

Title: Quantum Field Theory II - Lecture 13

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URL: <http://pirsa.org/11110019>

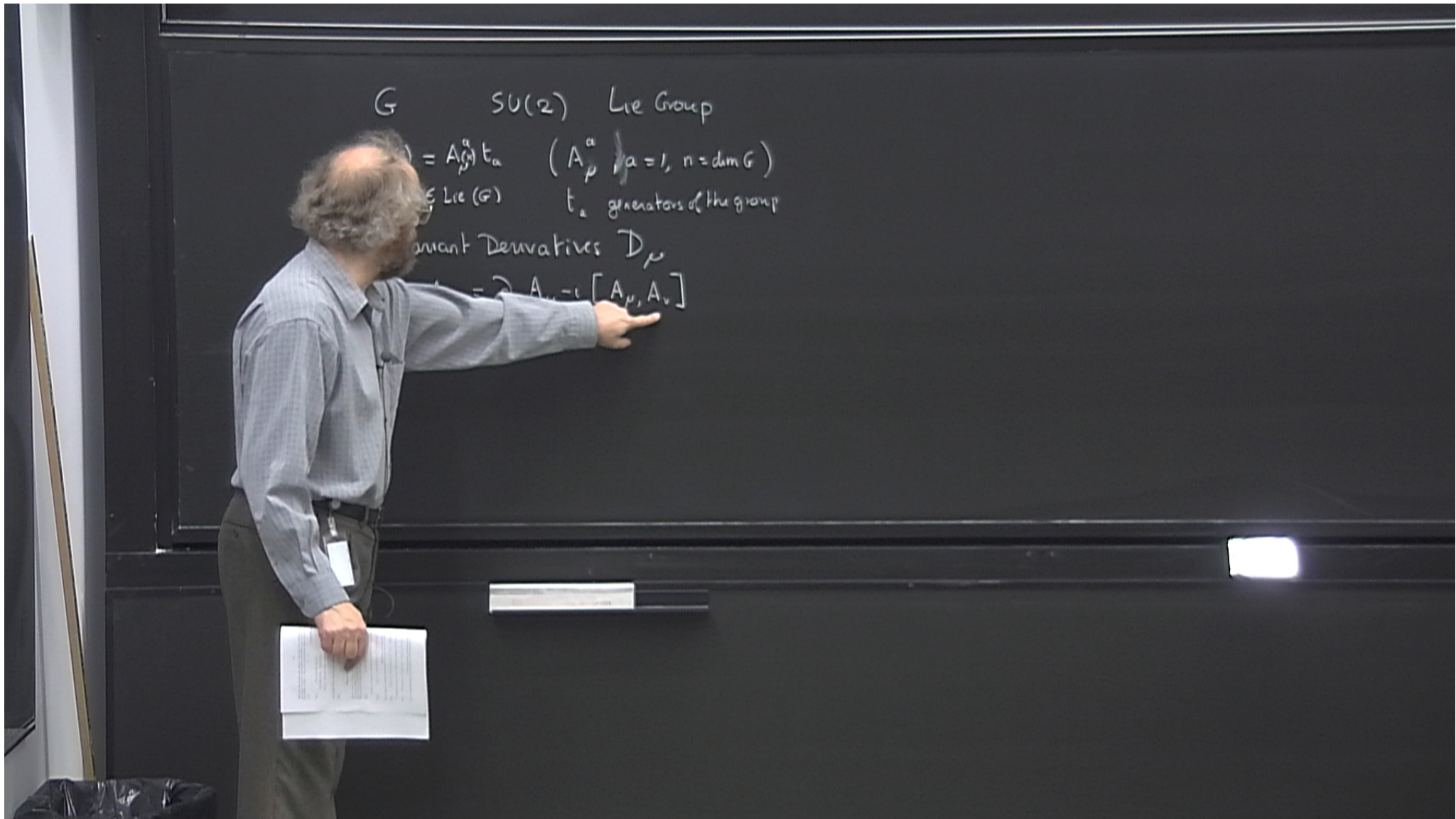
Abstract:

G $SU(2)$ Lie Group

$A_\mu = A_\mu^a t_a$ (A_μ^a $a=1, n = \dim G$)
 $t_a \in \text{Lie}(G)$ t_a generators of the group

Covariant Derivatives D_μ

$$D_\mu = \partial_\mu - i g A_\mu^a [A_\mu, A_a]$$



G $SU(2)$ Lie Group

$$A_\mu(x) = A_\mu^a(x) t_a \quad (A_\mu^a \mid a=1, n = \dim G)$$

$\in \mathfrak{L}$ generators of the group $= \frac{1}{2} \sigma_a$ Pauli Matrices

Covariant derivatives D_μ

$$D_\mu = \partial_\mu - i [A_\mu, \]$$

Local homomorphism $g(x) \in G$

G $SU(2)$ Lie Group

$$A_\mu^a = A_\mu^a t_a \quad (A_\mu^a \mid a=1, n = \dim G)$$

$\in \text{Lie}(G)$ t_a generators of the group $= \frac{1}{2} \sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Local Gauge Transformation $g(x) \in G$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1}$$

G $SU(2)$ Lie Group

$$A_\mu^a = A_\mu^a t_a \quad (A_\mu^a \mid a=1, n = \dim G)$$

$\in \text{Lie}(G)$ t_a generators of the group $= \frac{1}{2} \sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Gauge Transformation $g(x) \in G$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

$$A_\mu^a(x) = A_\mu^a(x) t_a \quad (A_\mu^a, a=1, n = \dim G)$$

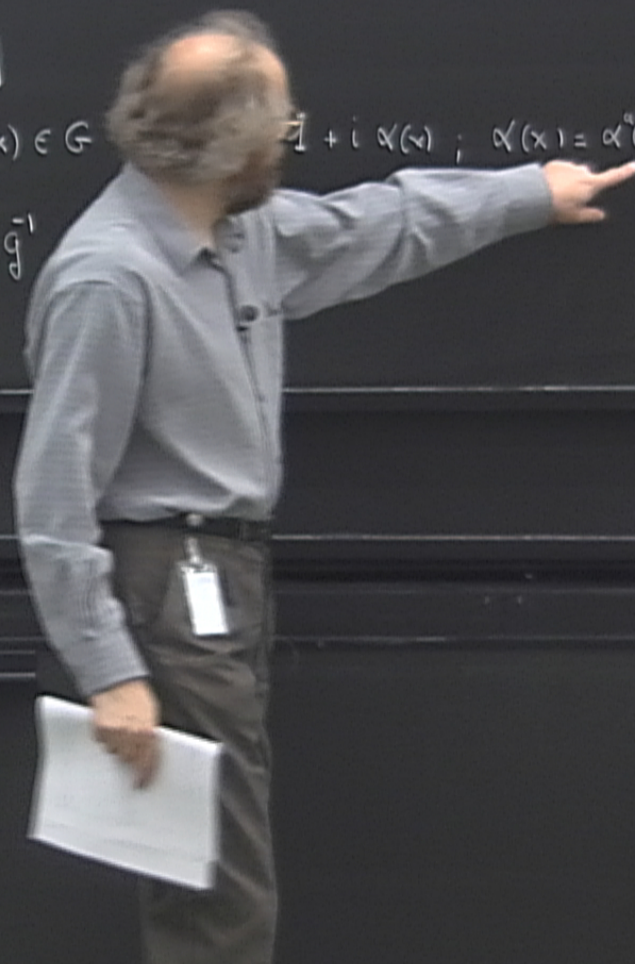
$\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2} \sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Local Gauge Transformation $g(x) \in G$ $U = 1 + i \alpha(x)$; $\alpha(x) = \alpha^a(x) t_a$

$$A_\mu \rightarrow g \cdot A_\mu g^{-1} + i g \partial_\mu g^{-1}$$



$$A_\mu^a(x) = A_\mu^a(x) t_a \quad (A_\mu^a, a=1, n = \dim G)$$

$\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2} \sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Local Gauge Transformation $g(x) \in G$ $1 + i \alpha(x)$; $\alpha(x) = \alpha^a(x) t_a$

$$A_\mu \rightarrow g \cdot A_\mu g^{-1} + i g \partial_\mu g^{-1}$$

$$A_\mu(x) = A_\mu^a(x) t_a \quad (A_\mu^a, a=1, n = \dim G)$$

$\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2} \sigma_a$ Pauli matrices

Covariant Derivatives \mathcal{D}_μ

$$A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

α small

Transformation $g(x) \in G$

$$g(x) = 1 + i \alpha(x); \quad \alpha(x) = \alpha^a(x) t_a$$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

$$A_\mu(x) \rightarrow A_\mu(x) + \mathcal{D}_\mu \alpha$$

$$A_\mu^a(x) = A_\mu^a(x) t_a \quad (A_\mu^a, a=1, n = \dim G)$$

$\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2} \sigma_a$ Pauli Matrices

Covariant Derivatives \mathcal{D}_μ

$$\mathcal{D}_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu] \quad \alpha \text{ small}$$

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i \alpha(x)$; $\alpha(x) = \alpha^a(x) t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} \quad A_\mu(x) \rightarrow A_\mu(x) + \mathcal{D}_\mu \alpha = A_\mu(x)$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly

→ $F_{\mu\nu} \cdot g^{-1}$

t_a

$$D_\mu \alpha(x) = \partial_\mu \alpha(x) - i [A_\mu, \alpha(x)]$$

G $SU(2)$ Lie Group

$A_\mu^a = A_\mu^a t_a$ (A_μ^a $a=1, n = \dim G$)
 $\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$= \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

transformation $g(x) \in G$

α small gauge transformation

$$g(x) = 1 + i \alpha(x), \quad \alpha(x) = \alpha^a(x) t_a$$

$$\rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

↑
invariant

$$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly

$$F_{\mu\nu} \rightarrow g \cdot F_{\mu\nu} \cdot g^{-1}$$

G $SU(2)$ Lie Group

$A_\mu^a = A_\mu^a t_a$ (A_μ^a $a=1, n = \dim G$)
 $\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Local Gauge Transformation $g(x) \in G$

α small gauge transformation

$$g(x) = 1 + i \alpha(x); \quad \alpha(x) = \alpha^a(x) t_a$$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

not covariant \uparrow

$$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly

$$g \cdot F_{\mu\nu} \cdot g^{-1}$$
$$F_{\mu\nu} = i [F_{\mu\nu}, \alpha]$$

G $SU(2)$ Lie Group

$$A_\mu^a = A_\mu^a t_a \quad (A_\mu^a, a=1, n = \dim G)$$

$t_a \in \text{Lie}(G)$ generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i \alpha(x)$; $\alpha(x) = \alpha^a(x) t_a$

$$A_\mu \rightarrow g \cdot A_\mu g^{-1} + i g \partial_\mu g^{-1}$$

not covariant \uparrow

$$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha - i [A_\mu(x), \alpha(x)]$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly inverse gauge transformations

$$F_{\mu\nu} \rightarrow F_{\mu\nu} \cdot g^{-1}$$

$$F_{\mu\nu} \rightarrow [F_{\mu\nu}, \alpha]$$

S

G $SU(2)$ Lie Group

$A_\mu^a = A_\mu^a t_a$ (A_μ^a $a=1, n = \dim G$)
 $\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$

Local Gauge Transformation $g(x) \in G$

α small gauge transformation

$g(x) = 1 + i \alpha(x)$; $\alpha(x) = \alpha^a(x) t_a$

$A_\mu \rightarrow g \cdot A_\mu g^{-1} + i g \partial_\mu g^{-1}$
not covariant \uparrow

$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$

Field strength

$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$

covariantly inverse gauge transformations

$F_{\mu\nu} \rightarrow g^{-1} F_{\mu\nu} g$

$F_{\mu\nu} \rightarrow F_{\mu\nu} + \partial_\mu \alpha_\nu - \partial_\nu \alpha_\mu$

$S_{\text{gauge}} = \frac{1}{4} \int d^4x \text{Tr} [F_{\mu\nu} F^{\mu\nu}]$

G $SU(2)$ Lie Group

$A_\mu^a = A_\mu^a t_a$ (A_μ^a $a=1, n = \dim G$)
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$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

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$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

not covariant \uparrow

α small gauge transformation

$$g(x) = 1 + i \alpha(x); \alpha(x) = \alpha^a(x) t_a$$

$$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + [A_\mu(x), \alpha(x)]$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly under gauge transformations

$$F_{\mu\nu} \rightarrow g \cdot F_{\mu\nu} \cdot g^{-1}$$

$$F_{\mu\nu} \rightarrow F_{\mu\nu} - i [F_{\mu\nu}, \alpha]$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} F^{\mu\nu}]$$

\uparrow Lorentz invariant

G $SU(2)$ Lie Group

$A_\mu^a = A_\mu^a t_a$ (A_μ^a $a=1, n = \dim G$)
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$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$

Local Gauge Transformation $g(x) \in G$

$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$
not covariant

α small gauge transformation

$g(x) = 1 + i \alpha(x)$, $\alpha(x) = \alpha^a(x) t_a$

$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$

Field strength

$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$

covariantly under gauge transformations

$F_{\mu\nu} \rightarrow g \cdot F_{\mu\nu} \cdot g^{-1}$

$F_{\mu\nu} \rightarrow F_{\mu\nu} - i [F_{\mu\nu}, \alpha]$

$F_{\mu\nu} = F_{\mu\nu}^a t_a$

$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}]$

\uparrow Lorentz invariant

G $SU(2)$ Lie Group

$A_\mu^a = A_\mu^a t_a$ (A_μ^a) $a=1, n = \dim G$
 $\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

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$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

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$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

not covariant \uparrow

α small gauge transformation

$$g(x) = 1 + i \alpha(x); \alpha(x) = \alpha^a(x) t_a$$

$$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly under gauge transformations

$$F_{\mu\nu} \rightarrow g \cdot F_{\mu\nu} \cdot g^{-1}$$

$$F_{\mu\nu} \rightarrow F_{\mu\nu} - i [F_{\mu\nu}, \alpha]$$

$$F_{\mu\nu} = F_{\mu\nu}^a t_a$$

$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{2g^2} \int d^4x \text{Tr}[F_{\mu\nu} \cdot F^{\mu\nu}]$$

\uparrow kinetic

G $SU(2)$ Lie Group, $SU(N)$, $SO(N)$. . .

$A_\mu^a = A_\mu^a t_a$ (A_μ^a $a=1, n = \dim G$)
 $\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$ α small gauge transformation

Local Gauge Transformation $g(x) \in G$ $U(x) = 1 + i\alpha(x)$, $\alpha(x) = \alpha^a(x)t_a$

$A_\mu \rightarrow g \cdot A_\mu g^{-1} + i g \partial_\mu g^{-1}$ $A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$
 not covariant

Field strength

$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$

covariantly under gauge transformations

$F_{\mu\nu} \rightarrow g \cdot F_{\mu\nu} \cdot g^{-1}$

$F_{\mu\nu} = F_{\mu\nu}^a t_a$

$F_{\mu\nu} \rightarrow F_{\mu\nu} - i [F_{\mu\nu}, \alpha]$

$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$

$S_{\text{gauge}}[A] = -\frac{1}{2g^2} \int d^4x \text{Tr}[F_{\mu\nu} \cdot F^{\mu\nu}]$
 \uparrow Lorentz invariant

G $SU(2)$ Lie Group, $SU(N)$, $SO(N)$. . .

$A_\mu^a = A_\mu^a t_a$ (A_μ^a) $a=1, n = \dim G$
 $\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Local Gauge Transformation $g(x) \in G$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

not covariant \uparrow

α small gauge transformation

$$g(x) = 1 + i \alpha(x), \quad \alpha(x) = \alpha^a(x) t_a$$

$$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly inverse gauge transformations

$$F_{\mu\nu} \rightarrow g \cdot F_{\mu\nu} \cdot g^{-1}$$

$$F_{\mu\nu} = F_{\mu\nu}^a t_a$$

$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$J = -\frac{1}{4g^2} \int d^4x \text{Tr}[F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

\uparrow Lorentz invariant

G $SU(2)$ Lie Group, $SU(N)$, $SO(N)$. . .

$A_\mu^a = A_\mu^a t_a$ (A_μ^a) $a=1, n = \dim G$
 $\in \text{Lie}(G)$ t_a generators of the group = $\frac{1}{2}\sigma_a$ Pauli Matrices

Covariant Derivatives D_μ

$$D_\mu A_\nu = \partial_\mu A_\nu - i [A_\mu, A_\nu]$$

Local Gauge Transformation $g(x) \in G$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1}$$

not covariant \uparrow

gauge transformation

$$A_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

Field strength

$$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$$

covariantly under gauge transformations

$$F_{\mu\nu} \rightarrow g \cdot F_{\mu\nu} \cdot g^{-1}$$

$$F_{\mu\nu} \rightarrow F_{\mu\nu} - i [F_{\mu\nu}, \alpha]$$

$$F_{\mu\nu} = F_{\mu\nu}^a t_a$$

$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr}[F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F^{\mu\nu a}$$

\uparrow gauge invariant \uparrow Lorentz invariant

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_{g,\mu}$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = U [F_{\mu\nu}, \alpha]$$

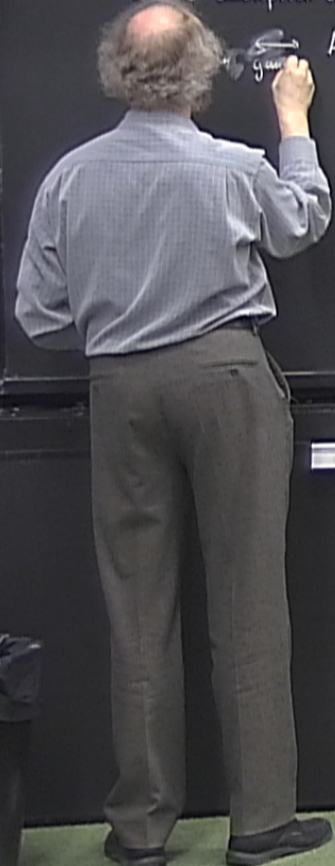
$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

↑ gauge invariant ↑ locality invariant

Local description of gauge Field in space-time

$$\vec{A}'_\mu$$



Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_{g,\mu}$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = U [F_{\mu\nu}, \alpha]$$

$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

↑ gauge invariant ↑ locality invariant

Local description of gauge Field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_{g,\mu}$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) + \mathcal{D}_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = U [F_{\mu\nu}, \alpha]$$

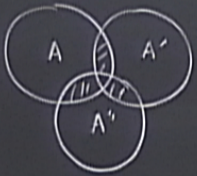
$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

↑ gauge invariant ↑ Lorentz invariant

Local description of gauge Field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_{g,\mu}$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = U [F_{\mu\nu}, \alpha]$$

$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

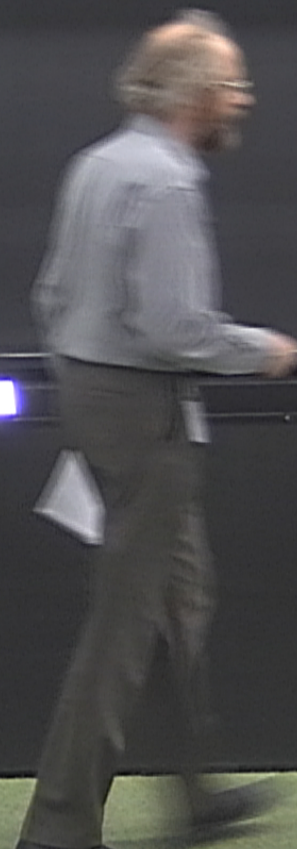
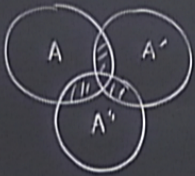
$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

↑ gauge invariant ↑ locality invariant

Local description of gauge Field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$

Topology & Geometry



Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_\mu$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) + \mathcal{D}_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

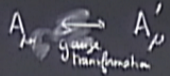
$$F_{\mu\nu} \rightarrow F_{\mu\nu} - i [F_{\mu\nu}, \alpha]$$

$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

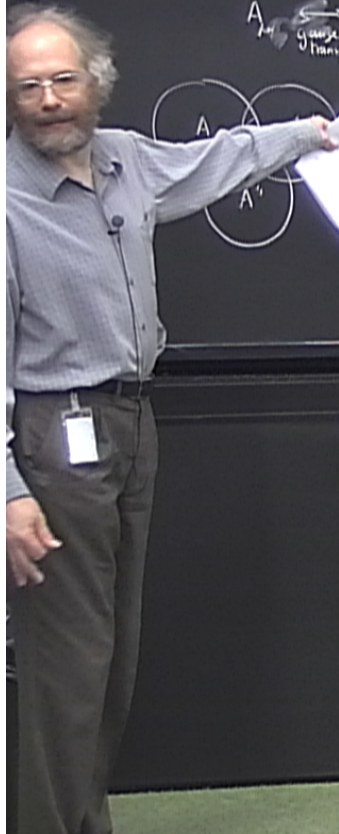
↑ gauge invariant ↑ locality invariant

Local description of gauge Field in space-time



Topology & Geometry

$$A = A_\mu dx^\mu \quad \text{1 form - connection on some Fiber Bundle}$$



Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_\mu$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = U [F_{\mu\nu}, U^\dagger]$$

$$\text{Tr}[t_a t_b] = \frac{1}{2} \delta_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

↑ gauge invariant ↑ Lorentz invariant

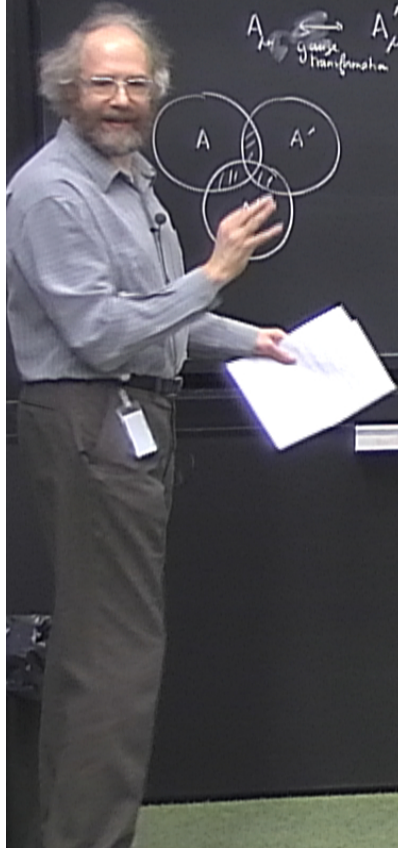
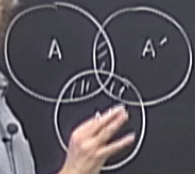
Local description of gauge Field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$

Topology & Geometry

$$A = A_\mu dx^\mu \quad \text{1 form, connection on some Fiber Bundle}$$

$$F = F_{\mu\nu} dx^\mu dx^\nu = dA + A \wedge A$$



Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_{g,\mu}$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = U^{-1} [F_{\mu\nu}, U]$$

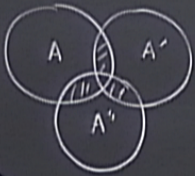
$$\text{Tr}[a^a a^b] = \frac{1}{2} \delta^{ab}$$

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↑ gauge invariant ↑ locality invariant

Local description of gauge Field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

$$A = A_\mu dx^\mu$$

1 form - connection on some Fiber Bundle

$$F = F_{\mu\nu} dx^\mu dx^\nu$$

\wedge exterior product

$$= dA + A \wedge A$$

$$S = \int d^4x \text{Tr} (*F \wedge F)$$

Hodge dual

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

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$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i [A_\mu(x), \alpha(x)]$$

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$$\text{Tr}[a^a b^b] = \frac{1}{2} \delta^{ab}$$

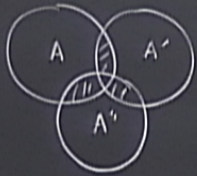
$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

↑ gauge invariant ↑ Lorentz invariant

Local description of gauge field in space-time

Quantization

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

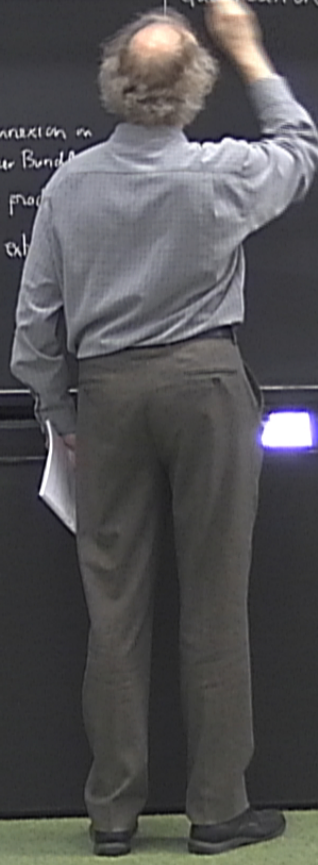
$$A = A_\mu dx^\mu$$

$$F = F_{\mu\nu} dx^\mu dx^\nu = dA + A \wedge A$$

$$S = \int d^4x \text{Tr} (\star F \wedge F)$$

Hodge dual

1 form - connection on some Fiber Bundle
 \wedge exterior product
 d exterior derivative



Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_{g,\mu}$$

↑
not covariant

$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - i[A_\mu(x), \alpha(x)]$$

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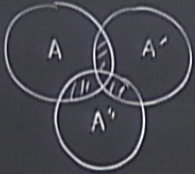
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↑ gauge invariant ↑ Lorentz invariant

Local description of gauge Field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

$$A = A_\mu dx^\mu$$

1 form, connection on some Fiber Bundle

$$F = F_{\mu\nu} dx^\mu dx^\nu$$

\wedge exterior product

$$= dA + A \wedge A$$

d exterior derivative

$$S = \int d^4x \text{Tr} (\star F \wedge F)$$

Hodge dual

Quantization . Functional Integral

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i \alpha(x)$; $\alpha(x) = \alpha^a(x) t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_\mu$$

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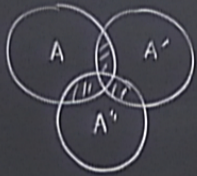
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↑ gauge invariant ↑ Lorentz invariant

Local description of gauge field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

$$A = A_\mu dx^\mu$$

1 form, connection on some Fiber Bundle

$$F = F_{\mu\nu} dx^\mu dx^\nu$$

\wedge exterior product

$$= dA + A \wedge A$$

d exterior derivat

$$S = \int d^4x \text{Tr} (\ast F \wedge F)$$

Hodge dual

Quantization . Functional Integral Gauge + Lorentz invariance

$$\int \mathcal{D}A \exp(i S_{\text{gauge}}[A])$$

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

$$A_\mu \rightarrow g \cdot A_\mu \cdot g^{-1} + i g \partial_\mu g^{-1} = A'_\mu$$

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$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - [A_\mu(x), \alpha(x)]$$

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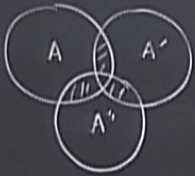
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↑ gauge invariant ↑ Lorentz invariant

Local description of gauge field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

$$A = A_\mu dx^\mu$$

1 form, connection on some Fiber Bundle

$$F = F_{\mu\nu} dx^\mu dx^\nu$$

exterior product

$$= dA + A \wedge A$$

d exterior derivative

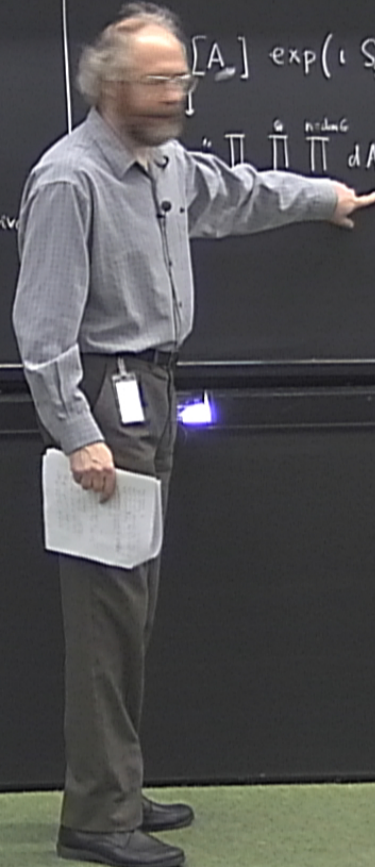
$$S = \int d^4x \text{Tr} (*F \wedge F)$$

Hodge dual

Quantization, Functional Integral, Gauge + Lorentz invariance

$$[A] \exp(i S_{\text{gauge}}[A])$$

$$\prod_{\mu, \nu} \prod_{a=1}^{\dim G} dA_\mu^a(x)$$



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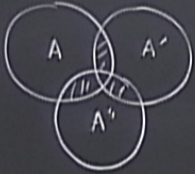
$$\text{Tr}[a^t b] = \frac{1}{2} a_{ab}$$

$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F_a^{\mu\nu}$$

gauge invariant Lorentz invariant

Local description of gauge field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

$\int dx^\mu$
 $\int dx^\nu$
 $\int dx^\rho$
 $\int dx^\sigma$

1 form - connection on some Fiber Bundle
 \wedge extensor product
 d extensor derivative

Quantization . Functional Integral Gauge + Lorentz invariance

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

$$\mathcal{D}[A] = \prod_{x \in M_4} \prod_{\mu=1}^4 \prod_{a=1}^{n-dim G} dA_\mu^a(x)$$

Local Gauge Transformation $g(x) \in G$ $g(x) = 1 + i\alpha(x)$; $\alpha(x) = \alpha^a(x)t_a$

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$$A_\mu(x) \rightarrow A'_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - [A_\mu(x), \alpha(x)]$$

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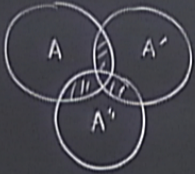
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$$S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} \cdot F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a \cdot F_a^{\mu\nu}$$

↑ gauge invariant ↑ Lorentz invariant

Local description of gauge Field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

$$A = A_\mu dx^\mu$$

$$F = F_{\mu\nu} dx^\mu dx^\nu = dA + A \wedge A$$

$$S = \int d^4x \text{Tr} (*F \wedge F)$$

↑
Hodge dual

1 form - connection on some Fiber Bundle
 \wedge exterior product
 d exterior derivative

Quantization . Functional Integral Gauge + Lorentz invariance

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

$$= \prod_{x \in M_4} \prod_{\mu=1}^4 \prod_{a=1}^{n \cdot \dim G} dA_\mu^a(x)$$

Quantization of Gauge theory

Quantization , Functional Integral

Gauge + Lorentz invariance

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

↓

$$\mathcal{D}[A] \equiv \prod_{x \in M_{1,3}} \prod_{\mu=1}^4 \prod_{a=1}^{n=\dim G} dA_{\mu}^a(x)$$

discretization of Gauge Theory

(K. Wilson)

e-time

Geom

connexion in
the Fiber Bundle
product

gauge invariant Lorentz invariant

Quantization , Functional Integral

Gauge + Lorentz invariance
Enormous redundancy

$$\int D[A] \exp(i S_{\text{gauge}}[A])$$

↓

$$D[A] \equiv \prod_{x \in \mathbb{H}_{1,3}} \prod_{\mu=1}^4 \prod_{a=1}^{n=\dim G} dA_{\mu}^a(x)$$

Lattice discretization of Gauge Theory
(K. Wilson)

m. connection in
the Fiber Bundle
exterior product

d exterior derivative

Quantization, Functional Integral

$$\int D[A] \exp(i S_{\text{gauge}}[A])$$

↓

$$D[A] \equiv \prod_{x \in M_{1,3}} \prod_{\mu=1}^4 \prod_{a=1}^{n=\dim G} dA_{\mu}^a(x)$$

Lattice discretization of Gauge theory
(K. Wilson)

Gauge + Lorentz invariance

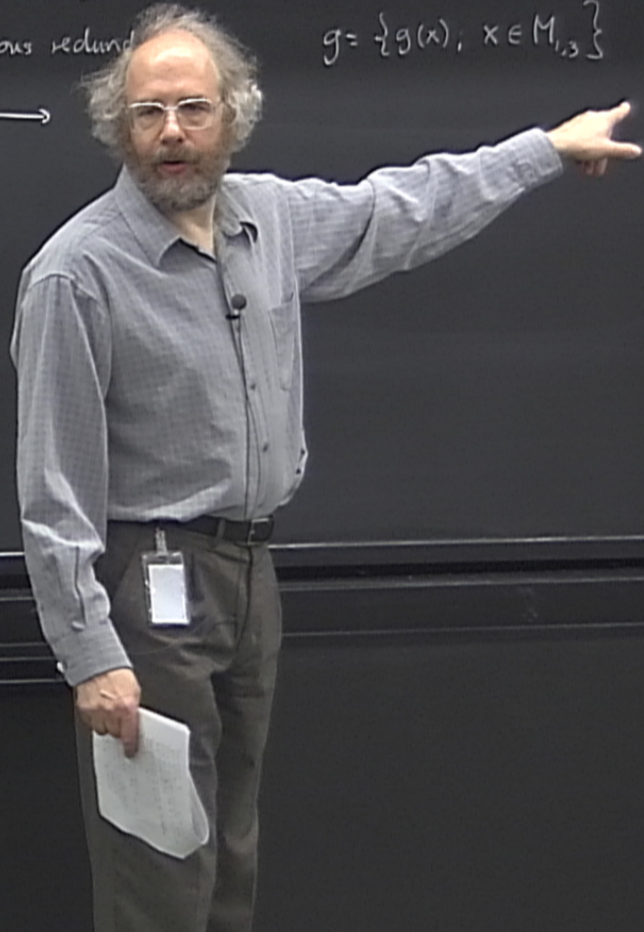
Enormous redundancy

$$A \xrightarrow{g}$$

$$g = \{g(x); x \in M_{1,3}\}$$

↑ gauge invariant

↑ Lorentz invariant



action on
Bundle
product
covariant derivative

Quantization, Functional Integral

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

↓

$$\mathcal{D}[A] \equiv \prod_{x \in M_{1,3}} \prod_{\mu=1}^4 \prod_{a=1}^{n=\dim G} dA_{\mu}^a(x)$$

Lattice discretization of Gauge Theory
(K. Wilson)

Gauge + Lorentz invariance

Enormous redundancy

$$A \xrightarrow{g}$$

$$g = \{g(x); x \in M_{1,3}\}$$

$$g \in \mathcal{G} = \bigotimes_{x \in M_{1,3}} G$$

↑ gauge invariant

↑ Lorentz invariant

action on
Bundle
product
tensor derivative

Quantization, Functional Integral

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

↓

$$\mathcal{D}[A] \equiv \prod_{x \in M_{1,3}} \prod_{\mu=1}^4 \prod_{a=1}^{n=\dim G} dA_{\mu}^a(x)$$

Lattice discretization of Gauge theory
(K. Wilson)

Gauge + Lorentz invariance

Enormous redundancy

$$A \xrightarrow{g} A_g$$

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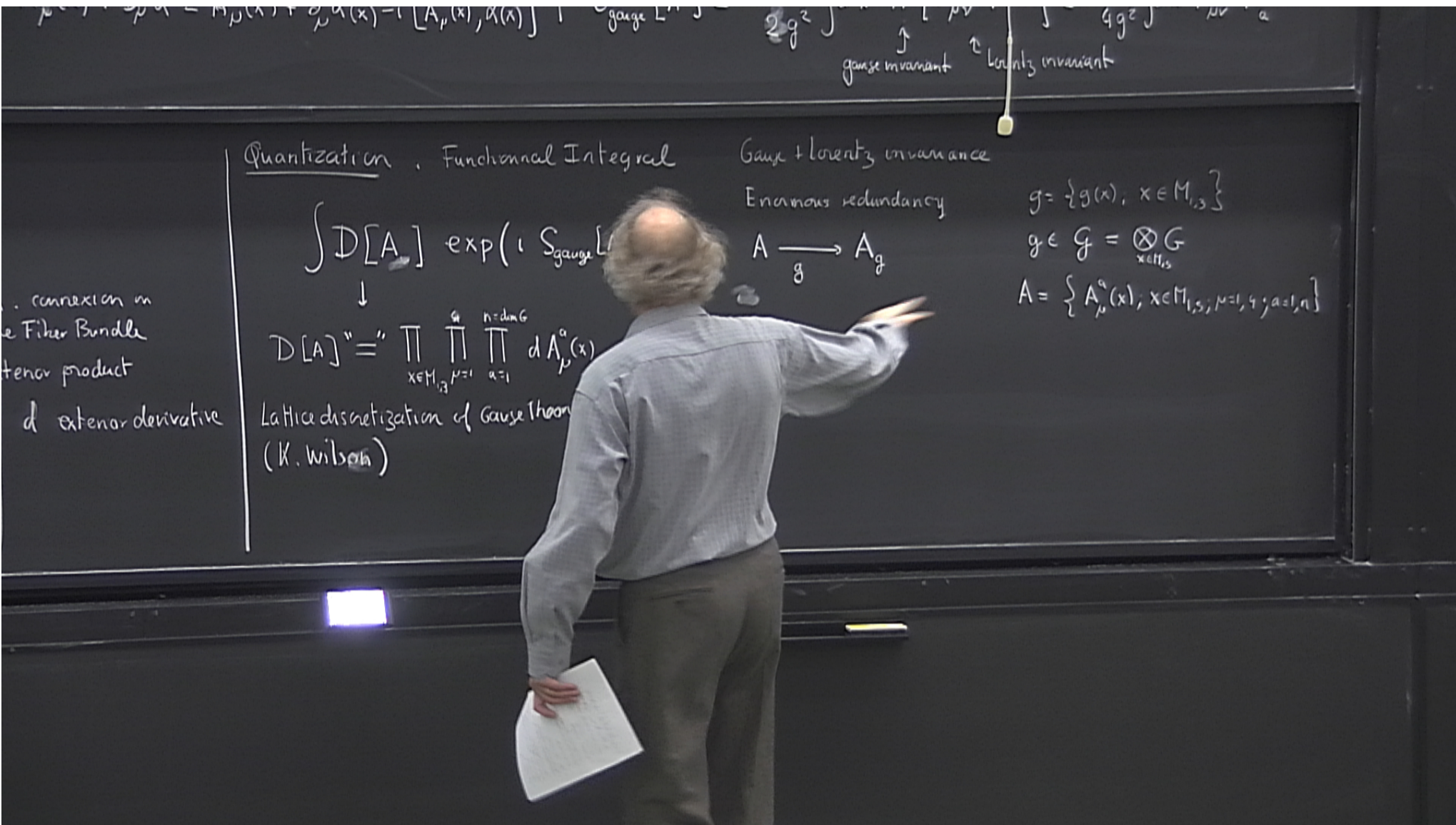
$$g \in \mathcal{G} = \bigotimes_{x \in M_{1,3}} G$$

↑ gauge invariant

↑ Lorentz invariant

action on
Bundle
product

tensor derivative



$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu] \quad \text{gauge invariant} \quad \text{Lorentz invariant}$$

Quantization . Functional Integral

Gauge + Lorentz invariance

Enormous redundancy

$$g = \{g(x); x \in M_{1,3}\}$$

$$g \in \mathcal{G} = \bigotimes_{x \in M_{1,3}} G$$

$$A = \{A_\mu^a(x), x \in M_{1,3}, \mu=1,2,3, a=1,2,3\}$$

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

$$A \xrightarrow{g} A_g$$

connection on the Fiber Bundle tensor product of exterior derivative

$$\mathcal{D}[A] \stackrel{?}{=} \prod_{x \in M_{1,3}} \prod_{\mu=1}^3 \prod_{a=1}^{n=\dim G} dA_\mu^a(x)$$

Lattice discretization of Gauge Theory (K. Wilson)

$$\mu^{\nu\alpha} = A_{\mu}^{\nu}(x) + g_{\mu}^{\nu\alpha}(x) = [A_{\mu}^{\nu}(x), \alpha(x)] \quad \text{gauge } L^{\mu\nu} \quad \text{gauge invariant} \quad \text{Lorentz invariant}$$

Quantization, Functional Integral

$$\int D[A_{\mu}] \exp(i S_{\text{gauge}}[A_{\mu}])$$

$$\downarrow$$

$$= \prod_{x \in M_{1,3}} \prod_{\mu=1}^4 \prod_{a=1}^{n=\dim G} dA_{\mu}^a(x)$$

Quantization of Gauge Theory

Gauge + Lorentz invariance

Enormous redundancy

$$A \xrightarrow{g} A_g$$

Same physics

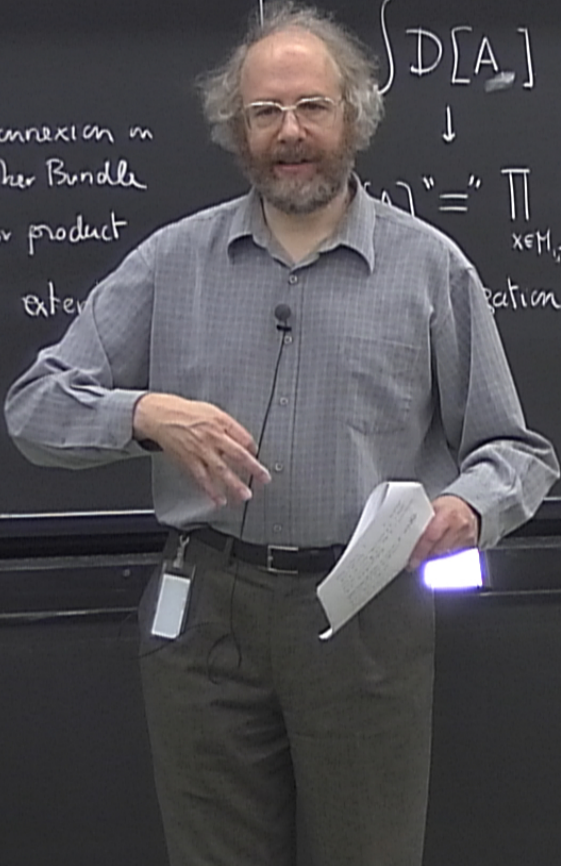
$$F_{\mu\nu} \rightarrow g F_{\mu\nu} g^{-1}$$

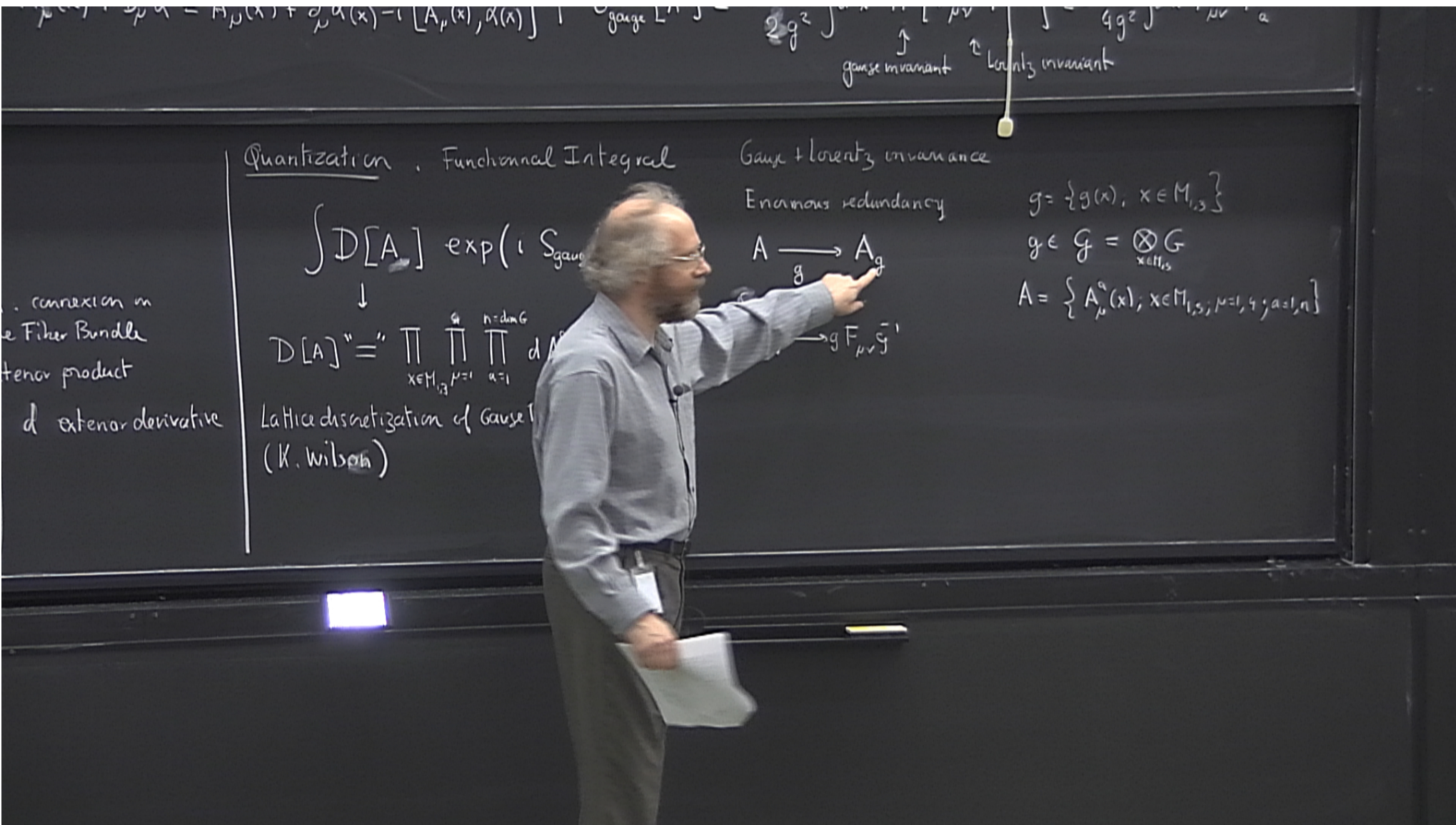
$$g = \{g(x), x \in M_{1,3}\}$$

$$g \in \mathcal{G} = \bigotimes_{x \in M_{1,3}} G$$

$$A = \{A_{\mu}^a(x), x \in M_{1,3}, \mu=1, 2, 3, 4, a=1, \dots, n\}$$

connection on
the Fiber Bundle
tensor product
d exten





$$\mu^{\nu\alpha} = A_{\mu}(x) + g_{\mu}(x) = i [A_{\mu}(x), \alpha(x)] \quad \text{gauge } L^{\mu\nu}$$

$$2g^2 \quad \uparrow \quad \uparrow$$

gauge invariant Lorentz invariant

Quantization . Functional Integral

Gauge + Lorentz invariance

Enormous redundancy

$$g = \{g(x), x \in M_{1,3}\}$$

$$g \in \mathcal{G} = \bigotimes_{x \in M_{1,3}} G$$

$$A = \{A_{\mu}^a(x), x \in M_{1,3}, \mu=1,2,3, a=1,2,3\}$$

$$\int D[A] \exp(i S_{\text{gauge}})$$

$$A \xrightarrow{g} A_g$$

$$F_{\mu\nu} \rightarrow g F_{\mu\nu} g^{-1}$$

$$D[A] \stackrel{?}{=} \prod_{x \in M_{1,3}} \prod_{\mu=1}^3 \prod_{a=1}^{n=\dim G} dA_{\mu}^a$$

Lattice discretization of Gauge Theory
(K. Wilson)

connection on
the Fiber Bundle
tensor product
d exterior derivative

$\rho_\mu(x) = A_\mu(x) + \partial_\mu \alpha(x) = [A_\mu(x), \alpha(x)]$ gauge invariant

$\int d^4x \frac{1}{2} g^2 F_{\mu\nu}^2$ Lorentz invariant

Quantization . Functional Integral

$$\int D[A] \exp(i S_{\text{gauge}}[A])$$

↓

$$D[A] \approx \prod_{x \in M_{1,3}} \prod_{\mu=1}^4 \prod_{a=1}^{n=\dim G} dA_\mu^a(x)$$

Lattice discretization of Gauge Theory
(K. Wilson) gauge invariant

connection in
the Fiber Bundle
tensor product
d exterior derivative

Gauge + Lorentz invariance

Enormous redundancy

$$A \xrightarrow{g} A_g$$

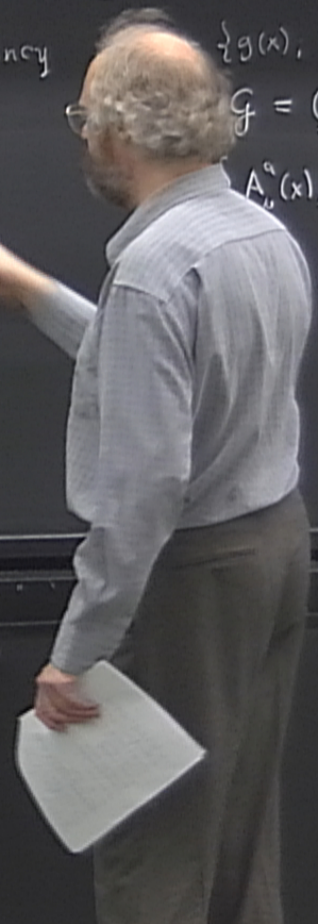
Same physics

$$F_{\mu\nu} \rightarrow g F_{\mu\nu} g^{-1}$$

$\{g(x), x \in M_{1,3}\}$

$$G = \bigotimes_{x \in M_{1,3}} G$$

$A_\mu^a(x), x \in M_{1,3}, \mu=1, 4, a=1, n$

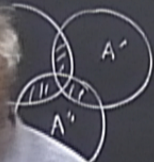


$$A'_\mu(x) \rightarrow A_\mu(x) + D_\mu \alpha = A_\mu(x) + \partial_\mu \alpha(x) - [A_\mu(x), \alpha(x)] \quad | \quad S_{\text{gauge}}[A] = -\frac{1}{4g^2} \int d^4x \text{Tr} [F_{\mu\nu} F^{\mu\nu}] = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F^{\mu\nu a}$$

↑ gauge invariant ↑ Lorentz invariant

Local description of gauge field in space-time

$$A_\mu \xrightarrow{\text{gauge transformation}} A'_\mu$$



Topology & Geometry

$$A = A_\mu dx^\mu$$

1 form - connection on some Fiber Bundle

$$F = F_{\mu\nu} dx^\mu \wedge dx^\nu$$

\wedge exterior product

$$= dA + A \wedge A$$

d exterior derivative

$$S = \int d^4x \text{Tr} (*F \wedge F)$$

Hodge dual

Quantization - Functional Integral

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

$$\mathcal{D}[A] \stackrel{?}{=} \prod_{x \in M_{1,3}} \prod_{\mu=1}^3 \prod_{a=1}^{n \cdot \dim G} dA_\mu^a(x)$$

Lattice discretization of Gauge Theory (K. Wilson) gauge invariant

Gauge + Lorentz invariance

Enormous redundancy

$$g = \{g(x), x \in M_{1,3}\}$$

$$A \xrightarrow{g} A_g$$

$$g \in \mathcal{G} = \otimes_{x \in M_{1,3}} G$$

Same physics

$$F_{\mu\nu} \rightarrow g F_{\mu\nu} g^{-1}$$

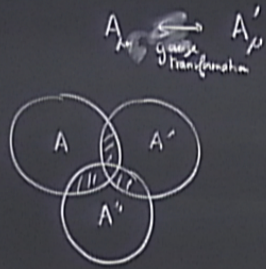
$$A = \{A_\mu^a(x), x \in M_{1,3}, \mu=1,2,3; a=1,2,3\}$$

U(1) : QED ; Gauge Fixing

$$A'_\mu(x) = A_\mu(x) + D_\mu \alpha(x) = A_\mu(x) + \partial_\mu \alpha(x) - [A_\mu(x), \alpha(x)]$$

not covariant \uparrow gauge transformation \uparrow gauge invariant \uparrow Lorentz invariant

Local description of gauge field in space-time



Topology & Geometry

$$A = A_\mu dx^\mu$$

1 form connection on some Fiber Bundle

$$F = F_{\mu\nu} dx^\mu \wedge dx^\nu$$

\wedge exterior product

$$S = \int d^4x \text{Tr}(*F \wedge F)$$

$*$ exterior derivative

Quantization, Functional Integral

$$\int \mathcal{D}[A] \exp(i S_{\text{gauge}}[A])$$

$$\mathcal{D}[A] = \prod_{x \in M_3} \prod_{\mu=1}^3 \prod_{a=1}^{\text{rank } G} dA_\mu^a(x)$$

Lattice discretization of Gauge Theory (K. Wilson) gauge invariant

Gauge + Lorentz invariance

Enormous redundancy $g = \{x \in M_{1,3}\}$

Same physics $A \rightarrow A_g$

$F_{\mu\nu} \rightarrow g F_{\mu\nu} g^{-1}$

$U(1) : \text{QED}; G$

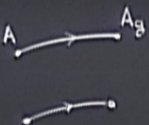


$$S = \int d^4x \text{Tr}(\kappa F \wedge F)$$

↑
Holger dual

(K. Wilson) gauge invariant

$\mathcal{A} = \{A\}$ All gauge configurations
 $\mathcal{G} = \{g\}$ All gauge transformations

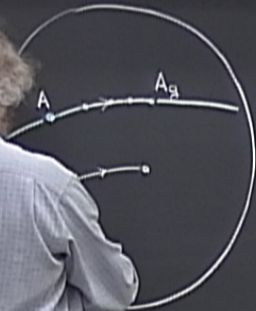


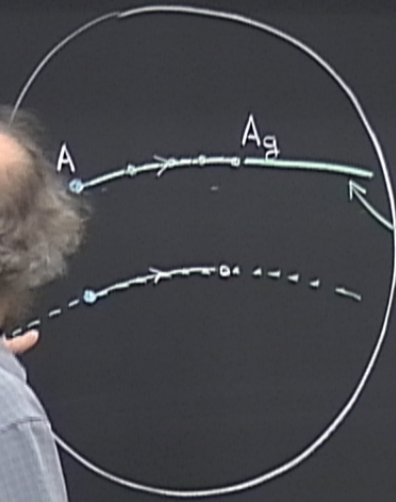
$$S = \int d^4x \text{Tr}(\kappa F \wedge F)$$

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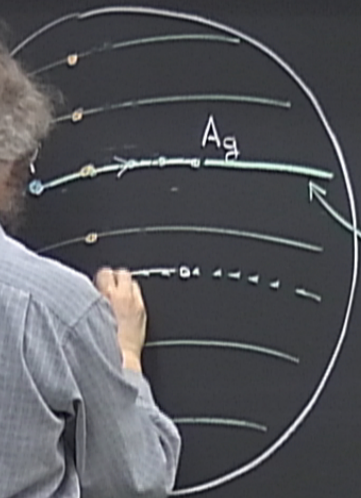


$\mathcal{A} = \{ A \}$ All gauge configurations

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orbit of A by the action of G



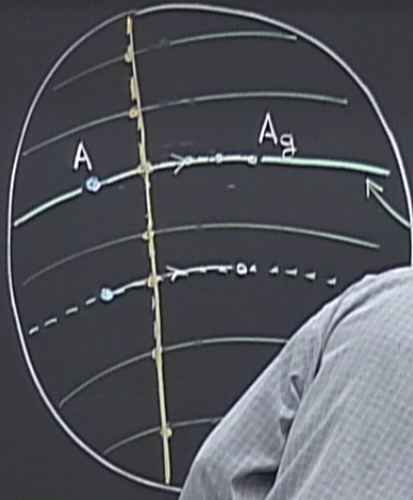


$\mathcal{A} = \{ A \}$ All gauge configurations

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orbit of A by the action of \mathcal{G}

$\mathcal{C} =$ space of physical (ly inequivalent) configuration
 $= \mathcal{A}/\mathcal{G}$



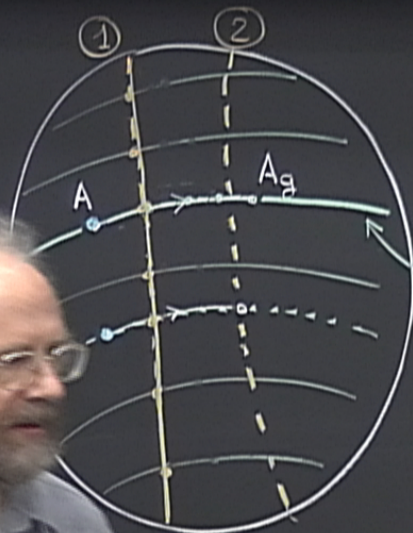
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orbit action of \mathcal{G}

→ of physical (ly inequivalent) configuration

\mathcal{G}



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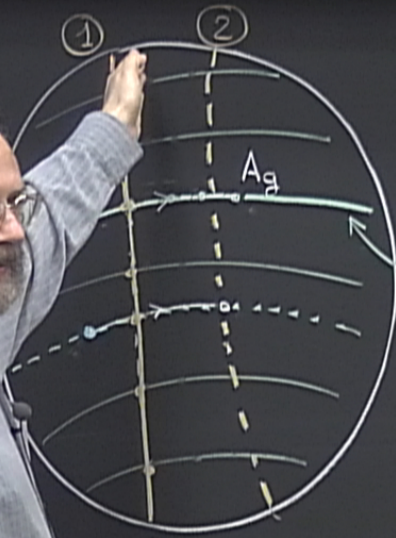
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↳ Gauge Fixing = choosing a "slice"

①



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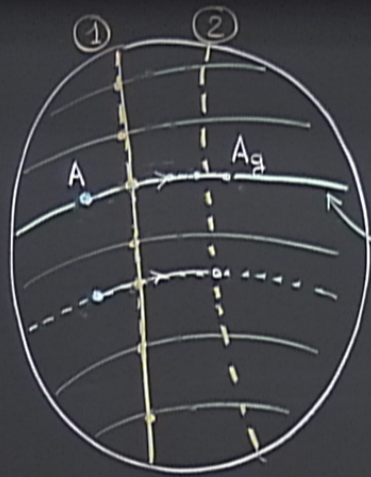
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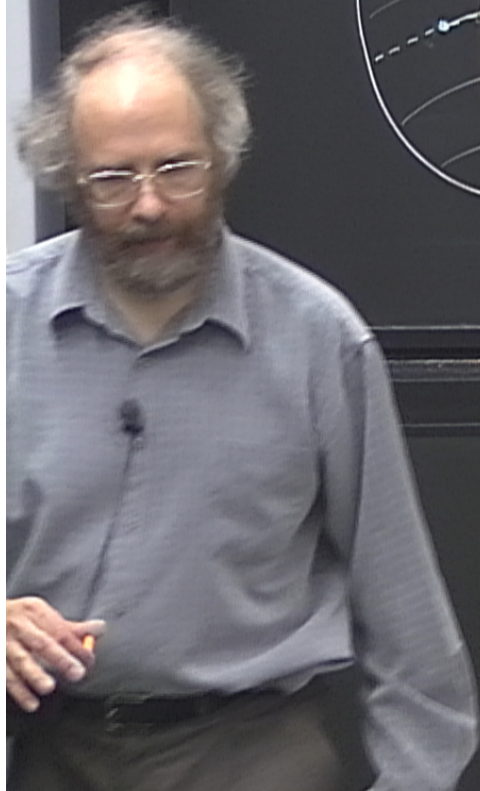
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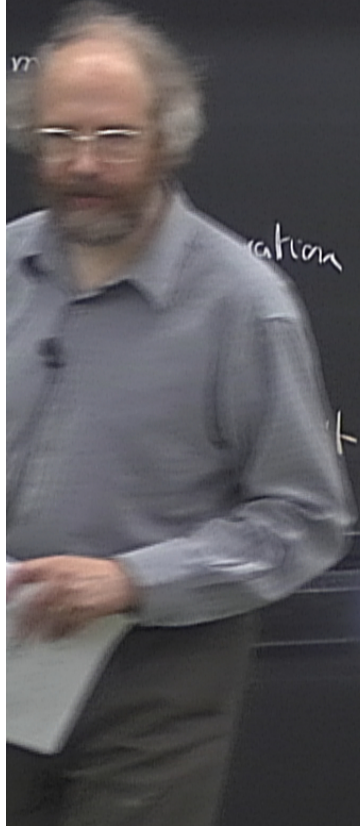
↑ Gauge Fixing = choosing a "slice" = choosing 1 configuration in each orbit
 ①

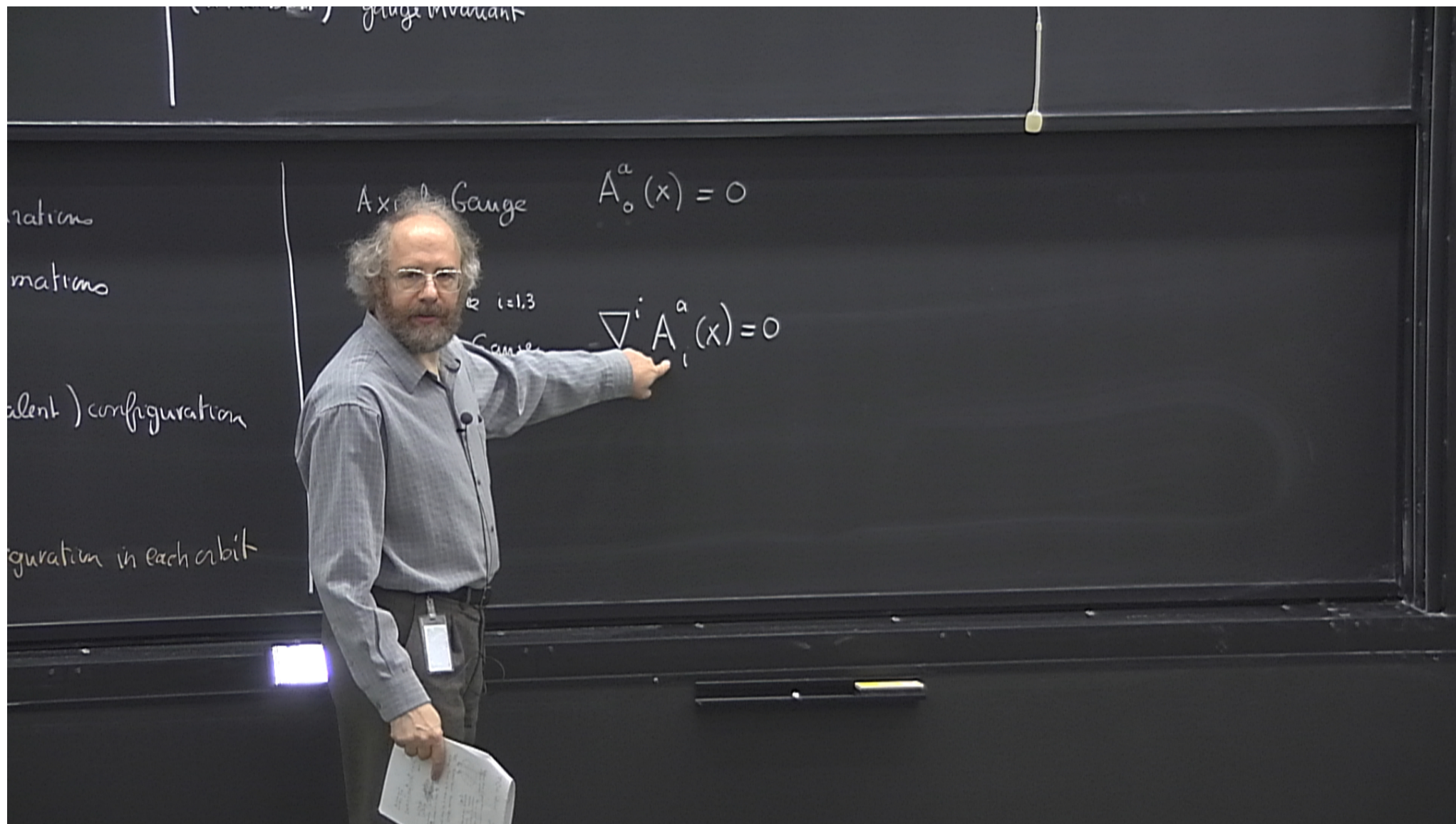


gauge invariant

Axial Gauge $A_0^a(x) = 0$

$X = (X^0, \vec{X})$
time ↑ space





gauge invariant

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alent) configuration

guration in each orbit

Axial Gauge

$$A_0^a(x) = 0$$

$l=1,3$

Gauge

$$\nabla^i A_i^a(x) = 0$$

(...) gauge invariant

rations

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alent) configuration

guration in each orbit

* Axial Gauge $A_0^a(x) = 0$

$X = (X^0, \vec{X})$
time ↑ space i=1,3

* Coulomb Gauge

$$\nabla^i A_i^a(x) = 0$$

*

gauge invariant

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* Axial Gauge $A_0^a(x) = 0$

(x^0, \vec{x})
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space $i=1,3$

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man-Landau... ∂^μ

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gauge invariant

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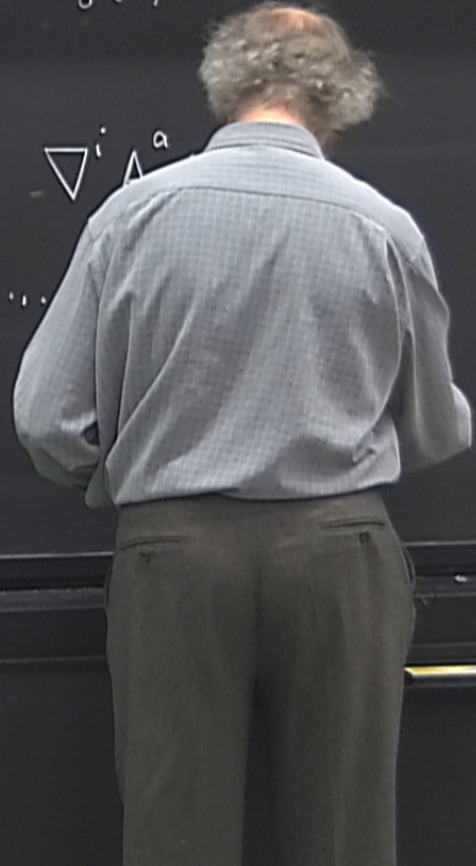
* Axial Gauge $A_0^a(x) = 0$

$X = (X^0, \vec{X})$
time ↑ space i=1,3

* Coulomb Gauge

$$\nabla_i A^a$$

* Feynman-Landau...
(Lorentz covariant)



gauge invariant

* Axial Gauge $A_0^a(x) = 0$

(t, \vec{x})
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$$\nabla^i A_i^a(x) = 0$$

omb Gauge

Landau... $\partial^\mu A_\mu^a(x) = \omega^a(x) = 0$

$\omega^a(x)$ some fixed function

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gauge invariant

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$$\partial^\mu A_\mu^a(x) = w^a(x) = 0$$

$w^a(x)$ some fixed function
ordinary derivative

gauge invariant

ations
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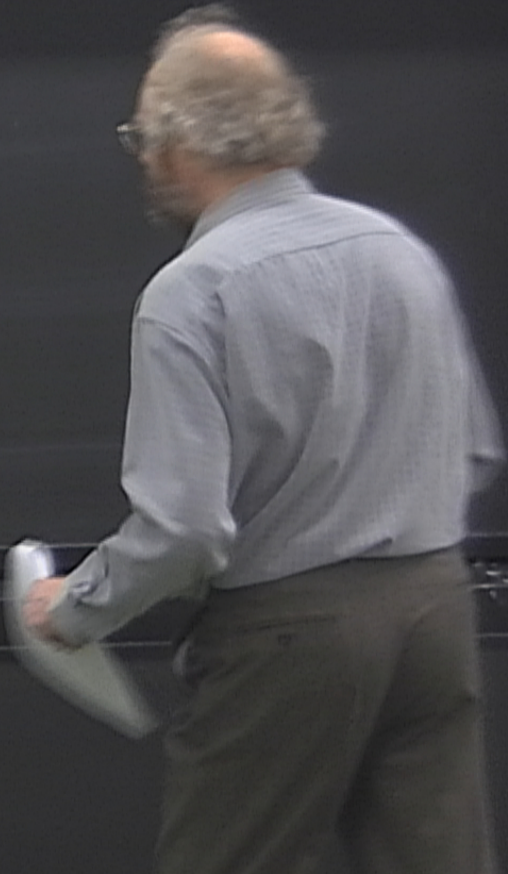
* Coulomb Gauge

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ordinary derivative



A^a

$$S = \int d^4x \text{Tr}(\star F \wedge F)$$

Hodge dual

d exterior derivative

Lattice discretization of Gauge theory
(K. Wilson) gauge invariant



$\mathcal{A} = \{A\}$ All gauge configurations

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the action of \mathcal{G}

space of physical (by inequivalent) configurations

\mathcal{P}

"gauge fixing" = choosing 1 configuration in each orbit

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$x = (x^0, \vec{x})$
time ↑ space ↑

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(Lorentz covariant)

$\omega^a(x)$ some fixed function
ordinary derivative

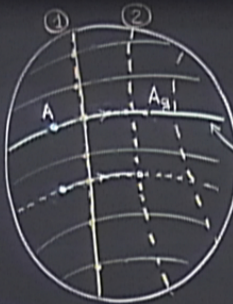
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Gauge Fixing condition

$x = (x^0, \vec{x})$

time ↑ space ↑

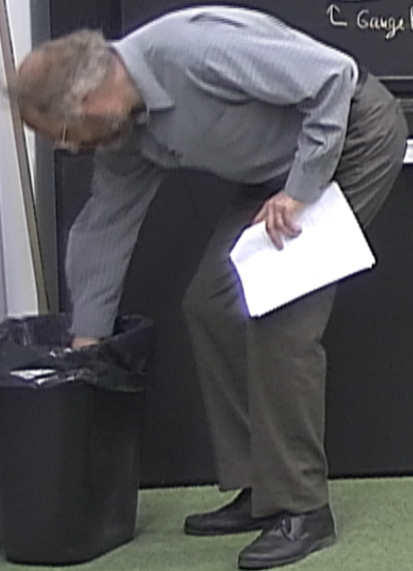
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$$\nabla^i A_i^a(x) = 0$$

* Feynman-Landau... (Lorentz covariant)

$$\partial^\mu A_\mu^a(x) = \omega^a(x) = 0$$

$\omega^a(x)$ some fixed function
ordinary derivative



$$A_0^a(x) = 0$$

$$\nabla^i A_i^a(x) = 0$$

$$\partial_\mu^a A_\mu^a(x) = \omega^a(x) = 0$$

$\omega^a(x)$ some fixed function
ordinary derivative

Gauge Fixing condition

$$F^a[A](x) = 0 \quad \text{all } a \text{ and } x$$

↑ Function involving A and its derivatives

* Axial Gauge

$$A_0^a(x) = 0$$

$$x = (x^0, \vec{x})$$

time ↑ space ↑ $i=1,2,3$

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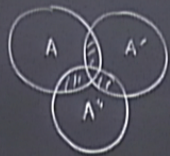
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Gauge Fixing condition

$$F^a[A](x) = 0 \quad \text{all } a \text{ and } x$$

↑ Functions involving A and its derivatives



Topology geometry

$$A = A_\mu dx^\mu$$

1 form - connection on some Fiber Bundle

$$F = F_{\mu\nu} dx^\mu dx^\nu = dA + A \wedge A$$

exterior product

$$S = \int d^4x \text{Tr}(\pm F \wedge F)$$

Holonomy

d exterior derivative

$$D[A] = \exp(i S_{\text{gauge}}[A])$$

$$\downarrow$$

$$D[A] = \prod_{x \in \Lambda} \prod_{\mu=1}^3 \prod_{a=1}^{n-1} dA_\mu^a(x)$$

Lattice discretization of gauge theory (K. Wilson) gauge invariant

$$A \xrightarrow{g} A_g$$

Same physics

$$F_\mu \rightarrow g F_\mu g^{-1}$$

U(1) : QED; Gauge Fixing

$g \in G = \prod_{x \in \Lambda} G$

$$A = \{A_\mu^a(x), x \in \Lambda, \mu=1,2,3, a=1,2\}$$

Classical & Quantum



$$\mathcal{A} = \{A\}$$

All gauge configurations

$$\mathcal{G} = \{g\}$$

All gauge transformations

orbit of A by the action of \mathcal{G}

$$\mathcal{E} = \text{Space of physically (inequivalent) configurations} = \mathcal{A}/\mathcal{G}$$

Gauge Fixing = choosing a "slice" = choosing 1 configuration in each orbit

- * Axial Gauge $A_0^a(x) = 0$
 $x = (x^0, \vec{x})$
time space cut
- * Coulomb Gauge $\nabla^i A_i^a(x) = 0$
- * Feynman-Landau... $\partial^\mu A_\mu^a(x) = \omega^a(x) = 0$
(Lorentz covariant)
 $\omega^a(x)$ some fixed function
ordinary derivative

Gauge Fixing condition

$$F^a[A](\omega) = 0$$

all a and x

↳ Functional over A and its derivatives (local in x)

(K. Wilson) gauge invariant

* Axial Gauge $A_0^a(x) = 0$

$X = (X^0, \vec{X})$
time \uparrow space \uparrow $i=1,3$

* Coulomb Gauge

$$\nabla^i A_i^a(x) = 0$$

* Feynman-Landau... $\partial^\mu A_\mu^a(x) = \omega^a(x) = 0$
(Lorentz covariant)

$\omega^a(x)$ some fixed function
ordinary derivative

Gauge Fixing condition

$$F^a[A](x)$$

all a and x

\uparrow Function involves A and its derivatives
(local)

configuration

in each orbit

(K. Wilson) gauge invariant

* Axial Gauge $A_0^a(x) = 0$

$X = (X^0, \vec{X})$
time ↑ ↑ space $i=1,3$

* Coulomb Gauge

$$\nabla^i A_i^a(x) = 0$$

* Feynman-Landau...
(Lorentz covariant)

$$\partial^\mu A_\mu^a$$

↑
 $w^{\mu\nu}$
ordinary

Gauge Fixing condition

$$F^a[A](x) = 0 \quad \text{all } a \text{ and } x$$

↑ Functions involving A and its derivatives
(local in x)

$$\int [F^a] = \prod_{x,a} \int [F^a[A](x)]$$

configuration

in each orbit

(K. Wilson) gauge invariant

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$X = (X^0, \vec{X})$
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Gauge Fixing condition

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↑ Function involving A and its derivatives
(local in x)

$$\int [F^a] = \prod_x^a \int [F^a[A](x)]$$

BAD IDEA

configuration

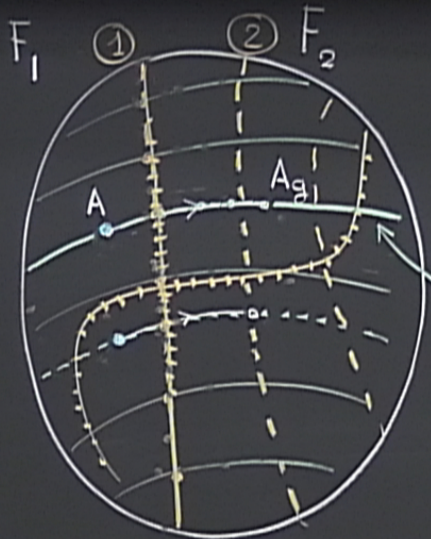
in each orbit

Faddeev-Popov + Feynman -

a and X

its derivatives

$F[A](x)$



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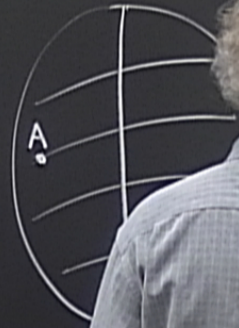
* Axial

$X = (X, \text{time})$

* Coulomb

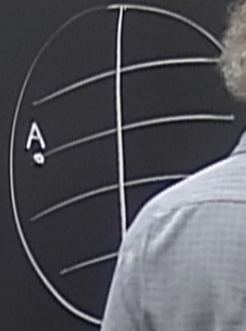
* Feynman (Lorentz)

Faddeev-Popov + Feynman-DeWitt



sea $F[A]$
ixing

Faddeev-Popov + Feynman-DeWitt



near $F[A]$

fixing
condition

on

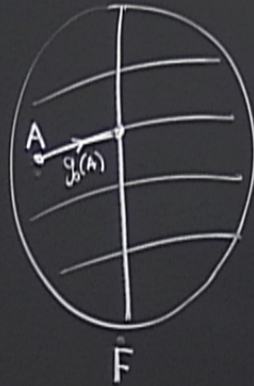
Faddeev-Popov + Feynman-DeWitt



Choose a $F[A]$
Gauge Fixing
condition

for any A , there is a $g = g_0[A]$
such that $F[A_{g_0[A]}] = 0$

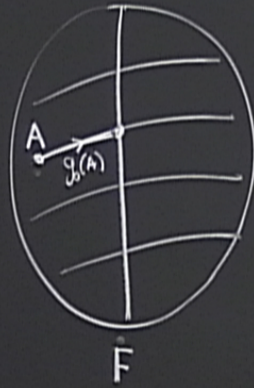
Faddeev-Popov + Feynman-DeWitt



Choose a $F[A]$
Gauge Fixing
condition

for any A , there is a $g(h)$
such that $F[A + g(h)] = 0$

Faddeev-Popov + Feynman-DeWitt



Choose a $F[A]$
Gauge Fixing
condition

for any A , there is a g (only one)
such that $F[A_g] = 0$

Faddeev-Popov + Feynman-DeWitt



Choose a $F[A]$
Gauge Fixing
condition

for any A , there is a g (only one)
such that $F[A_g] = 0$

let me call this $g = g(A)$

Faddeev-Popov + Feynman-DeWitt

Choose a $F[A]$
Gauge Fixing
condition

for any A , there is a g (only one)

such that $F[A_g] = 0$

let me call this g ; $g_0[A]$

$$\int_A D[A]$$

Faddeev-Popov + Feynman-DeWitt



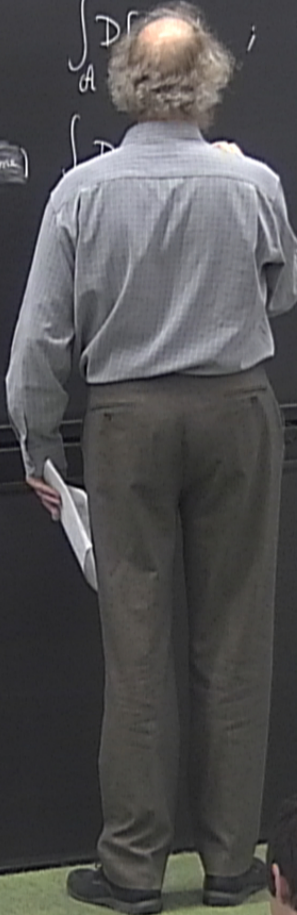
choose a $F[A]$
Gauge Fixing
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for any A , there is a g (only one)

such that $F[A_g] = 0$

let me call this g ; $g_0[A]$

$$\int_A Dg ; 1 = \int_g D[g] \delta[g - g_0[A]] \text{ for any } g_0[A]$$



Faddeev-Popov + Feynman-DeWitt



choose a $F[A]$
Gauge Fixing
condition

for any A , there is a g (only one)

such that $F[A, g] = 0$

let me call this g ; $g_0[A]$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1 \quad ; \quad (\mathcal{D}[g] \delta[g - g_0[A]] \text{ for any } g_0[A])$$

$$\int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{G}} \mathcal{D}[g]$$

Faddeev-Popov + Feynman-DeWitt



choose a $F[A]$
Gauge Fixing
condition

for any A , there is a g (unique)

such that $F[A, g] = 0$

let me call this g ; $g[A]$

$$D[A] \equiv 1 ; 1 = \int_{\mathcal{G}} D[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$D[A] \equiv \int_{\mathcal{G}} D[g] \delta[g - g_0[A]]$$

Faddeev-Popov + Feynman-DeWitt

choose a $F[A]$

Gauge Fixing
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for any A , there is a g (only one)

such that $F[A_g] = 0$

let me call this g ; $g_0[A]$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1 ; 1 = \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$= \int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]]$$

Faddeev-Popov + Feynman-DeWitt



choose a $F[A]$

Gauge Fixing
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for any A , there is a g (only one)

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call this g ; $g_0[A]$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1 ; 1 = \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

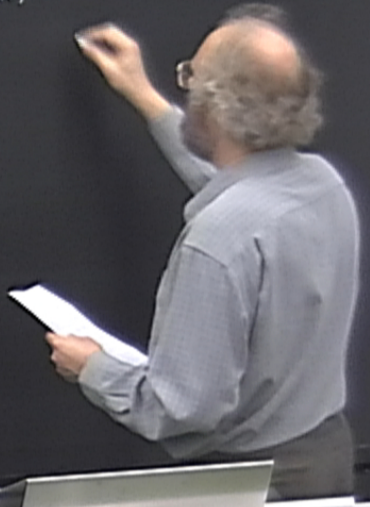
$$= \int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]]$$

$$\int_{\mathcal{A}} \mathcal{D}[A] \, 1 \quad ; \quad 1 = \int_{\mathcal{g}} \mathcal{D}[g] \, \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$= \int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{g}} \mathcal{D}[g] \, \delta[g - g_0[A]]$$

$g = \text{(only one)}$
 $g = 0$
 $g[A]$

$x \in \mathbb{R} \quad f(x)$

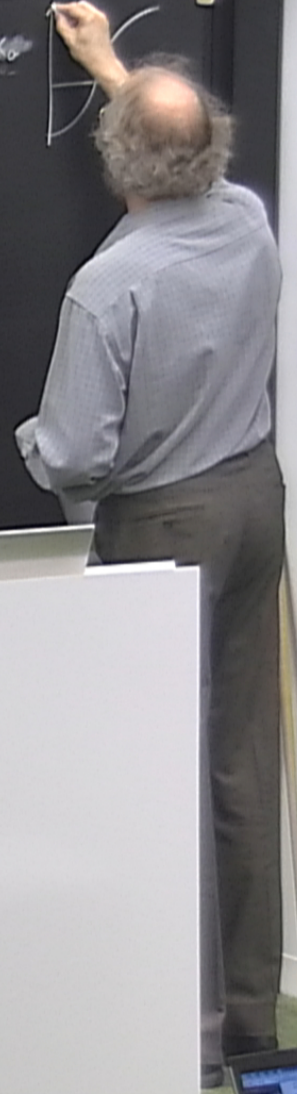


$$\int_A \mathcal{D}[A] 1 ; 1 = \int_g \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$= \int_A \mathcal{D}[A] \int_g \mathcal{D}[g] \delta[g - g_0[A]]$$

$g = (\text{only one})$
 $g = 0$
 $g[A]$

$$x \in \mathbb{R} \quad f(x) = 0 \Rightarrow x = x_0$$



Faddeev-Popov + Feynman-DeWitt



choose $F[A]$
 Gauge fixing
 condition
 for any A , there is a
 such that
 let me call
 $g_0(A)$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1, \quad 1 = \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$= \int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]]$$

$x \in \mathbb{R} \quad f(x) = 0 \Rightarrow x = x_0$

$\delta(x - x_0)$

Faddeev-Popov + Feynman-DeWitt



choose $F[A]$

for any $g_0[A]$, there is a $g = f(g_0)$ such that $F[A, g] = 0$.
 All this $g, g_0[A]$
 g such that $F(A, g) = 0$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1 \quad ; \quad 1 = \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$= \int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]]$$

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$\delta(x - x_0)$

Faddeev-Popov + Feynman-DeWitt



choose a $F[A]$
Gauge Fixing
condition

for any A , there is a g such that $F[A, g] = 0$
let me call this g , $g_0[A]$
 $g_0(A) = g$ such that $F(A, g) = 0$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1 \quad ; \quad 1 = \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$= \int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]]$$

$x \in \mathbb{R} \quad f(x) = 0 \Rightarrow x = x_0$

$$\delta(x - x_0) = \delta(f(x))$$

Faddeev-Popov + Feynman-DeWitt



choose $F[A]$
Gauge Fixing
condition

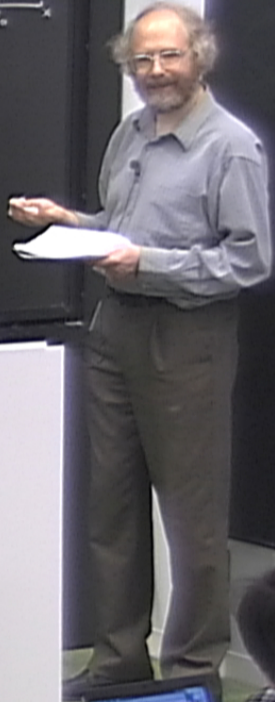
for any A , there is a g such that $F[A, g] = 0$
let me call this g , $g_0[A]$
 $g_0(A) = g$ such that $F(A, g_0) = 0$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1 \quad ; \quad 1 = \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

$$= \int_{\mathcal{A}} \mathcal{D}[A] \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]]$$

$x \in \mathbb{R} \quad f(x) = 0 \Rightarrow x = x_0$

$$\delta(x - x_0) = \delta(f(x)) \left| \frac{df}{dx} \right|$$



Faddeev-Popov + Feynman-DeWitt



choose a $F[A]$
Gauge Fixing
condition

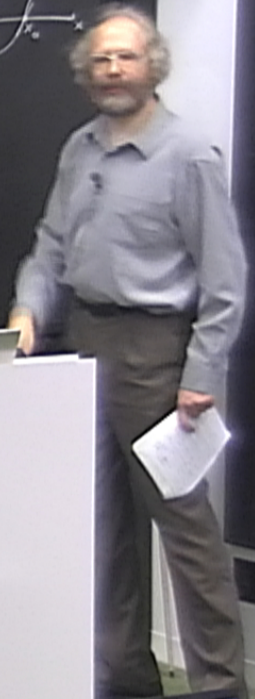
for any A , there is a g such that $F[A_g] = 0$
let me call this g , $g_0[A]$
 $g_0(A) = g$ such that $F(A_{g_0}) = 0$

$$\int_{\mathcal{A}} \mathcal{D}[A] 1 \quad ; \quad 1 = \int_{\mathcal{G}} \mathcal{D}[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

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$\delta(x - x_0) = \delta(f(x)) \left| \frac{df}{dx} \right|$
 $\delta(ax) = \frac{1}{|a|} \delta(x) \quad a = \text{constant}$



Faddeev-Popov + Feynman-DeWitt



Choose a $F[A]$
Gauge Fixing
condition
for any A , there is a $g = g_0[A]$
such that $F[A, g_0] = 0$
let me call this g_0 ; $g_0[A]$
 $g_0(A) \neq g$ such that $F(A, g_0) = 0$

$$\int_A D[A] 1 ; 1 = \int_g D[g] \delta[g - g_0[A]] \quad \text{for any } g_0[A]$$

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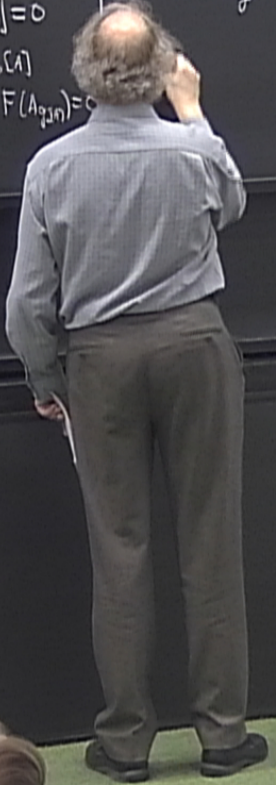
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$\vec{x} = (x_1, \dots, x_n) \in \mathbb{R}^n \quad \{f_i(\vec{x}), i=1, \dots, n\} \quad N \text{ constraints}$

$$\delta(\vec{x}) = 0$$

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Faddeev-Popov + Feynman-DeWitt



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$$= \int_A \mathcal{D}[A] \int_g \mathcal{D}[g] \delta[F[A_g]] \cdot \left| \frac{F[A_g]}{g} \right|$$

$x \in \mathbb{R} \quad f(x) = 0 \Rightarrow x = x_0$

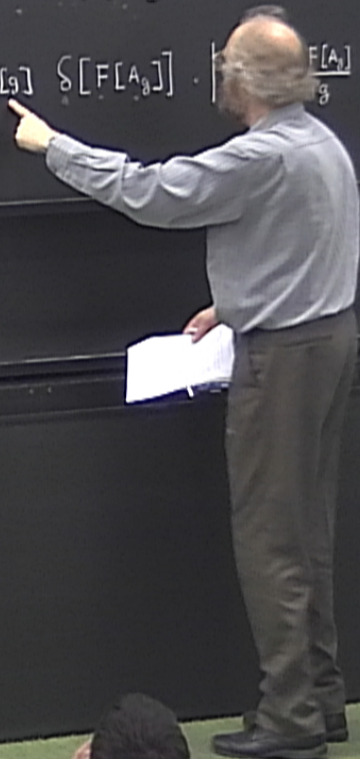
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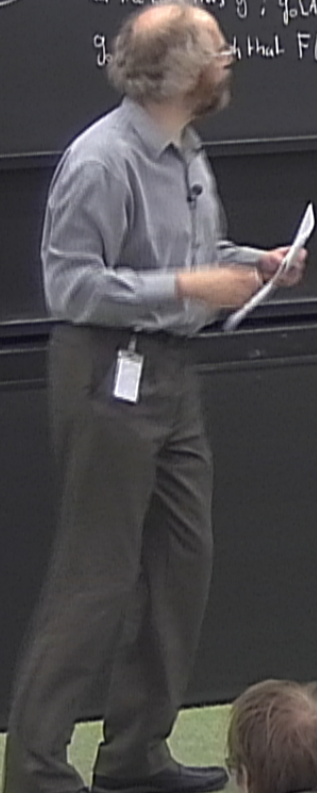
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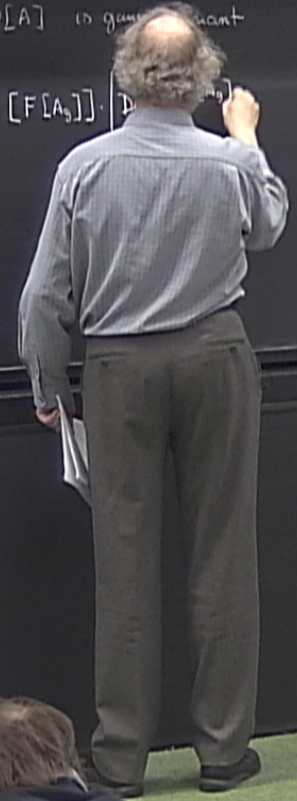
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$\bar{x} = (x_1, \dots, x_n) \in \mathbb{R}^n$ $\bar{y} = (y_1, \dots, y_m)$ No constraints
 $\bar{y}(\bar{x}) = 0$
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Use the fact that $\int_A D[A]$ is gauge invariant

$$\int_g D[g] \int_A D[A] \delta[F[A, g]] \cdot \left| \text{Det} \left[\frac{\partial F[A, g]}{\partial g} \right] \right|$$





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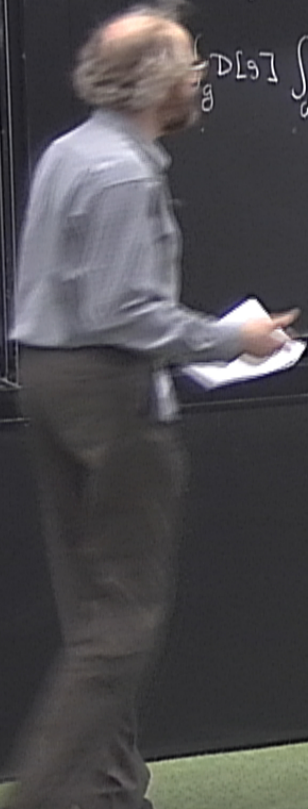
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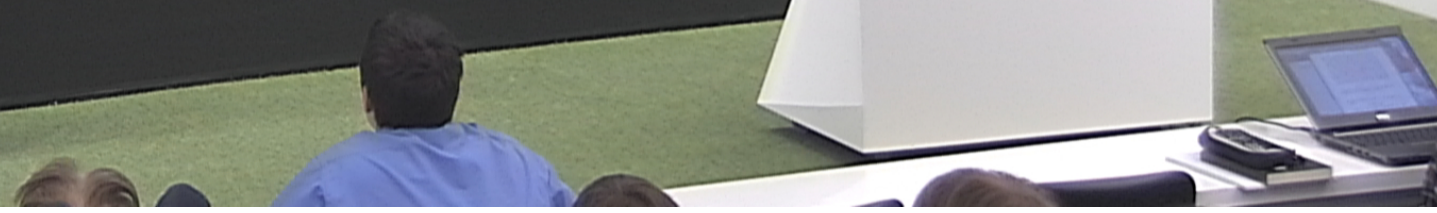
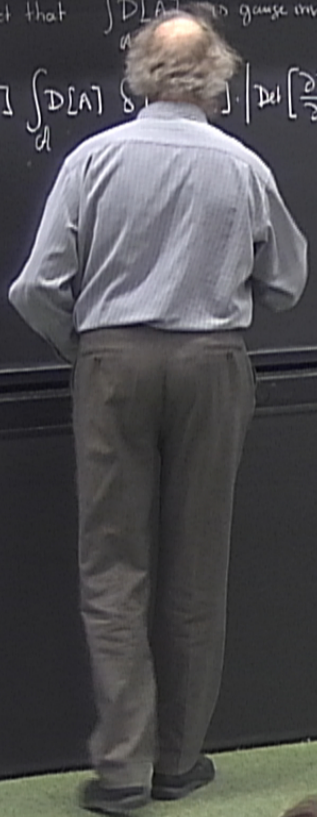
$$= \int_A D[A] \int_g D[g] \delta[g - g_0[A]]$$

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Use the fact that $\int_A D[A]$ is gauge invariant and $\exp(i S[A])$ also

$$\int_g D[g] \int_d D[A] \delta[F[A_g]] \cdot \left| \text{Det} \left[\frac{\partial F[A_g]}{\partial g} \right] \right|$$





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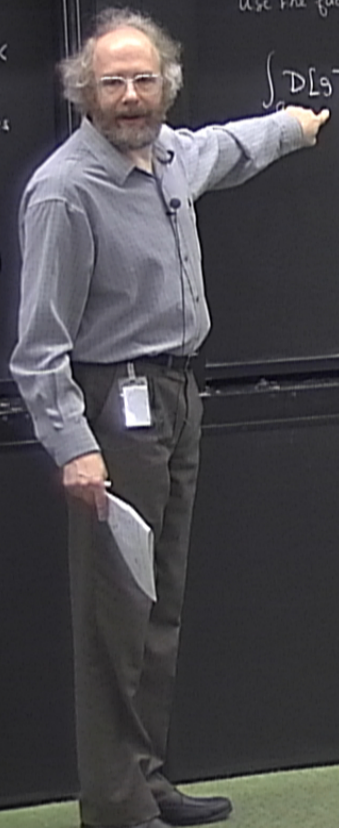
$$= \int_A D[A] \int_g D[g] \delta[F[A_g]] \cdot \left| \text{Det} \left[\frac{\partial F[A_g]}{\partial g} \right] \right|$$

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$$\int D[g] \int D[A] \delta[F[A_g]] \cdot \left| \text{Det} \left[\frac{\partial F[A_g]}{\partial g} \right] \right|$$

change of variable where $A \rightarrow A' = A_g$





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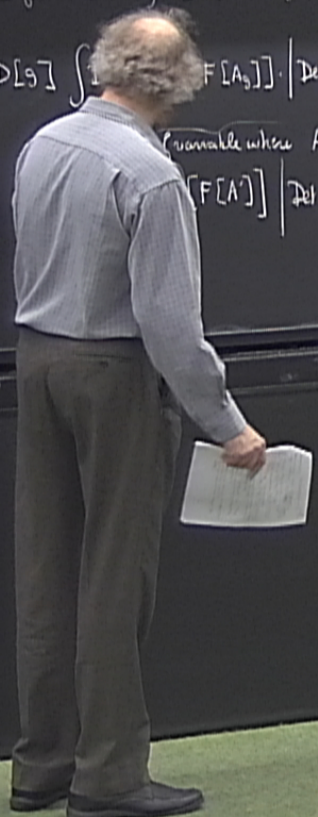
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Use the fact that $D[A]$ is gauge invariant and $\exp(i S[A])$ also

$$\int_g D[g] \int_A D[A] \delta[F[A_g]] \cdot \left| \text{Det} \left[\frac{\partial F[A_g]}{\partial g} \right] \right|$$

invariant when $A \rightarrow A' = A_g$

$$\int D[A] \delta[F[A]] \cdot \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right|$$





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$$\int_g D[g] \int_A D[A] \delta[F[A_g]] \cdot \left| \text{Det} \left[\frac{\partial F[A_g]}{\partial g} \right] \right|$$

change of variable where $A \rightarrow A' = A_g$

$$\int_g D[g] \times \int_{A'} D[A'] \delta[F[A']] \cdot \left| \text{Det} \left[\frac{\partial F[A_g]}{\partial h} \right] \right|$$



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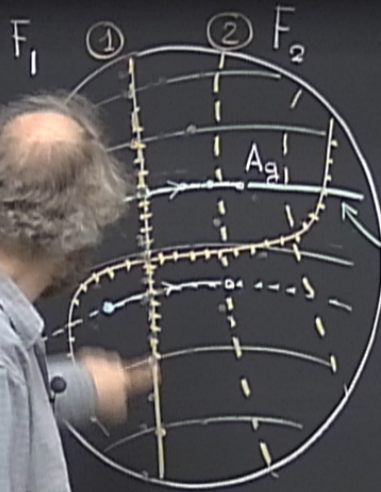
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change of variable where $A \rightarrow A' = A_g$

$$\int_g D[g] \times \int_A D[A] \delta[F[A]] \cdot \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right|$$

$S = \int \dots$
 Hodge dual



$\mathcal{A} = \{A\}$ All gauge configurations

$\mathcal{G} = \{g\}$ All gauge transformations

orbit of A by the action of \mathcal{G}

$\mathcal{C} =$ space of physical (ly inequivalent) configuration
 $= \mathcal{A}/\mathcal{G}$

Gauge Fixing = choosing a "slice" = choosing 1 configuration in each orbit
 ①

- * Axial
- $X = (X, \text{time})$
- * Coulomb
- * Feynman (Lorentz)



Use the fact that $\int_{\mathcal{A}} \mathcal{D}[A]$ is gauge invariant and $\exp(iS[A])$ also

$$\int_{\mathcal{g}} \mathcal{D}[g] \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A_g]] \cdot \left| \text{Det} \left[\frac{\partial F[A_h]}{\partial h} \right] \right|$$

change of variable where $A \rightarrow A' = A_g$

$$\int_{\mathcal{g}} \mathcal{D}[g] \times \int_{\mathcal{A}} \mathcal{D}[A'] \delta[F[A']] \left| \text{Det} \left[\frac{\partial F[A'_h]}{\partial h} \right] \right| = \text{Vol}(\mathcal{g})$$

Use the fact that $\int_{\mathcal{A}} \mathcal{D}[A]$ is gauge invariant and $\exp(iS[A])$ also

$$\int_{\mathcal{g}} \mathcal{D}[g] \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A_g]] \cdot \left| \text{Det} \left[\frac{\partial F[A_h]}{\partial h} \right] \right|$$

change of variable where $A \rightarrow A' = A_g$

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$\mu=1, \dots, a, n$

Quantum

a and X

derivatives

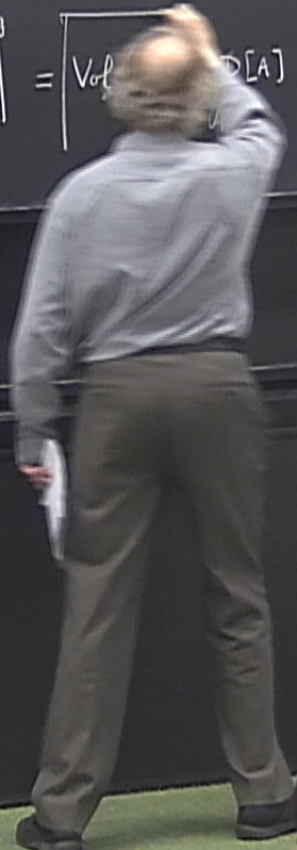
$[A](x)$

Use the fact that $\int_{\mathcal{A}} \mathcal{D}[A]$ is gauge invariant and $\exp(iS[A])$ also

$$\int_{\mathcal{G}} \mathcal{D}[\xi] \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A_\mu]] \left| \text{Det} \left[\frac{\partial F[A_\mu]}{\partial h} \right] \right|$$

change of variable where $A \rightarrow A' = A_\xi$

$$\int_{\mathcal{G}} \mathcal{D}[\xi] \times \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A']] \left| \text{Det} \left[\frac{\partial F[A_\mu]}{\partial h} \right] \right| = \text{Vol}(\mathcal{U}) \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A_\mu]}{\partial h} \right] \right| \exp(iS[A])$$



$\mu=1, \dots, n$

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change of variable where $A \rightarrow A' = A_\mu \xi$

$$\int_{\mathcal{G}} \mathcal{D}[\xi] \times \int_A \mathcal{D}[A] \delta[F[A']] \left| \text{Det} \left[\frac{\partial F[A_\mu]}{\partial A_\mu} \right] \right|$$

$$= \text{Vol}_{\mathcal{G}}(\mathcal{G}) \int_A \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A_\mu]}{\partial A_\mu} \right] \right| \exp(iS[A])$$

Functional Integral for A



$\mu=1, \dots, n$

Quantum

a and X

derivatives

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Use the fact that $\int_{\mathcal{A}} \mathcal{D}[A]$ is gauge invariant and $\exp(iS[A])$ also

$$\int_{\mathcal{G}} \mathcal{D}[\xi] \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A_\mu]] \left| \text{Det} \left[\frac{\partial F[A_\mu]}{\partial A_\mu} \right] \right|$$

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$$\int_{\mathcal{G}} \mathcal{D}[\xi] \times \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A']] \left| \text{Det} \left[\frac{\partial F[A_\mu]}{\partial A_\mu} \right] \right|$$

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Integral for field, using a given



$\mu=1, 4; a=1, n$

Quantum

a and X

derivatives

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Use the fact that $\int_{\mathcal{A}} \mathcal{D}[A]$ is gauge invariant and $\exp(iS[A])$ also

$$\int_{\mathcal{G}} \mathcal{D}[\xi] \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A_\xi]] \left| \text{Det} \left[\frac{\partial F[A_\xi]}{\partial h} \right] \right|$$

change of variable where $A \rightarrow A_\xi$

$$\int_{\mathcal{G}} \mathcal{D}[\xi] \times \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right|$$

$$= \text{Vol}_{\mathcal{G}}(\mathcal{G}) \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right| \exp(iS[A])$$

Functional Integral for a gauge Field, using a given gauge Fixing condition $F[A]=0$

d exterior derivative

Lattice discretization of Gauge theory
(K. Wilson) gauge invariant



gauge configurations
gauge transformations

g
cal(ly inequivalent) configuration

choosing 1 configuration in each orbit

* Axial Gauge $A_0^a(x) = 0$

$X = (x^0, \vec{x})$
time ↑
space ↓

* Coulomb $A_i^a(x) = 0$

* Feynman $A_\mu^a(x) = \omega^a(x) = 0$
(Lorenz)

$\omega^a(x)$ some fixed function
any derivative

Gauge Fixing condition

$$F^a[A](x) = 0 \quad \text{all } a \text{ and } x$$

↑ Function involving A and its derivatives
(local in x)

$$\mathcal{D}[F^a] = \prod_x \prod_a \mathcal{D}[F^a(x)]$$

BAD IDEA

d exterior derivative

Lattice discretization of Gauge theory
(K. Wilson) gauge invariant



finite number of points

gauge configurations
gauge transformations

g
cal(ly inequivalent) configuration

choosing 1 configuration in each orbit

* Axial Gauge

$$X = (X^0, \vec{X})$$

time ↑ space = 1,2,3

* Coulomb Gauge

* Feynman-Landau
(Lorentz covariant)

$$A_0^a(x) = 0$$

$$A(x) = 0$$

fixed function

Gauge Fixing condition

$$F^a[A](x) = 0 \quad \text{all } a \text{ and } x$$

Function involving A and its derivatives
(local in x)

$$\int \mathcal{D}[F^a] = \prod_x \int \mathcal{D}[F^a(x)]$$

BAD IDEA

Use the fact that $\int_A \mathcal{D}[A]$ is gauge invariant and $\exp(iS[A])$ also

$$\int_g \mathcal{D}[g] \int_A \mathcal{D}[A] \delta[F[A_g]] \left| \text{Det} \left[\frac{\partial F[A_h]}{\partial h} \right] \right|$$

change of variable where $A \rightarrow A' = A_g$

$$\int_g \mathcal{D}[g] \times \int_A \mathcal{D}[A'] \delta[F[A']] \left| \text{Det} \left[\frac{\partial F[A'_h]}{\partial h} \right] \right|$$

Haar measure

$\text{Vol}(G)$

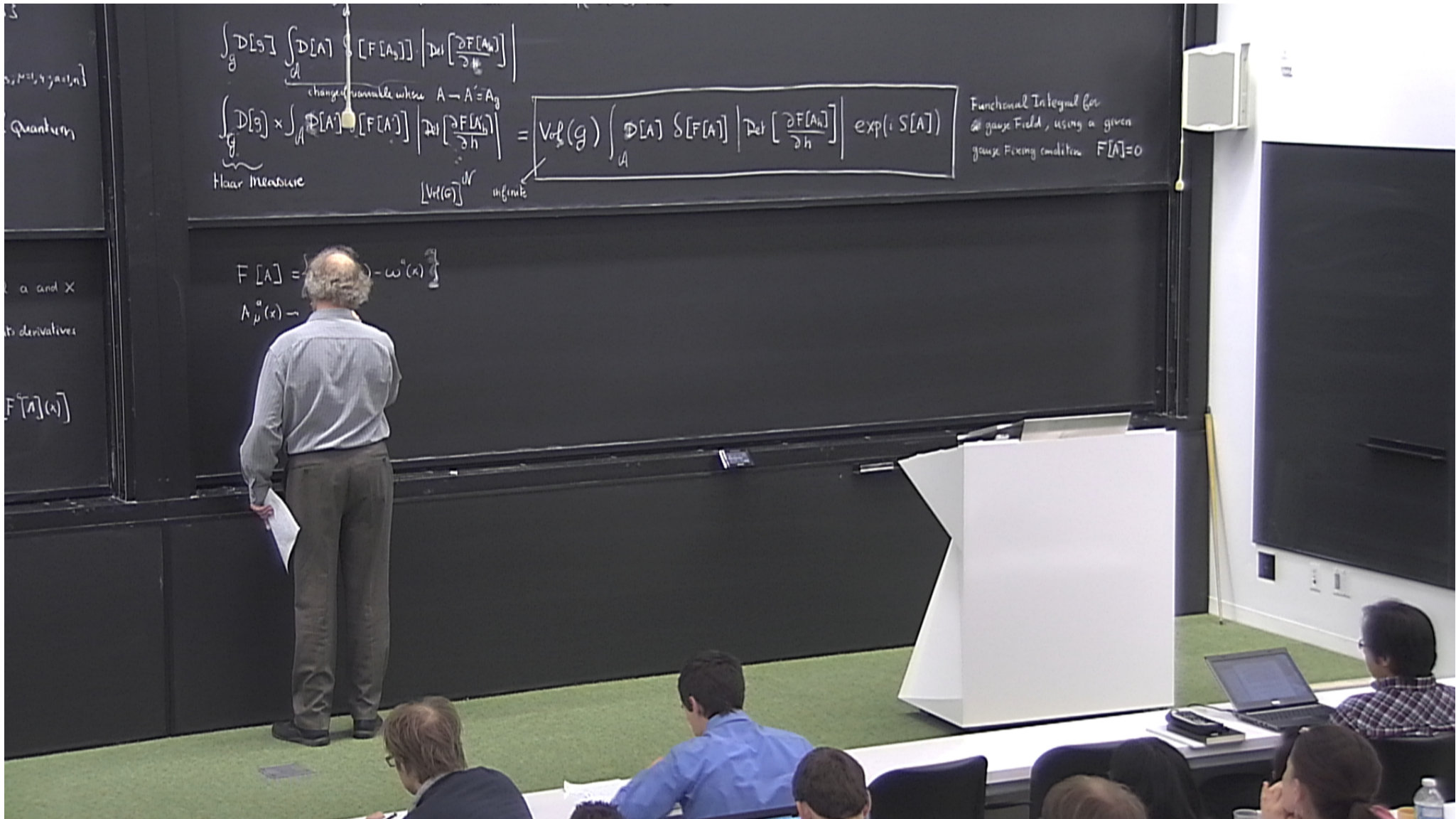
infinite

$$= \text{Vol}(g) \int_A \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A_h]}{\partial h} \right] \right| \exp(iS[A])$$

all a and x

nd





Quantum
 a and X
 derivatives
 $F[A](x)$

$$\int_{\mathcal{G}} \mathcal{D}[S] \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right|$$

change of variable when $A \rightarrow A' = A_g$

$$\int_{\mathcal{G}} \mathcal{D}[S] \times \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right| = \text{Vol}_{\mathcal{G}}(\mathcal{G}) \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right| \exp(i S[A])$$

Haar Measure $[\text{Vol}(\mathcal{G})]^{dV}$ infinite

Functional Integral for
 gauge field, using a given
 gauge fixing condition $F[A]=0$

$$F[A] = \int \dots - \omega(x)$$

$$A_{\mu}^a(x) \rightarrow$$

$$\int_{\mathcal{G}} \mathcal{D}[S] \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right|$$

change of variables when $A \rightarrow A' = A_g$

$$\int_{\mathcal{G}} \mathcal{D}[S] \times \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right| = \text{Vol}(\mathcal{G}) \int_{\mathcal{A}} \mathcal{D}[A] \delta[F[A]] \left| \text{Det} \left[\frac{\partial F[A]}{\partial h} \right] \right| \exp(iS[A])$$

Haar Measure [Vol(G)]^{dim} infinite

Functional Integral for gauge field, using a given gauge fixing condition $F[A]=0$

$$F[A] = \left\{ \partial^\mu A_\nu - \omega^\mu(\alpha) \right\}$$

$$h = 1 + (\alpha^\mu(\alpha) t_a \dots) A$$

$F[A](x)$

Quantum

a and x

derivatives

