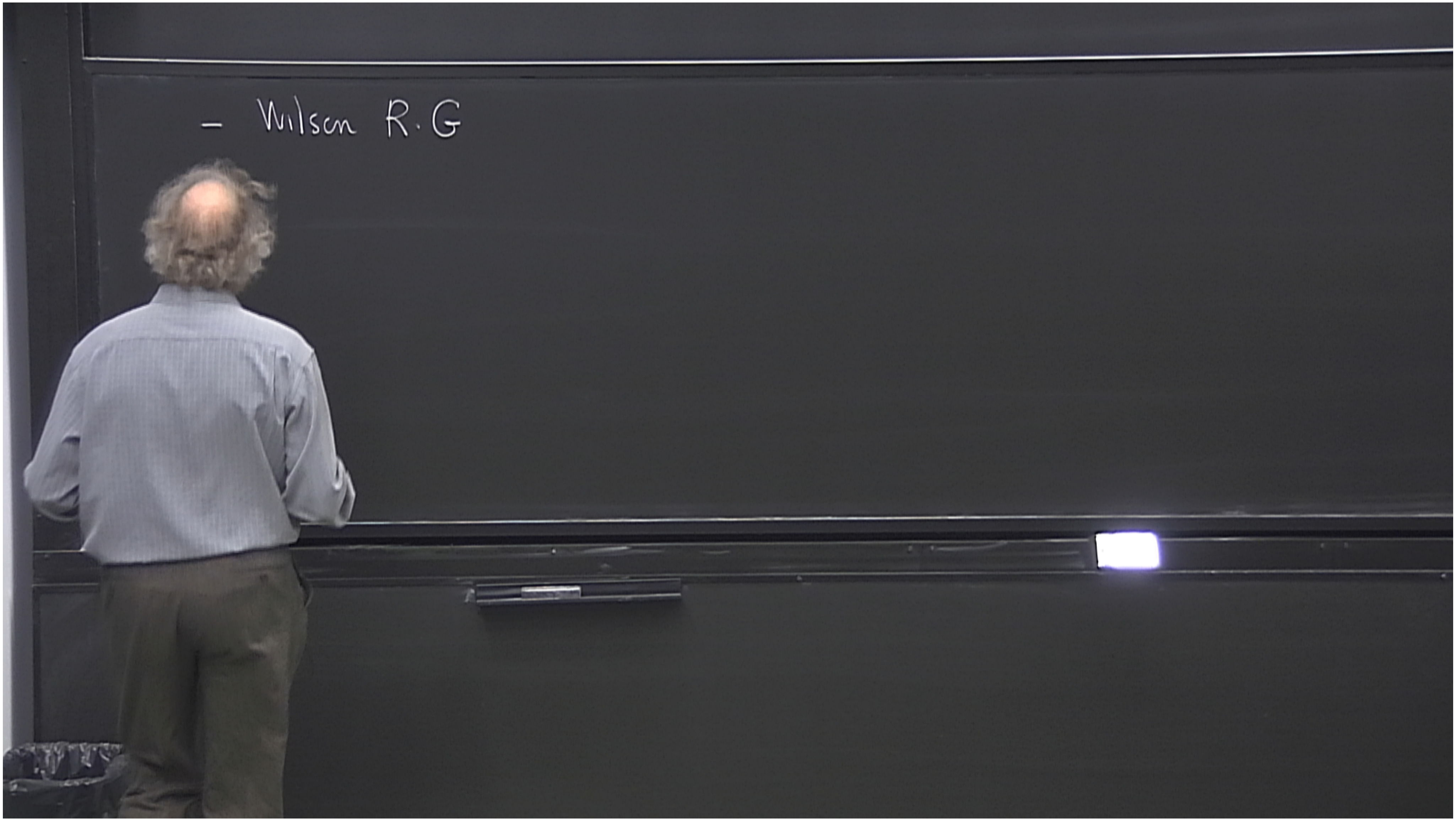
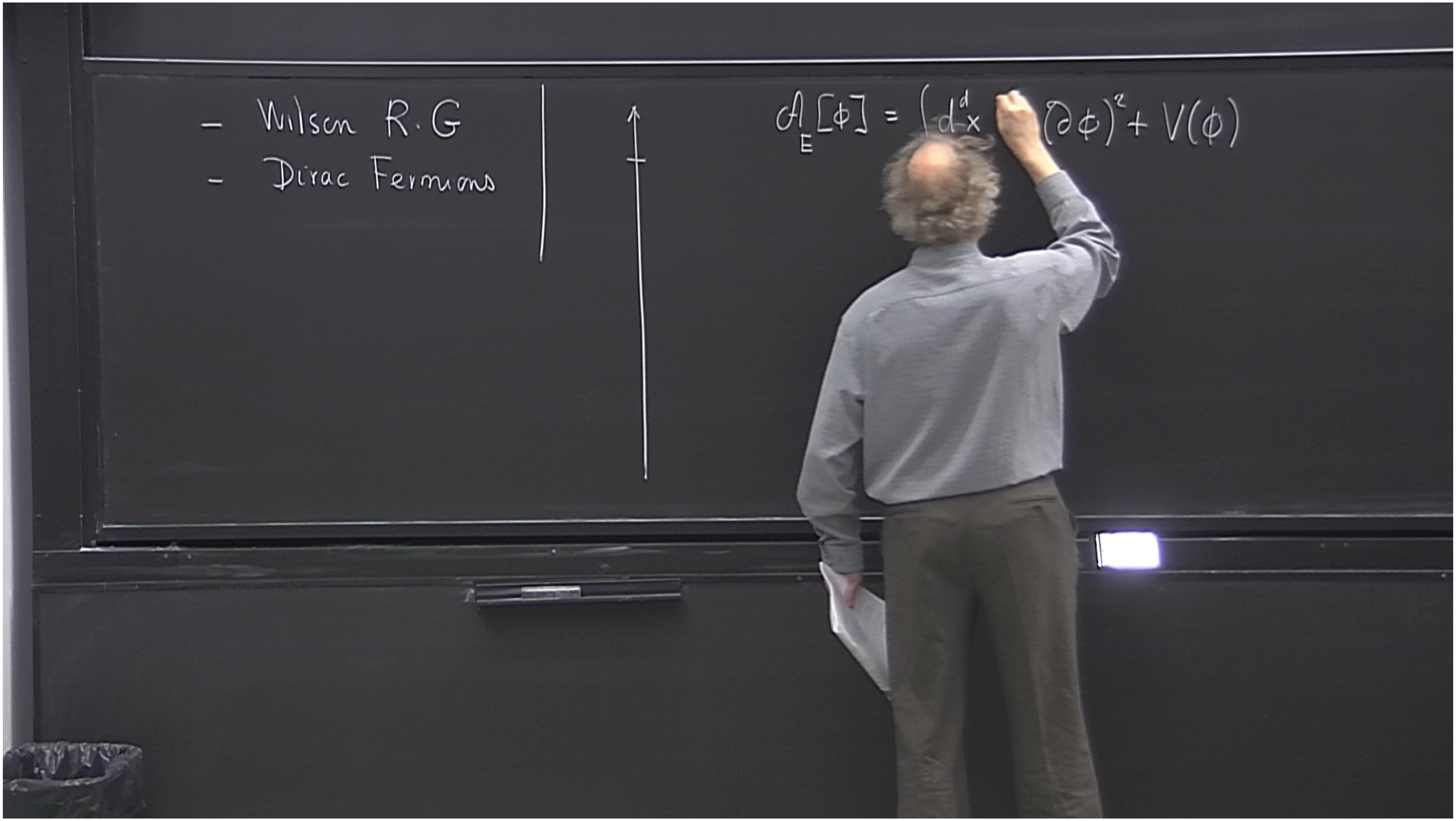
Title: Quantum Field Theory II - Lecture 10

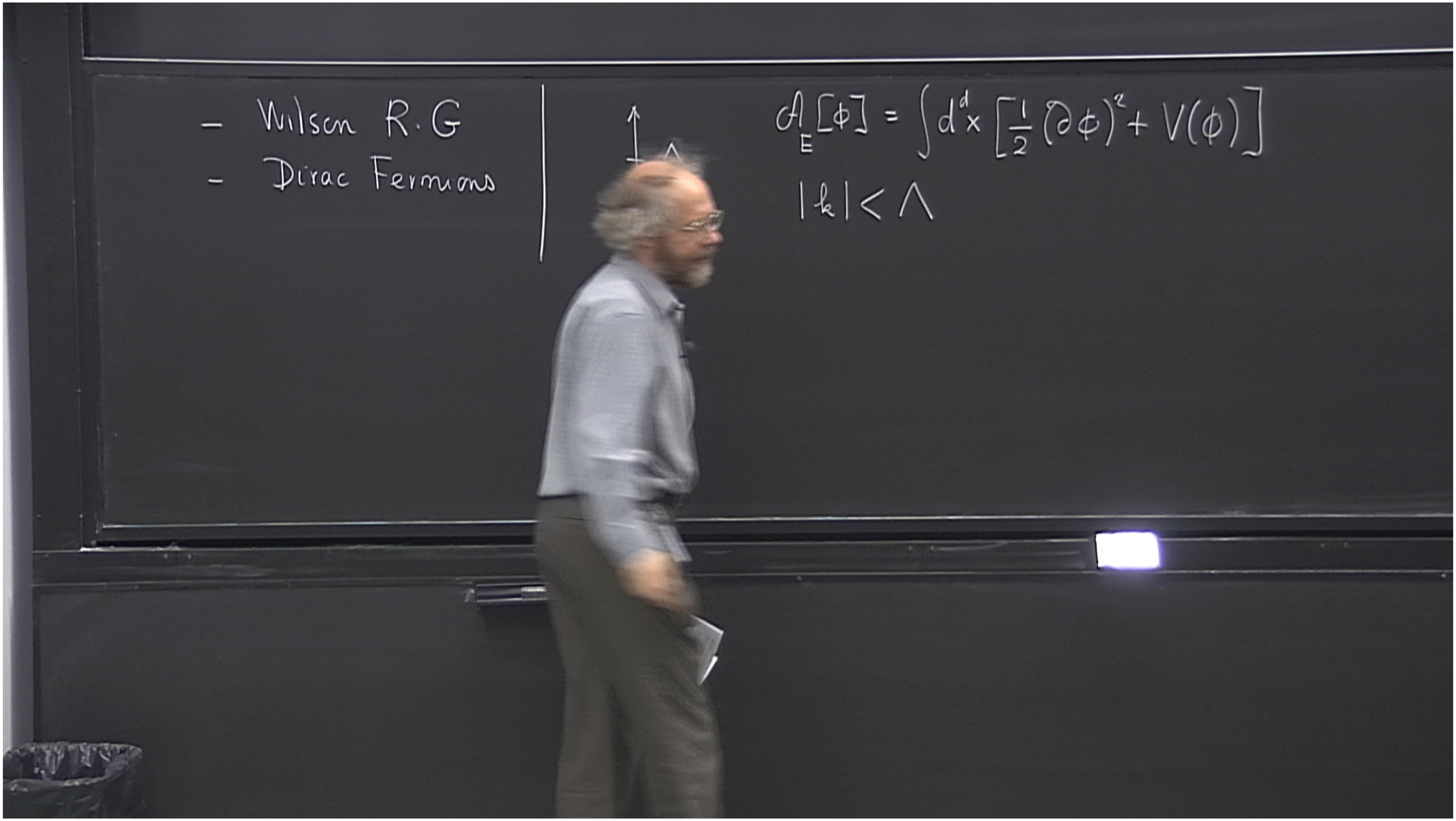
Date: Nov 11, 2011 10:30 AM

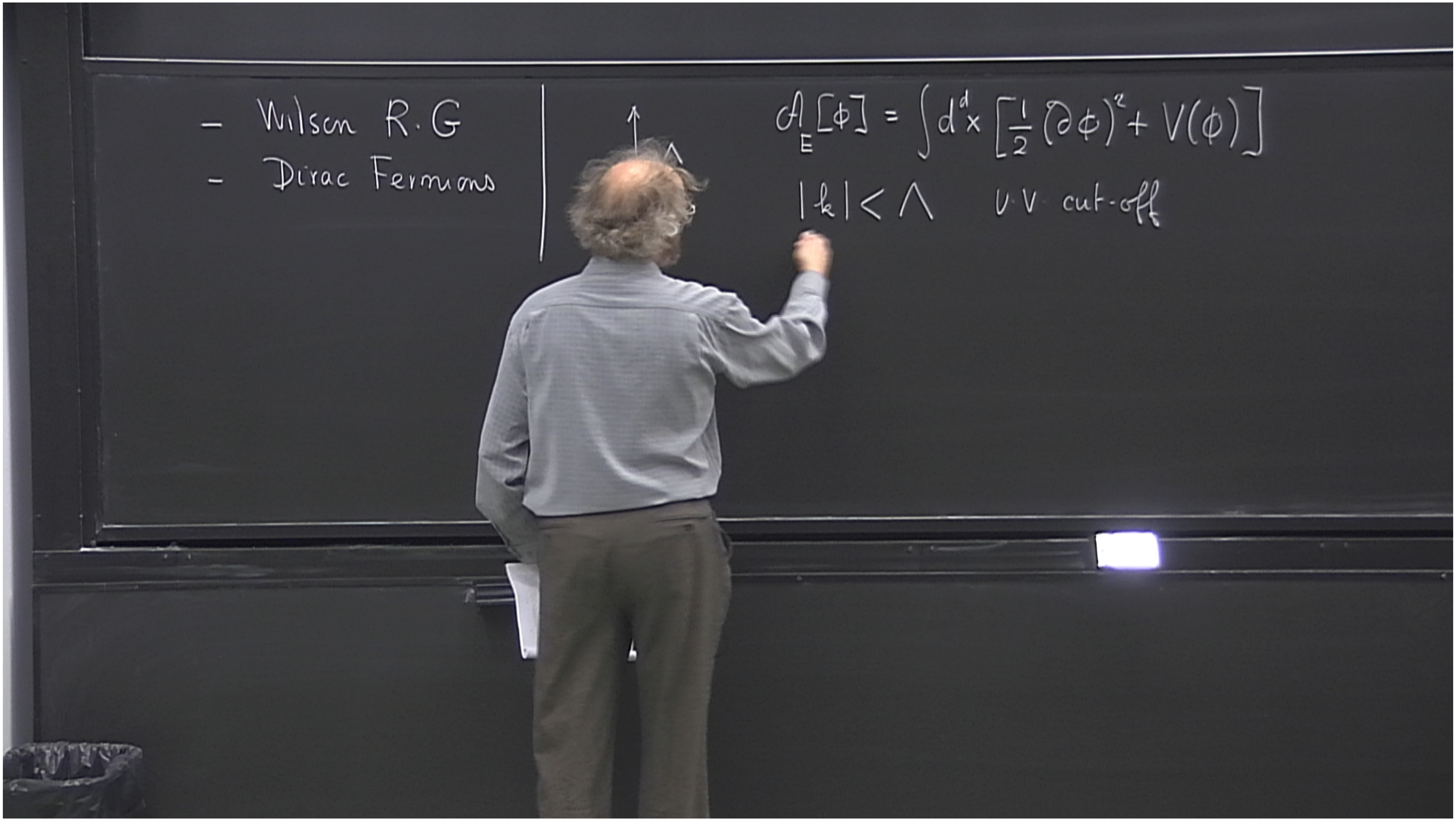
URL: http://pirsa.org/11110014

Abstract:


– Wilson R.G


- Wilson R.G
- Dirac Fermions
$S_E[\phi] = \int d^dx$
$(\partial\phi)^2 + V(\phi)$


- Wilson R.G
- Dirac Fermions
$\mathcal{A}_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial\phi)^2 + V(\phi) \right]$
$|k| < \Lambda$


- Wilson R.G
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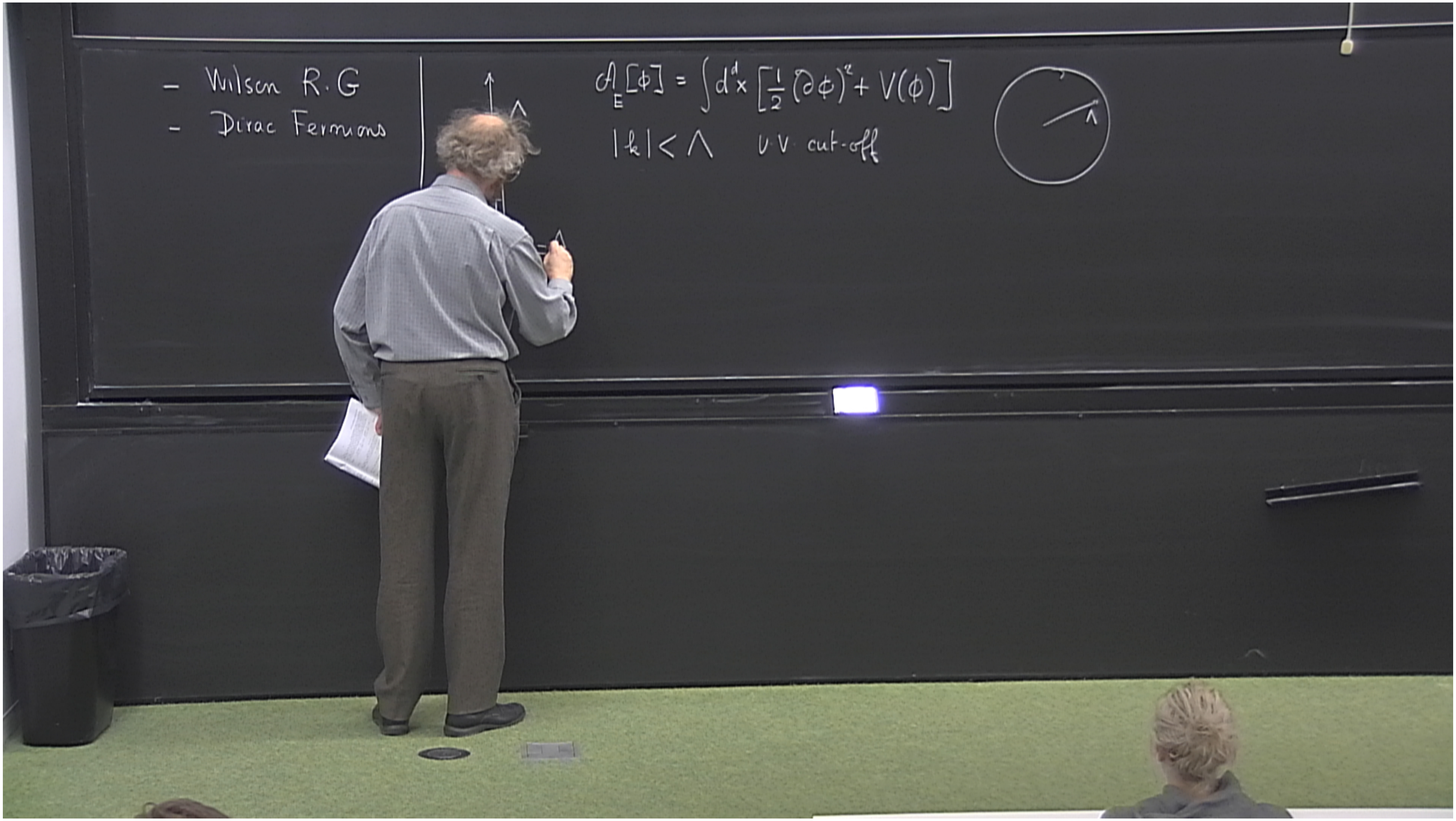
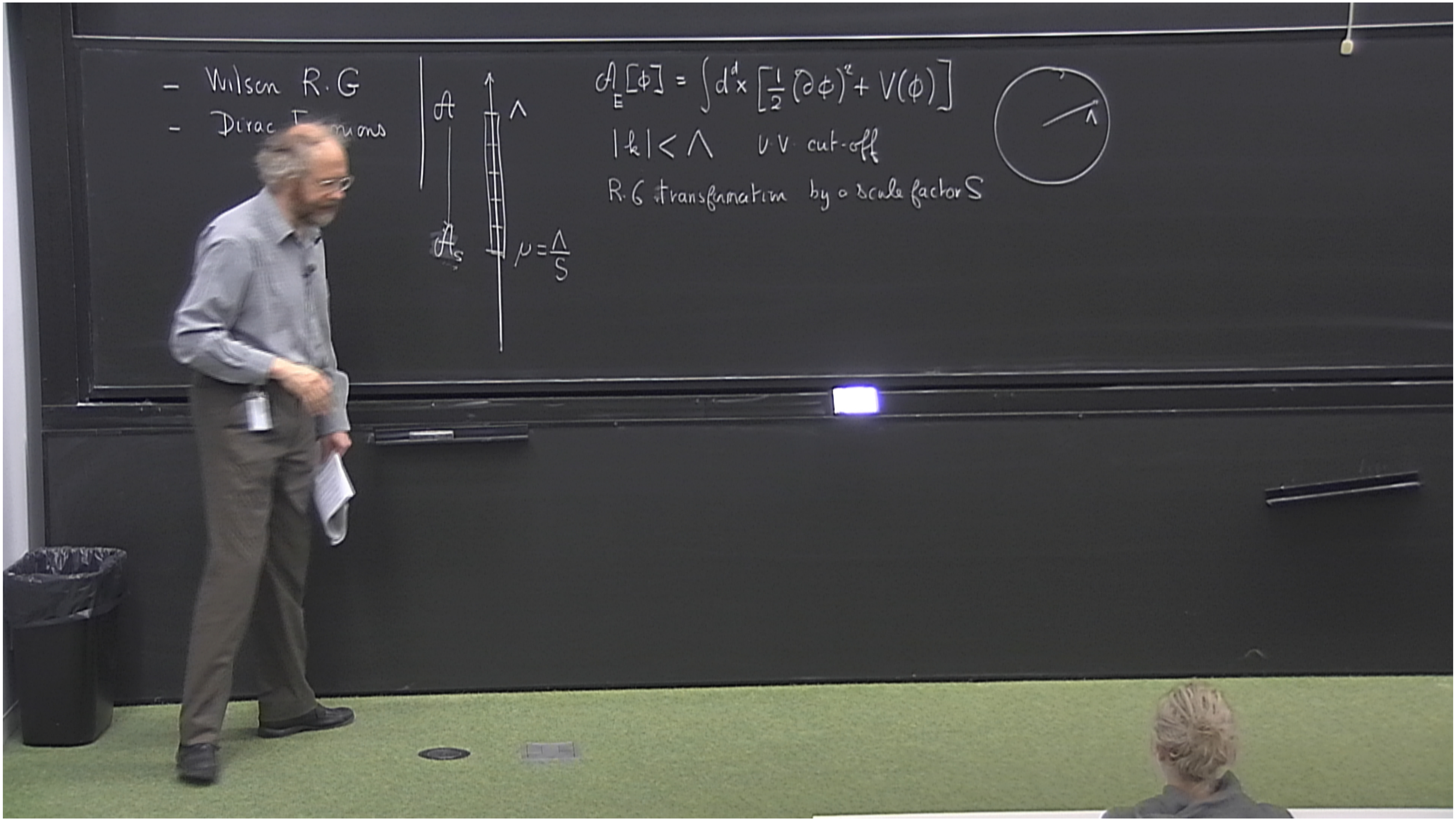
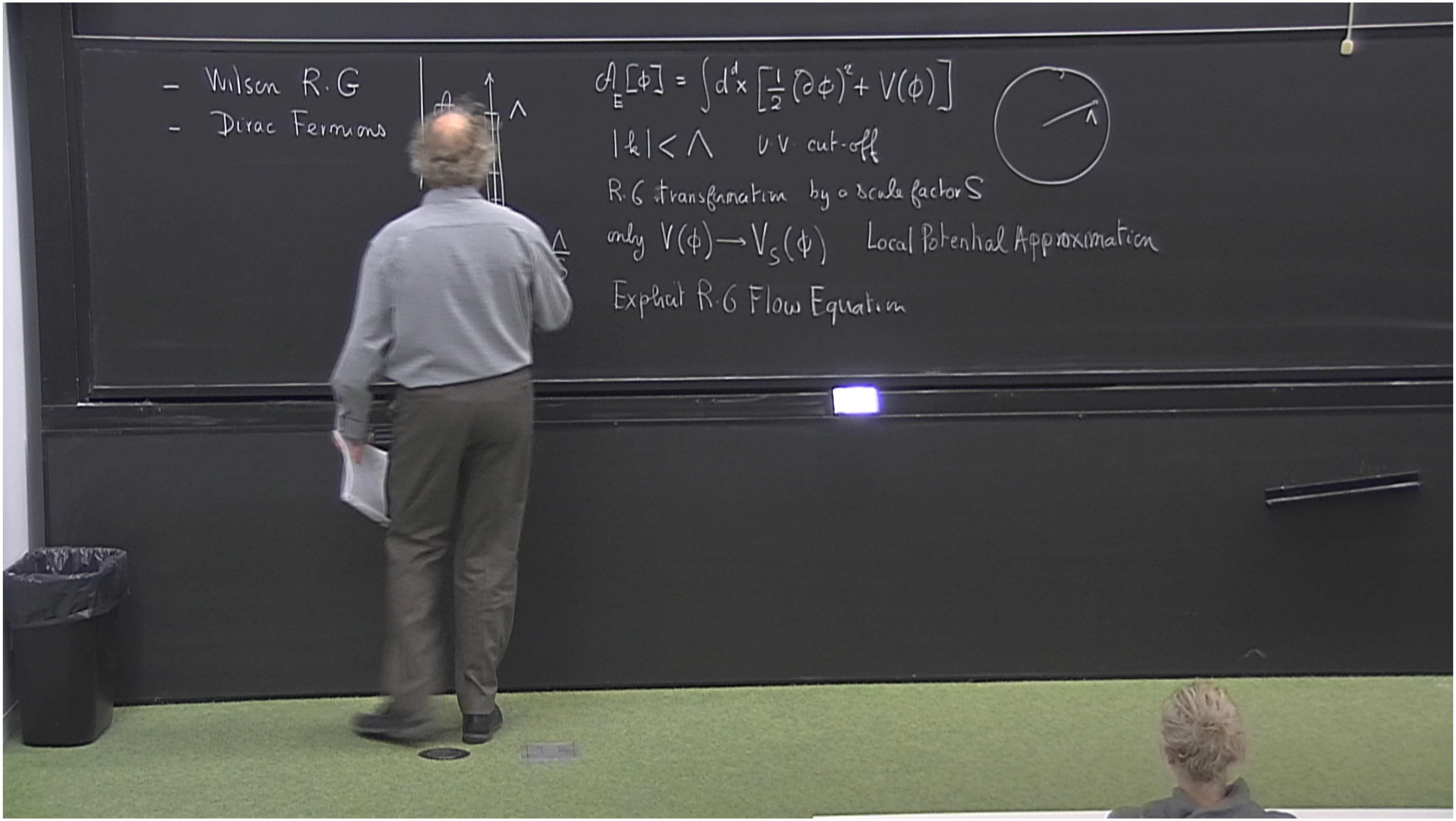
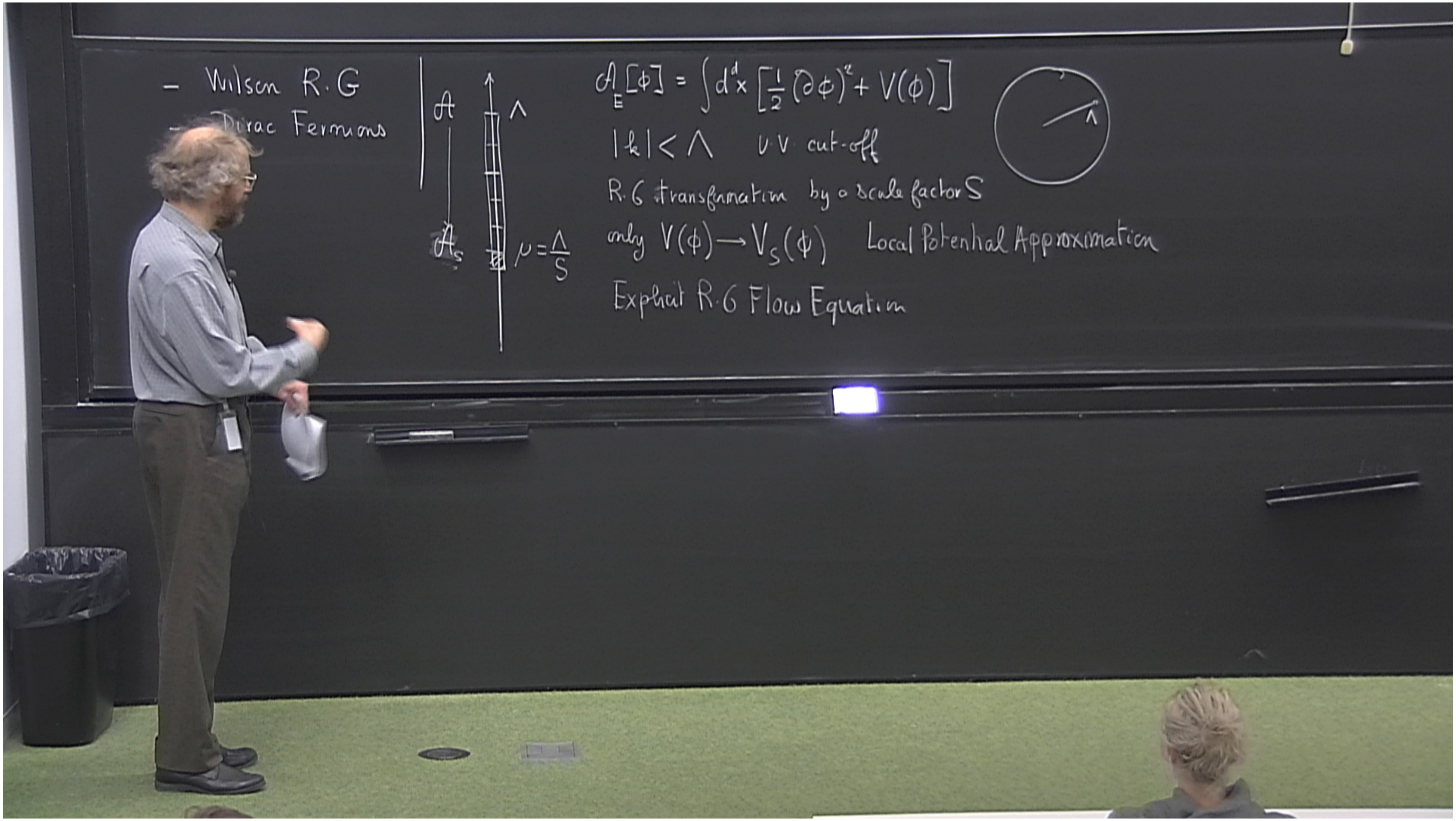
$S_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V. cut-off

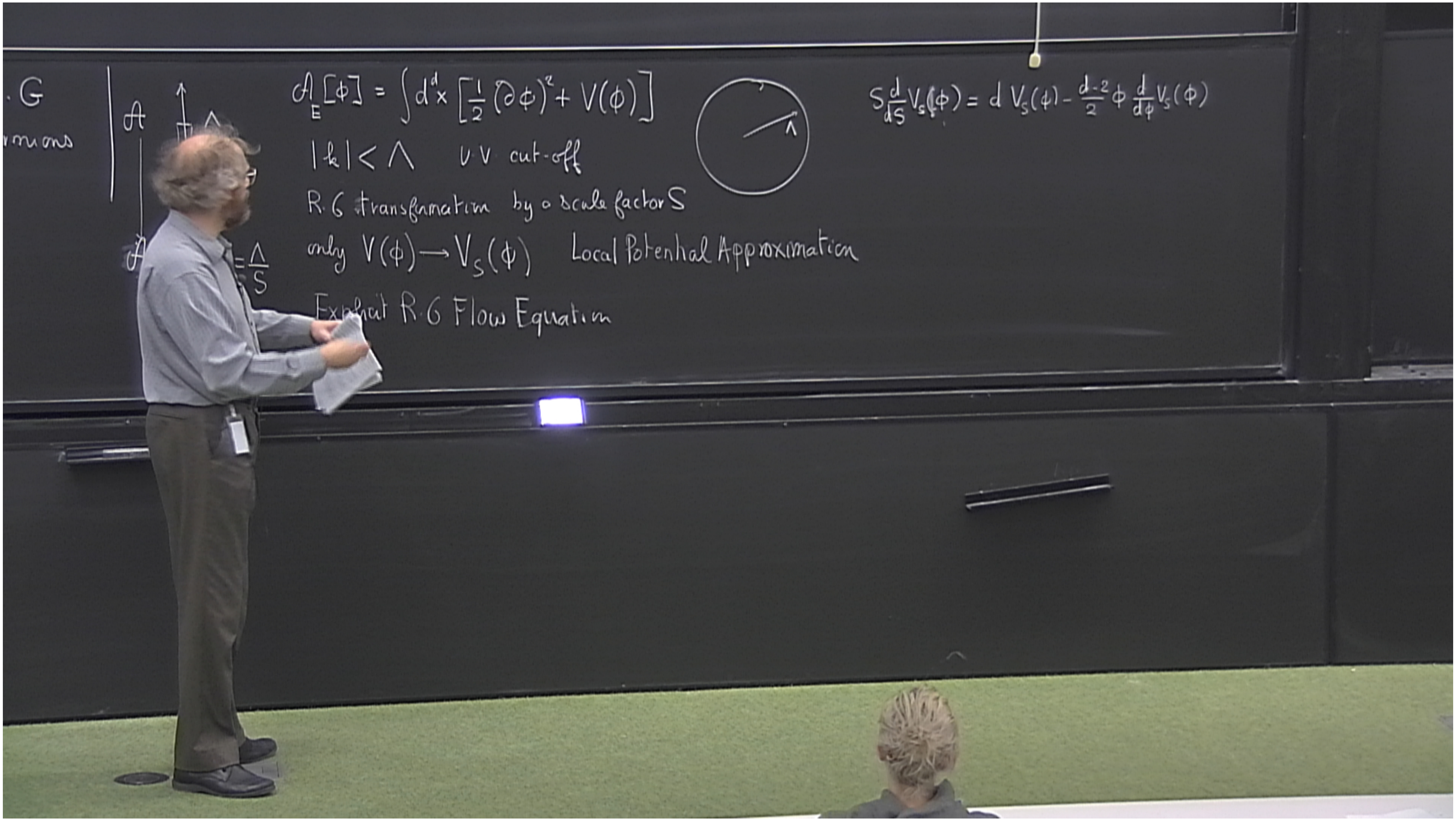


- Wilson R.G
- Dirac Fermions
$S_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V. cut-off
Λ

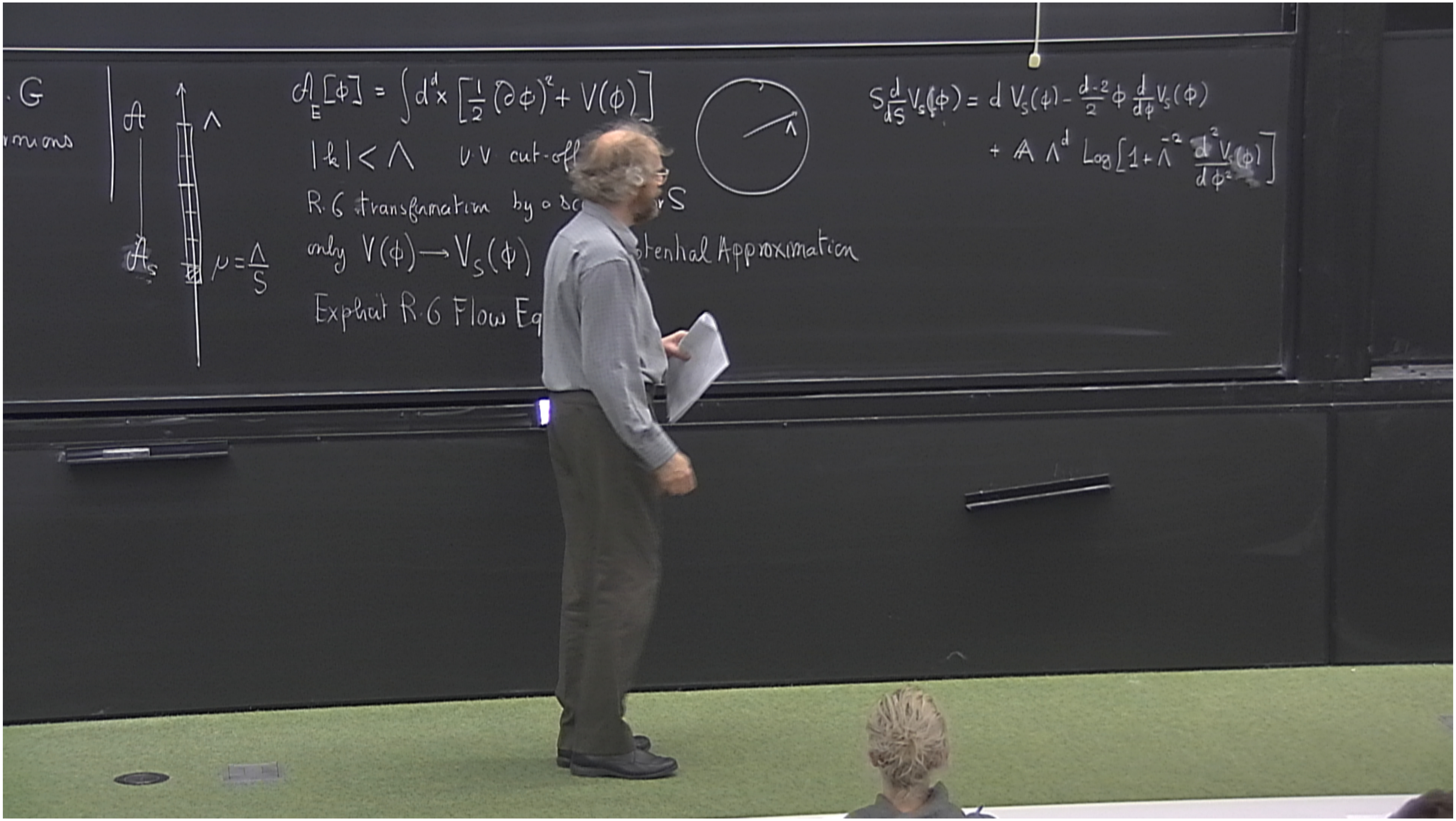

- Wilson R.G
- Dirac Fermions
$\mathcal{A}$
$\mathcal{A}_S$
Λ
$\mu = \frac{\Lambda}{S}$
$\mathcal{A}_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V. cut-off
R.G transformation by a scale factor S
Λ


- Wilson R.G
- Dirac Fermions
$S_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V. cut-off
R.G transformation by a scale factor S
only $V(\phi) \longrightarrow V_S(\phi)$ Local Potential Approximation
Explicit R.G Flow Equation

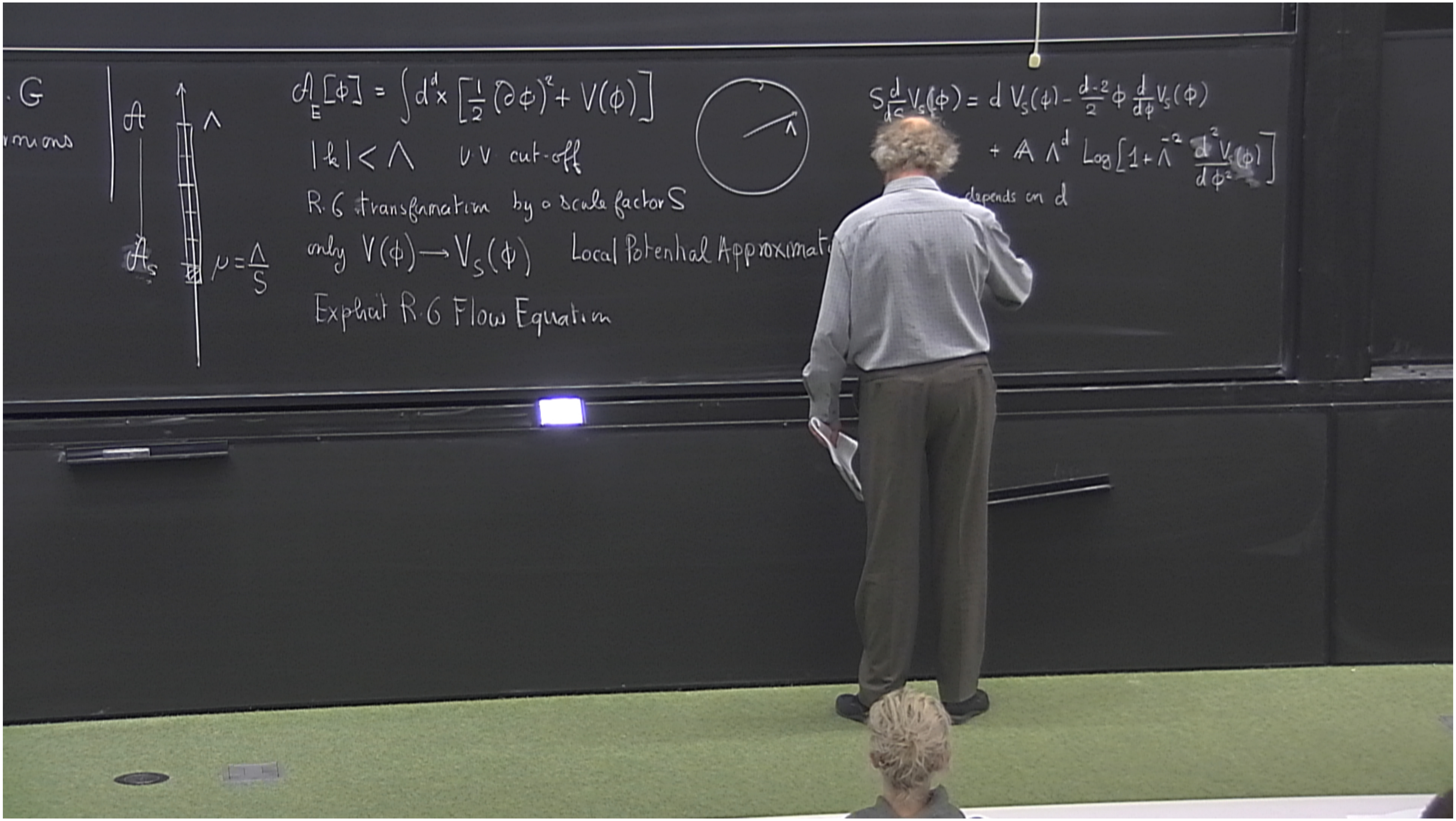

- Wilson R.G
- Dirac Fermions
A
Λ
A_s
μ = Λ/S
A_E[φ] = ∫d^d x [½(∂φ)² + V(φ)]
|k| < Λ U.V. cut-off
R.G transformation by a scale factor S
only V(φ) → V_S(φ) Local Potential Approximation
Explicit R.G Flow Equation
Λ

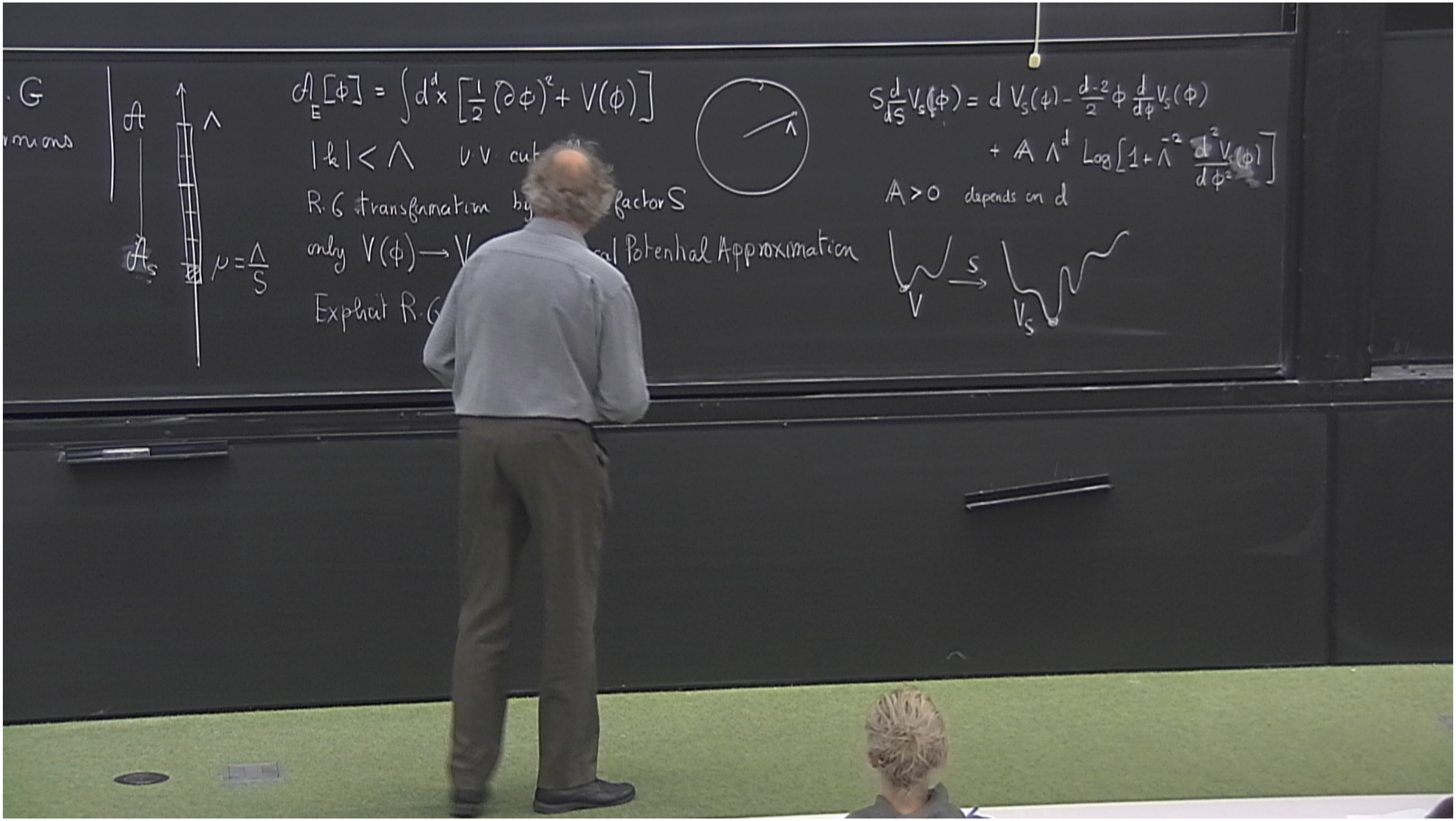


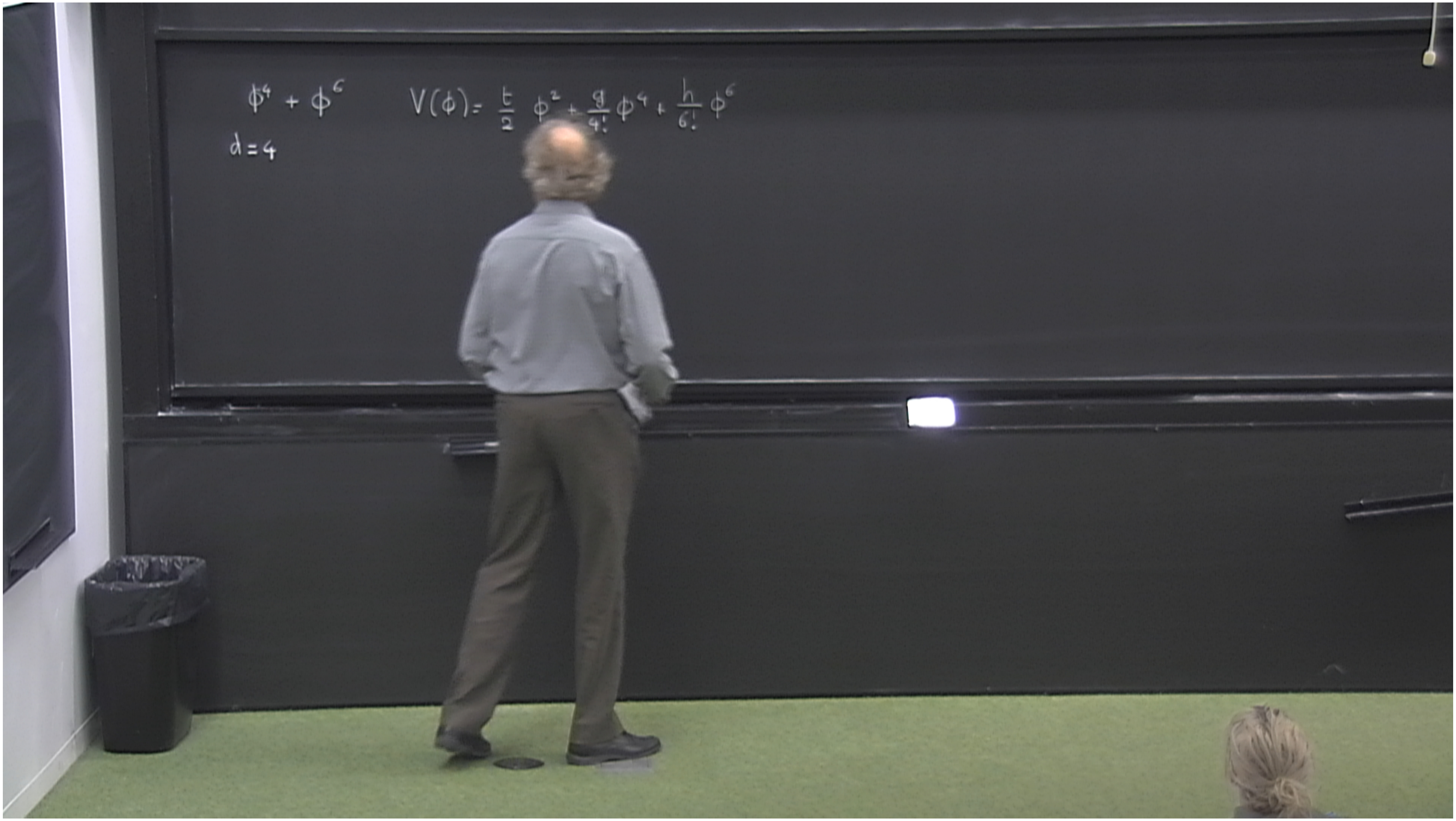
$\mathcal{A}_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V. cut-off
R.G transformation by a scale factor S
only $V(\phi) \longrightarrow V_S(\phi)$ Local Potential Approximation
Explicit R.G Flow Equation
$S\frac{d}{dS}V_S(\phi) = d\,V_S(\phi) - \frac{d-2}{2}\phi\frac{d}{d\phi}V_S(\phi)$



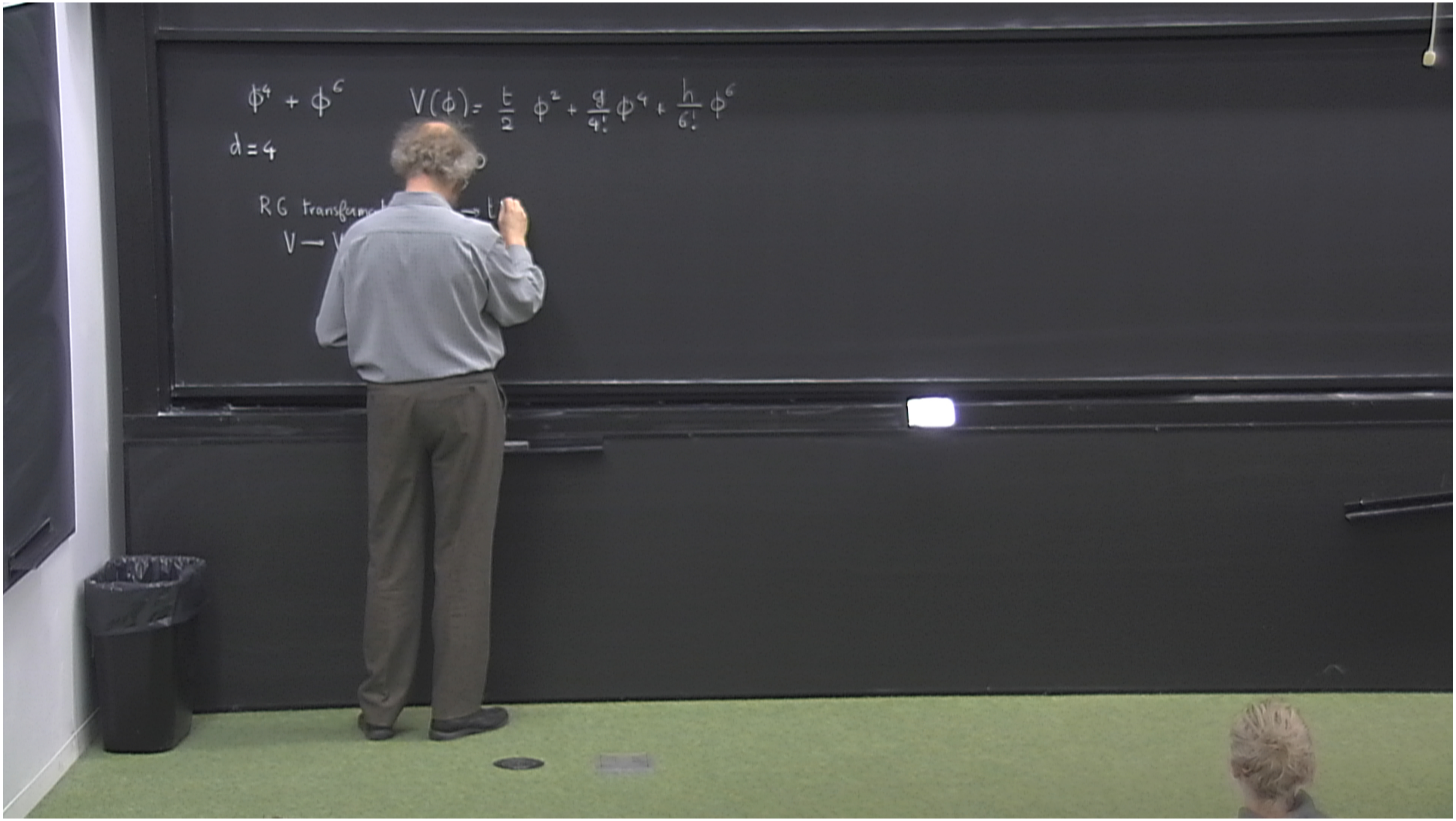
$A_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V cut-off
R.G transformation by a sc
only $V(\phi) \longrightarrow V_s(\phi)$
tential Approximation
Explicit R.G Flow Eq
$\mu = \frac{\Lambda}{S}$
$S\frac{d}{dS}V_s(\phi) = d\,V_s(\phi) - \frac{d-2}{2}\phi\frac{d}{d\phi}V_s(\phi) + A\,\Lambda^d \,\mathrm{Log}\left[1 + \Lambda^{-2}\frac{d^2V_s(\phi)}{d\phi^2}\right]$

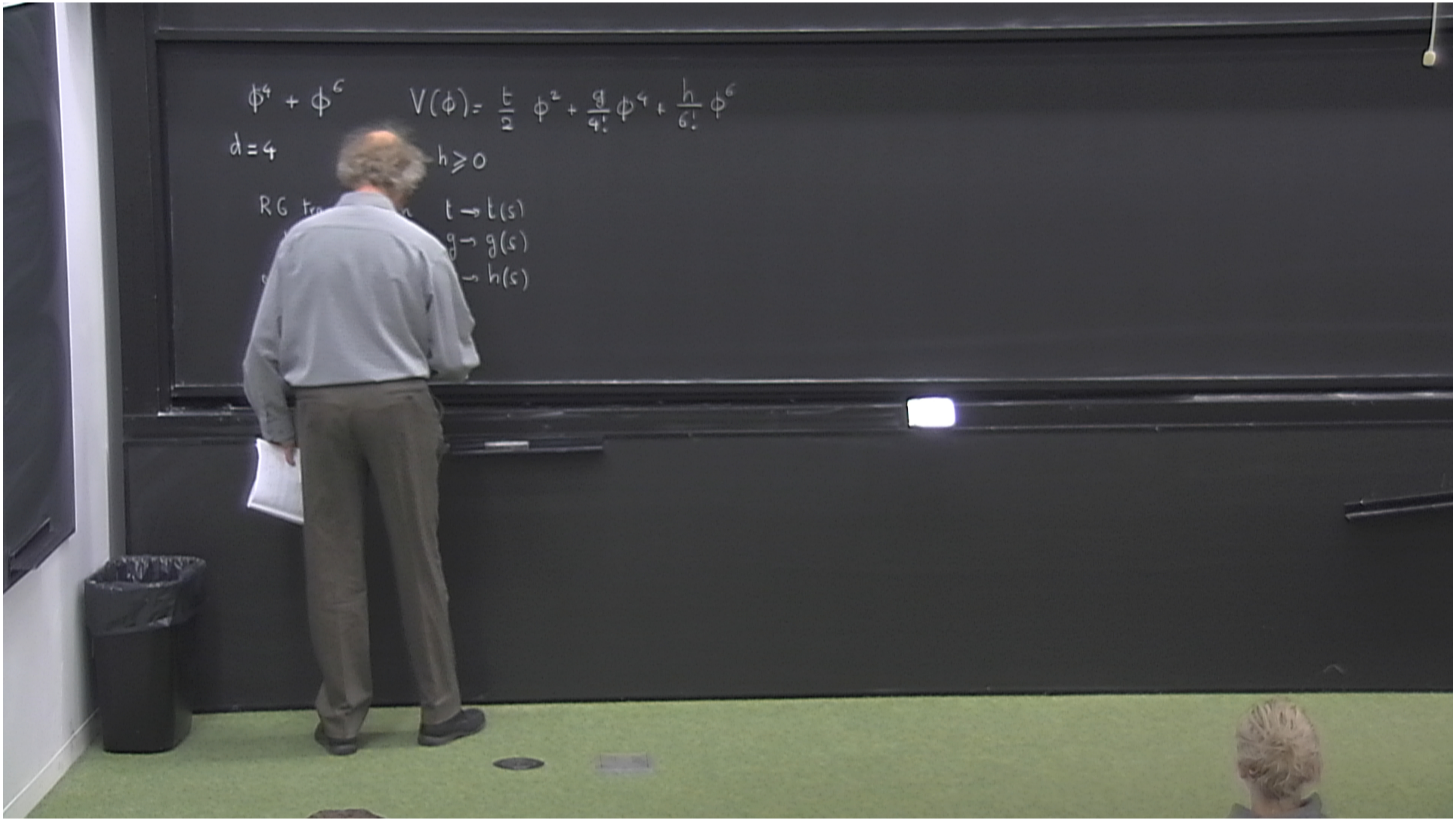

R.G
$\mathcal{A}_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ UV cut-off
R.G transformation by a scale factor S
only $V(\phi) \longrightarrow V_s(\phi)$ Local Potential Approximat
Explicit R.G Flow Equation
$\mu = \frac{\Lambda}{S}$
$S\frac{d}{dS}V_s(\phi) = d\,V_s(\phi) - \frac{d-2}{2}\phi\frac{d}{d\phi}V_s(\phi)$
$+ A\,\Lambda^d\,\mathrm{Log}\left[1 + \Lambda^{-2}\frac{d^2V_s(\phi)}{d\phi^2}\right]$
depends on d

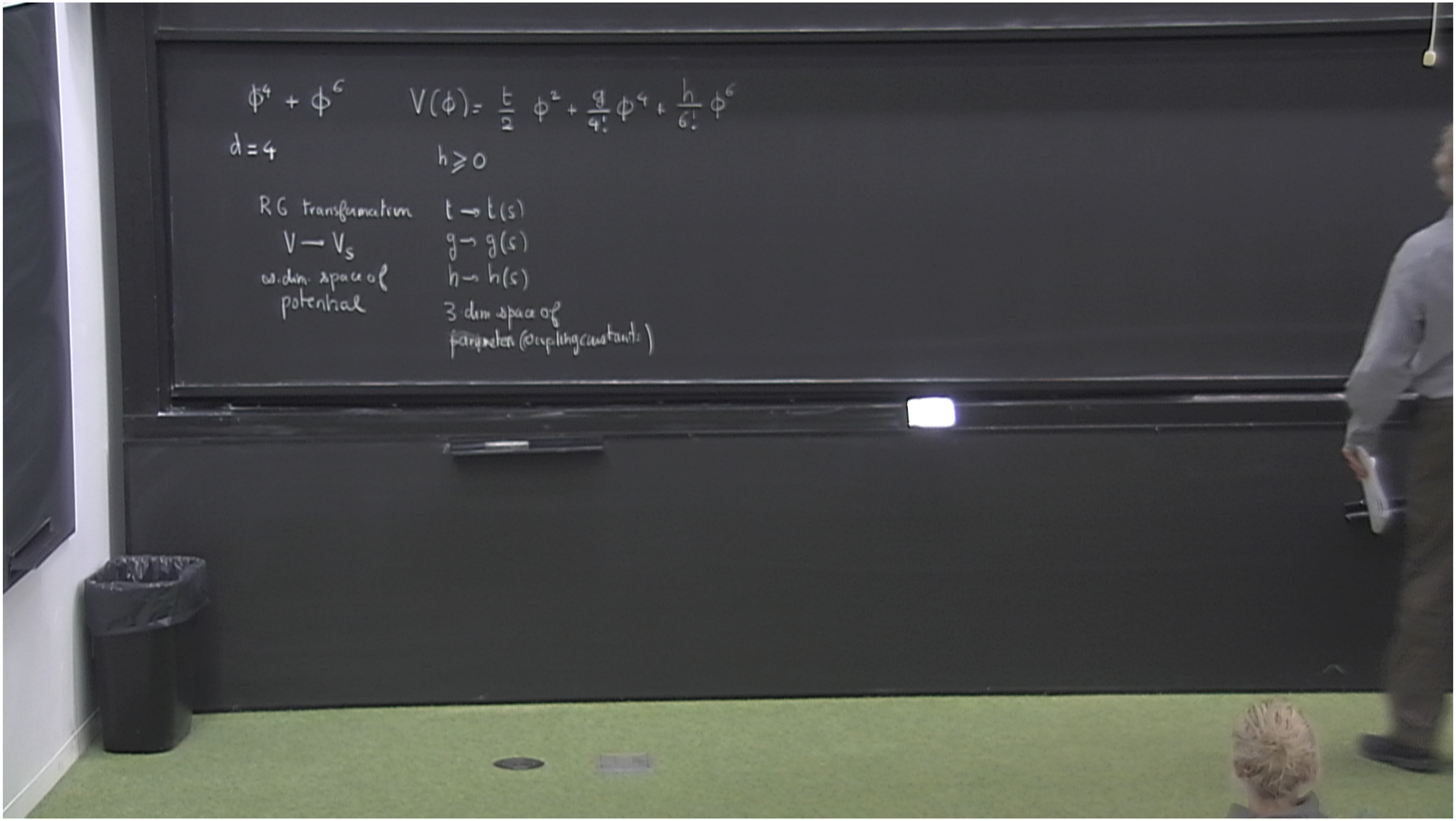

$\mathcal{A}_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V. cut
R.G transformation by factor S
only $V(\phi) \longrightarrow V$ al Potential Approximation
Explicit R.G
$\mu = \frac{\Lambda}{S}$
$S\frac{d}{dS}V_S(\phi) = d\,V_S(\phi) - \frac{d-2}{2}\phi\frac{d}{d\phi}V_S(\phi)$
$+ A\,\Lambda^d \,\mathrm{Log}\left[1 + \Lambda^{-2}\frac{d^2 V_S(\phi)}{d\phi^2}\right]$
$A > 0$ depends on d

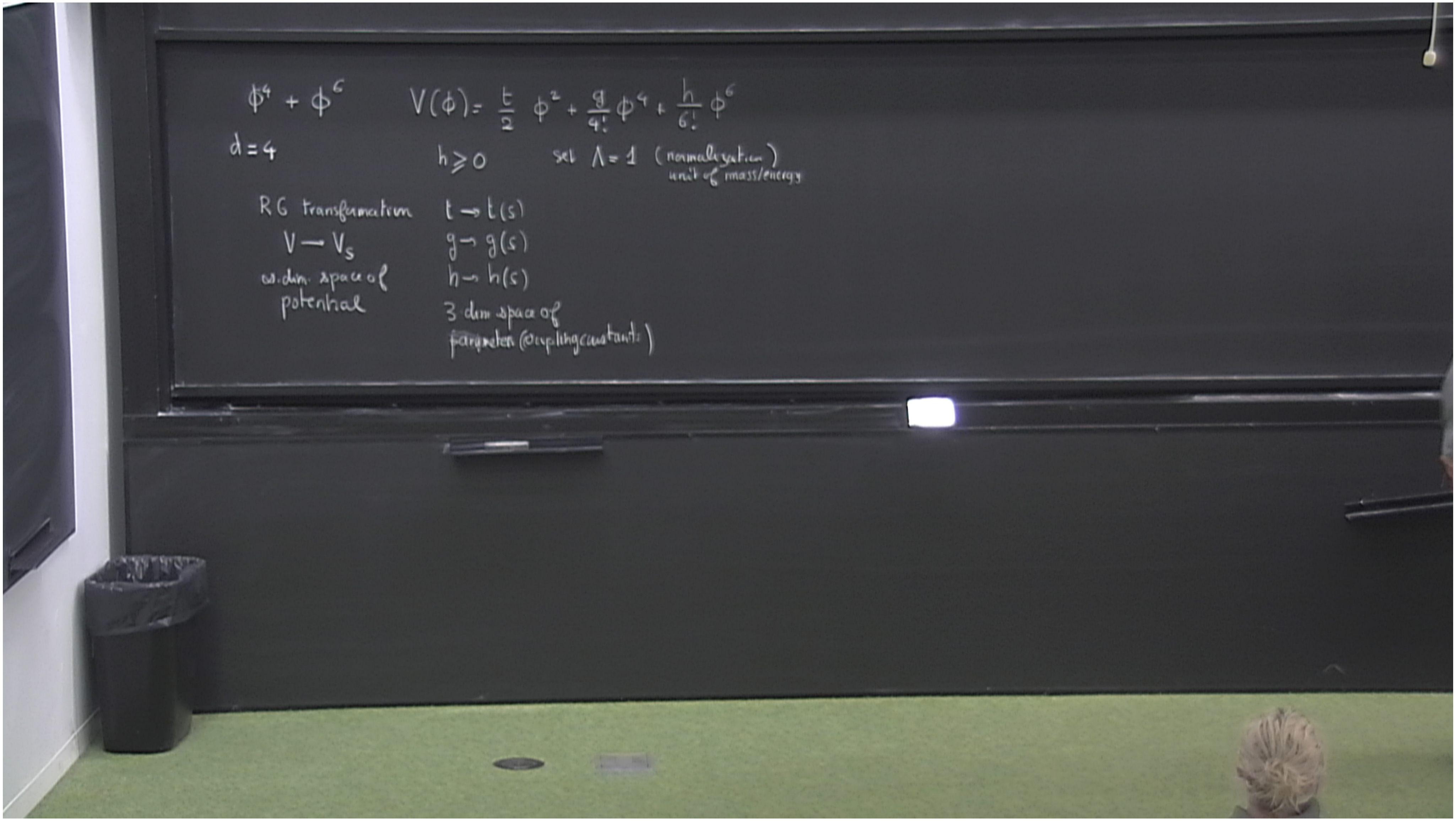


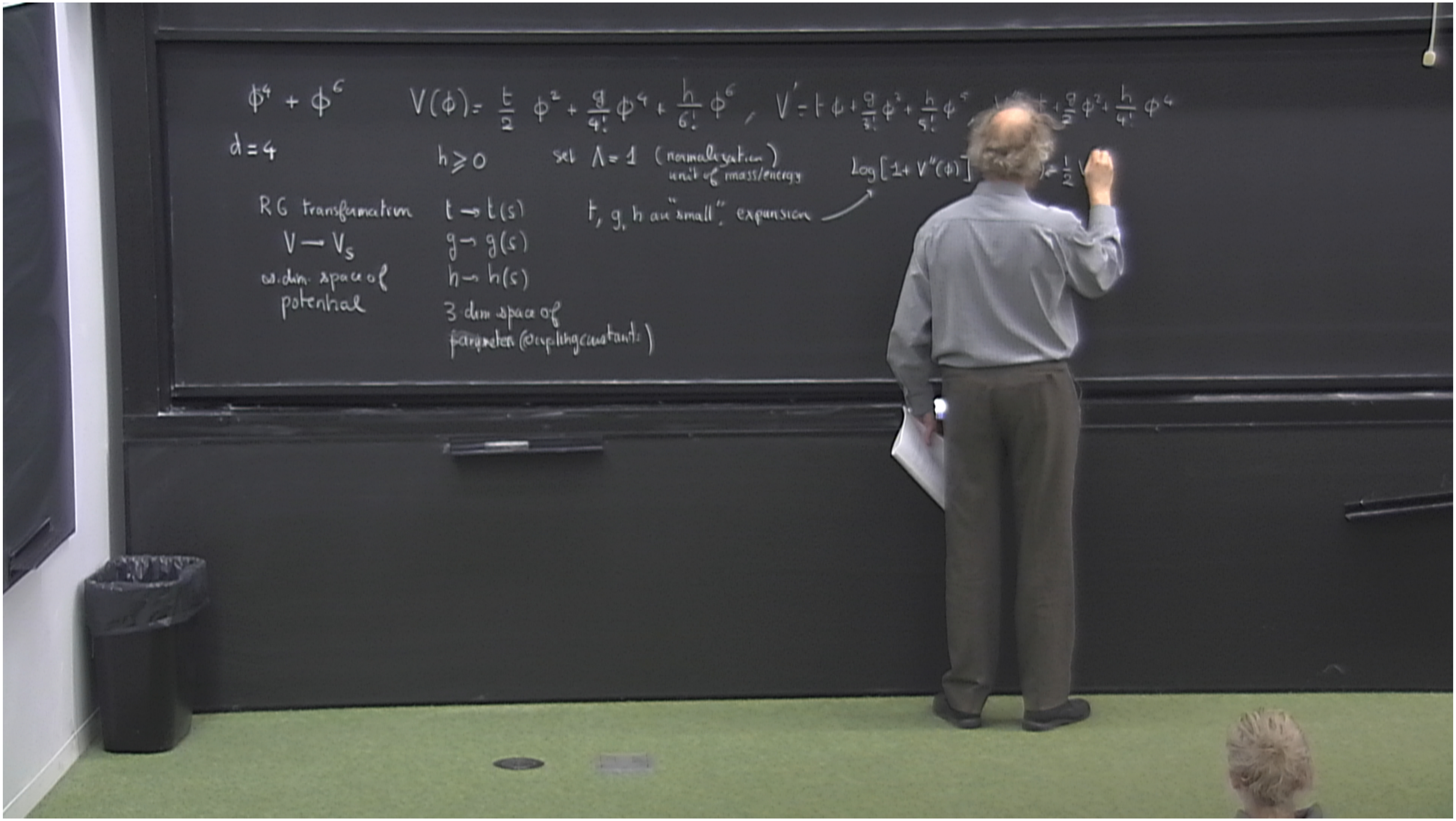
$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
$d = 4$

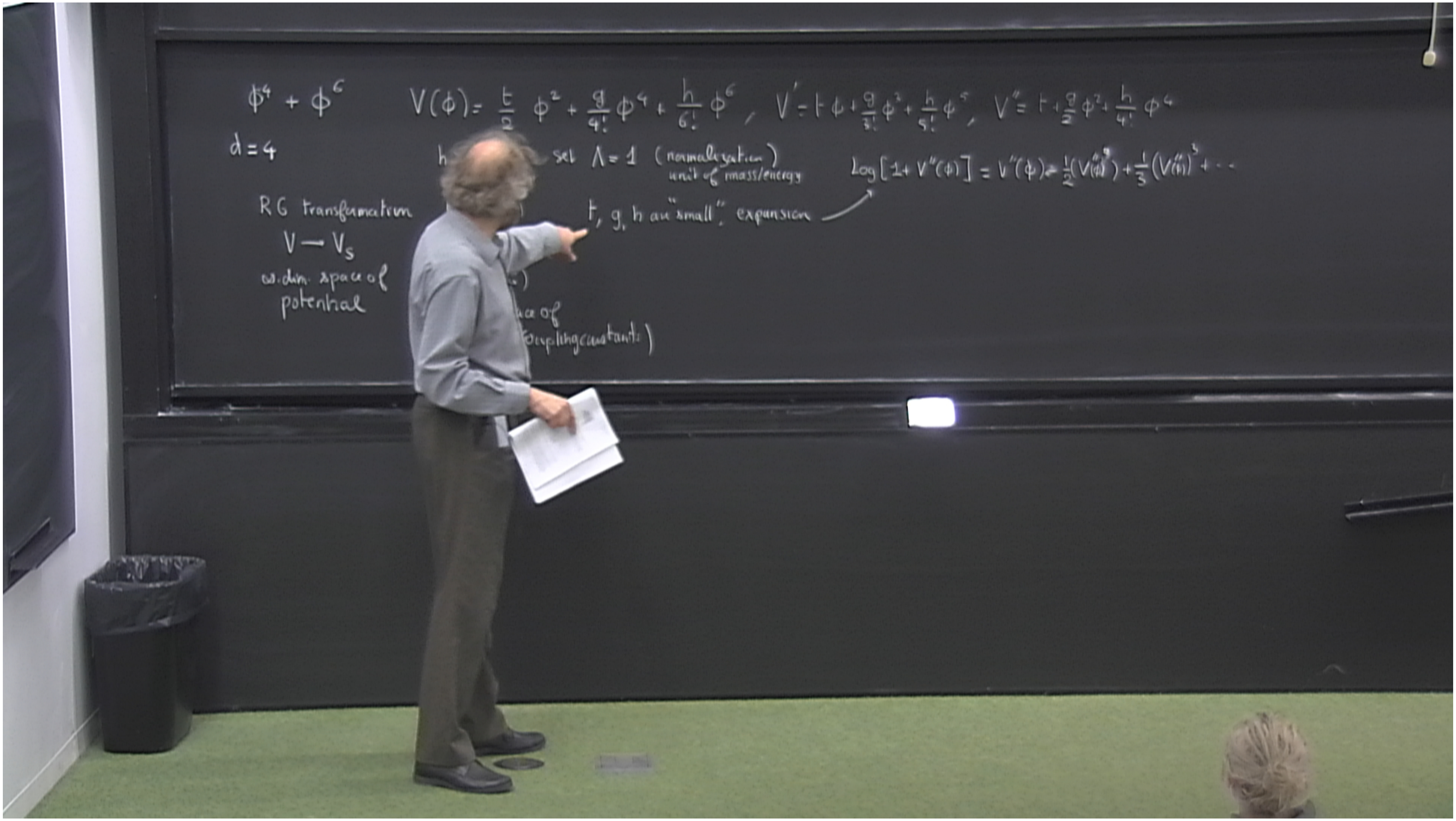

$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
$d = 4$
RG transform


$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
$d=4$
$h \geq 0$
RG
$t \to t(s)$
$g \to g(s)$
$\to h(s)$

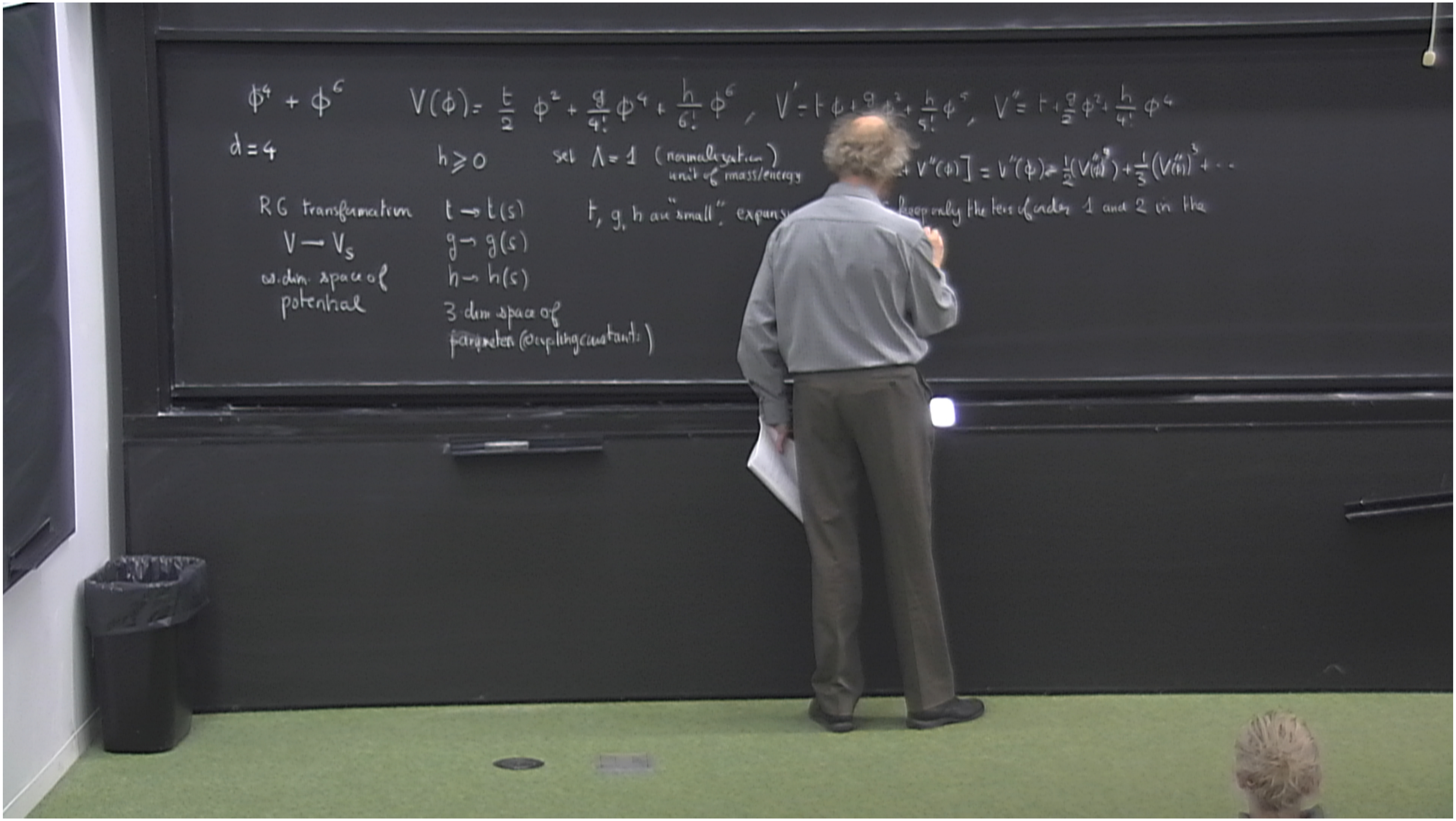

$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
$d = 4$
$h \geq 0$
RG transformation
$V \to V_s$
∞. dim. space of
potential
$t \to t(s)$
$g \to g(s)$
$h \to h(s)$
3 dim space of
parameters (coupling constants)

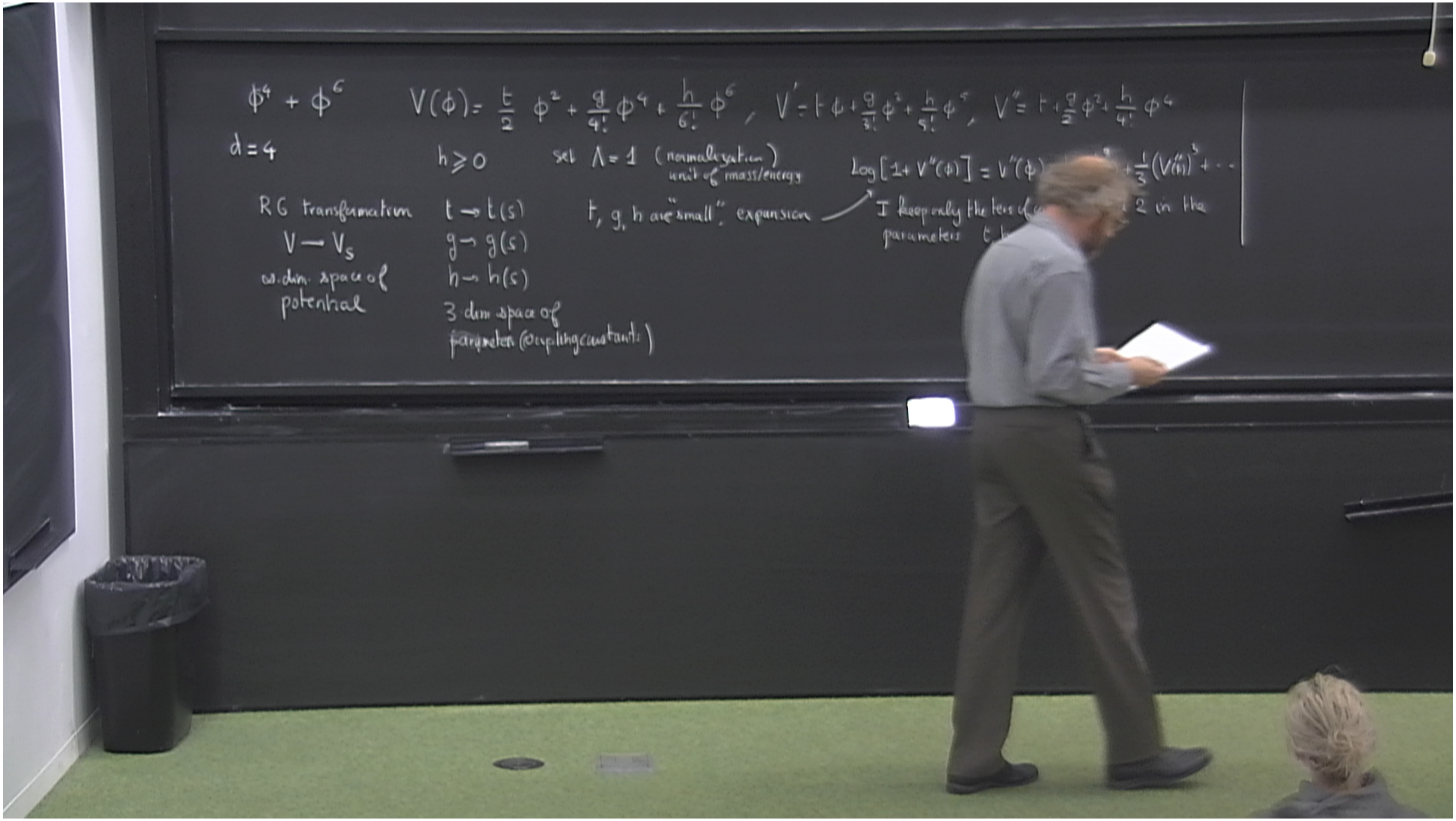

$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
$d = 4$
$h \geq 0$
set $\Lambda = 1$ (normalization)
unit of mass/energy
RG transformation
$t \to t(s)$
$V \to V_s$
$g \to g(s)$
∞-dim. space of potential
$h \to h(s)$
3 dim space of parameter (coupling constants)

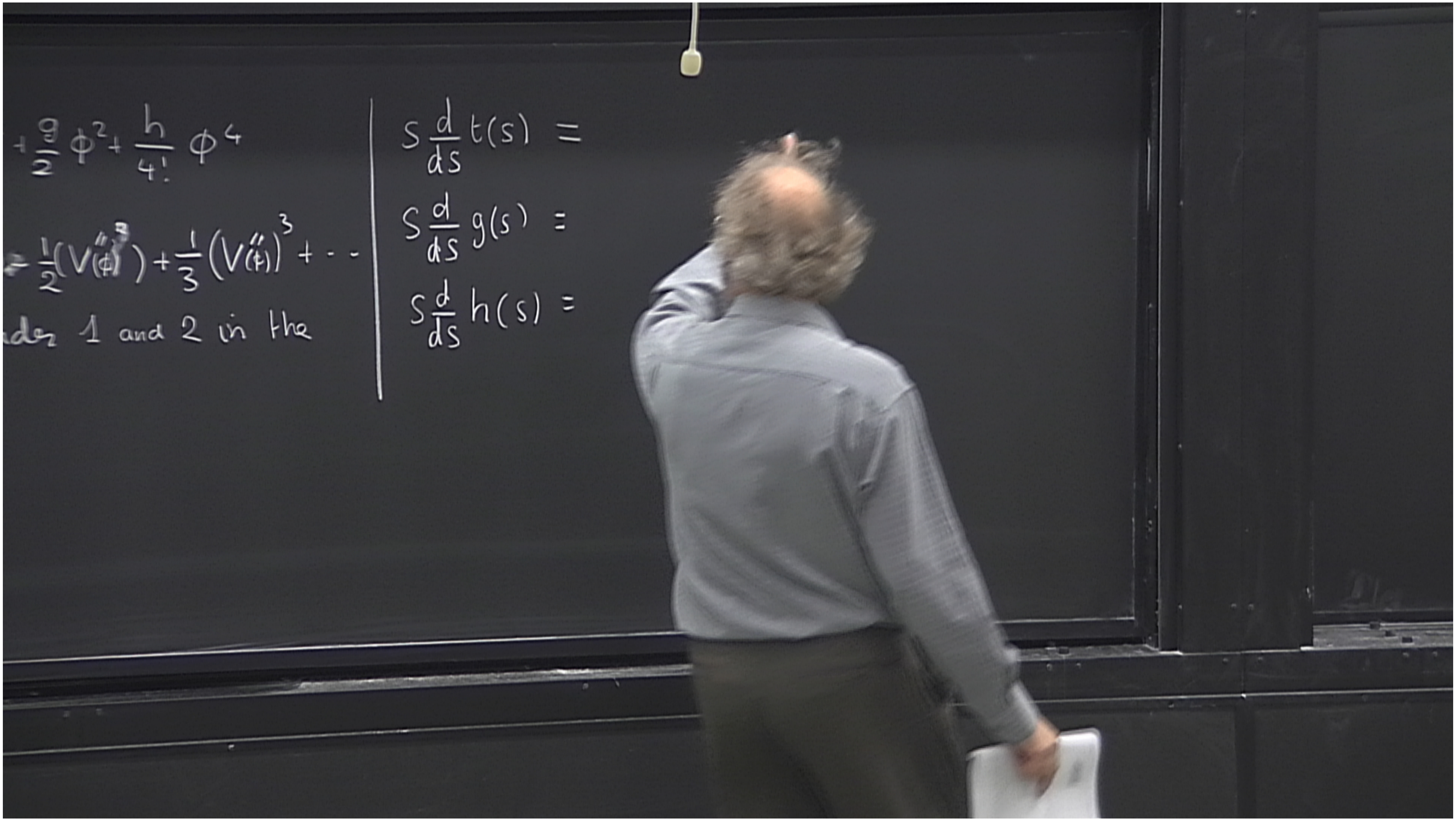

$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
d = 4
$h \geq 0$
set $\Lambda = 1$ (normalization)
unit of mass/energy
RG transformation
$V \to V_s$
$t \to t(s)$
$g \to g(s)$
$h \to h(s)$
3 dim space of
potential
t, g, h are "small", expansion

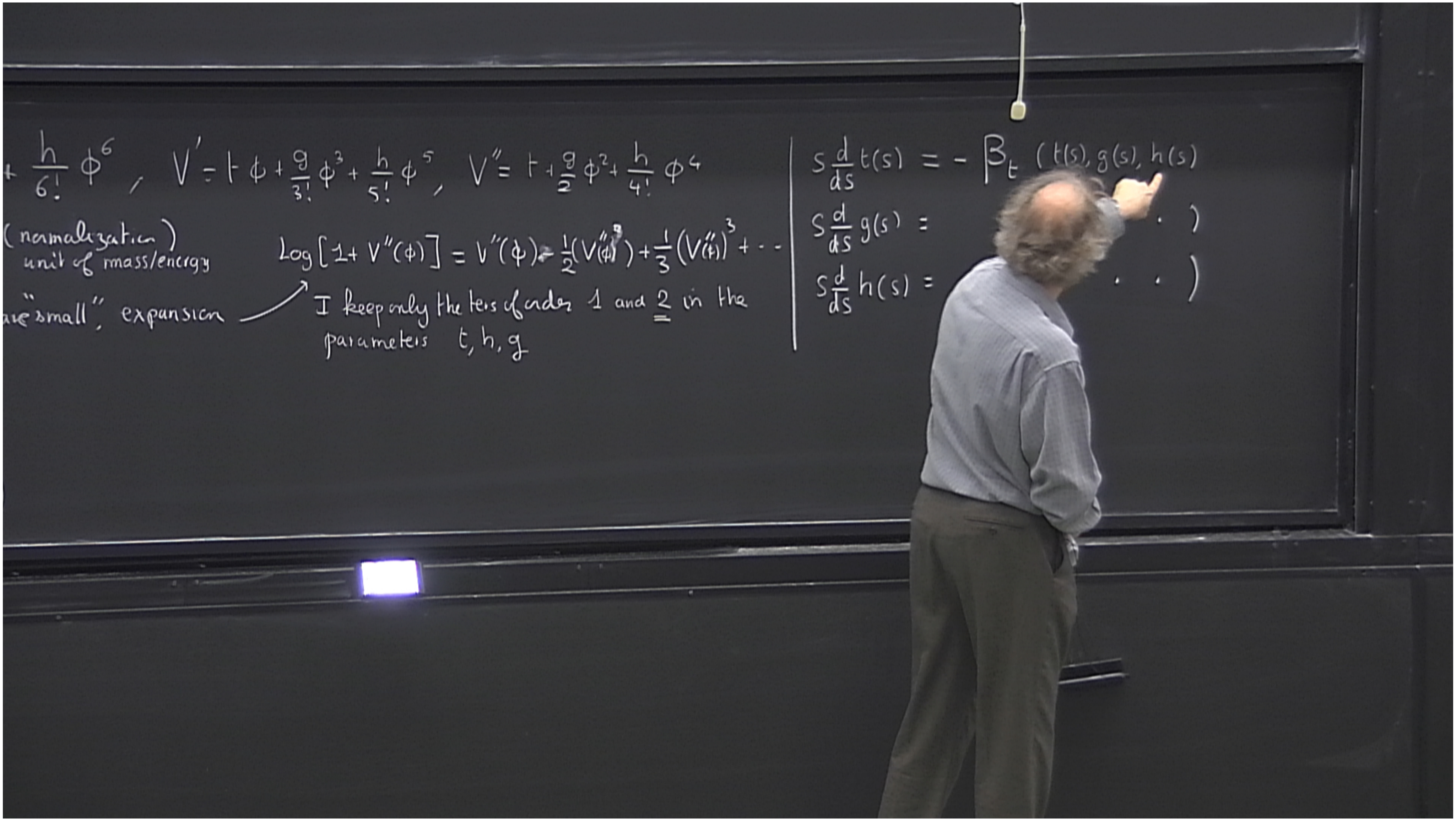


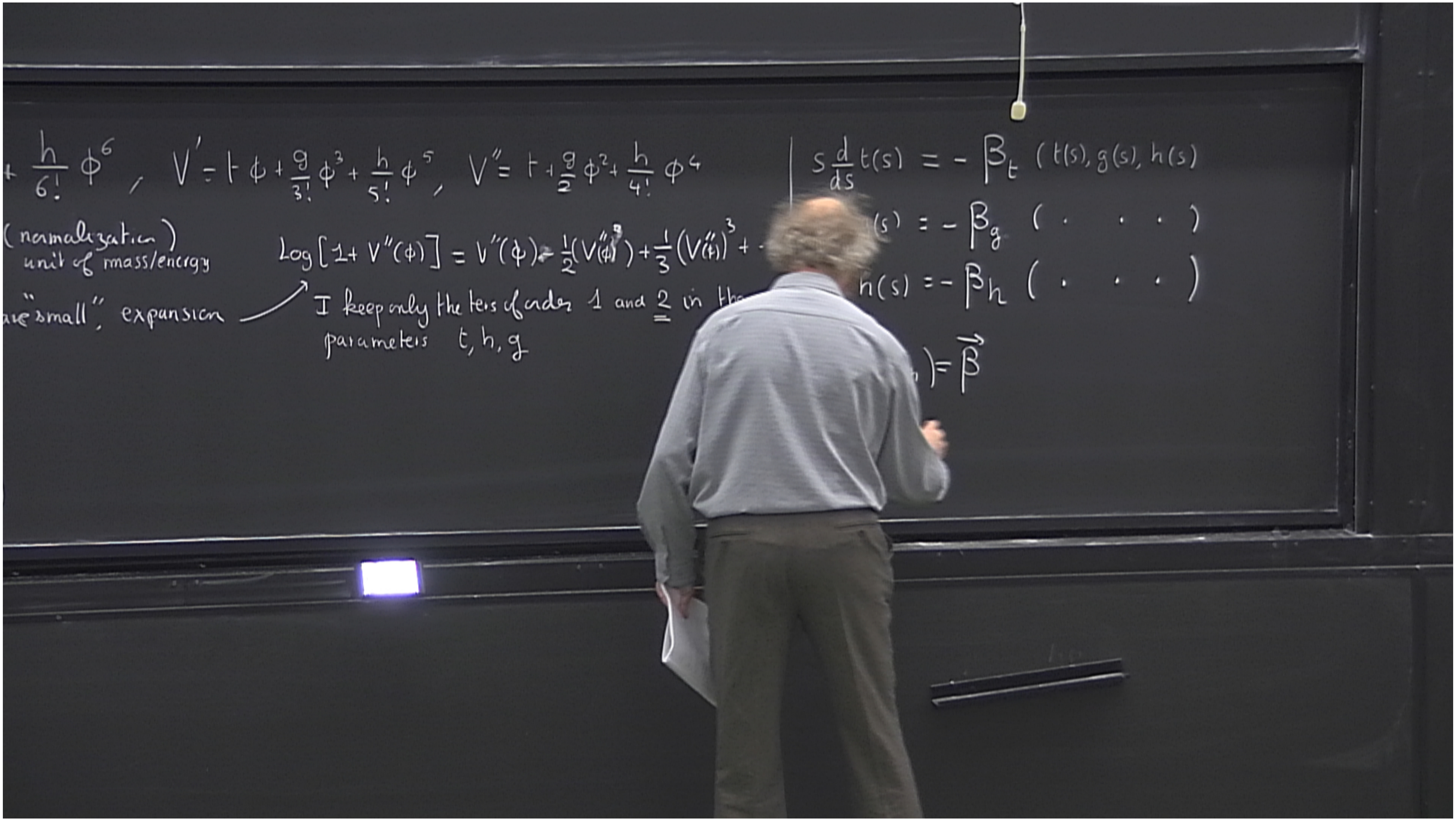
$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$, $V' = t\phi + \frac{g}{3!}\phi^3 + \frac{h}{5!}\phi^5$, $V'' = t + \frac{g}{2}\phi^2 + \frac{h}{4!}\phi^4$
d = 4
set Λ = 1 (normalization) unit of mass/energy
$\log[1 + V''(\phi)] = V''(\phi) - \frac{1}{2}(V'')^2 + \frac{1}{3}(V'')^3 + \cdots$
RG transformation
$V \to V_s$
t, g, h are "small", expansion
potential
coupling constants)

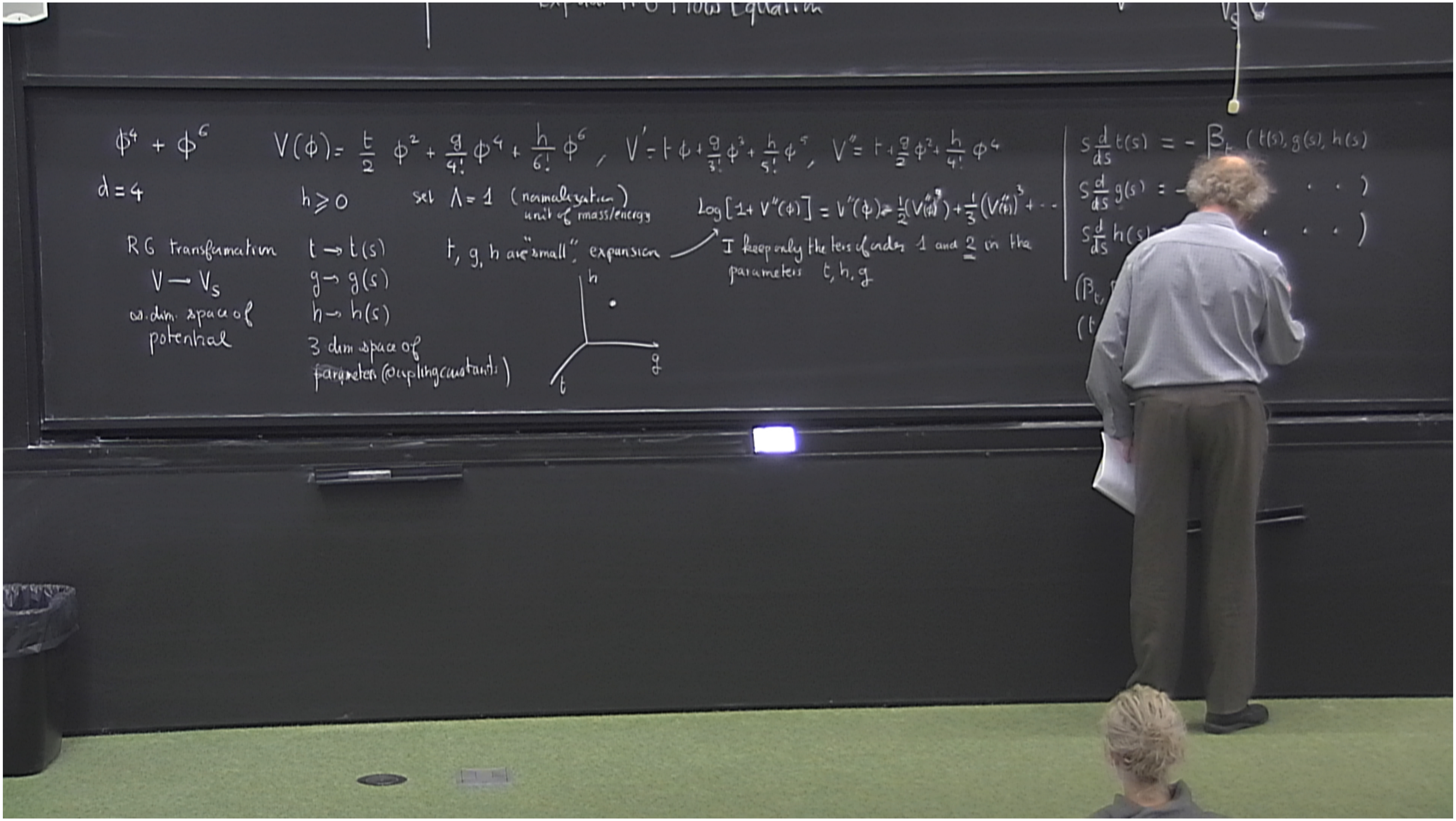

$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
d = 4
h ≥ 0
set Λ = 1 (normalization)
unit of mass/energy
RG transformation
V → V_s
∞ dim. space of potential
t → t(s)
g → g(s)
h → h(s)
3 dim space of parameters (coupling constants)


$\phi^4 + \phi^6$
$V(\phi) = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4 + \frac{h}{6!}\phi^6$
$d = 4$
$h \geq 0$
set $\Lambda = 1$ (normalization)
unit of mass/energy
RG transformation
$V \to V_s$
$t \to t(s)$
$g \to g(s)$
$h \to h(s)$
potential
3 dim space of
t, g, h are "small" expansion
I keep only the
parameters

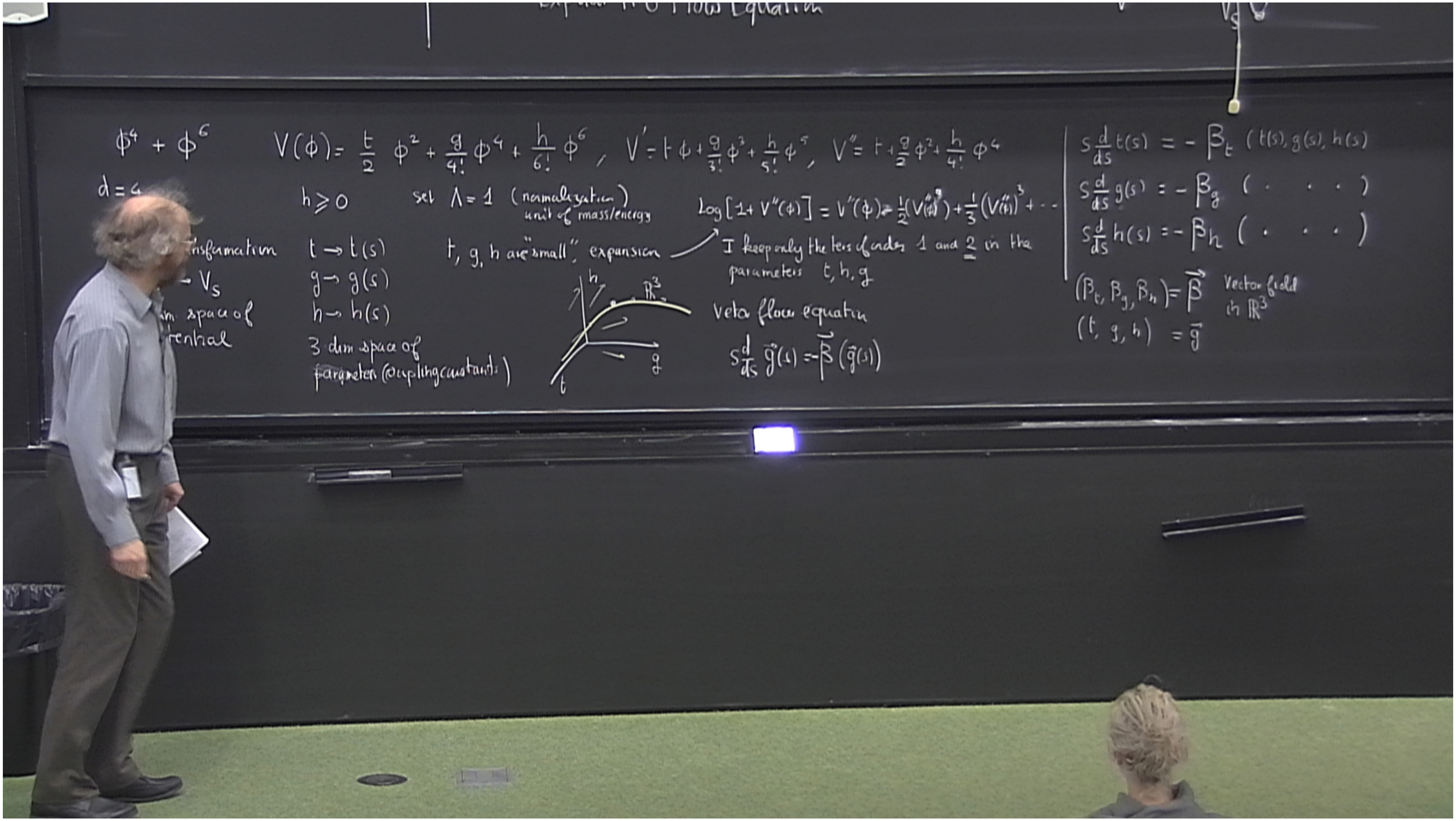

+ g/2 φ² + h/4! φ⁴
1/2 (V''(φ))² + 1/3 (V''(φ))³ + - -
1 and 2 in the
s d/ds t(s) =
s d/ds g(s) =
s d/ds h(s) =


+ h/6! φ^6 , V' = t φ + g/3! φ^3 + h/5! φ^5 , V'' = t + g/2 φ^2 + h/4! φ^4
(normalization)
unit of mass/energy
Log [1 + V''(φ)] = V''(φ) − 1/2 (V''(φ))^2 + 1/3 (V''(φ))^3 + . . .
are "small", expansion
I keep only the te of order 1 and 2 in the
parameters t, h, g
s d/ds t(s) = − β_t (t(s), g(s), h(s))
s d/ds g(s) =
s d/ds h(s) =


+ h/6! φ^6 , V' = t φ + g/3! φ^3 + h/5! φ^5 , V'' = t + g/2 φ^2 + h/4! φ^4
(normalization)
unit of mass/energy
Log [1 + V''(φ)] = V''(φ) − 1/2 (V''(φ))^2 + 1/3 (V''(φ))^3 + ...
are "small", expansion
I keep only the terms of order 1 and 2 in the
parameters t, h, g
s d/ds t(s) = − β_t (t(s), g(s), h(s))
(s) = − β_g (. . .)
h(s) = − β_h (. . .)
) = β⃗



ϕ⁴ + ϕ⁶
d = 4
V(ϕ) = t/2 ϕ² + g/4! ϕ⁴ + h/6! ϕ⁶ , V′ = tϕ + g/3! ϕ³ + h/5! ϕ⁵ , V″ = t + g/2 ϕ² + h/4! ϕ⁴
h ≥ 0
set Λ = 1 (normalization)
unit of mass/energy
Log [1 + V″(ϕ)] = V″(ϕ) − ½(V″(ϕ))² + ⅓(V″(ϕ))³ + . . .
RG transformation
V → V_s
∞ dim. space of potential
t → t(s)
g → g(s)
h → h(s)
3 dim space of parameters (coupling constants)
t, g, h are "small". expansion
I keep only the terms of order 1 and 2 in the parameters t, h, g
h
g
t
s d/ds t(s) = − β_t (t(s), g(s), h(s))
s d/ds g(s) =
s d/ds h(s)



φ⁴ + φ⁶
d = 4
V(φ) = t/2 φ² + g/4! φ⁴ + h/6! φ⁶ , V′ = t φ + g/3! φ³ + h/5! φ⁵ , V″ = t + g/2 φ² + h/4! φ⁴
h ≥ 0
set Λ = 1 (normalization) unit of mass/energy
Log[1 + V″(φ)] = V″(φ) − 1/2 (V″(φ))² + 1/3 (V″(φ))³ + · · ·
t → t(s)
g → g(s)
h → h(s)
→ V_s
3 dim space of parameters (coupling constants)
t, g, h are "small" expansion
I keep only the terms of order 1 and 2 in the parameters t, h, g
Vector flow equation
s d/ds g(s) = −β(g(s))
s d/ds t(s) = − β_t (t(s), g(s), h(s))
s d/ds g(s) = − β_g (. . .)
s d/ds h(s) = − β_h (. . .)
(β_t, β_g, β_h) = β Vector field in ℝ³
(t, g, h) = g

+

$V''(t))^3 + \cdots$

in the

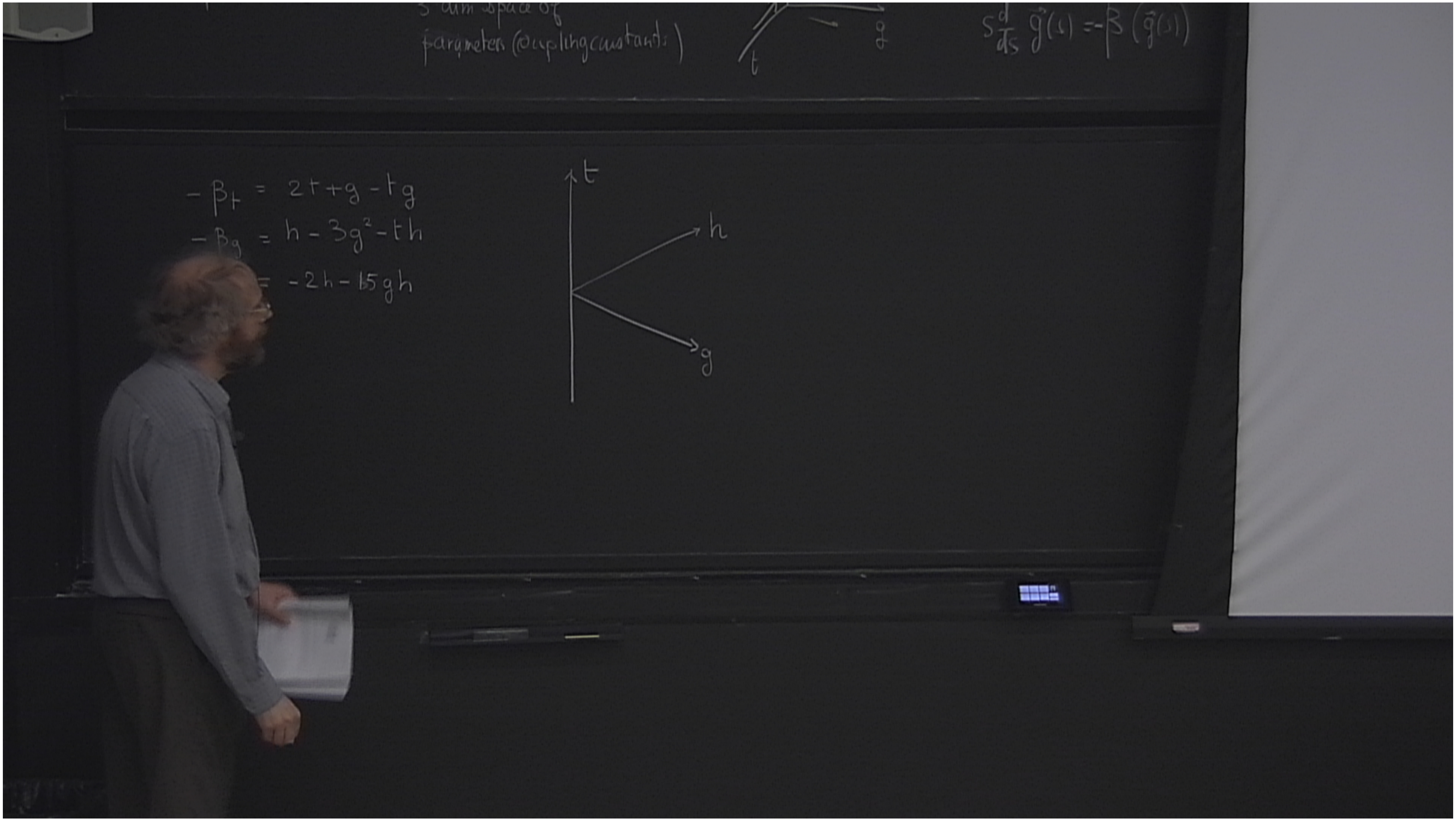
$$s\frac{d}{ds}t(s) = -\beta_t\,(t(s), g(s), h(s))$$

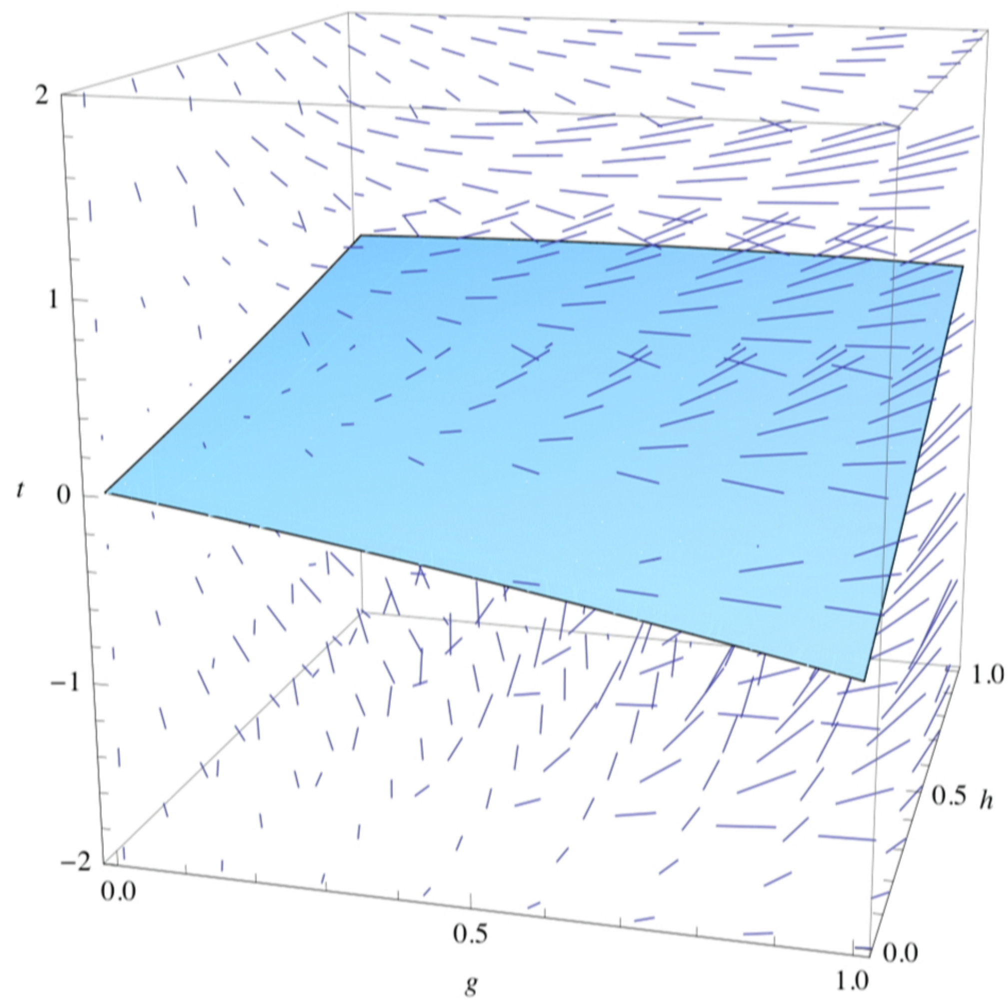
$$s\frac{d}{ds}g(s) = -\beta_g\,(\;\cdot\quad\cdot\quad\cdot\;)$$

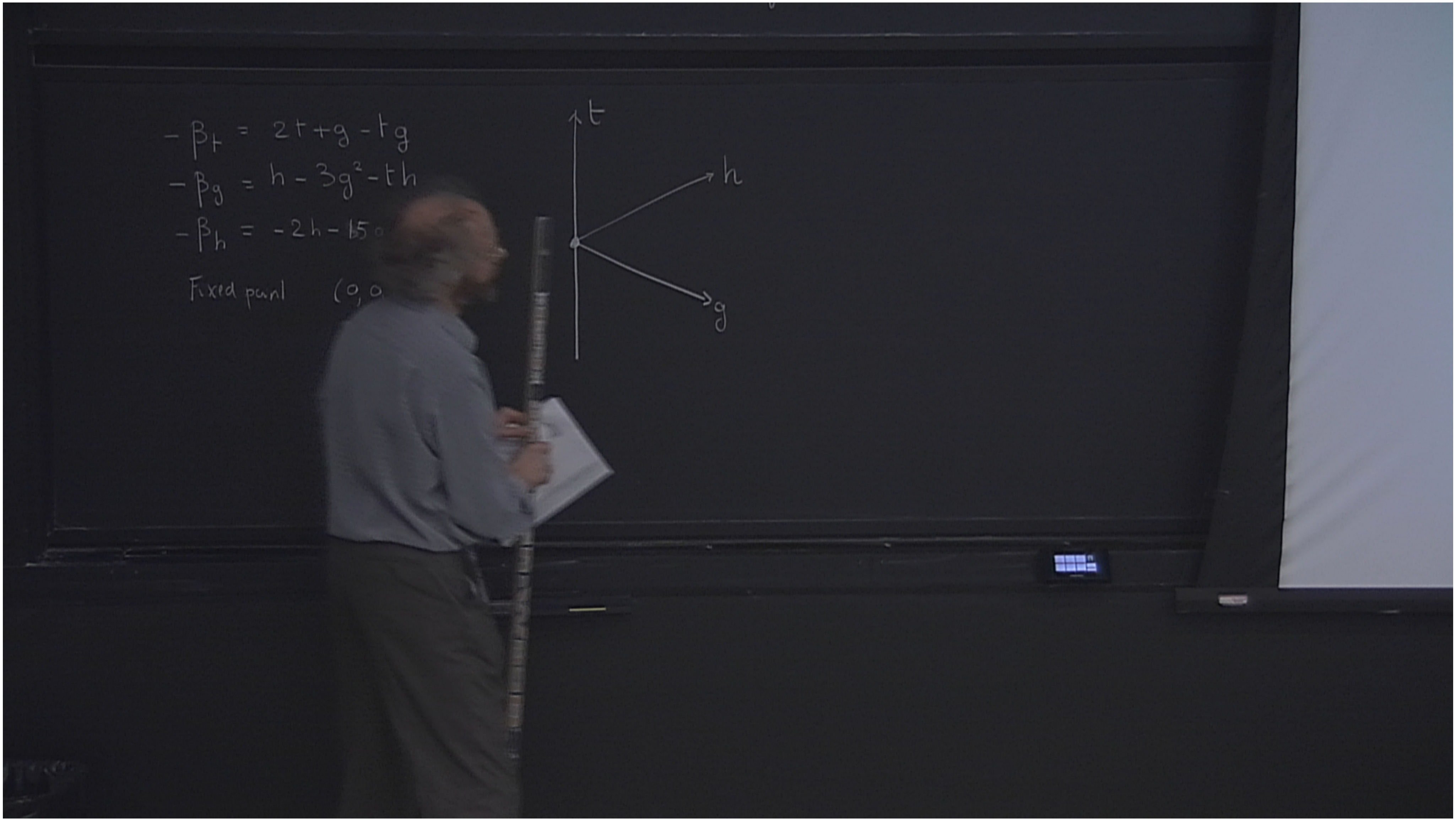
$$s\frac{d}{ds}h(s) = -\beta_h\,(\;\cdot\quad\cdot\quad\cdot\;)$$

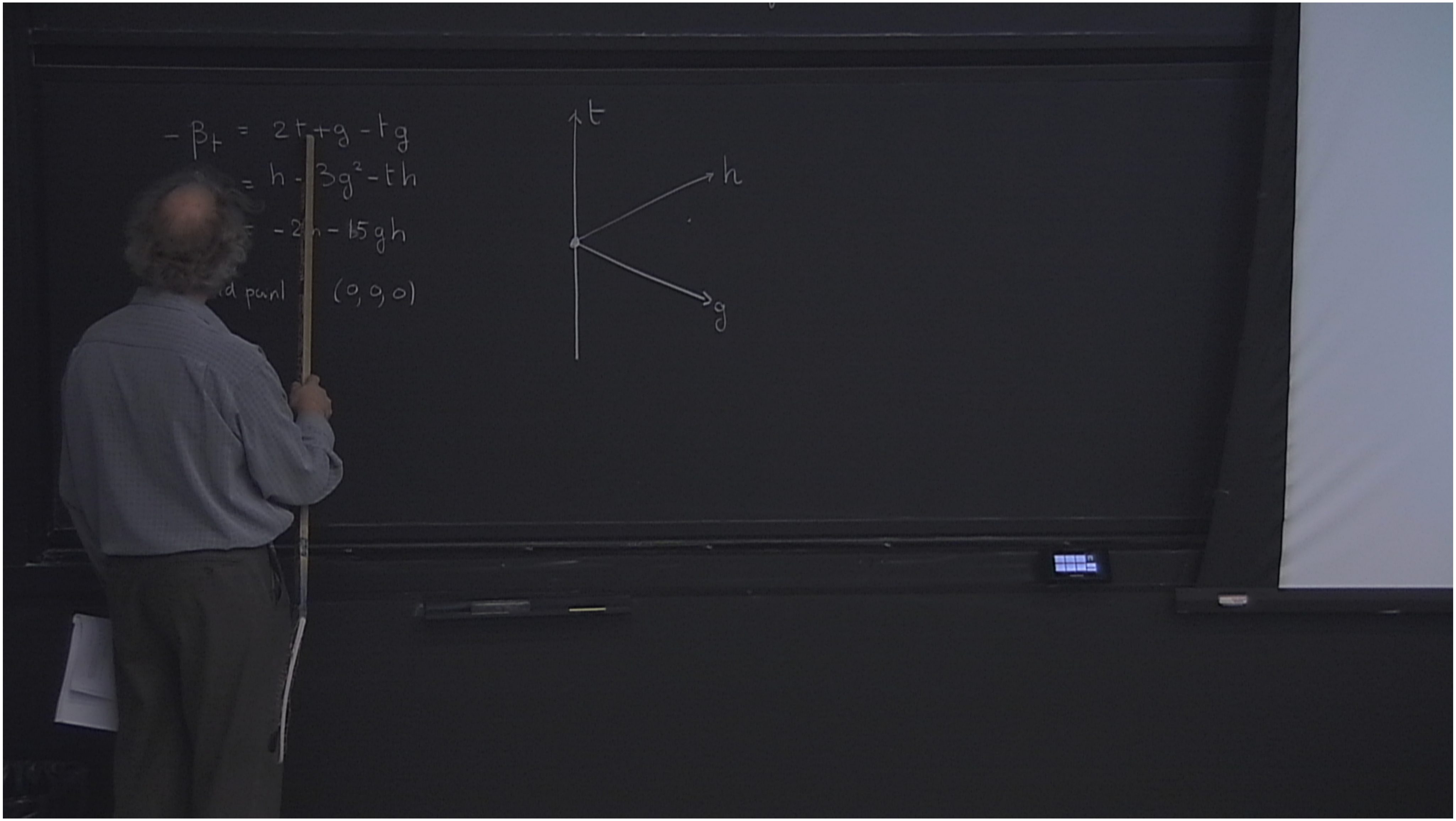
$(\beta_t, \beta_g, \beta_h) = \vec{\beta}$ Vector field in $\mathbb{R}^3$ ← components of a

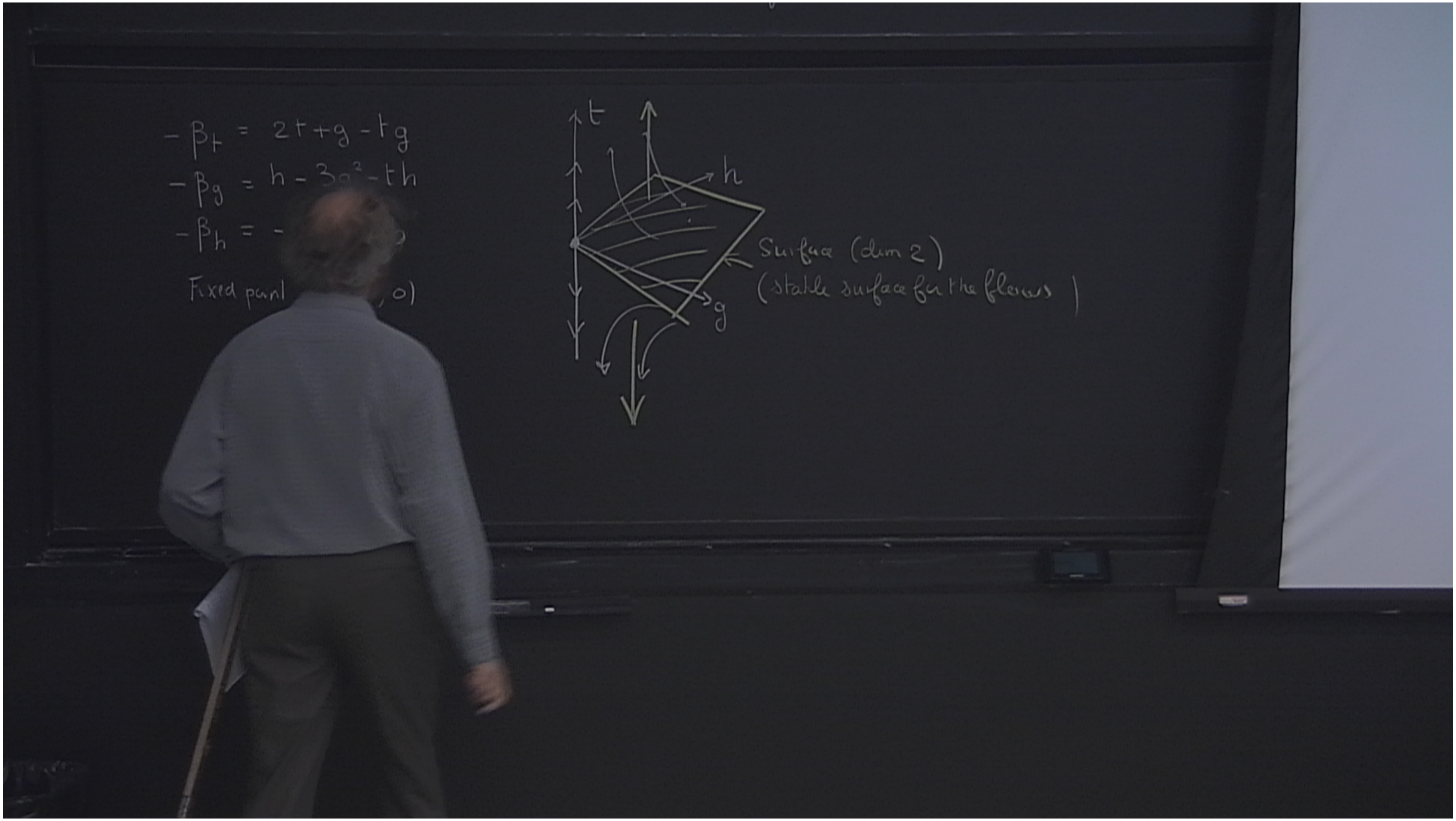
$(t, g, h) = \vec{g}$


−β_t = 2t + g − tg
−β_g = h − 3g² − th
= −2h − 15gh
t
h
g

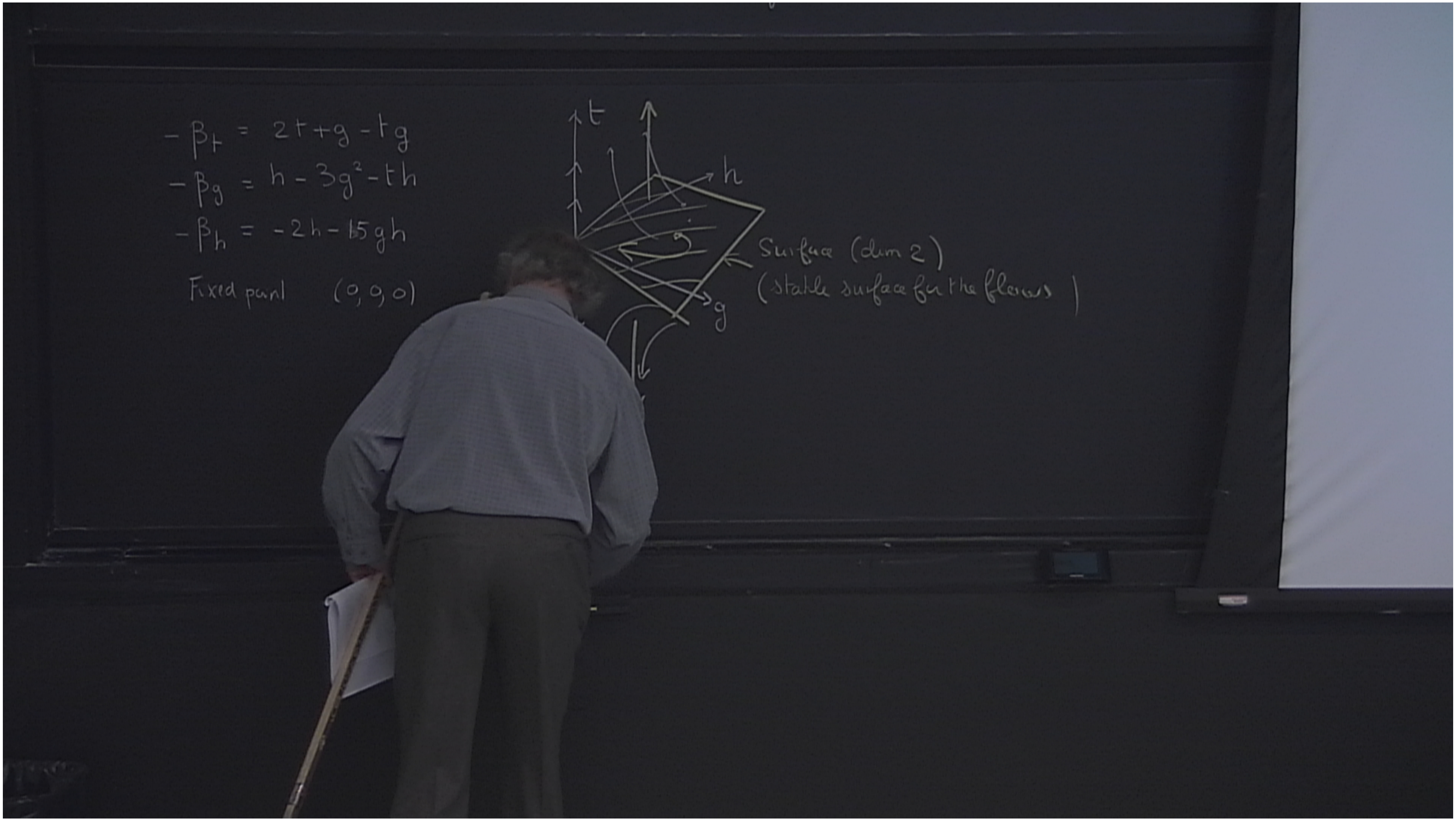

2
1
t
0
−1
−2
0.0
0.5
g
1.0
0.0
0.5
h
1.0

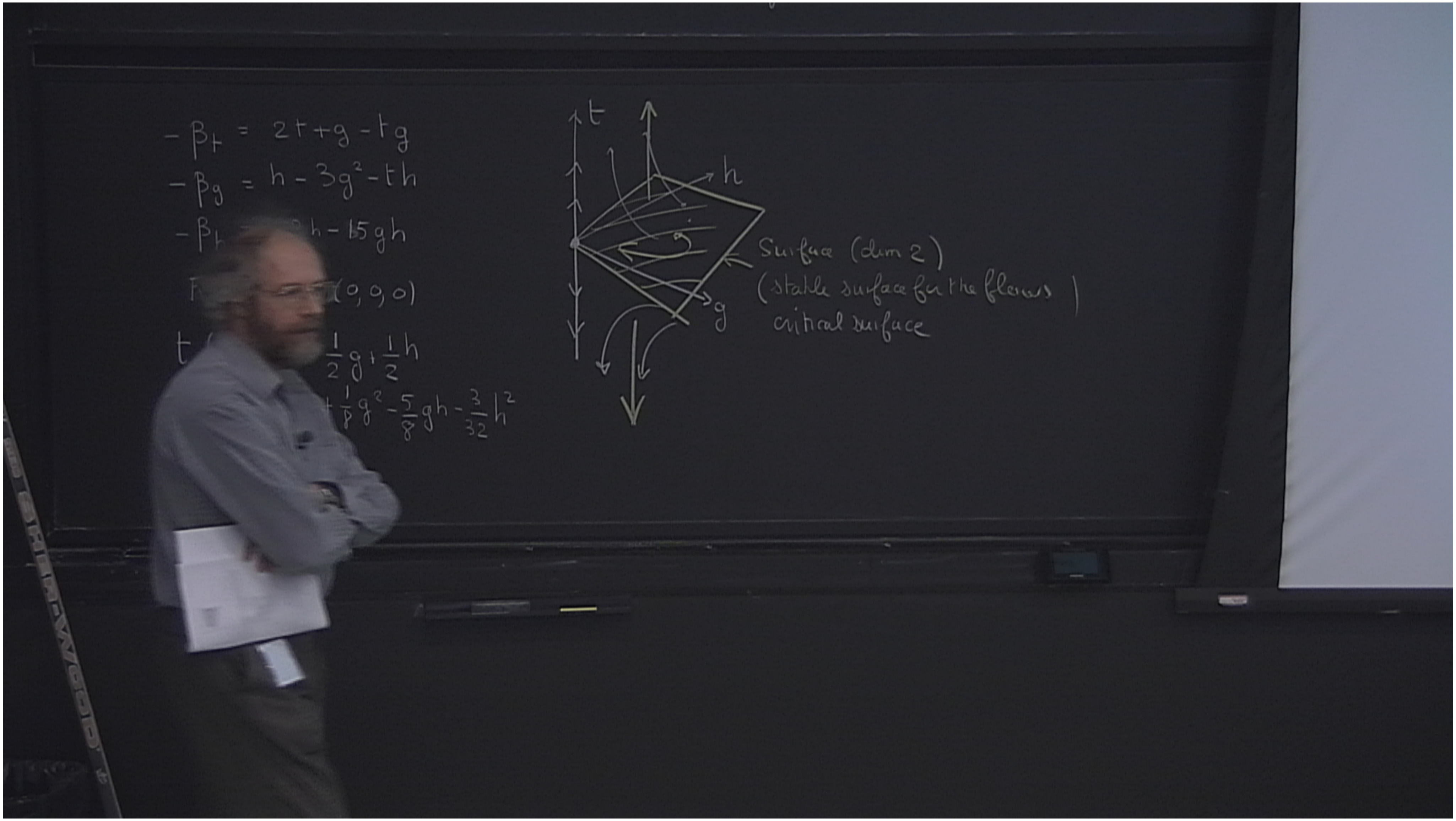

-βt = 2t+g-tg
-βg = h-3g²-th
-βh = -2h-
Fixed point
t
h
g


$-\beta_t = 2t + g - tg$
$= h - 3g^2 - th$
$= -2h - 15gh$
(0, 0, 0)
t
h
g

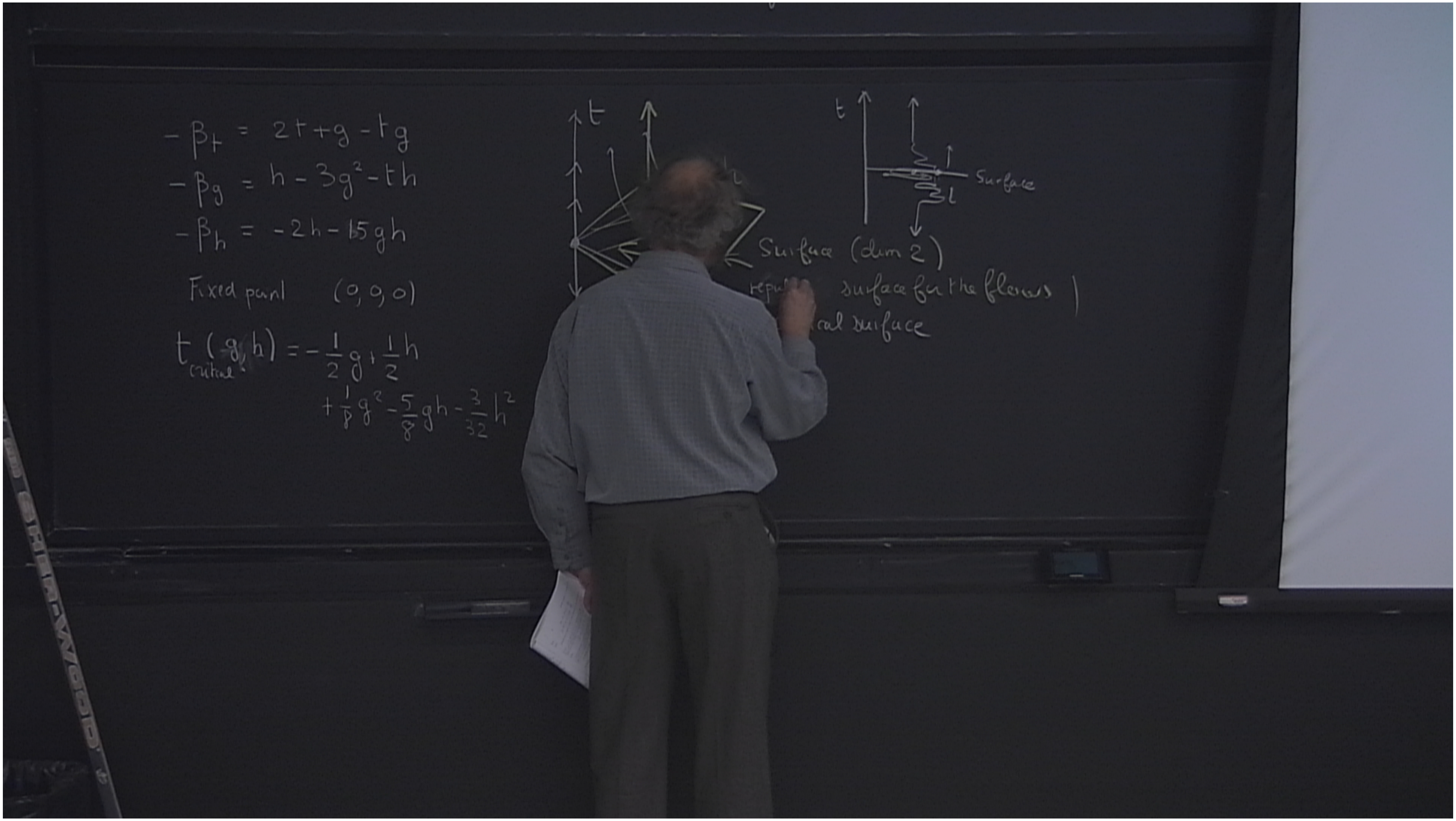


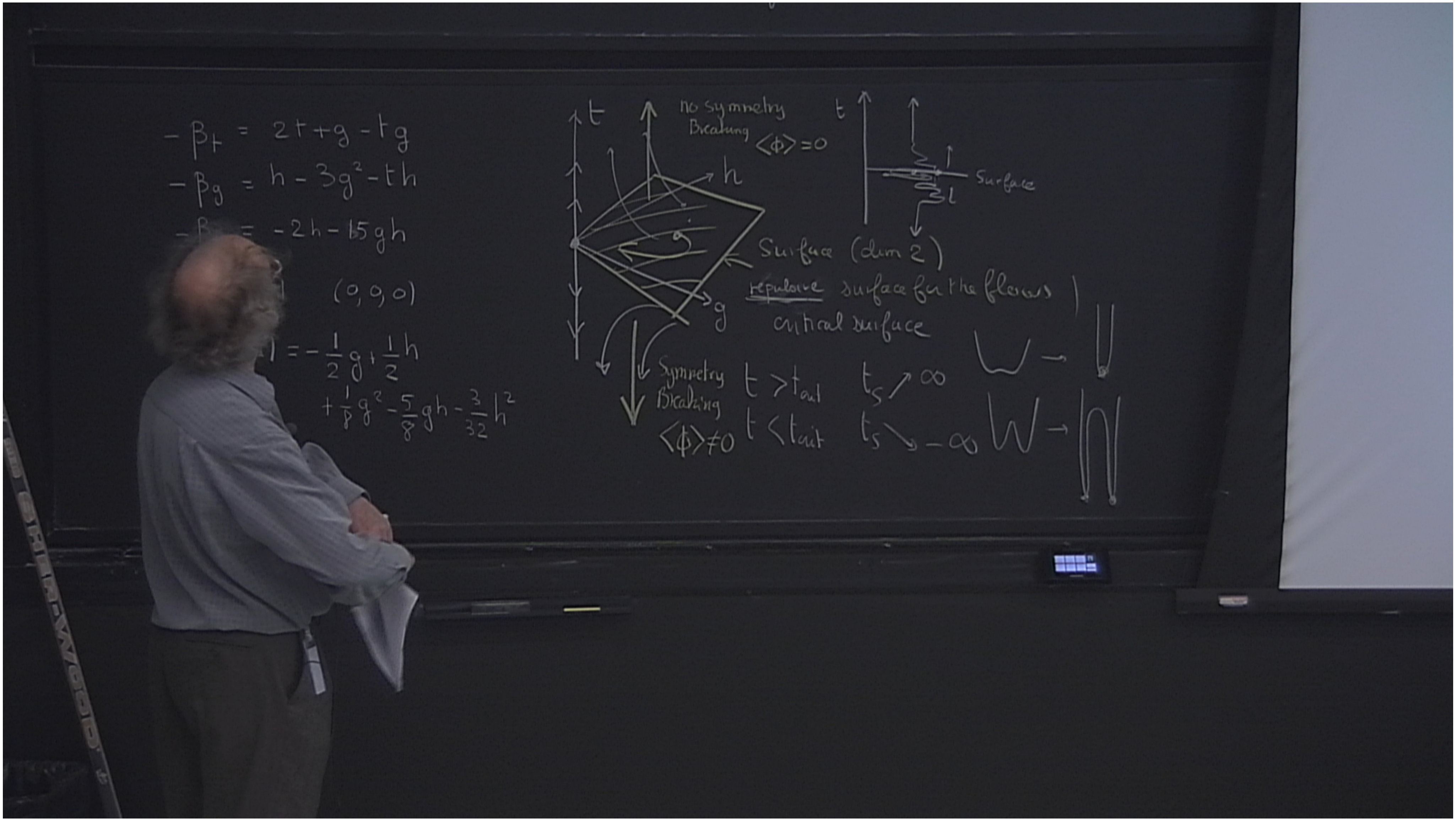
$-\beta_t = 2t + g - tg$
$-\beta_g = h - 3g^2 - th$
Surface (dim 2)
(stable surface for the flows)


-βt = 2t + g - tg
-βg = h - 3g² - th
-βh = -2h - 15gh
Fixed point (0,0,0)
t
h
g
Surface (dim 2)
(stable surface for the flows)

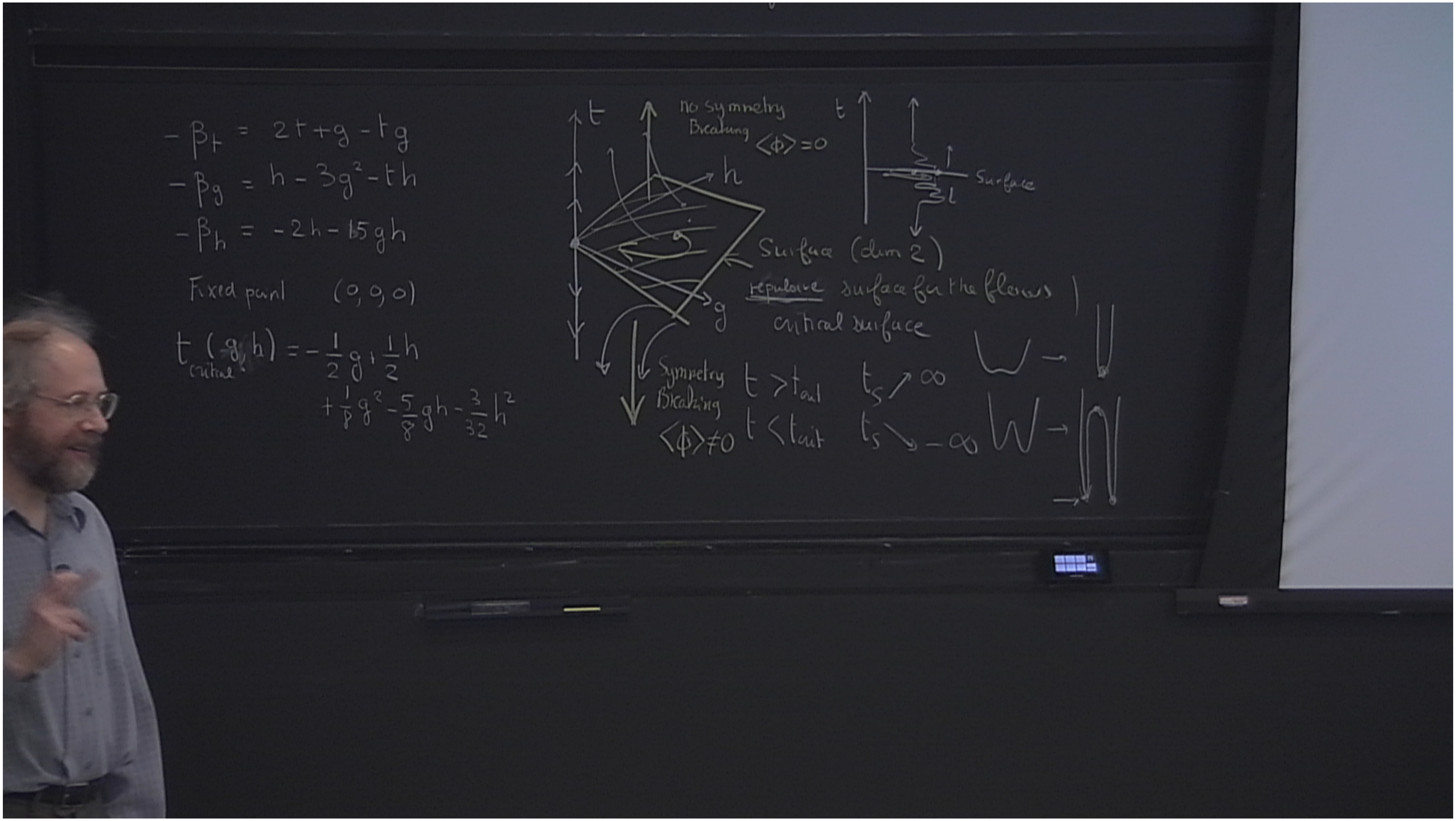


$-\beta_t = 2t + g - tg$
$-\beta_g = h - 3g^2 - th$
Surface (dim 2)
(stable surface for the flows)
critical surface

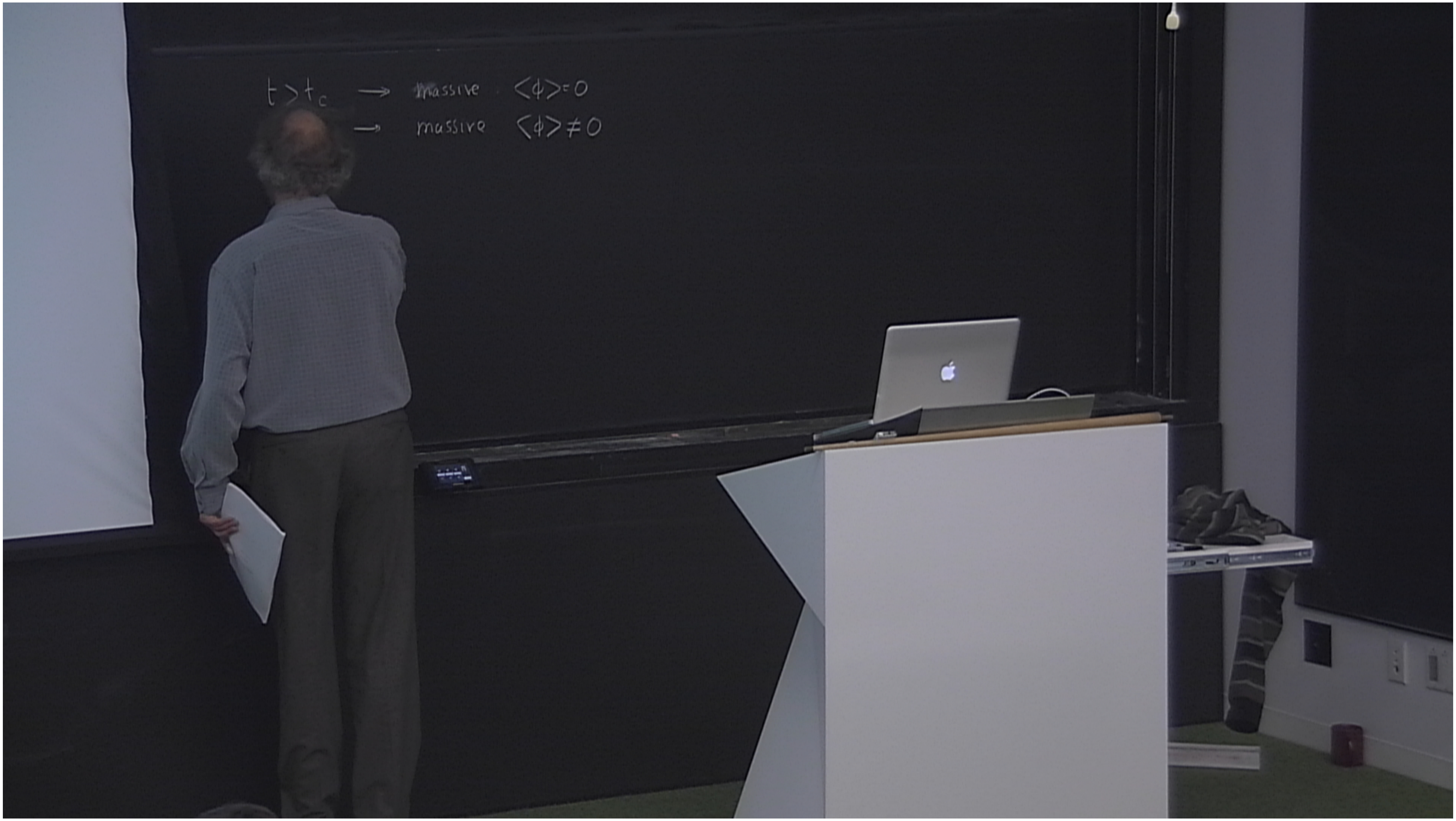

−β_t = 2t + g − tg
−β_g = h − 3g² − th
−β_h = −2h − 15gh
Fixed point (0,0,0)
t_critical(g,h) = −1/2 g + 1/2 h
+ 1/8 g² − 5/8 gh − 3/32 h²
Surface (dim 2)
surface for the flows
Surface

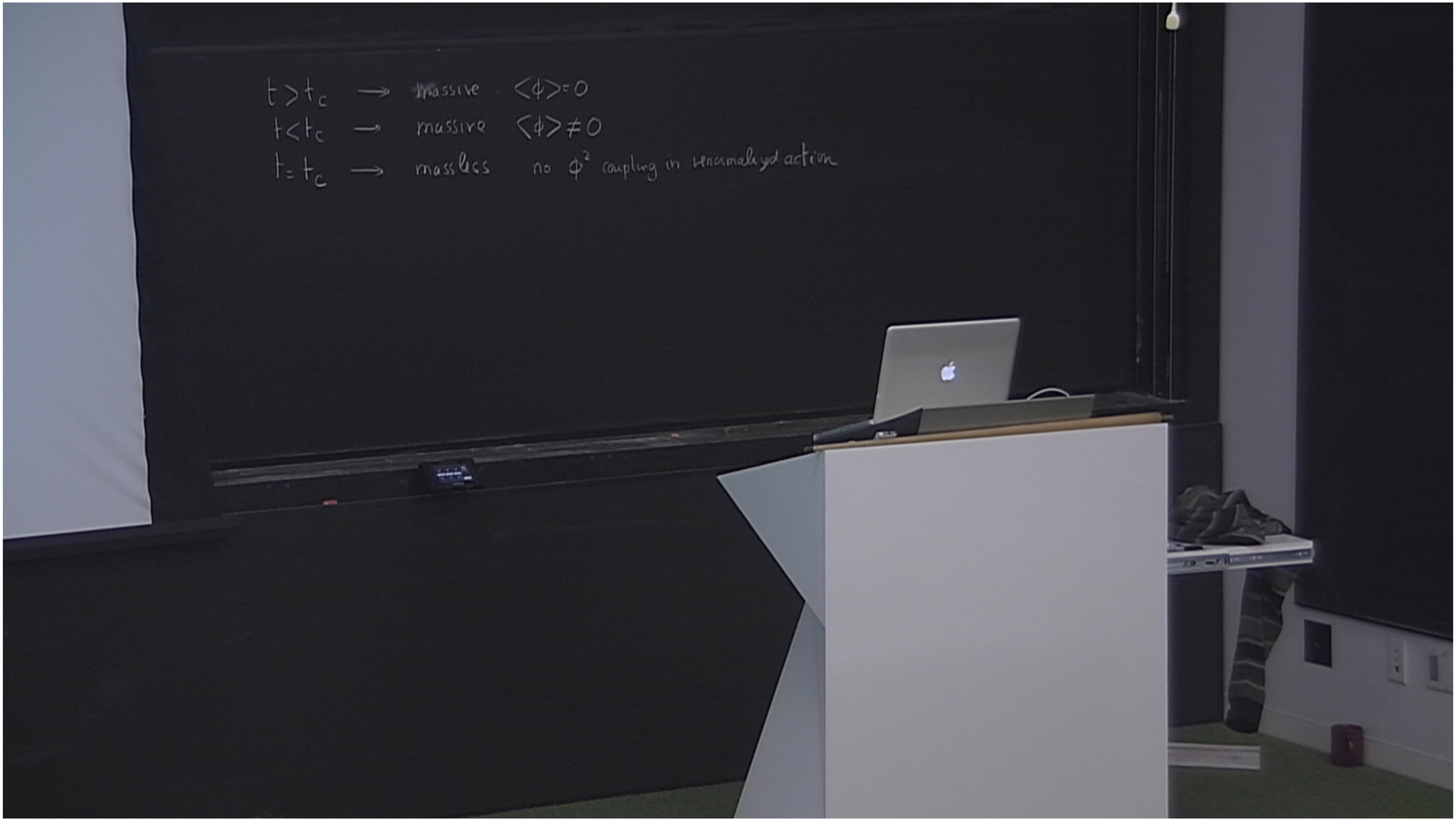


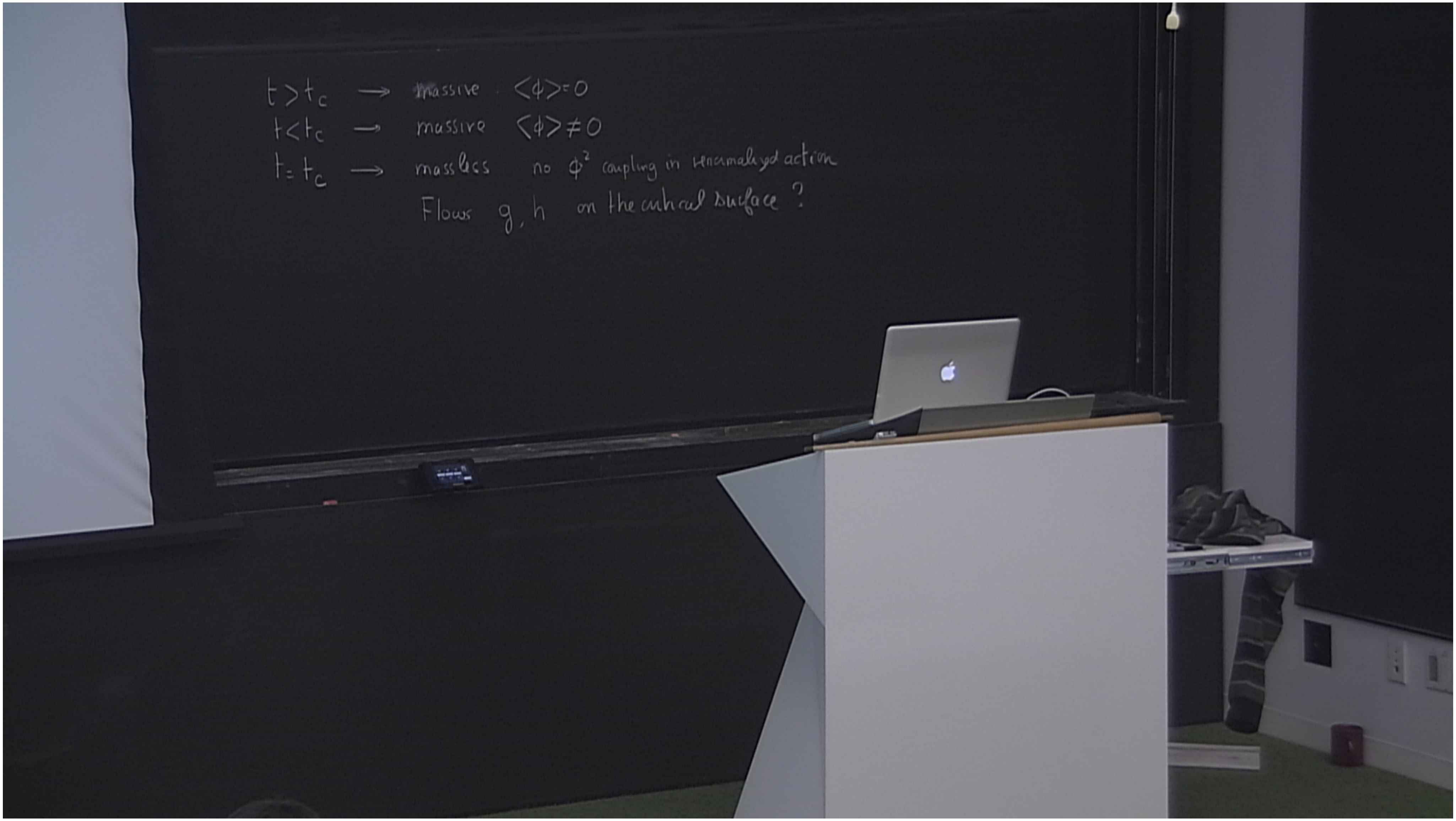
$-\beta_t = 2t + g - tg$
$-\beta_g = h - 3g^2 - th$
$= -2h - 15gh$
$(0,0,0)$
$= -\frac{1}{2}g + \frac{1}{2}h$
$+ \frac{1}{8}g^2 - \frac{5}{8}gh - \frac{3}{32}h^2$
no symmetry Breaking
$\langle\phi\rangle = 0$
Surface
Surface (dim 2)
repulsive surface for the flows
critical surface
Symmetry Breaking
$\langle\phi\rangle \neq 0$
$t > t_{crit}$ $t_s \nearrow \infty$
$t < t_{crit}$ $t_s \searrow -\infty$

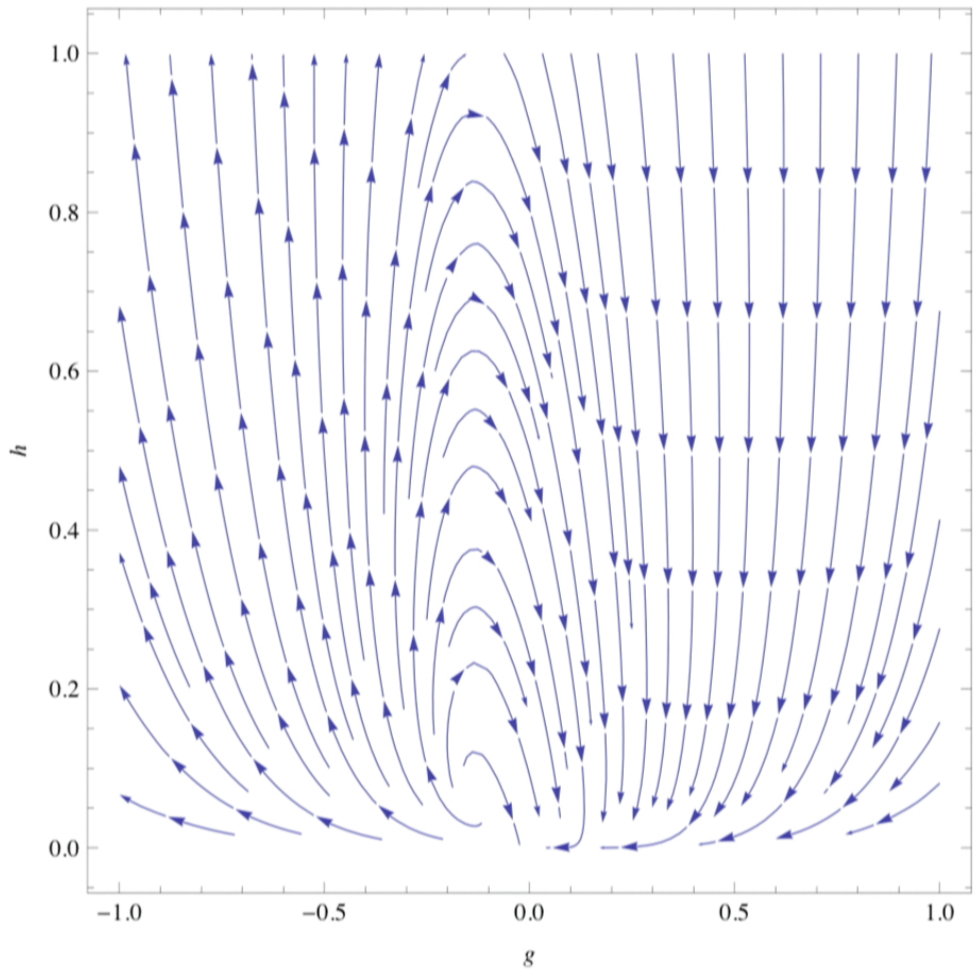


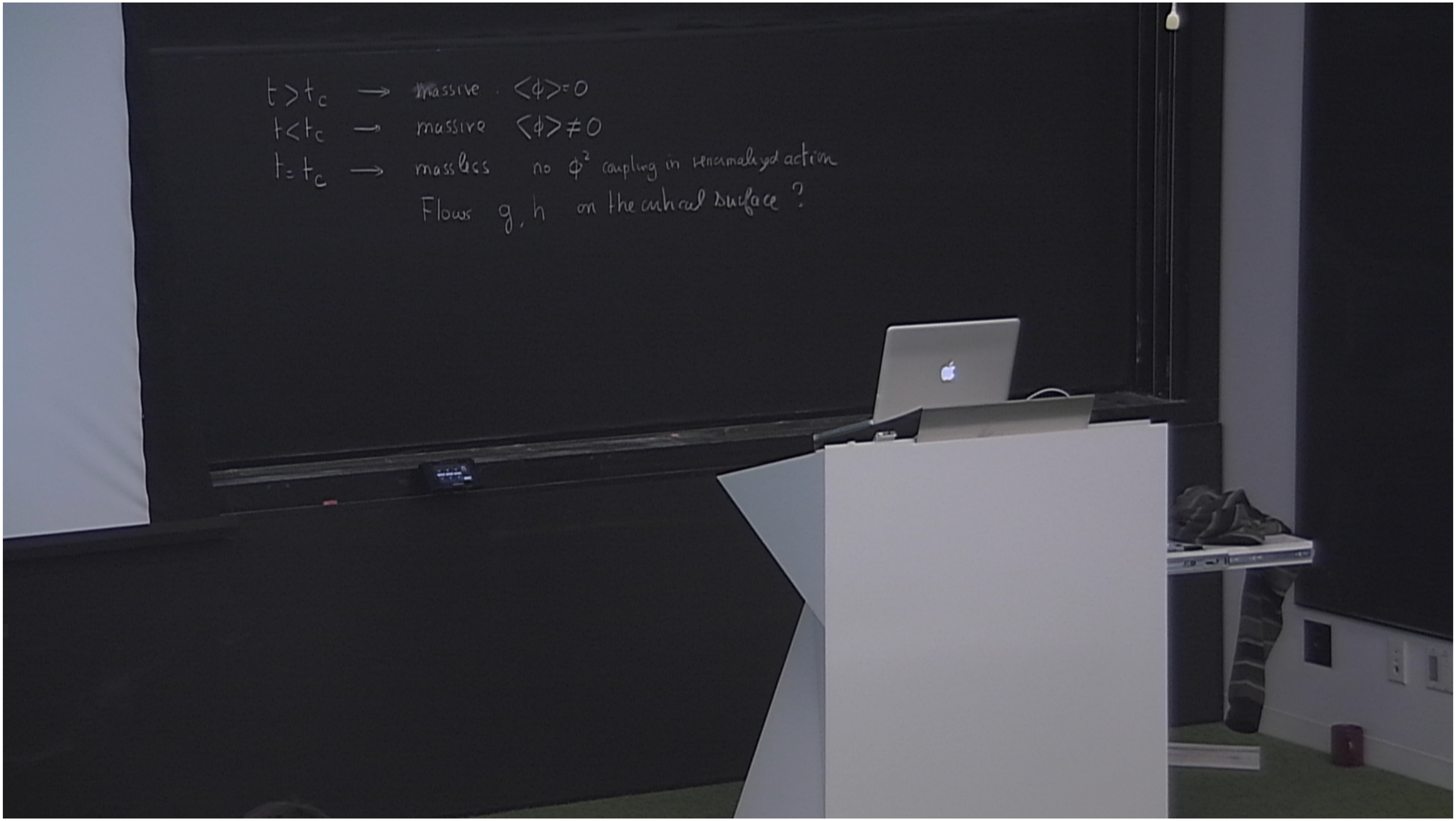
$-\beta_t = 2t + g - tg$
$-\beta_g = h - 3g^2 - th$
$-\beta_h = -2h - 15gh$
Fixed point (0, 0, 0)
$t_{critical}(g, h) = -\frac{1}{2}g + \frac{1}{2}h + \frac{1}{8}g^2 - \frac{5}{8}gh - \frac{3}{32}h^2$
t
h
g
no symmetry Breaking
$\langle\phi\rangle = 0$
Surface (dim 2)
repulsive surface for the flows
critical surface
Symmetry Breaking
$\langle\phi\rangle \neq 0$
$t > t_{crit}$ $t_s \nearrow \infty$
$t < t_{crit}$ $t_s \searrow -\infty$
Surface


$t > t_c \longrightarrow$ massive $\langle\phi\rangle = 0$
$\longrightarrow$ massive $\langle\phi\rangle \neq 0$

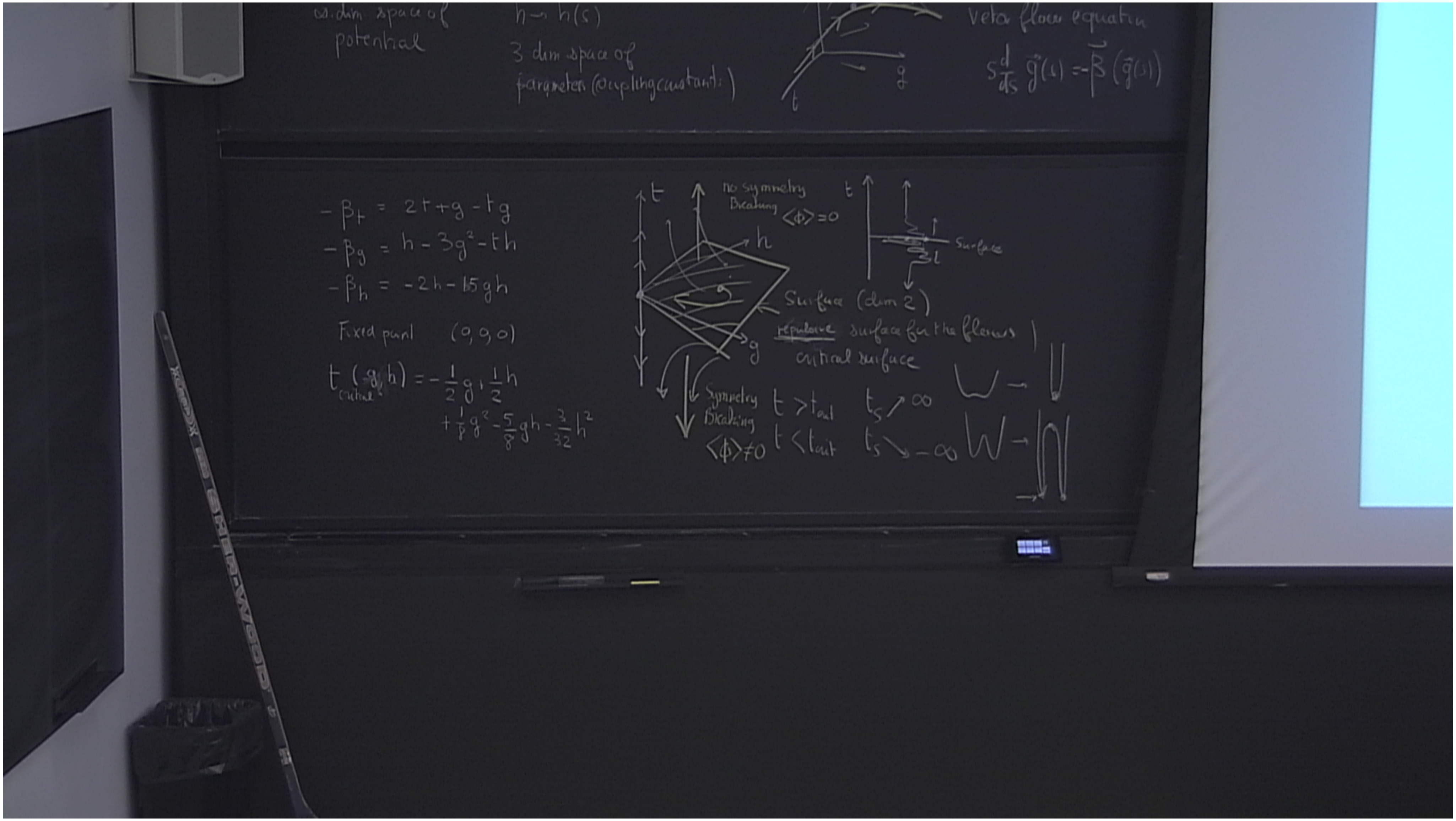

$t > t_c \longrightarrow$ massive $\langle \phi \rangle = 0$
$t < t_c \longrightarrow$ massive $\langle \phi \rangle \neq 0$
$t = t_c \longrightarrow$ massless no ϕ^2 coupling in renormalized action


t > t_c → massive <φ>=0
t < t_c → massive <φ> ≠ 0
t = t_c → massless no φ² coupling in renormalized action
Flows g, h on the critical surface ?

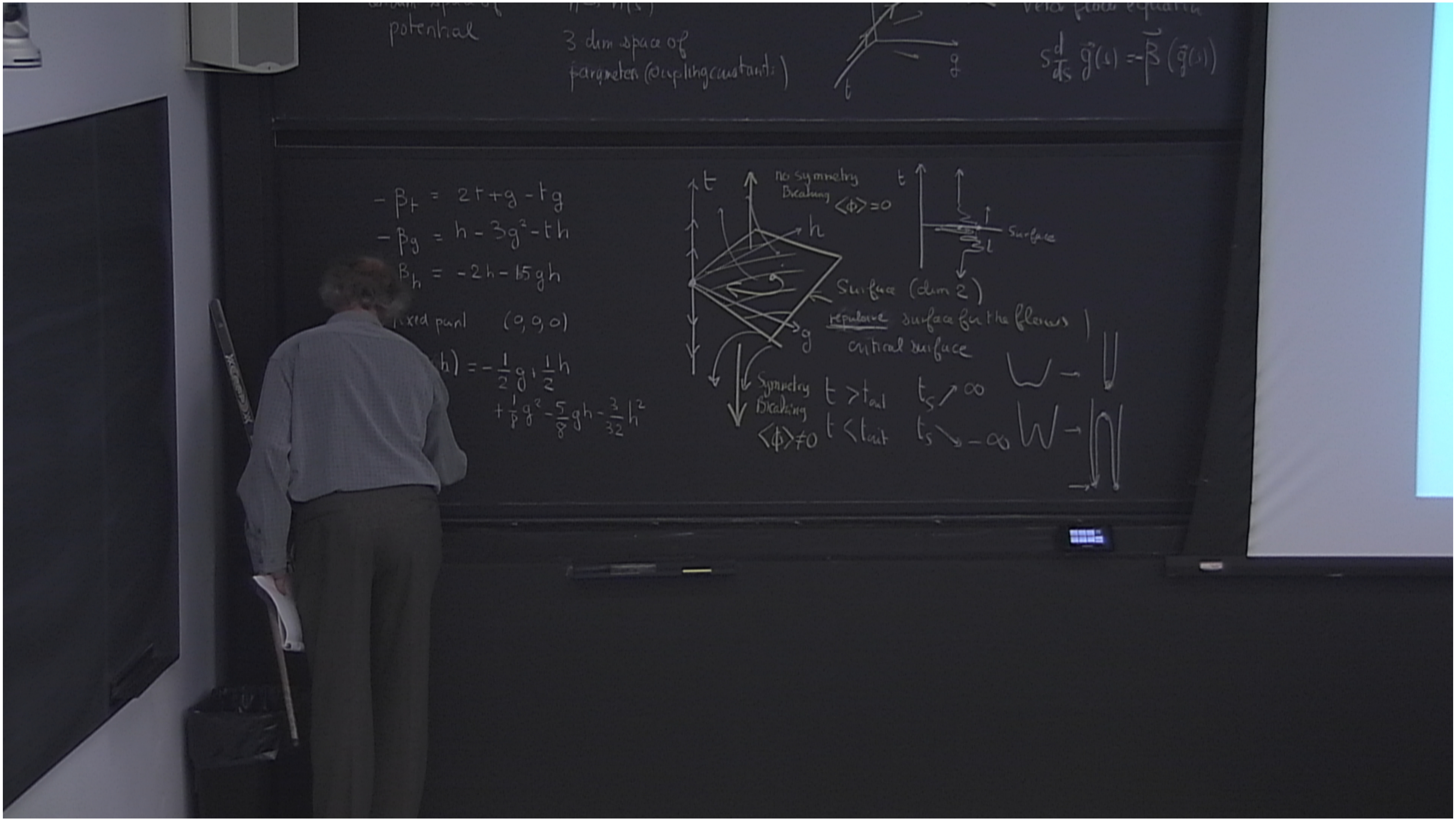

1.0
0.8
0.6
0.4
0.2
0.0
h
−1.0
−0.5
0.0
0.5
1.0
g

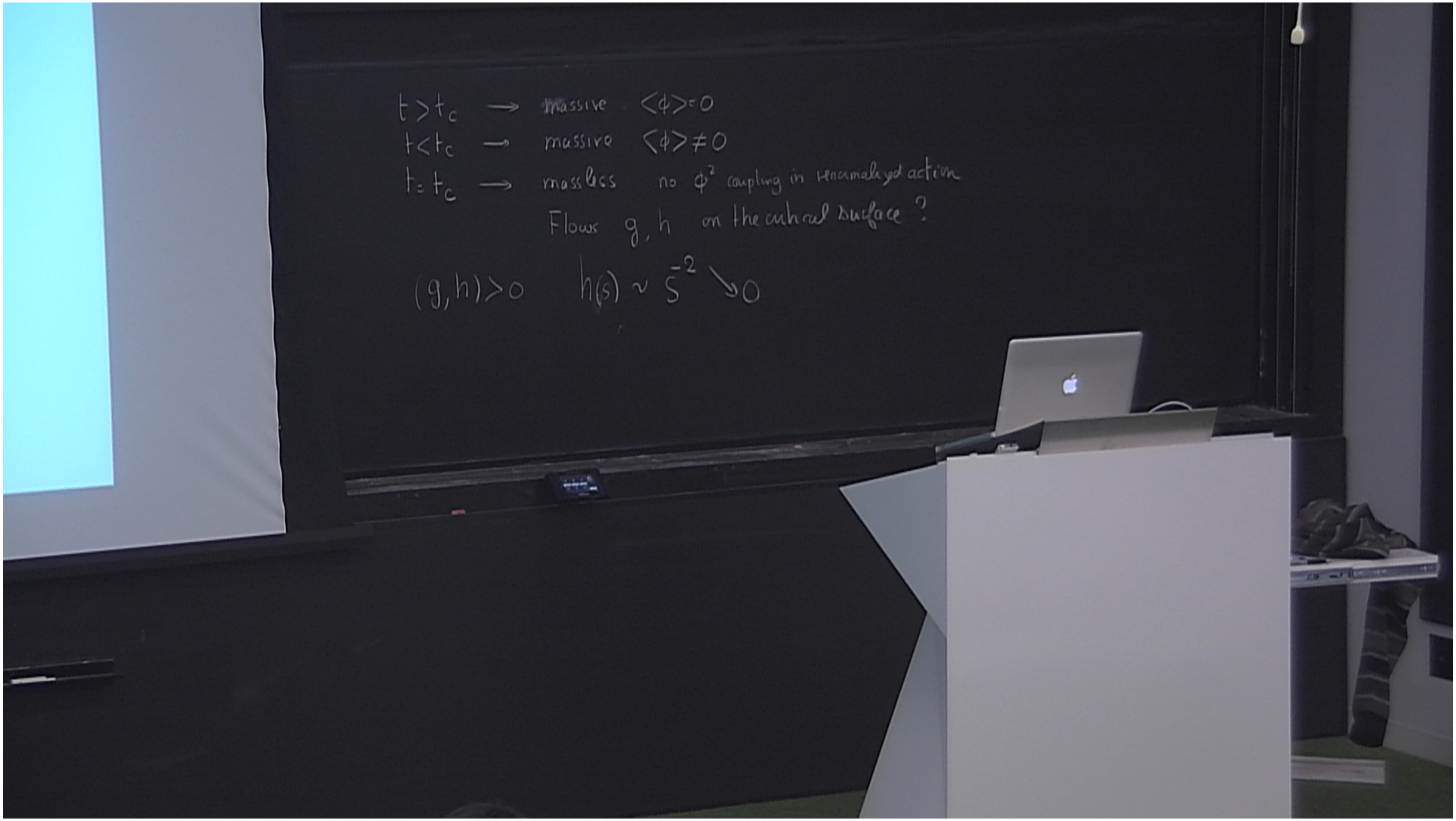


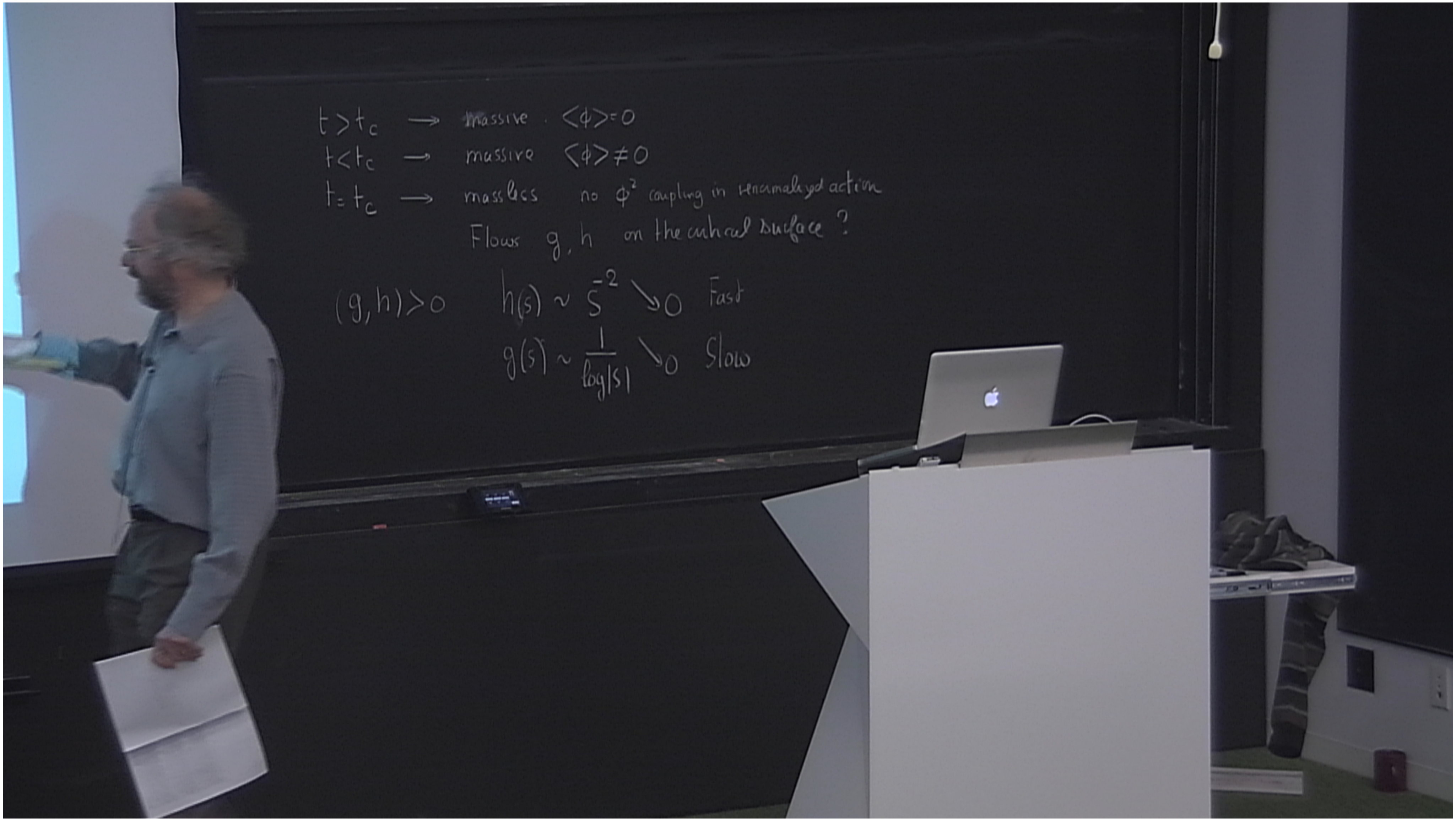
$t > t_c \longrightarrow$ massive $\langle\phi\rangle = 0$
$t < t_c \longrightarrow$ massive $\langle\phi\rangle \neq 0$
$t = t_c \longrightarrow$ massless no ϕ^2 coupling in renormalized action
Flows g, h on the critical surface ?

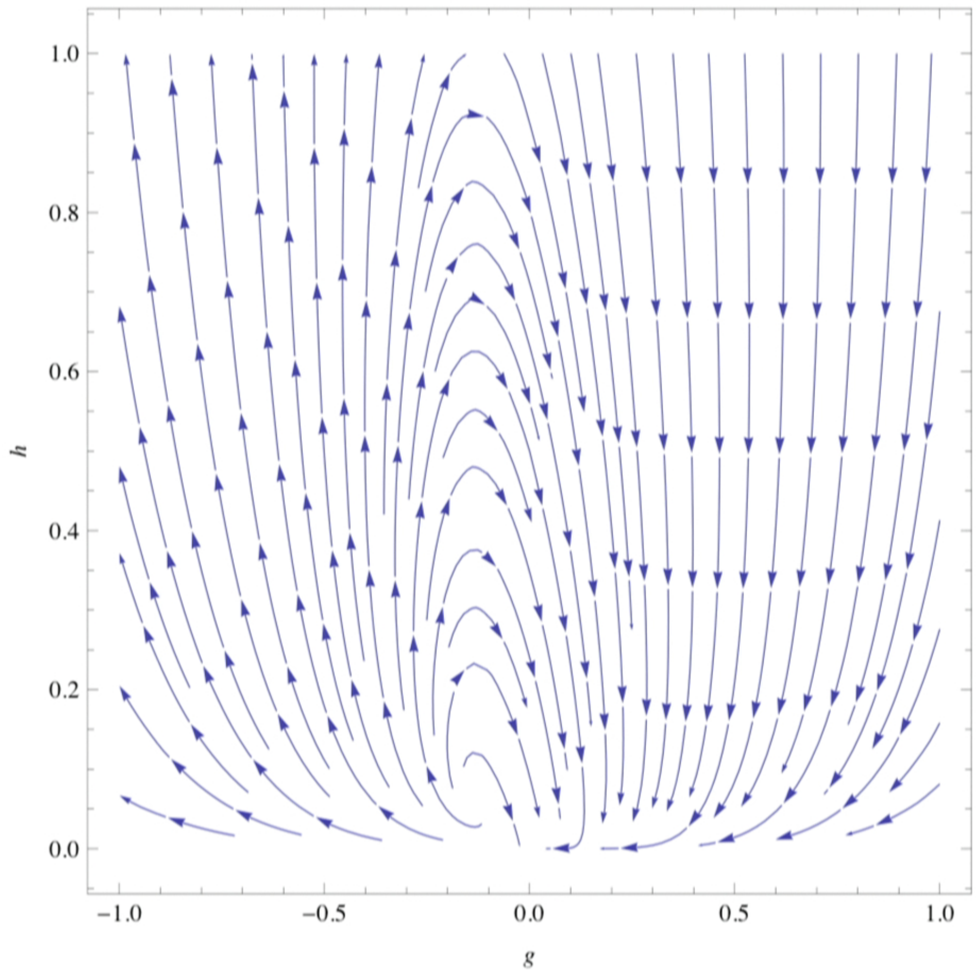


potential
$h \to h(s)$
3 dim space of parameters (coupling constants)
beta flow equation
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$-\beta_t = 2t + g - tg$
$-\beta_g = h - 3g^2 - th$
$-\beta_h = -2h - 15gh$
Fixed point (0,0,0)
$t_{critical}(g,h) = -\frac{1}{2}g + \frac{1}{2}h + \frac{1}{8}g^2 - \frac{5}{8}gh - \frac{3}{32}h^2$
no symmetry Breaking $\langle\phi\rangle = 0$
Surface (dim 2)
repulsive surface for the flows
critical surface
Symmetry Breaking $\langle\phi\rangle \neq 0$
$t > t_{crit}$ $t_s \nearrow \infty$
$t < t_{crit}$ $t_s \searrow -\infty$

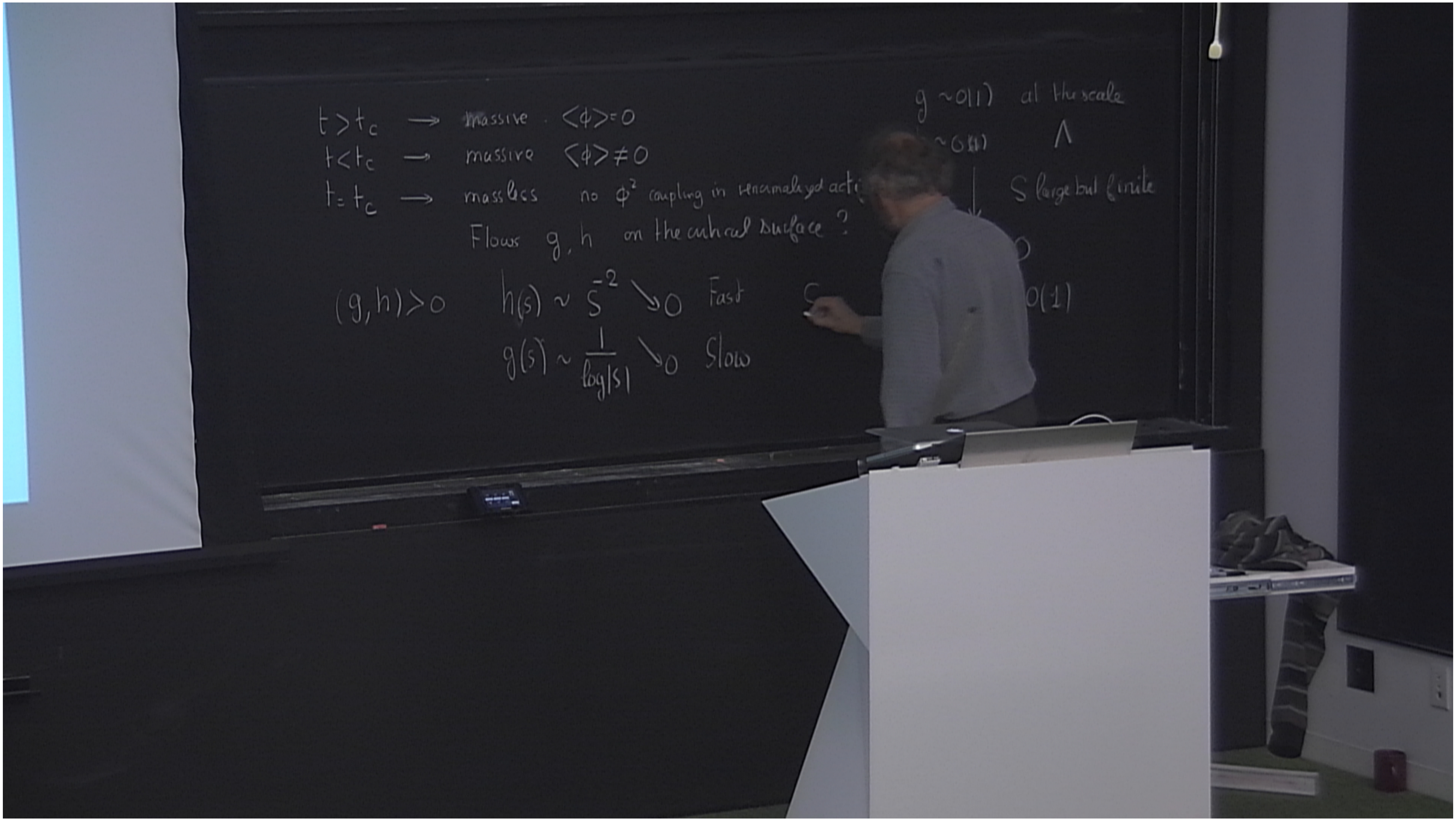

potential
3 dim space of
parameter (coupling constants)
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$-\beta_t = 2t + g - tg$
$-\beta_g = h - 3g^2 - th$
$\beta_h = -2h - 15gh$
fixed point (0,0,0)
no symmetry Breaking
$\langle\phi\rangle = 0$
Surface (dim 2)
repulsive surface for the flows
critical surface
Symmetry Breaking
$\langle\phi\rangle \neq 0$
$t > t_{crit}$
$t < t_{crit}$
$t_s \nearrow \infty$
$t_s \searrow -\infty$

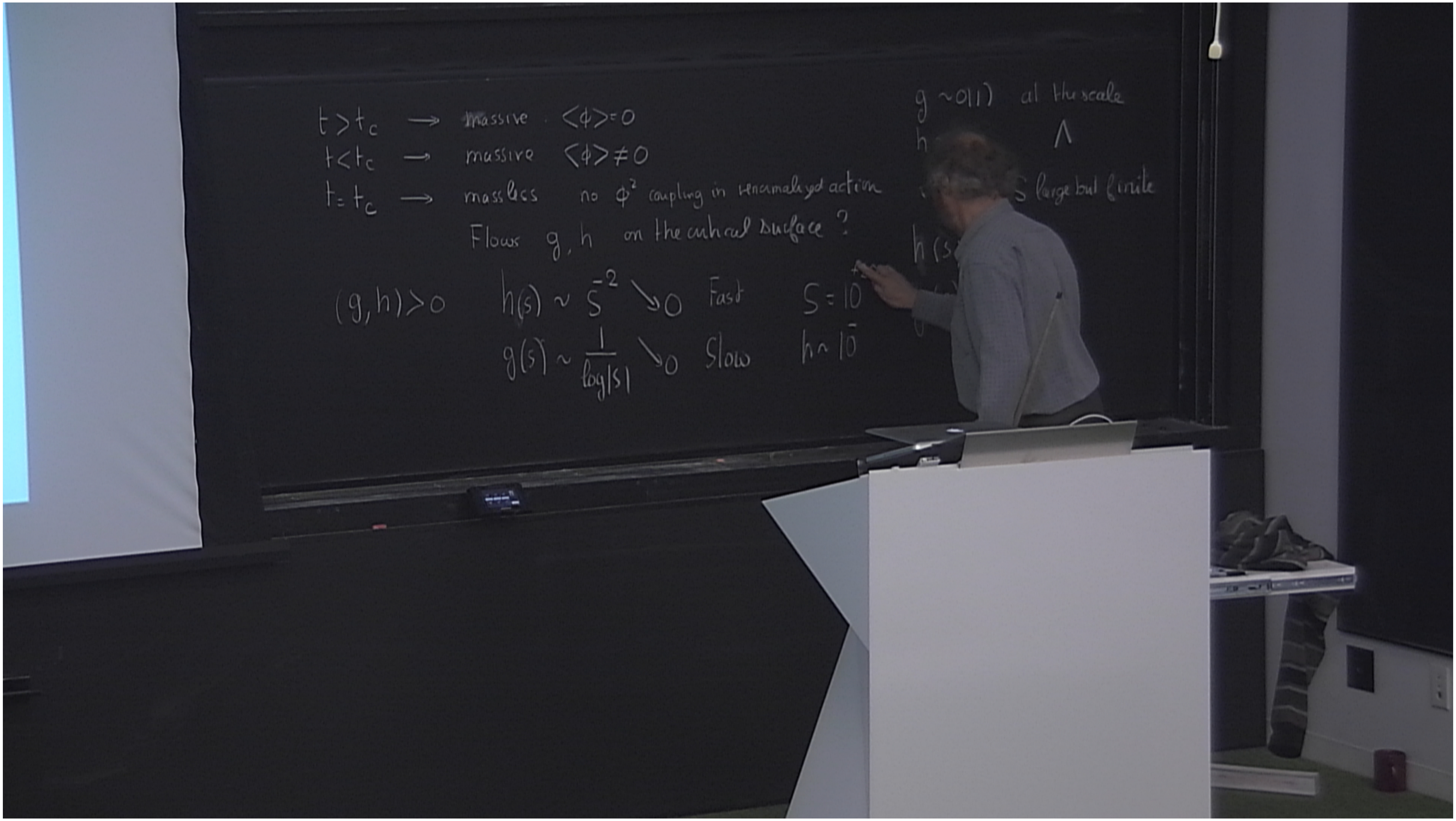

t > t_c → massive ⟨φ⟩ = 0
t < t_c → massive ⟨φ⟩ ≠ 0
t = t_c → massless no φ² coupling in renormalized action
Flows g, h on the critical surface ?
(g,h) > 0 h(s) ~ s^-2 ↘ 0

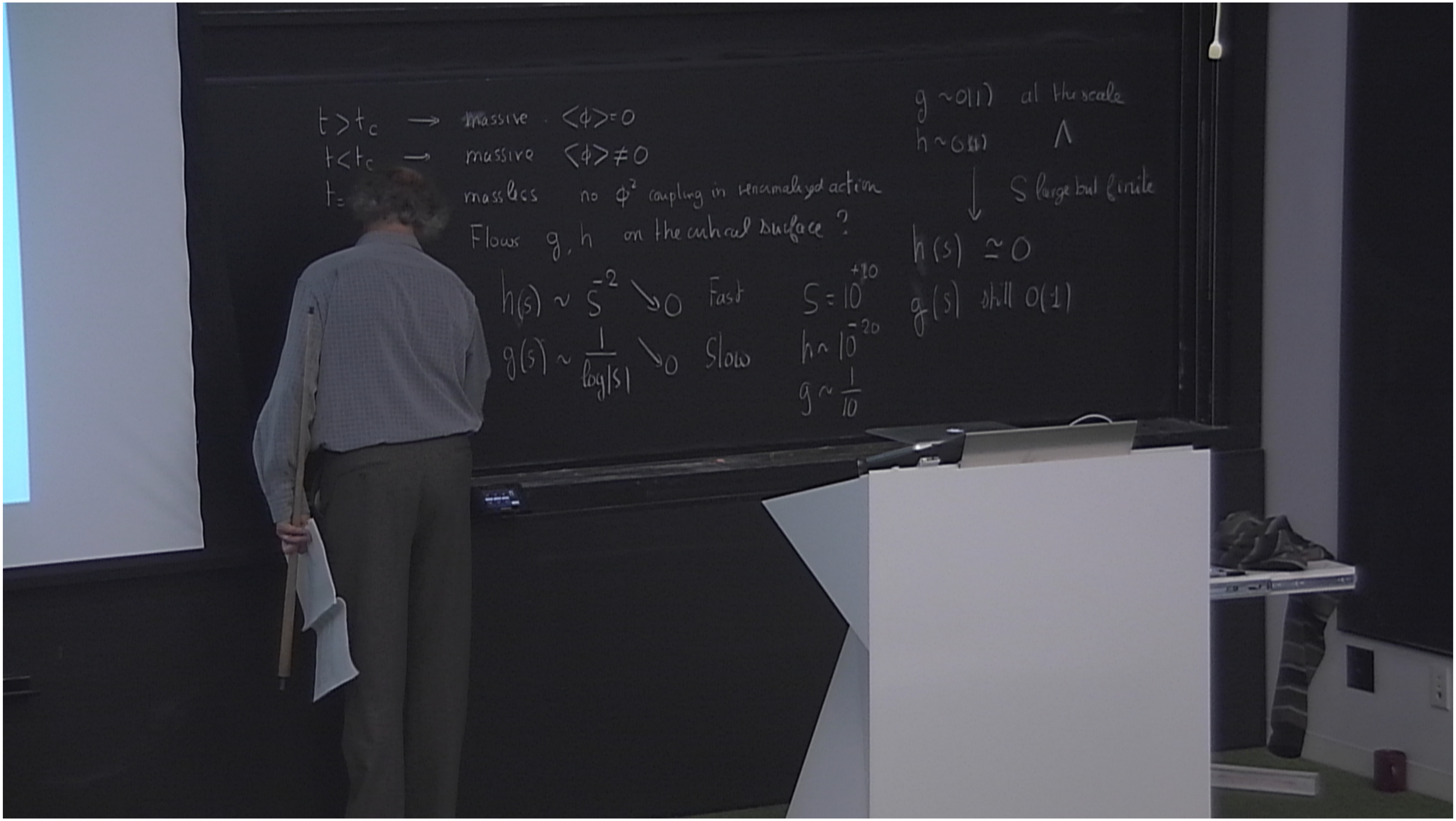

t > t_c → massive . <φ>=0
t<t_c → massive <φ>≠0
t=t_c → massless no φ^2 coupling in renormalized action
Flows g, h on the critical surface ?
(g,h)>0 h(s) ~ s^-2 ↘0 Fast
g(s) ~ 1/log|s| ↘0 Slow

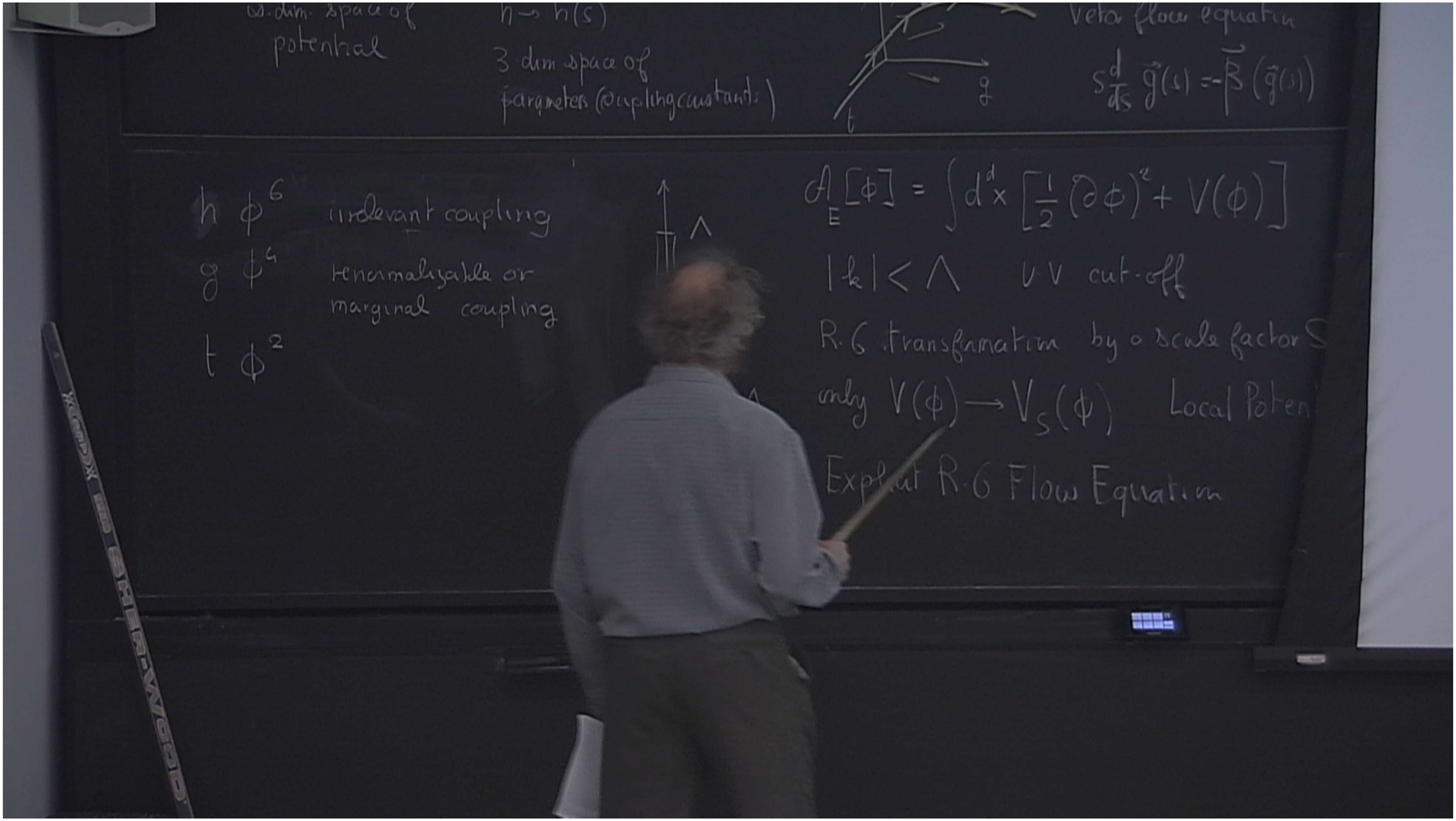


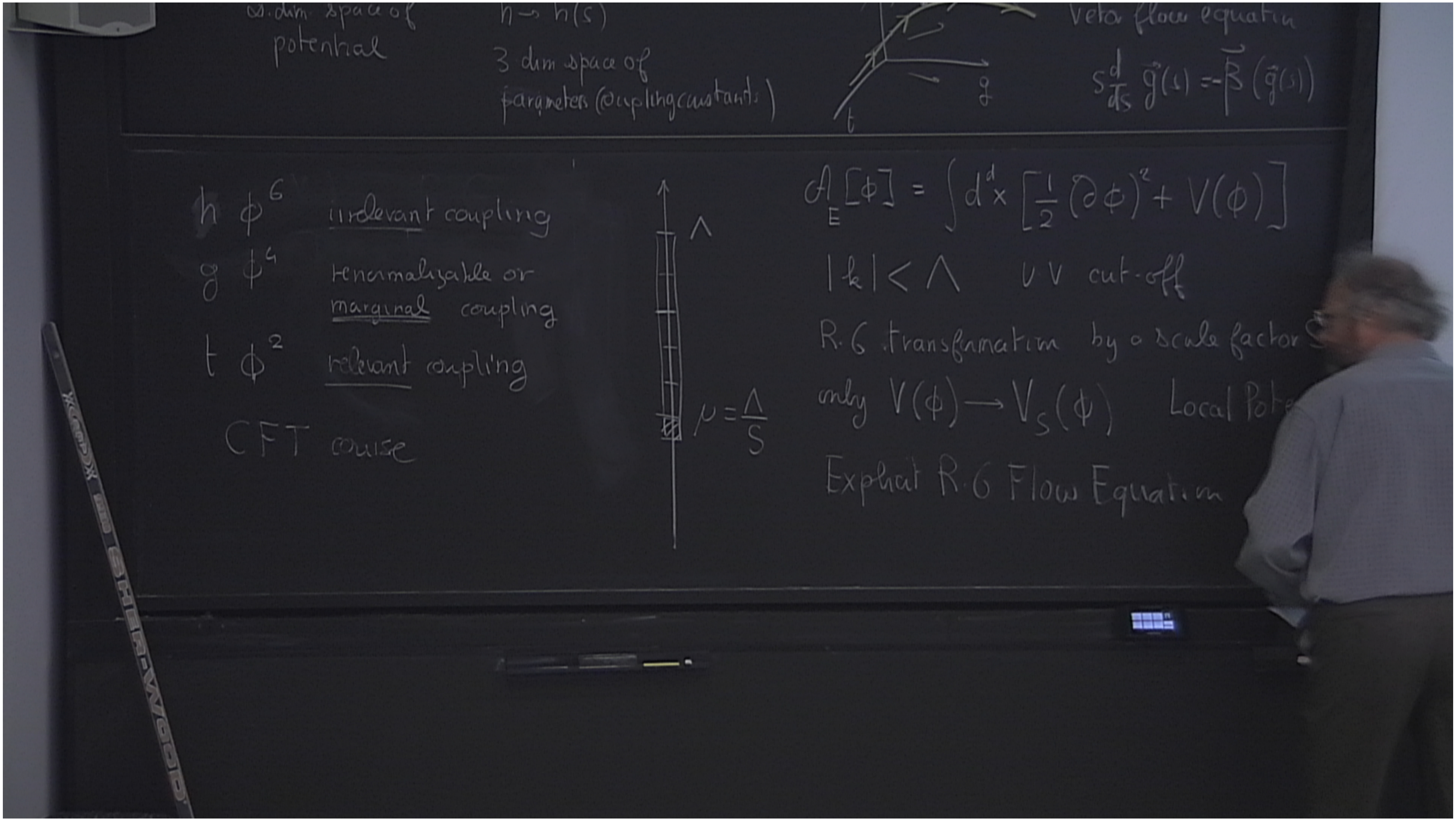
1.0
0.8
0.6
0.4
0.2
0.0
h
−1.0
−0.5
0.0
0.5
1.0
g

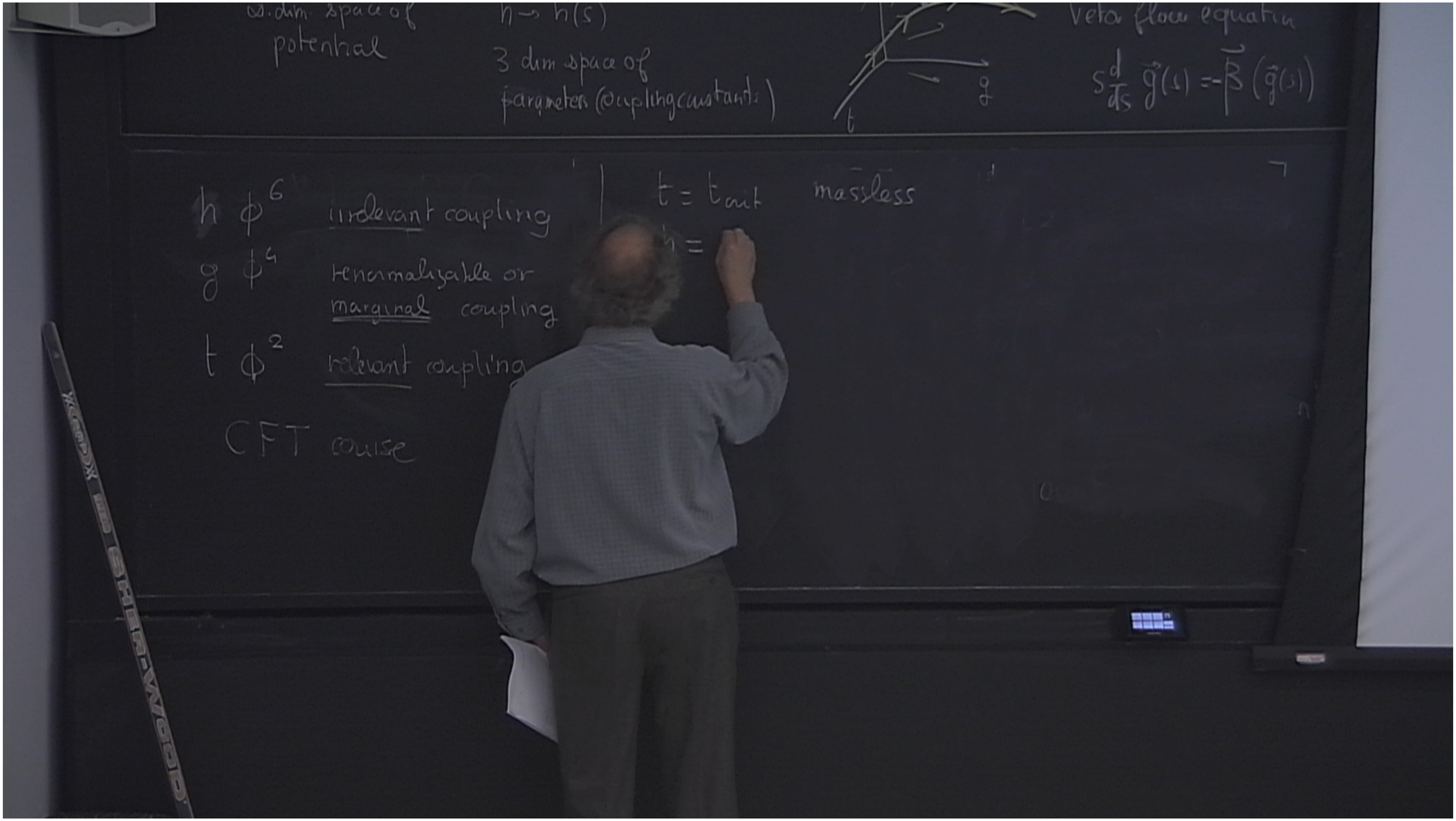

t > t_c → massive <φ> = 0
t < t_c → massive <φ> ≠ 0
t = t_c → massless no φ² coupling in renormalized
Flows g, h on the critical surface ?
(g,h) > 0 h(s) ~ s^-2 ↘ 0 Fast
g(s) ~ 1/log|s| ↘ 0 Slow
g ~ O(1) at the scale
Λ
S large but finite
O(1)

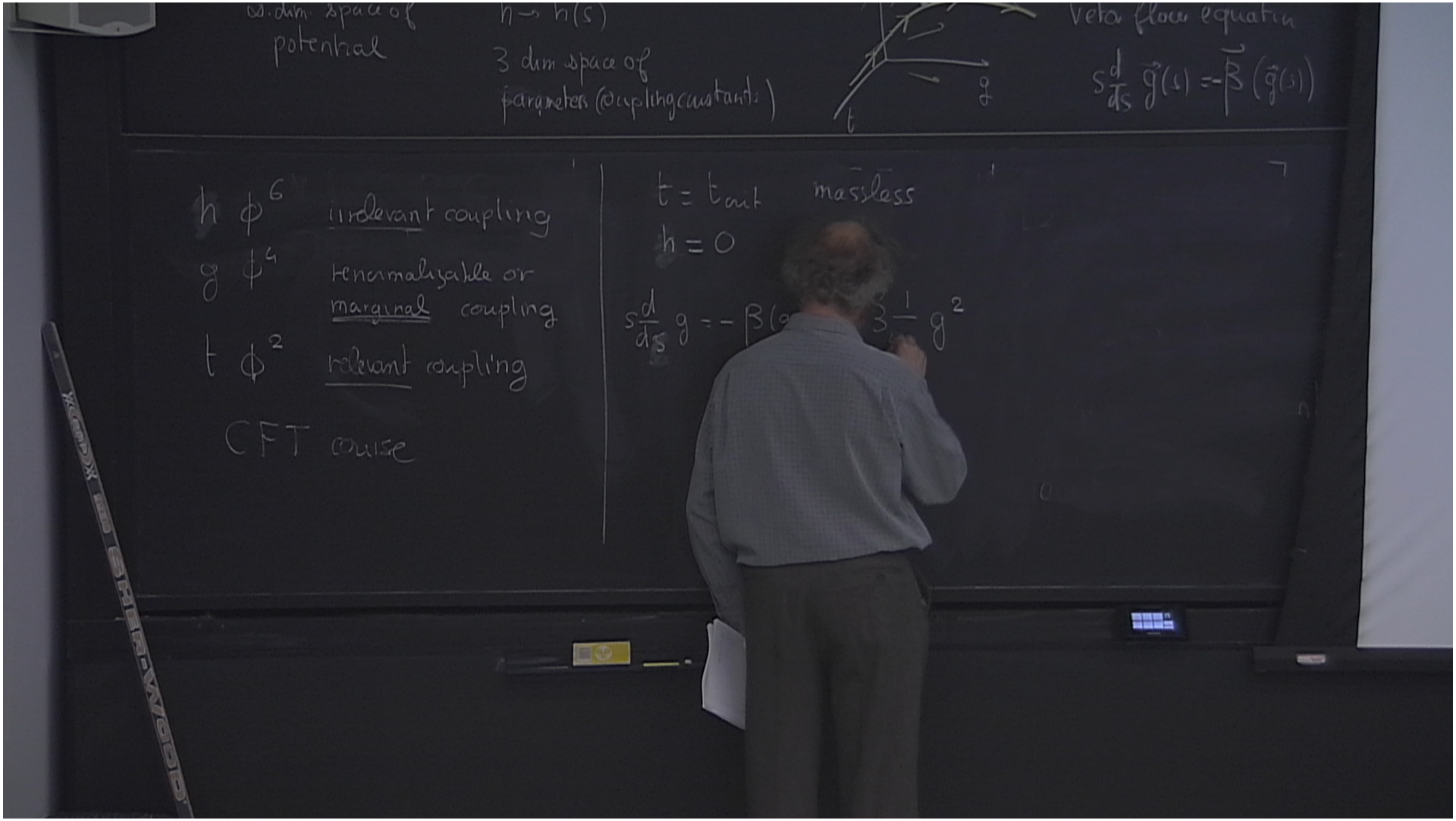

t > tc → massive <φ>=0
t < tc → massive <φ>≠0
t = tc → massless no φ² coupling in renormalized action
Flows g, h on the critical surface ?
(g, h) > 0
h(s) ~ s⁻² ↘ 0 Fast
g(s) ~ 1/log|s| ↘ 0 Slow
g ~ O(1) at the scale
S large but finite

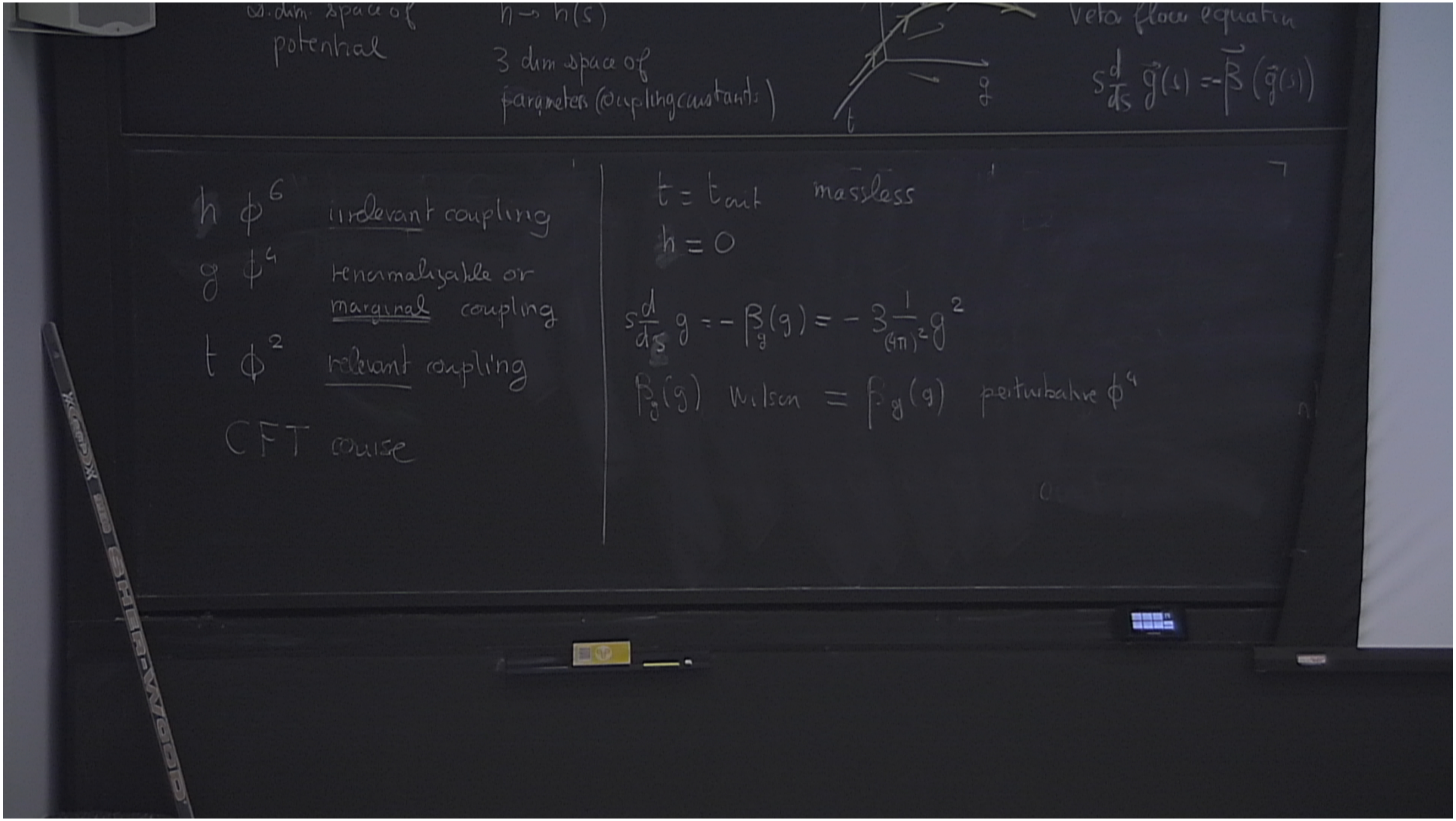

t > t_c → massive <φ>=0
t < t_c → massive <φ>≠0
massless no φ² coupling in renormalized action
Flows g, h on the critical surface ?
h(s) ~ s^-2 ↘ 0 Fast
g(s) ~ 1/log|s| ↘ 0 Slow
s = 10^+10
h ~ 10^-20
g ~ 1/10
g ~ O(1) at the scale
h ~ O(1)
Λ
s large but finite
h(s) ≃ 0
g(s) still O(1)

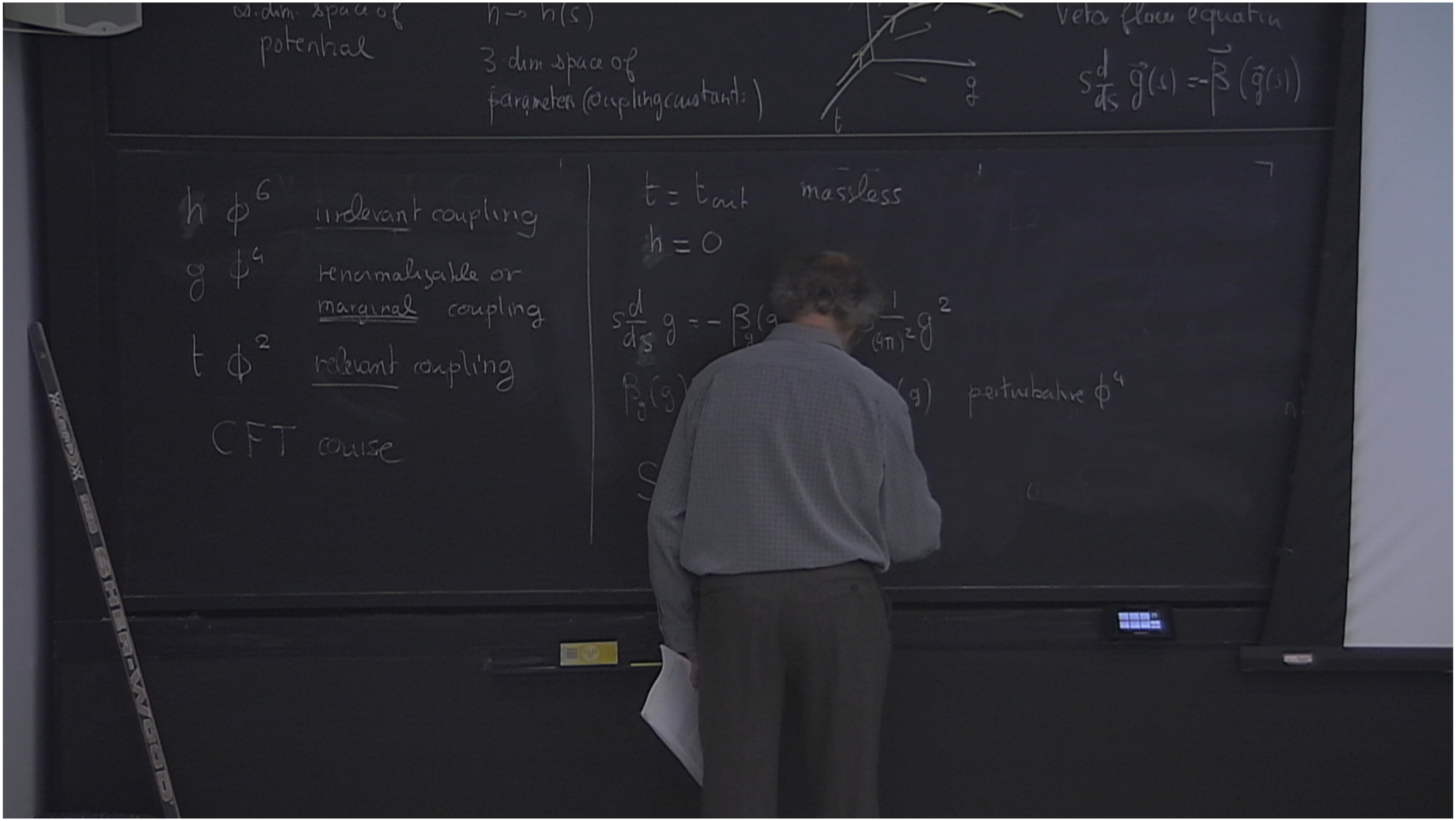

potential
3 dim space of
parameters (coupling constants)
Beta flow equation
$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$h\,\phi^6$ irrelevant coupling
$g\,\phi^4$ renormalizable or marginal coupling
$t\,\phi^2$
$A_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V cut-off
R.G transformation by a scale factor S
only $V(\phi) \to V_S(\phi)$ Local Poten
Explicit R.G Flow Equation

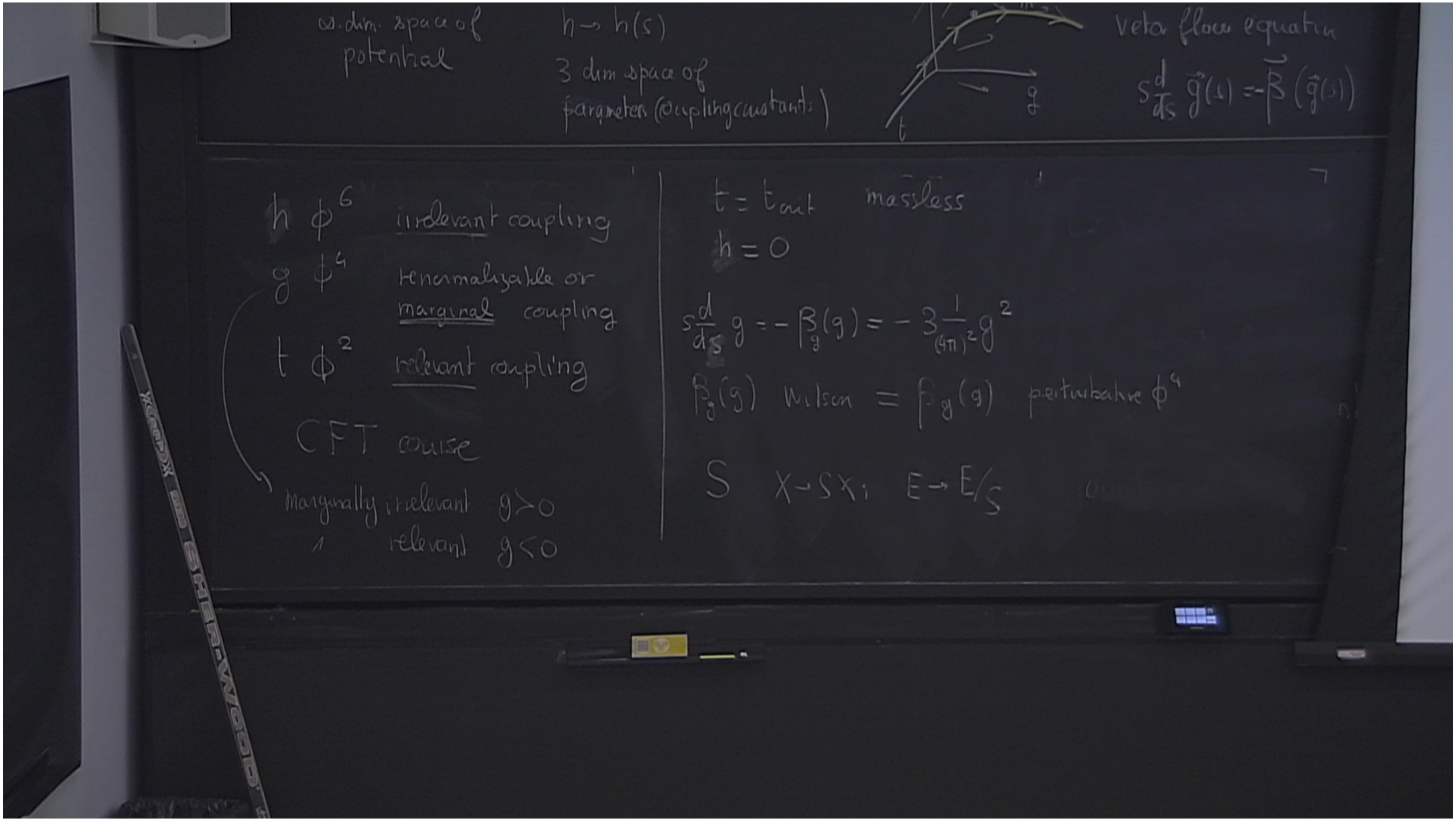

potential
3 dim space of
parameters (coupling constants)
$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$h\ \phi^6$ irrelevant coupling
$g\ \phi^4$ renormalizable or marginal coupling
$t\ \phi^2$ relevant coupling
CFT course
Λ
$\mu = \frac{\Lambda}{S}$
$A_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$
$|k| < \Lambda$ U.V cut-off
R.G transformation by a scale factor
only $V(\phi) \rightarrow V_S(\phi)$ Local Pote
Explicit R.G Flow Equation

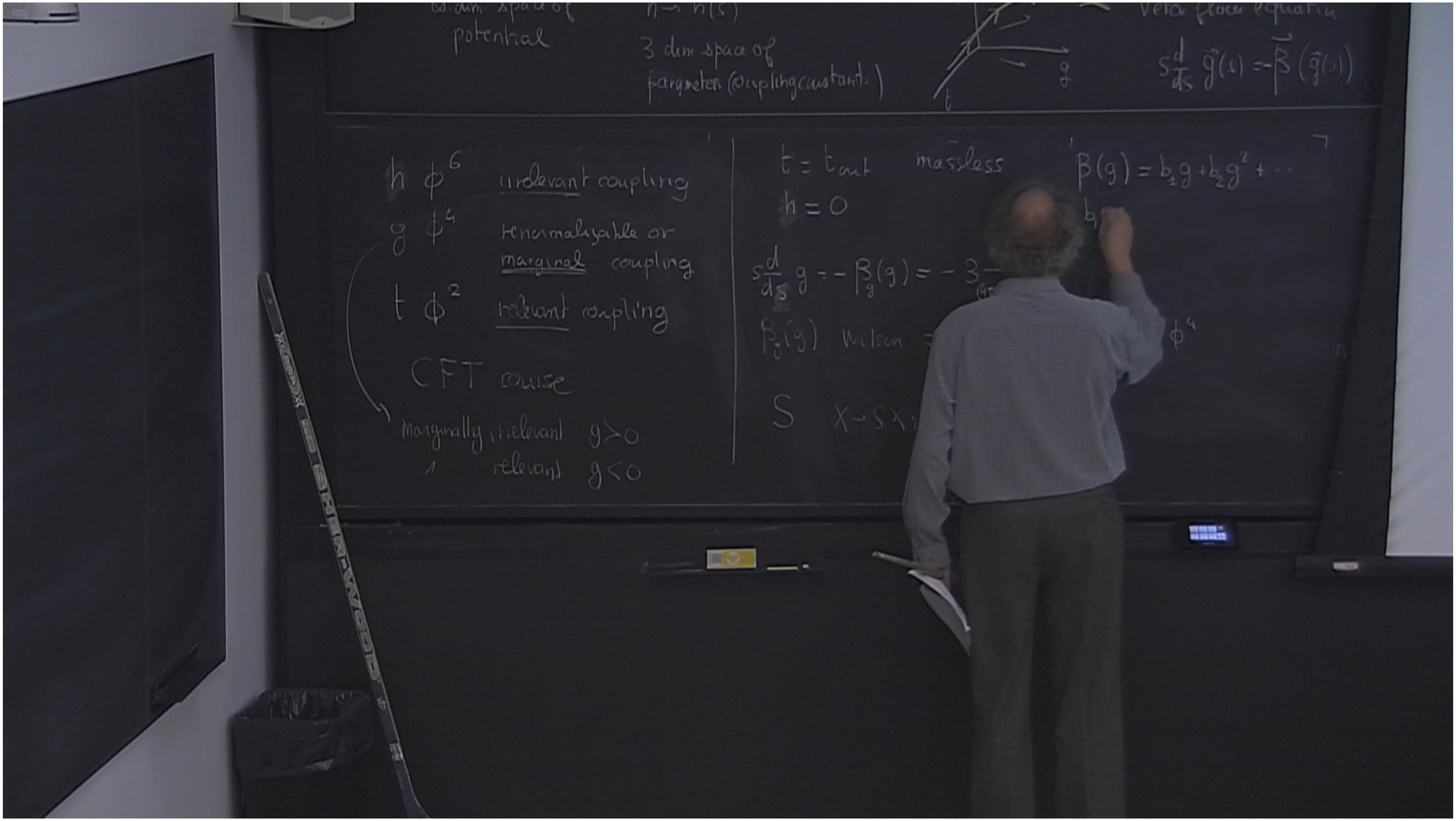

potential
3 dim space of
parameters (coupling constants)
beta flow equation
h φ^6 irrelevant coupling
g φ^4 renormalizable or
marginal coupling
t φ^2 relevant coupling
CFT course
t = t_crit massless

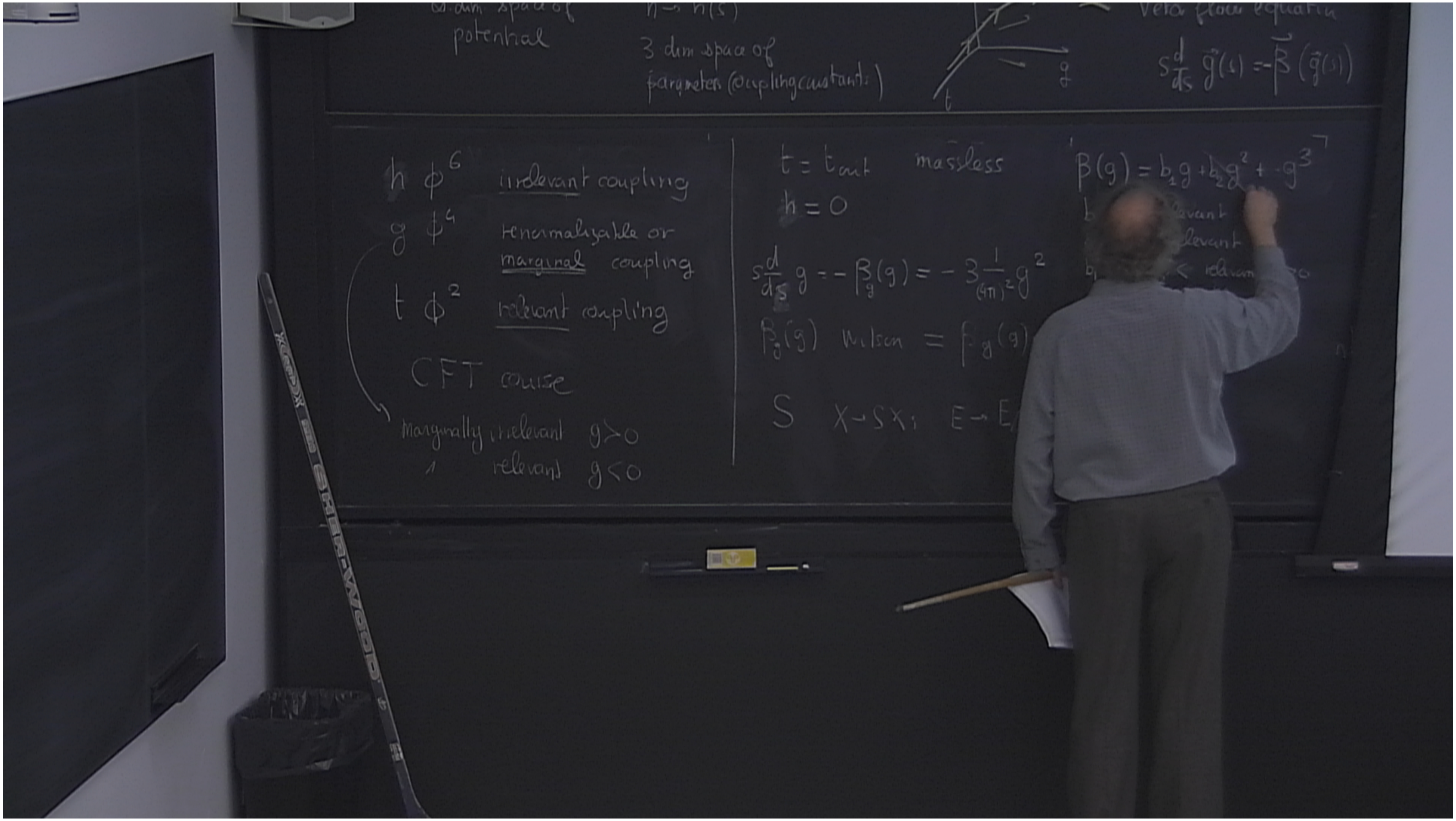

potential
3 dim space of
parameters (coupling constants)
beta flow equation
irrelevant coupling
renormalizable or
marginal coupling
relevant coupling
CFT course
massless
h = 0


potential
h → h(s)
3 dim space of
parameters (coupling constants)
beta flow equation
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$h\,\phi^6$ irrelevant coupling
$g\,\phi^4$ renormalizable or marginal coupling
$t\,\phi^2$ relevant coupling
CFT course
$t = t_{crit}$ massless
$h = 0$
$s\frac{d}{ds}g = -\beta_g(g) = -3\frac{1}{(4\pi)^2}g^2$
$\beta_g(g)$ Wilson $= \beta_g(g)$ perturbative ϕ^4


∞ dim. space of potential
h → h(s)
3 dim space of parameters (coupling constants)
Vector flow equation
h φ^6 irrelevant coupling
g φ^4 renormalizable or marginal coupling
t φ^2 relevant coupling
CFT course
t = t_crit massless
h = 0
perturbative φ^4


∞ dim. space of potential
h → h(s)
3 dim space of parameters (coupling constants)
g
t
Vector flow equation
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$h\,\phi^6$ irrelevant coupling
$g\,\phi^4$ renormalizable or marginal coupling
$t\,\phi^2$ relevant coupling
CFT course
marginally irrelevant $g>0$
relevant $g<0$
$t = t_{crit}$ massless
$h = 0$
$s\frac{d}{ds}g = -\beta_g(g) = -3\frac{1}{(4\pi)^2}g^2$
$\beta_g(g)$ Wilson $= \beta_g(g)$ perturbative ϕ^4
S $x \to sx;\ E \to E/s$


potential
3 dim space of
parameters (coupling constants)
g
t
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$h\ \phi^6$ irrelevant coupling
$g\ \phi^4$ renormalizable or marginal coupling
$t\ \phi^2$ relevant coupling
CFT course
marginally irrelevant $g>0$
relevant $g<0$
$t = t_{crit}$ massless
$h = 0$
$s\frac{d}{ds} g = -\beta_g(g) = -3$
$\beta_g(g)$ Wilson
S
$\beta(g) = b_1 g + b_2 g^2 + \cdots$
ϕ^4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$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$h\ \phi^6$ irrelevant coupling
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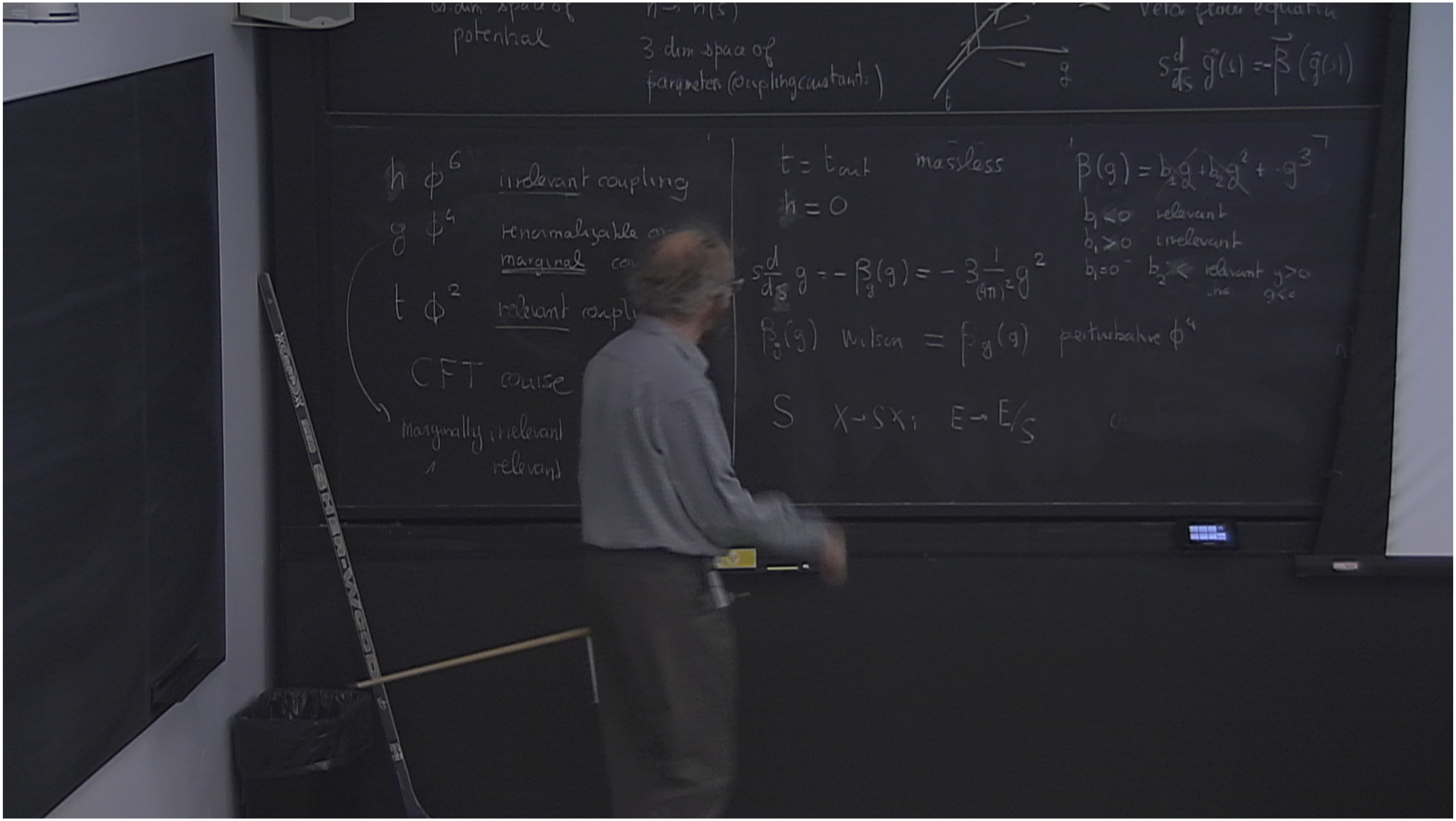
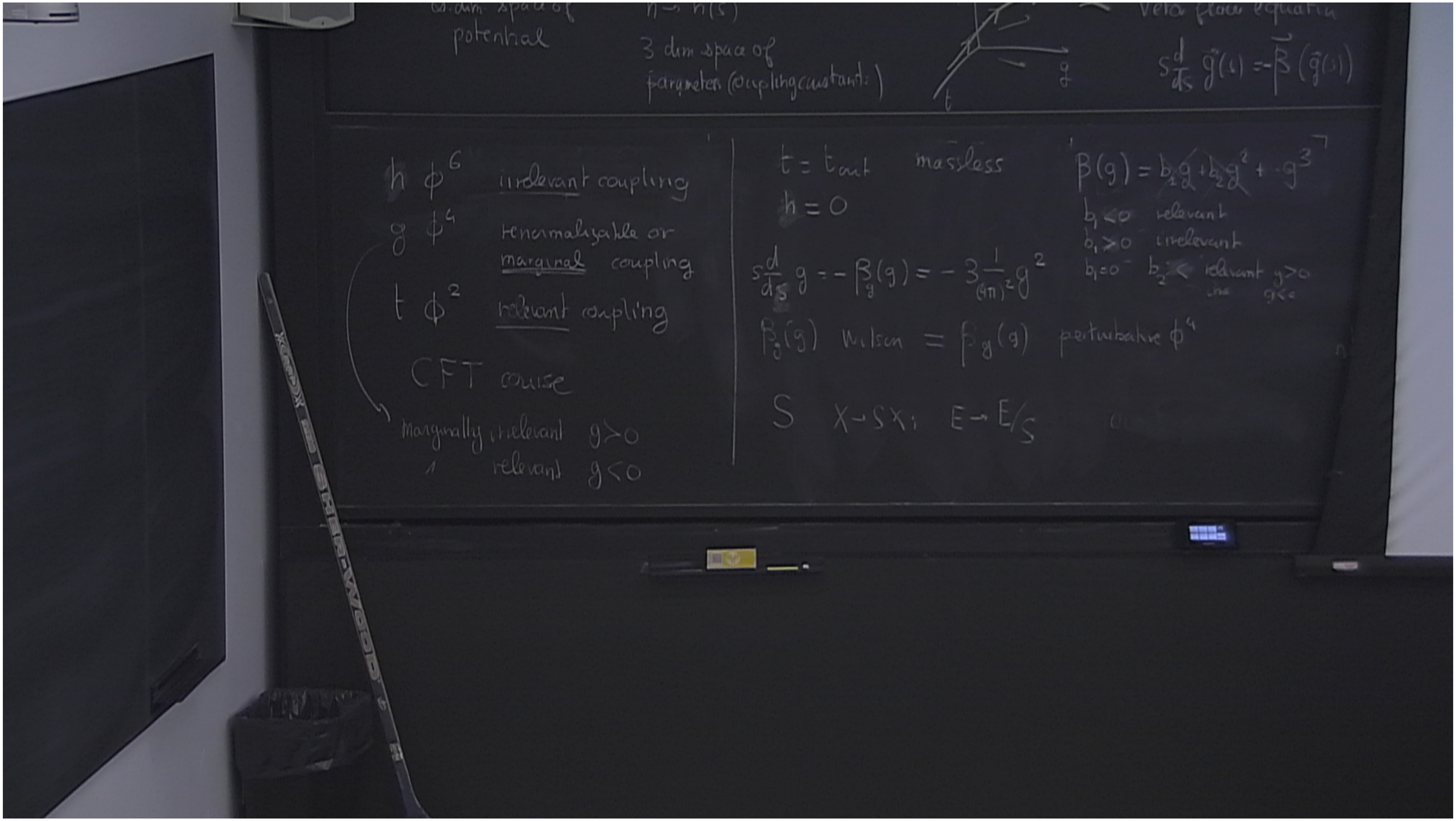
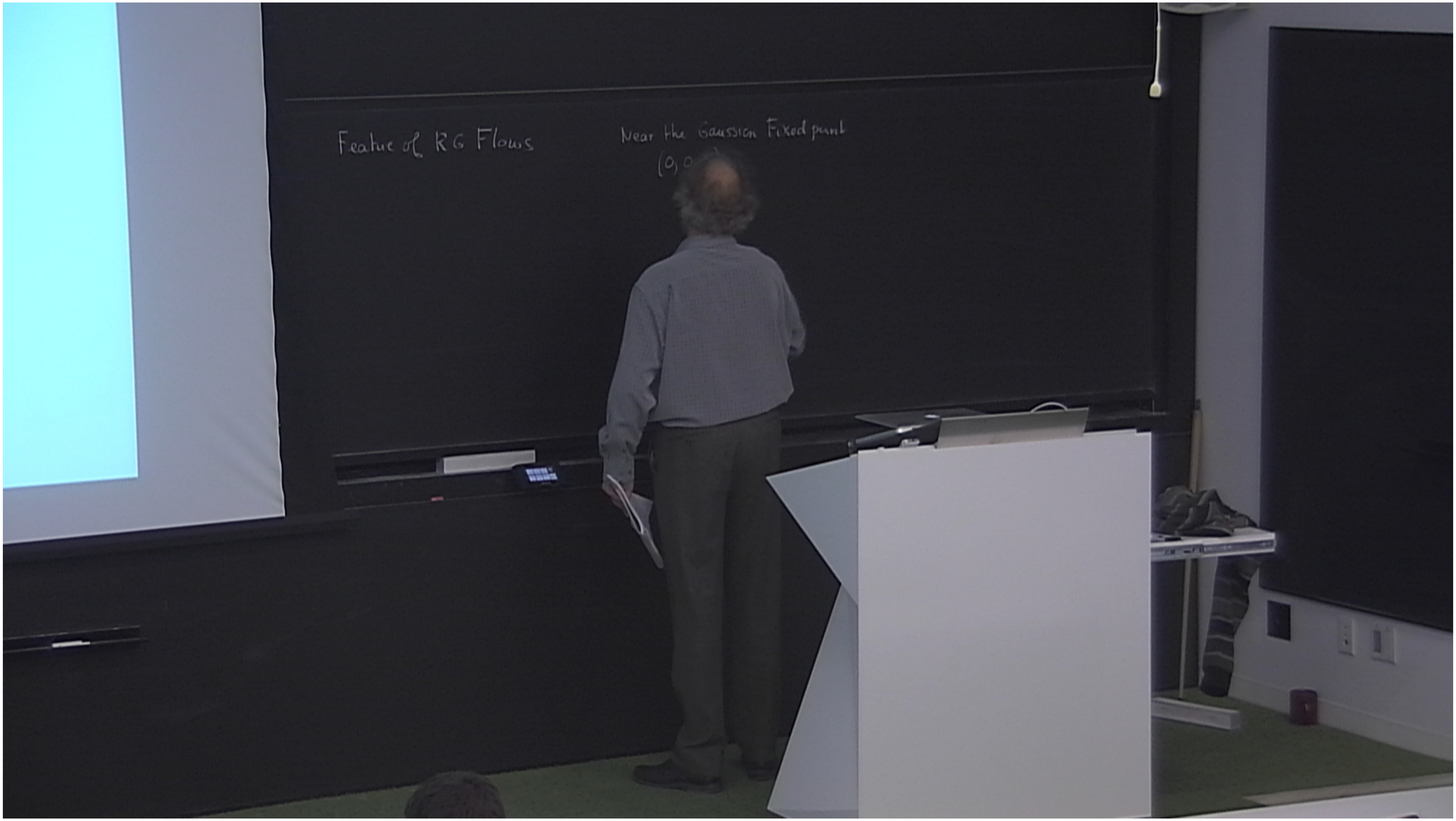
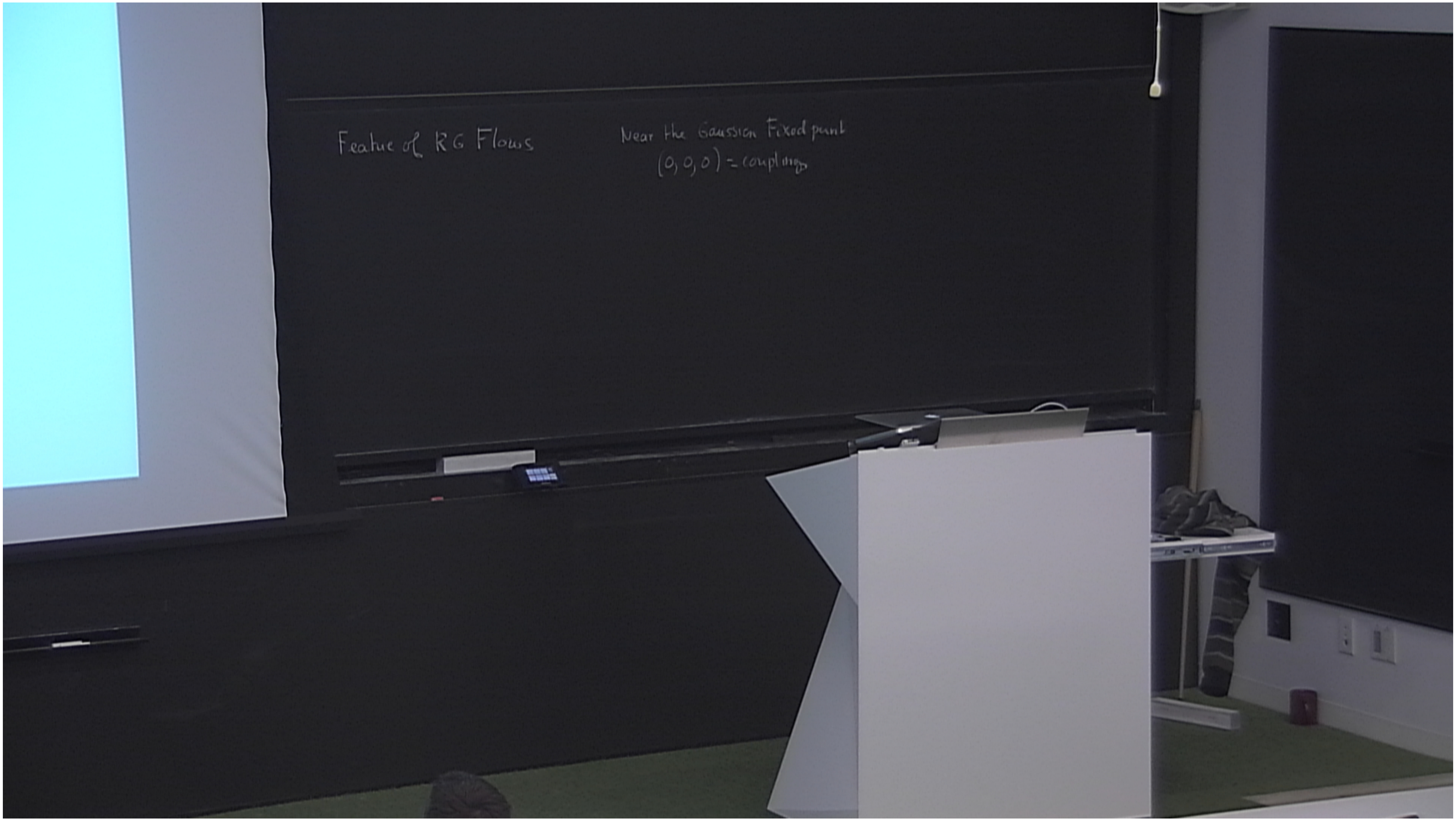
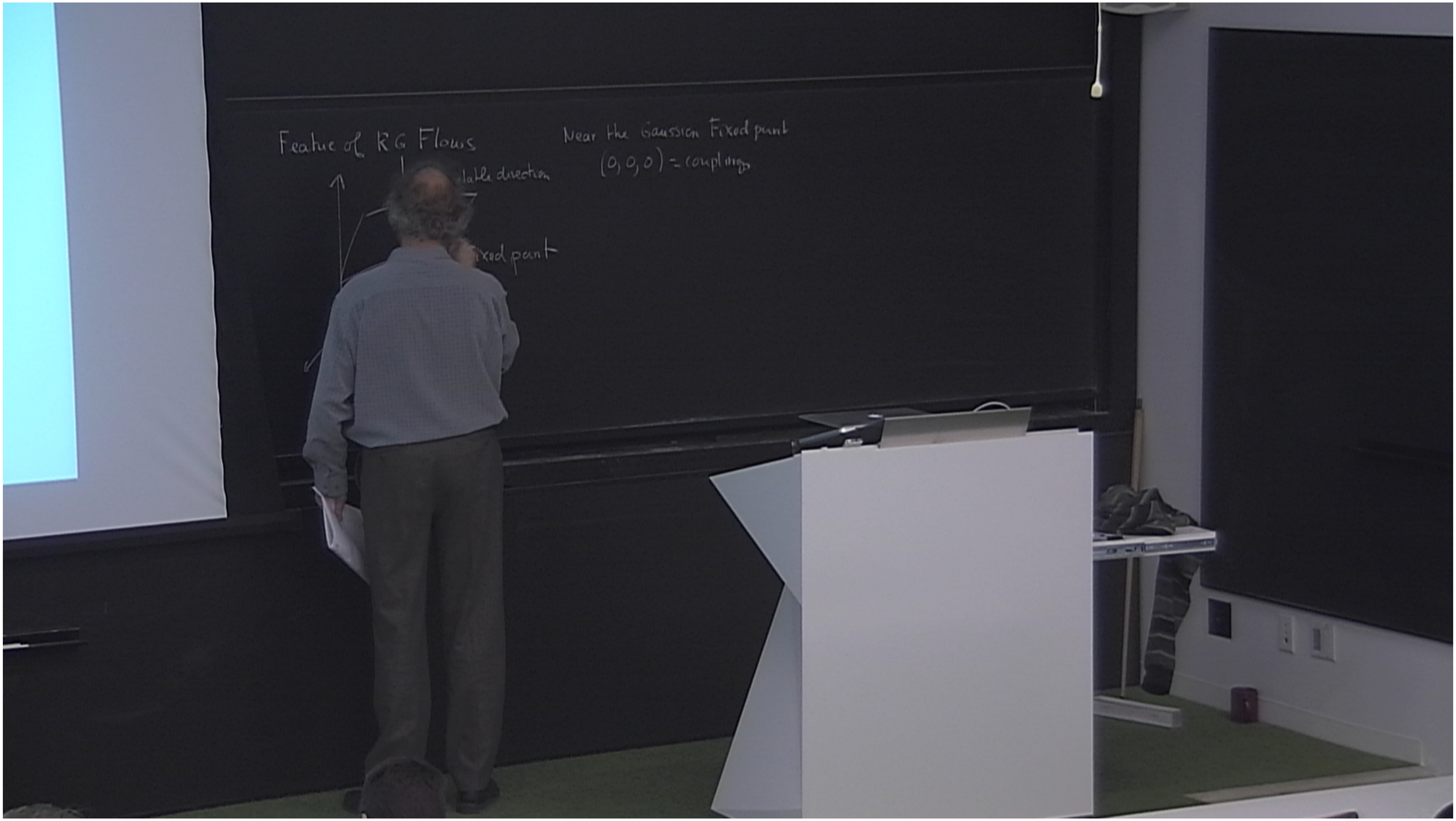
$t\ \phi^2$ relevant coupling
CFT course
marginally irrelevant $g>0$
relevant $g<0$
$t = t_{crit}$ massless
$h = 0$
$s\frac{d}{ds}g = -\beta_g(g) = -3\frac{1}{(4\pi)^2}g^2$
$\beta_g(g)$ Wilson $= \beta_g(g)$
$\beta(g) = b_1 g + b_2 g^2 + \cdot g^3$

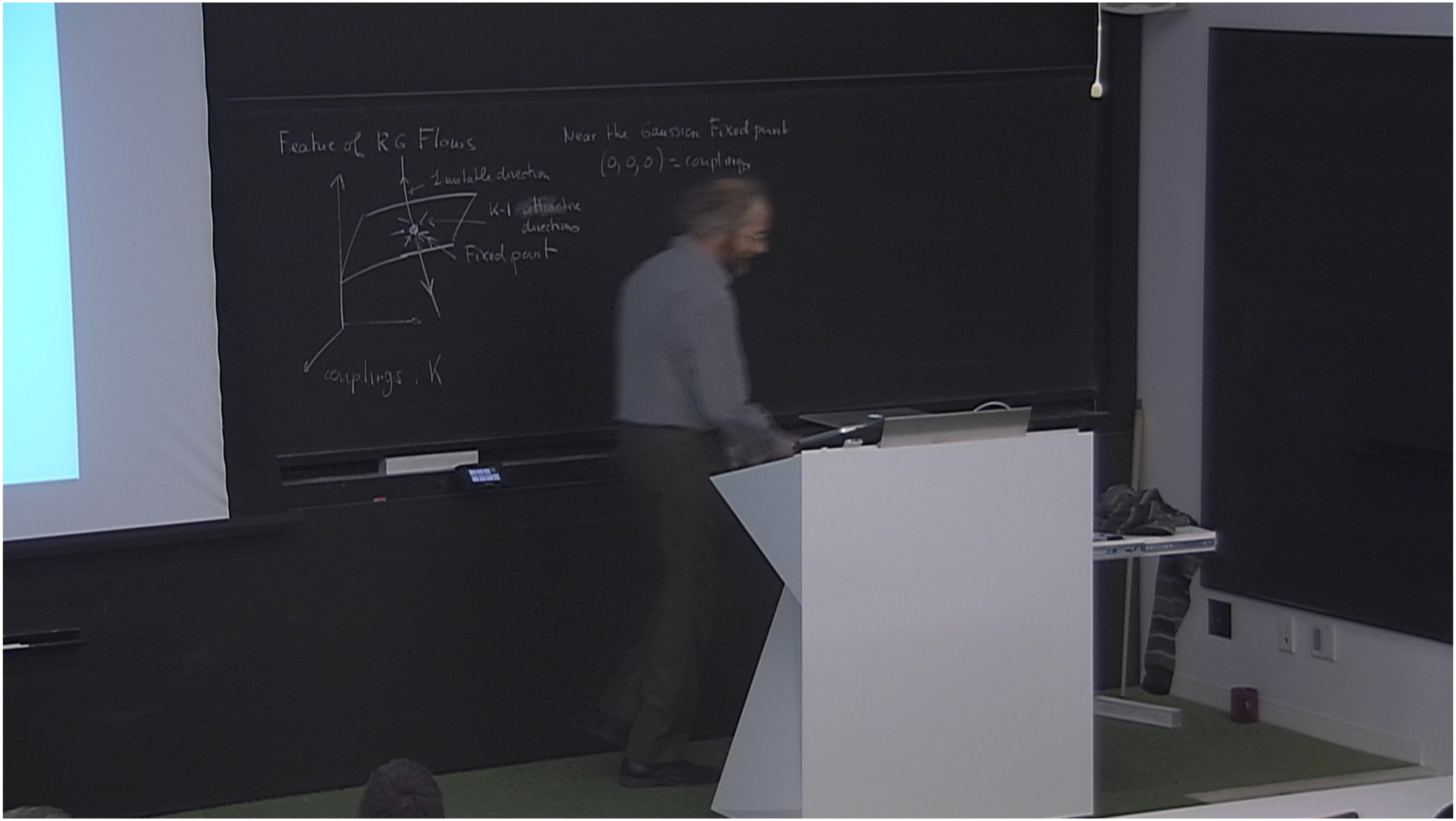

potential
3 dim space of
parameters (coupling constants)
Vector flow equation
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
h φ^6 irrelevant coupling
g φ^4 renormalizable or
marginal
t φ^2 relevant coupling
CFT course
marginally irrelevant
relevant
t = t_crit massless
h = 0
$s\frac{d}{ds} g = -\beta_g(g) = -3\frac{1}{(4\pi)^2}g^2$
β_g(g) Wilson = β_g(g) perturbative φ^4
S x→sx; E→E/s
β(g) = b_1 g + b_2 g^2 + · g^3
b_1 < 0 relevant
b_1 > 0 irrelevant
b_1 = 0 b_2 < relevant g>0

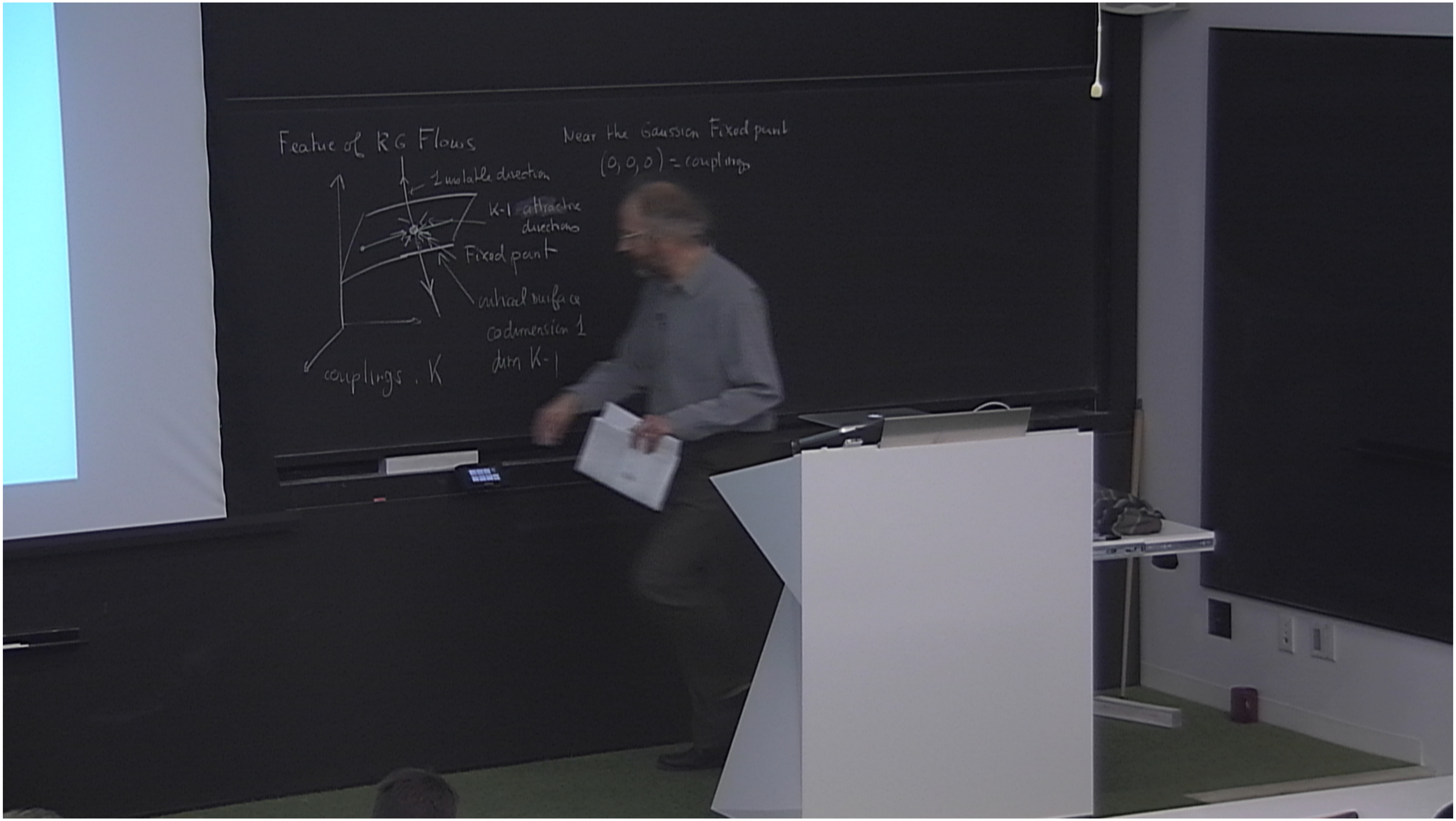

potential
3 dim space of
parameters (coupling constants)
g
t
s d/ds g(s) = -β(g(s))
h φ^6 irrelevant coupling
g φ^4 renormalizable or
marginal coupling
t φ^2 relevant coupling
CFT course
marginally irrelevant g>0
relevant g<0
t = t_crit massless
h = 0
s d/ds g = - β_g(g) = - 3 1/(4π)^2 g^2
β_g(g) Wilson = β_g(g) perturbative φ^4
S X → sX; E → E/s
β(g) = b_1 g + b_2 g^2 + g^3
b_1 < 0 relevant
b_1 > 0 irrelevant
b_1 = 0 b_2 < relevant g>0

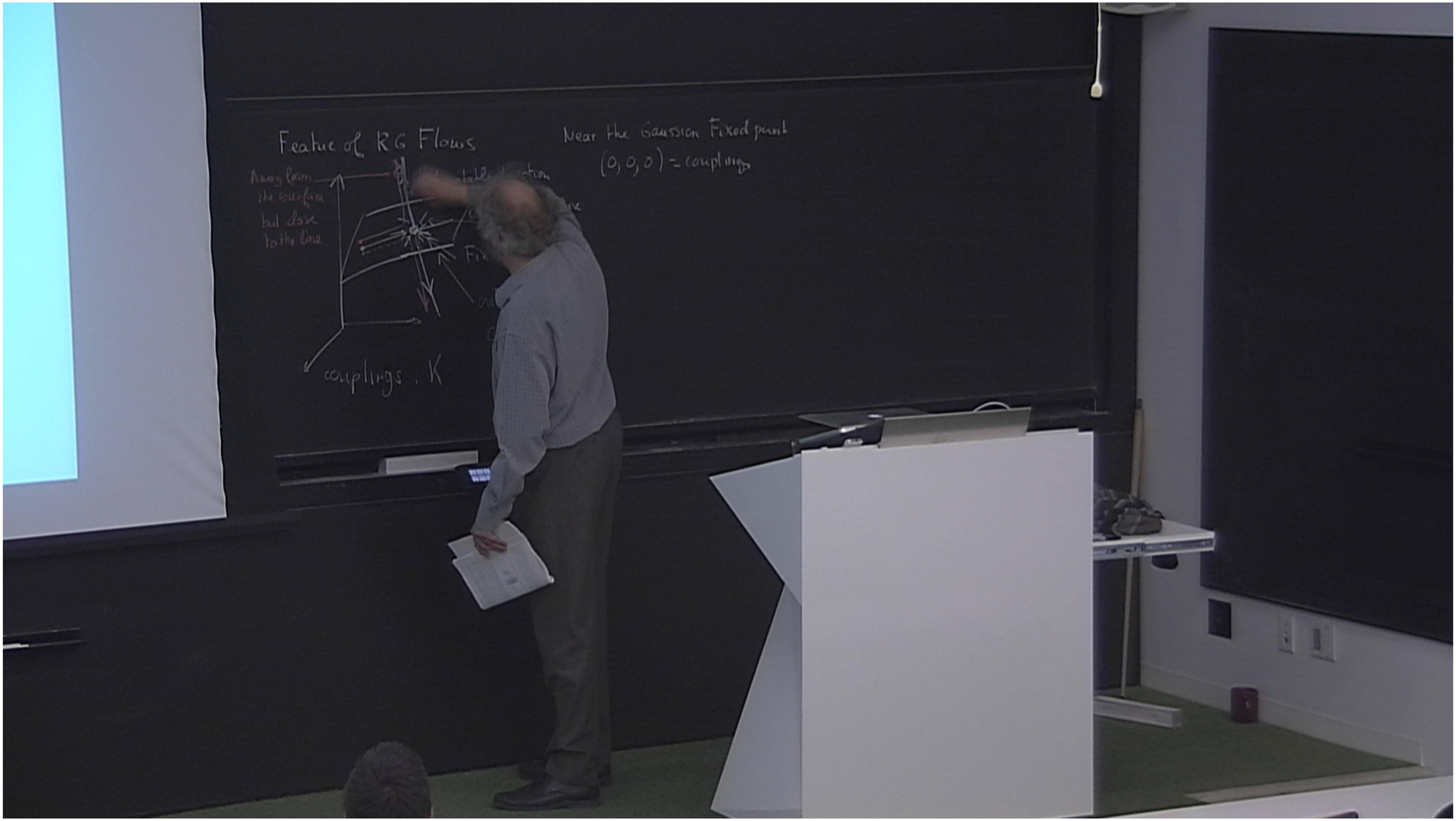

Feature of RG Flows
Near the Gaussian Fixed point

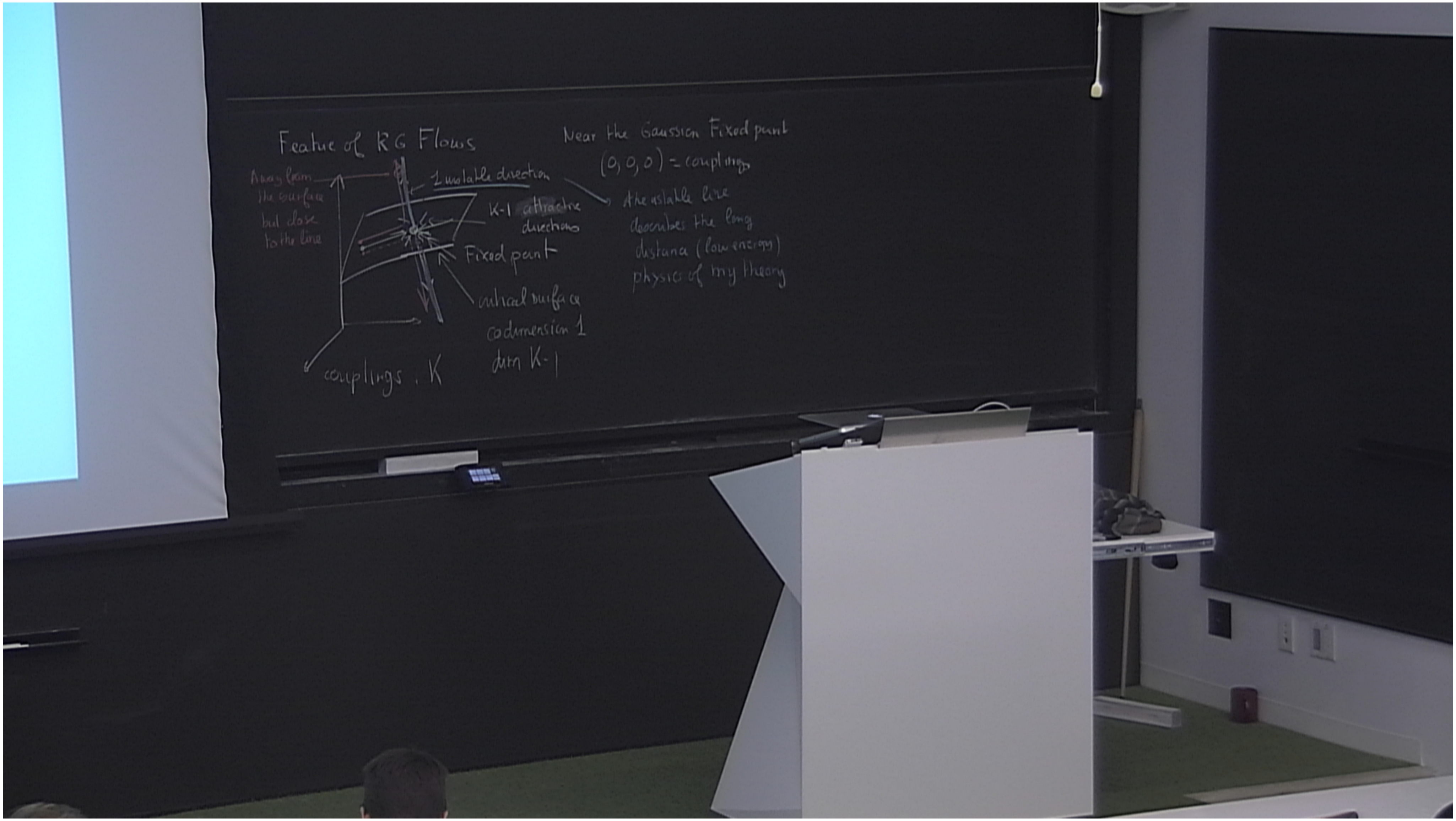

Feature of RG Flows
Near the Gaussian Fixed point
(0, 0, 0) = couplings

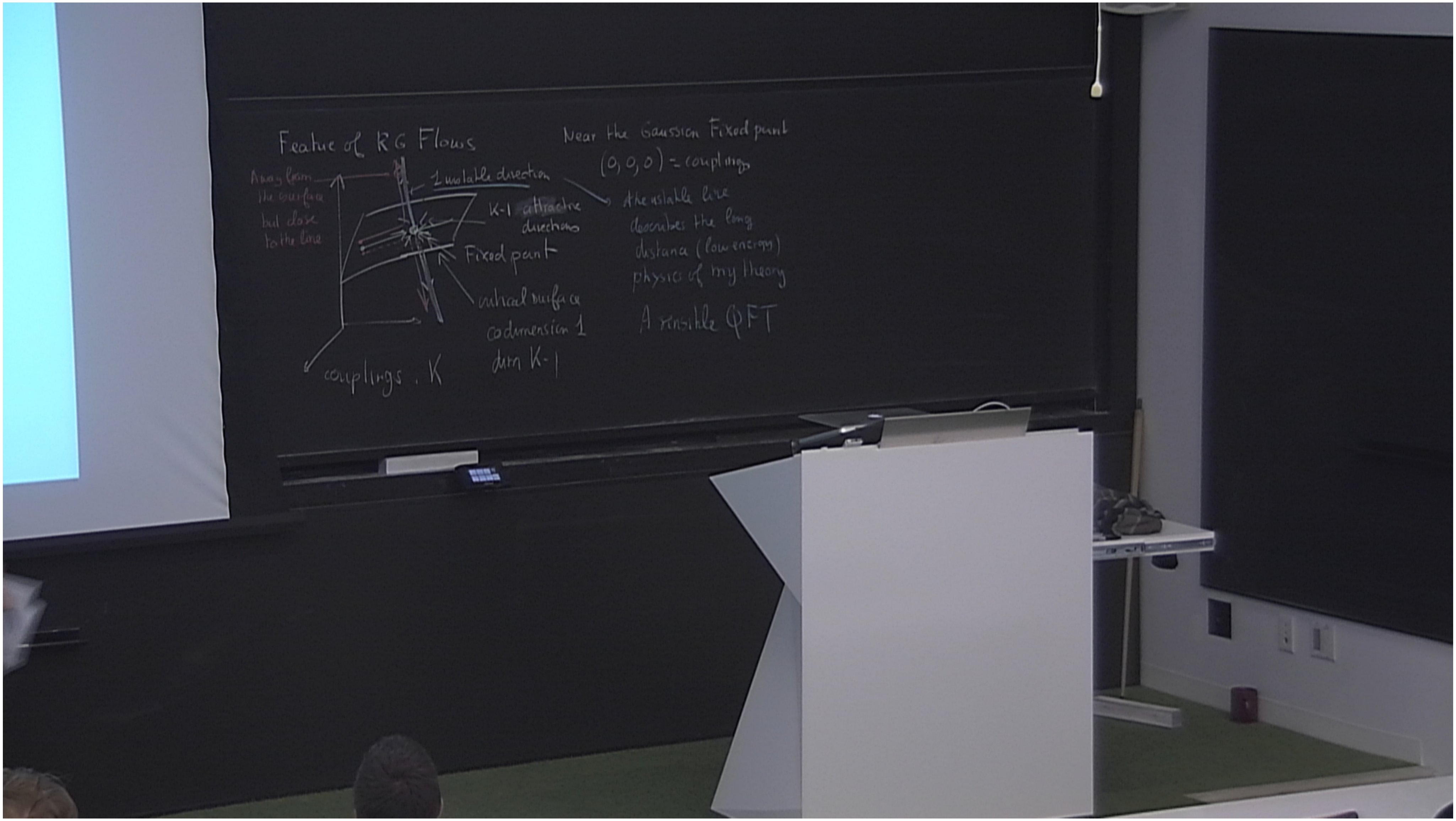

Feature of RG Flows
Near the Gaussian Fixed point
(0, 0, 0) = couplings
Fixed point


Feature of RG Flows
1 unstable direction
K-1 attractive directions
Fixed point
couplings : K
Near the Gaussian Fixed point
(0, 0, 0) = couplings

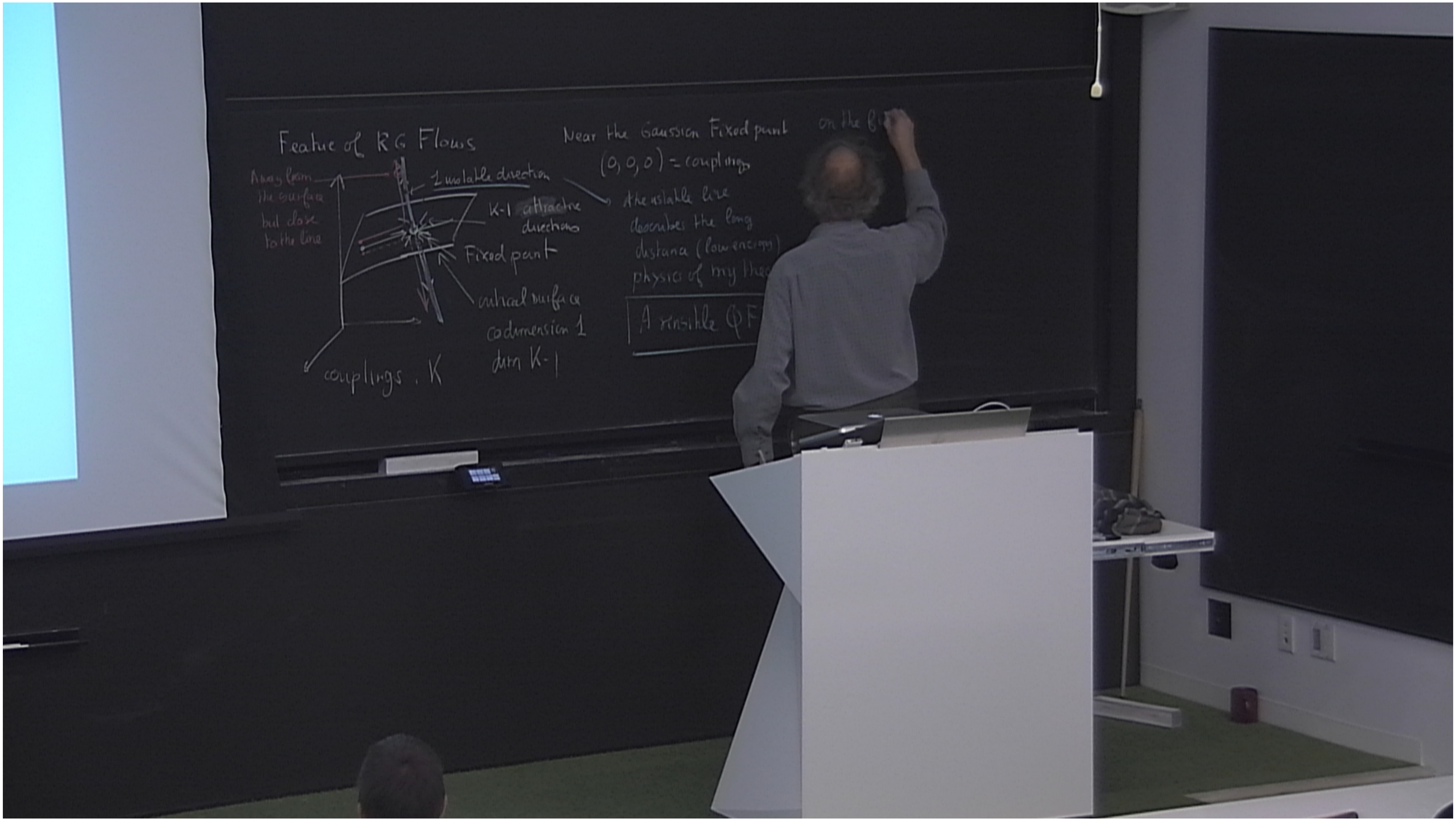

Feature of RG Flows
1 unstable direction
K-1 attractive directions
Fixed point
critical surface
codimension 1
dim K-1
couplings . K
Near the Gaussian Fixed point
(0, 0, 0) = couplings

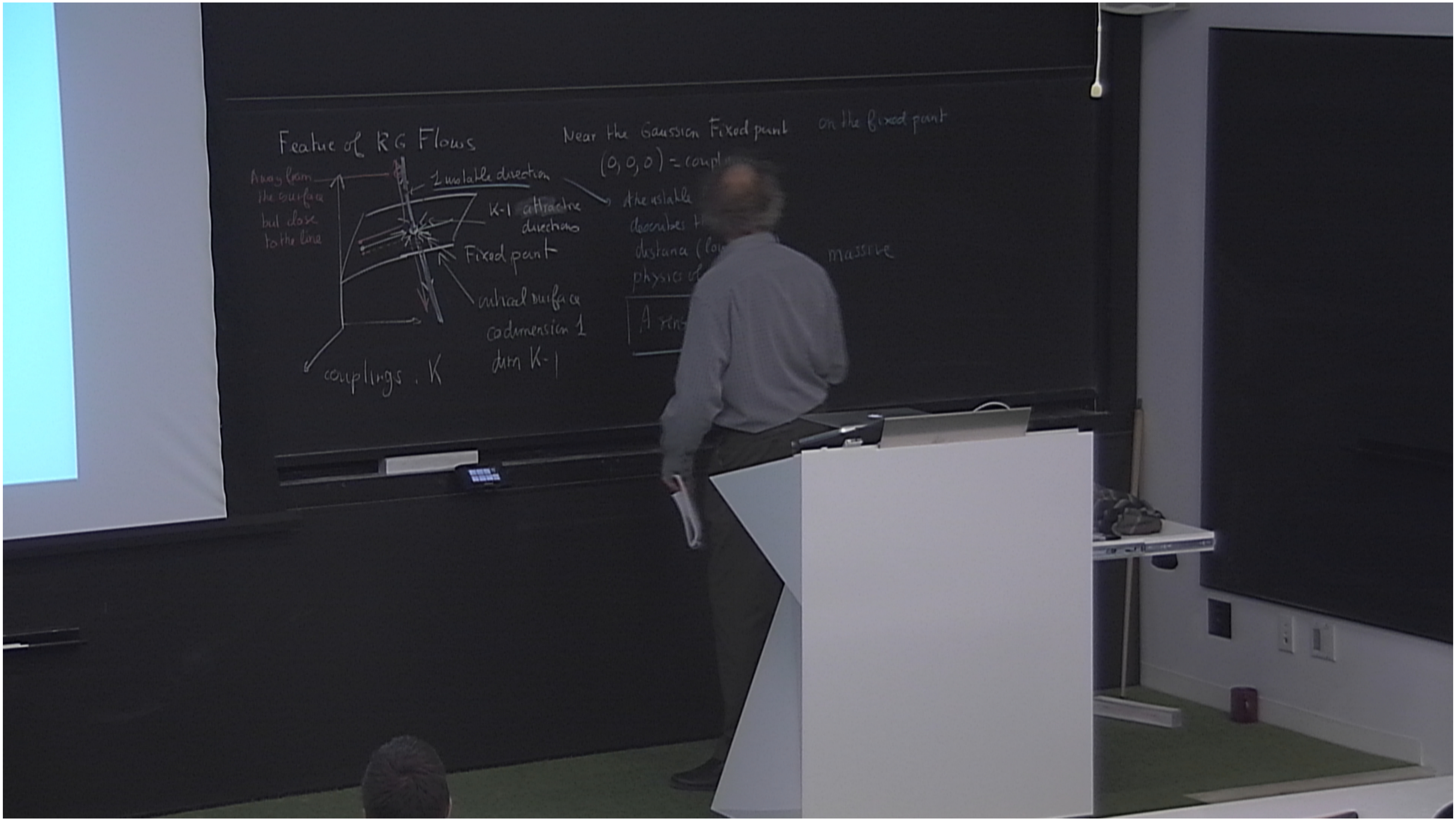

Feature of RG Flows
Near the Gaussian Fixed point
(0,0,0) = couplings
Away from the surface but close to the line
couplings . K

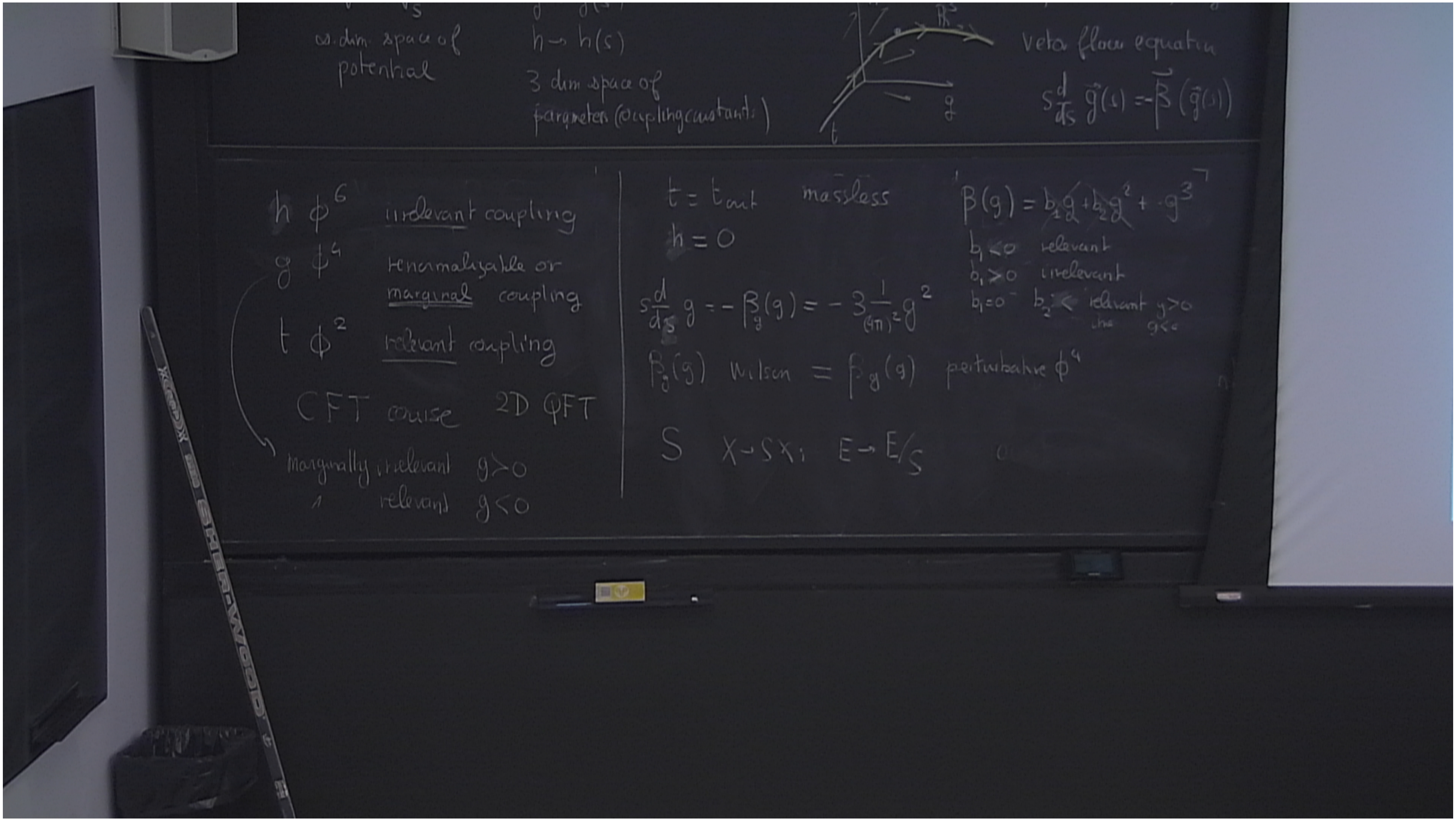

Feature of RG Flows
Away from the surface but close to the line
1 unstable direction
K-1 attractive directions
Fixed point
critical surface codimension 1 dim K-1
couplings : K
Near the Gaussian Fixed point
(0, 0, 0) = couplings
the unstable line describes the long distance (low energy) physics of my theory

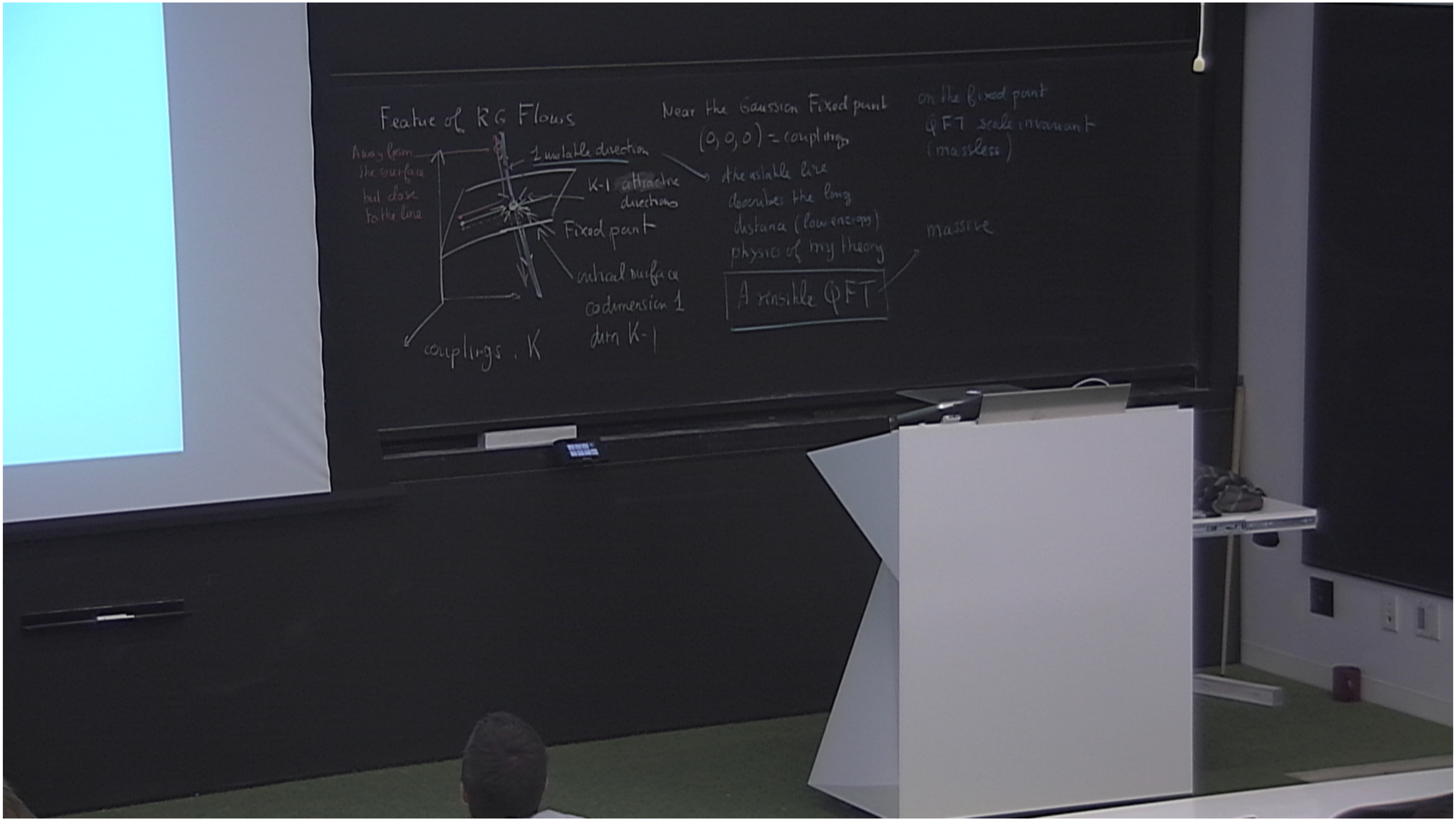


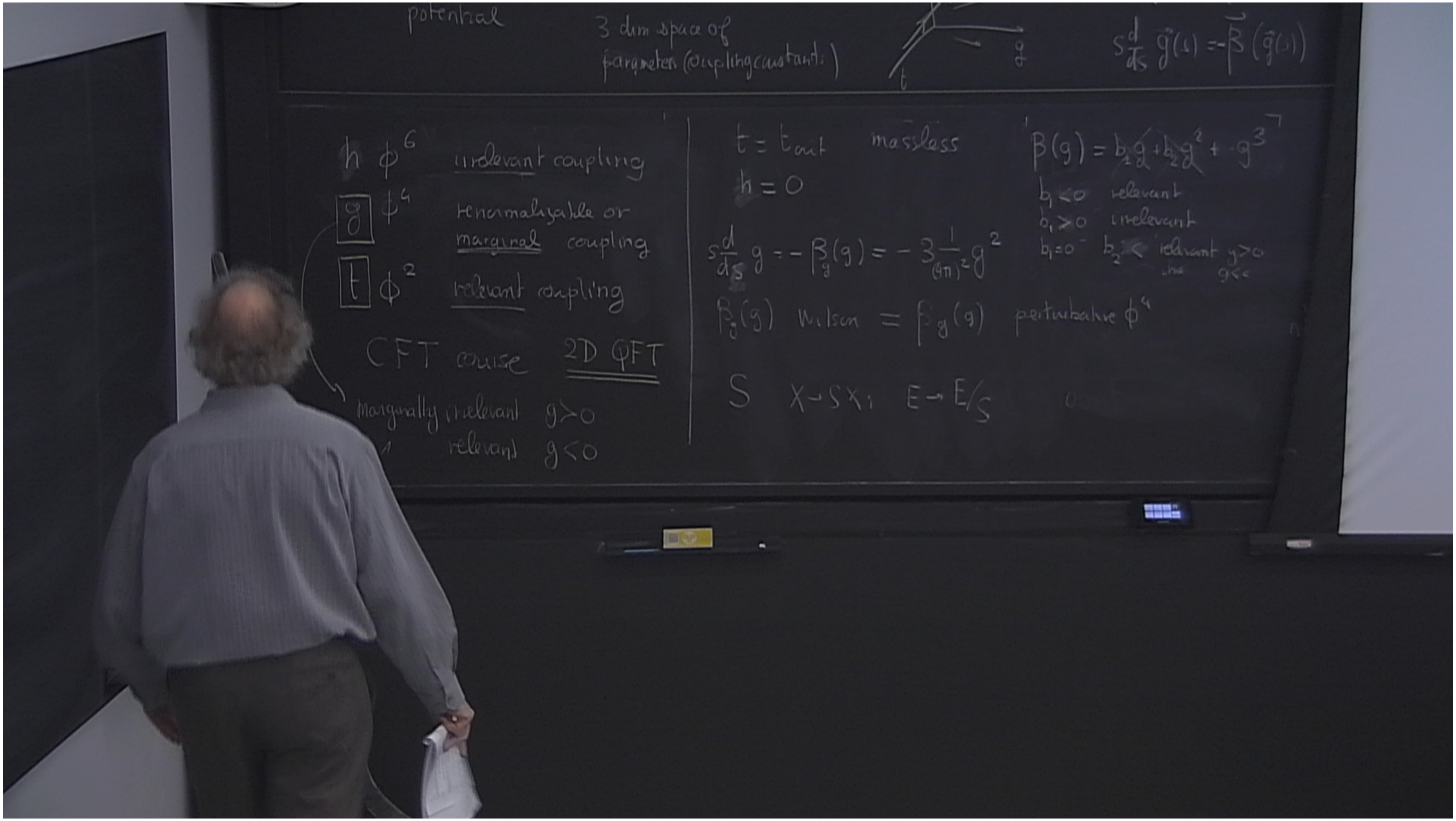
Feature of RG Flows
Away from the surface but close to the line
1 unstable direction
K-1 attractive directions
Fixed point
critical surface
codimension 1
dim K-1
couplings . K
Near the Gaussian Fixed point
(0, 0, 0) = couplings
the unstable line describes the long distance (low energy) physics of my theory
A sensible QFT

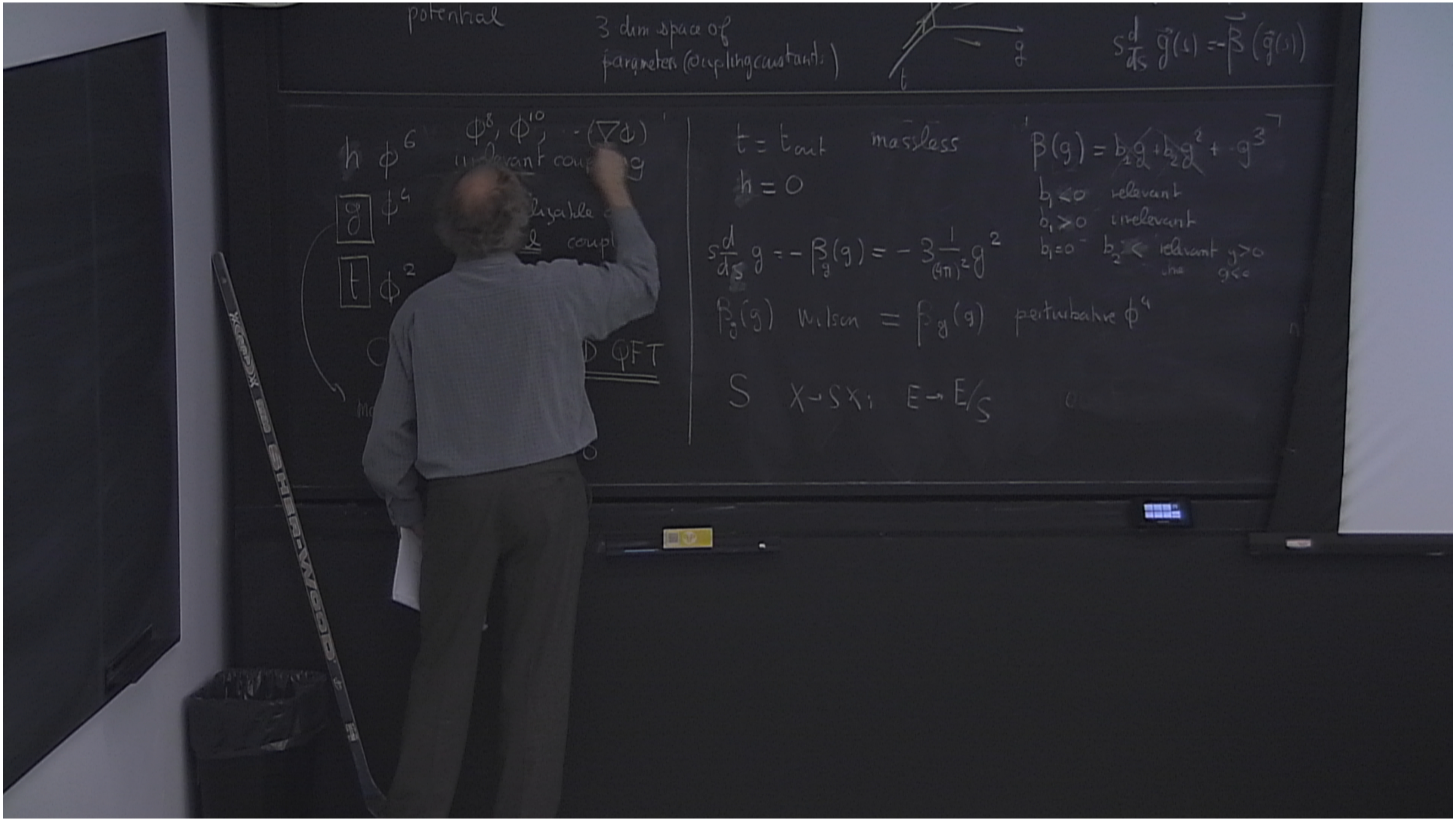

Feature of RG Flows
Away from the surface but close to the line
1 unstable direction
K-1 attractive directions
Fixed point
critical surface codimension 1 dim K-1
couplings . K
Near the Gaussian Fixed point
(0, 0, 0) = couplings
the unstable line describes the long distance (low energy) physics of my theo
A sensible φF
on the


Feature of RG Flows
Near the Gaussian Fixed point
on the fixed point
Away from the surface but close to the line
1 unstable direction
K-1 attractive directions
Fixed point
critical surface
codimension 1
dim K-1
couplings . K
massive

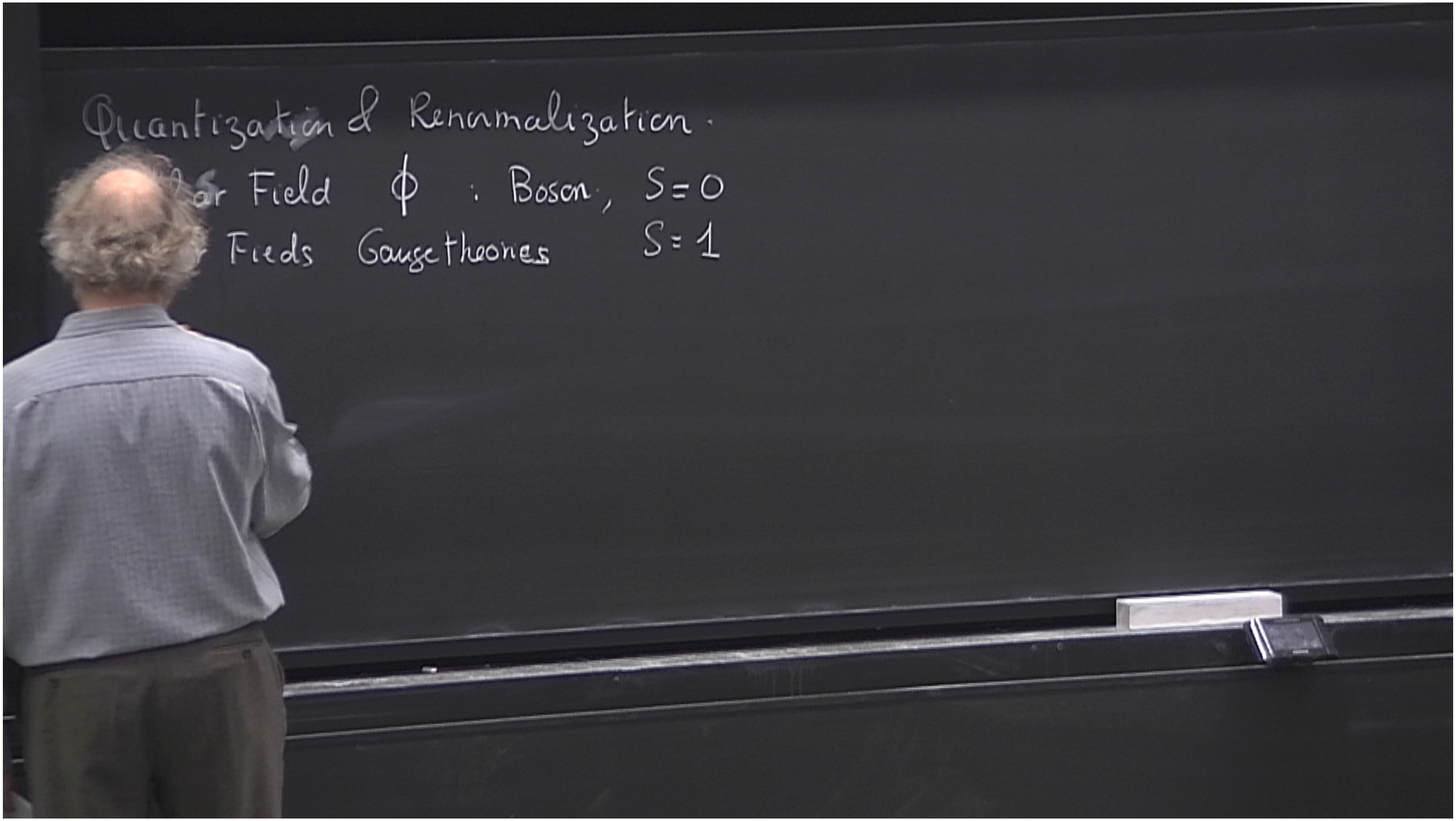

∞ dim space of
potential
h → h(s)
3 dim space of
parameter (coupling constants)
vector flow equation
h φ6 irrelevant coupling
g φ4 renormalizable or
marginal coupling
t φ2 relevant coupling
CFT course 2D QFT
marginally irrelevant g>0
relevant g<0
t = tcrit massless
h = 0
b1<0 relevant
b1>0 irrelevant
Wilson
perturbative φ4

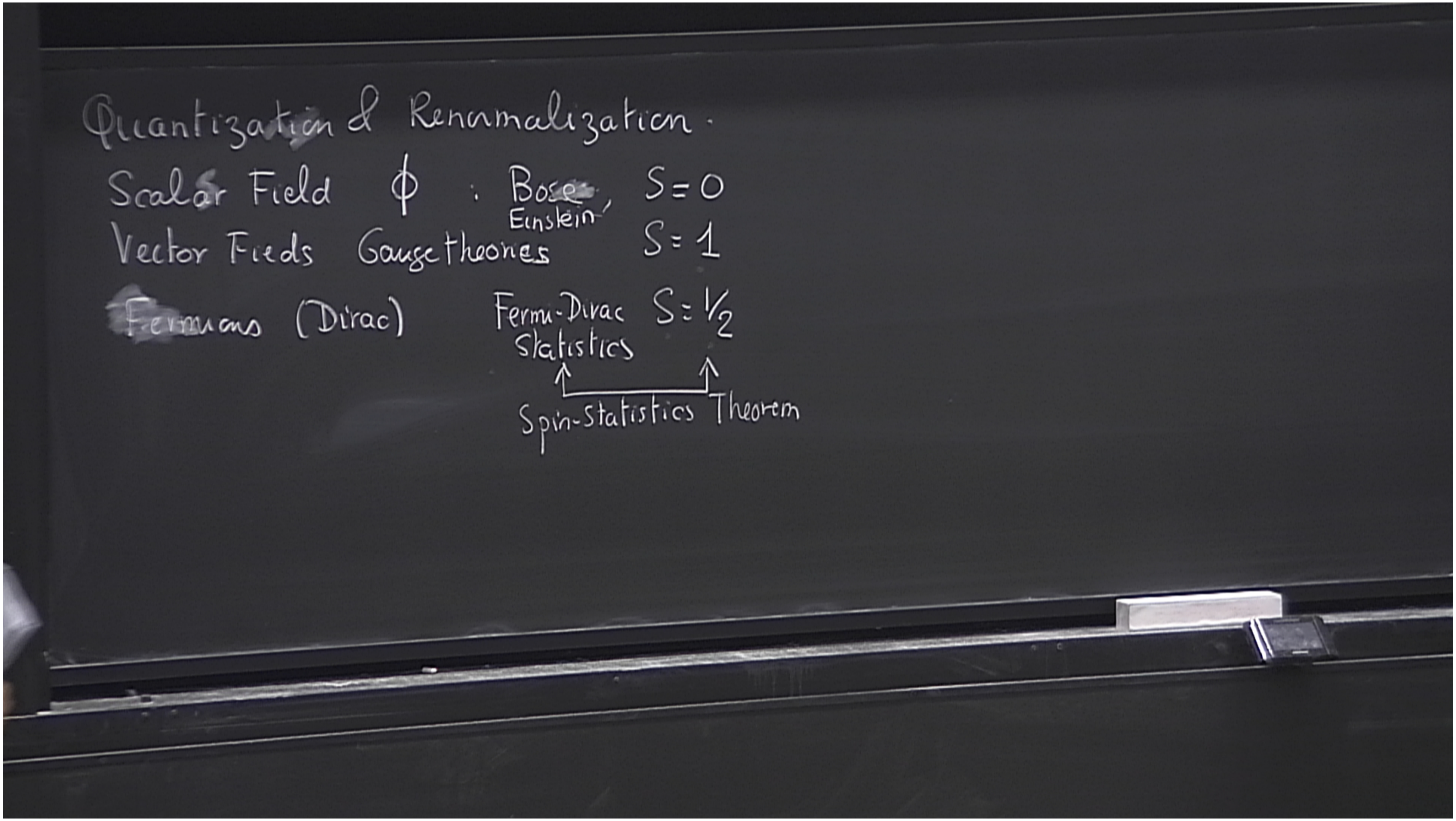

Feature of RG Flows
Away from the surface but close to the line
1 unstable direction
K-1 attractive directions
Fixed point
critical surface
codimension 1
dim K-1
couplings . K
Near the Gaussian Fixed point
(0,0,0) = couplings
the unstable line describes the long distance (low energy) physics of my theory
A sensible QFT
On the fixed point
QFT scale invariant
(massless)
massive

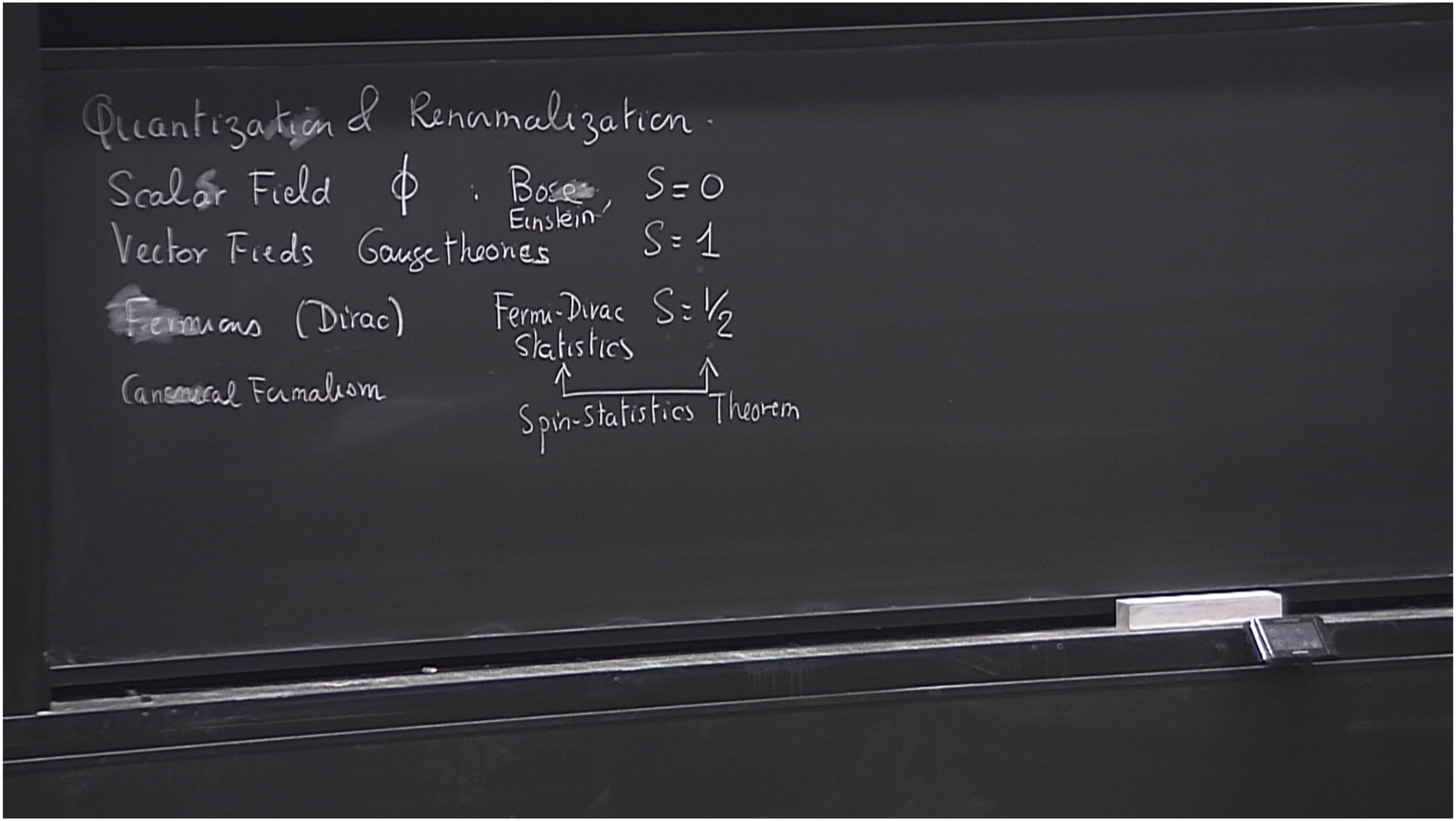




potential
3 dim space of
parameters (coupling constants)
relevant
irrelevant
QFT
t = t_crit
massless
h = 0
Wilson
perturbative


Quantization & Renormalization.
ar Field φ : Boson, S=0
Fieds Gauge theories S=1


Quantization & Renormalization.
Scalar Field ϕ : Bose Einstein, S=0
Vector Fieds Gauge theories S=1
Fermions (Dirac) Fermi-Dirac Statistics S=1/2
Spin-Statistics Theorem


Quantization & Renormalization.
Scalar Field ϕ : Bose-Einstein, S=0
Vector Fields Gauge theories S=1
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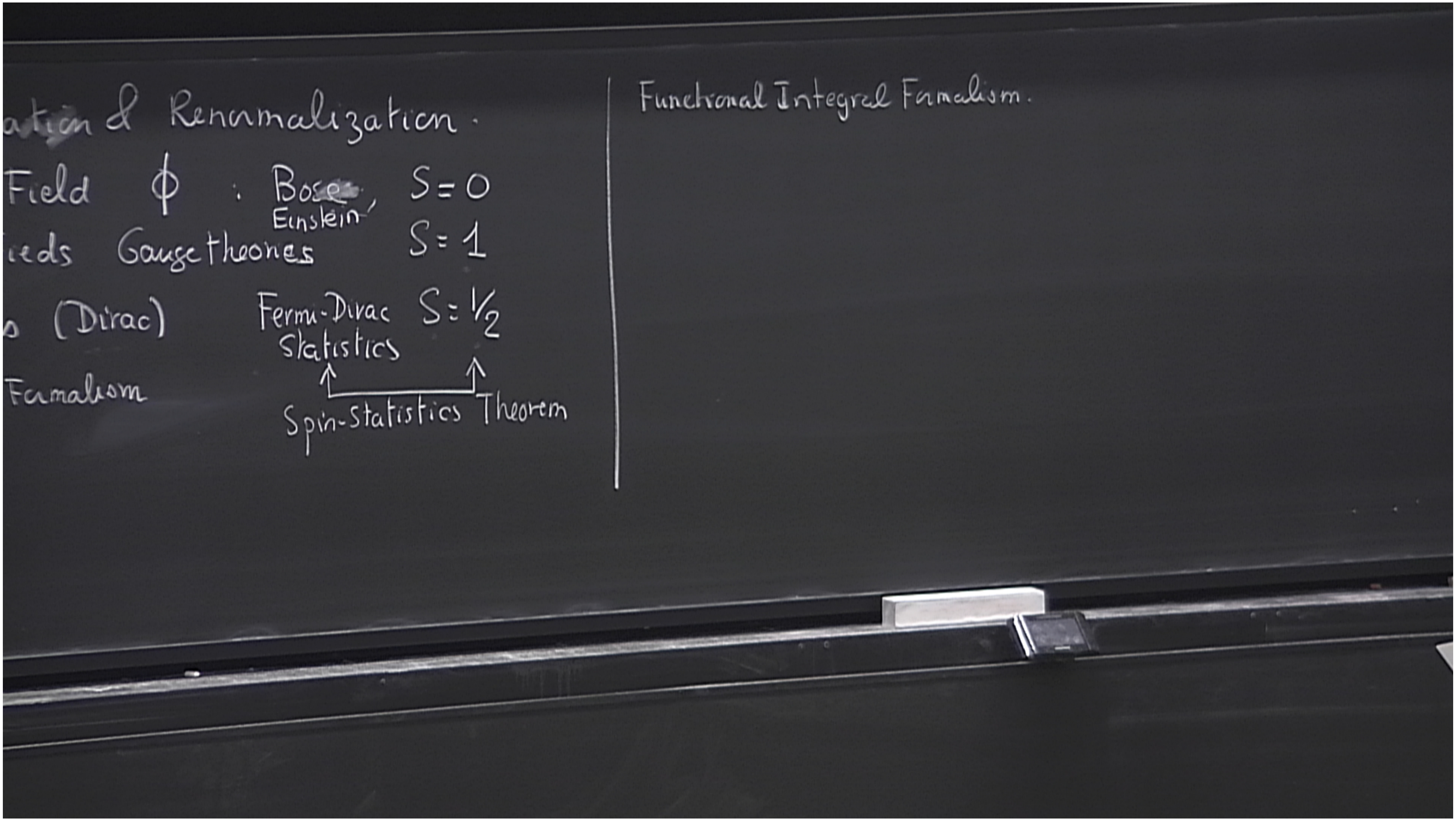
Canonical Formalism
Spin-Statistics Theorem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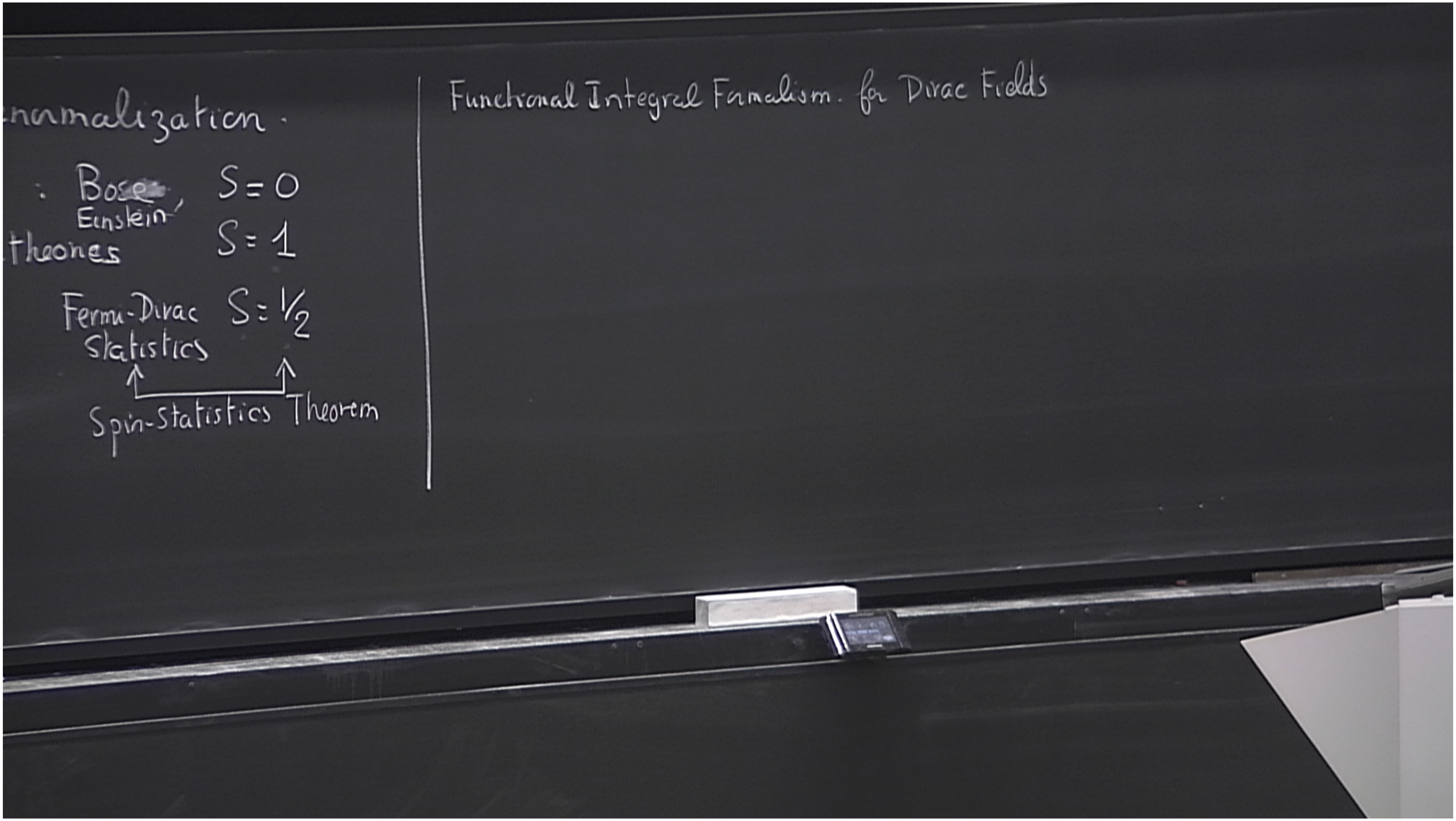
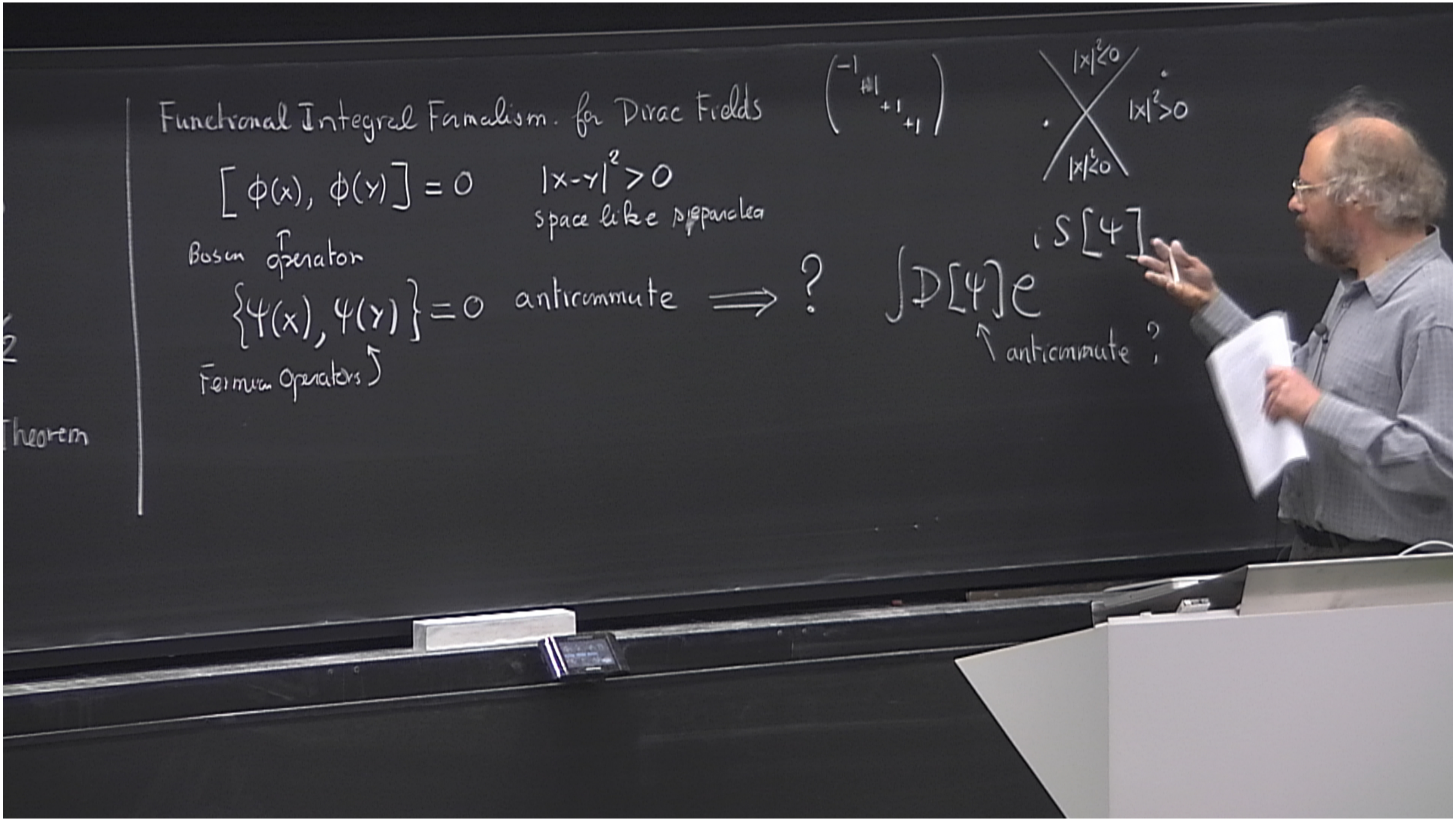
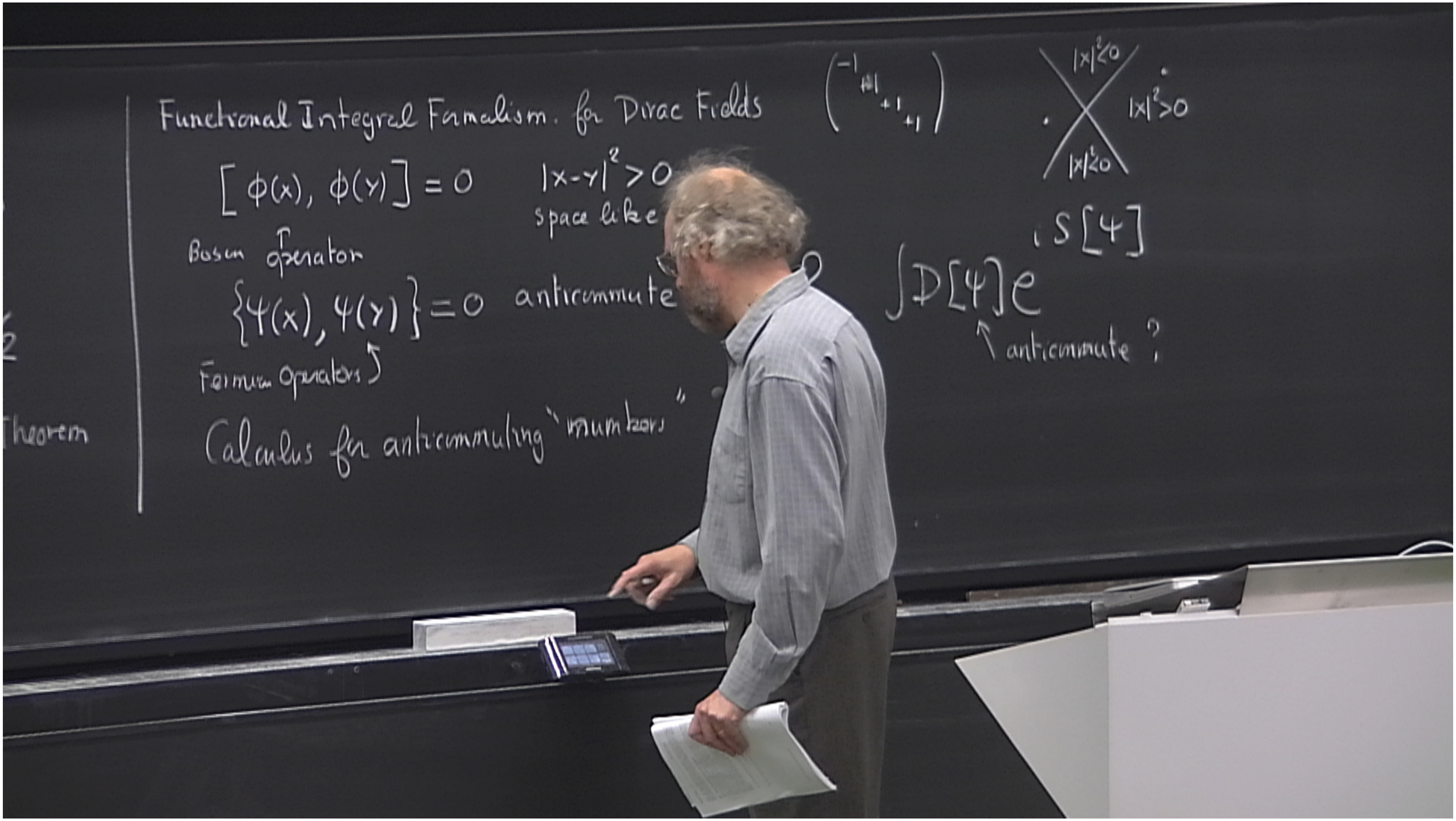
Formalism
Spin-Statistics Theorem
Functional Integral Formalism.


Bose-Einstein, S = 0
S = 1
Fermi-Dirac Statistics S = 1/2
Spin-Statistics Theorem
Functional Integral Formalism. for Dirac Fields


Functional Integral Formalism. for Dirac Fields
[φ(x), φ(y)] = 0 |x-y|² > 0
space like separation
Boson operator
{ψ(x), ψ(y)} = 0 antirommute ⟹ ?
Fermion Operators
∫D[ψ] e^{iS[ψ]}
anticommute ?
|x|²<0
|x|²>0
Theorem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Calculus for anticommuting "numbers"
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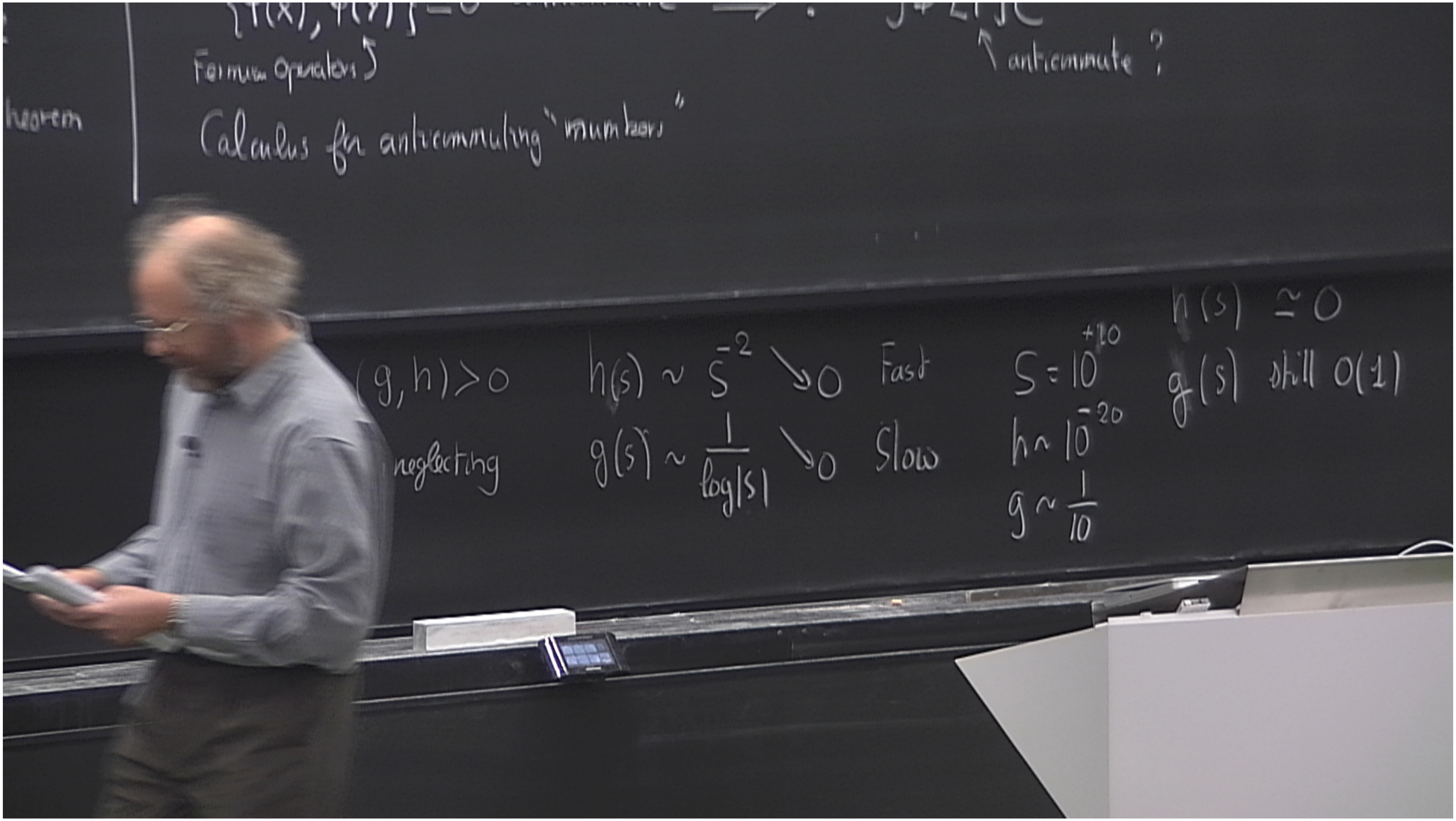
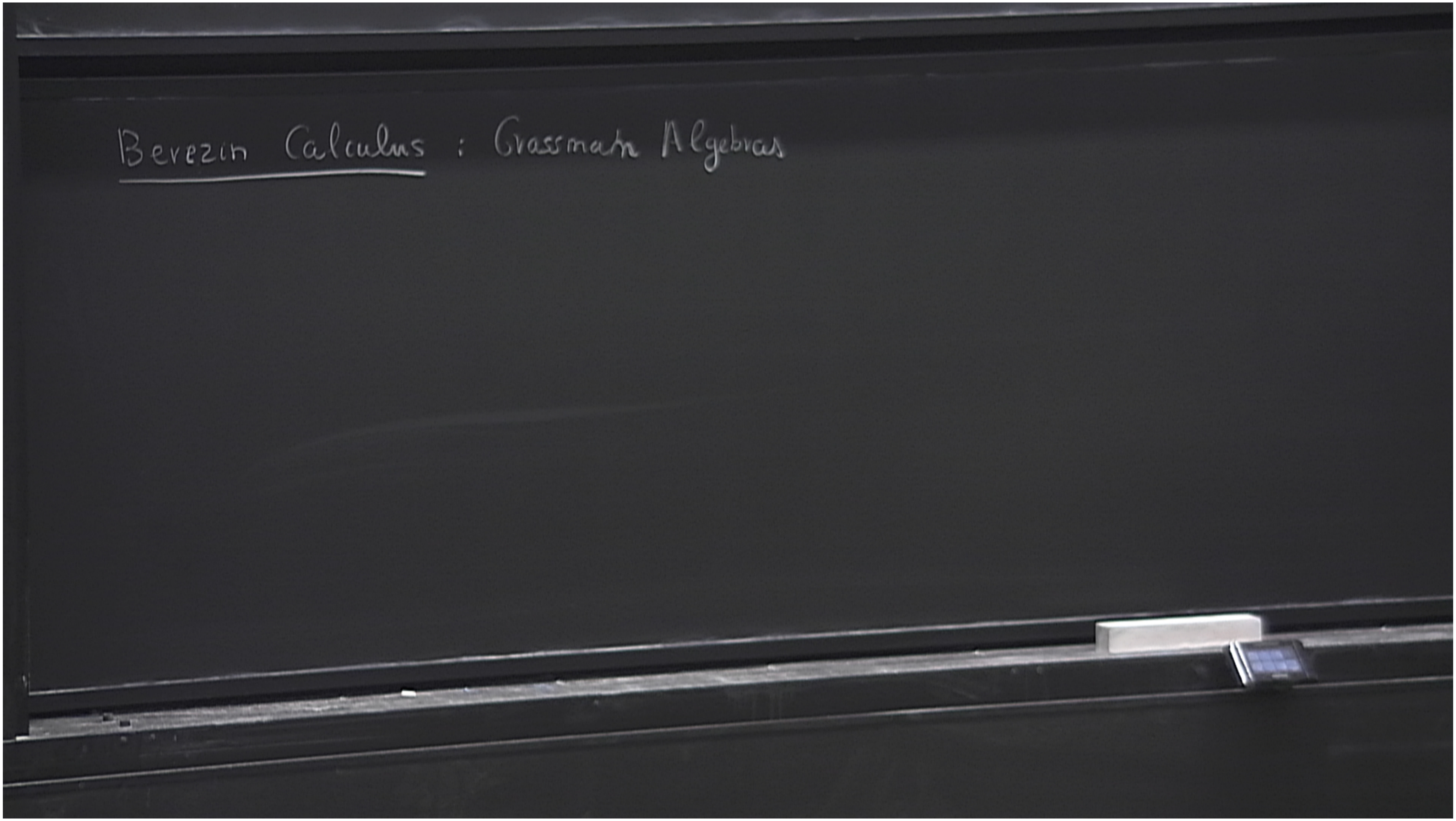
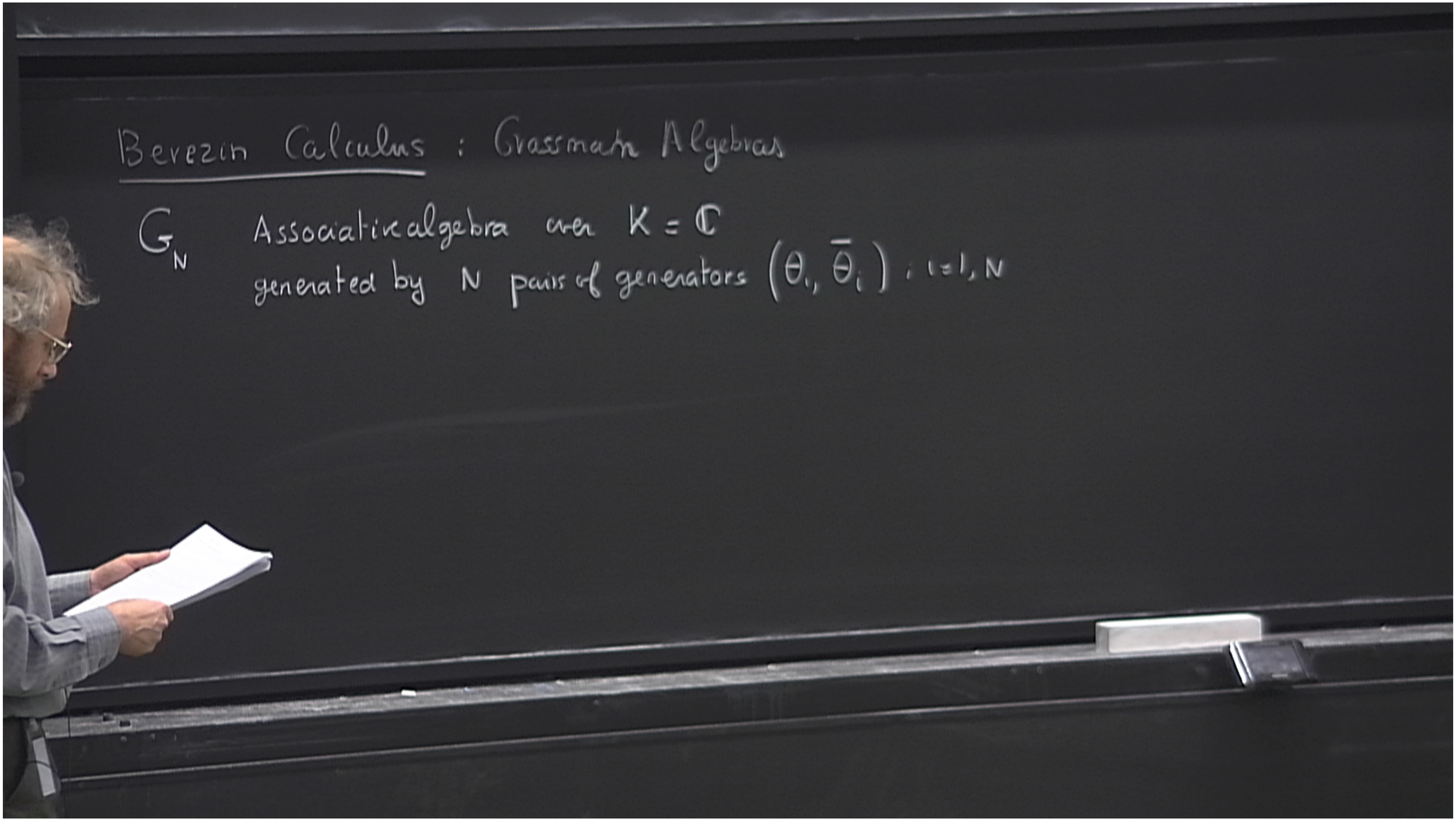
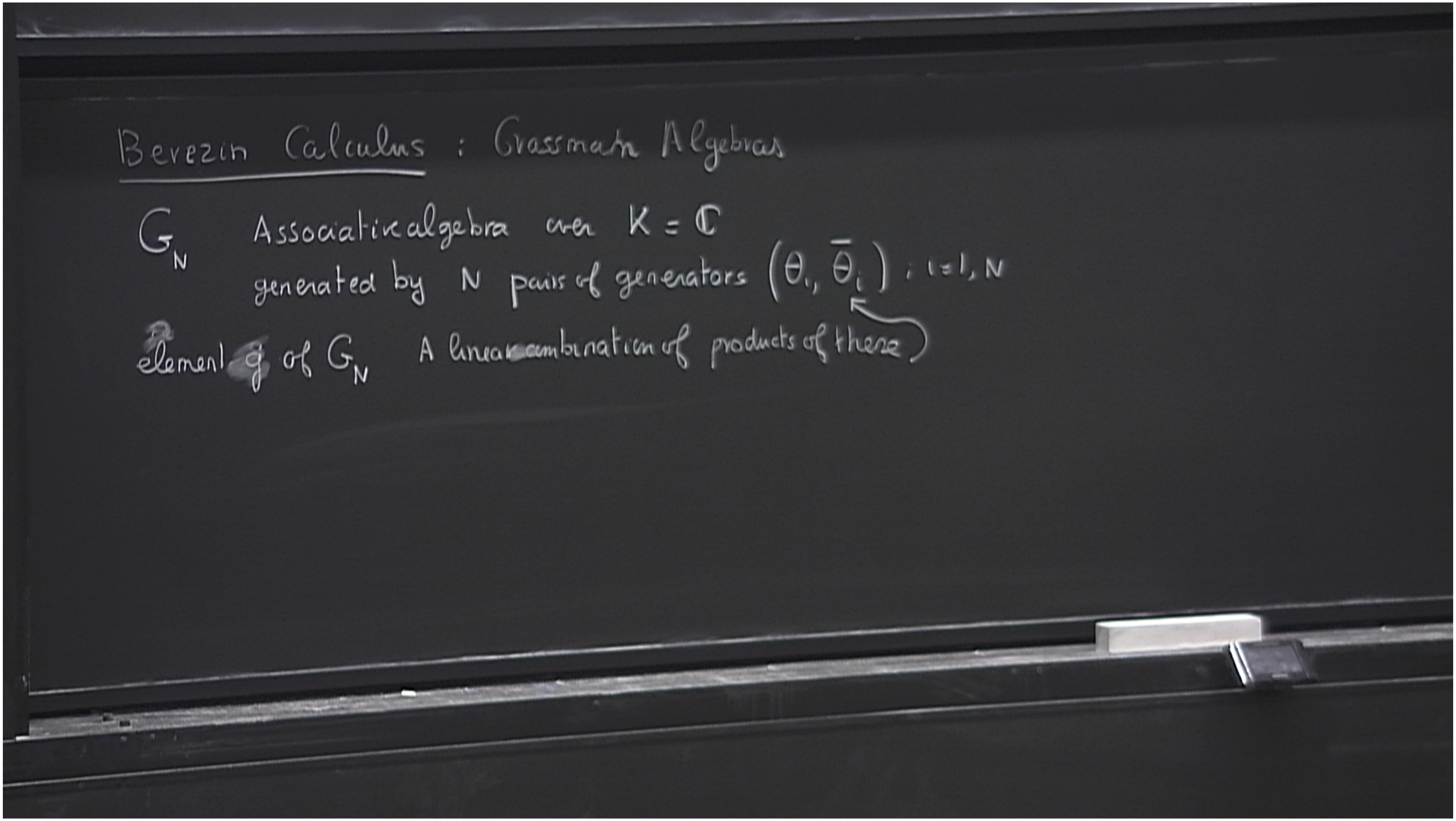

Fermion Operators
anticommute ?
Calculus for anticommuting "numbers"
(g, h) > 0
h(s) ~ s^-2 ↘ 0 Fast
neglecting
g(s) ~ 1/log|s| ↘ 0 Slow
S = 10^+10
h ~ 10^-20
g ~ 1/10
h(s) ≃ 0
g(s) still O(1)


Berezin Calculus : Grassmann Algebras


Berezin Calculus : Grassmann Algebras
G_N Associative algebra over K = ℂ
generated by N pairs of generators (θ_i, θ̄_i) ; i = 1, N


Berezin Calculus : Grassmann Algebras
G_N Associative algebra over $K = \mathbb{C}$
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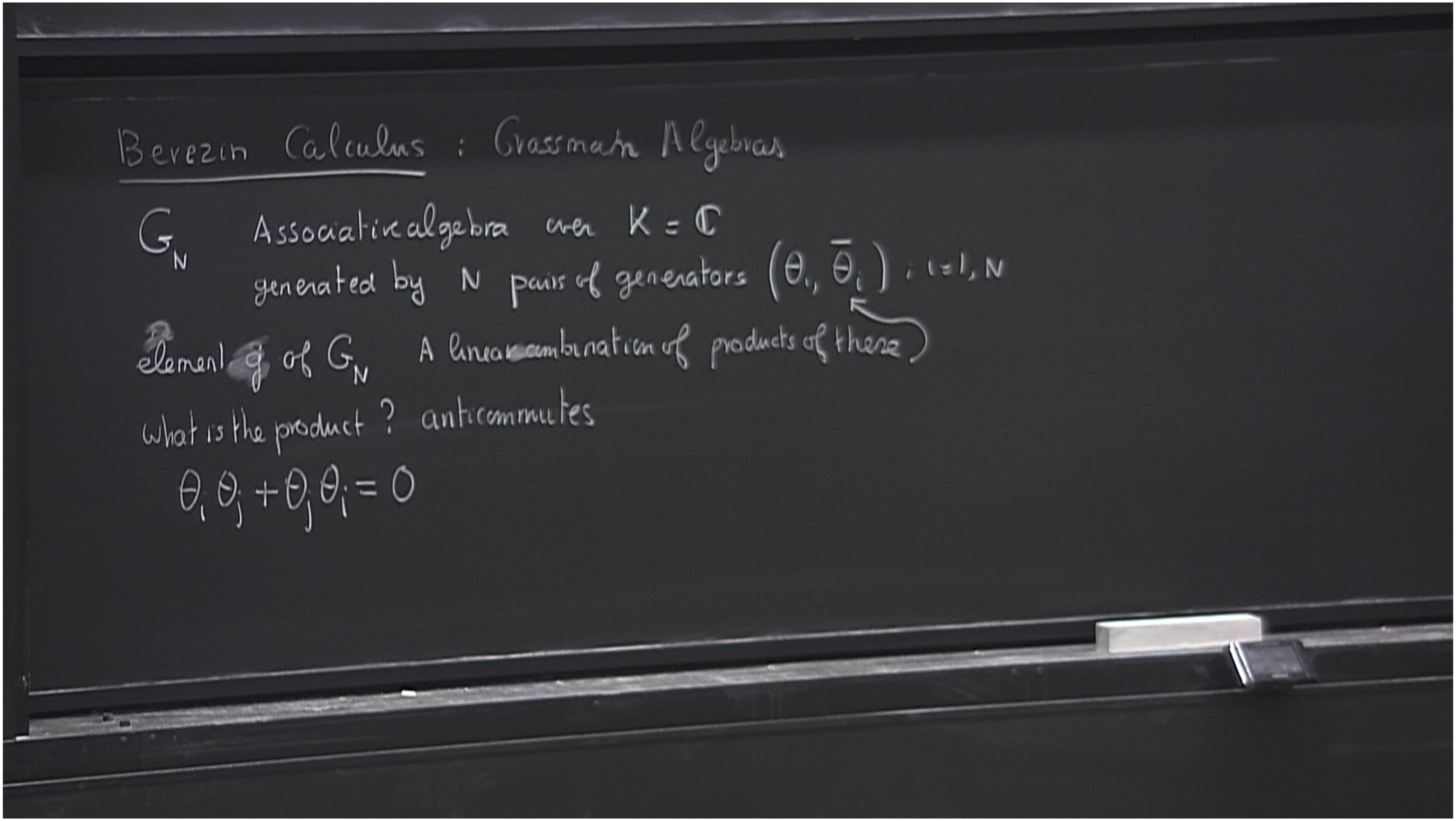
element g of G_N A linear combination of products of these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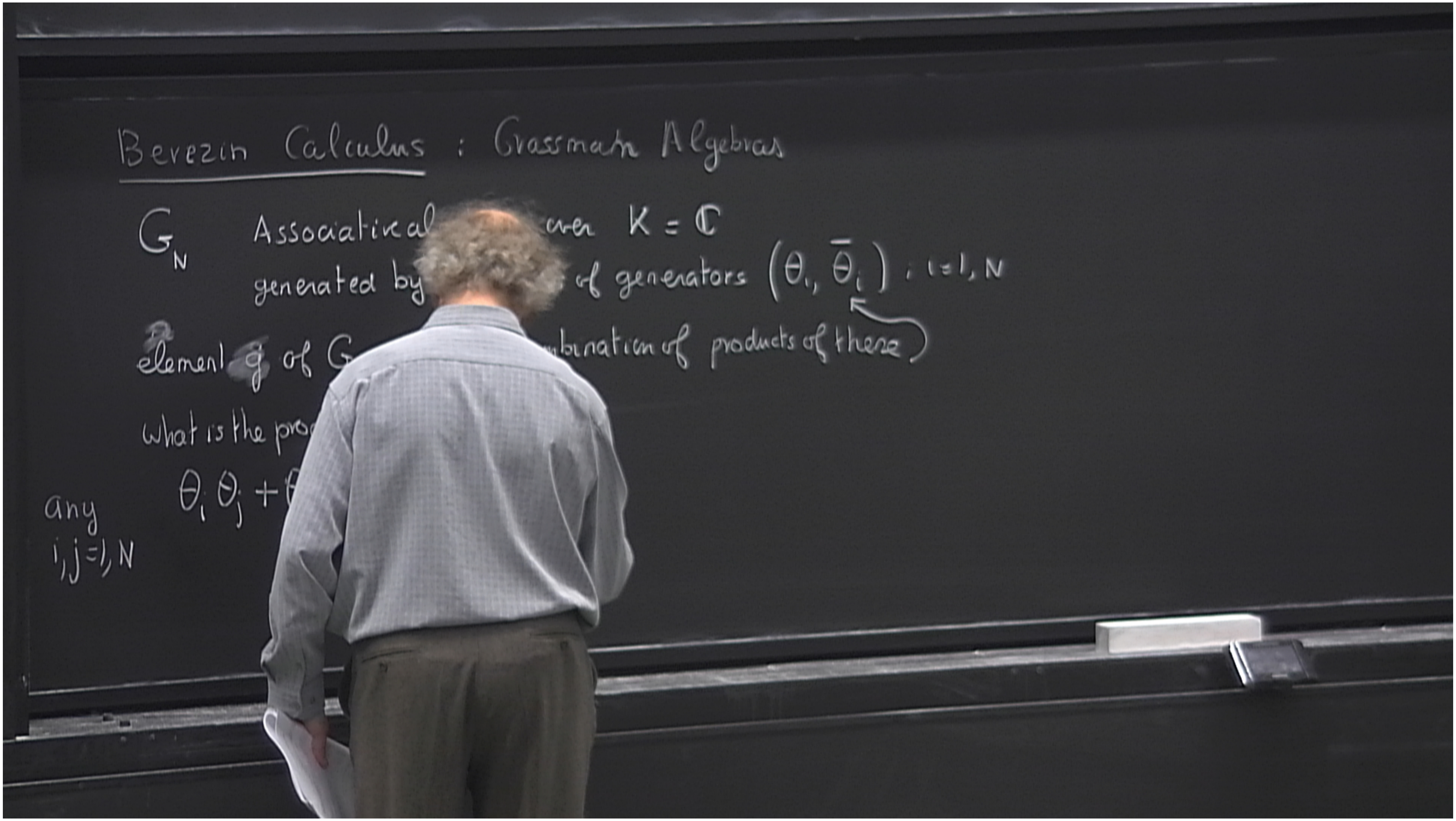
What is the product? anticommutes
$\theta_i \theta_j + \theta_j \theta_i = 0$


Berezin Calculus : Grassmann Algebras
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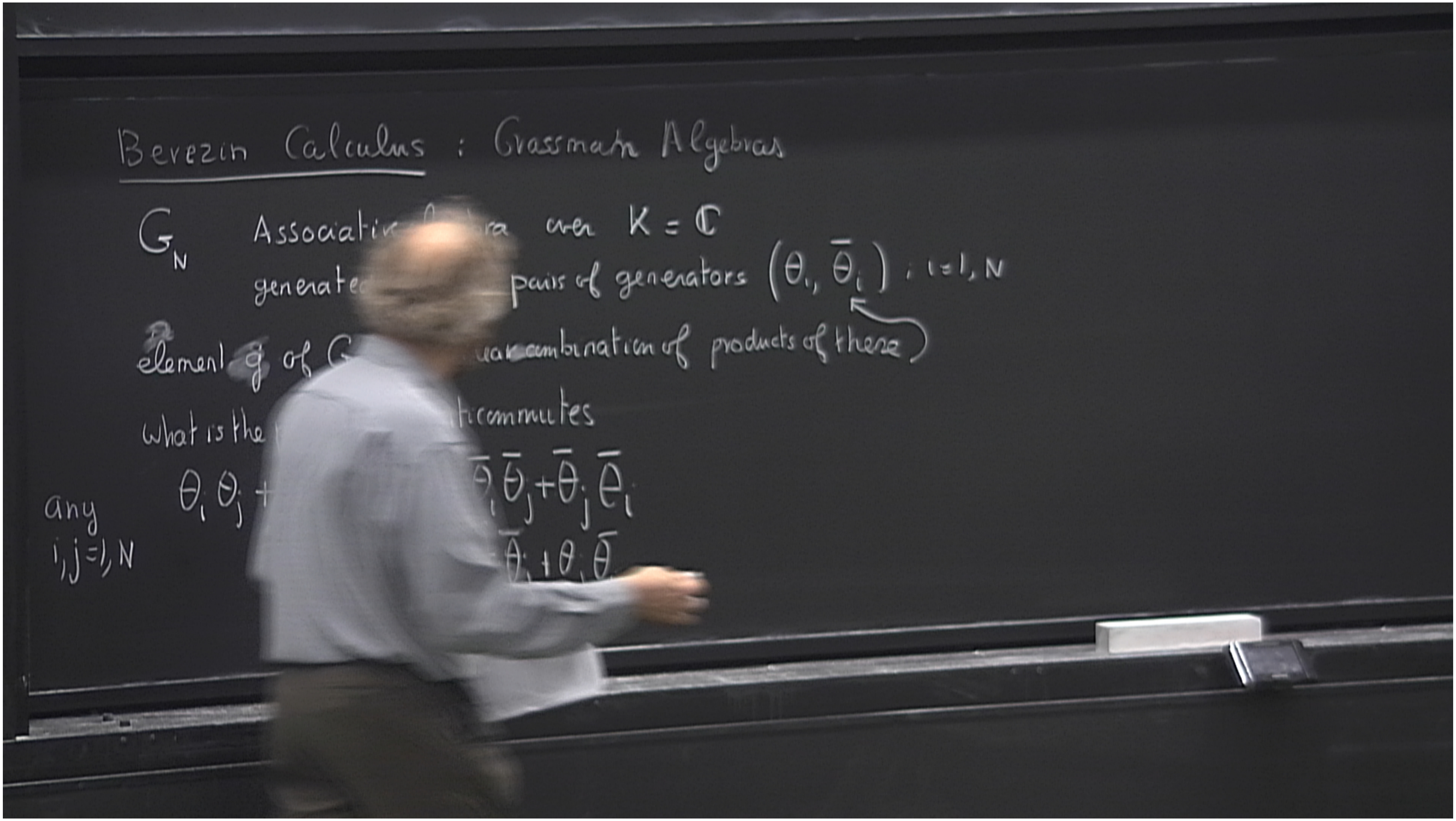
θ_i θ_j +
any
i, j = 1, N


Berezin Calculus : Grassmann Algebras
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Berezin Calculus : Grassmann Algebras

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$$\theta_i \theta_j + \theta_j \theta_i = 0 = \bar{\theta}_i \bar{\theta}_j + \bar{\theta}_j \bar{\theta}_i$$

$$= \bar{\theta}_i \theta_j + \theta_j \bar{\theta}_i$$

Berezin Calculus : Grassmann Algebras

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What is the product ? anticommutes (the rule!)

any $i, j = 1, N$

$$\theta_i \theta_j + \theta_j \theta_i = 0 = \bar{\theta}_i \bar{\theta}_j + \bar{\theta}_j \bar{\theta}_i \Rightarrow \theta_i^2 = 0$$

$$= \bar{\theta}_i \theta_j + \theta_j \bar{\theta}_i \qquad \bar{\theta}_j^2 = 0$$

n Algebras

$K = \mathbb{C}$

generators $(\theta_i, \bar{\theta}_i)$; $i = 1, N$

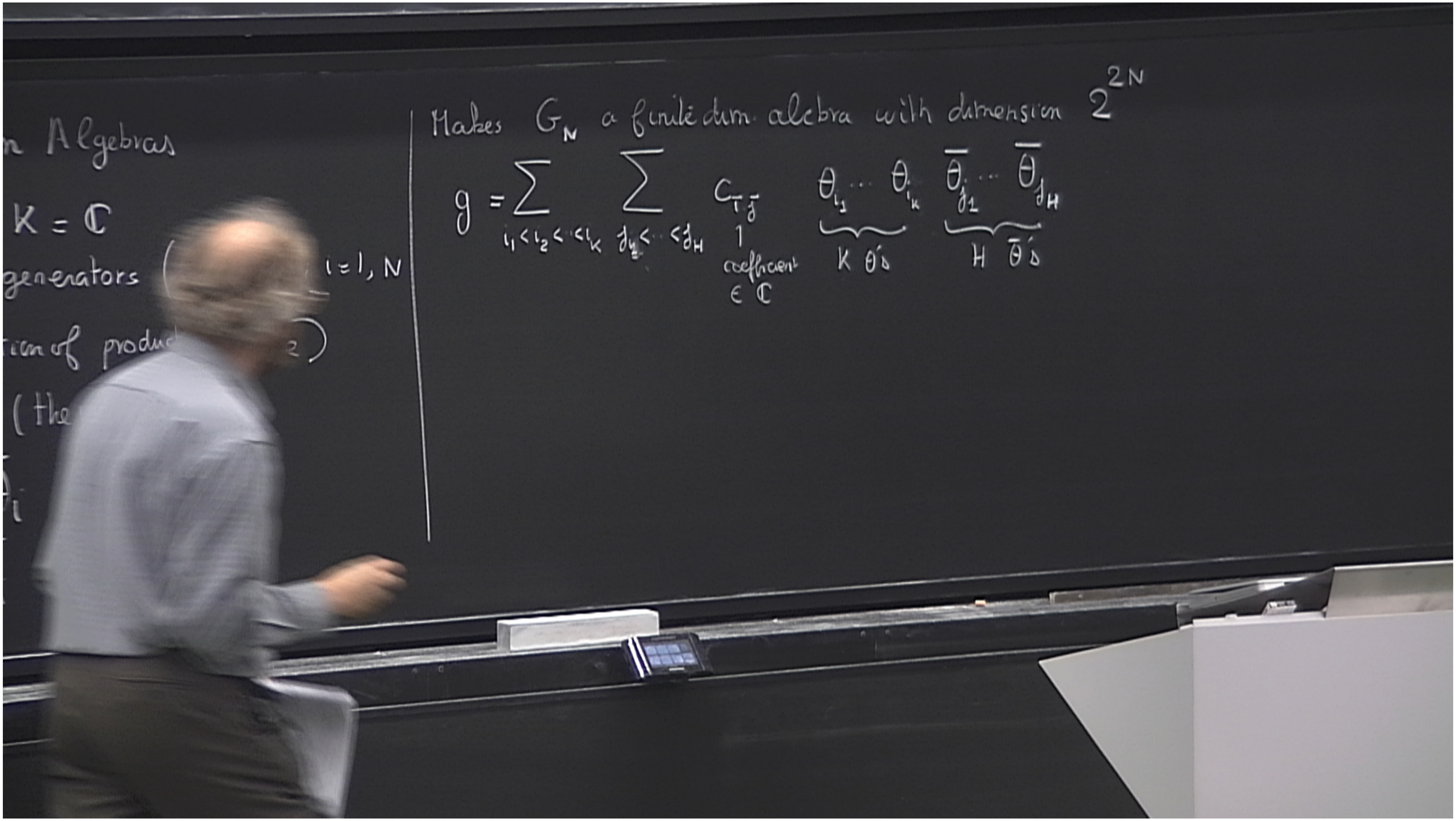
tion of products of these)

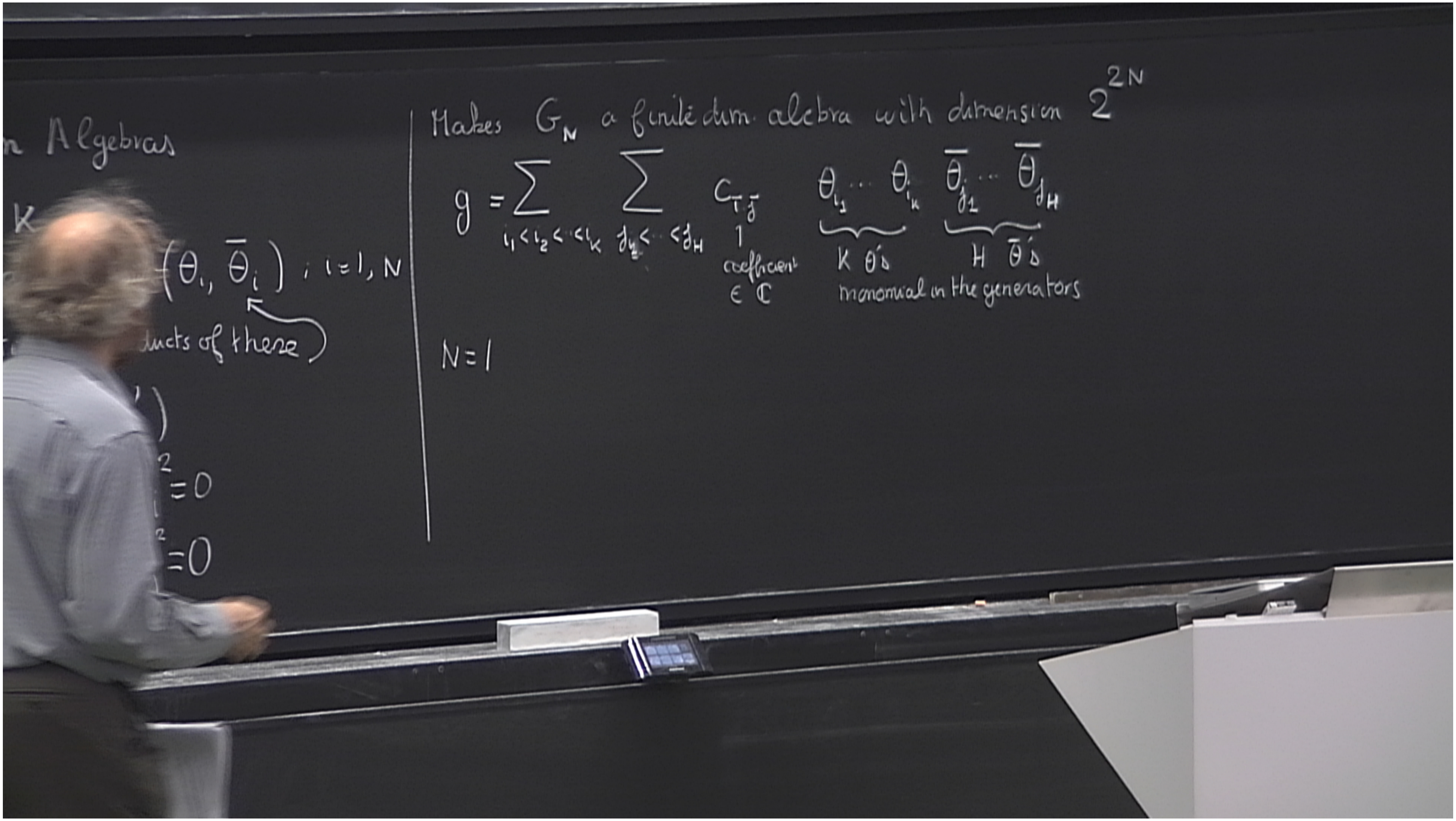
(the rule!)

$\theta_i \Rightarrow \theta_i^2 = 0$

$\bar{\theta}_j^2 = 0$

Makes G_N a finite dim. alcbra with dimension 2^{2N}


n Algebras
K = ℂ
generators
i = 1, N
Makes G_N a finite dim. alcebra with dimension 2^2N
coefficient ∈ ℂ
K θ's
H θ̄'s


n Algebras
K
$(\theta_i, \bar{\theta}_i)$; $i=1, N$
ducts of these
$=0$
$=0$
Makes G_N a finite dim. alcebra with dimension 2^{2N}
$g = \sum_{i_1<i_2<\cdots<i_K} \sum_{j_1<\cdots<j_H} C_{\vec{i}\,\vec{j}} \; \theta_{i_1}\cdots\theta_{i_K}\;\bar{\theta}_{j_1}\cdots\bar{\theta}_{j_H}$
coefficient $\in \mathbb{C}$
K θ's
H $\bar{\theta}$'s
monomial in the generators
$N=1$

...n Algebras

$K = \mathbb{C}$

generators $(\theta_i, \bar{\theta}_i)$; $i = 1, N$

...tion of products of these

(the rule!)

$\Rightarrow \theta_i^2 = 0$

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Makes G_N a finite dim. alcebra with dimension 2^{2N}

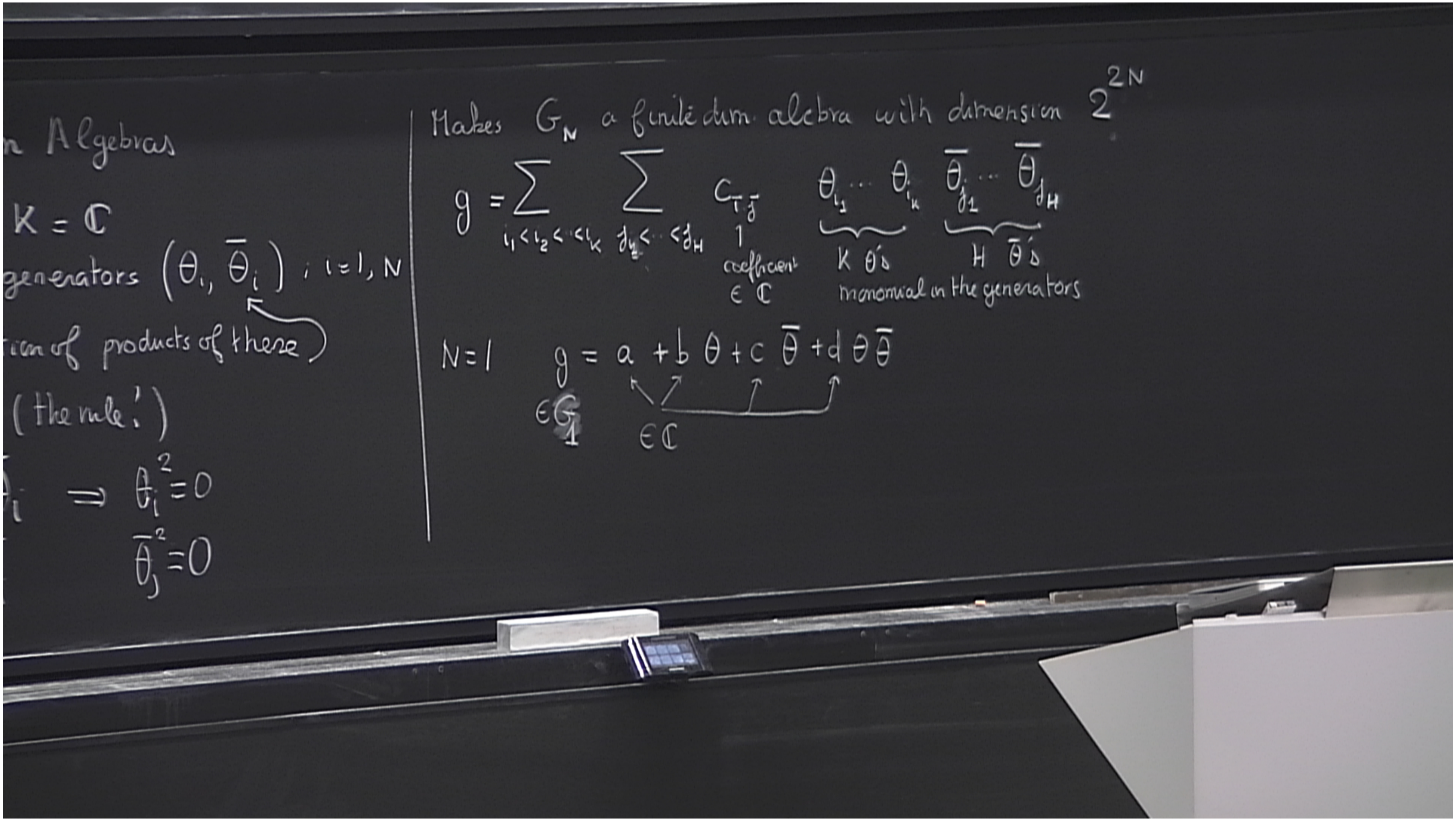
$$g = \sum_{i_1 < i_2 < \dots < i_K} \; \sum_{j_1 < \dots < j_H} C_{\vec{I}\vec{J}} \; \underbrace{\theta_{i_1} \cdots \theta_{i_K}}_{K\ \theta\text{'s}} \; \underbrace{\bar{\theta}_{j_1} \cdots \bar{\theta}_{j_H}}_{H\ \bar{\theta}\text{'s}}$$

$C_{\vec{I}\vec{J}}$: coefficient $\in \mathbb{C}$

monomial in the generators

$N = 1$ $\quad g = a + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$

$g \in G_1$ $\quad a, b, c, d \in \mathbb{C}$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coefficient $\in \mathbb{C}$
K θ's
H $\bar{\theta}$'s
monomial in the generators
N = 1
$g = a + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$
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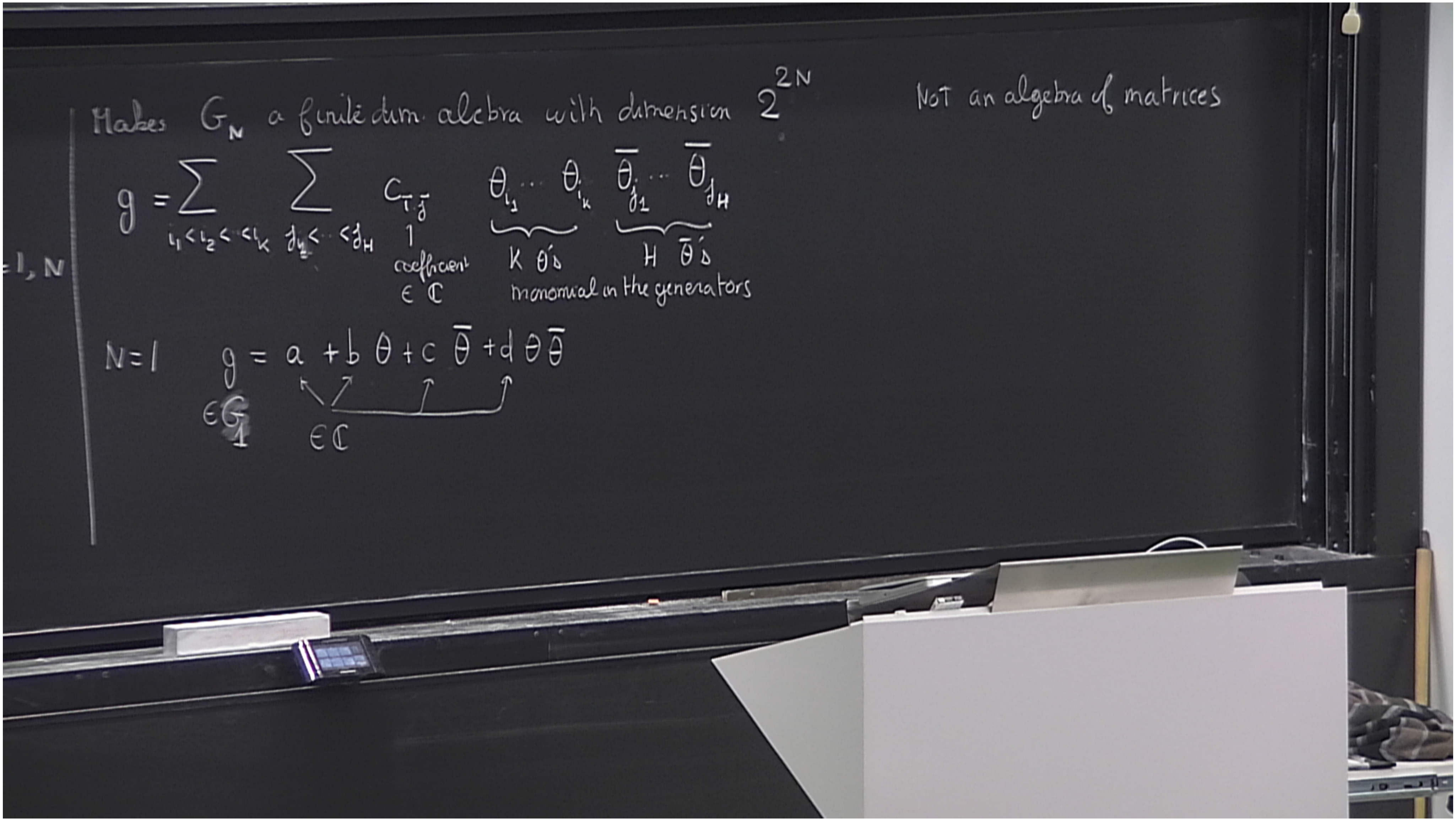
Makes G_N a finite dim. alcebra with dimension 2^{2N}
NOT an algebra of matrices
$g = \sum_{i_1<i_2<\dots<i_K} \sum_{j_1<\dots<j_H} C_{\bar{i}\bar{j}} \, \theta_{i_1} \cdots \theta_{i_K} \bar{\theta}_{j_1} \cdots \bar{\theta}_{j_H}$
coefficient $\in \mathbb{C}$
K θ's
H $\bar{\theta}$'s
monomial in the generators
$N=1$ $g = a + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$
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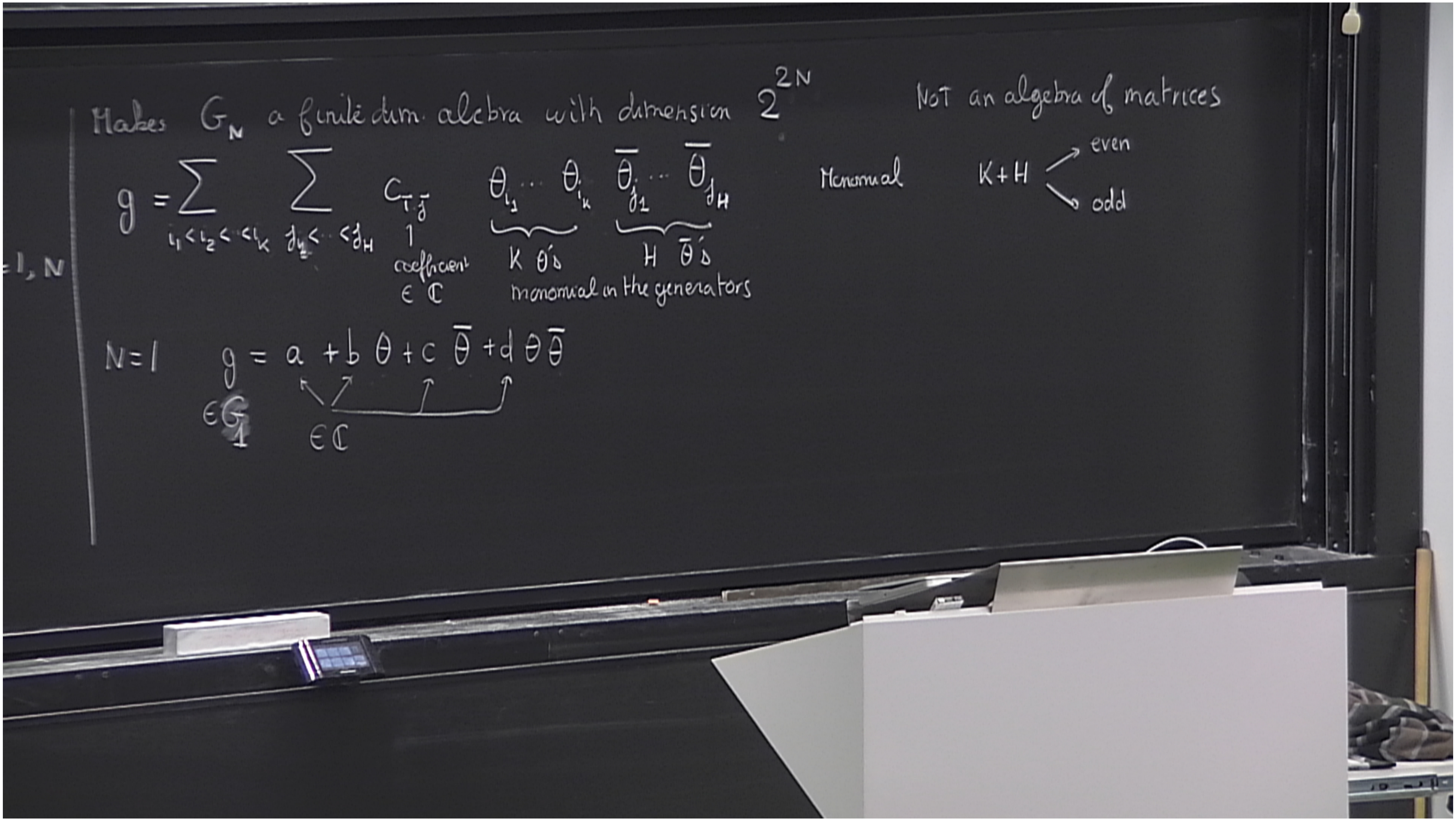
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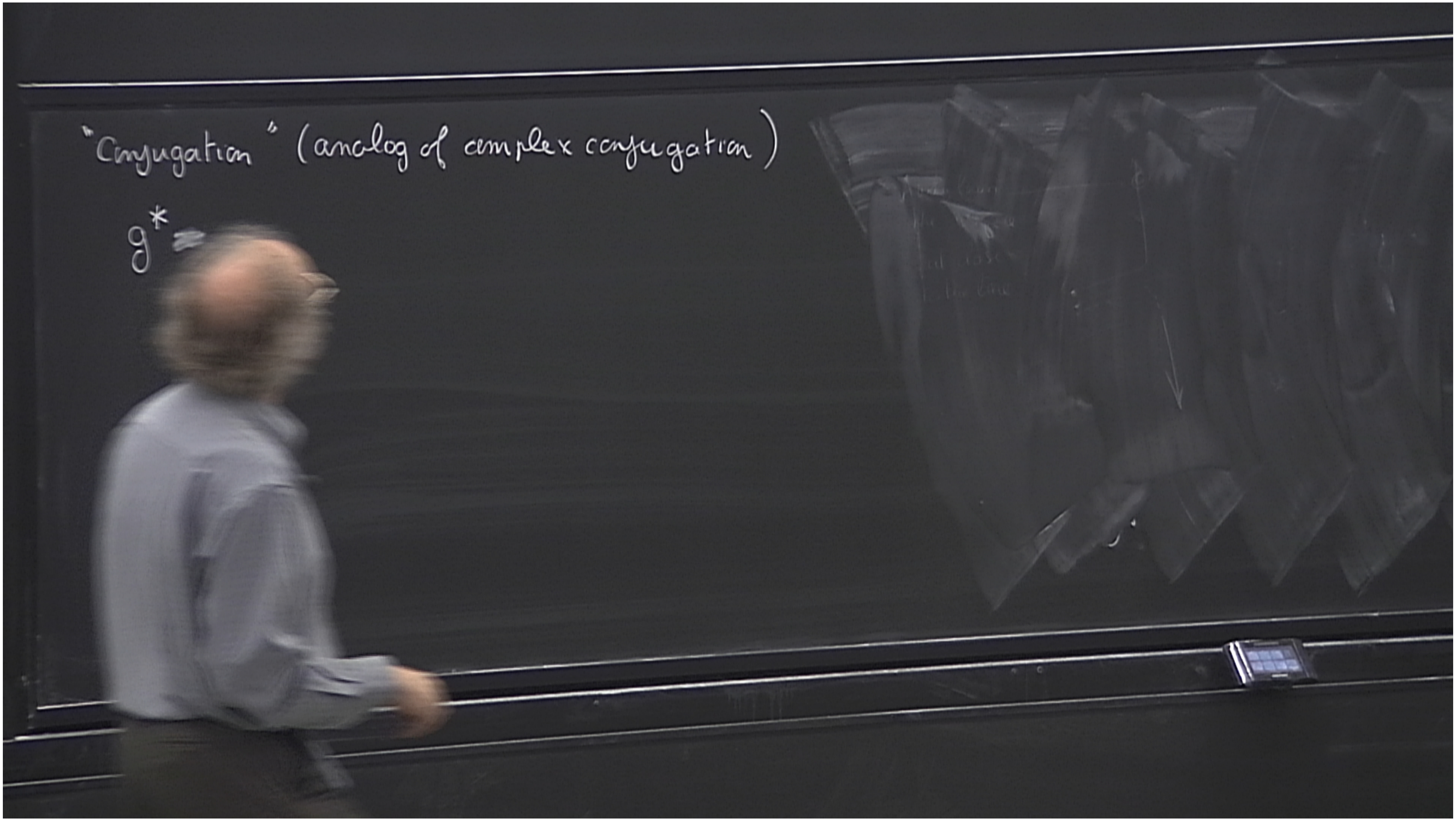
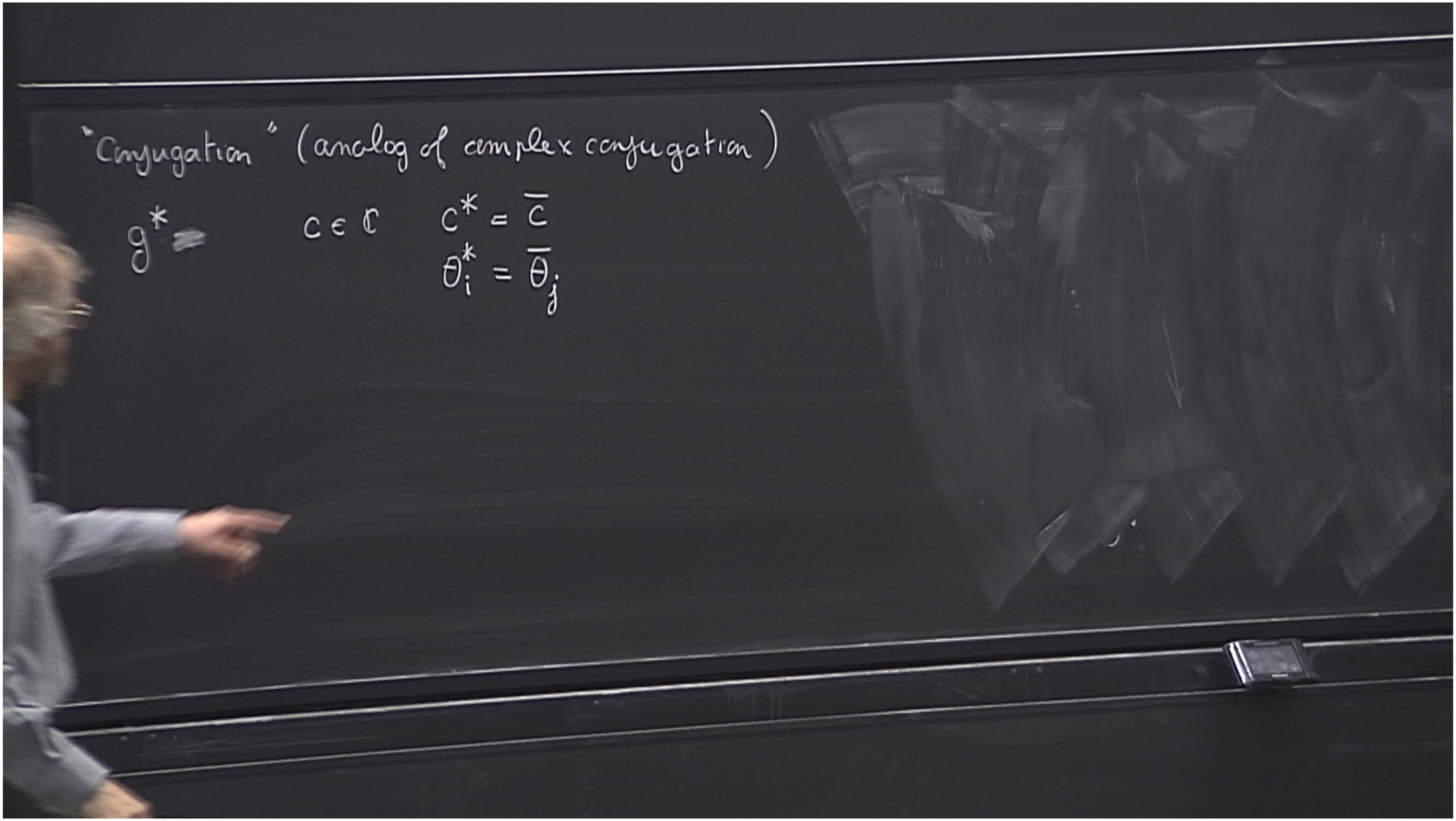
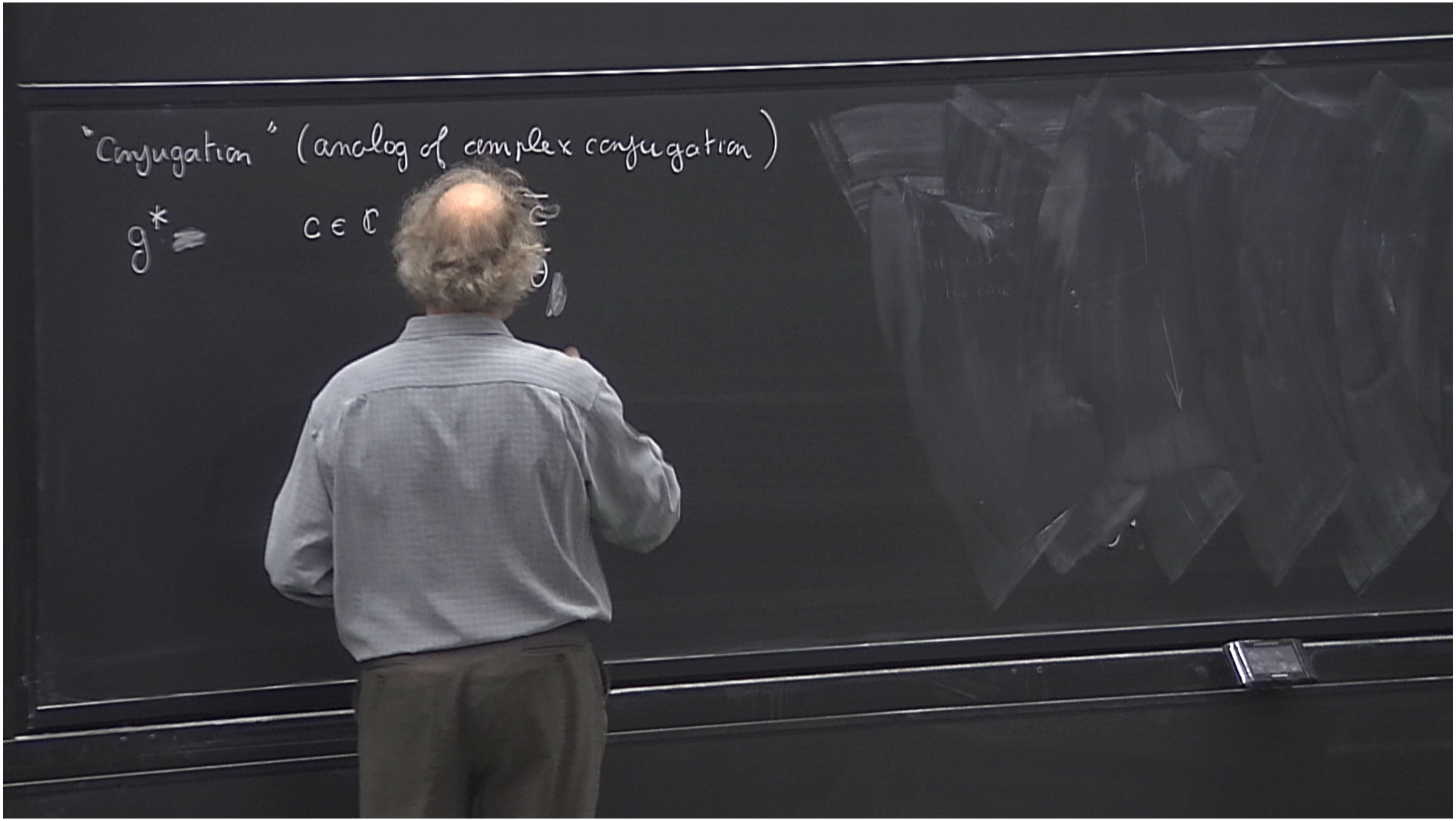
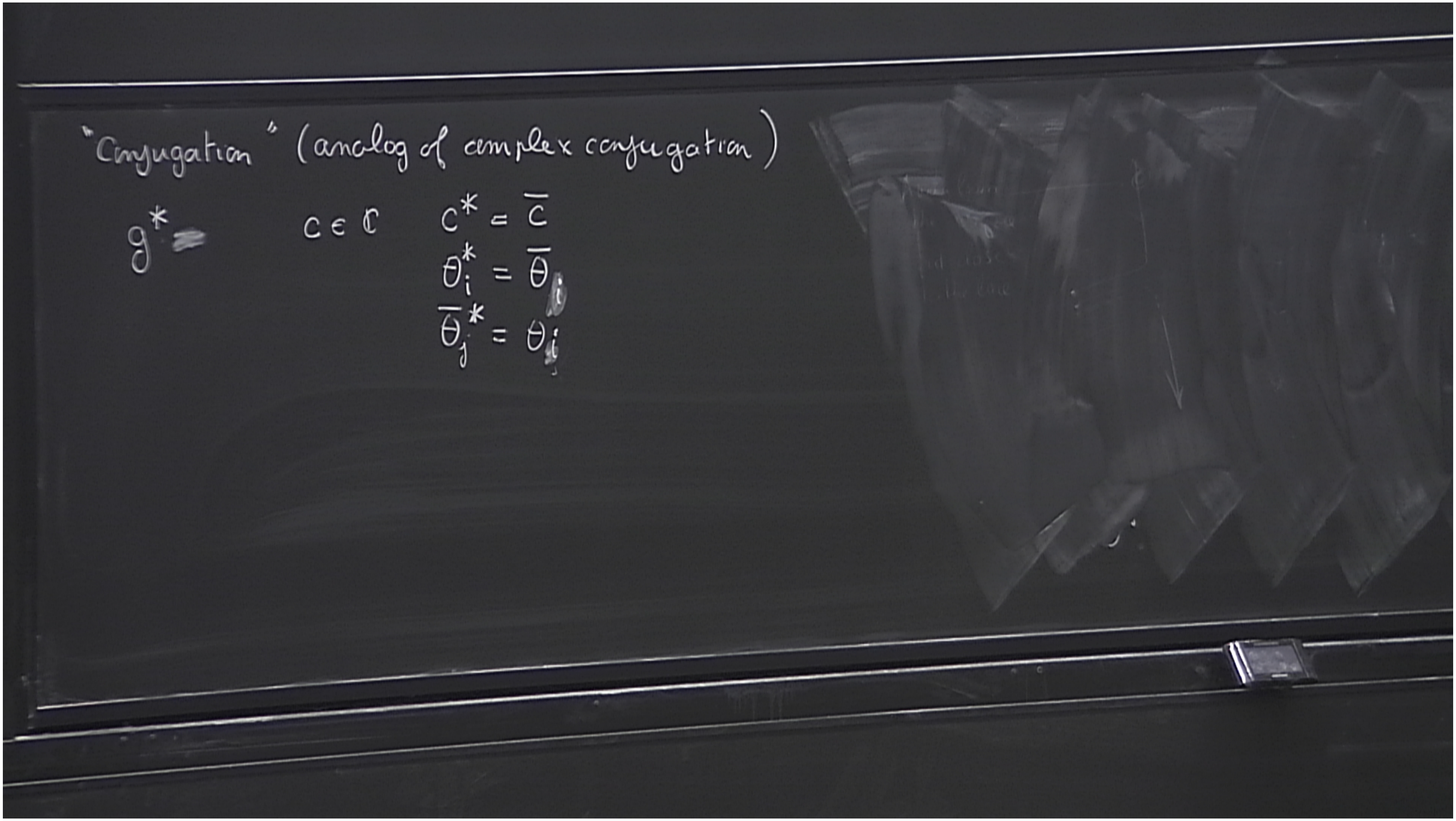
coefficient $\in \mathbb{C}$
K θ's
H $\bar{\theta}$'s
monomial in the generators
Monomial $K+H$ even / odd
$=1, N$
$N=1$ $g = a + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$
$\in G_1$
$\in \mathbb{C}$


"Conjugation" (analog of complex conjugation)
g* =


"Conjugation" (analog of complex conjugation)
g^*
$c \in \mathbb{C}$ $c^* = \bar{c}$
$\theta_i^* = \bar{\theta}_j$


"Conjugation" (analog of complex conjugation)
g* =
c ∈ ℂ


"Conjugation" (analog of complex conjugation)
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$c^* = \bar{c}$
$\theta_i^* = \bar{\theta}_i$
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"Conjugation" (analog of complex conjugation)

$g^* =$ $\qquad c \in \mathbb{C} \qquad c^* = \bar{c}$

$$\theta_i^* = \bar{\theta}_i$$

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$g_1, g_2 \in G_N \qquad (g_1 \cdot g_2)^* =$

"Conjugation" (analog of complex conjugation & hermitian conjugation for matrices)

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"Conjugation" (analog of complex conjugation & hermitian conjugation for matrices)

$g^* =$

$c \in \mathbb{C} \qquad c^* = \bar{c}$

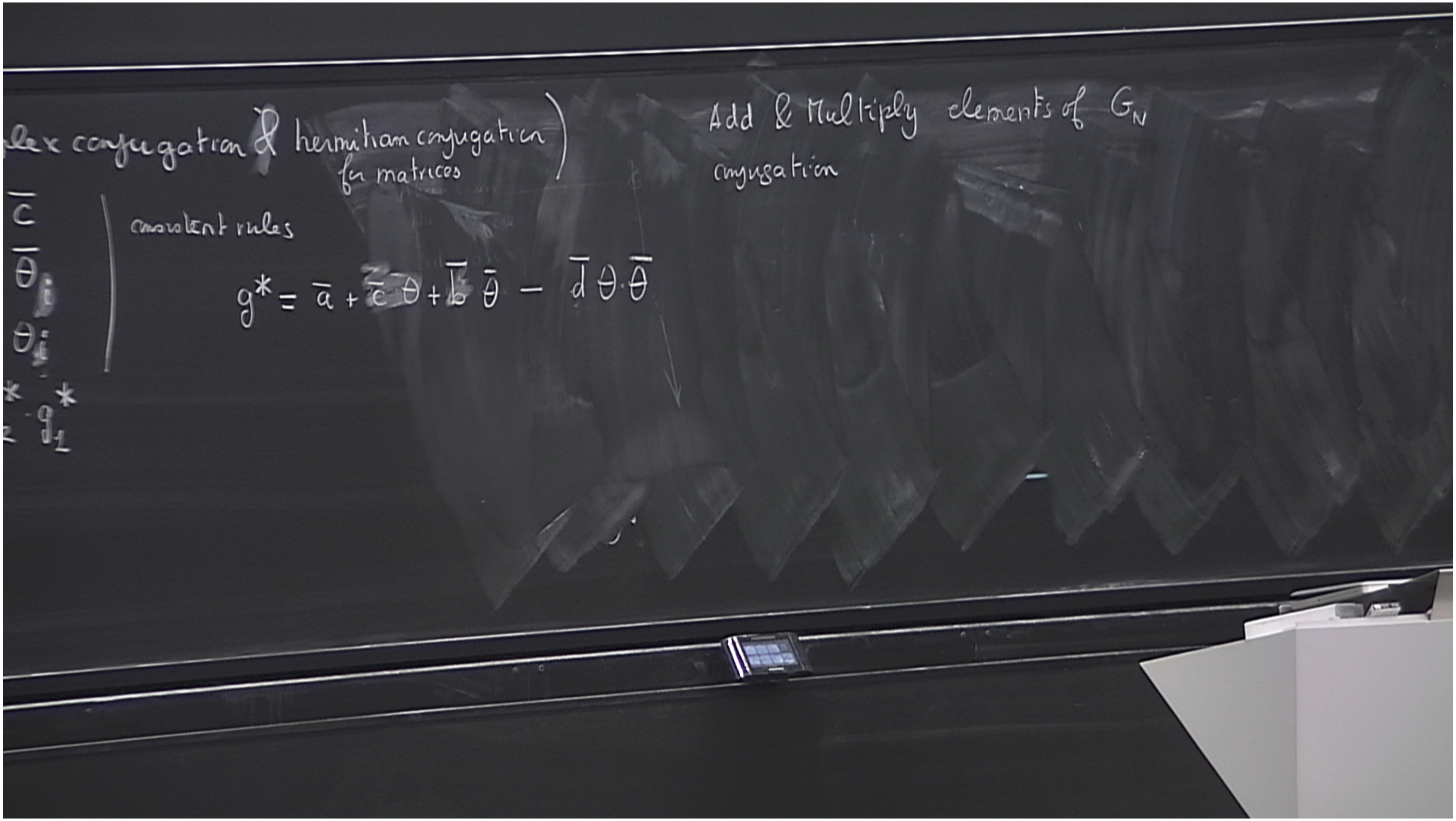
$\theta_i^* = \bar{\theta}_i$

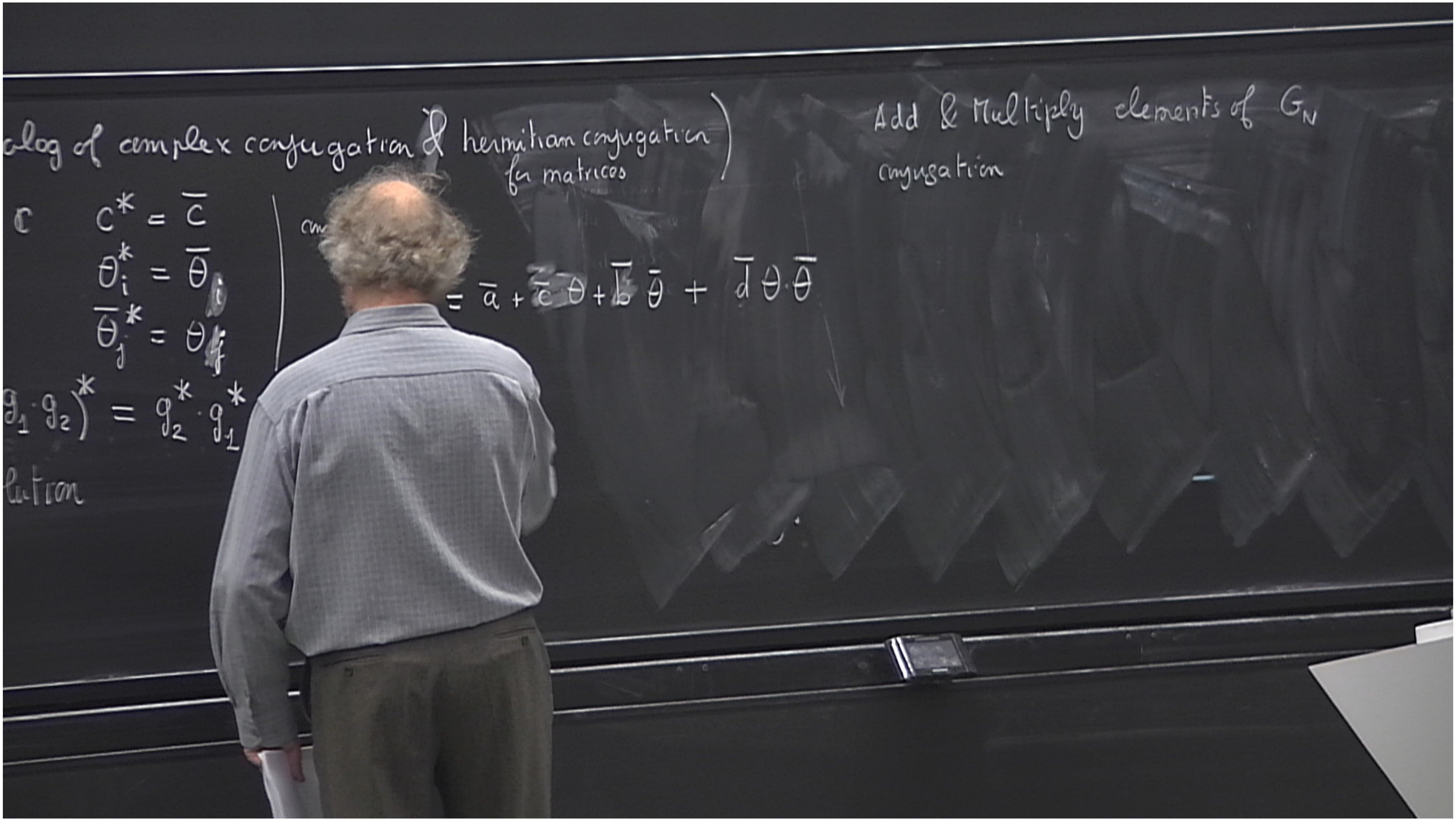
$\bar{\theta}_j^* = \theta_i$

consistent rules

$$g^* = \bar{a} + \bar{c}\,\theta + \bar{b}\,\bar{\theta} - \bar{d}\,\theta \cdot \bar{\theta}$$

$g_1, g_2 \in G_N \qquad (g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$


lex conjugation & hermitian conjugation
for matrices
consistent rules
$g^* = \bar{a} + \bar{c}\,\theta + \bar{b}\,\bar{\theta} - \bar{d}\,\theta\,\bar{\theta}$
Add & Multiply elements of G_N
conjugation



alog of complex conjugation & hermitian conjugation
for matrices
Add & Multiply elements of G_N
conjugation
$c^* = \bar{c}$
$\theta_i^* = \bar{\theta}_i$
$\bar{\theta}_j^* = \theta_j$
$(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$
lution
$= \bar{a} + \bar{c}\,\theta + \bar{b}\,\bar{\theta} + \bar{d}\,\theta \cdot \bar{\theta}$

…alog of complex conjugation & hermitian conjugation for matrices)

Add & Multiply elements of G_N

conjugation

$\mathbb{C}$ $c^* = \bar{c}$

$\theta_i^* = \bar{\theta}_i$

$\bar{\theta}_j^* = \theta_j$

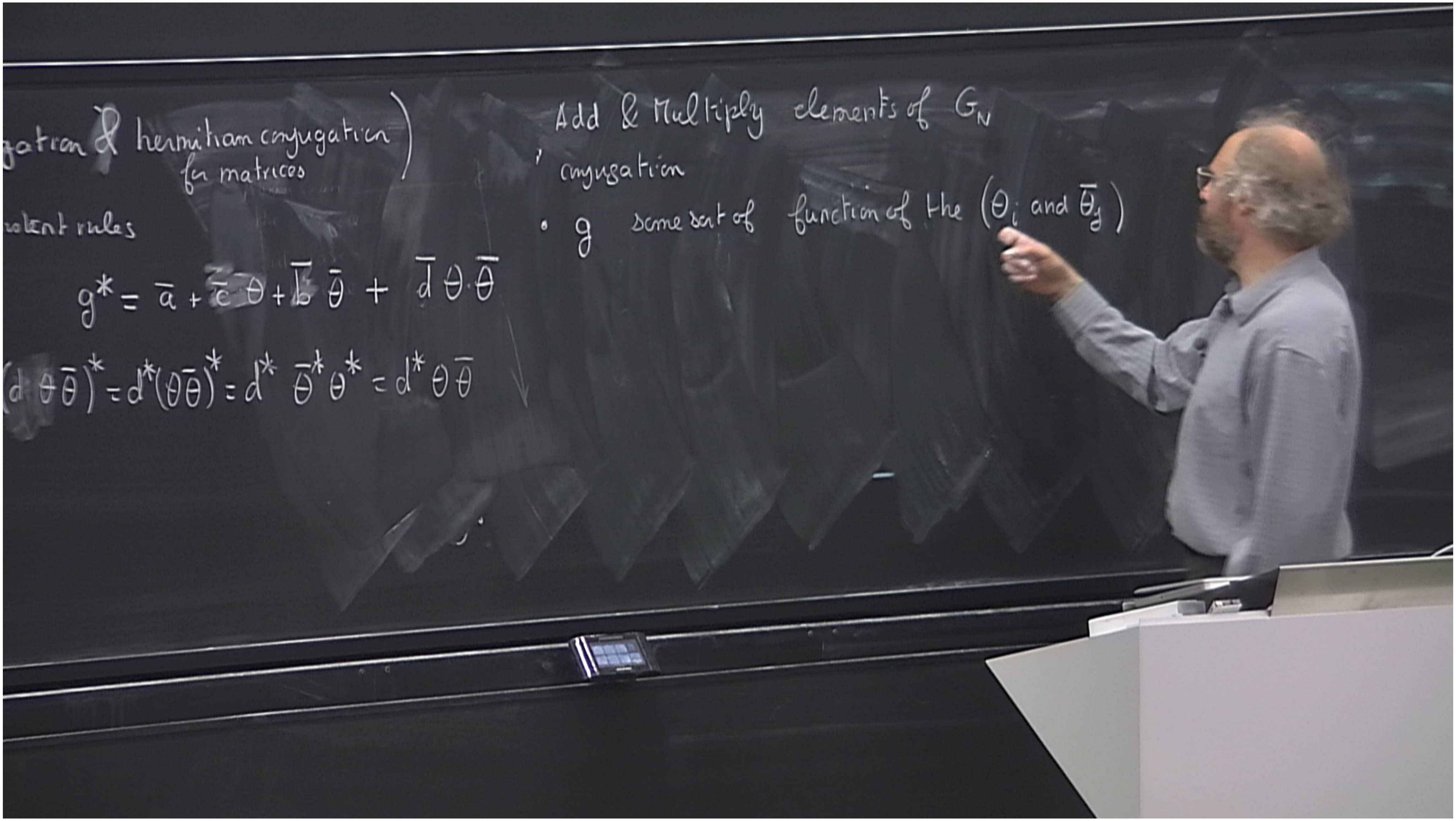
$(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$

…lution

consistent rules

$$g^* = \bar{a} + \bar{c}\,\theta + \bar{b}\,\bar{\theta} + \bar{d}\,\theta\cdot\bar{\theta}$$

$$(d\,\theta\bar{\theta})^* = d^*(\theta\bar{\theta})^* = d^*\,\bar{\theta}^*\theta^* = d^*\,\theta\bar{\theta}$$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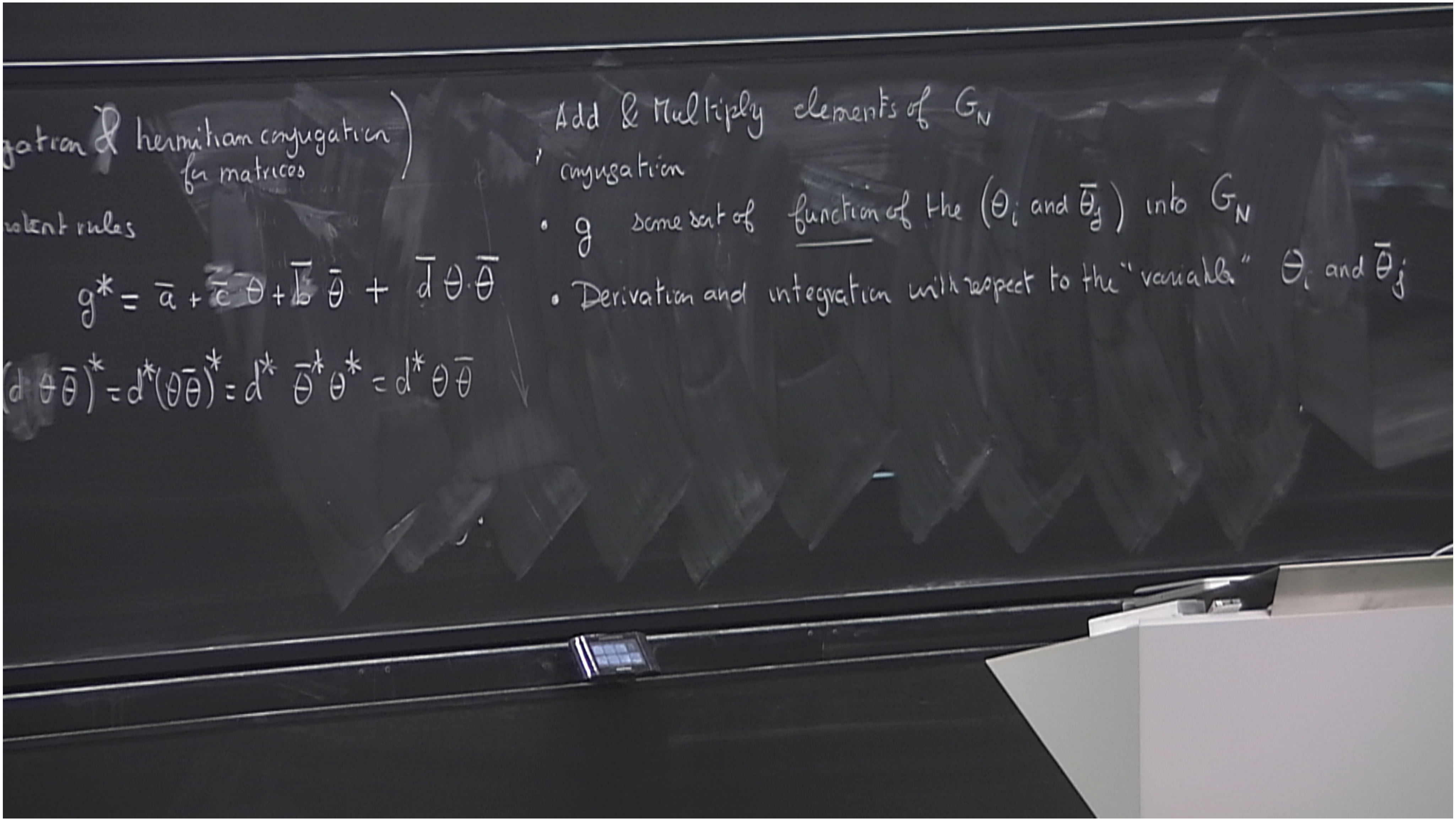
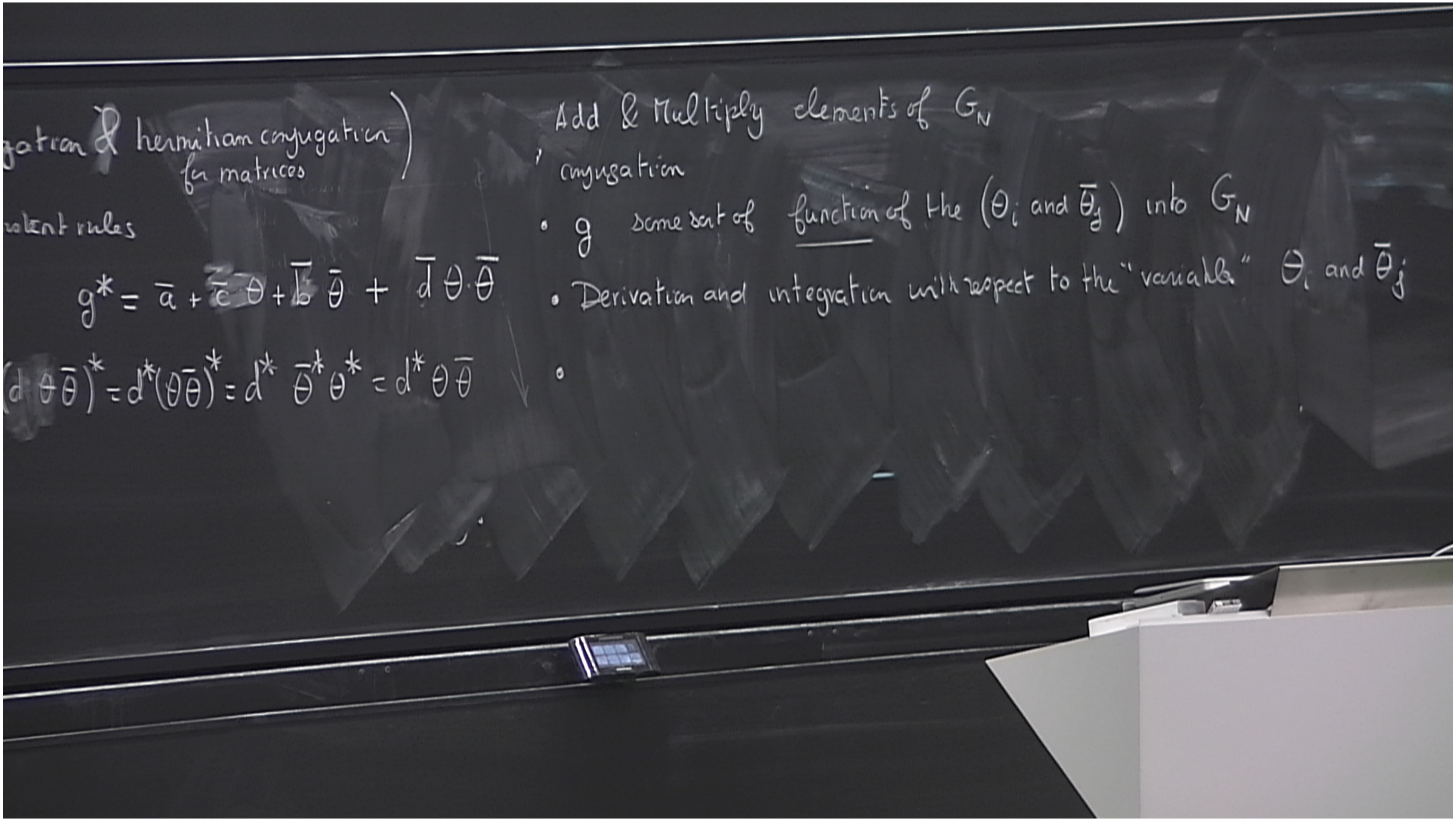
conjugation
• g some sort of function of the (θ_i and $\bar{\theta}_j$)


gation & hermitian conjugation for matrices)
valent rules
g* = ā + c̄ θ + b̄ θ̄ + d̄ θ·θ̄
(d θθ̄)* = d*(θθ̄)* = d* θ̄* θ* = d* θ θ̄
Add & Multiply elements of G_N
conjugation
• g some sort of function of the (θ_i and θ̄_j) into G_N
• Derivation and integration with respect to the "variables" θ_i and θ̄_j


gation & hermitian conjugation
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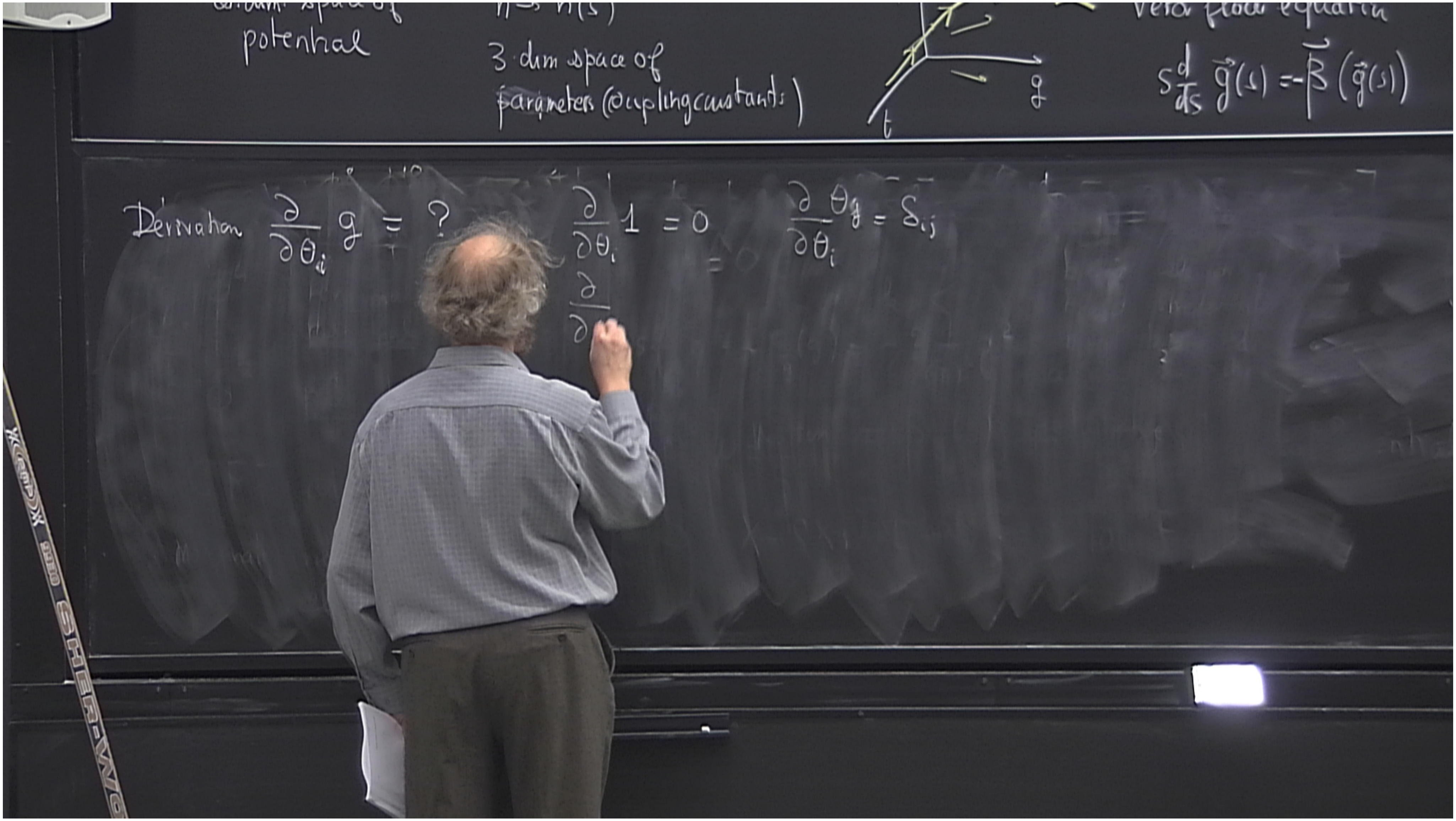
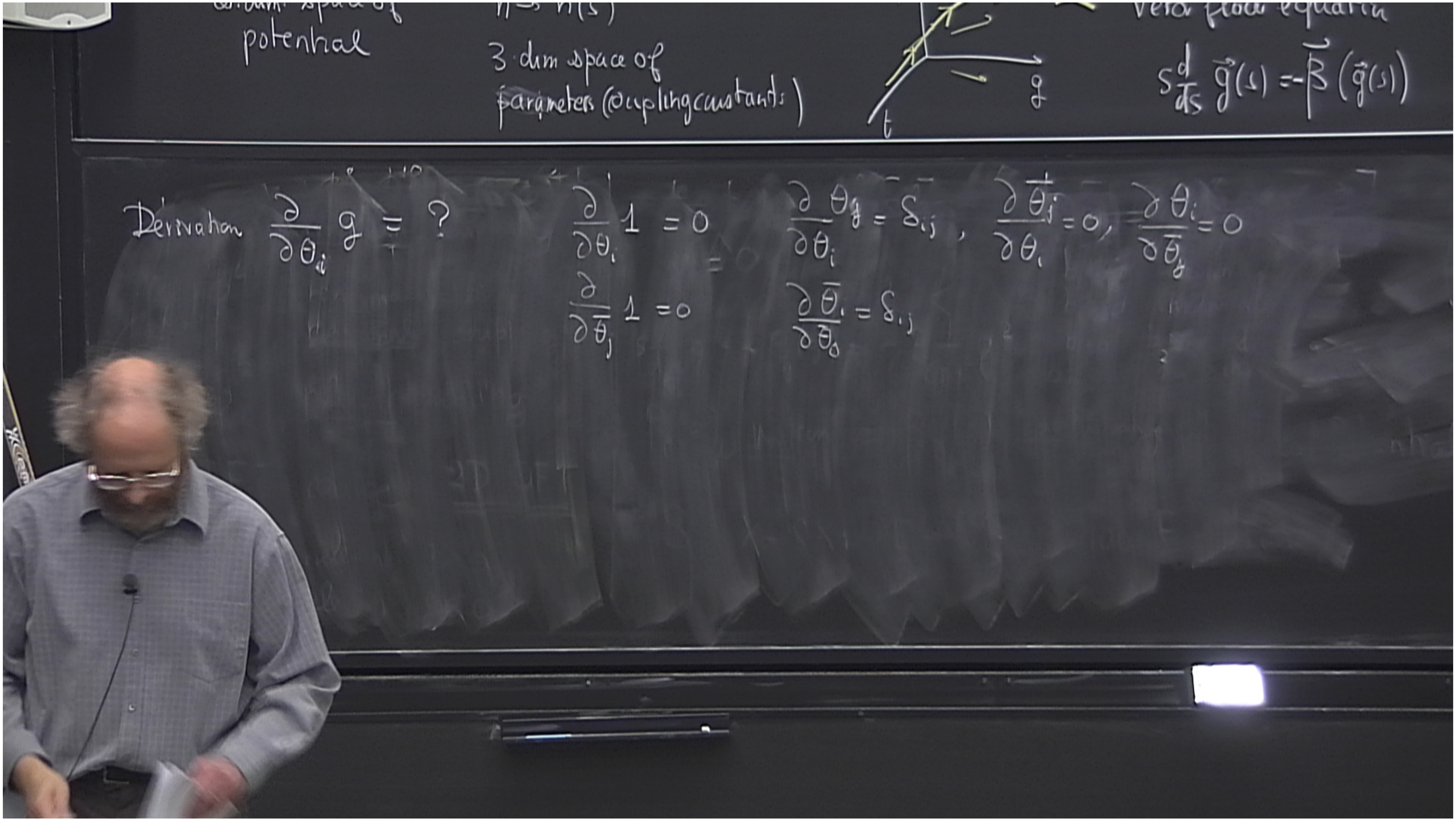
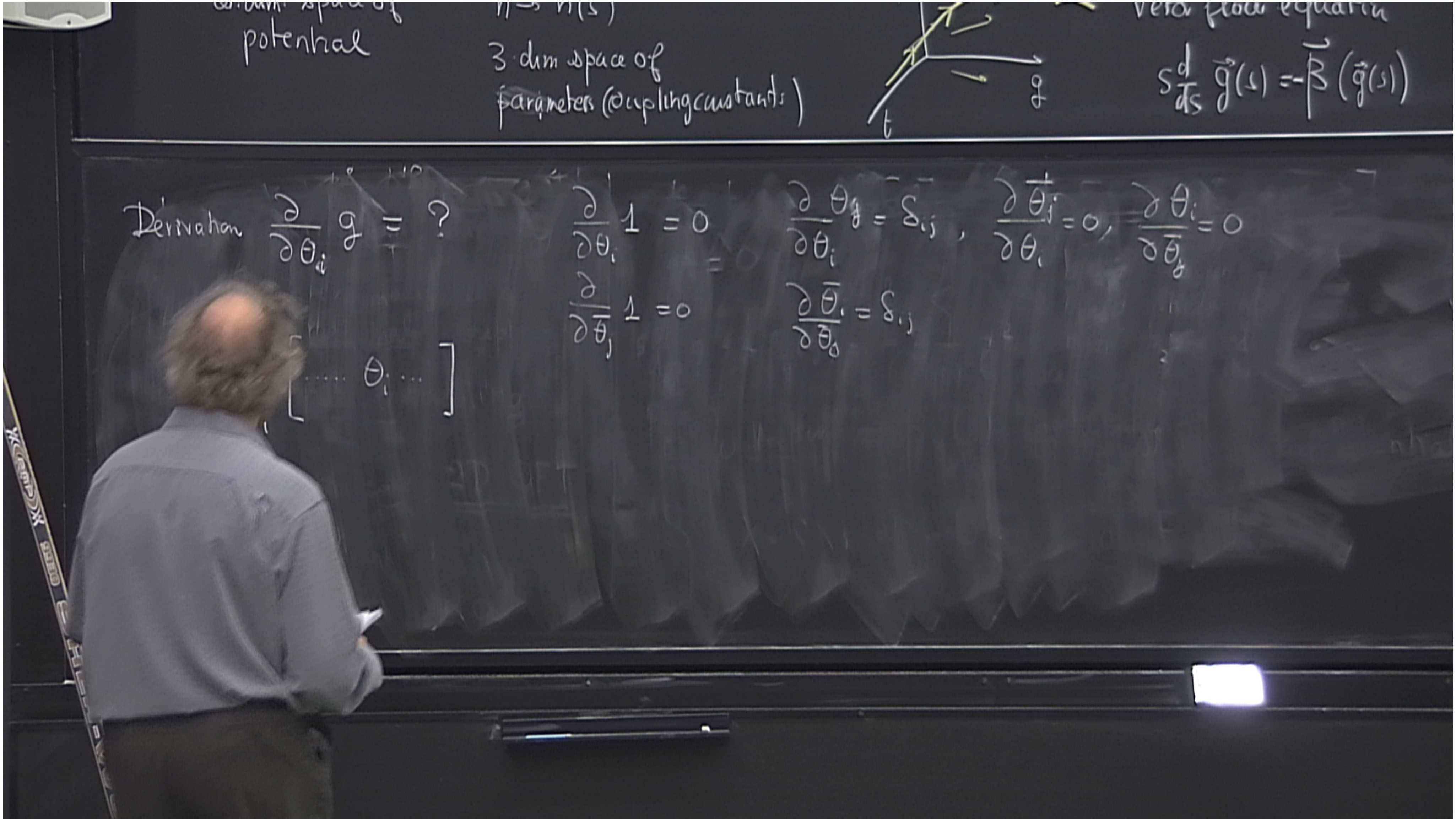
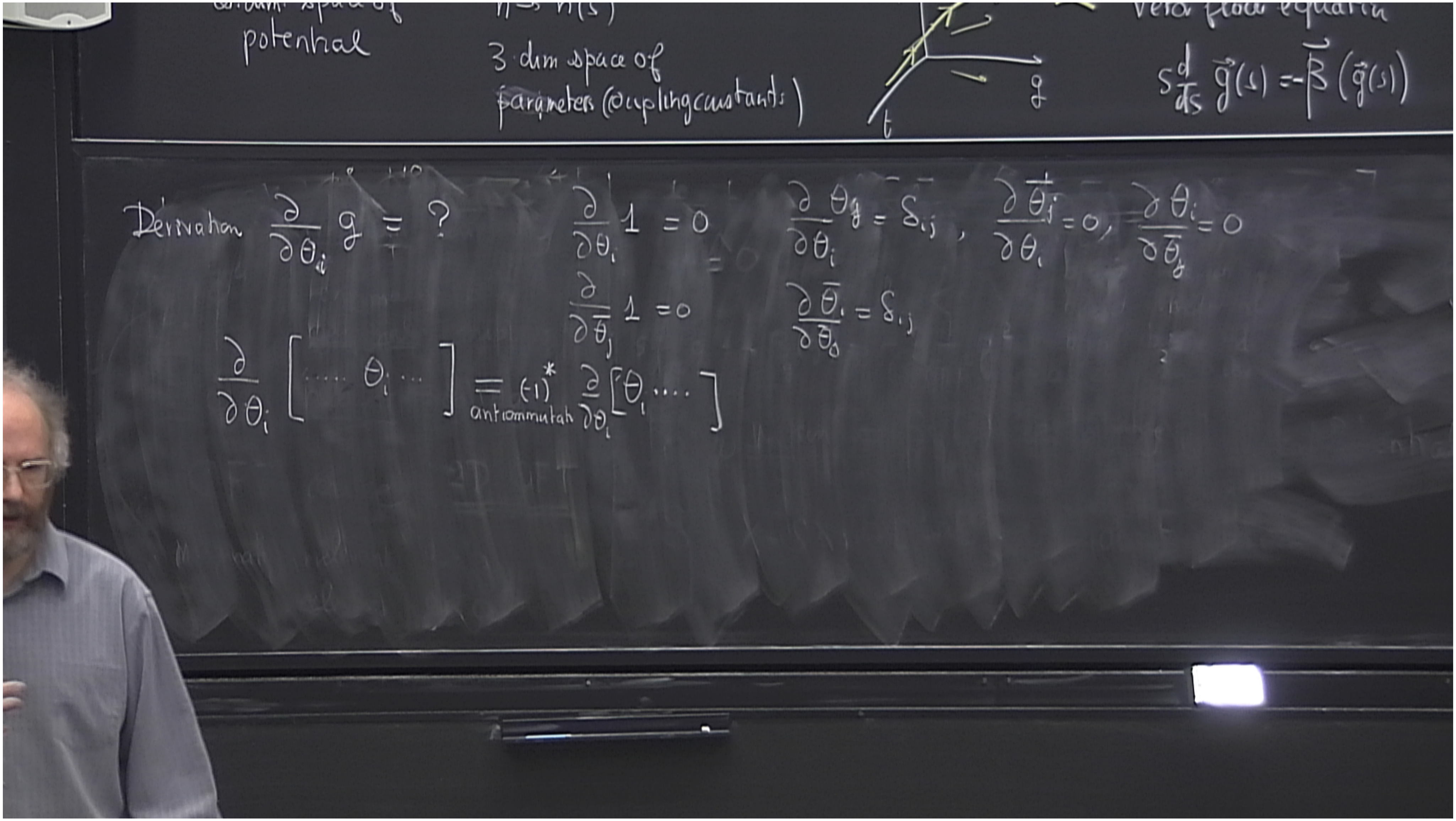
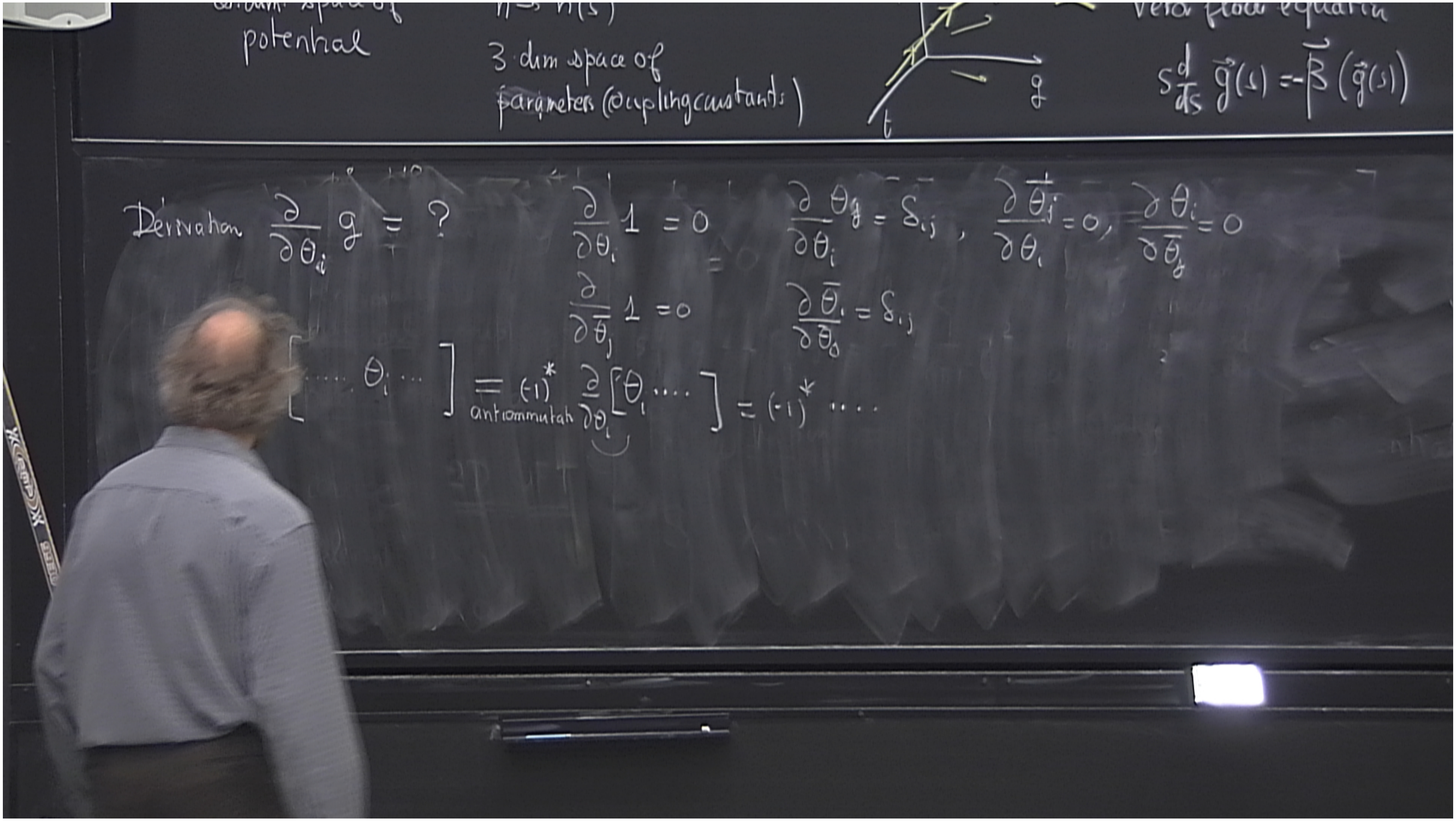
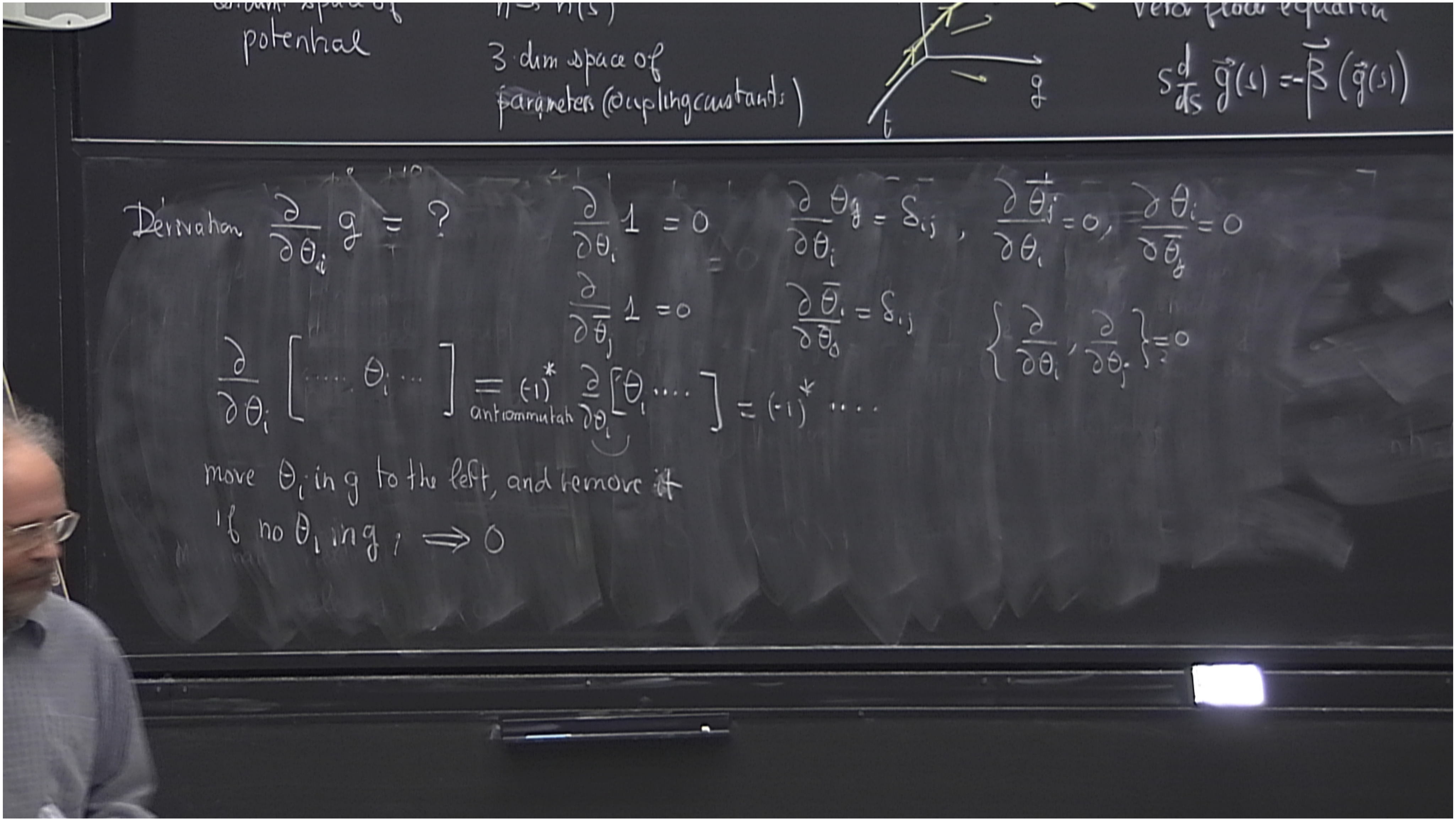

potential
3 dim space of
parameters (couplingconstants)
g
t
s d/ds g(s) = -β(g(s))
Derivation ∂/∂θi g = ?
∂/∂θi 1 = 0
∂/∂θi θj = δij

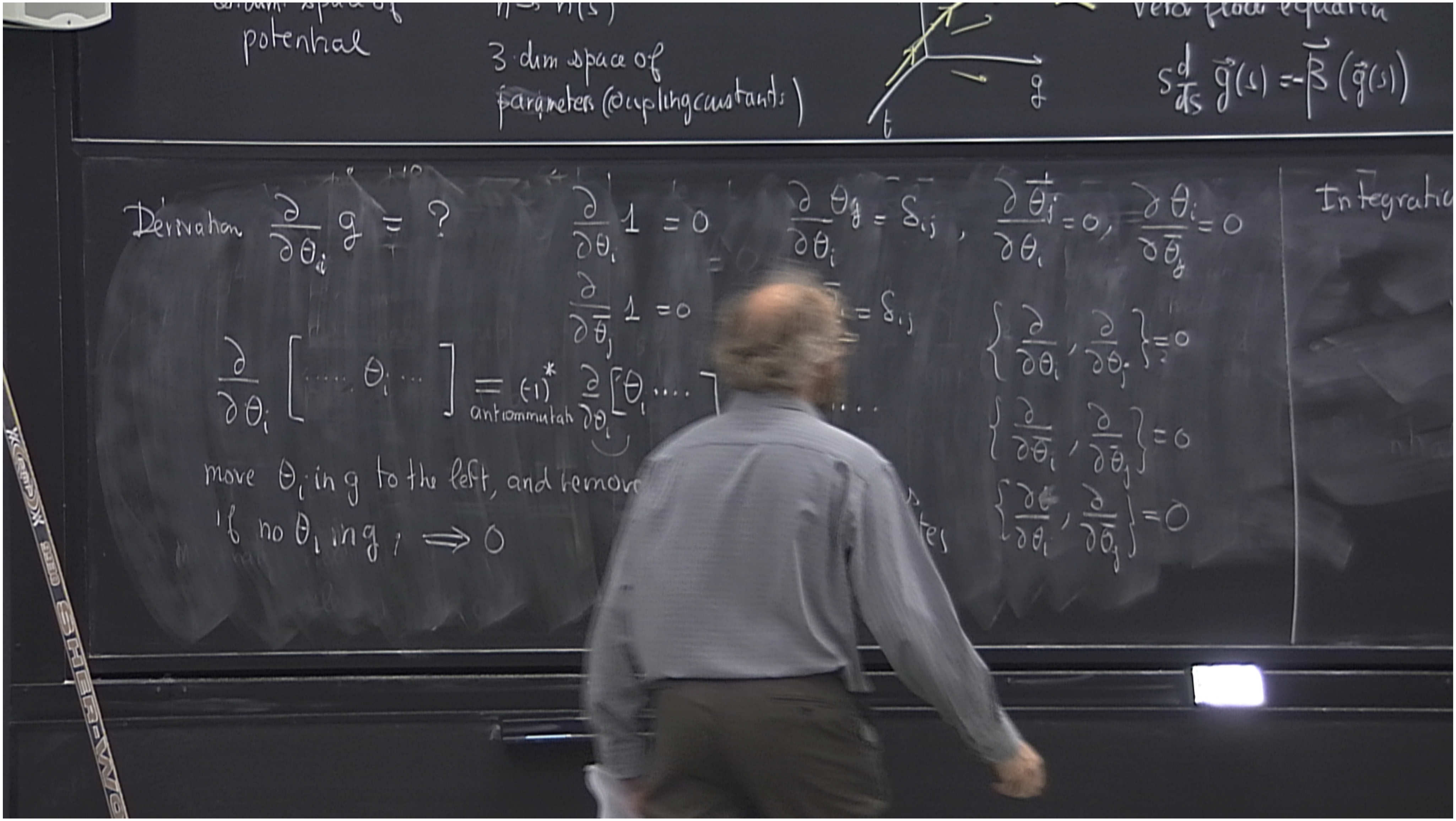

potential
3 dim space of
parameters (coupling constants)
g
t
$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
Derivation $\frac{\partial}{\partial \theta_i} g = ?$
$\frac{\partial}{\partial \theta_i} 1 = 0$
$\frac{\partial}{\partial \bar{\theta}_j} 1 = 0$
$\frac{\partial}{\partial \theta_i} \theta_j = \delta_{ij}$, $\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0$, $\frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\frac{\partial \bar{\theta}_i}{\partial \bar{\theta}_j} = \delta_{ij}$

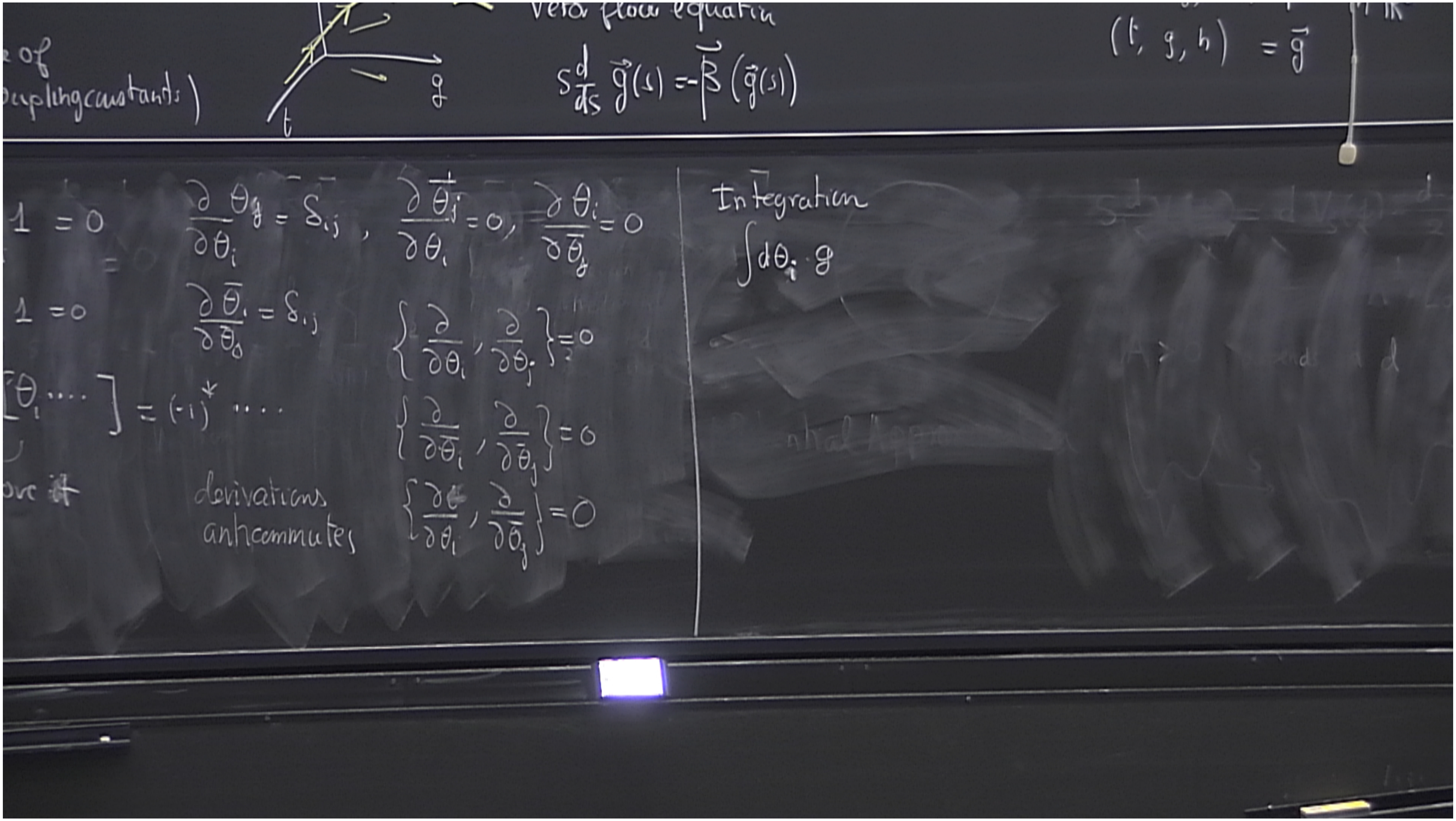

potential
3 dim space of
parameters (couplingconstants)
g
t
$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
Derivation $\frac{\partial}{\partial \theta_i} g = ?$
$\frac{\partial}{\partial \theta_i} 1 = 0$
$\frac{\partial}{\partial \bar{\theta}_j} 1 = 0$
$\frac{\partial \theta_j}{\partial \theta_i} = \delta_{ij}$, $\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0$, $\frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\frac{\partial \bar{\theta}_i}{\partial \bar{\theta}_j} = \delta_{ij}$
$[\cdots\cdots \theta_i \cdots]$

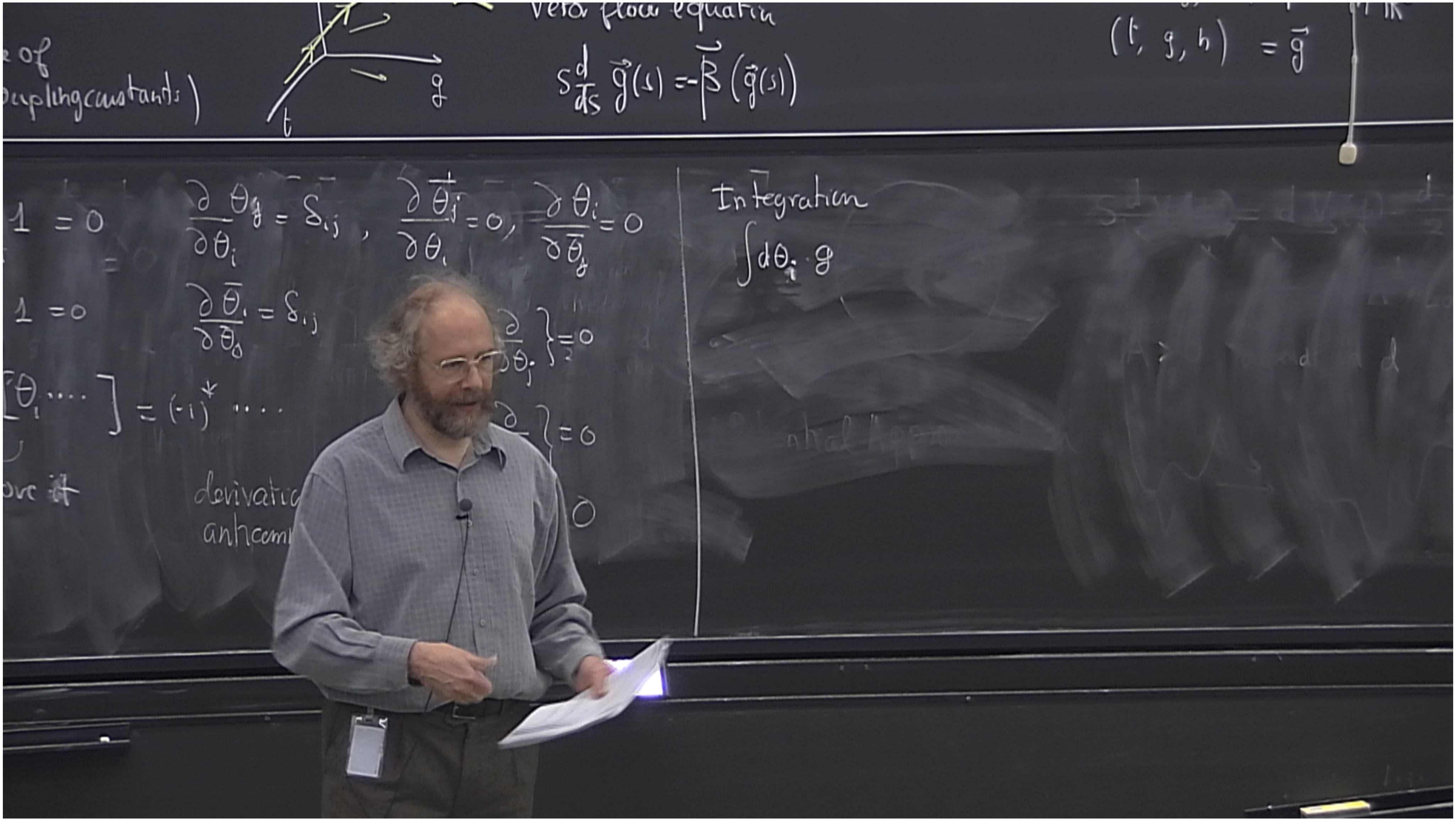

potential
3 dim space of
parameters (coupling constants)
g
t
$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
Derivation $\frac{\partial}{\partial \theta_i} g = ?$
$\frac{\partial}{\partial \theta_i} 1 = 0$
$\frac{\partial}{\partial \bar{\theta}_j} 1 = 0$
$\frac{\partial \theta_j}{\partial \theta_i} = \delta_{ij}$, $\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0$, $\frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\frac{\partial \bar{\theta}_i}{\partial \bar{\theta}_j} = \delta_{ij}$
$\frac{\partial}{\partial \theta_i} [\cdots \theta_i \cdots] = (-1)^* \frac{\partial}{\partial \theta_i} [\theta_i \cdots]$
anticommutation

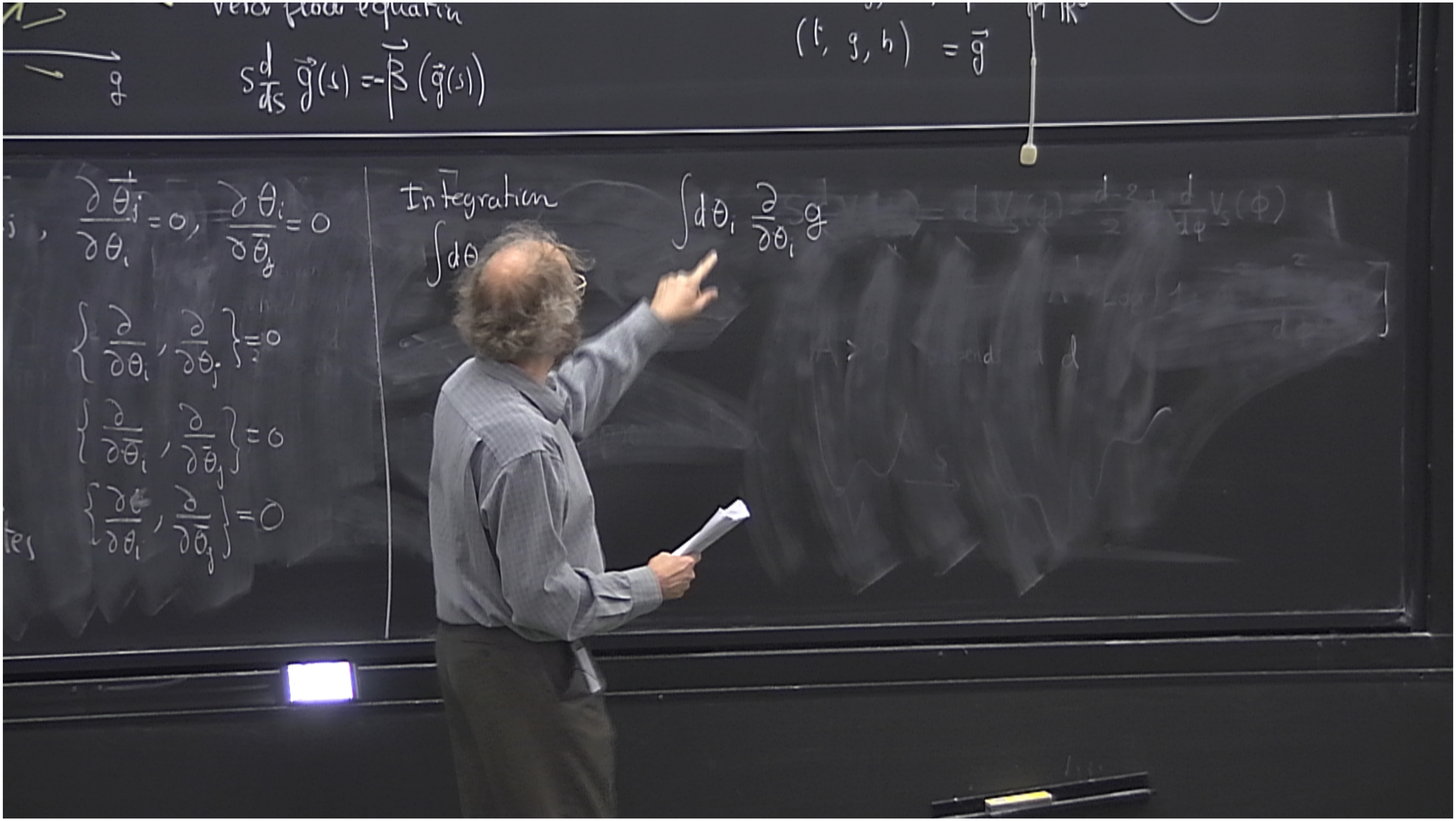

potential
3 dim space of
parameters (coupling constants)
t
g
$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
Derivation $\frac{\partial}{\partial \theta_i} g = ?$
$\frac{\partial}{\partial \theta_i} 1 = 0$
$\frac{\partial}{\partial \theta_i} \theta_j = \delta_{ij}$, $\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0$, $\frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\frac{\partial}{\partial \bar{\theta}_j} 1 = 0$
$\frac{\partial \bar{\theta}_i}{\partial \bar{\theta}_j} = \delta_{ij}$
$[\cdots \theta_i \cdots] = (-1)^* \frac{\partial}{\partial \theta_i}[\theta_i \cdots] = (-1)^* \cdots$
anticommutation

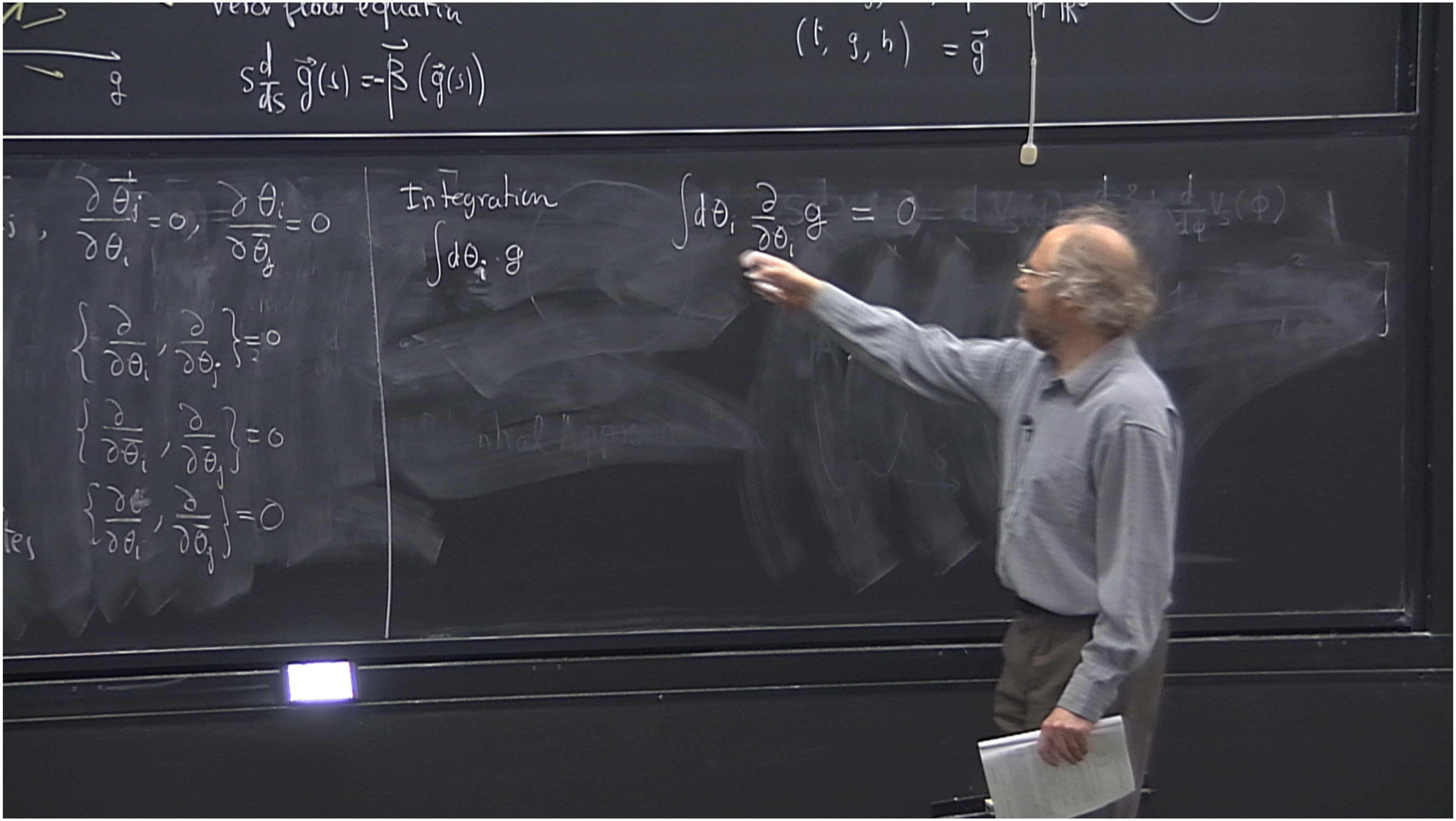

potential
3 dim space of
parameters (coupling constants)
g
t
$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
Derivation $\frac{\partial}{\partial \theta_i} g = ?$
$\frac{\partial}{\partial \theta_i} 1 = 0$
$\frac{\partial \theta_j}{\partial \theta_i} = \delta_{ij}$, $\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0$, $\frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\frac{\partial}{\partial \bar{\theta}_j} 1 = 0$
$\frac{\partial \bar{\theta}_i}{\partial \bar{\theta}_j} = \delta_{ij}$
$\left\{ \frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \theta_j} \right\} = 0$
$\frac{\partial}{\partial \theta_i} [\cdots \theta_i \cdots] = (-1)^* \frac{\partial}{\partial \theta_i} [\theta_i \cdots] = (-1)^* \cdots$
anticommutation
move θ_i in g to the left, and remove it
if no θ_i in g, $\Rightarrow 0$

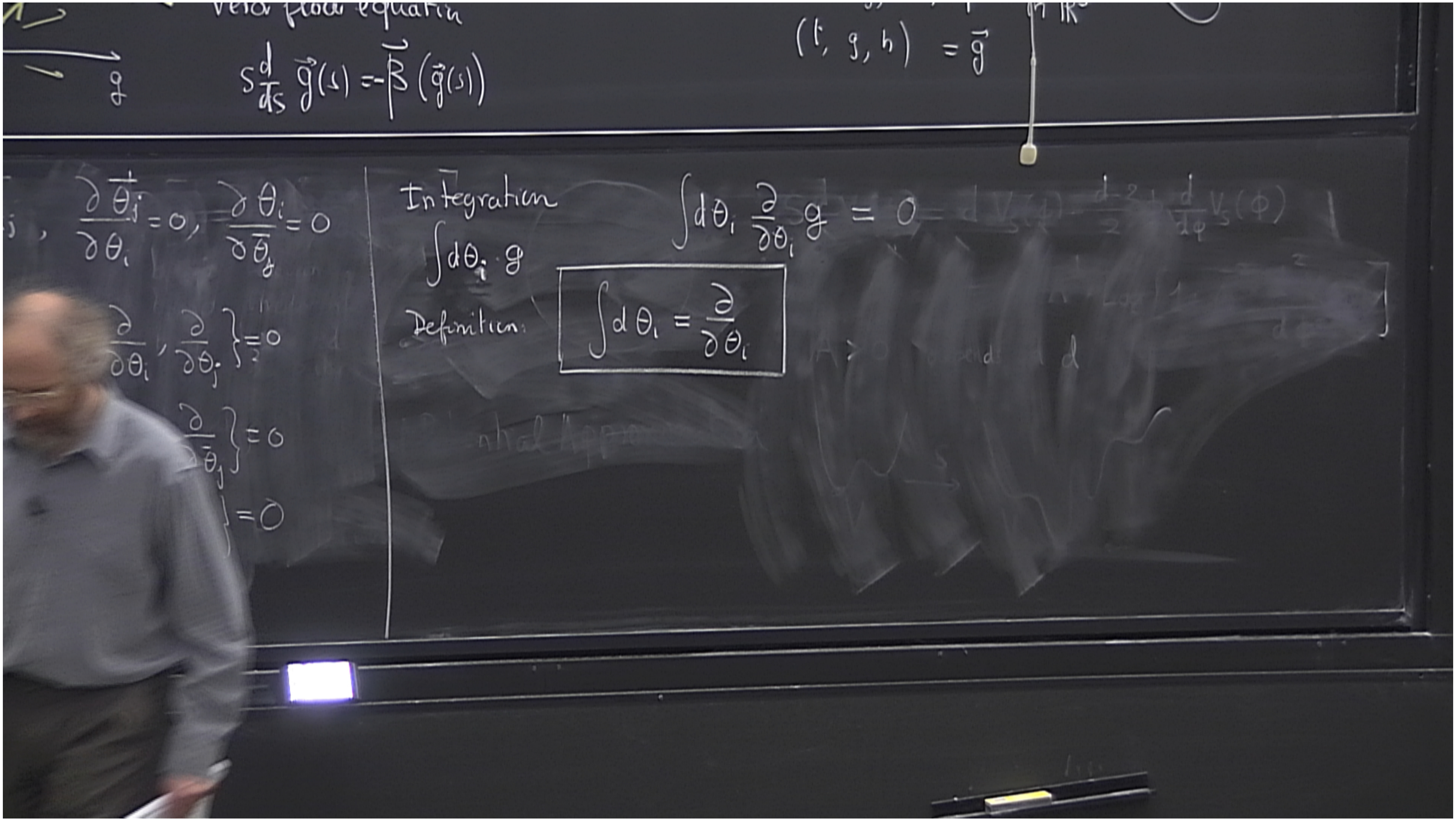

potential
3 dim space of
parameters (couplingconstants)
g
t
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
Derivation $\frac{\partial}{\partial\theta_i} g = ?$
$\frac{\partial}{\partial\theta_i} 1 = 0$
$\frac{\partial}{\partial\bar{\theta}_j} 1 = 0$
$\frac{\partial}{\partial\theta_i}\theta_j = \delta_{ij}$,
$\frac{\partial\bar{\theta}_j}{\partial\theta_i} = 0$, $\frac{\partial\theta_i}{\partial\bar{\theta}_j} = 0$
$\left\{\frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\theta_j}\right\} = 0$
$\left\{\frac{\partial}{\partial\bar{\theta}_i}, \frac{\partial}{\partial\bar{\theta}_j}\right\} = 0$
$\left\{\frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\bar{\theta}_j}\right\} = 0$
$\frac{\partial}{\partial\theta_i}[\cdots\theta_i\cdots] = (-1)^* \frac{\partial}{\partial\theta_i}[\theta_i \cdots$
anticommutation
move θ_i in g to the left, and remove
if no θ_i in g, $\Rightarrow 0$
Integratio

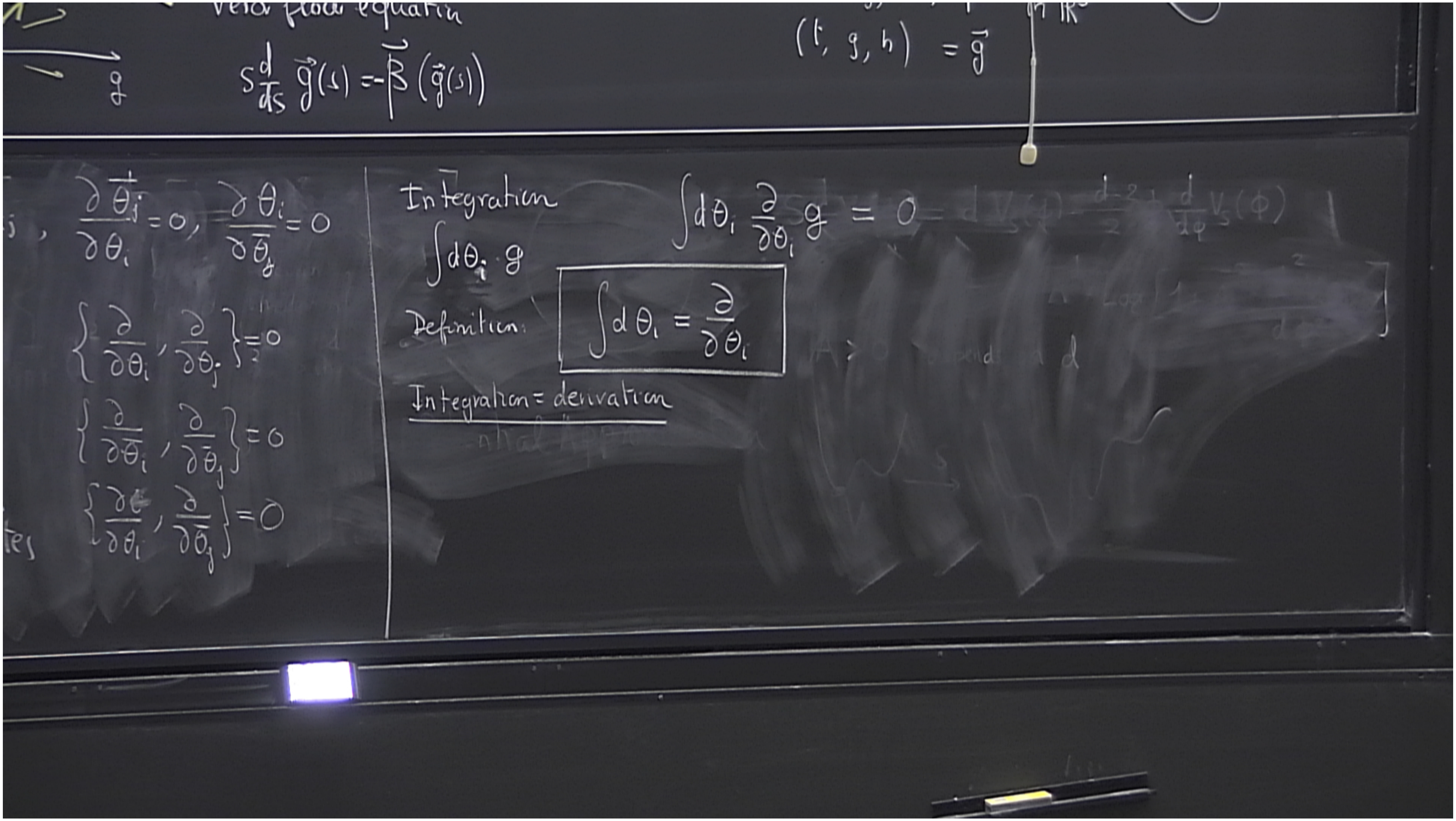

Veta flow equation
(couplingconstants)
(t, g, h) = g
Integration
derivations
anticommutes

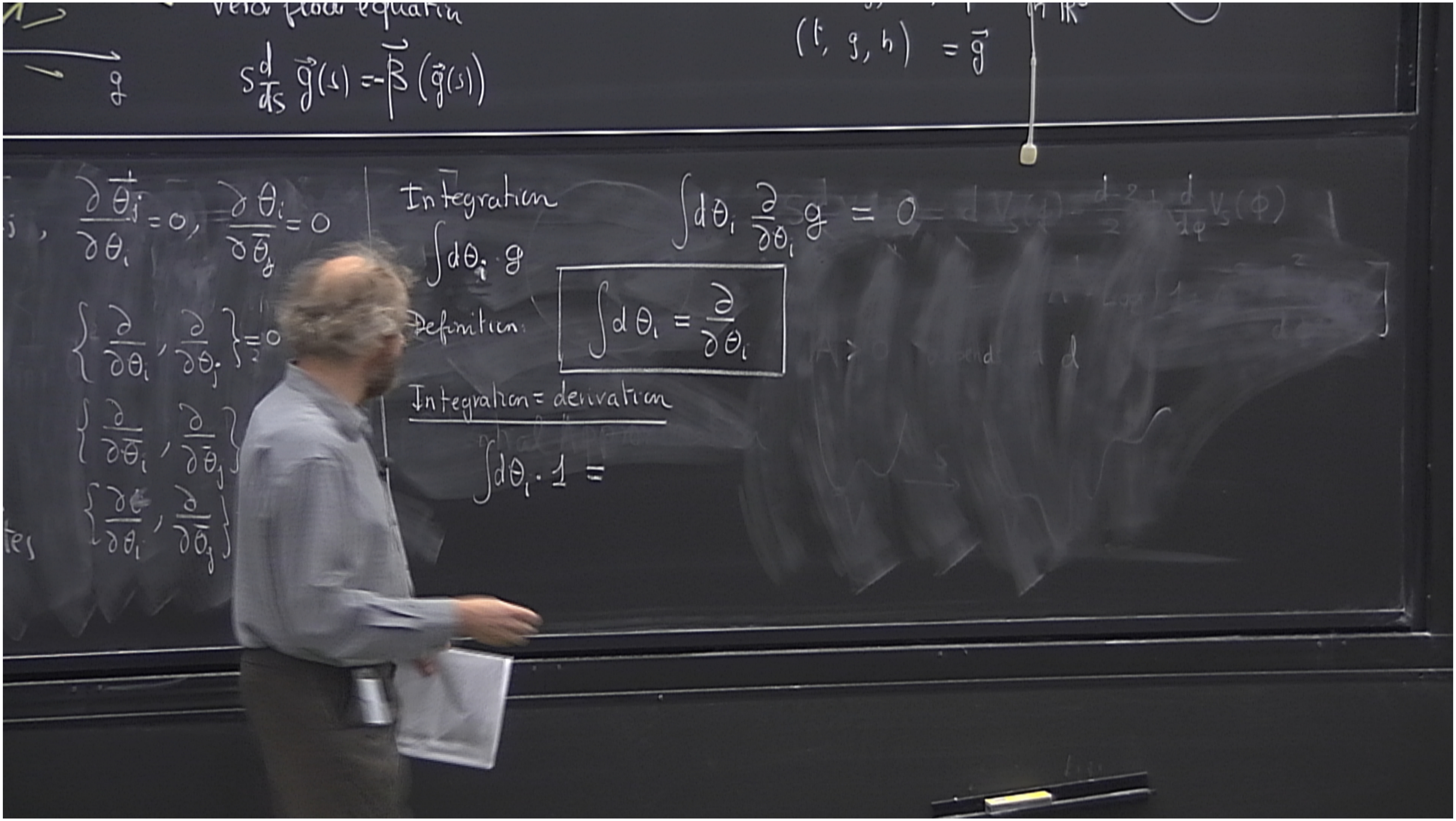



Integration
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(t, g, h) = \vec{g}$
$\int d\theta_i \frac{\partial}{\partial\theta_i} g$
$\left\{\frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\theta_j}\right\} = 0$
$\left\{\frac{\partial}{\partial\bar{\theta}_i}, \frac{\partial}{\partial\bar{\theta}_j}\right\} = 0$

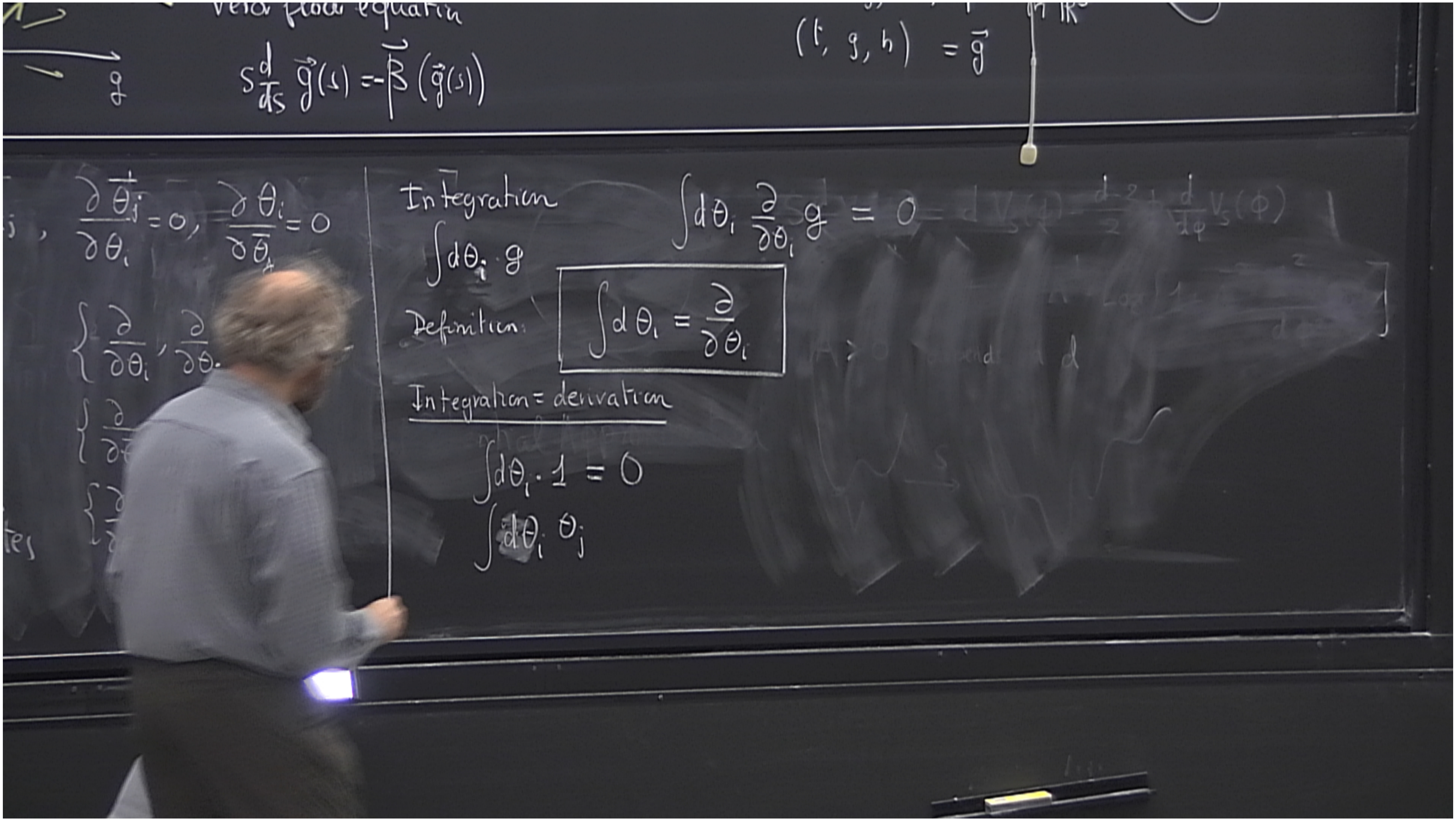

$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(t, g, h) = \vec{g}$
Integration
$\int d\theta_i \cdot g$
$\int d\theta_i \frac{\partial}{\partial\theta_i} g = 0$
$\frac{\partial\bar{\theta}_j}{\partial\theta_i} = 0, \frac{\partial\theta_i}{\partial\bar{\theta}_j} = 0$
$\left\{\frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\theta_j}\right\} = 0$
$\left\{\frac{\partial}{\partial\bar{\theta}_i}, \frac{\partial}{\partial\bar{\theta}_j}\right\} = 0$
$\left\{\frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\bar{\theta}_j}\right\} = 0$


$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(t, g, h) = \vec{g}$
Integration
$\int d\theta_i \cdot g$
$\int d\theta_i \frac{\partial}{\partial\theta_i} g = 0$
Definition:
$\int d\theta_i = \frac{\partial}{\partial\theta_i}$

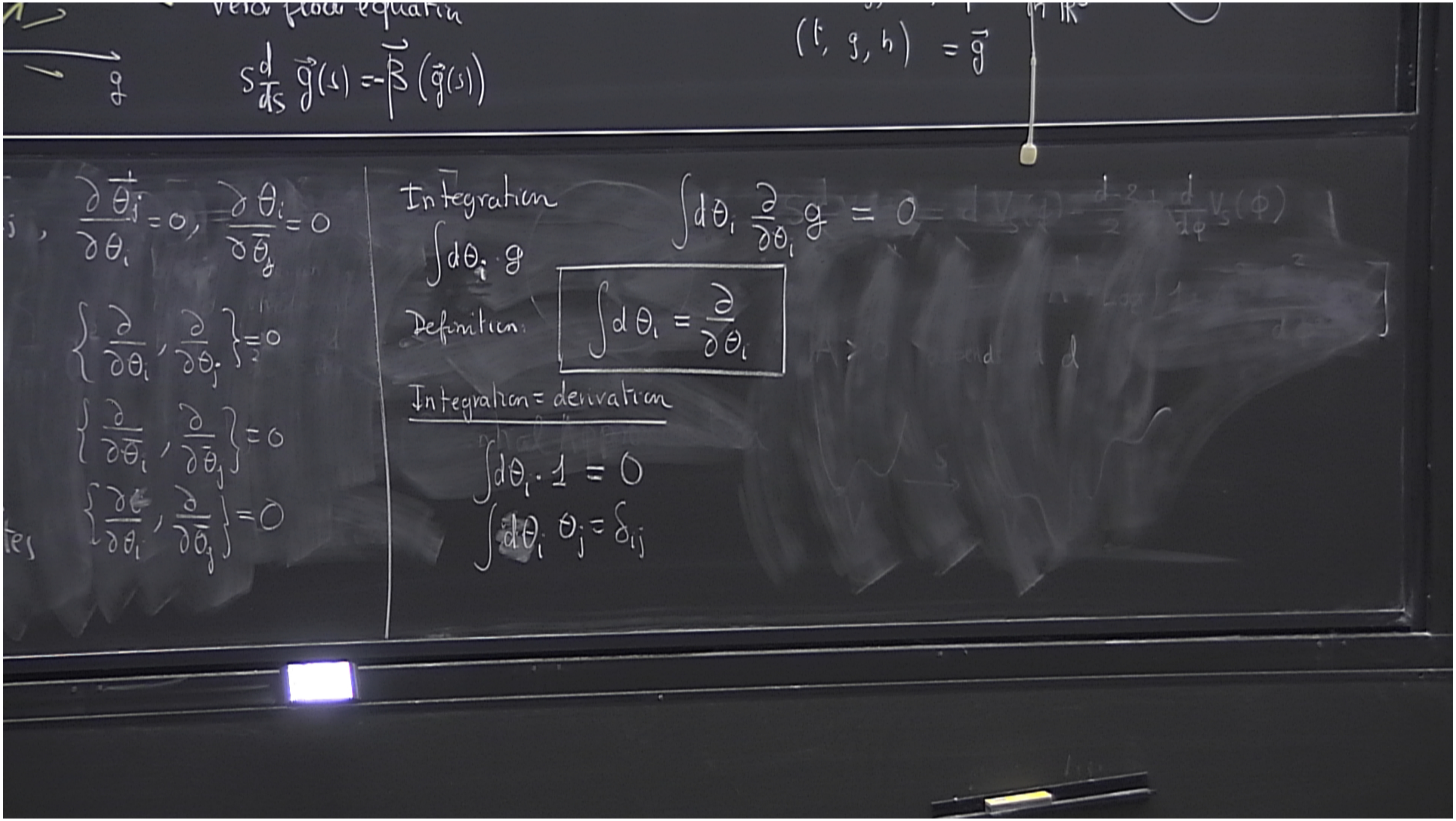

s d/ds g(s) = -β(g(s))
(t, g, h) = g
Integration
∫dθi · g
Definition:
∫dθi = ∂/∂θi
Integration = derivation
∫dθi ∂/∂θi g = 0
{∂/∂θi, ∂/∂θj} = 0

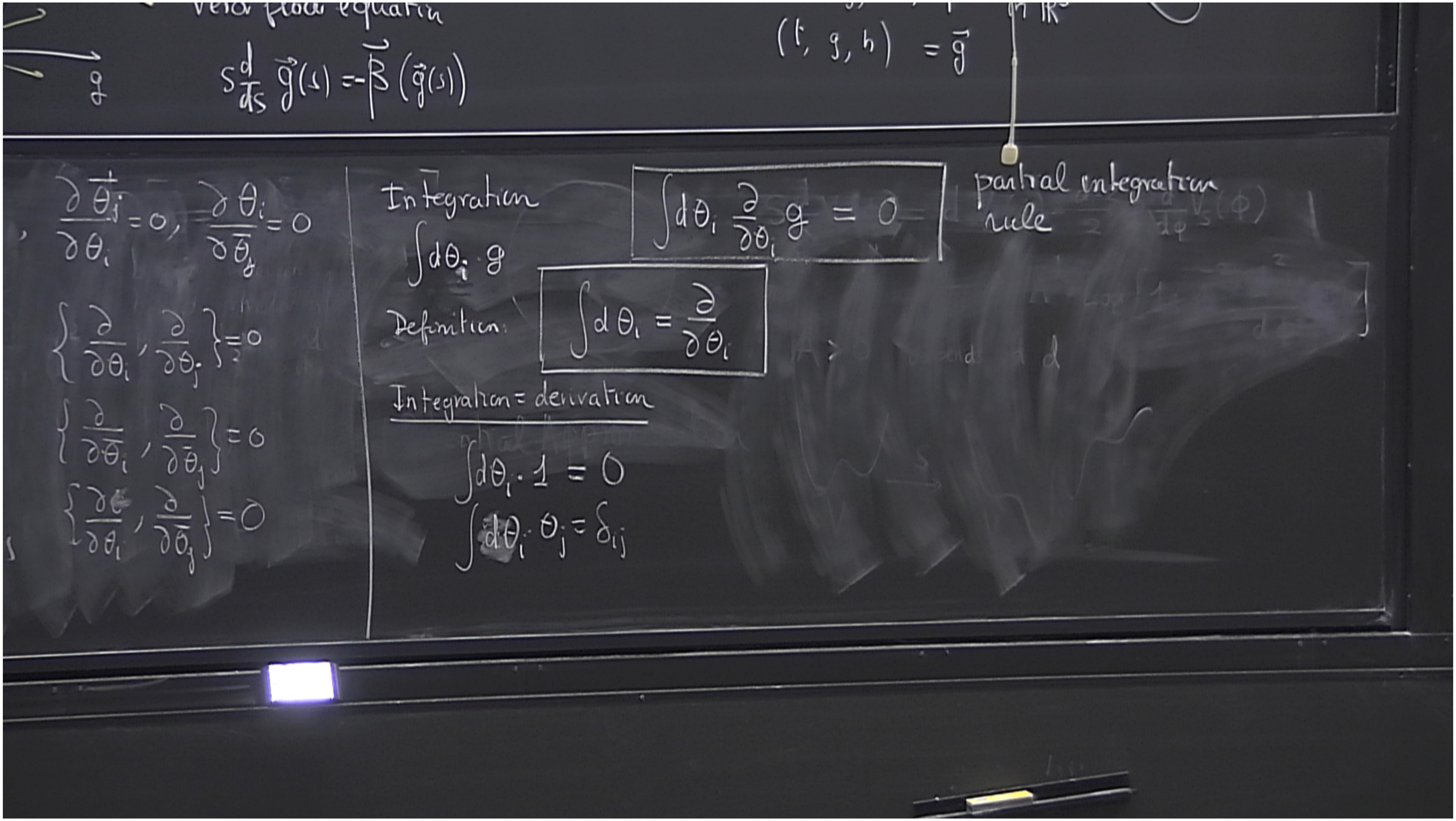


$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(t, g, h) = \vec{g}$
$\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0, \quad \frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\left\{\frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \theta_j}\right\} = 0$
Integration
$\int d\theta_i \cdot g$
Definition
$\int d\theta_i = \frac{\partial}{\partial \theta_i}$
Integration = derivation
$\int d\theta_i \cdot 1 =$
$\int d\theta_i \frac{\partial}{\partial \theta_i} g = 0$

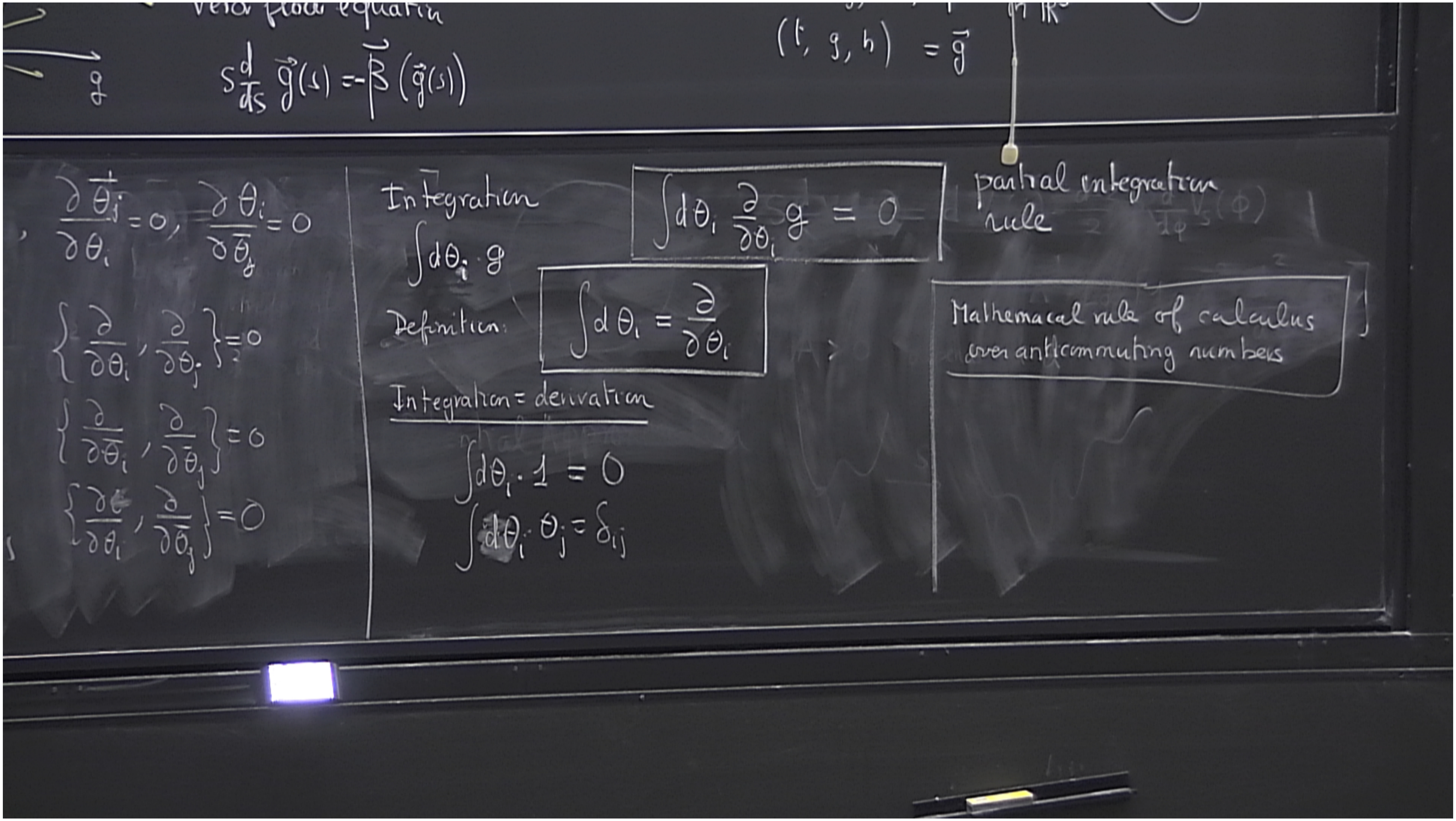


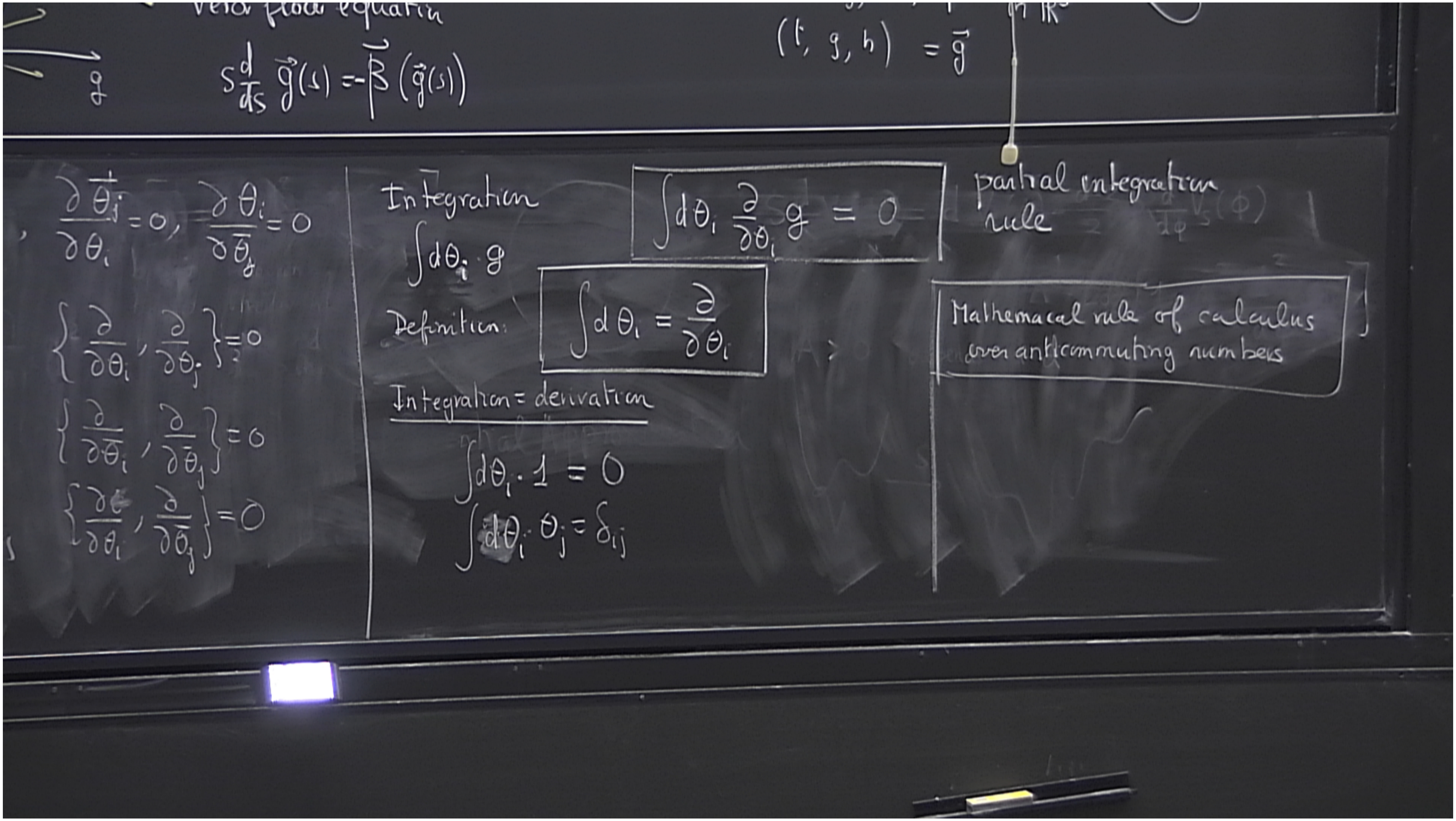
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(t, g, h) = \vec{g}$
Integration
$\int d\theta_i \cdot g$
$\int d\theta_i \frac{\partial}{\partial\theta_i} g = 0$
Definition:
$\int d\theta_i = \frac{\partial}{\partial\theta_i}$
Integration = derivation
$\int d\theta_i \cdot 1 = 0$
$\int d\theta_i \, \theta_j$


$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(f, g, h) = \vec{g}$
$\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0, \frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\{\frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \theta_j}\} = 0$
$\{\frac{\partial}{\partial \bar{\theta}_i}, \frac{\partial}{\partial \bar{\theta}_j}\} = 0$
$\{\frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \bar{\theta}_j}\} = 0$
Integration
$\int d\theta_i \cdot g$
$\int d\theta_i \frac{\partial}{\partial \theta_i} g = 0$
Definition
$\int d\theta_i = \frac{\partial}{\partial \theta_i}$
Integration = derivation
$\int d\theta_i \cdot 1 = 0$
$\int d\theta_i \, \theta_j = \delta_{ij}$



$s \frac{d}{ds} \vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(t, g, h) = \vec{g}$
$\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0, \quad \frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\left\{ \frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \theta_j} \right\} = 0$
$\left\{ \frac{\partial}{\partial \bar{\theta}_i}, \frac{\partial}{\partial \bar{\theta}_j} \right\} = 0$
$\left\{ \frac{\partial \theta}{\partial \theta_i}, \frac{\partial}{\partial \bar{\theta}_j} \right\} = 0$
Integration
$\int d\theta_i \cdot g$
Definition:
$\int d\theta_i = \frac{\partial}{\partial \theta_i}$
Integration = derivation
$\int d\theta_i \cdot 1 = 0$
$\int d\theta_i \cdot \theta_j = \delta_{ij}$
$\int d\theta_i \frac{\partial}{\partial \theta_i} g = 0$
partial integration rule


Integration
$\int d\theta_i \cdot g$
Definition:
$\int d\theta_i = \frac{\partial}{\partial\theta_i}$
Integration = derivation
$\int d\theta_i \cdot 1 = 0$
$\int d\theta_i \cdot \theta_j = \delta_{ij}$
$\int d\theta_i \frac{\partial}{\partial\theta_i} g = 0$
partial integration rule
Mathemacal rule of calculus over anticommuting numbers
$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(f, g, h) = \vec{g}$


$s\frac{d}{ds}\vec{g}(s) = -\vec{\beta}(\vec{g}(s))$
$(f, g, h) = \vec{g}$
$\frac{\partial \bar{\theta}_j}{\partial \theta_i} = 0, \frac{\partial \theta_i}{\partial \bar{\theta}_j} = 0$
$\{\frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \theta_j}\} = 0$
$\{\frac{\partial}{\partial \bar{\theta}_i}, \frac{\partial}{\partial \bar{\theta}_j}\} = 0$
$\{\frac{\partial}{\partial \theta_i}, \frac{\partial}{\partial \bar{\theta}_j}\} = 0$
Integration
$\int d\theta_i \cdot g$
Definition:
$\int d\theta_i = \frac{\partial}{\partial \theta_i}$
Integration = derivation
$\int d\theta_i \cdot 1 = 0$
$\int d\theta_i \cdot \theta_j = \delta_{ij}$
$\int d\theta_i \frac{\partial}{\partial \theta_i} g = 0$
partial integration rule
Mathemacal rule of calculus over anticommuting numbers