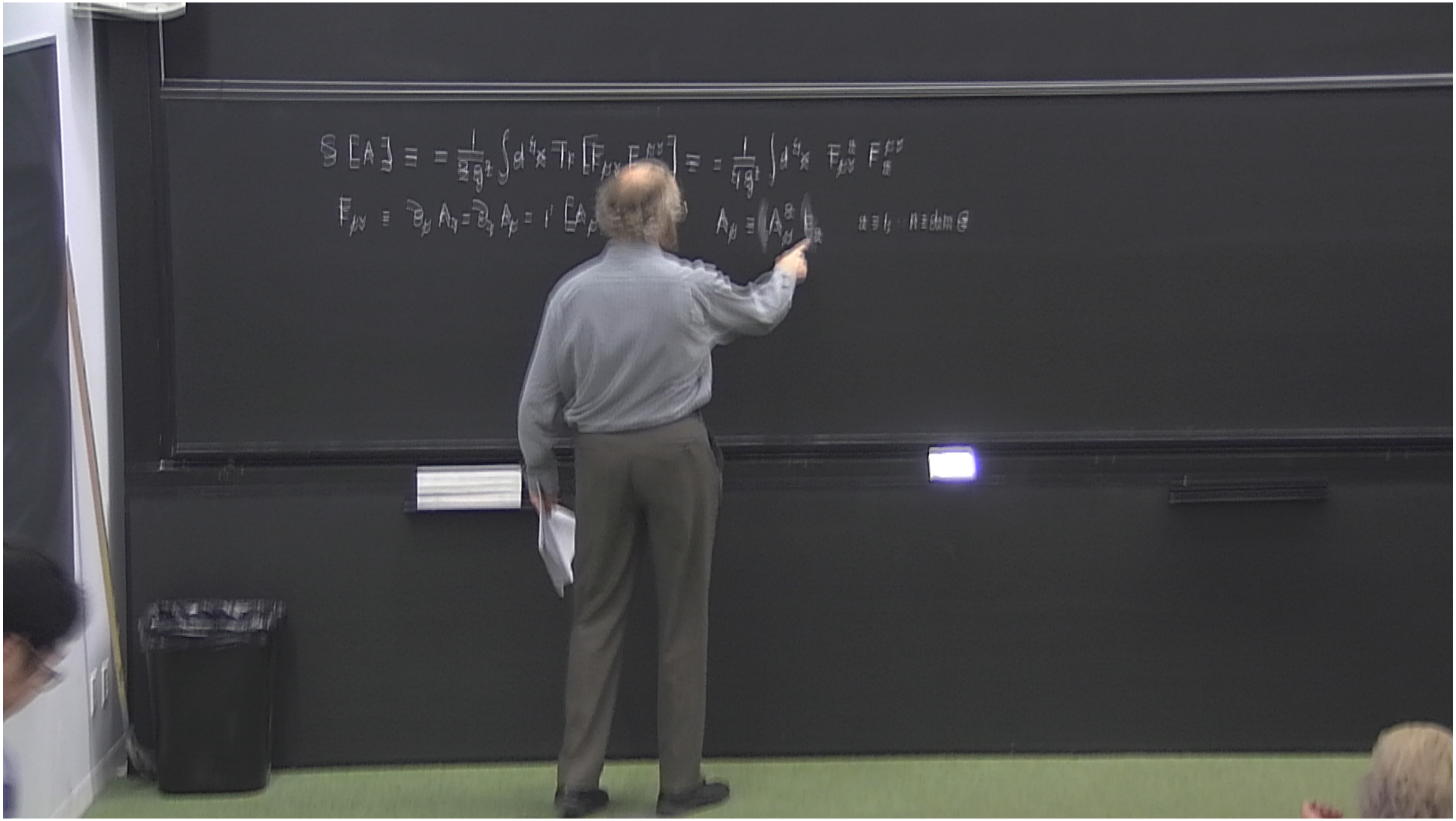
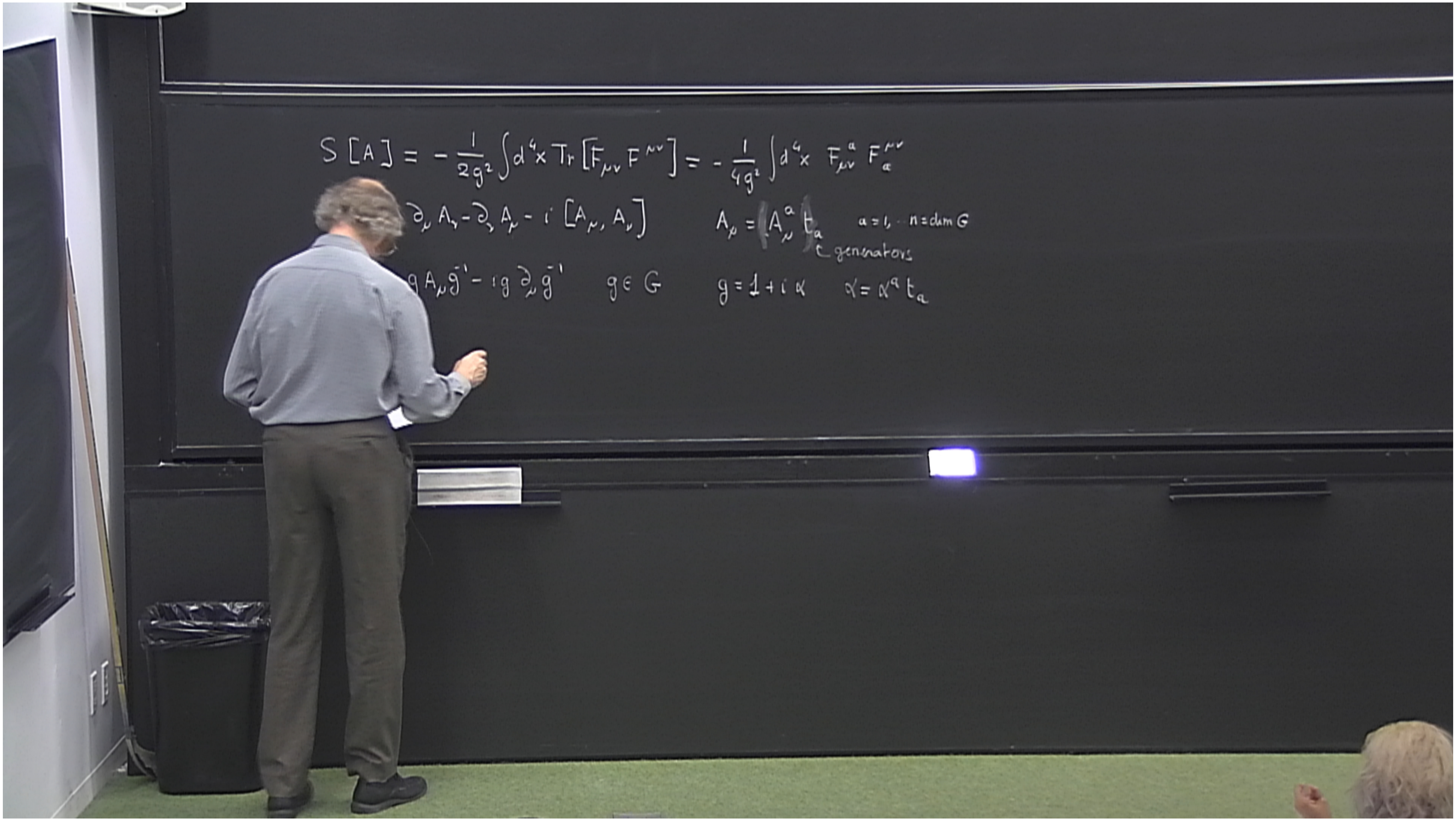
Title: Quantum Field Theory II - Lecture 14

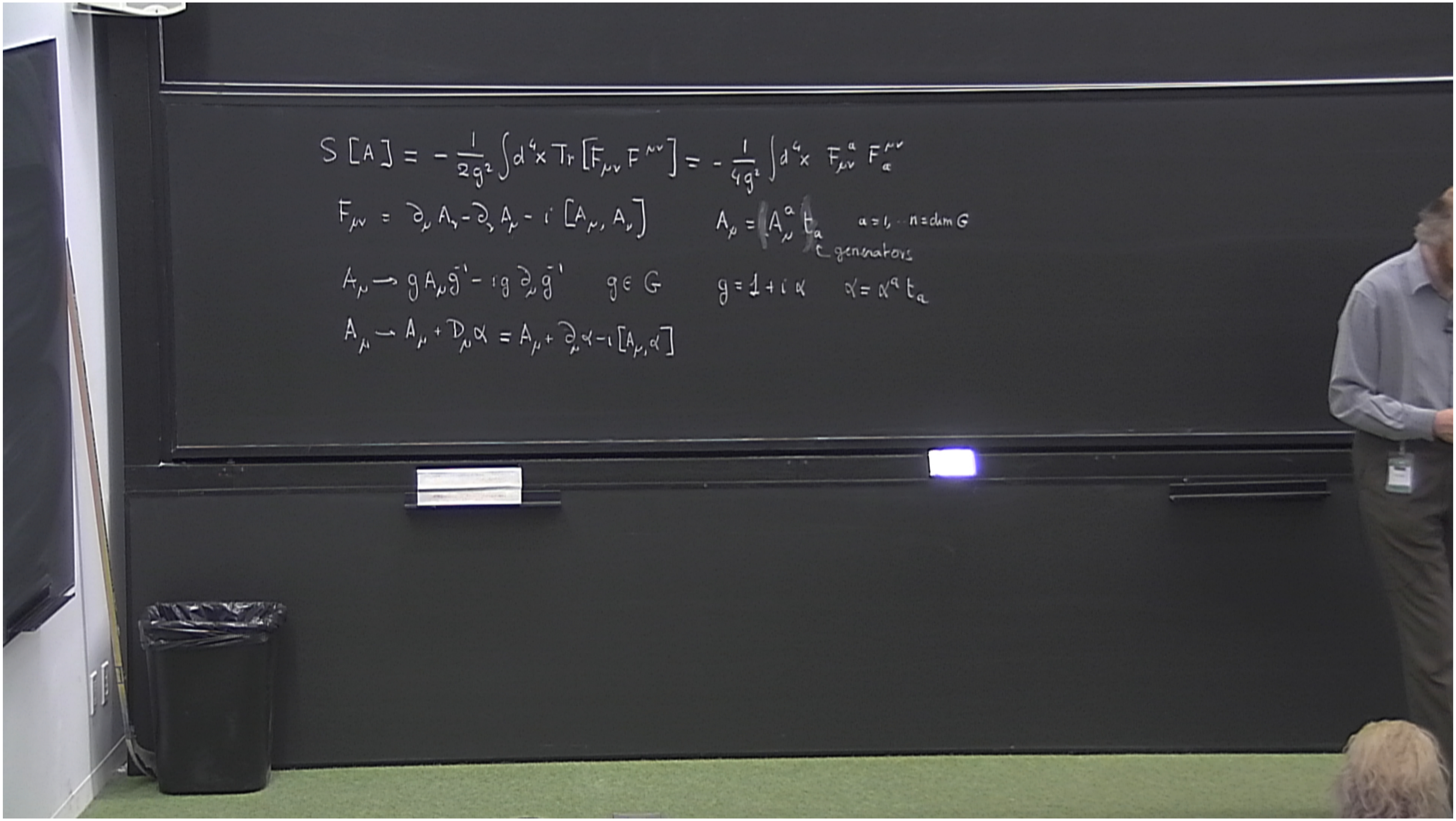
Date: Nov 17, 2011 09:00 AM

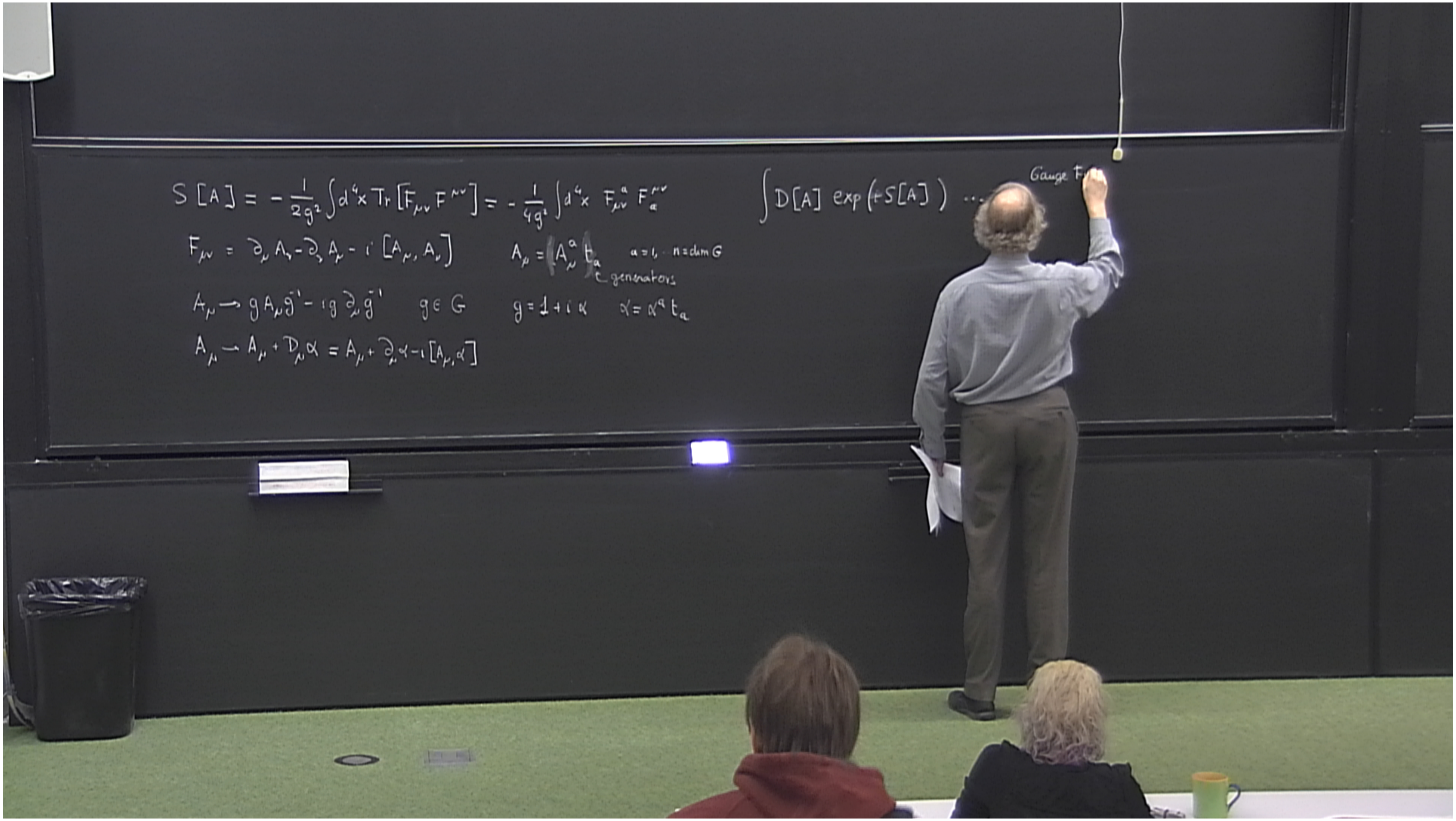
URL: http://pirsa.org/11110020

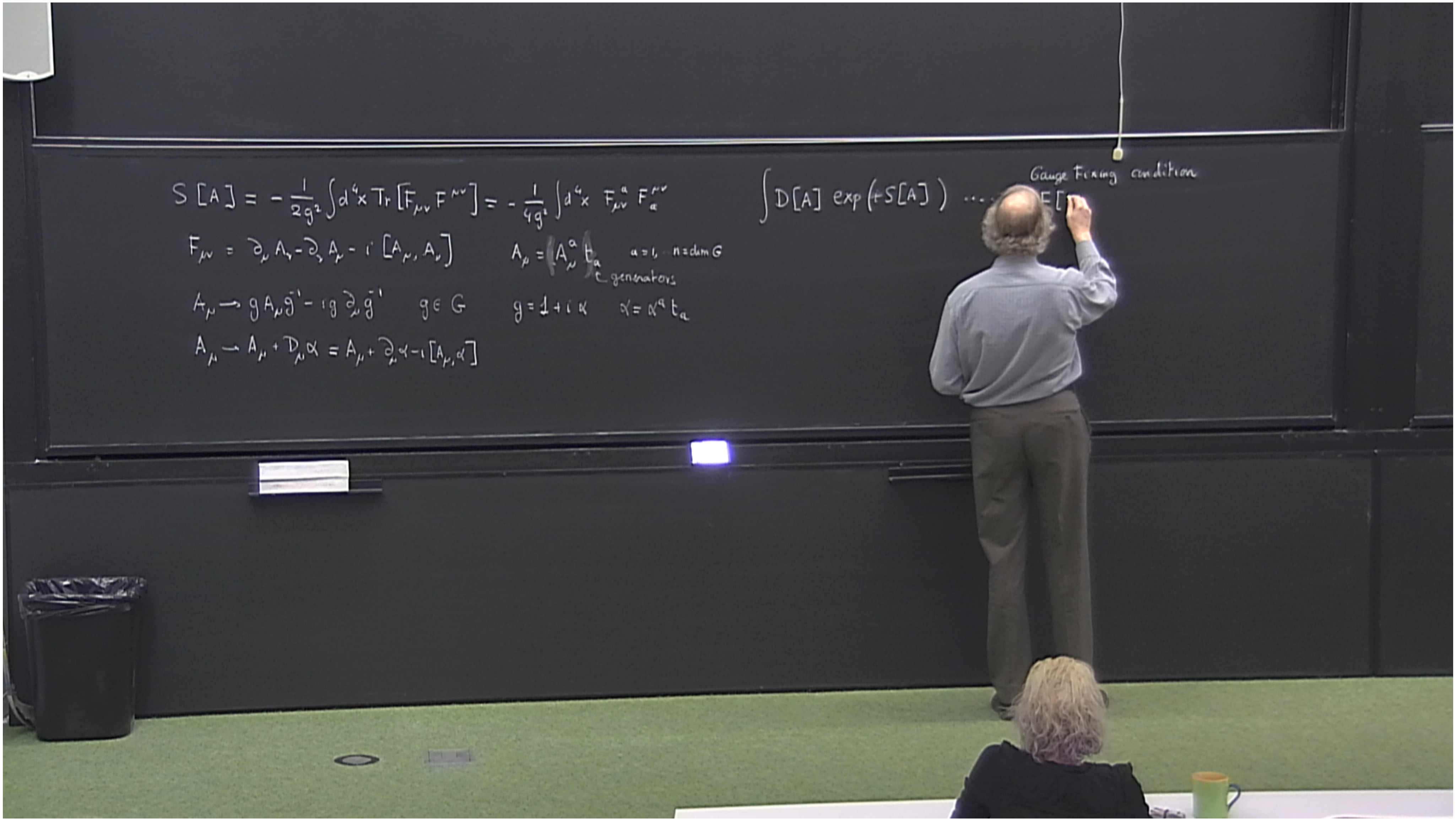
Abstract:

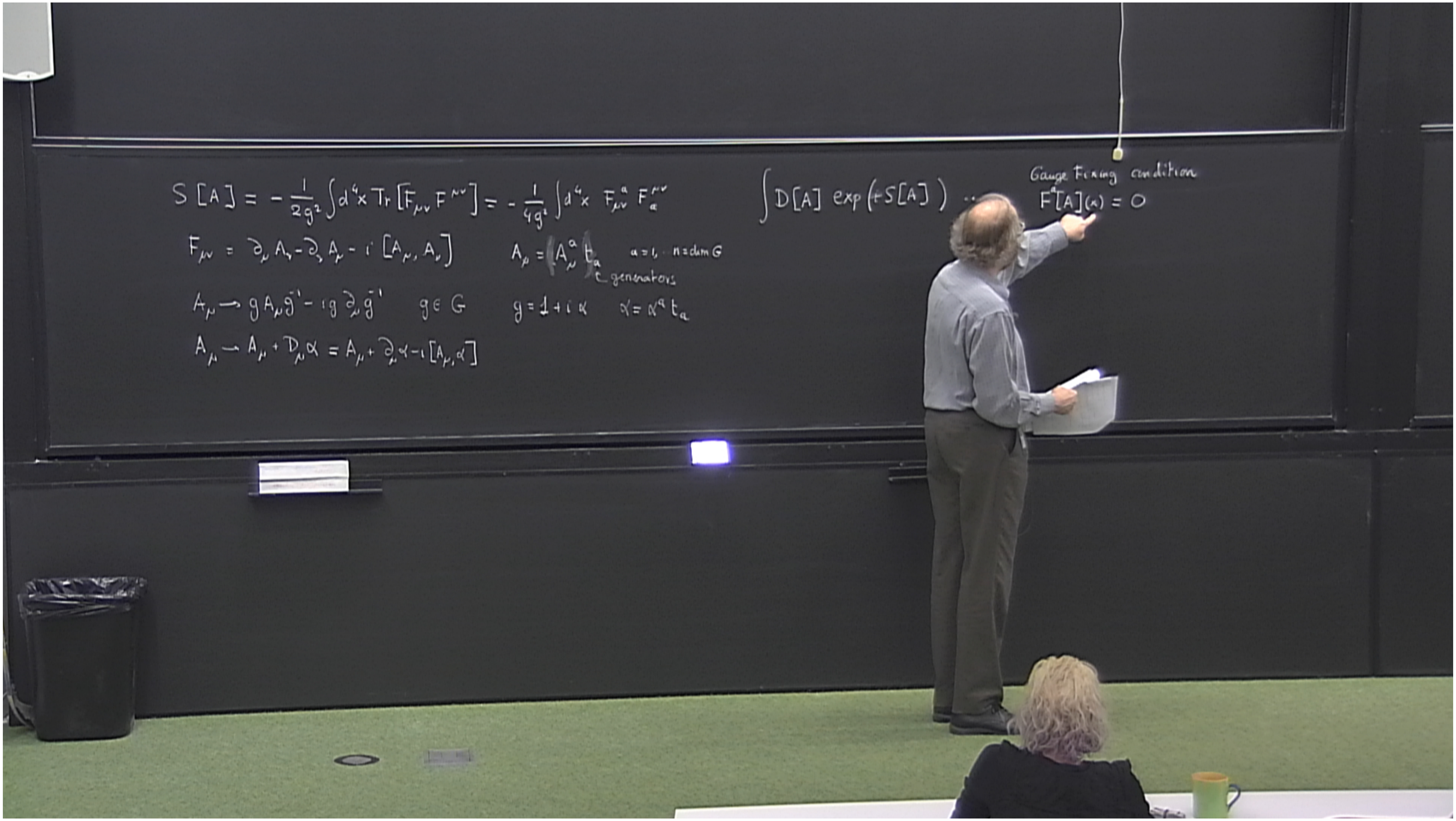

S[A] ≡ -1/(2g^2) ∫d^4x Tr[F_μν F^μν] = -1/(4g^2) ∫d^4x F^a_μν F^μν_a


$S[A] = -\frac{1}{2g^2}\int d^4x \, Tr[F_{\mu\nu}F^{\mu\nu}] = -\frac{1}{4g^2}\int d^4x \, F^a_{\mu\nu} F^{\mu\nu}_a$
$\partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
$A_\mu = A^a_\mu t_a$
$a = 1, \ldots n = \dim G$
generators
$g A_\mu g^{-1} - i g \partial_\mu g^{-1}$
$g \in G$
$g = 1 + i\alpha$
$\alpha = \alpha^a t_a$

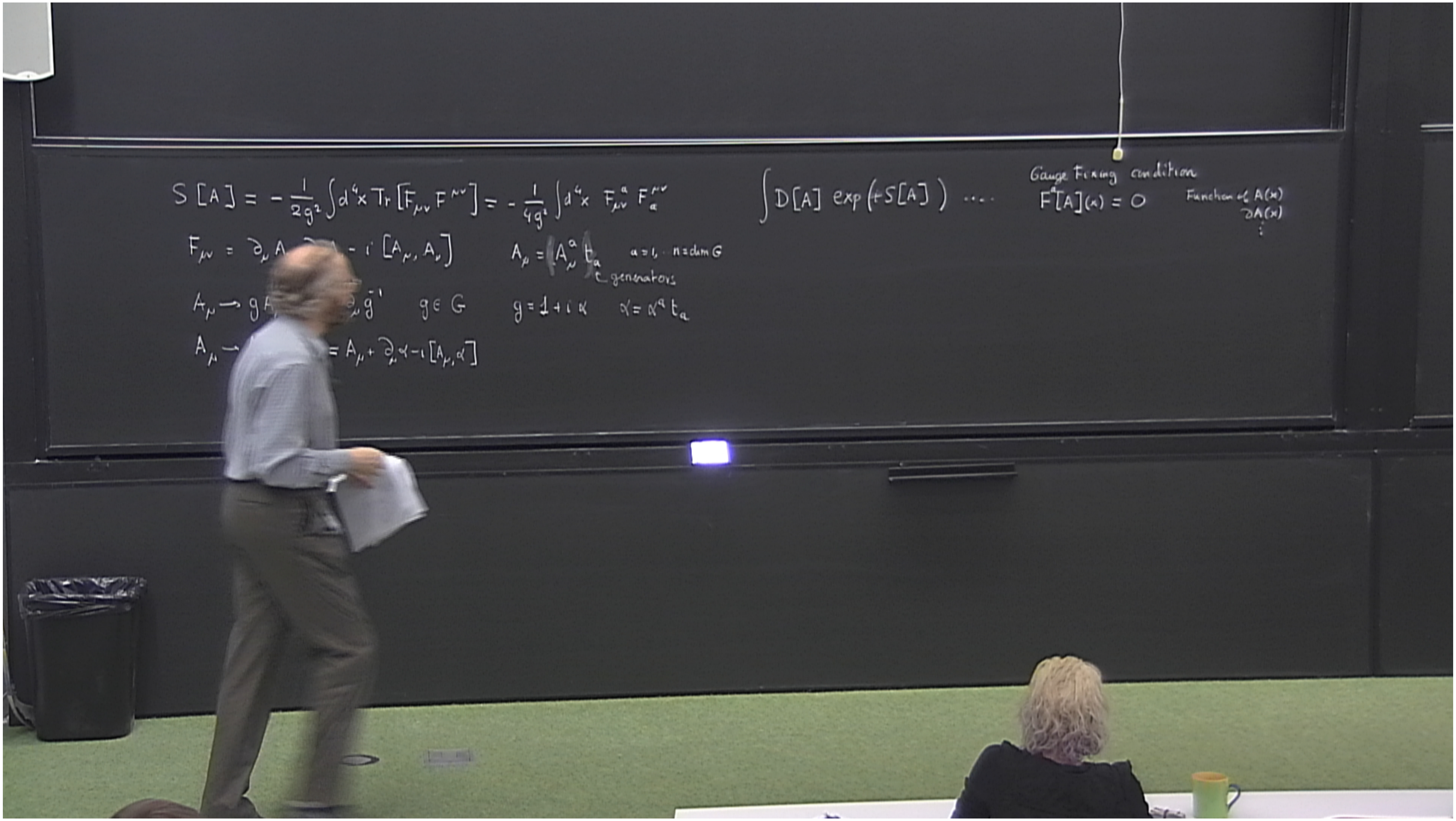

$S[A] = -\frac{1}{2g^2}\int d^4x \, Tr[F_{\mu\nu}F^{\mu\nu}] = -\frac{1}{4g^2}\int d^4x \, F^a_{\mu\nu} F_a^{\mu\nu}$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
$A_\mu = A^a_\mu t_a$
$a = 1, \dots n = \dim G$
generators
$A_\mu \to g A_\mu g^{-1} - i g \partial_\mu g^{-1}$
$g \in G$
$g = 1 + i\alpha$
$\alpha = \alpha^a t_a$
$A_\mu \to A_\mu + D_\mu \alpha = A_\mu + \partial_\mu \alpha - i[A_\mu, \alpha]$


$S[A] = -\frac{1}{2g^2}\int d^4x \, Tr[F_{\mu\nu}F^{\mu\nu}] = -\frac{1}{4g^2}\int d^4x \, F^a_{\mu\nu} F^{\mu\nu}_a$
$\int D[A] \exp(iS[A]) \ldots$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
$A_\mu = A^a_\mu t_a$ $a = 1, \ldots n = \dim G$
generators
$A_\mu \to g A_\mu g^{-1} - ig\partial_\mu g^{-1}$ $g \in G$ $g = 1 + i\alpha$ $\alpha = \alpha^a t_a$
$A_\mu \to A_\mu + D_\mu\alpha = A_\mu + \partial_\mu\alpha - i[A_\mu, \alpha]$

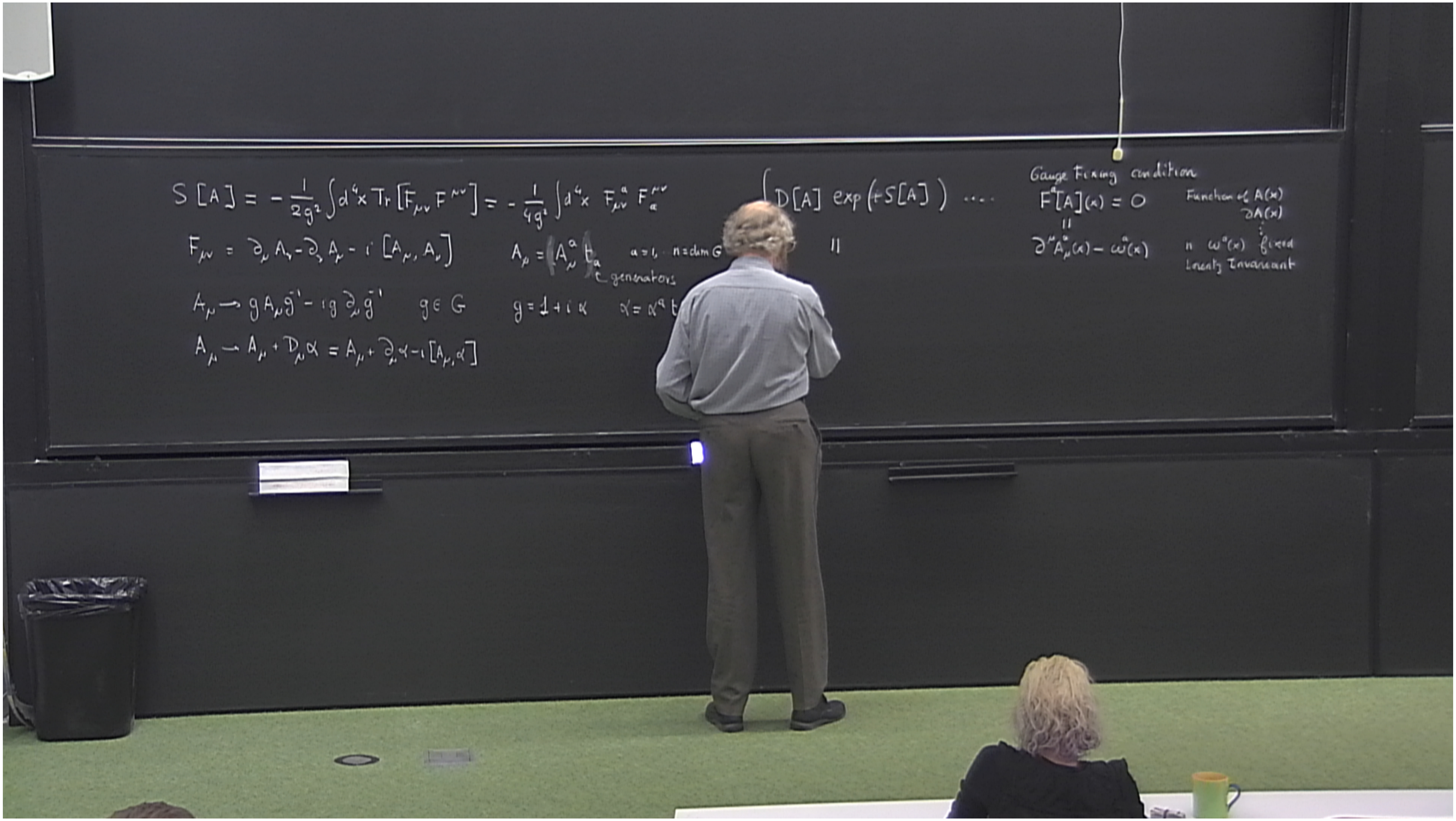

$S[A] = -\frac{1}{2g^2}\int d^4x \, \mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] = -\frac{1}{4g^2}\int d^4x \; F^a_{\mu\nu} F^{\mu\nu}_a$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
$A_\mu = A^a_\mu t_a$
$a = 1, \dots n = \dim G$
generators
$A_\mu \to g A_\mu g^{-1} - ig\,\partial_\mu g^{-1}$
$g \in G$
$g = 1 + i\alpha$
$\alpha = \alpha^a t_a$
$A_\mu \to A_\mu + D_\mu \alpha = A_\mu + \partial_\mu \alpha - i[A_\mu, \alpha]$
$\int D[A] \exp(+S[A]) \dots$
Gauge Fixing condition

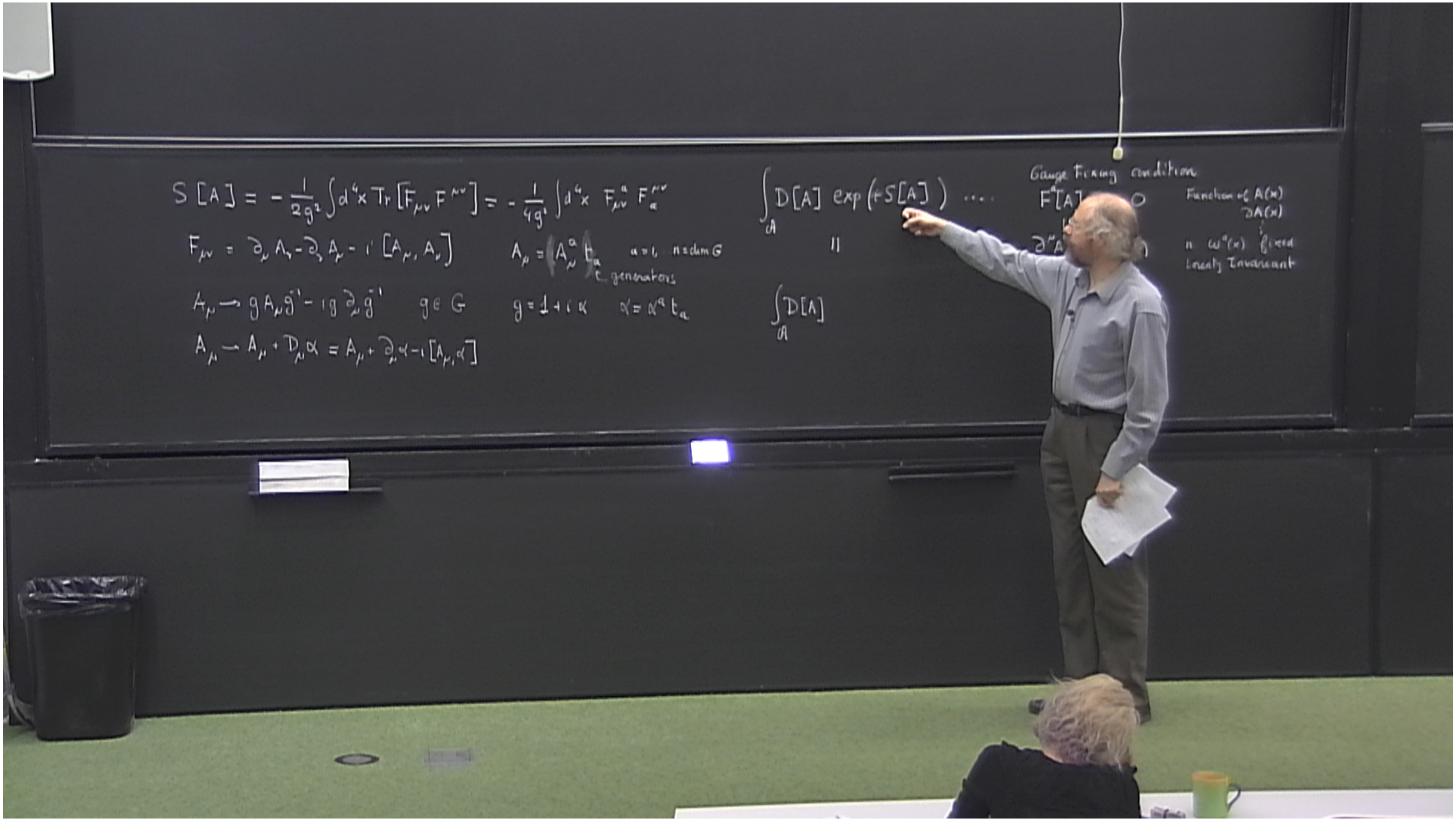


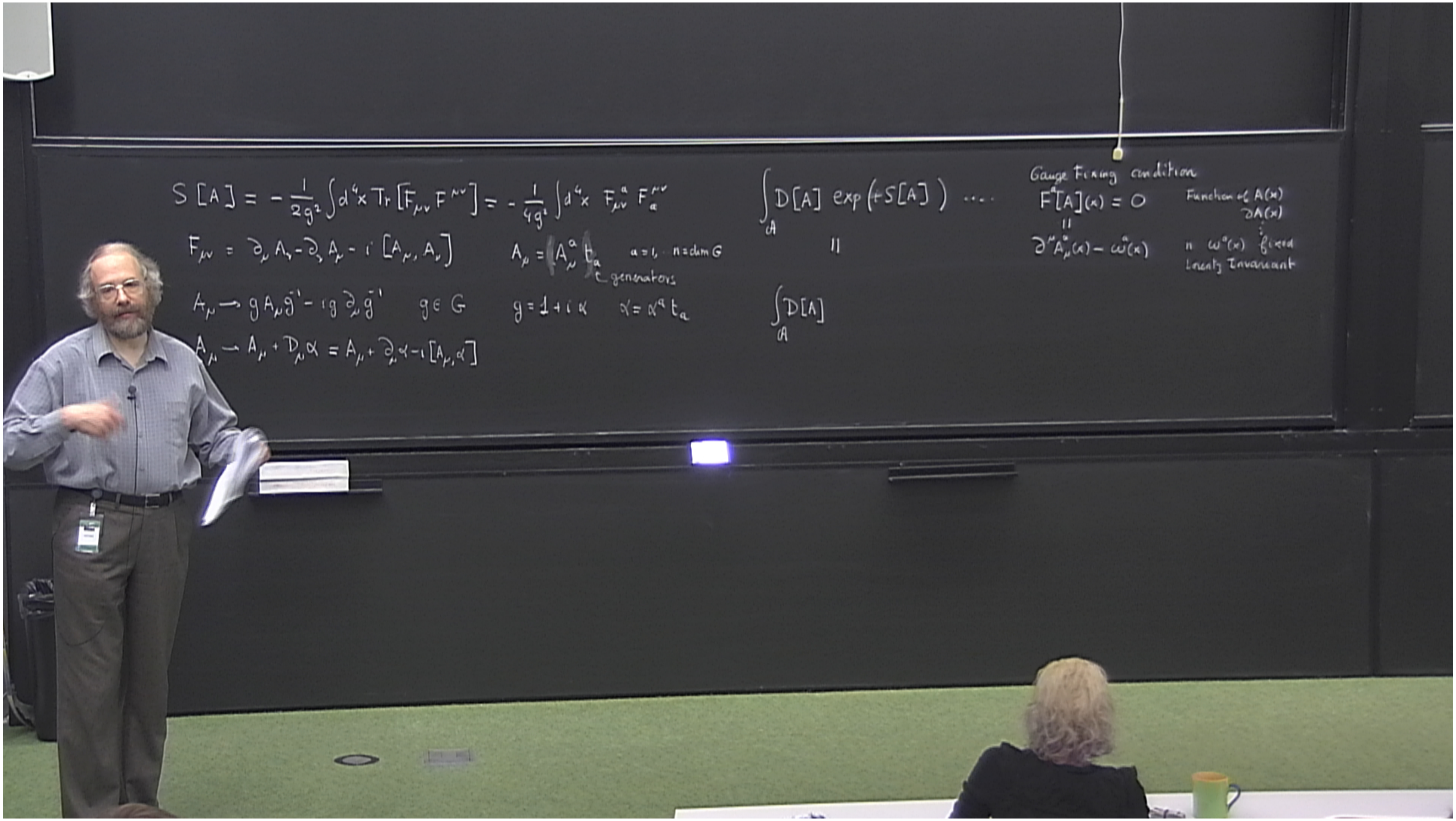
S[A] = − 1/2g² ∫d⁴x Tr[F_μν F^μν] = − 1/4g² ∫d⁴x F^a_μν F_a^μν
F_μν = ∂_μ A_ν − ∂_ν A_μ − i [A_μ, A_ν]
A_μ = A^a_μ t_a
a = 1, .. n = dim G
generators
A_μ → g A_μ g⁻¹ − i g ∂_μ g⁻¹
g ∈ G
g = 1 + i α
α = α^a t_a
A_μ → A_μ + D_μ α = A_μ + ∂_μ α − i [A_μ, α]
∫D[A] exp(−S[A]) ..
Gauge Fixing condition
F^a[A](x) = 0



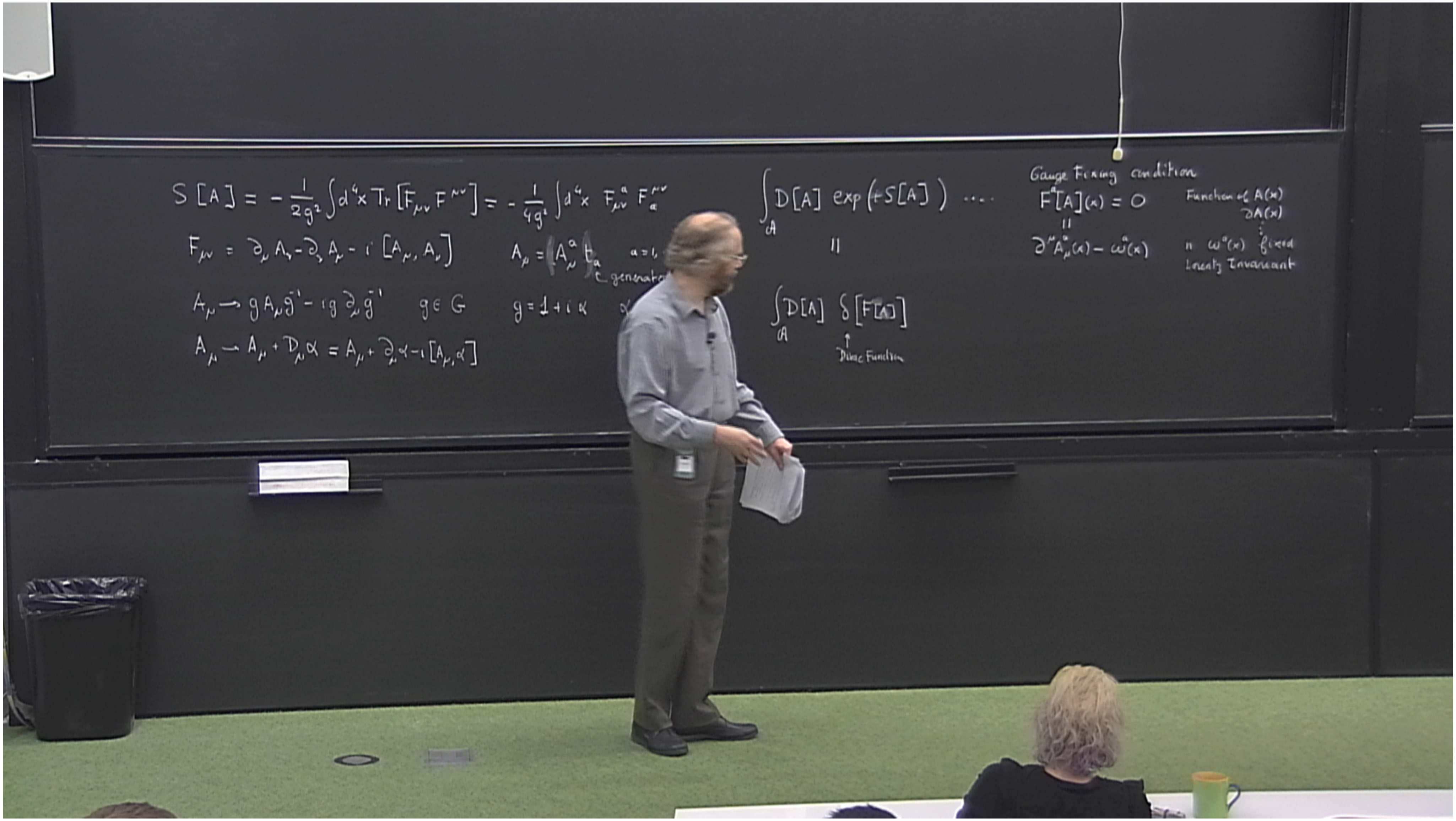
Gauge Fixing condition
Function of A(x)
∂A(x)
generators

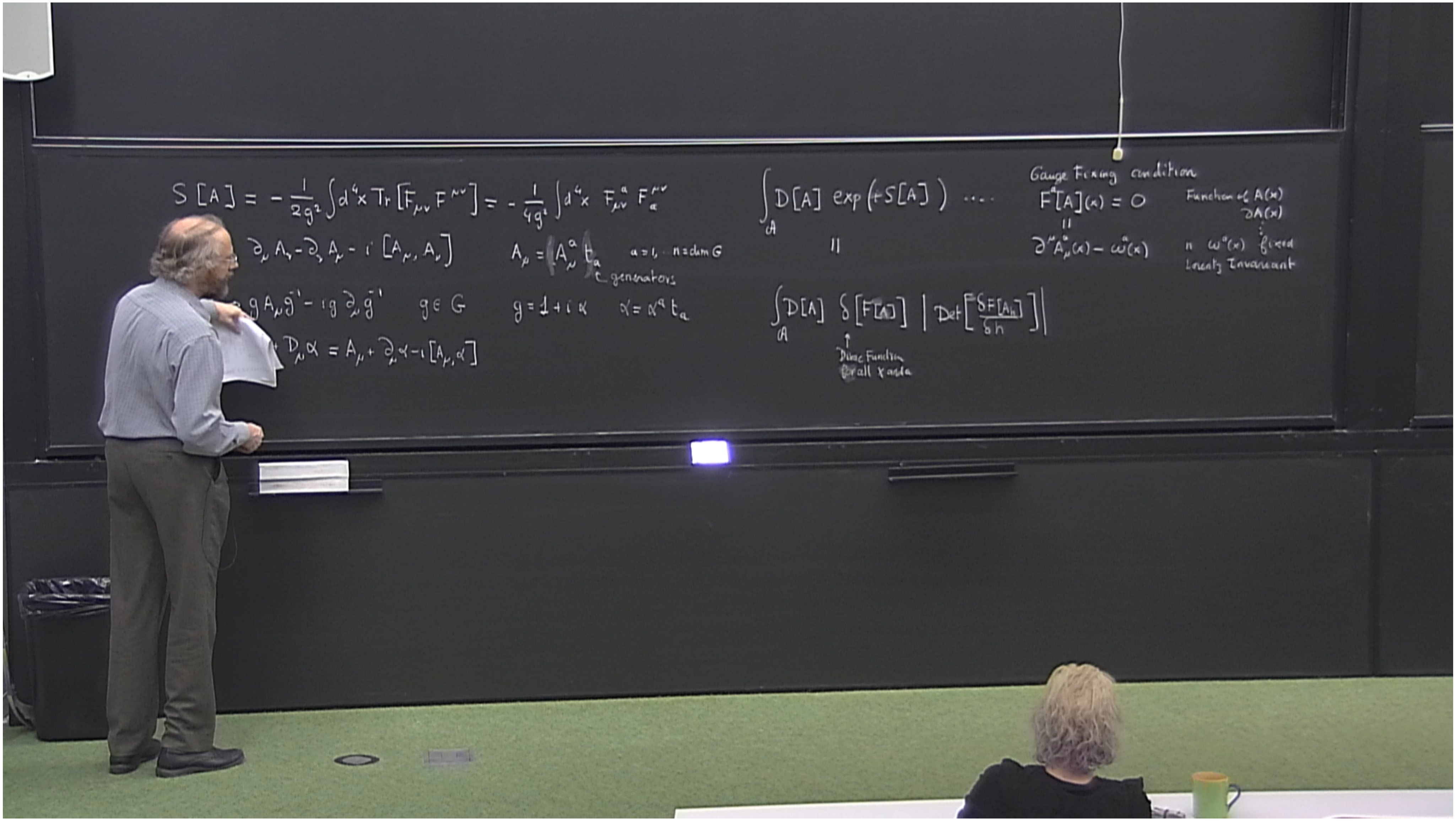

S [A] = - 1/2g² ∫d⁴x Tr [F_μν F^μν] = - 1/4g² ∫d⁴x F^a_μν F^μν_a
∫D[A] exp(+S[A]) ....
Gauge Fixing condition
F^a[A](x) = 0
Function of A(x) ∂A(x)
F_μν = ∂_μ A_ν - ∂_ν A_μ - i [A_μ, A_ν]
A_μ = A^a_μ t_a
a = 1, ... n = dim G
generators
∂^μ A^a_μ(x) - ω^a(x)
ω^a(x) fixed
Lorentz Invariant
A_μ → g A_μ g⁻¹ - i g ∂_μ g⁻¹  g ∈ G
g = 1 + i α  α = α^a t
A_μ → A_μ + D_μ α = A_μ + ∂_μ α - i [A_μ, α]


S[A] = - 1/2g² ∫d⁴x Tr[F_μν F^μν] = - 1/4g² ∫d⁴x F^a_μν F^μν_a
F_μν = ∂_μ A_ν - ∂_ν A_μ - i [A_μ, A_ν]
A_μ = A^a_μ t_a
generators
a = 1, ... n = dim G
A_μ → g A_μ g⁻¹ - i g ∂_μ g⁻¹
g ∈ G
g = 1 + iα
α = α^a t_a
A_μ → A_μ + D_μ α = A_μ + ∂_μ α - i [A_μ, α]
∫ D[A] exp(iS[A]) ...
∫ D[A]
Gauge Fixing condition
Function of A(x)
∂A(x)
Locally Invariant

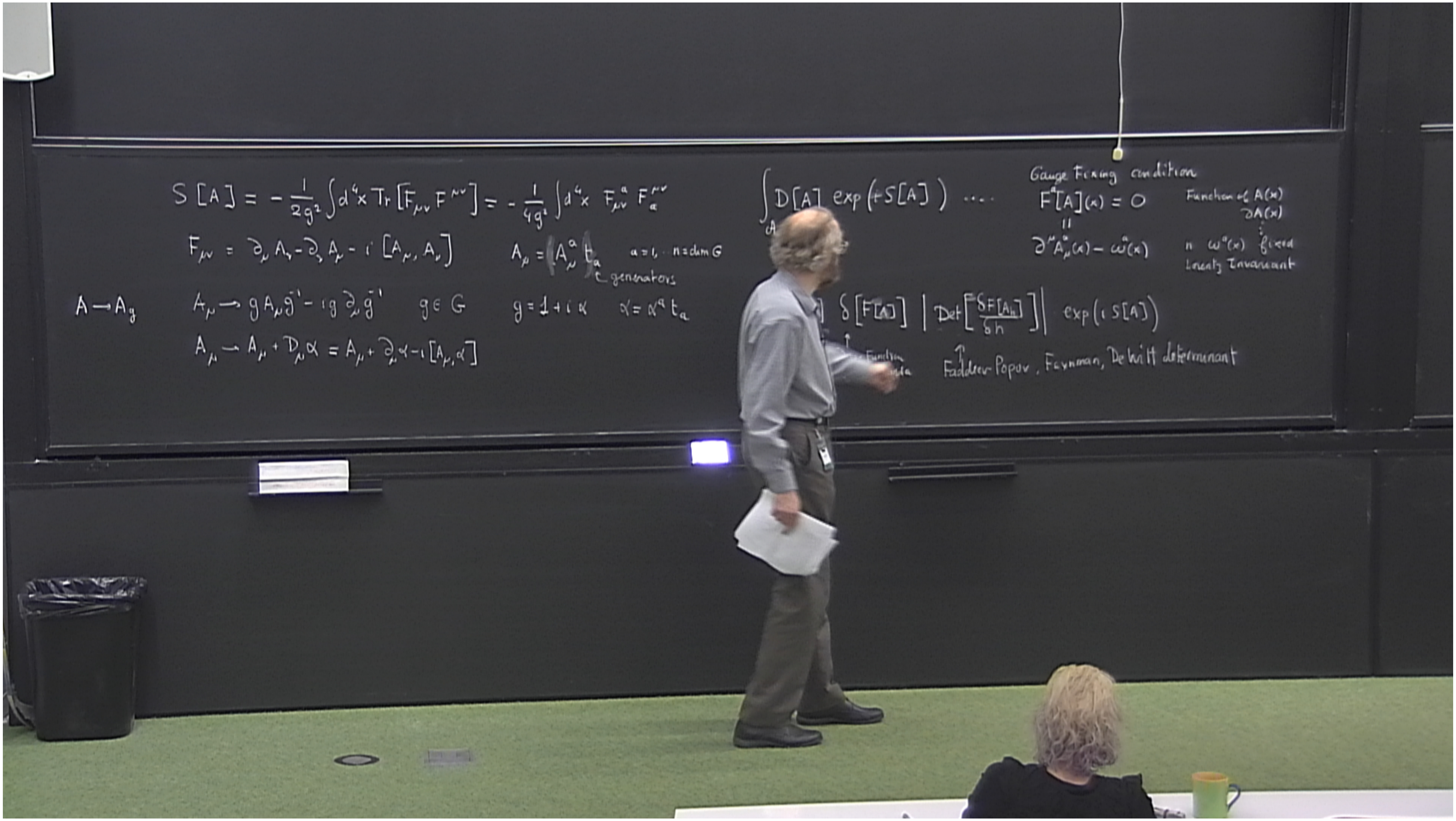


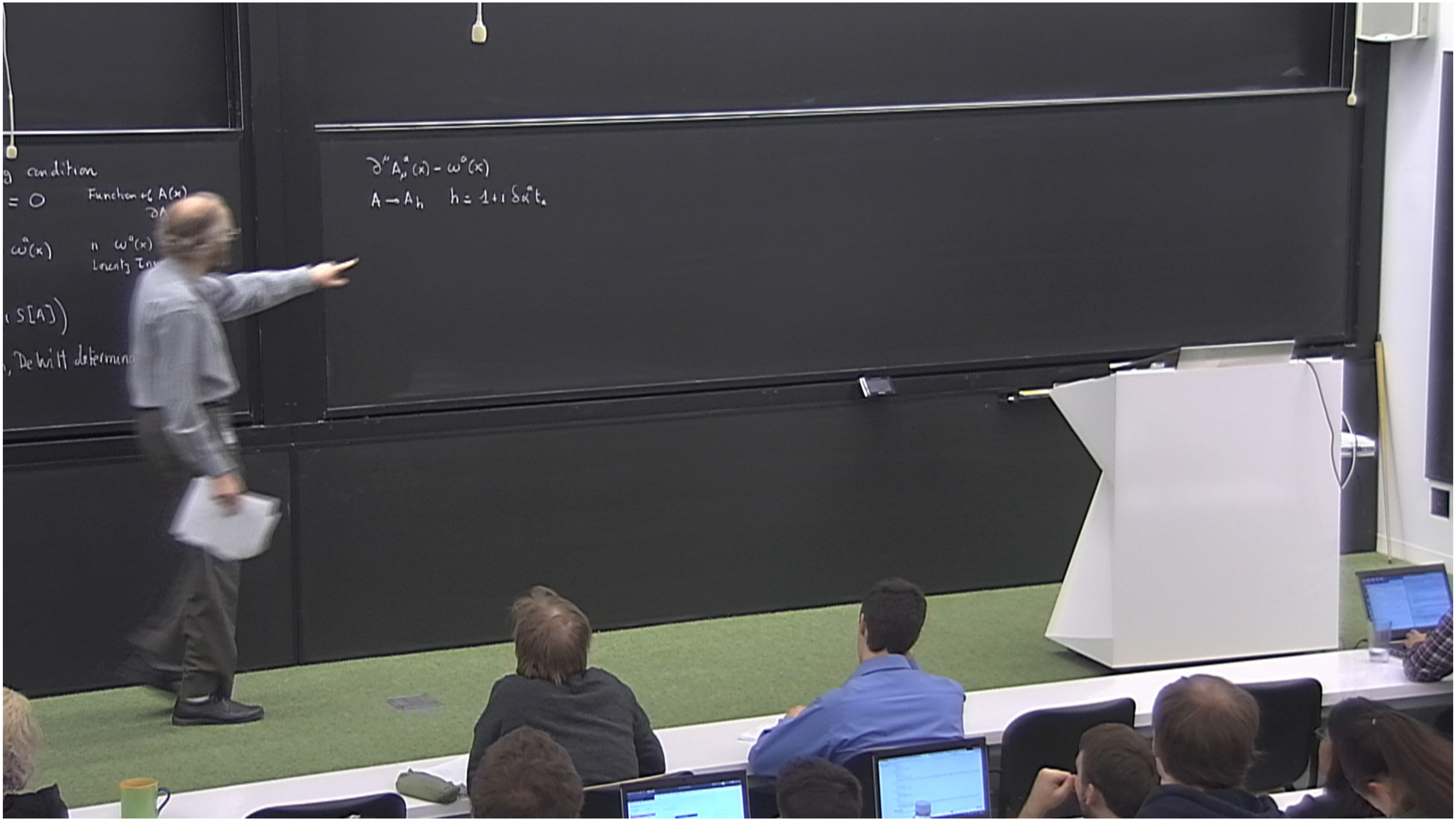
Gauge Fixing condition
generators

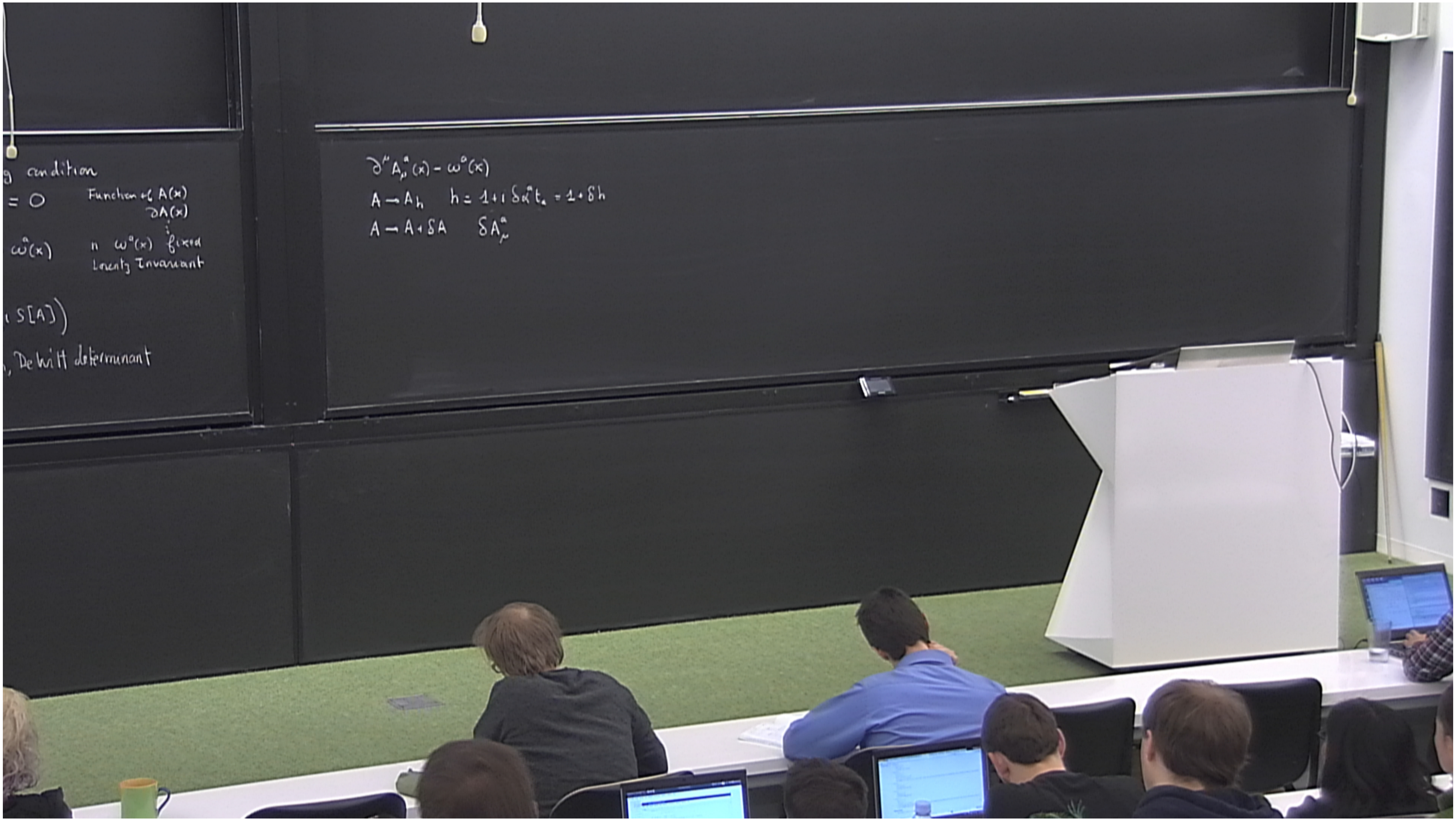

S [A] = - 1/2g² ∫d⁴x Tr [F_μν F^μν] = - 1/4g² ∫d⁴x F^a_μν F^μν_a
F_μν = ∂_μ A_ν - ∂_ν A_μ - i [A_μ, A_ν]
A_μ = A^a_μ t_a
generato
A_μ → g A_μ g⁻¹ - i g ∂_μ g⁻¹   g ∈ G
g = 1 + i α
A_μ → A_μ + D_μ α = A_μ + ∂_μ α - i [A_μ, α]
∫_A D[A] exp (i S[A]) ...
∫_A D[A] δ [F[A]]
Dirac Function
Gauge Fixing condition
F^a[A](x) = 0
Function of A(x)
∂A(x)
∂^μ A^a_μ(x) - ω^a(x)
ω^a(x) fixed
Locally Invariant

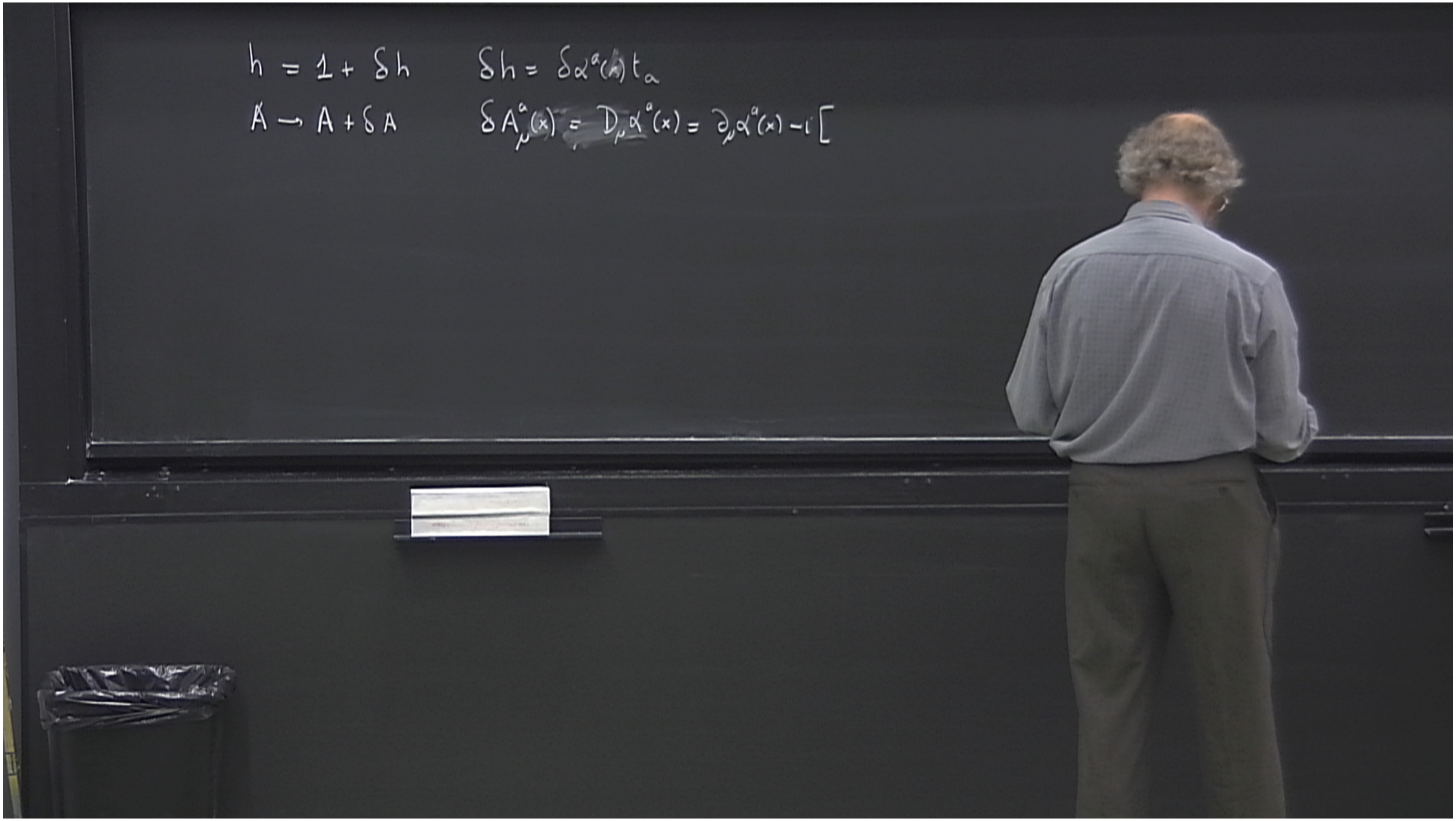


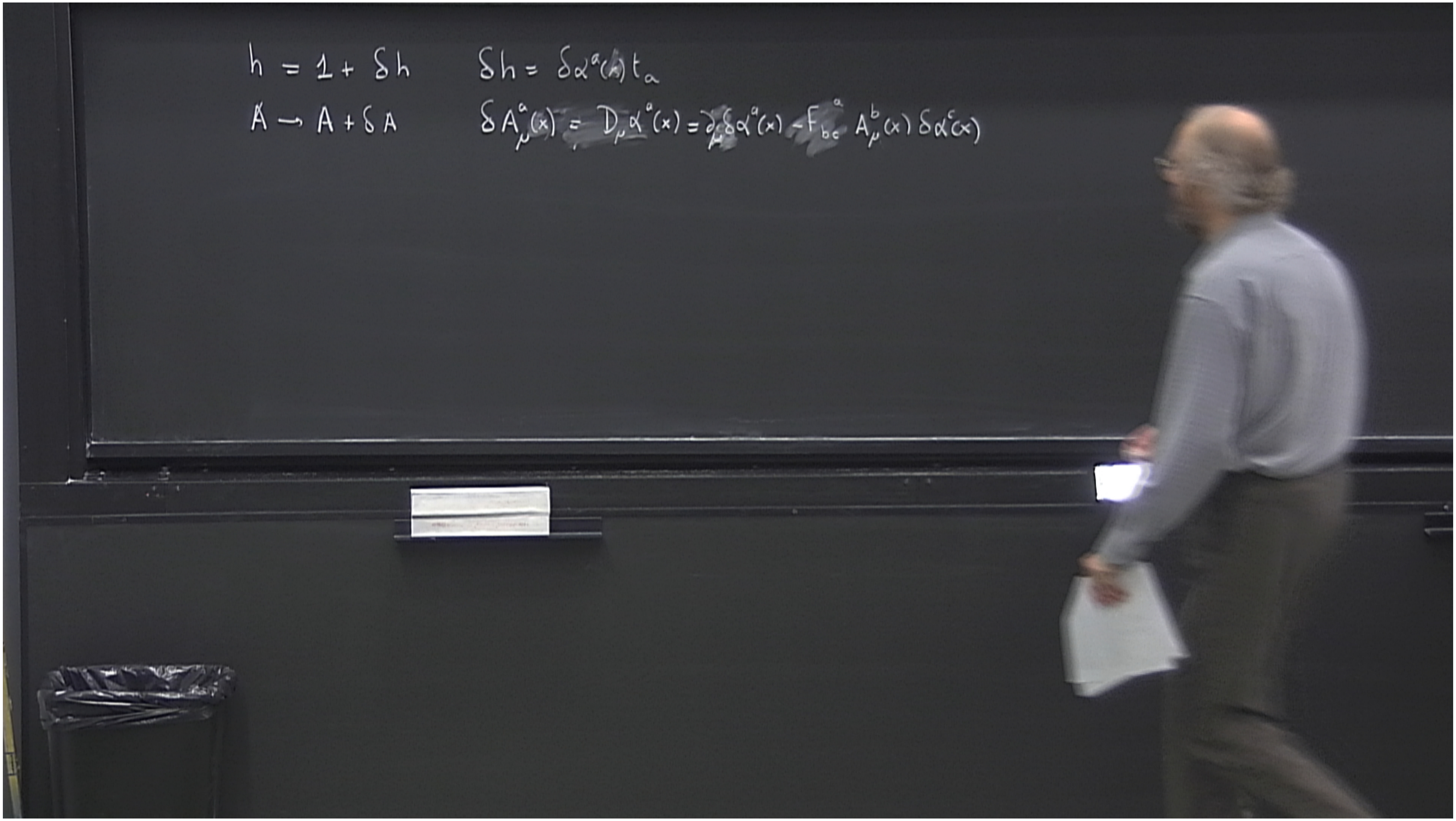
Gauge Fixing condition
Function of A(x)
Locally Invariant
generators
Dirac Function

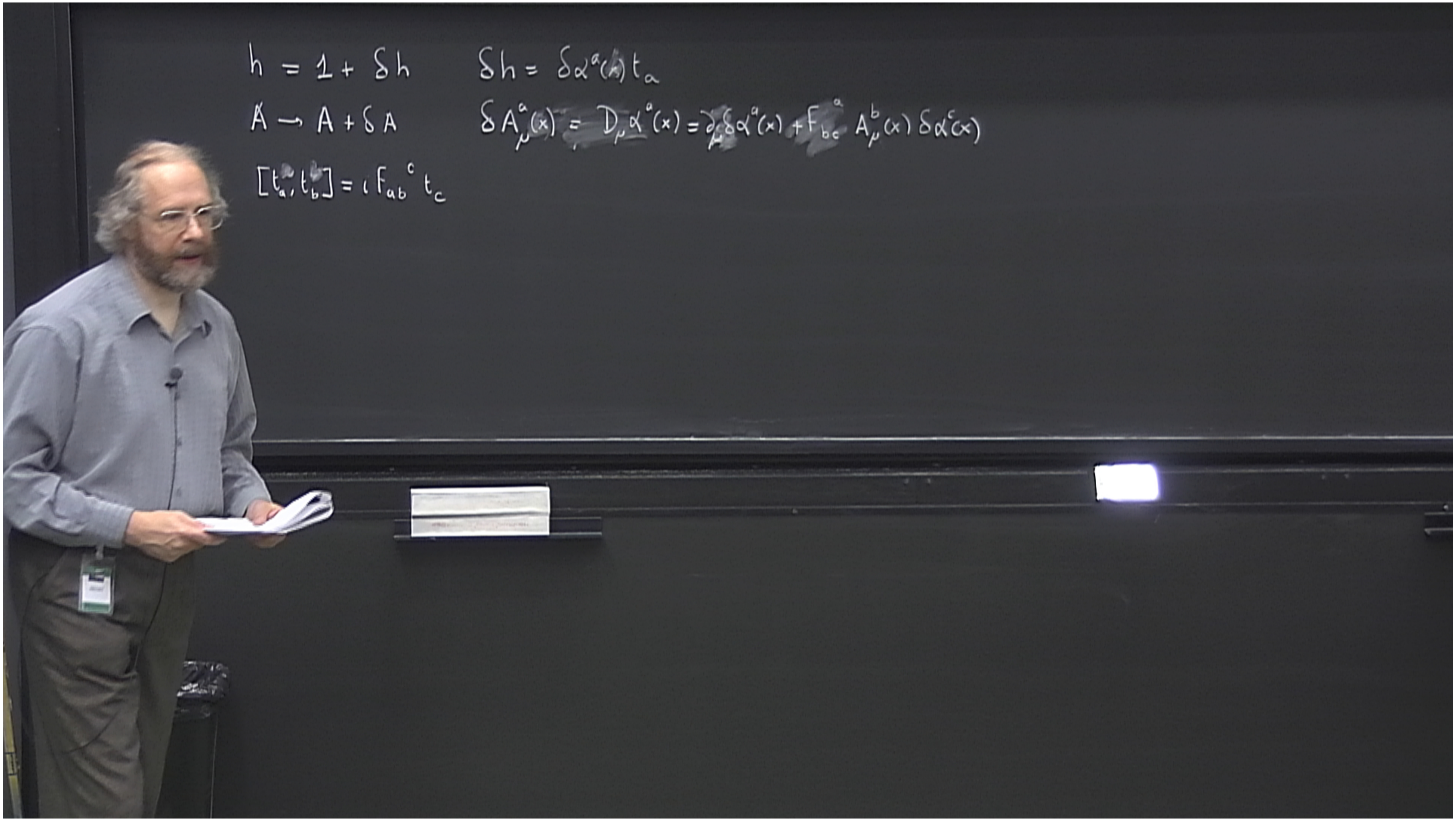

S[A] = - 1/2g² ∫d⁴x Tr[F_μν F^μν] = - 1/4g² ∫d⁴x F^a_μν F^μν_a
F_μν = ∂_μ A_ν - ∂_ν A_μ - i [A_μ, A_ν]
A_μ = A^a_μ t_a
a = 1, … n = dim G
generators
A → A_g
A_μ → g A_μ g⁻¹ - i g ∂_μ g⁻¹
g ∈ G
g = 1 + i α
α = α^a t_a
A_μ → A_μ + D_μ α = A_μ + ∂_μ α - i [A_μ, α]
∫D[A] exp(i S[A]) …
Gauge Fixing condition
F^a[A](x) = 0
∂^μ A^a_μ(x) - ω^a(x)
Function of A(x)
∂A(x)
n ω^a(x) fixed
Locally Invariant
δ[F[A]] | Det[δF[A_h]/δh] | exp(i S[A])
Faddeev-Popov, Feynman, DeWitt determinant

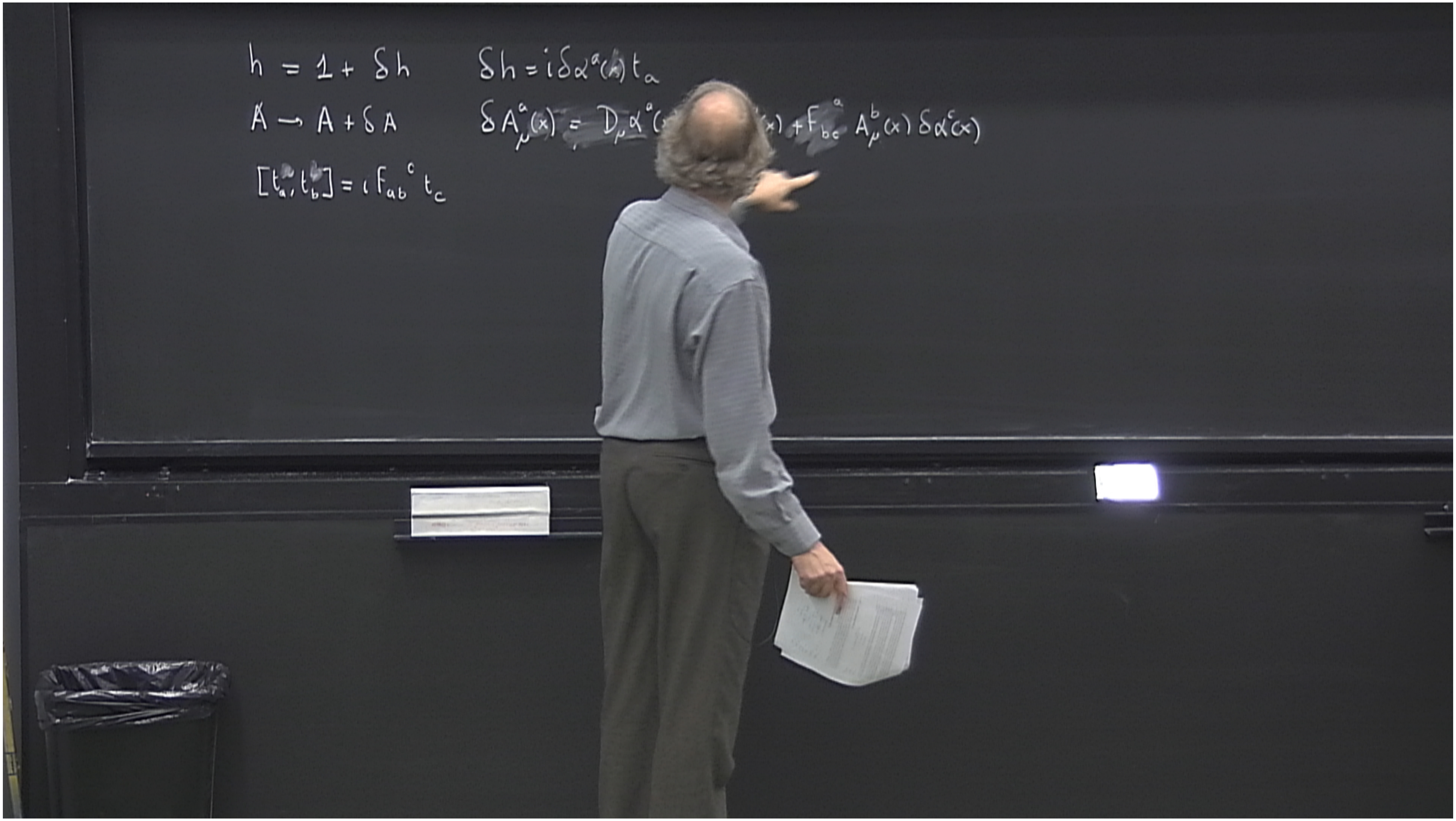

condition
Function of A(x)
S[A]
De Witt determin
$\partial^\mu A_\mu^a(x) - \omega^a(x)$
$A \to A_h \quad h = 1 + i\,\delta\alpha^a t_a$


g condition
= 0
Function of A(x)
∂A(x)
ω^a(x)
n ω^a(x) fixed
Lorentz Invariant
S[A])
, DeWitt determinant
∂^μ A^a_μ(x) - ω^a(x)
A → A_h h = 1 + i δα^a t_a = 1 + δh
A → A + δA δA^a_μ


h = 1 + δh    δh = δα^a(x) t_a
A → A + δA    δA^a_μ(x) = D_μ α^a(x) = ∂_μ α^a(x) − i [


h = 1 + δh   δh = δα^a(x) t_a
A → A + δA   δA^a_μ(x) = D_μ α^a(x) = ∂_μ δα^a(x) − f_bc^a A^b_μ(x) δα^c(x)


$h = 1 + \delta h$
$\delta h = \delta\alpha^a(x) t_a$
$A \to A + \delta A$
$\delta A^a_\mu(x) = D_\mu \alpha^a(x) = \partial_\mu \delta\alpha^a(x) + f_{bc}{}^a A^b_\mu(x) \delta\alpha^c(x)$
$[t_a, t_b] = i f_{ab}{}^c t_c$

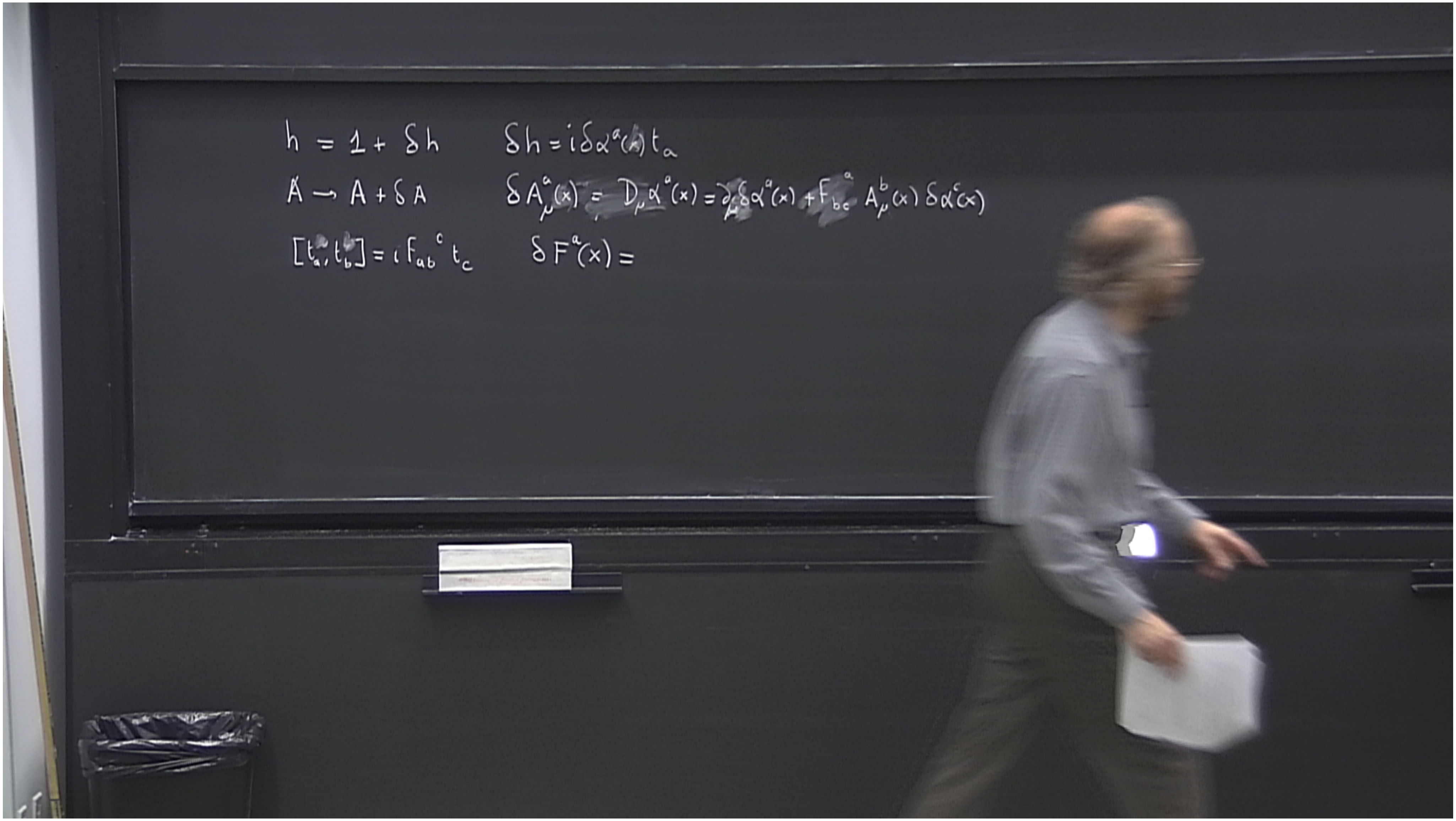


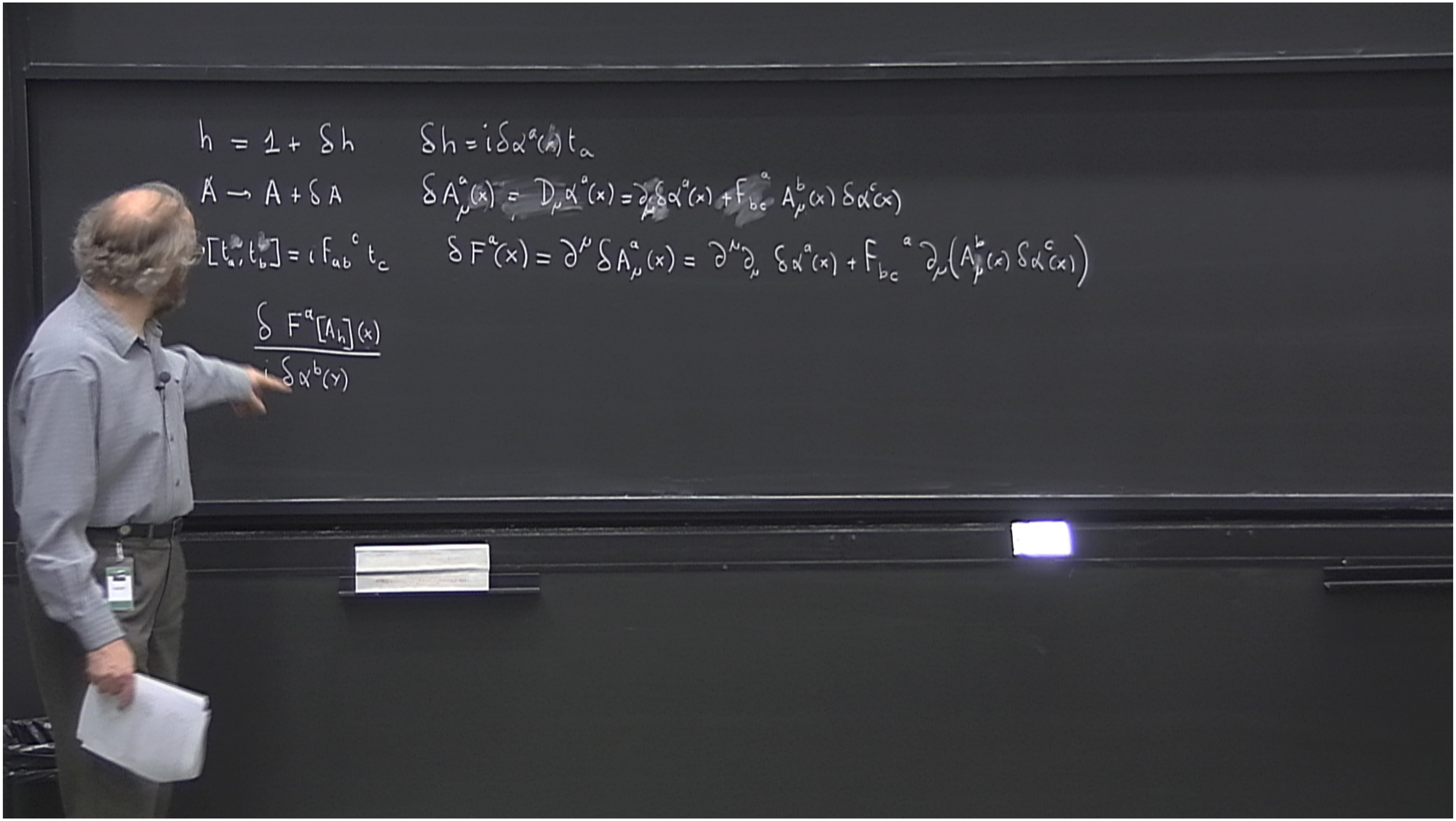
h = 1 + δh    δh = iδα^a(x) t_a
A → A + δA    δA^a_μ(x) = D_μ α^a(x) + F_bc^a A^b_μ(x) δα^c(x)
[t_a, t_b] = i F_ab^c t_c

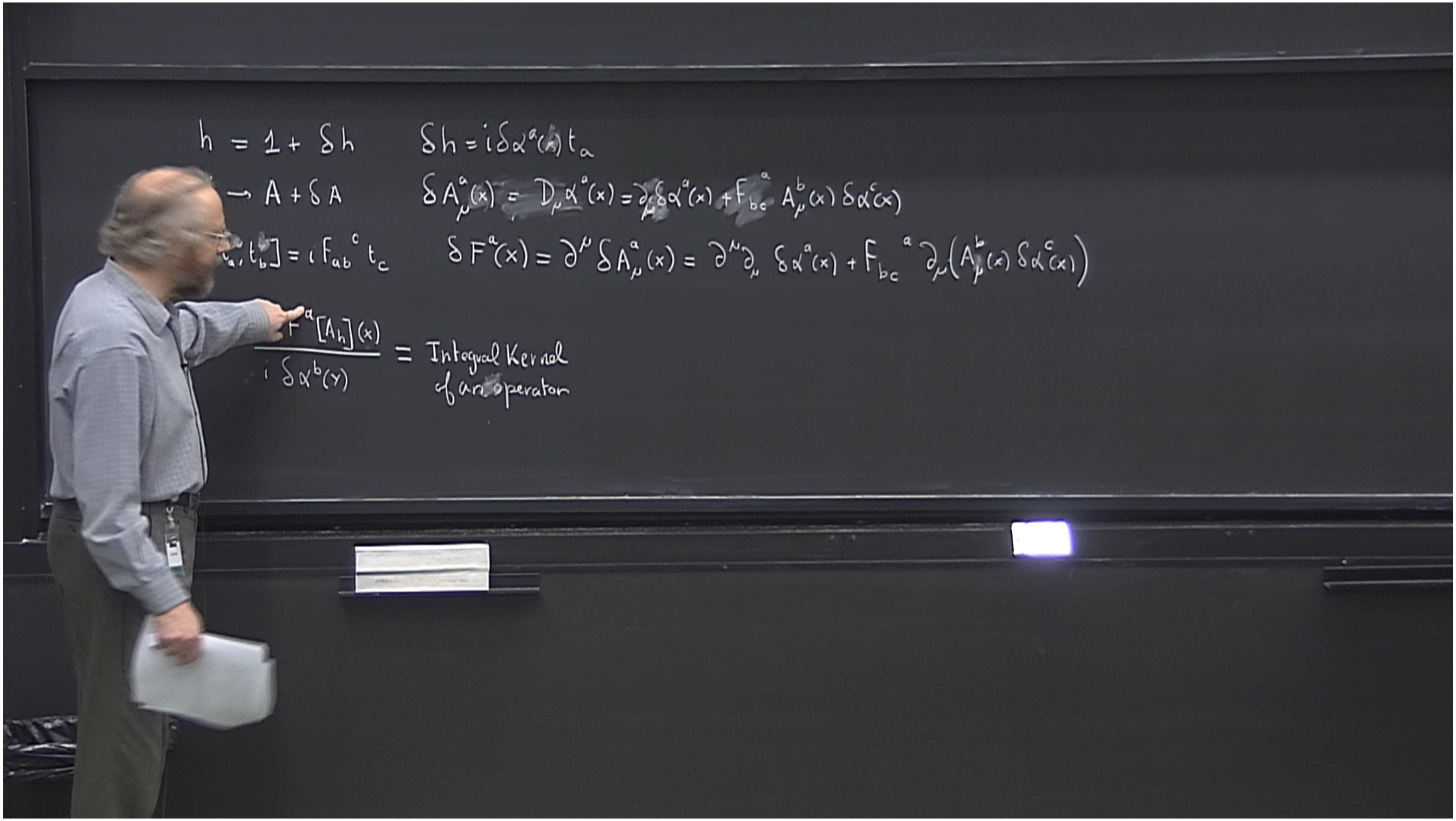
$$F^a(x) = \partial^\mu A_\mu^a(x) - \omega^a(x)$$

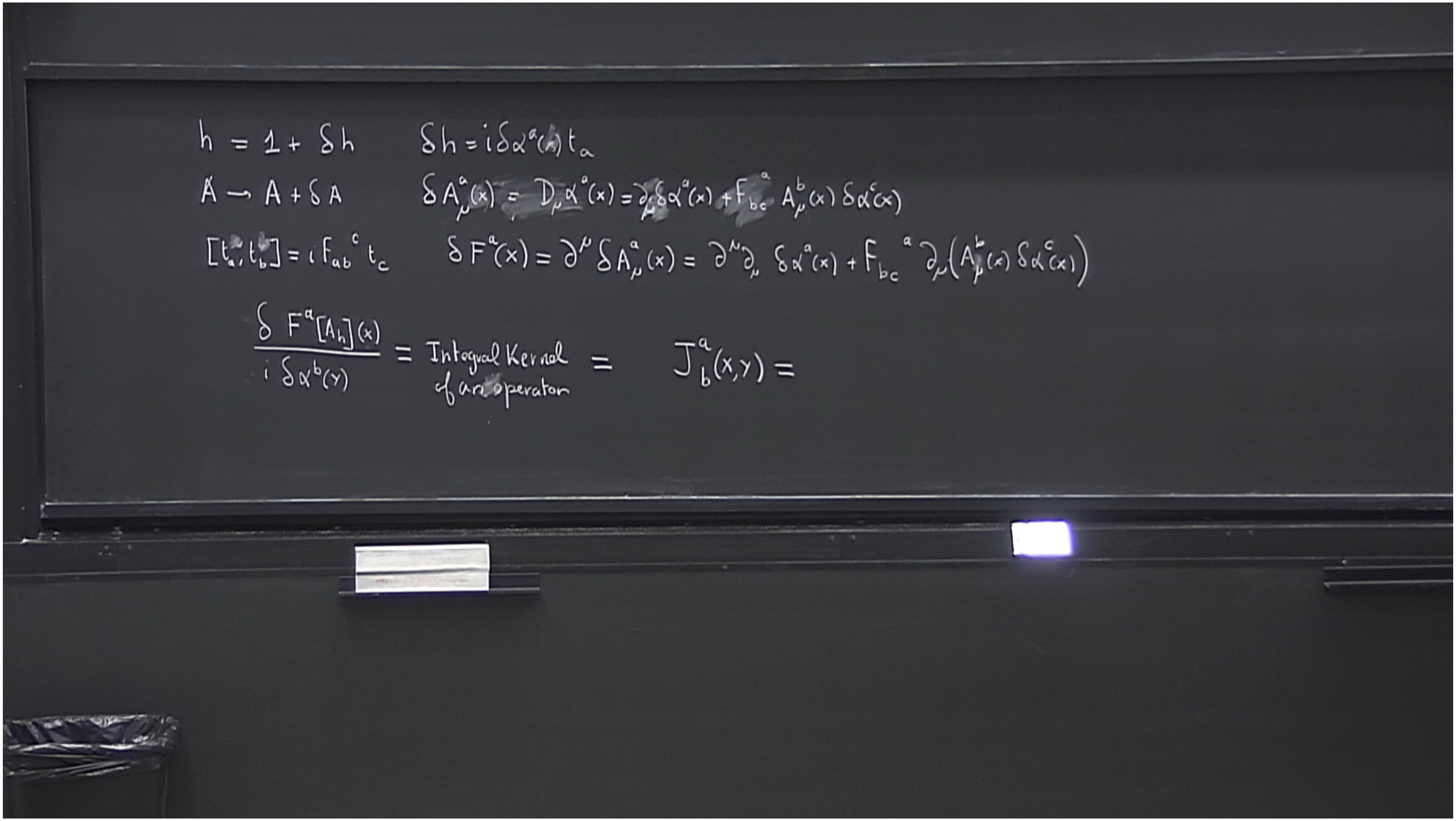
$$A \rightarrow A_h \qquad h = 1 + i\,\delta\alpha^a t_a = 1 + \delta h$$

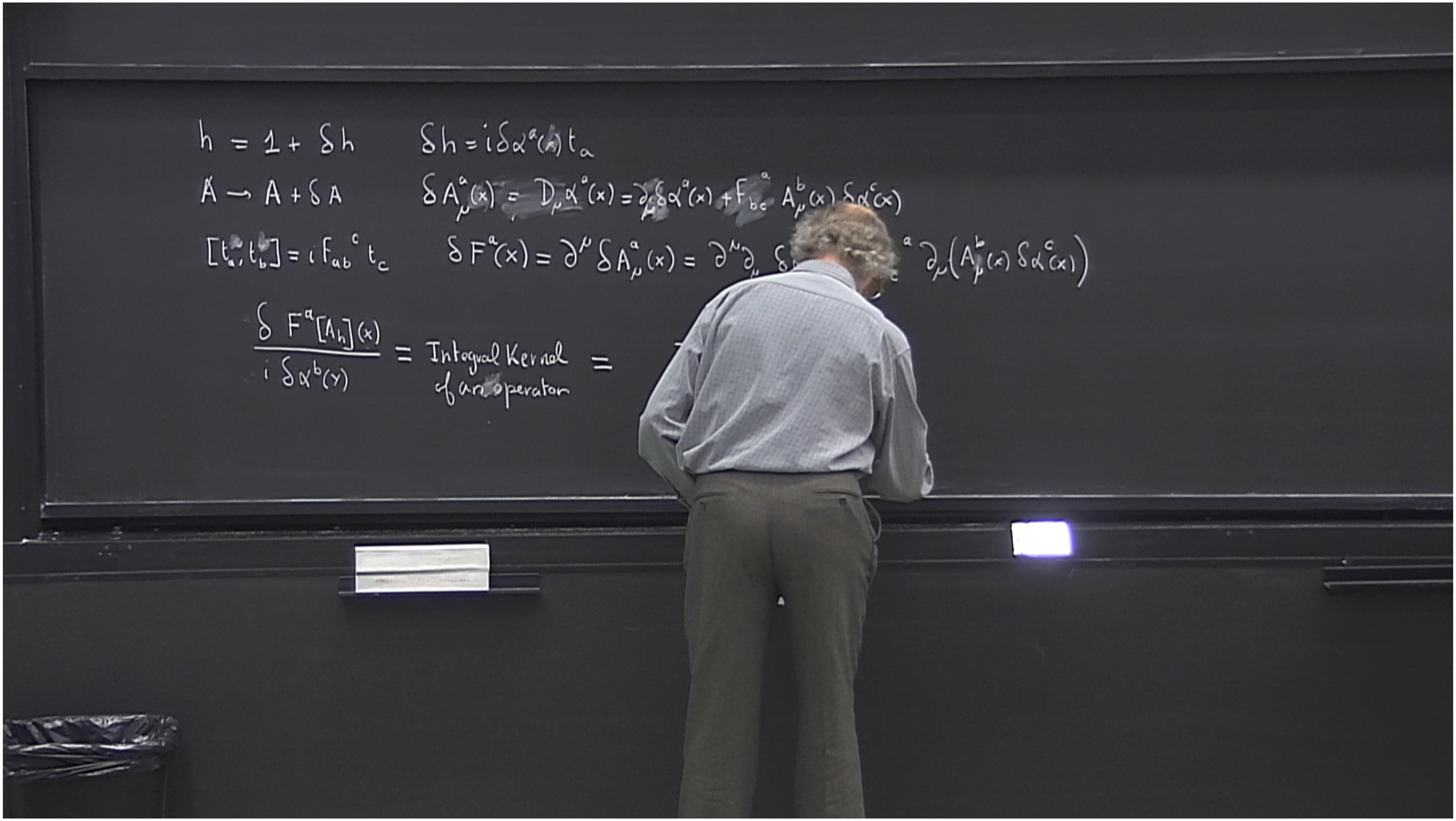
$$A \rightarrow A + \delta A \qquad \delta A_\mu^a$$

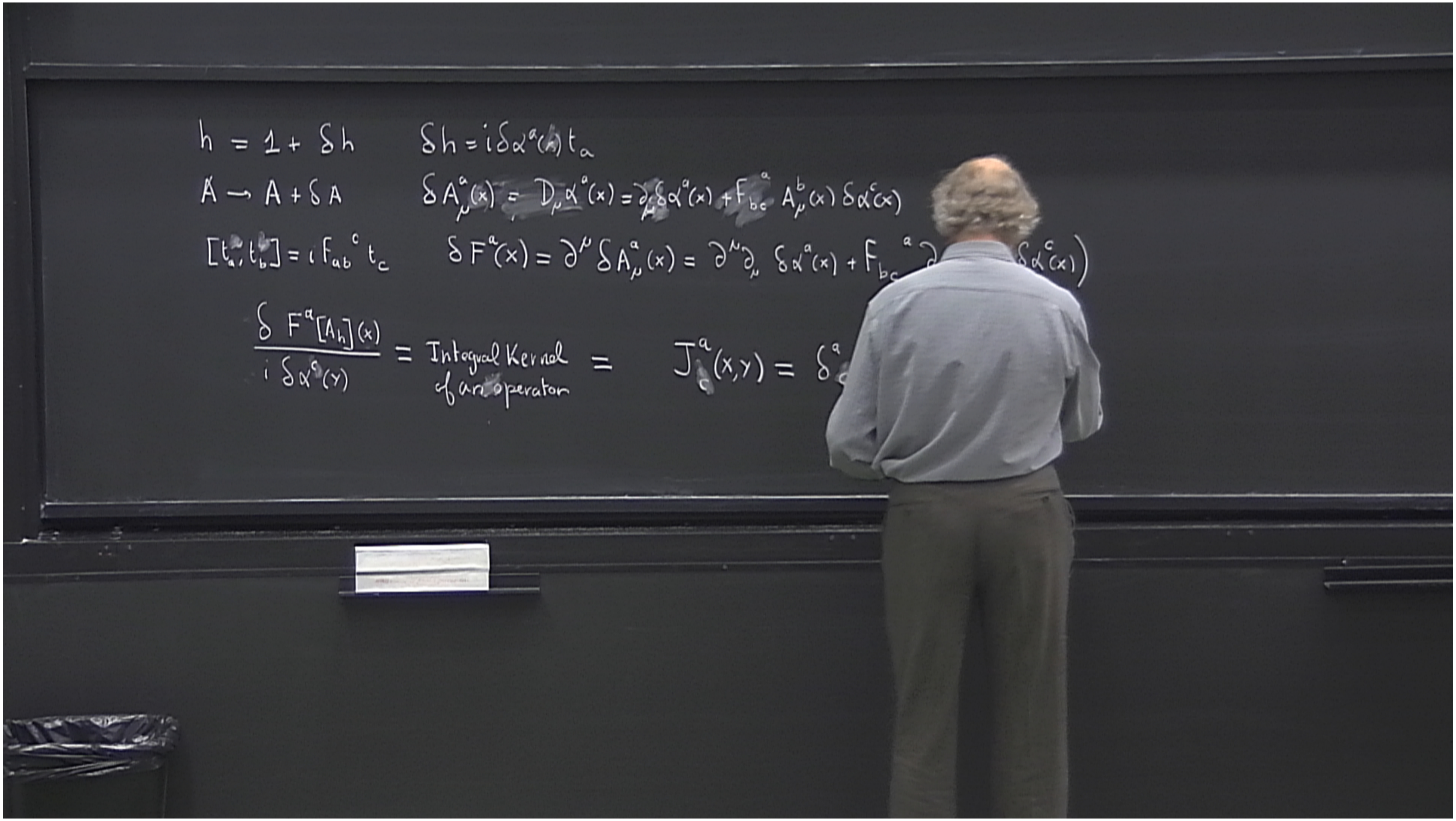

$h = 1 + \delta h$ $\delta h = i\delta\alpha^a(x) t_a$
$A \to A + \delta A$ $\delta A^a_\mu(x) = D_\mu \alpha^a(x) = \partial_\mu \delta\alpha^a(x) + f_{bc}{}^a A^b_\mu(x)\,\delta\alpha^c(x)$
$[t_a, t_b] = i f_{ab}{}^c t_c$ $\delta F^a(x) =$

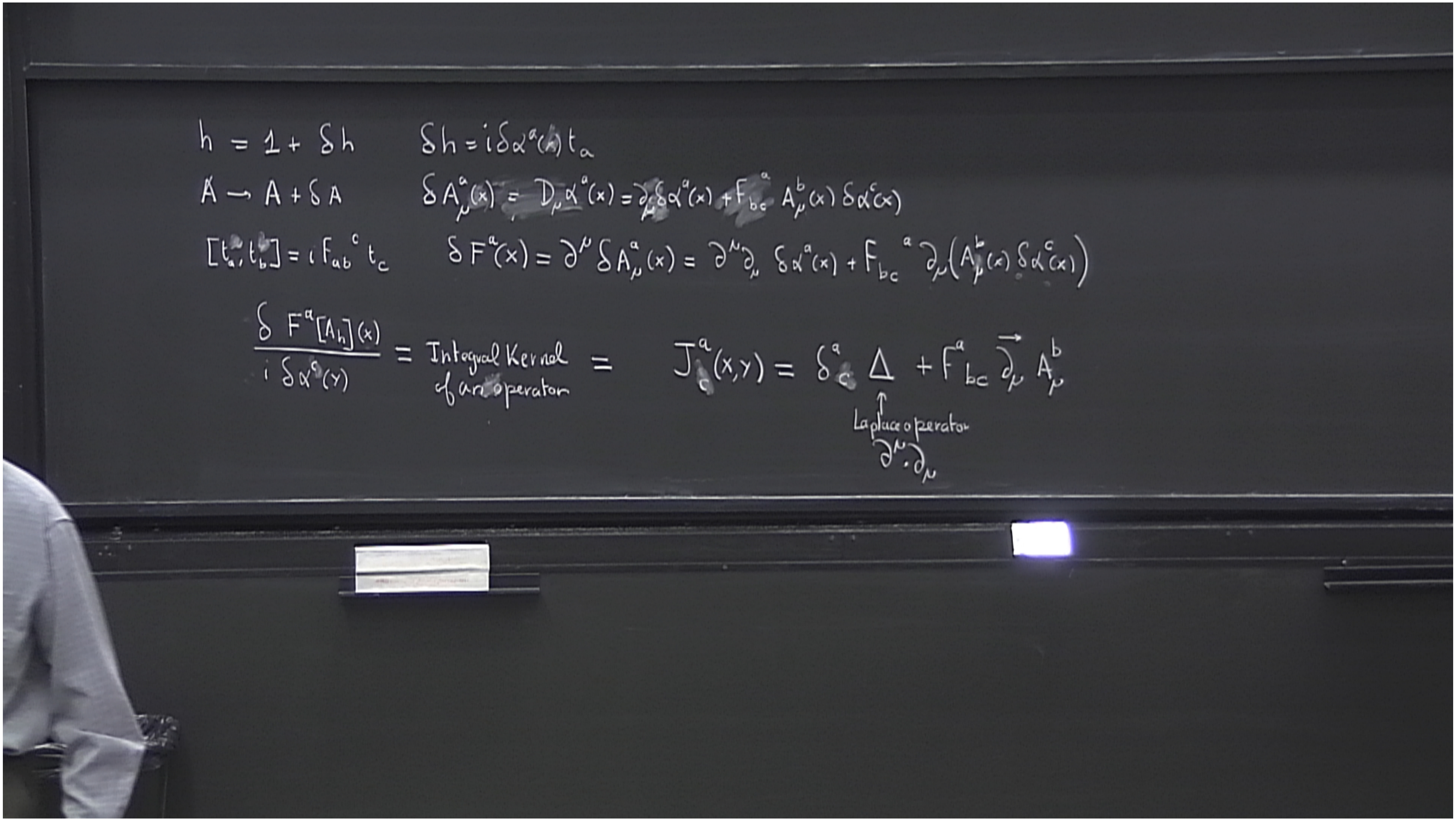

$h = 1 + \delta h$
$\delta h = i\delta\alpha^a(x) t_a$
$A \to A + \delta A$
$\delta A^a_\mu(x) = D_\mu \alpha^a(x) = \partial_\mu \delta\alpha^a(x) + F_{bc}{}^a A^b_\mu(x)\delta\alpha^c(x)$
$[t_a, t_b] = i F_{ab}{}^c t_c$
$\delta F^a(x) = \partial^\mu \delta A^a_\mu(x) = \partial^\mu\partial_\mu \delta\alpha^a(x) + F_{bc}{}^a \partial_\mu (A^b_\mu(x)\delta\alpha^c(x))$
$\frac{\delta F^a[A_h](x)}{i\delta\alpha^b(y)}$


h = 1 + δh    δh = iδα^a(x) t_a
→ A + δA    δA^a_μ(x) = D_μ α^a(x) = ∂_μ δα^a(x) + F_bc^a A^b_μ(x) δα^c(x)
t_b] = i f_ab^c t_c    δF^a(x) = ∂^μ δA^a_μ(x) = ∂^μ ∂_μ δα^a(x) + F_bc^a ∂_μ(A^b_μ(x) δα^c(x))
F^a[A_h](x)
i δα^b(y)
= Integral Kernel
of an operator

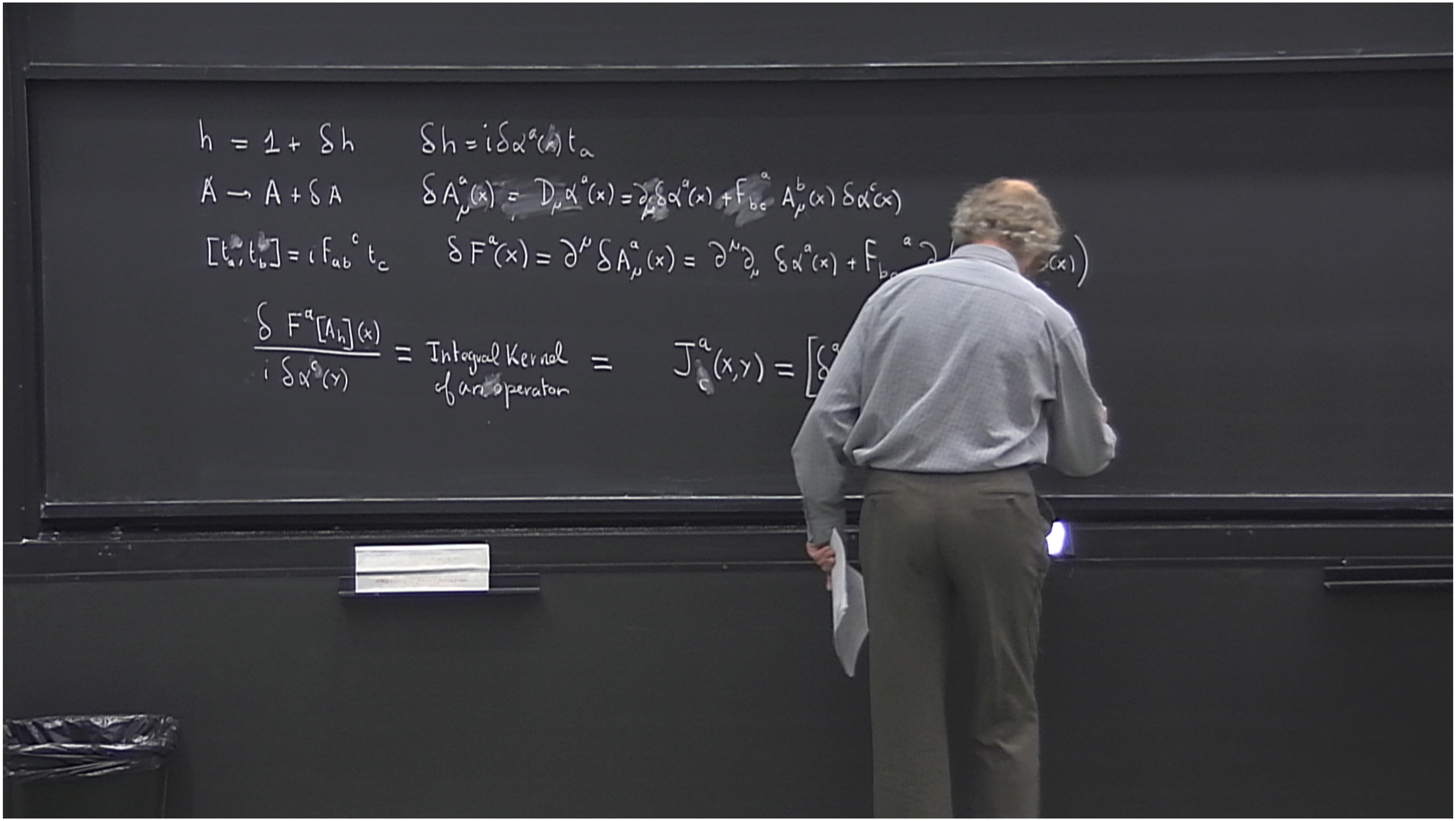

$h = 1 + \delta h$
$\delta h = i\delta\alpha^a(x) t_a$
$A \to A + \delta A$
$\delta A^a_\mu(x) = D_\mu \alpha^a(x) = \partial_\mu \delta\alpha^a(x) + F_{bc}{}^a A^b_\mu(x)\delta\alpha^c(x)$
$[t_a, t_b] = i f_{ab}{}^c t_c$
$\delta F^a(x) = \partial^\mu \delta A^a_\mu(x) = \partial^\mu \partial_\mu \delta\alpha^a(x) + F_{bc}{}^a \partial_\mu (A^b_\mu(x)\delta\alpha^c(x))$
$\frac{\delta F^a[A_h](x)}{i\delta\alpha^b(y)}$ = Integral Kernel of an operator = $J^a_b(x,y) =$

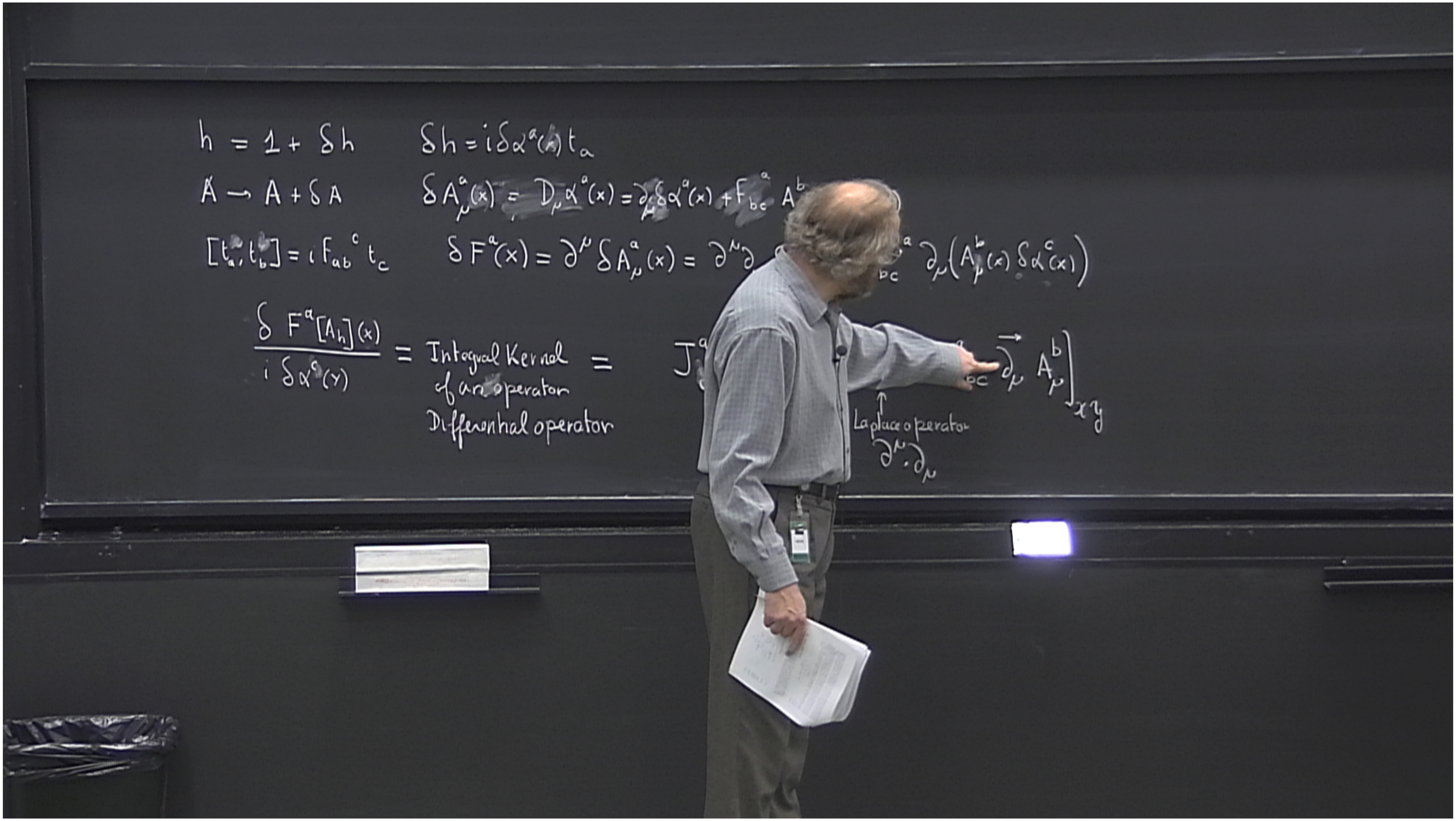

h = 1 + δh    δh = iδα^a(x) t_a
A → A + δA    δA^a_μ(x) = D_μ α^a(x) = ∂_μ δα^a(x) + F_bc^a A^b_μ(x) δα^c(x)
[t_a, t_b] = i F_ab^c t_c    δF^a(x) = ∂^μ δA^a_μ(x) = ∂^μ ∂_μ δ
∂_μ(A^b_μ(x) δα^c(x))
δ F^a[A_h](x) / i δα^b(y) = Integral Kernel of an operator =

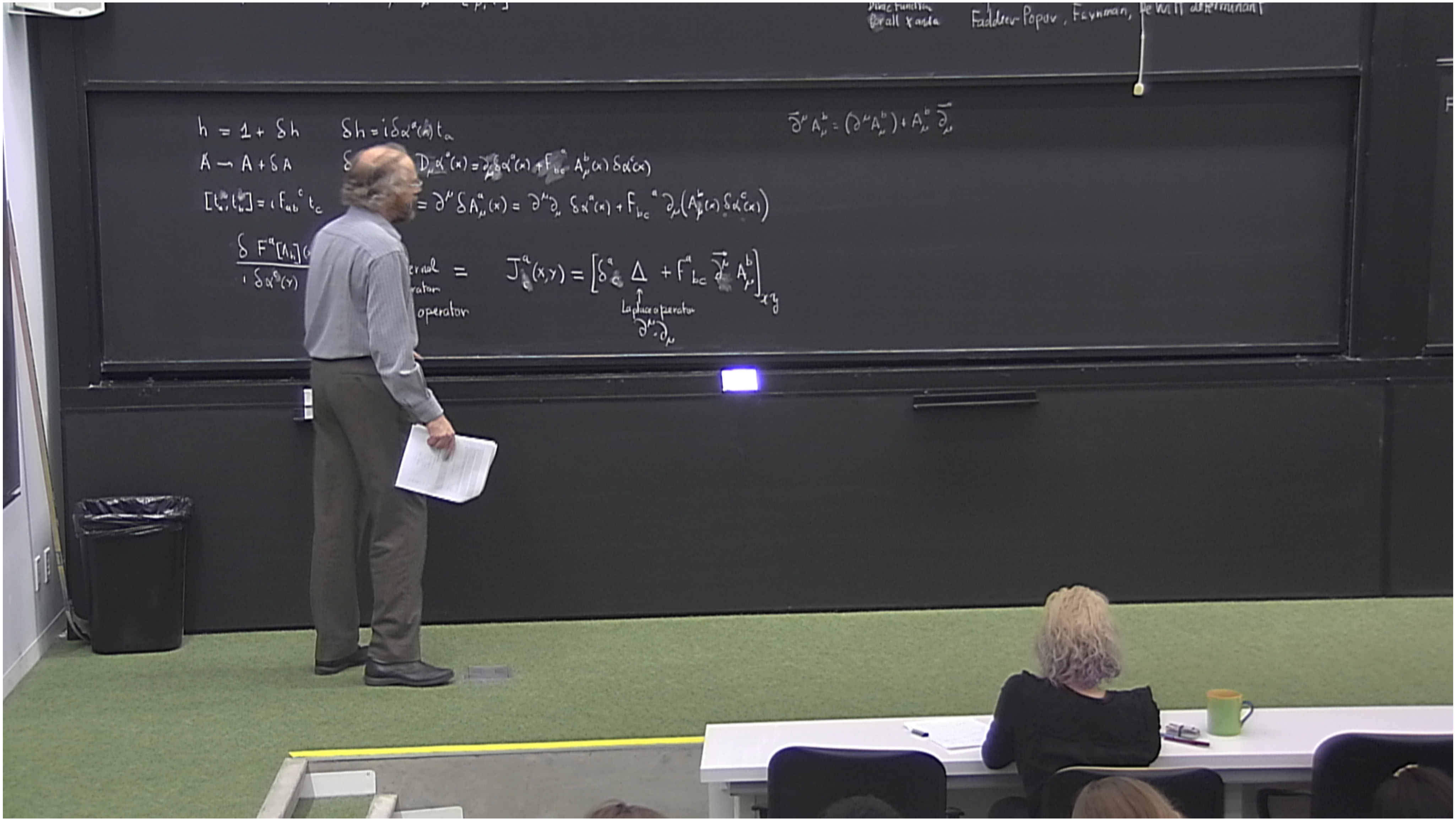

h = 1 + δh
δh = iδα^a(x) t_a
A → A + δA
δA^a_μ(x) = D_μ α^a(x) = ∂_μ δα^a(x) + F_bc^a A^b_μ(x) δα^c(x)
[t_a, t_b] = i f_ab^c t_c
δF^a(x) = ∂^μ δA^a_μ(x) = ∂^μ ∂_μ δα^a(x) + F_bc^a
δF^a[A_h](x) / i δα^c(y) = Integral Kernel of an operator = J^a_c(x,y) = δ^a

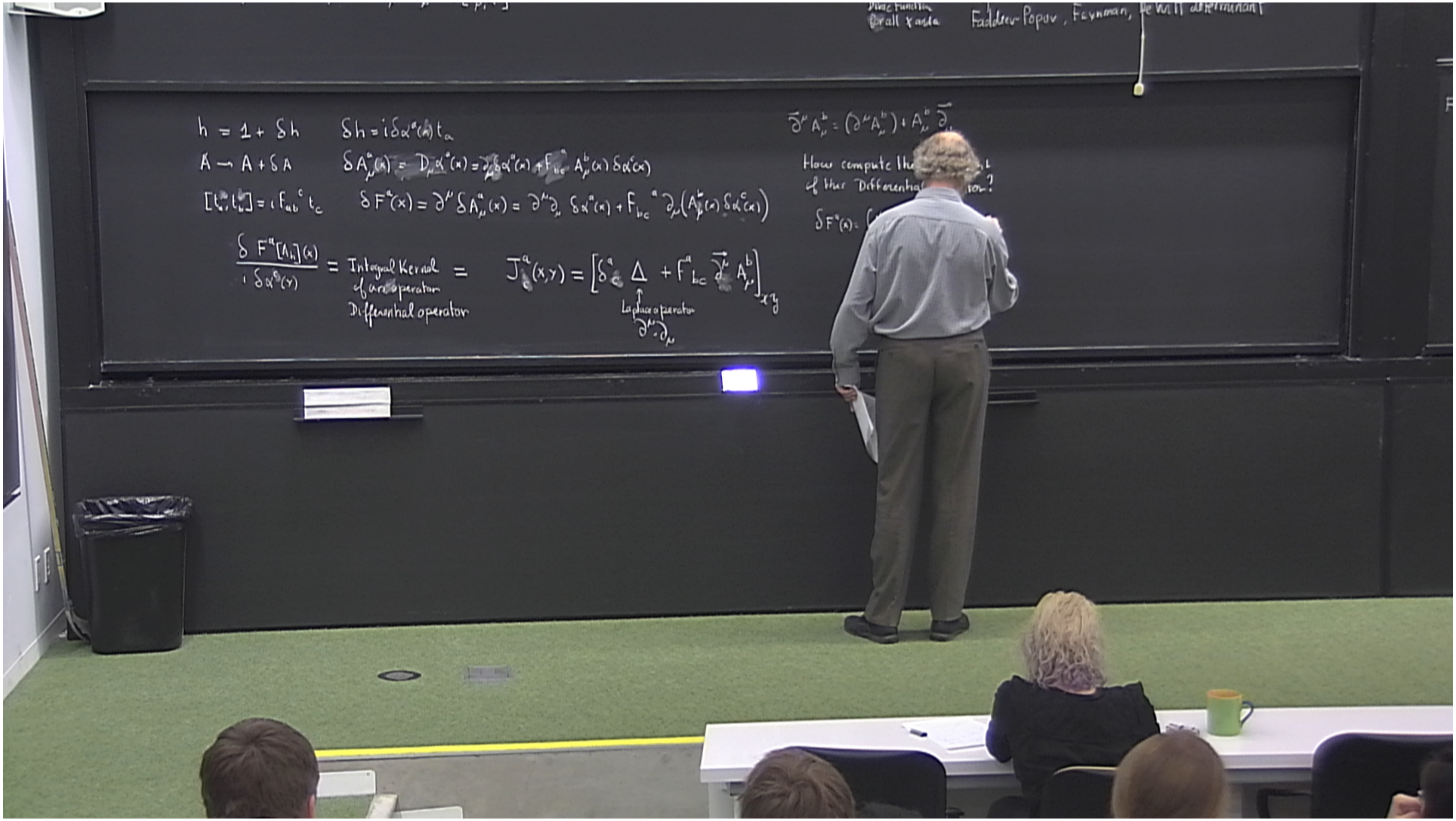


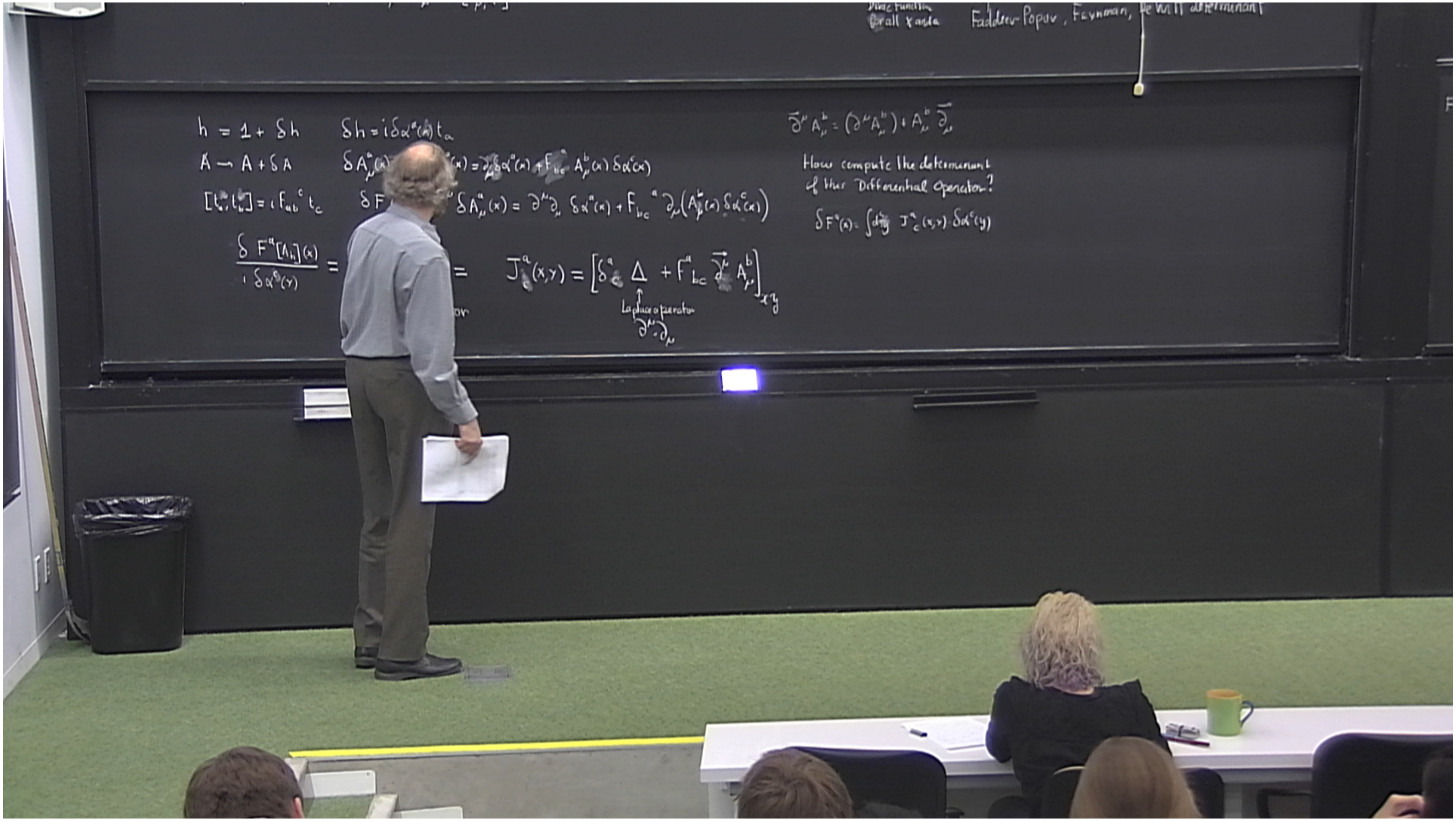
$h = 1 + \delta h \qquad \delta h = i\,\delta\alpha^a(x)\, t_a$
$A \to A + \delta A \qquad \delta A^a_\mu(x) = D_\mu \alpha^a(x) = \partial_\mu \delta\alpha^a(x) + F_{bc}{}^a A^b_\mu(x)\, \delta\alpha^c(x)$
$[t_a, t_b] = i F_{ab}{}^c t_c \qquad \delta F^a(x) = \partial^\mu \delta A^a_\mu(x) = \partial^\mu \partial_\mu \delta\alpha^a(x) + F_{bc}{}^a \partial_\mu \left(A^b_\mu(x)\, \delta\alpha^c(x)\right)$
$\frac{\delta F^a[A_h](x)}{i\,\delta\alpha^c(y)}$ = Integral Kernel of an operator = $J^a_c(x,y) = \delta^a_c \Delta + F^a{}_{bc} \vec{\partial}_\mu A^b_\mu$
Laplace operator
$\partial^\mu \cdot \partial_\mu$


h = 1 + δh
A → A + δA
Integral Kernel
of an operator

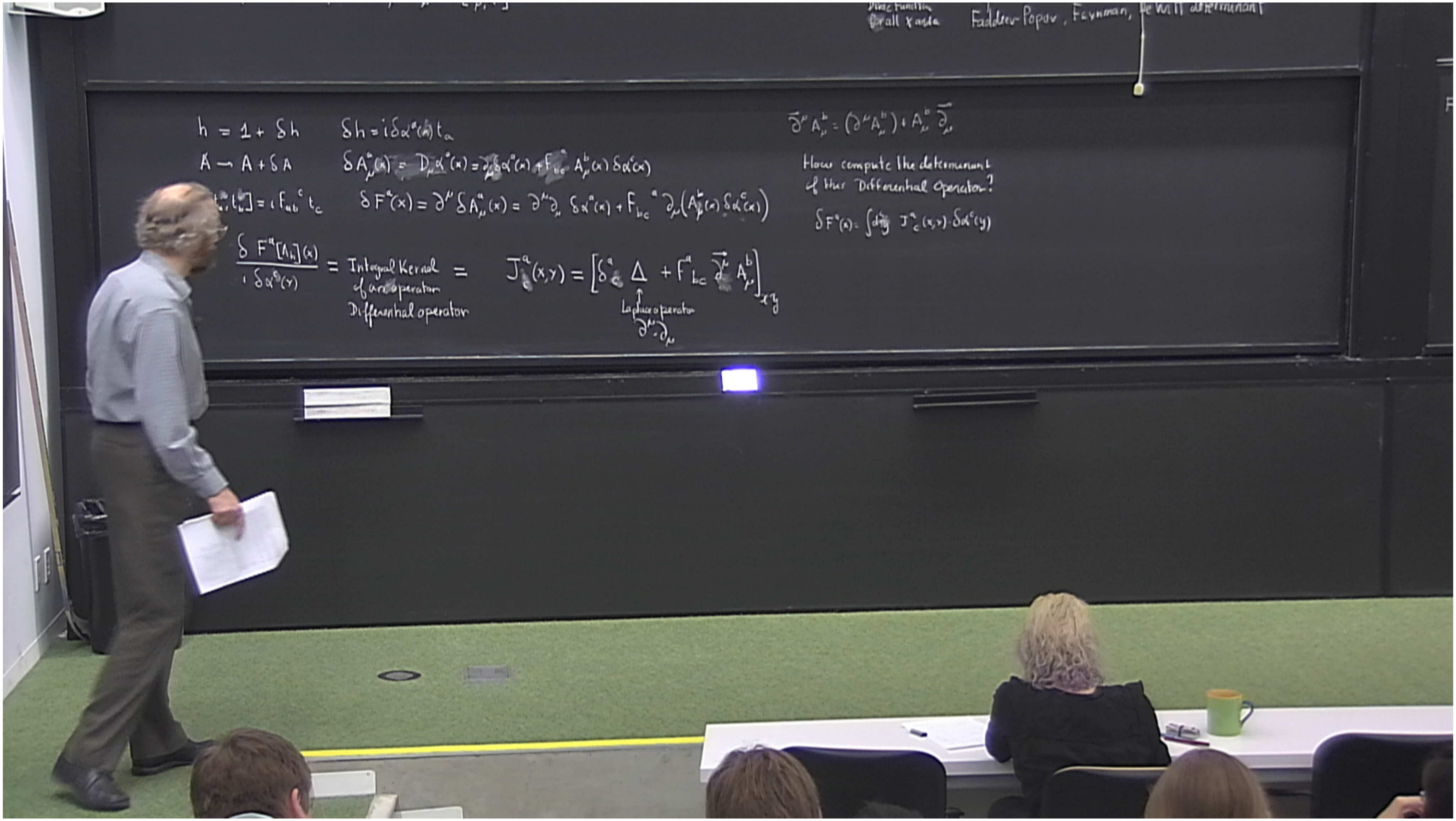

h = 1 + δh
δh = iδα^a(x) t_a
A → A + δA
δA^a_μ(x) = D_μ α^a(x) = ∂_μ δα^a(x) + F_bc^a A^b
[t_a, t_b] = i F_ab^c t_c
δF^a(x) = ∂^μ δA^a_μ(x) = ∂^μ ∂
∂_μ (A^b_μ(x) δα^c(x))
δF^a[A_h](x) / iδα^c(y) = Integral Kernel of an operator = J
Differential operator
Laplace operator
∂^μ · ∂_μ

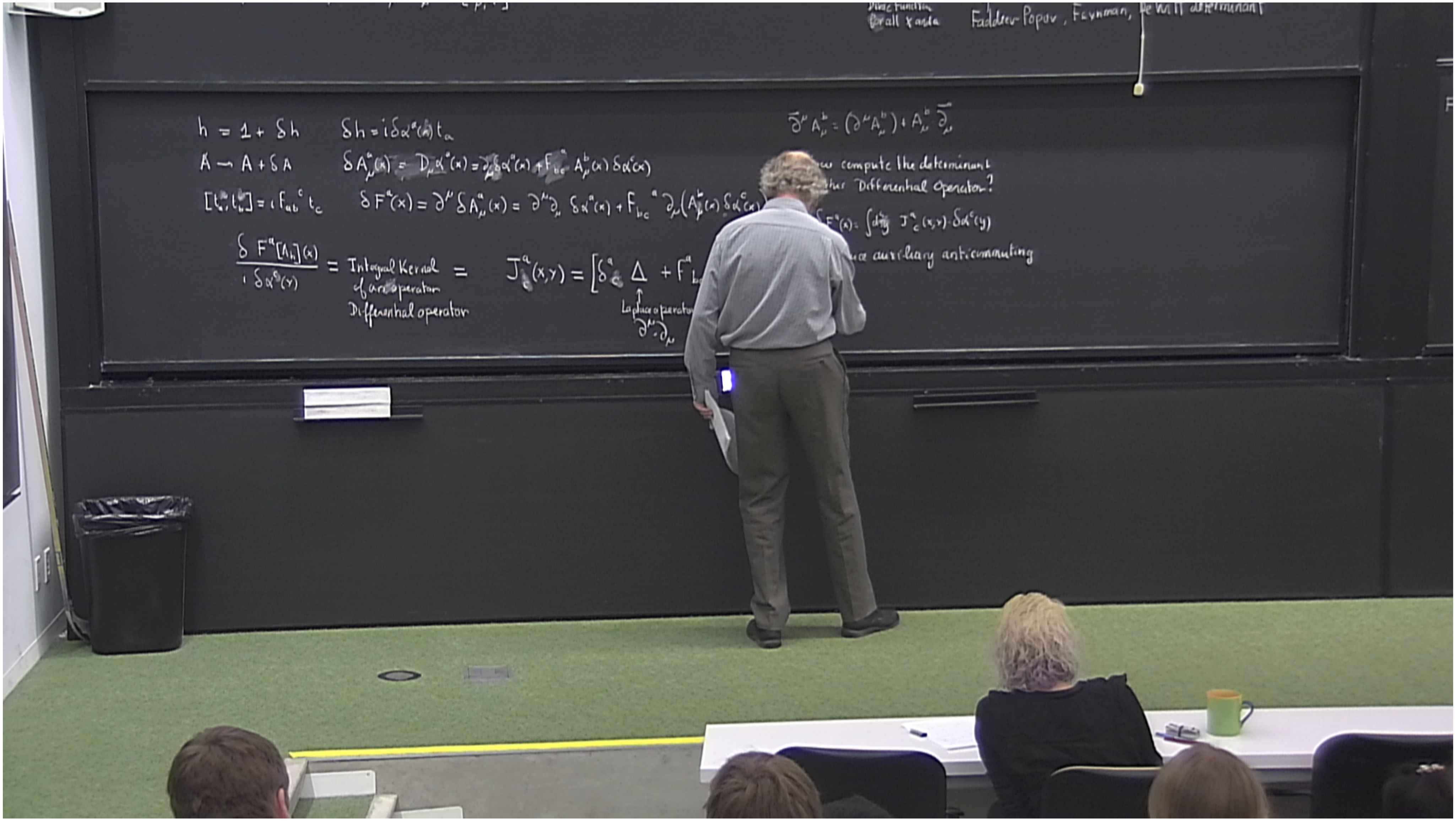

Faddeev-Popov, Feynman, DeWitt determinant
Laplace operator
operator


Faddeev-Popov, Feynman, DeWitt determinant
How compute the
of this Differential
Integral Kernel
of an operator
Differential operator
Laplace operator



Faddeev-Popov, Feynman, DeWitt determinant
How compute the determinant
of this Differential Operator?
Laplace operator


Faddeev-Popov, Feynman, DeWitt determinant
How compute the determinant
of this Differential Operator?
Integral Kernel
of an operator
Differential operator
Laplace operator


Faddeev-Popov, Feynman, De Witt determinant
h = 1 + δh   δh = iδα^a(x) t_a
A → A + δA
δF^a(x) = ∂^μ δA^a_μ(x)
Integral Kernel
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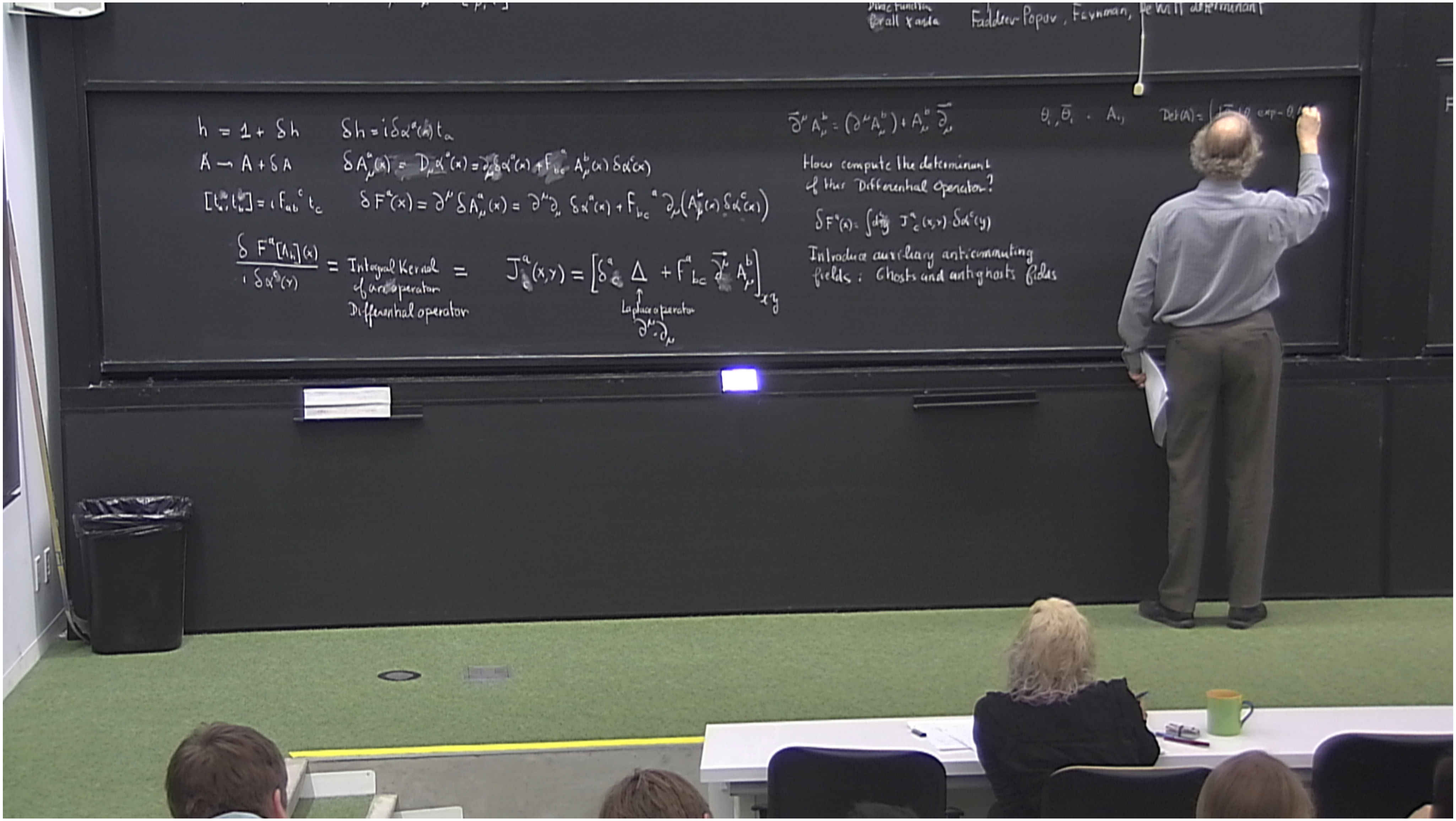
Differential operator
Laplace operator
compute the determinant
Differential Operator?
auxiliary anticommuting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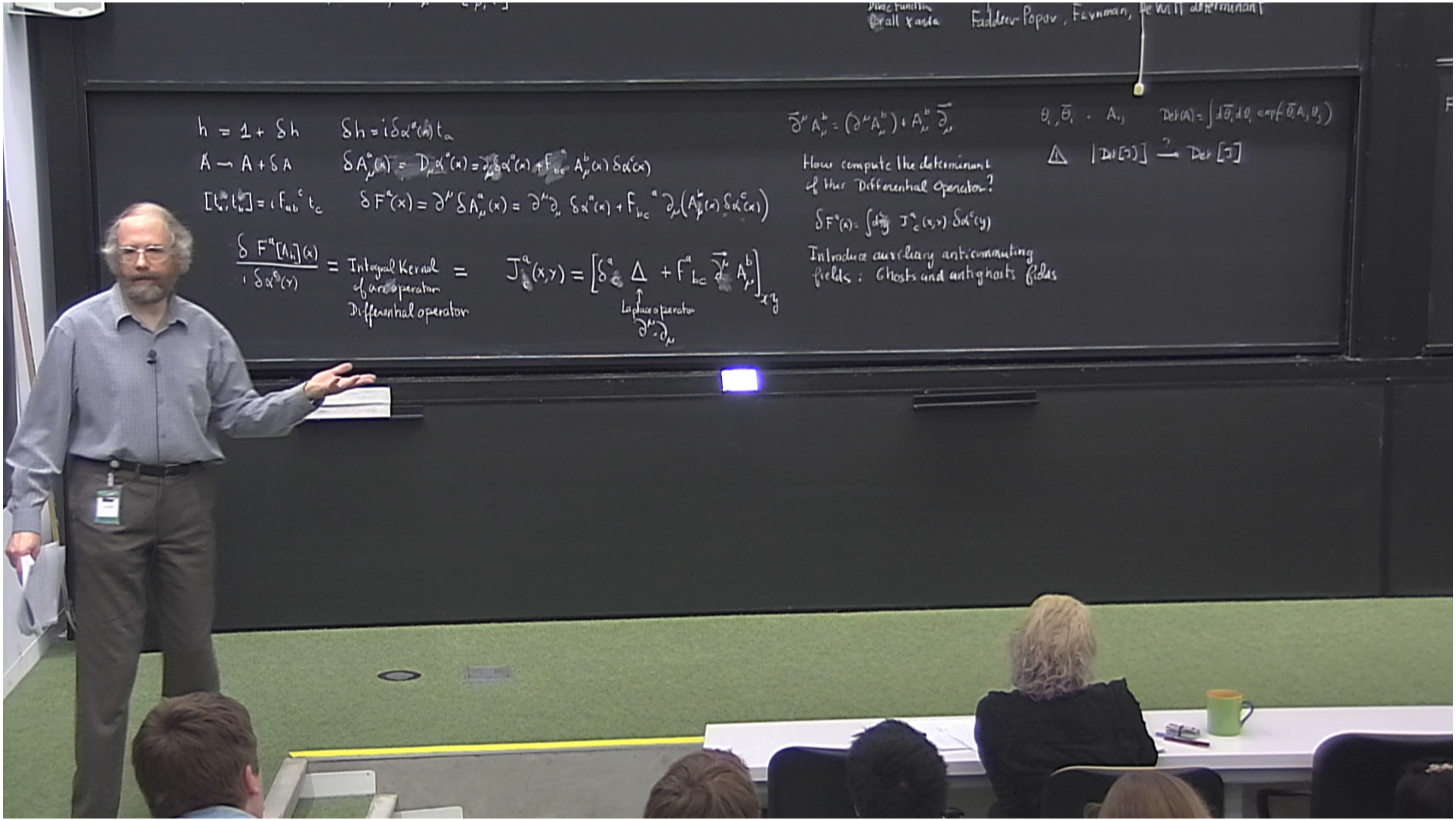
= Integral Kernel of an operator
Differential operator
Laplace operator
How compute the determinant of this Differential Operator?
Introduce auxiliary anticommuting fields: Ghosts and antighosts fields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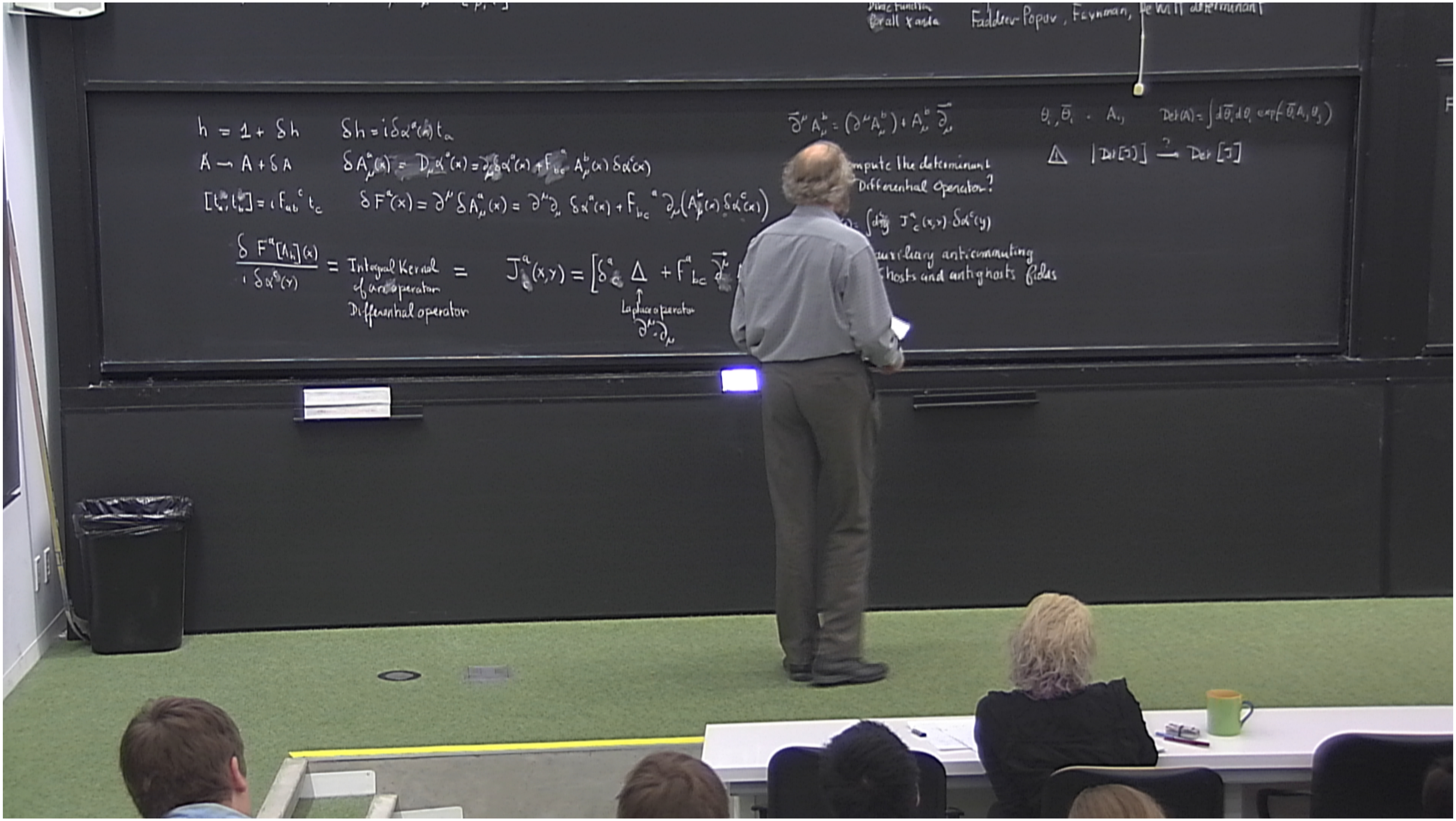
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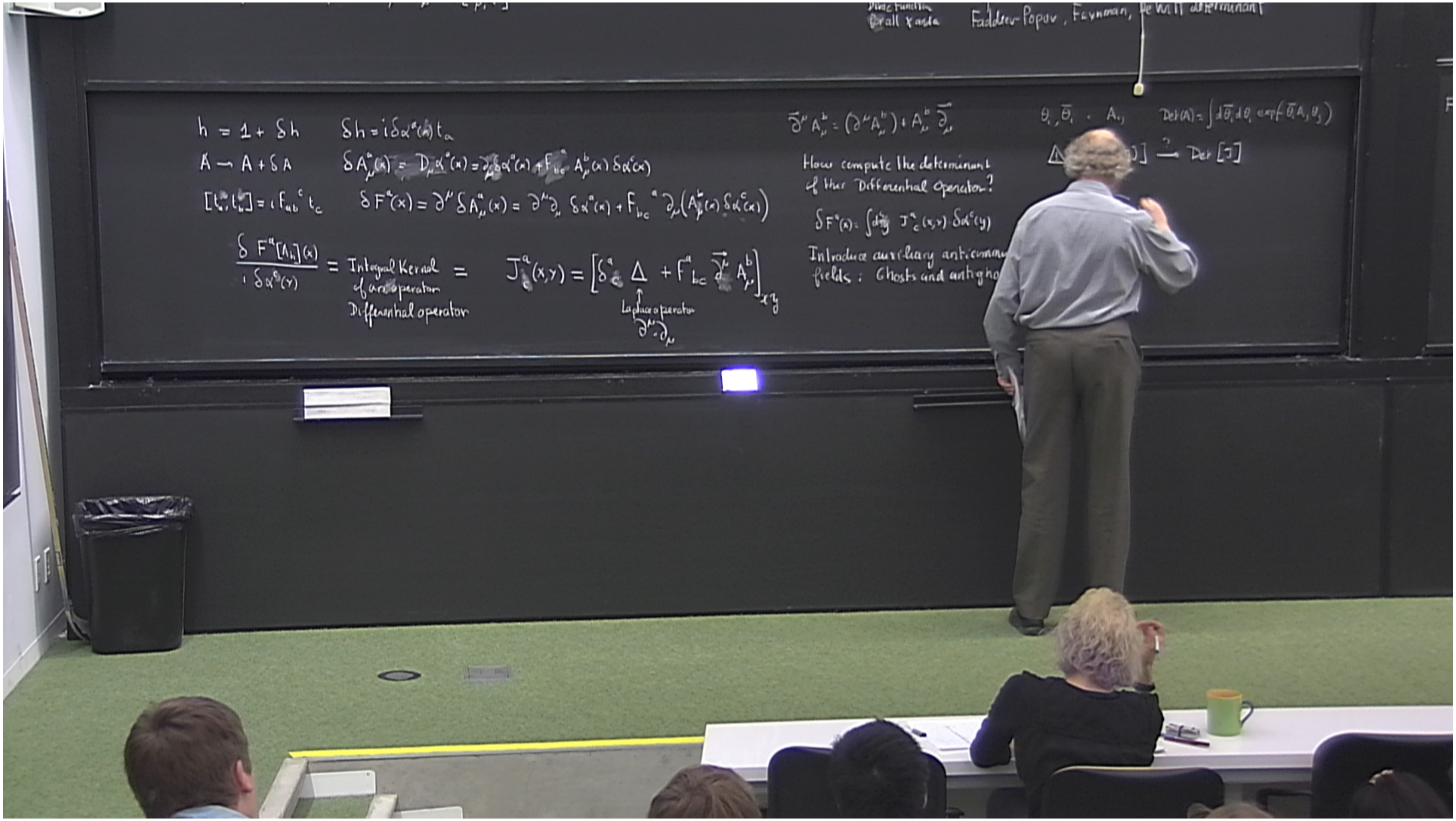
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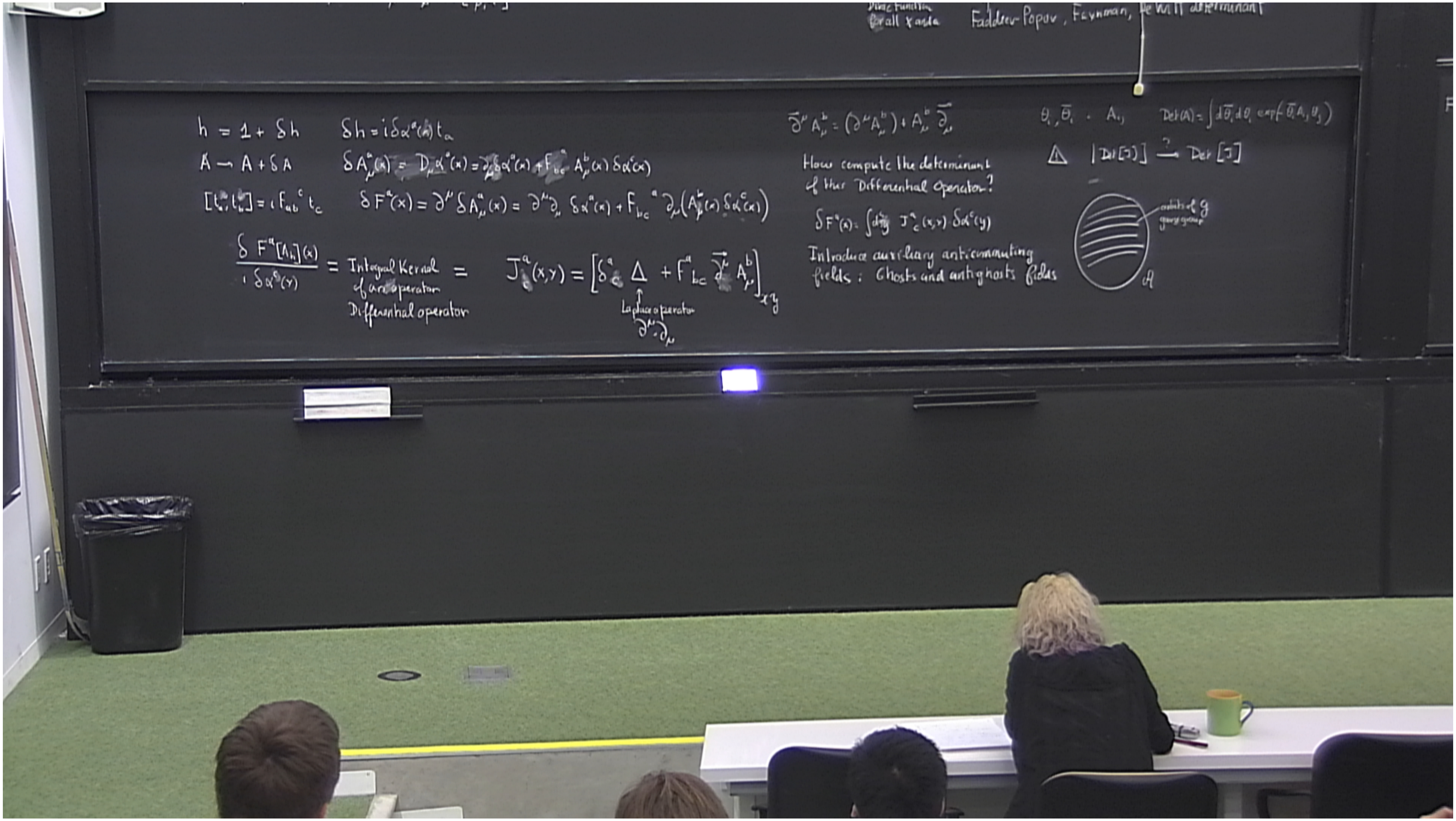
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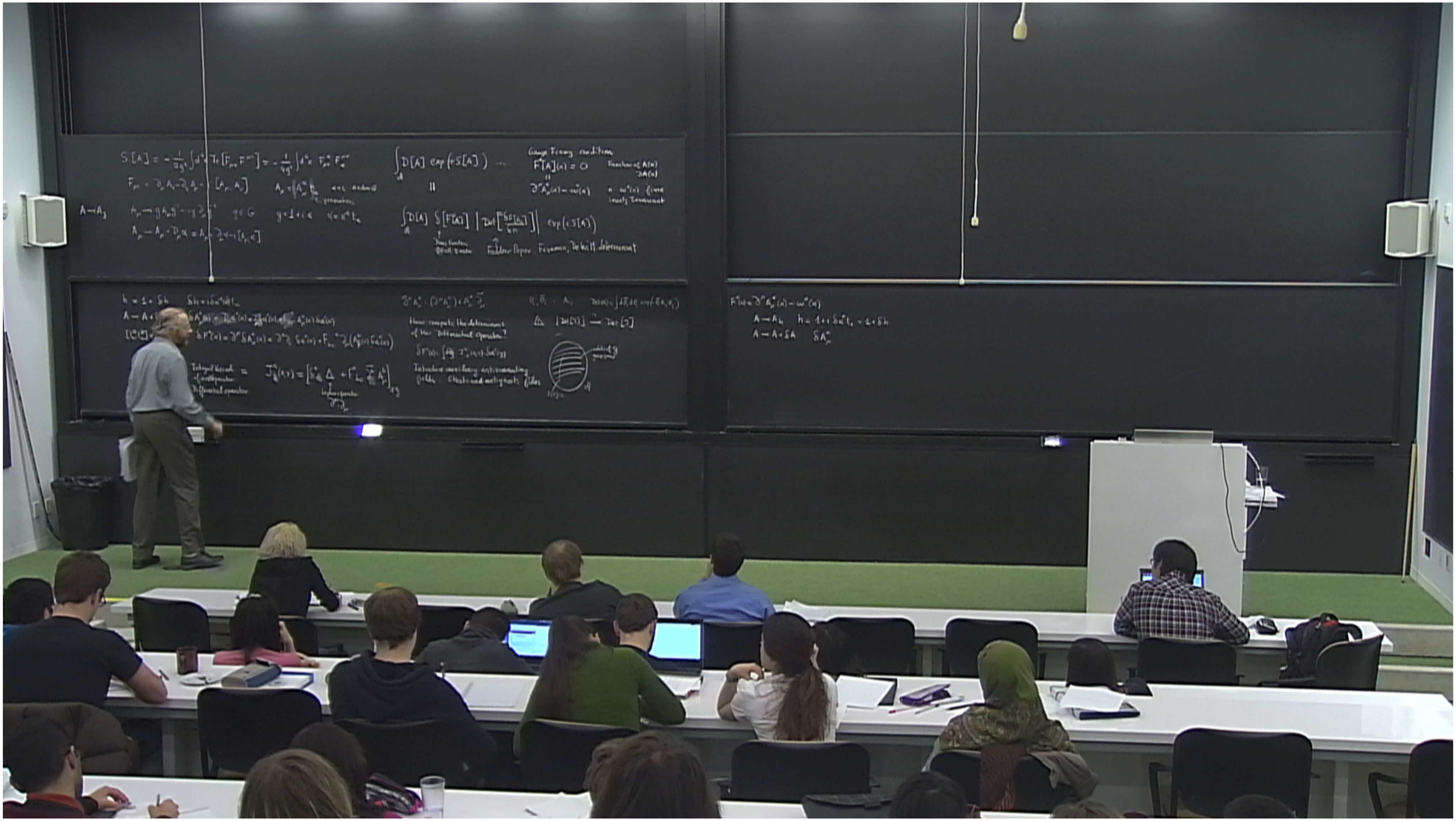

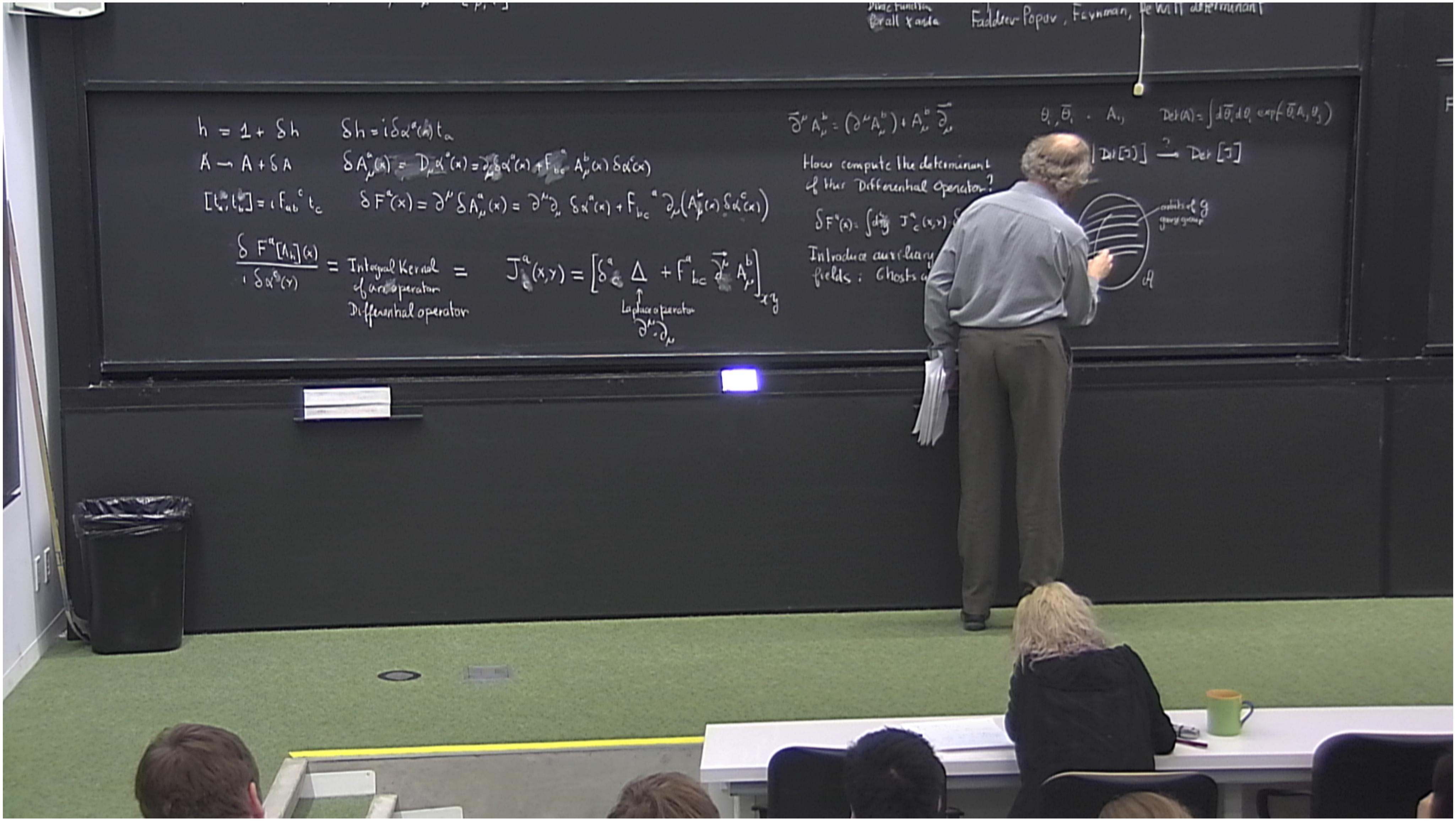
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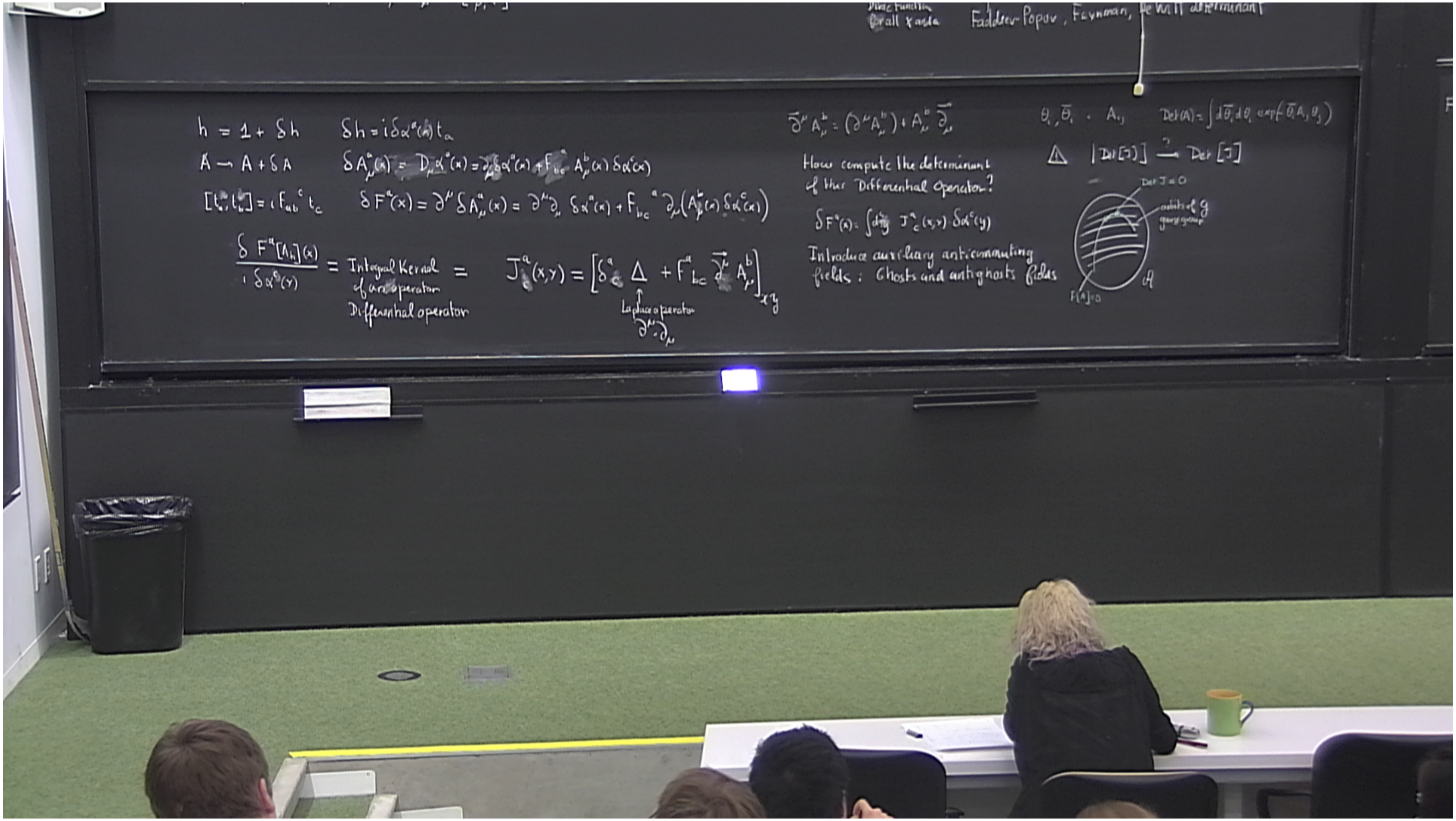
of an operator
Differential operator
Laplace operator
∂^μ ∂_μ
How compute the determinant
of this Differential Operator?
Introduce auxiliary
fields : Ghosts
Det[J]
orbits of g
gauge group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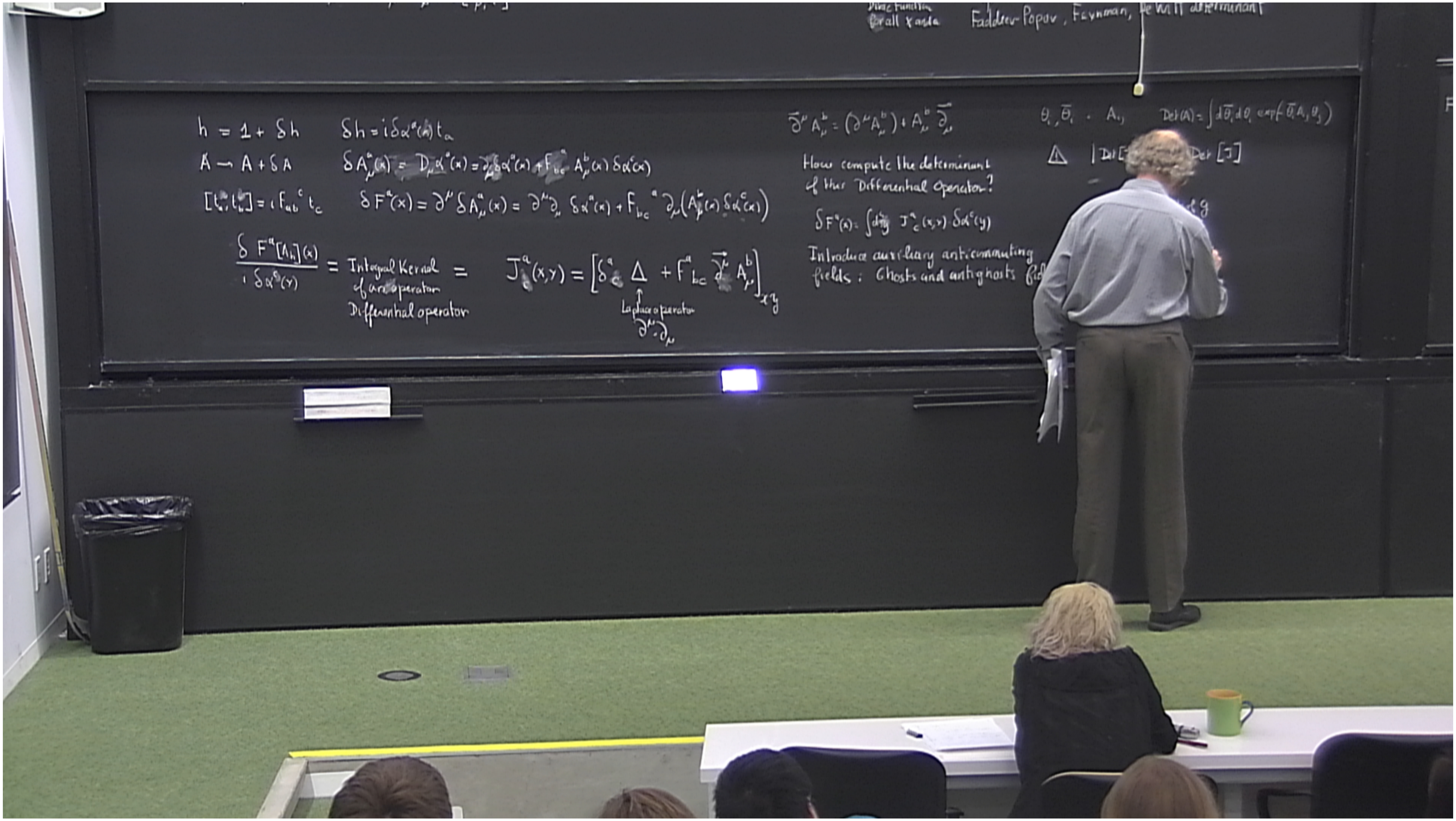
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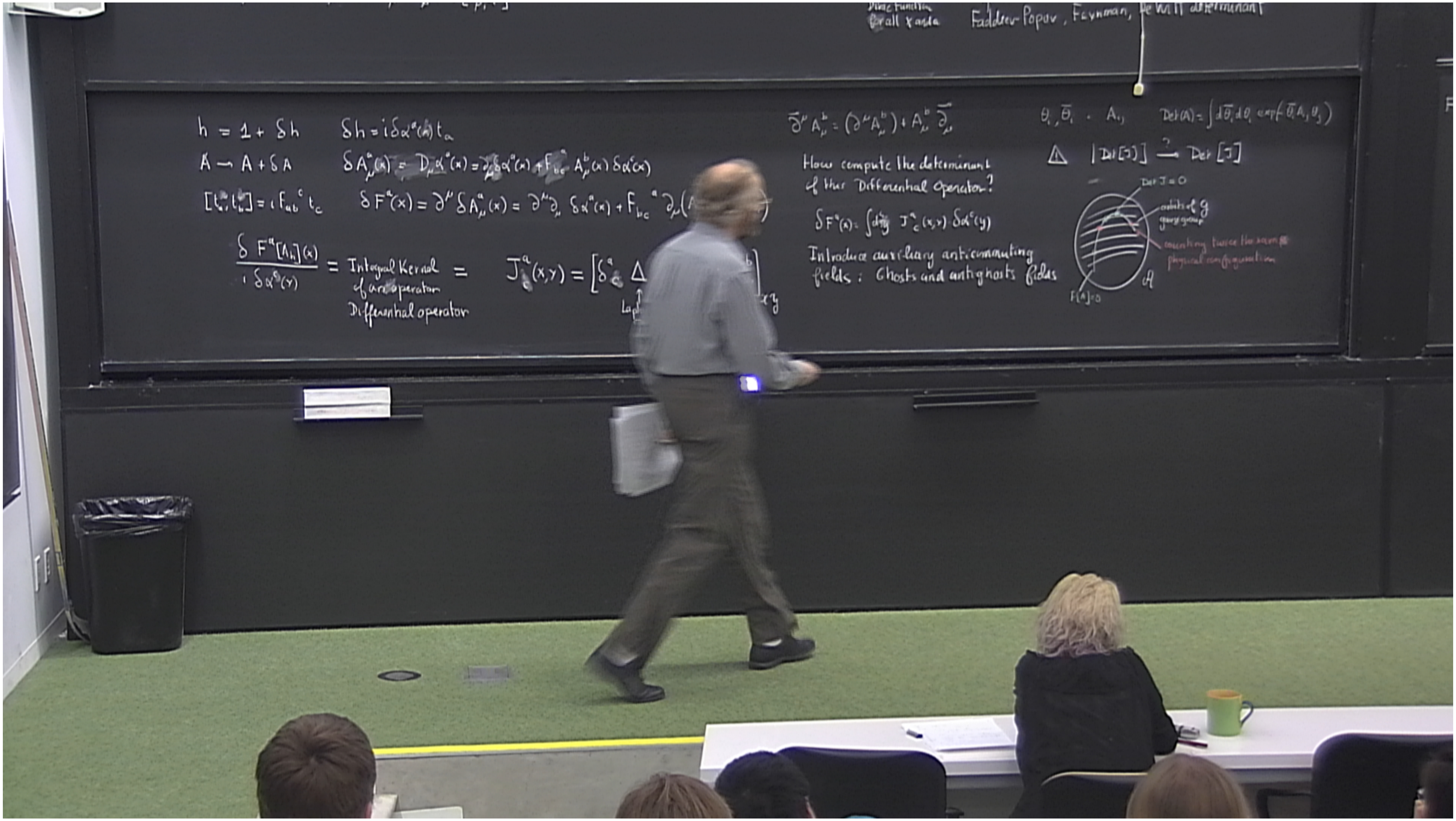
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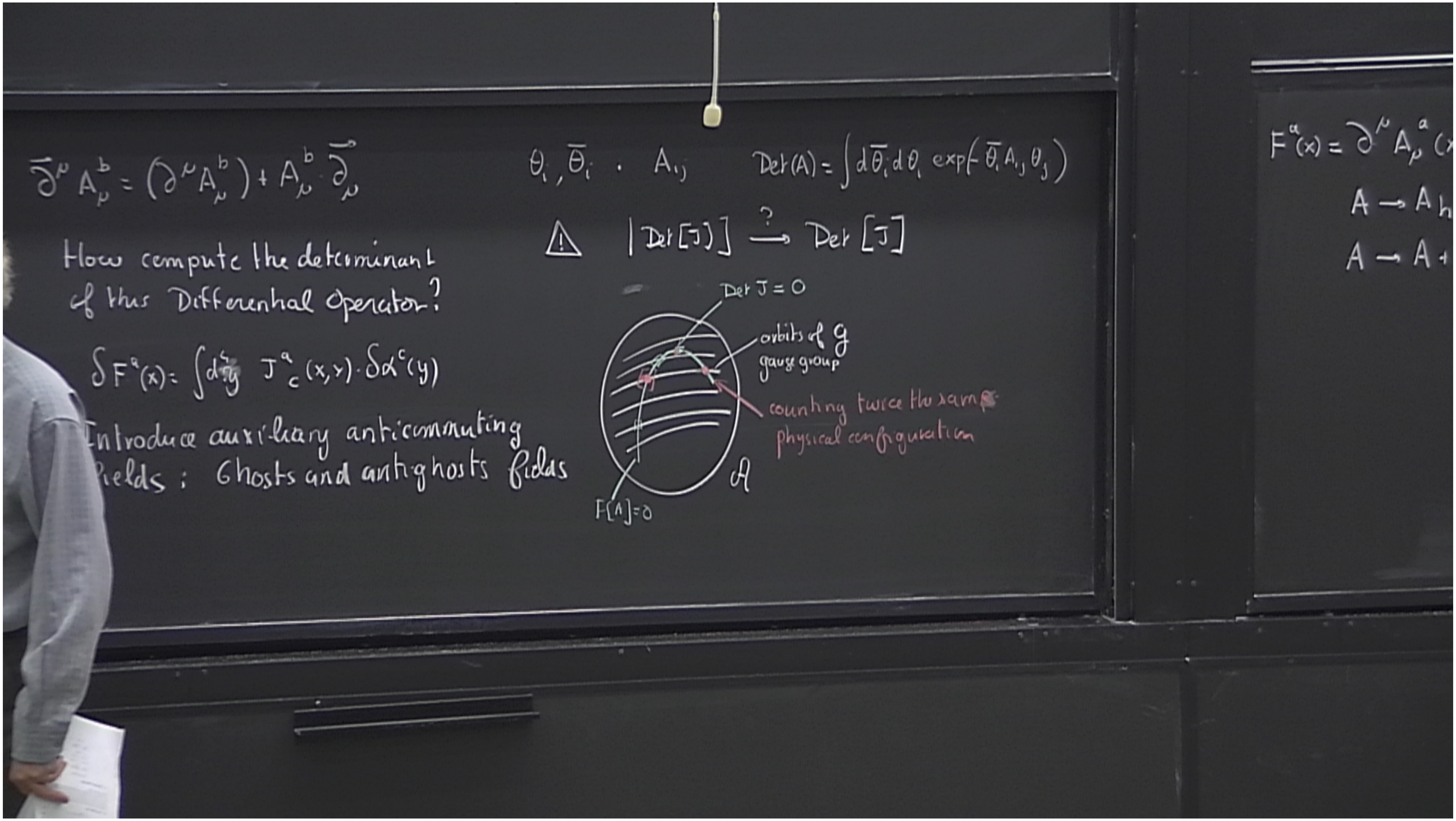
counting twice the same physical configuration


How compute the determinant
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Introduce auxiliary anticommuting
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Det J = 0
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gauge group
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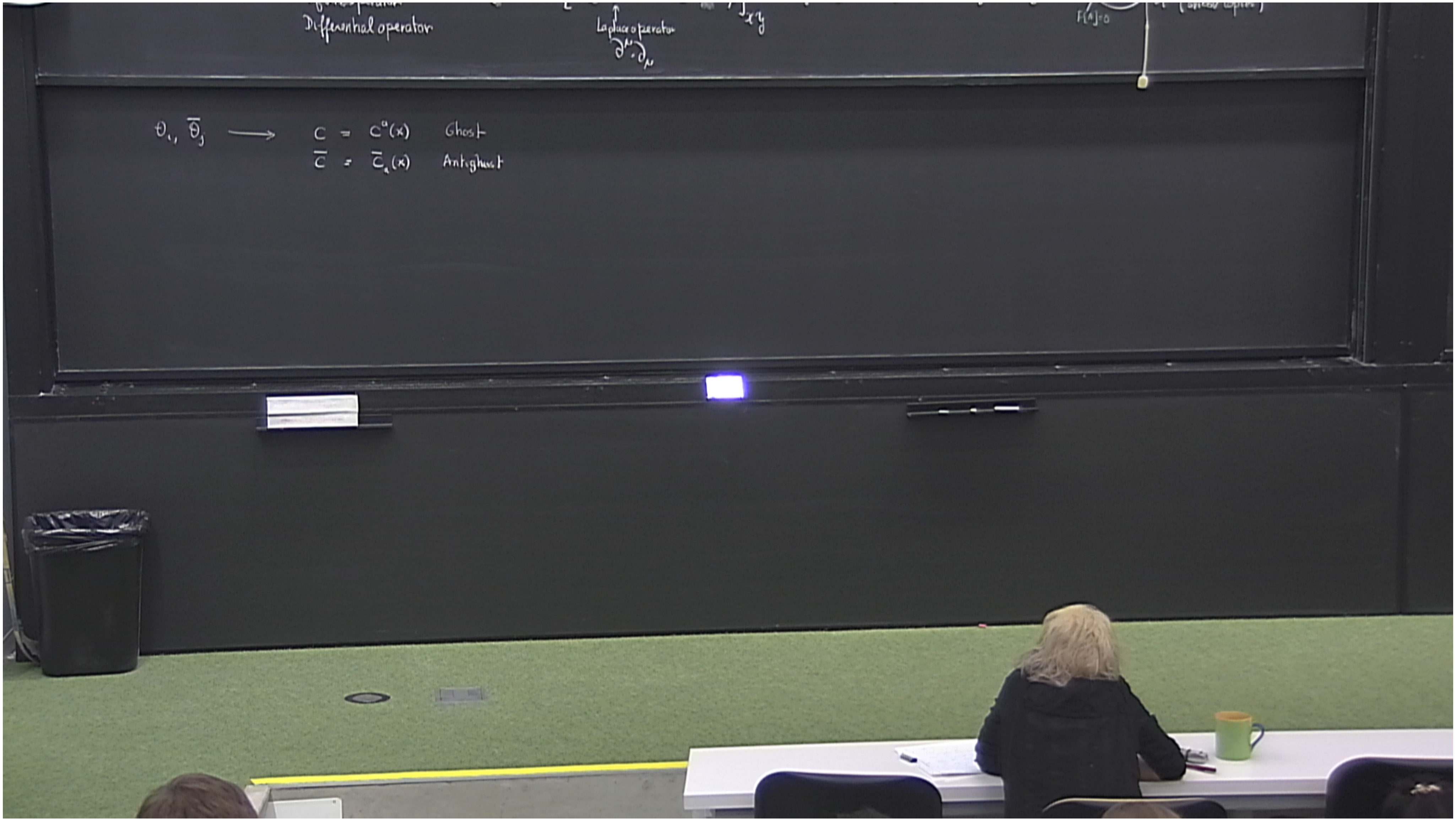
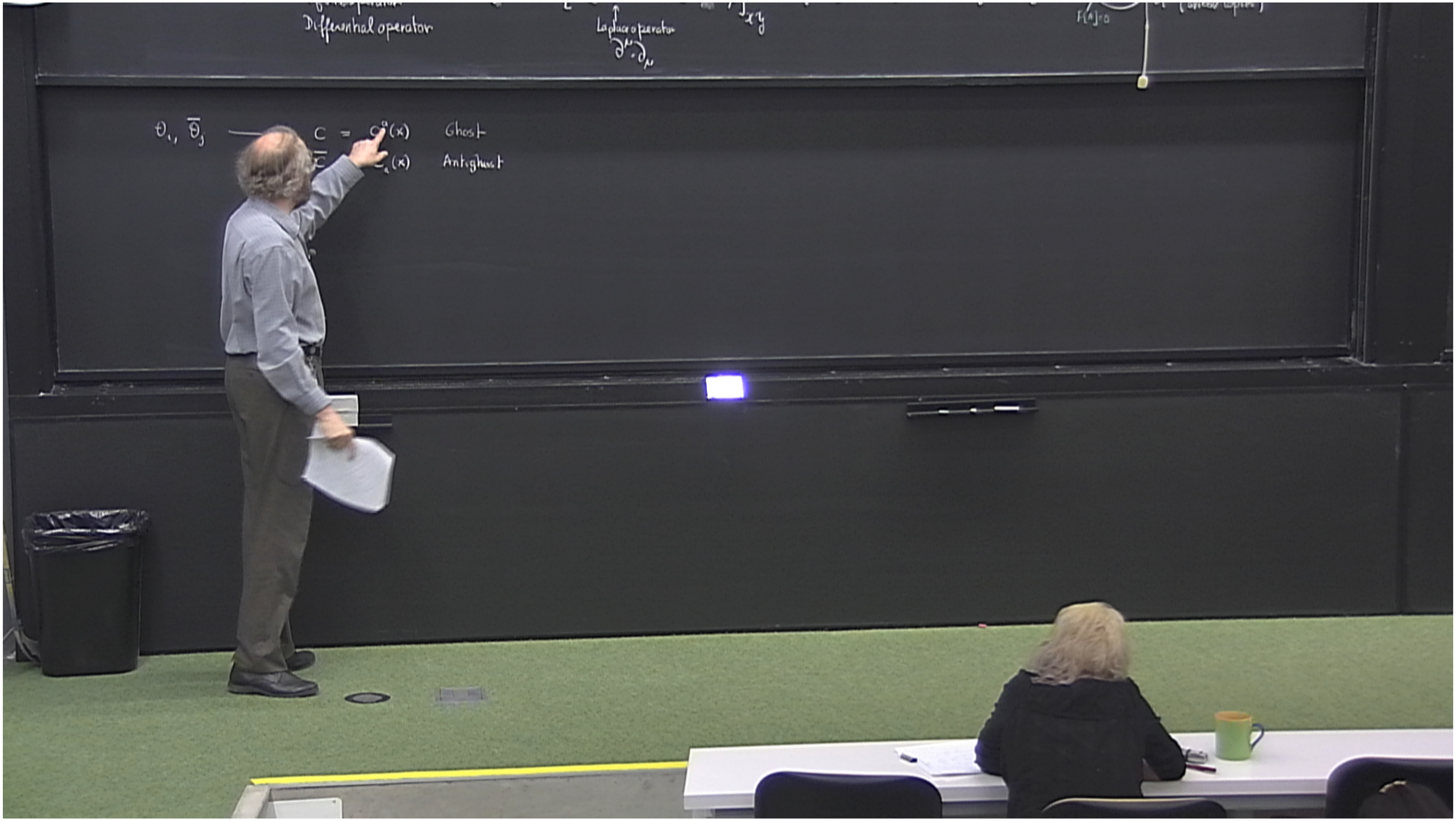
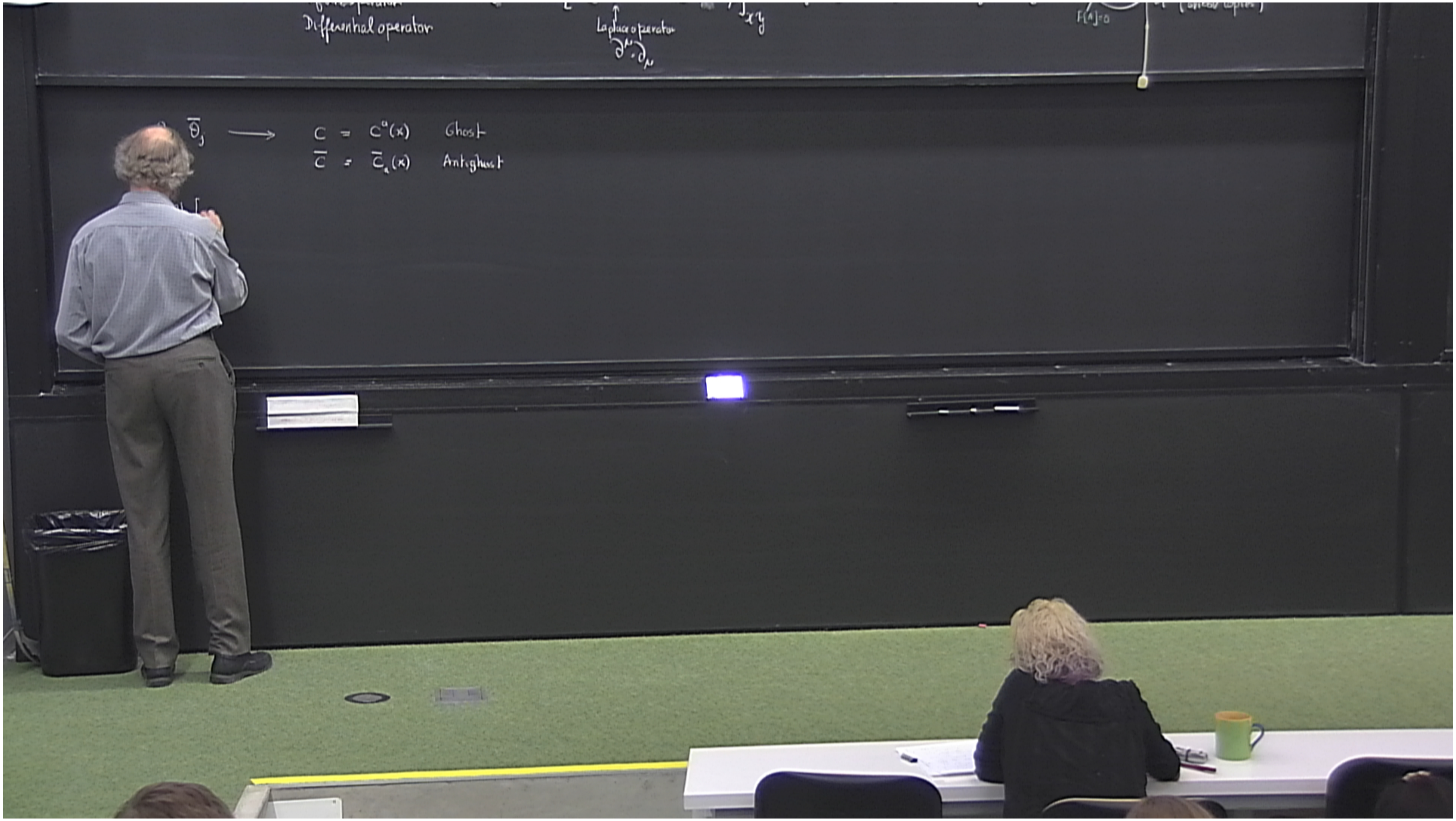
F[A]=0


Differential operator
Laplace operator
$\partial^\mu \cdot \partial_\mu$
$\theta_i, \bar{\theta}_j \longrightarrow C = C^a(x)$ Ghost
$\bar{C} = \bar{C}_a(x)$ Antighost


Differential operator
Laplace operator
Ghost
Antighost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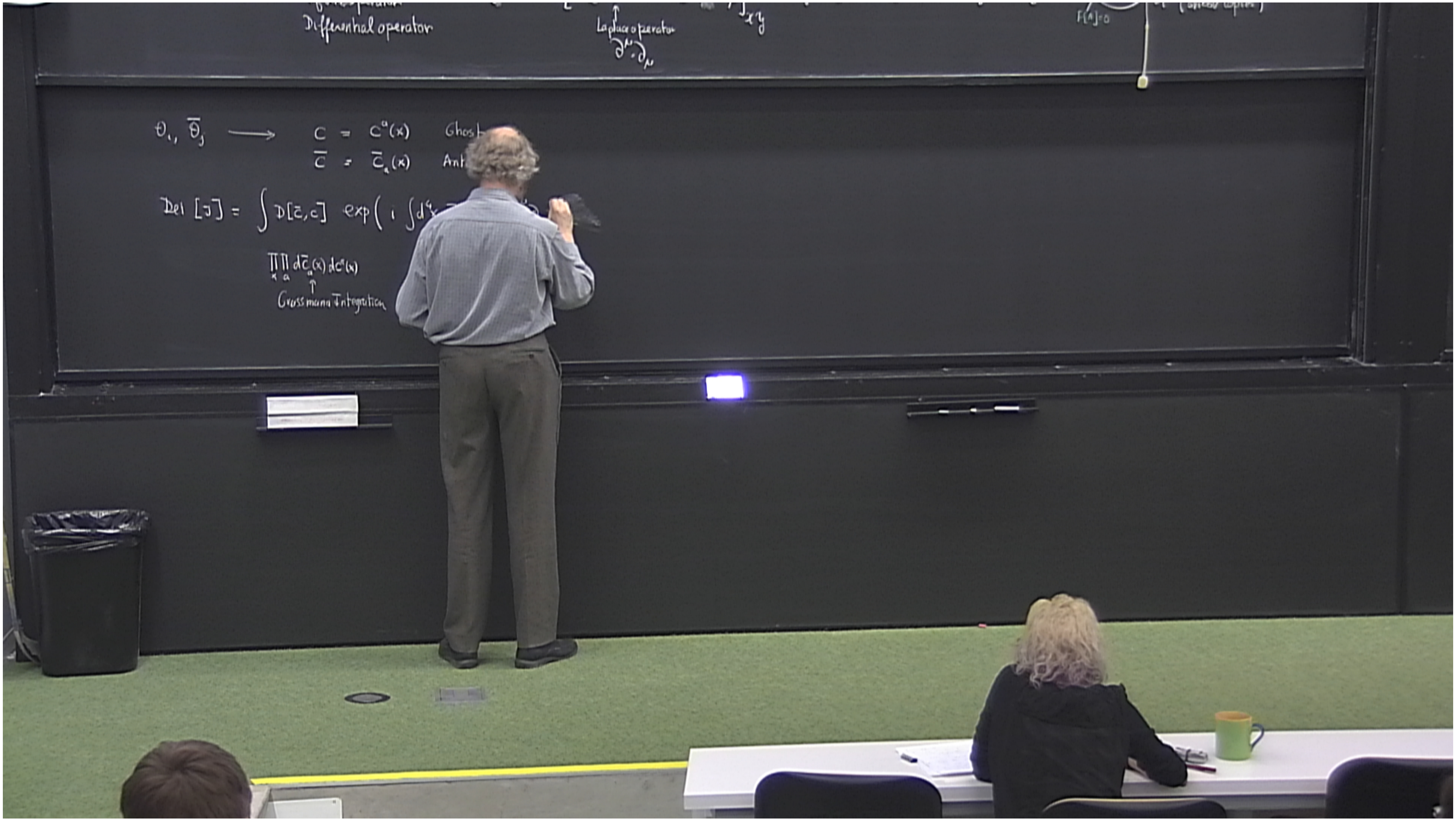
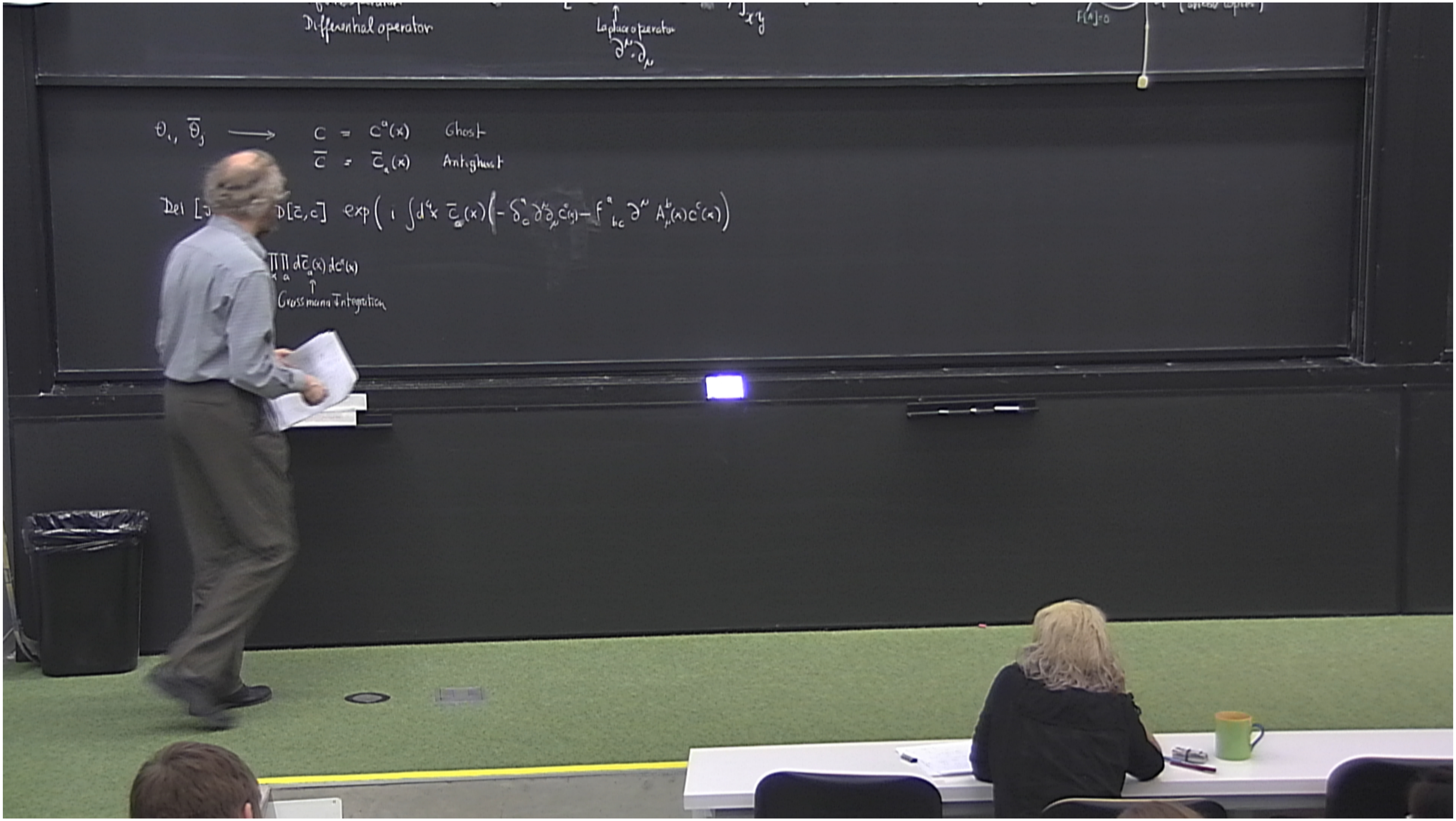

Differential operator
Laplace operator
Ghost
Grassmann Integration


Differential operator
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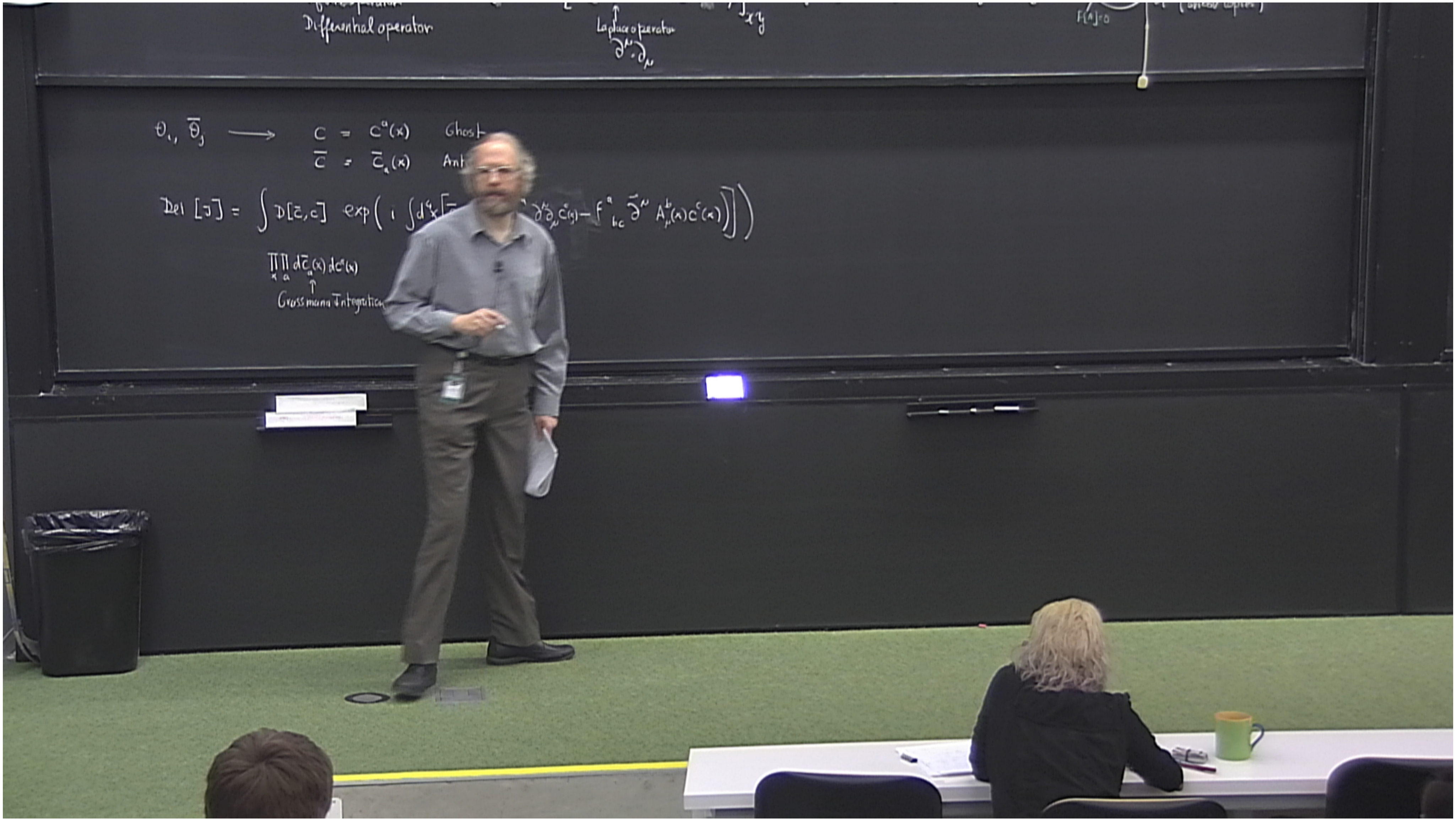
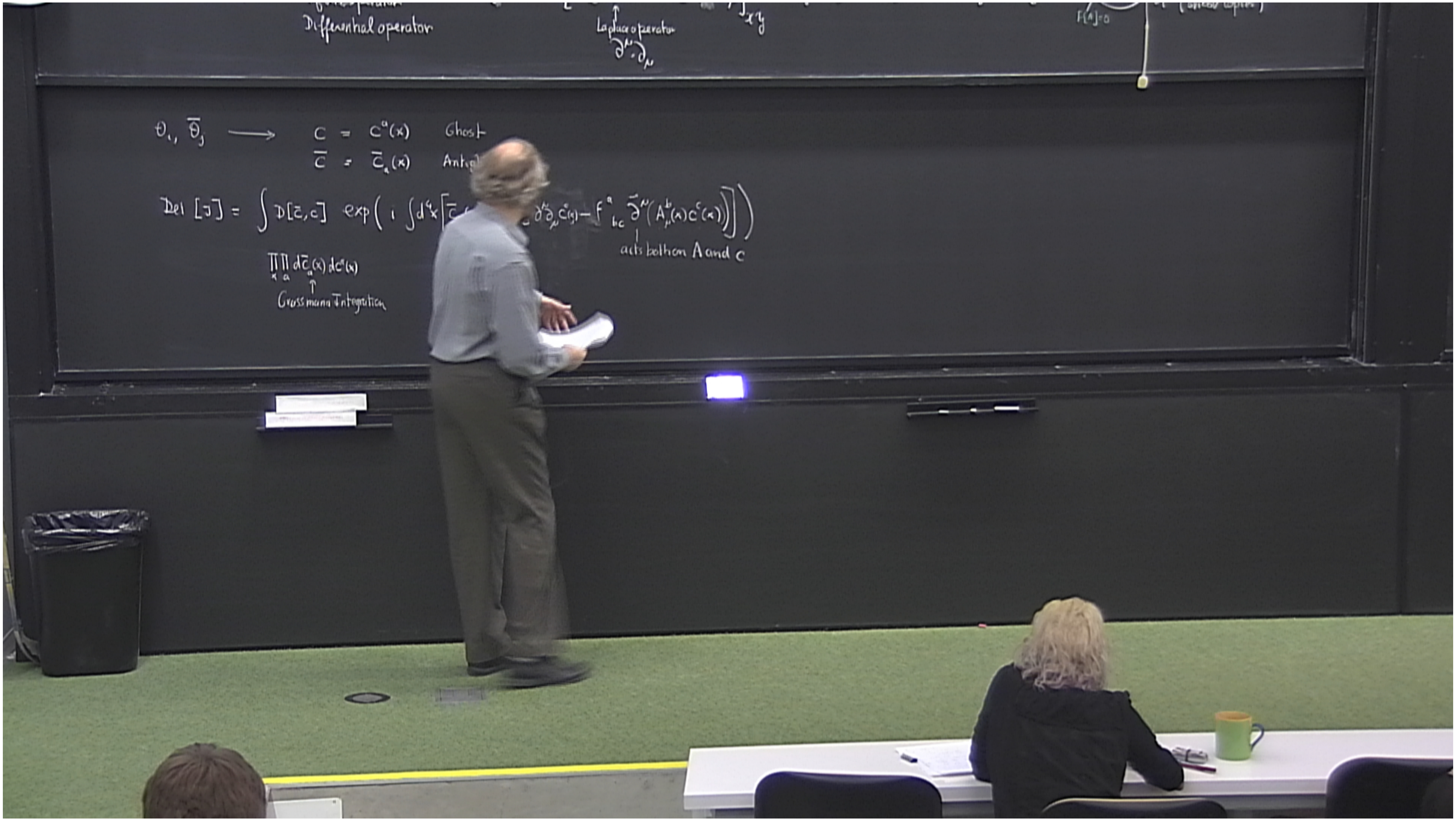
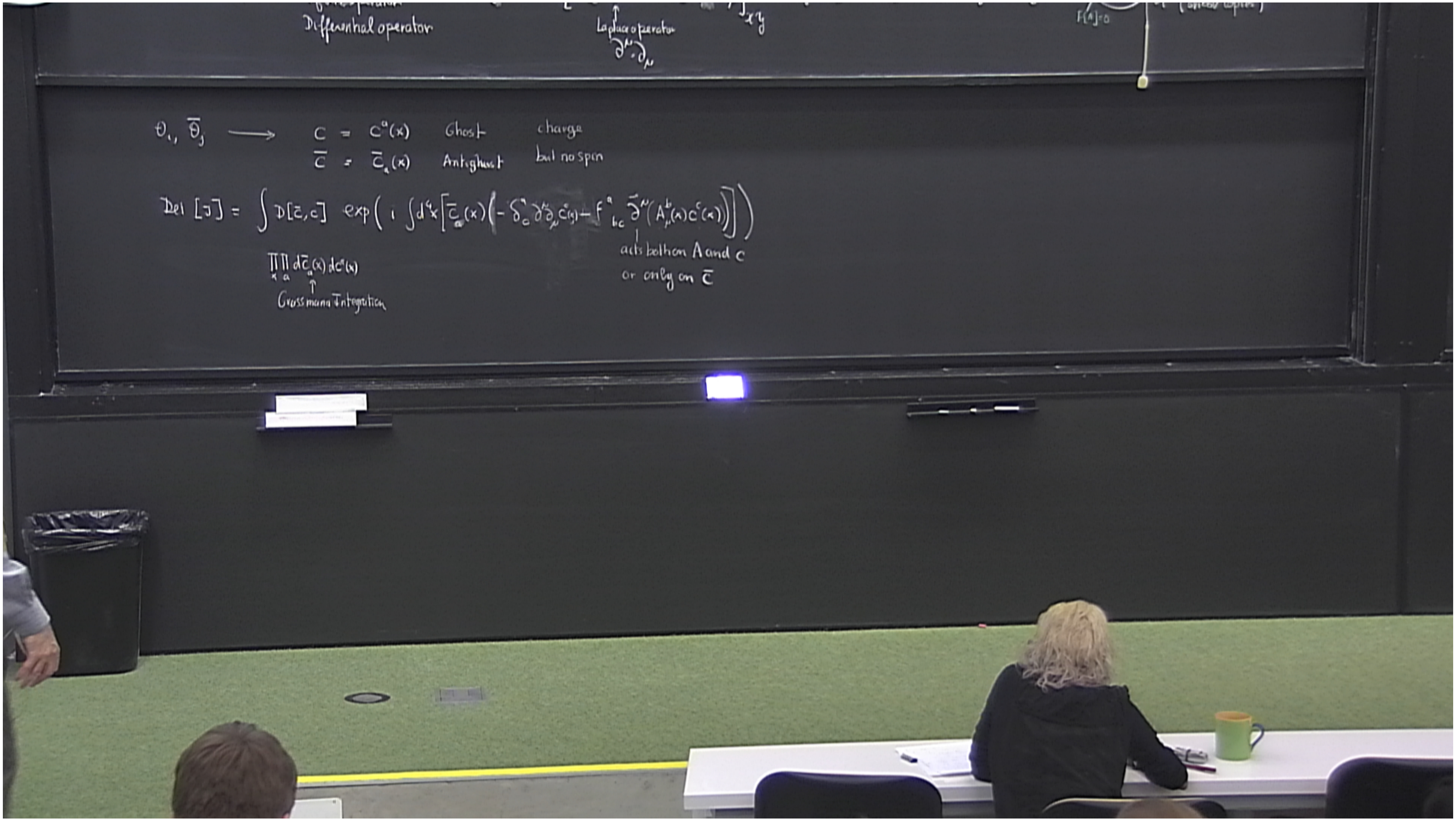
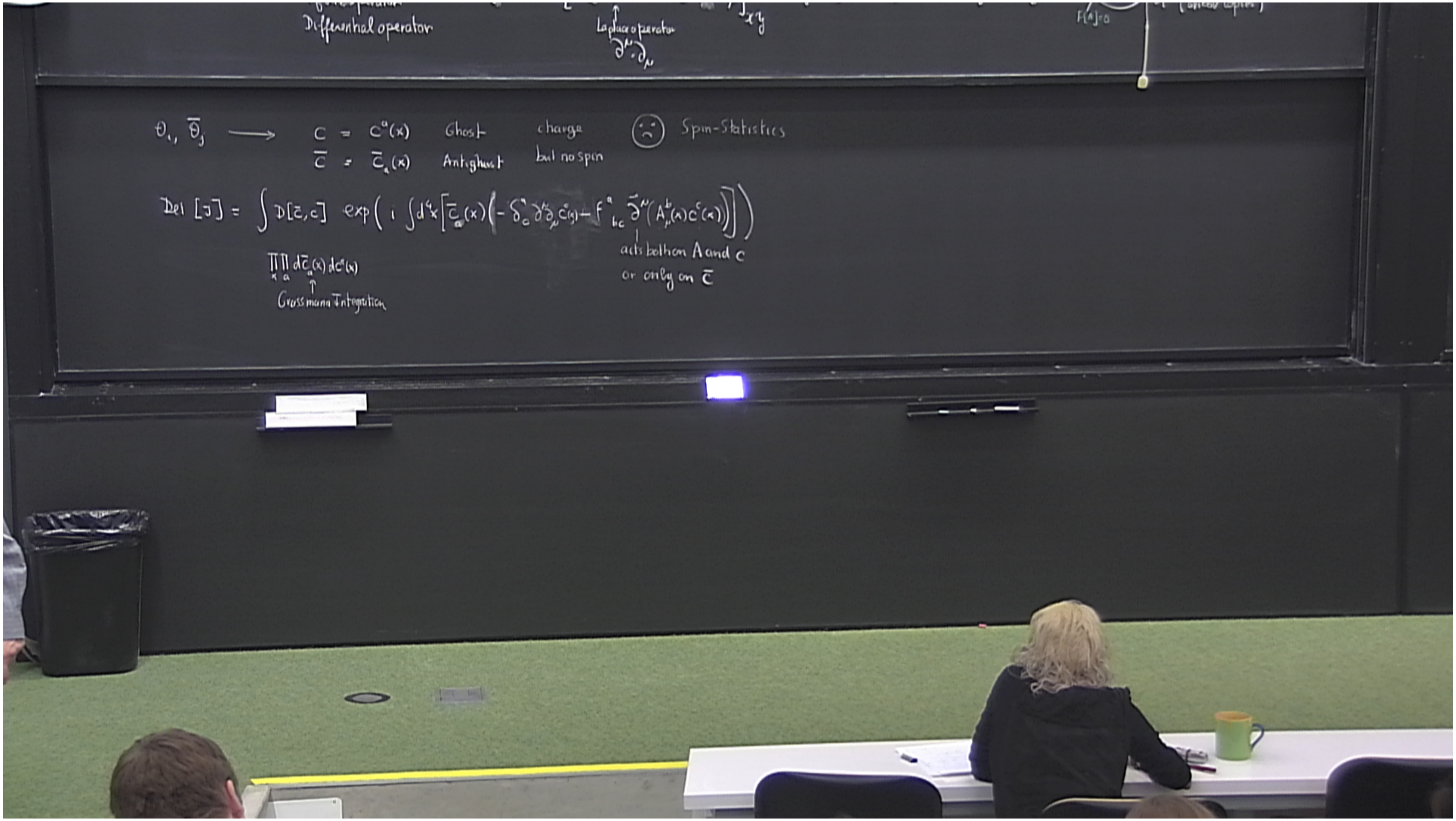


Differential operator
Laplace operator
Ghost
Grassmann Integration


Differential operator
Laplace operator
Ghost
acts both on A and c
Grassmann Integration


Differential operator
Laplace operator
θ_i, θ̄_j → C = C^a(x) Ghost charge
C̄ = C̄_a(x) Antighost but no spin
Det [J] = ∫ D[c̄, c] exp( i ∫d^4x [c̄_a(x)(− δ^a_c ∂^μ∂_μ c^c(x) − f^a_bc ∂^μ(A^b_μ(x) c^c(x))]])
∏_x ∏_a dc̄_a(x) dc^a(x)
Grassmann Integration
acts both on A and c
or only on c̄


Differential operator
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Ghost
charge
Antighost
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Spin-Statistics
acts both on A and c
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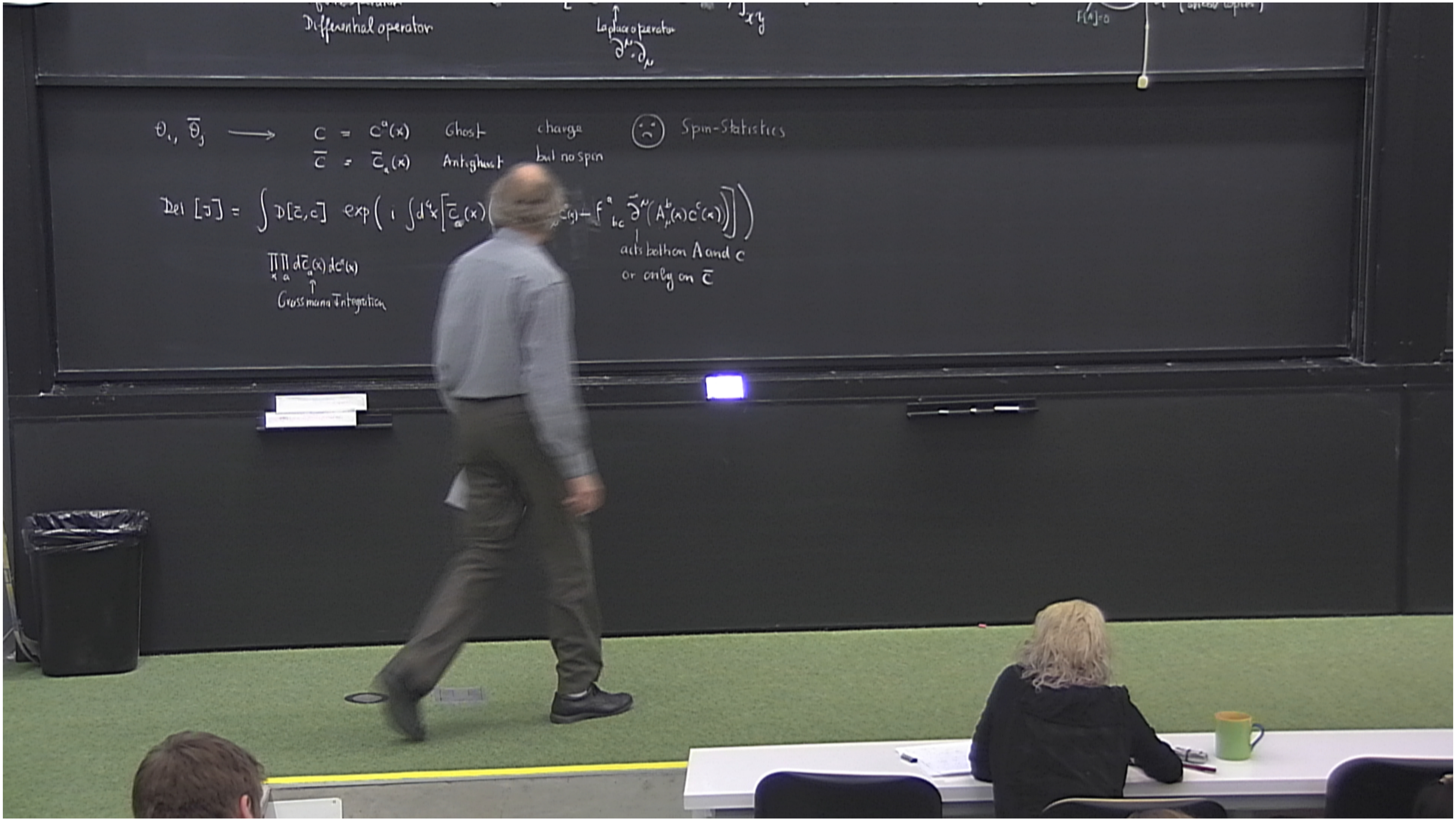

Differential operator
Laplace operator
Spin-Statistics
Ghost
Antighost
charge
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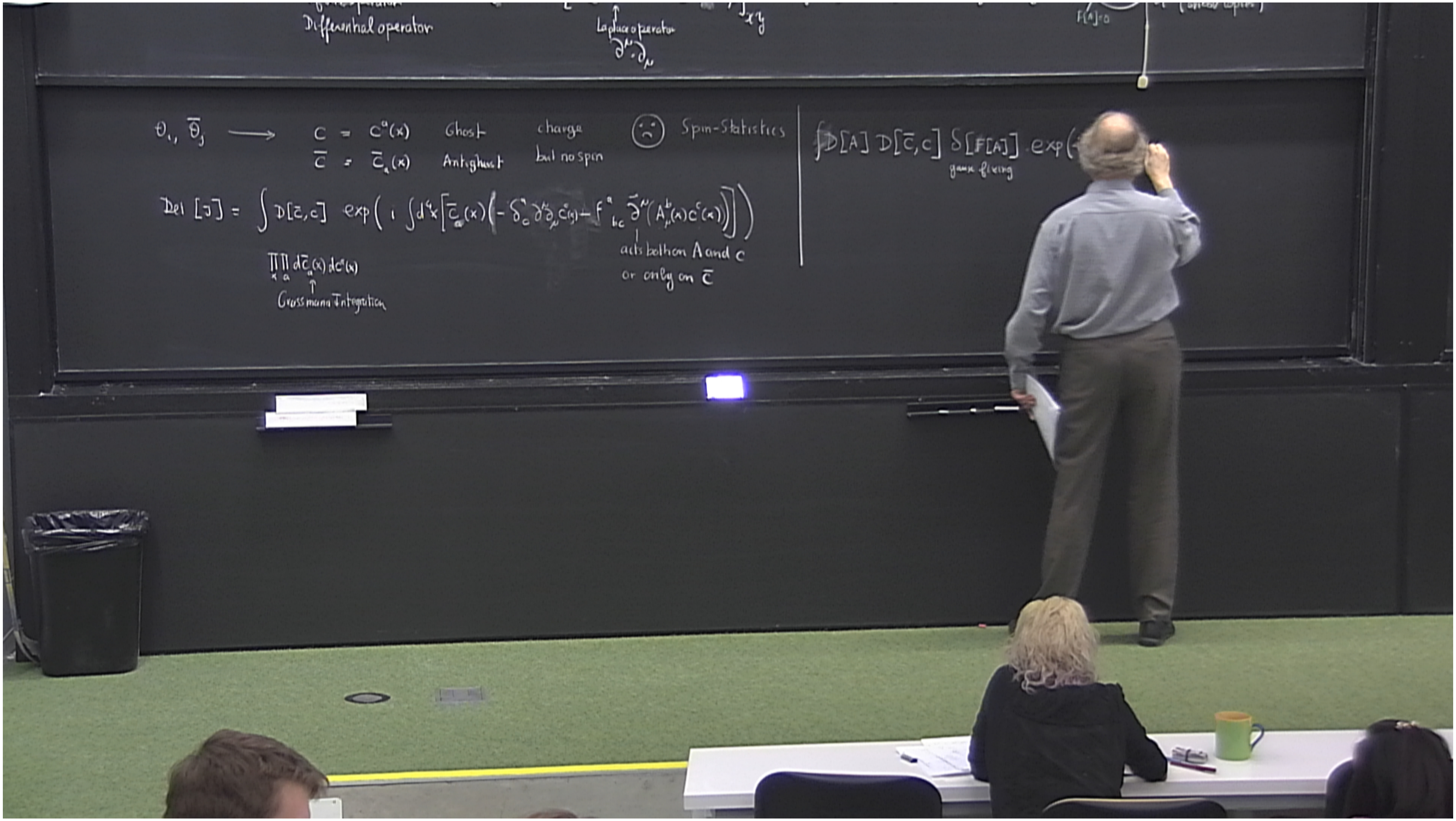
Grassmann Integration


Differential operator
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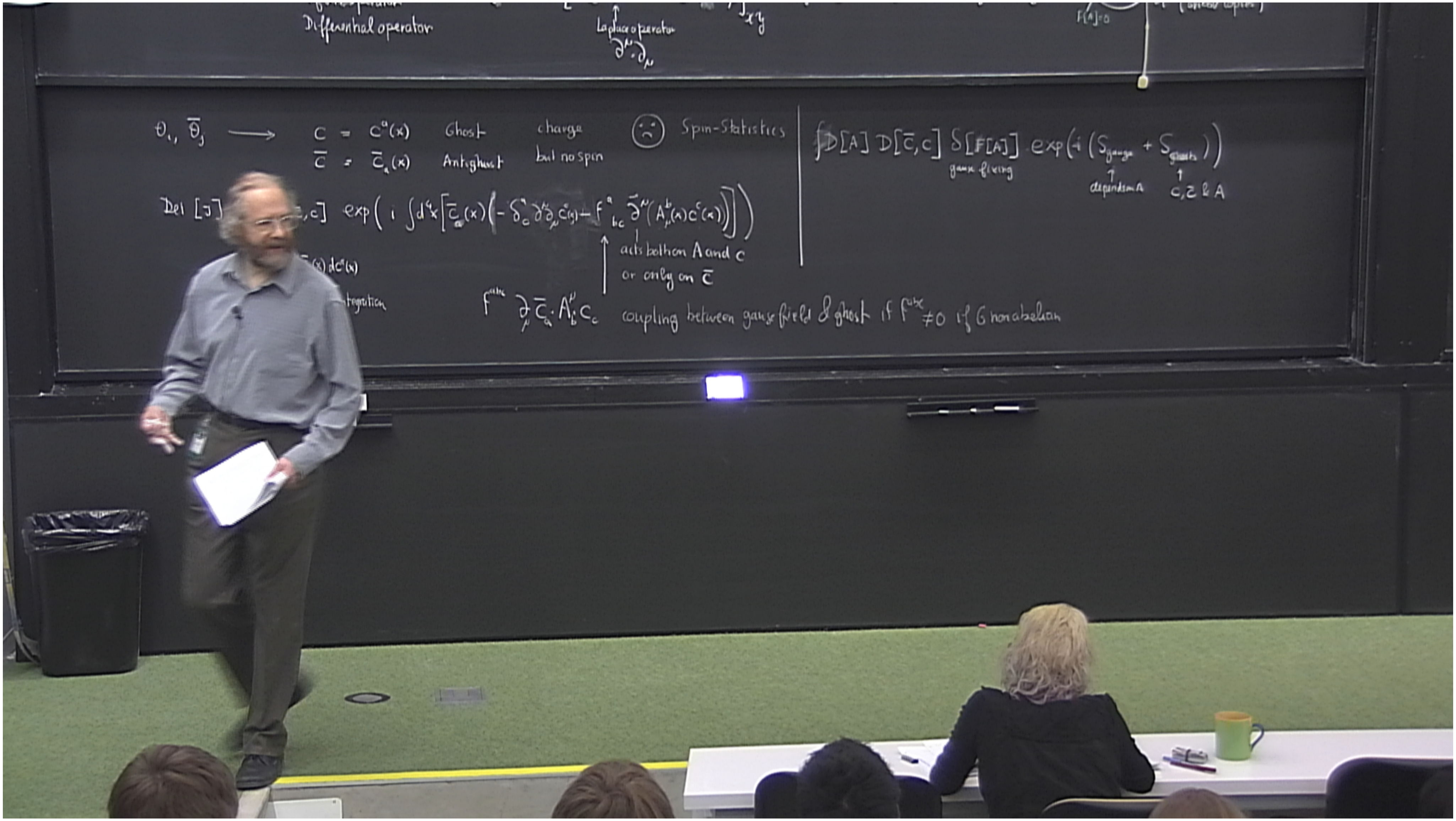
gauge fixing


Differential operator
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Spin-Statistics
gauge fixing
depends on A
acts both on A and c
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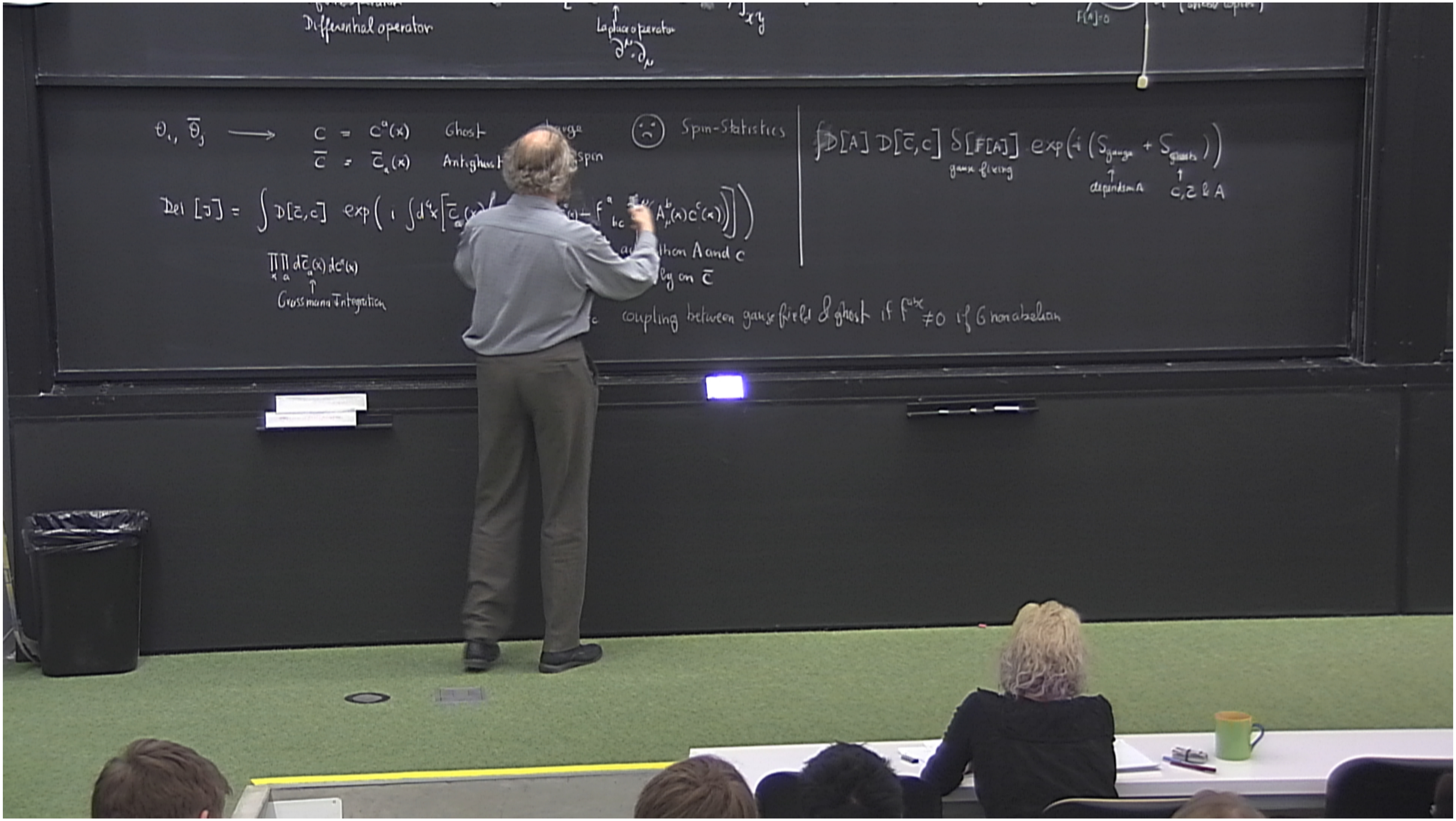
coupling between gauge field & ghost if f^abc ≠ 0 if G nonabelian


Differential operator
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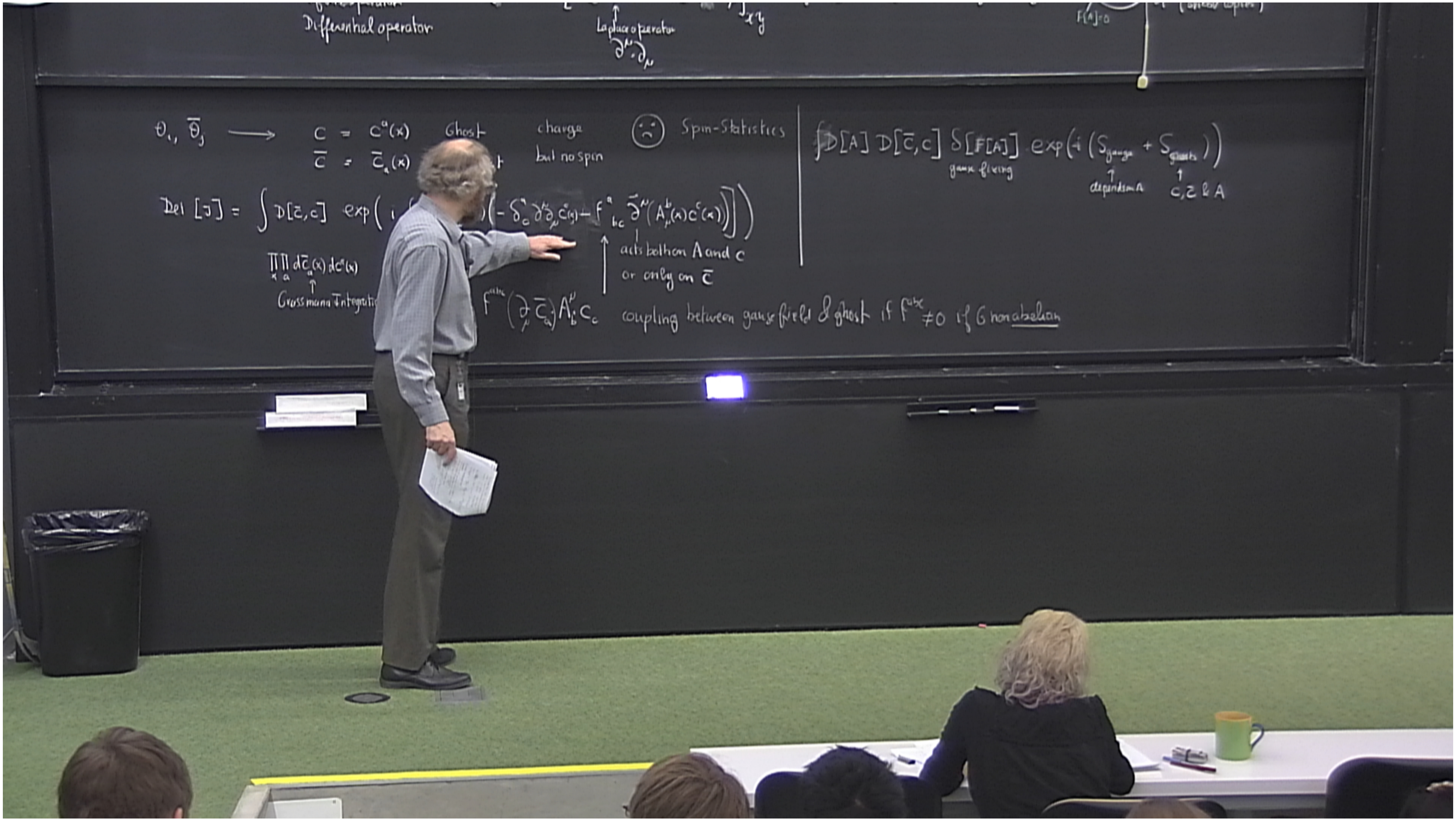
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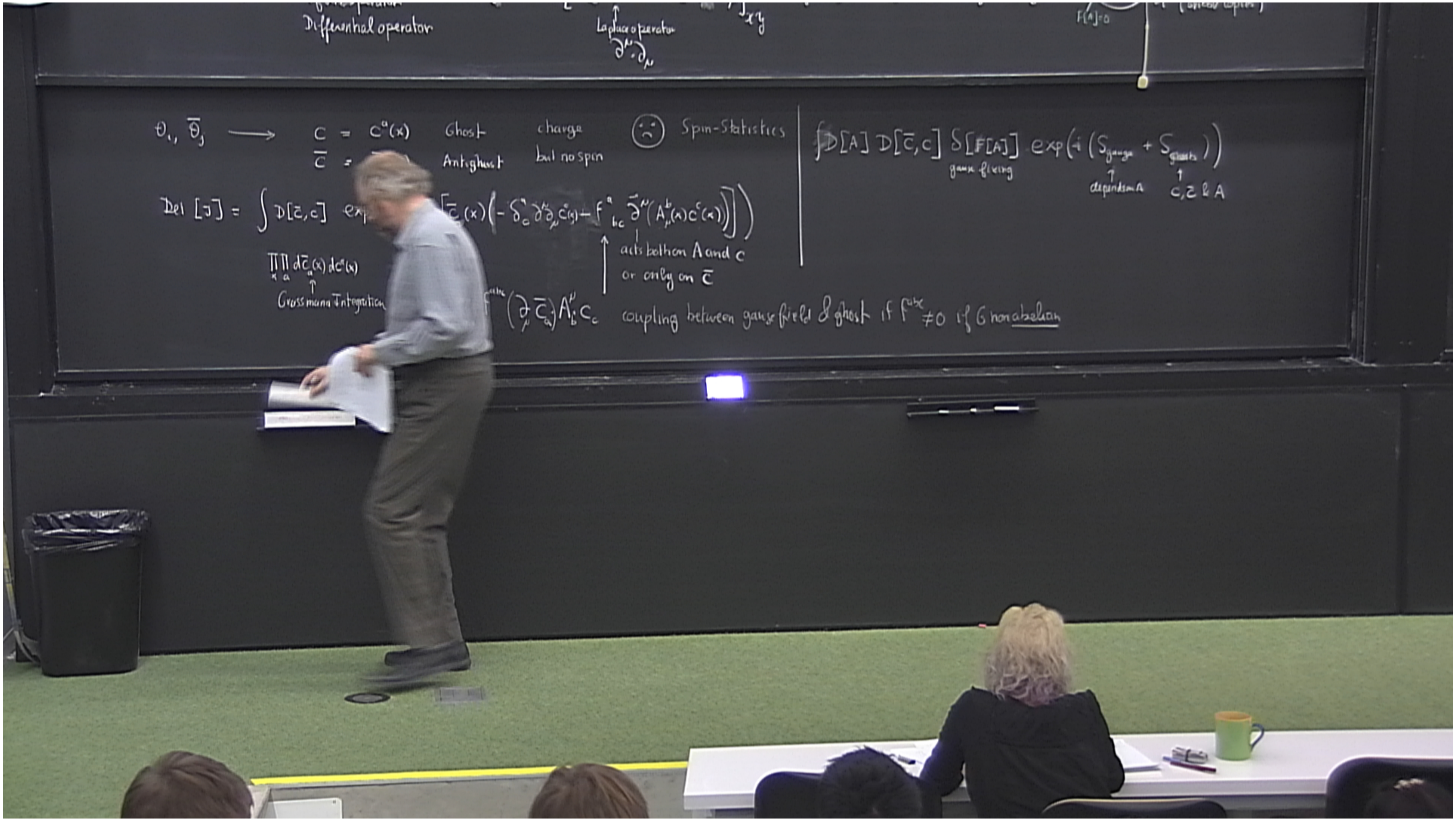

Differential operator
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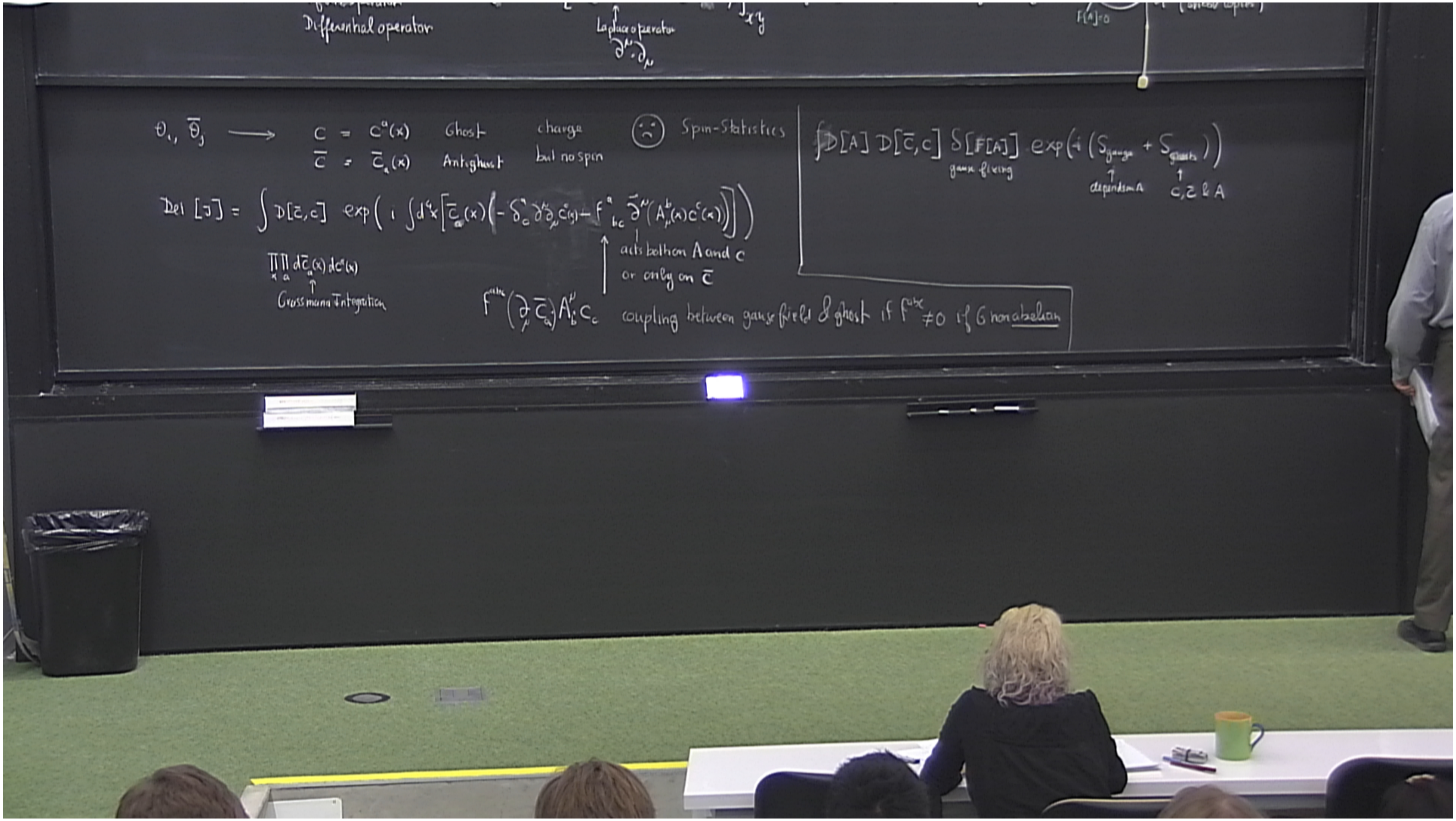
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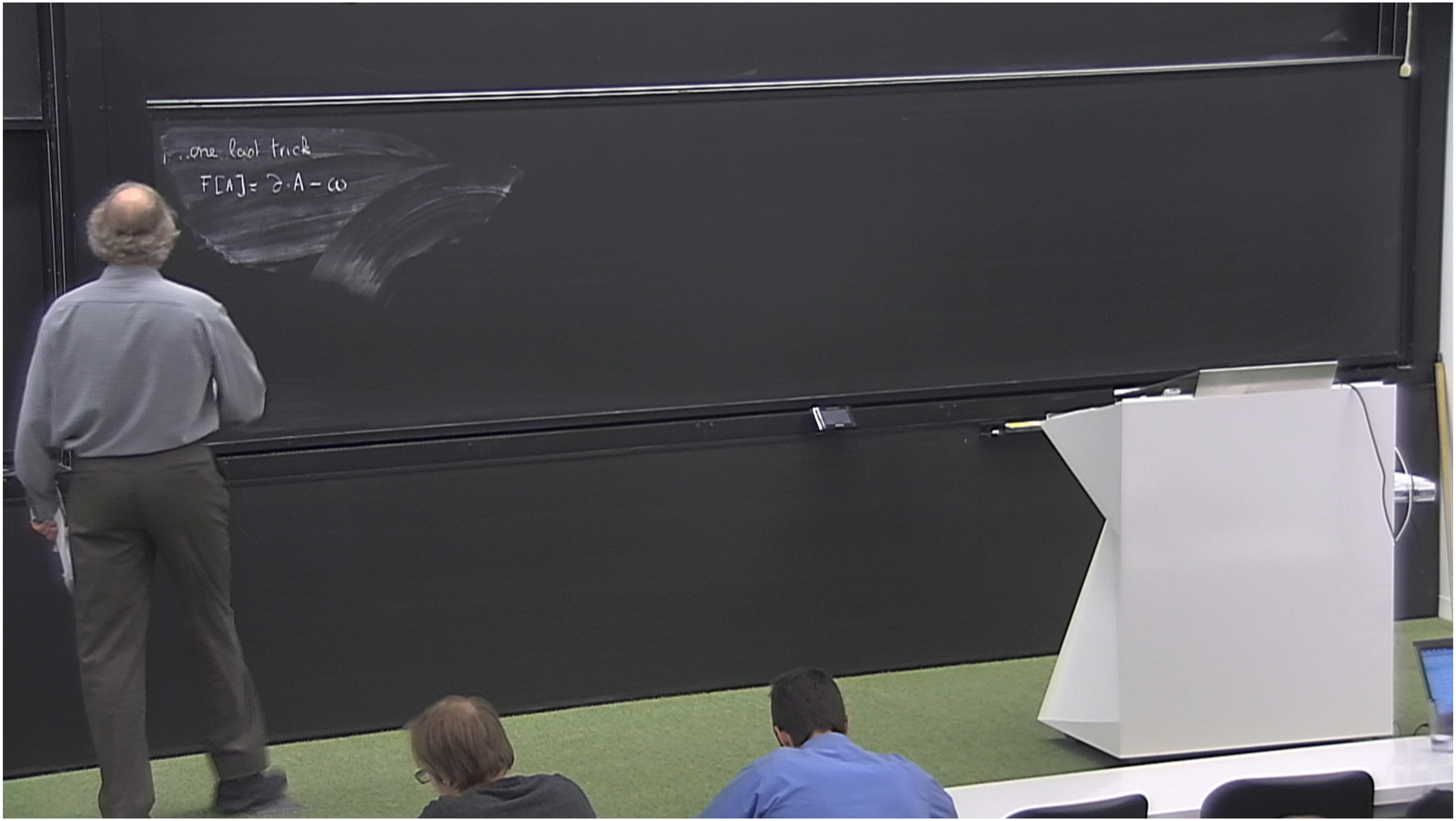
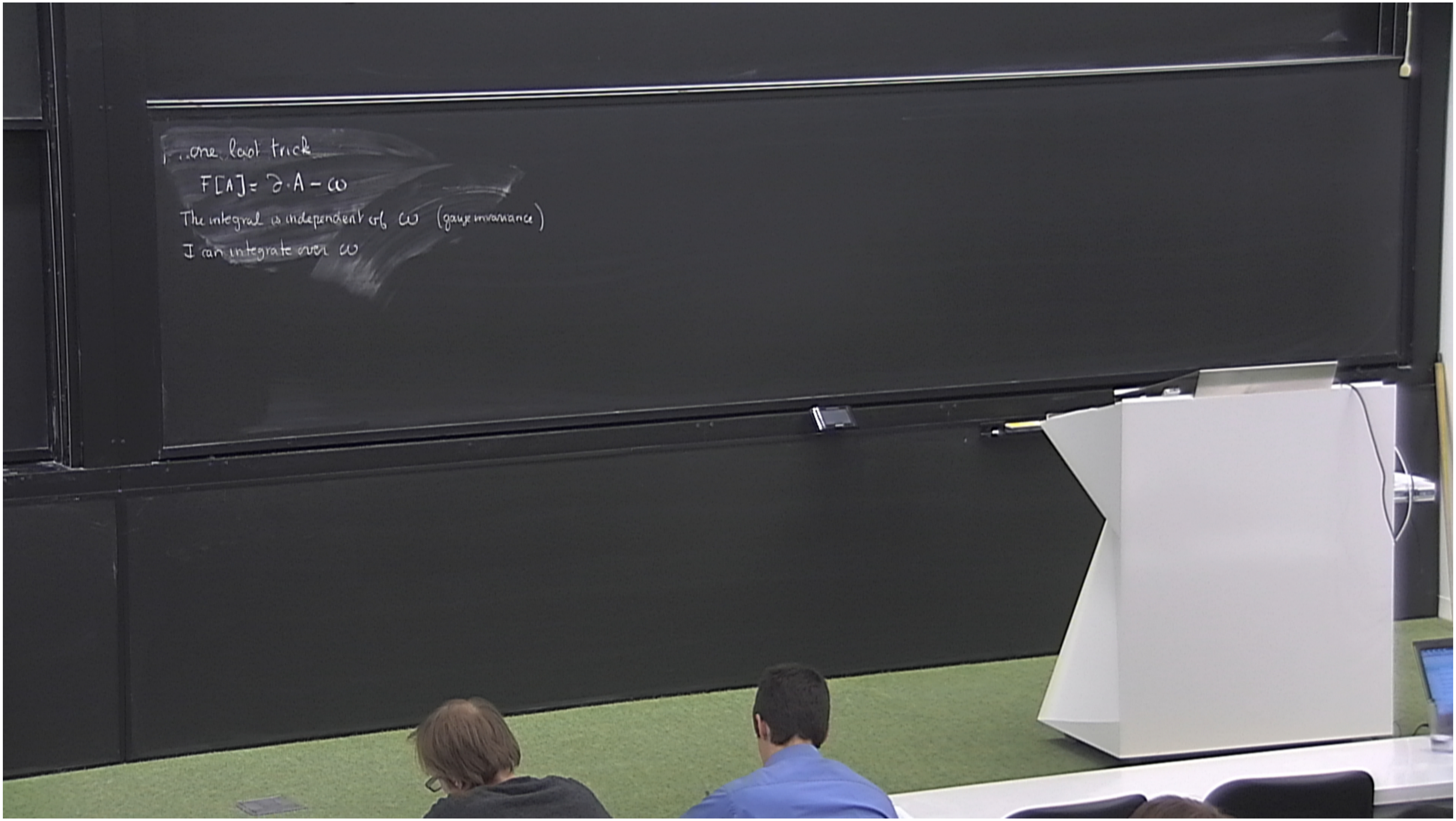


...one last trick
F[A] = ∂·A − ω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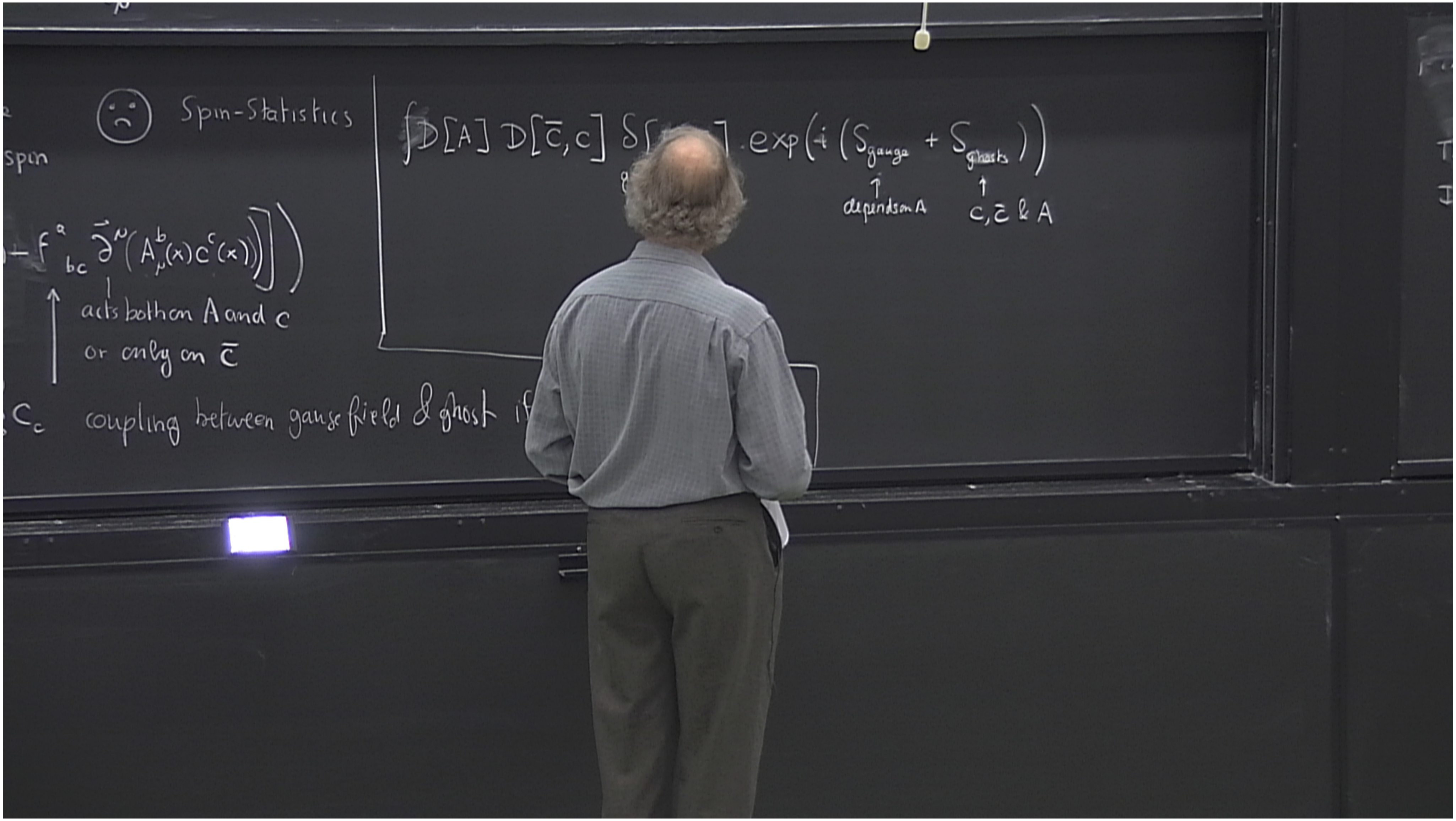
The integral is independent of ω (gauge invariance)
I can integrate over ω


Spin-Statistics
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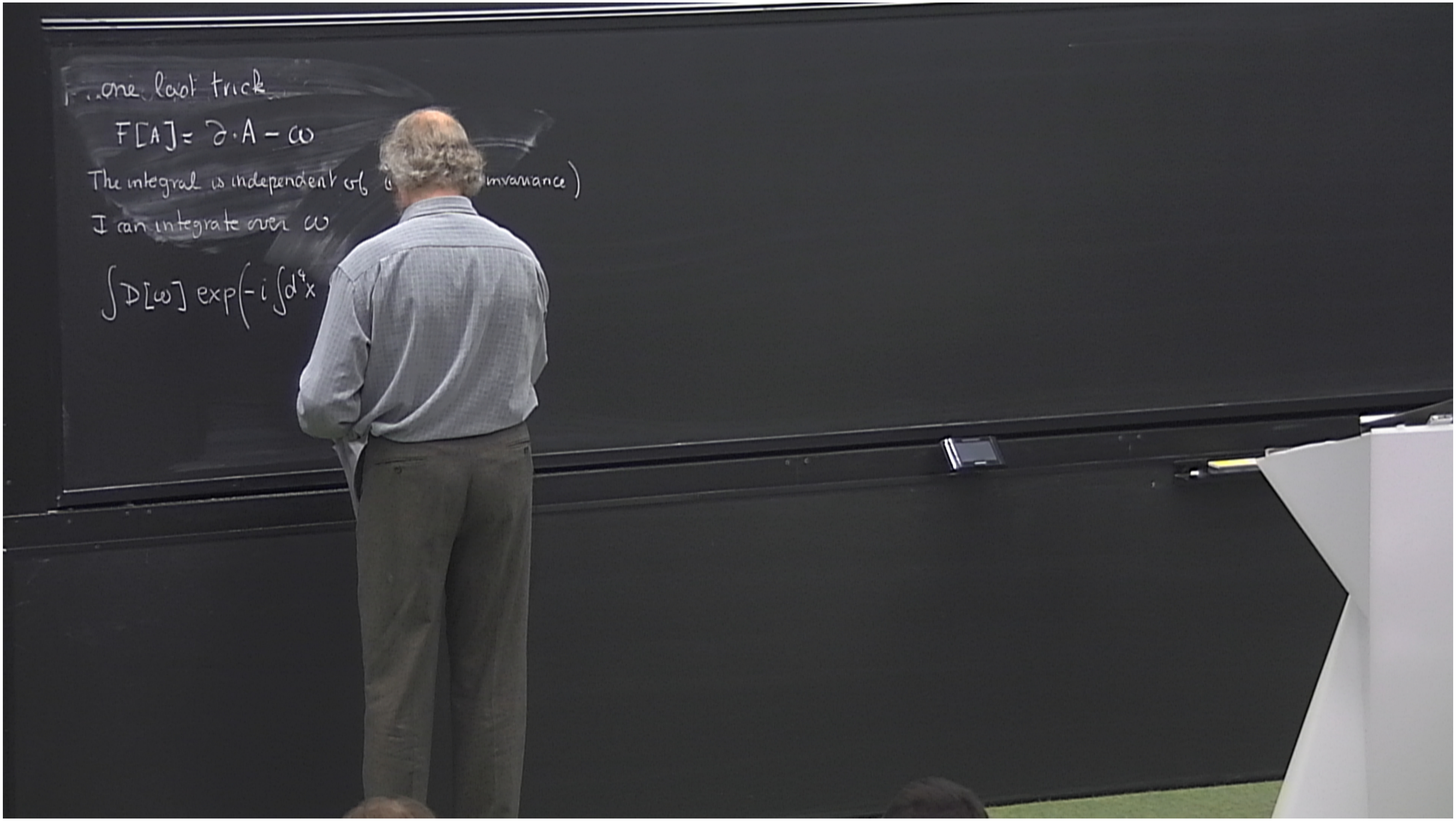
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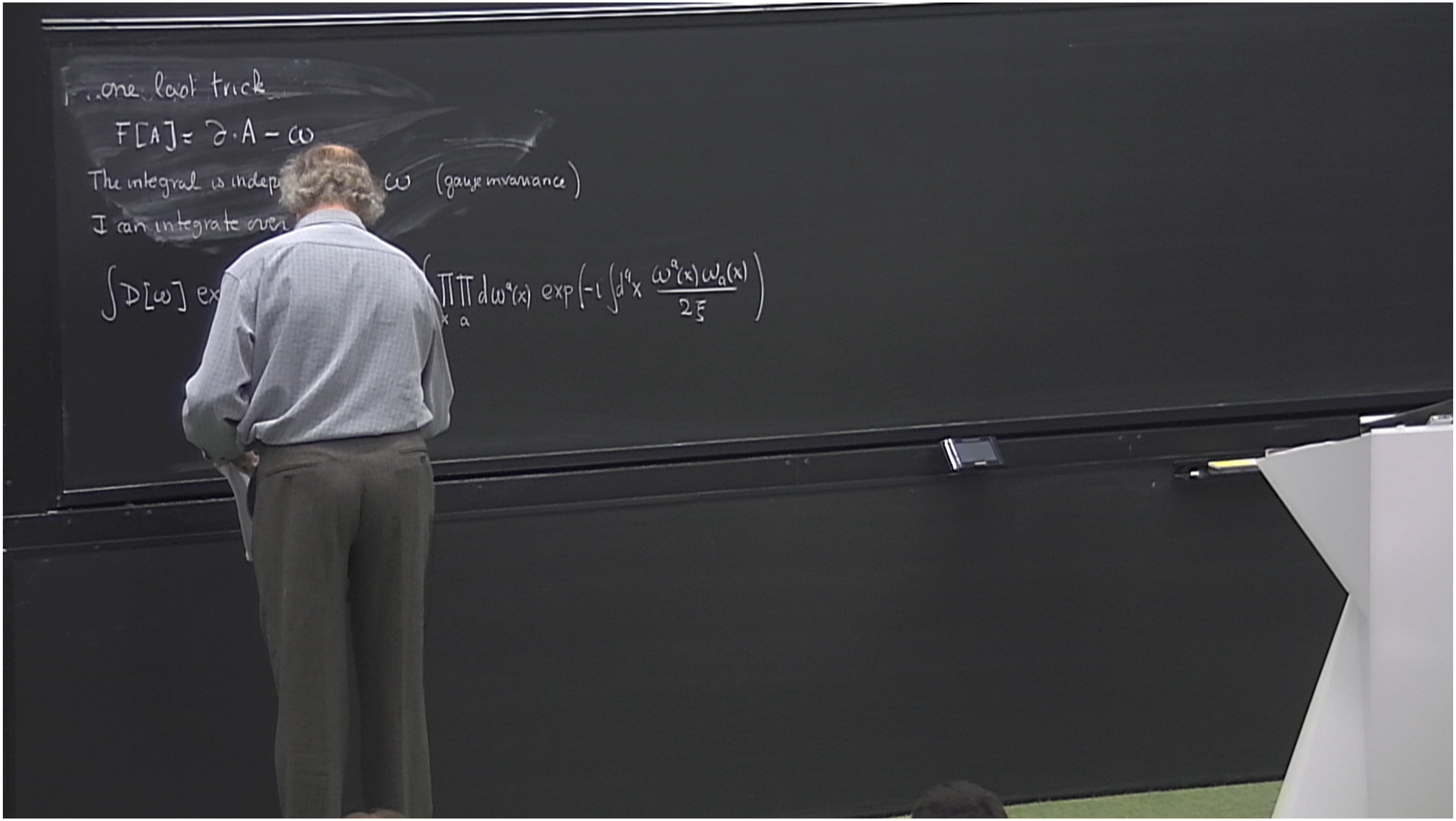
∫D[ω] exp(−i ∫d⁴x



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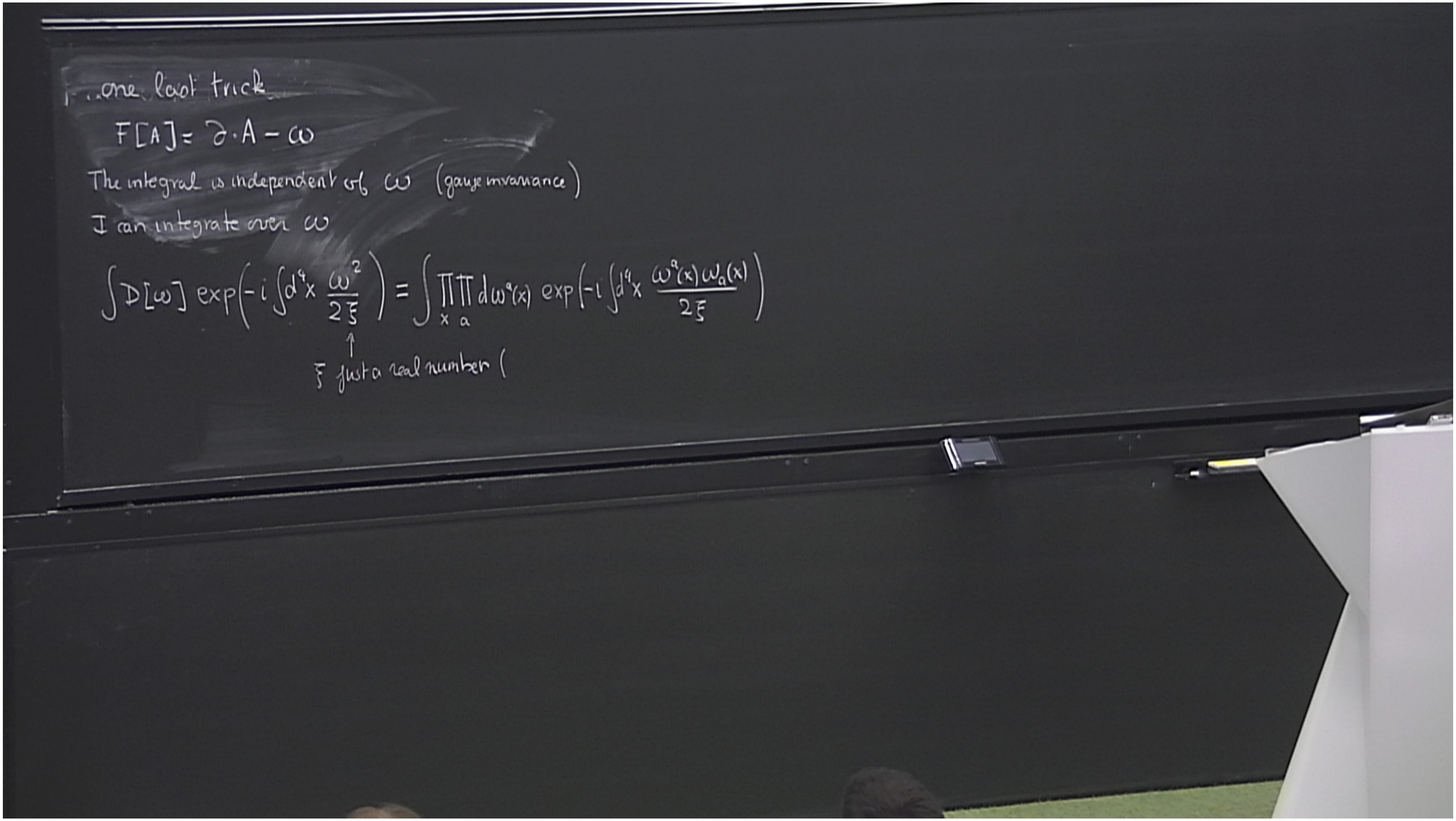
∫D[ω] ex ∏∏ dω^a(x) exp(−i ∫d^4x ω^a(x)ω_a(x) / 2ξ)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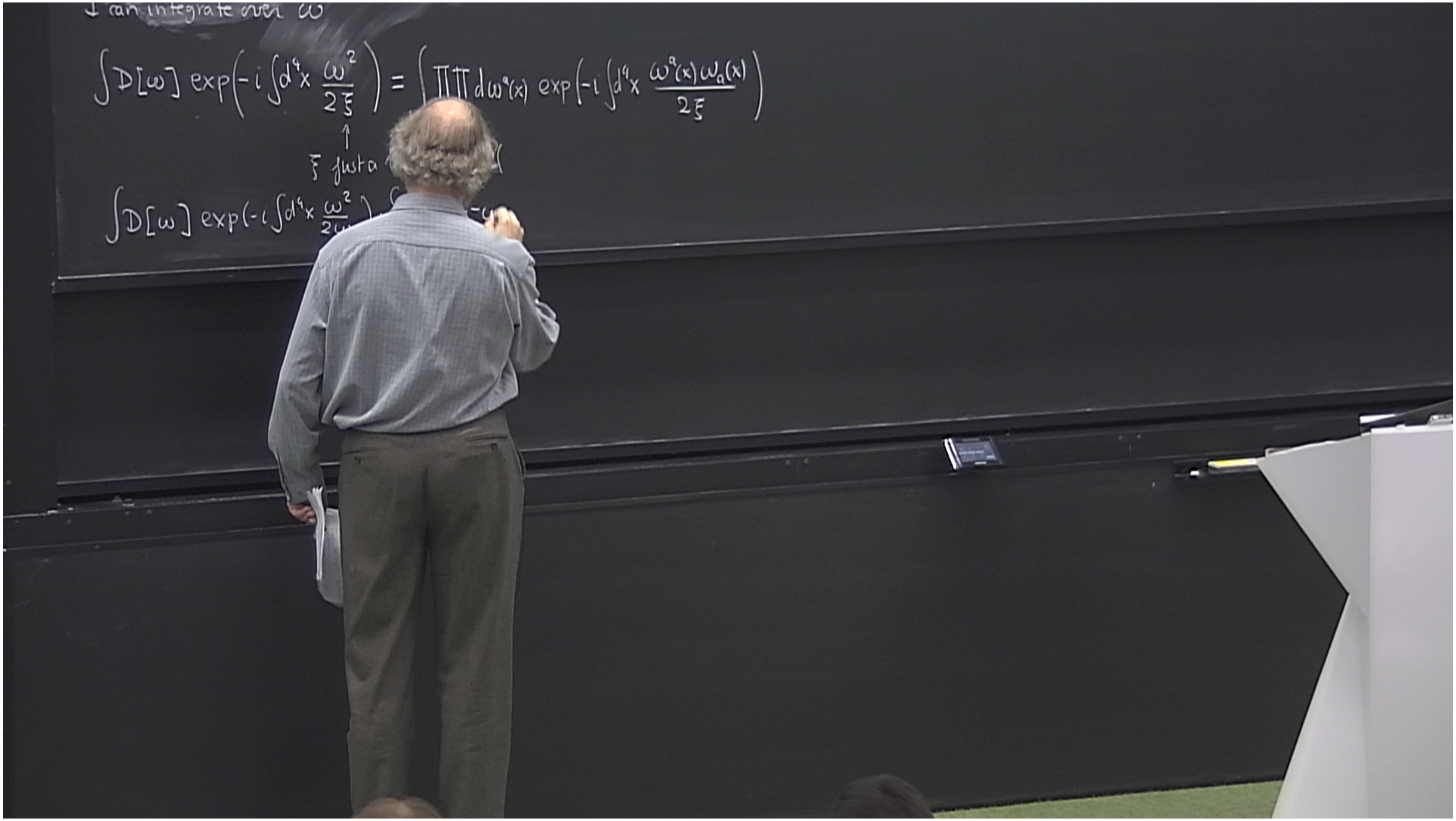
ξ just a real number (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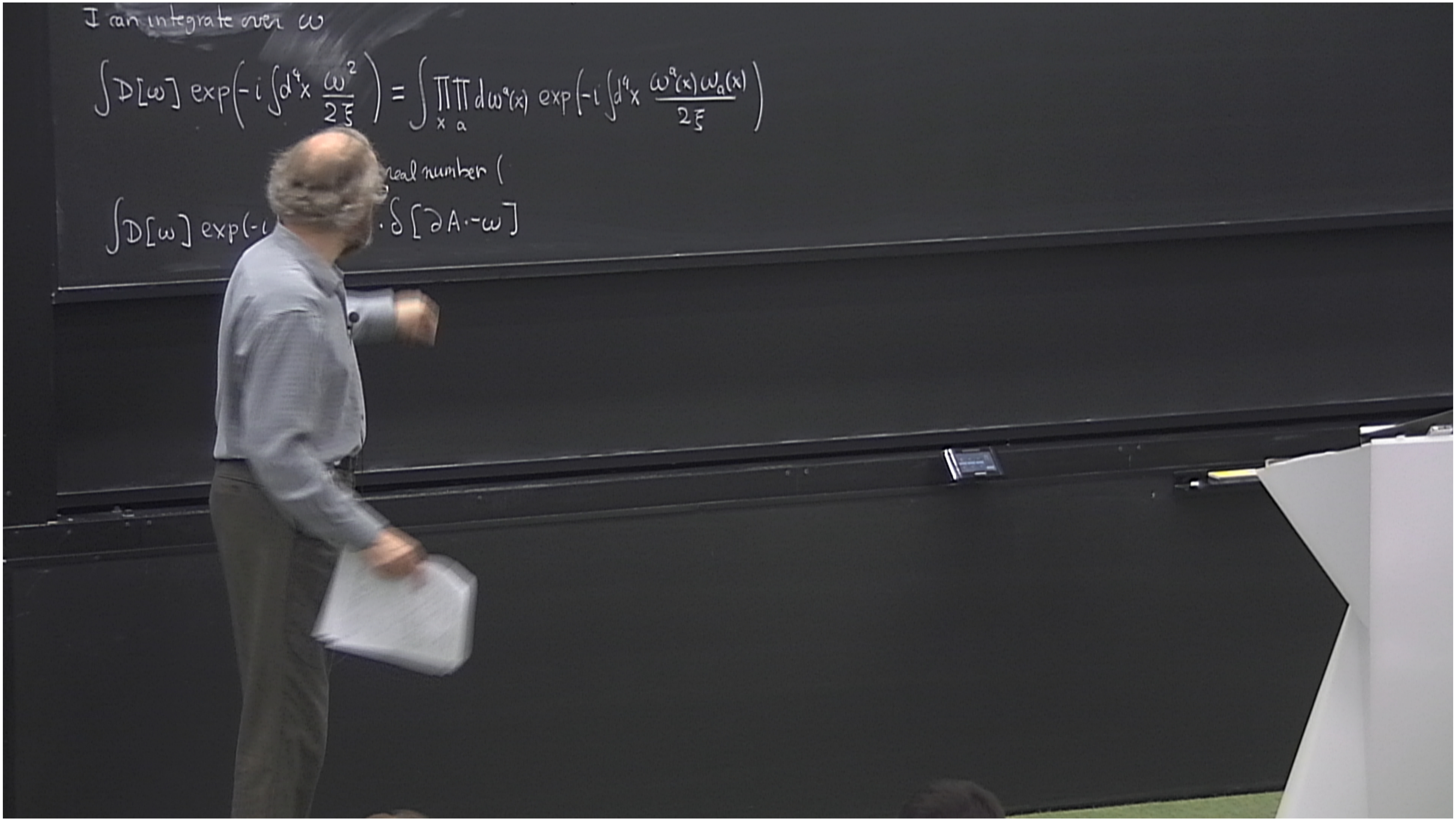
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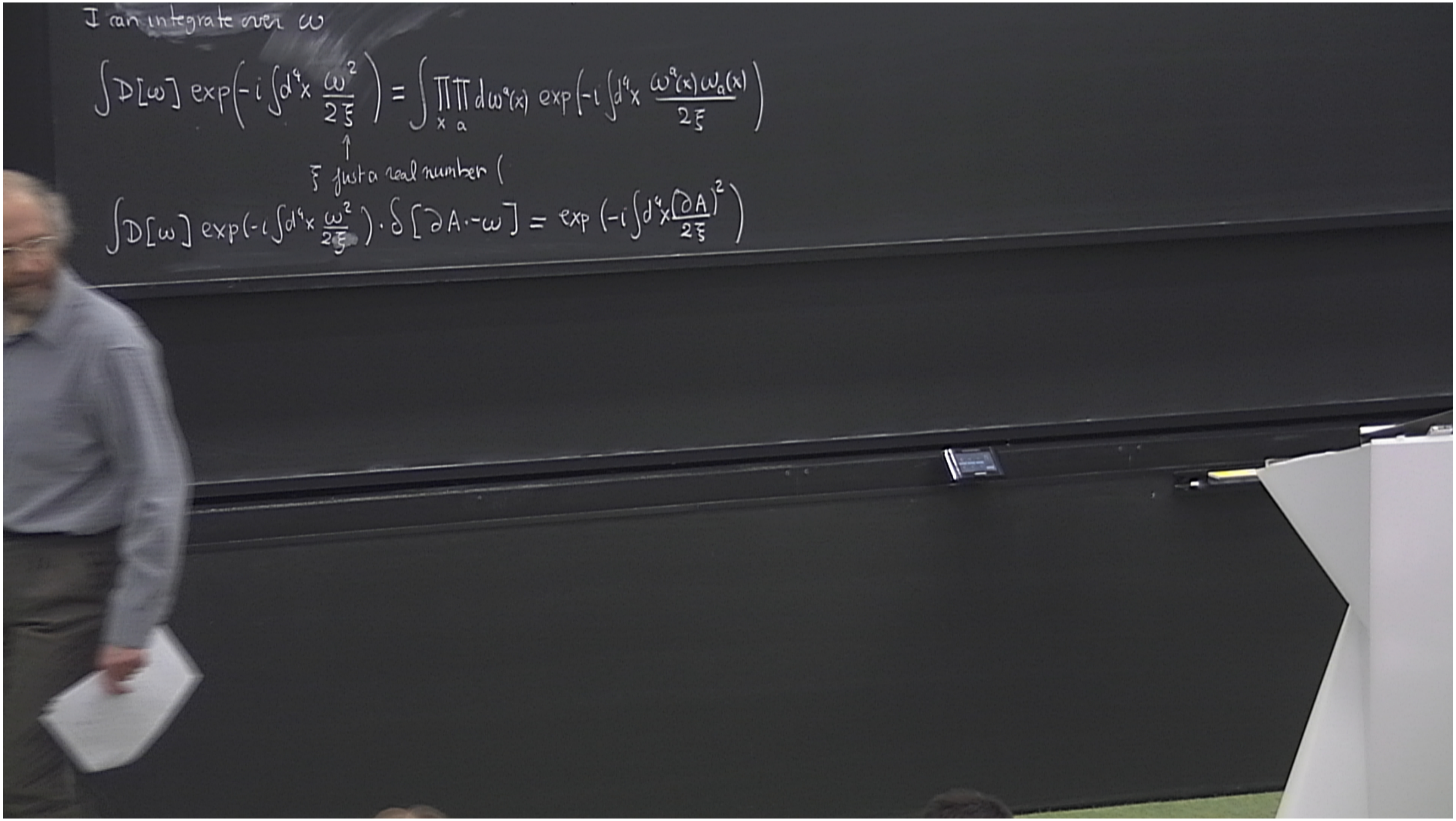
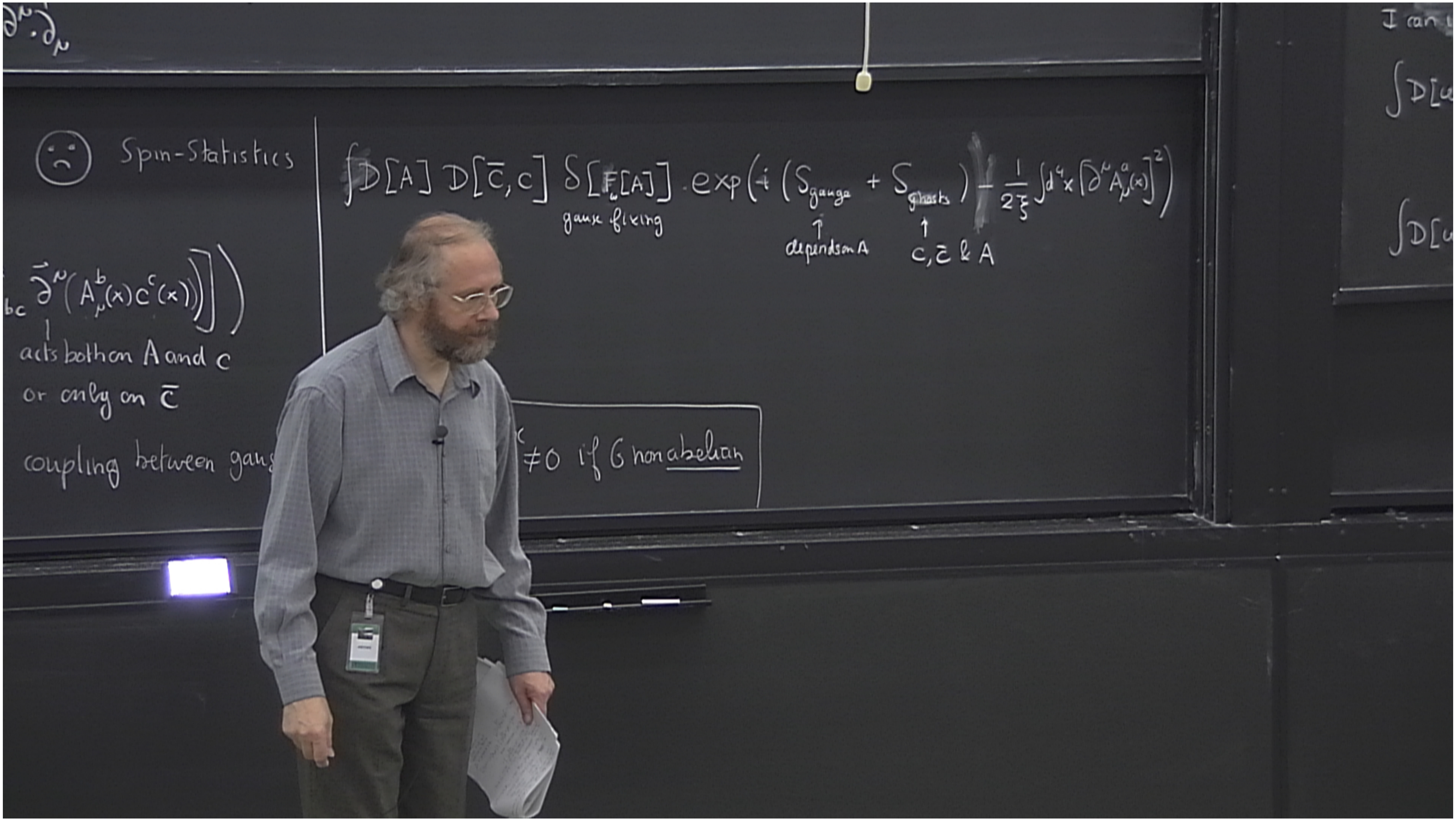
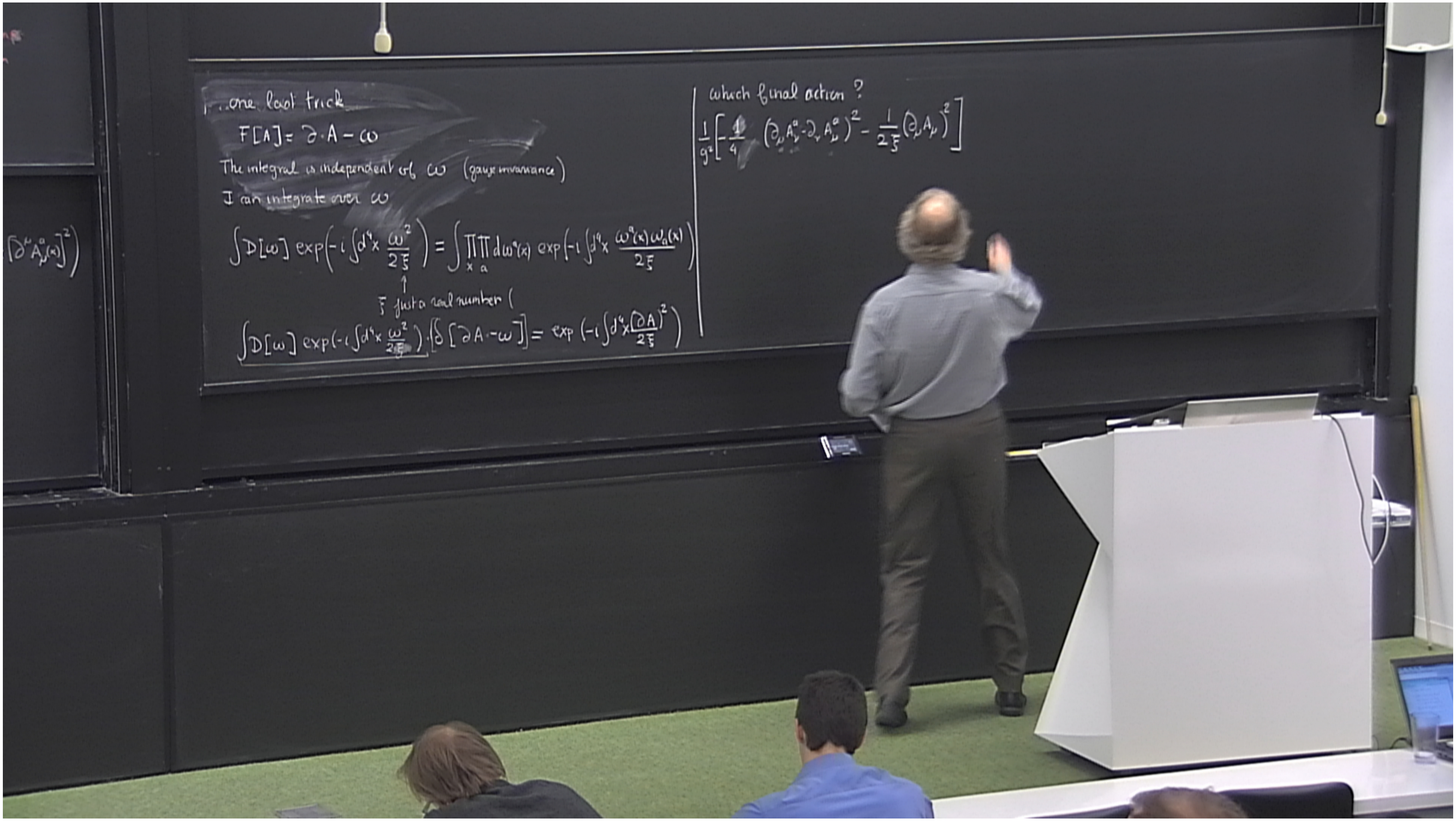
$\int D[\omega] \exp(-i$ … $\cdot\, \delta[\partial A \cdot - \omega]$


I can integrate over $\omega$
$\int D[\omega] \exp\left(-i\int d^4x \frac{\omega^2}{2\xi}\right) = \int \prod_x \prod_a d\omega^a(x) \exp\left(-i\int d^4x \frac{\omega^a(x)\omega_a(x)}{2\xi}\right)$
$\xi$ just a real number (
$\int D[\omega] \exp\left(-i\int d^4x \frac{\omega^2}{2\xi}\right) \cdot \delta[\partial A \cdot - \omega] = \exp\left(-i\int d^4x \frac{(\partial A)^2}{2\xi}\right)$


Spin-Statistics
$\int D[A]\, D[\bar{c}, c]\, \delta[F_\omega[A]]\, \exp\left(i\left(S_{gauge} + S_{ghosts}\right) - \frac{1}{2\xi}\int d^4x\, [\partial^\mu A^a_\mu(x)]^2\right)$
gauge fixing
depends on A
$c, \bar{c}$ & A
acts both on A and c
or only on $\bar{c}$
$\neq 0$ if G nonabelian



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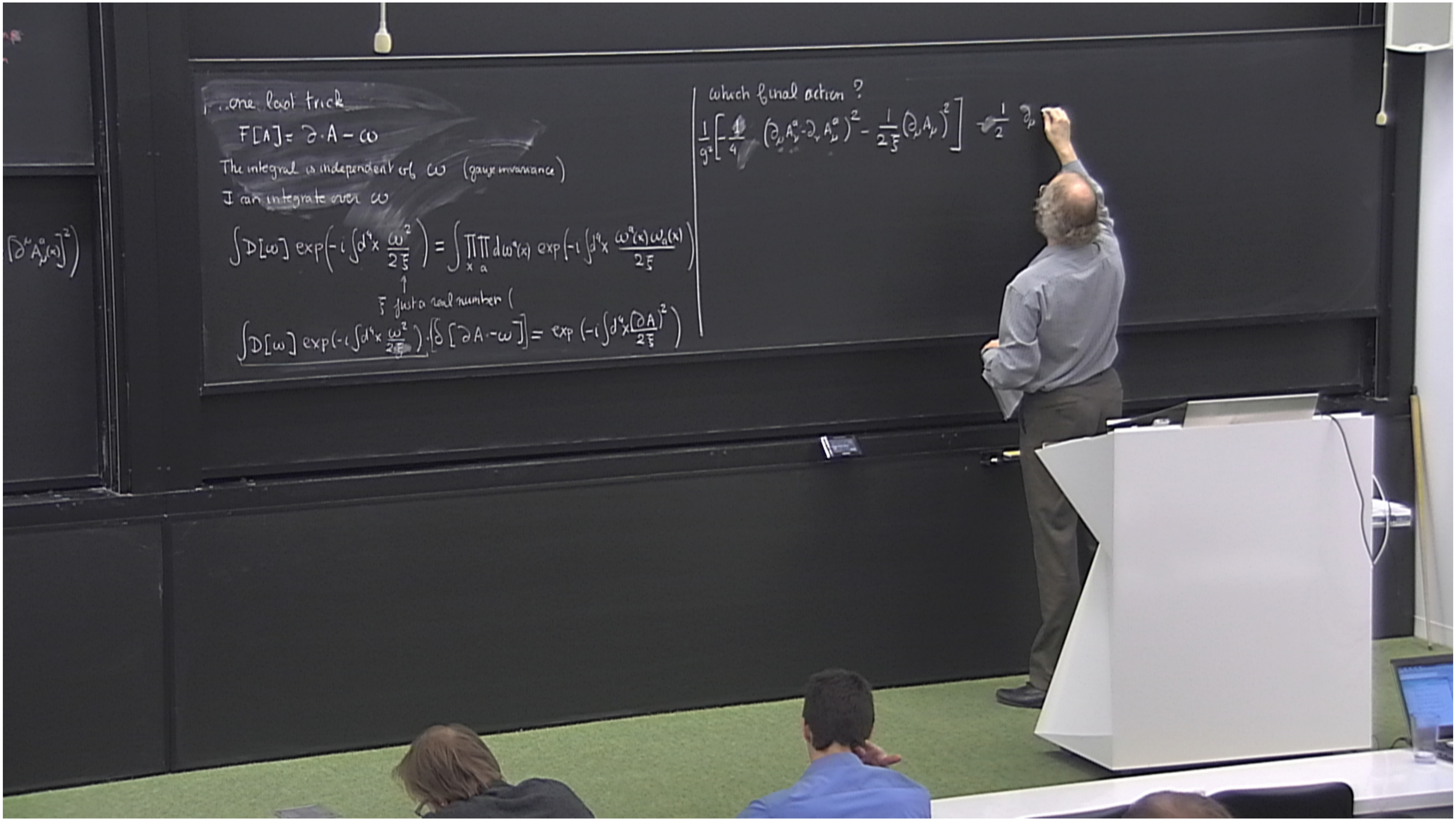
F[A] = ∂·A − ω
The integral is independent of ω (gauge invariance)
I can integrate over ω
ξ just a real number
which final action ?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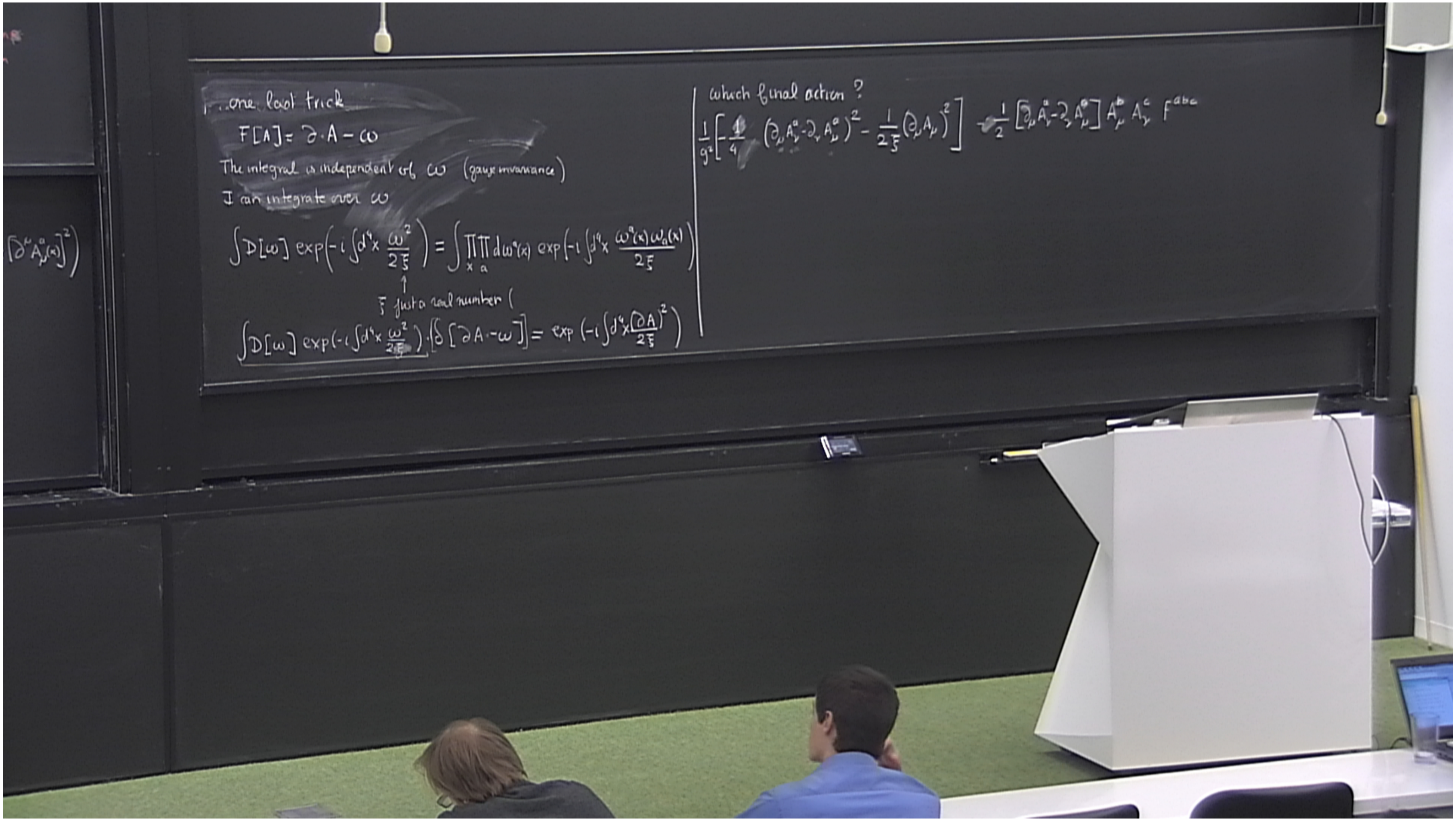
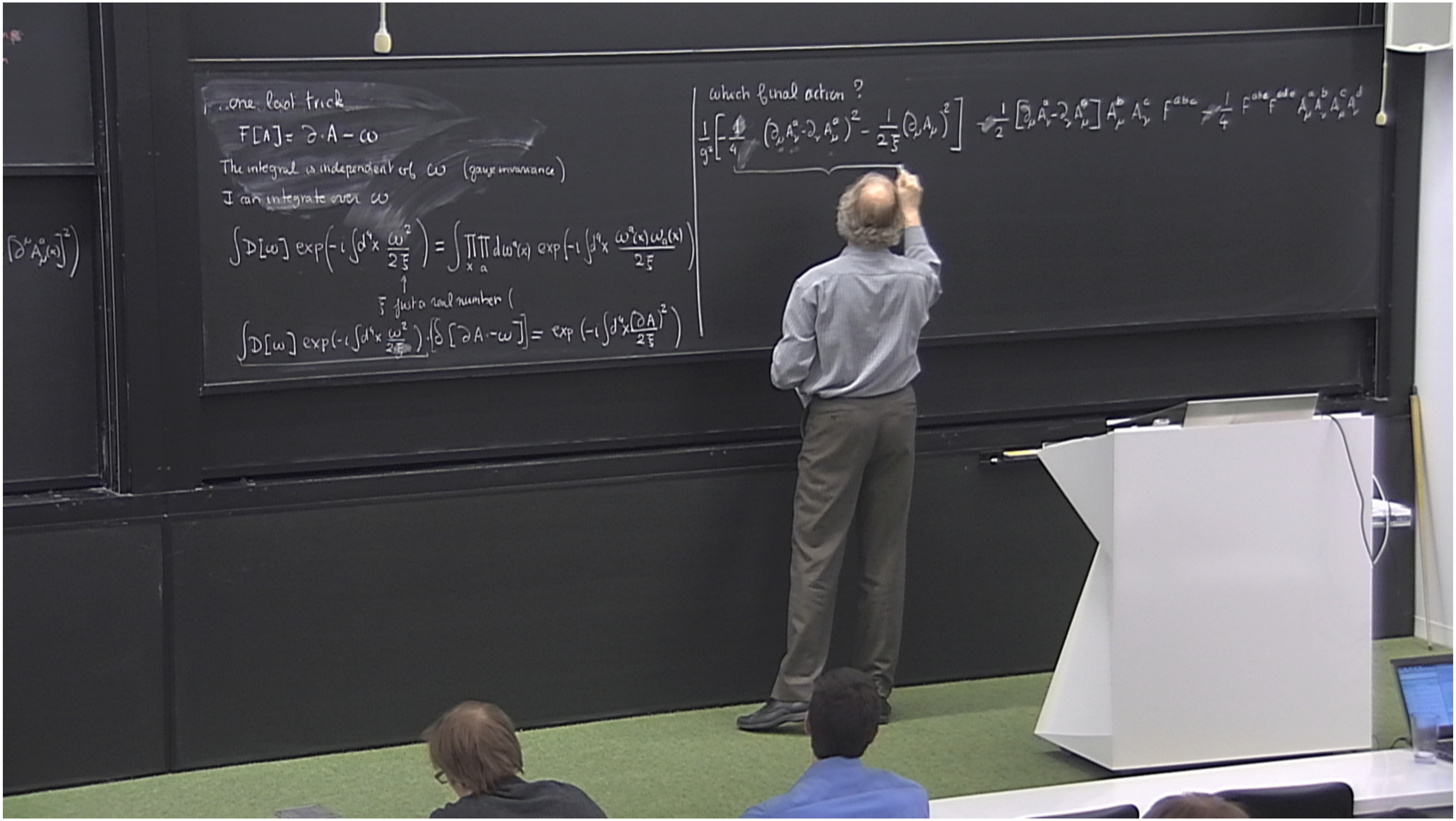

...one last trick
$F[A] = \partial \cdot A - \omega$
The integral is independent of $\omega$ (gauge invariance)
I can integrate over $\omega$
$\int D[\omega] \exp\left(-i \int d^4x \frac{\omega^2}{2\xi}\right) = \int \prod_x \prod_a d\omega^a(x) \exp\left(-i \int d^4x \frac{\omega^a(x)\omega_a(x)}{2\xi}\right)$
$\xi$ just a real number
$\int D[\omega] \exp\left(-i \int d^4x \frac{\omega^2}{2\xi}\right) \cdot \delta[\partial A - \omega] = \exp\left(-i \int d^4x \frac{(\partial A)^2}{2\xi}\right)$
which final action ?
$\frac{1}{g^2}\left[-\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 - \frac{1}{2\xi}(\partial_\mu A_\mu)^2\right] - \frac{1}{2}[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu] A^b_\mu A^c_\nu F^{abc}$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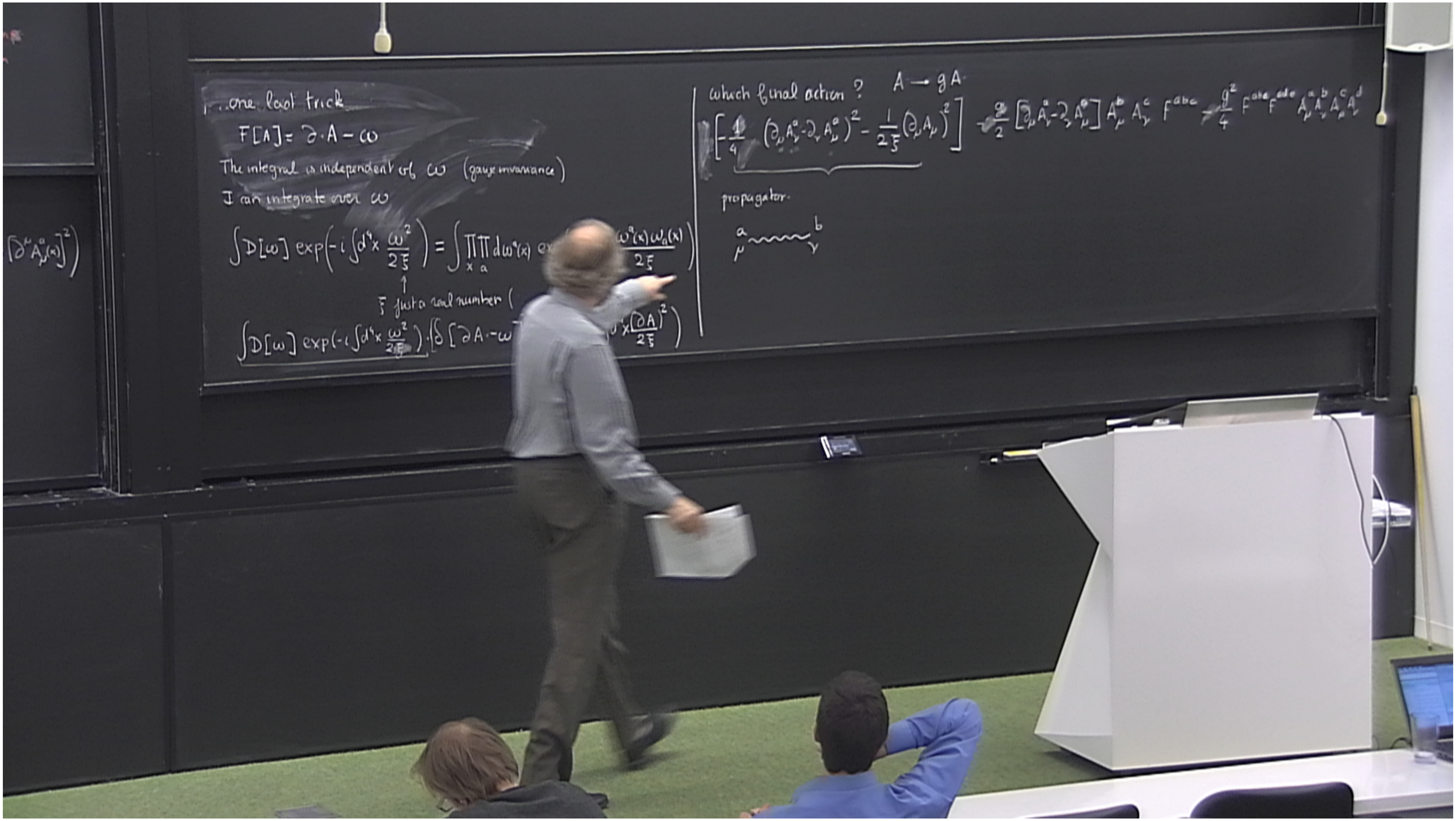

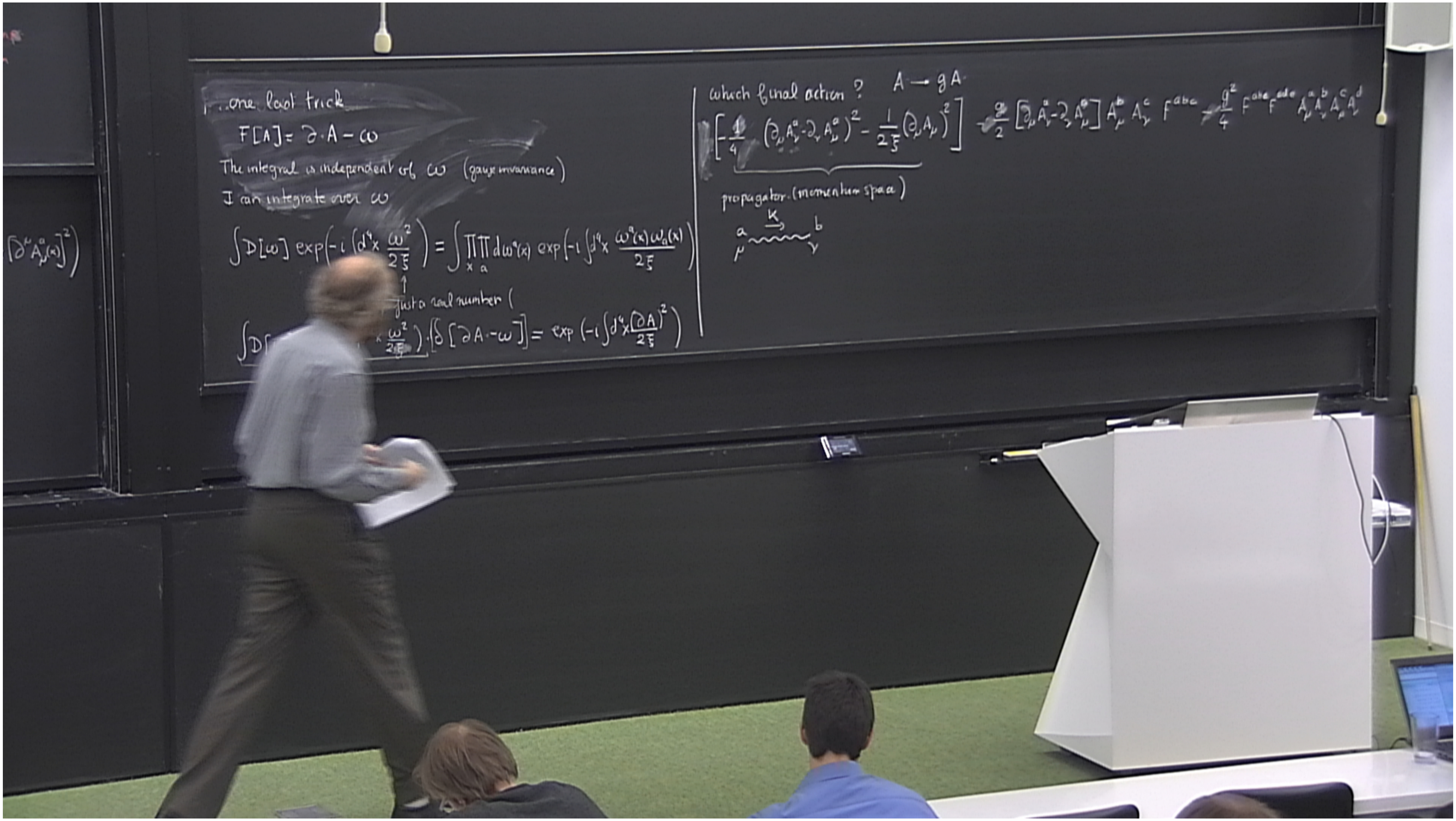

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The integral is independent of ω (gauge invariance)
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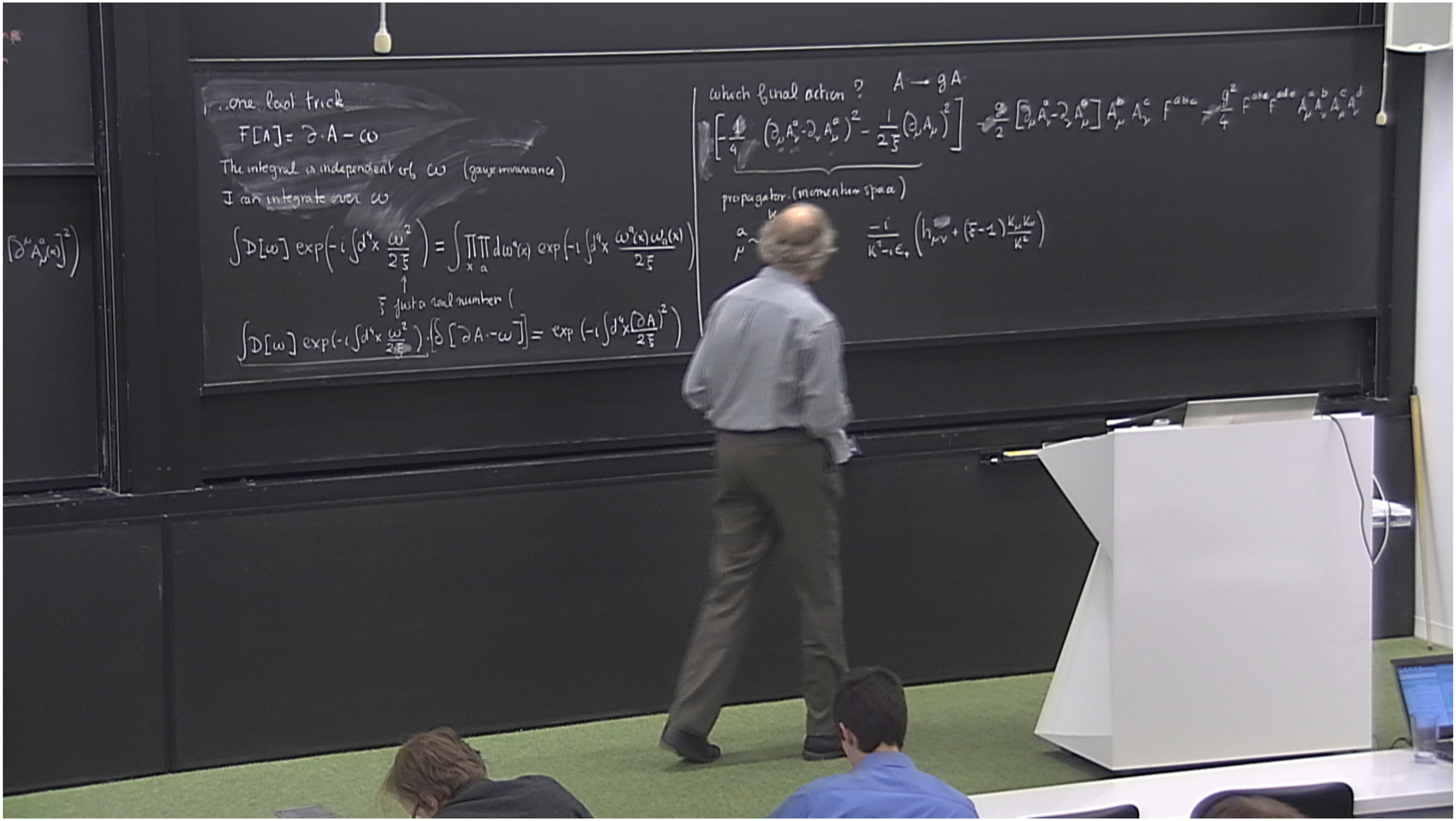
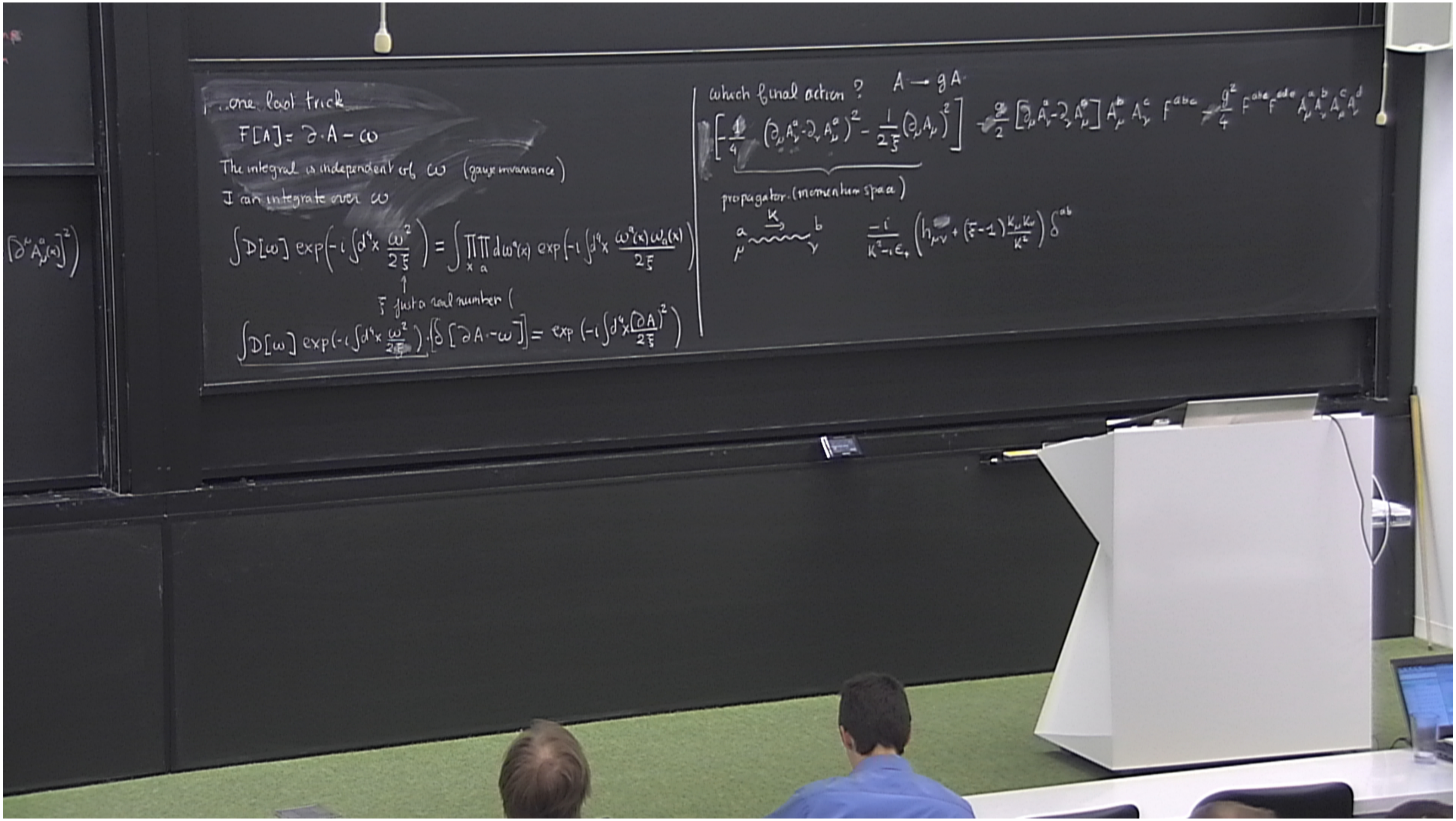
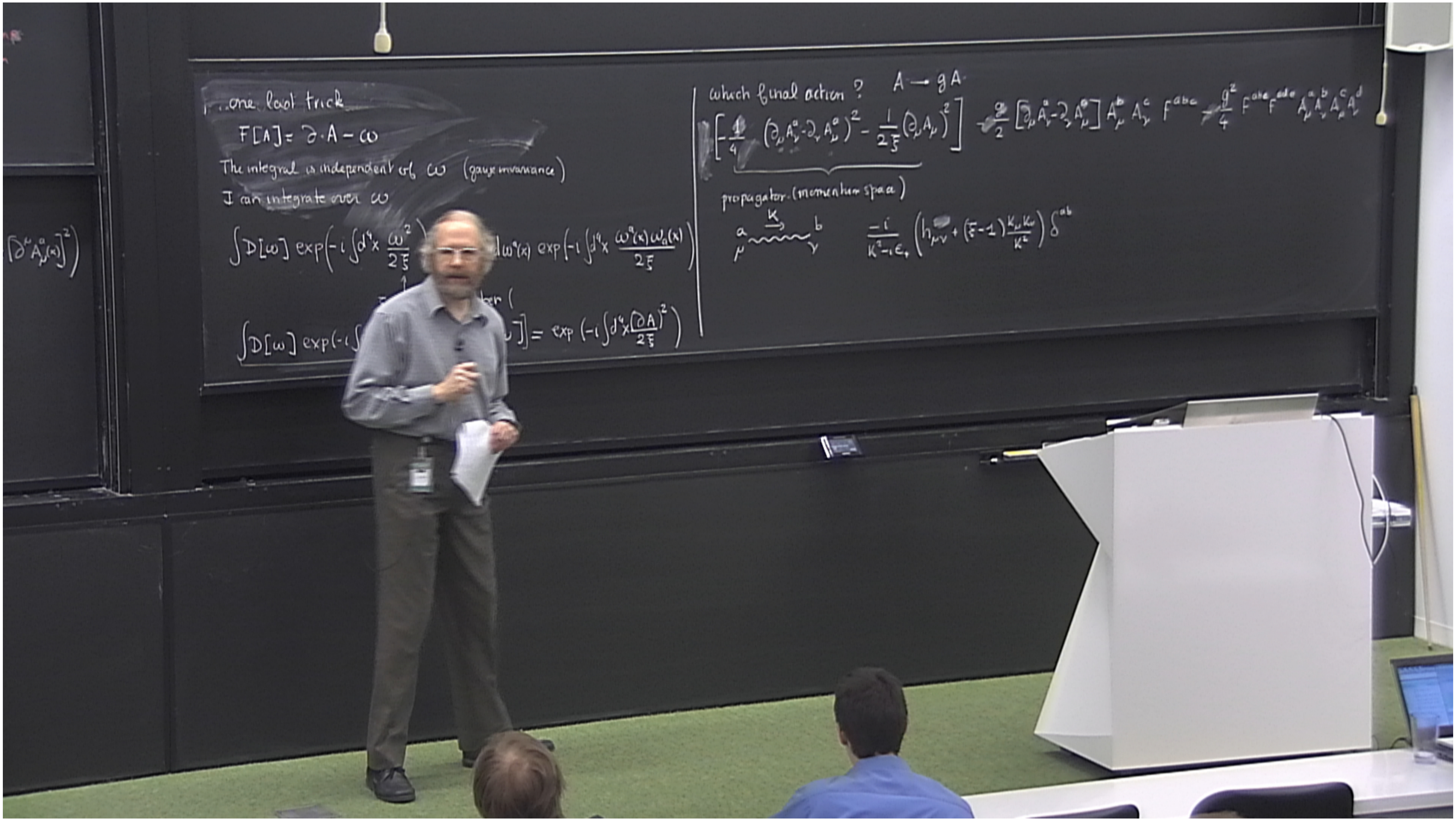
which final action ? A → gA
propagator (momentum space)


...one last trick
F[A] = ∂·A − ω
The integral is independent of ω (gauge invariance)
I can integrate over ω
ξ just a real number
which final action ? A → gA
propagator (momentum space)


... one last trick
$F[A] = \partial \cdot A - \omega$
The integral is independent of $\omega$ (gauge invariance)
I can integrate over $\omega$
$\int D[\omega] \exp\left(-i \int d^4x \frac{\omega^2}{2\xi}\right) = \int \prod_x \prod_a d\omega^a(x) \exp\left(-i \int d^4x \frac{\omega^a(x)\omega_a(x)}{2\xi}\right)$
$\xi$ just a real number (
$\int D[\omega] \exp\left(-i \int d^4x \frac{\omega^2}{2\xi}\right) \cdot \left(\delta[\partial A - \omega]\right) = \exp\left(-i \int d^4x \frac{(\partial A)^2}{2\xi}\right)$
which final action ? $A \to gA$
$\left[-\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 - \frac{1}{2\xi}(\partial_\mu A_\mu)^2\right] - \frac{g}{2}\left[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right] A^b_\mu A^c_\nu F^{abc} - \frac{g^2}{4} F^{abe} F^{cde} A^a_\mu A^b_\nu A^c_\mu A^d_\nu$
propagator (momentum space)
$\frac{-i}{k^2 - i\epsilon_+}\left(h_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}$



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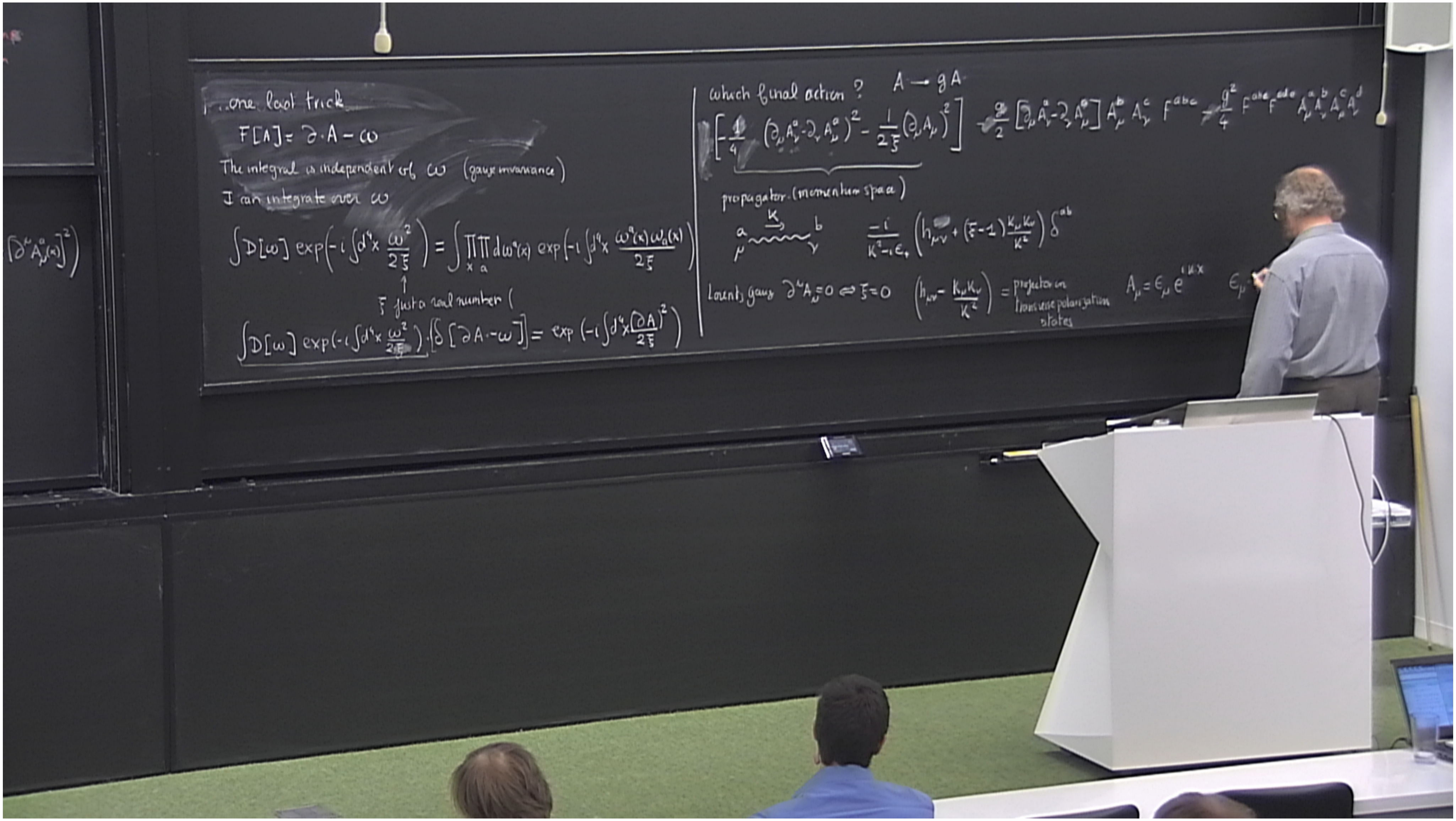
one last trick
F[A] = ∂·A − ω
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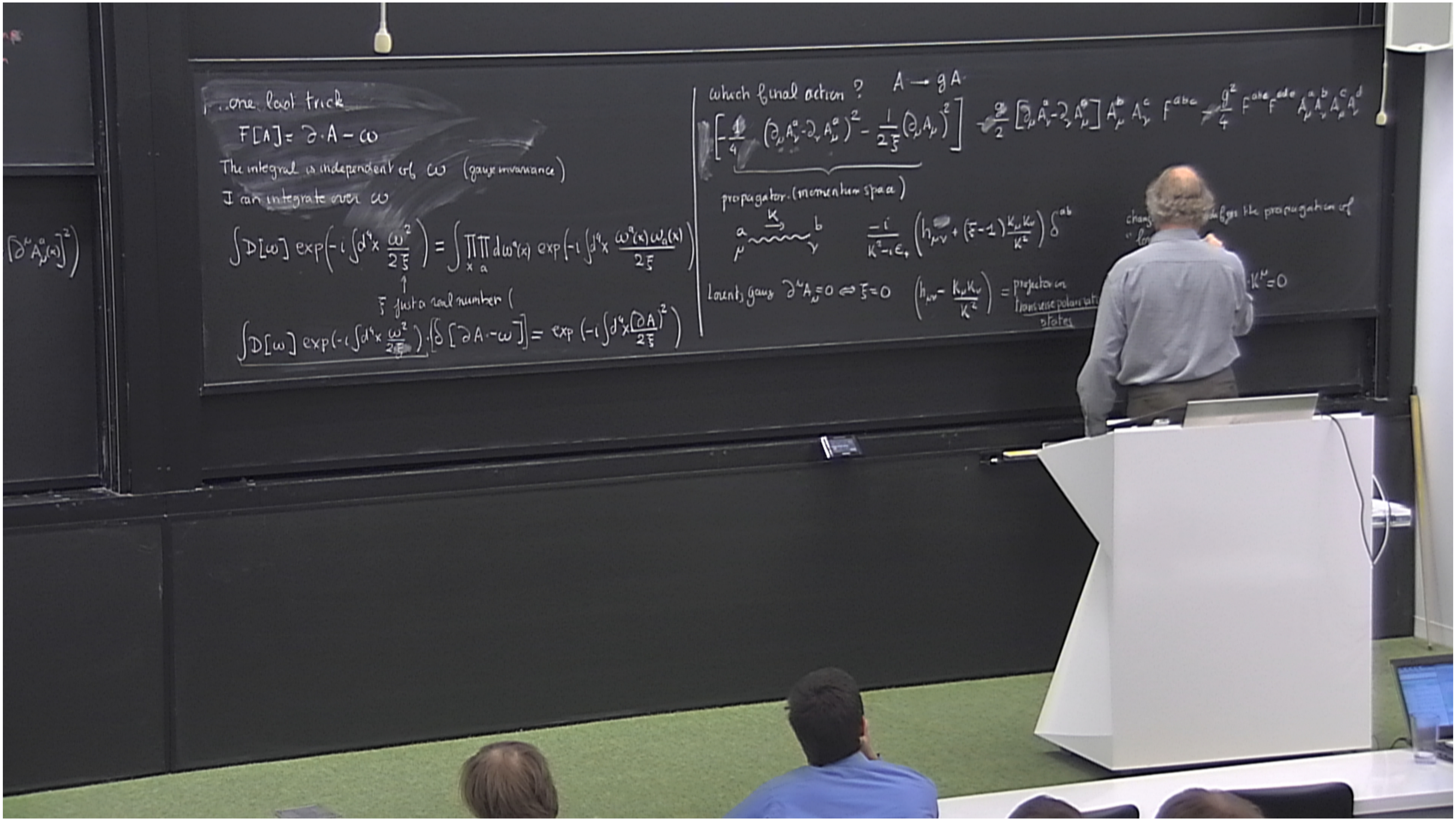
which final action ? A → gA
propagator (momentum space)
Lorentz gauge ∂^μ A_μ = 0 ⇔ ξ = 0
= projector on transverse polarization states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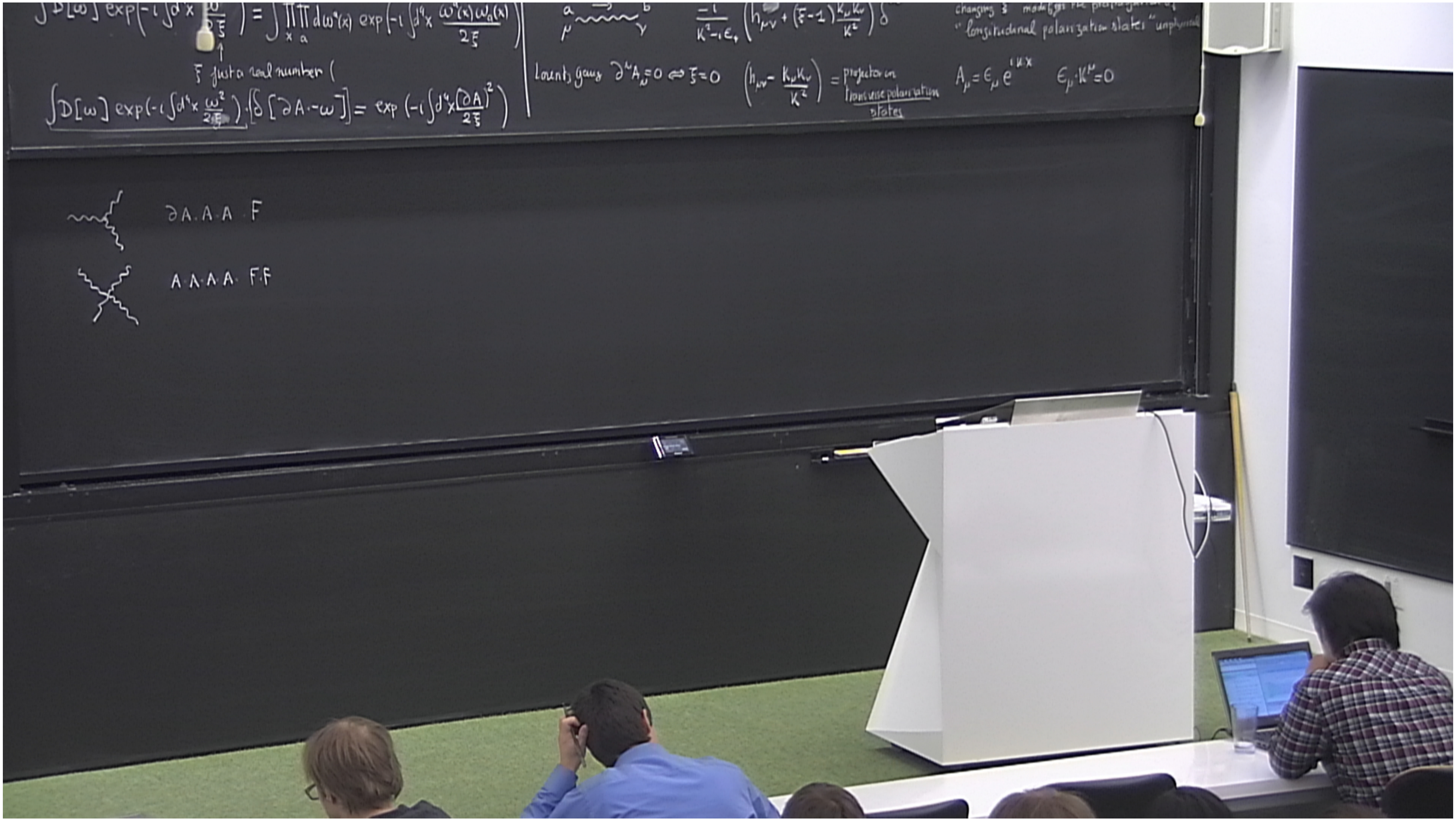
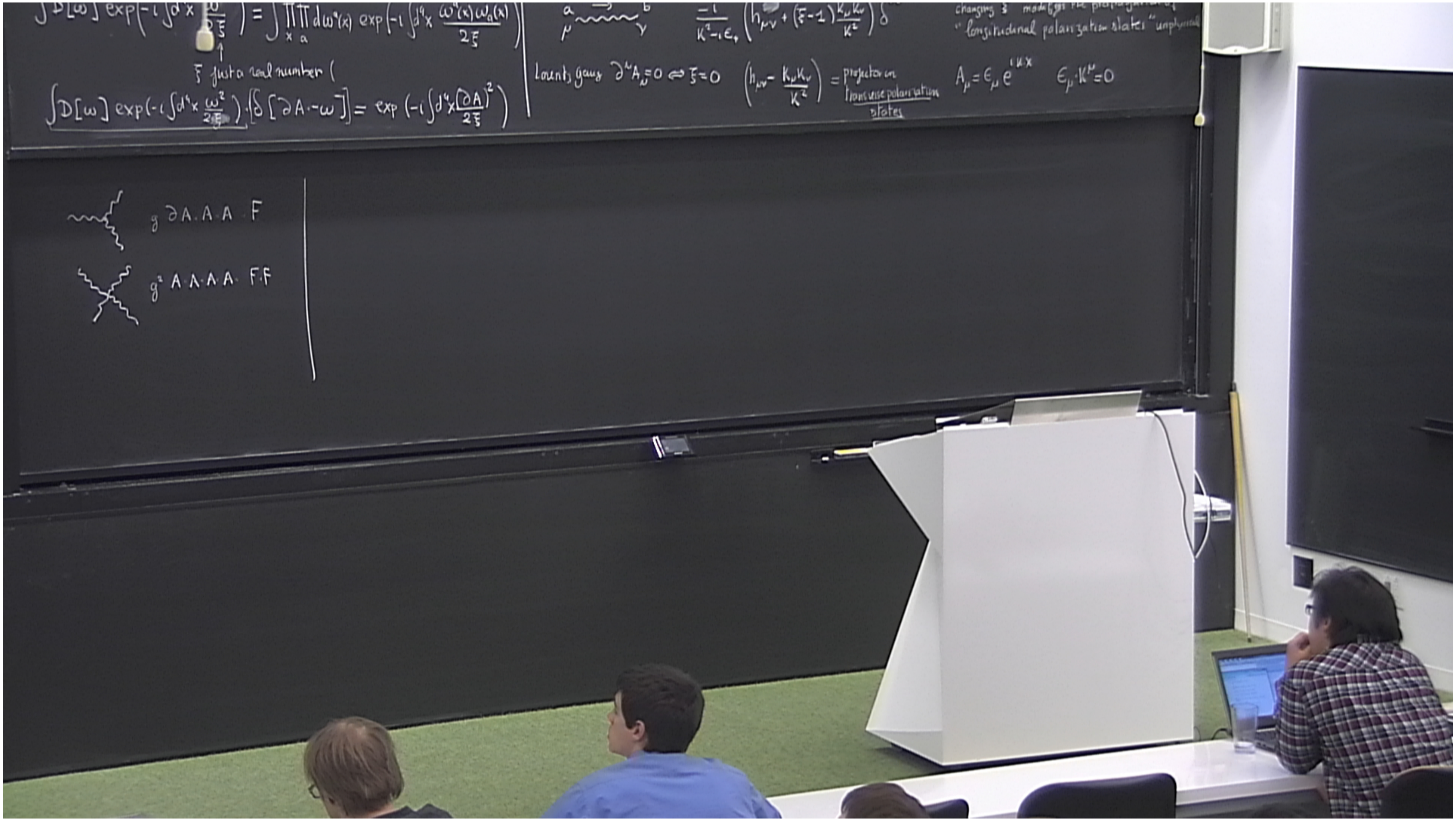
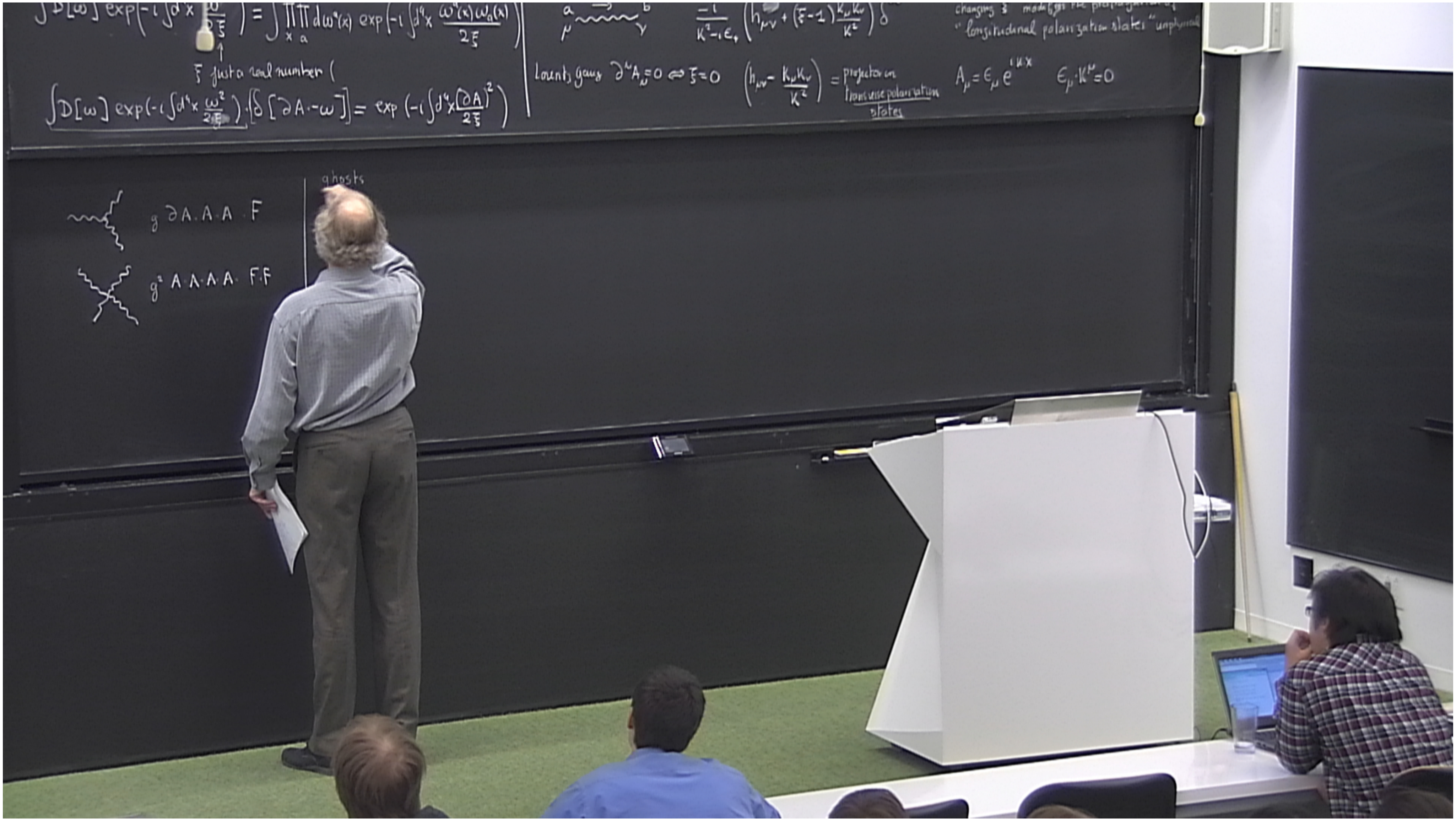
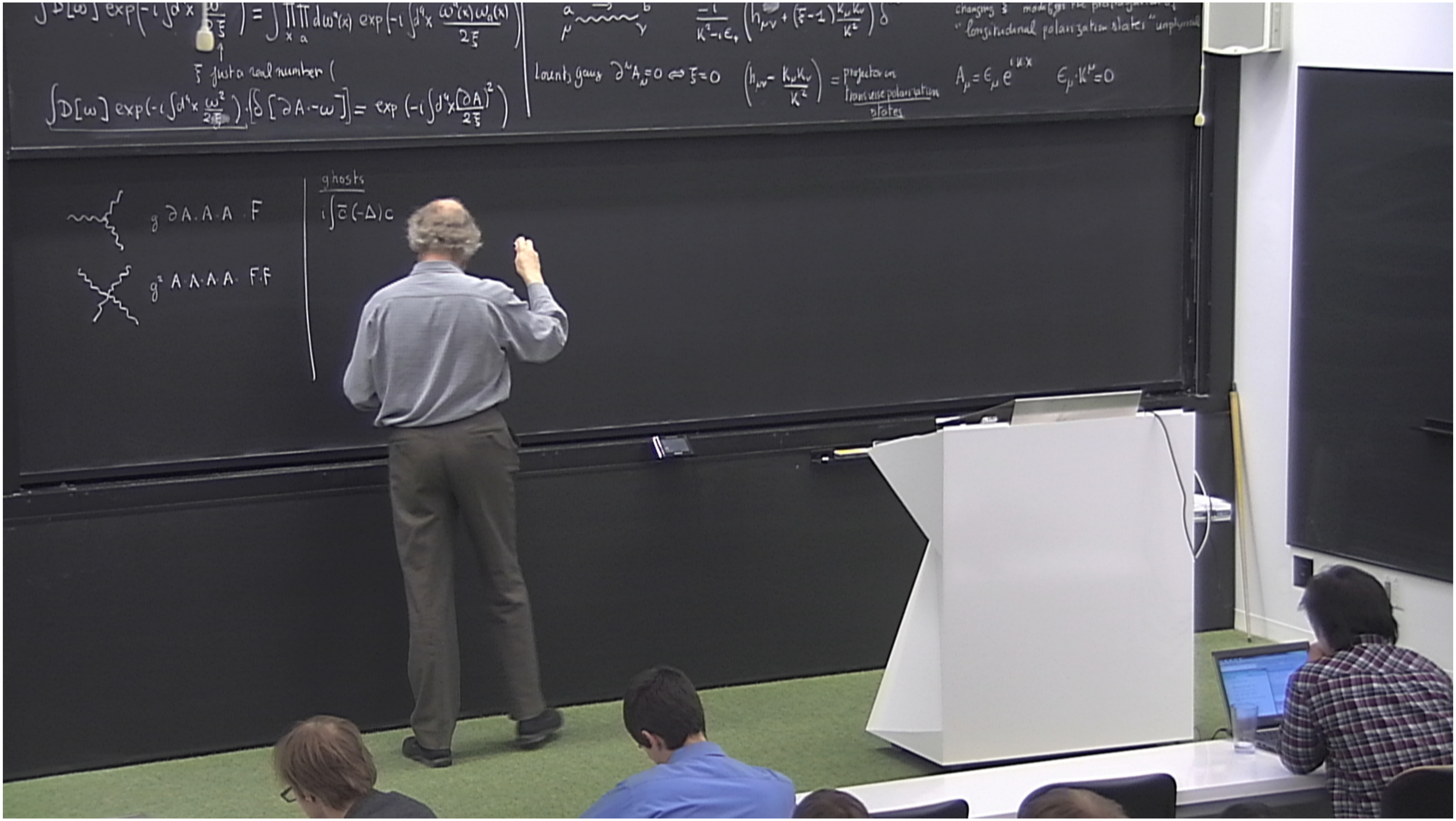
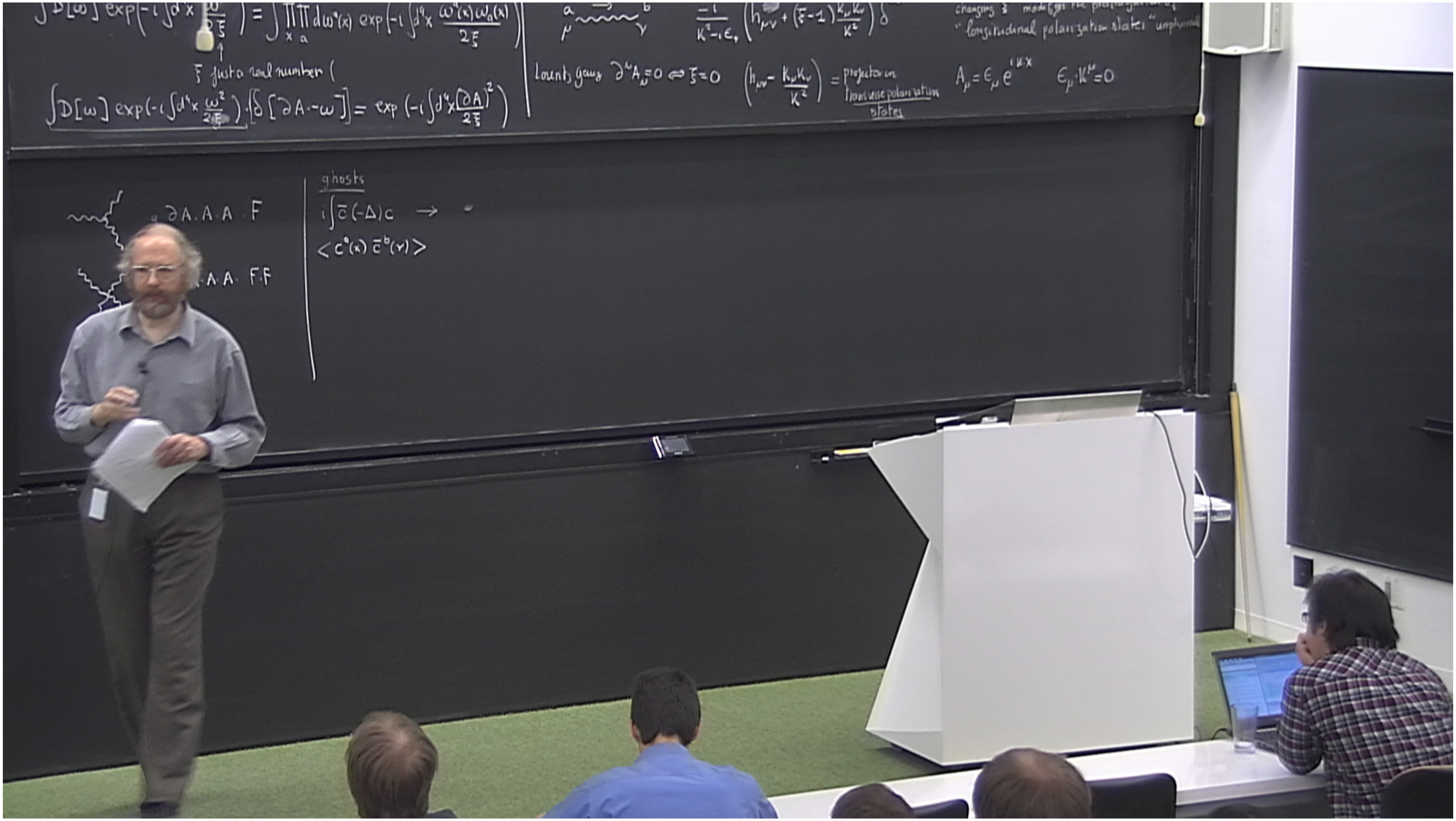

ξ just a real number (
Lorentz gauge ∂^μ A_μ = 0 ⇔ ξ = 0
= projector on transverse polarization states
longitudinal polarization states
∂A·A·A  F
A·A·A·A·  F·F

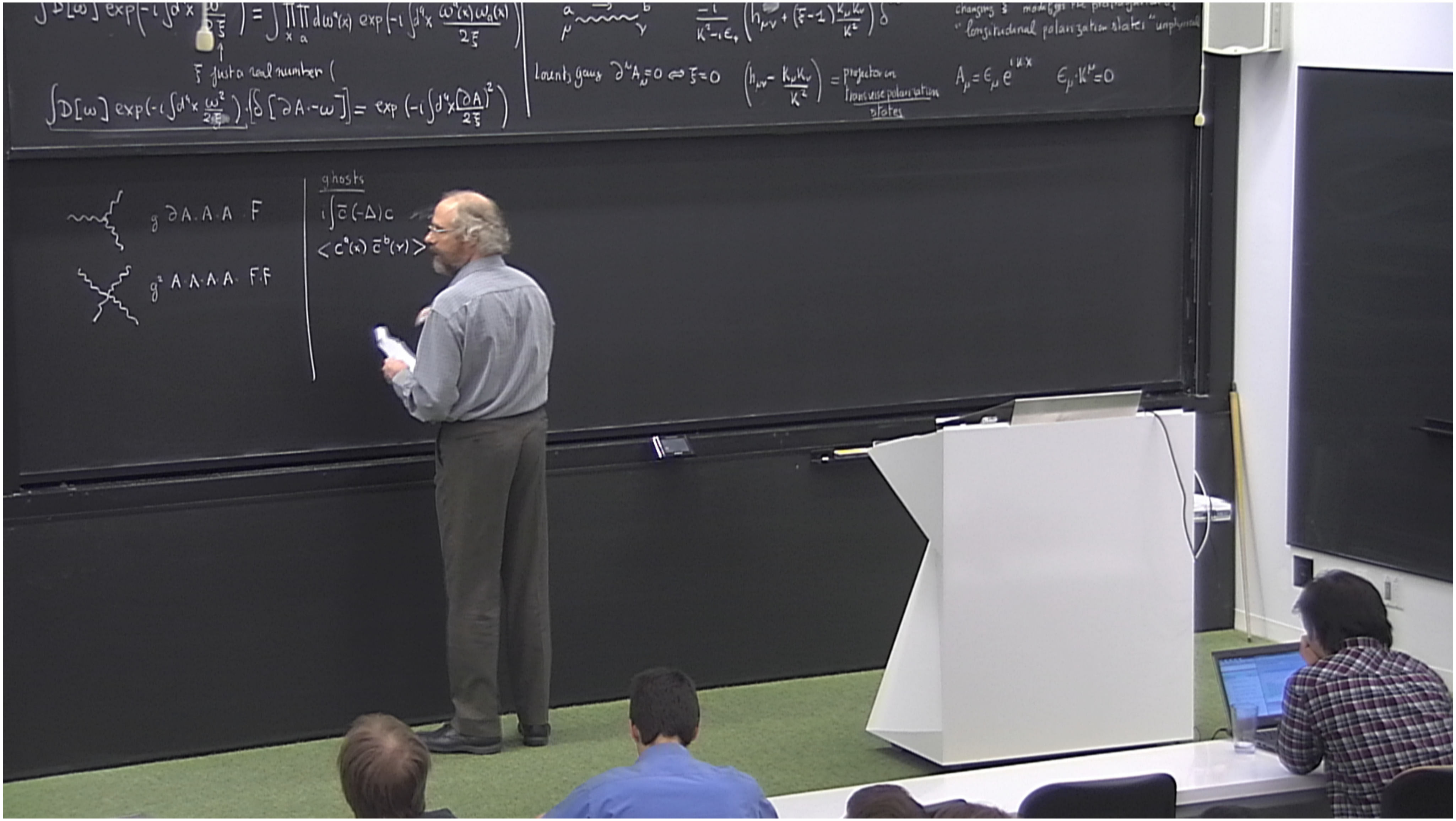

ξ just a real number (
"longitudinal polarization states" "unphysical"
projector on transverse polarization states
g ∂A·A·A F
g² A·A·A·A F·F

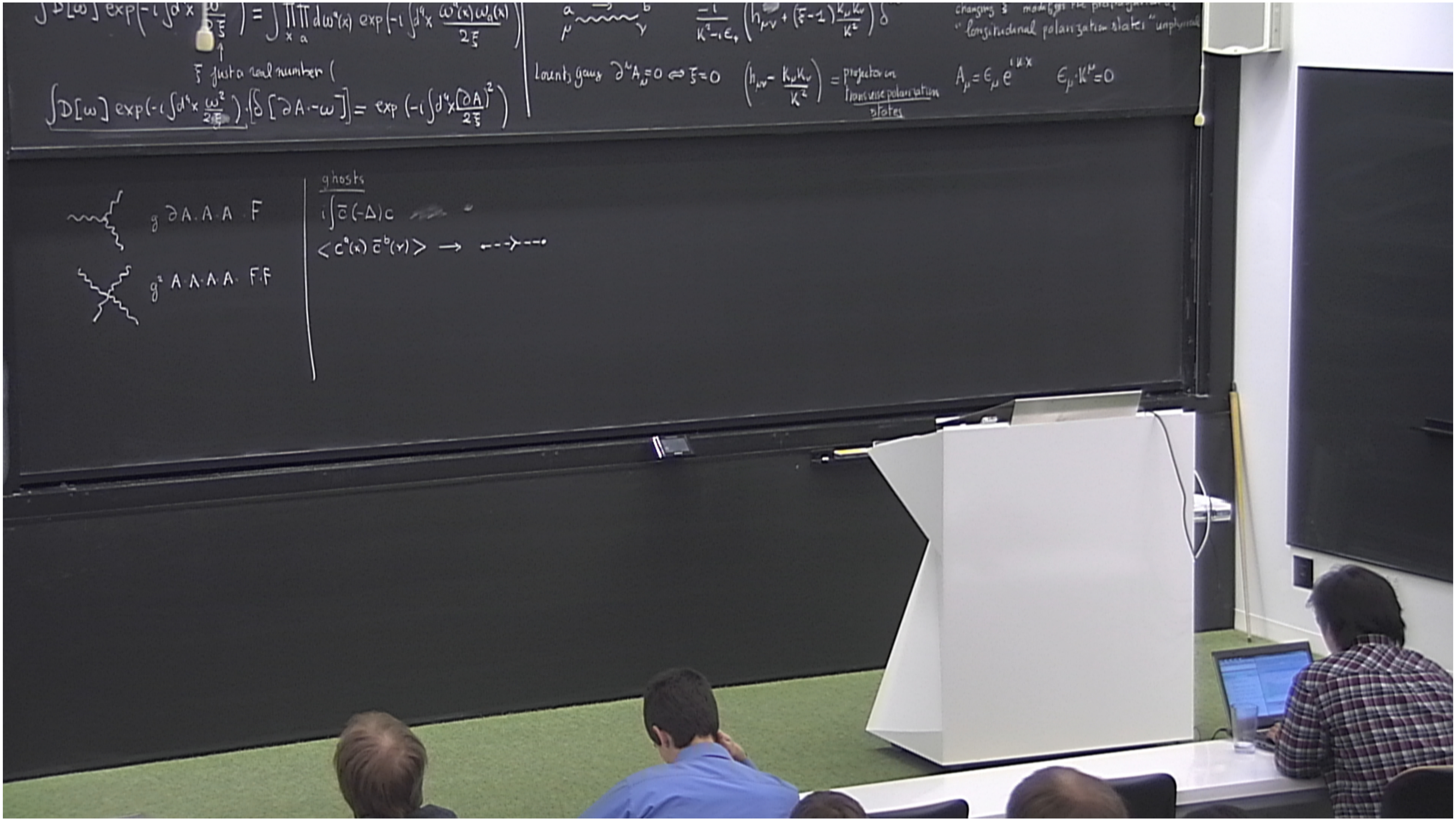


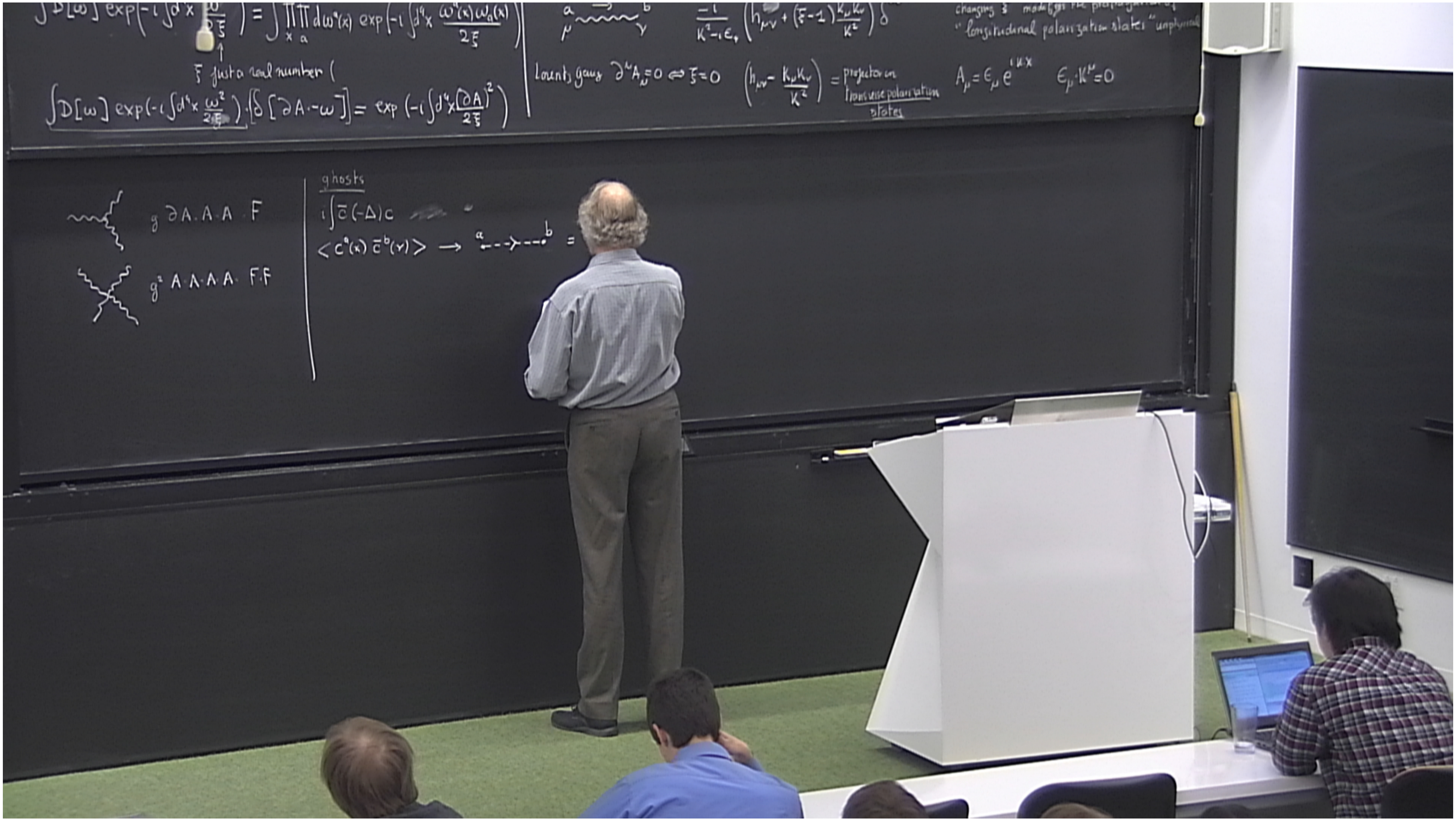
ξ just a real number
Lorentz gauge
projector on transverse polarization states
"longitudinal polarization states" unphysical
ghosts
g ∂A.A.A F
g² A.A.A.A F.F

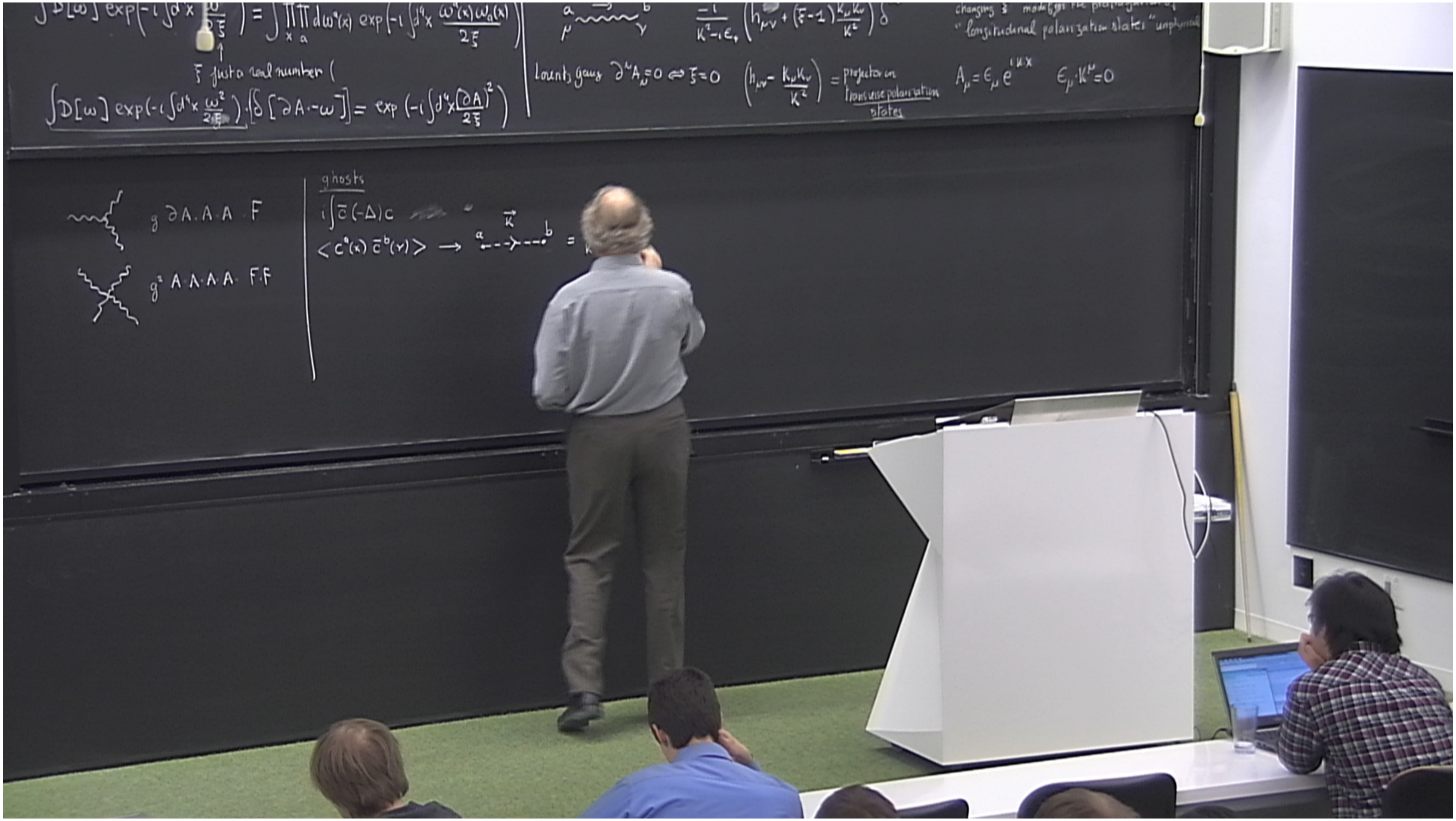





ghosts
Lorentz gauge
projector on transverse polarization states
longitudinal polarization states "unphysical"


ξ just a real number (
Lorentz gauge
projector on transverse polarization states
ghosts
g ∂A·A·A · F
g² A·A·A·A · F·F




ξ just a real number
Lorentz gauge
projector on transverse polarization states
longitudinal polarization states
ghosts
g ∂A.A.A F
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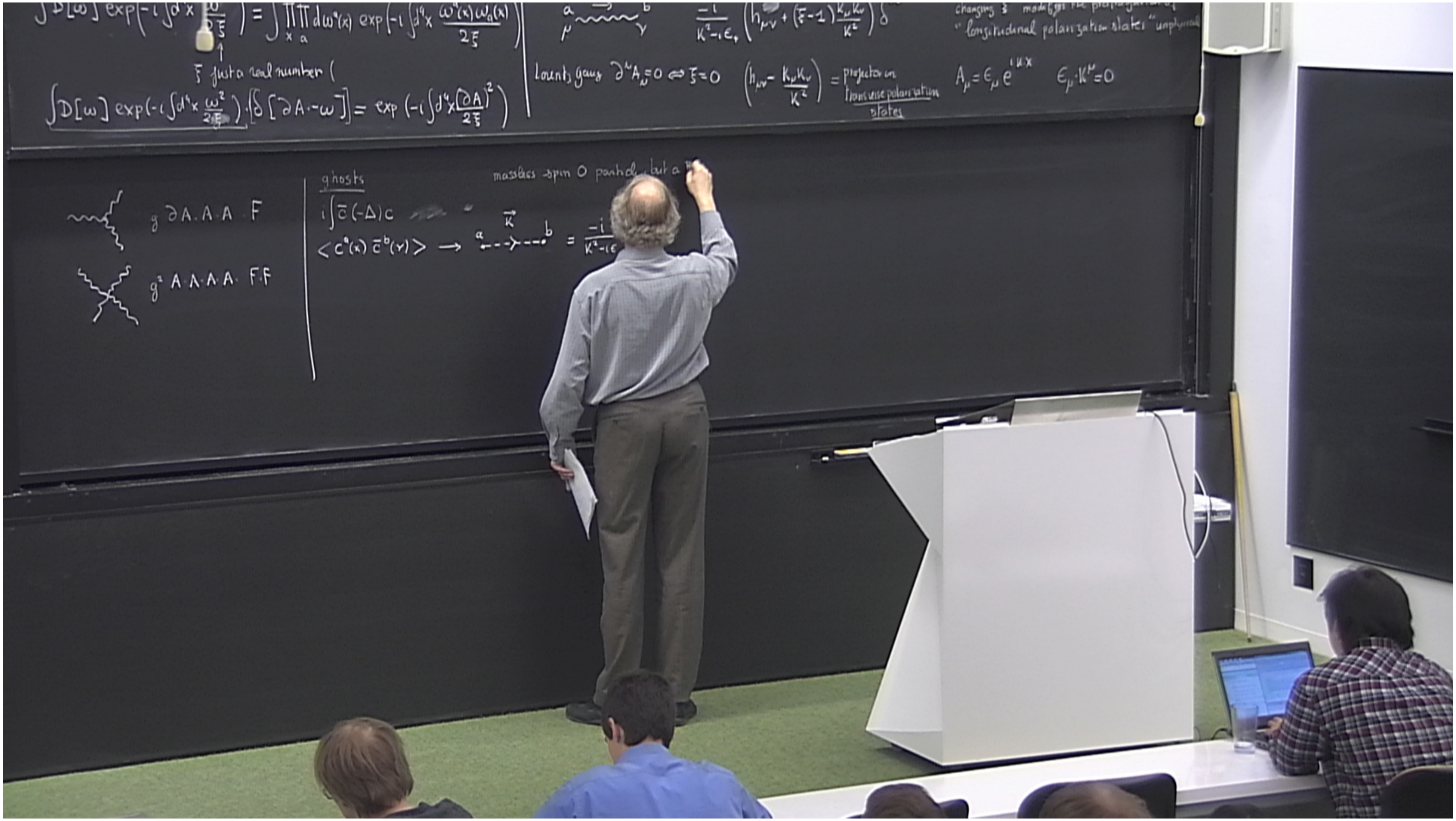
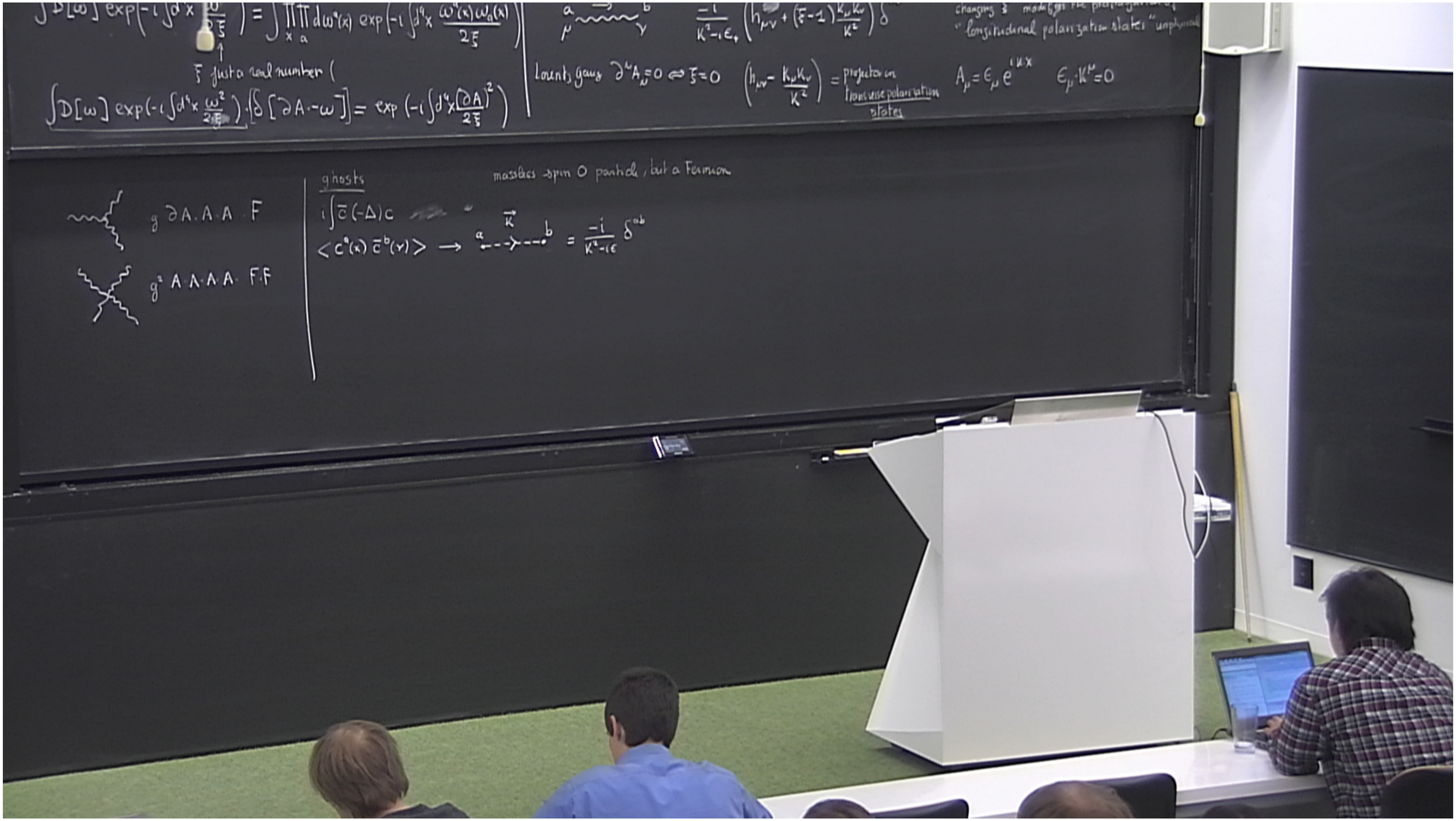
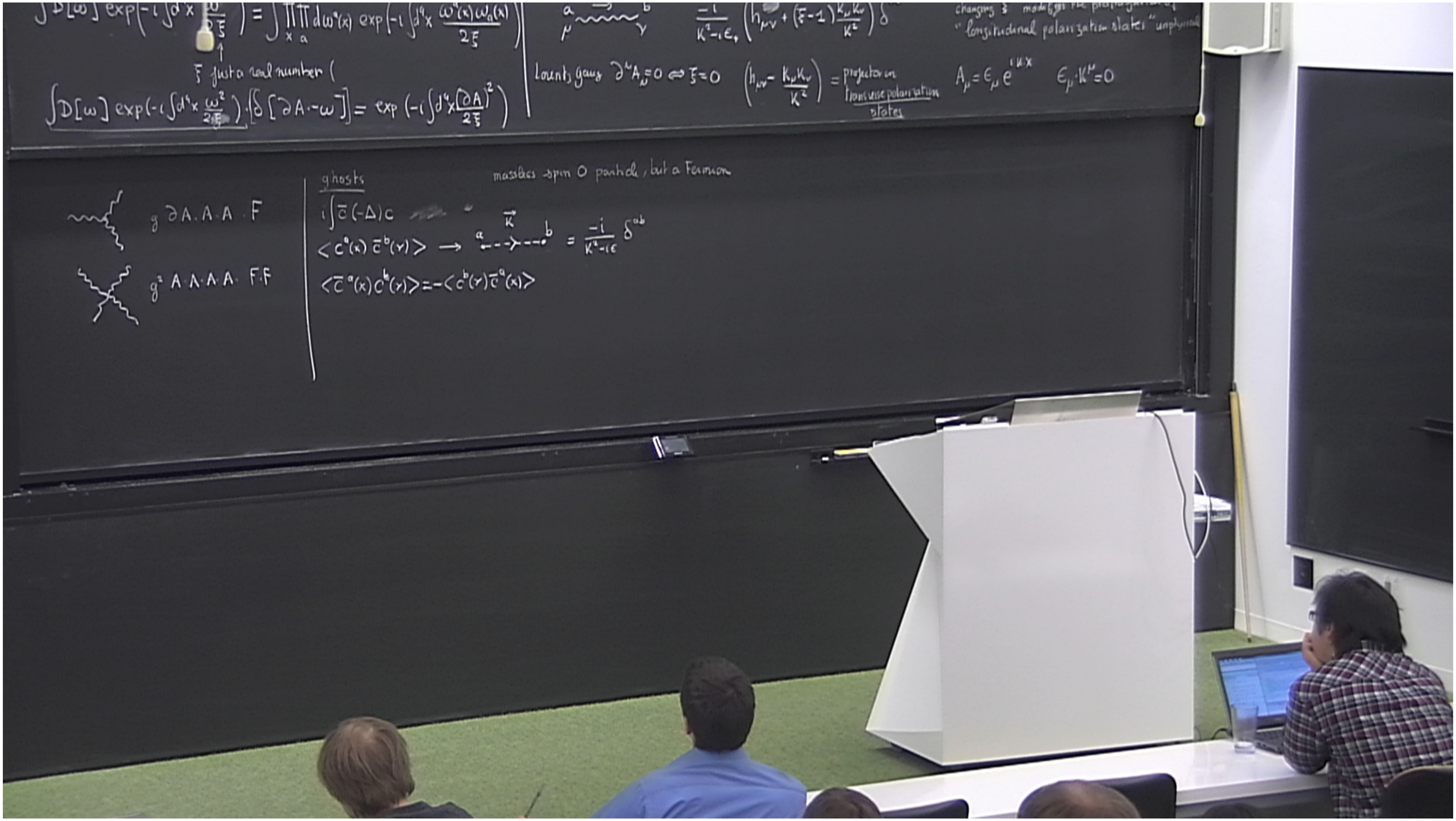
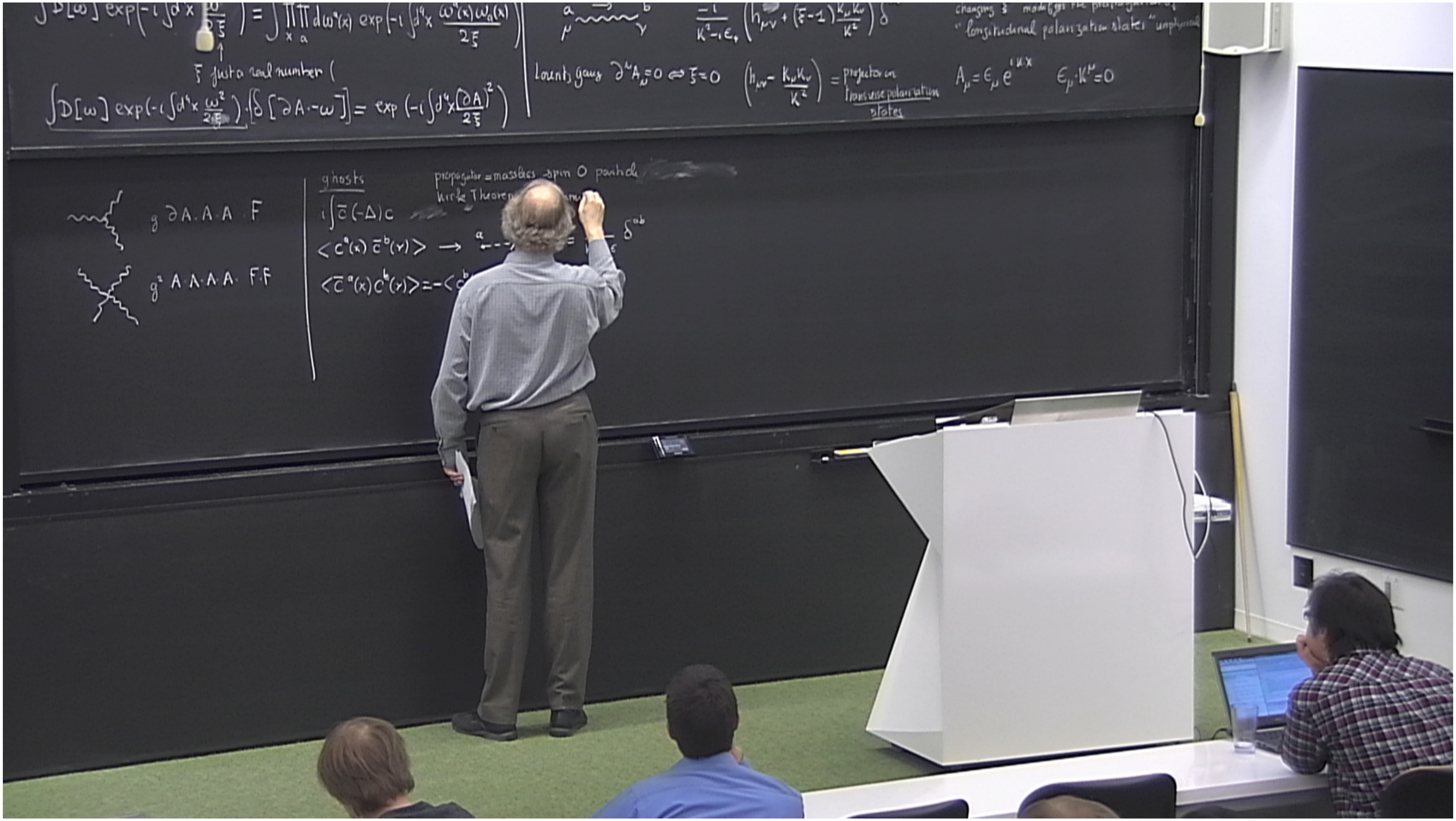
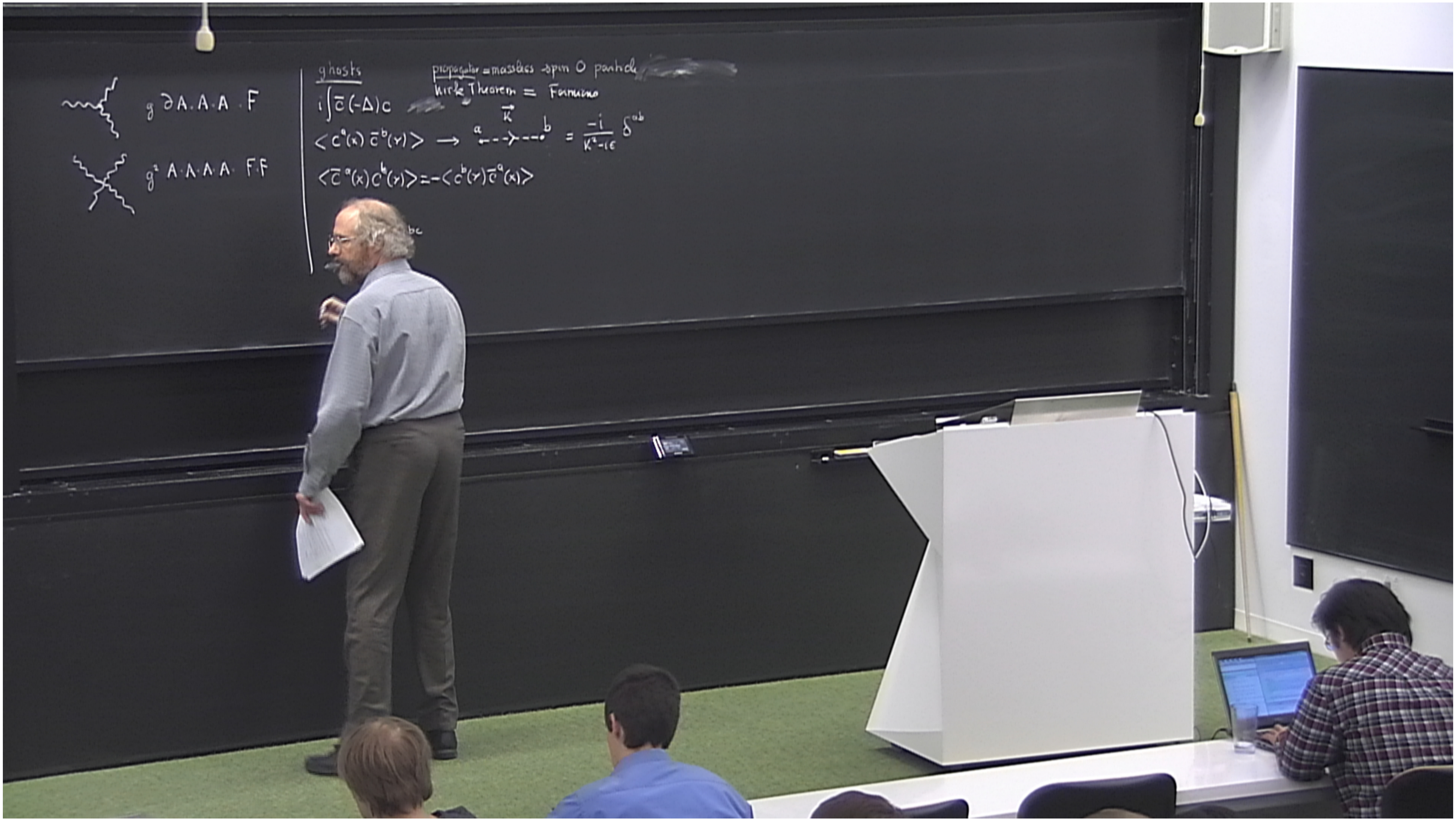


ξ just a real number
Lorentz gauge $\partial^\mu A_\mu = 0 \Leftrightarrow \xi = 0$
= projector on transverse polarization states
$A_\mu = \epsilon_\mu e^{ik\cdot x}$ $\epsilon_\mu \cdot k^\mu = 0$
longitudinal polarization states "unphysical"
$g\, \partial A \cdot A \cdot A \quad F$
$g^2\, A \cdot A \cdot A \cdot A \quad F \cdot F$
ghosts
$i \int \bar{c}(-\Delta) c$
$\langle c^a(x)\, \bar{c}^b(y) \rangle$
massless spin 0 particle but a

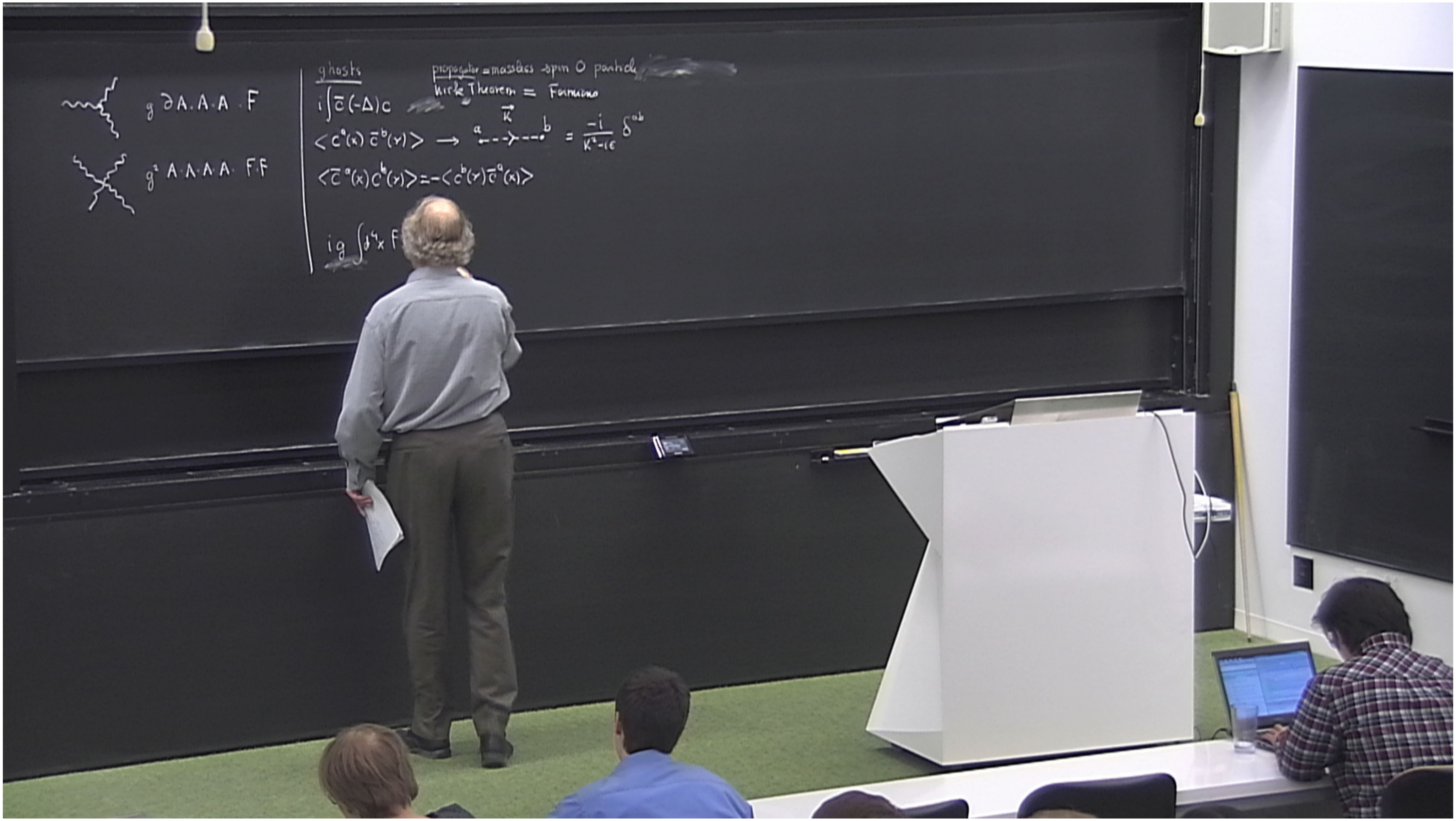


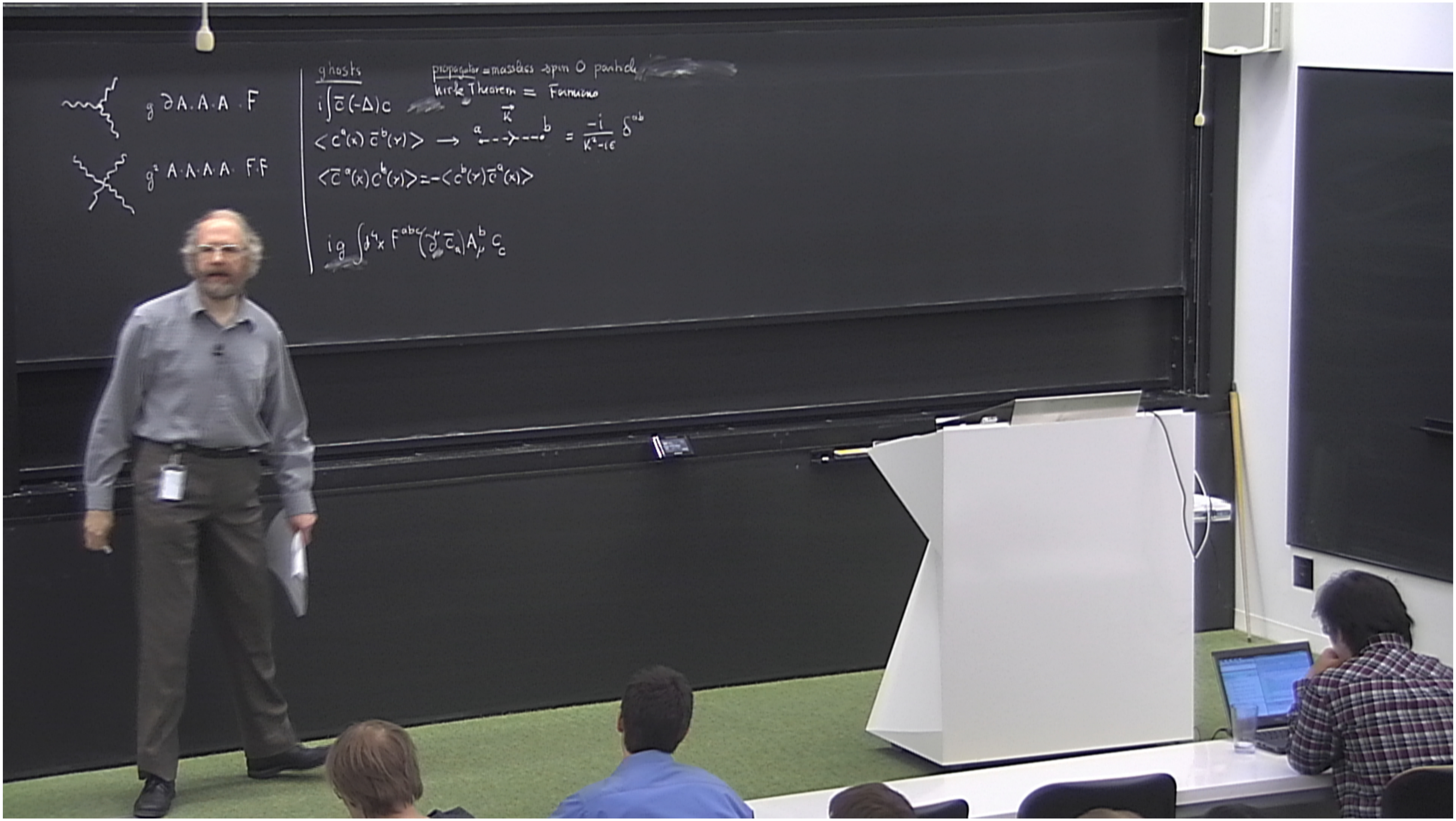

ξ just a real number (
Lorentz gauge ∂^μ A_μ = 0 ⇔ ξ = 0
= projector on transverse polarization states
"longitudinal polarization states" unphysical
ghosts
massless spin 0 particle, but a Fermion
g ∂A.A.A F
g² A.A.A.A. F.F
i∫c̄(−Δ)c
<c^a(x) c̄^b(y)>
<c̄^a(x)c^b(y)> = −<c^b(y)c̄^a(x)>





ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions


ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions
$g\,\partial A \cdot A \cdot A \cdot F$
$g^2\, A \cdot A \cdot A \cdot A \cdot F \cdot F$
$i\int \bar{c}(-\Delta)c$
$\langle c^a(x)\,\bar{c}^b(y)\rangle \longrightarrow$ = $\frac{-i}{k^2 - i\epsilon}\,\delta^{ab}$
$\langle \bar{c}^a(x)\,c^b(y)\rangle = -\langle c^b(y)\,\bar{c}^a(x)\rangle$
$ig\int d^4x\, F$


ghosts
propagator = massless spin 0 particle
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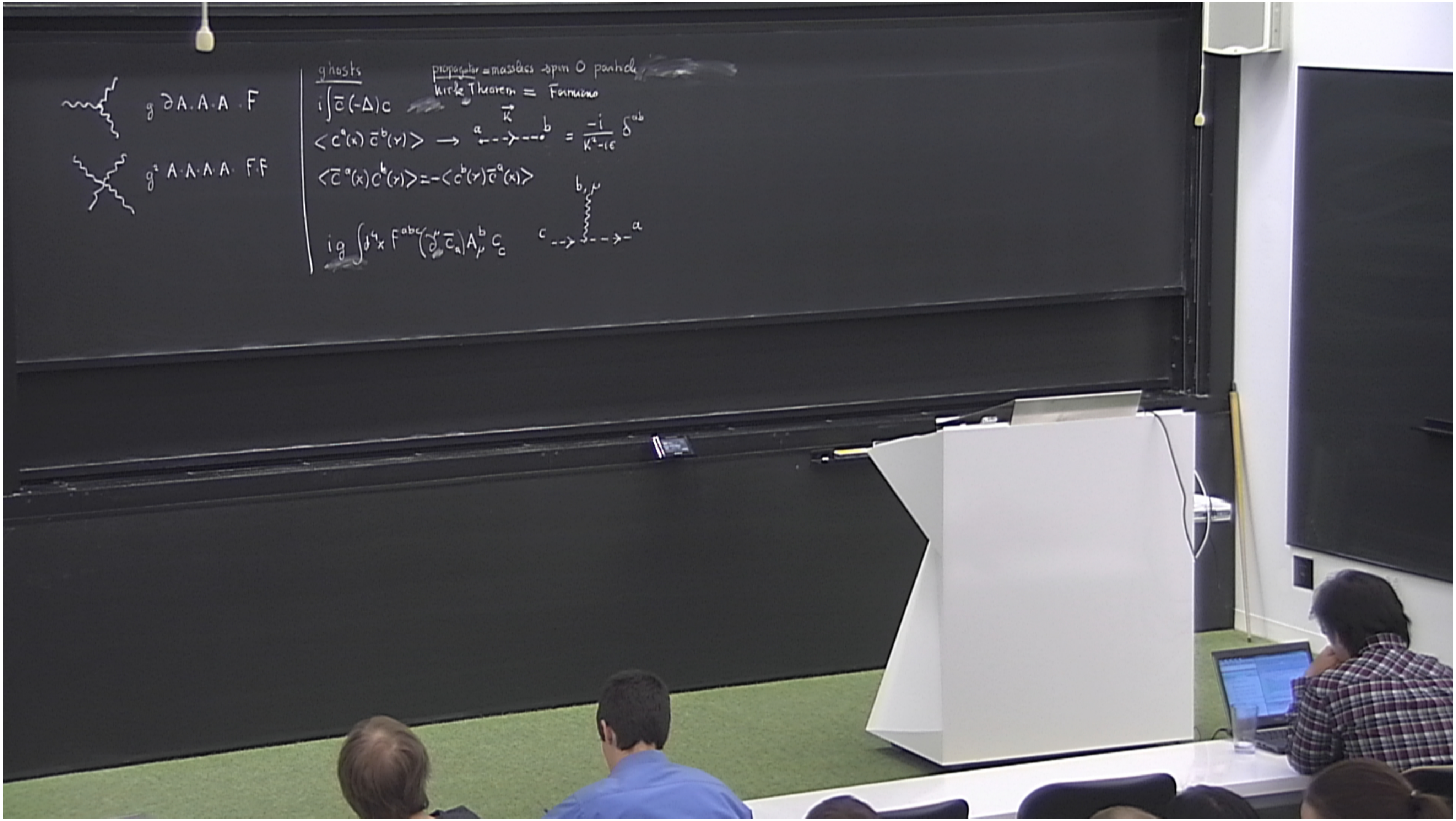
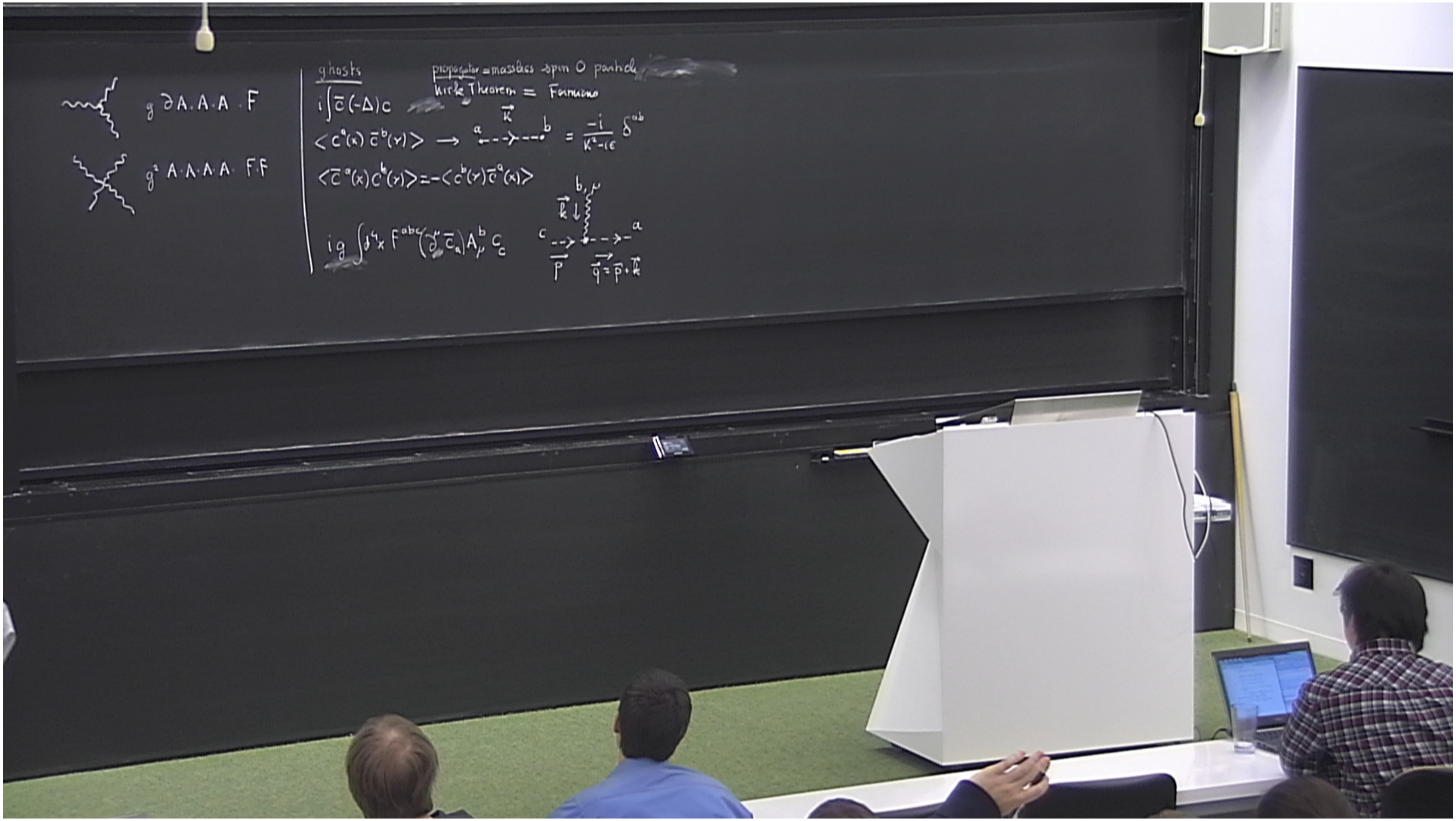
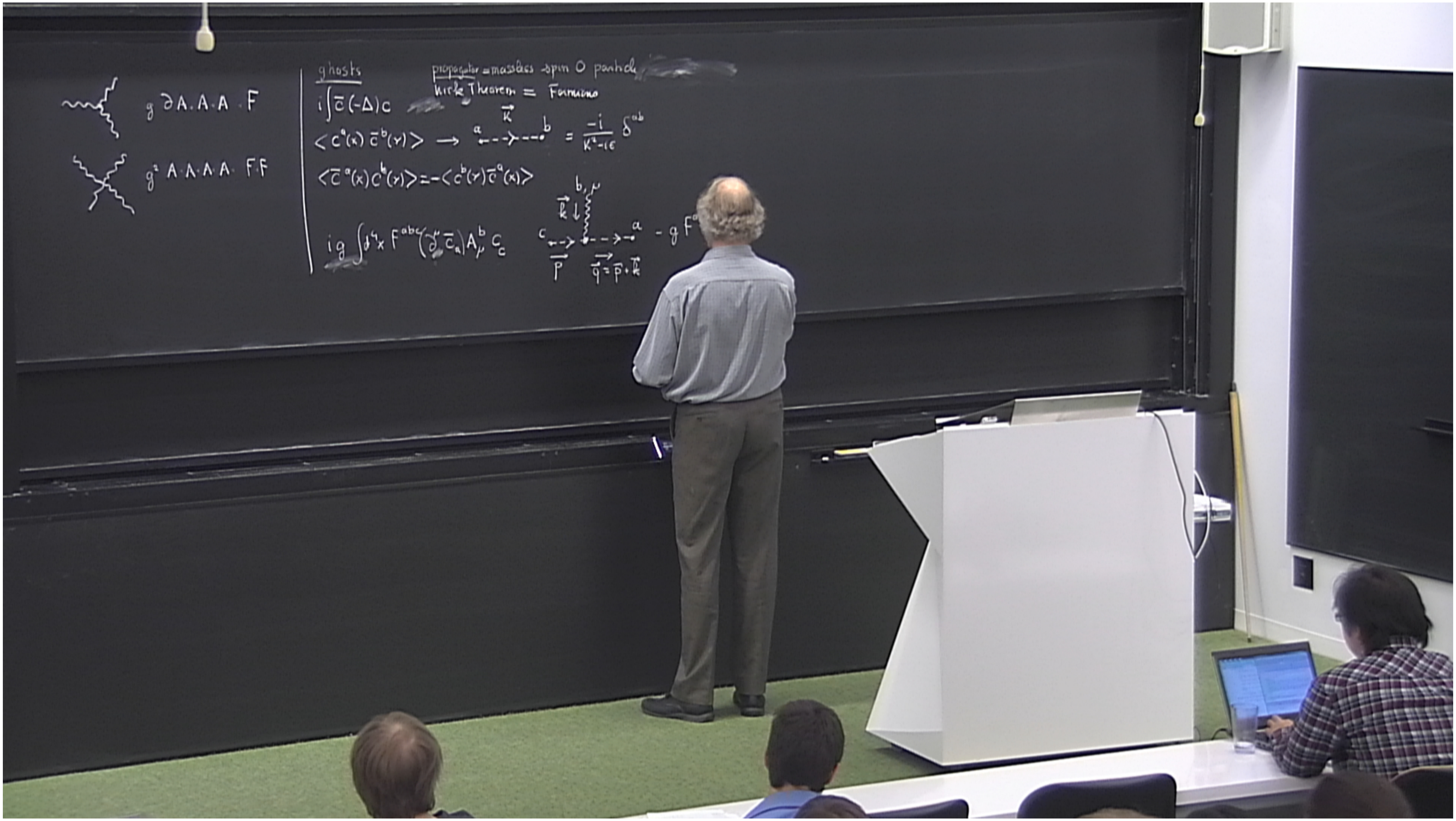
$g\,\partial A.A.A\;.\;F$
$g^2\,A.A.A.A.\;F.F$
$i\int \bar{c}(-\Delta)c$
$\langle c^a(x)\,\bar{c}^b(y)\rangle \longrightarrow$ a - - -> - - - b $= \frac{-i}{k^2-i\epsilon}\,\delta^{ab}$
$\langle \bar{c}^a(x)\,c^b(y)\rangle = -\langle c^b(y)\,\bar{c}^a(x)\rangle$
$ig\int d^4x\, f^{abc}(\partial^\mu \bar{c}_a)A^b_\mu\, c_c$



g ∂A.A.A . F
g² A.A.A.A. F.F
ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions



ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions
g ∂A.A.A . F
g² A.A.A.A. F.F
i∫c̄(-Δ)c
<c^a(x) c̄^b(y)> ⟶
= -i/(k²-iε) δ^ab
<c̄^a(x)c^b(y)> = -<c^b(y)c̄^a(x)>
ig∫d⁴x f^abc (∂^μ c̄_a) A_μ^b c_c
q = p + k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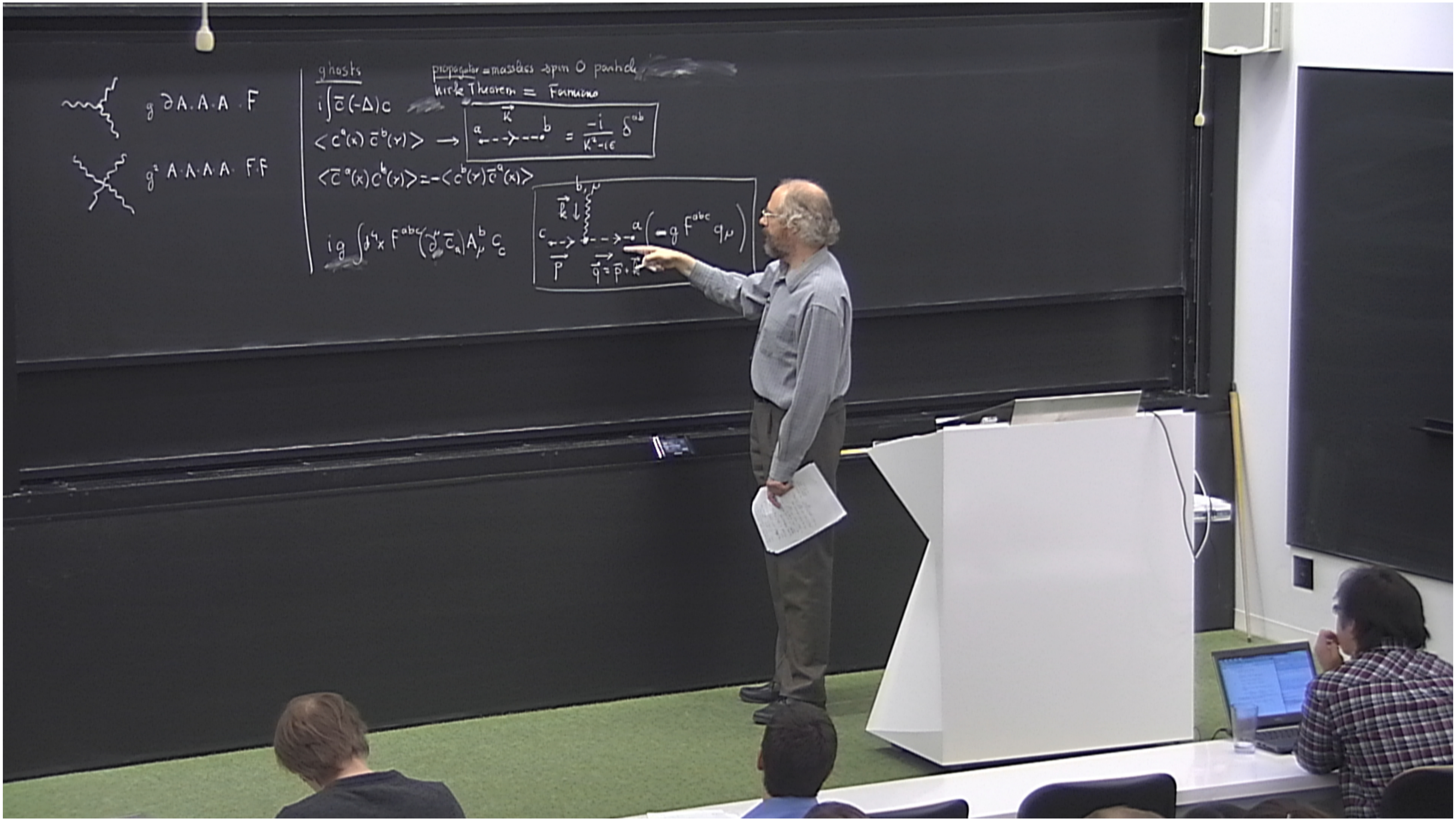
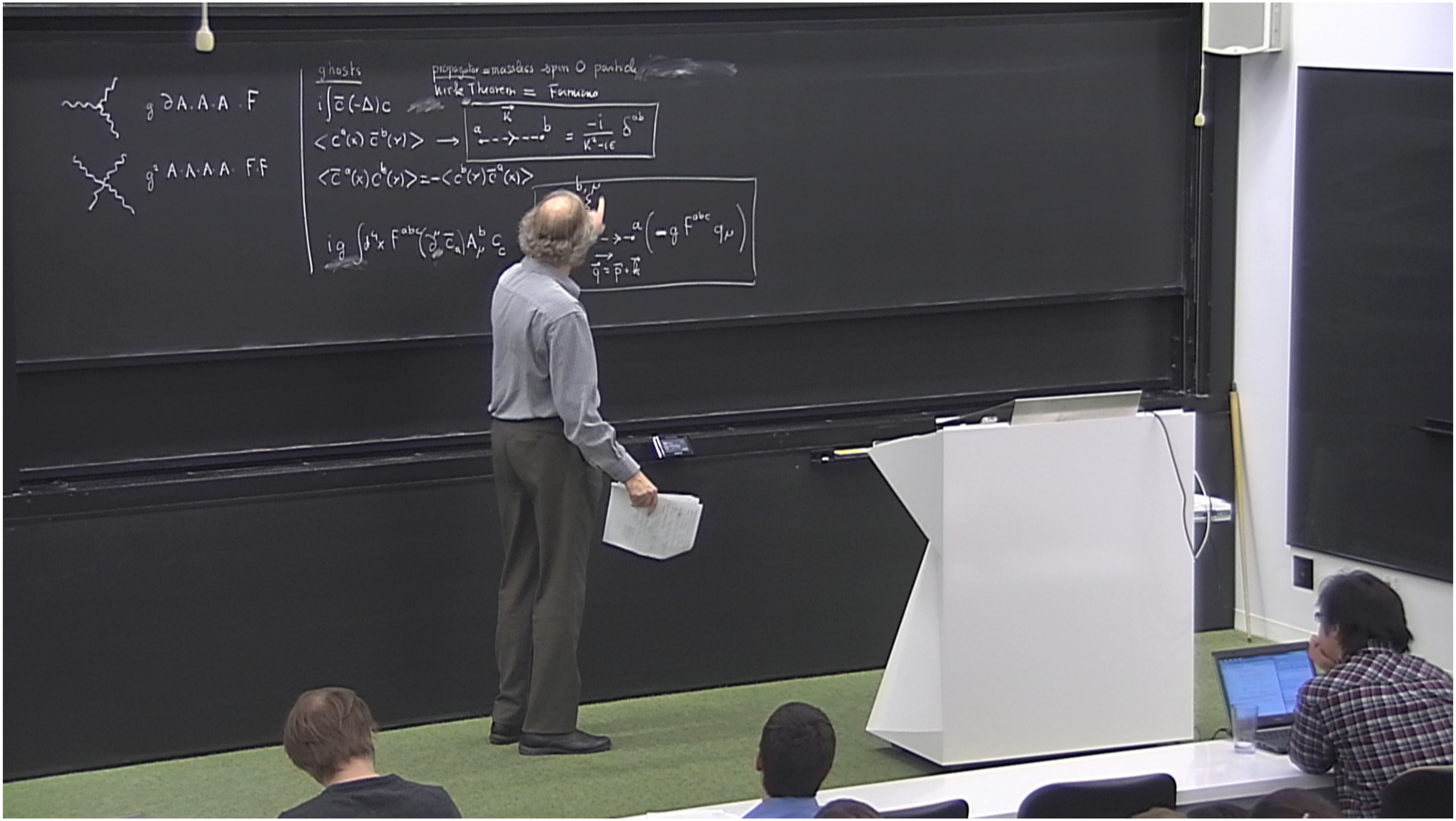
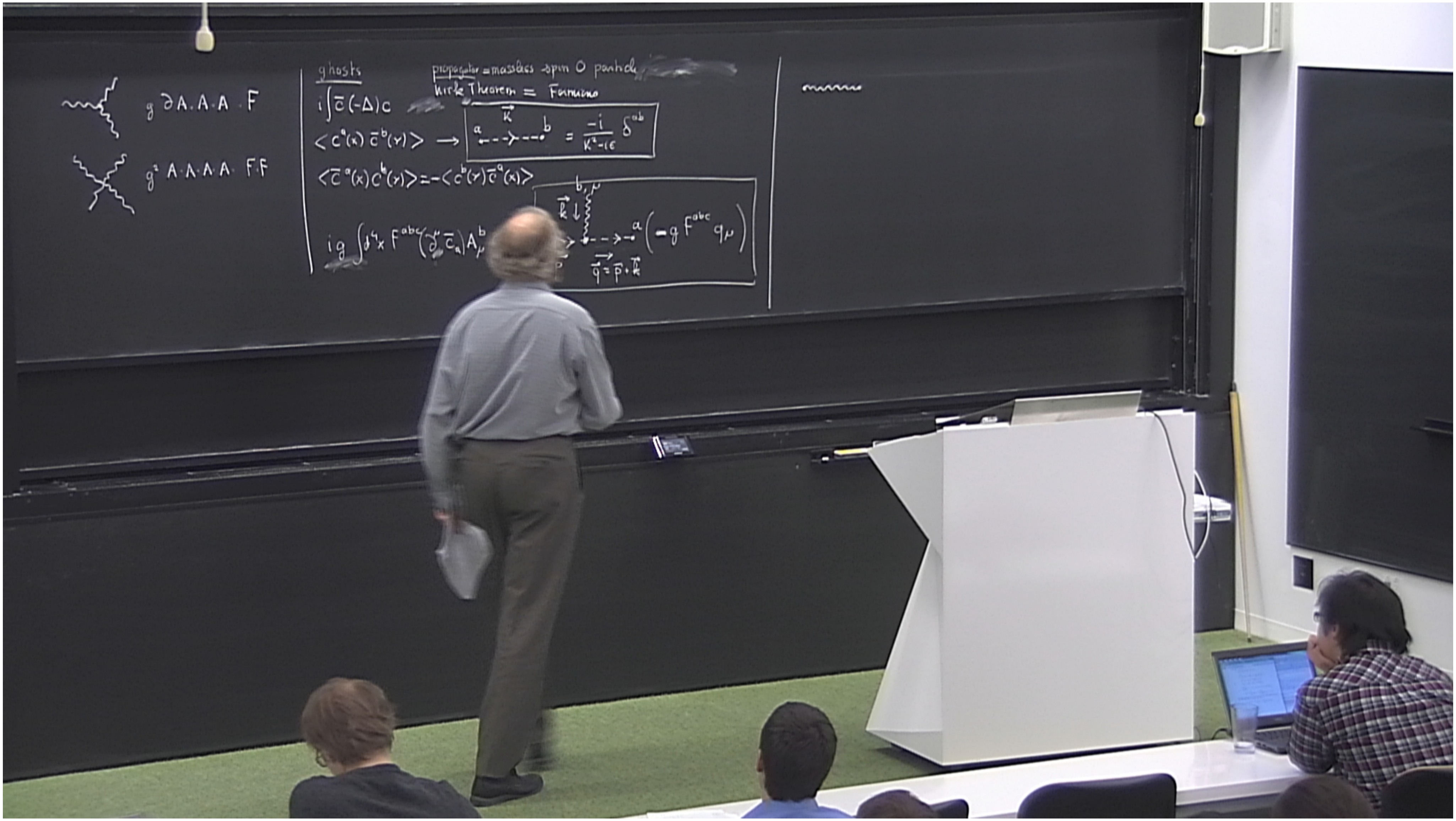
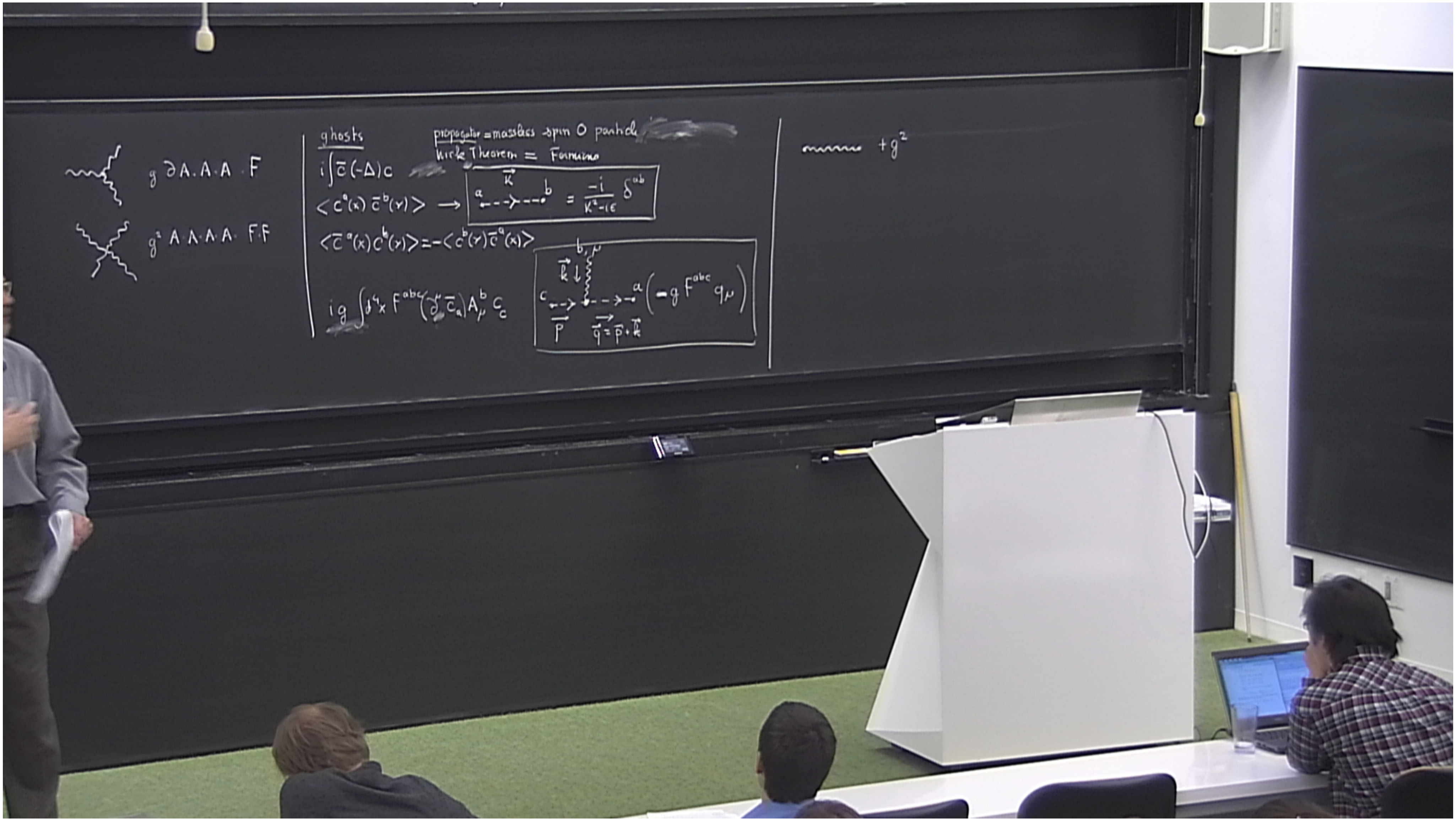
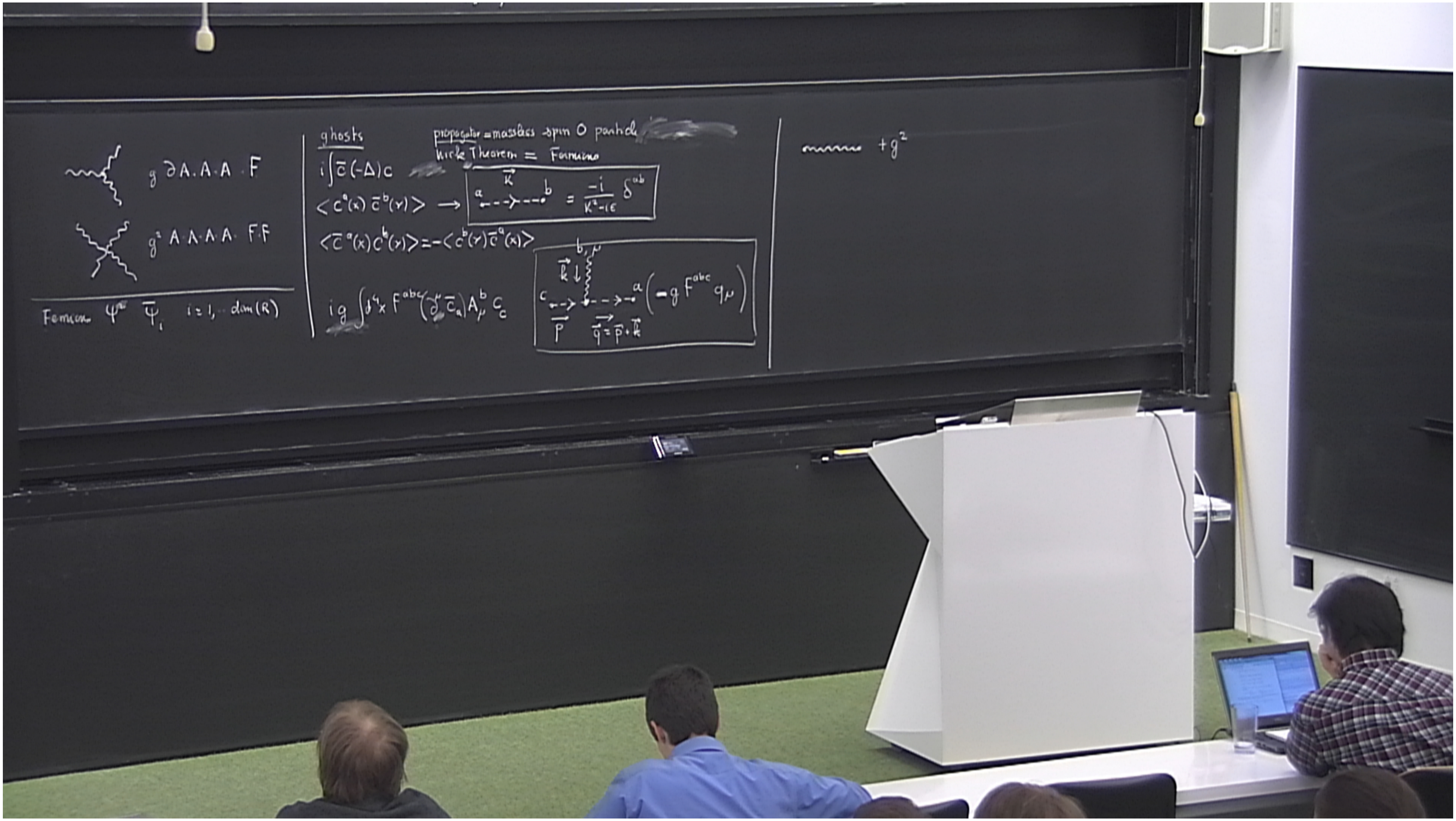
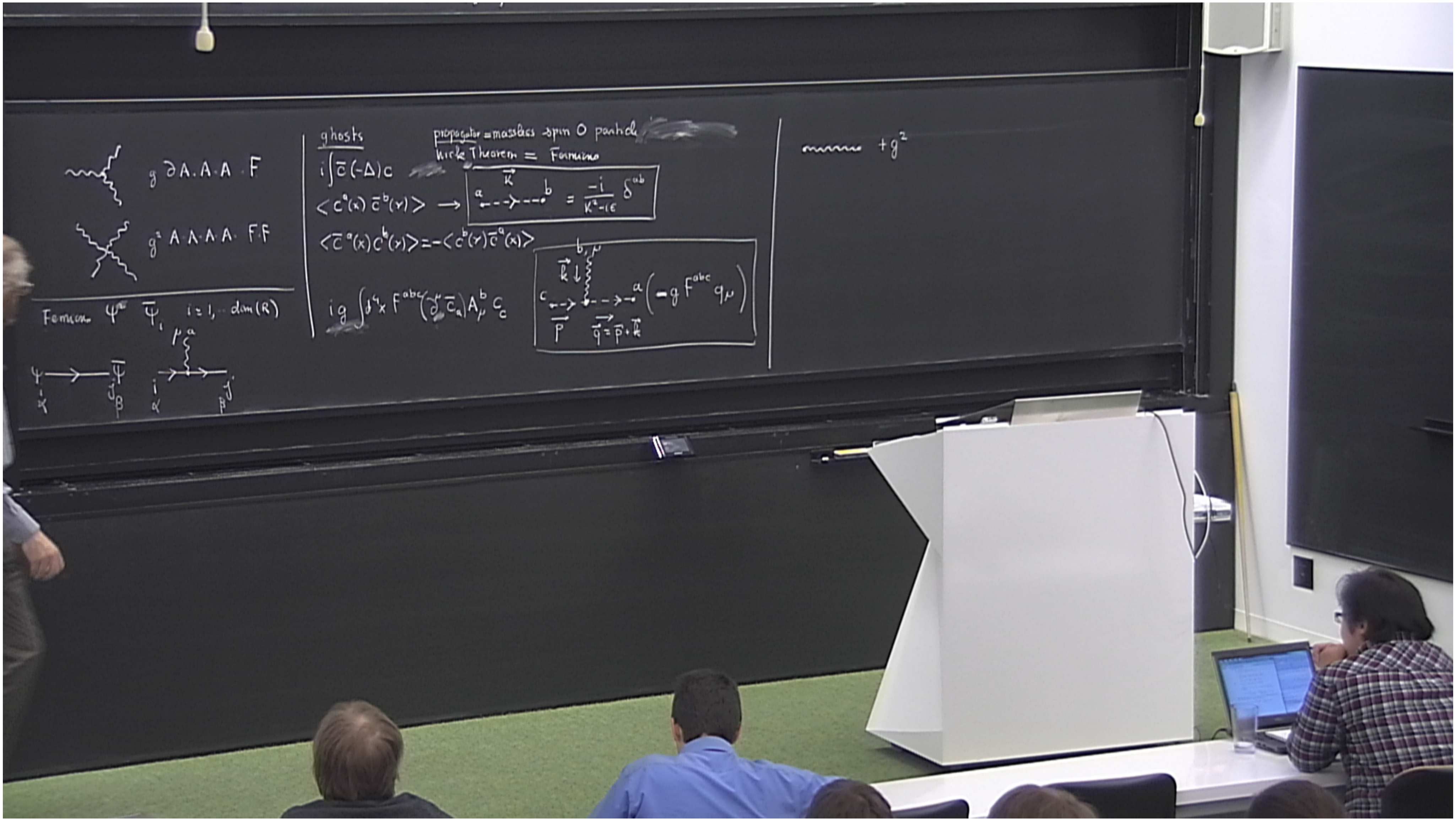





ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions


$g\, \partial A . A . A \quad F$
$g^2\, A . A . A . A \quad F . F$
ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions
$i \int \bar{c} (-\Delta) c$
$\langle c^a(x)\, \bar{c}^b(y) \rangle \longrightarrow$
$= \frac{-i}{k^2 - i\epsilon} \delta^{ab}$
$\langle \bar{c}^a(x)\, c^b(y) \rangle = - \langle c^b(y)\, \bar{c}^a(x) \rangle$
$ig \int d^4x\, f^{abc} (\partial^\mu \bar{c}_a) A^b_\mu c_c$
$\left(-g f^{abc} q_\mu\right)$
$\vec{q} = \vec{p} + \vec{k}$
$+g^2$


ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions
Fermion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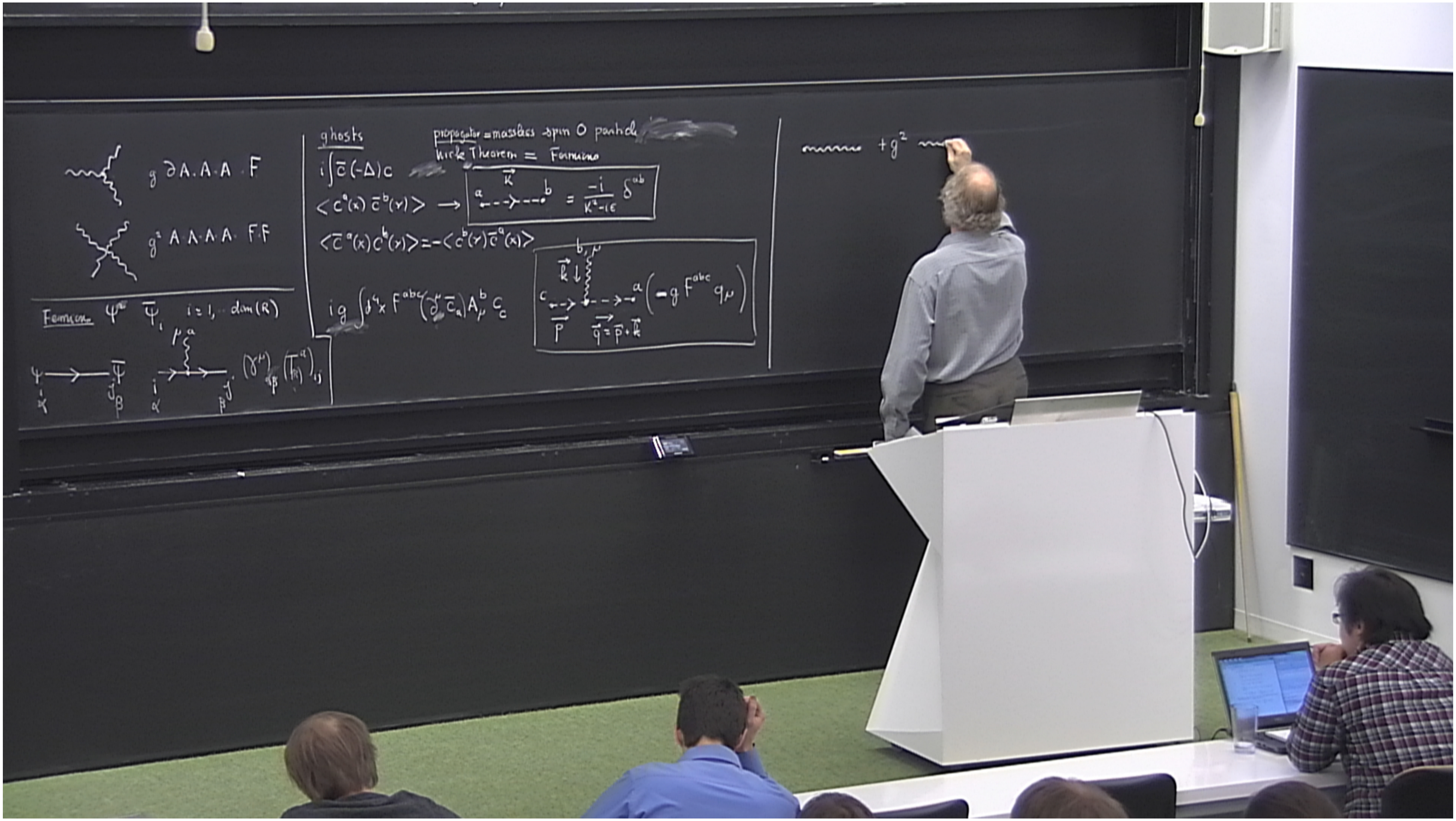
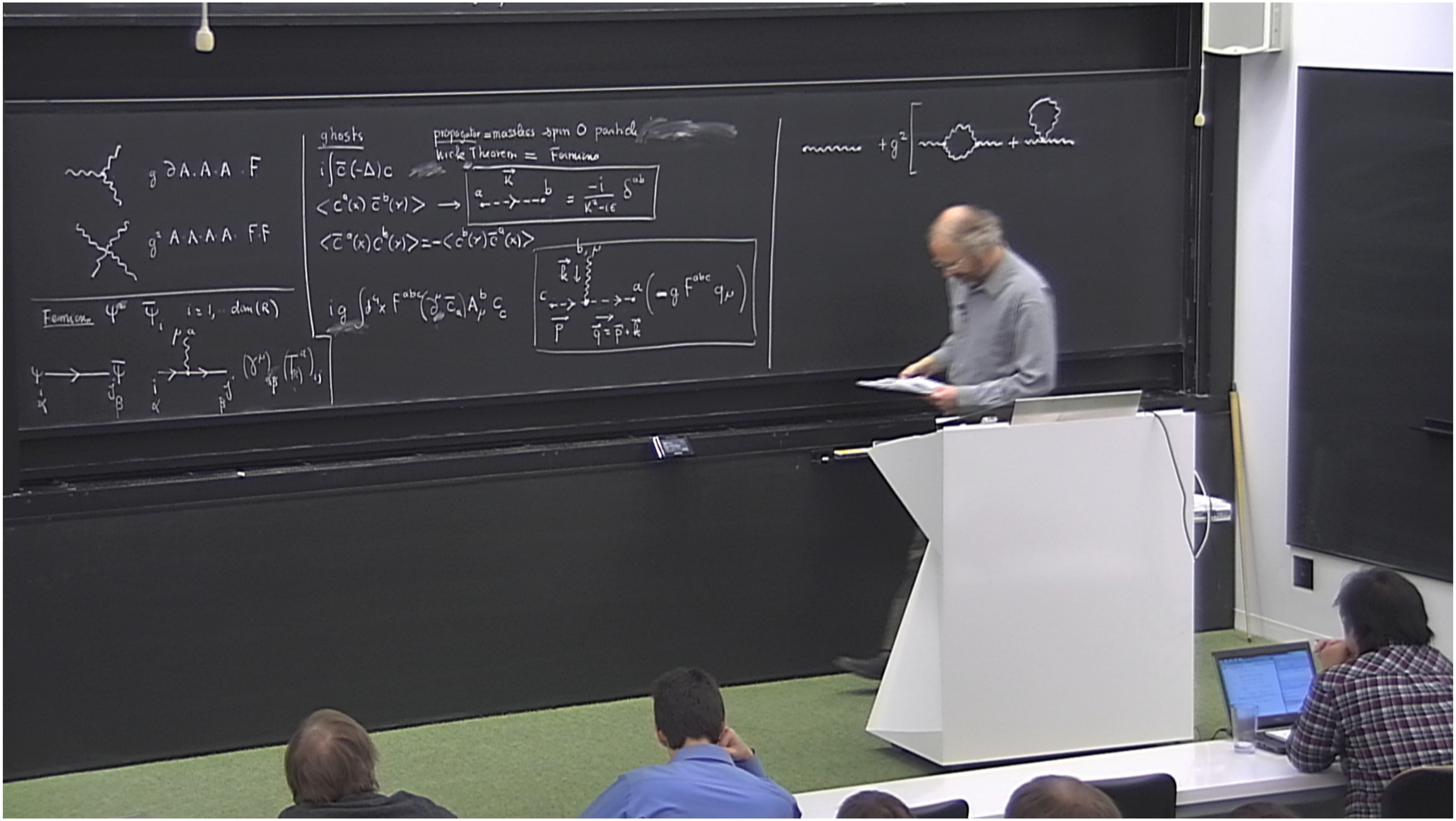
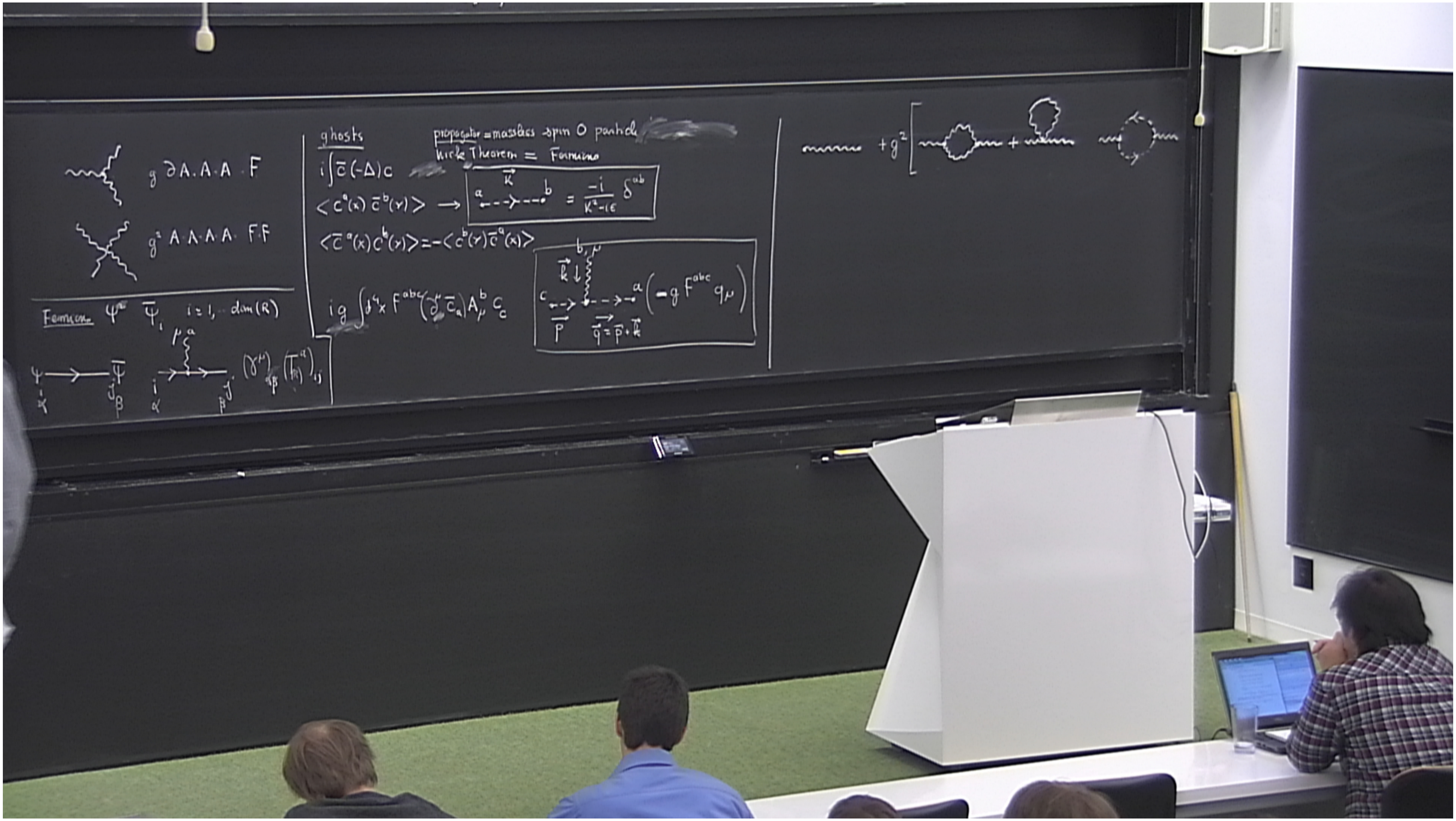
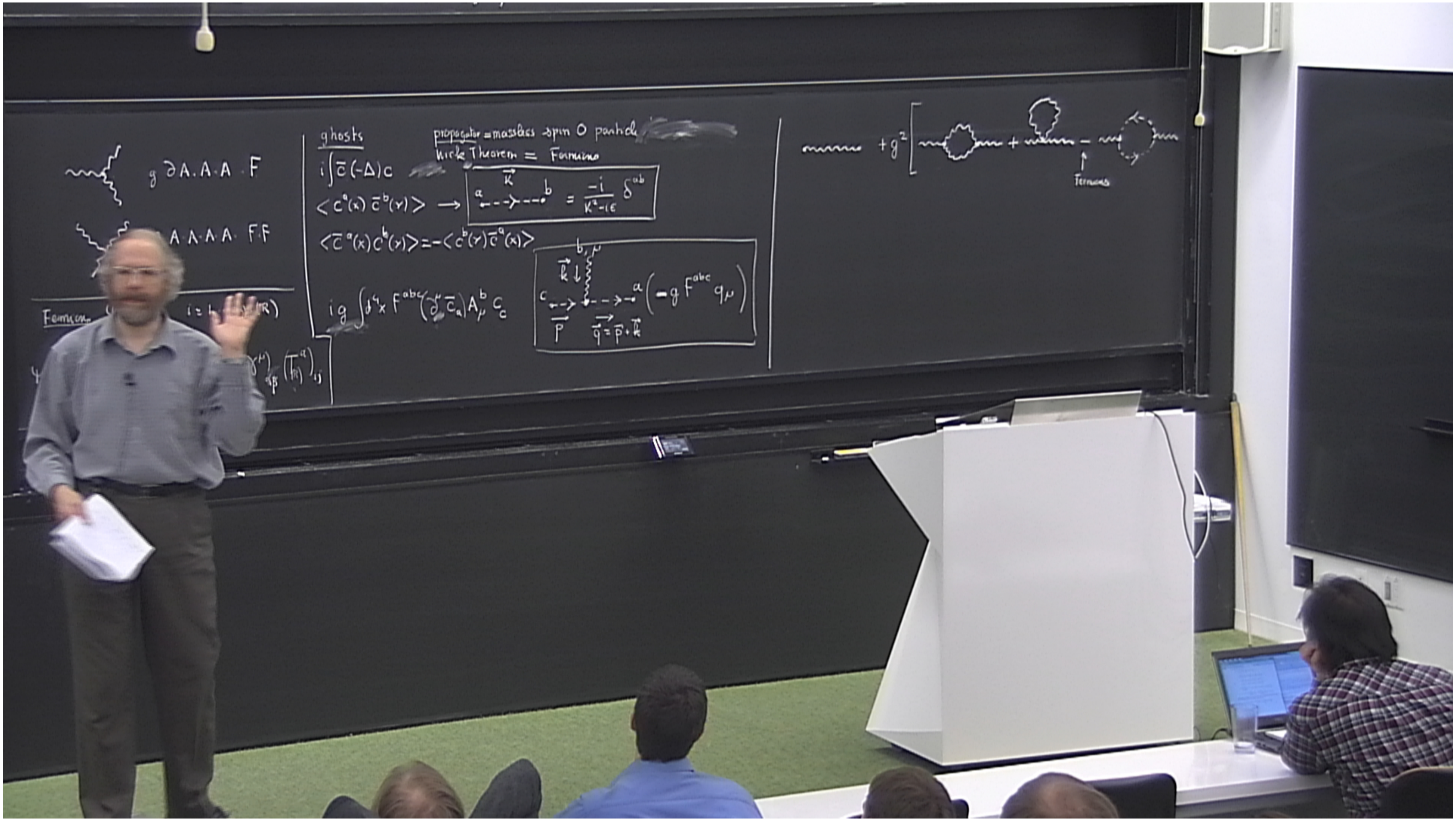
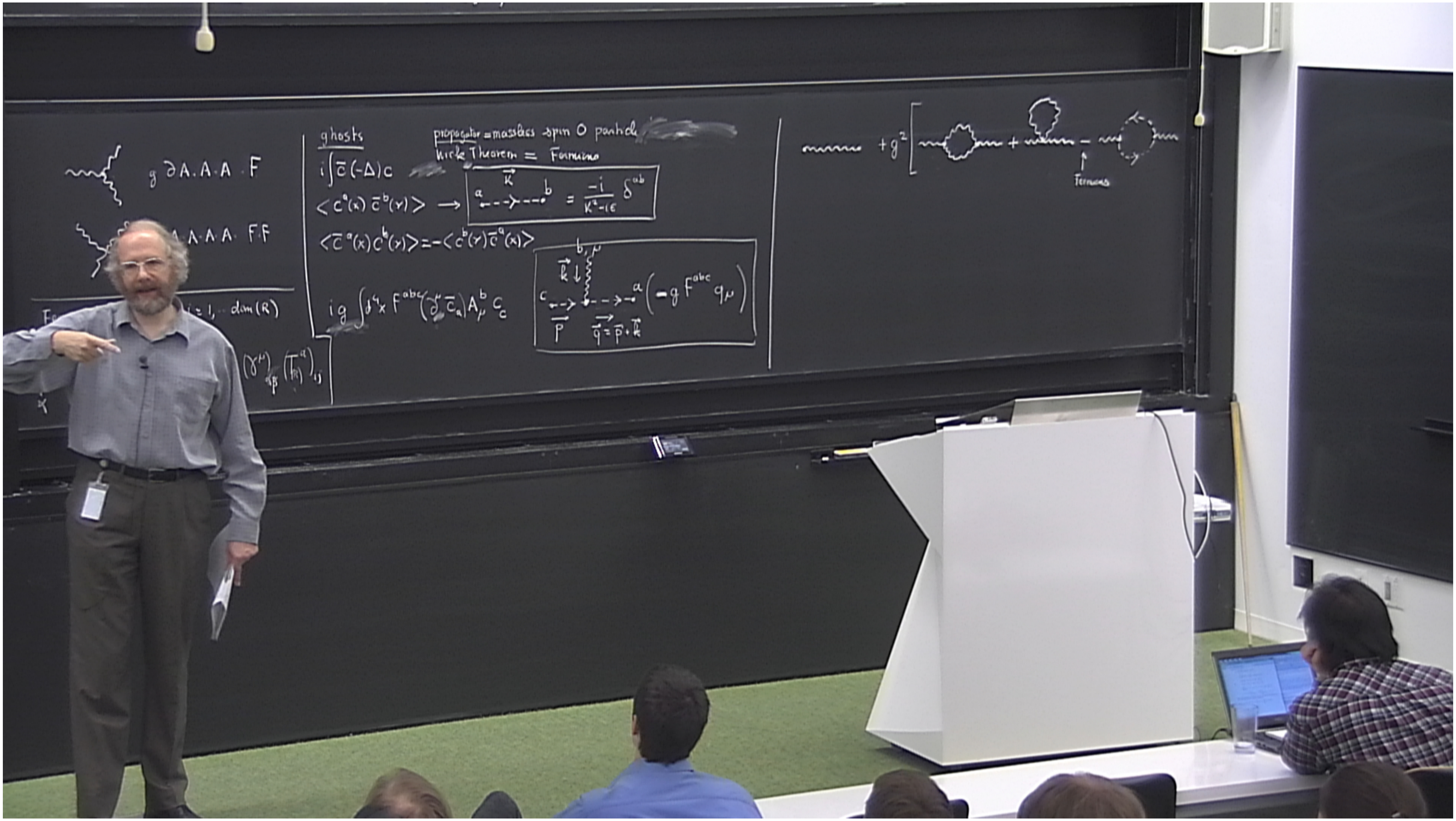
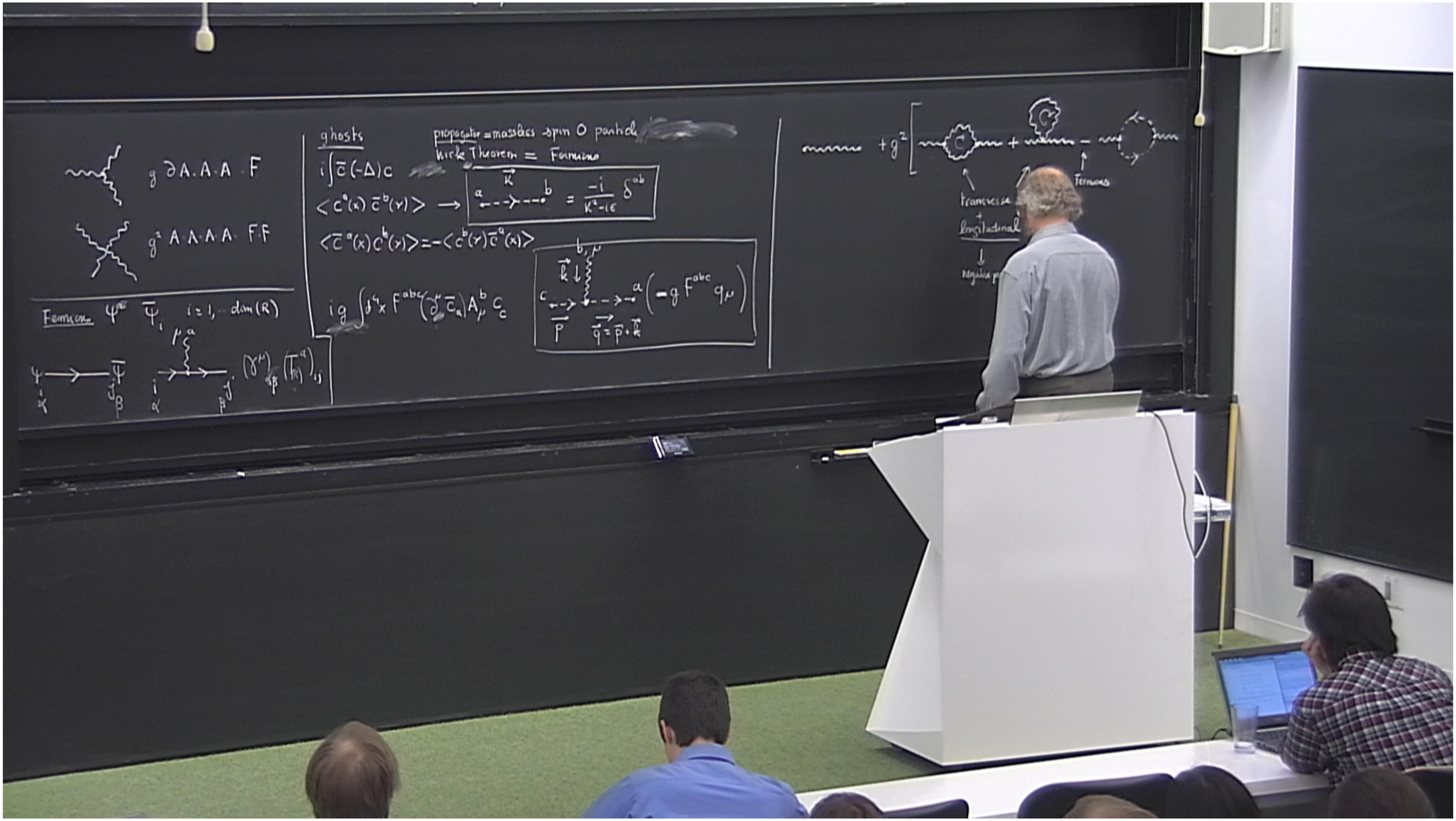


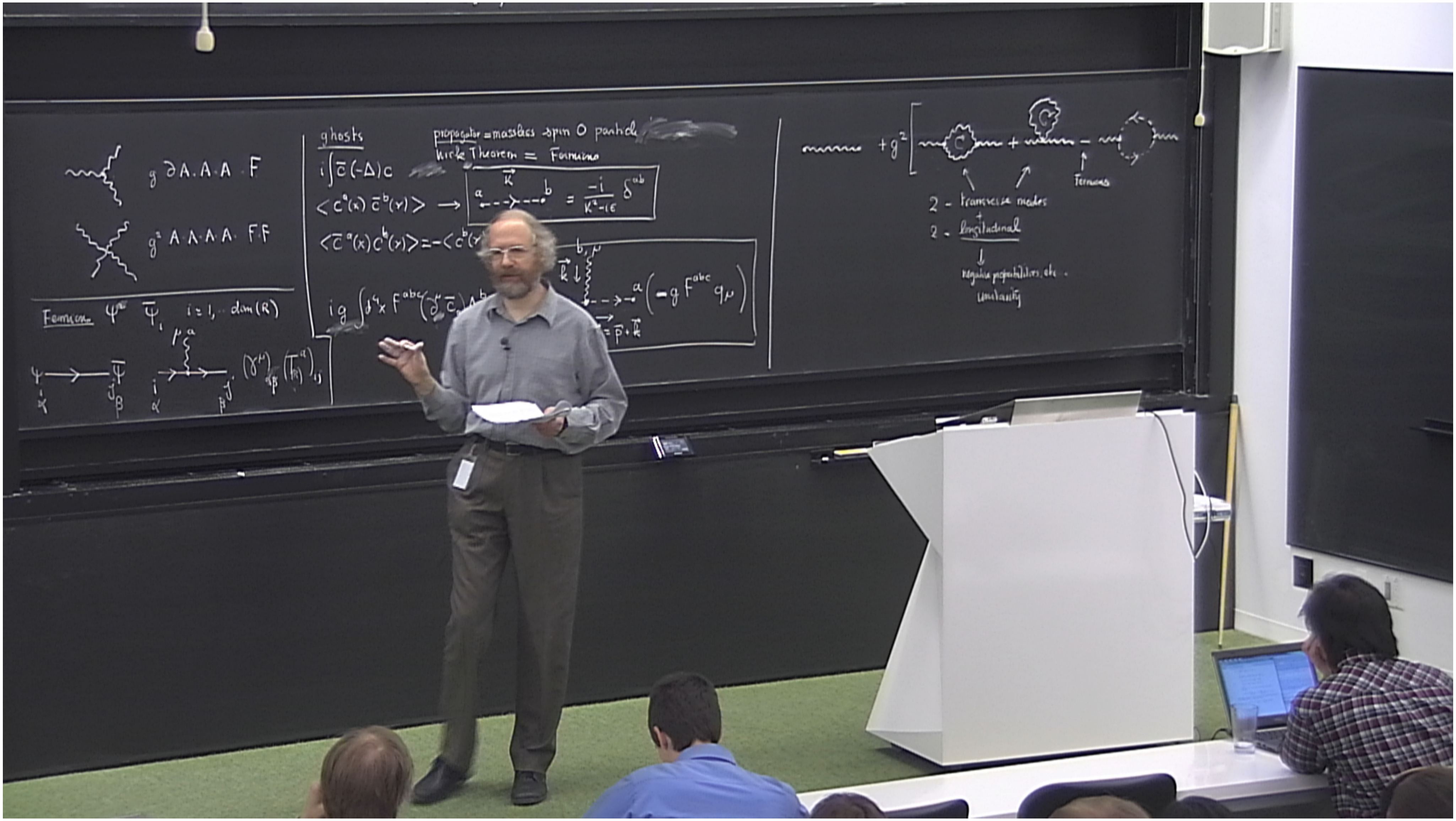


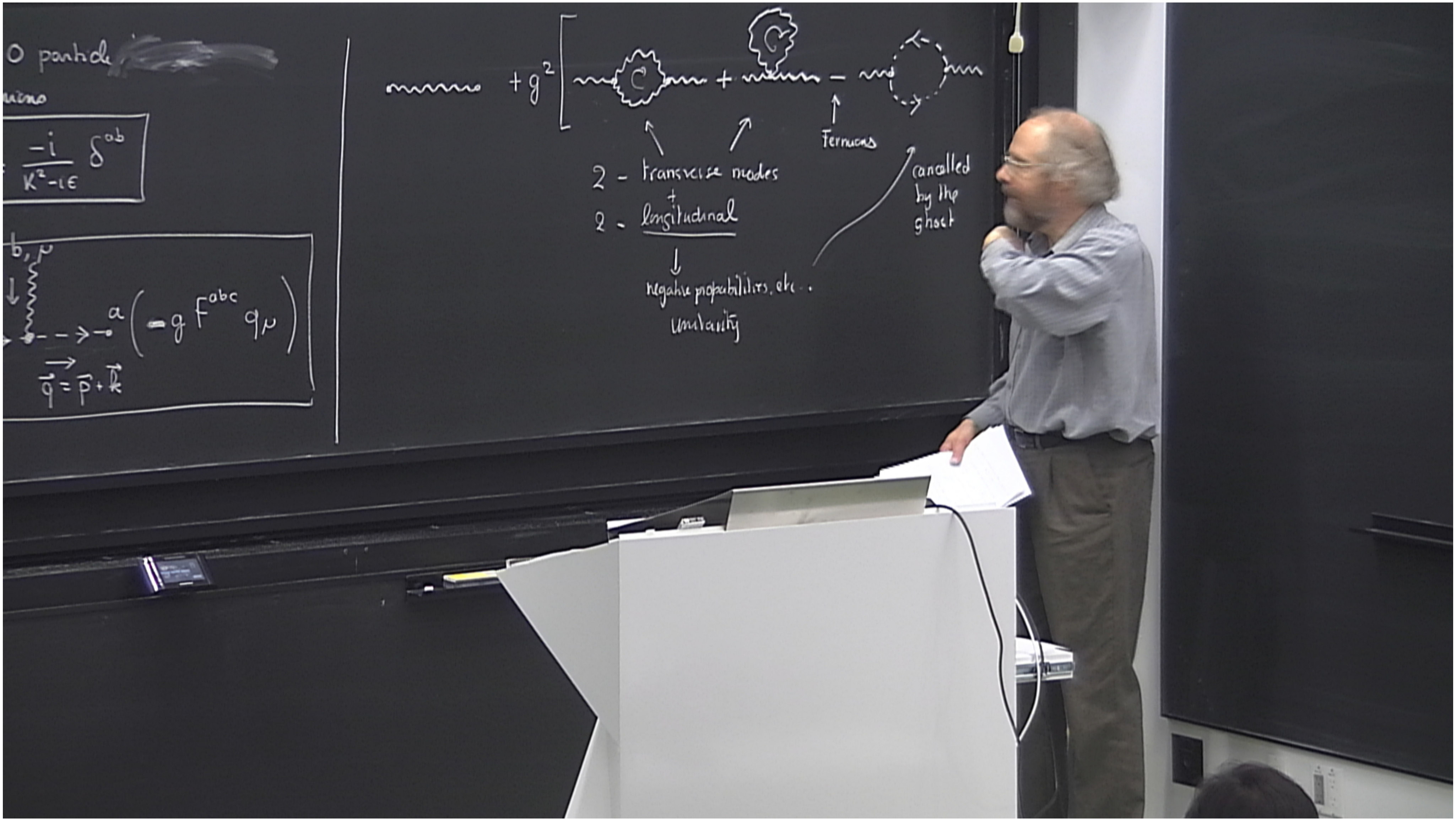



ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions
g ∂A.A.A F
A.A.A.A F.F
Fermions
Fermions

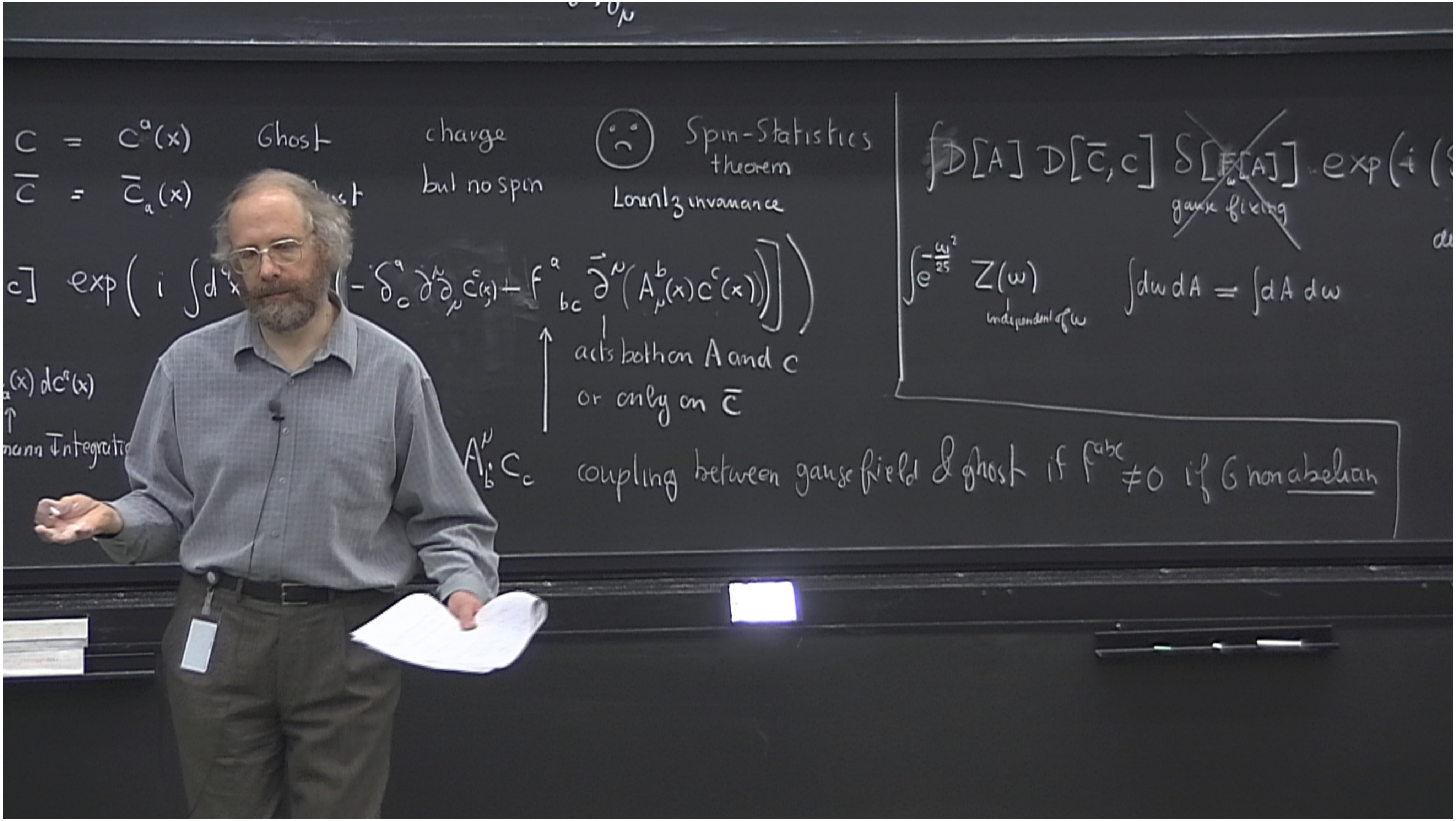

ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions
Fermions

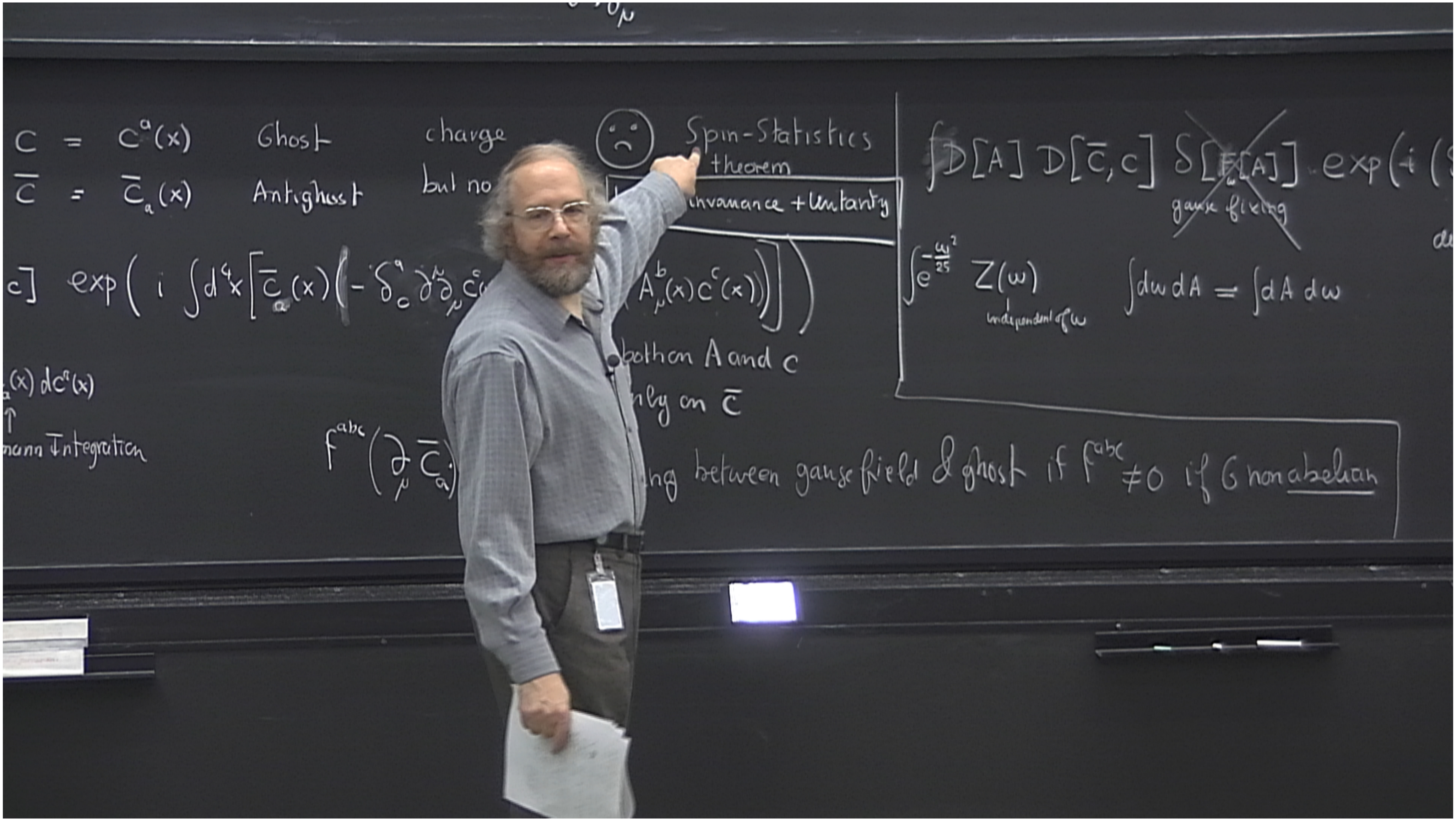

ghosts
propagator = massless spin 0 particle
Wick Theorem = Fermions
Fermion
transverse
longitudinal
Fermions

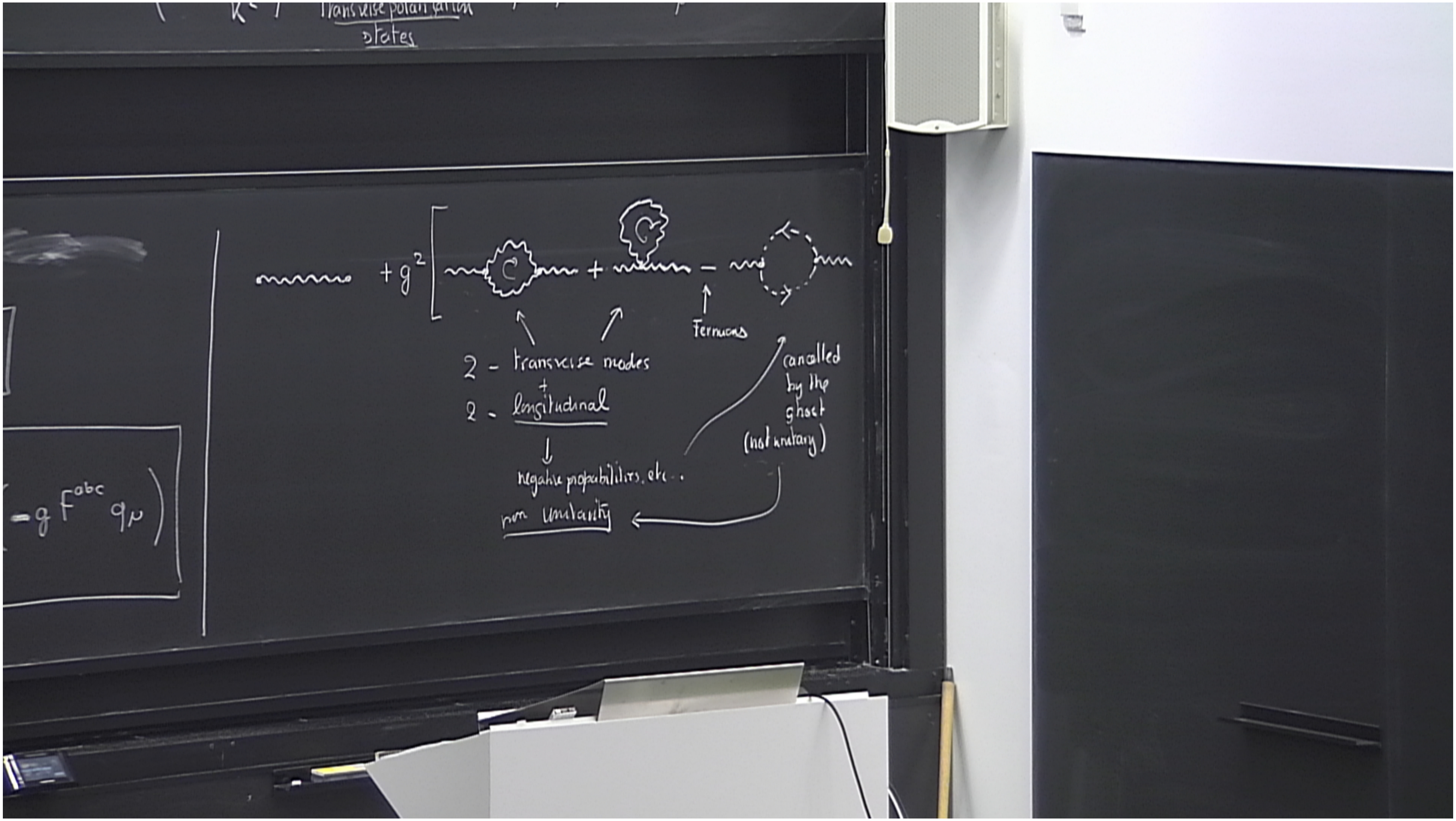

ghosts
propagator = massless spin 0 particle
Fermions
2 - transverse modes
2 - longitudinal
negative probabilities, etc.
unitarity



O particle
$+g^2$
2 - transverse modes
+
2 - longitudinal
negative probabilities, etc..
unitarity
Fermions
cancelled by the ghost
$\frac{-i}{K^2 - i\epsilon}\delta^{ab}$
$(-g F^{abc} q_\mu)$
$\vec{q} = \vec{p} + \vec{k}$


Ghost
charge
but no spin
Spin-Statistics
theorem
Lorentz invariance
gauge fixing
independent of ω
acts both on A and c
or only on c̄
coupling between gauge field & ghost if f^abc ≠ 0 if G non abelian


c = c^a(x) Ghost charge
c̄ = c̄_a(x) Antighost but no
Spin-Statistics theorem
invariance + Unitarity
gauge fixing
independent of ω
∫dω dA = ∫dA dω
bothon A and c
nly on c̄
Integration
between gauge field & ghost if f^abc ≠ 0 if G non abelian


states
+g²
Fermions
2 - transverse modes
+
2 - longitudinal
negative probabilities, etc..
non unitarity
cancelled
by the
ghost
(not unitary)