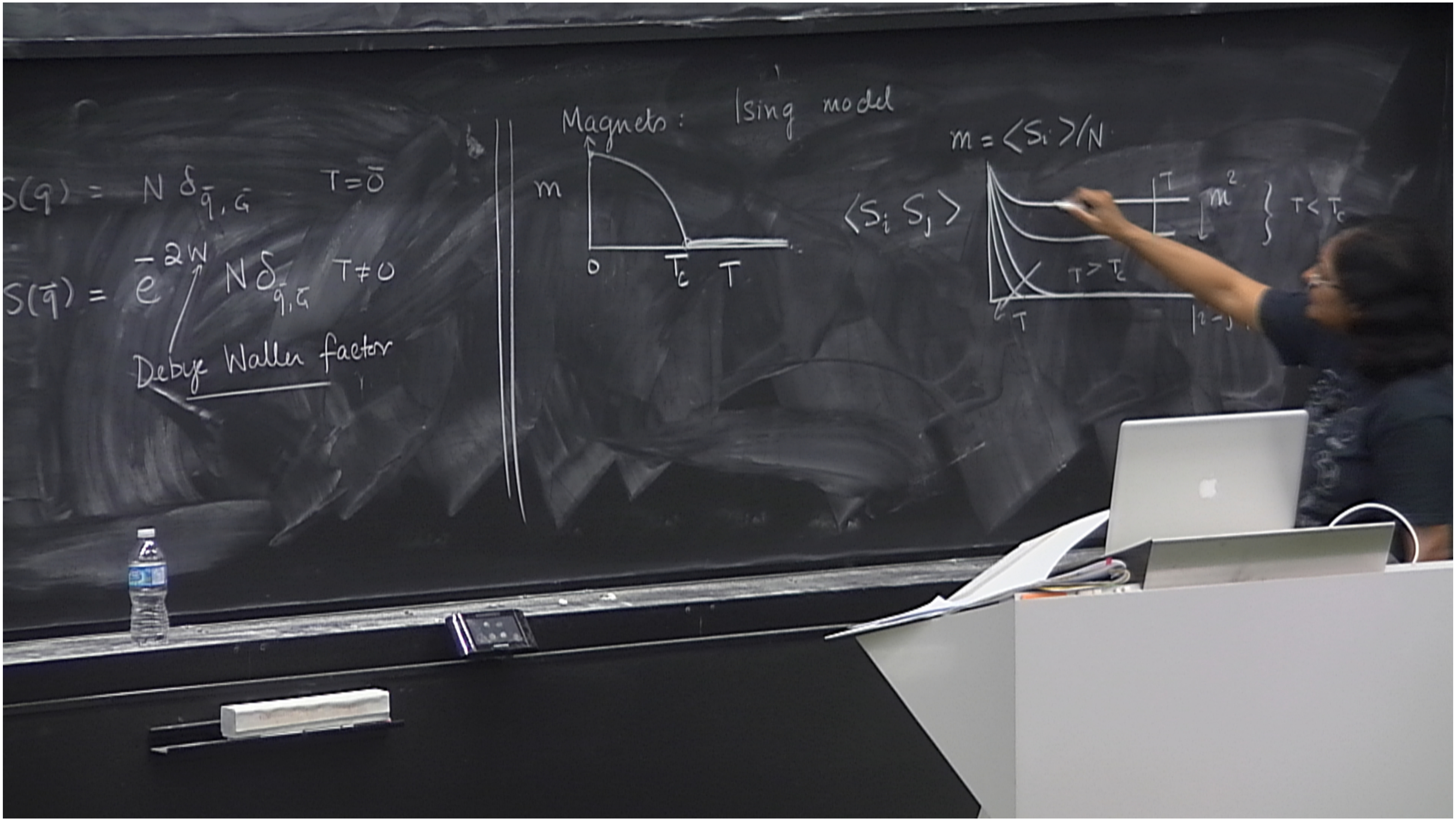


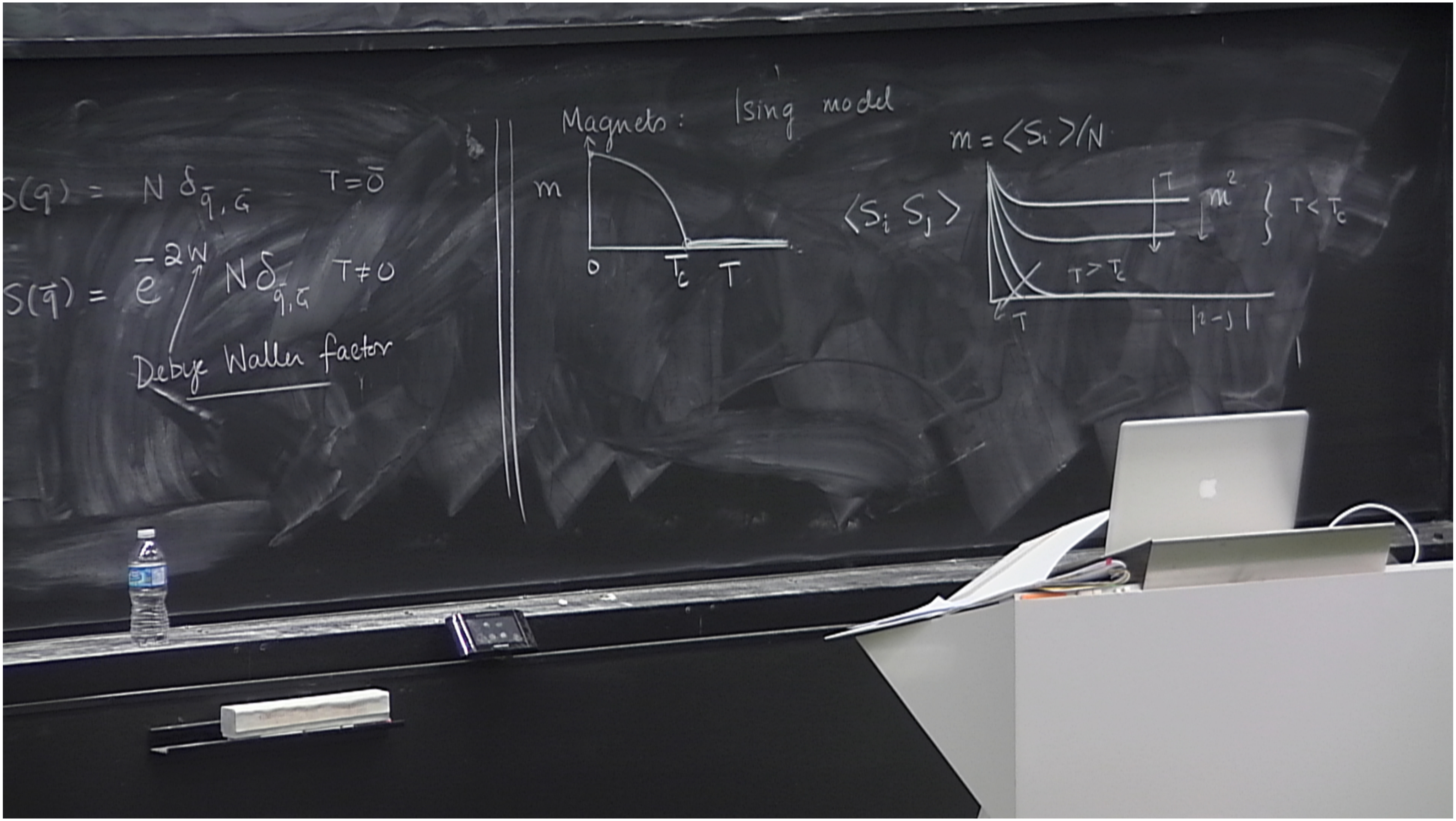
Magnets: Ising model
m = <S_i>/N
m
T_c T
S(q) = N δ_{q,G} T=0
S(q) = e^{-2W} N δ_{q,G} T≠0
Debye Waller factor

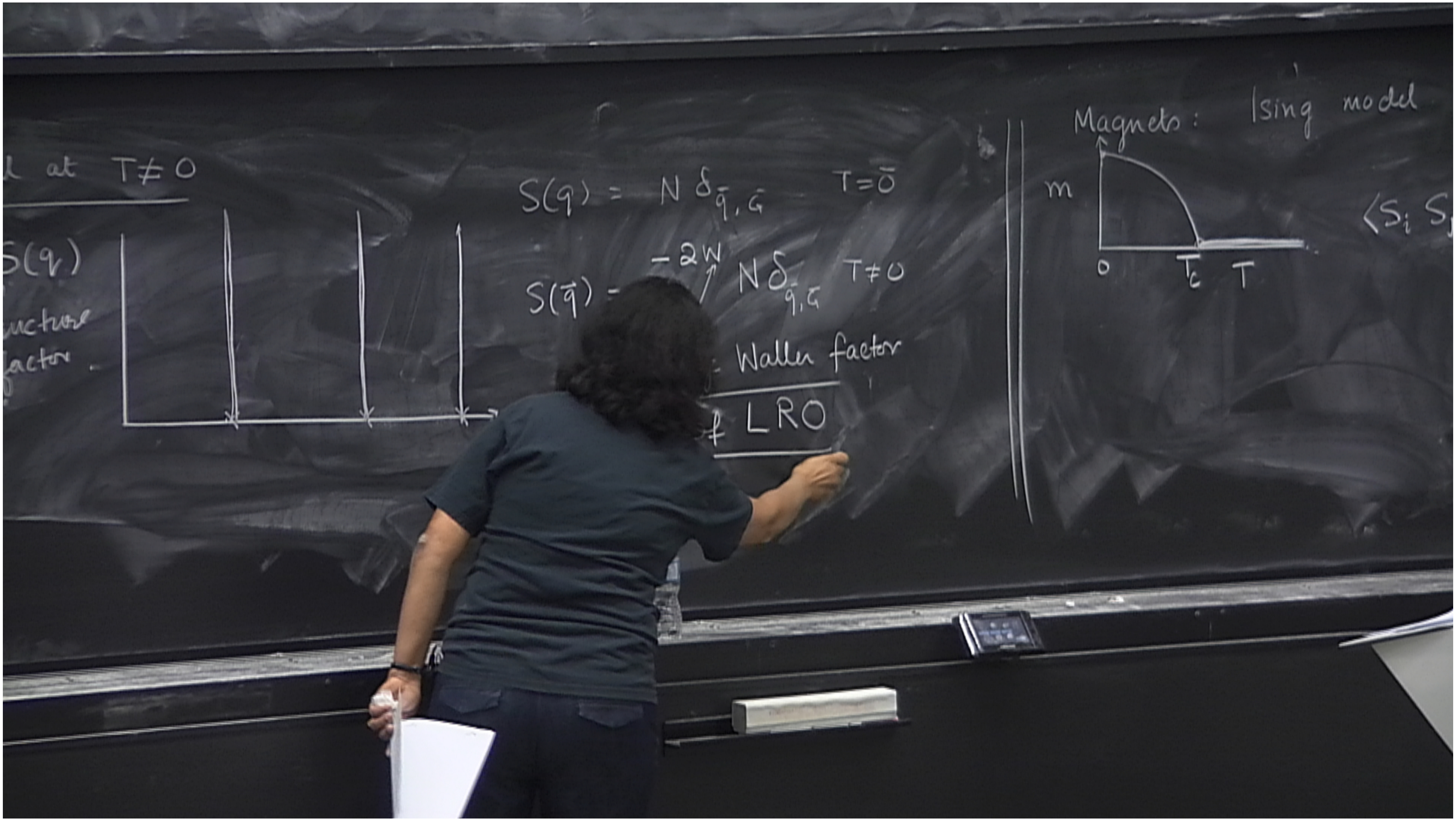


Magnets : Ising model
$m = \langle S_i \rangle / N$
$S(\bar{q}) = N\delta_{\bar{q},\bar{G}}$ T=0
$S(\bar{q}) = e^{-2W} N\delta_{\bar{q},\bar{G}}$ T≠0
Debye Waller factor
m
0
T_c
T
$\langle S_i S_j \rangle$
m^2
$|i-j|$

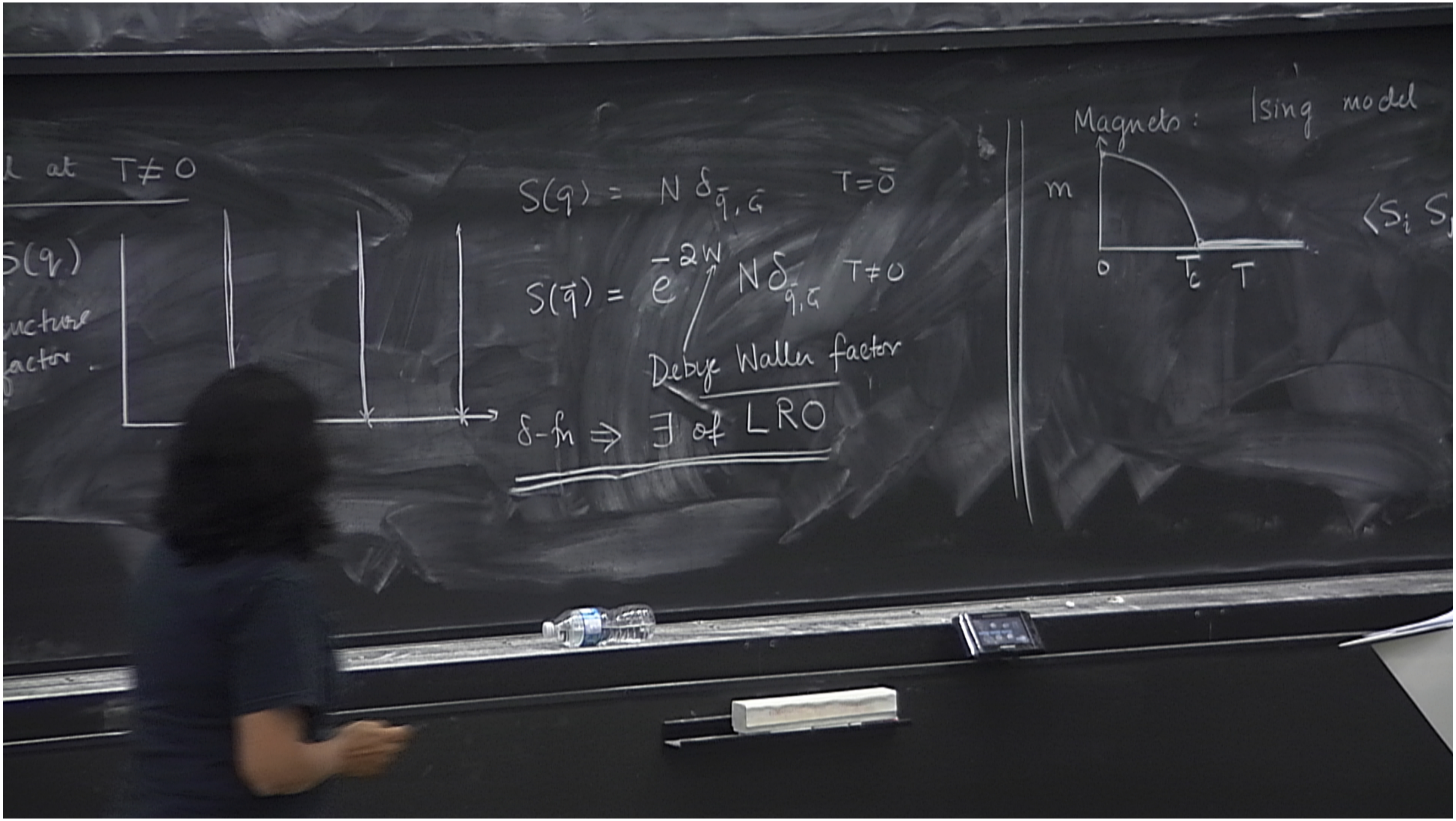

Magnets : Ising model
$S(q) = N\delta_{\bar{q},\bar{G}}$ $T=0$
$S(\vec{q}) = e^{-2W} N\delta_{\bar{q},\bar{G}}$ $T\neq 0$
Debye Waller factor
m
0
T_c
T
$m = \langle S_i \rangle / N$
$\langle S_i S_j \rangle$
m^2

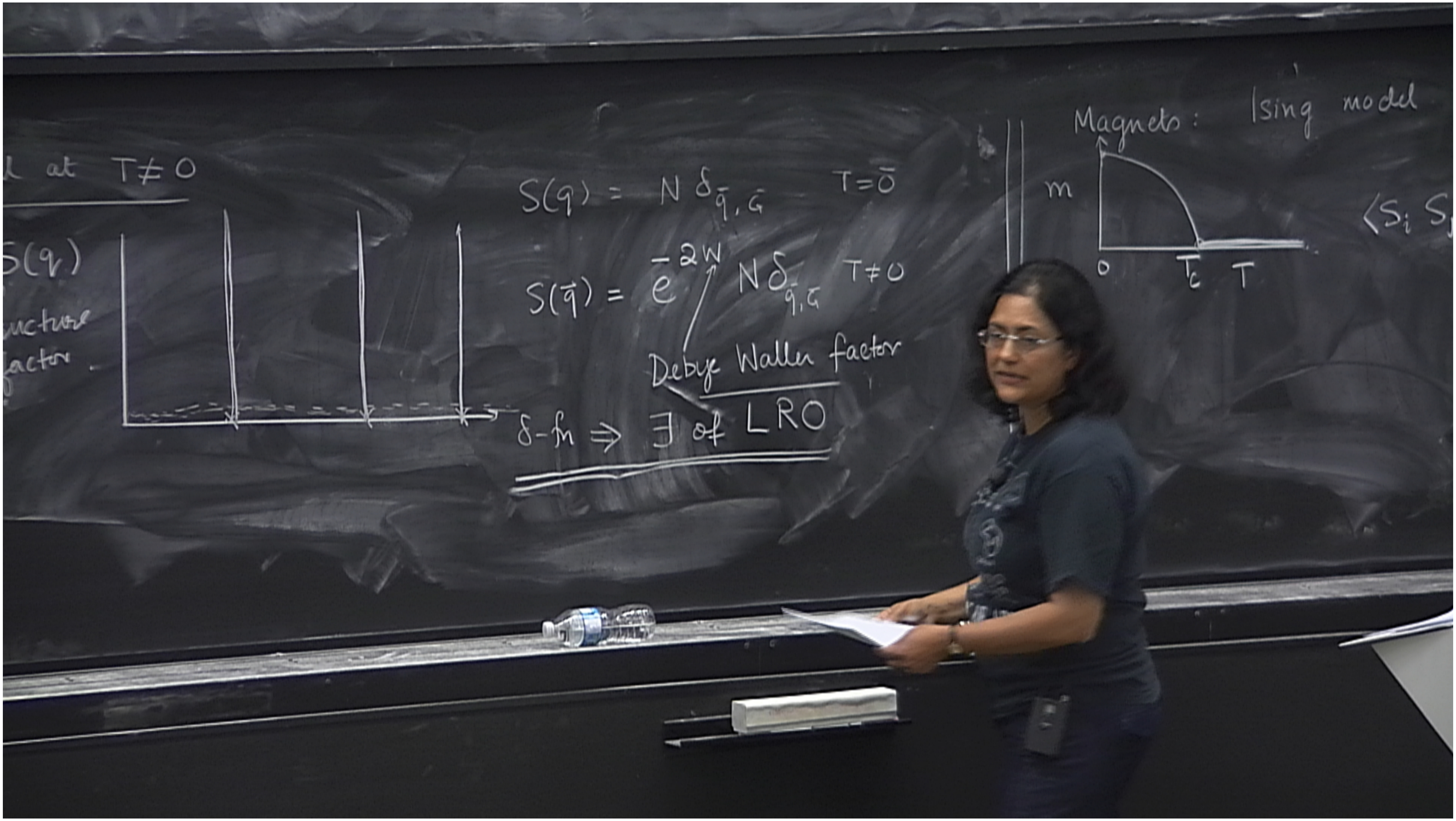

Magnets : Ising model
m = ⟨Sᵢ⟩/N
S(q) = N δ_{q,G} T=0
S(q) = e^{-2W} N δ_{q,G} T≠0
Debye Waller factor
m
0 Tc T
⟨Si Sj⟩
m²
T<Tc
T>Tc

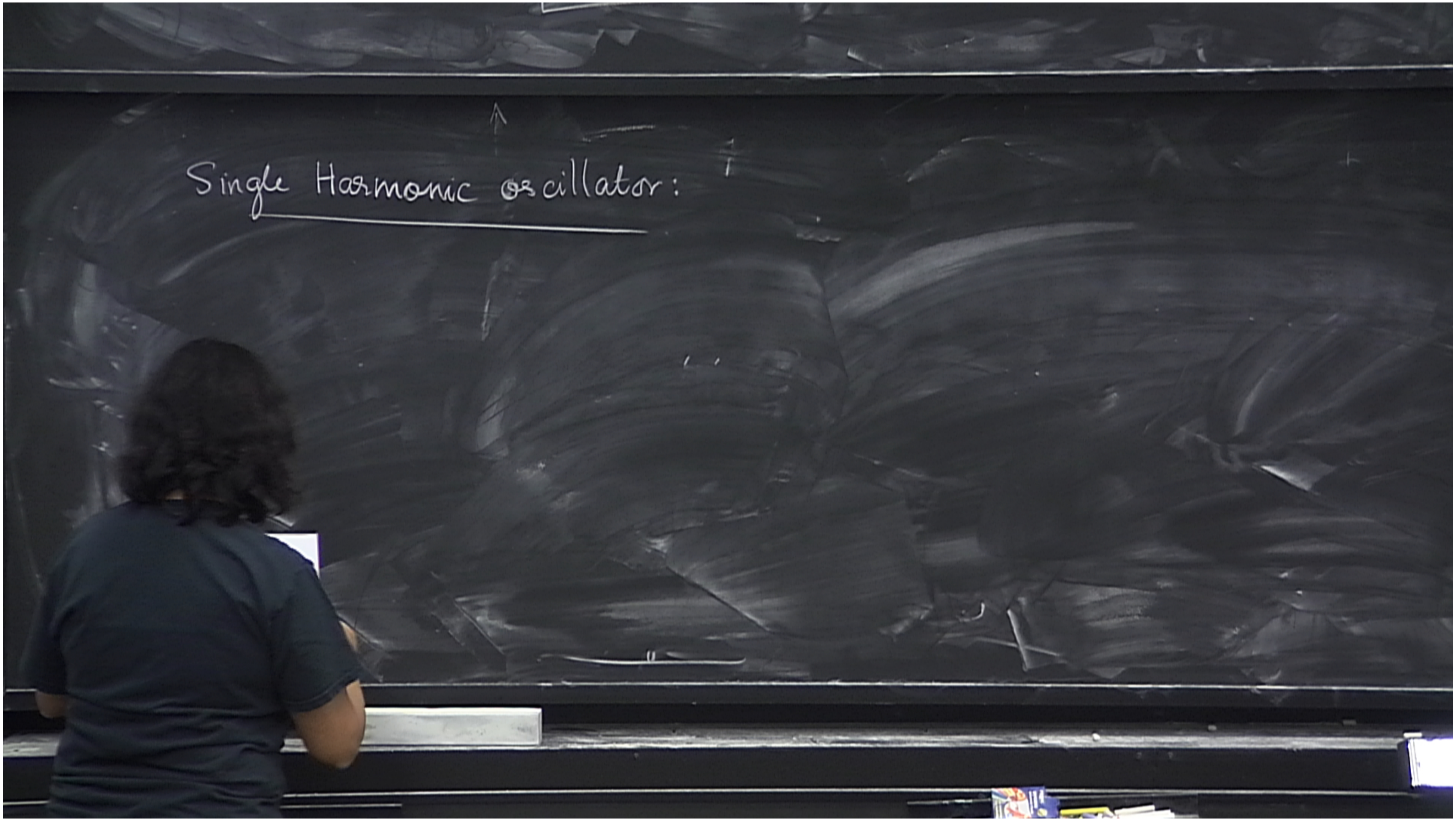

S(q) = N δ q,G T=0
S(q) = e^-2W N δ q,G T≠0
Debye Waller factor
Magnets: Ising model
m
0 Tc T
m = ⟨Si⟩/N
⟨Si Sj⟩
m²
T<Tc
T>Tc
|i-j|

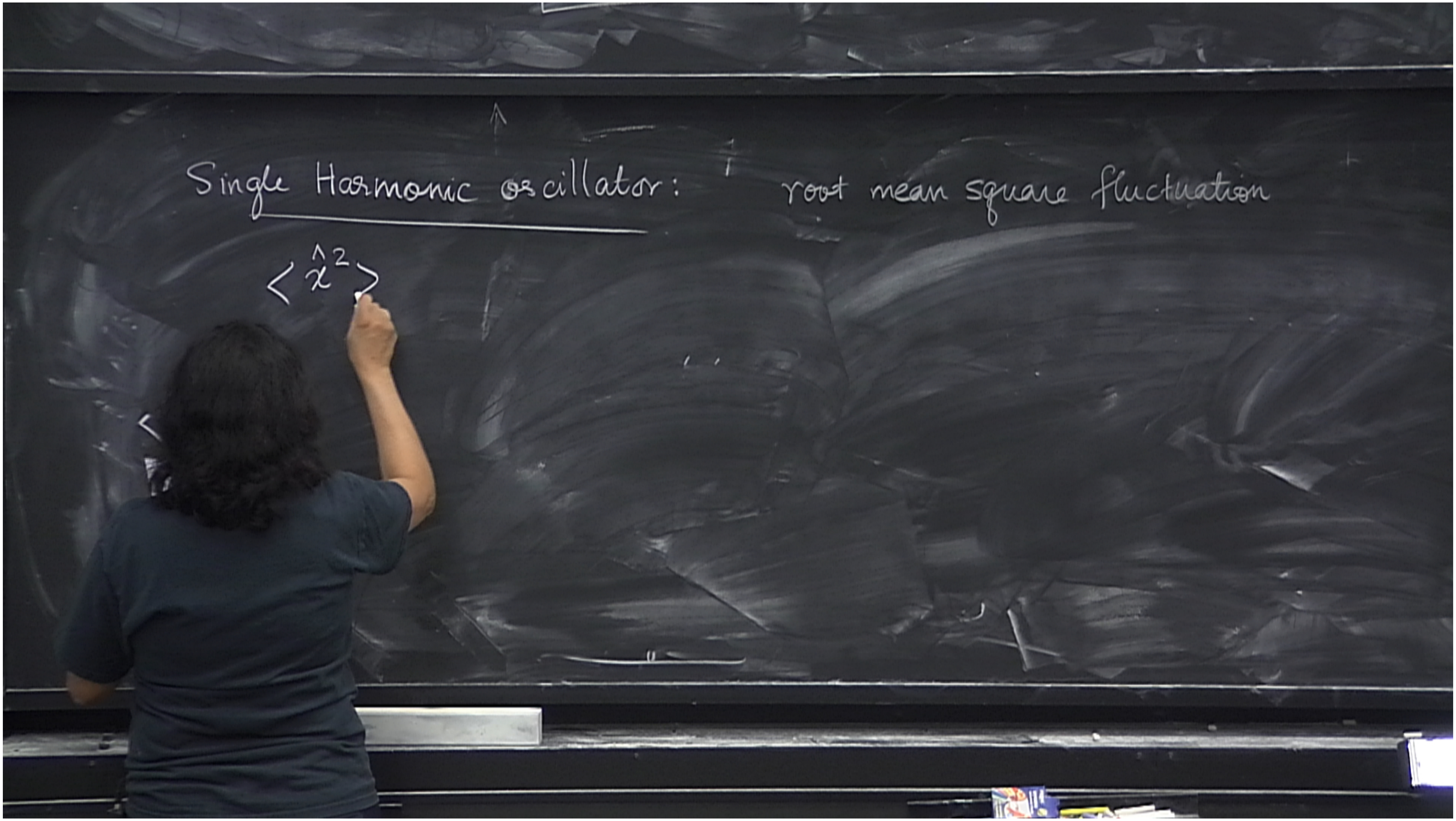


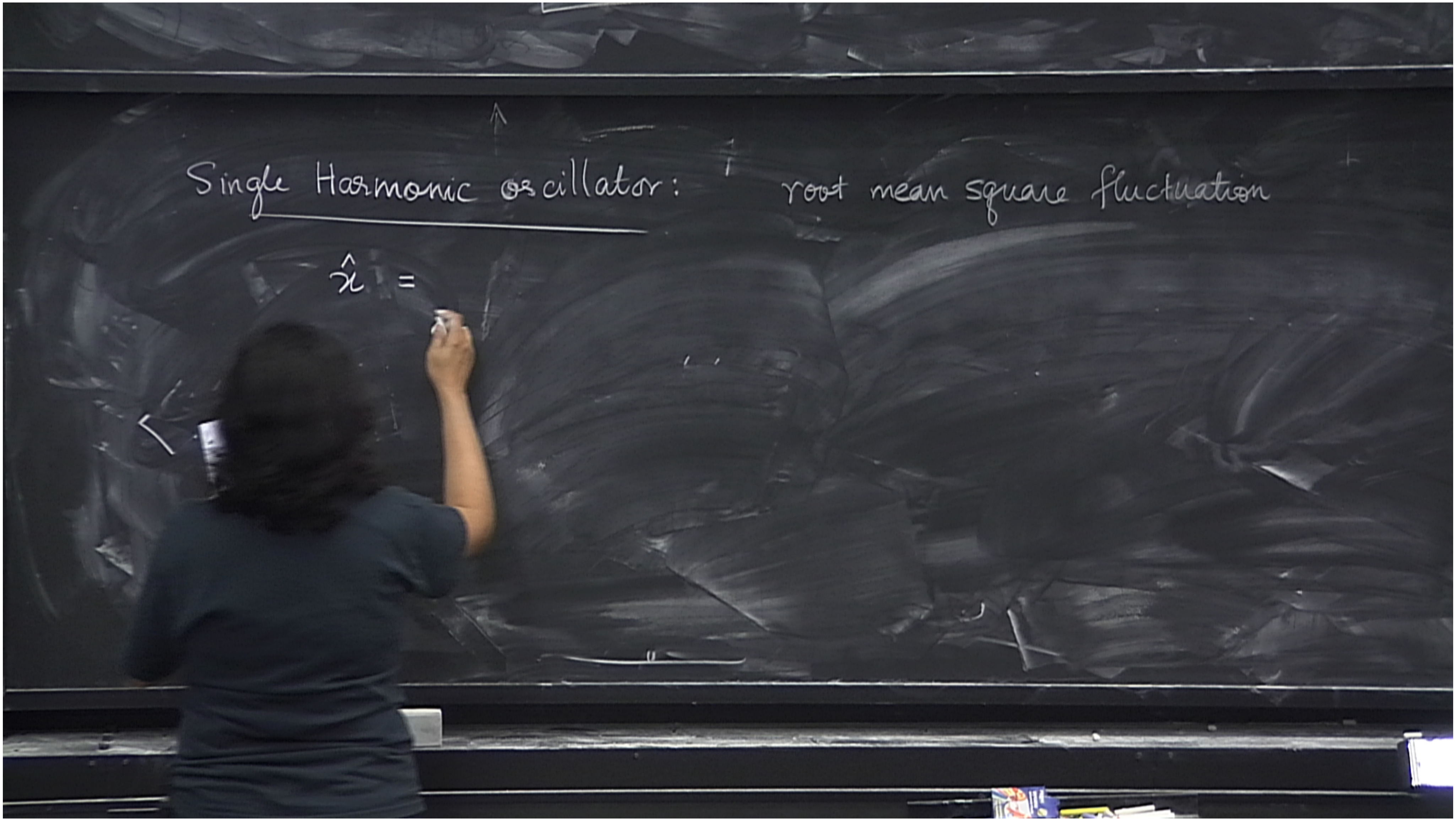
l at T≠0
S(q)
ucture
factor
S(q) = N δ q,G T=0
−2W
S(q) = N δ q,G T≠0
Waller factor
f LRO
Magnets : Ising model
m
0 Tc T

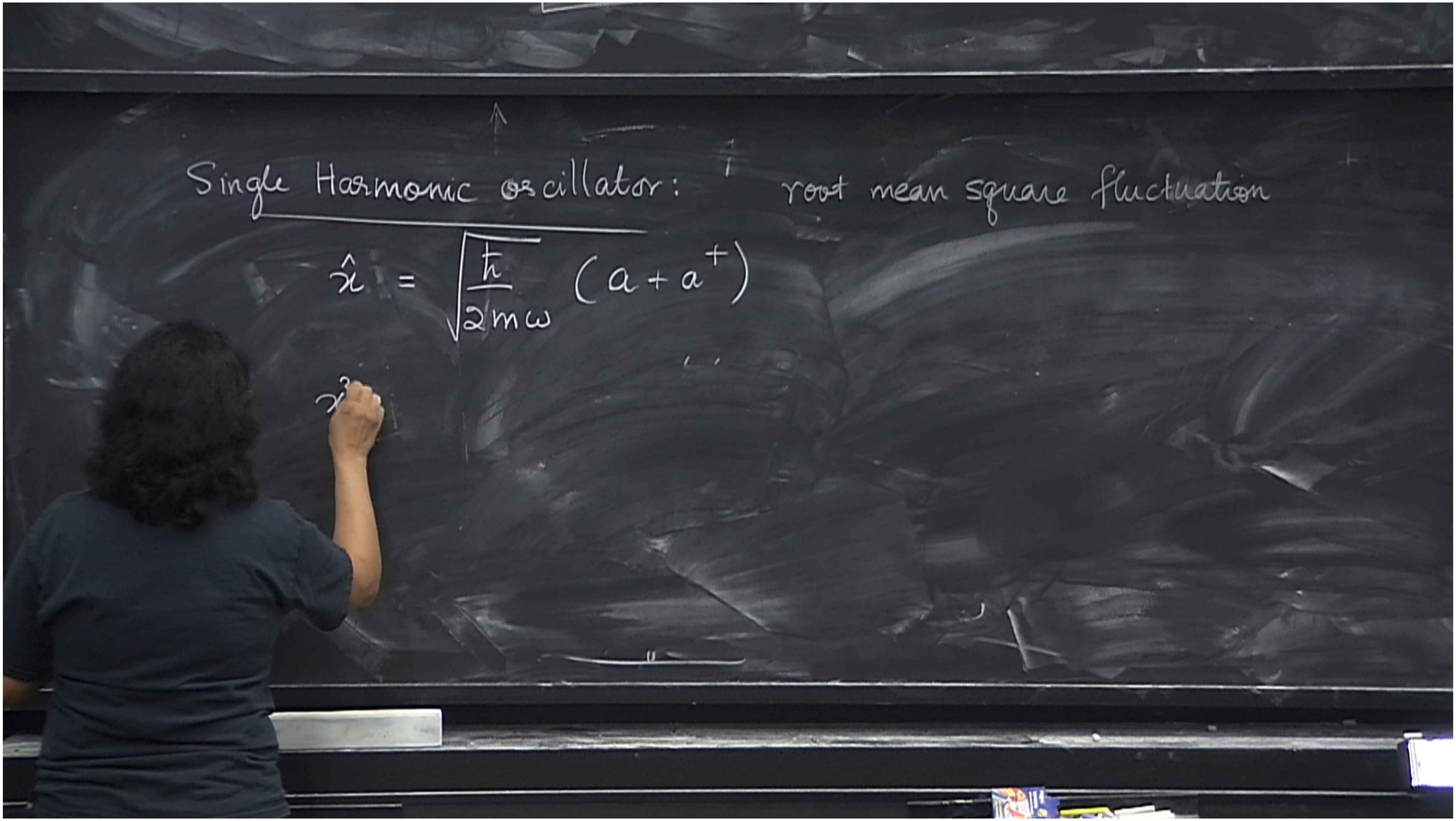

l at T≠0
S(q)
ucture
factor
S(q) = N δ q, G   T=0
S(q) = e^-2W N δ q, G   T≠0
Debye Waller factor
δ-fn ⇒ ∃ of LRO
Magnets: Ising model
m
0
Tc
T
⟨Si Sj


at T≠0
S(q)
structure
factor
S(q) = N δ q̄,Ḡ T=0
S(q̄) = e^{-2W} N δ q̄,Ḡ T≠0
Debye Waller factor
δ-fn ⇒ ∃ of LRO
Magnets: Ising model
m
0
Tc
T

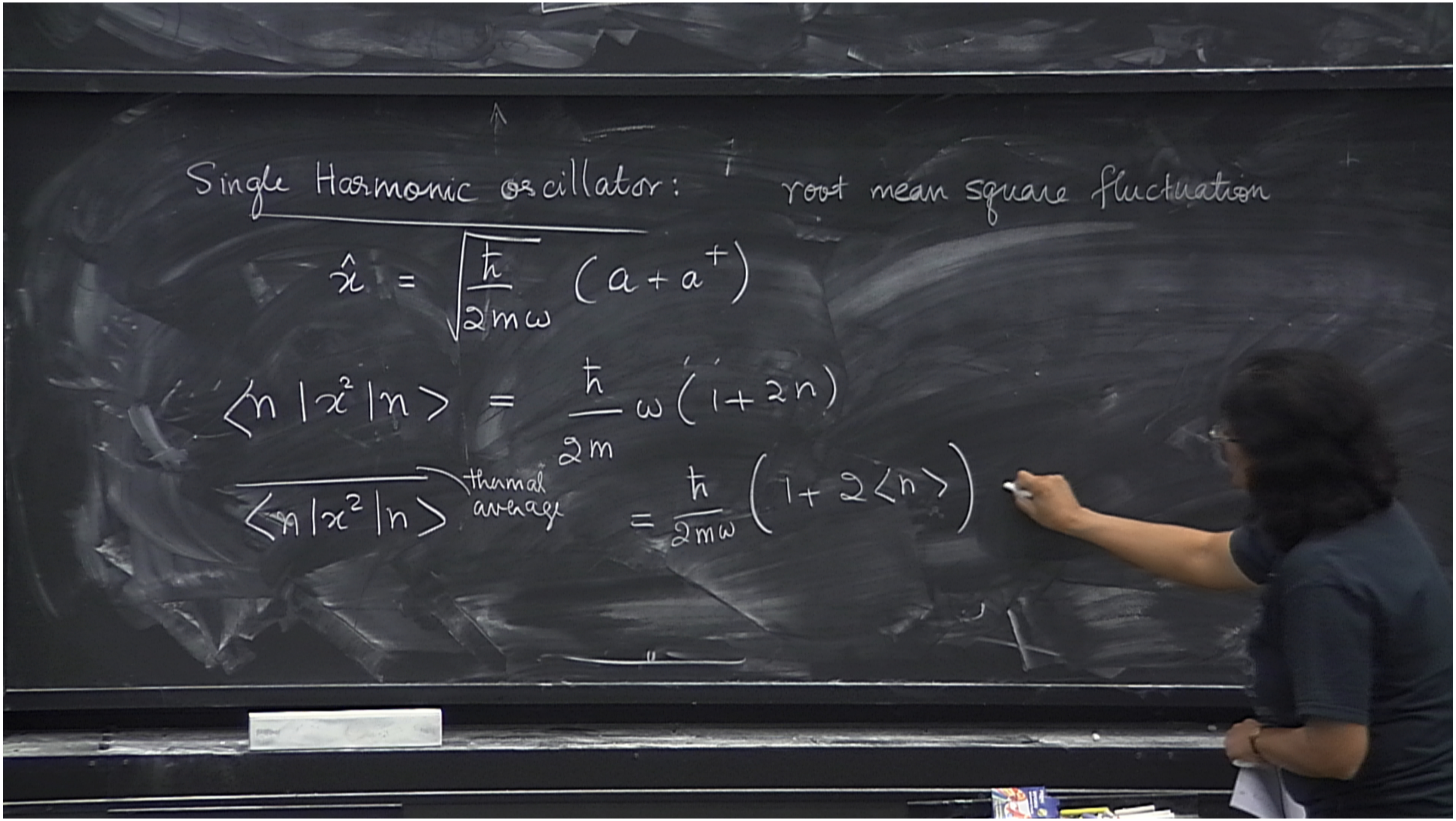

Single Harmonic oscillator:

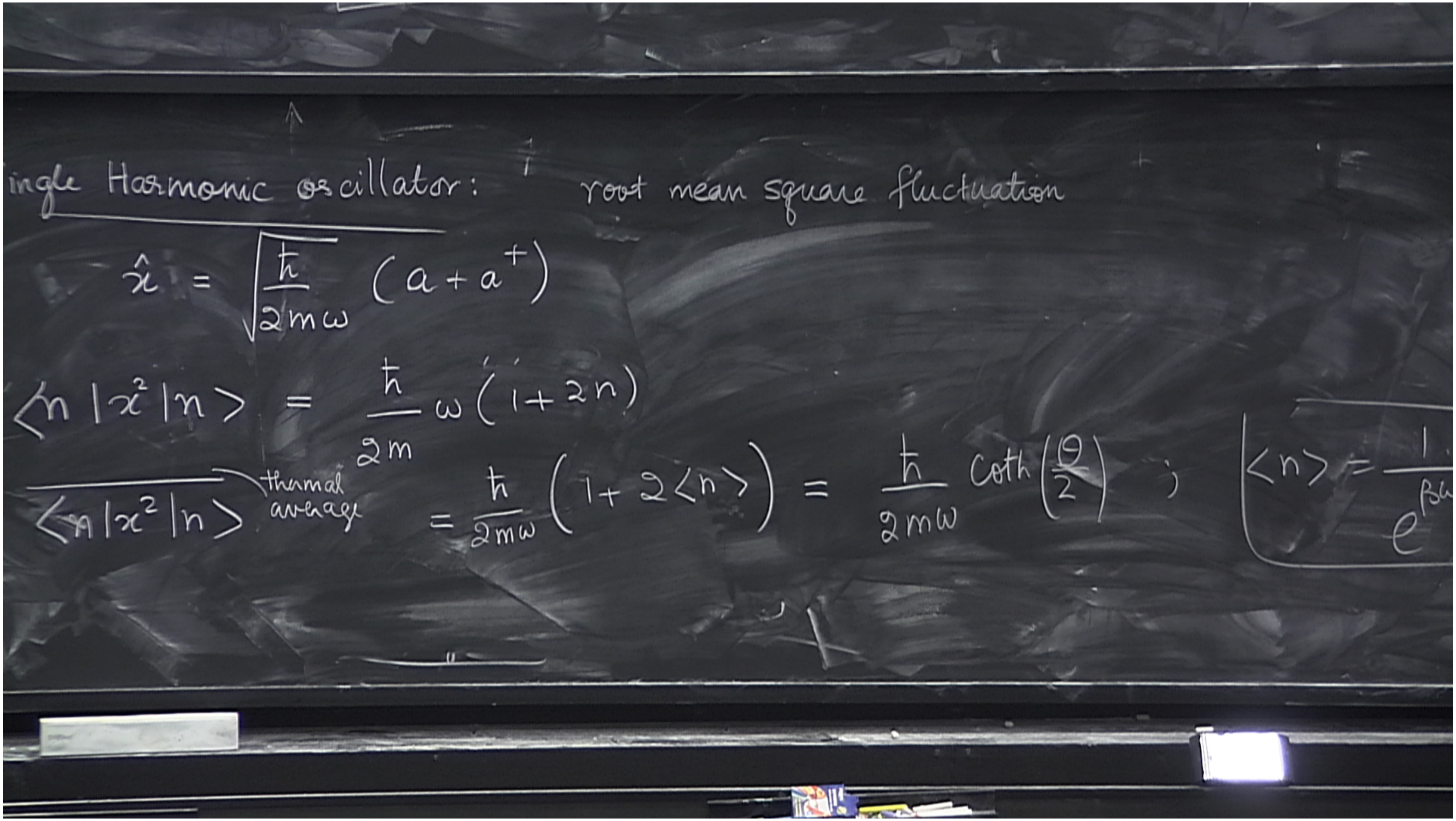

Single Harmonic oscillator:
root mean square fluctuation
$\langle \hat{x}^2 \rangle$

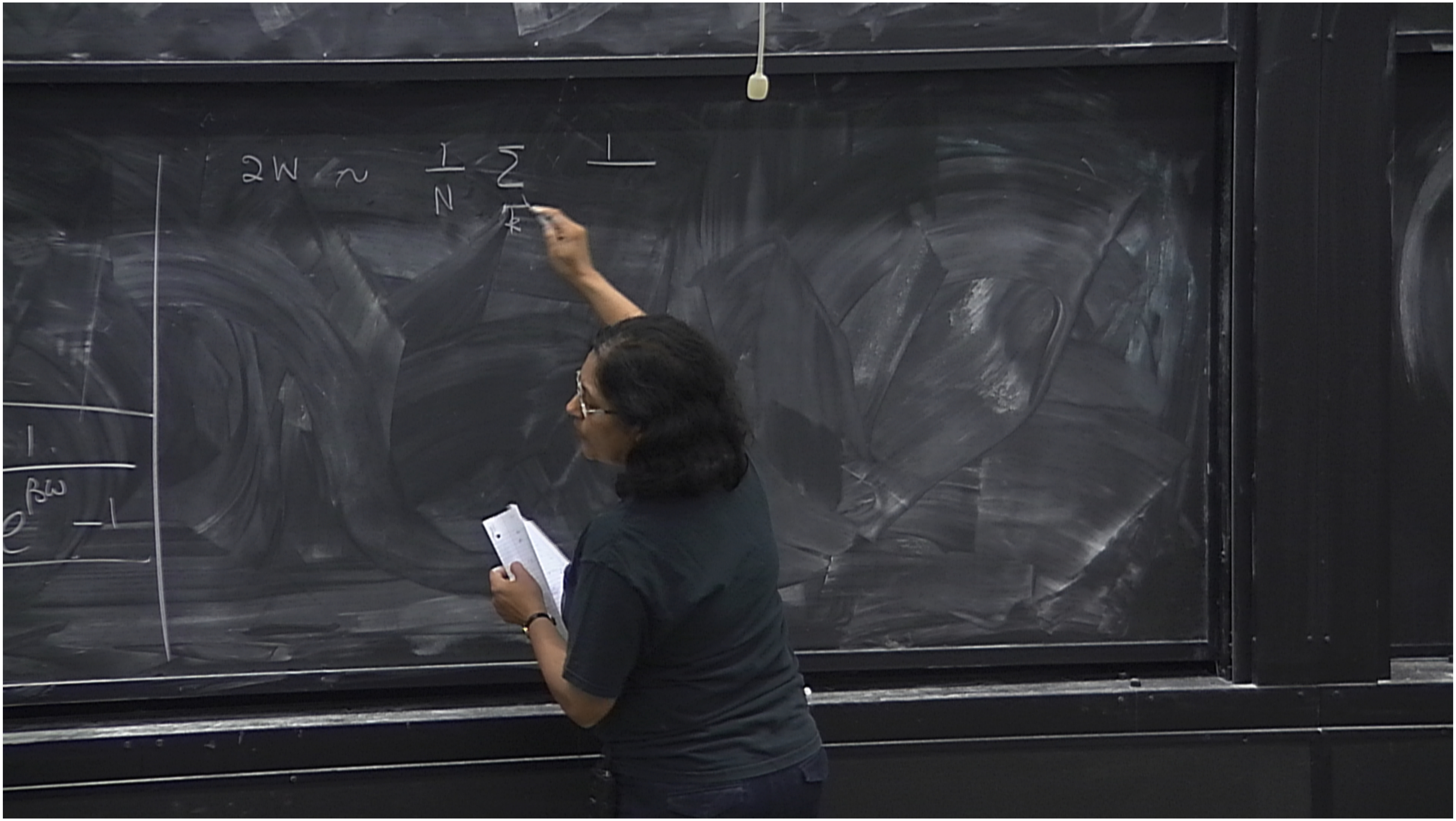

Single Harmonic oscillator:
root mean square fluctuation
$\hat{x} =$

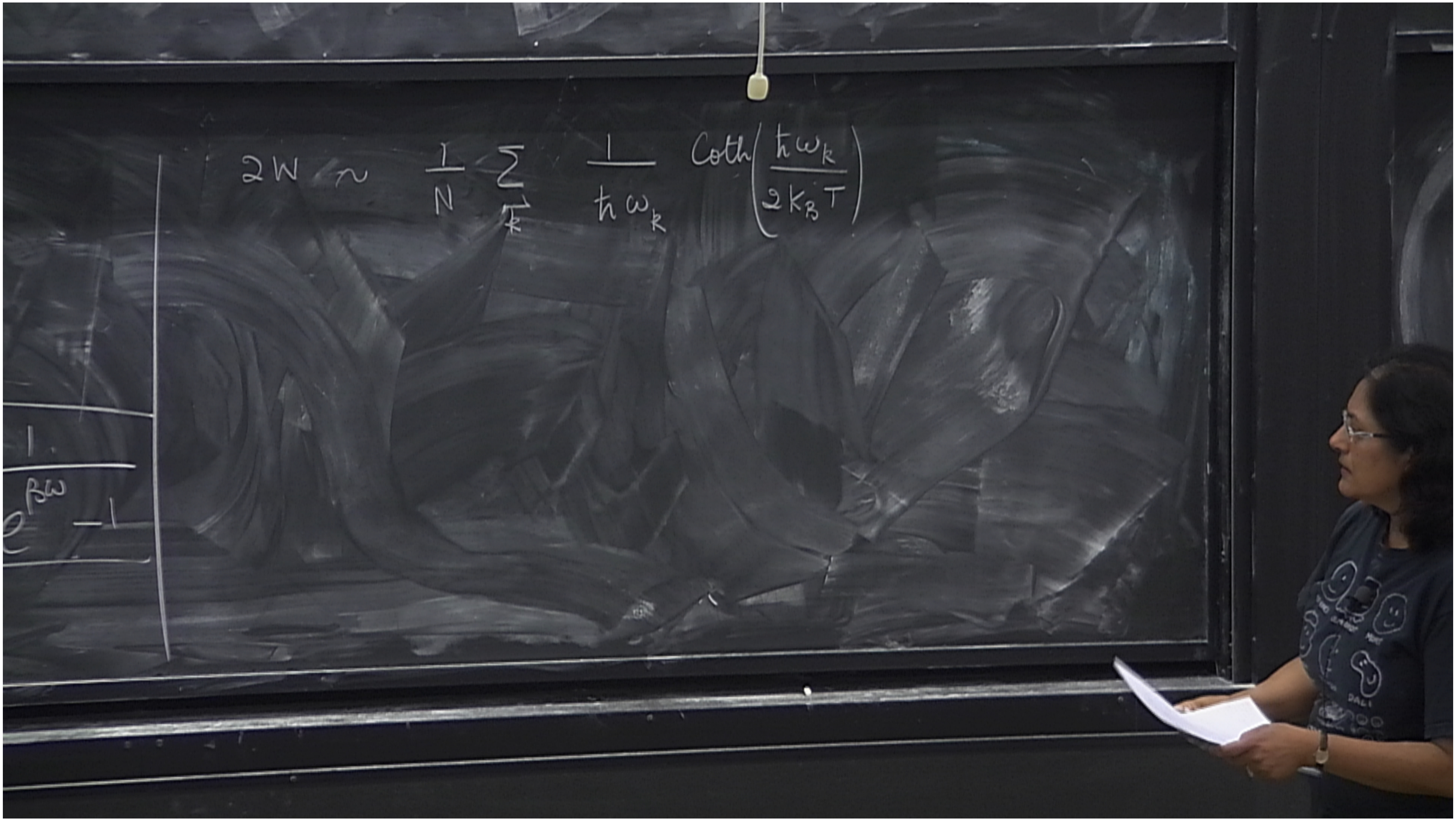


Single Harmonic oscillator:
root mean square fluctuation
$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} \, (a + a^{+})$


Single Harmonic oscillator:
root mean square fluctuation
$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^{+})$
$\langle n | x^2 | n \rangle = \frac{\hbar}{2m}\,\omega\,(1 + 2n)$
$\overline{\langle n | x^2 | n \rangle}$
thermal average
$= \frac{\hbar}{2m\omega}\left(1 + 2\langle n \rangle\right)$


ingle Harmonic oscillator:
root mean square fluctuation
$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^{+})$
$\langle n | x^2 | n \rangle = \frac{\hbar}{2m}\omega(1 + 2n)$
$\overline{\langle n | x^2 | n \rangle}$ thermal average
$= \frac{\hbar}{2m\omega}\left(1 + 2\langle n \rangle\right) = \frac{\hbar}{2m\omega}\coth\left(\frac{\theta}{2}\right)$;
$\langle n \rangle = \frac{1}{e^{\beta\hbar\omega}}$


2W ~ 1/N Σ_k 1/
1/(e^βω − 1)


$2W \sim \frac{1}{N} \sum_{\vec{k}} \frac{1}{\hbar \omega_k} \coth\left(\frac{\hbar \omega_k}{2 k_B T}\right)$
$\frac{1}{e^{\beta\omega} - 1}$

Single Harmonic oscillator: root mean square fluctuation

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^{+})$$

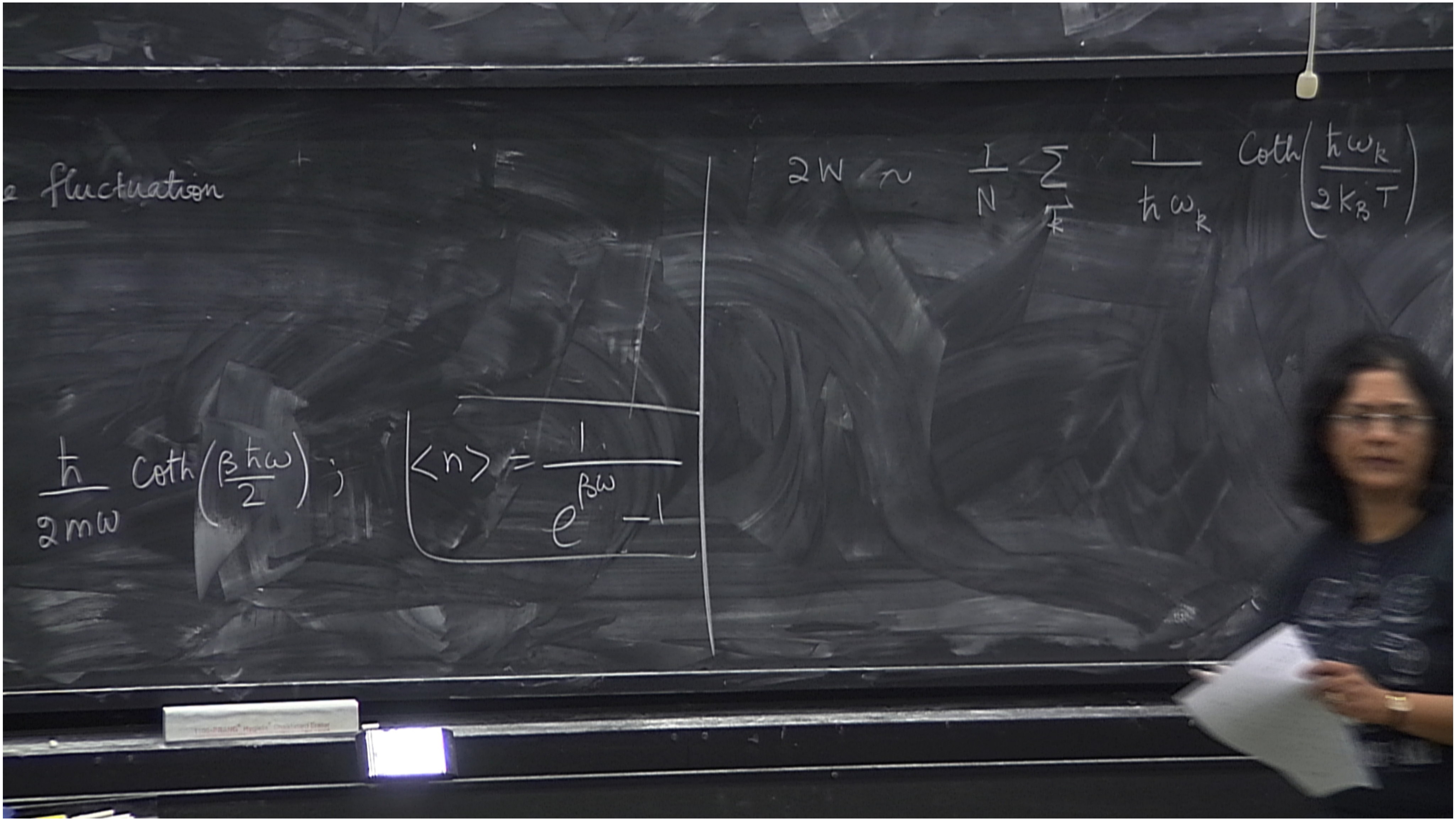
$$\langle n | x^2 | n \rangle = \frac{\hbar}{2m\omega}(1 + 2n)$$

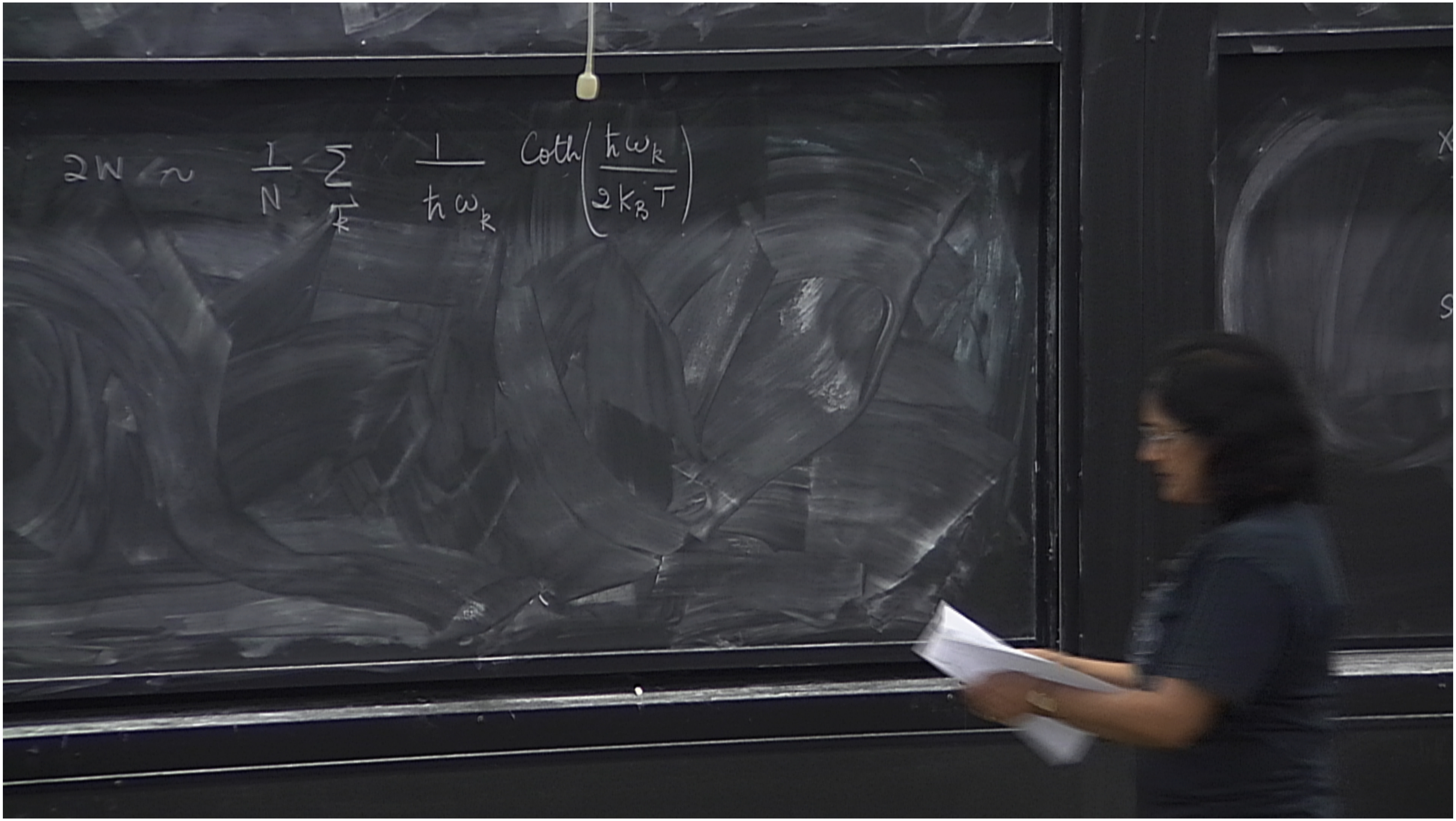
$$\overline{\langle n | x^2 | n \rangle}^{\text{thermal average}} = \frac{\hbar}{2m\omega}\left(1 + 2\langle n \rangle\right) = \frac{\hbar}{2m\omega}\coth\left(\frac{\theta}{2}\right) \; ; \quad \langle n \rangle$$

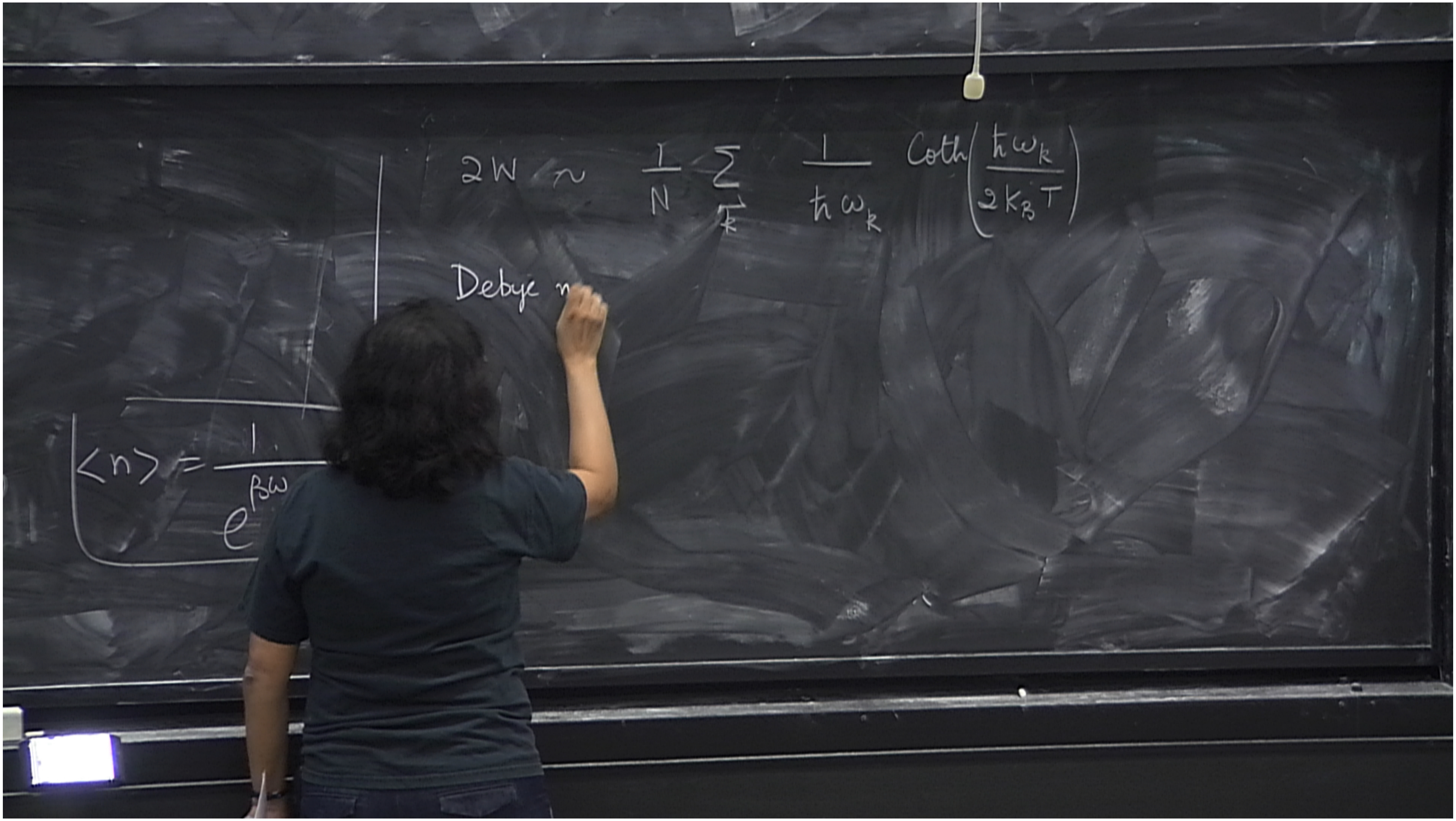
e fluctuation

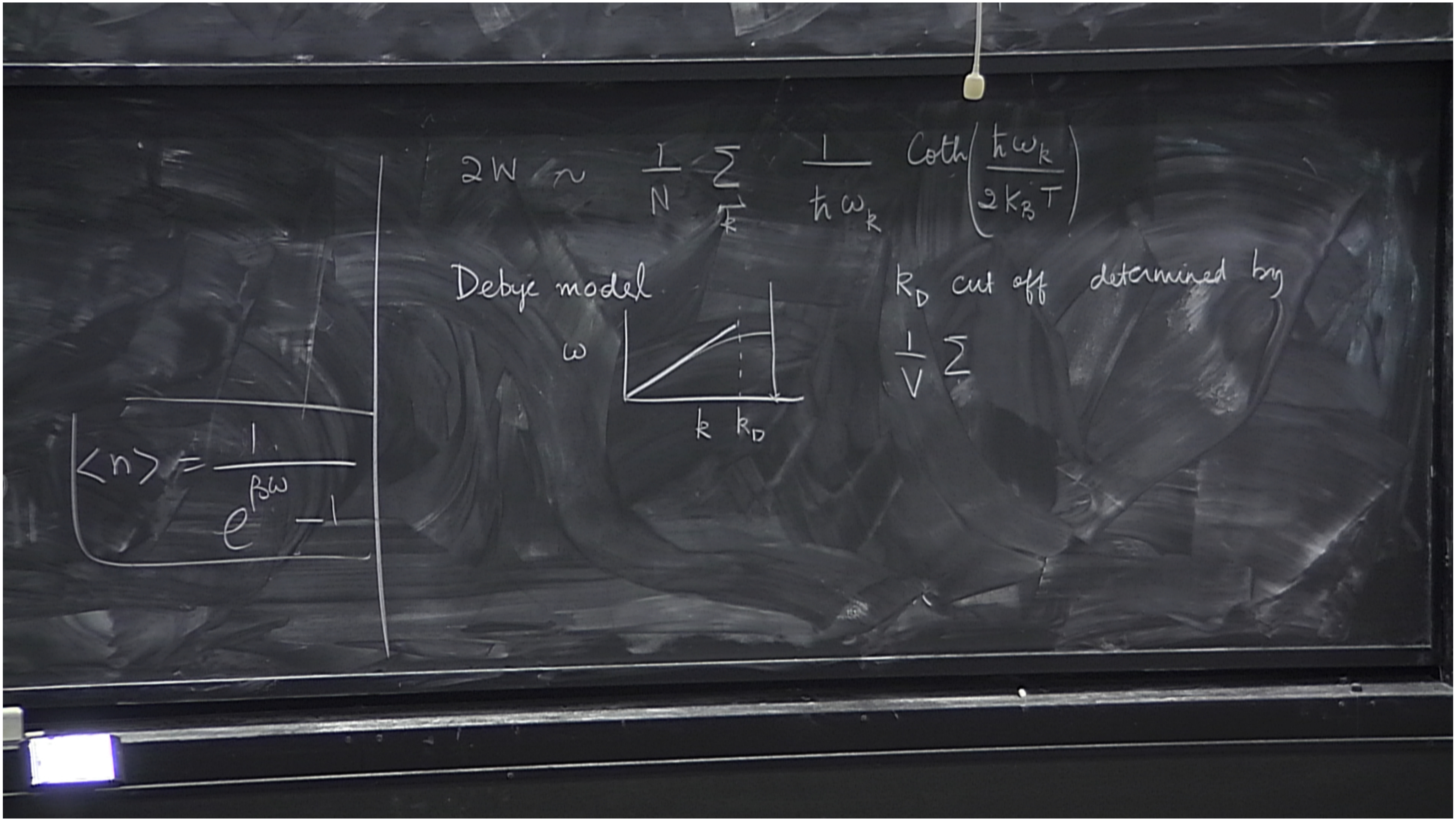
$$\frac{\hbar}{2m\omega}\coth\left(\frac{\quad}{2}\right) \; ; \quad \langle n\rangle = \frac{1}{e^{\beta\omega}-1}$$

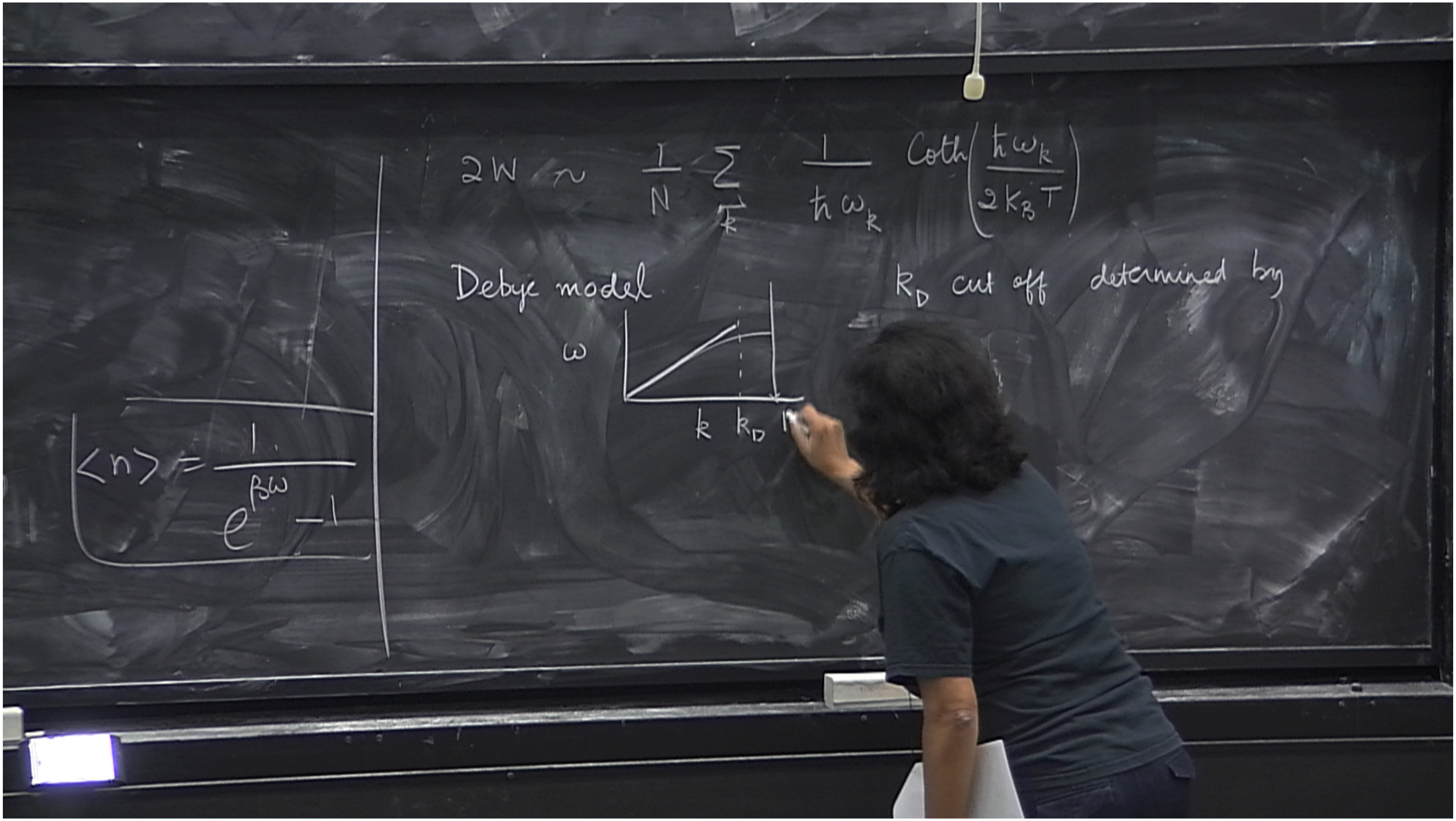
$$2W \sim \frac{1}{N}\sum_k \frac{1}{\hbar\omega_k}\coth\left(\frac{\hbar\omega_k}{2k_B T}\right)$$

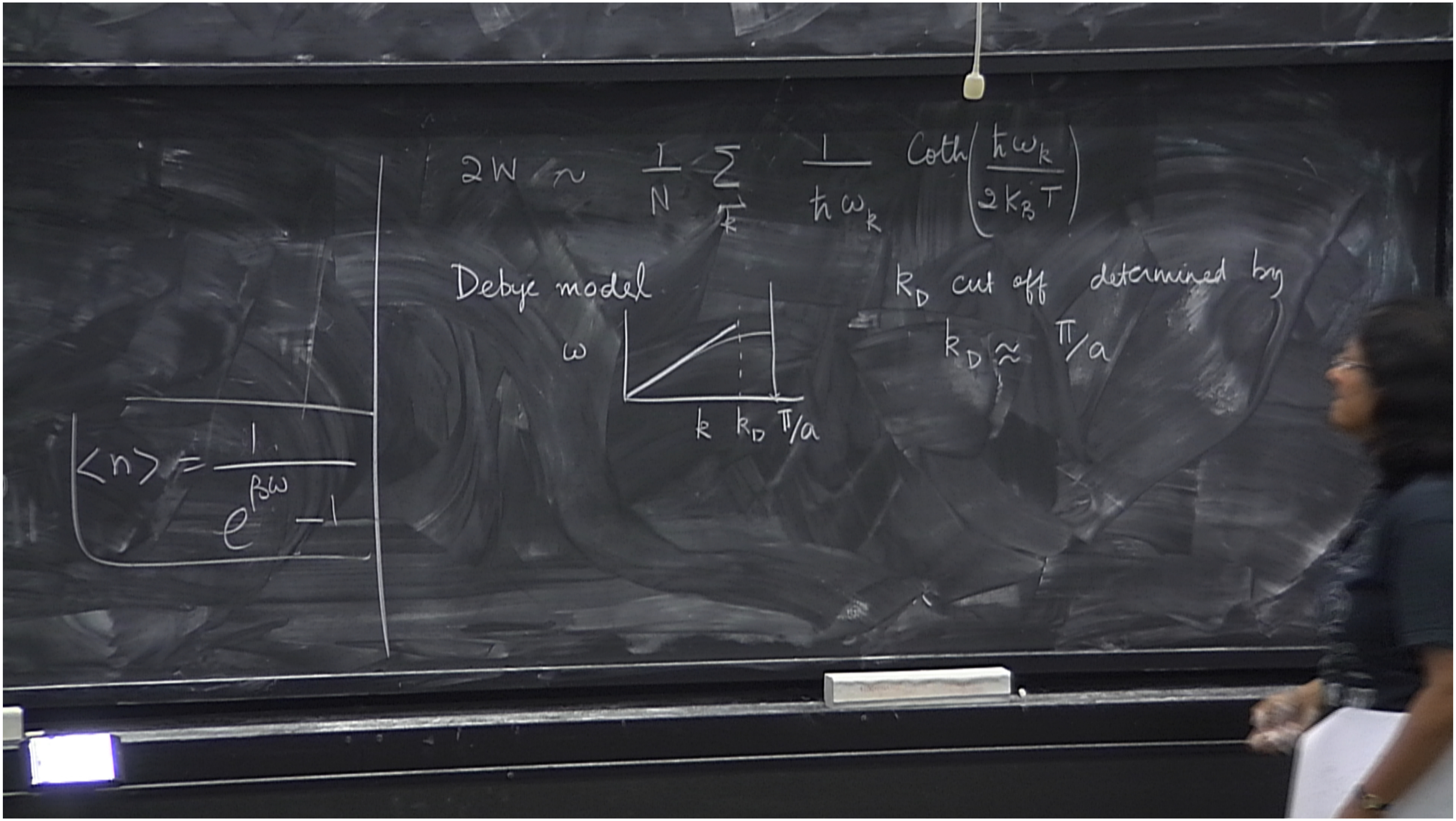

fluctuation
$2W \sim \frac{1}{N} \sum_k \frac{1}{\hbar \omega_k} \mathrm{Coth}\left(\frac{\hbar \omega_k}{2 k_B T}\right)$
$\frac{\hbar}{2m\omega} \mathrm{Coth}\left(\frac{\beta \hbar \omega}{2}\right)$; $\langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$


$2W \sim \frac{1}{N} \sum_k \frac{1}{\hbar \omega_k} \mathrm{Coth}\left(\frac{\hbar \omega_k}{2 k_B T}\right)$


2W ~ 1/N Σ_k 1/ħω_k Coth(ħω_k / 2k_B T)
Debye


$2W \sim \frac{1}{N} \sum_k \frac{1}{\hbar\omega_k} \mathrm{Coth}\left(\frac{\hbar\omega_k}{2k_B T}\right)$
Debye model
ω
k k_D
k_D cut off determined by
$\frac{1}{V} \sum$
$\langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$

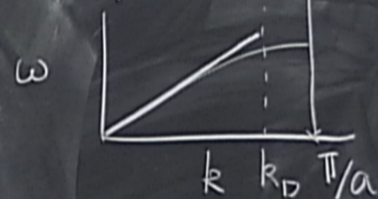

$2W \sim \frac{1}{N} \sum_k \frac{1}{\hbar \omega_k} \mathrm{Coth}\left(\frac{\hbar \omega_k}{2 k_B T}\right)$
Debye model
k_D cut off determined by
ω
k k_D
$\langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$


$2W \sim \frac{1}{N} \sum_k \frac{1}{\hbar \omega_k} \mathrm{Coth}\left(\frac{\hbar \omega_k}{2 k_B T}\right)$
Debye model
ω
k k_D π/a
k_D cut off determined by
$k_D \approx \pi/a$
$\langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$

$$\langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$$

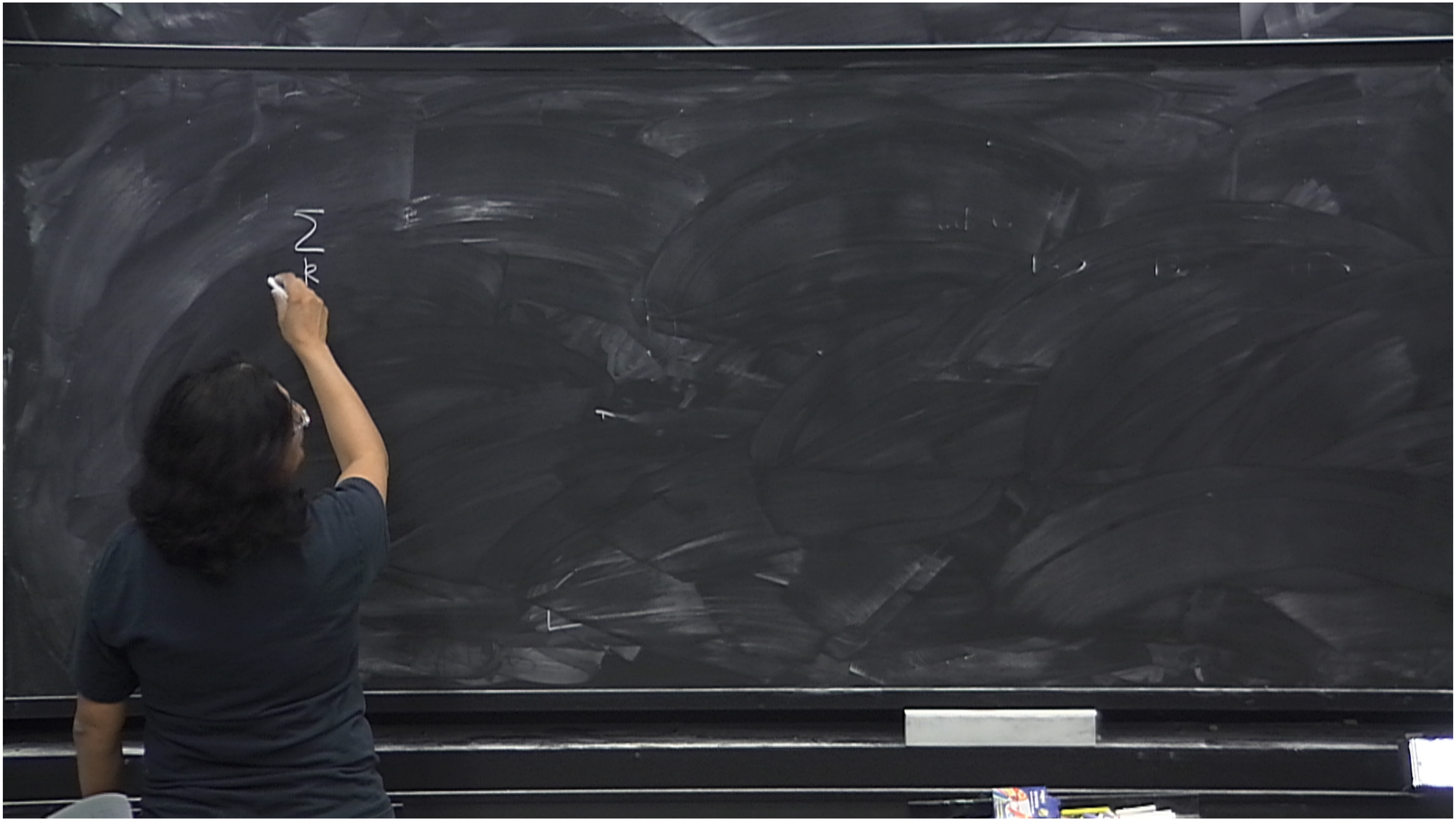
$$2W \sim \frac{1}{N} \sum_k \frac{1}{\hbar\omega_k} \coth\left(\frac{\hbar\omega_k}{2k_B T}\right)$$

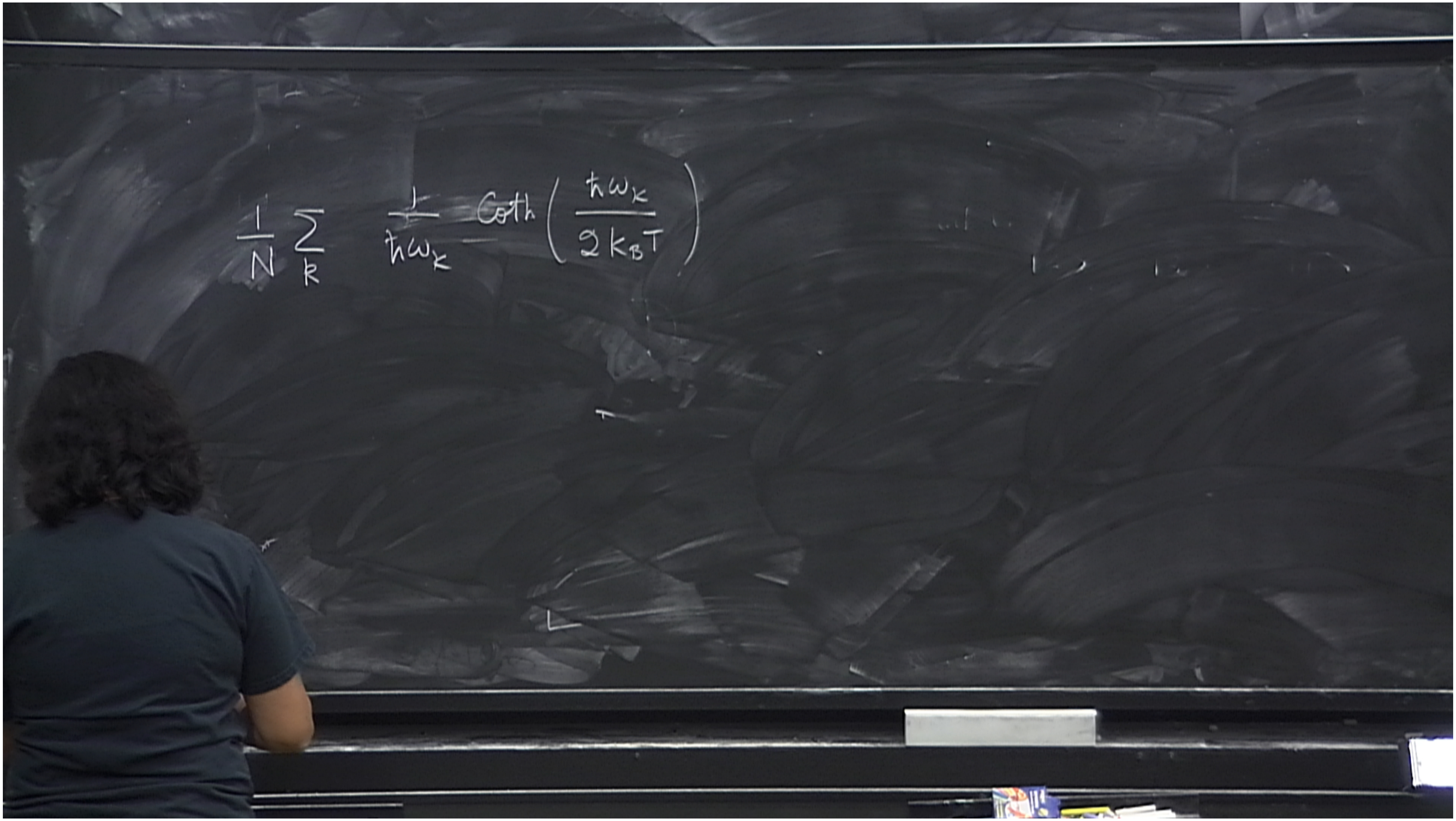
Debye model

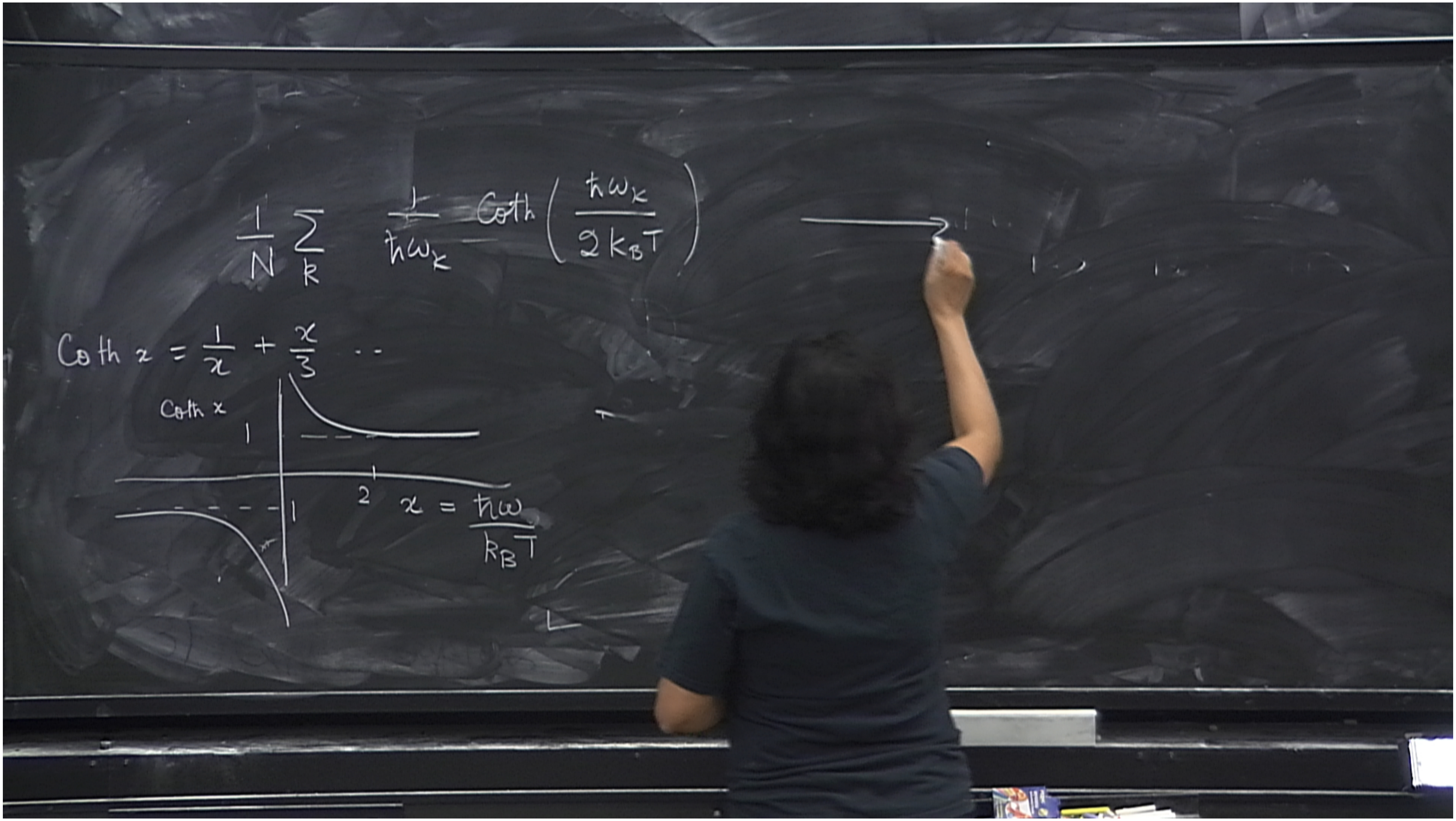


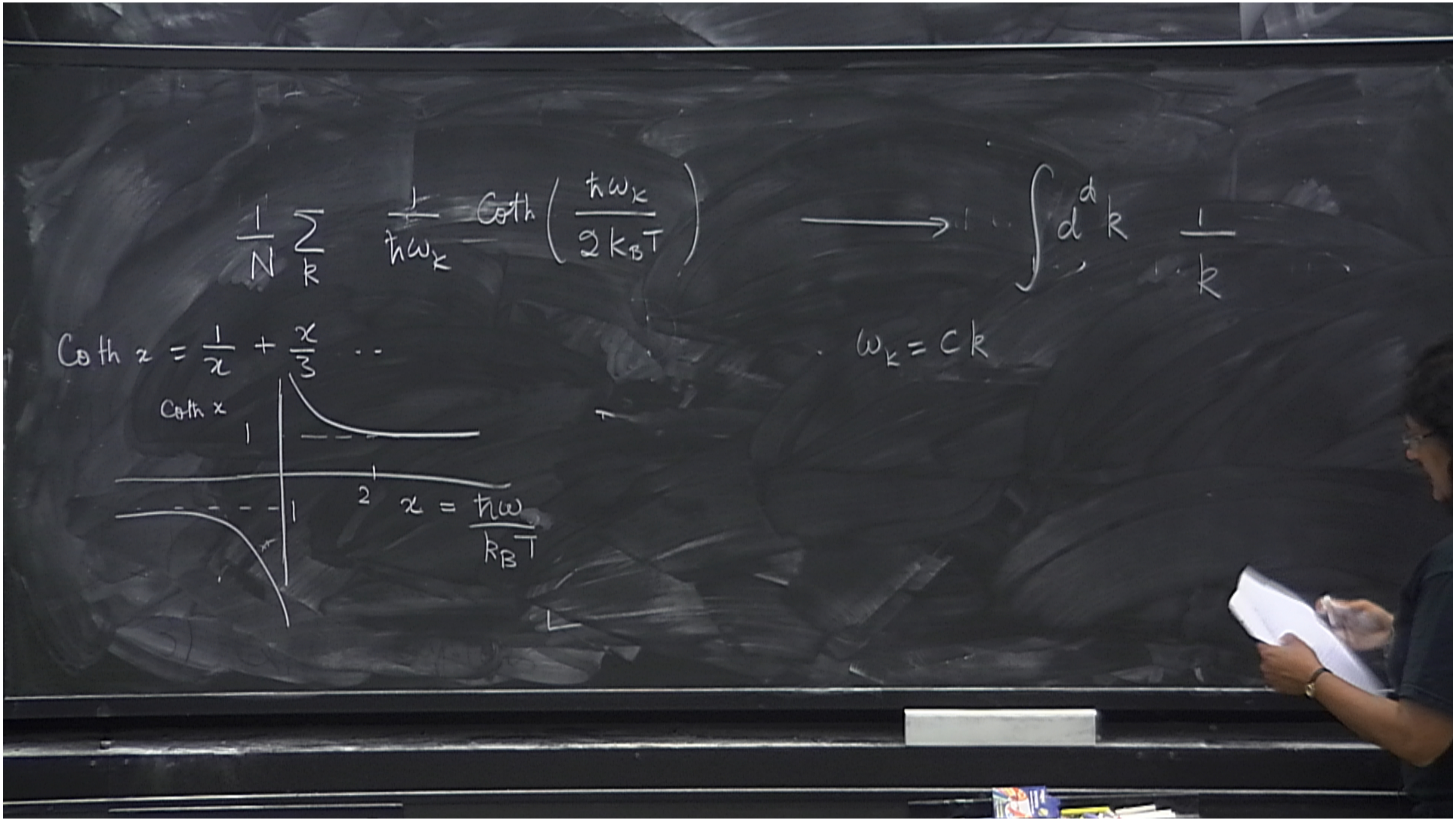
k_D cut off determined by adding up all modes

$$k_D \approx \pi/a$$

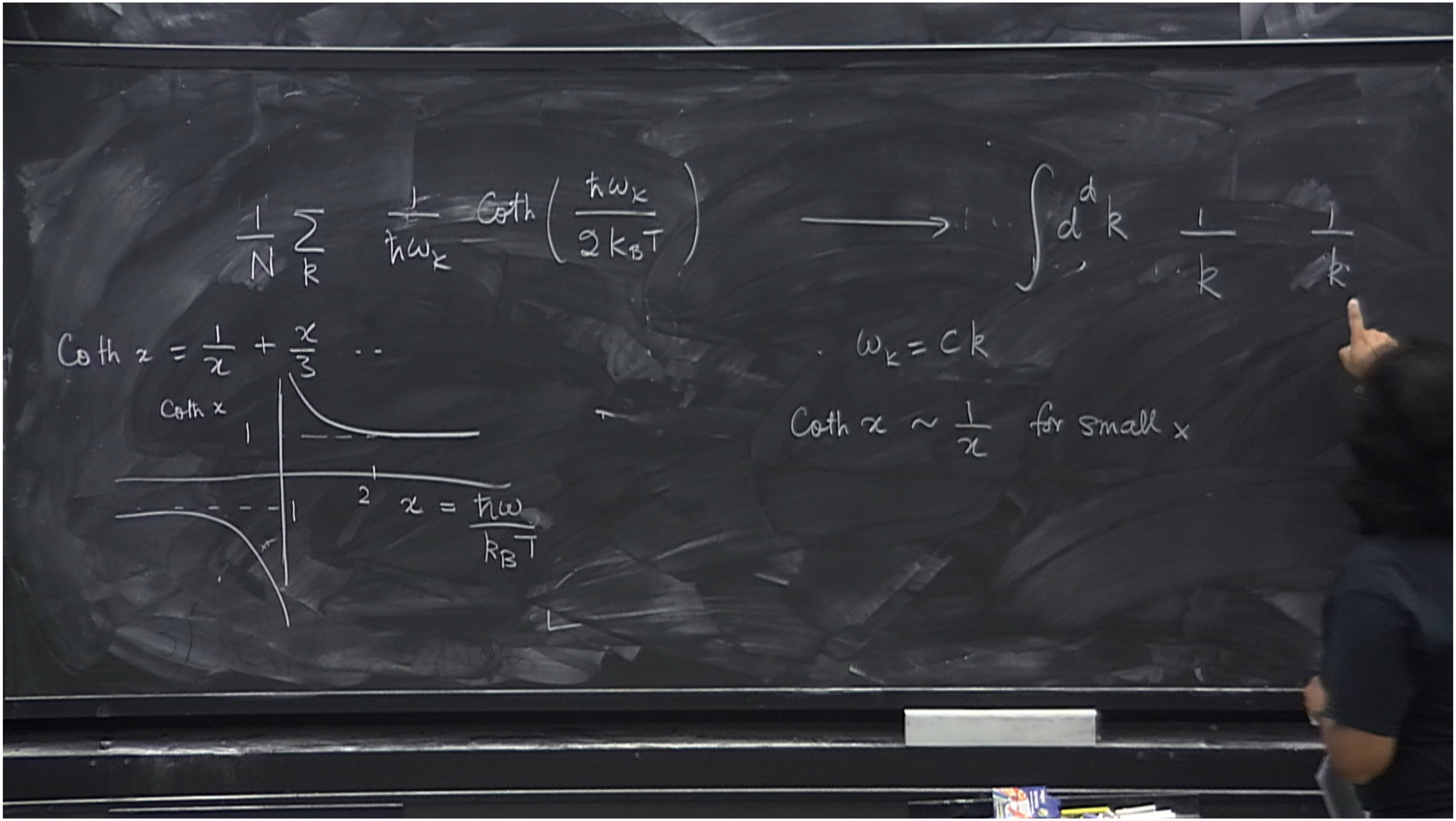


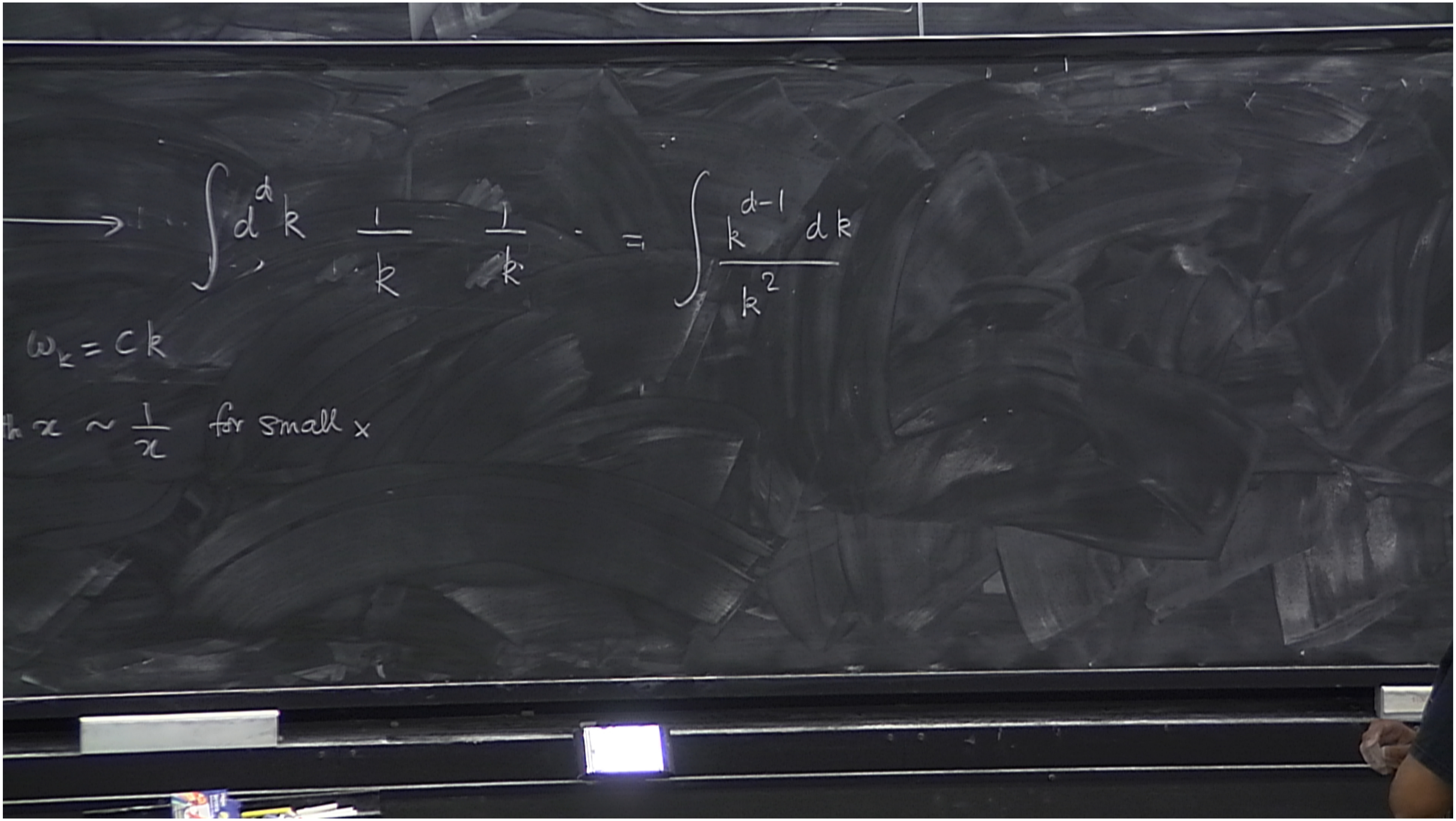

$\frac{1}{N}\sum_{k} \frac{1}{\hbar\omega_k} \coth\left(\frac{\hbar\omega_k}{2k_BT}\right)$

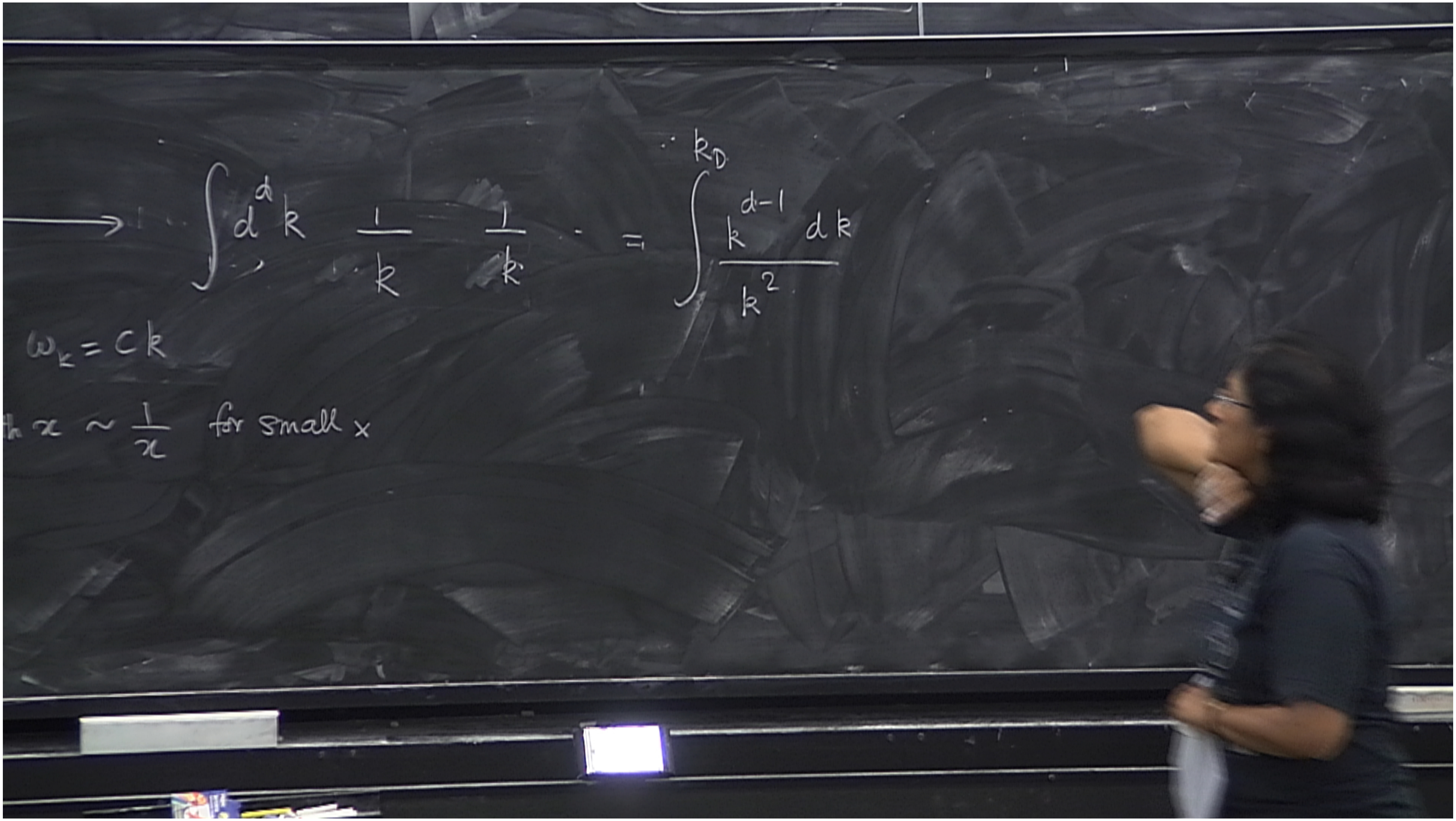

1/N Σ_k 1/ħω_k Coth(ħω_k / 2k_B T)
Coth x = 1/x + x/3 ..
Coth x
1
2
x = ħω / k_B T

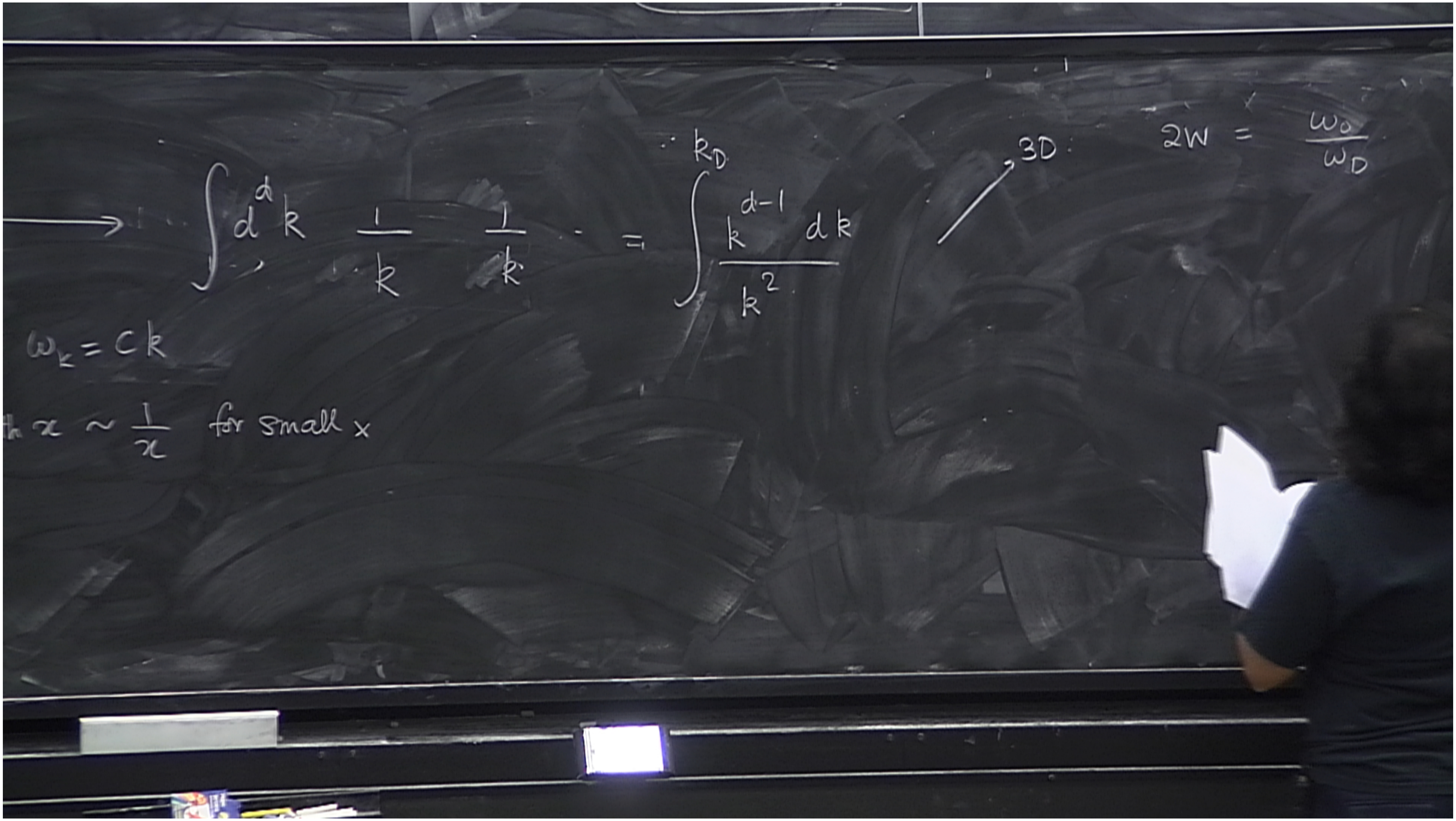


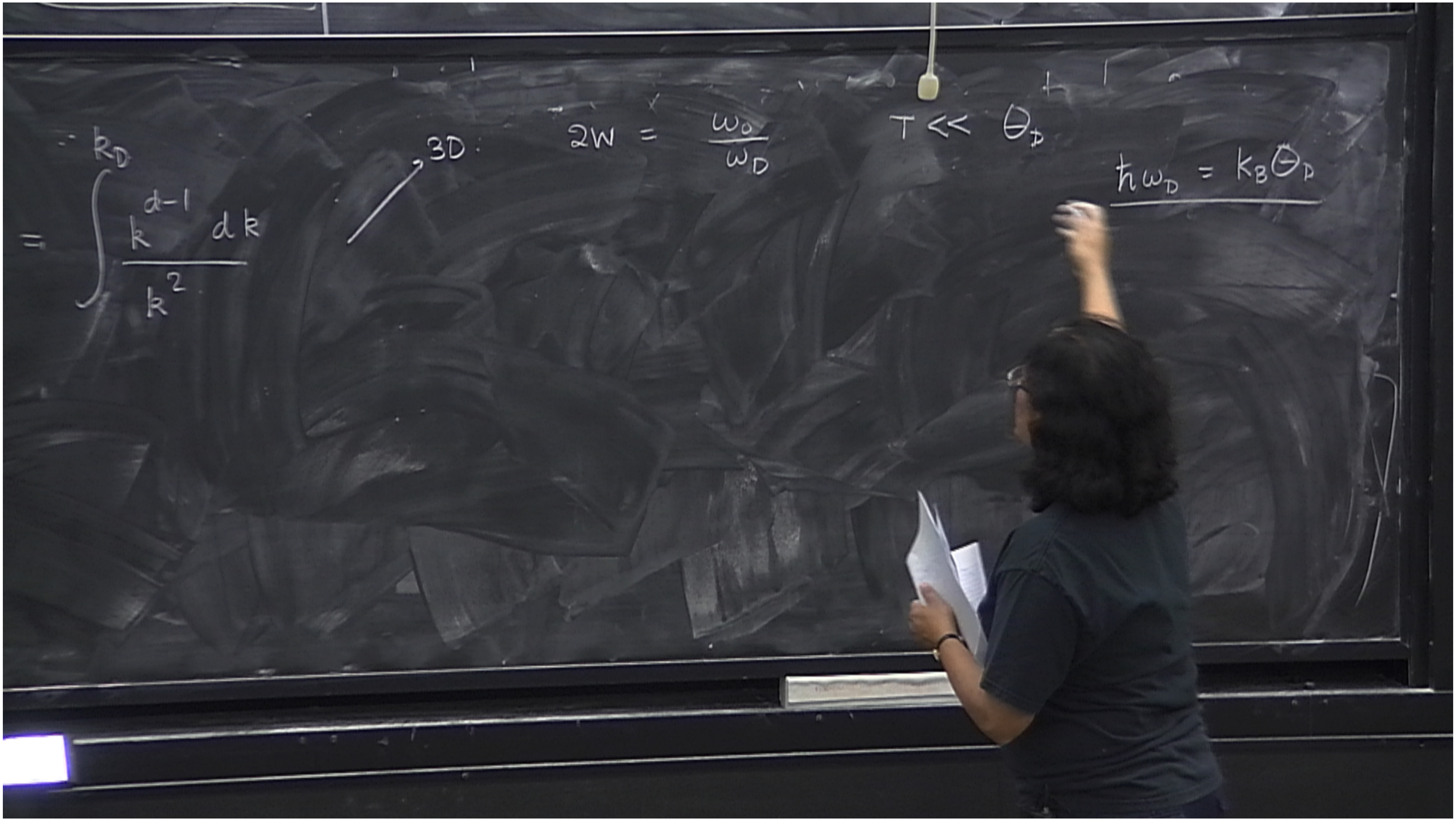
$\frac{1}{N}\sum_k \frac{1}{\hbar\omega_k} \mathrm{Coth}\left(\frac{\hbar\omega_k}{2k_BT}\right) \longrightarrow \int d^dk \, \frac{1}{k}$
$\mathrm{Coth}\, x = \frac{1}{x} + \frac{x}{3} \cdots$
$\omega_k = ck$
Coth x
1
2
$x = \frac{\hbar\omega}{k_BT}$


1/N Σ_k 1/ħω_k Coth(ħω_k / 2k_BT) ⟶ ∫d^d k 1/k 1/k
Coth x = 1/x + x/3 . .
ω_k = ck
Coth x
1
2
x = ħω / k_BT
Coth x ~ 1/x for small x

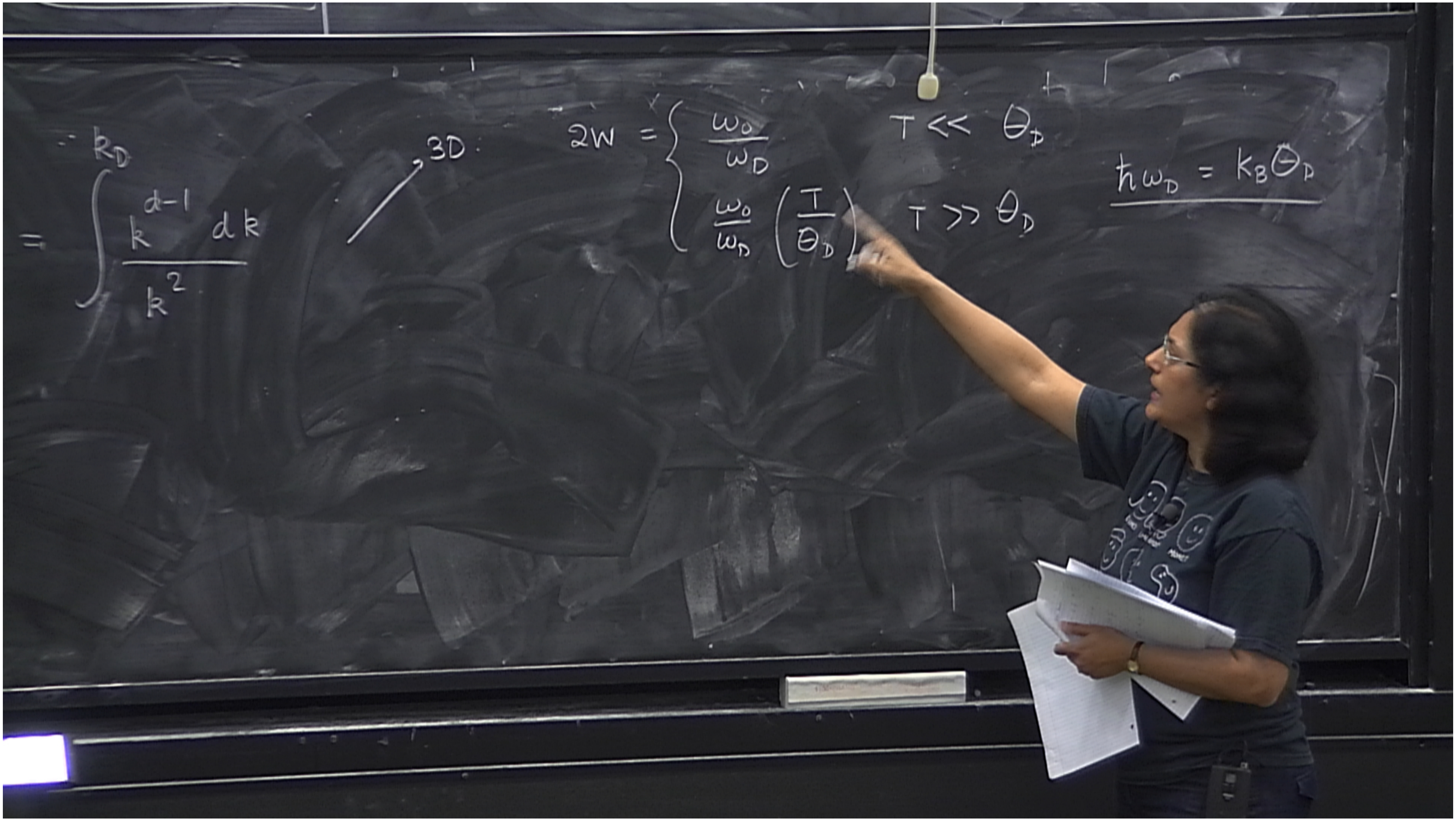

$\longrightarrow \int d^d k \, \frac{1}{k} \, \frac{1}{k} = \int \frac{k^{d-1} \, dk}{k^2}$
$\omega_k = c k$
$\text{th}\, x \sim \frac{1}{x}$ for small x

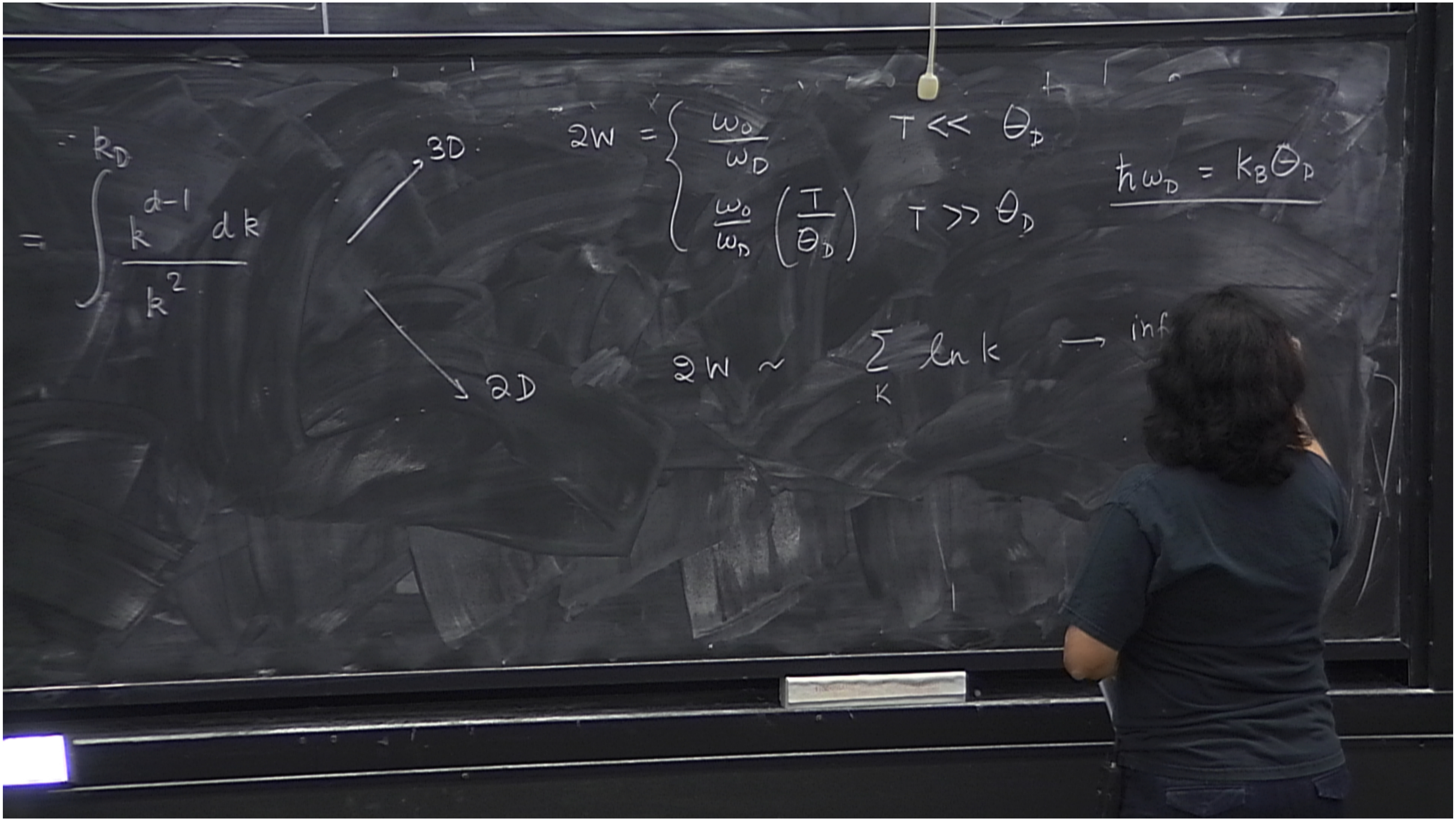

$\int d^d k \; \frac{1}{k} \; \frac{1}{k} = \int^{k_D} \frac{k^{d-1}\, dk}{k^2}$
$\omega_k = c k$
$x \sim \frac{1}{x}$ for small x

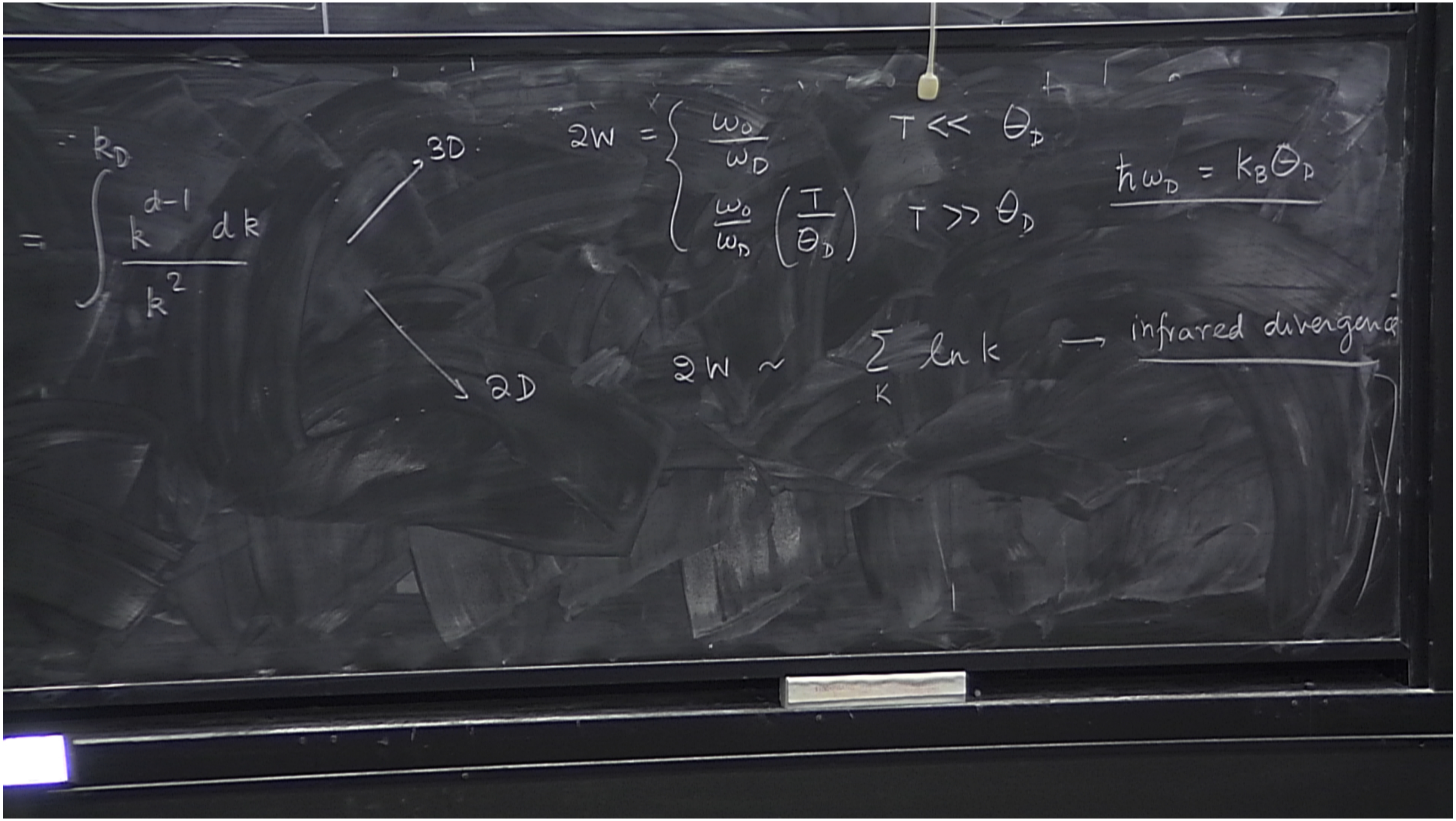

$\int d^d k \, \frac{1}{k} \, \frac{1}{k} = \int^{k_D} \frac{k^{d-1} \, dk}{k^2}$
3D
$2W = \frac{\omega_0}{\omega_D}$
$\omega_k = ck$
$x \sim \frac{1}{x}$ for small x

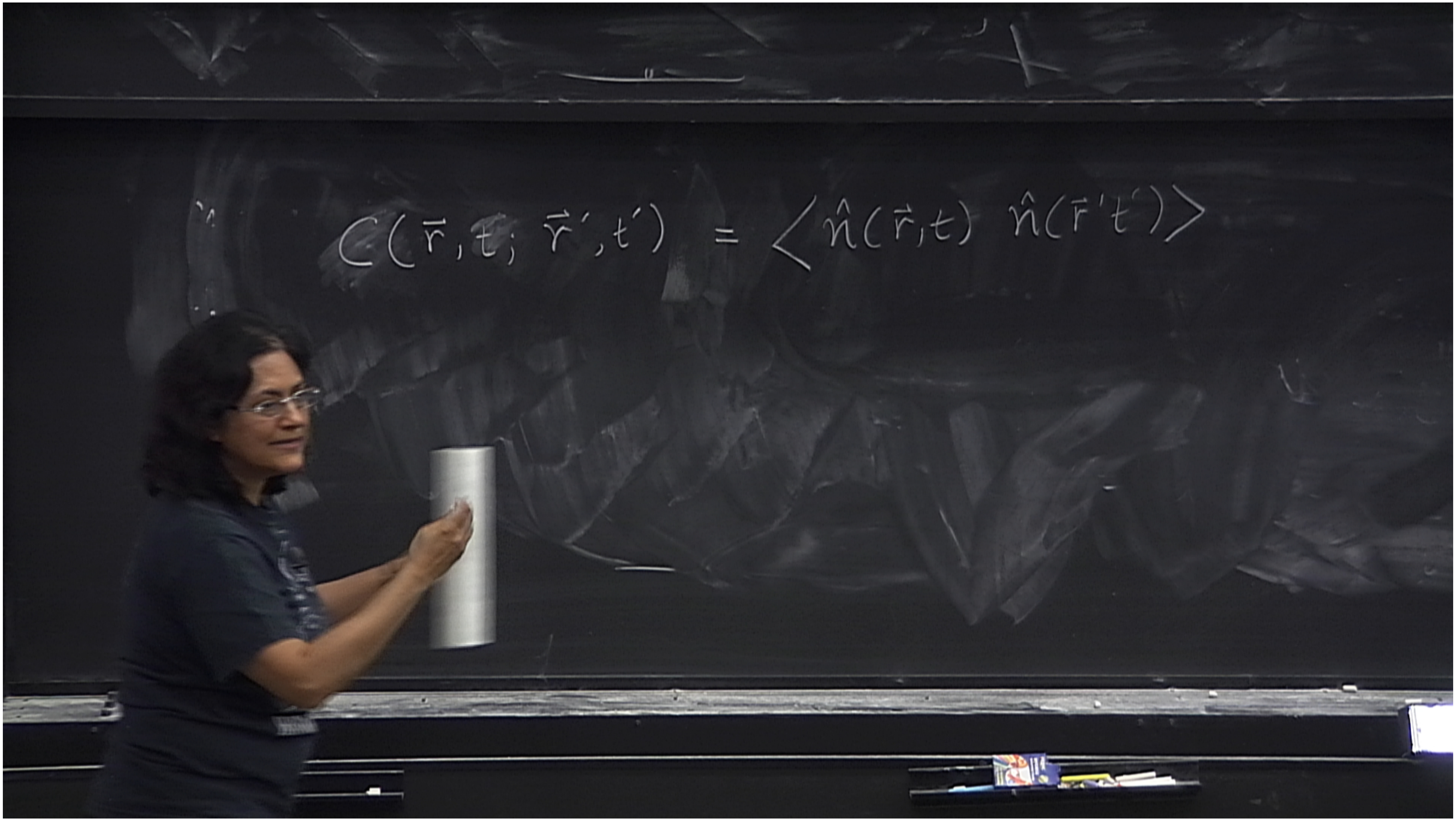


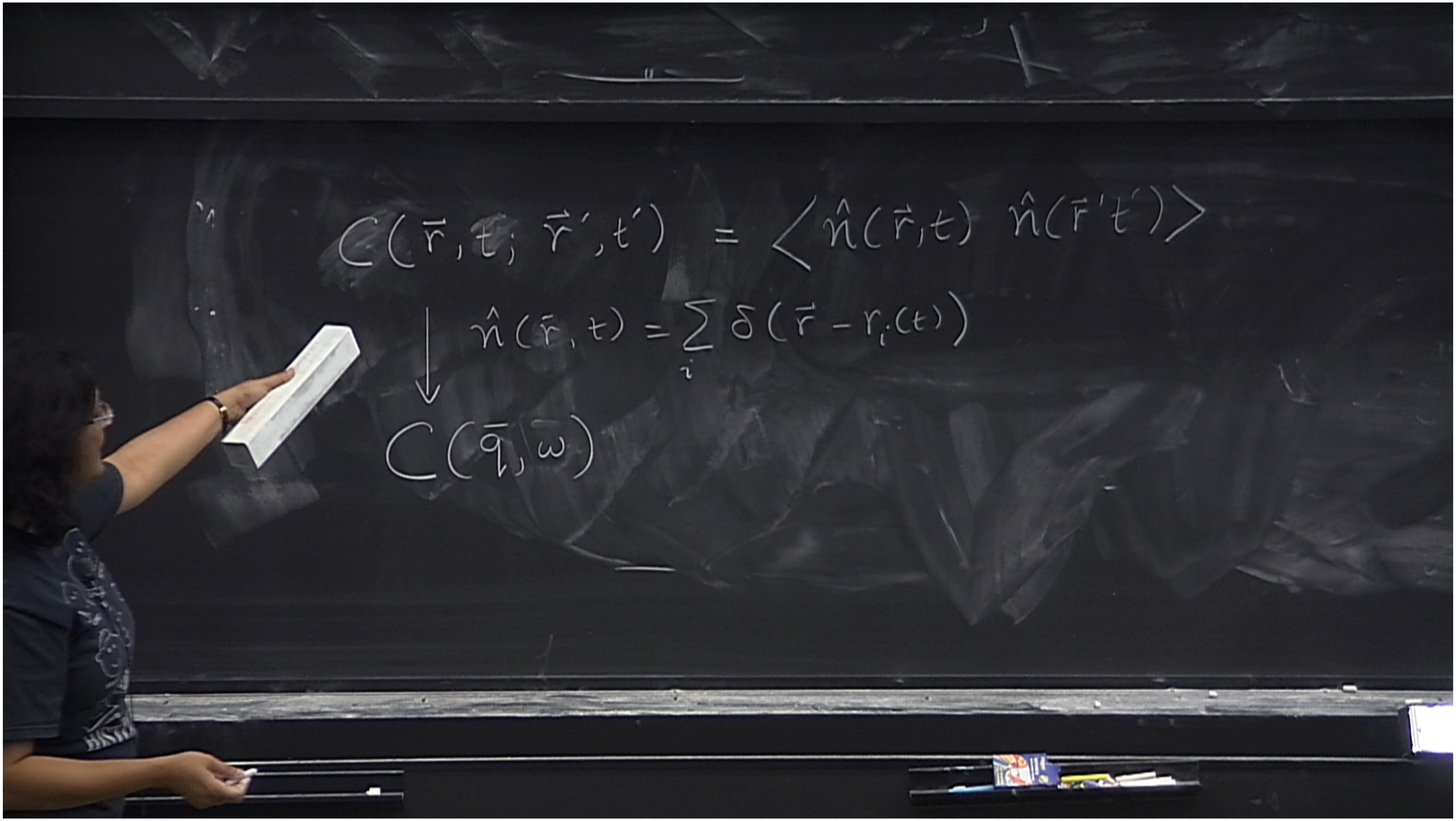
$= \int^{k_D} \frac{k^{d-1}\, dk}{k^2}$
3D
$2W = \frac{\omega_0}{\omega_D}$
$T \ll \Theta_D$
$\hbar\omega_D = k_B\Theta_D$


$= \int^{k_D} \frac{k^{d-1}\, dk}{k^2}$
3D
$2W = \begin{cases} \frac{\omega_0}{\omega_D} & T \ll \Theta_D \\ \frac{\omega_0}{\omega_D}\left(\frac{T}{\Theta_D}\right) & T \gg \Theta_D \end{cases}$
$\hbar \omega_D = k_B \Theta_D$


= ∫^kD k^(d-1) dk / k^2
3D
2D
2W = { ω0/ωD   T << θD
ω0/ωD (T/θD)   T >> θD
ħωD = kBθD
2W ~ Σ_k ln k → inf


$= \int^{k_D} \frac{k^{d-1}\, dk}{k^2}$
3D
2D
$2W = \begin{cases} \frac{\omega_0}{\omega_D} & T \ll \Theta_D \\ \frac{\omega_0}{\omega_D}\left(\frac{T}{\Theta_D}\right) & T \gg \Theta_D \end{cases}$
$\hbar \omega_D = k_B \Theta_D$
$2W \sim \sum_k \ln k$ → infrared divergence


C(r⃗, t; r⃗', t') = ⟨n̂(r⃗, t) n̂(r⃗' t')⟩


$C(\vec{r}, t; \vec{r}', t') = \langle \hat{n}(\vec{r}, t)\ \hat{n}(\vec{r}' t') \rangle$
$\hat{n}(\vec{r}, t) = \sum_i \delta(\vec{r} - r_i(t))$
$C(\vec{q}, \omega)$

$$C(\vec{r},t;\vec{r}\,',t') = \langle \hat{n}(\vec{r},t)\, \hat{n}(\vec{r}\,'t') \rangle$$

$$\downarrow \quad \hat{n}(\vec{r},t) = \sum_i \delta(\vec{r} - r_i(t))$$

$$C(\vec{q},\omega) = N \underbrace{S(\vec{q},\omega)}_{\text{dynamical structure factor}}$$

ron will
pecified

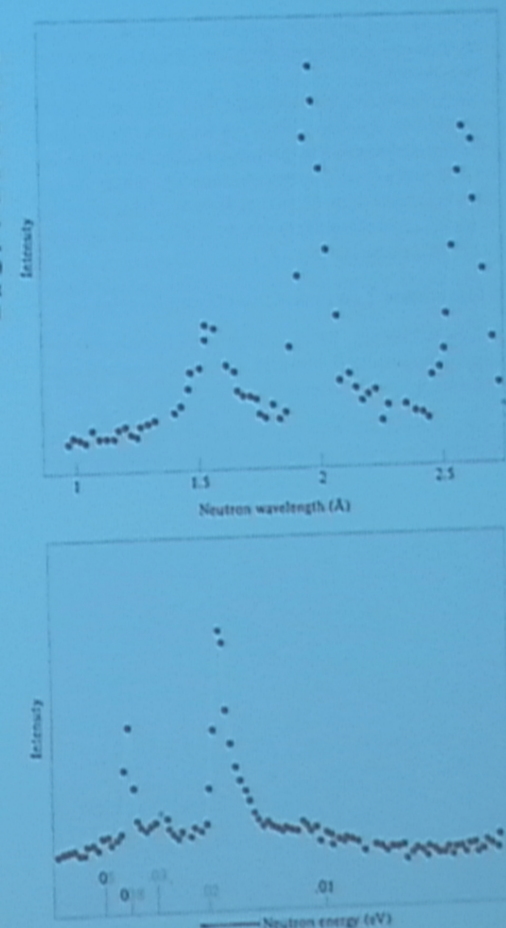
phonon
phonon
crete set
changed
ficiently
neutron

the rest
gle scat-
neutrons
rp peaks
us back-
processes

of scat-
a given
nction of
e smooth
round due
ocesses. In
crystal the
sses would
eaks. In a
peaks are
d curves)
ifetime ef-

although
tly sharp,

Figure 24.4
Some typical experimental neutron groups. In all cases the number of neutrons emerging in a fixed direction for a fixed incident energy is plotted against a variable that distinguishes scattered neutron energies. (a) Copper. (G. Gobert and B. Jacrot, *J. Phys. Radium* 19 (1959).) (b) Germanium. (I. Pelah et al., *Phys. Rev.* 108, 1091 (1957).)



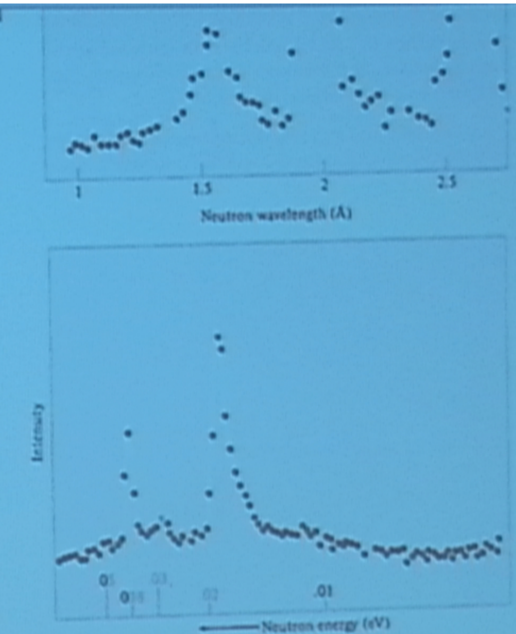
to the phonons finite lifetimes, reflecting the eventual decay of the approximate

gle scat-
ıeutrons
rp peaks
us back-
rocesses

of scat-
a given
nction of
ıe smooth
ound due
ocesses. In
crystal the
ises would
eaks. In a
peaks are
d curves)
ifetime ef-

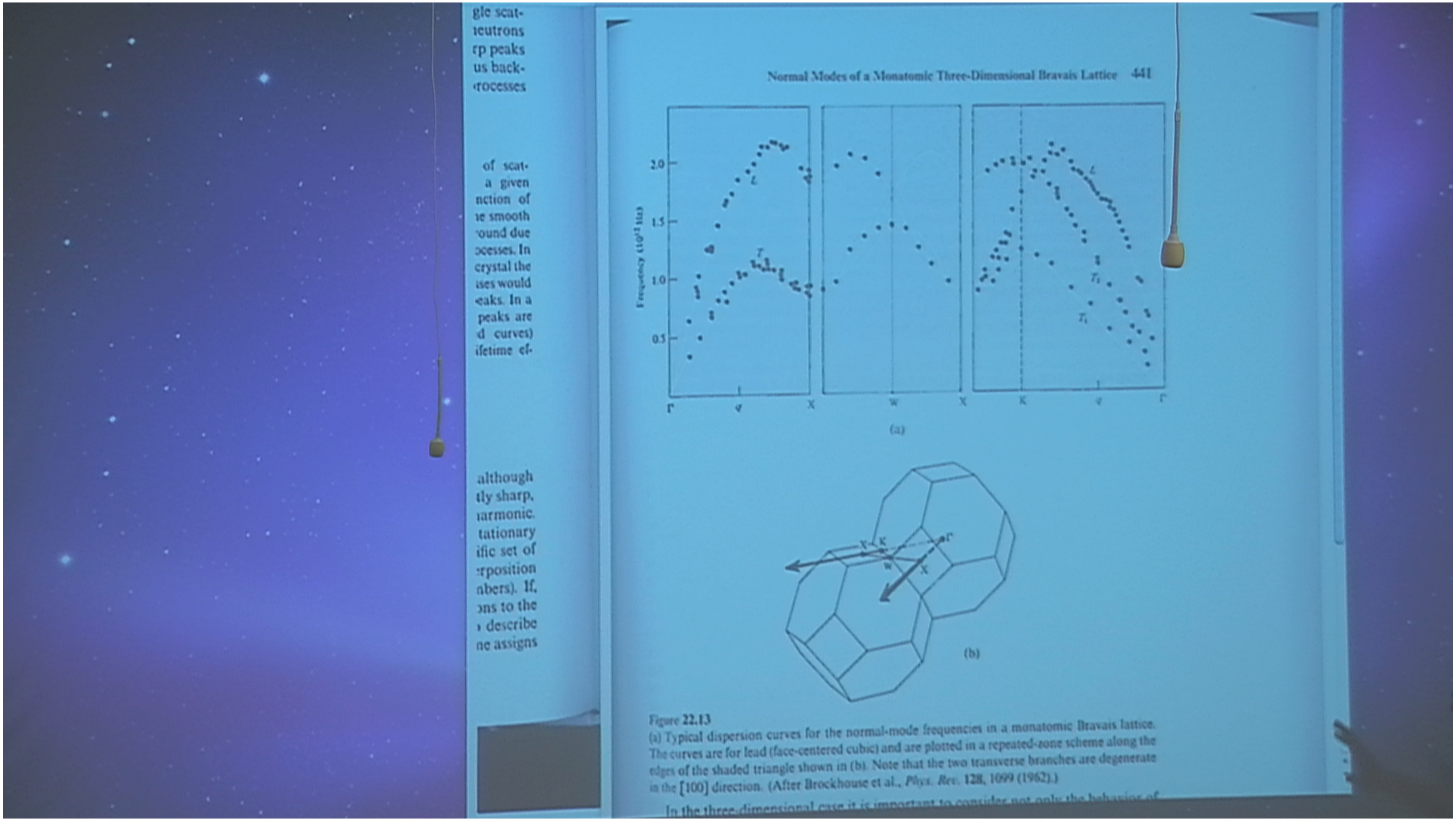
although
tly sharp,
ıarmonic.
tationary
ific set of
rposition
nbers). If,
ons to the
describe
ne assigns

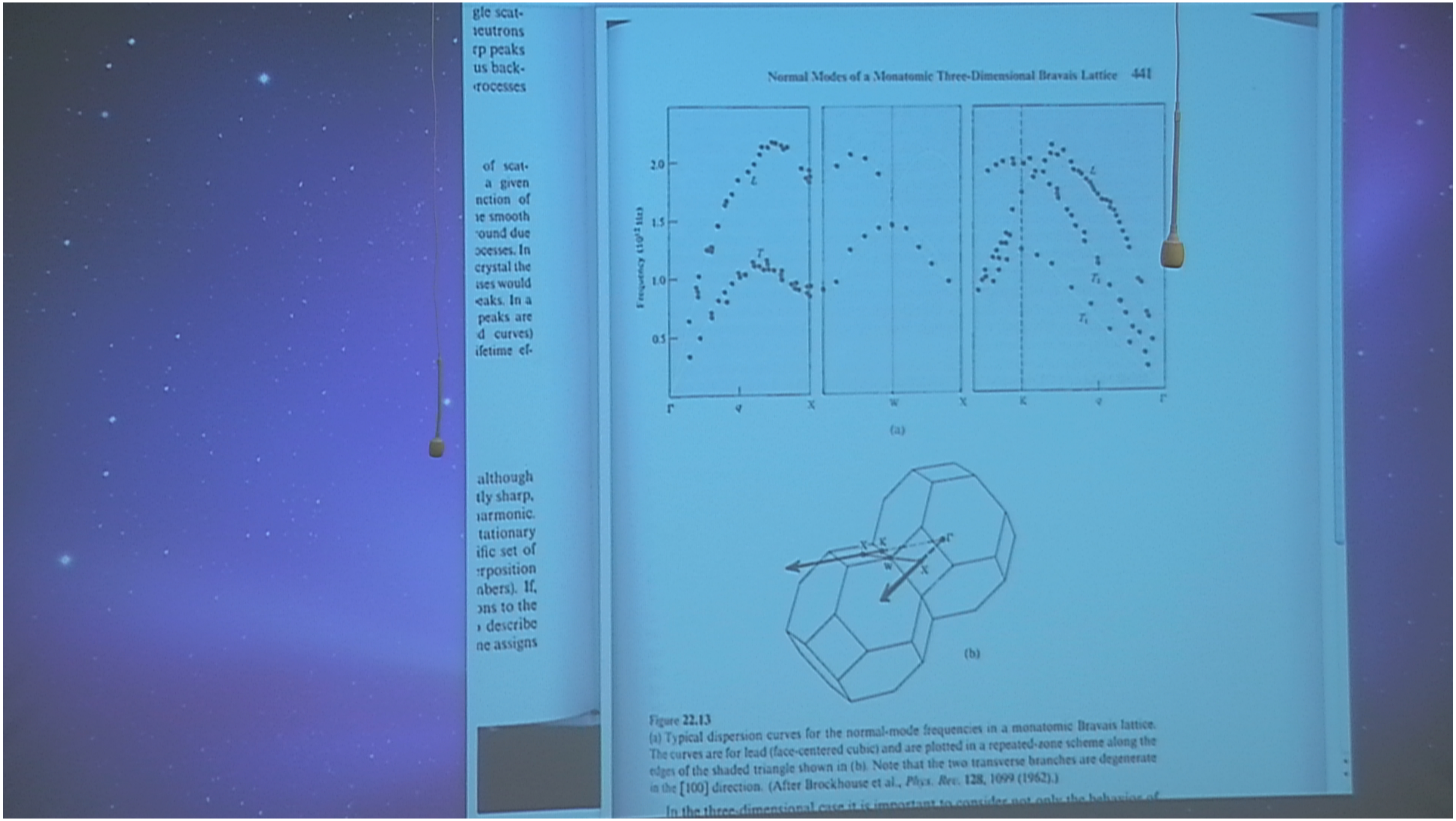
al., *Phys. Rev.* 108, 1091 (1957).)



to the phonons finite lifetimes, reflecting the eventual decay of the approximate harmonic stationary state. Associated with a phonon of lifetime τ, there will be an uncertainty $\hbar/\tau$ in the phonon energy. The energy conservation law determining the one-phonon peaks will then be correspondingly weakened.

These points will be taken up in more detail in Chapter 25. Here we merely note that the one-phonon peaks, though broadened, are still clearly identifiable. The fact that they are indeed due to one-phonon processes is strikingly confirmed by the consistency of the $\omega_s(\mathbf{k})$ curves inferred from their positions, for there is considerable redundancy in the data furnished by the one-phonon peaks. One can extract infor-


Normal Modes of a Monatomic Three-Dimensional Bravais Lattice 441
2.0
1.5
1.0
0.5
L
T
T₁
Γ
q
X
W
X
K
q
Γ
(a)
X
K
W
X
Γ
(b)
Figure 22.13
(a) Typical dispersion curves for the normal-mode frequencies in a monatomic Bravais lattice. The curves are for lead (face-centered cubic) and are plotted in a repeated-zone scheme along the edges of the shaded triangle shown in (b). Note that the two transverse branches are degenerate in the [100] direction. (After Brockhouse et al., Phys. Rev. 128, 1099 (1962).)



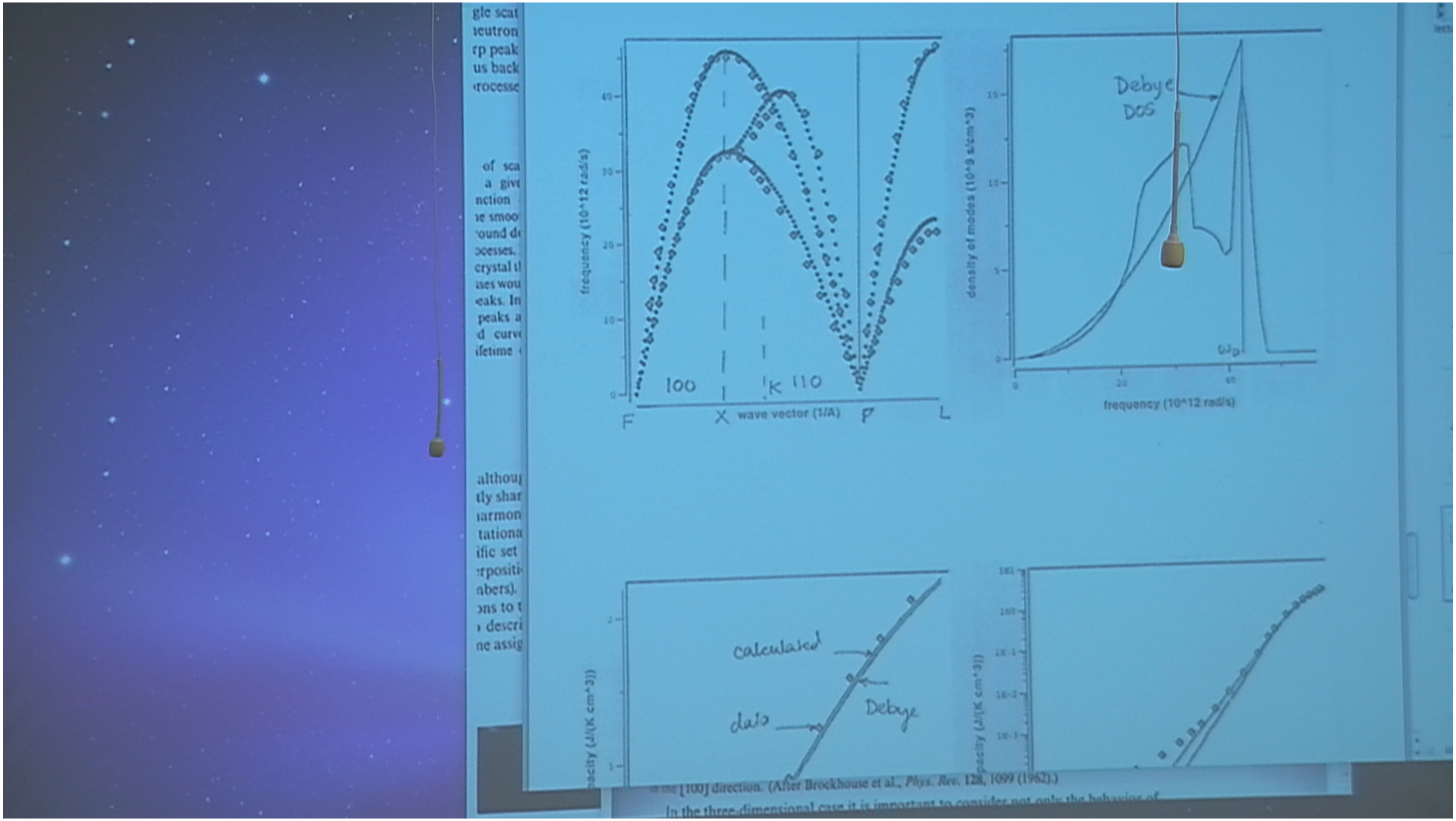
gle scat-
ıeutrons
rp peaks
us back-
rocesses

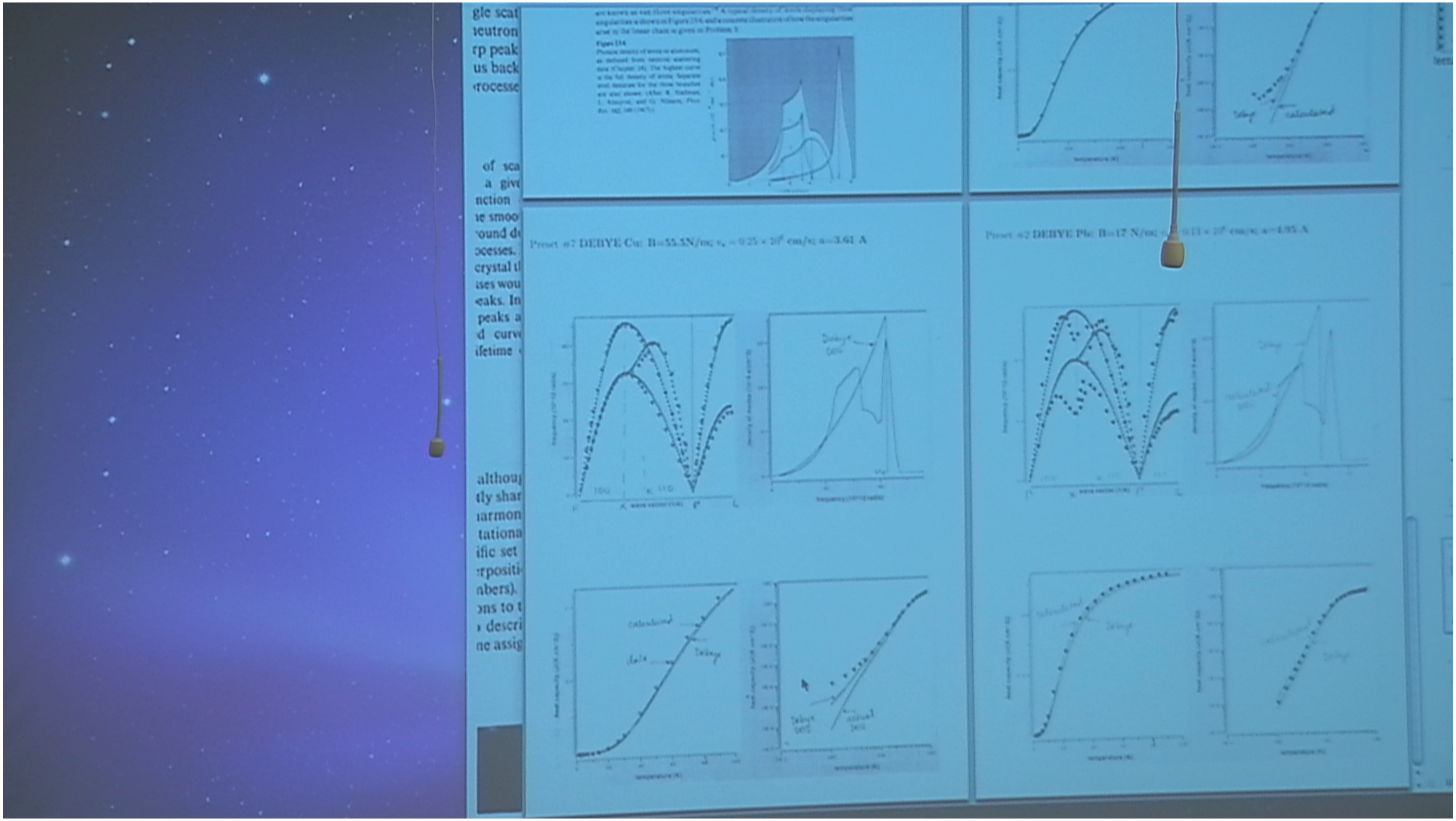
of scat-
a given
nction of
ıe smooth
ound due
ocesses. In
crystal the
ıses would
eaks. In a
peaks are
d curves)
ifetime ef-

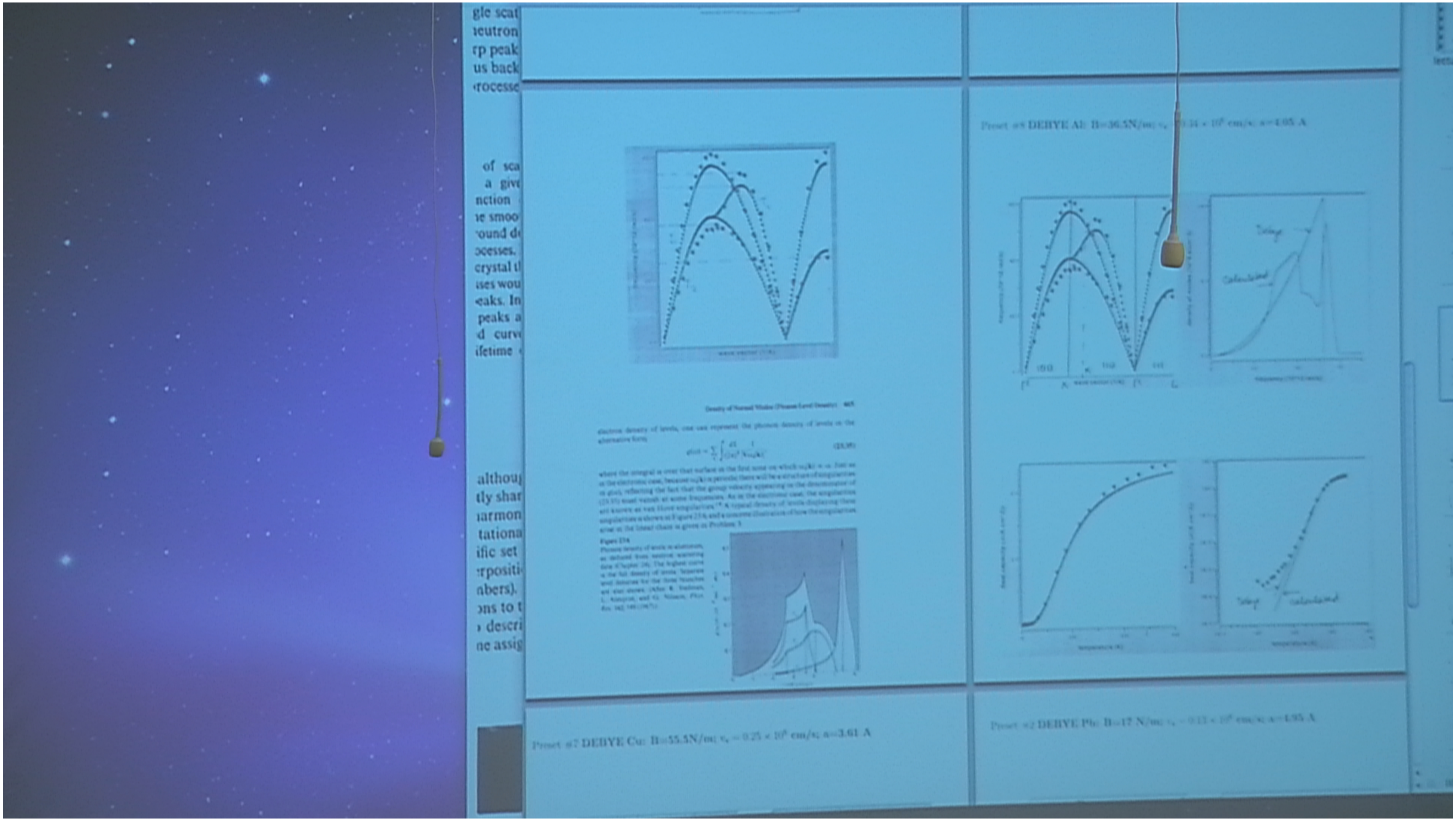
although
tly sharp,
ıarmonic.
tationary
ific set of
rposition
nbers). If,
ons to the
describe
ne assigns

Figure 22.13
(a) Typical dispersion curves for the normal-mode frequencies in a monatomic Bravais lattice. The curves are for lead (face-centered cubic) and are plotted in a repeated-zone scheme along the edges of the shaded triangle shown in (b). Note that the two transverse branches are degenerate in the [100] direction. (After Brockhouse et al., *Phys. Rev.* **128**, 1099 (1962).)

In the three-dimensional case it is important to consider not only the behavior of


frequency (10^12 rad/s)
100
K 110
wave vector (1/A)
Debye
DOS
frequency (10^12 rad/s)
calculated
data
Debye
[100] direction. (After Brockhouse et al., Phys. Rev. 128, 1099 (1962).)


Preset #7 DEBYE Cu: B=55.5N/m; a=3.61 A
Preset #2 DEBYE Pb: B=17 N/m; a=4.95 A


Preset #7 DEBYE Cu: B=55.5N/m
DEBYE Al
Debye
Calculated