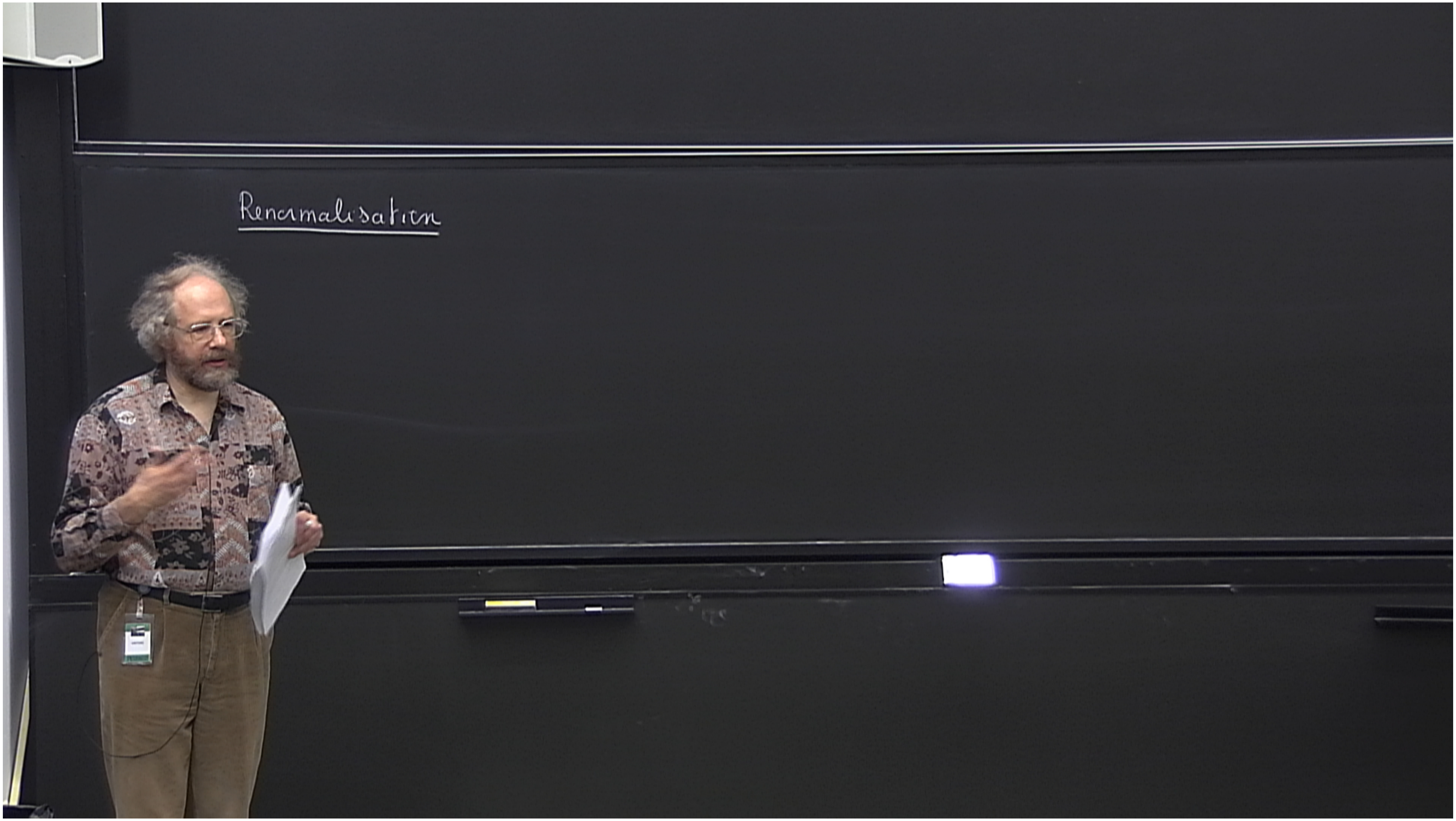
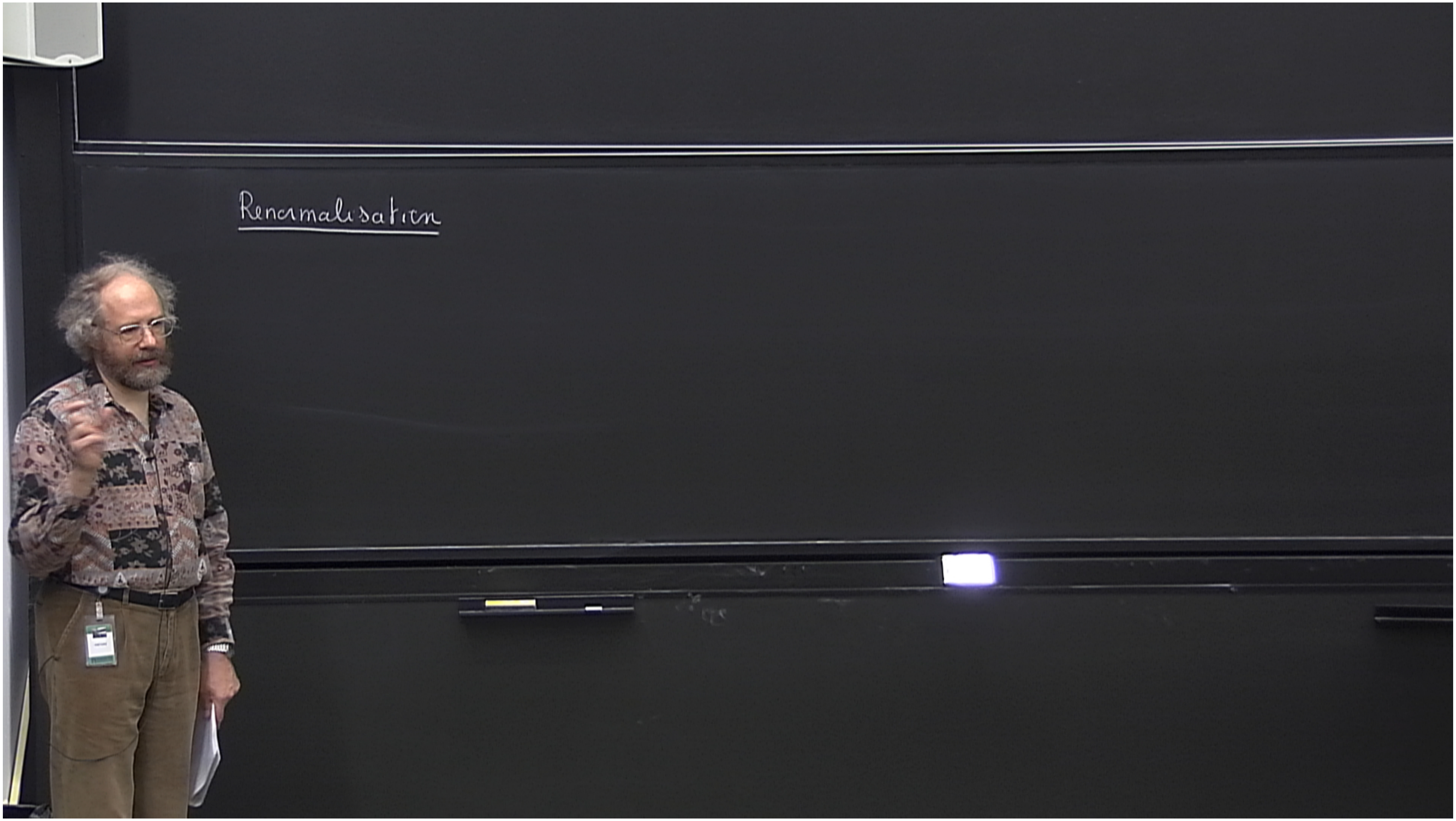
Title: Quantum Field Theory II - Lecture 6

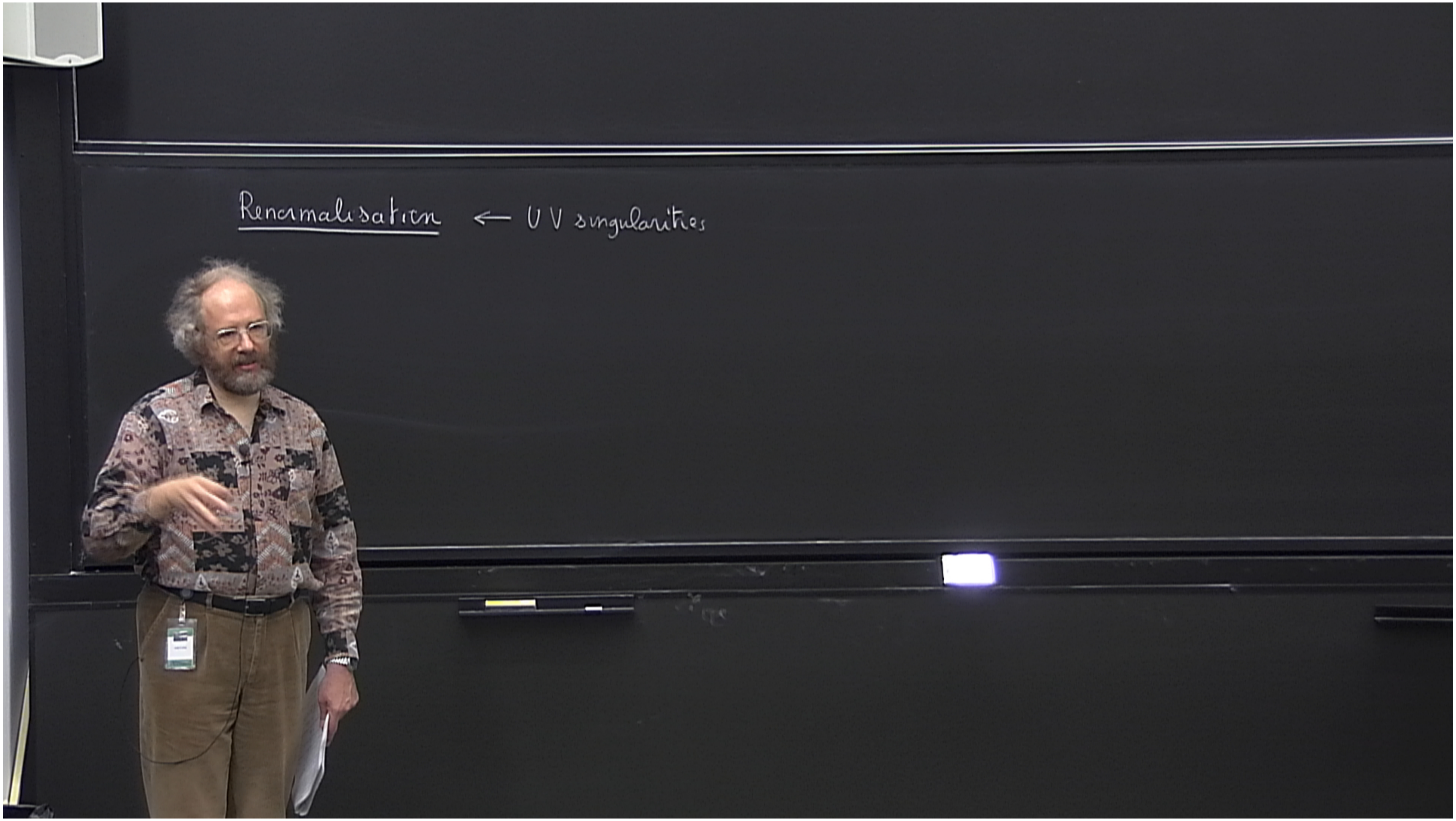
Date: Nov 07, 2011 09:00 AM

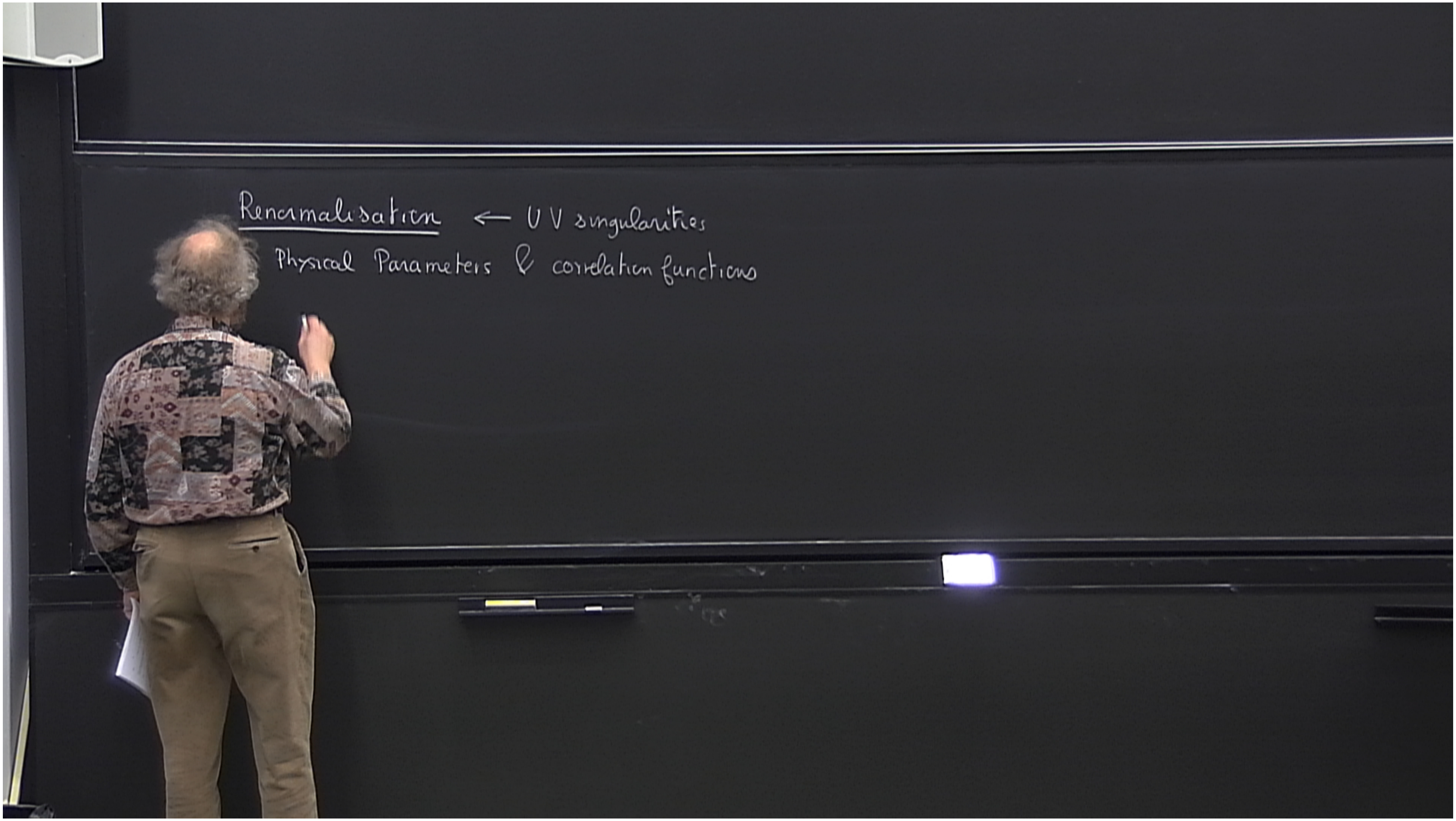
URL: http://pirsa.org/11110010

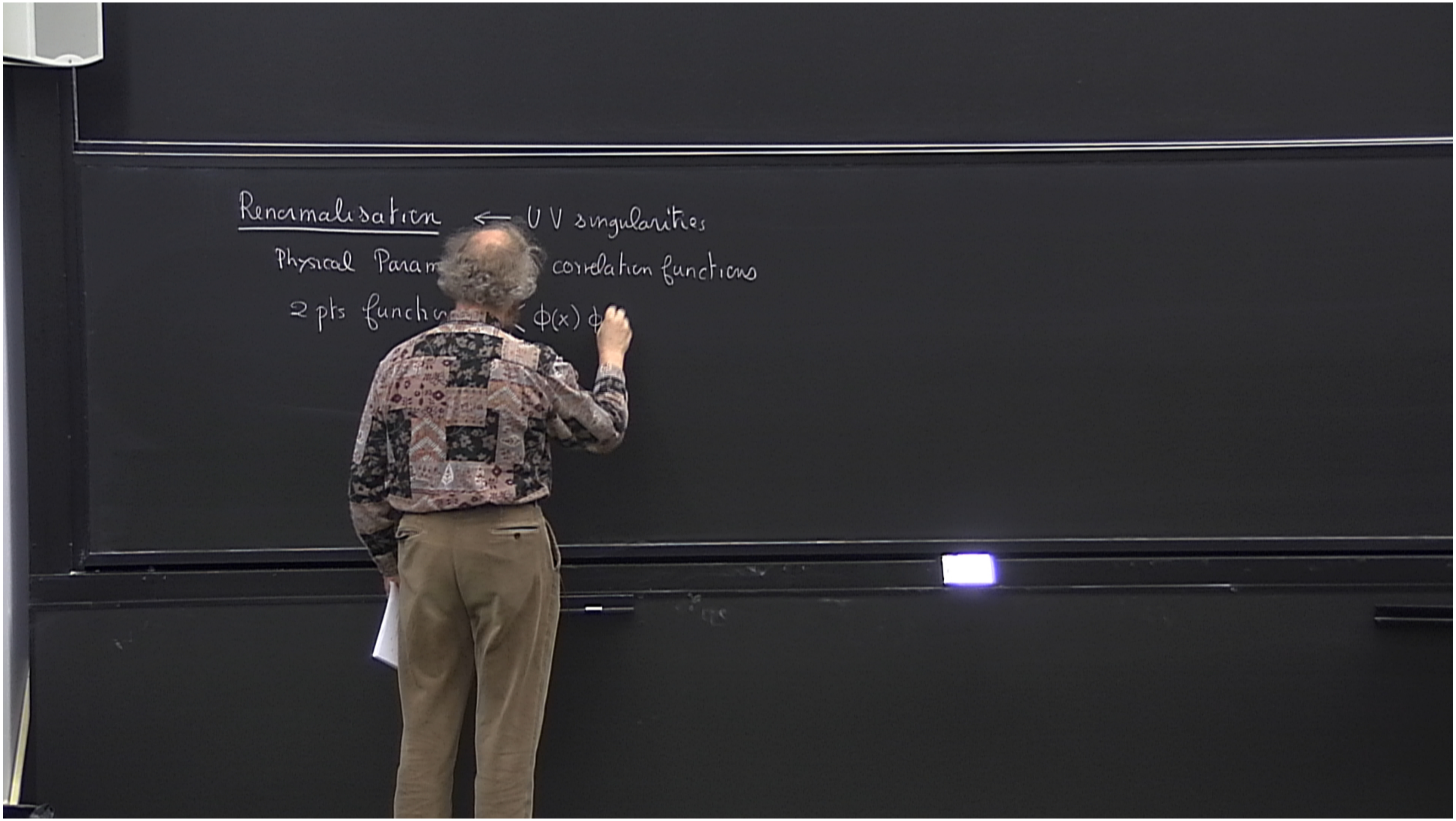
Abstract:

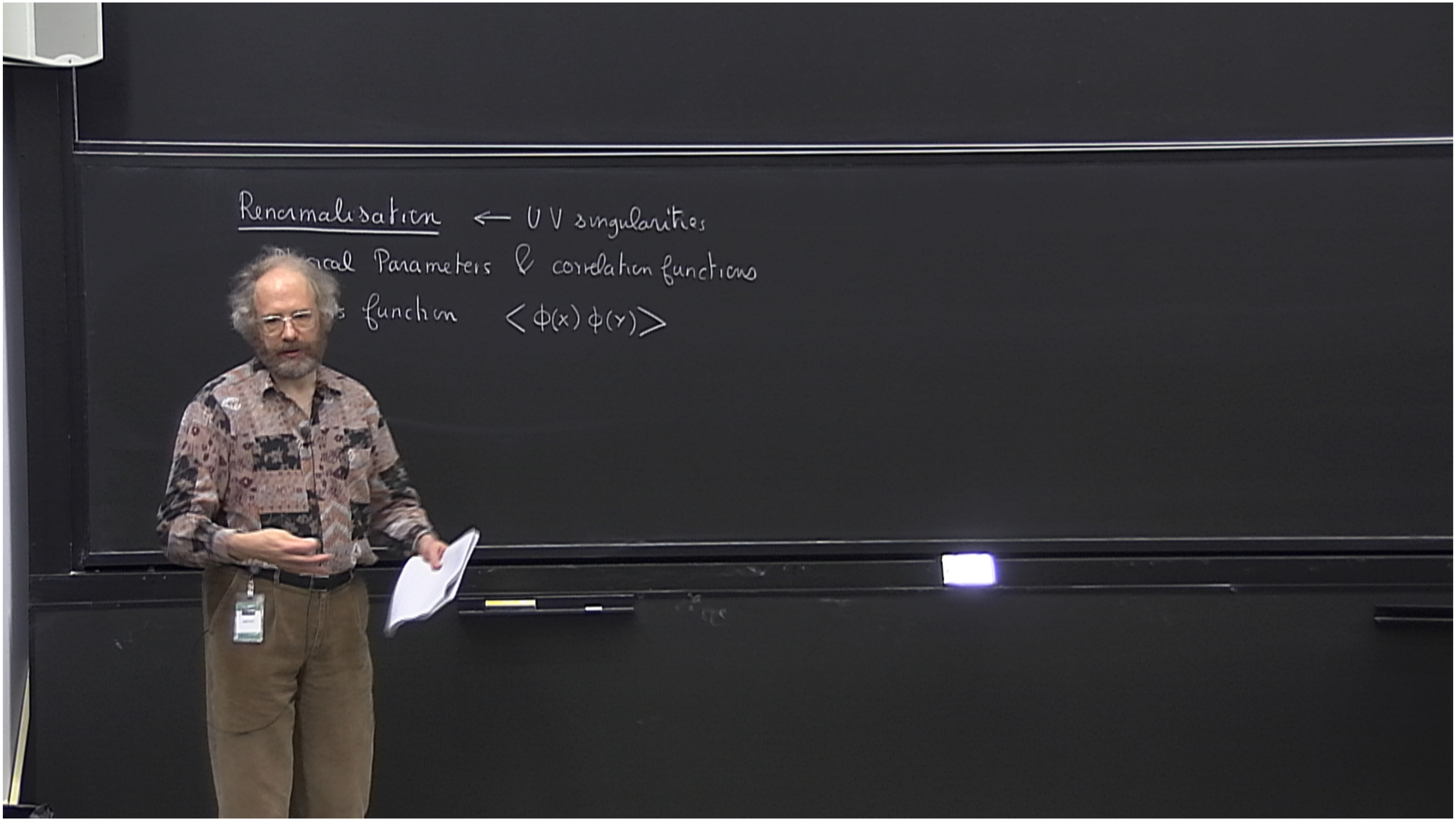

Renormalisation

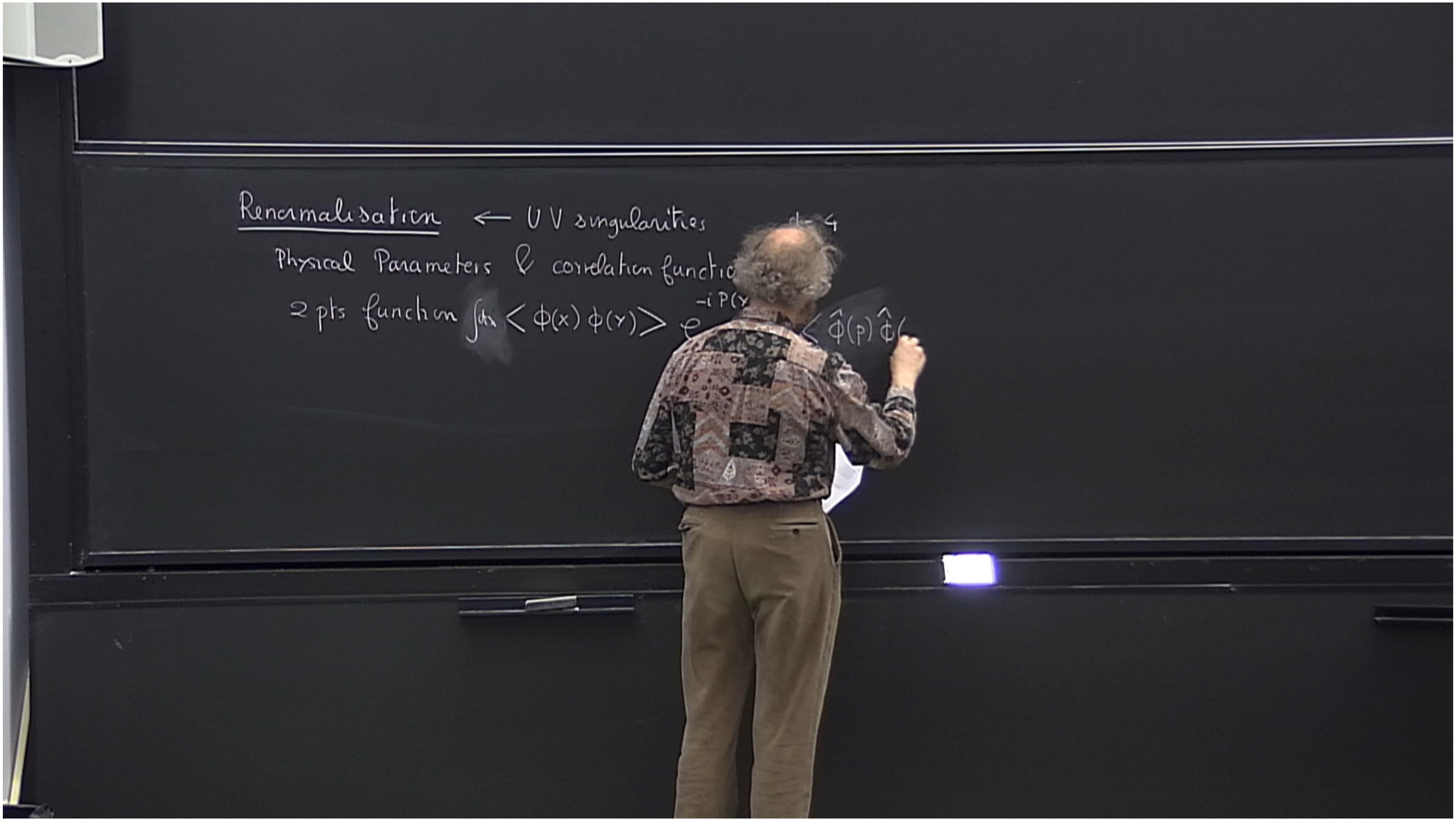

Renormalisation

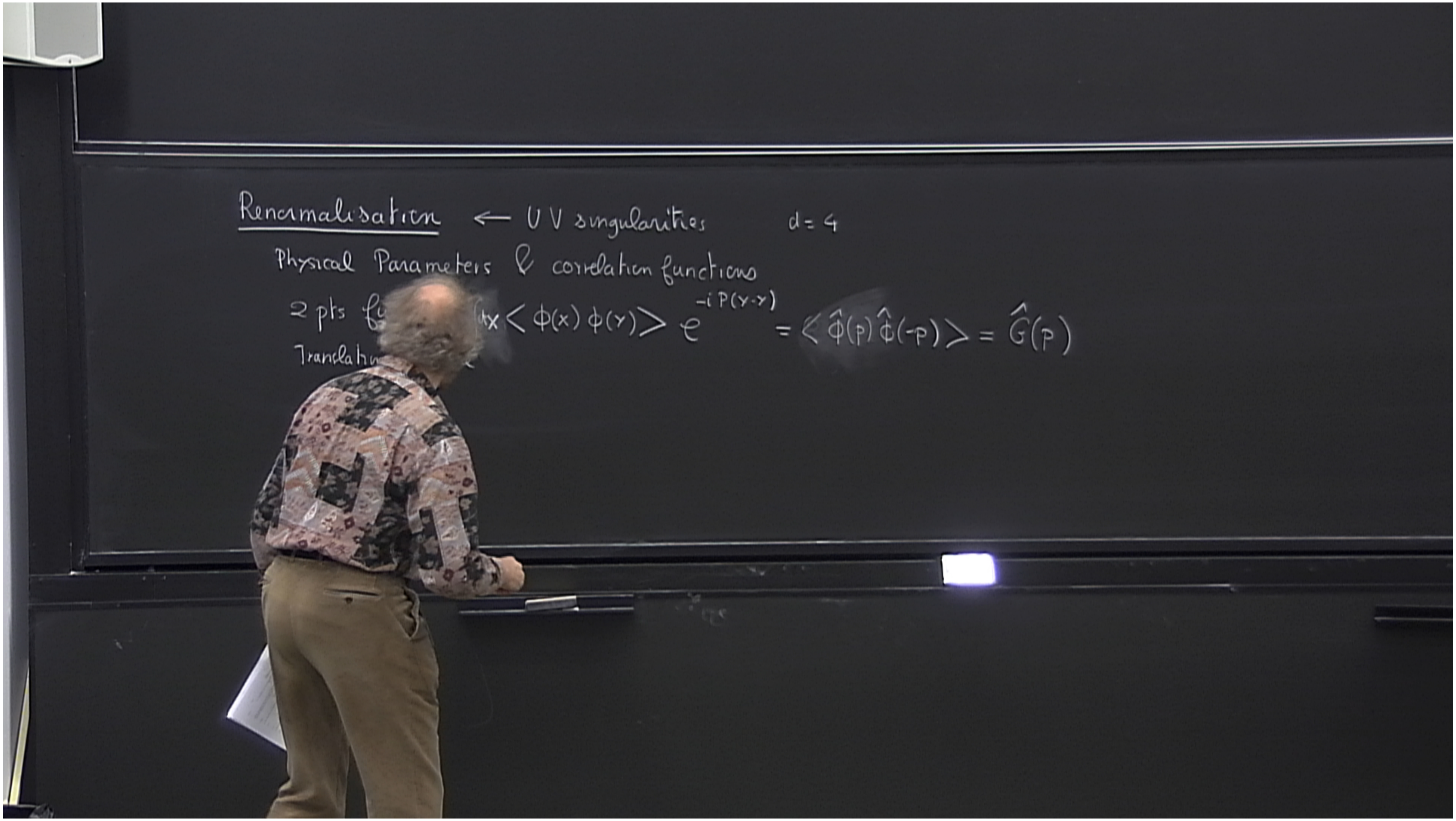

Renormalisation ← UV singularities


Renormalisation ← UV singularities
Physical Parameters & correlation functions

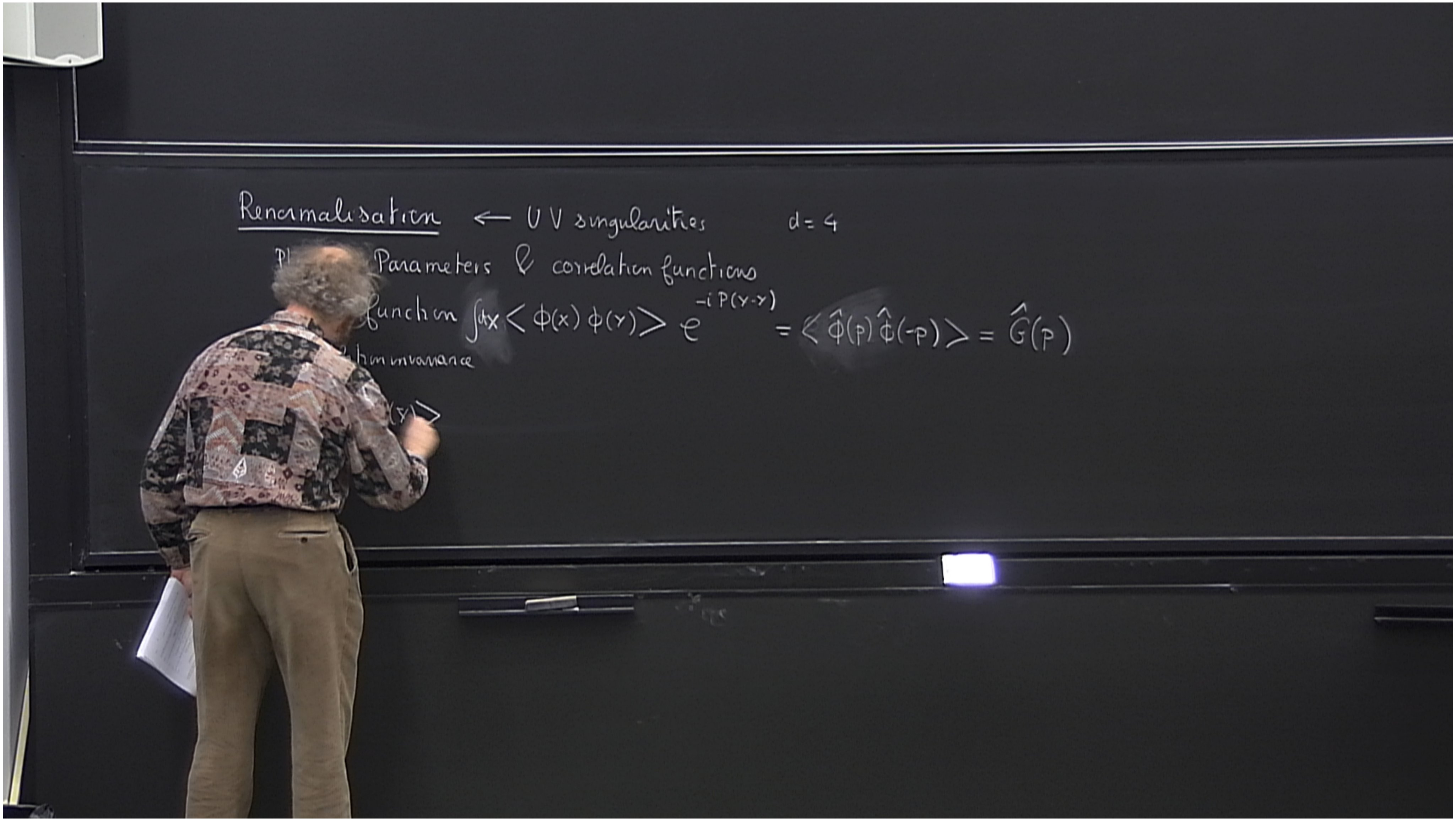

Renormalisation ← U V singularities
Physical Param
correlation functions
2 pts
φ(x) φ

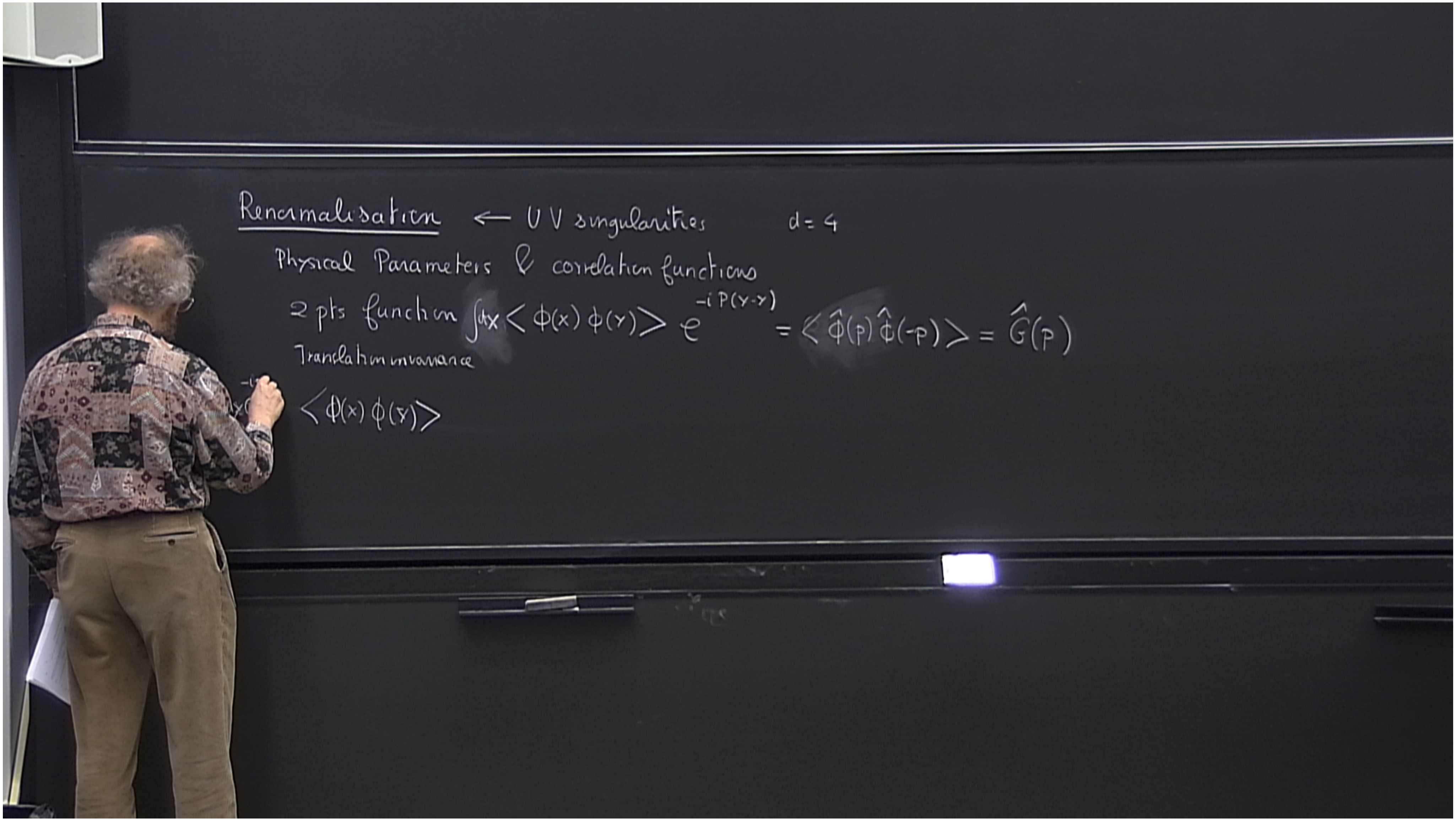

Renormalisation ← UV singularities
Parameters & correlation functions
function $\langle \phi(x)\, \phi(y) \rangle$

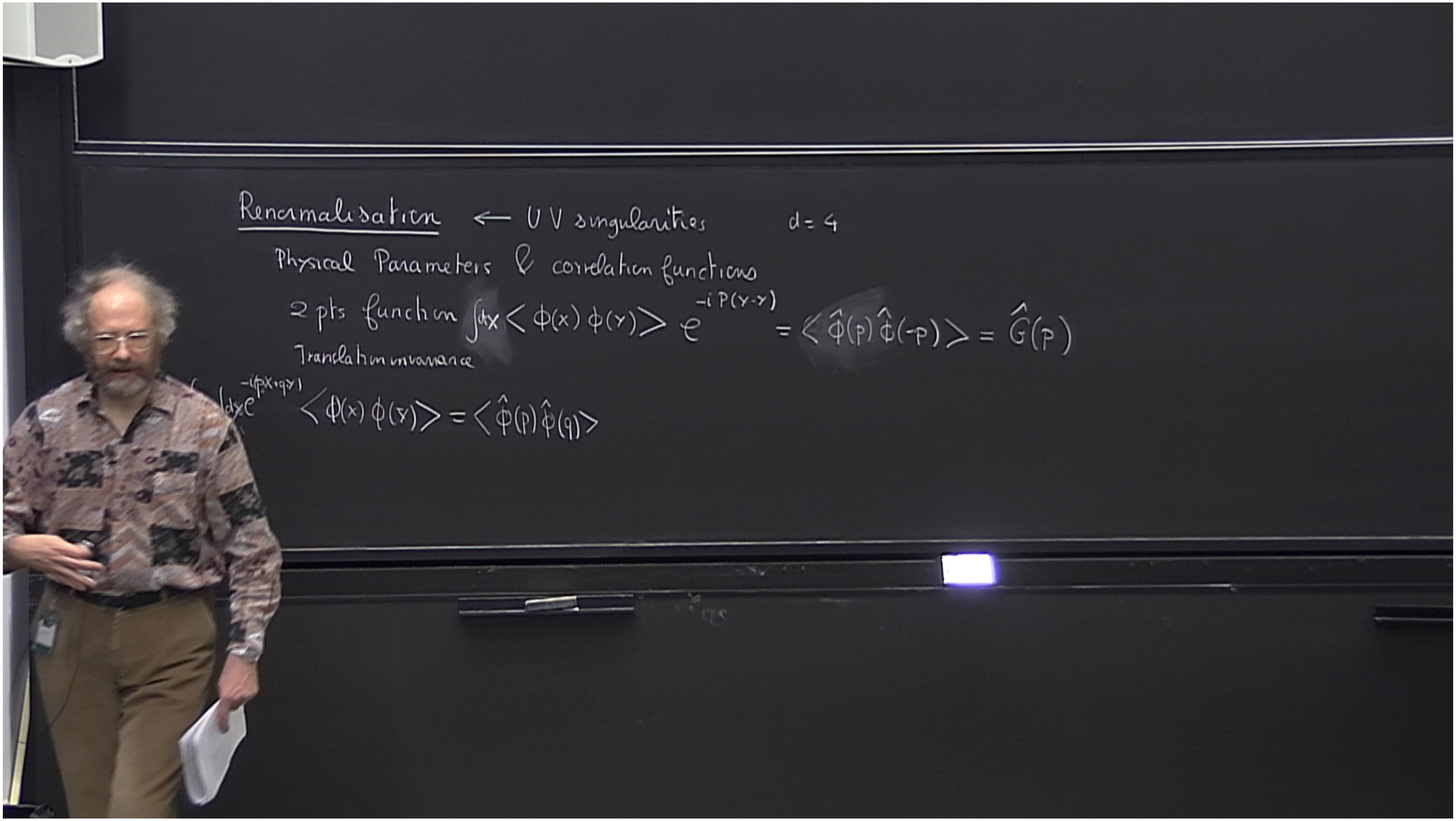

Renormalisation ← UV singularities
Physical Parameters & correlation functio
2 pts function ∫dx < φ(x) φ(y) > e

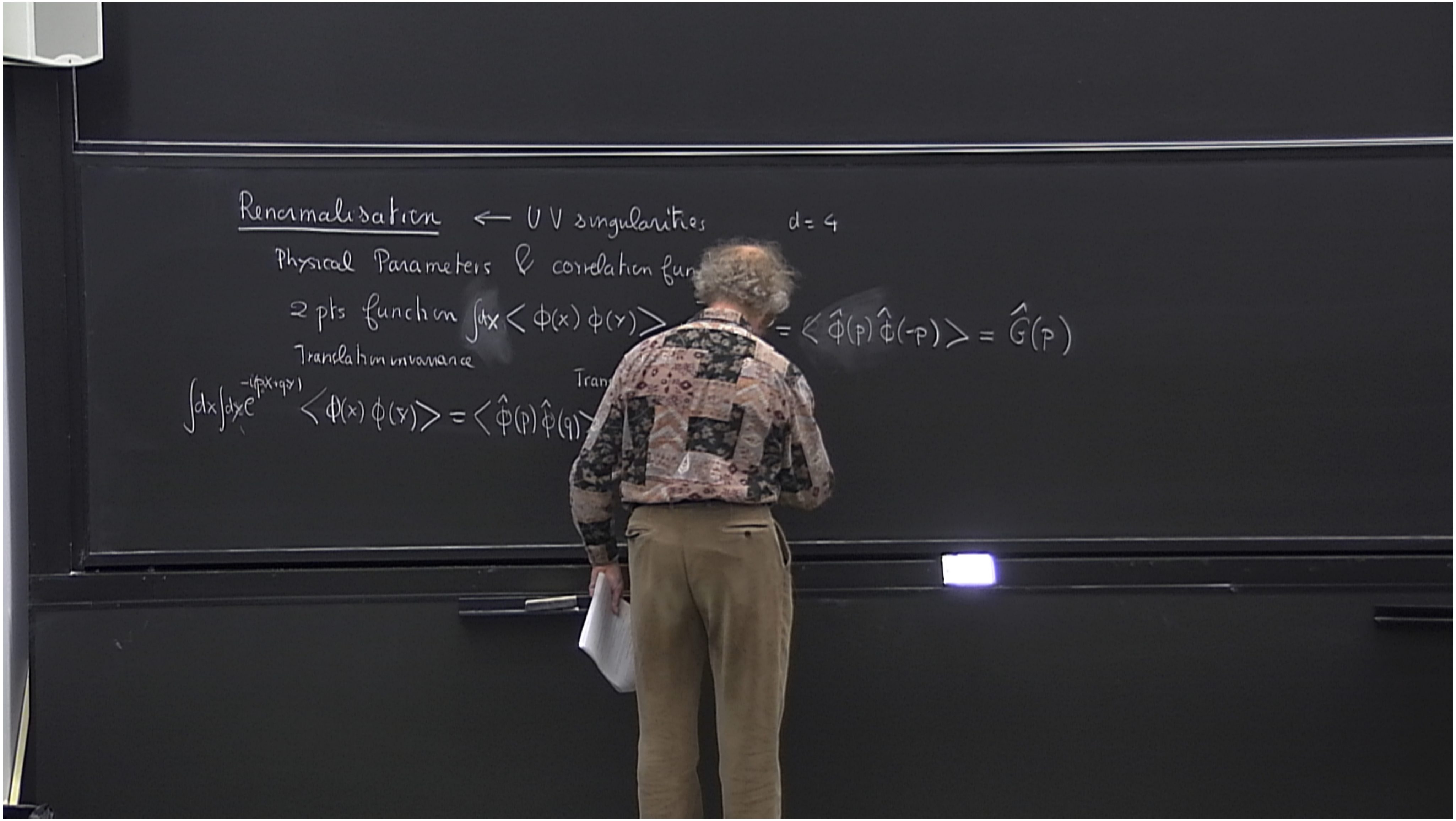


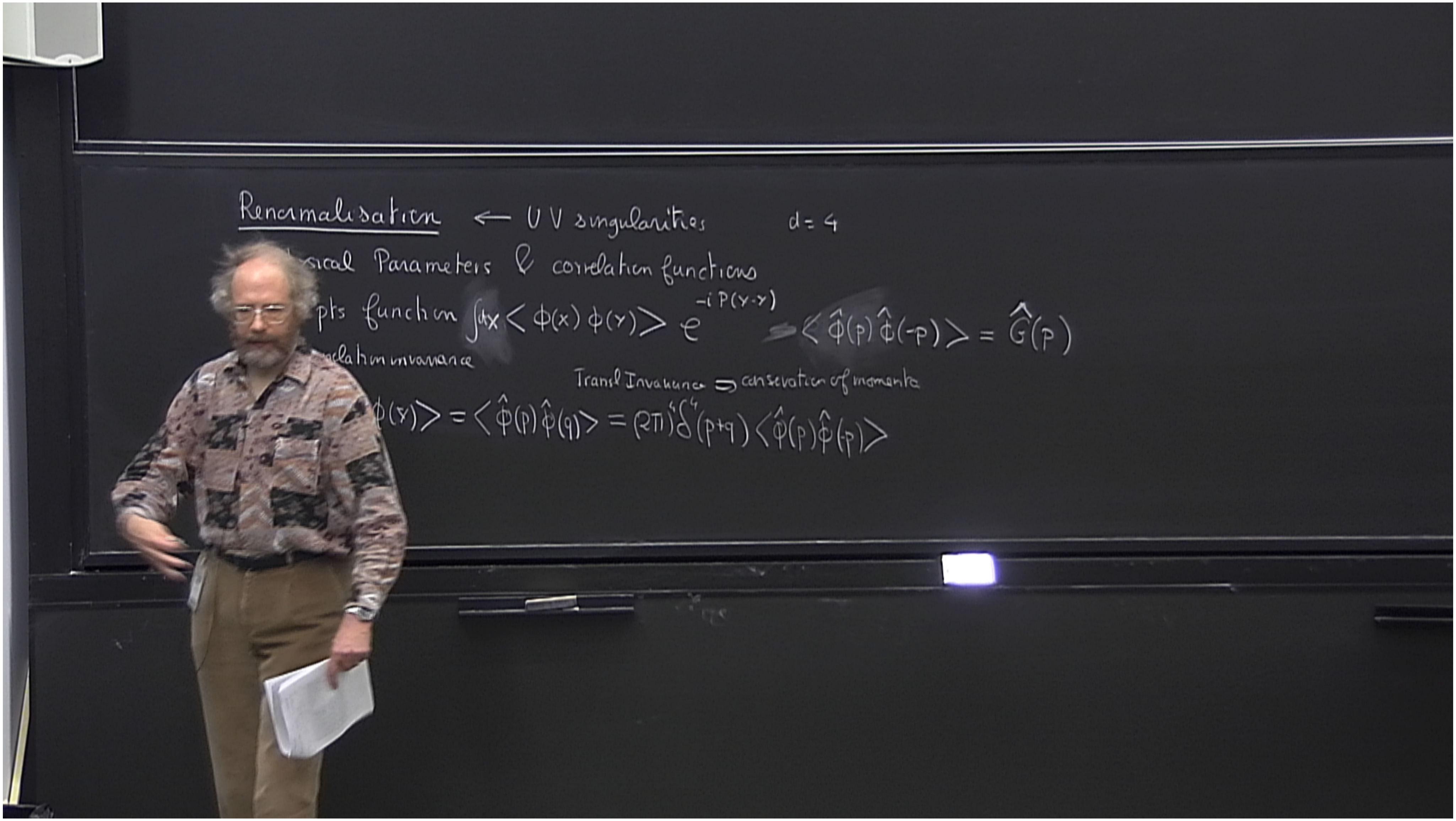
Renormalisation ← UV singularities
d = 4
Physical Parameters & correlation functions
2 pts
$\langle \phi(x)\, \phi(y) \rangle\, e^{-i P(x-y)} = \langle \hat{\phi}(p)\, \hat{\phi}(-p) \rangle = \hat{G}(p)$

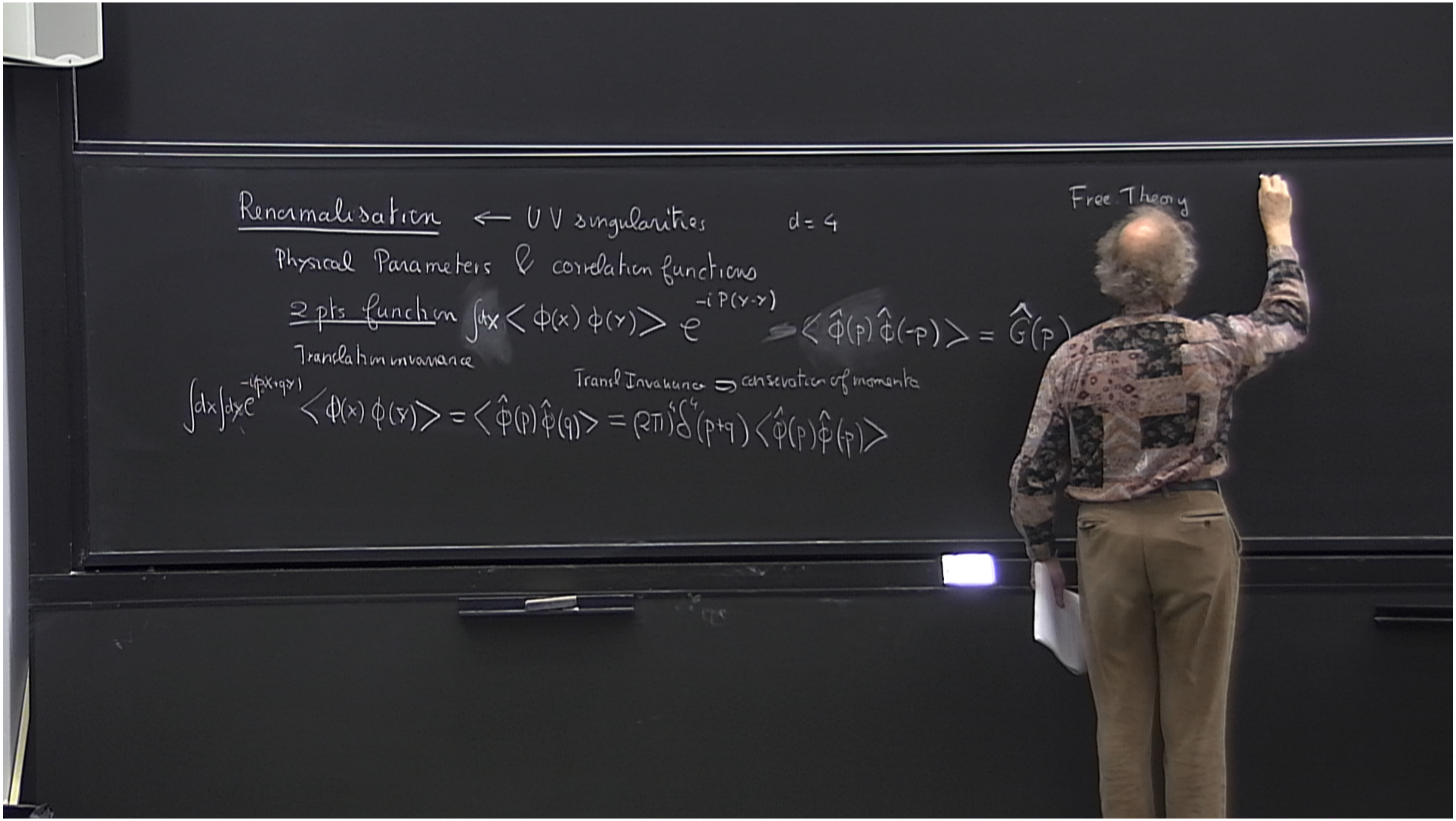

Renormalisation ← UV singularities
d = 4
Parameters & correlation functions
function ∫dx < φ(x) φ(y) > e^{-ip(x-y)} = < φ̂(p) φ̂(-p) > = Ĝ(p)

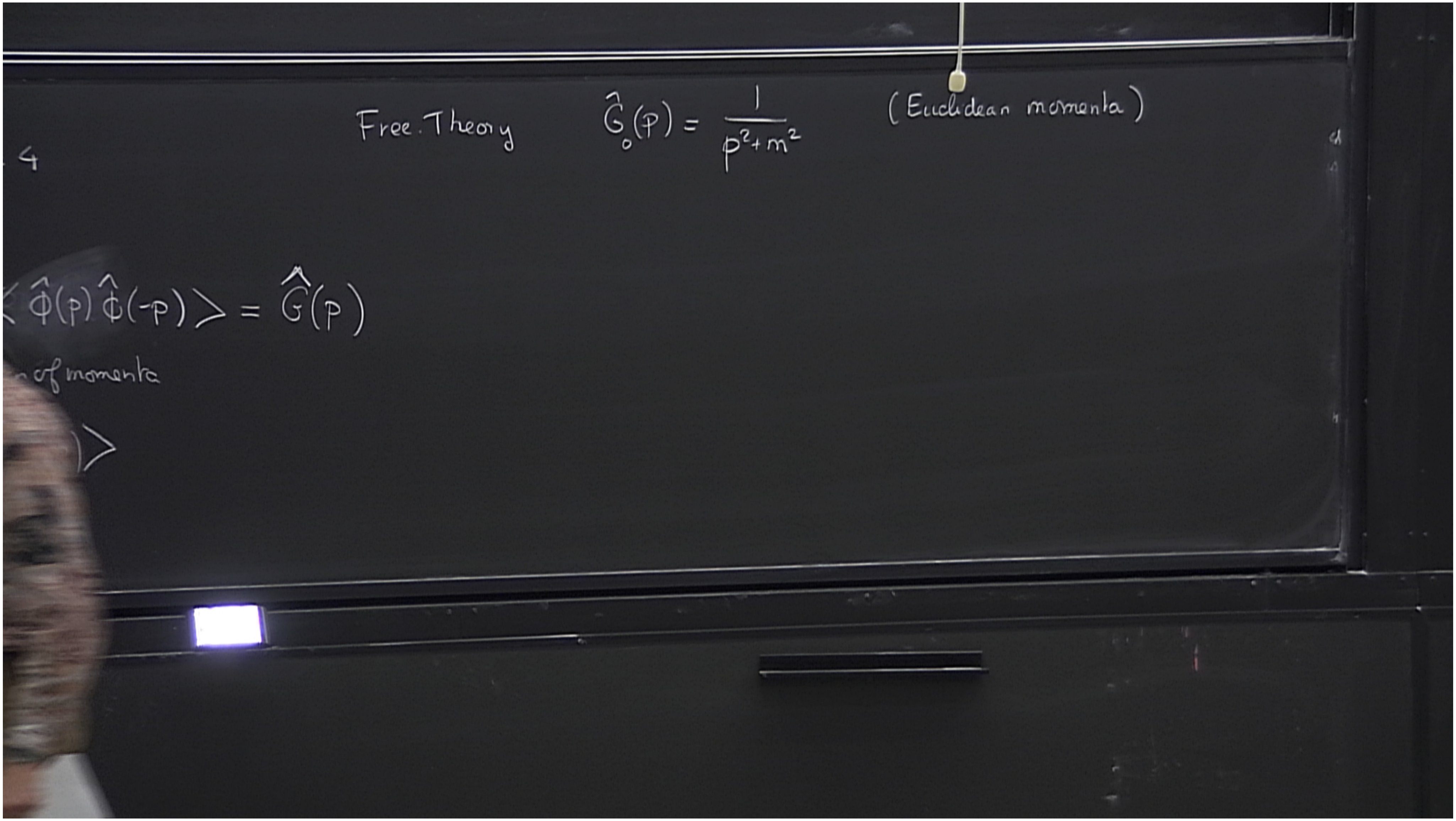

Renormalisation ← UV singularities
d = 4
Physical Parameters & correlation functions
2 pts function ∫dx < φ(x) φ(y) > e^{-i P(x-y)} = < φ̂(p) φ̂(-p) > = Ĝ(p)
Translation invariance
< φ(x) φ(y) >

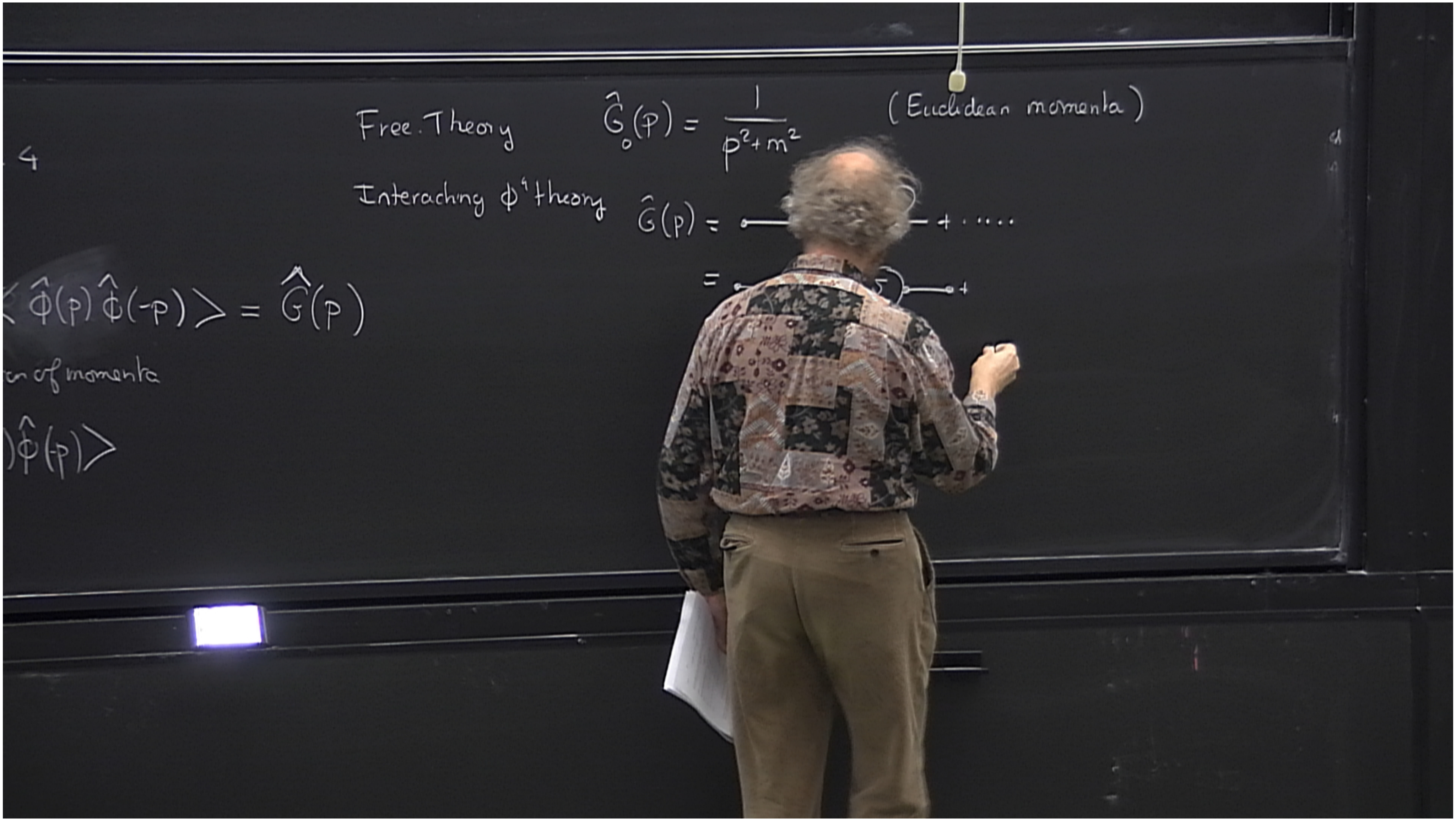

Renormalisation ← UV singularities
d = 4
Physical Parameters & correlation functions
2 pts function $\int dx \langle \phi(x)\phi(y)\rangle e^{-ip(x-y)} = \langle \hat\phi(p)\hat\phi(-p)\rangle = \hat G(p)$
Translation invariance
$\int dx\,dy\, e^{-i(px+qy)} \langle \phi(x)\phi(y)\rangle = \langle \hat\phi(p)\hat\phi(q)\rangle$

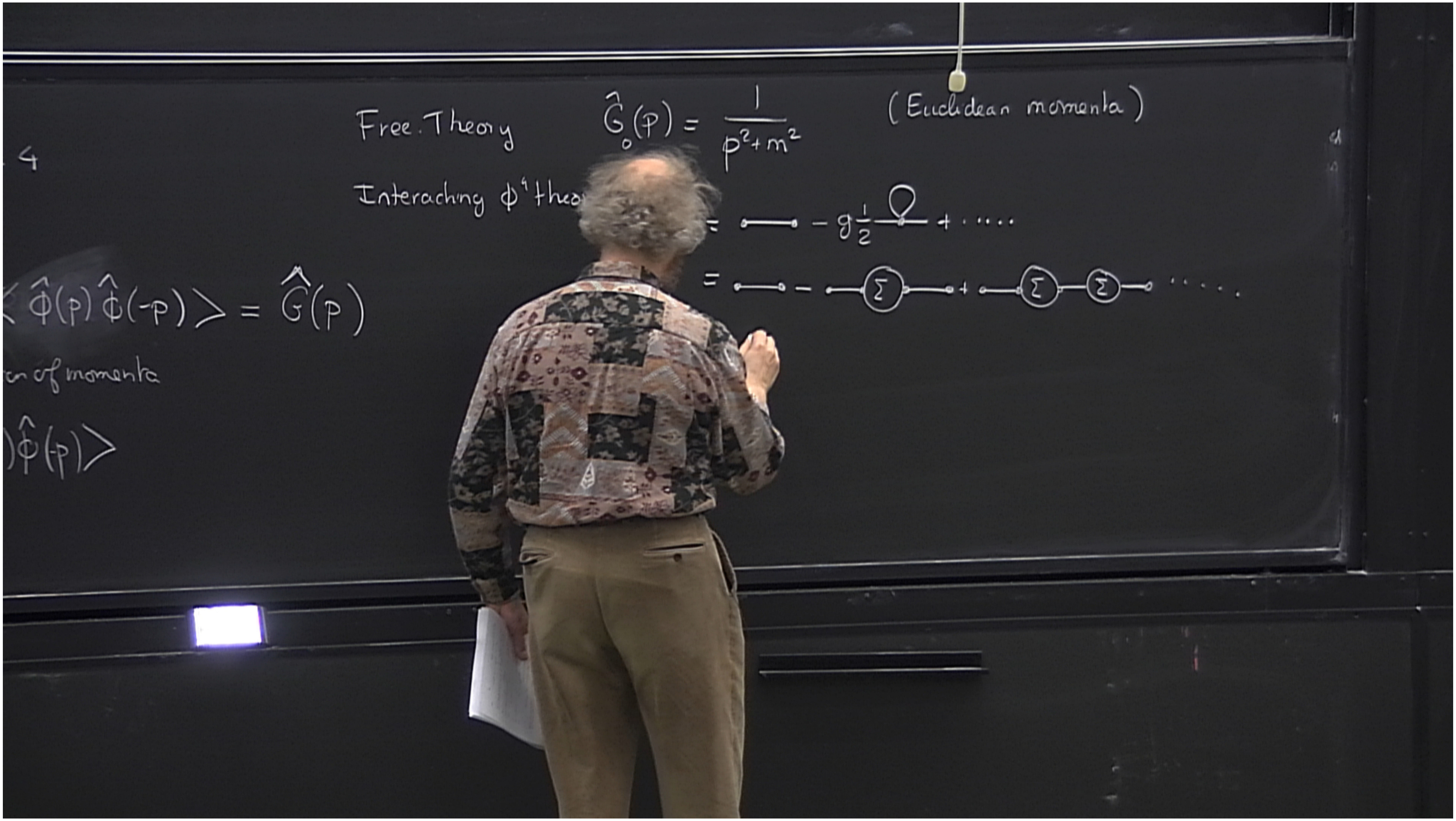

Renormalisation ← UV singularities
d = 4
Physical Parameters & correlation fun
2 pts function $\int dx \langle \phi(x)\,\phi(y) \rangle$
$= \langle \hat{\phi}(p)\,\hat{\phi}(-p) \rangle = \hat{G}(p)$
Translation invariance
$\int dx \int dy\, e^{-i(px+qy)} \langle \phi(x)\,\phi(y) \rangle = \langle \hat{\phi}(p)\,\hat{\phi}(q) \rangle$

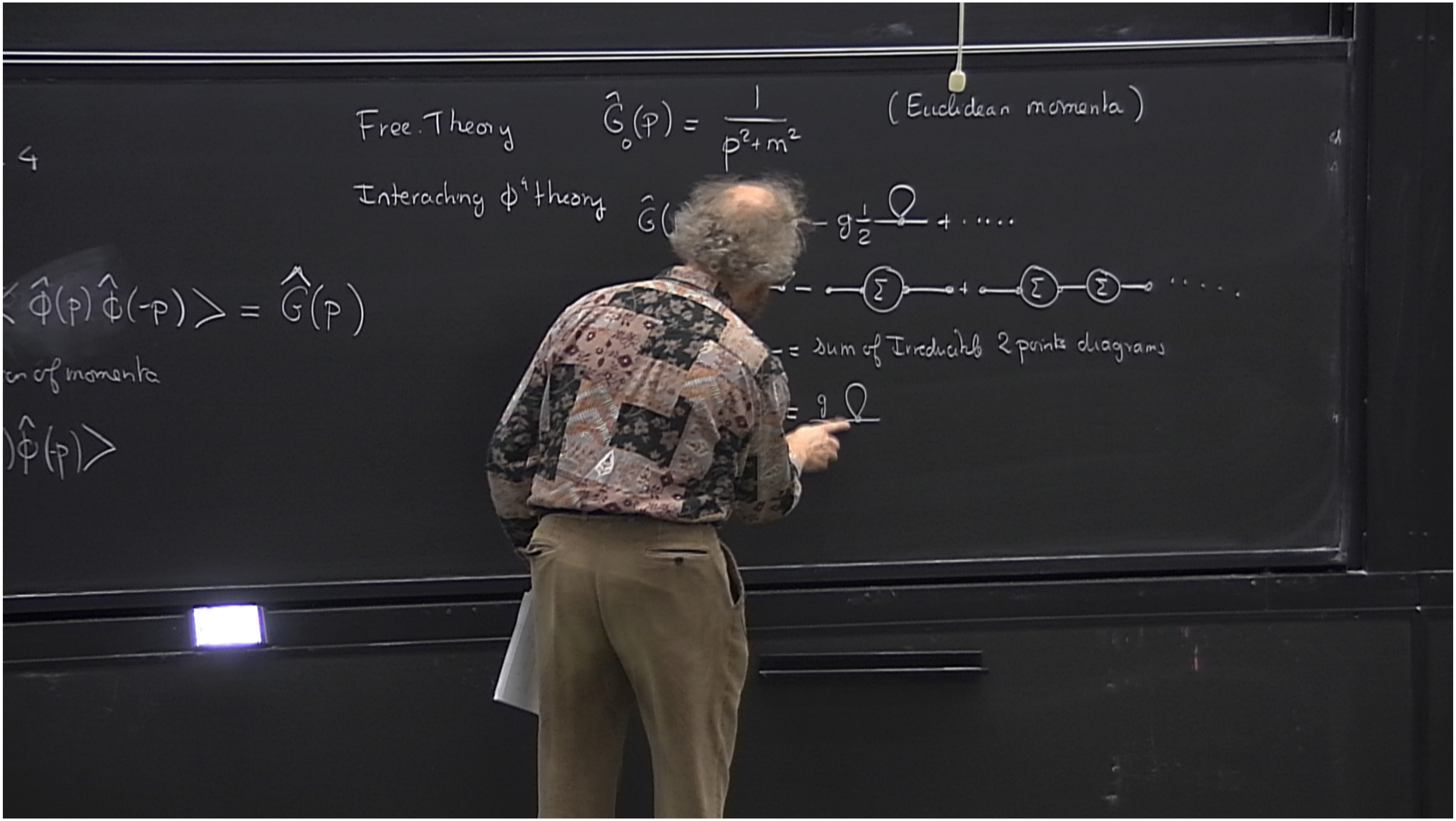

Renormalisation ← UV singularities d = 4
Parameters & correlation functions
pts function
Translation invariance
Transl Invariance ⇒ conservation of momenta

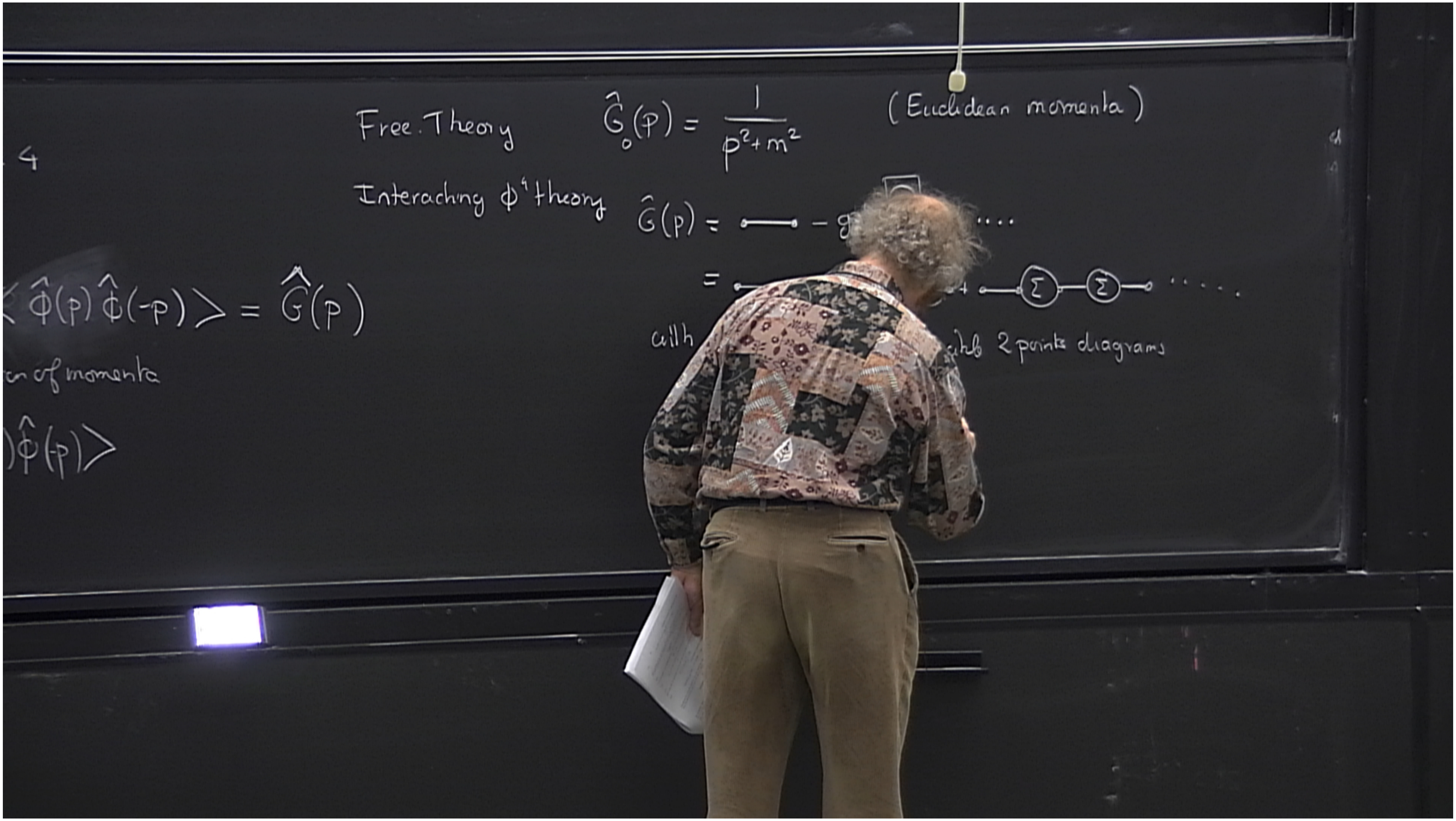

Renormalisation ← U V singularities d = 4
Physical Parameters & correlation functions
2 pts function $\int dx \langle \phi(x)\,\phi(y) \rangle\, e^{-ip(x-y)} = \langle \hat\phi(p)\,\hat\phi(-p) \rangle = \hat G(p)$
Translation invariance
$\int dx \int dy\, e^{-i(px+qy)} \langle \phi(x)\,\phi(y) \rangle = \langle \hat\phi(p)\,\hat\phi(q) \rangle = (2\pi)^4 \delta^4(p+q) \langle \hat\phi(p)\,\hat\phi(-p) \rangle$
Transl Invariance ⇒ conservation of momenta
Free Theory

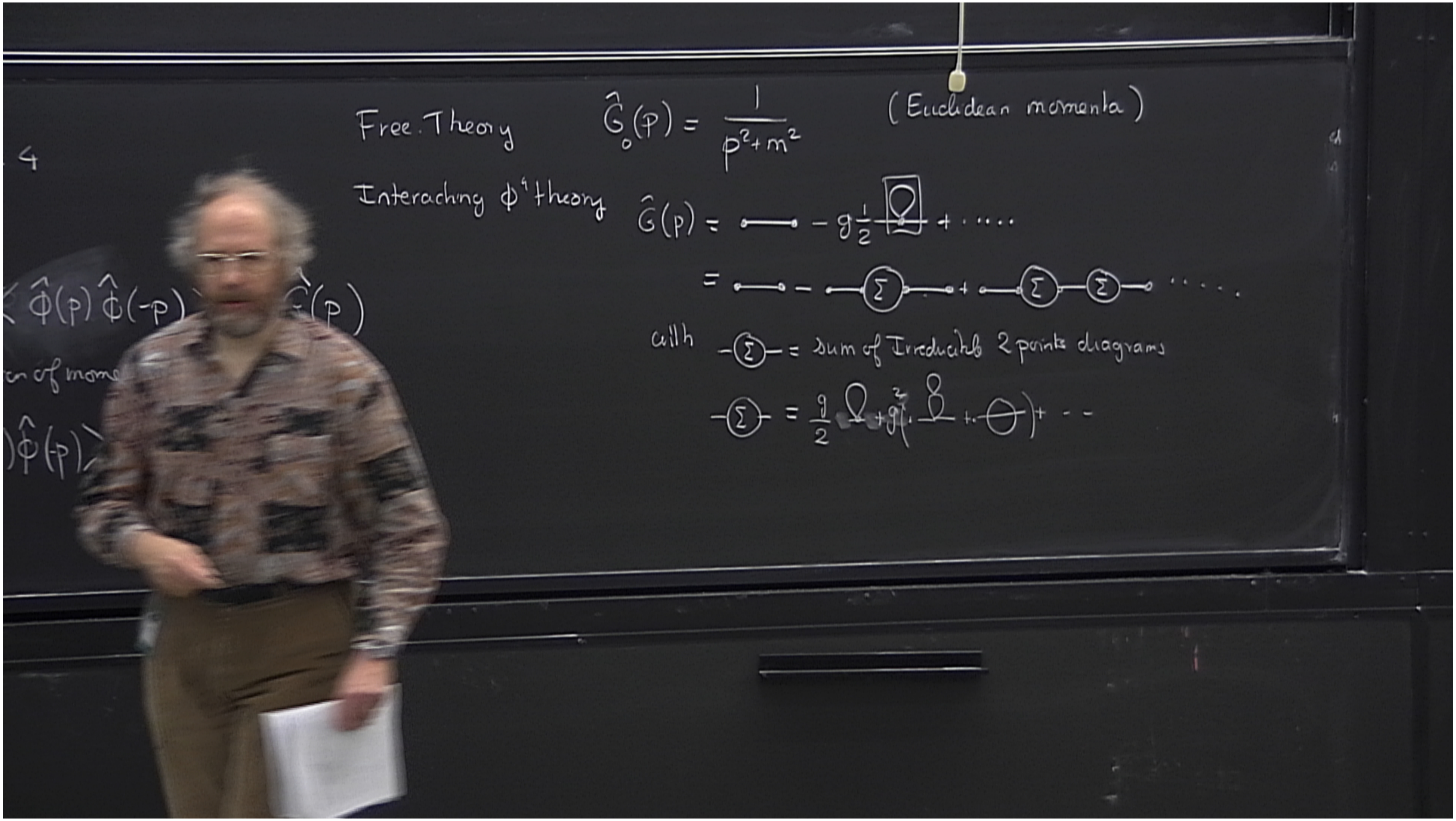

Free. Theory
$\hat{G}_0(p) = \frac{1}{p^2+m^2}$
(Euclidean momenta)
4
$\langle \hat{\Phi}(p)\hat{\Phi}(-p) \rangle = \hat{G}(p)$
of momenta

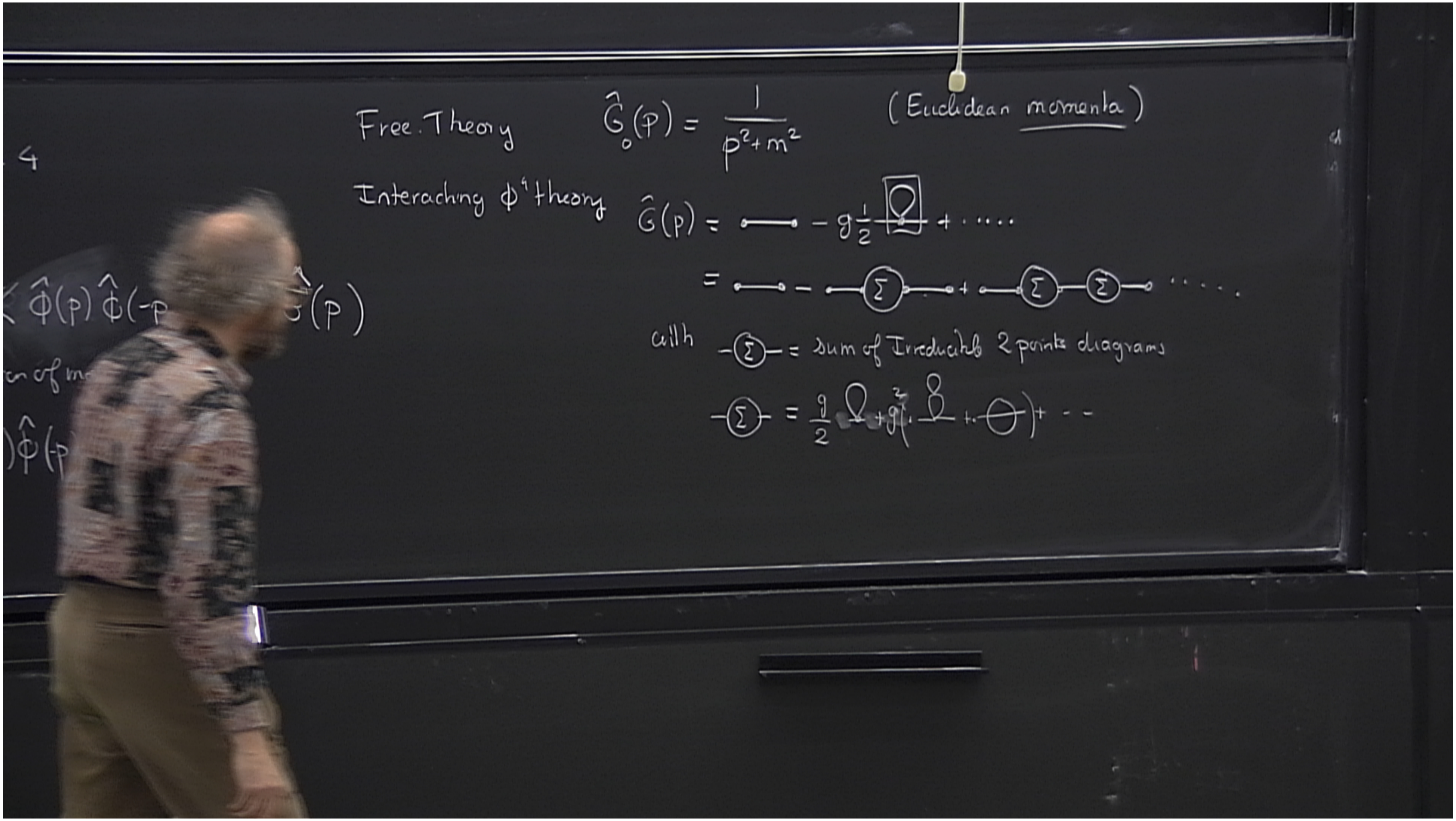

Free. Theory
Ĝ₀(p) = 1 / (p²+m²)
(Euclidean momenta)
Interacting φ⁴ theory
Ĝ(p) =
+ · · · · ·
< Φ̂(p) Φ̂(-p) > = Ĝ(p)
of momenta

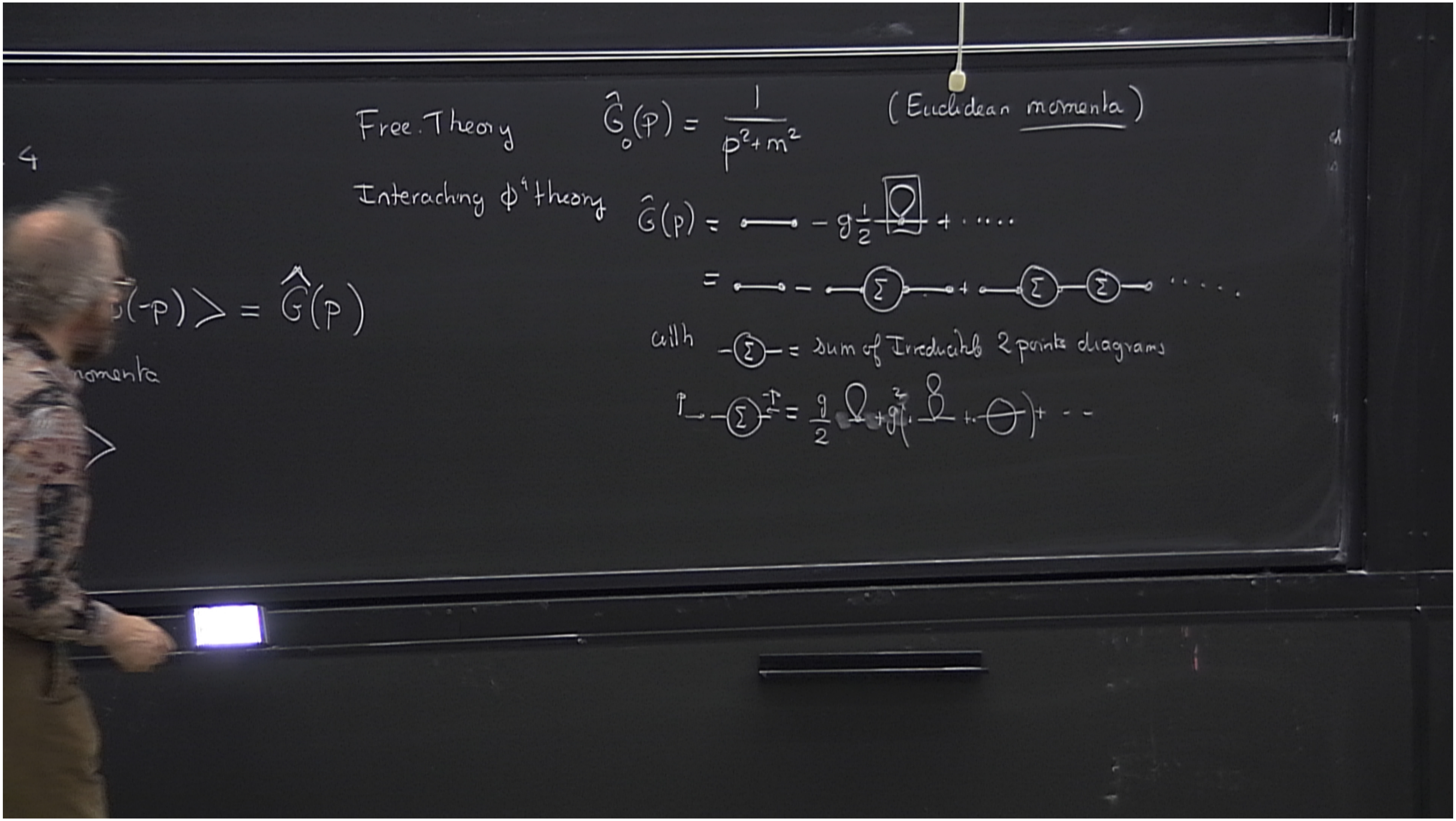

Free. Theory
(Euclidean momenta)
Interacting ϕ⁴ theo
of momenta

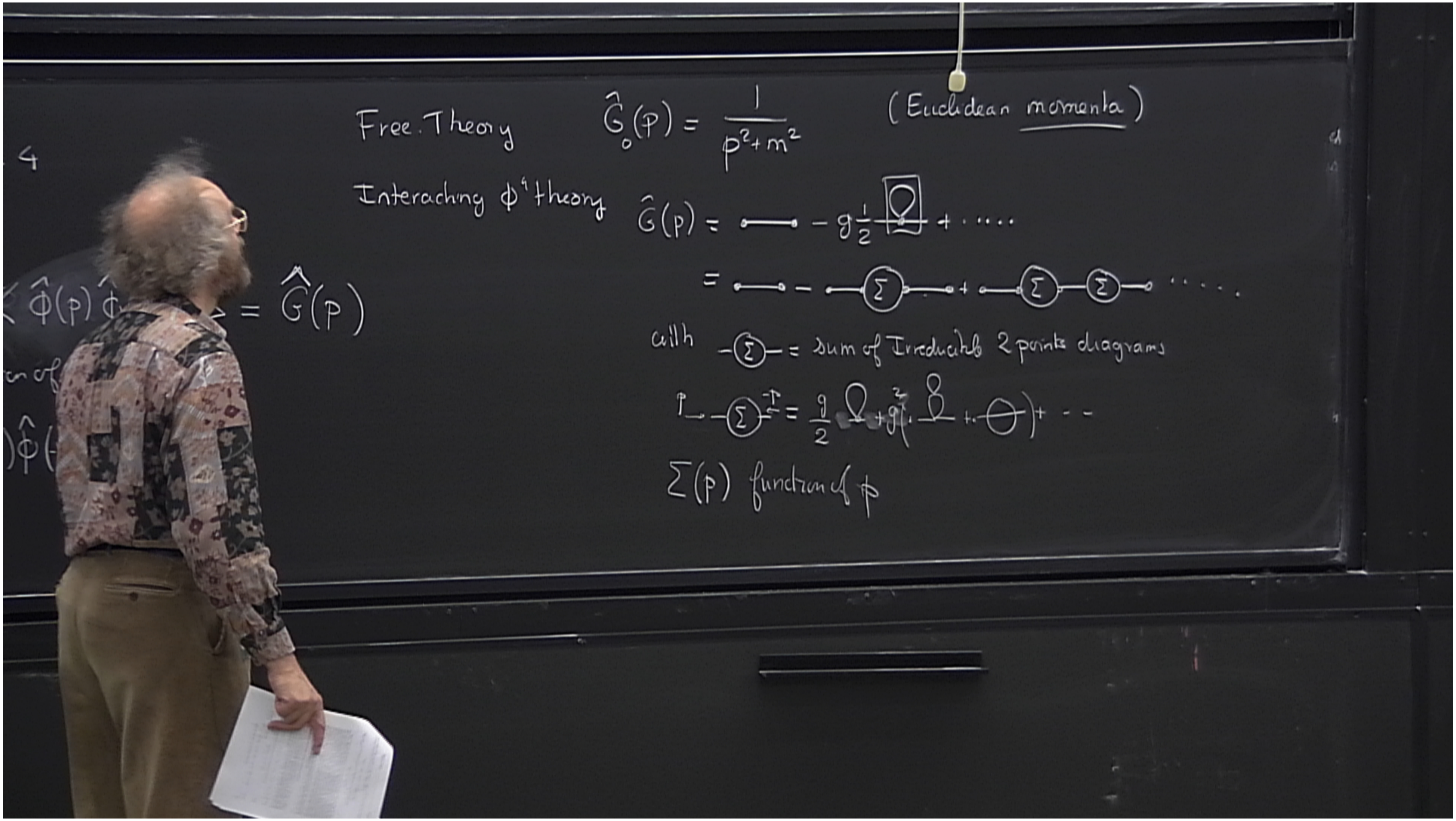

Free Theory
$\hat{G}_0(p) = \frac{1}{p^2+m^2}$
(Euclidean momenta)
Interacting ϕ^4 theory
$\langle \hat{\phi}(p)\hat{\phi}(-p) \rangle = \hat{G}(p)$
sum of Irreducible 2 points diagrams

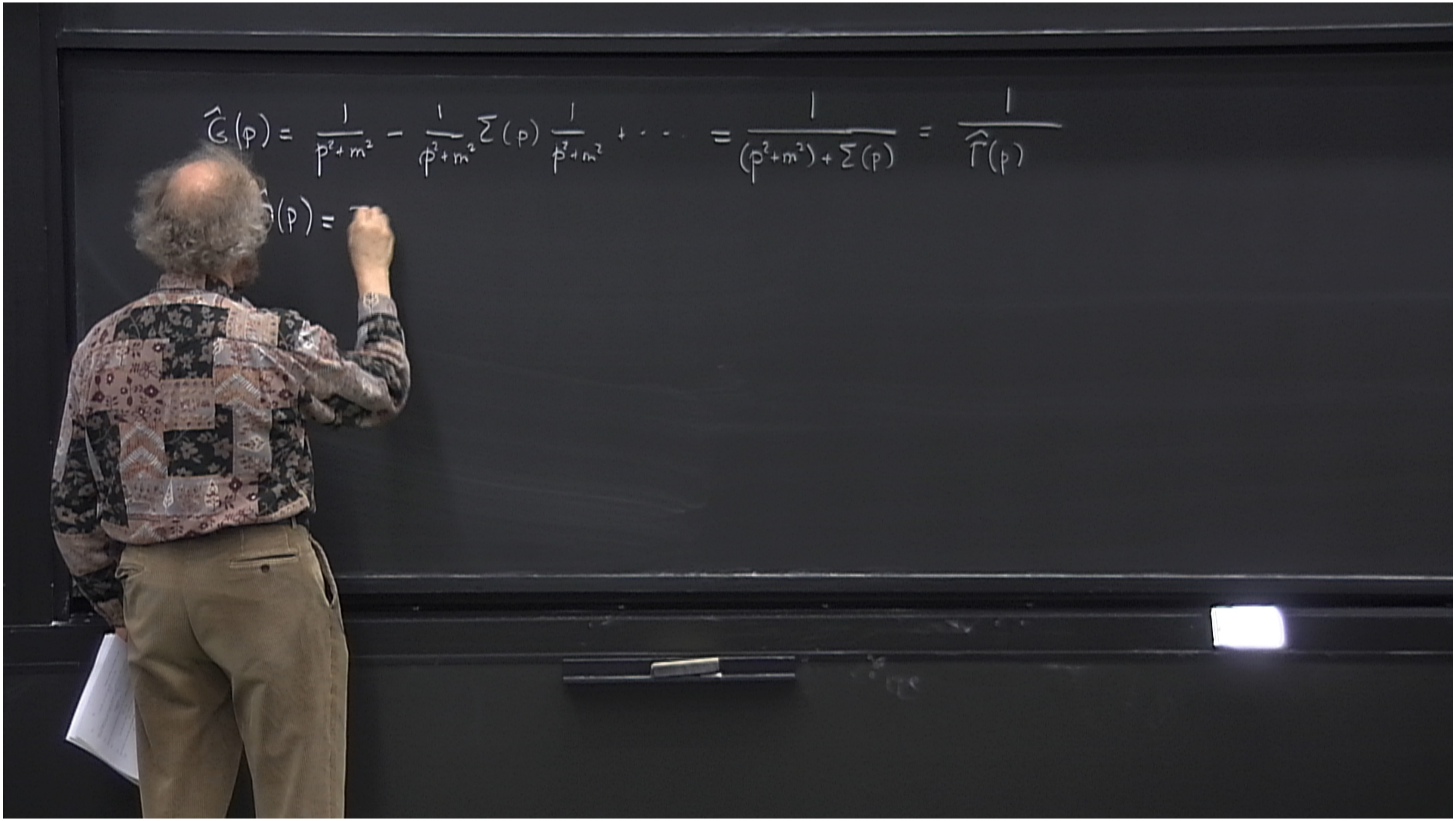

Free. Theory
(Euclidean momenta)
Interacting ϕ⁴ theory
2 points diagrams
of momenta

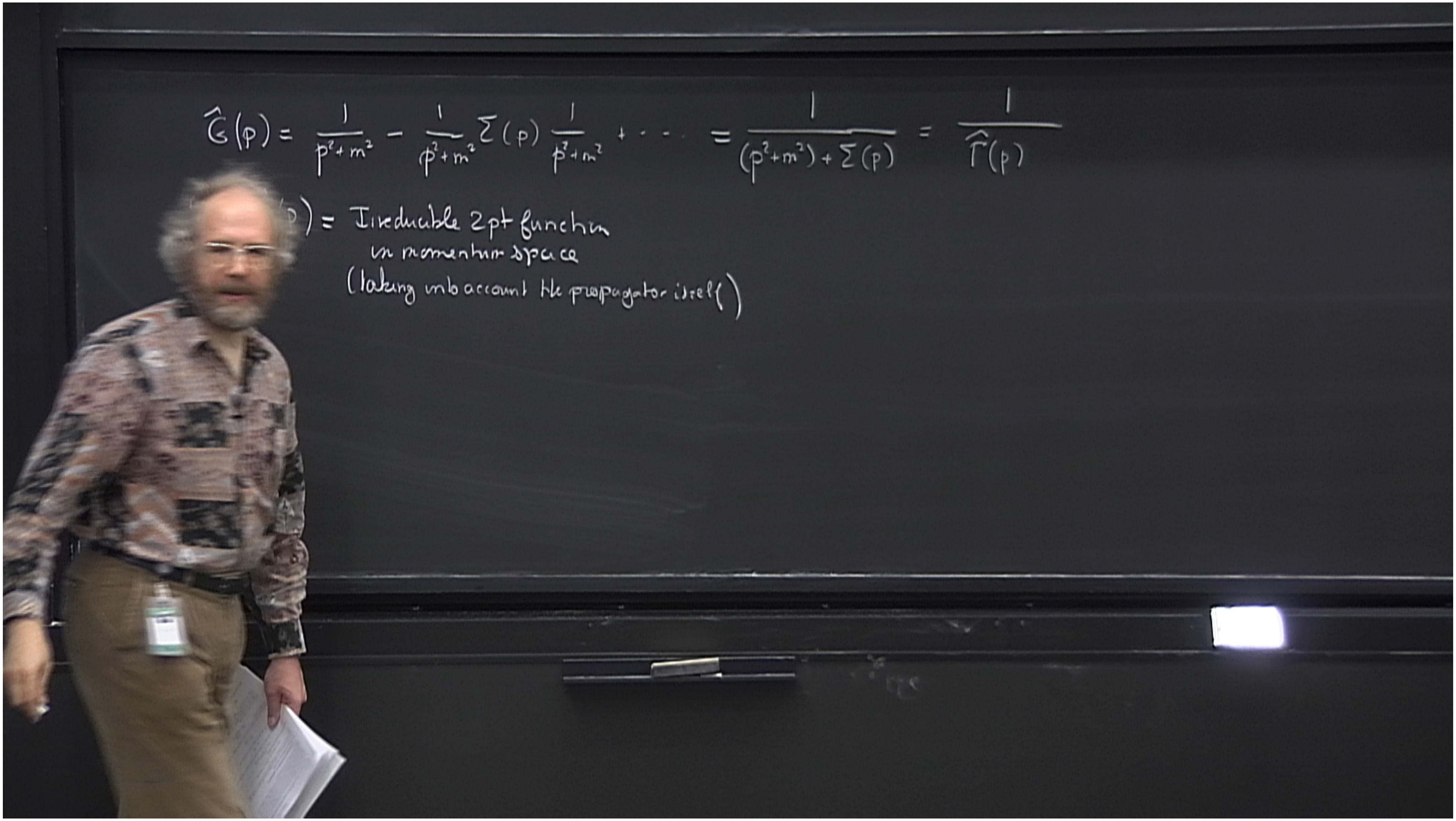

Free Theory
(Euclidean momenta)
Interacting φ⁴ theory
with
= sum of Irreducible 2 points diagrams

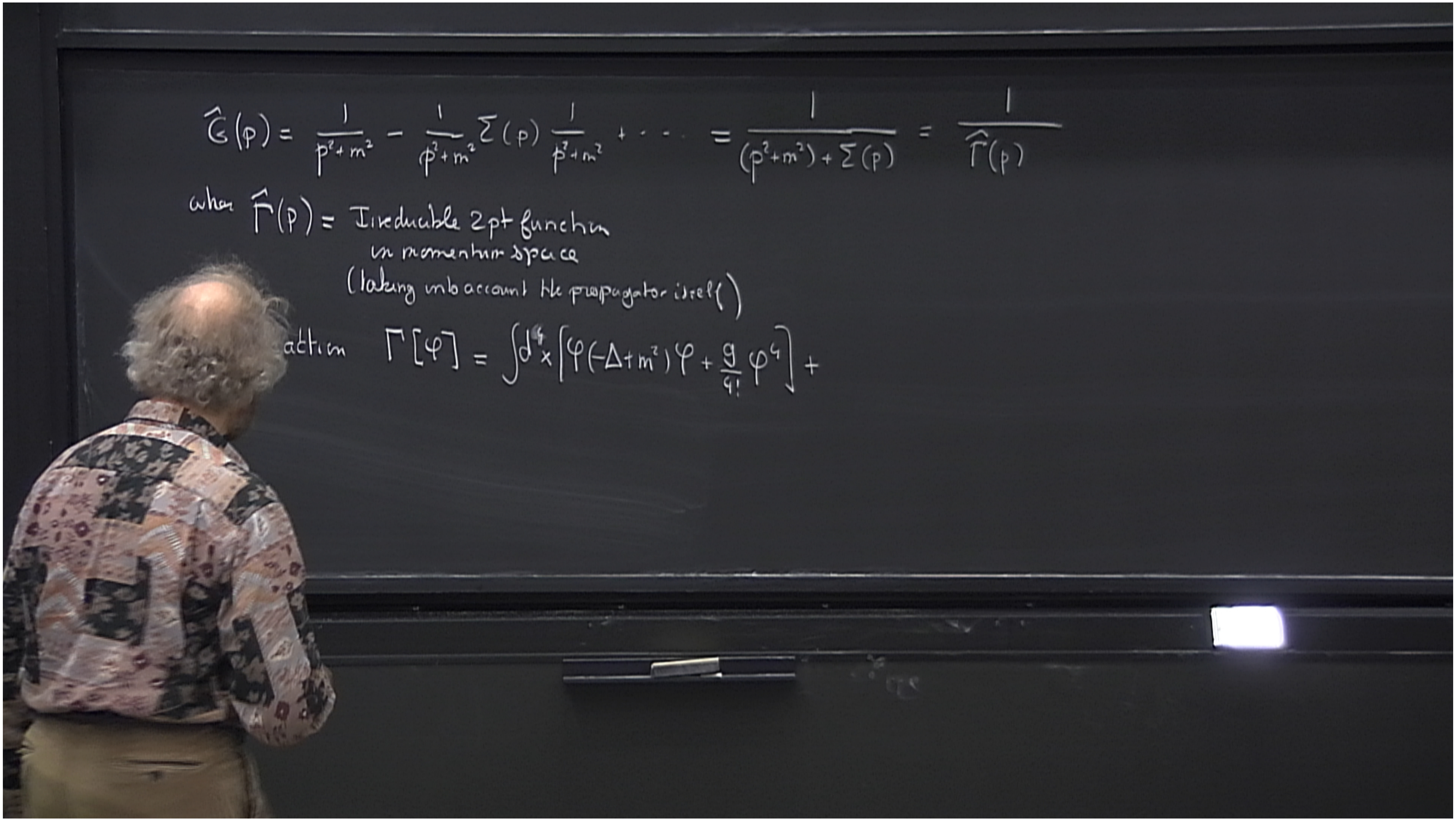

Free. Theory
$\hat{G}_0(p) = \frac{1}{p^2+m^2}$
(Euclidean momenta)
Interacting ϕ^4 theory
$\hat{G}(p) =$
with
= sum of Irreducible 2 points diagrams

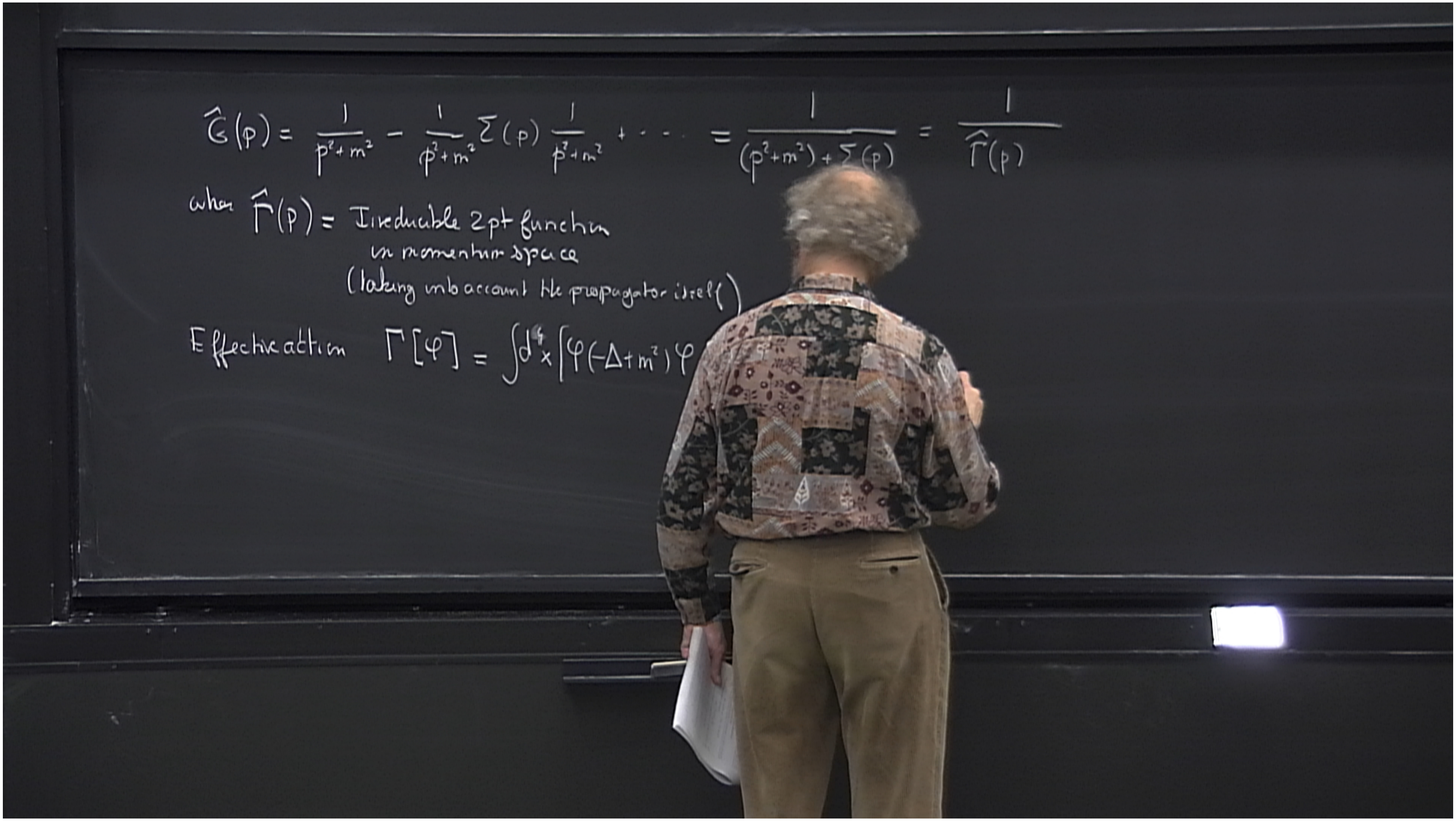

Free. Theory
(Euclidean momenta)
Interacting φ⁴ theory
= sum of Irreducible 2 points diagrams
momenta

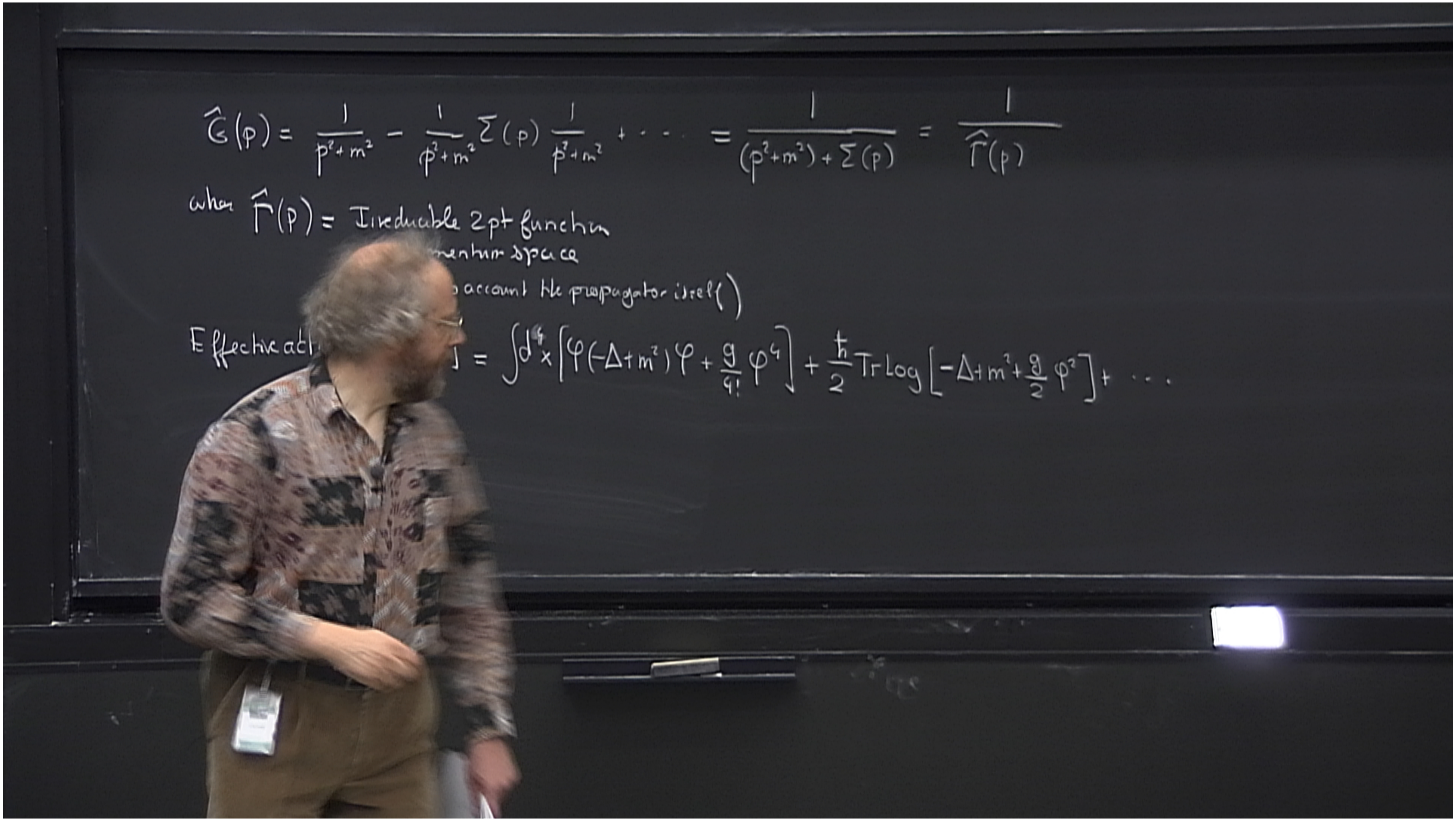

Free. Theory
$\hat{G}_0(p) = \frac{1}{p^2+m^2}$
(Euclidean momenta)
Interacting ϕ^4 theory
$\hat{G}(p) =$ [diagram] $- g\frac{1}{2}$ [diagram] $+ \cdots$
$= \hat{G}(p)$
with $-(\Sigma)- =$ sum of Irreducible 2 points diagrams
$\Sigma(p)$ function of p


Ĝ(p) = 1/(p²+m²) − 1/(p²+m²) Σ(p) 1/(p²+m²) + · · · = 1/((p²+m²) + Σ(p)) = 1/Γ̂(p)

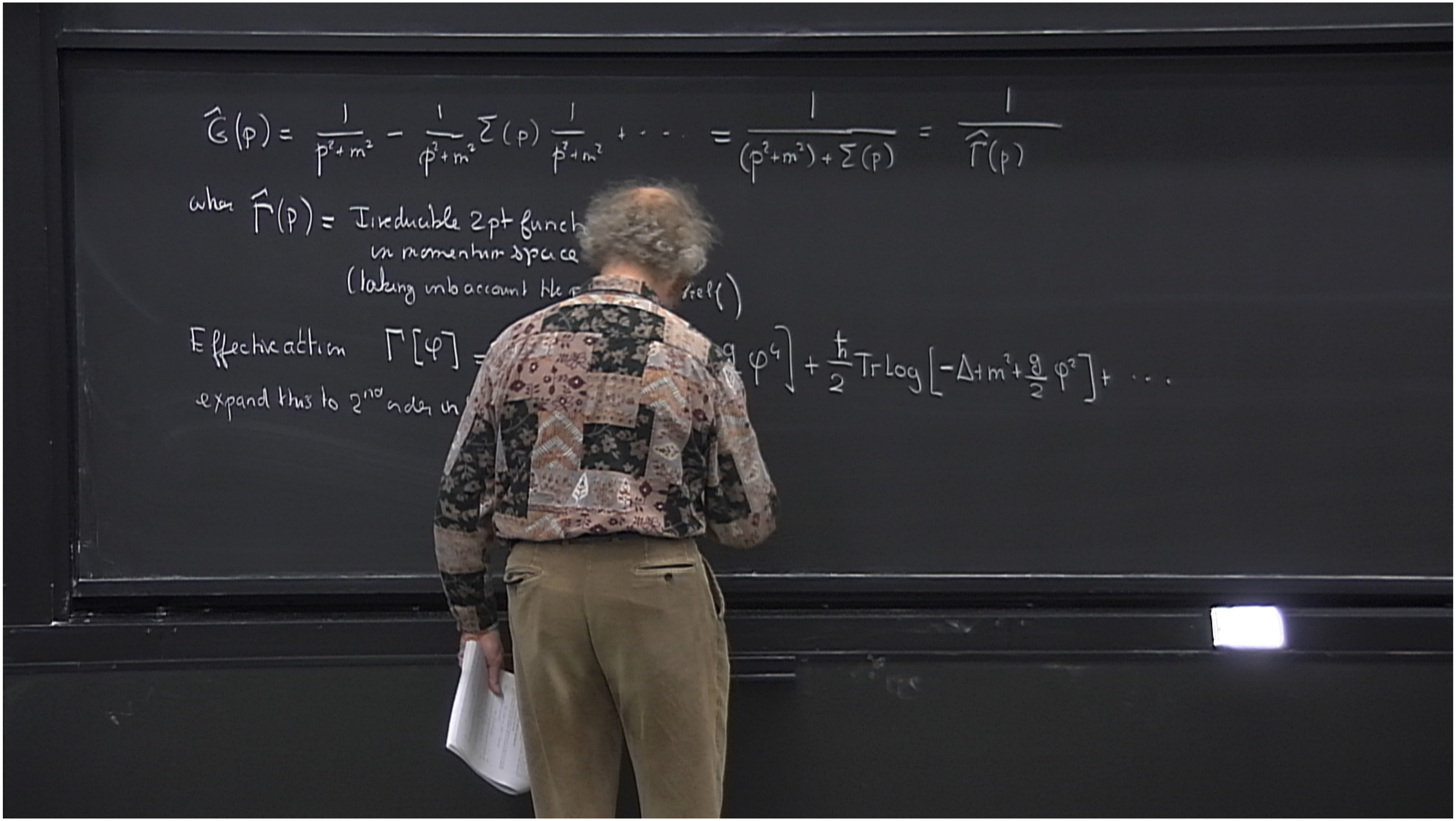

Ĝ(p) = 1/(p²+m²) − 1/(p²+m²) Σ(p) 1/(p²+m²) + · · · = 1/((p²+m²) + Σ(p)) = 1/Γ̂(p)
p) = Irreducible 2pt function
in momentum space
(taking into account the propagator itself)

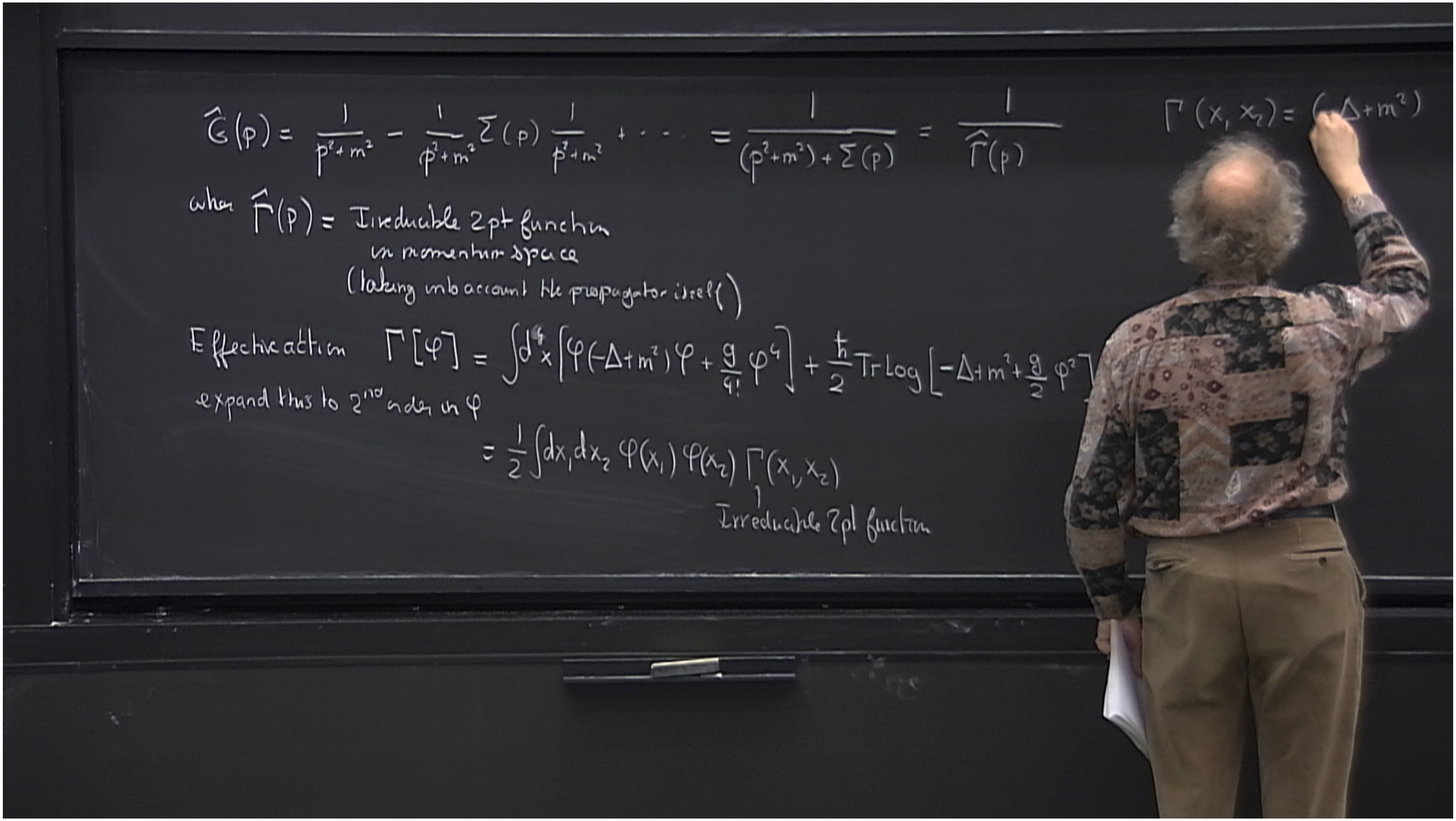

$\hat{G}(p) = \frac{1}{p^2+m^2} - \frac{1}{p^2+m^2}\Sigma(p)\frac{1}{p^2+m^2} + \cdots = \frac{1}{(p^2+m^2)+\Sigma(p)} = \frac{1}{\hat{\Gamma}(p)}$
when $\hat{\Gamma}(p)$ = Irreducible 2pt function
in momentum space
(taking into account the propagator itself)
action $\Gamma[\varphi] = \int d^4x \left[\varphi(-\Delta+m^2)\varphi + \frac{g}{4!}\varphi^4\right] +$

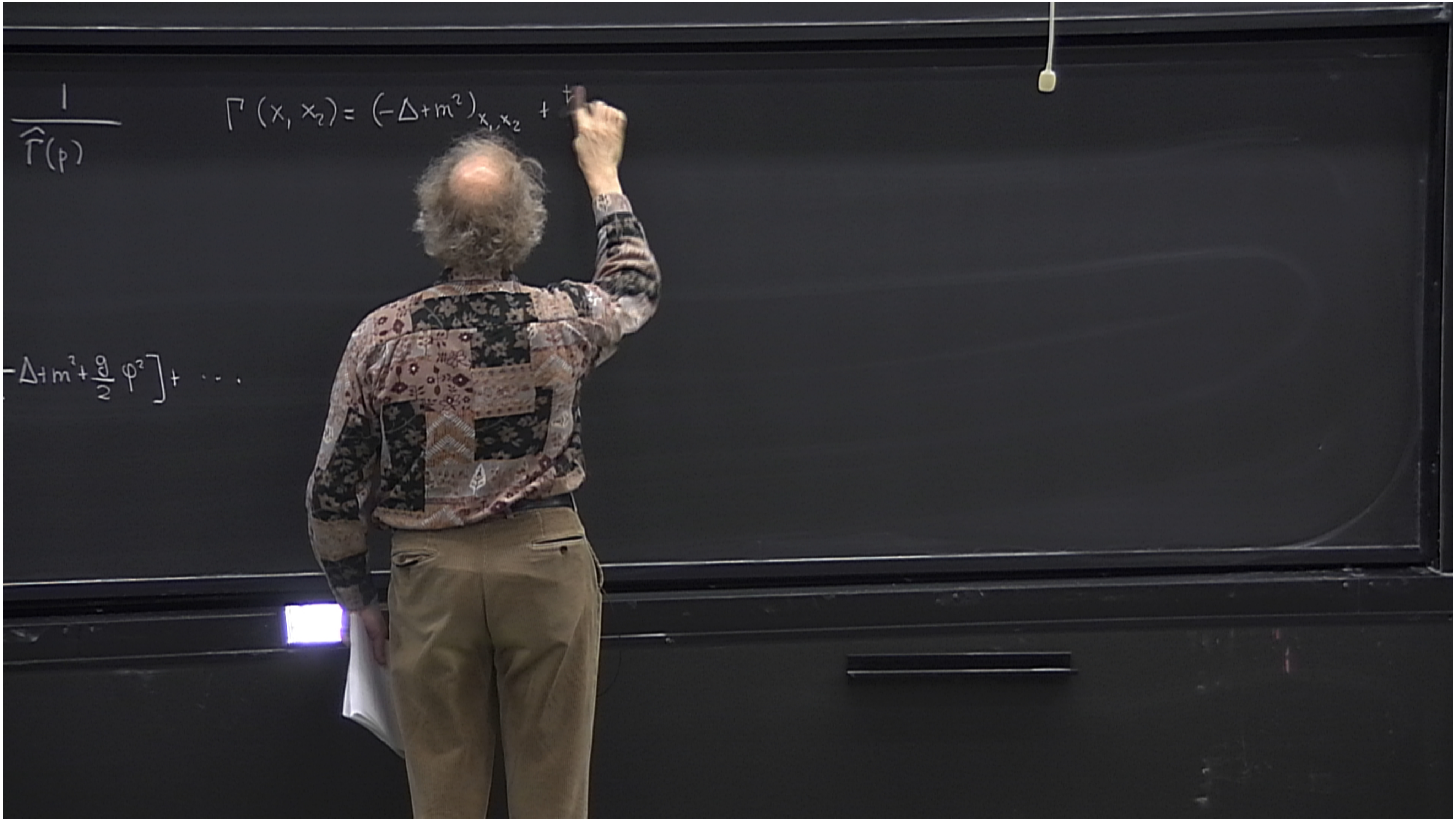

$\hat{G}(p) = \frac{1}{p^2+m^2} - \frac{1}{p^2+m^2}\Sigma(p)\frac{1}{p^2+m^2} + \cdots = \frac{1}{(p^2+m^2)+\Sigma(p)} = \frac{1}{\hat{\Gamma}(p)}$
where $\hat{\Gamma}(p)$ = Irreducible 2pt function
in momentum space
(taking into account the propagator itself)
Effective action $\Gamma[\varphi] = \int d^4x \,[\varphi(-\Delta+m^2)\varphi$

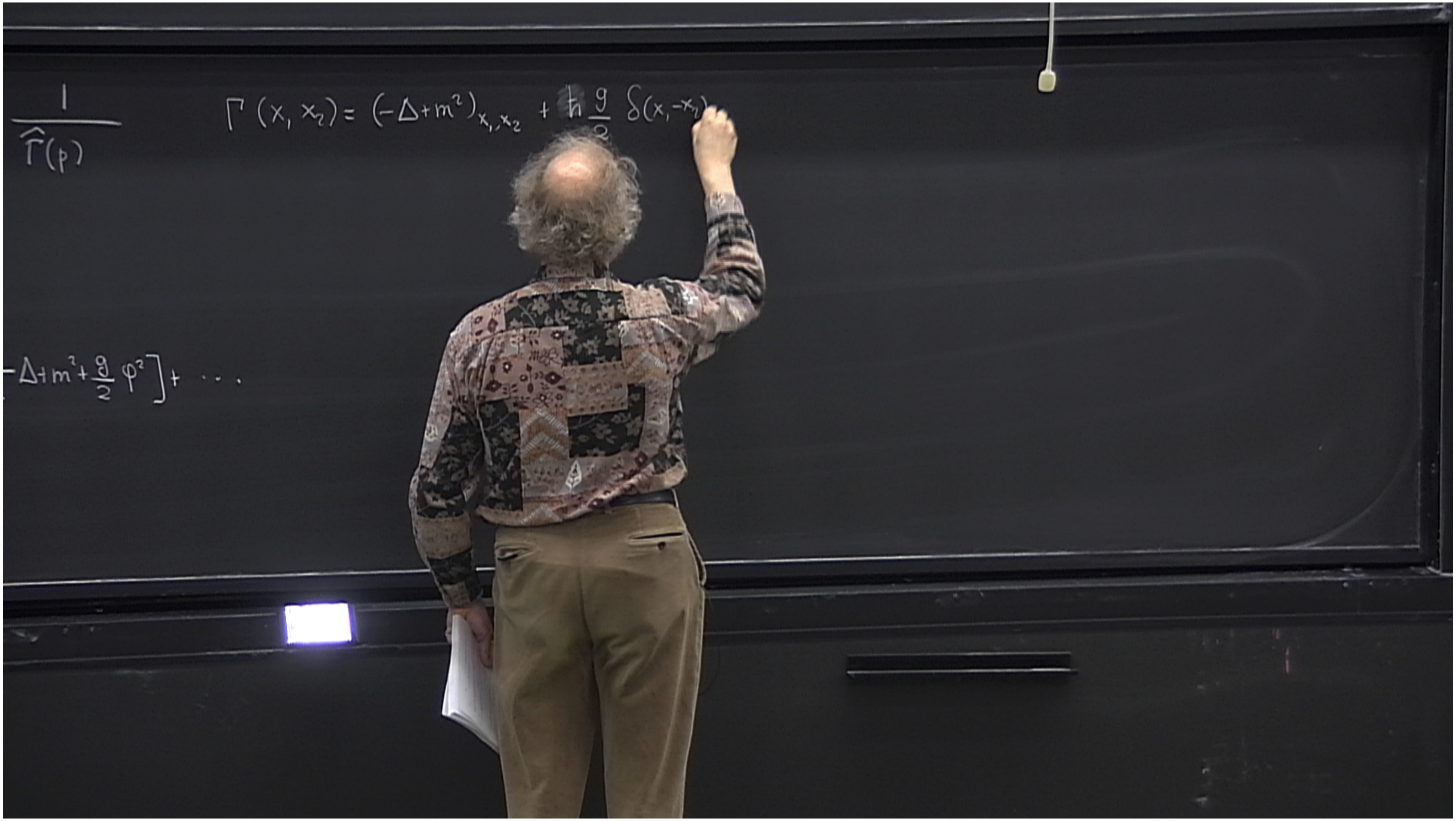


$\hat{G}(p) = \frac{1}{p^2+m^2} - \frac{1}{p^2+m^2}\Sigma(p)\frac{1}{p^2+m^2} + \cdots = \frac{1}{(p^2+m^2)+\Sigma(p)} = \frac{1}{\hat{\Gamma}(p)}$
where $\hat{\Gamma}(p)$ = Irreducible 2pt function
mentum space
account the propagator itself)
Effective act
$= \int d^4x \left[\varphi(-\Delta+m^2)\varphi + \frac{g}{4!}\varphi^4\right] + \frac{\hbar}{2}\mathrm{Tr}\,\mathrm{Log}\left[-\Delta+m^2+\frac{g}{2}\varphi^2\right] + \cdots$


$\hat{G}(p) = \frac{1}{p^2+m^2} - \frac{1}{p^2+m^2}\Sigma(p)\frac{1}{p^2+m^2} + \cdots = \frac{1}{(p^2+m^2)+\Sigma(p)} = \frac{1}{\hat{\Gamma}(p)}$
where $\hat{\Gamma}(p)$ = Irreducible 2pt funct
in momentum space
(taking into account the
itself)
Effective action $\Gamma[\varphi]$ =
$\varphi^4] + \frac{\hbar}{2}\mathrm{Tr}\,\mathrm{Log}[-\Delta + m^2 + \frac{g}{2}\varphi^2] + \cdots$
expand this to 2nd order in


$\hat{G}(p) = \frac{1}{p^2+m^2} - \frac{1}{p^2+m^2}\Sigma(p)\frac{1}{p^2+m^2} + \cdots = \frac{1}{(p^2+m^2)+\Sigma(p)} = \frac{1}{\hat{\Gamma}(p)}$
$\Gamma(x_1, x_2) = (-\Delta + m^2)$
where $\hat{\Gamma}(p)$ = Irreducible 2pt function
in momentum space
(taking into account the propagator itself)
Effective action $\Gamma[\varphi] = \int d^4x \left[\varphi(-\Delta+m^2)\varphi + \frac{g}{4!}\varphi^4\right] + \frac{\hbar}{2} \mathrm{Tr}\,\mathrm{Log}\left[-\Delta + m^2 + \frac{g}{2}\varphi^2\right]$
expand this to 2nd order in φ
$= \frac{1}{2}\int dx_1 dx_2\, \varphi(x_1)\varphi(x_2)\Gamma(x_1, x_2)$
Irreducible 2pt function


1 / Γ̂(p)
Γ(x1, x2) = (−Δ+m²)x1, x2 +
−Δ+m²+ g/2 φ²] + . . .

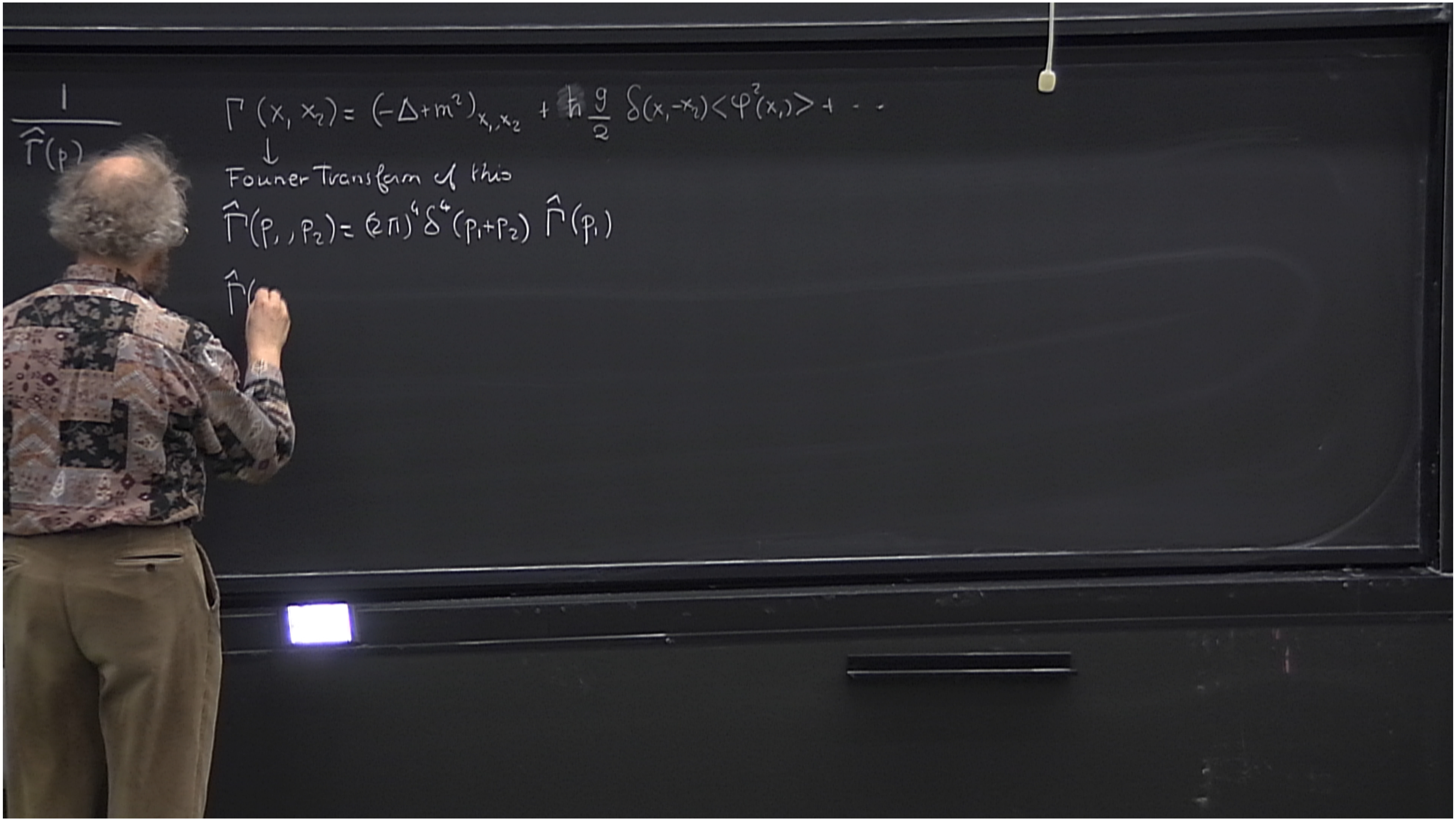


$\frac{1}{\hat{\Gamma}(p)}$
$\Gamma(x_1, x_2) = (-\Delta + m^2)_{x_1, x_2} + \hbar \frac{g}{2} \delta(x_1 - x_2)$
$-\Delta + m^2 + \frac{g}{2} \varphi^2] + \cdots$

$$\frac{1}{\hat{\Gamma}(p)}$$

$$\Gamma(x_1, x_2) = (-\Delta + m^2)_{x_1, x_2} + \hbar \frac{g}{2} \delta(x_1 - x_2) \langle \varphi^2(x_1) \rangle + \cdots$$

$$-\Delta + m^2 + \frac{g}{2}\varphi^2] + \cdots$$



$\frac{1}{\hat{\Gamma}(p)}$
$\Gamma(x_1, x_2) = (-\Delta + m^2)_{x_1, x_2} + \hbar \frac{g}{2} \delta(x_1 - x_2) \langle \varphi^2(x_1) \rangle + \ldots$
Fourier Transform of this
$\hat{\Gamma}(p_1, p_2) = (2\pi)^4 \delta^4(p_1 + p_2) \hat{\Gamma}(p_1)$

$$\frac{1}{\hat{\Gamma}(p)}$$

$$\Gamma(x_1, x_2) = (-\Delta + m^2)_{x_1, x_2} + \hbar \frac{g}{2} \delta(x_1 - x_2) \langle \varphi^2(x_1) \rangle + \cdots$$

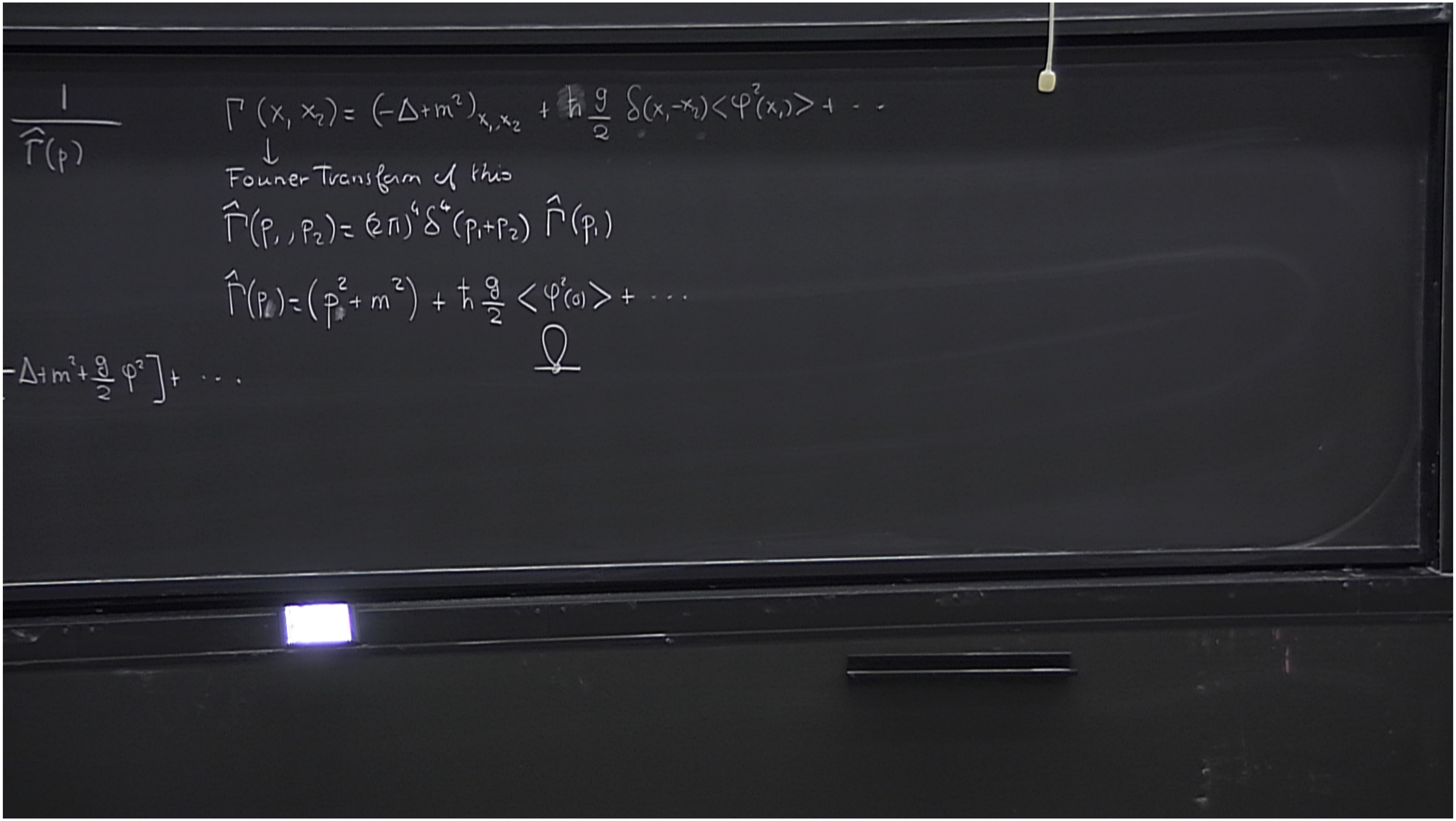
$\downarrow$

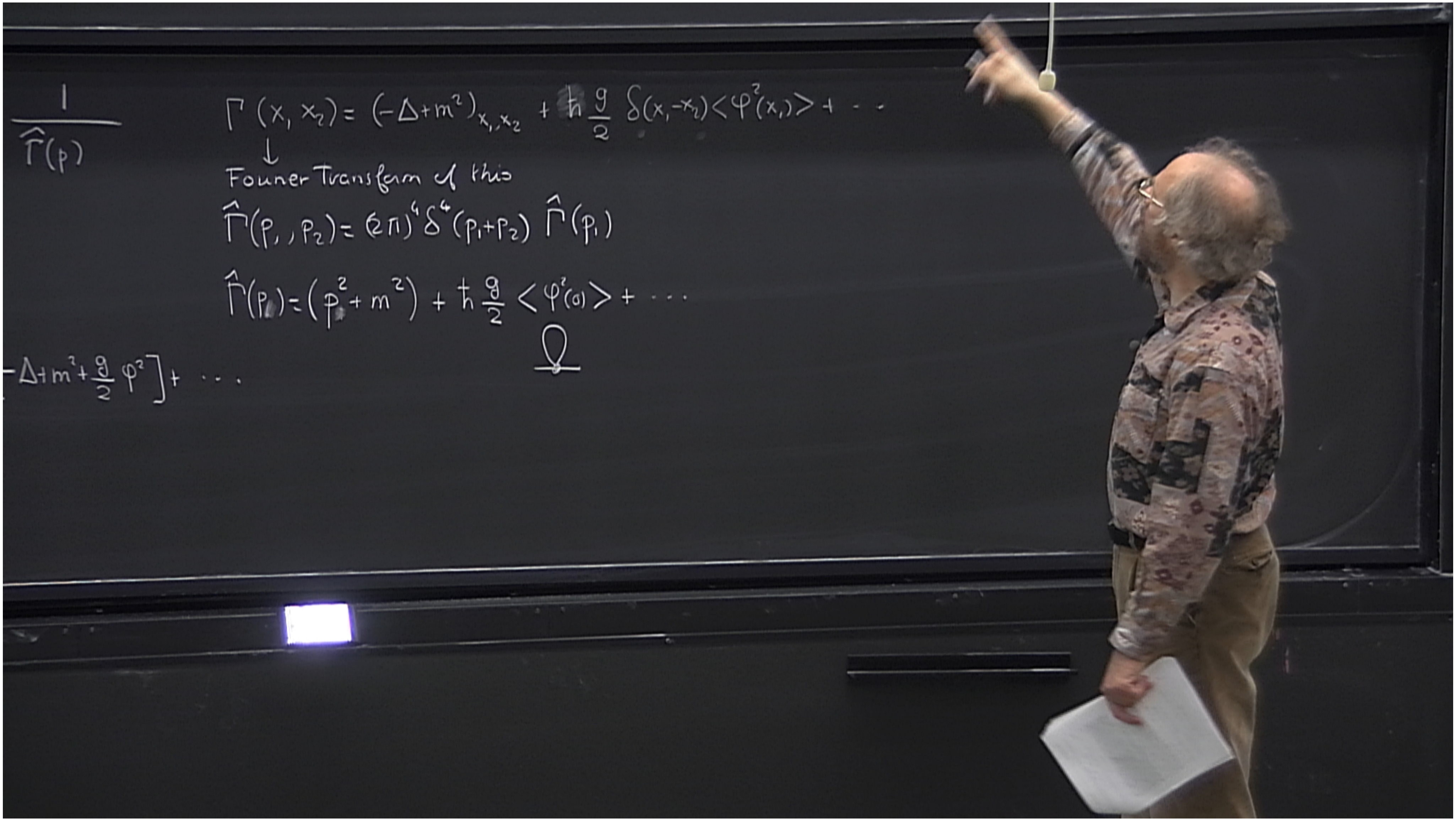
Fourier Transform of this

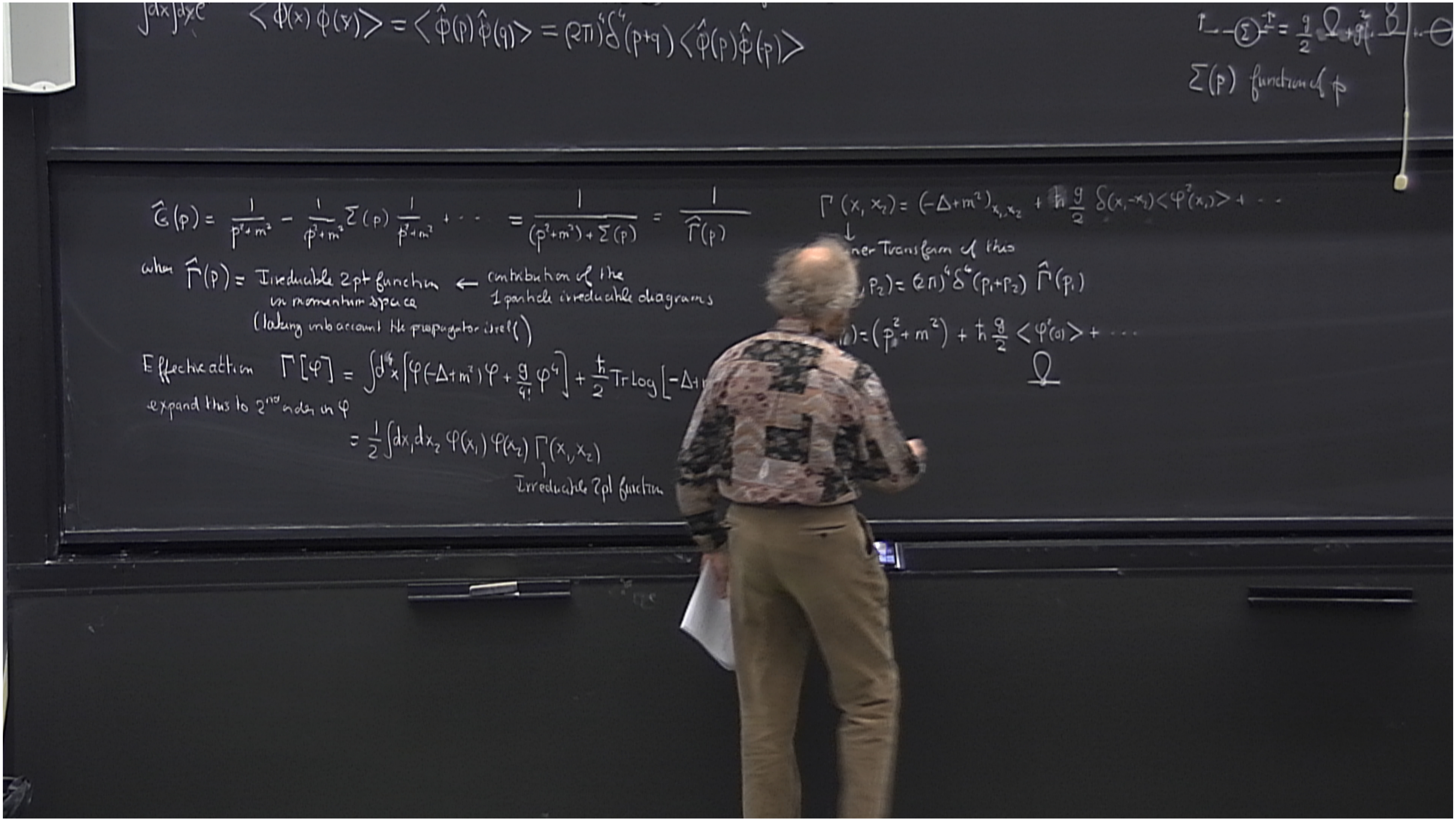
$$\hat{\Gamma}(p_1, p_2) = (2\pi)^4 \delta^4(p_1 + p_2)\, \hat{\Gamma}(p_1)$$

$$\hat{\Gamma}(p_1) = (p_1^2 + m^2) + \hbar \frac{g}{2} \langle \varphi^2(0) \rangle + \cdots$$

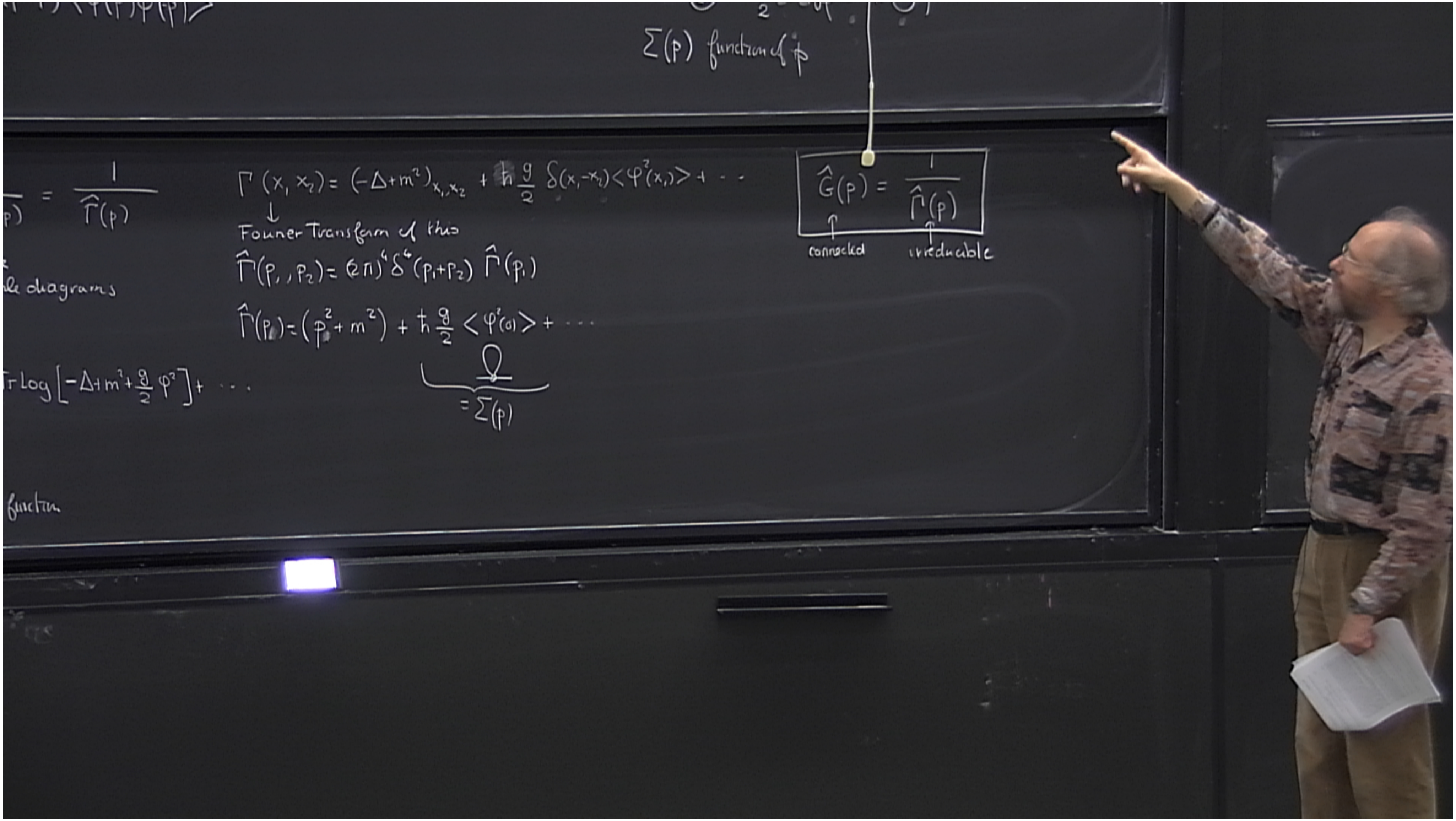
$$-\Delta + m^2 + \frac{g}{2}\varphi^2 \Big] + \cdots$$

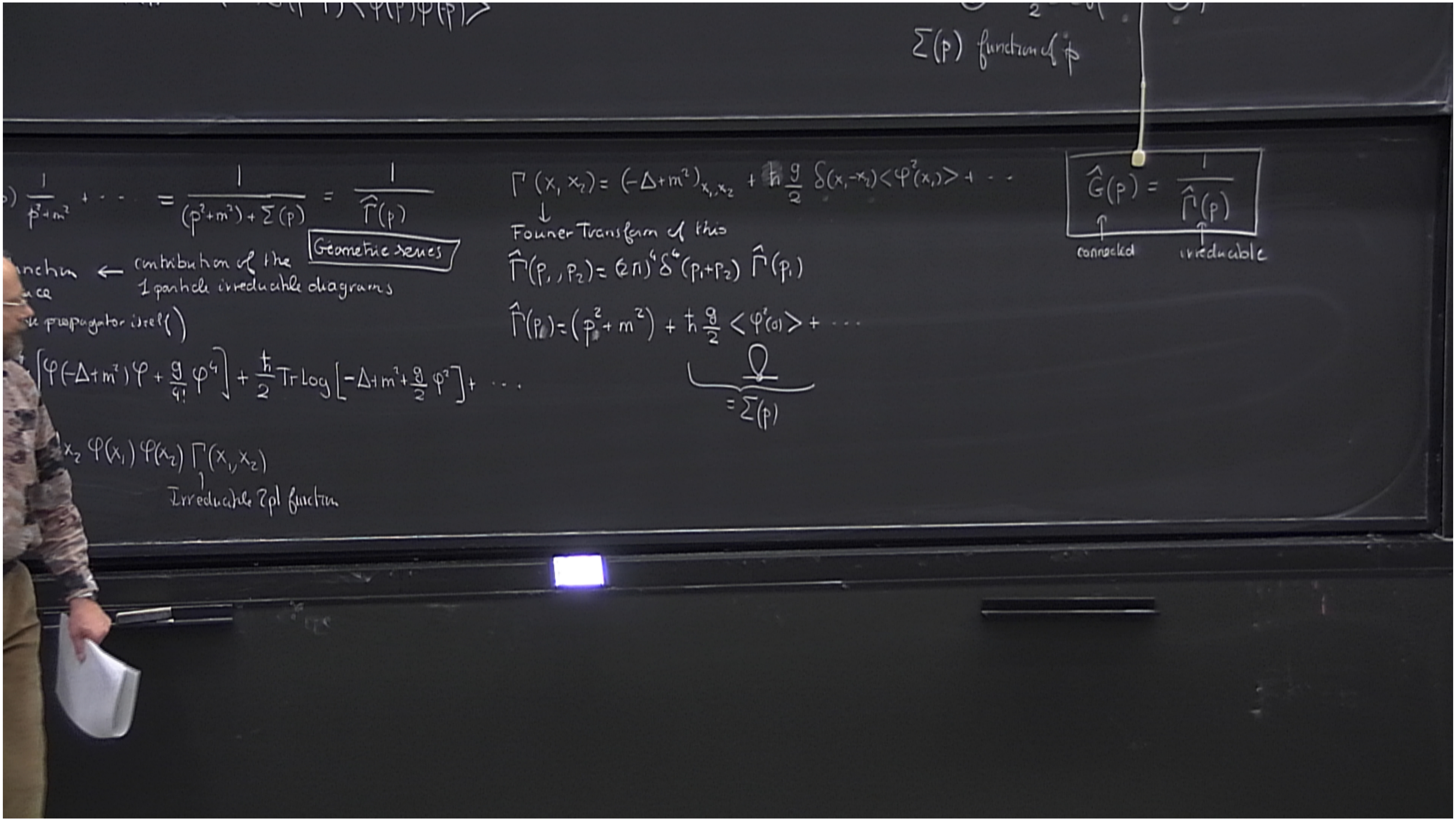

$\frac{1}{\hat{\Gamma}(p)}$
$\Gamma(x_1, x_2) = (-\Delta + m^2)_{x_1, x_2} + \hbar \frac{g}{2} \delta(x_1 - x_2) \langle \varphi^2(x_1) \rangle + \dots$
Fourier Transform of this
$\hat{\Gamma}(p_1, p_2) = (2\pi)^4 \delta^4(p_1 + p_2) \hat{\Gamma}(p_1)$
$\hat{\Gamma}(p_1) = (p_1^2 + m^2) + \hbar \frac{g}{2} \langle \varphi^2(0) \rangle + \dots$
$-\Delta + m^2 + \frac{g}{2} \varphi^2] + \dots$

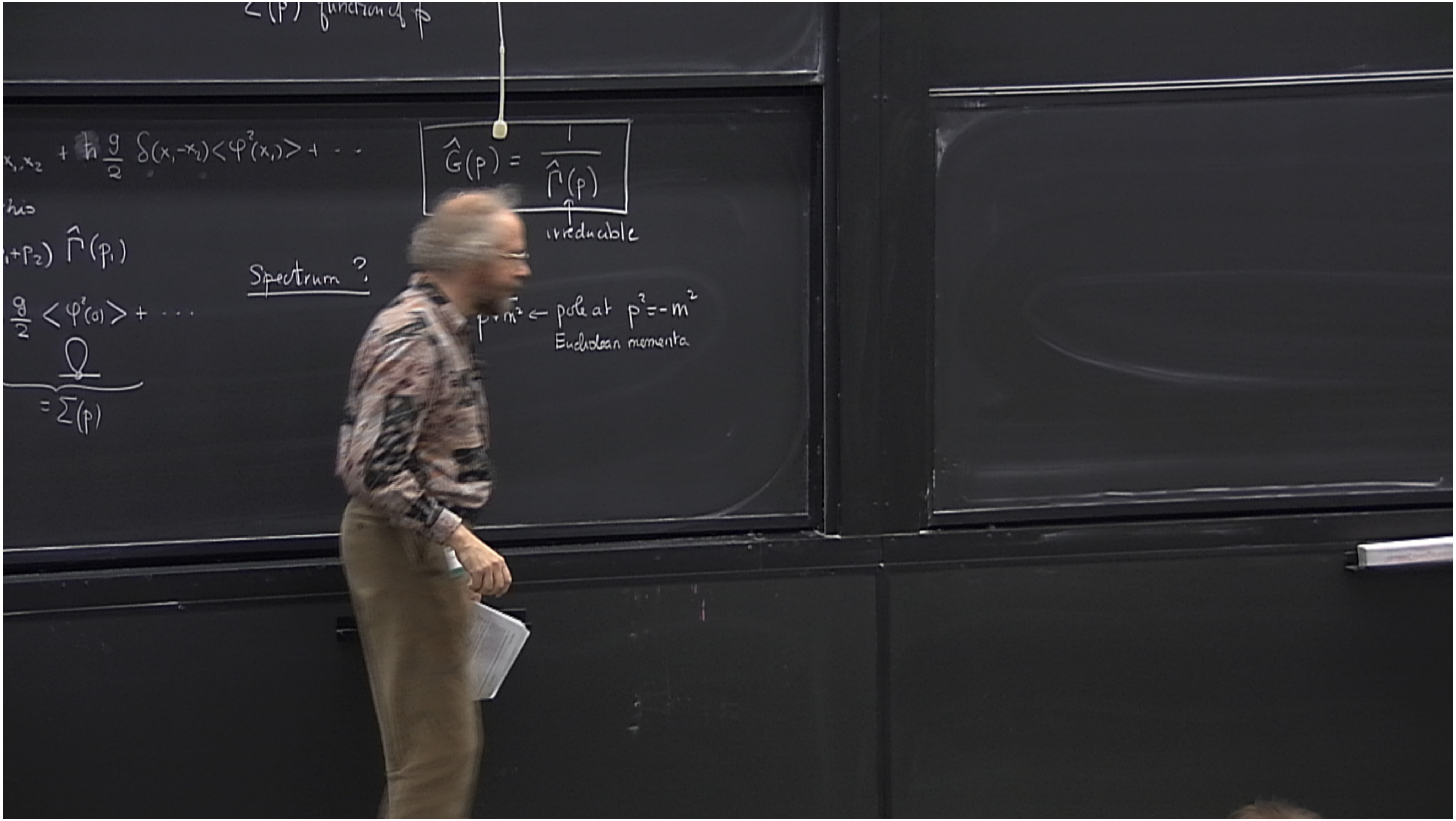

Γ (x₁, x₂) = (−Δ + m²)_{x₁, x₂} + ħ g/2 δ(x₁ − x₂) ⟨φ²(x₁)⟩ + . . .
Fourier Transform of this
Γ̂(p₁, p₂) = (2π)⁴ δ⁴(p₁ + p₂) Γ̂(p₁)
Γ̂(p₁) = (p₁² + m²) + ħ g/2 ⟨φ²(0)⟩ + . . .
1 / Γ̂(p)
−Δ + m² + g/2 φ²] + . . .

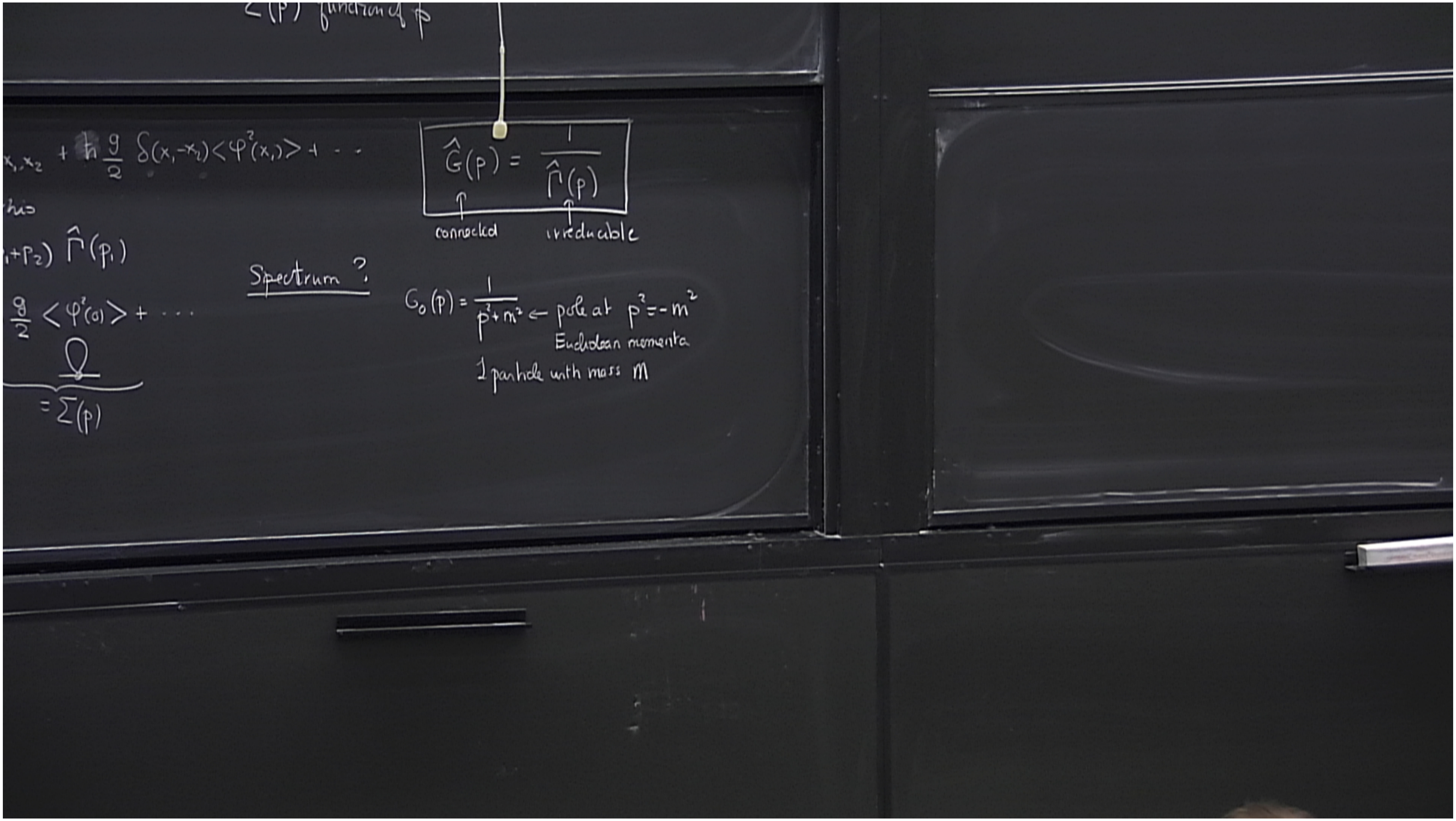

Σ(p) function of p
where Γ̂(p) = Irreducible 2pt function in momentum space
contribution of the 1 particle irreducible diagrams
(taking into account the propagator itself)
Effective action
expand this to 2nd order in φ
Irreducible 2pt function
Fourier Transform of this

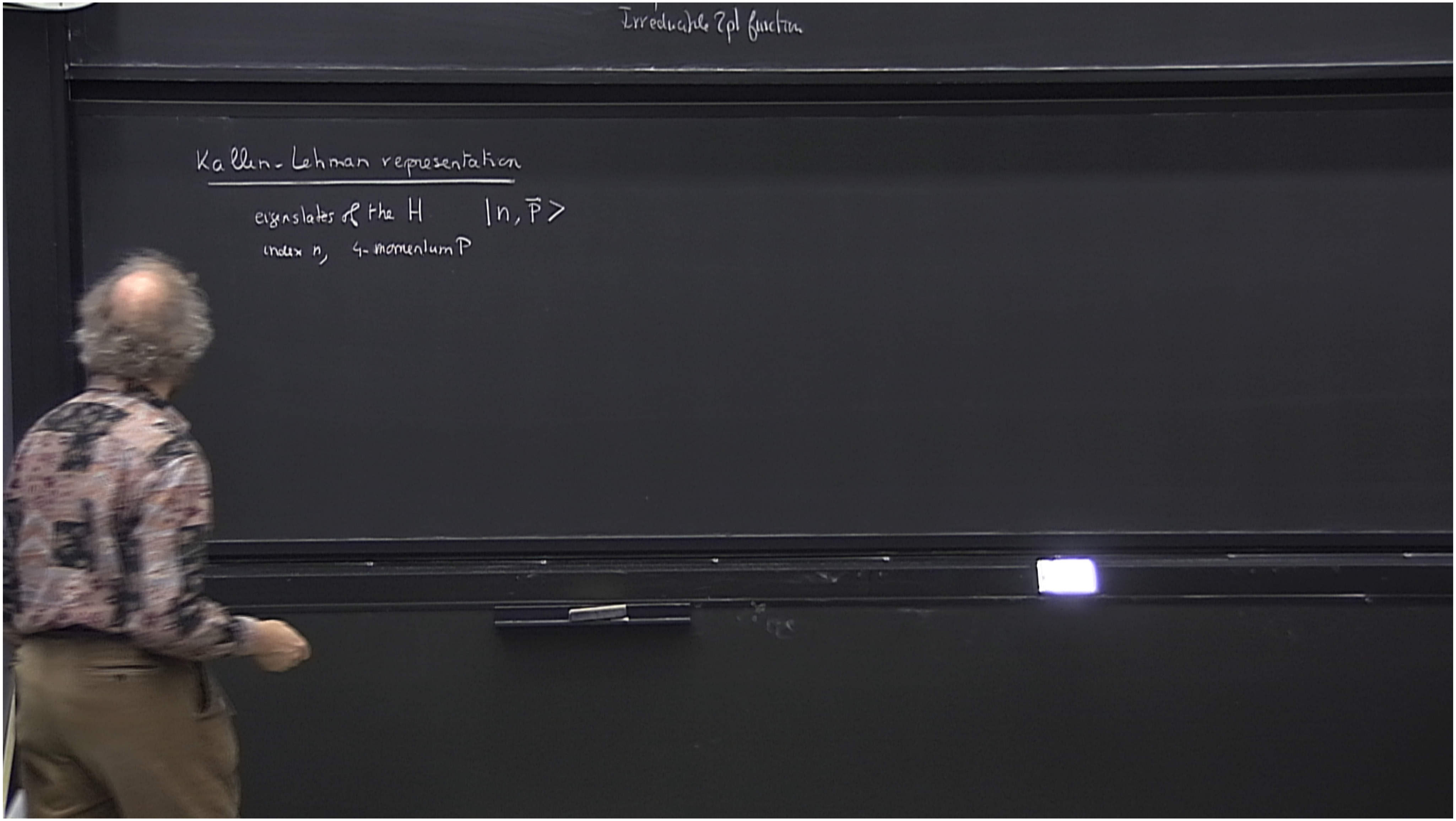

EXIT

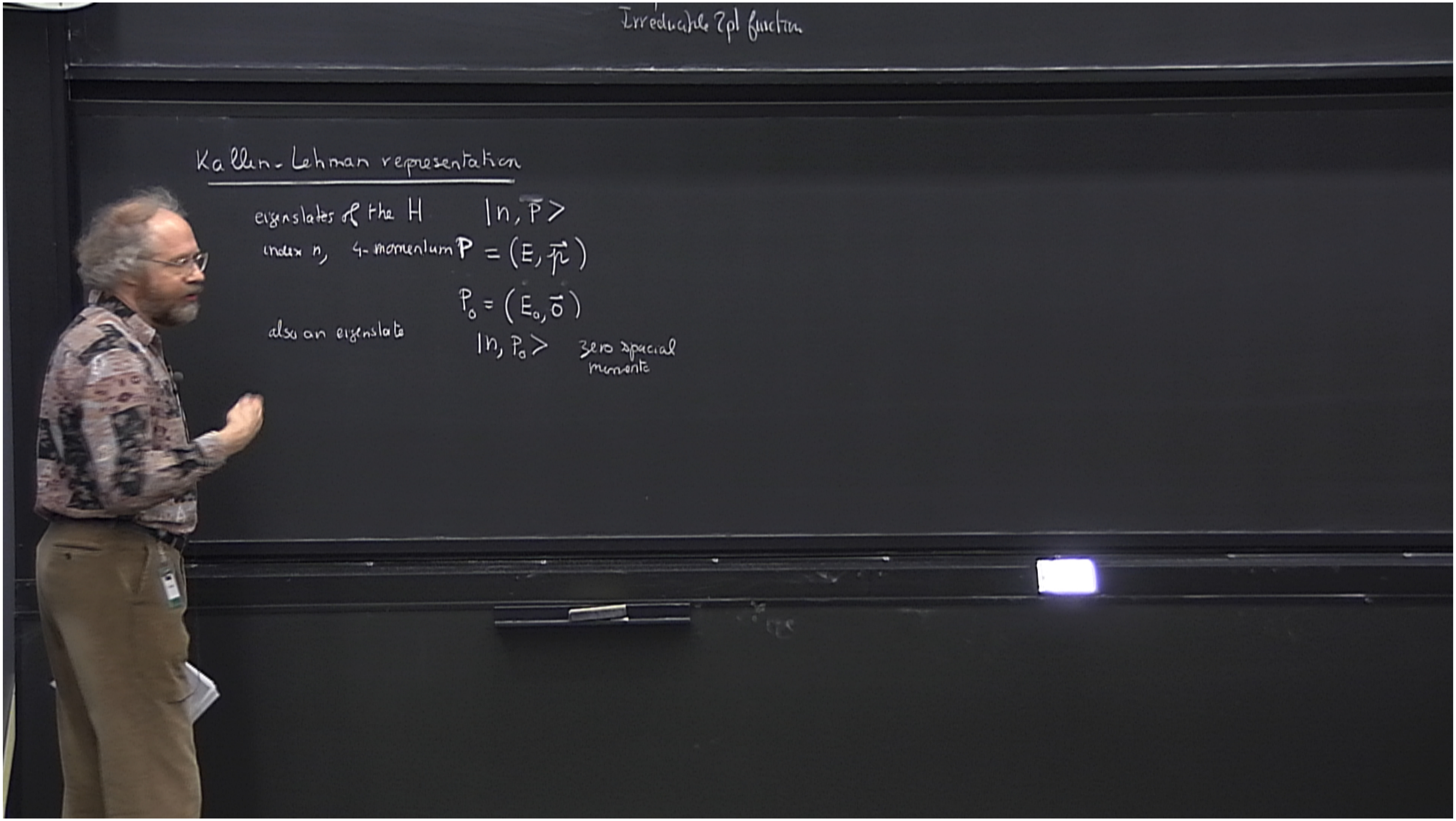

Σ(p) function of p
Γ(x₁, x₂) = (−Δ+m²)_{x₁,x₂} + ħ g/2 δ(x₁−x₂)<φ²(x₁)> + · · ·
Fourier Transform of this
Γ̂(p₁, p₂) = (2π)⁴ δ⁴(p₁+p₂) Γ̂(p₁)
Γ̂(p₁) = (p² + m²) + ħ g/2 <φ²(0)> + · · ·
= Σ(p)
Ĝ(p) = 1/Γ̂(p)
connected
irreducible
Tr Log [−Δ+m²+g/2 φ²] + · · ·
diagrams
function

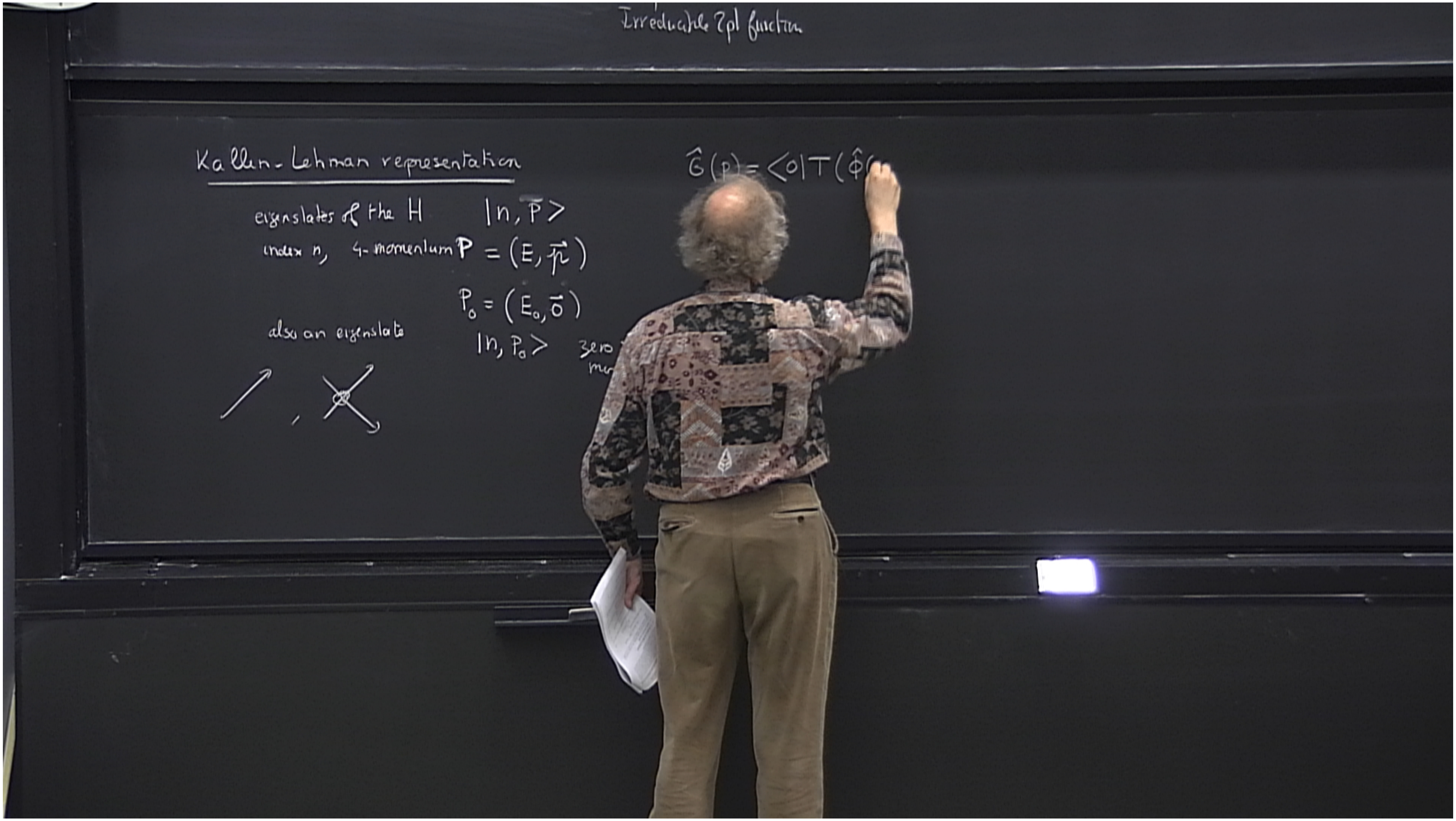

Σ(p) function of p
1/(p²+m²) + · · · = 1/((p²+m²) + Σ(p)) = 1/Γ̂(p)
Geometric series
contribution of the
1 particle irreducible diagrams
propagator itself)
[φ(−Δ+m²)φ + g/4! φ⁴] + ħ/2 Tr Log[−Δ+m² + g/2 φ²] + · · ·
x₂ φ(x₁) φ(x₂) Γ(x₁, x₂)
Irreducible 2pl function
Γ(x₁, x₂) = (−Δ+m²)_{x₁,x₂} + ħ g/2 δ(x₁−x₂)<φ²(x₁)> + · · ·
Fourier Transform of this
Γ̂(p₁, p₂) = (2π)⁴ δ⁴(p₁+p₂) Γ̂(p₁)
Γ̂(p₁) = (p² + m²) + ħ g/2 <φ²(0)> + · · ·
= Σ(p)
Ĝ(p) = 1/Γ̂(p)
connected
irreducible

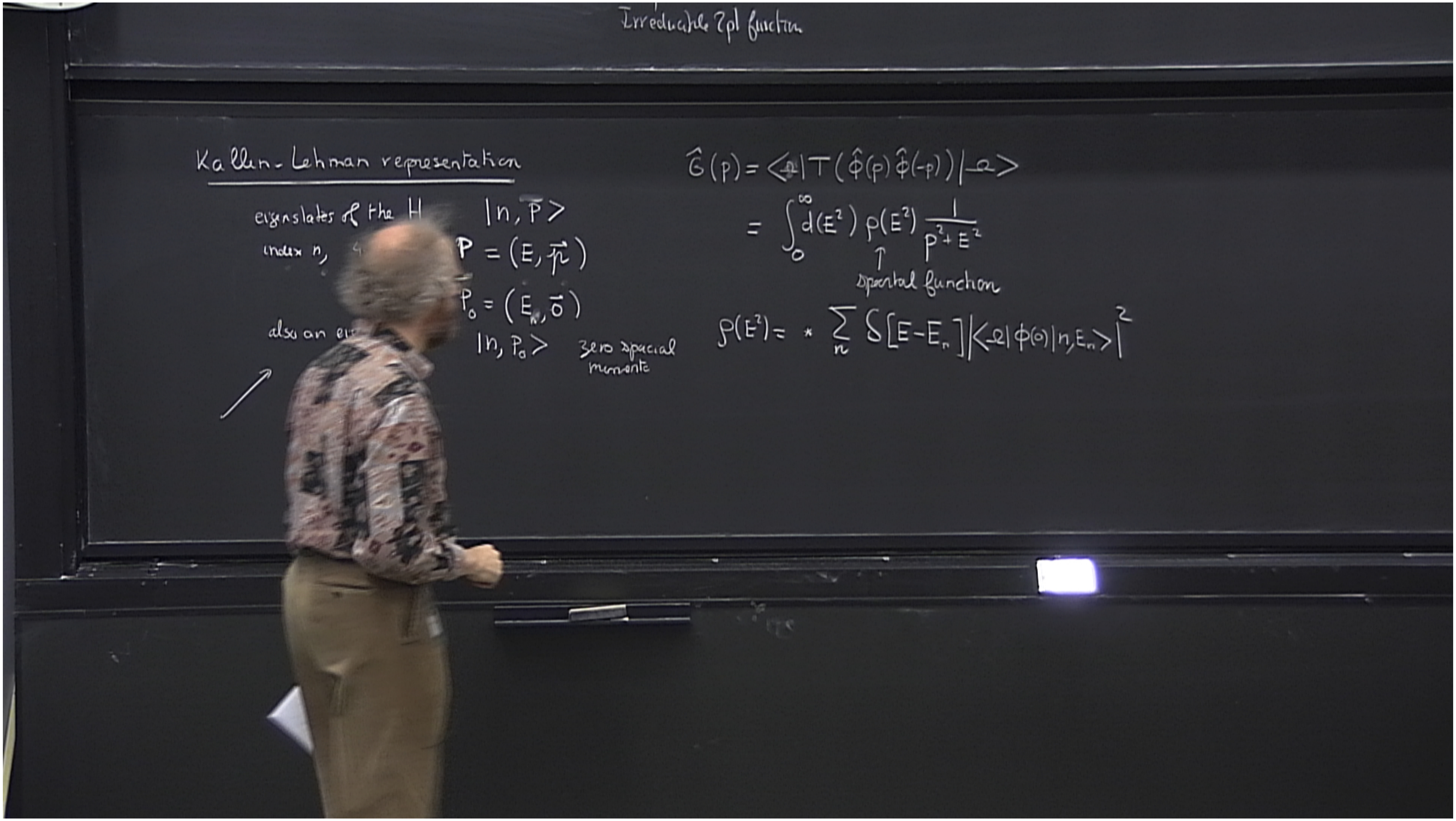

Spectrum ?
irreducible
pole at p² = -m²
Euclidean momenta

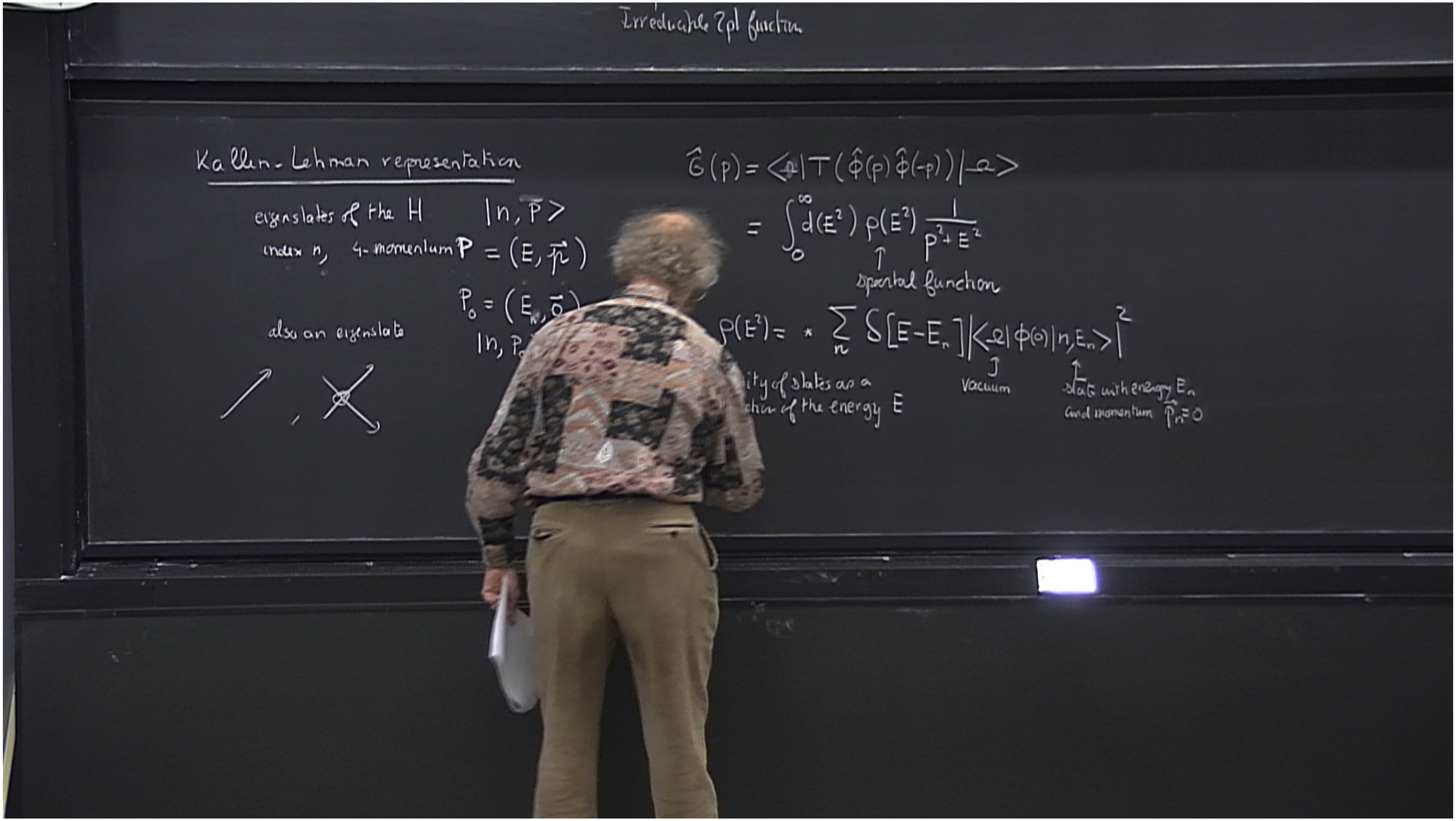

Ĝ(p) = 1 / Γ̂(p)
connected
irreducible
Spectrum ?
G₀(p) = 1 / (p² + m²) ← pole at p² = −m²
Euclidean momenta
1 particle with mass m
= Σ(p)

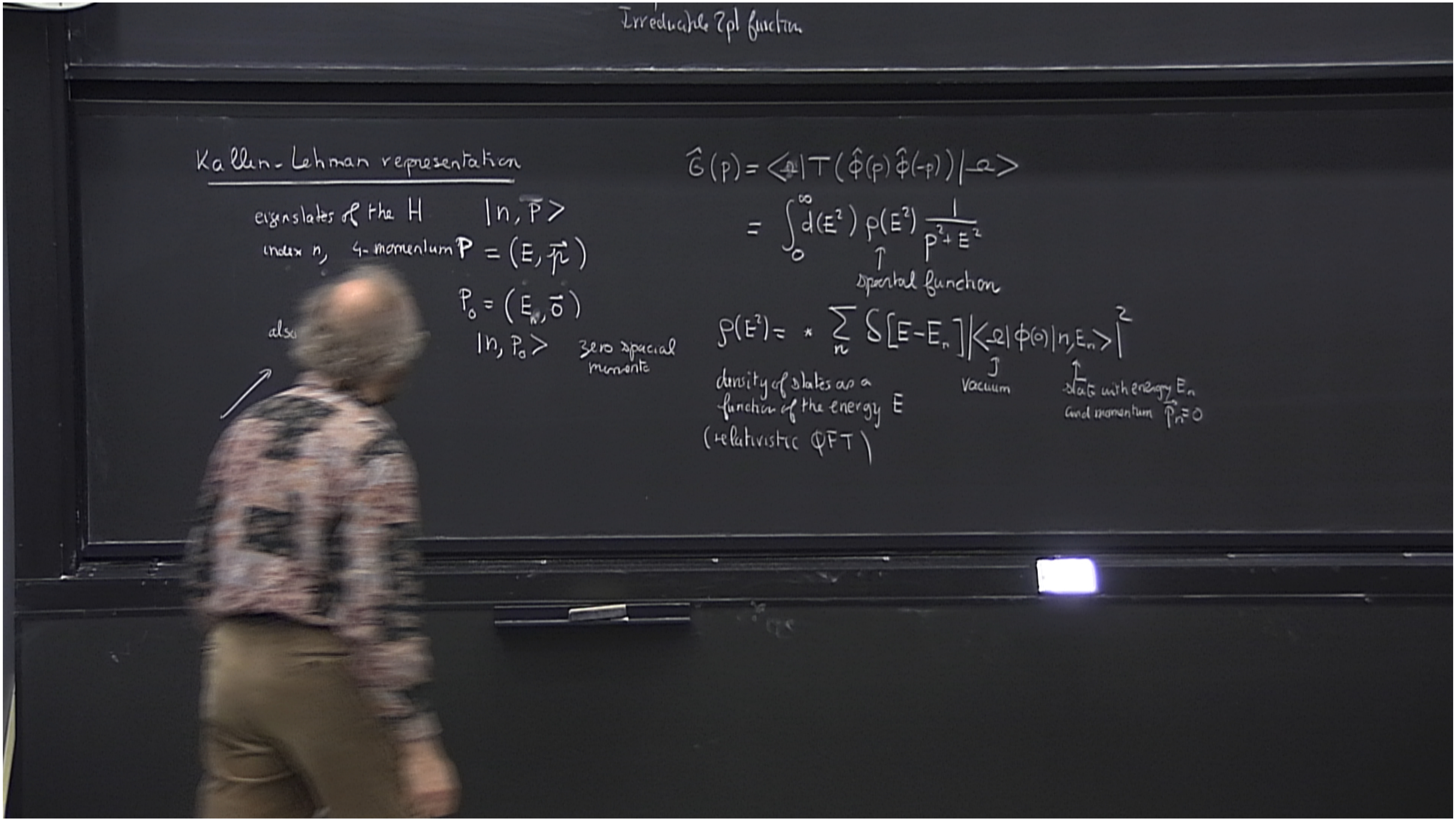

Irreducible 2pt function
Kallen-Lehman representation
eigenstates of the H |n, P⃗>
index n, 4-momentum P

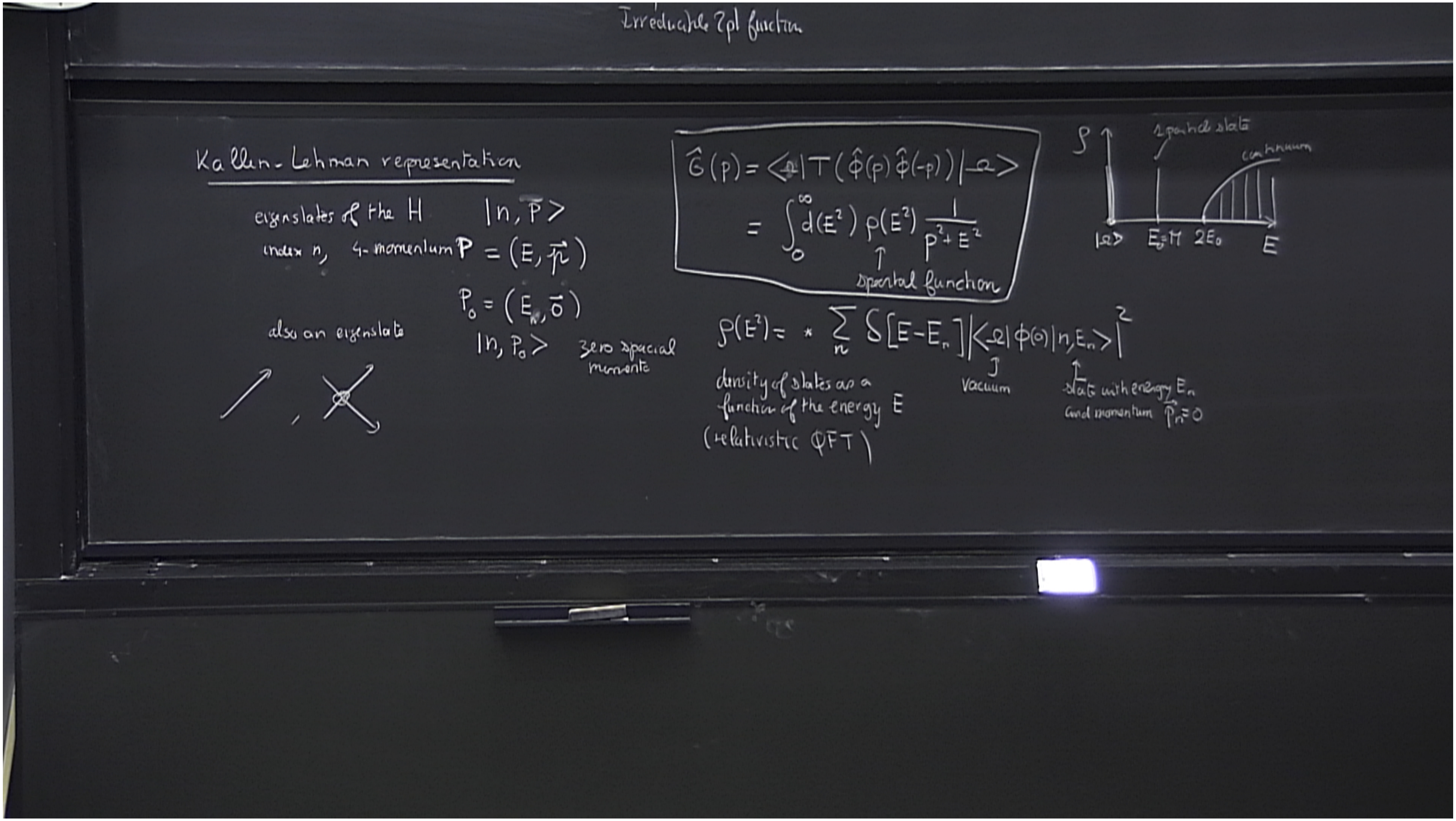

Irreducible 2pt function
Kallen-Lehman representation
eigenstates of the H |n, P>
index n, 4-momentum P = (E, p)
P0 = (E0, 0)
also an eigenstate |n, P0>
zero spacial momenta


Irreducible 2pt function
Kallen-Lehman representation
eigenstates of the H
|n, P>
index n, 4-momentum P = (E, p)
P0 = (E0, 0)
also an eigenstate
|n, P0>
zero
Ĝ(p) = <0|T(Φ̂(

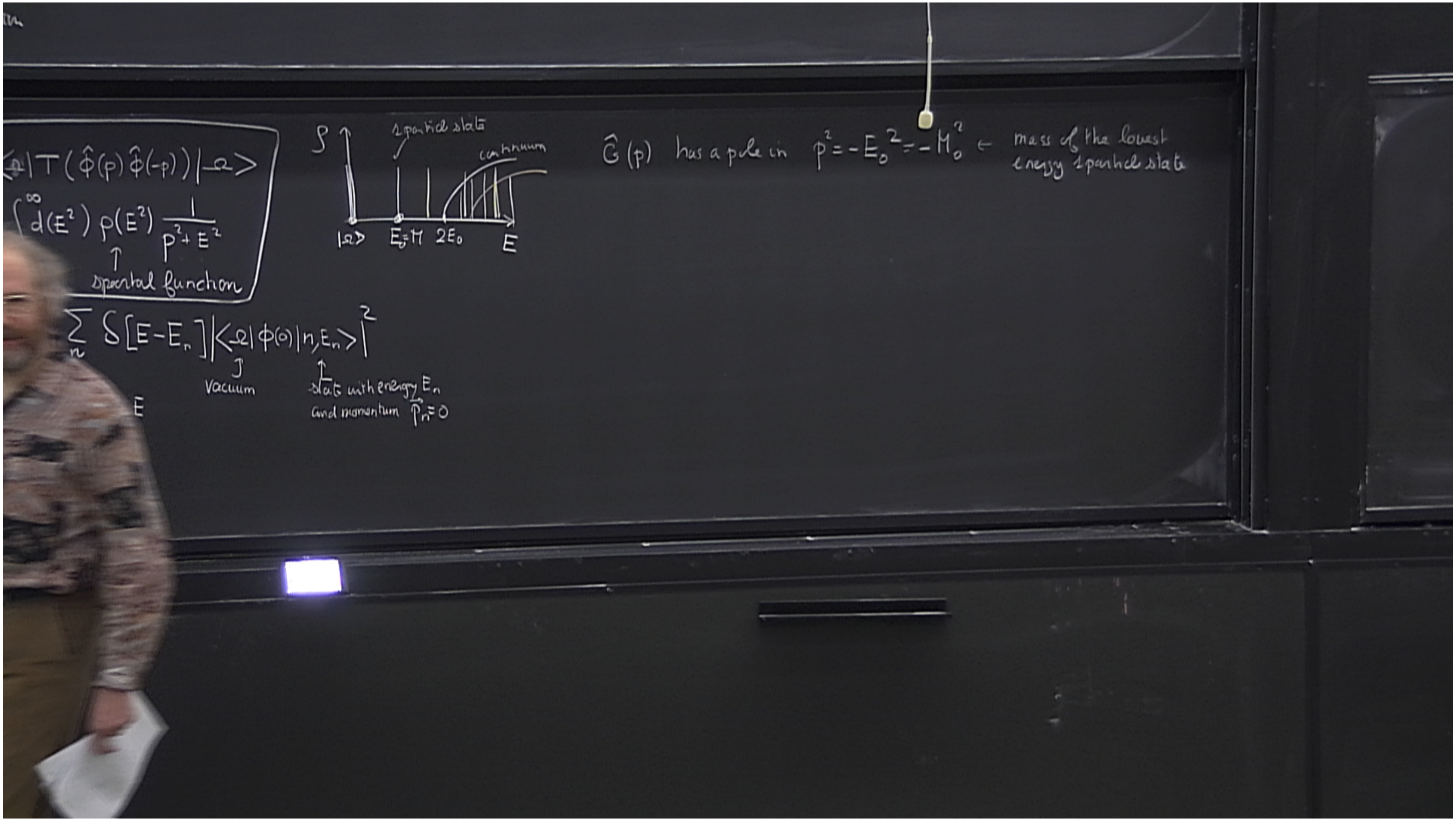

Irreducible 2pt function
Kallen-Lehman representation
eigenstates of the H |n, P>
index n,
P = (E, p)
P_0 = (E_n, 0)
also an
|n, P_0>
zero spacial momenta
Ĝ(p) = <Ω|T(Φ̂(p) Φ̂(-p))|Ω>
= ∫_0^∞ d(E²) ρ(E²) 1/(p² + E²)
spectral function
ρ(E²) = * Σ_n δ[E - E_n] |<Ω|φ(0)|n, E_n>|²

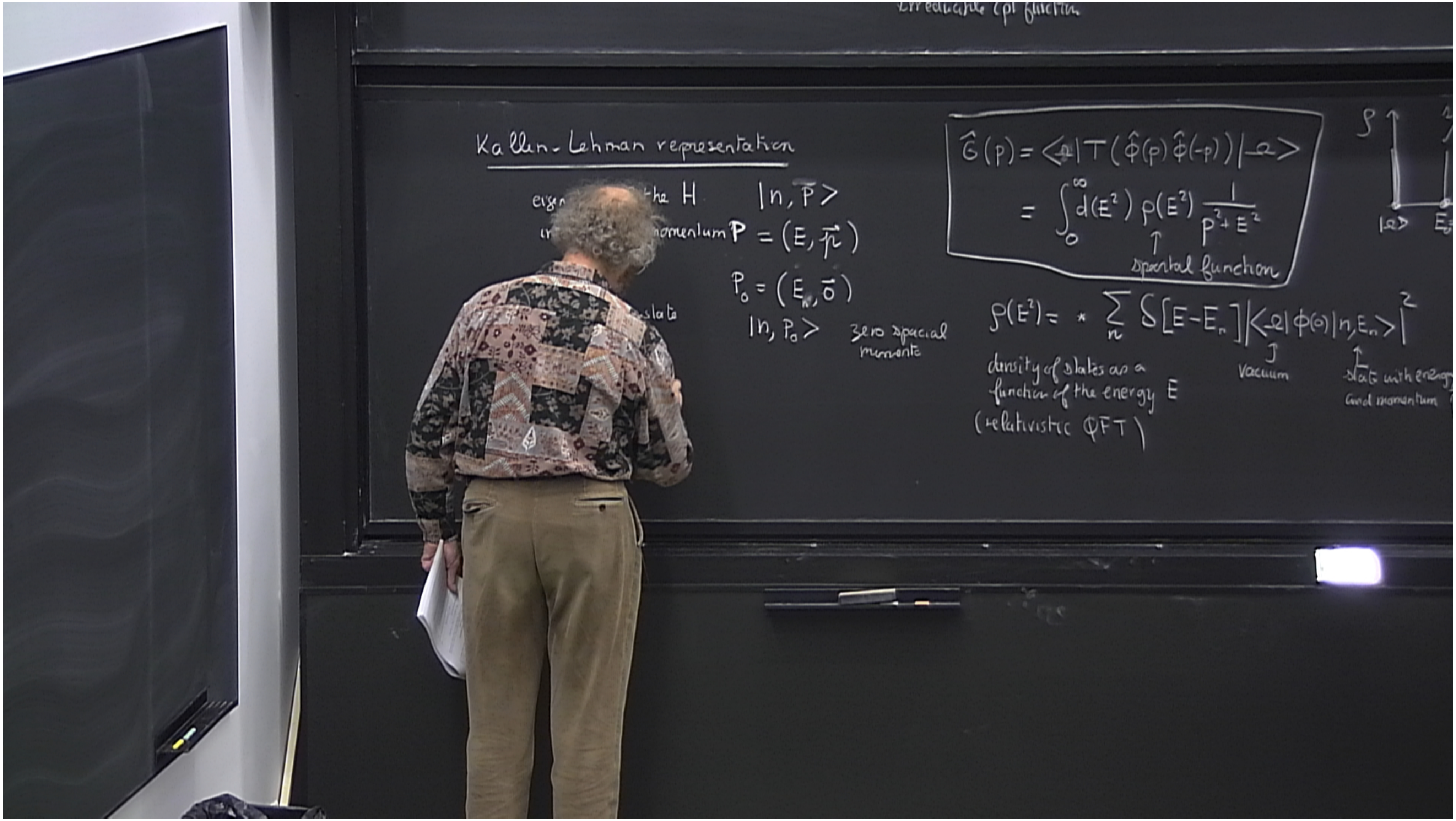

Irreducible 2pt function
Kallen-Lehman representation
eigenstates of the H |n, P>
index n, 4-momentum P = (E, p)
also an eigenstate
spectral function
Vacuum
state with energy E_n and momentum p_n = 0
ity of states as a
ction of the energy E

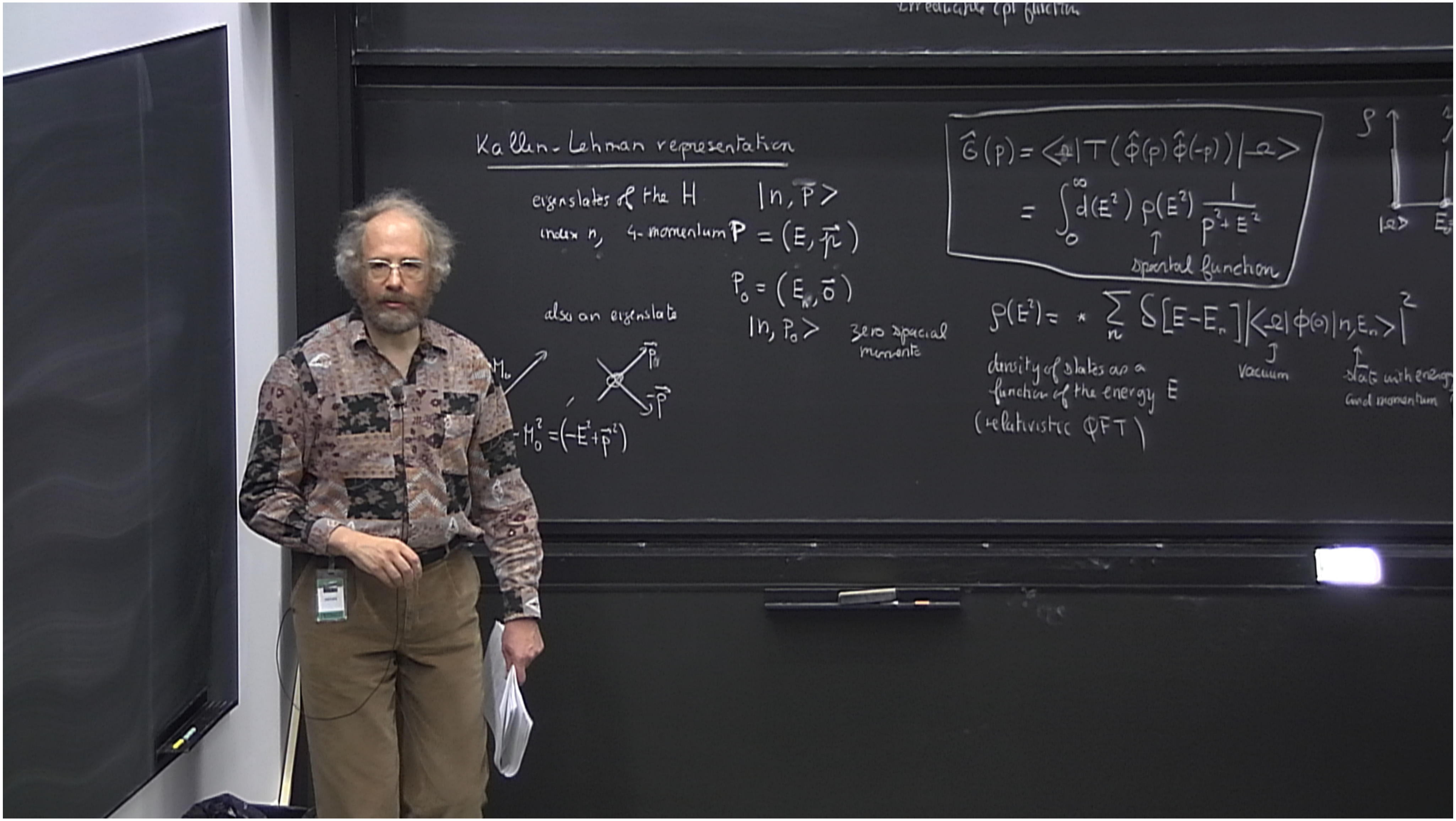

Irreducible 2pt function
Kallen-Lehman representation
eigenstates of the H |n, P>
index n, 4-momentum P = (E, p)
P_0 = (E_n, 0)
|n, P_0> zero spacial momenta
G(p) = <Ω| T (φ(p) φ(-p)) |Ω>
= ∫_0^∞ d(E²) ρ(E²) 1/(p² + E²)
spectral function
ρ(E²) = ∑_n δ[E - E_n] |<Ω| φ(0) |n, E_n>|²
density of states as a function of the energy E
(relativistic QFT)
vacuum
state with energy E_n and momentum p_n = 0

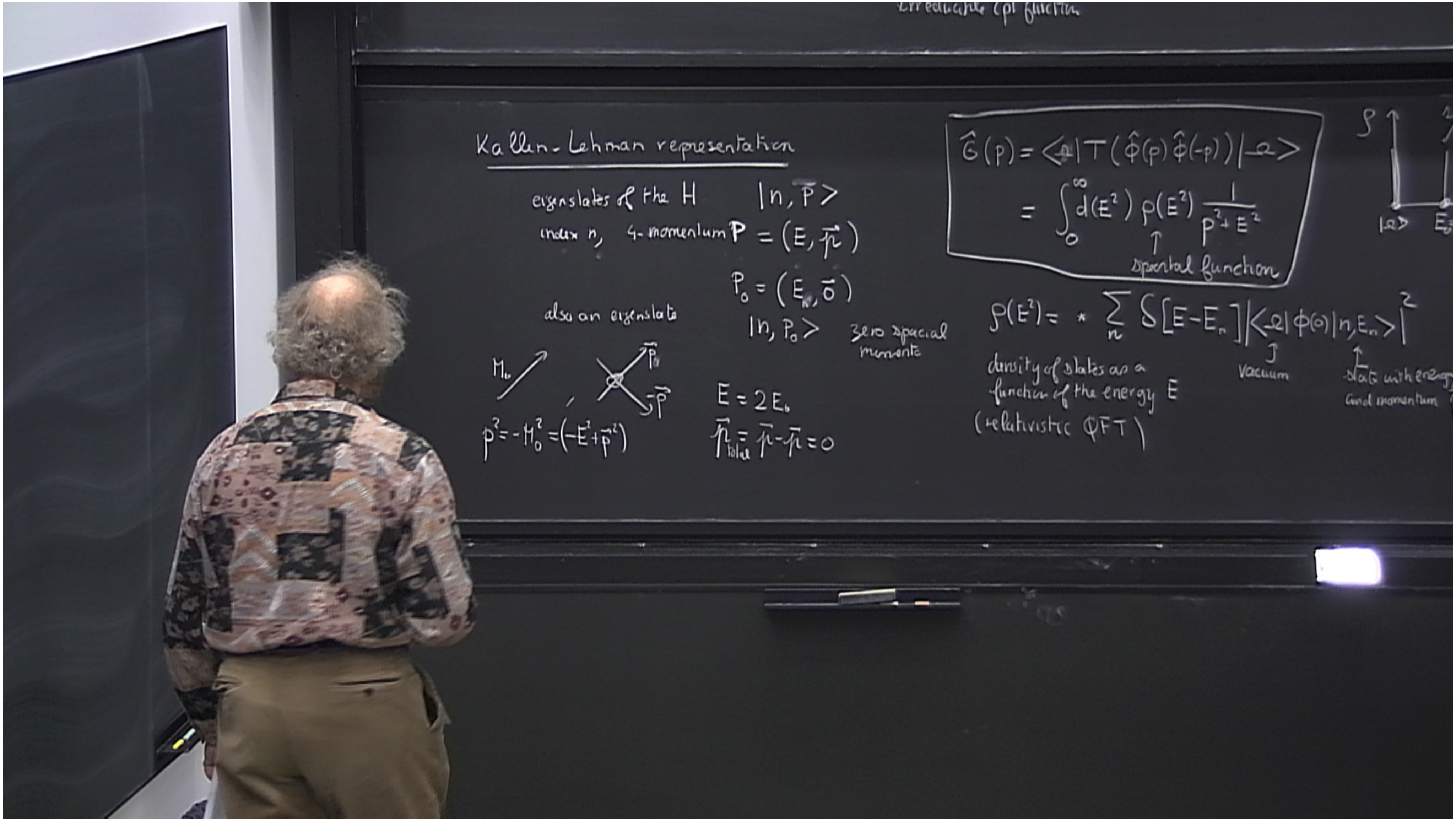


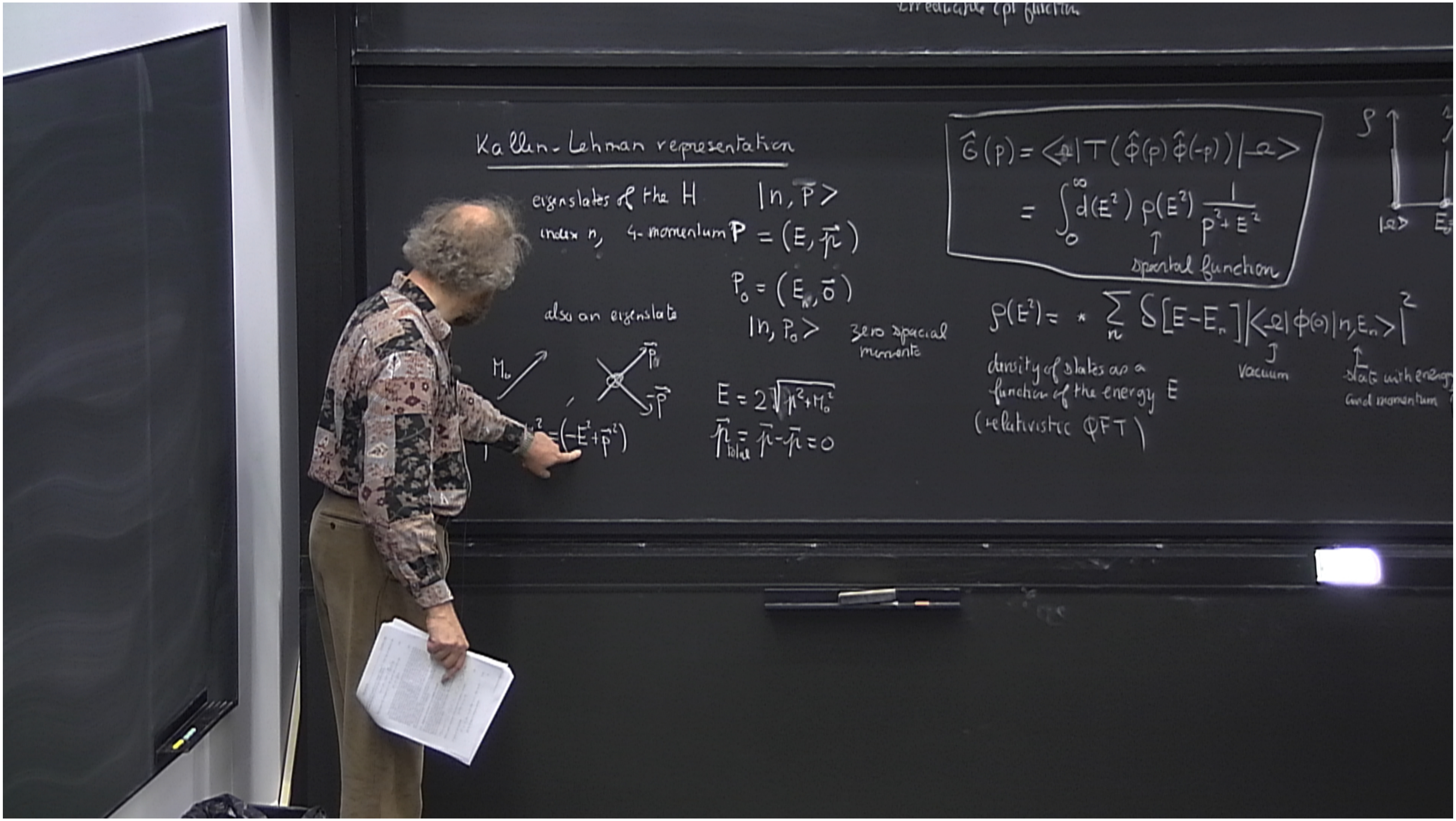
Irreducible 2pt function
Kallen-Lehman representation
eigenstates of the H |n, P>
index n, 4-momentum P = (E, p)
P0 = (En, 0)
also an eigenstate
|n, P0>
zero spacial momenta
Ĝ(p) = <Ω| T(φ̂(p) φ̂(-p)) |Ω>
= ∫0∞ d(E²) ρ(E²) 1/(p² + E²)
spectral function
ρ(E²) = Σn δ[E - En] |<Ω| φ(0) |n, En>|²
density of states as a function of the energy E
(relativistic QFT)
Vacuum
state with energy En and momentum pn = 0
ρ
1 particle state
continuum
|Ω>
E0 = M
2E0
E

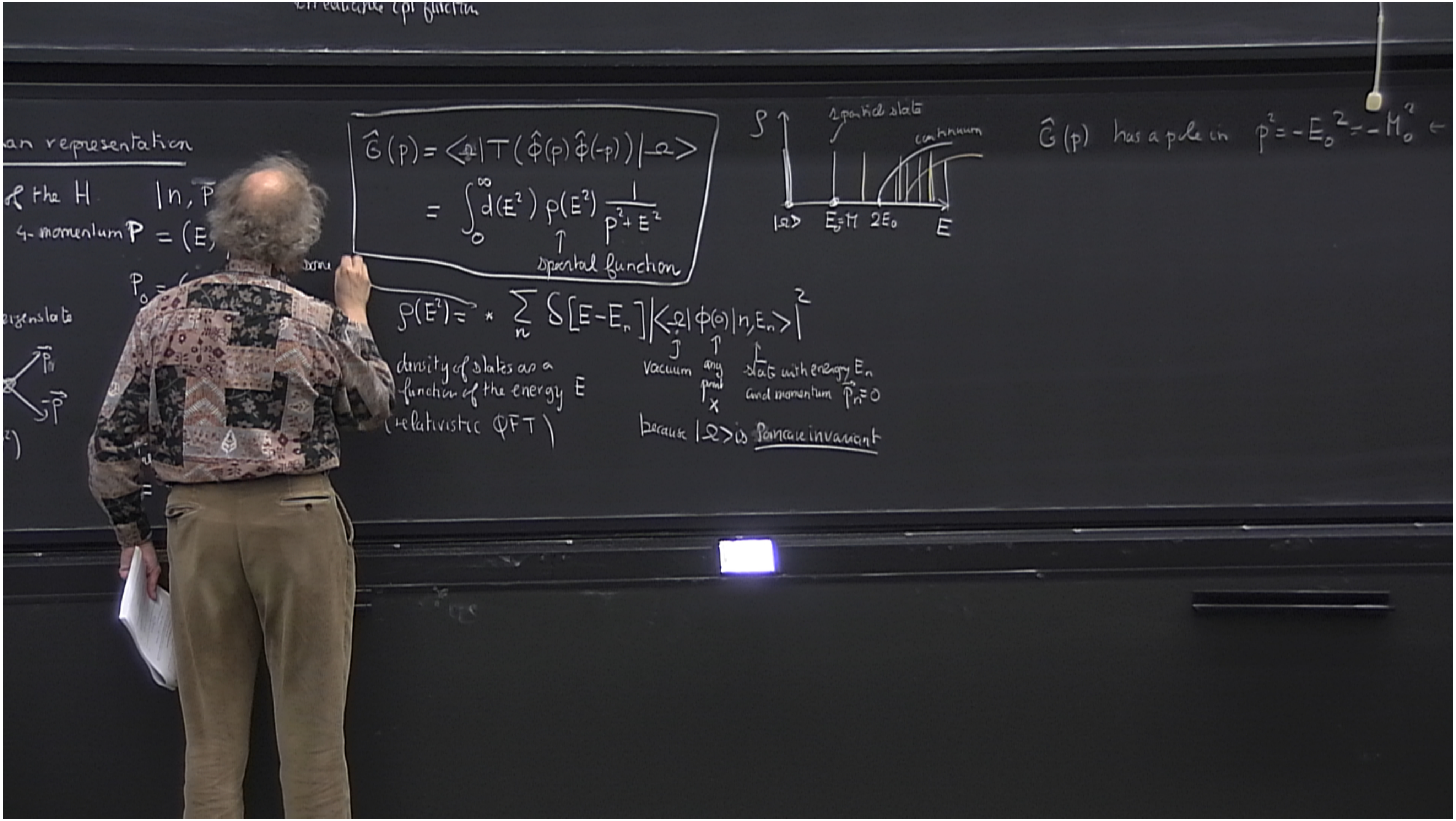

⟨Ω|T(Φ̂(p)Φ̂(-p))|Ω⟩
∫ d(E²) ρ(E²) 1/(p²+E²)
spectral function
∑ₙ δ[E−Eₙ] |⟨Ω|φ(0)|n,Eₙ⟩|²
Vacuum
state with energy Eₙ
and momentum p⃗ₙ=0
ρ
1 particle state
continuum
|Ω⟩
E₀=M
2E₀
E
Ĝ(p) has a pole in p² = −E₀² = −M₀²
mass of the lowest
energy 1 particle state

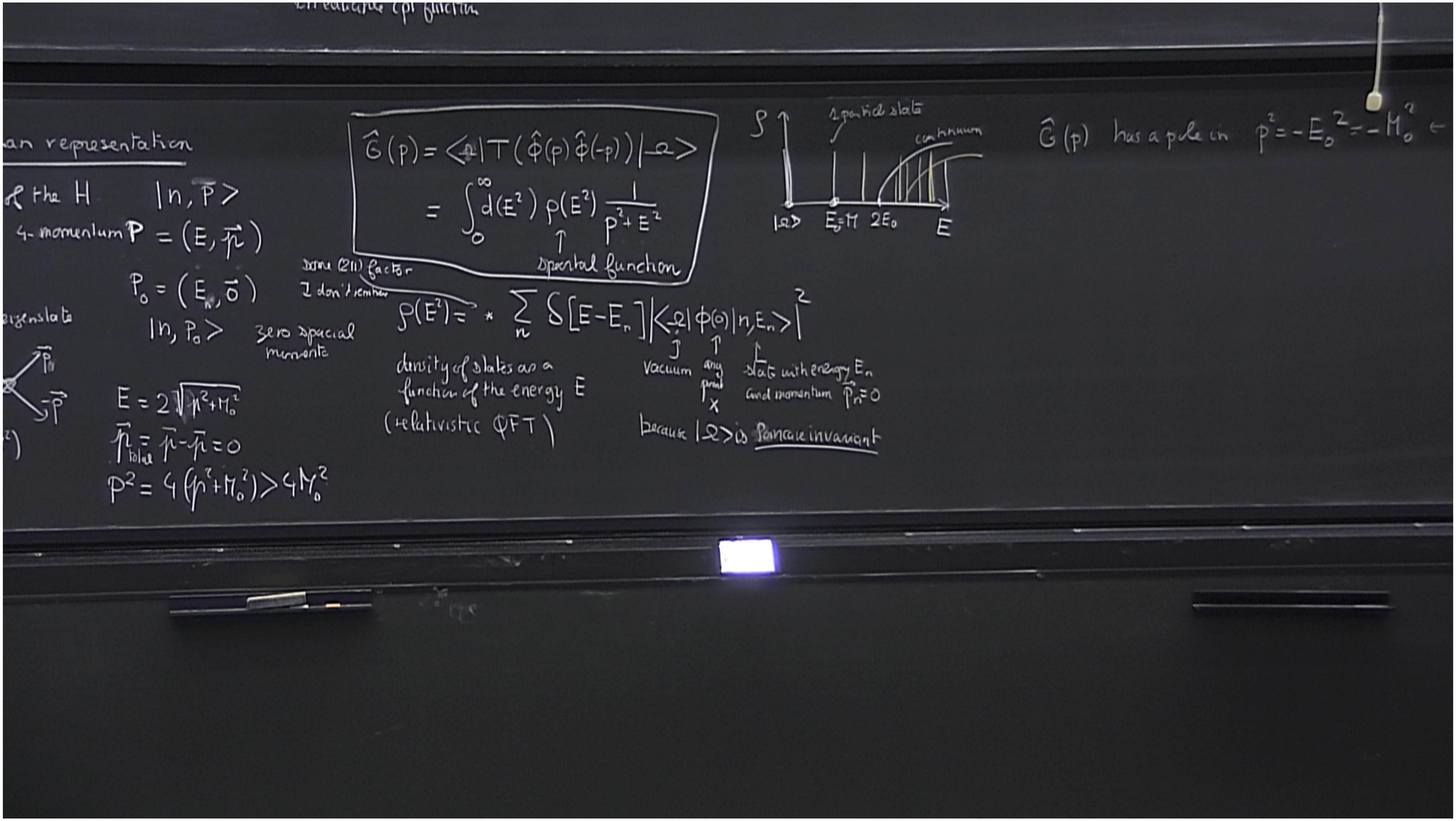

Kallen-Lehman representation
|n, P>
P = (E, p)
P_0 = (E_n, 0)
|n, P_0>
zero spacial momenta
Ĝ(p) = <Ω| T(φ̂(p) φ̂(-p)) |Ω>
= ∫_0^∞ d(E²) ρ(E²) 1/(p² + E²)
spectral function
ρ(E²) = Σ_n δ[E - E_n] |<Ω| φ(0) |n, E_n>|²
density of states as a function of the energy E
(relativistic QFT)
vacuum

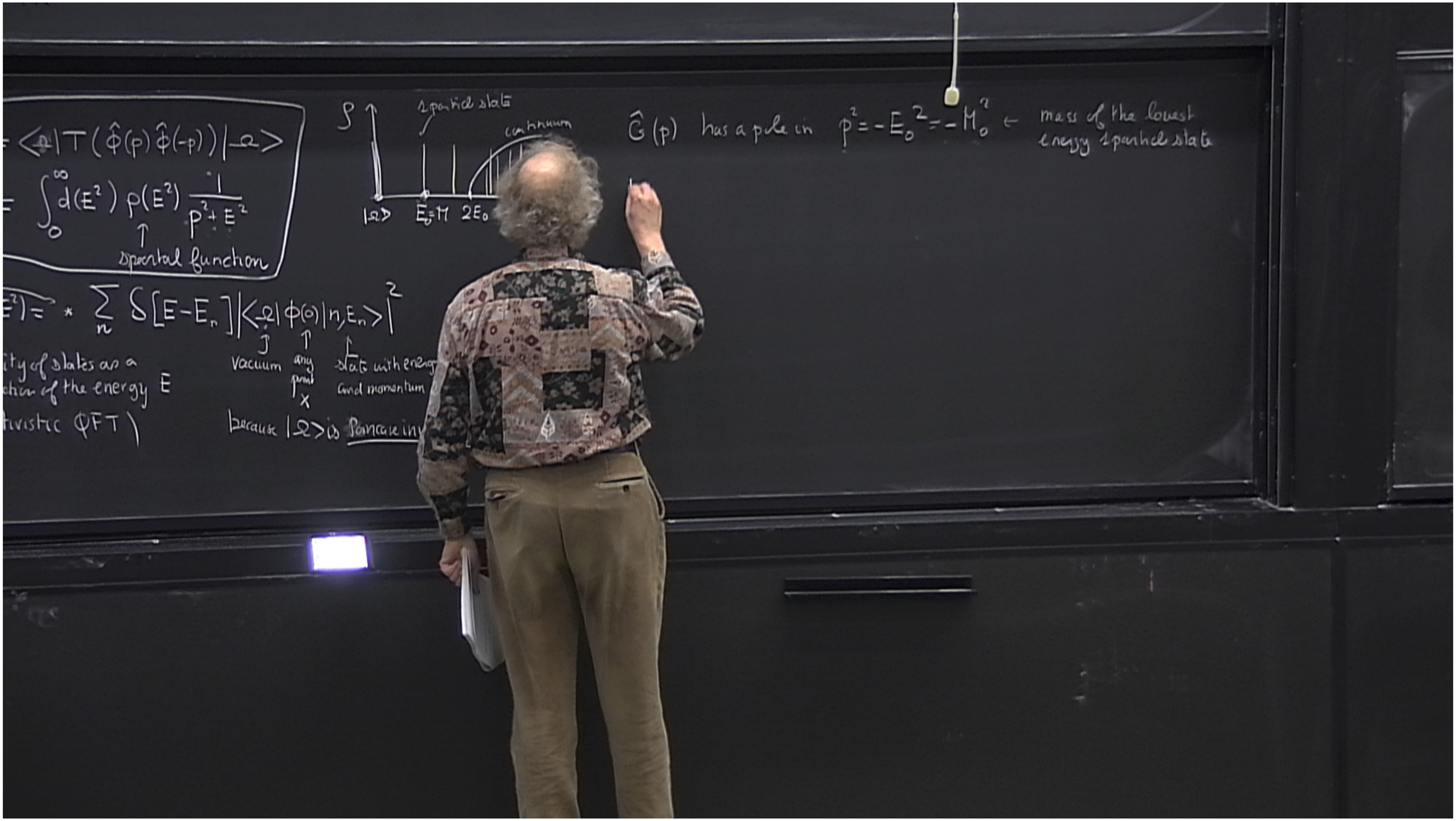

Kallen-Lehman representation
eigenstates of the H |n, p>
index n, 4-momentum P = (E, p)
P0 = (En, 0)
also an eigenstate
|n, P0>
zero spacial momenta
Ĝ(p) = <Ω|T(φ̂(p) φ̂(-p))|Ω>
spectral function
density of states as a function of the energy E
(relativistic QFT)
vacuum
state with energy and momentum

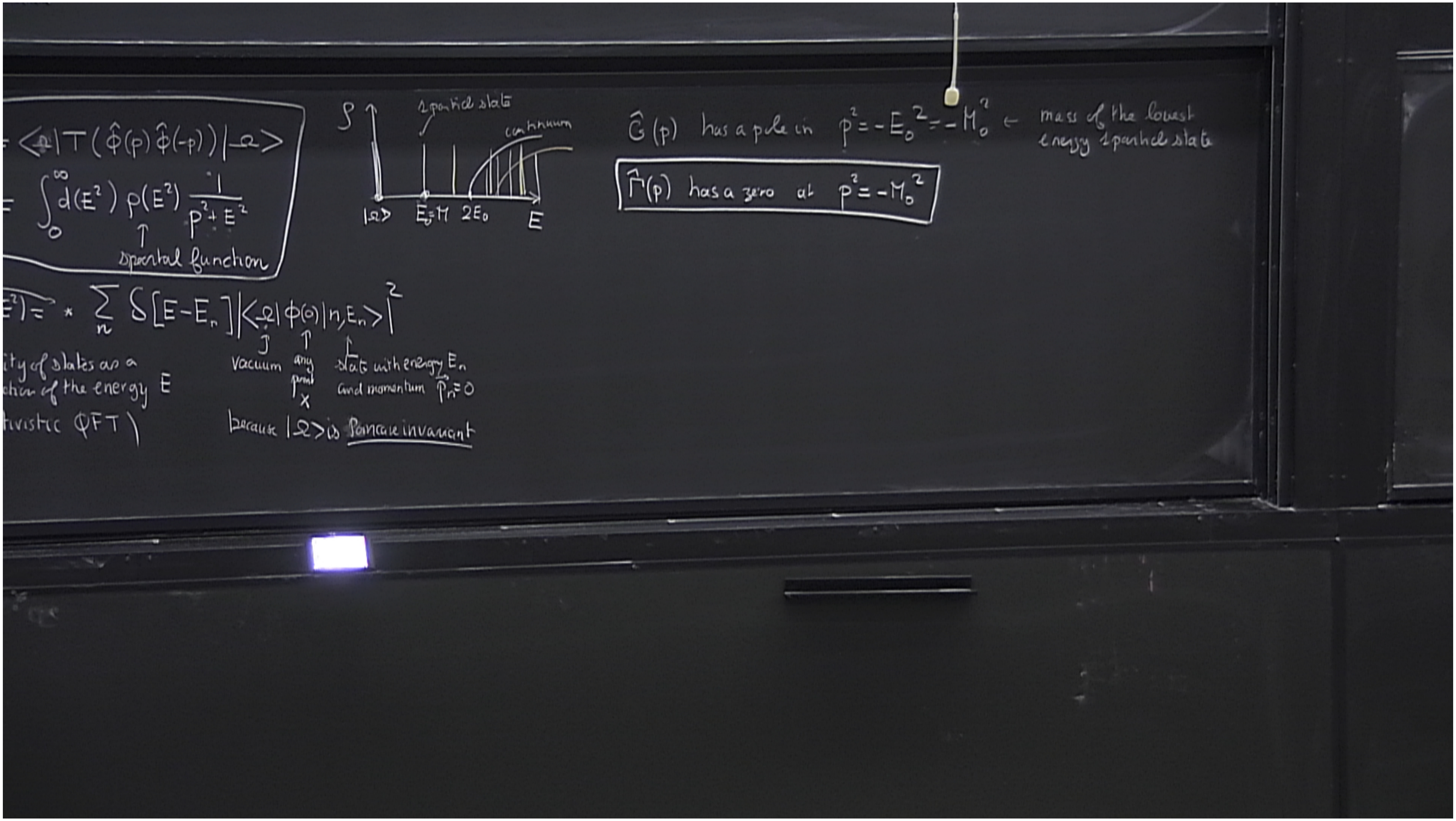

Kallen-Lehman representation
eigenstates of the H |n, P>
index n, 4-momentum P = (E, p)
P0 = (E, 0)
also an eigenstate |n, P0>
zero spacial momenta
E = 2E0
spectral function
density of states as a function of the energy E
(relativistic QFT)
vacuum
state with energy and momentum

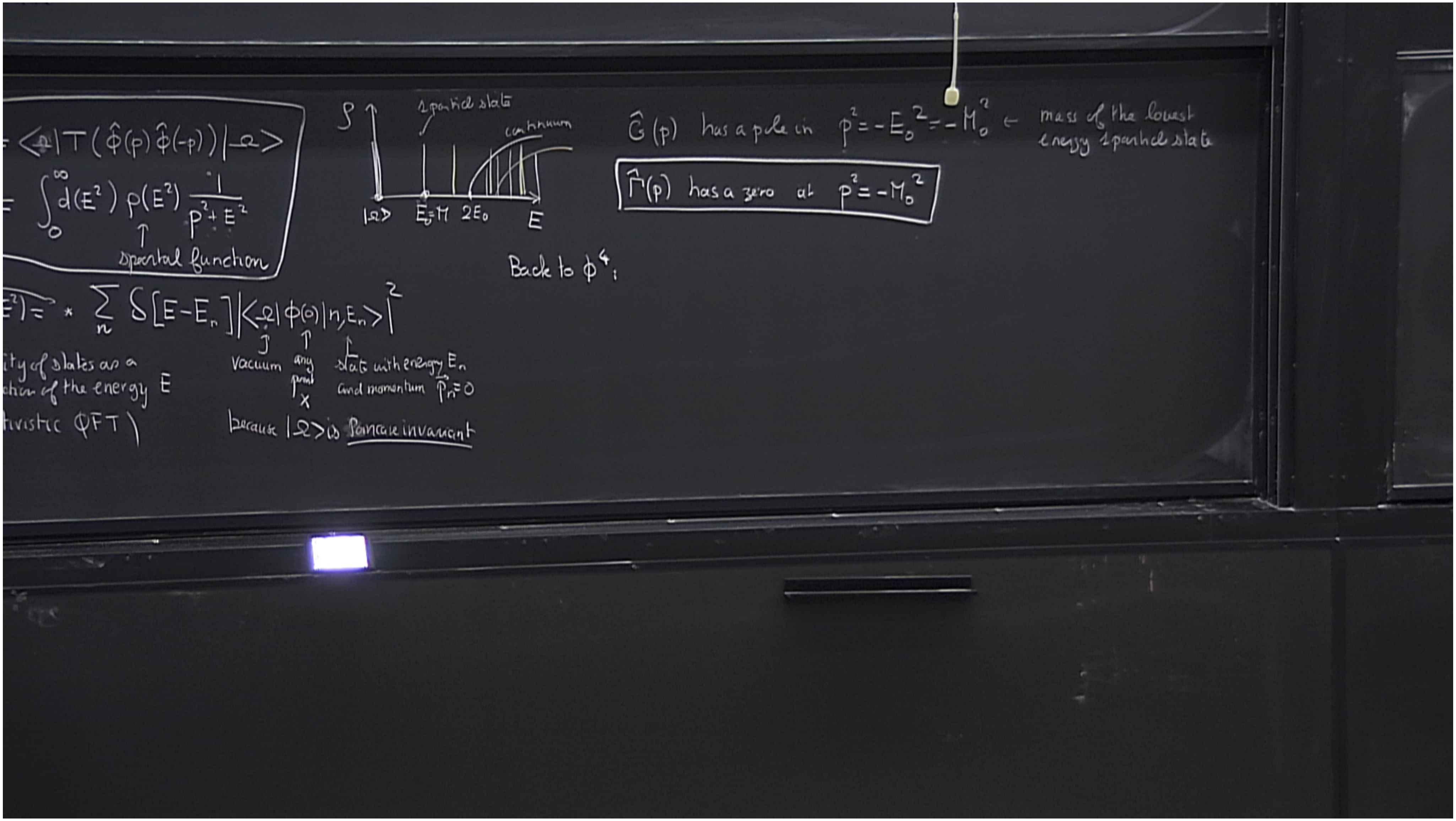

Kallen-Lehman representation
eigenslates of the H |n, P>
index n, 4-momentum P = (E, p)
P_0 = (E_n, 0)
also an eigenslate |n, P_0>
zero spacial momenta
E = 2√(p²+M_0²)
p_total = p - p = 0
Ĝ(p) = <Ω|T(φ̂(p) φ̂(-p))|Ω>
spectral function
ρ(E²) = Σ_n δ[E-E_n] |<Ω|φ(0)|n, E_n>|²
density of states as a function of the energy E
(relativistic φFT)
vacuum

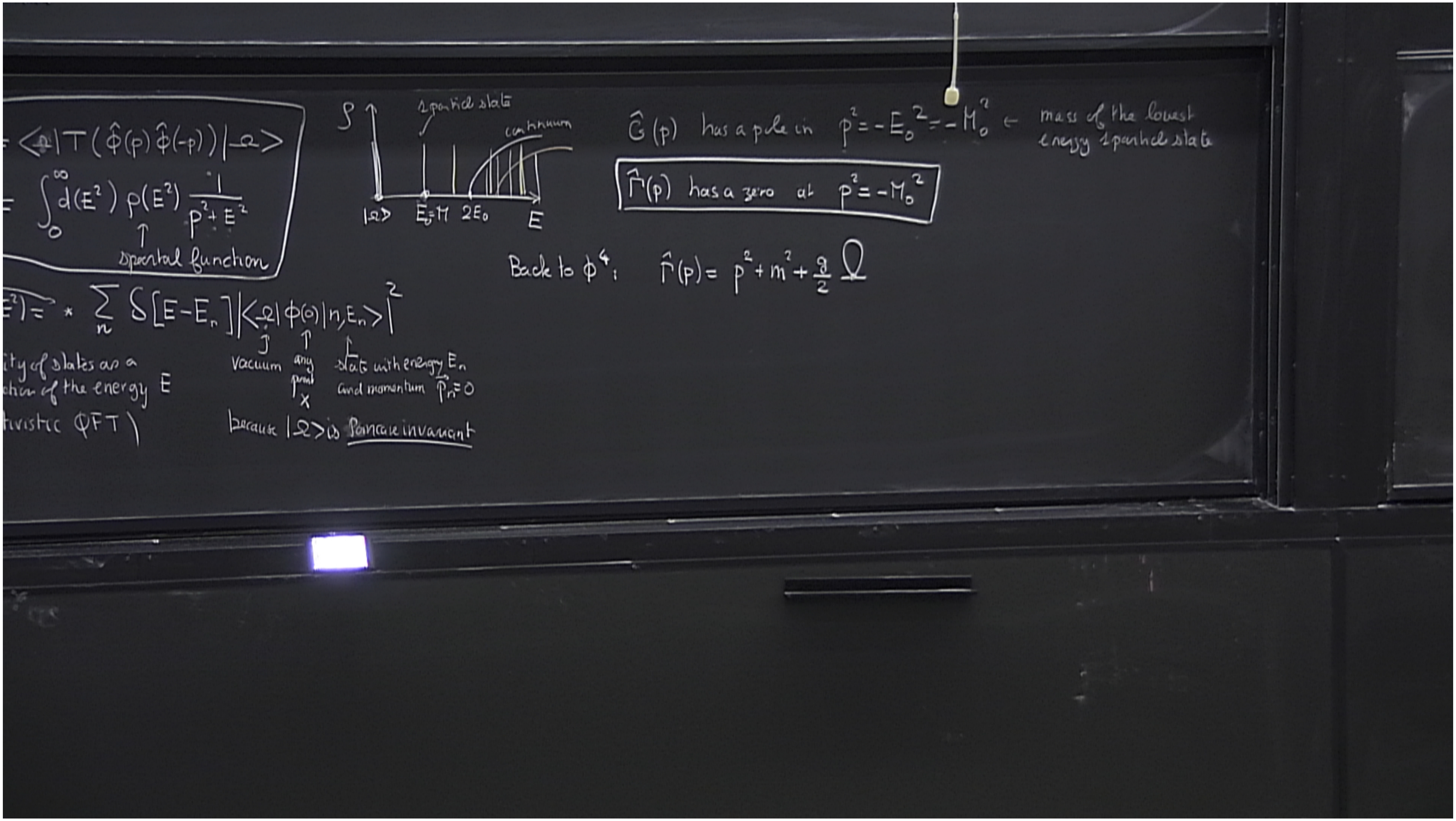

an representation
the H
4-momentum P = (E,
eigenstate
Ĝ(p) = <Ω|T(φ̂(p) φ̂(-p))|Ω>
= ∫₀^∞ d(E²) ρ(E²) 1/(p²+E²)
spectral function
ρ(E²) = Σₙ δ[E−Eₙ] |<Ω|φ(0)|n,Eₙ>|²
density of states as a function of the energy E
(relativistic QFT)
vacuum
any point x
state with energy Eₙ and momentum pₙ = 0
because |Ω> is Poincaré invariant
ρ
2 particle state
continuum
|Ω>
E₀=M
2E₀
E
Ĝ(p) has a pole in p² = −E₀² = −M₀²

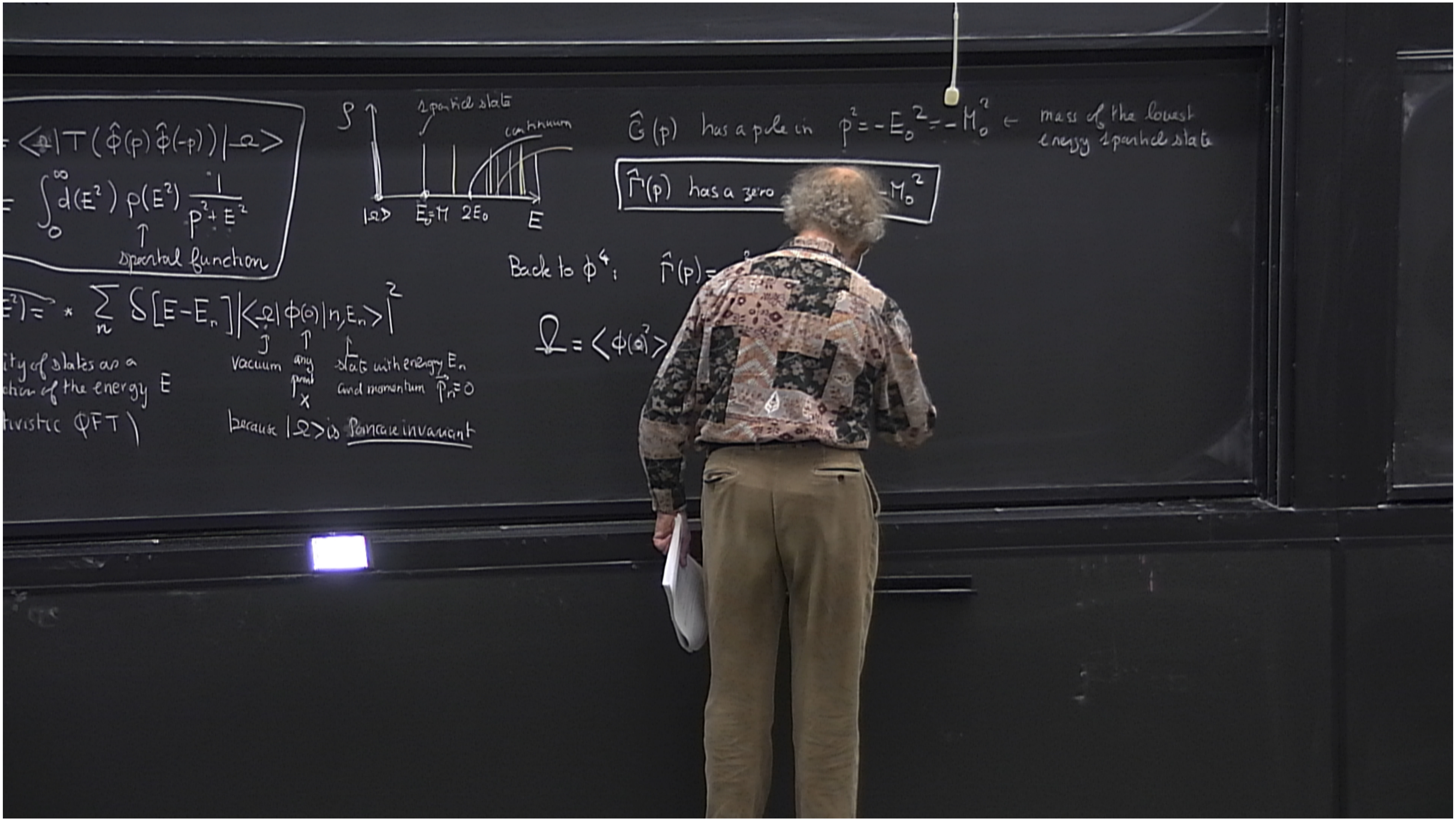

an representation
the H |n, P>
4-momentum P = (E, p)
P_0 = (E, 0)
eigenstate
|n, P_0>
zero spacial momenta
E = 2√(p²+M_0²)
p_total = p - p = 0
P² = 4(p²+M_0²) > 4M_0²
Ĝ(p) = <Ω|T(φ̂(p) φ̂(-p))|Ω>
= ∫_0^∞ d(E²) ρ(E²) 1/(p²+E²)
spectral function
some (2π) factor I don't remember
ρ(E²) = * Σ_n δ[E-E_n] |<Ω|φ(0)|n,E_n>|²
density of states as a function of the energy E
(relativistic QFT)
vacuum
any point x
state with energy E_n and momentum p_n = 0
because |Ω> is Poincaré invariant
ρ
2 particle state
continuum
|Ω>
E_0 = M
2E_0
E
Ĝ(p) has a pole in p² = -E_0² = -M_0²

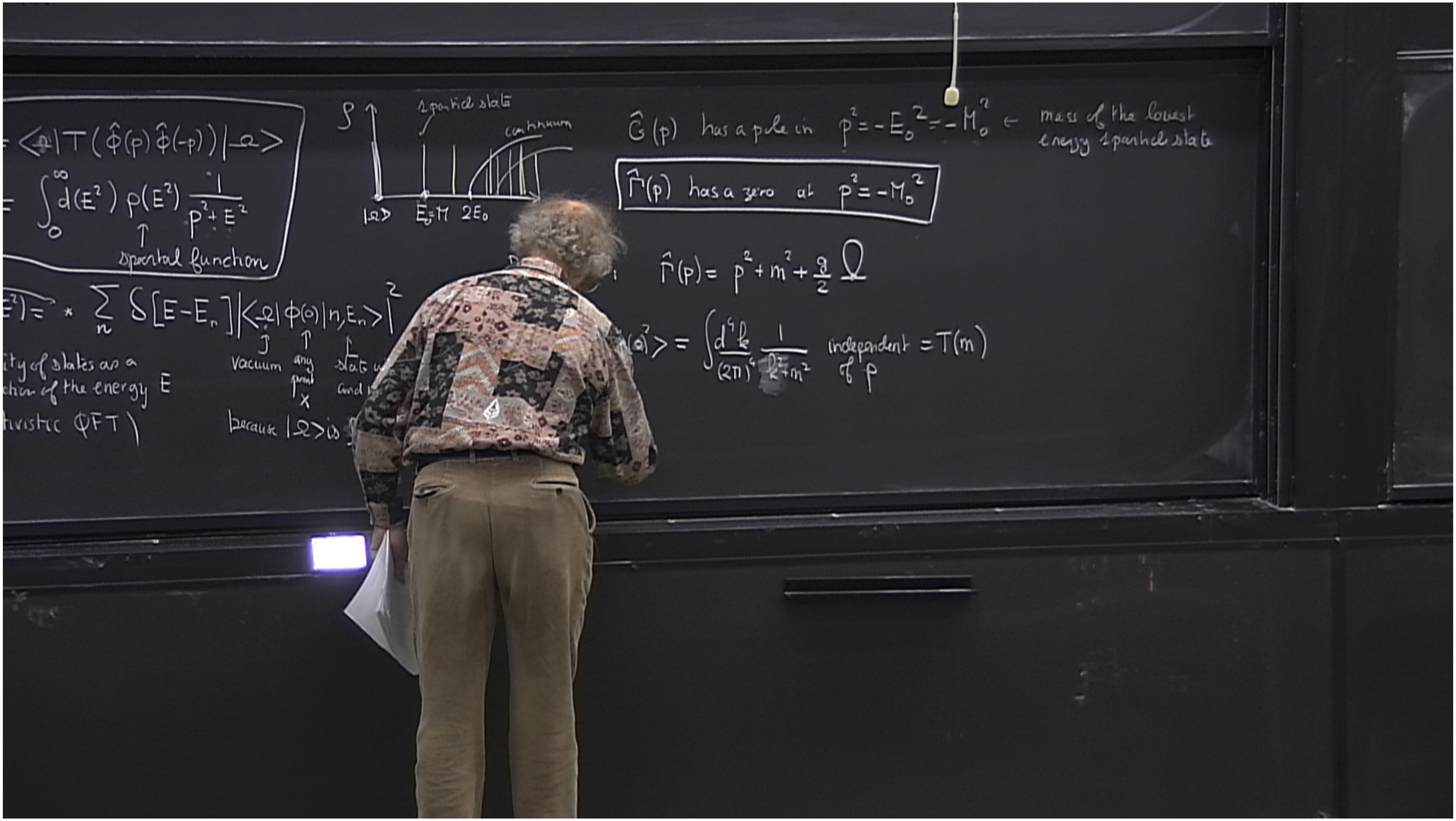

1 particle state
continuum
Ĝ(p) has a pole in p² = −E₀² = −M₀² ← mass of the lowest energy 1 particle state
spectral function
vacuum
any point x
state with energy and momentum
because |Ω> is Poincare inv

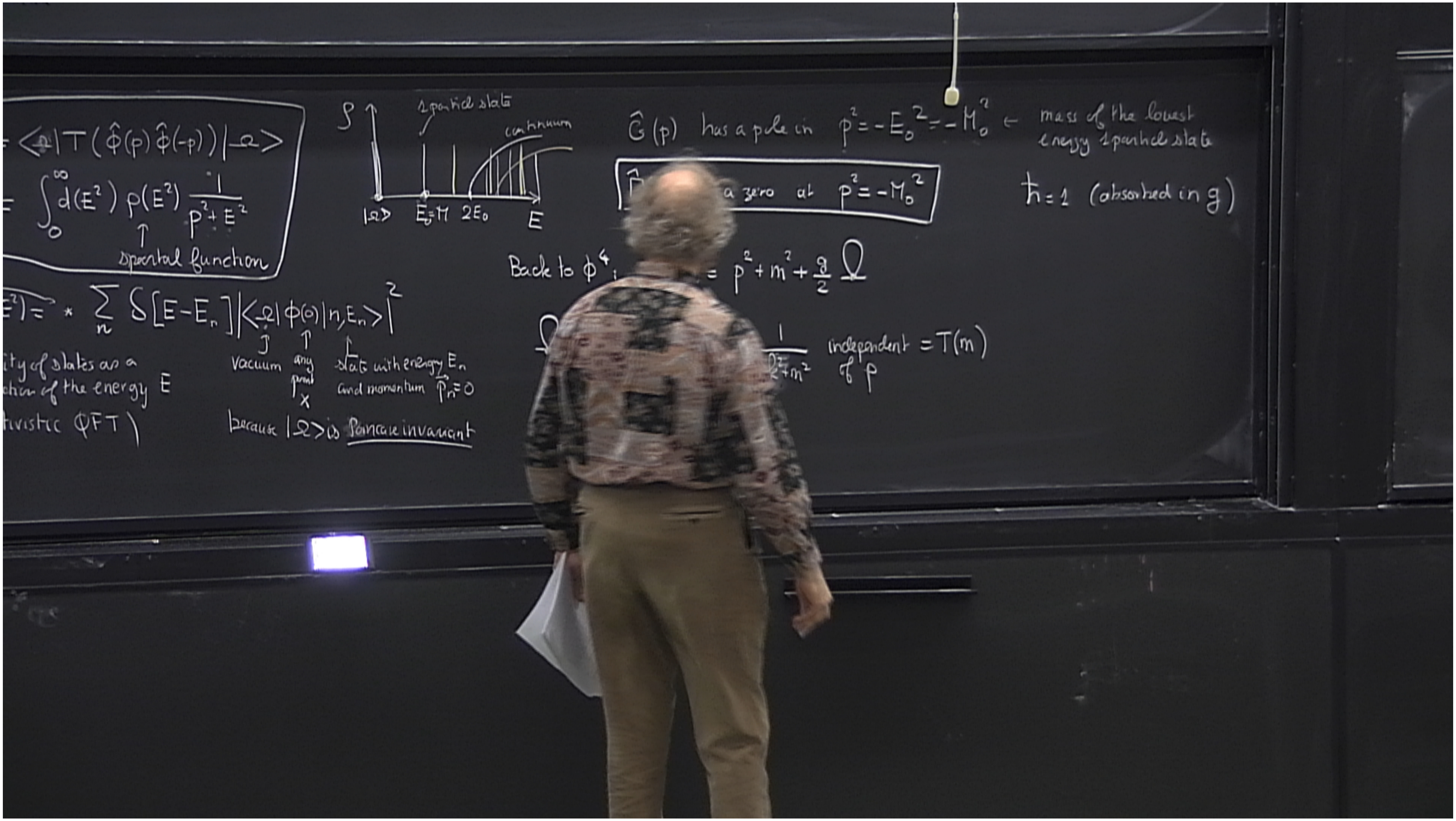

spectral function
1 particle state
continuum
Ĝ(p) has a pole in p² = −E₀² = −M₀² ← mass of the lowest energy 1 particle state
Γ̂(p) has a zero at p² = −M₀²
vacuum
any point x
state with energy Eₙ and momentum p⃗ₙ = 0
because |Ω> is Poincaré invariant
ity of states as a
ction of the energy E
tivistic QFT)

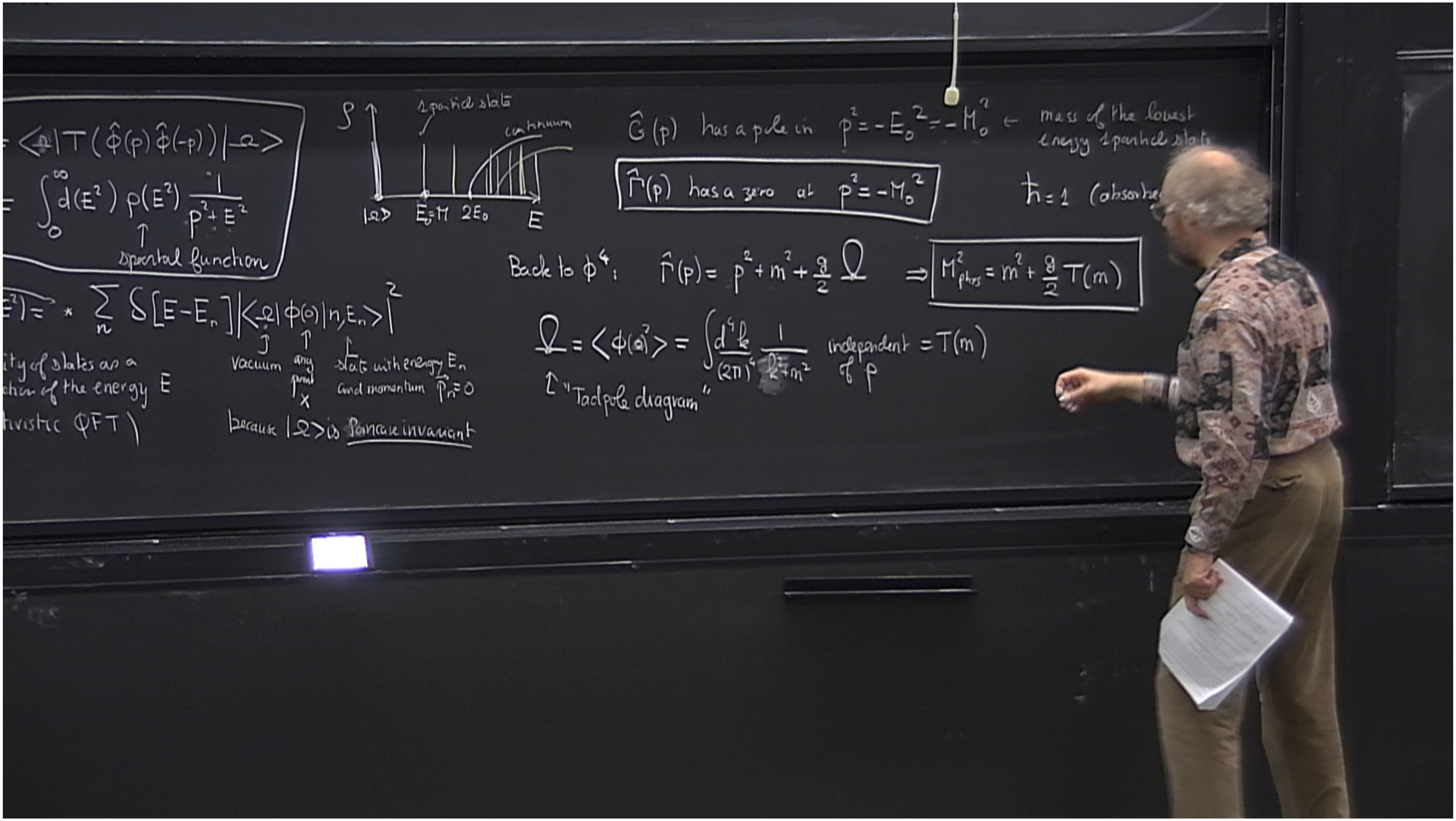

1 particle state
continuum
|Ω> E0=M 2E0 E
spectral function
Ĝ(p) has a pole in p² = −E0² = −M0² ← mass of the lowest energy 1 particle state
Γ̂(p) has a zero at p² = −M0²
Back to φ⁴:
vacuum
any point x
state with energy En and momentum pn = 0
because |Ω> is Poincaré invariant
ity of states as a
ction of the energy E
tivistic φFT)

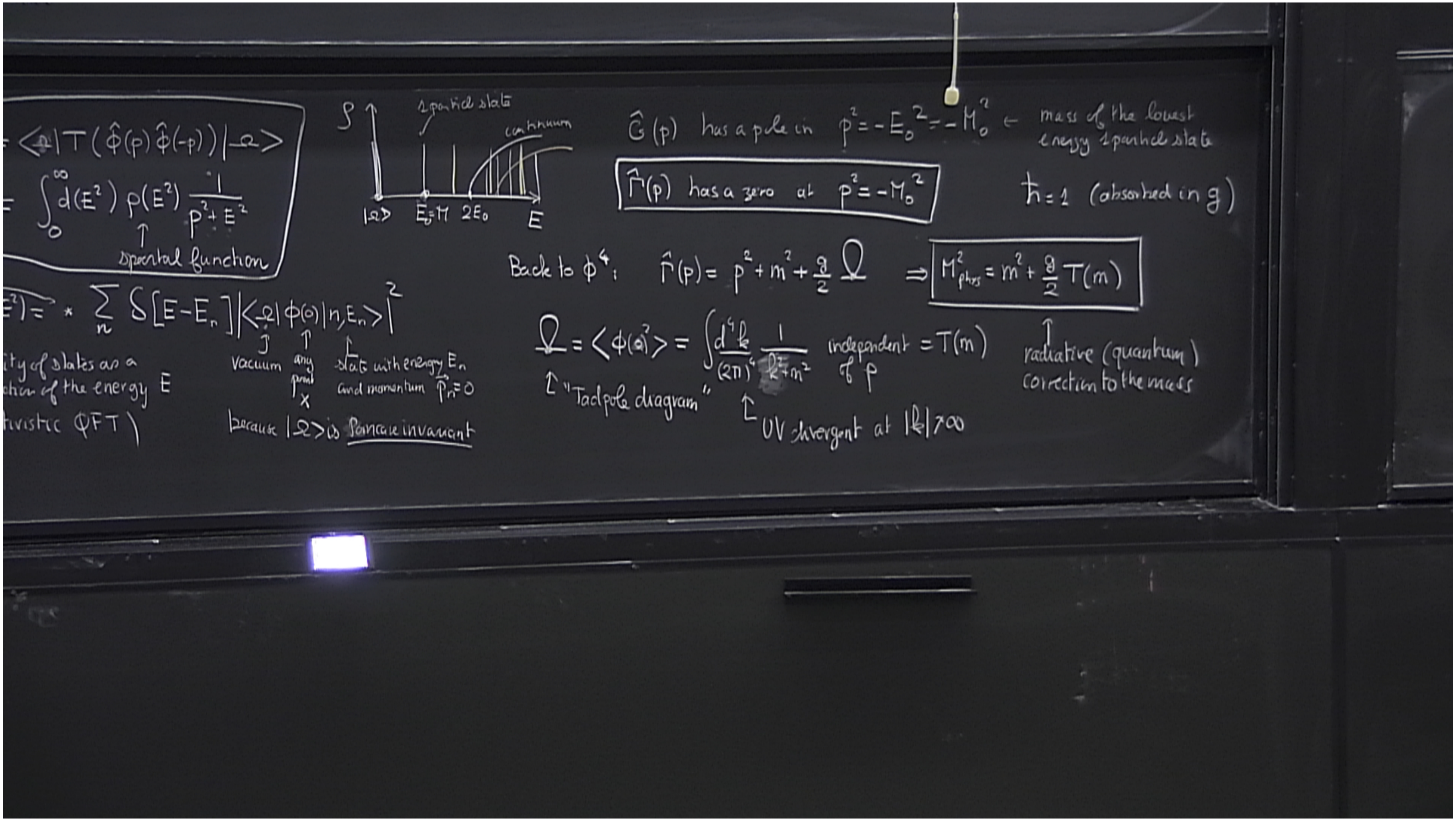

= <Ω|T(φ̂(p) φ̂(-p))|Ω>
= ∫₀^∞ d(E²) ρ(E²) 1/(p² + E²)
spectral function
1 particle state
continuum
|Ω> E₀=M 2E₀ E
Ĝ(p) has a pole in p² = -E₀² = -M₀² ← mass of the lowest energy 1 particle state
Γ̂(p) has a zero at p² = -M₀²
Back to φ⁴: Γ̂(p) = p² + m² + g/2 Ω
E²) = * Σₙ δ[E-Eₙ] |<Ω|φ(0)|n,Eₙ>|²
vacuum
any point x
state with energy Eₙ and momentum p⃗ₙ = 0
because |Ω> is Poincare invariant

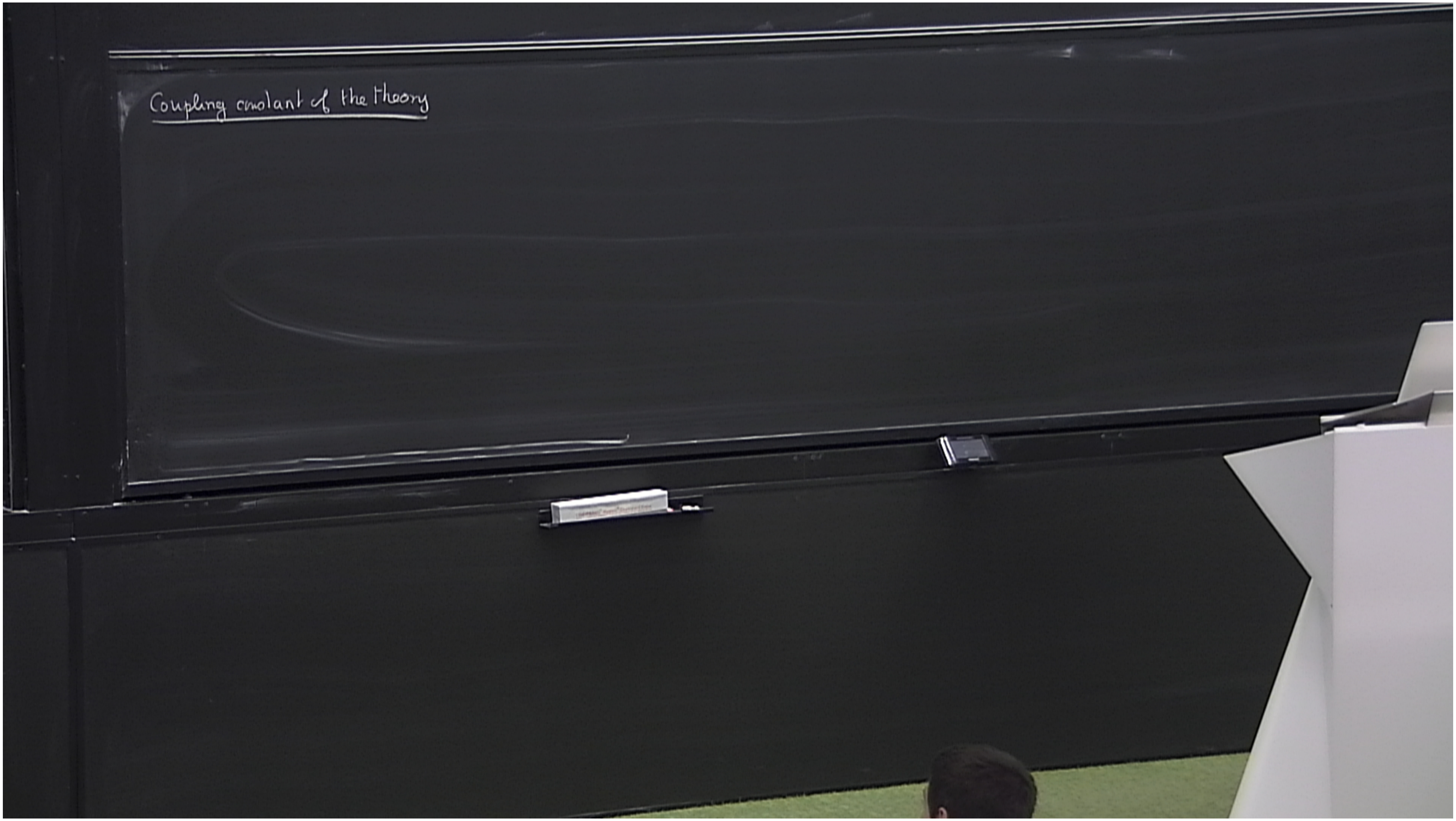

1 particle state
continuum
|Ω> E_0=M $2E_0$ E
spectral function
$\hat{G}(p)$ has a pole in $p^2 = -E_0^2 = -M_0^2$ ← mass of the lowest energy 1 particle state
$\hat{\Gamma}(p)$ has a zero
Back to ϕ^4:
vacuum
any point x
state with energy E_n and momentum $\vec{p}_n = 0$
because |Ω> is Poincaré invariant
ity of states as a
ction of the energy E
tivistic QFT)

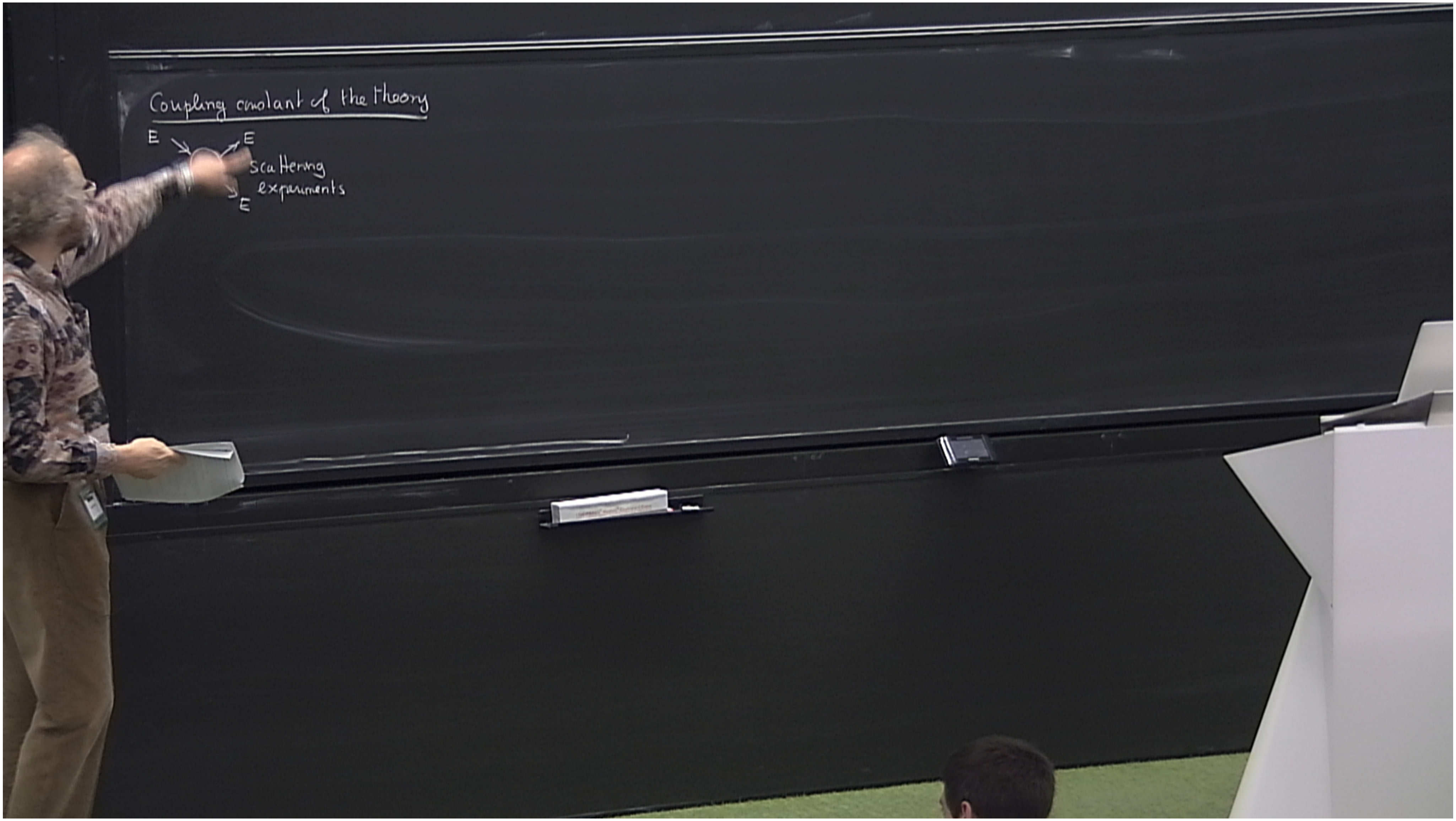

1 particle state
continuum
|Ω⟩ E₀=M 2E₀
Ĝ(p) has a pole in p² = −E₀² = −M₀² ← mass of the lowest energy 1 particle state
Γ̂(p) has a zero at p² = −M₀²
spectral function
Γ̂(p) = p² + m² + g/2
independent of p = T(m)
vacuum
any point x
because |Ω⟩ is
ity of states as a
ction of the energy E
tivistic QFT)

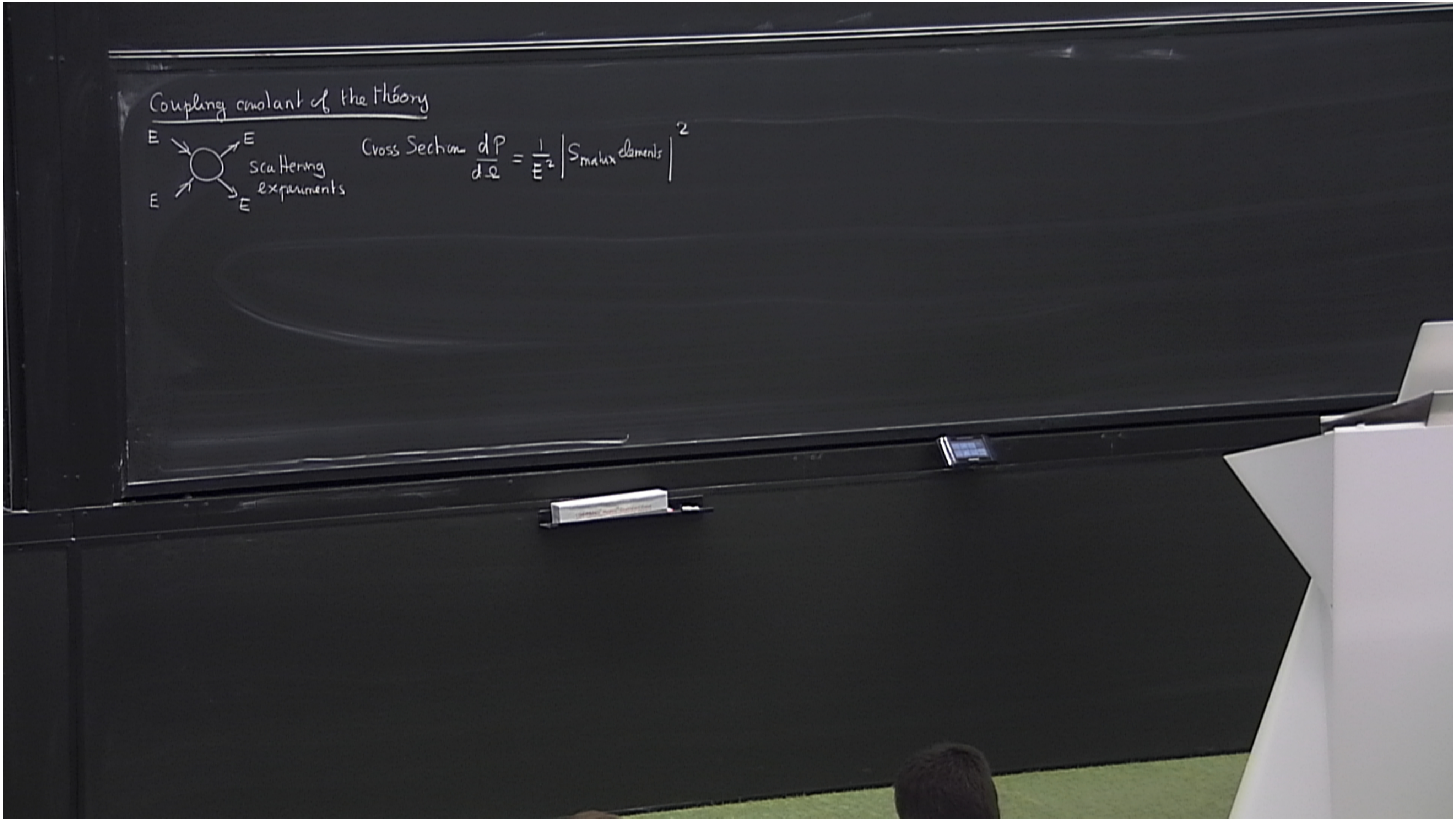

spectral function
1 particle state
continuum
Ĝ(p) has a pole in p² = -E₀² = -M₀² ← mass of the lowest energy 1 particle state
a zero at p² = -M₀²
ħ = 1 (absorbed in g)
Back to φ⁴:
independent of p = T(m)
Vacuum
any point x
state with energy Eₙ and momentum p⃗ₙ = 0
because |Ω> is Poincare invariant
density of states as a function of the energy E

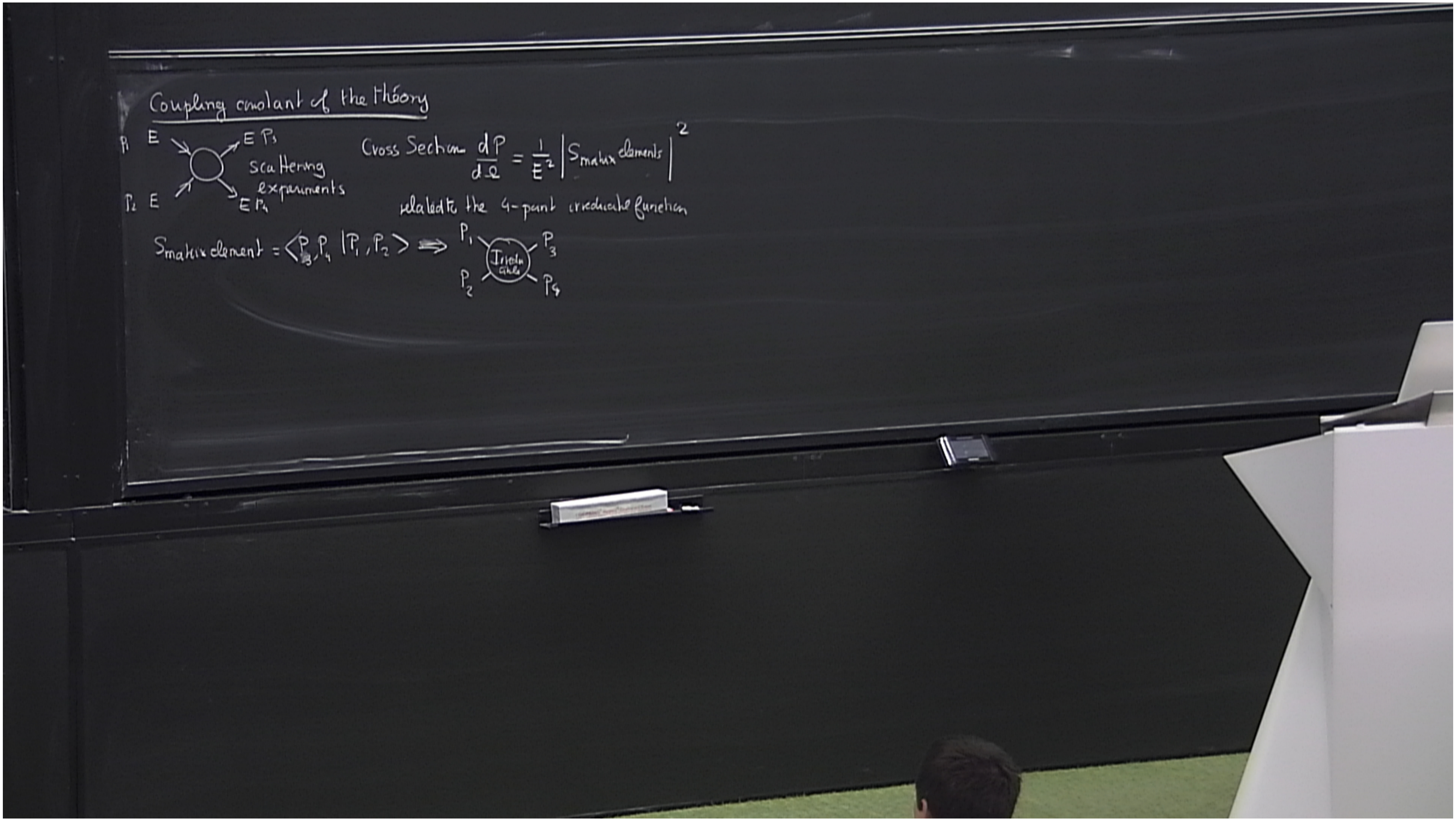

1 particle state
continuum
|Ω>
E
spectral function
Ĝ(p) has a pole in p² = −E₀² = −M₀² ← mass of the lowest energy 1 particle state
Γ̂(p) has a zero at p² = −M₀²
Back to φ⁴: Γ̂(p) = p² + m² + g/2 Ω ⇒ M²phys = m² + g/2 T(m)
Ω = ⟨φ(0)²⟩ = ∫ d⁴k/(2π)⁴ 1/(k²+m²) independent of p = T(m)
"Tadpole diagram"
vacuum
any point x
state with energy En and momentum pn = 0
because |Ω> is Poincaré invariant

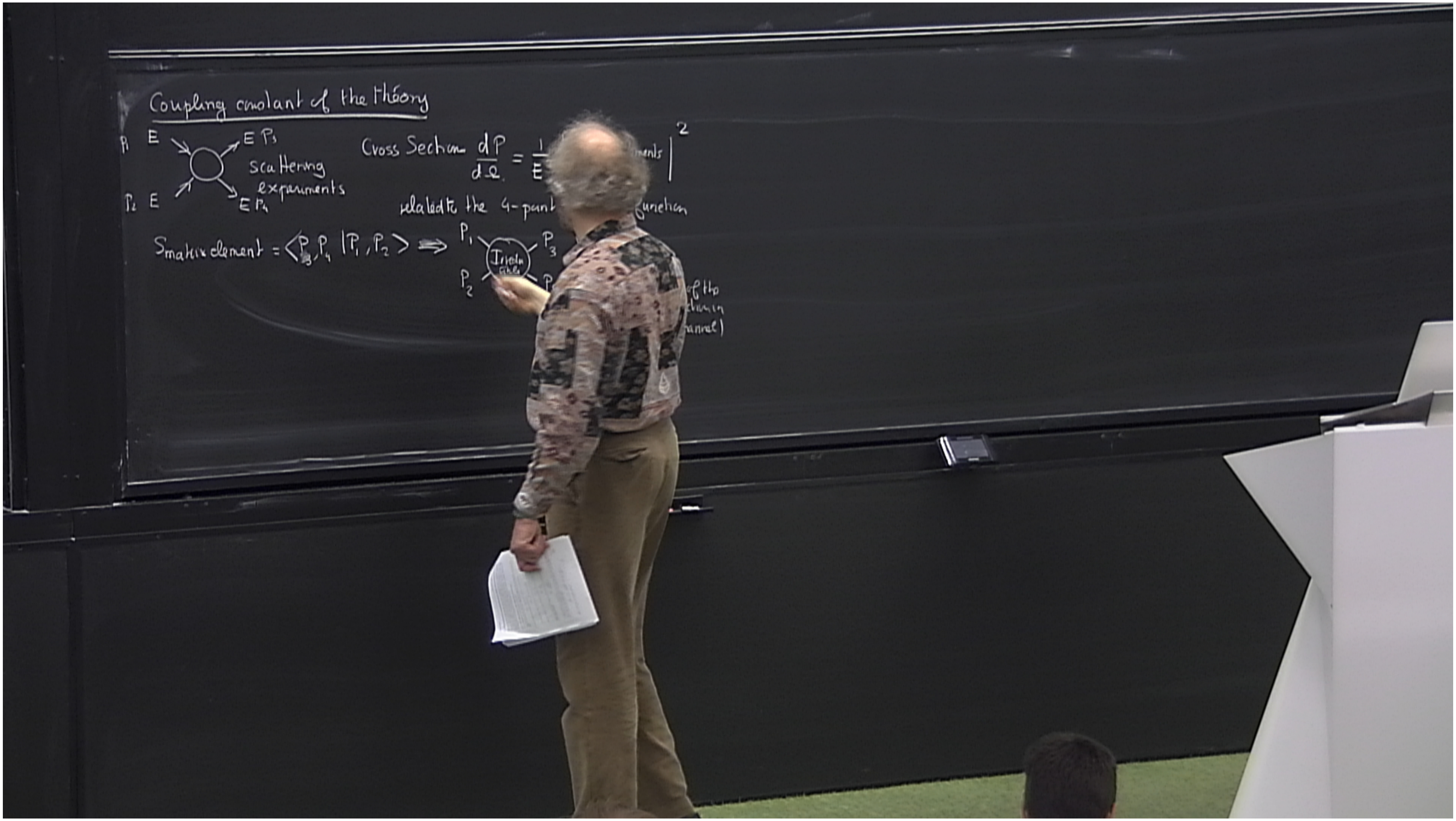

⟨Ω|T(φ̂(p)φ̂(-p))|Ω⟩
∫₀^∞ d(E²) ρ(E²) 1/(p²+E²)
spectral function
1 particle state
continuum
|Ω⟩ E₀=M 2E₀ E
Ĝ(p) has a pole in p² = -E₀² = -M₀² ← mass of the lowest energy 1 particle state
Γ̂(p) has a zero at p² = -M₀²
ħ = 1 (absorbed in g)
Back to φ⁴: Γ̂(p) = p² + m² + g/2 Ω ⇒ M²phys = m² + g/2 T(m)
Ω = ⟨φ(0)²⟩ = ∫ d⁴k/(2π)⁴ 1/(k²+m²) independent of p = T(m)
radiative (quantum) correction to the mass
"Tadpole diagram"
UV divergent at |k|→∞
ity of states as a
ction of the energy E
tivistic QFT)
vacuum
any point x
state with energy Eₙ and momentum p⃗ₙ = 0
because |Ω⟩ is Poincaré invariant

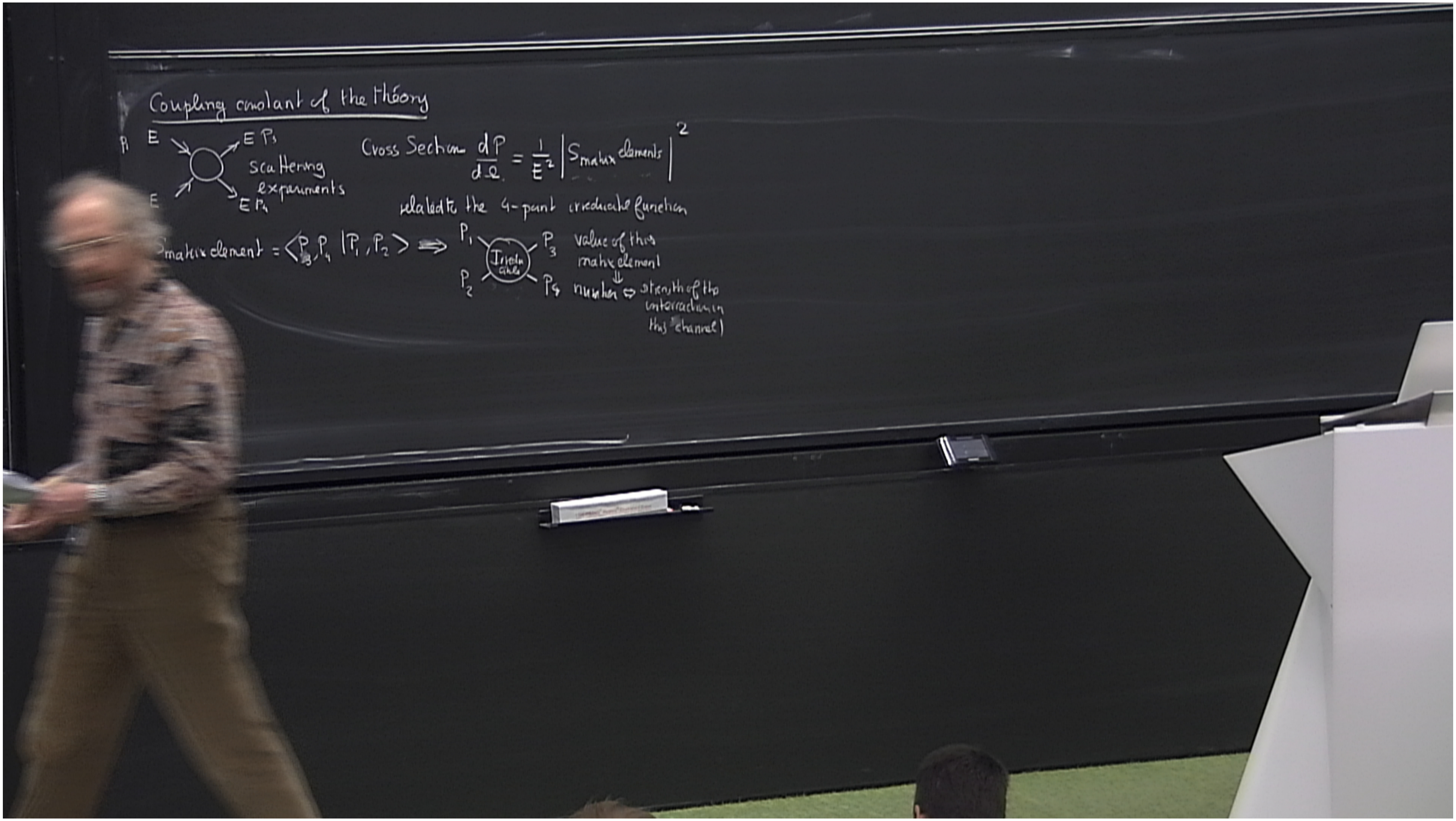

Coupling constant of the theory

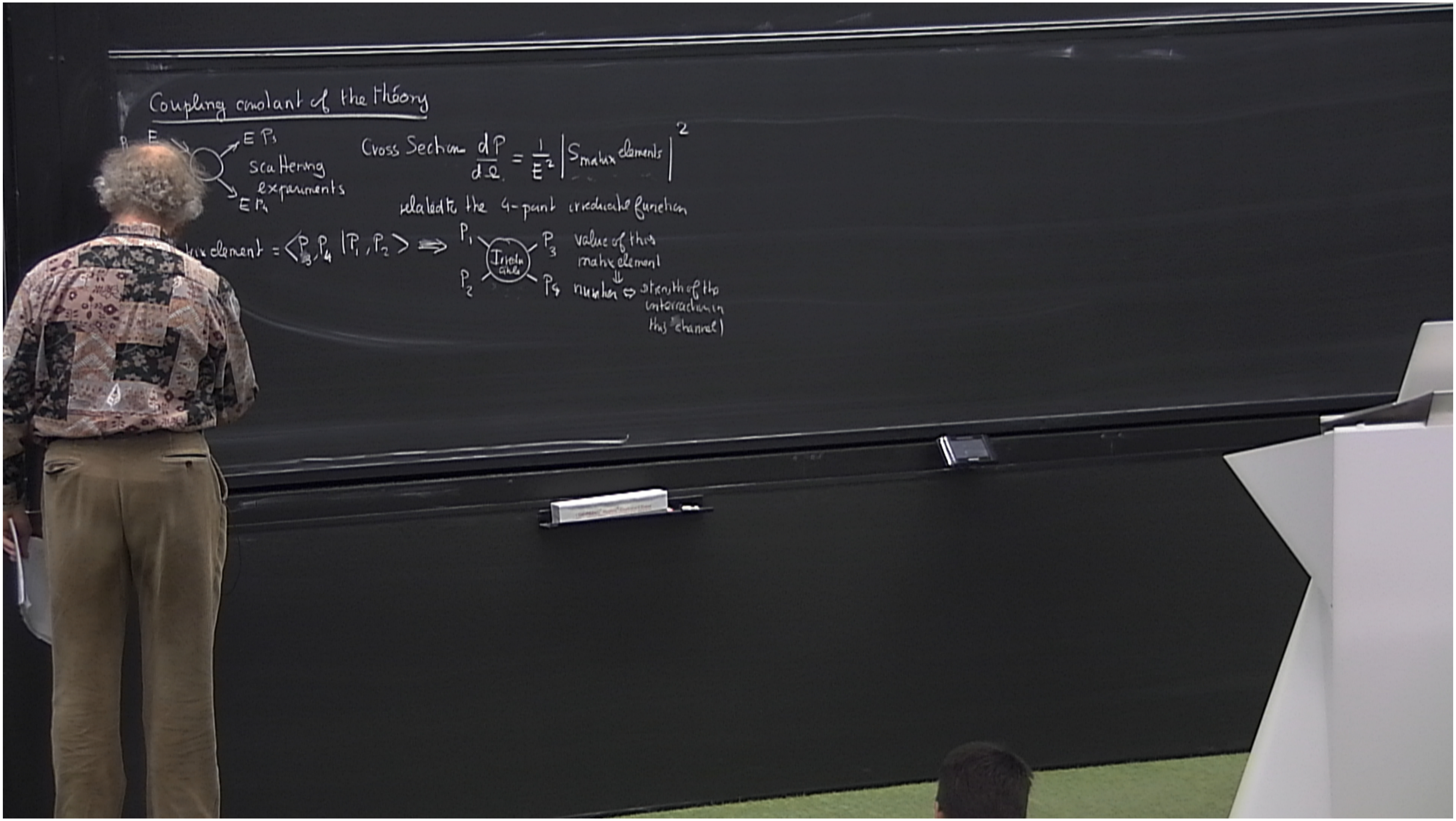

Coupling constant of the theory
E
E
scattering
experiments
E

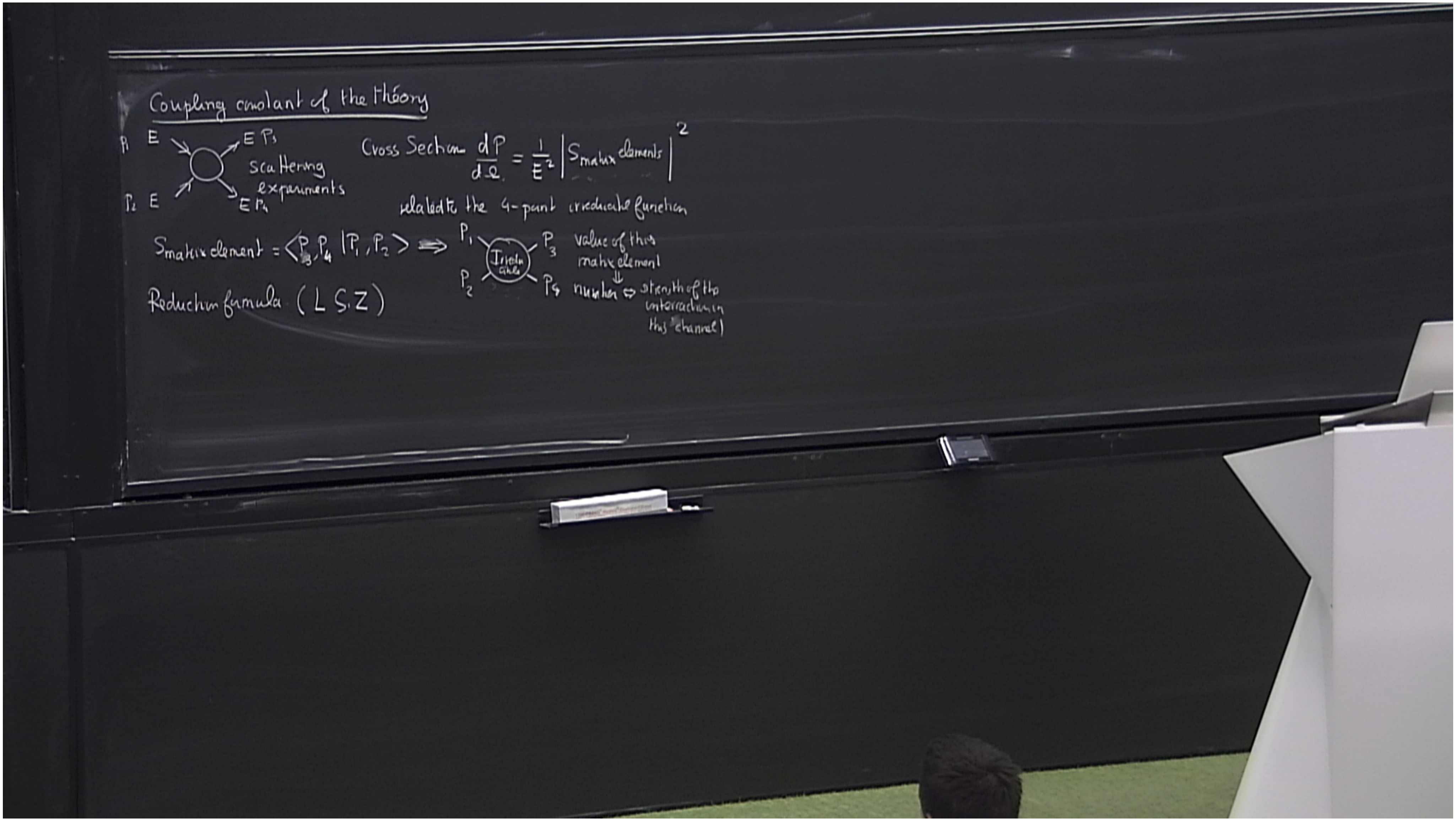

Coupling constant of the theory
E
E
E
E
scattering
experiments
Cross Section dP/dΩ = 1/E² |S matrix elements|²


Coupling constant of the theory
scattering experiments
Cross Section $\frac{dP}{d\Omega} = \frac{1}{E^2} \left| S_{matrix} \text{ elements} \right|^2$
related to the 4-point irreducible function
S_{matrix} element $= \langle P_3, P_4 | P_1, P_2 \rangle \Longrightarrow$
P_1
P_2
P_3
P_4

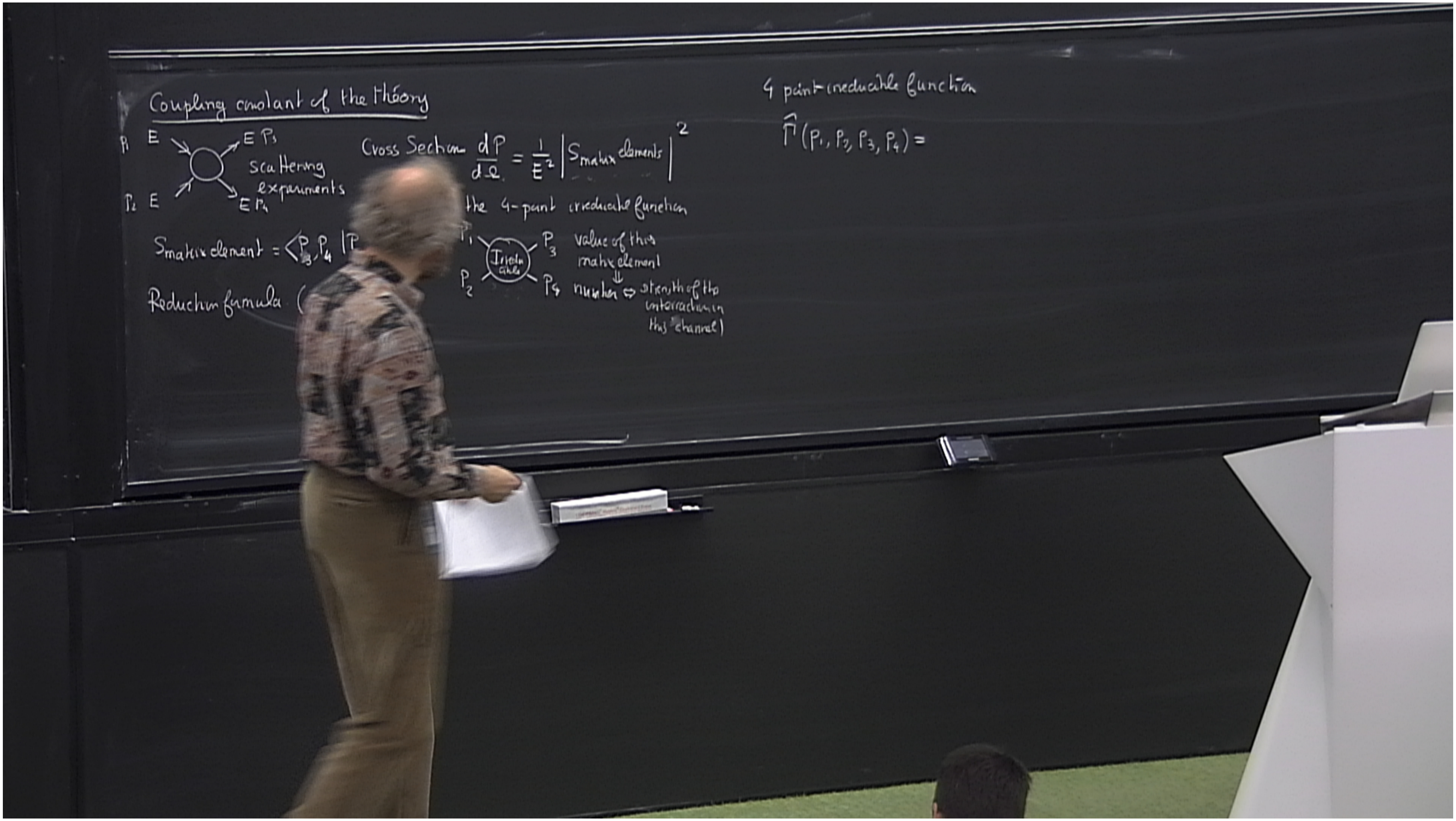

Coupling constant of the theory
Cross Section
scattering
experiments
related to the 4-point
function
S matrix element


Coupling constant of the theory
scattering experiments
Cross Section
related to the 4-point irreducible function
S matrix element
value of this matrix element
number
strength of the interaction in this channel

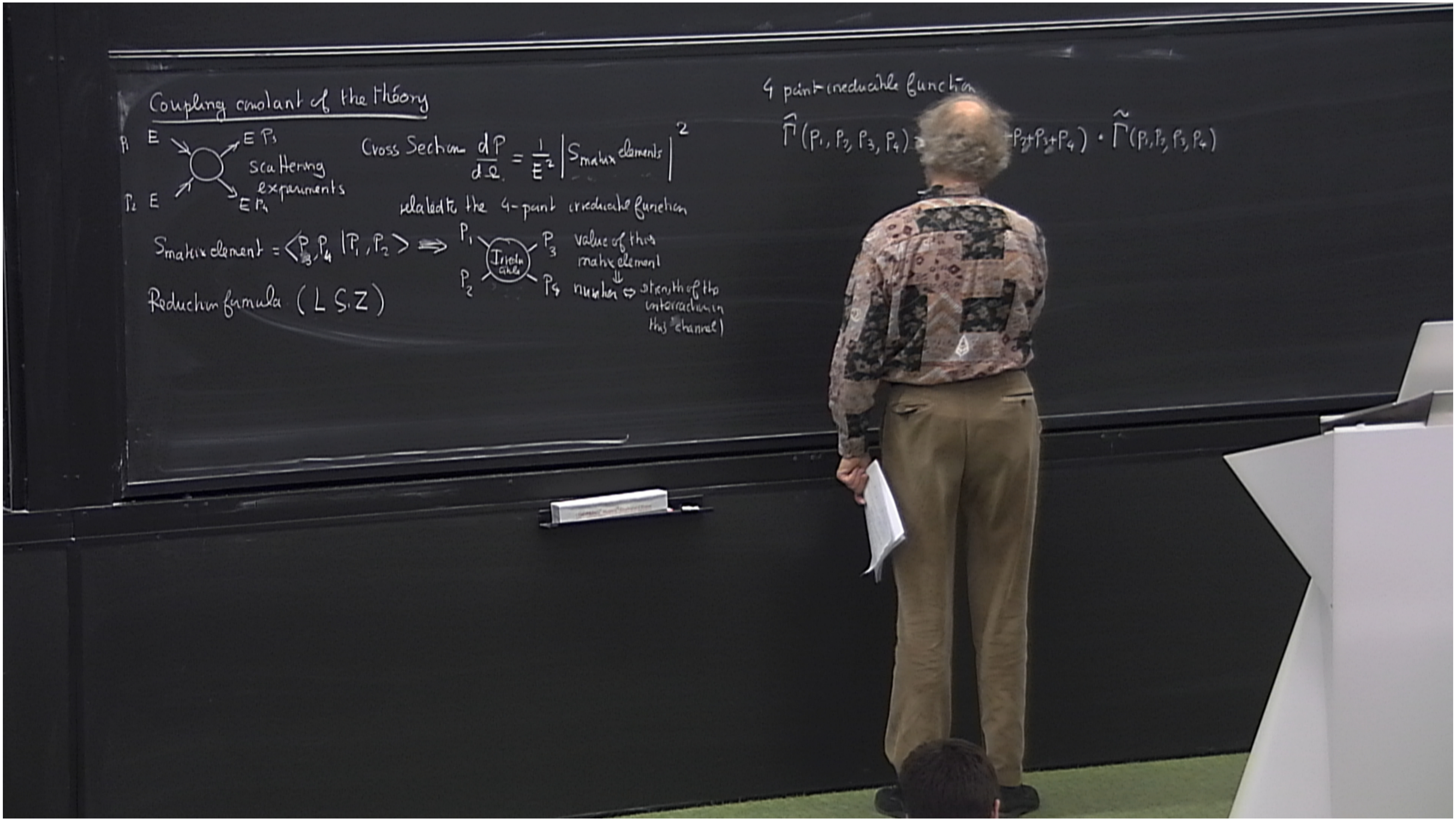

Coupling constant of the theory
scattering experiments
Cross Section
value of this matrix element
strength of the interaction in this channel)



Coupling constant of the theory
Cross Section
scattering experiments
related to the 4-point irreducible function
S matrix element = <P3, P4 | P1, P2> ⇒
value of this matrix element
number ⇔ strength of the interaction in this channel)
Reduction formula (L S Z)



Coupling constant of the theory
scattering experiments
Cross Section
the 4-point irreducible function
Smatrix element
Reduction formula
value of this matrix element
number ⇔ strength of the interaction in this channel)
4 point irreducible function

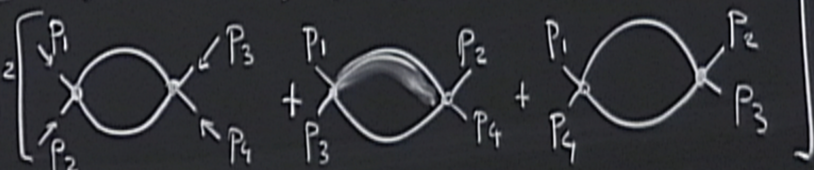


Coupling constant of the theory
scattering experiments
Cross Section
related to the 4-point irreducible function
S matrix element
Reduction formula (L S.Z)
value of this matrix element
number
strength of the interaction in this channel)
4 point irreducible function

4 point irreducible function

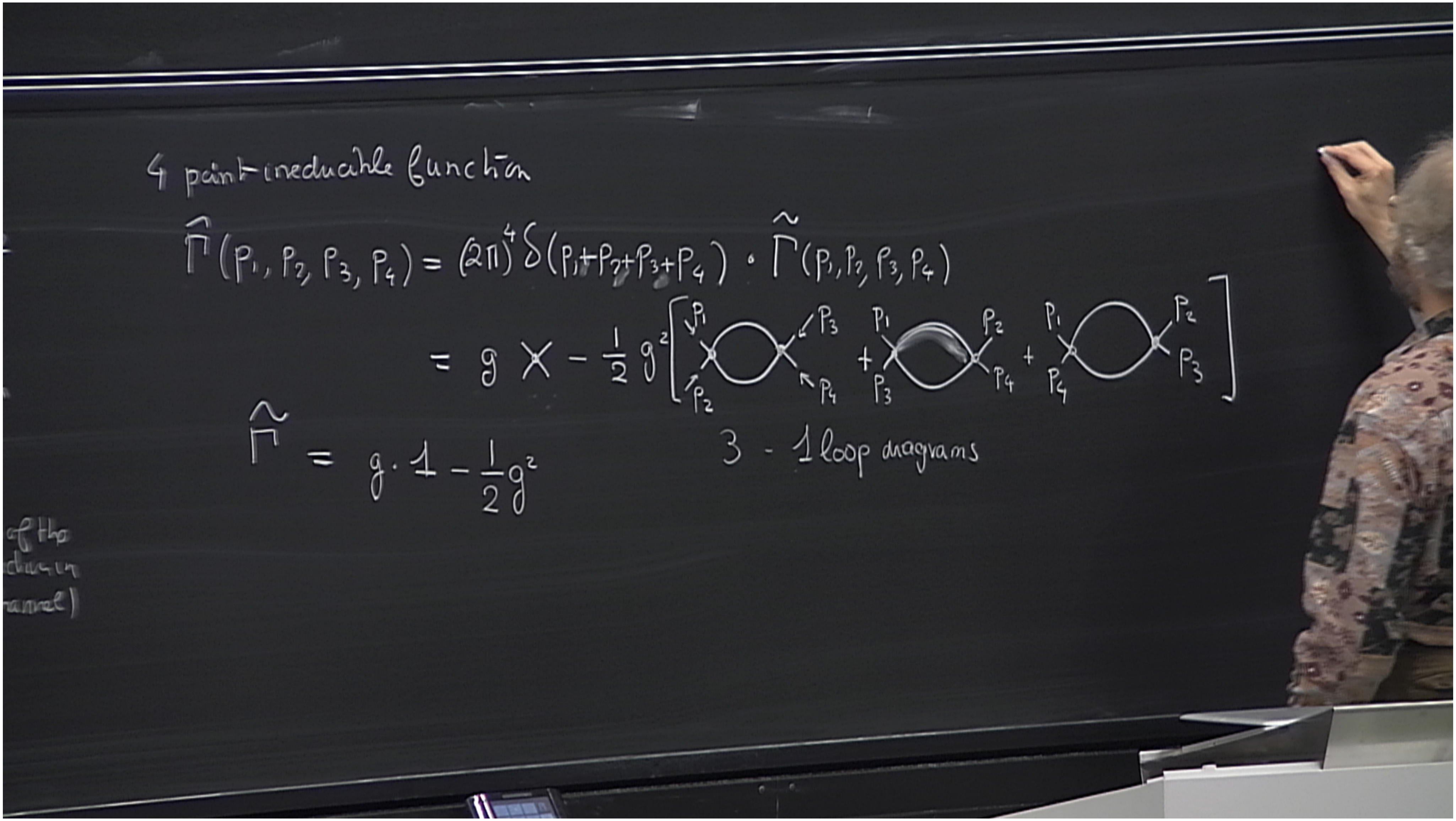
$$\hat{\Gamma}(p_1, p_2, p_3, p_4) = (2\pi)^4 \delta(p_1+p_2+p_3+p_4) \cdot \tilde{\Gamma}(p_1, p_2, p_3, p_4)$$

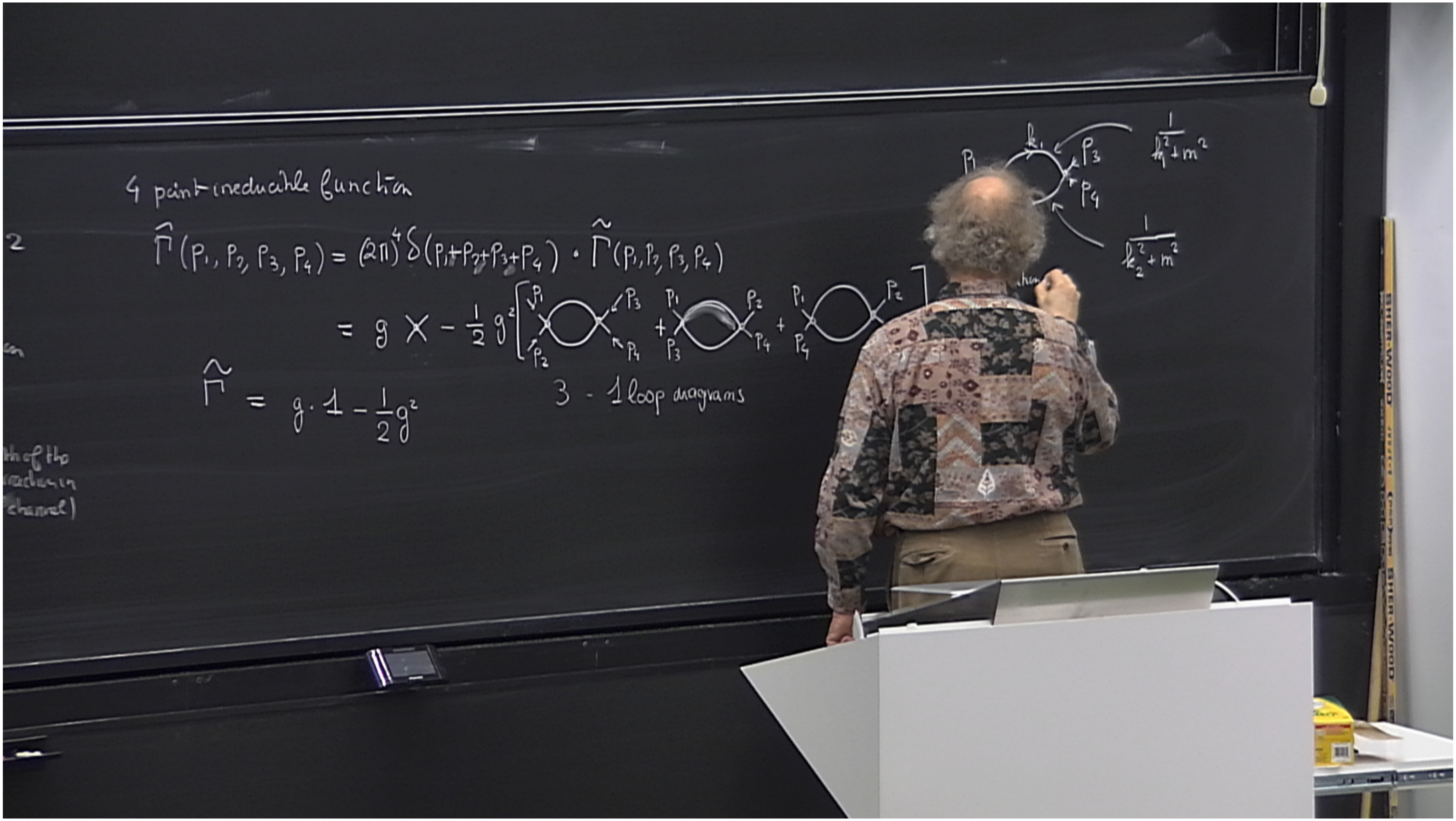
$$= g \times - \frac{1}{2} g^2 [\ldots]$$

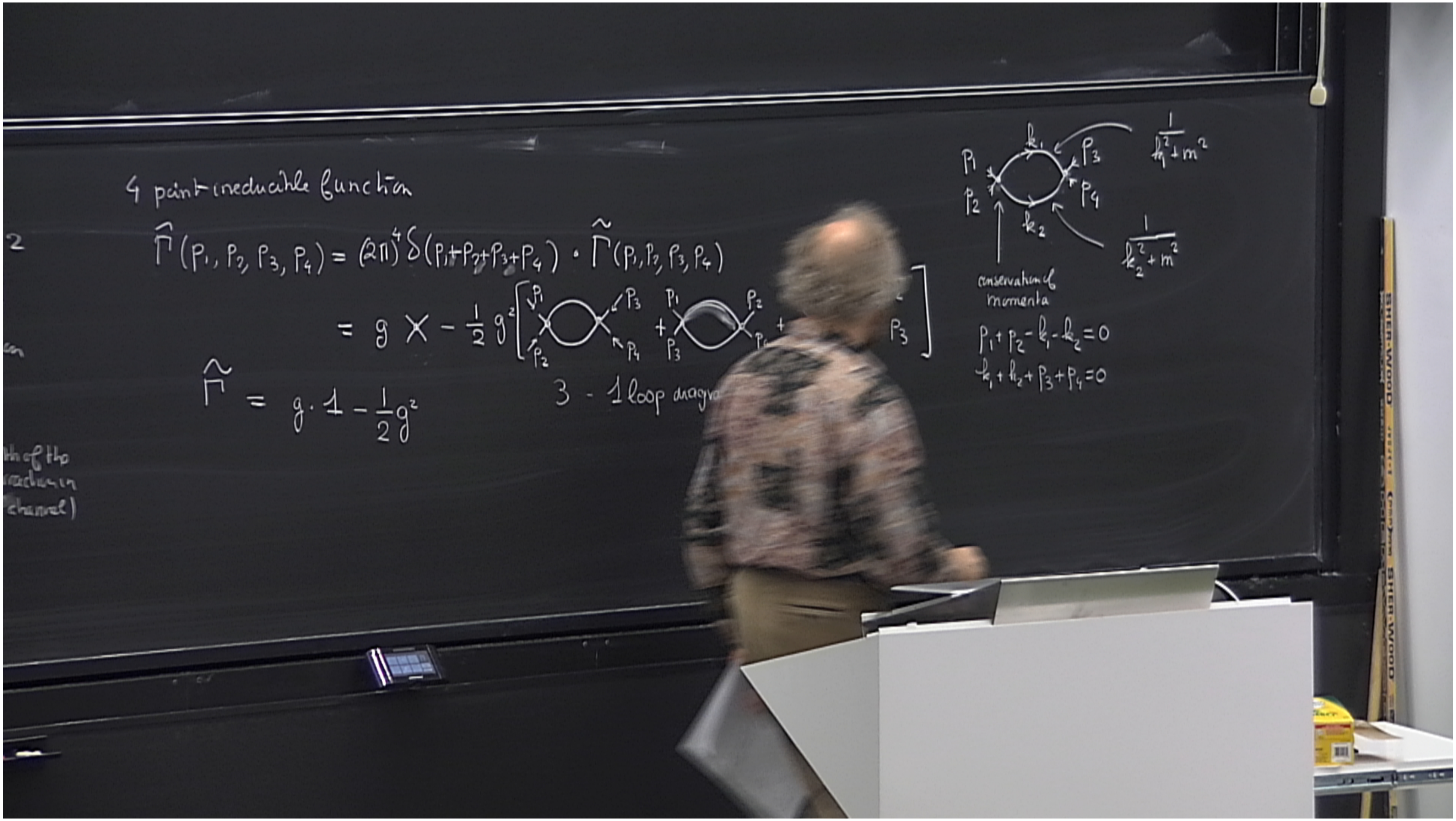


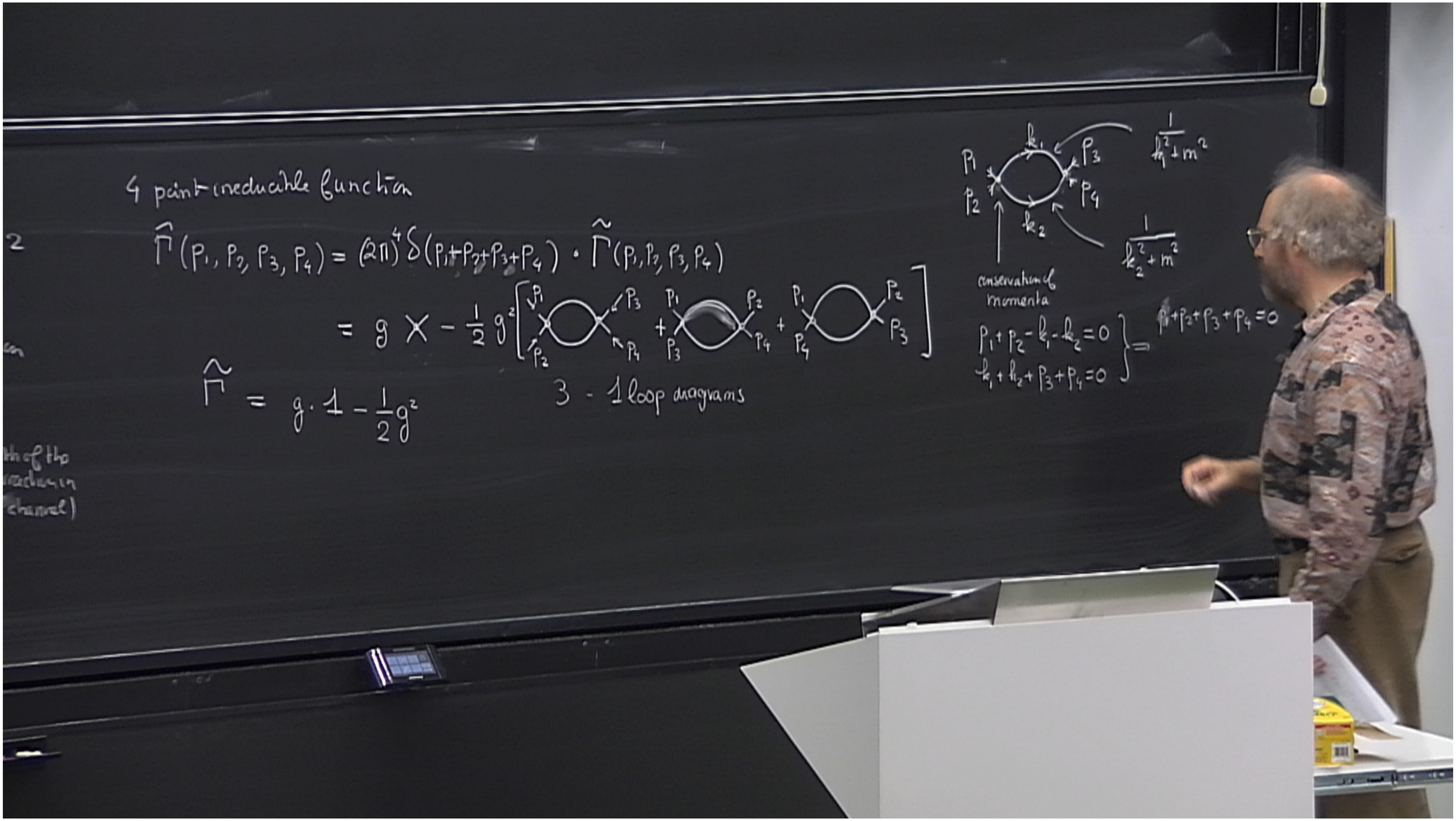
3 - 1 loop diagrams

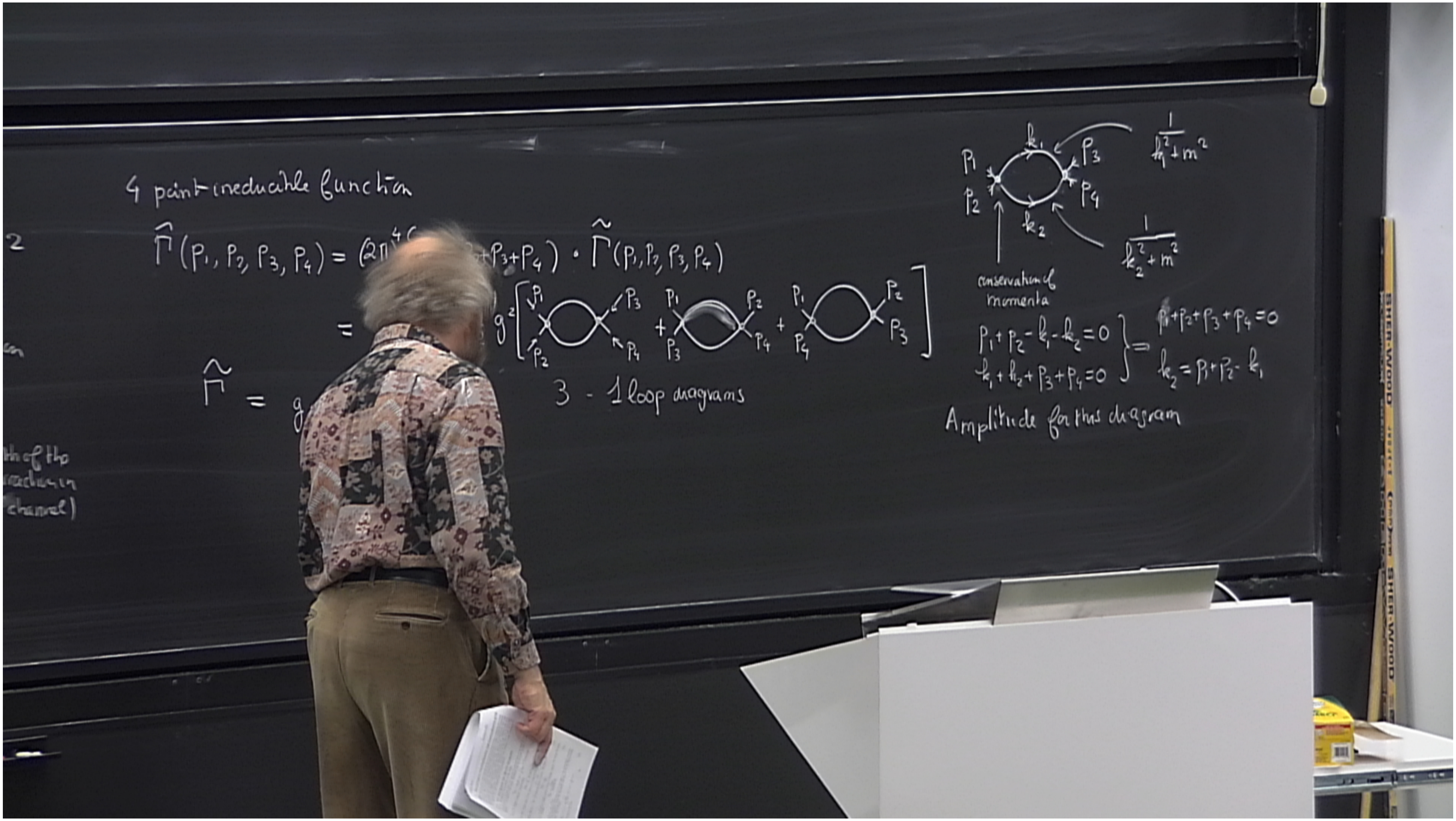
of the
chain in
annel)

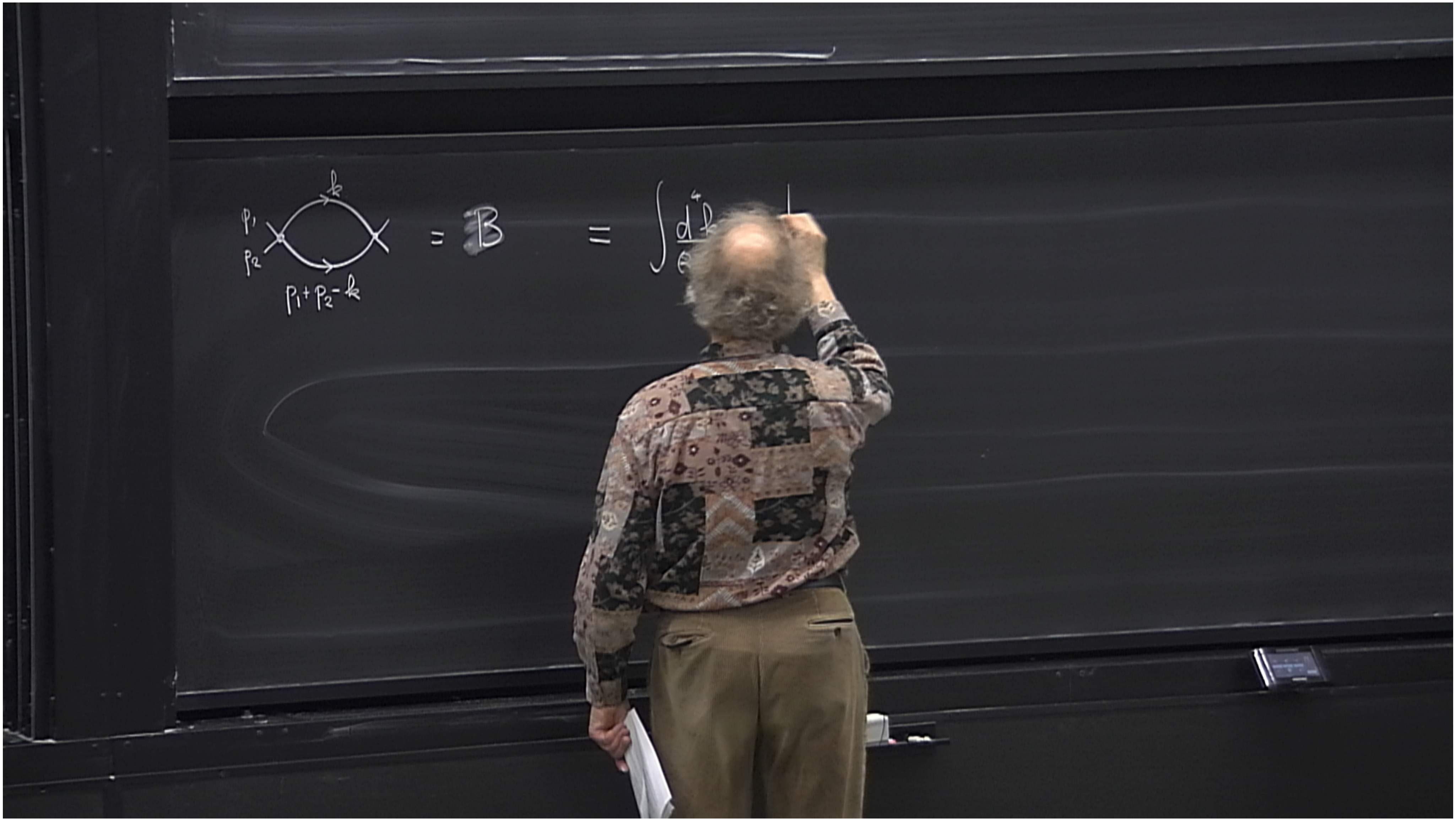

4 point irreducible function
$\hat{\Gamma}(p_1, p_2, p_3, p_4) = (2\pi)^4 \delta(p_1+p_2+p_3+p_4) \cdot \tilde{\Gamma}(p_1, p_2, p_3, p_4)$
$= g \times - \frac{1}{2} g^2 [\ \ + \ \ + \ \ ]$
p_1 p_2 p_3 p_4
$\tilde{\hat{\Gamma}} = g \cdot 1 - \frac{1}{2} g^2$
3 - 1 loop diagrams
of the
channel)

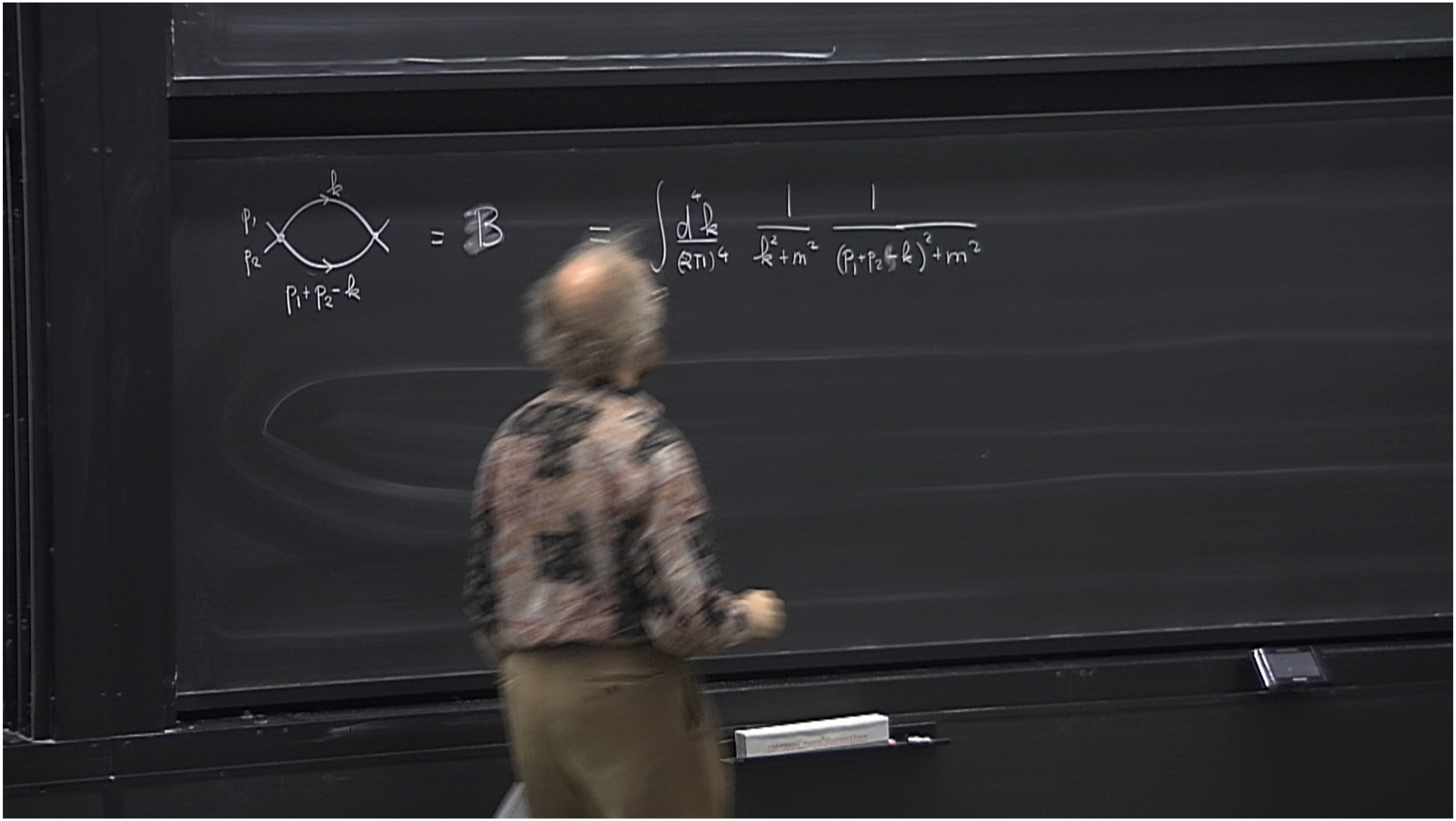

4 point irreducible function
Γ̂(p1, p2, p3, p4) = (2π)^4 δ(p1+p2+p3+p4) · Γ̃(p1, p2, p3, p4)
= g X − 1/2 g² [+ +]
Γ̃ = g · 1 − 1/2 g²
3 - 1 loop diagrams
1/(k1² + m²)
1/(k2² + m²)

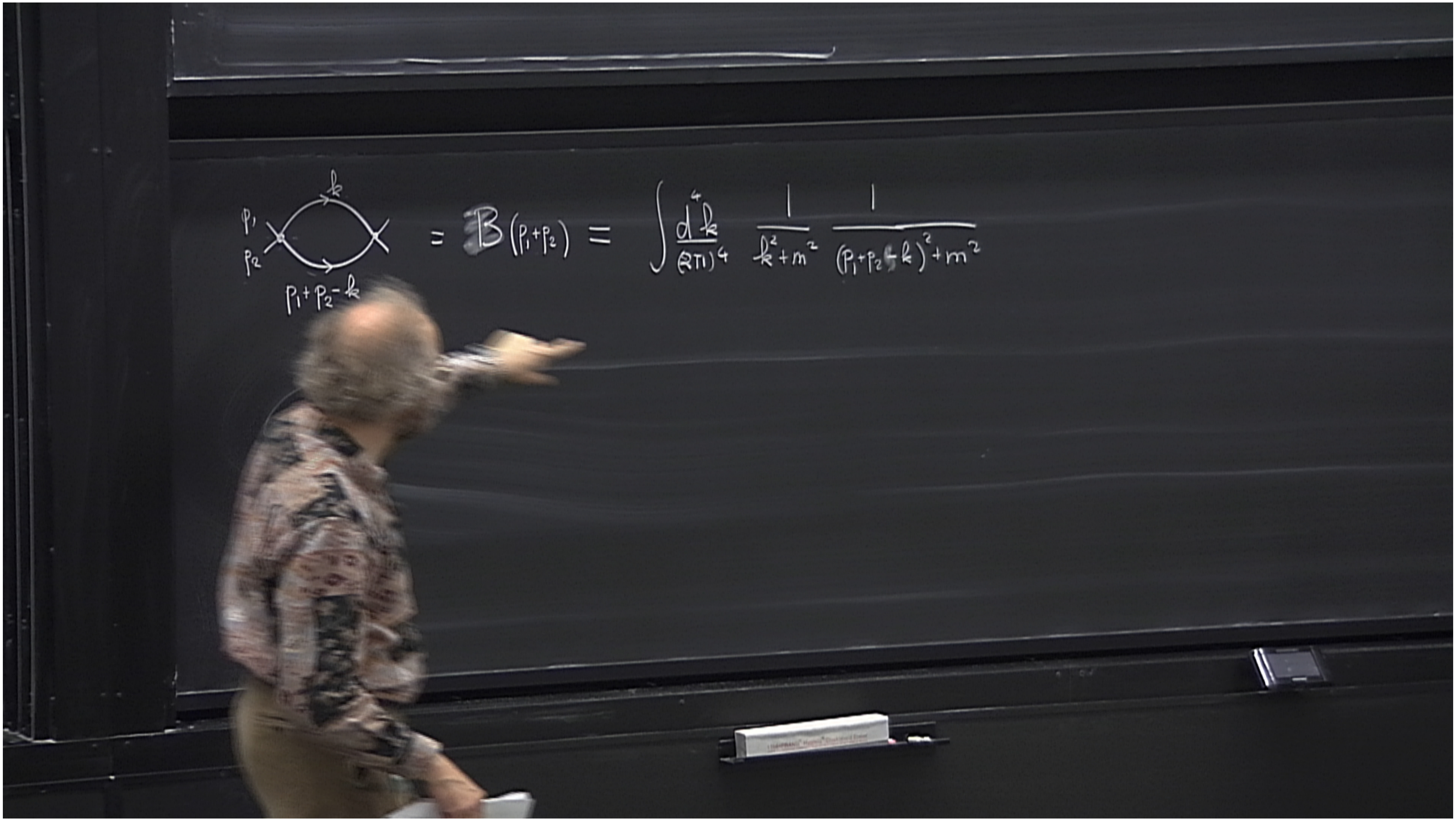

4 point irreducible function
Γ̂(p1, p2, p3, p4) = (2π)^4 δ(p1+p2+p3+p4) · Γ̃(p1, p2, p3, p4)
= g X − 1/2 g² [...]
Γ̃ = g · 1 − 1/2 g²
3 - 1 loop
conservation of momenta
p1 + p2 − k1 − k2 = 0
k1 + k2 + p3 + p4 = 0
1/(k1² + m²)
1/(k2² + m²)

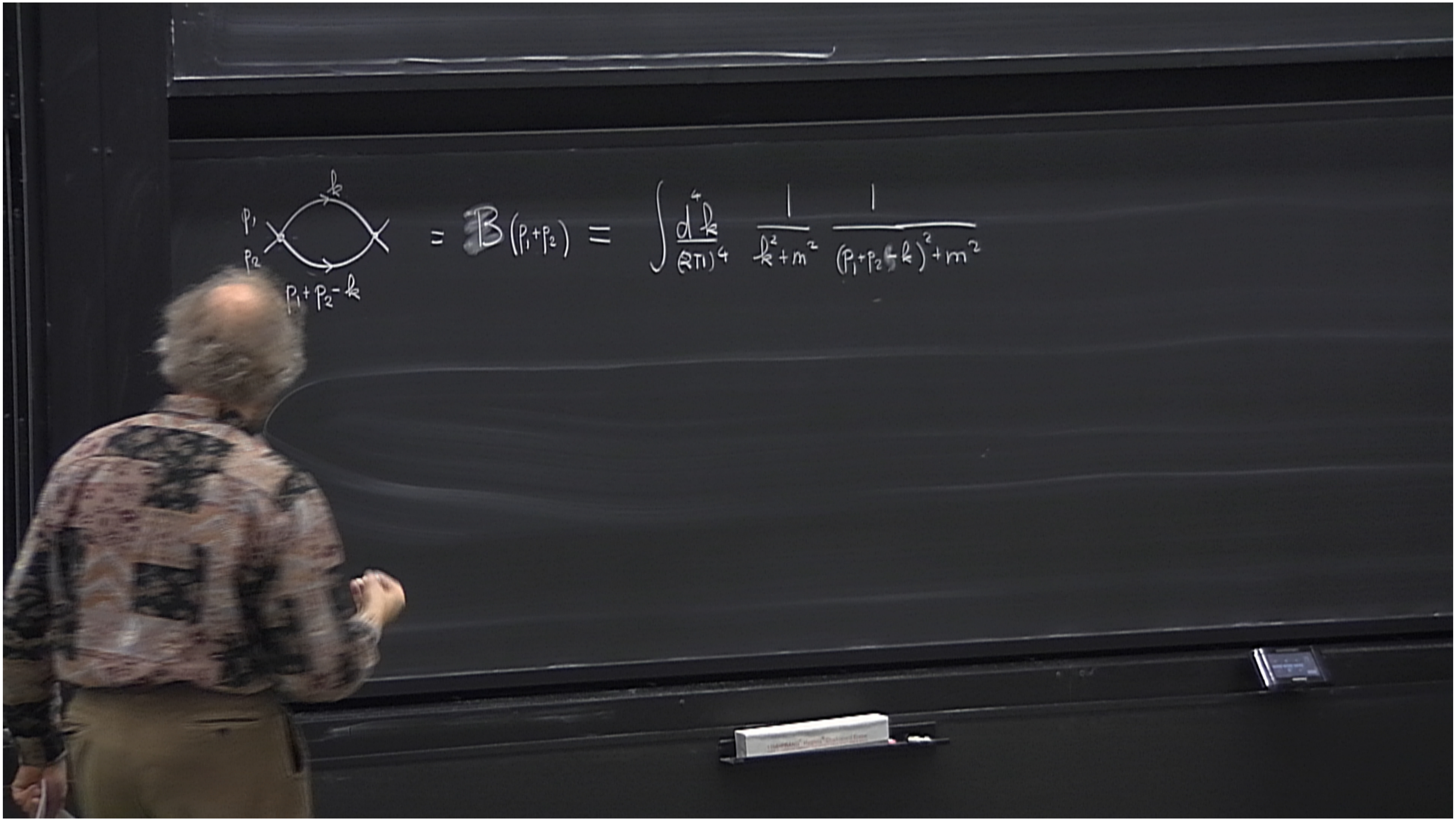

4 point irreducible function
Γ̂(p1, p2, p3, p4) = (2π)^4 δ(p1+p2+p3+p4) · Γ̃(p1, p2, p3, p4)
= g X − 1/2 g² [+ +]
Γ̃ = g · 1 − 1/2 g²
3 - 1 loop diagrams
conservation of momenta
p1 + p2 − k1 − k2 = 0
k1 + k2 + p3 + p4 = 0
⇒ p1+p2+p3+p4 = 0
1/(k1² + m²)
1/(k2² + m²)

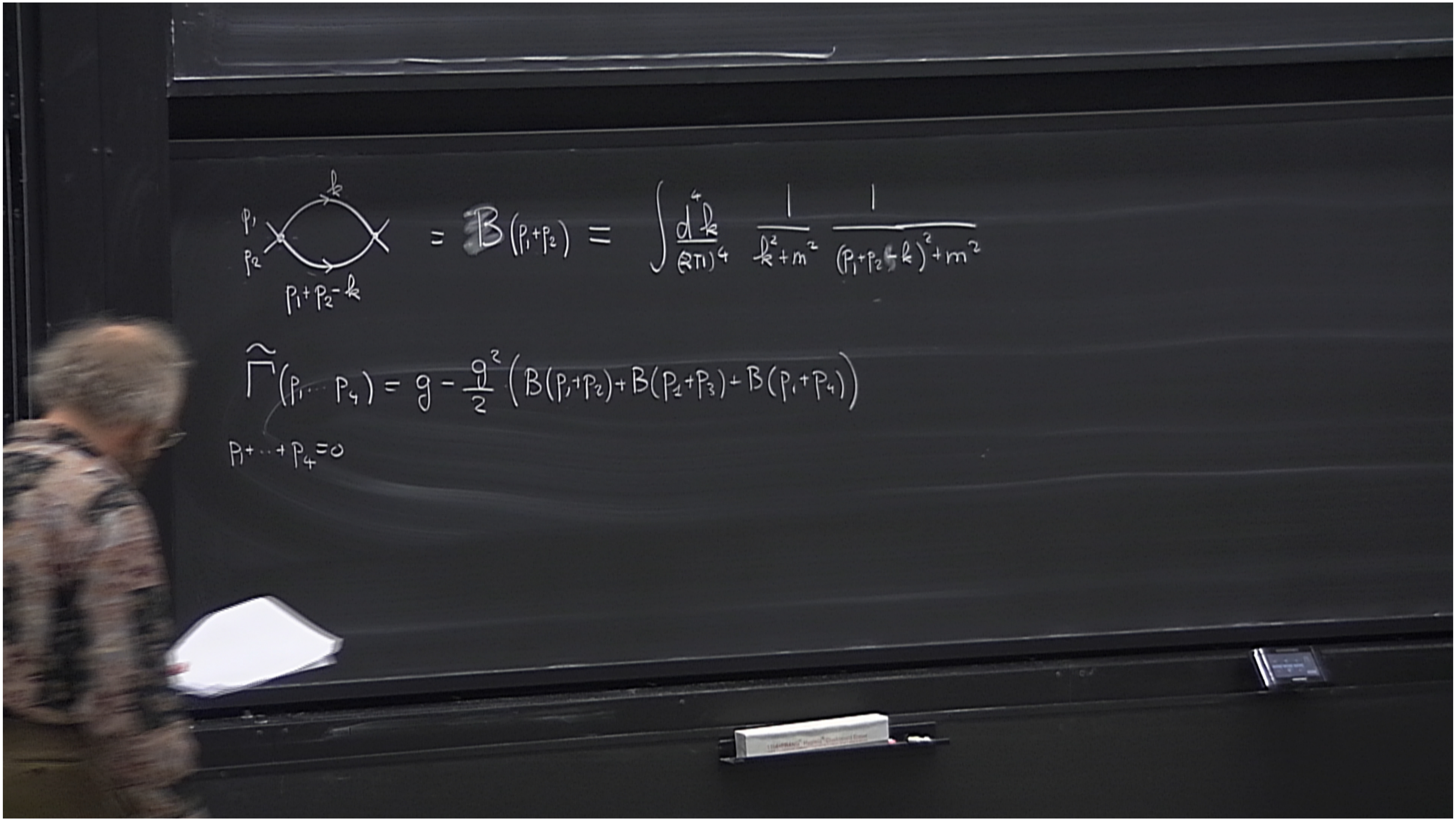

4 point irreducible function
3 - 1 loop diagrams
conservation of momenta
Amplitude for this diagram

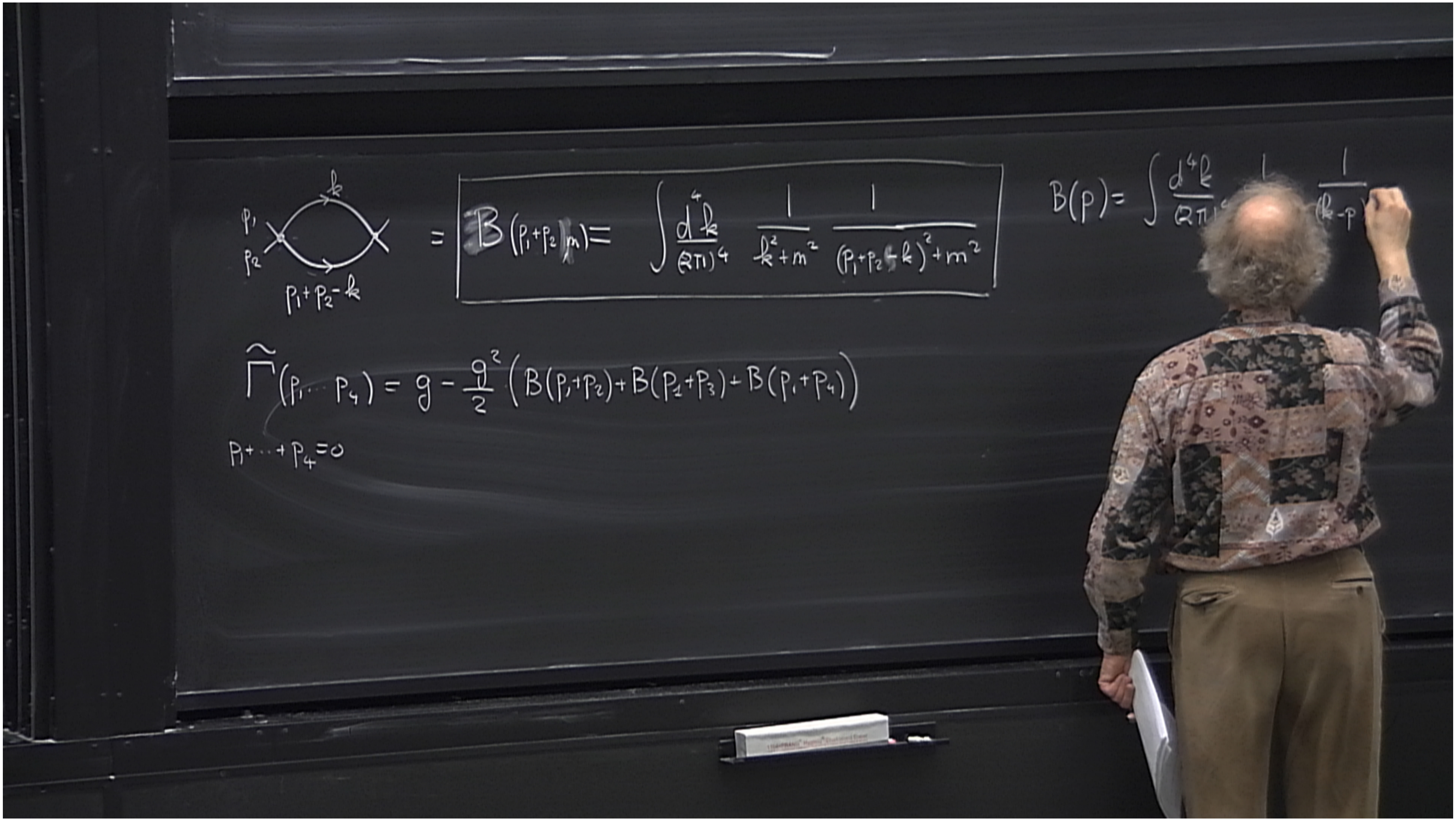

= B =

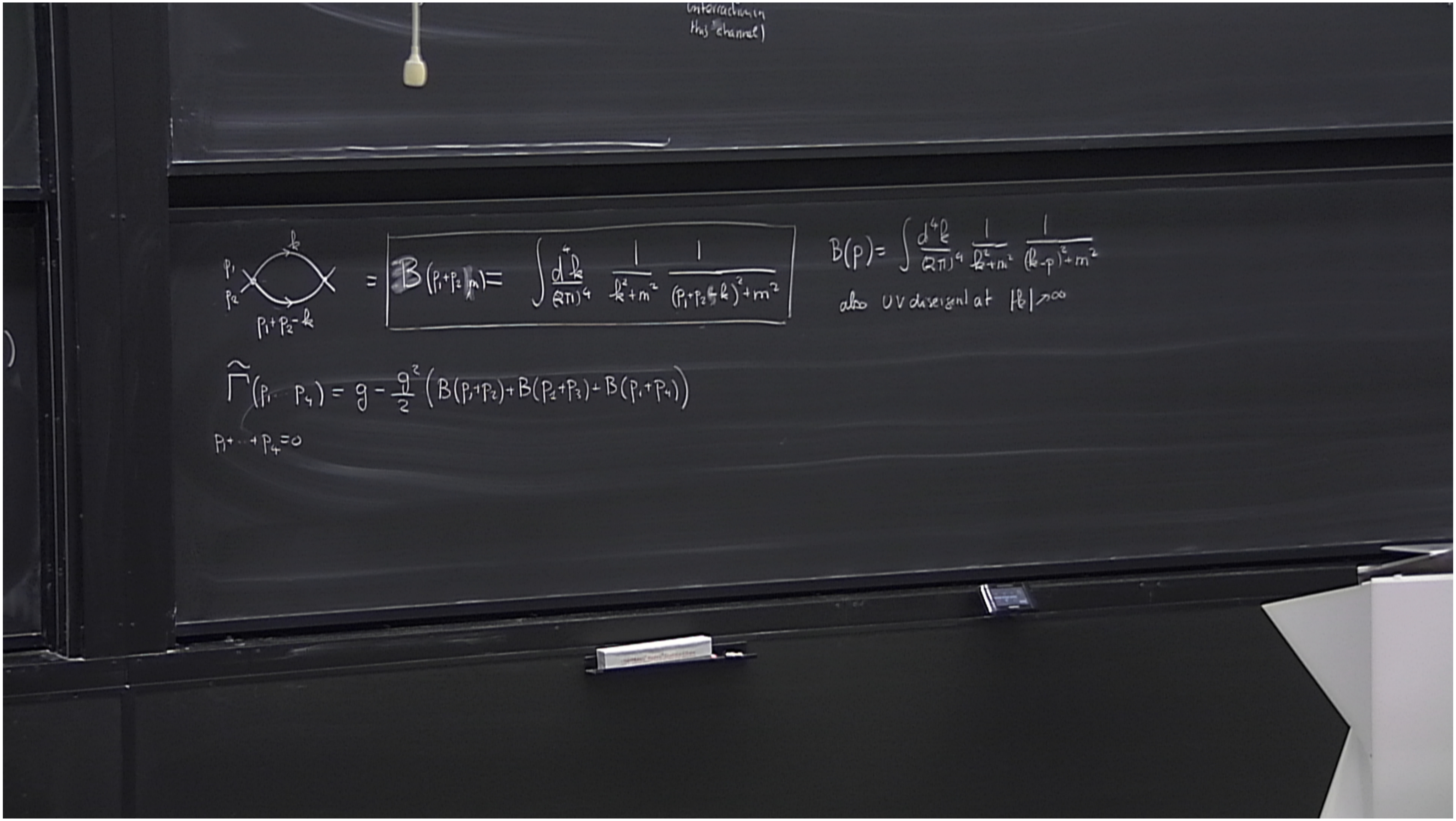

p₁
p₂
k
p₁+p₂-k
= B = ∫ d⁴k/(2π)⁴ 1/(k²+m²) 1/((p₁+p₂-k)²+m²)


k
p1
p2
p1+p2-k
= B(p1+p2) = ∫ d^4k/(2π)^4 1/(k^2+m^2) 1/((p1+p2-k)^2+m^2)

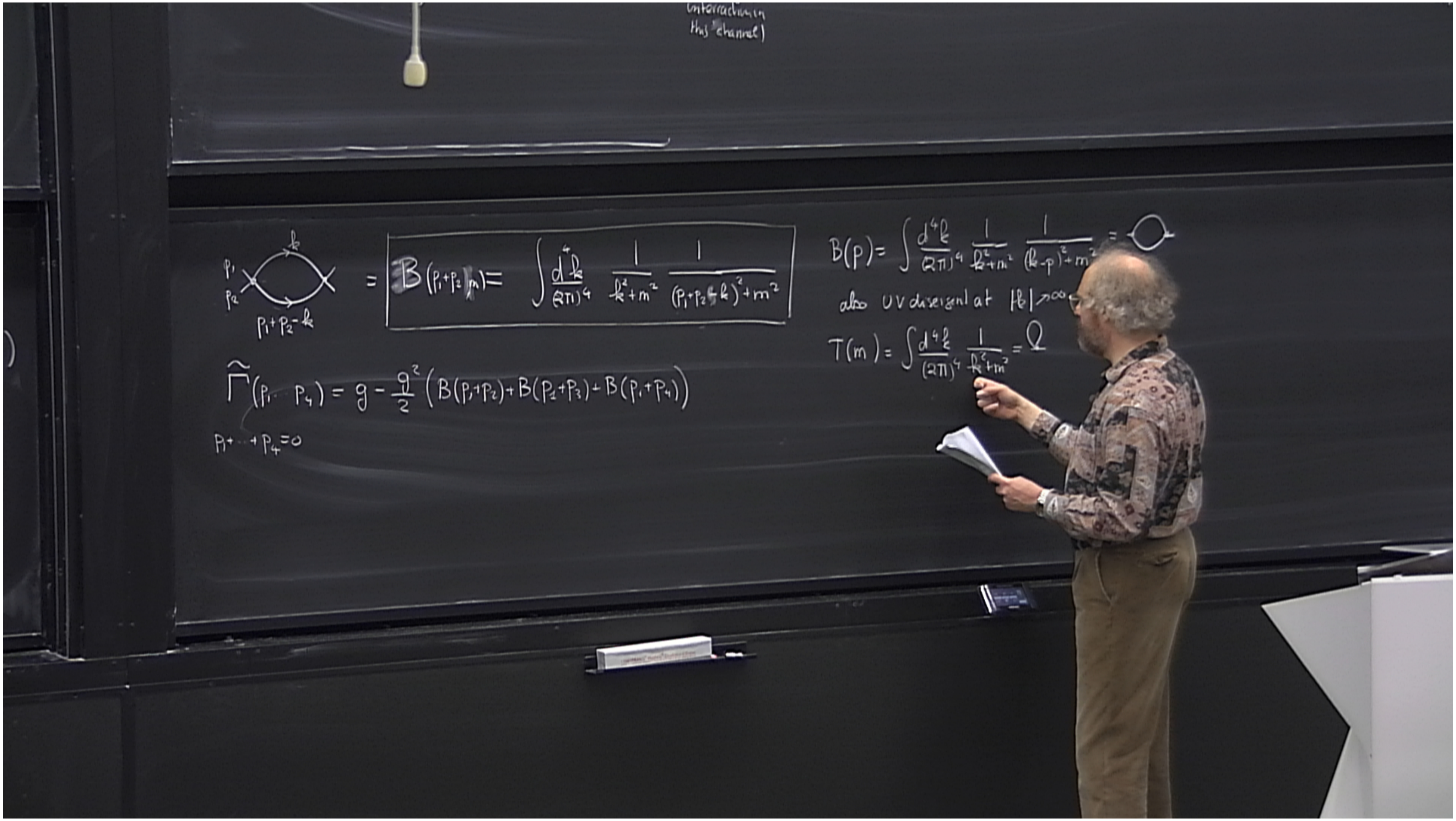

p_1
p_2
k
p_1+p_2-k
$= B(p_1+p_2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+m^2} \frac{1}{(p_1+p_2-k)^2+m^2}$

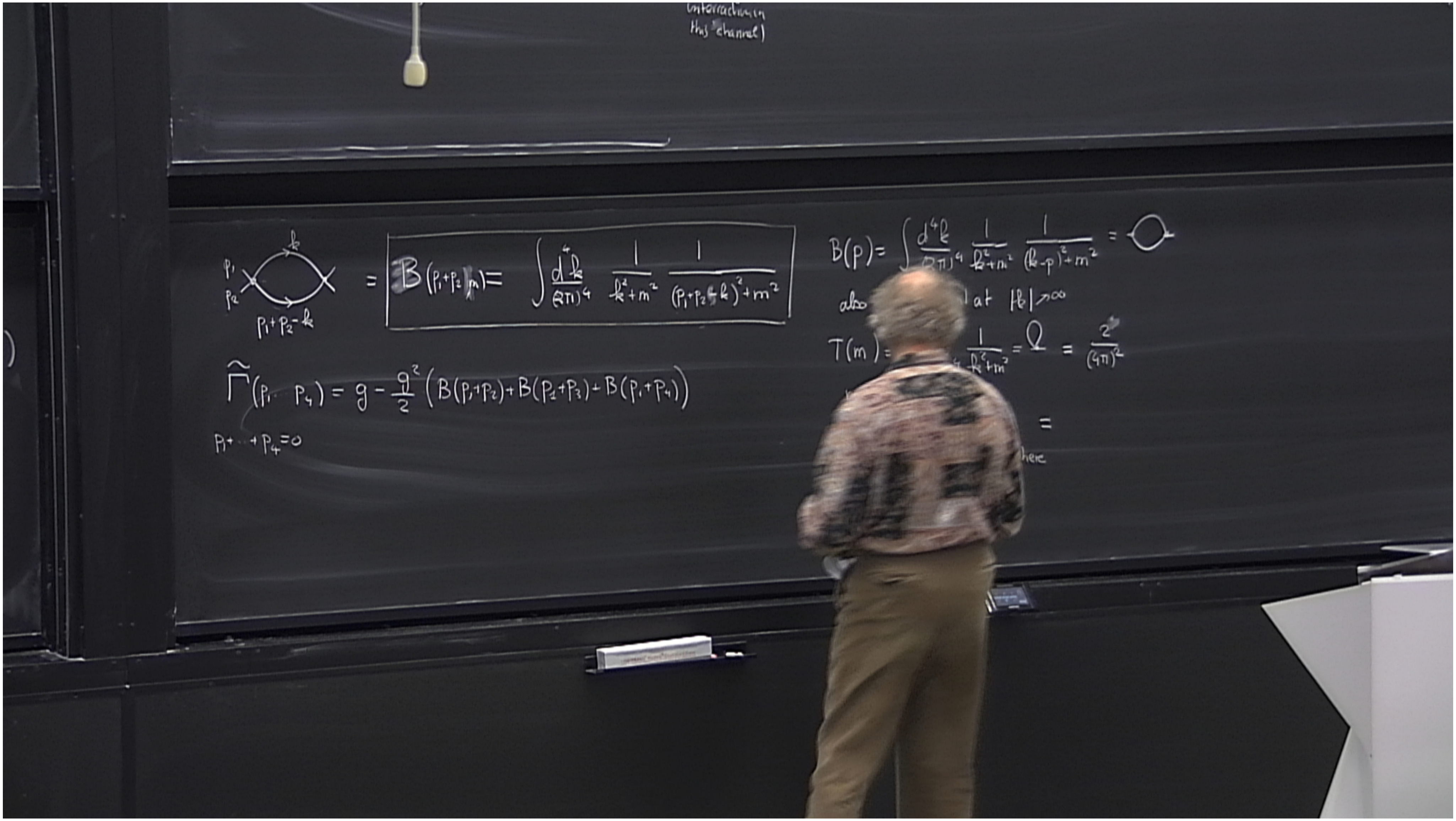

p_1
p_2
k
p_1+p_2-k
$= B(p_1+p_2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+m^2} \frac{1}{(p_1+p_2-k)^2+m^2}$
$\tilde{\Gamma}(p_1 \cdots p_4) = g - \frac{g^2}{2}\left(B(p_1+p_2)+B(p_2+p_3)+B(p_1+p_4)\right)$
$p_1+\cdots+p_4=0$


p1
p2
k
p1+p2-k
= B(p1+p2, m) = ∫ d^4k/(2π)^4 1/(k^2+m^2) 1/((p1+p2-k)^2+m^2)
Γ̃(p1 ⋯ p4) = g − g^2/2 (B(p1+p2)+B(p1+p3)+B(p1+p4))
p1+⋯+p4=0
B(p) = ∫ d^4k

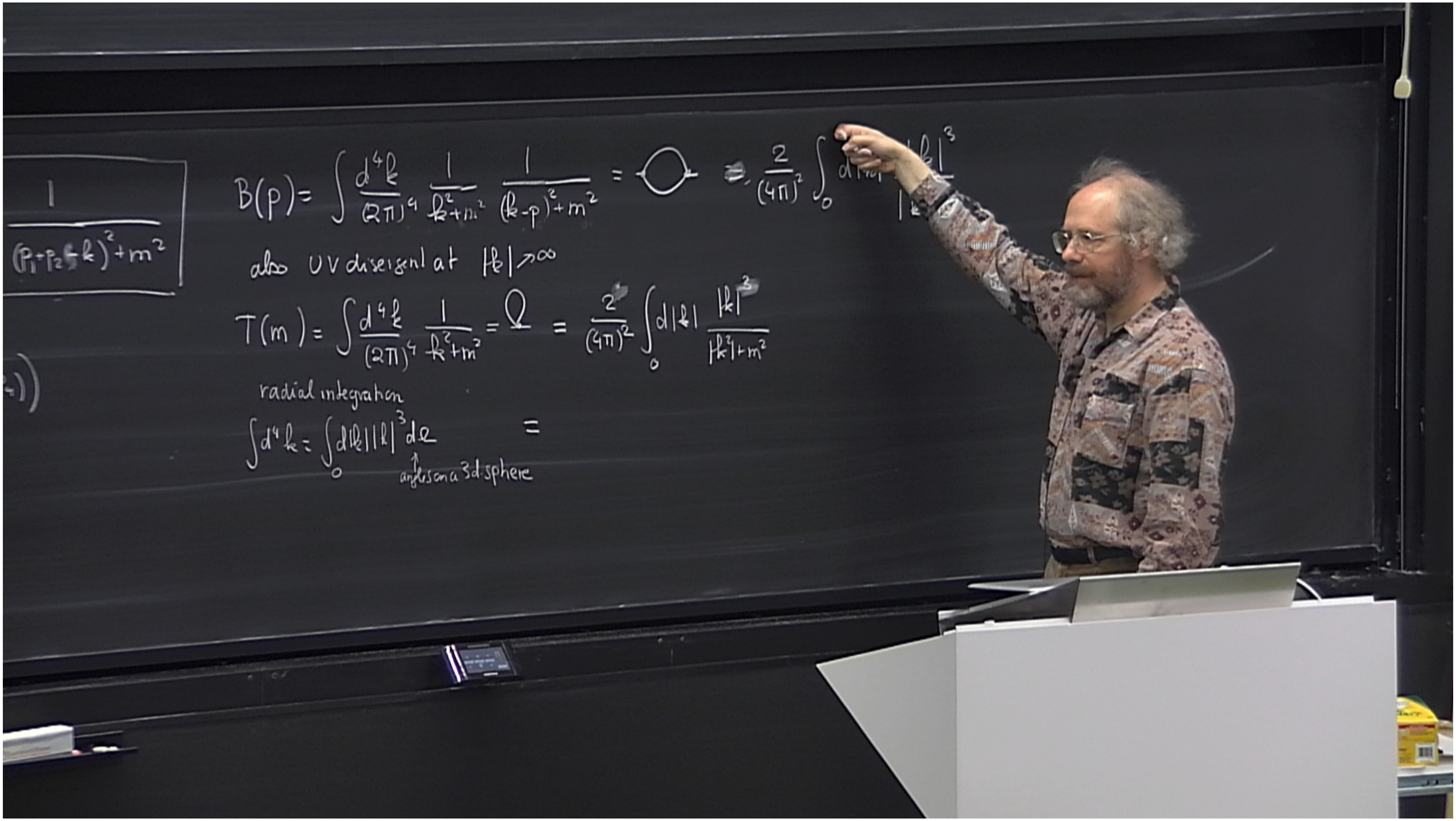


interaction in
this channel)
p1
p2
k
p1+p2−k
= B(p1+p2; m) = ∫ d^4k/(2π)^4 1/(k^2+m^2) 1/((p1+p2−k)^2+m^2)
B(p) = ∫ d^4k/(2π)^4 1/(k^2+m^2) 1/((k−p)^2+m^2)
also UV divergent at |k|→∞
Γ̃(p1 … p4) = g − g^2/2 (B(p1+p2)+B(p2+p3)+B(p1+p4))
p1+…+p4=0

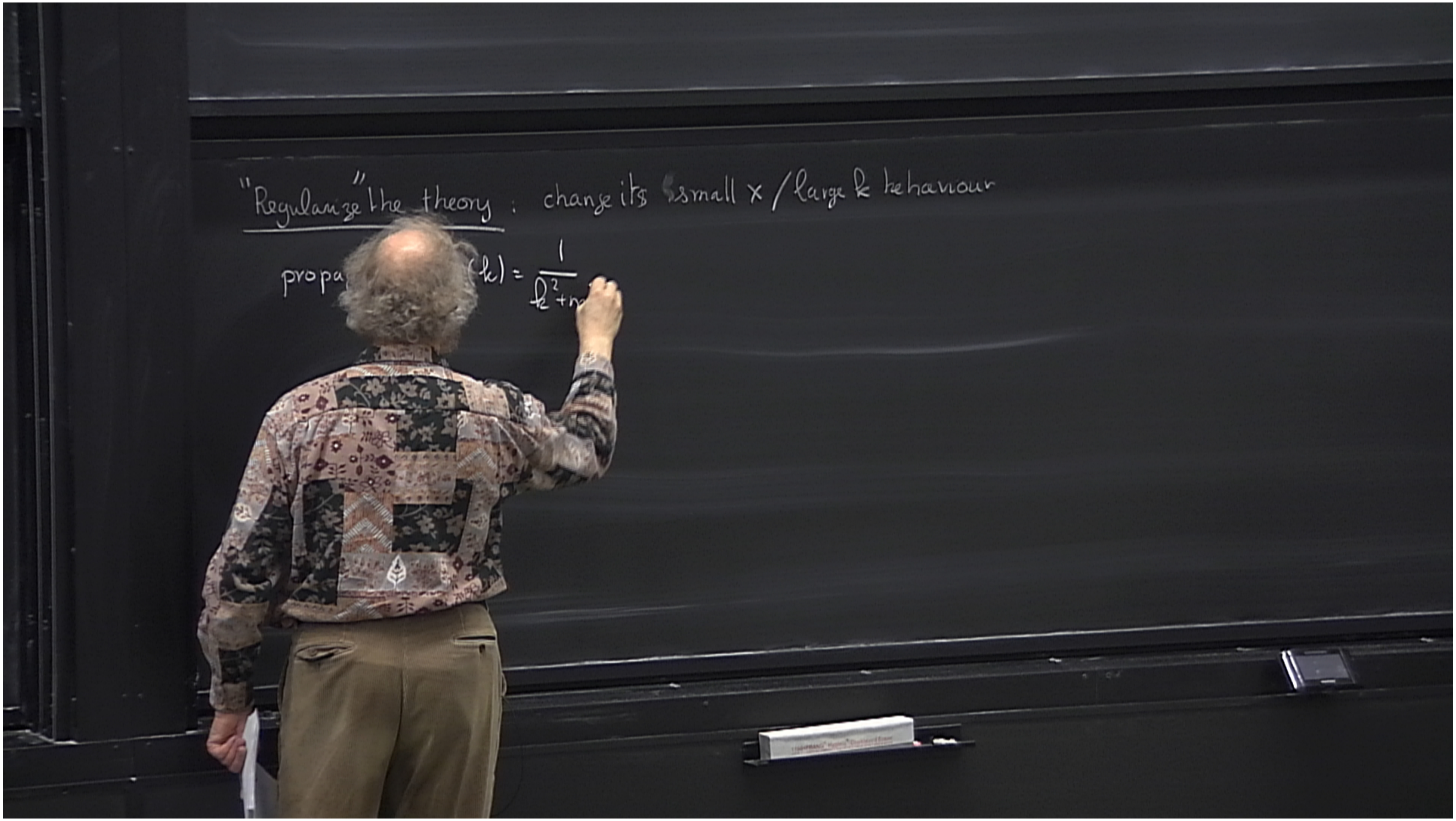

interaction in this channel)
B(p) = ∫ d^4k/(2π)^4 1/(k^2+m^2) 1/((k-p)^2+m^2) =
also UV divergent at |k|→∞
T(m) = ∫ d^4k/(2π)^4 1/(k^2+m^2) =
= B(p1+p2, m) = ∫ d^4k/(2π)^4 1/(k^2+m^2) 1/((p1+p2-k)^2+m^2)
p1+p2-k
Γ̃(p1 ... p4) = g - g^2/2 (B(p1+p2) + B(p2+p3) + B(p1+p4))
p1+ ... +p4=0

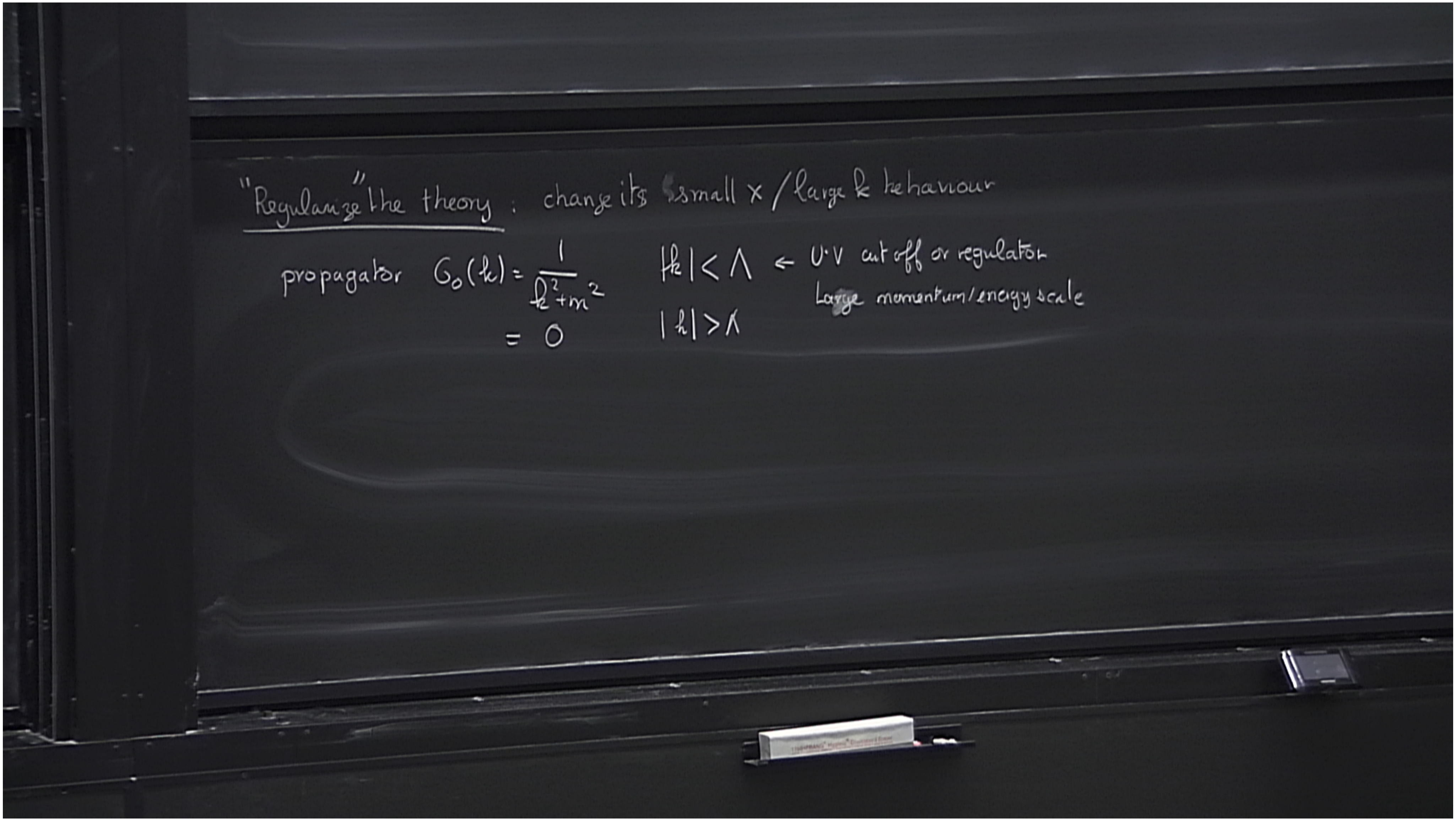


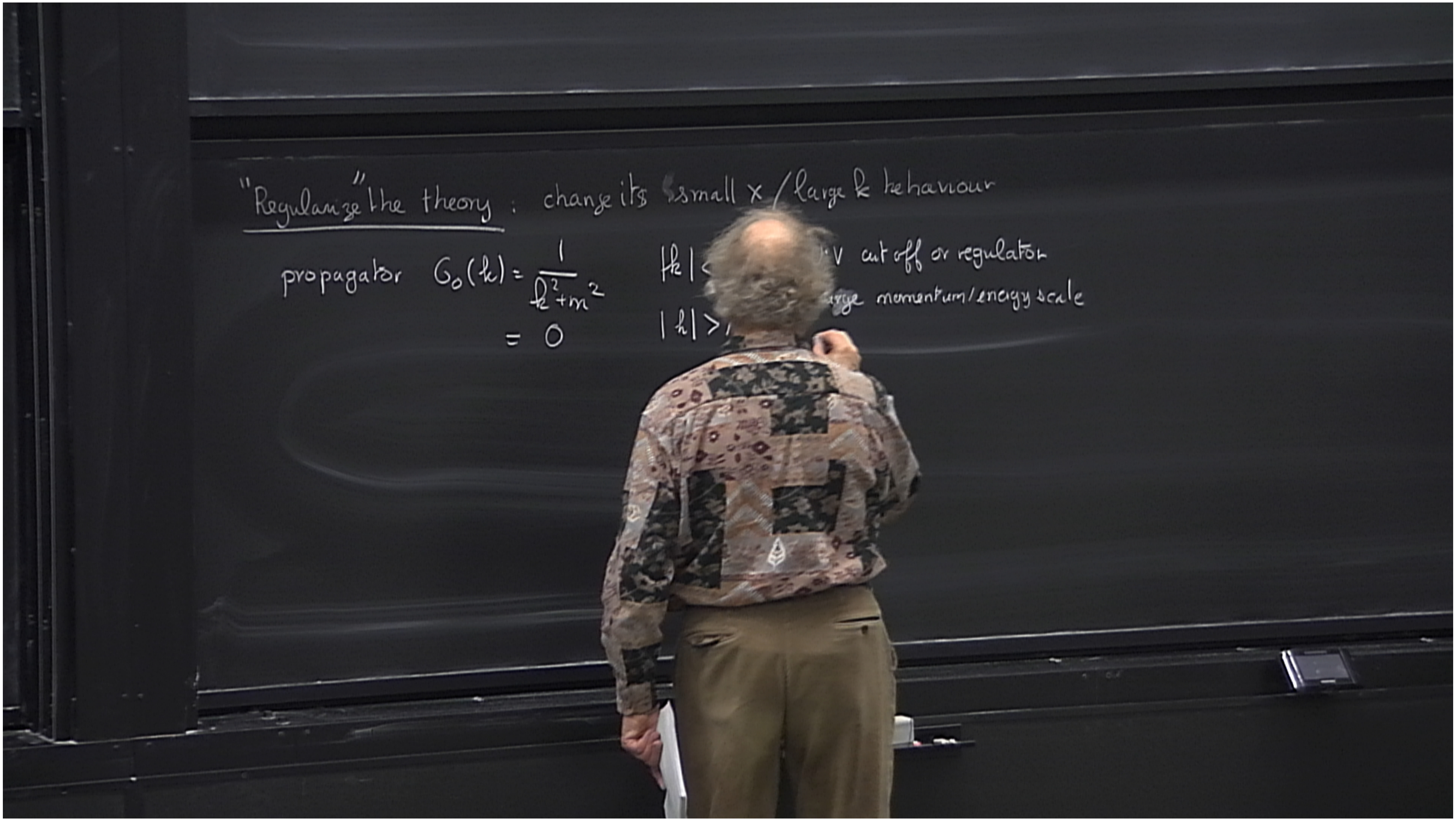
B(p₁+p₂, m) = ∫ d⁴k/(2π)⁴ 1/(k²+m²) 1/((p₁+p₂−k)²+m²)
Γ̃(p₁ … p₄) = g − g²/2 (B(p₁+p₂) + B(p₂+p₃) + B(p₁+p₄))
p₁+ … +p₄=0
B(p) = ∫ d⁴k/(2π)⁴ 1/(k²+m²) 1/((k−p)²+m²)
at |k|→∞
T(m)

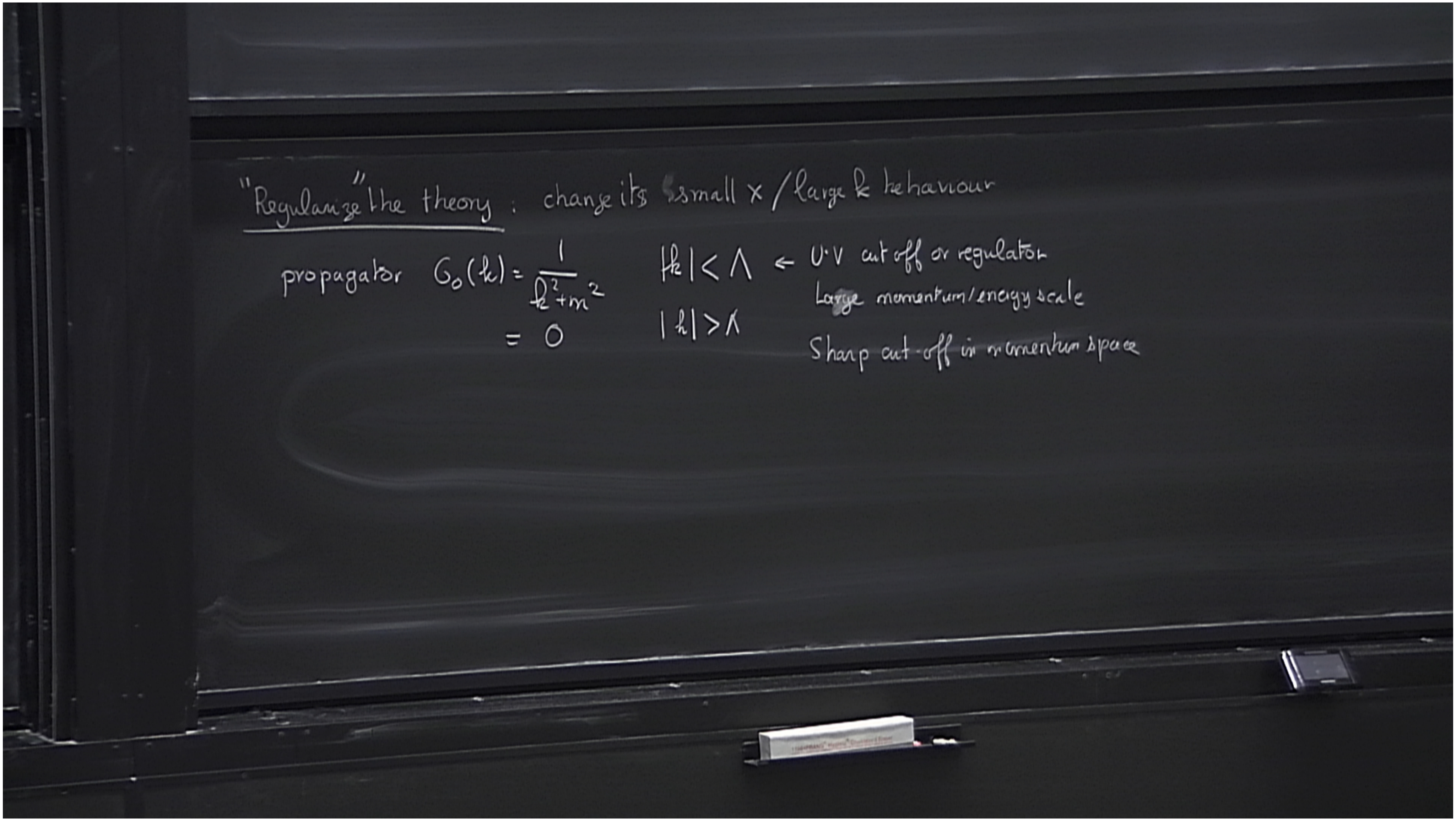


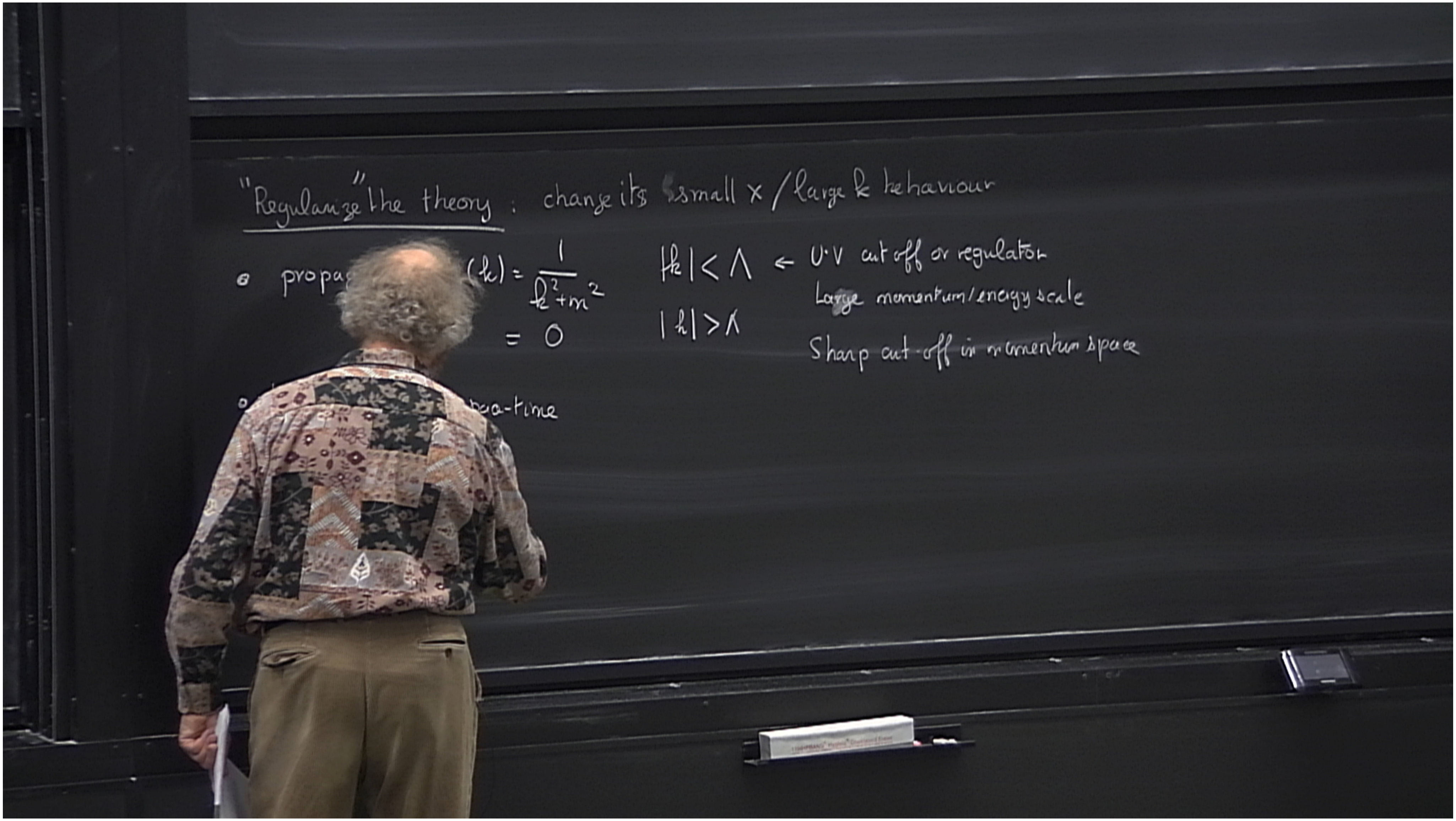
B(p) = ∫ d⁴k/(2π)⁴ 1/(k²+m²) 1/((k-p)²+m²)
also UV divergent at |k| → ∞
T(m) = ∫ d⁴k/(2π)⁴ 1/(k²+m²)
radial integration
angles on a 3d sphere

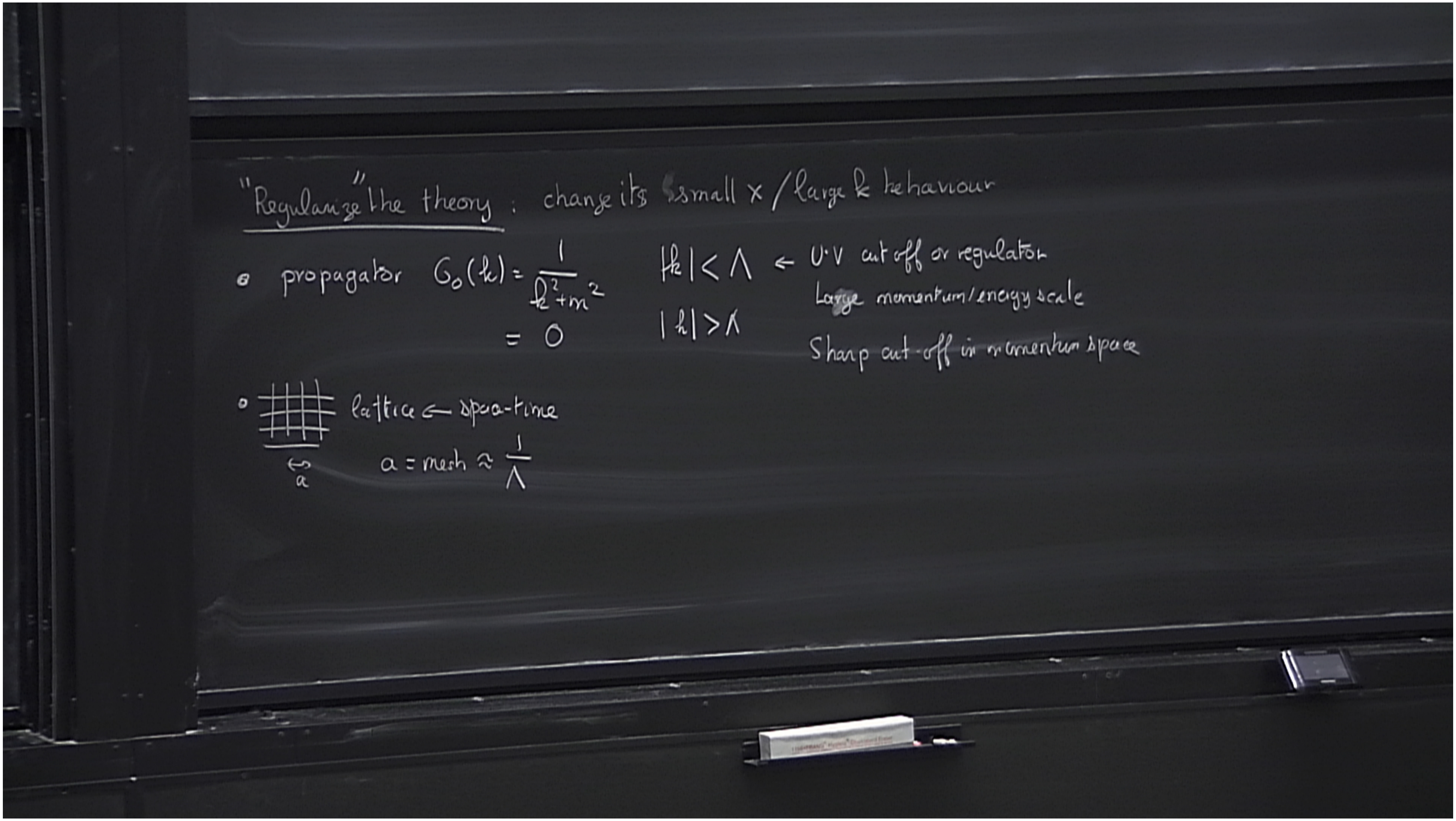

"Regularize" the theory : change its small x / large k behaviour
propa
(k) =
1
k² + m

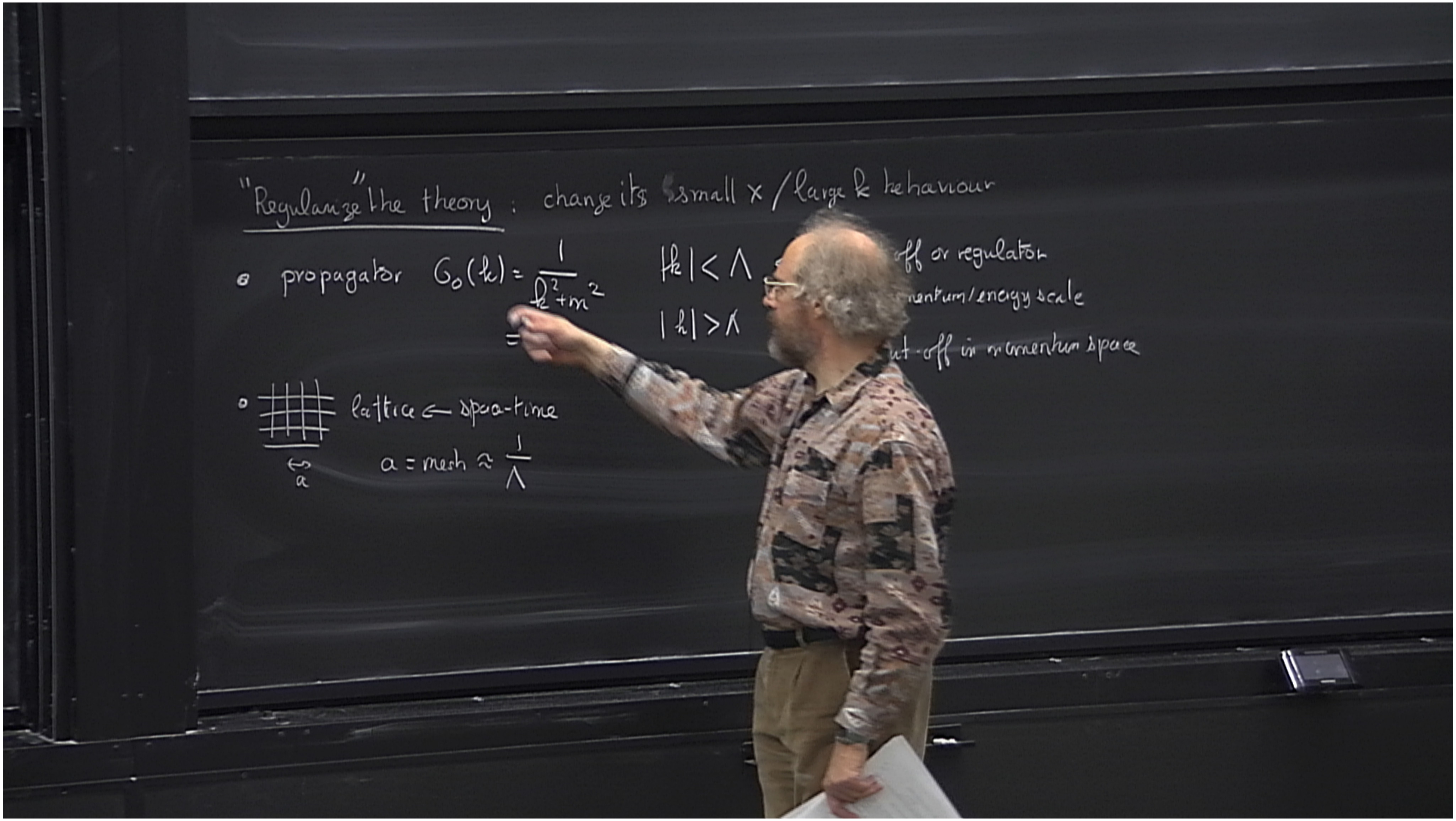

"Regularize" the theory : change its small x / large k behaviour
propagator $G_0(k) = \frac{1}{k^2+m^2}$ $|k| < \Lambda$ ← U.V cut off or regulator
Large momentum/energy scale
$= 0$ $|k| > \Lambda$

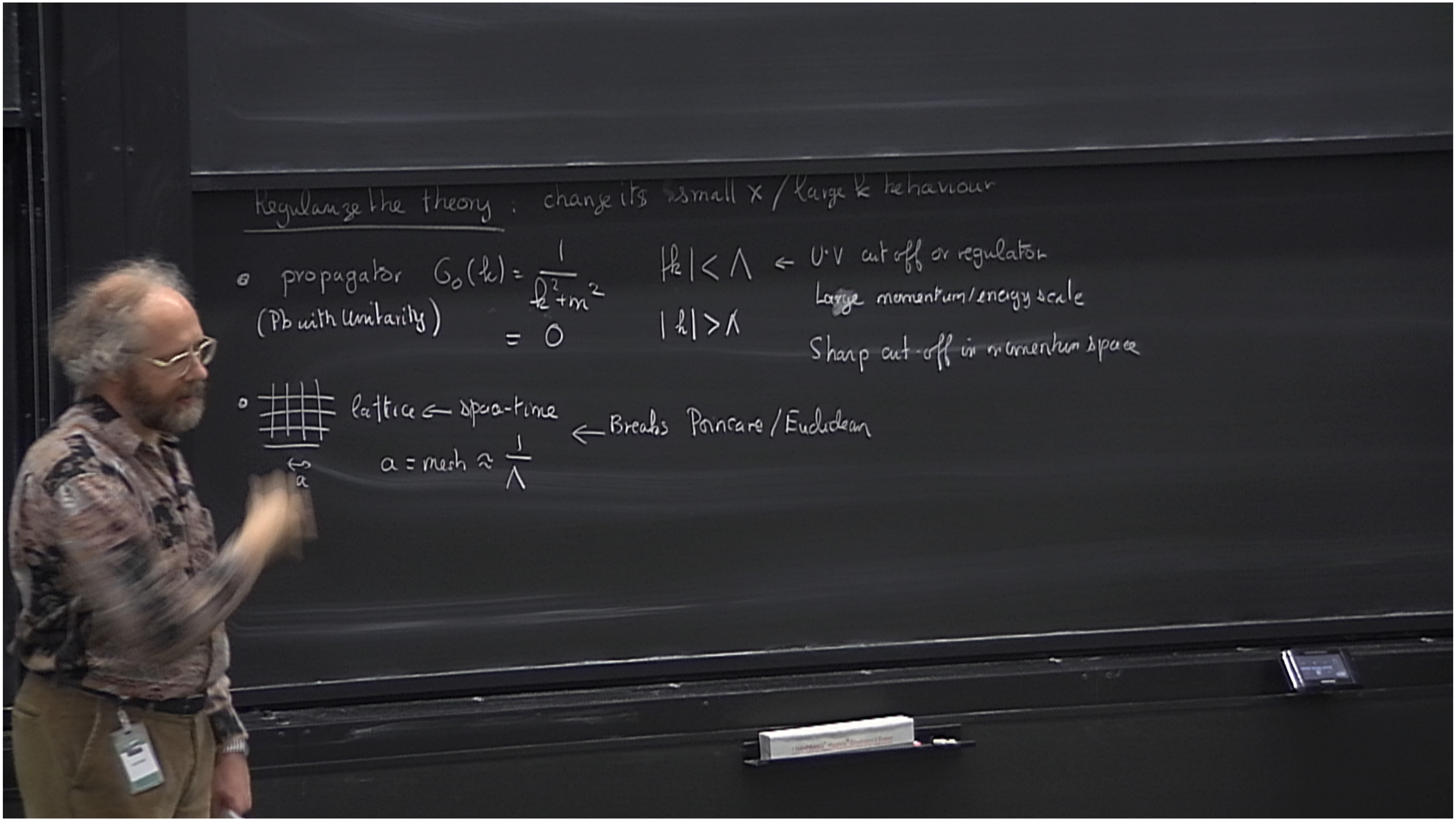

"Regularize" the theory : change its small x / large k behaviour
propagator $G_0(k) = \frac{1}{k^2+m^2}$
$|k| <$
cut off or regulator
momentum/energy scale
$= 0$
$|k| >$

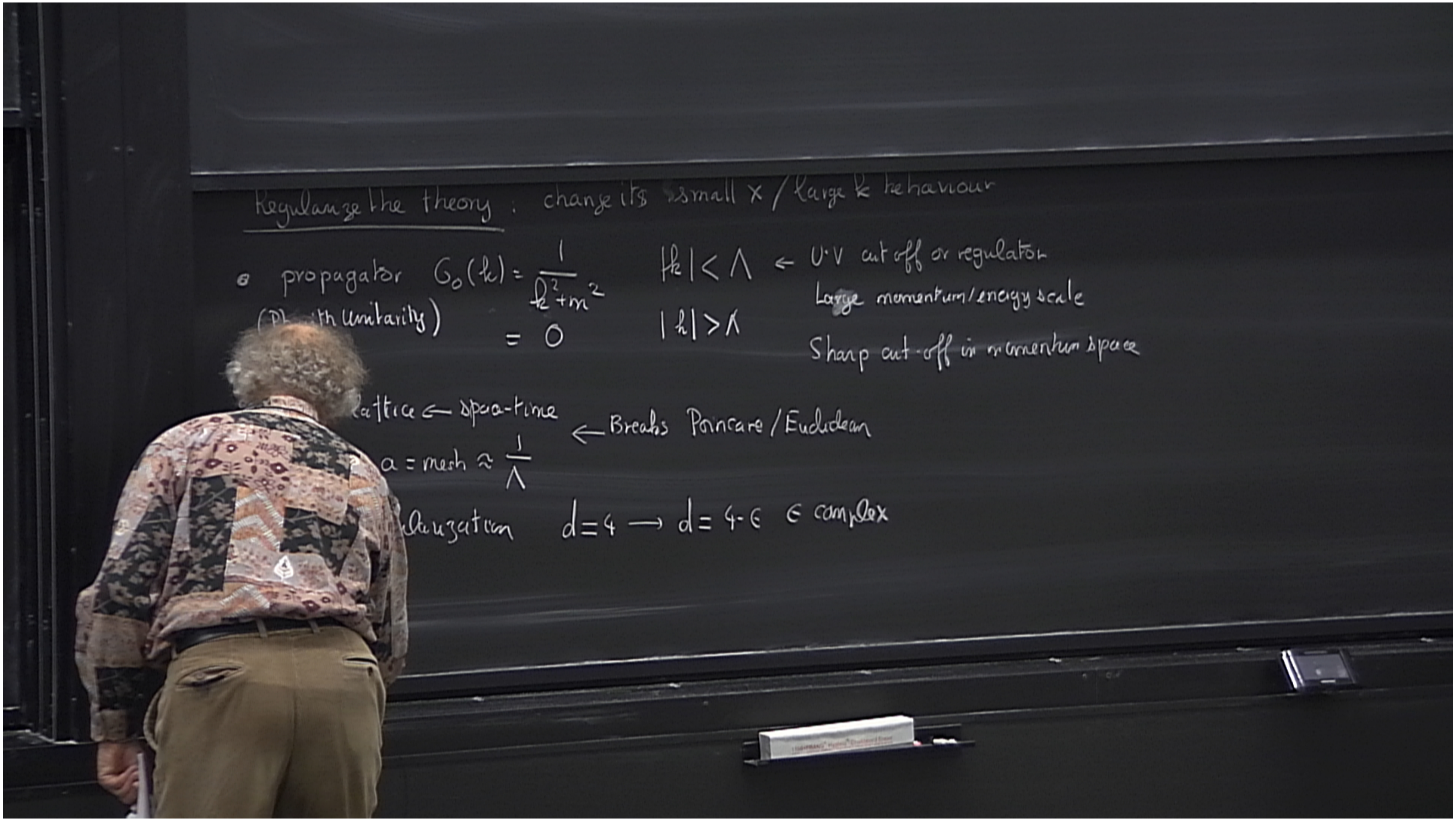

"Regularize" the theory : change its small x / large k behaviour
propagator $G_0(k) = \frac{1}{k^2+m^2}$ $|k| < \Lambda$ ← U.V cut off or regulator
Large momentum/energy scale
$= 0$ $|k| > \Lambda$
Sharp cut-off in momentum space


"Regularize" the theory : change its small x / large k behaviour
propa
(k) = 1 / k2+m2
= 0
|k| < Λ ← U.V cut off or regulator
Large momentum/energy scale
|k| > Λ
Sharp cut-off in momentum space
pace-time

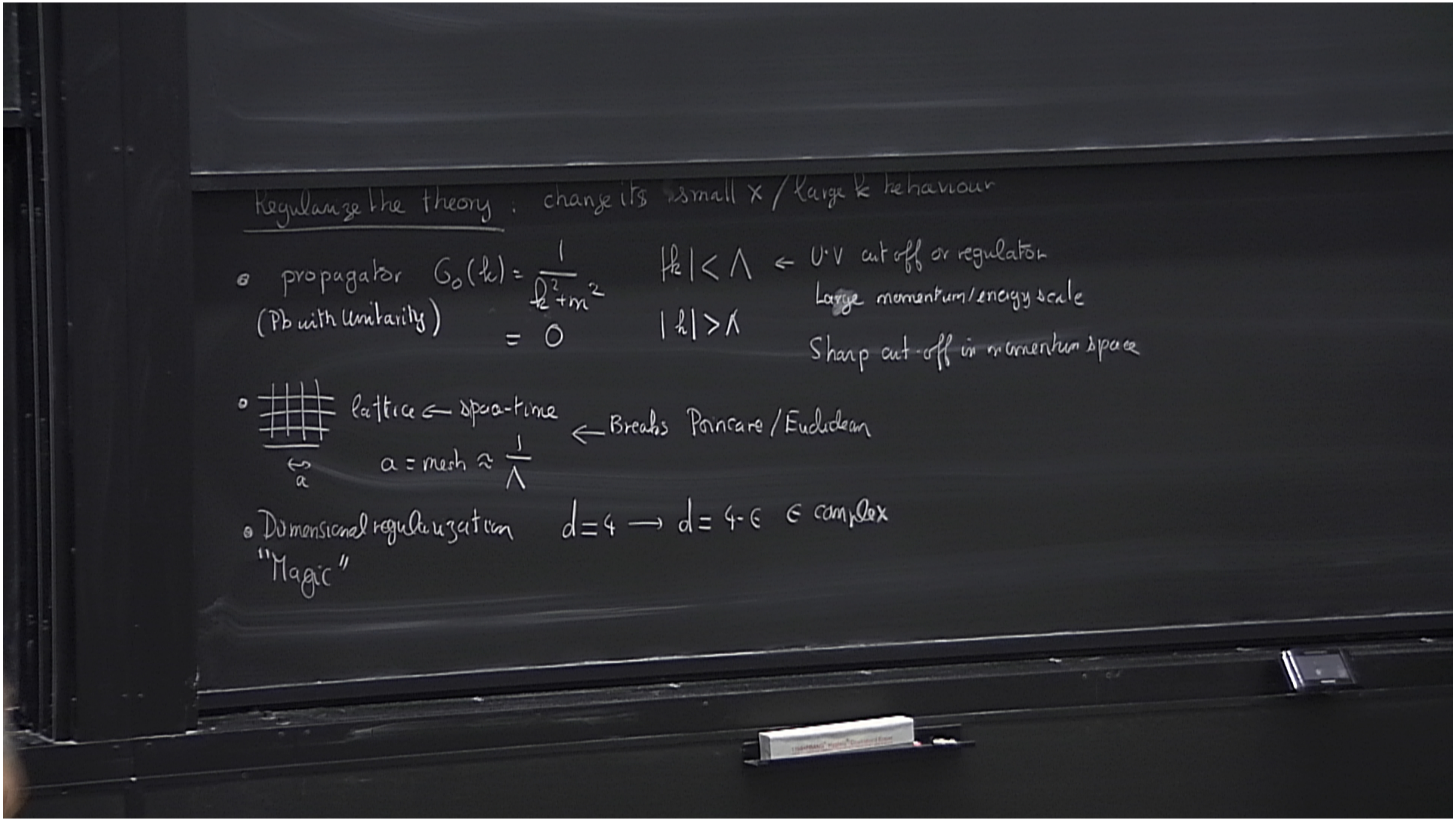

"Regularize" the theory : change its small x / large k behaviour
propagator G0(k) = 1/(k^2+m^2)
= 0
|k| < Λ ← U.V cut off or regulator
Large momentum/energy scale
|k| > Λ
Sharp cut-off in momentum space
lattice ← space-time
a
a = mesh ≈ 1/Λ

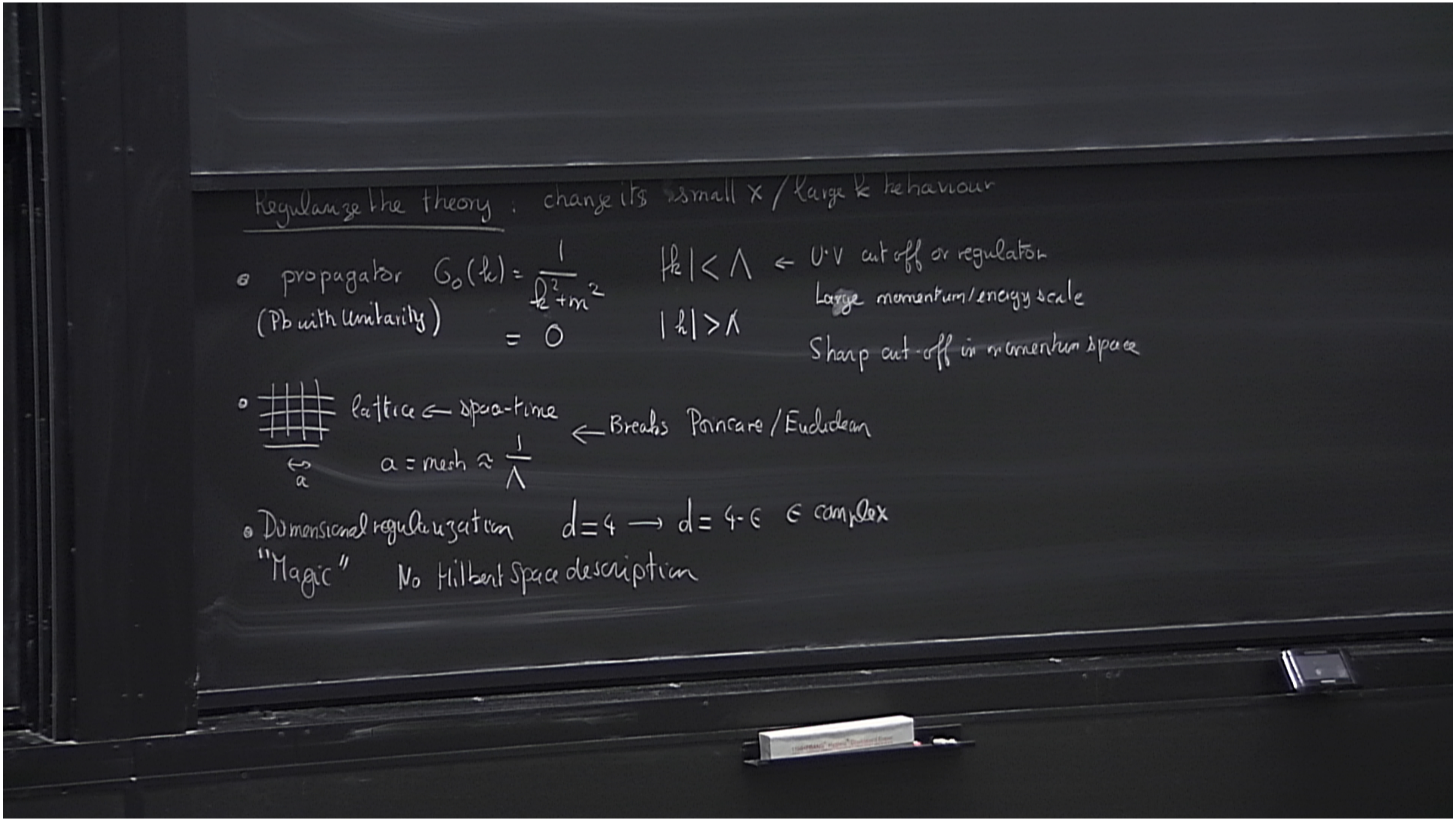

"Regularize" the theory : change its small x / large k behaviour
propagator $G_0(k) = \frac{1}{k^2+m^2}$
$|k| < \Lambda$
ff or regulator
nentum/energy scale
$|k| > \Lambda$
ut-off in momentum space
lattice ← space-time
a = mesh $\approx \frac{1}{\Lambda}$

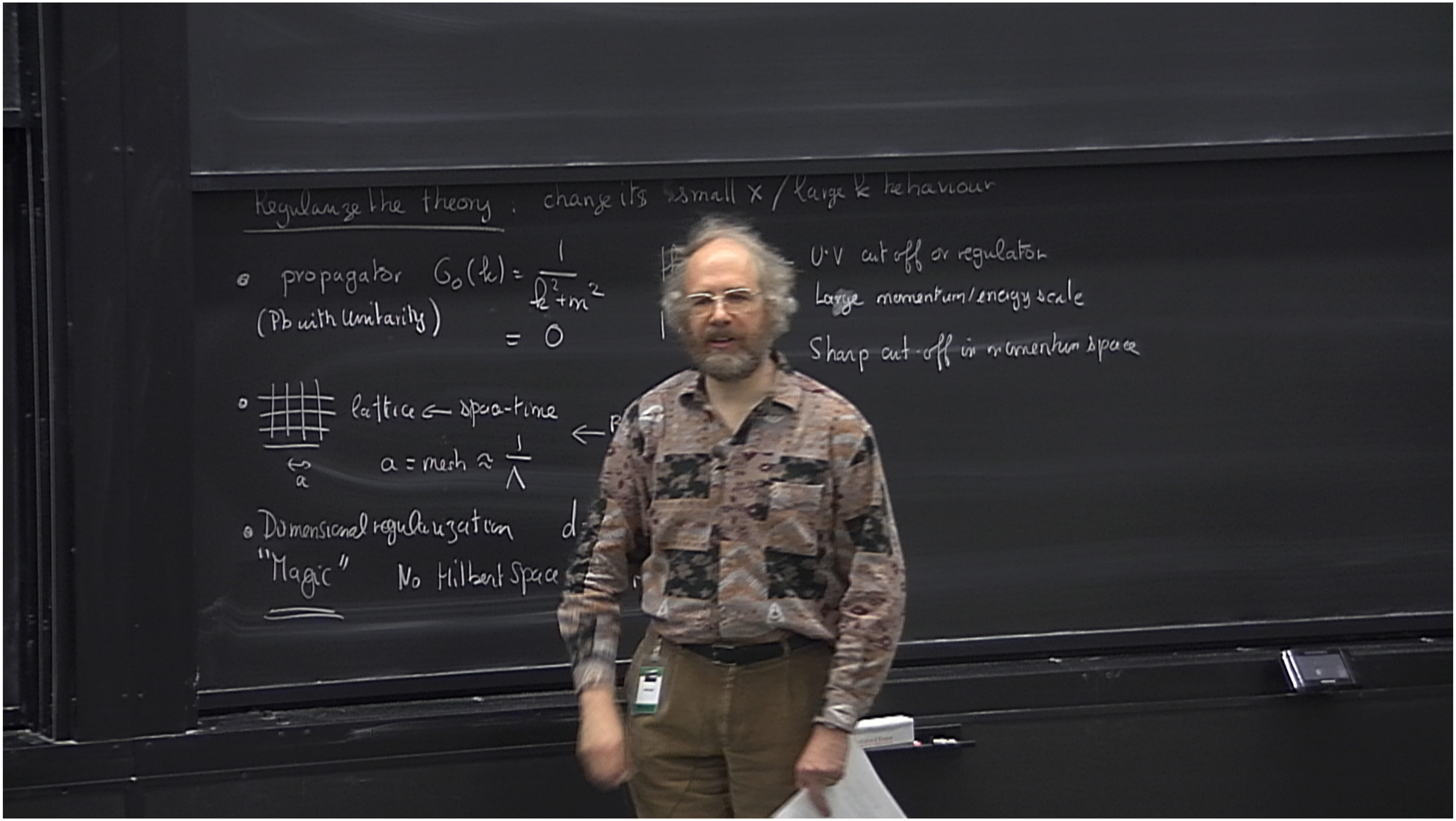

Regularize the theory : change its small x / large k behaviour
propagator $G_0(k) = \frac{1}{k^2+m^2}$
(Pb with Unitarity)
$= 0$
$|k| < \Lambda$ ← U.V cut off or regulator
Large momentum/energy scale
$|k| > \Lambda$
Sharp cut-off in momentum space
lattice ← space-time
← Breaks Poincare / Euclidean
$a = \text{mesh} \approx \frac{1}{\Lambda}$
a

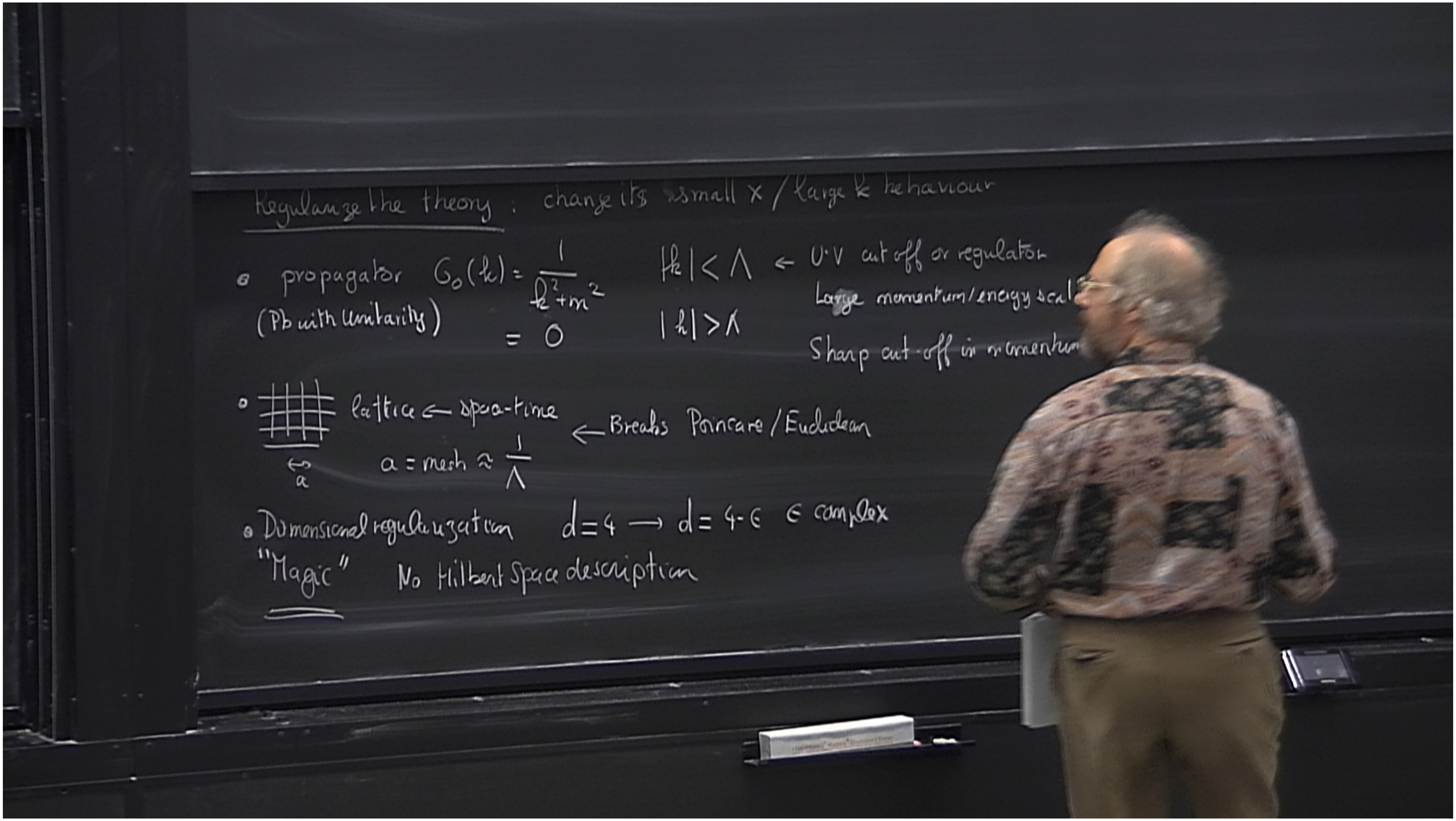


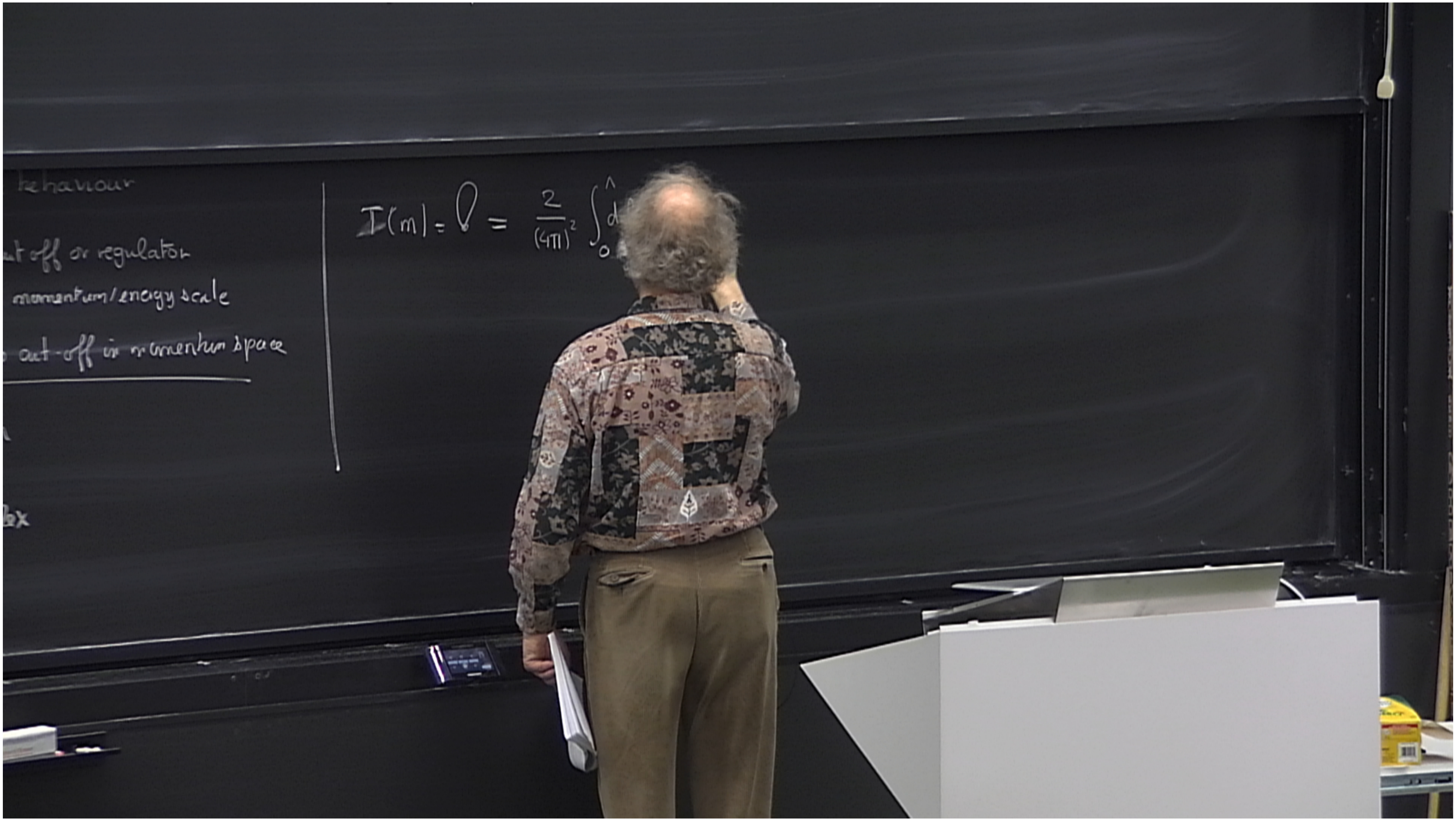
Regularize the theory : change its small x / large k behaviour
propagator G₀(k) = 1 / k² + m²
= 0
|k| < Λ ← U·V cut off or regulator
|k| > Λ
Large momentum/energy scale
Sharp cut-off in momentum space
lattice ← space-time
← Breaks Poincare / Euclidean
a = mesh ≈ 1/Λ
d=4 ⟶ d= 4-ϵ ϵ complex

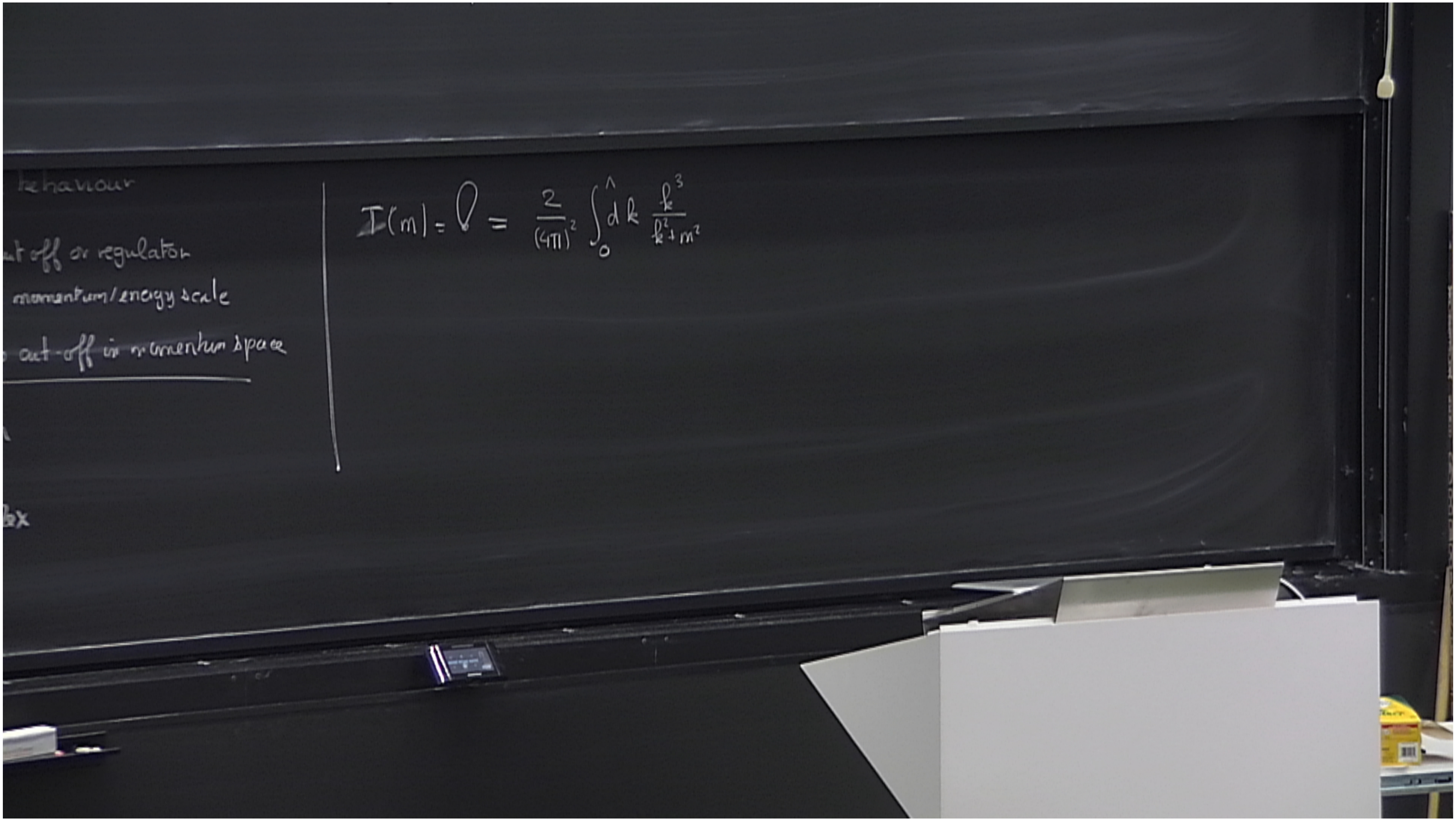

Regularize the theory : change its small x / large k behaviour
• propagator $G_0(k) = \frac{1}{k^2+m^2}$
(Pb with Unitarity)
$= 0$
$|k| < \Lambda$ ← U.V cut off or regulator
Large momentum/energy scale
$|k| > \Lambda$
Sharp cut-off in momentum space
• lattice ← space-time ← Breaks Poincare / Euclidean
a = mesh $\approx \frac{1}{\Lambda}$
• Dimensional regularization $d=4 \longrightarrow d = 4-\epsilon$ ϵ complex
"Magic"

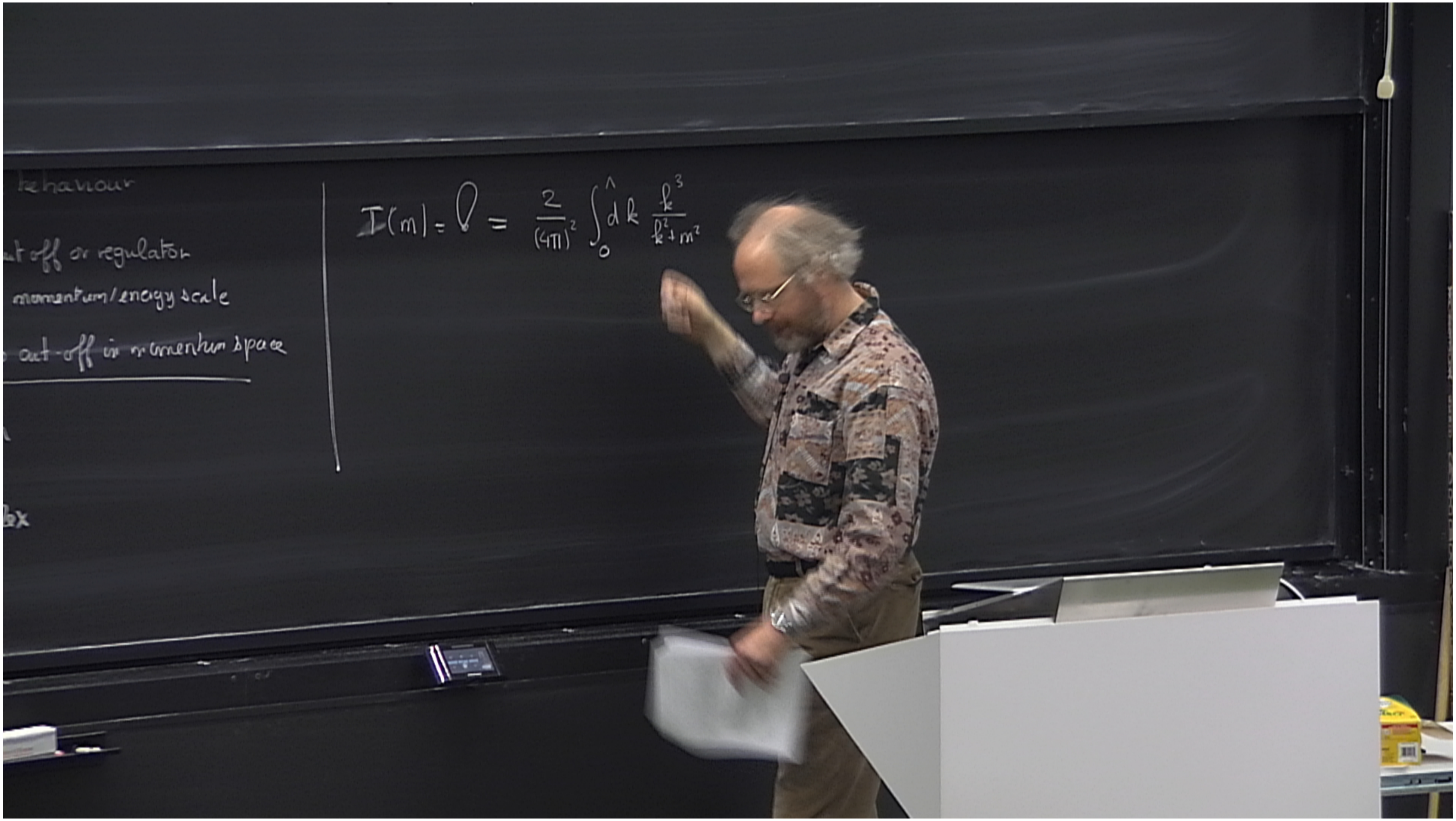

Regularize the theory : change its small x / large k behaviour
• propagator $G_0(k) = \frac{1}{k^2+m^2}$ $|k| < \Lambda$ ← U.V cut off or regulator
(Pb with Unitarity)
Large momentum/energy scale
$= 0$ $|k| > \Lambda$
Sharp cut-off in momentum space
• lattice ← space-time ← Breaks Poincare / Euclidean
a
a = mesh $\approx \frac{1}{\Lambda}$
• Dimensional regularization $d=4 \longrightarrow d = 4-\epsilon$ ϵ complex
"Magic" No Hilbert Space description

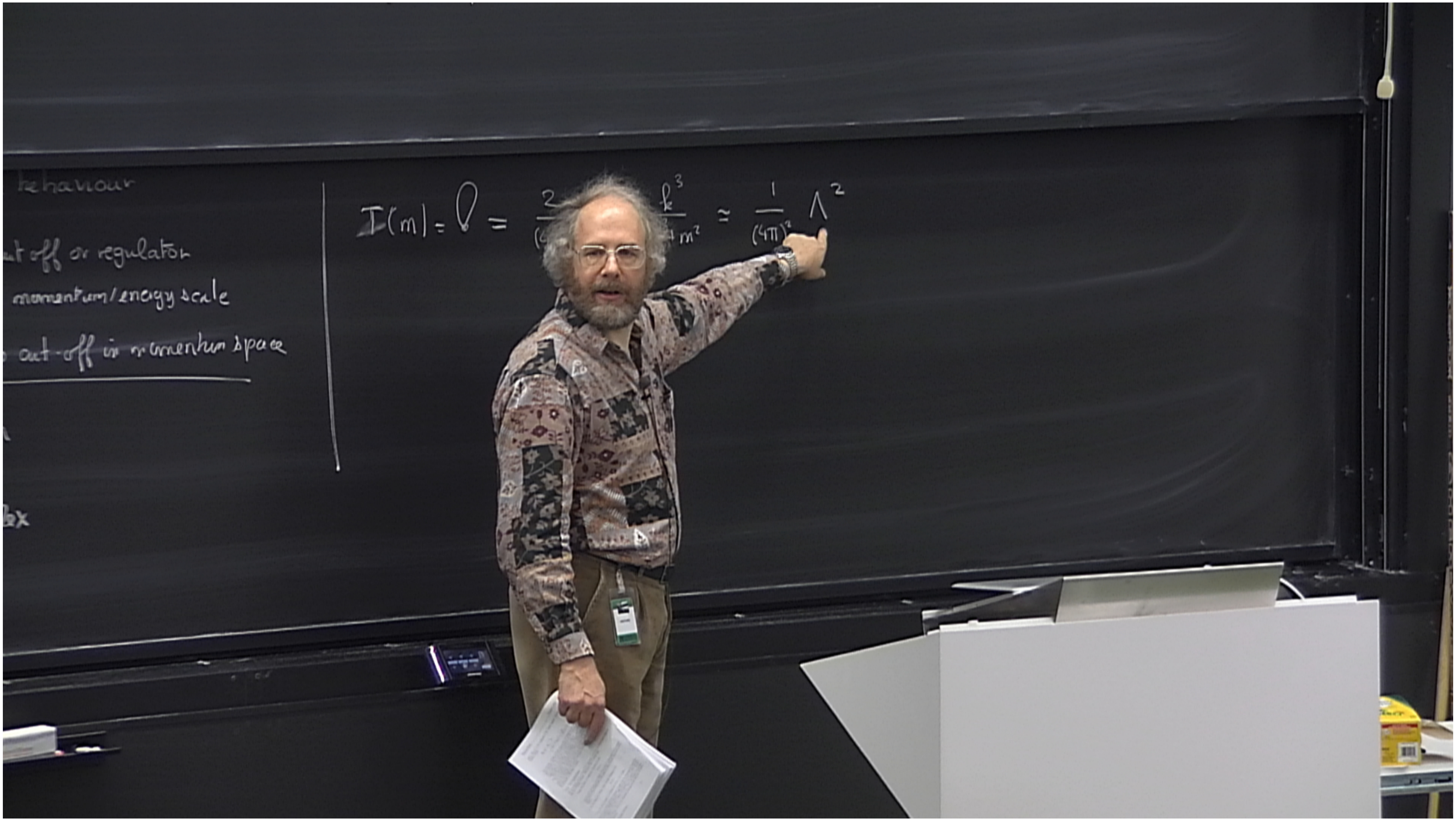

Regularize the theory : change its small x / large k behaviour
propagator $G_0(k) = \frac{1}{k^2+m^2}$
(Pb with Unitarity)
$= 0$
U.V cut off or regulator
Large momentum/energy scale
Sharp cut-off in momentum space
lattice ← space-time
a = mesh $\approx \frac{1}{\Lambda}$
Dimensional regularization
"Magic"
No Hilbert Space

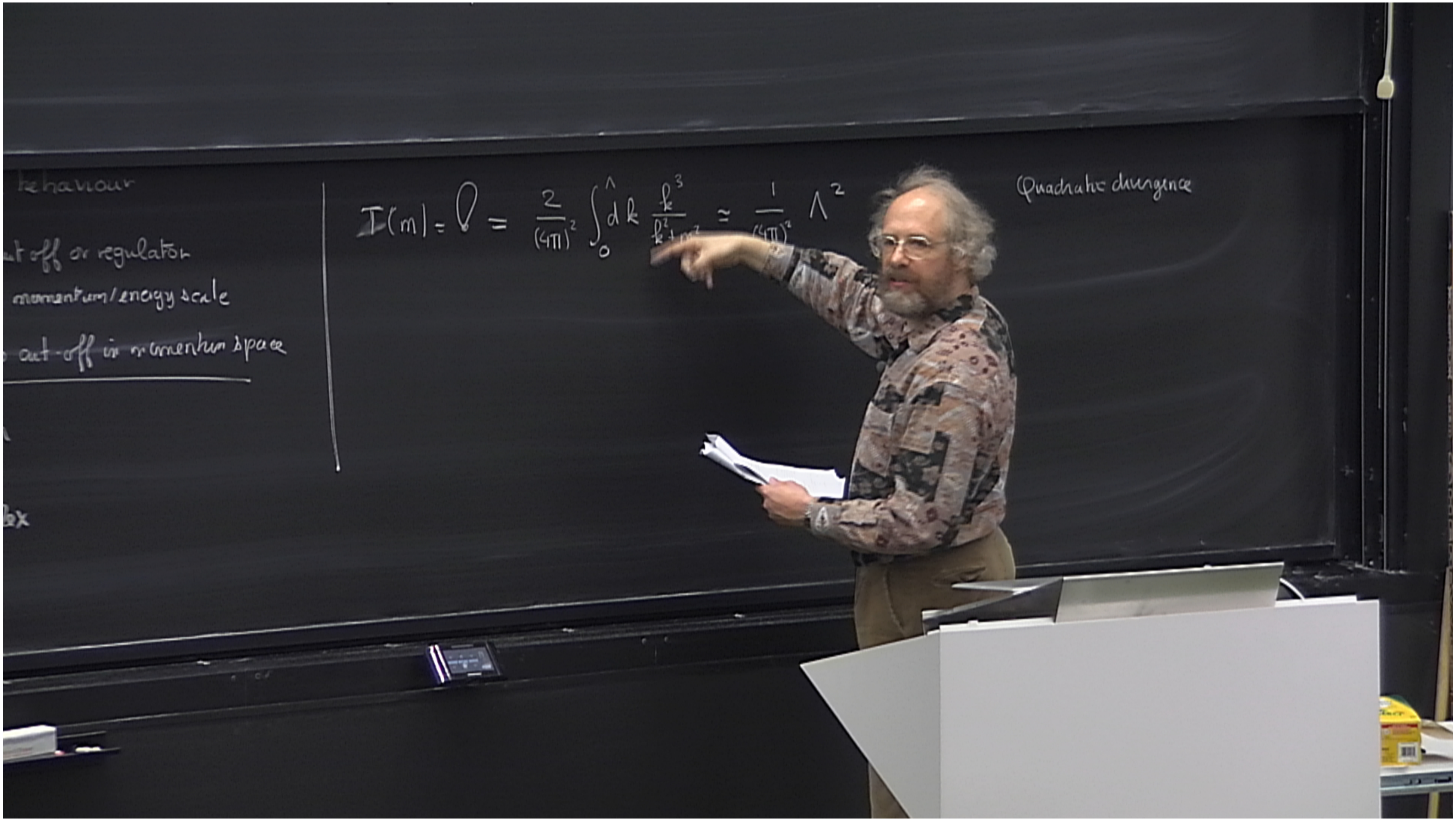

Regularize the theory : change its small x / large k behaviour
propagator G₀(k) = 1 / (k² + m²)
|k| < Λ ← U.V cut off or regulator
Large momentum/energy scal
(Pb with Unitarity)
= 0
|k| > Λ
Sharp cut-off in momentum
lattice ← space-time
← Breaks Poincare / Euclidean
a = mesh ≈ 1/Λ
Dimensional regularization
d = 4 → d = 4 - ε
ε complex
"Magic"
No Hilbert Space description

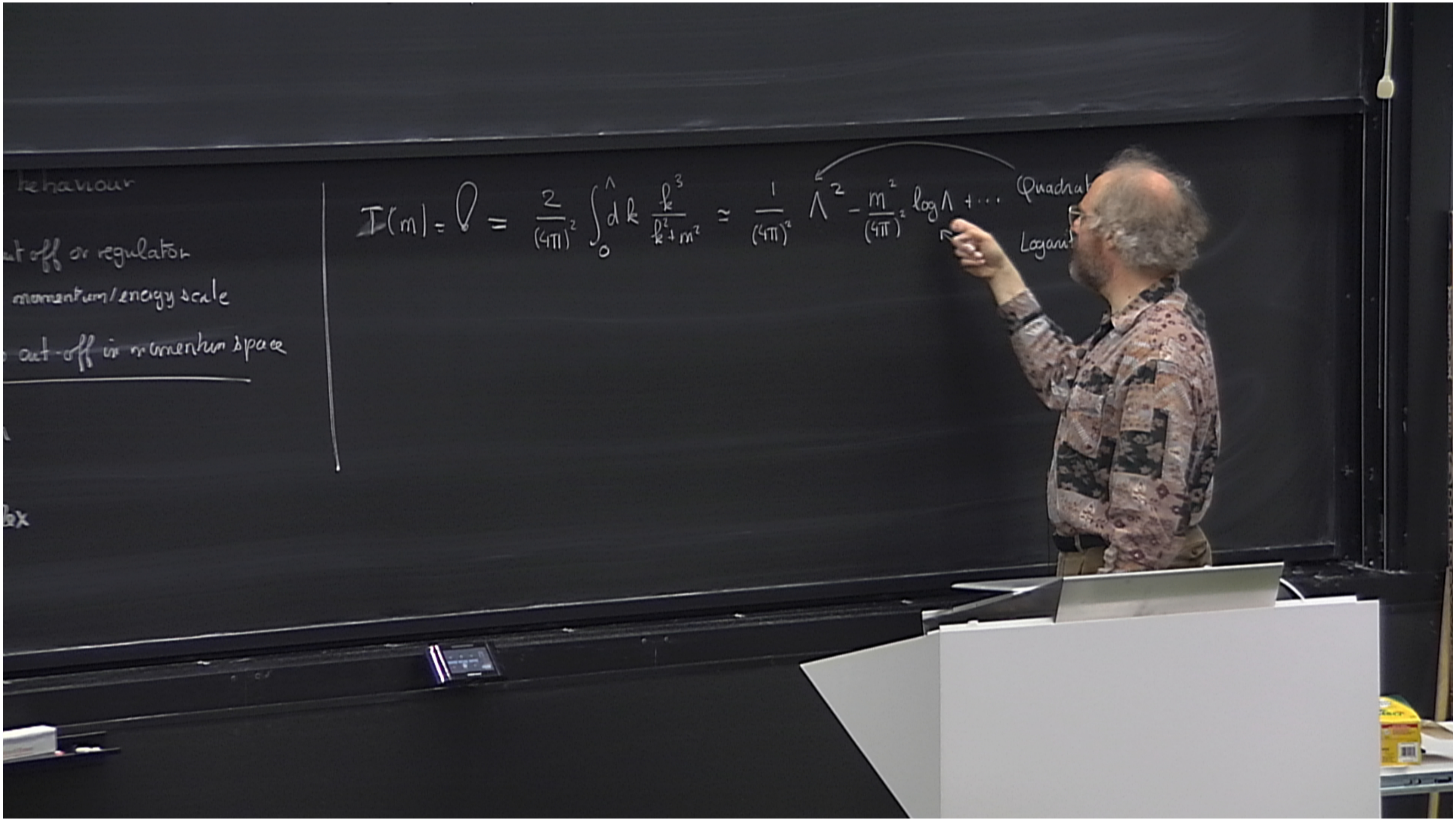

behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space

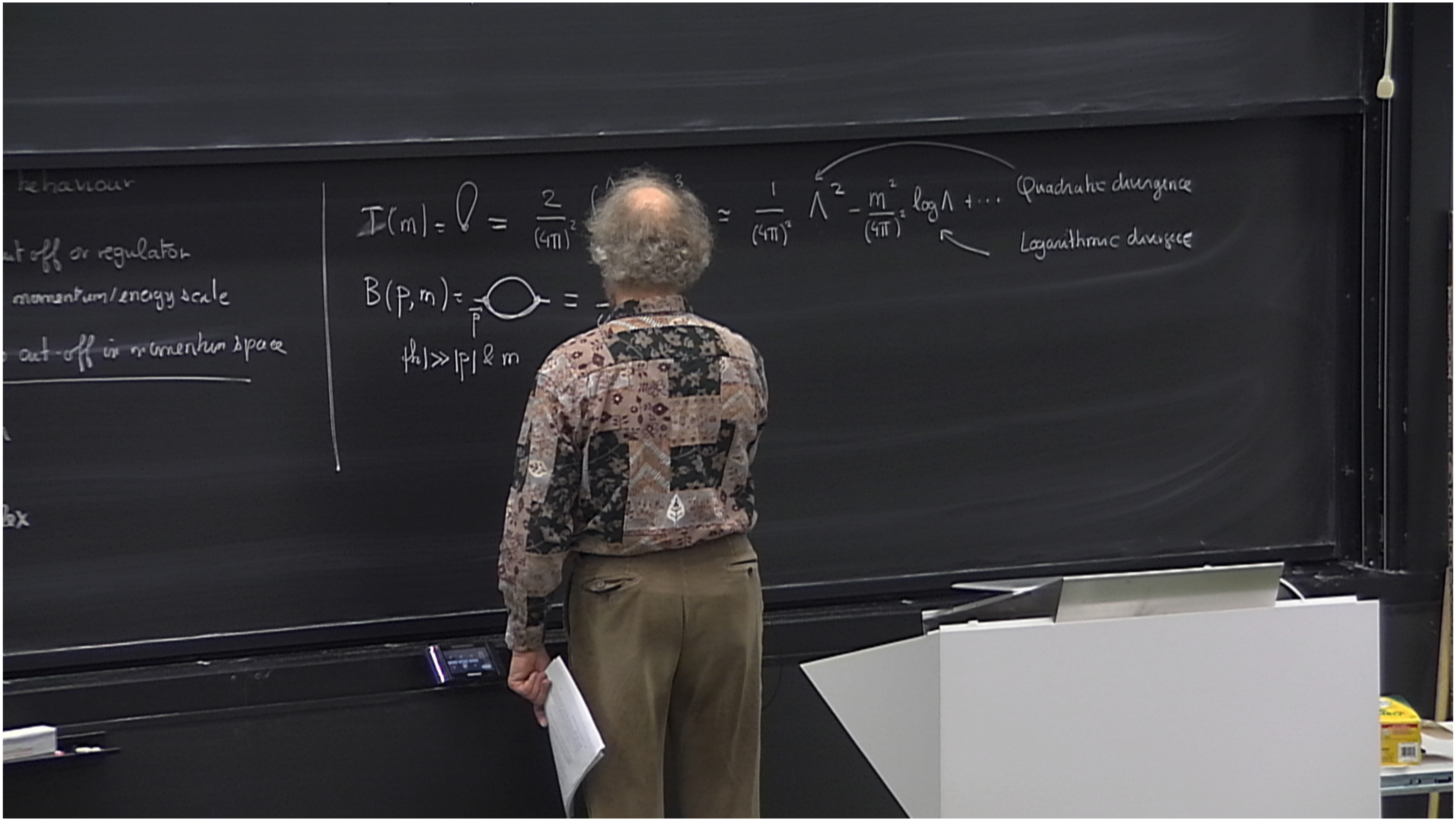

behaviour
off or regulator
momentum/energy scale
out-off in momentum space
I(m) = = 2/(4π)^2 ∫_0^Λ dk k^3/(k^2+m^2)

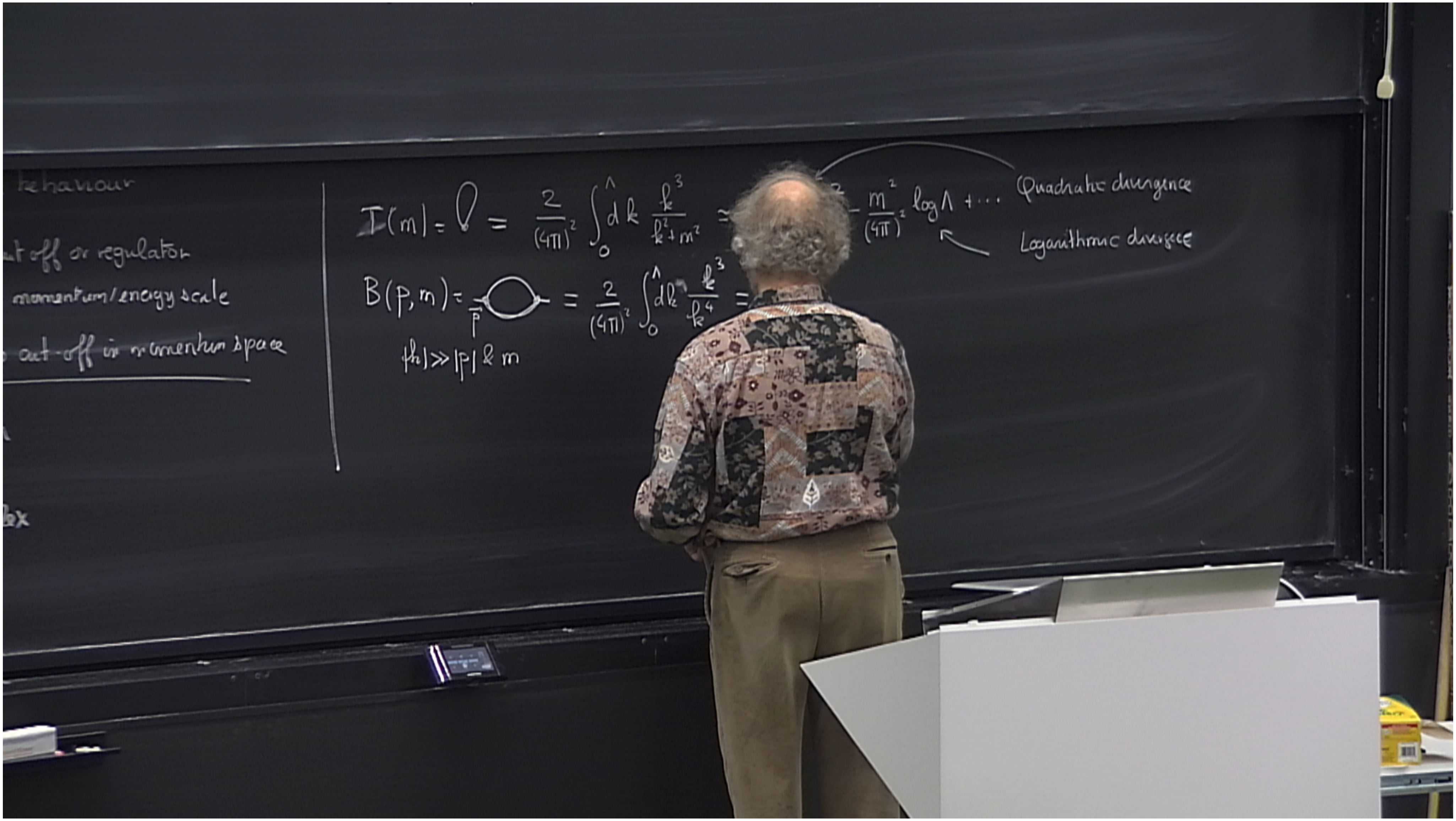

behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space
$I(m) = \frac{2}{(4\pi)^2}\int_0^{\Lambda} dk\, \frac{k^3}{k^2+m^2}$

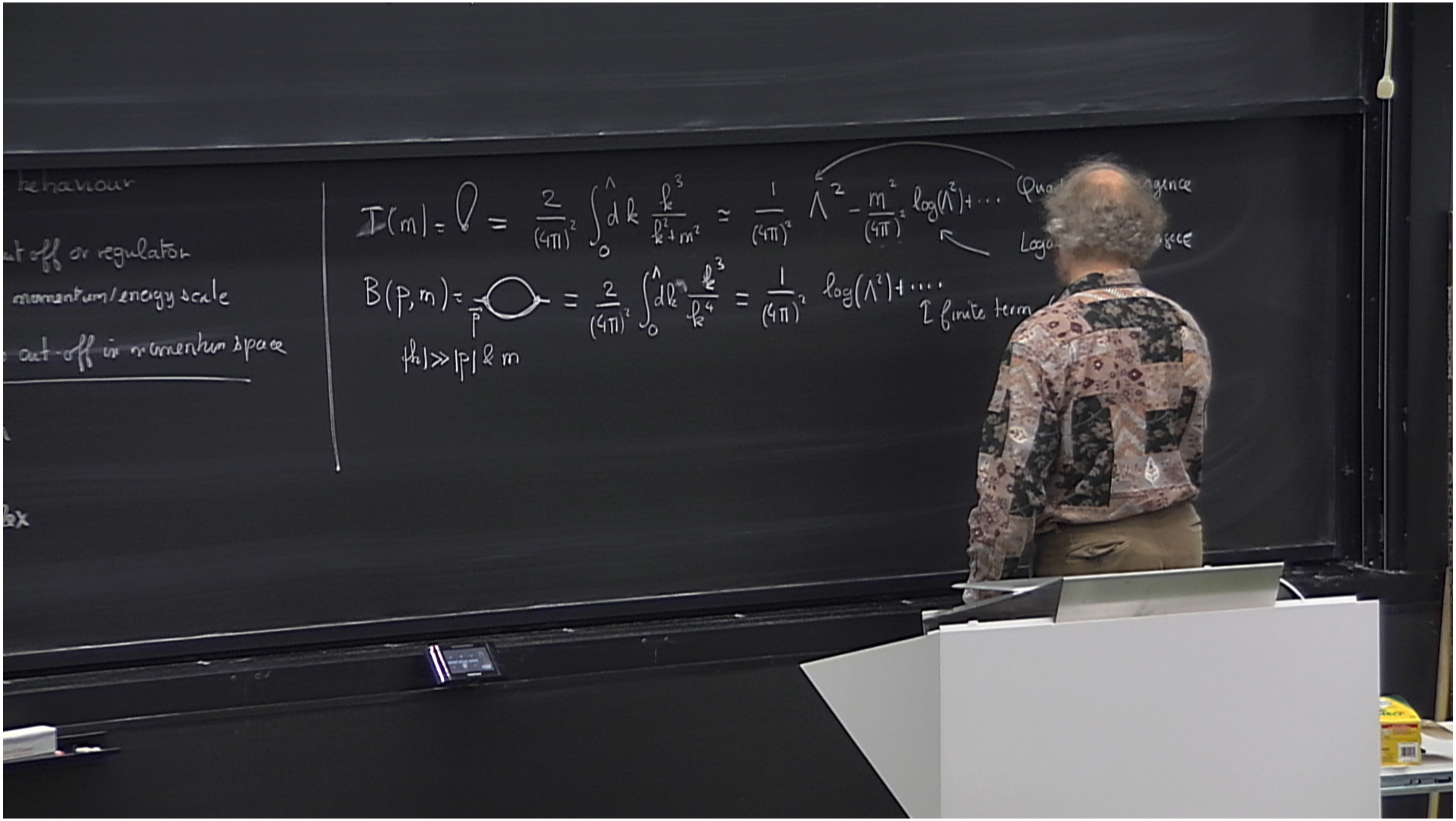

behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space

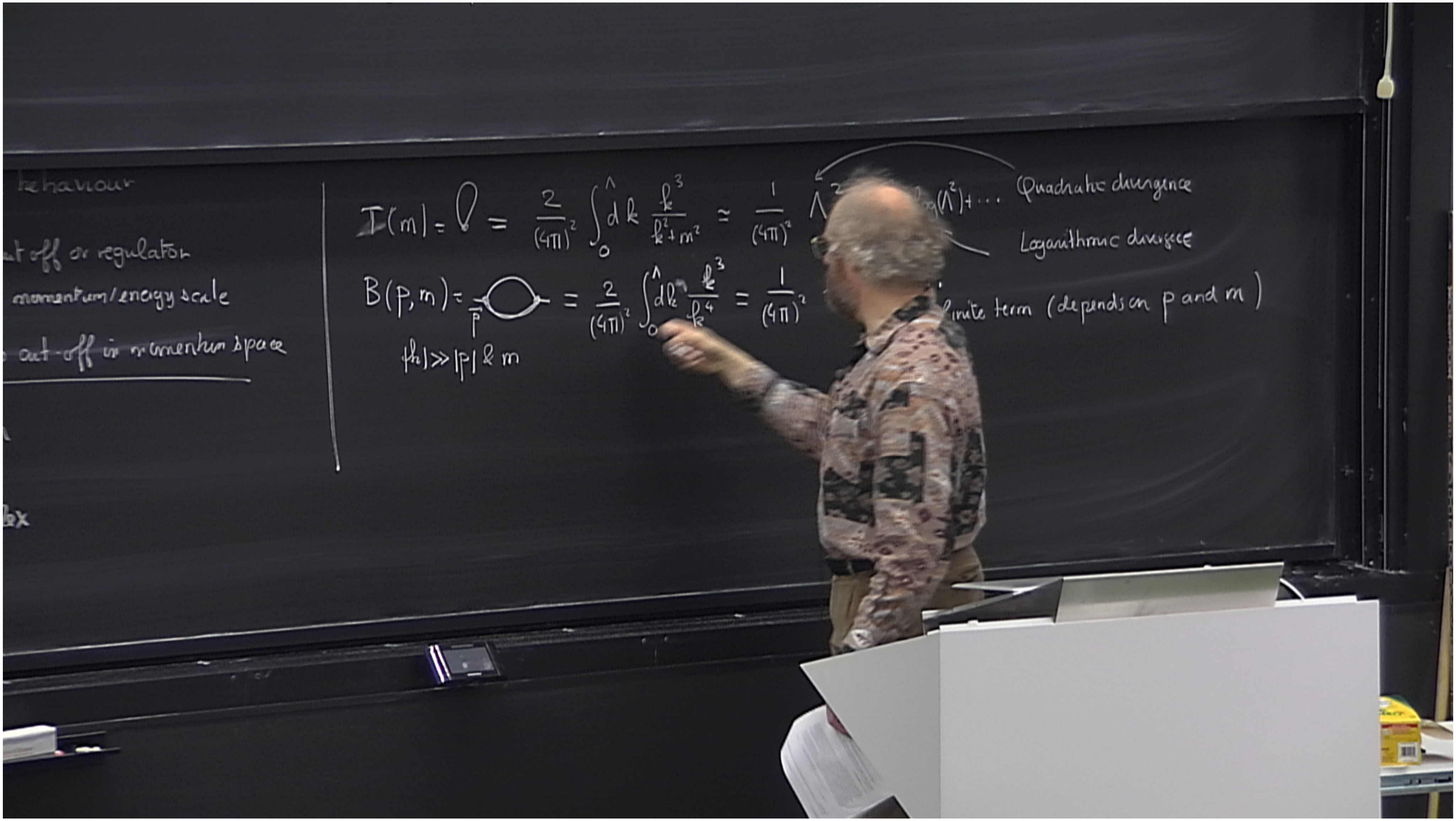

behaviour
ut off or regulator
momentum/energy scale
cut-off in momentum space
Quadratic divergence


behaviour
ut off or regulator
momentum/energy scale
cut-off in momentum space
$I(m) = \frac{2}{(4\pi)^2}\int_0^\Lambda dk \frac{k^3}{k^2+m^2} \simeq \frac{1}{(4\pi)^2}\Lambda^2 - \frac{m^2}{(4\pi)^2}\log\Lambda + \cdots$
Quadrat
Logari

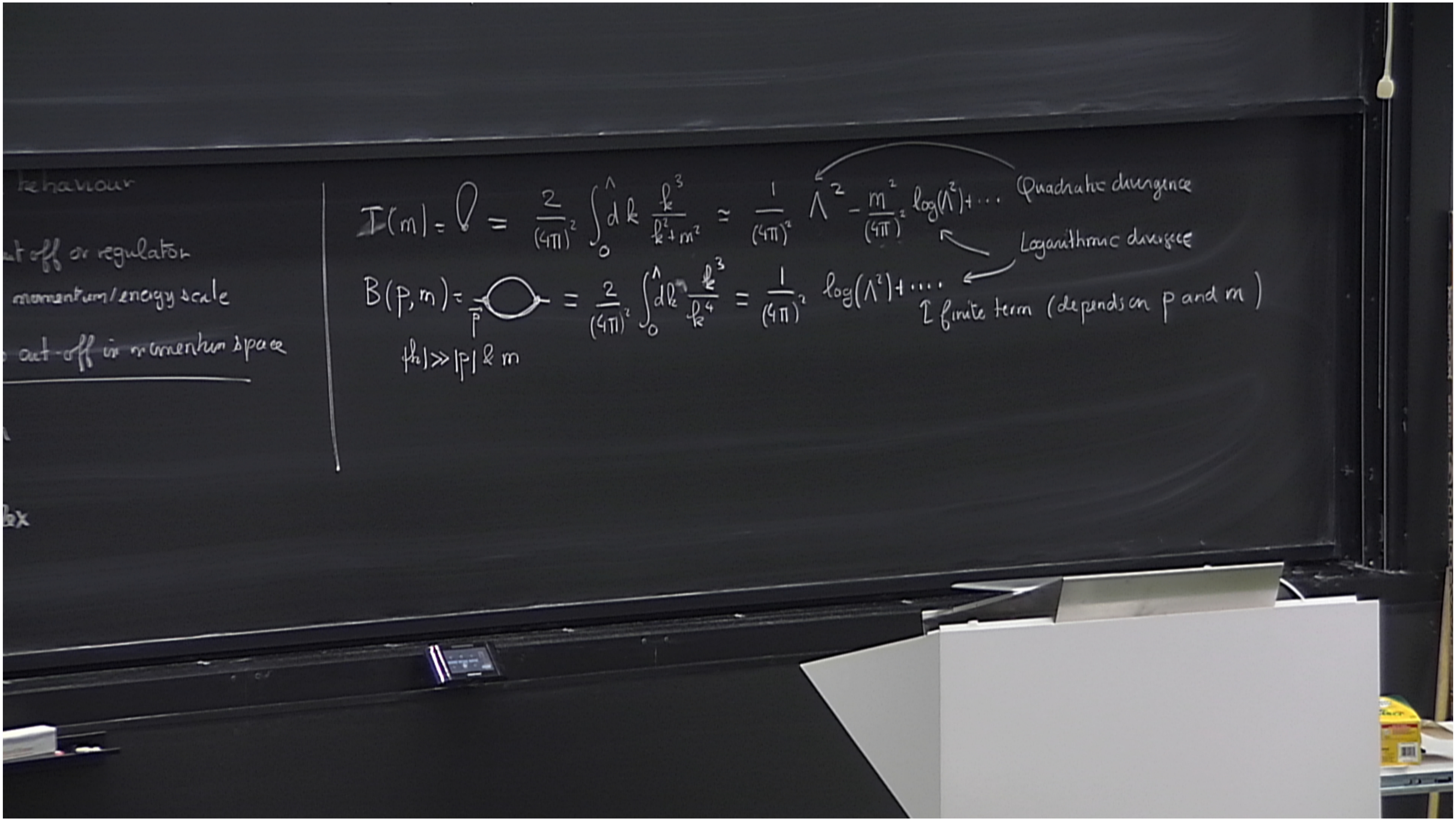

behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space
I(m) =
B(p,m) =
Quadratic divergence
Logarithmic divergence

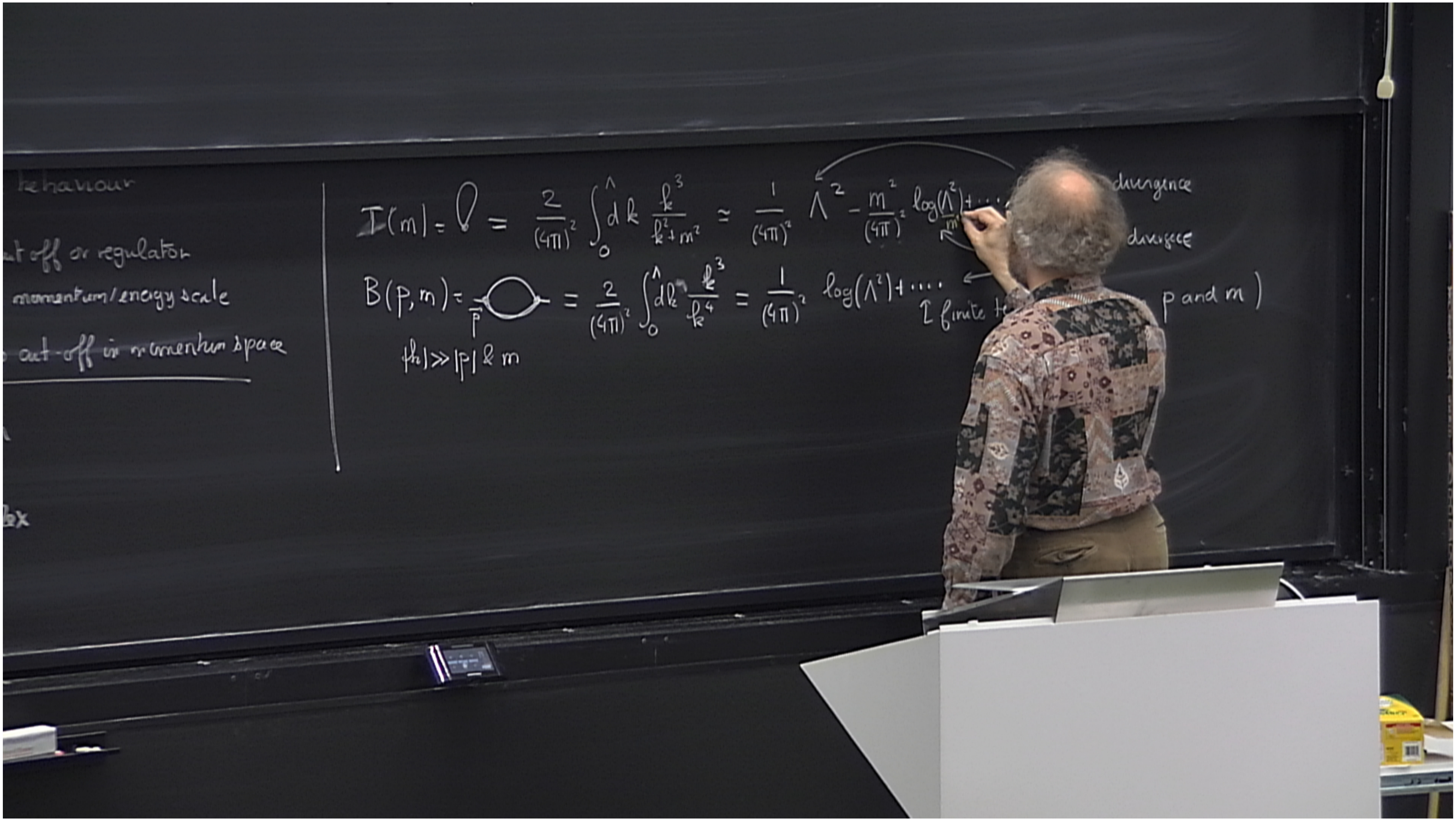

behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space
Quadratic divergence
Logarithmic divergence
|k| ≫ |p| & m


behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space
I(m) = = 2/(4π)^2 ∫_0^Λ dk k^3/(k^2+m^2) ≃ 1/(4π)^2 Λ^2 − m^2/(4π)^2 log(Λ^2) + ...
B(p,m) = = 2/(4π)^2 ∫_0^Λ dk k^3/k^4 = 1/(4π)^2 log(Λ^2) + ...
|k| ≫ |p| & m
finite term



Quadratic divergence
Logarithmic divergence
finite term (depends on p and m)


behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space
I(m) = = 2/(4π)^2 ∫_0^Λ dk k^3/(k^2+m^2) ≃ 1/(4π)^2 Λ^2 − m^2/(4π)^2 log(Λ^2) + …
Quadratic divergence
Logarithmic divergence
B(p, m) = = 2/(4π)^2 ∫_0^Λ dk k^3/k^4 = 1/(4π)^2 log(Λ^2) + ….
finite term (depends on p and m)
|k| ≫ |p| & m


behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space
I(m) = = 2/(4π)^2 ∫_0^Λ dk k^3/(k^2+m^2) ≃ 1/(4π)^2 Λ^2 − m^2/(4π)^2 log(Λ^2/m^2) + ...
divergence
divergence
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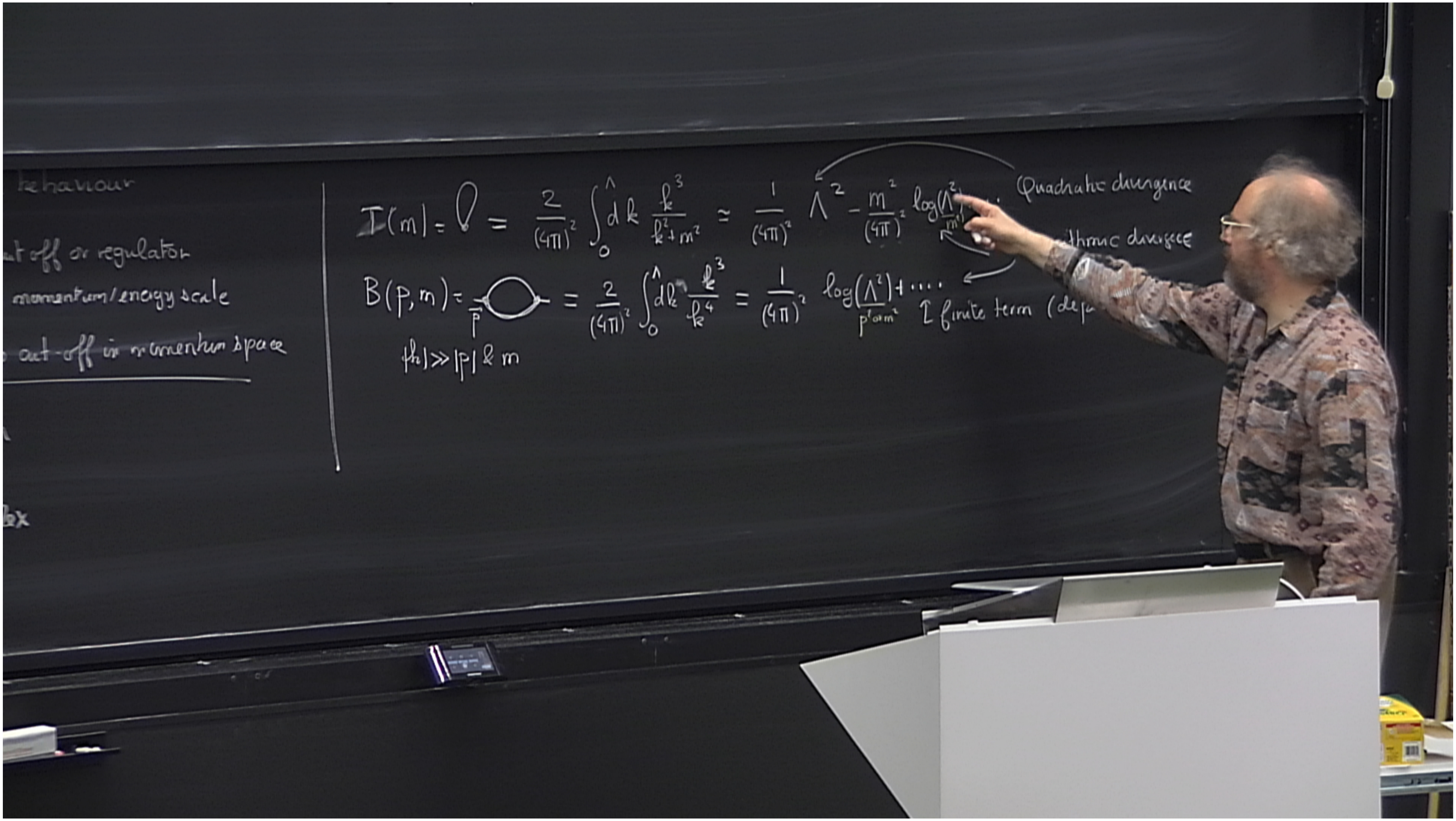
p
|k| ≫ |p| & m
finite
p and m)


behaviour
cut off or regulator
momentum/energy scale
cut-off in momentum space
Quadratic divergence
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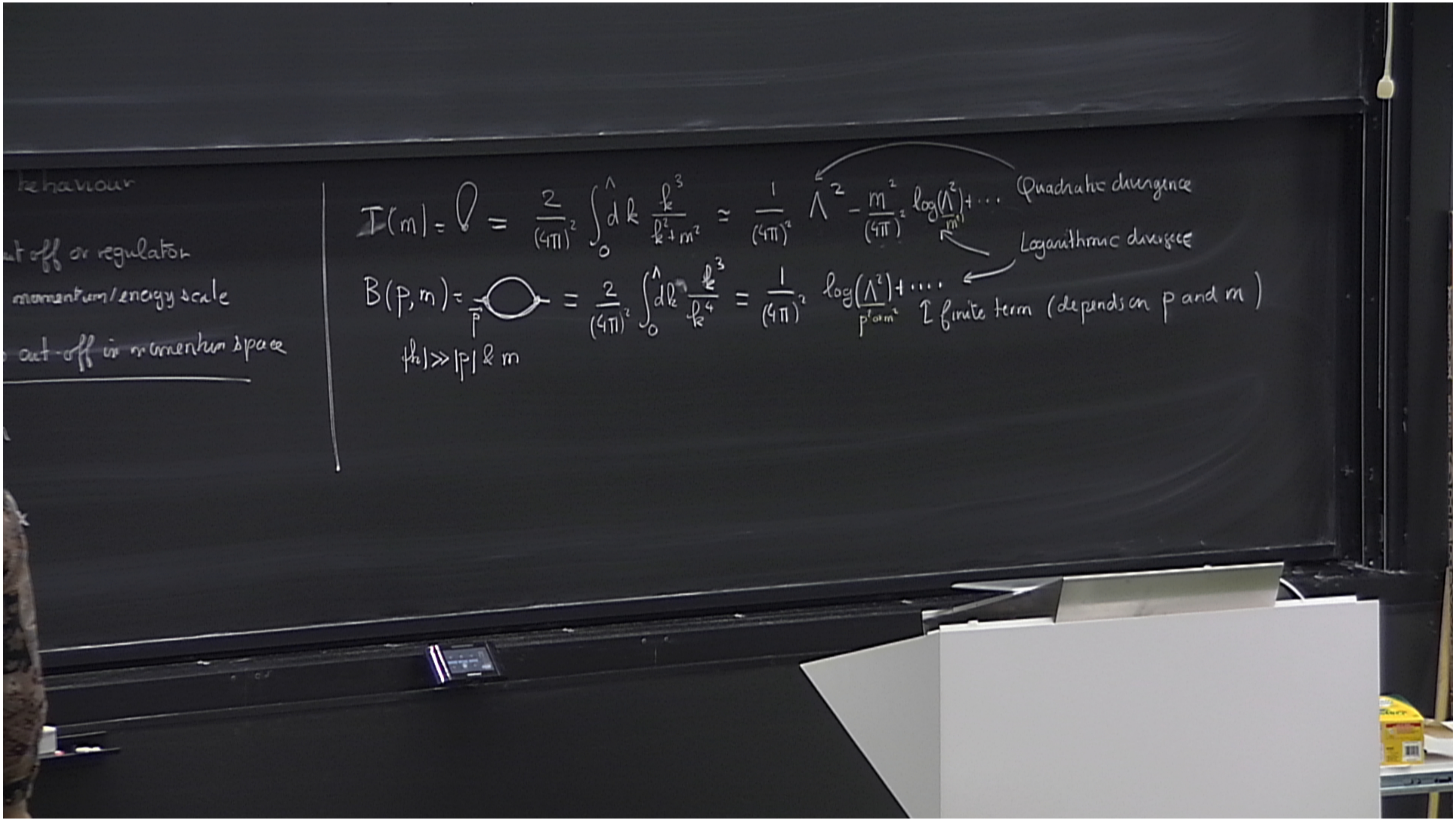
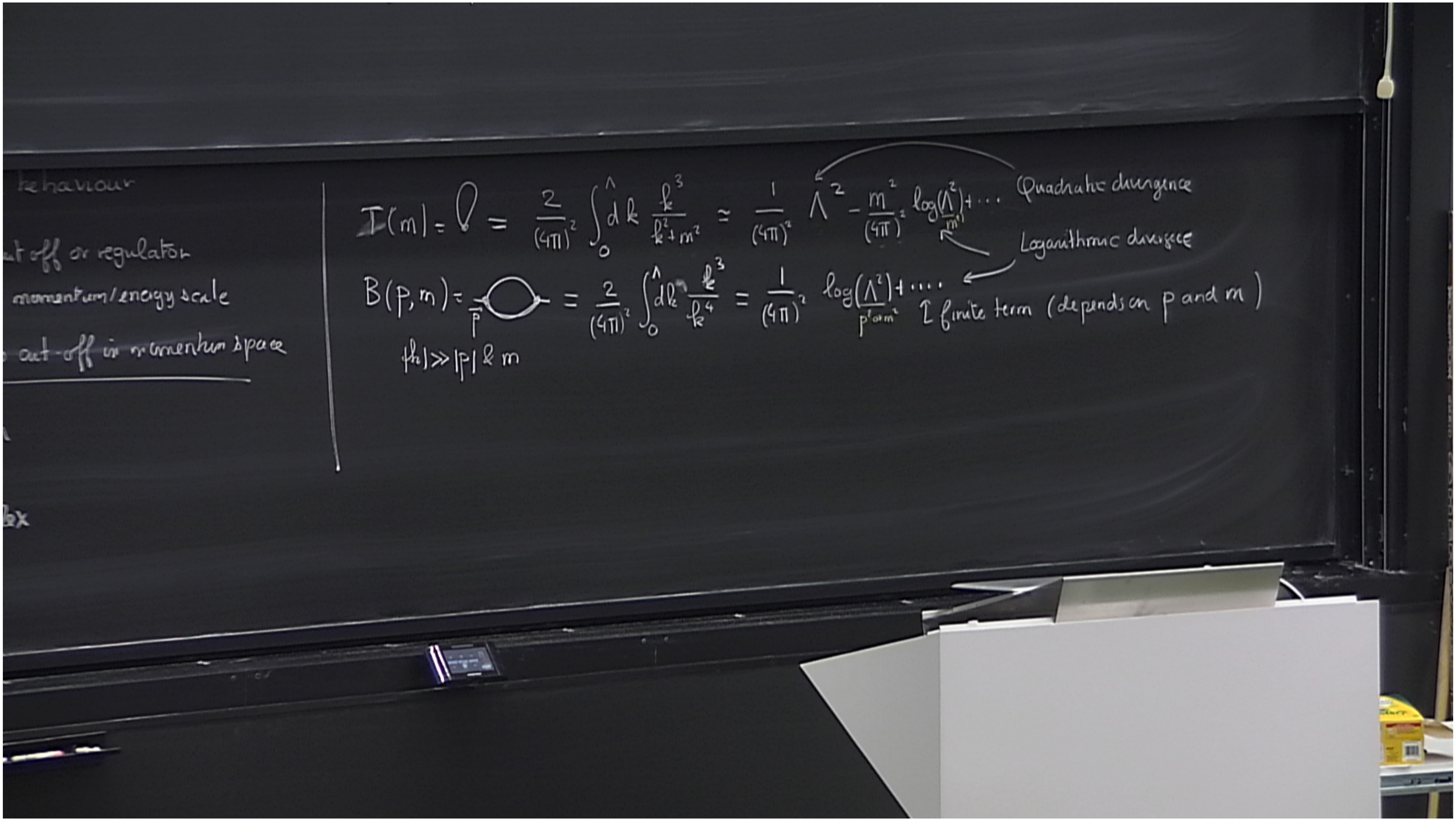
finite term (dep
|k| >> |p| & m


behaviour
ut off or regulator
momentum/energy scale
cut-off in momentum space
$I(m) = \frac{2}{(4\pi)^2}\int_0^\Lambda dk \frac{k^3}{k^2+m^2} \simeq \frac{1}{(4\pi)^2}\Lambda^2 - \frac{m^2}{(4\pi)^2}\log\left(\frac{\Lambda^2}{m^2}\right) + \cdots$
Quadratic divergence
Logarithmic divergence
$B(p,m) = \frac{2}{(4\pi)^2}\int_0^\Lambda dk \frac{k^3}{k^4} = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2 \text{ or } m^2}\right) + \cdots$
finite term (depends on p and m)
$|k| \gg |p|$ & m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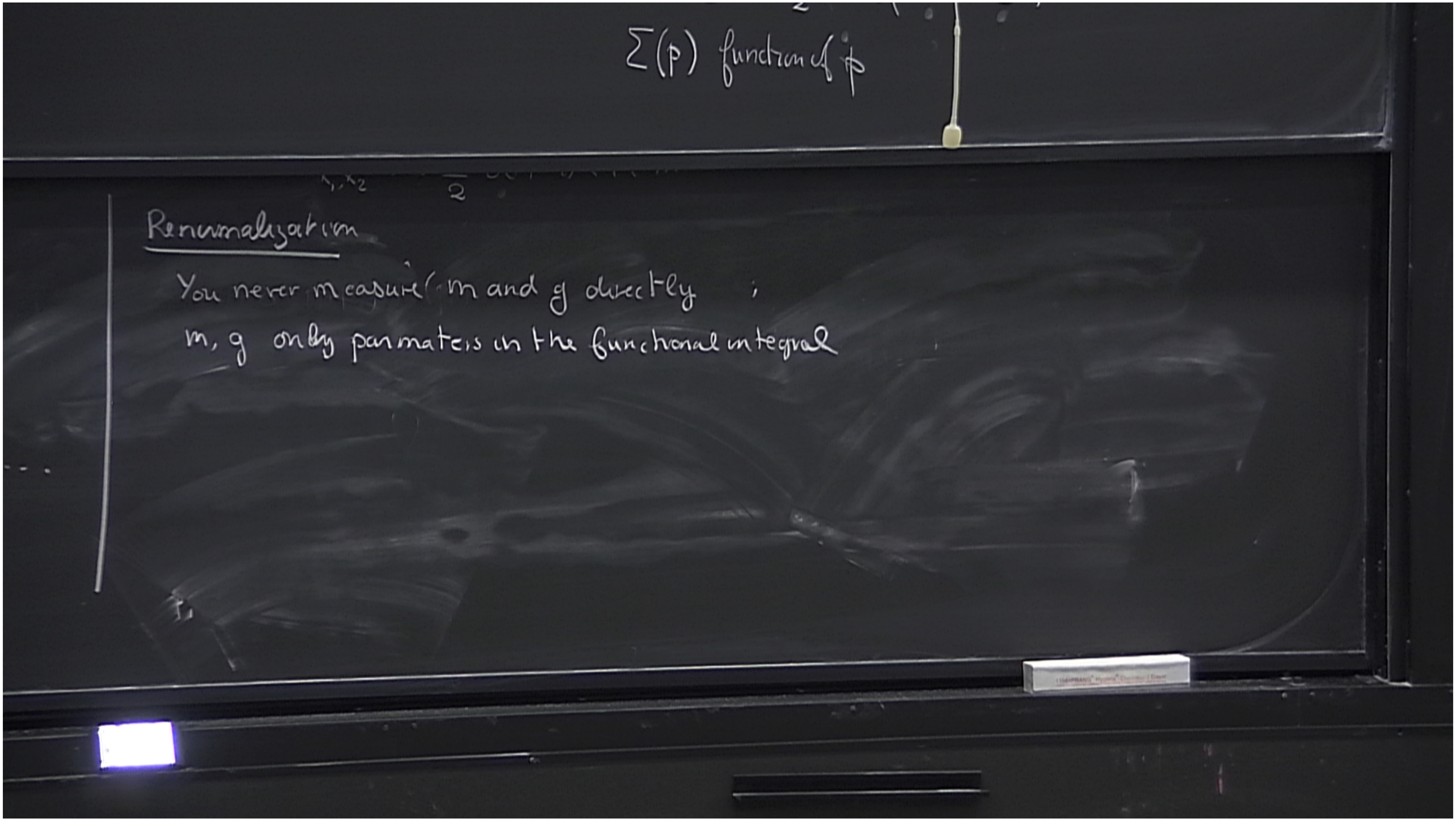
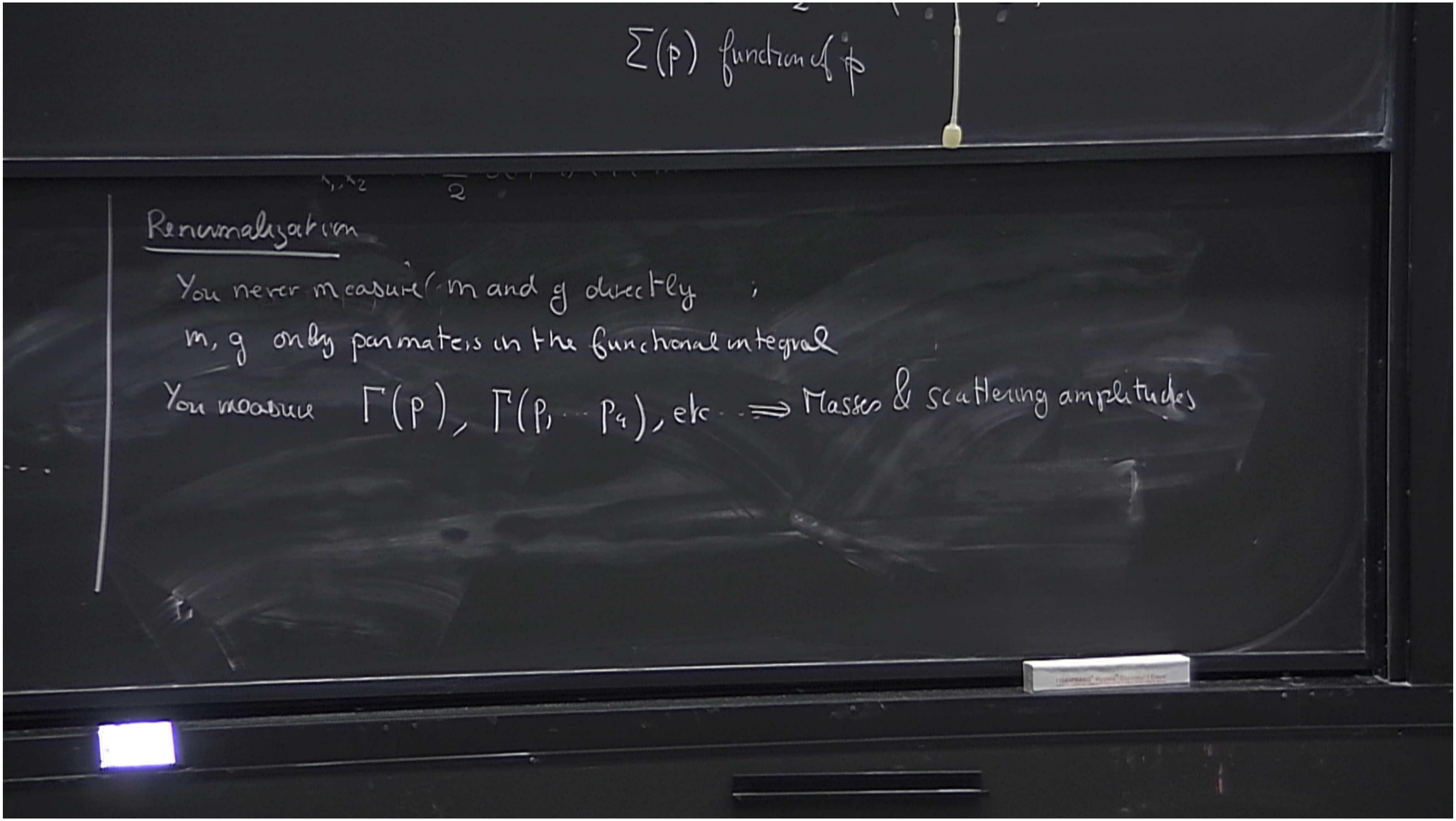
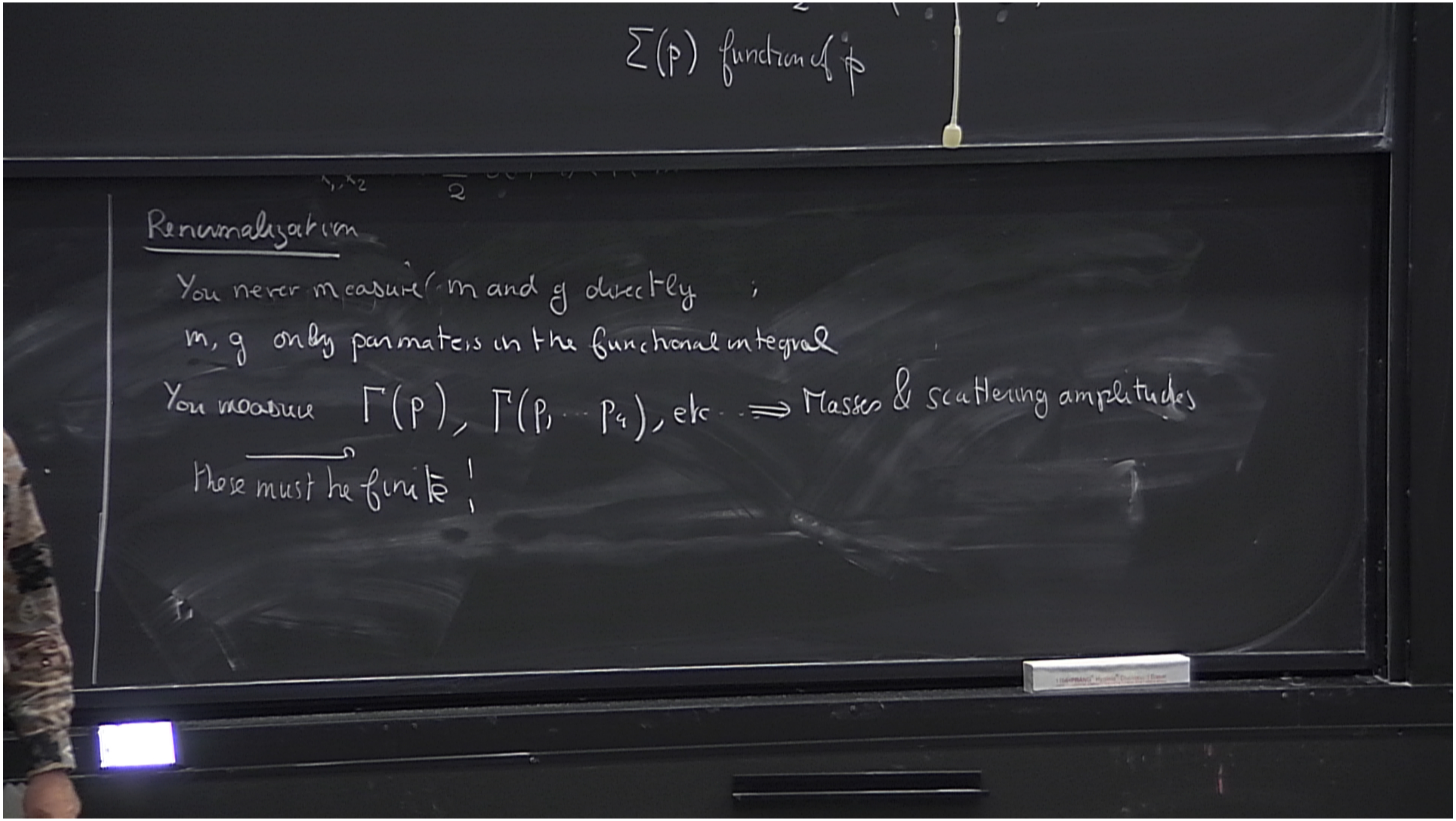

Σ(p) function of p
Renormalization
You never measure m and g directly ;
m, g only parameters in the functional integral


$\Sigma(p)$ function of p
Renormalization
You never measure m and g directly ;
m, g only parameters in the functional integral
You measure $\Gamma(p)$, $\Gamma(p_1 \cdots p_4)$, etc $\Rightarrow$ Masses & scattering amplitudes


Σ(p) function of p
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You never measure m and g directly ;
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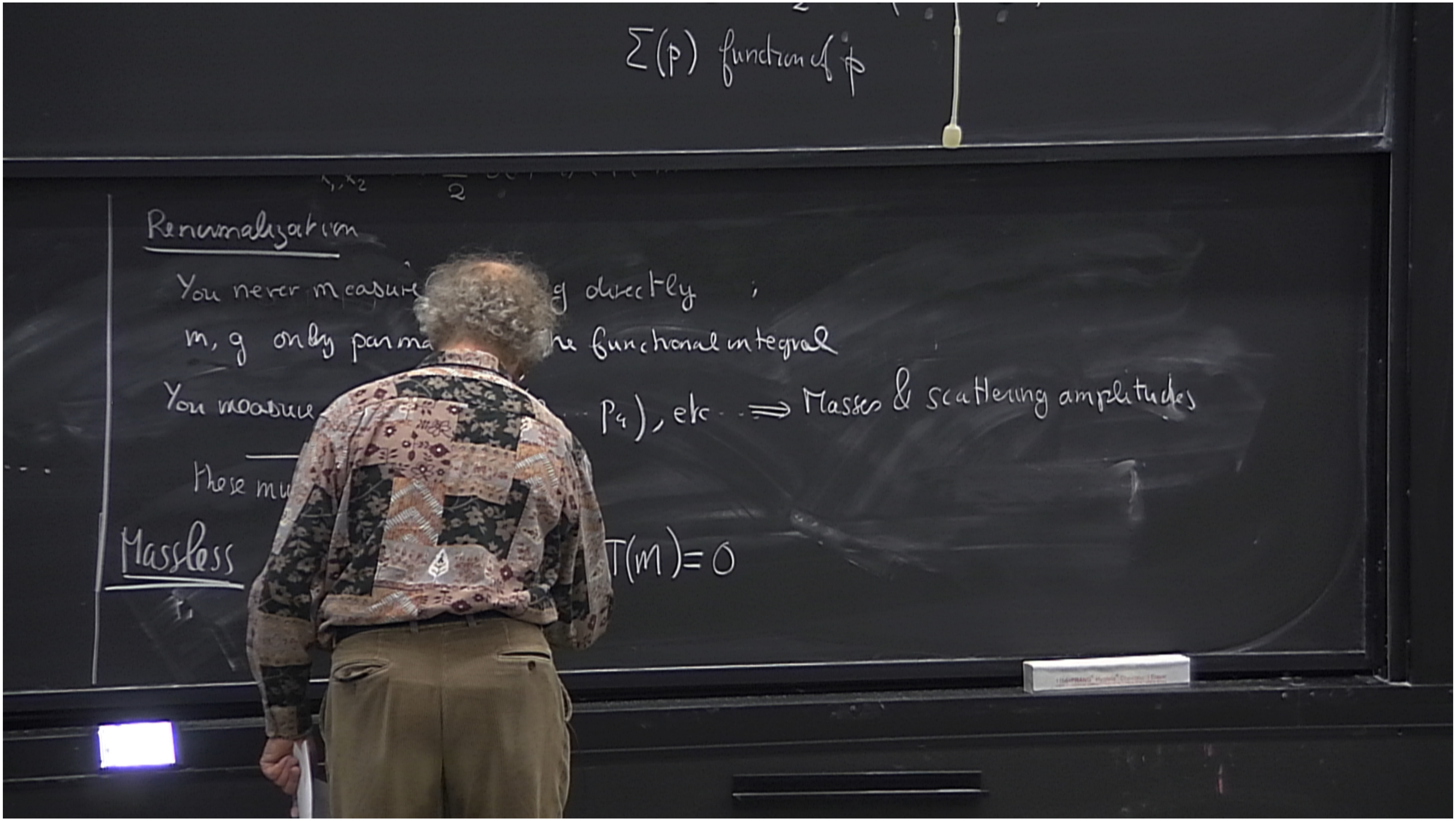
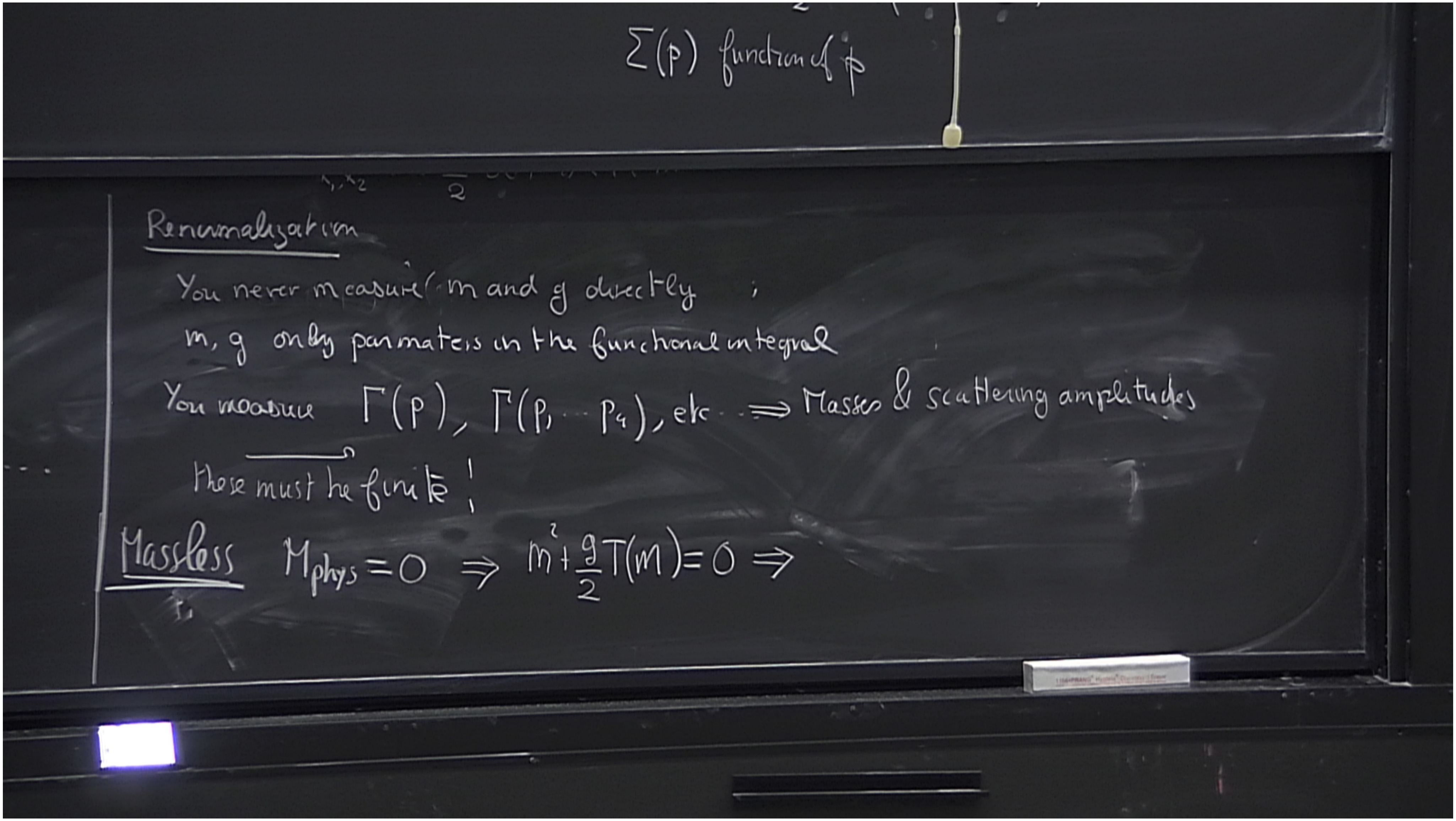
You measure Γ(p), Γ(p1 ... p4), etc ⇒ Masses & scattering amplitudes
These must be finite!



Σ(p) function of p
Renormalization
You never measure g directly ;
m, g only parm the functional integral
You measure p4), etc ⇒ Masses & scattering amplitudes
These mu
Massless
T(m)=0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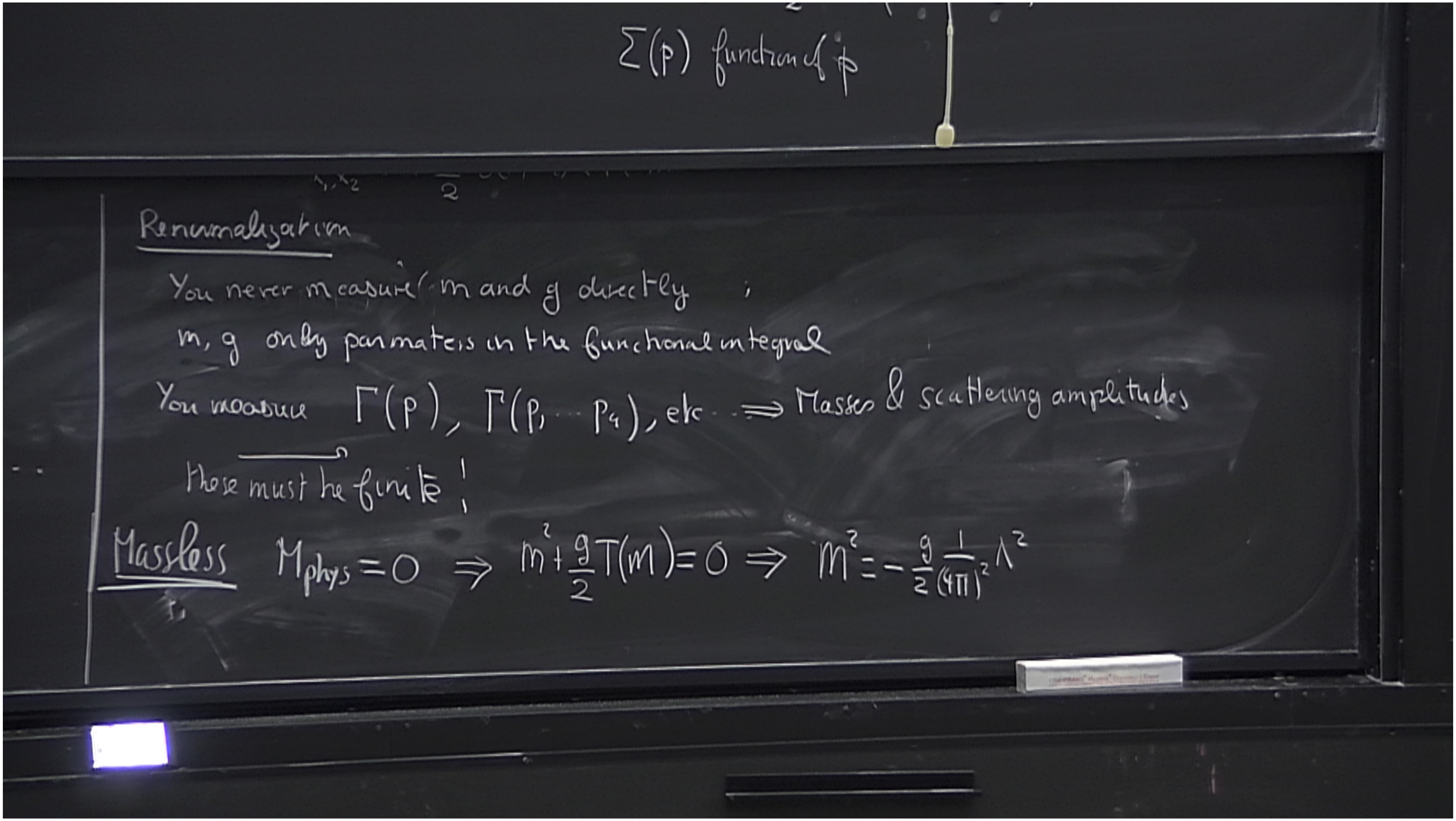
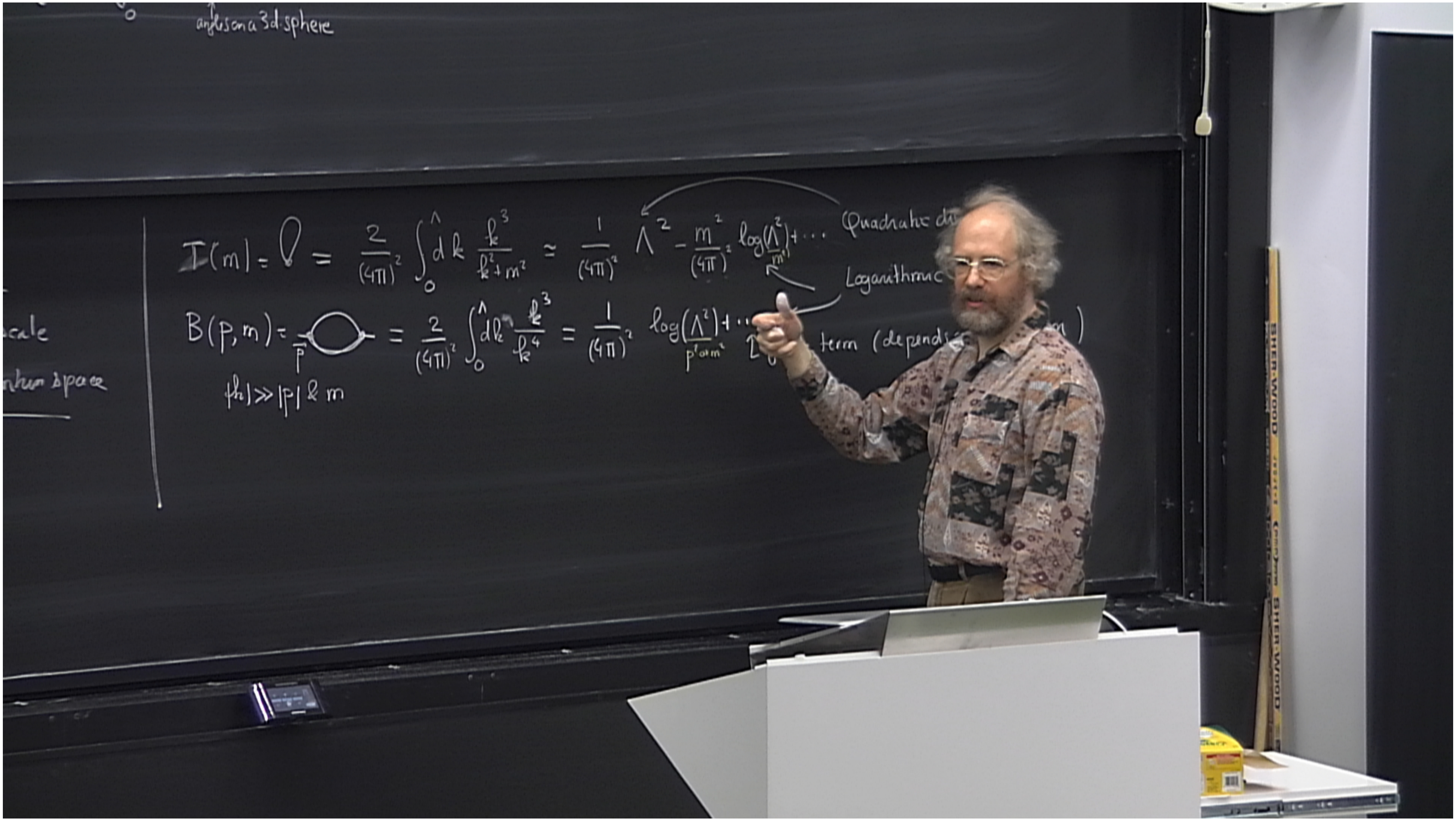
Massless $M_{phys} = 0 \Rightarrow m^2 + \frac{g}{2} T(m) = 0 \Rightarrow$


$\Sigma(p)$ functional of p
Renormalization
You never measure m and g directly ;
m, g only parameters in the functional integral
You measure $\Gamma(p)$, $\Gamma(p_1 \cdots p_4)$, etc ⇒ Masses & scattering amplitudes
These must be finite !
Massless $M_{phys} = 0 \Rightarrow m^2 + \frac{g}{2} T(m) = 0 \Rightarrow m^2 = -\frac{g}{2} \frac{1}{(4\pi)^2} \Lambda^2$


angles on a 3d sphere
Quadratic di
Logarithmic
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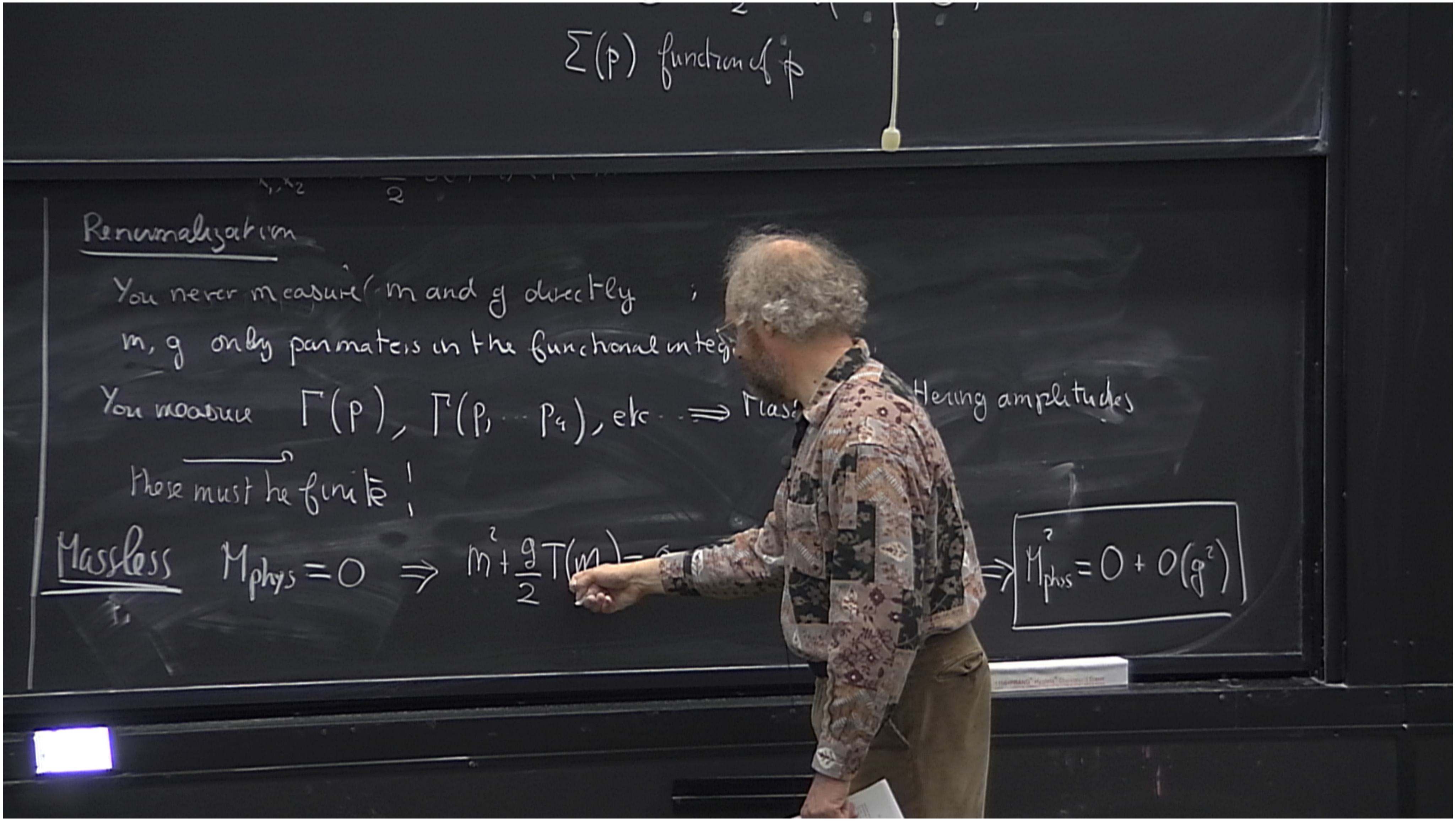
|k| ≫ |p| & m
SHER-WOOD



Σ(p) function of p
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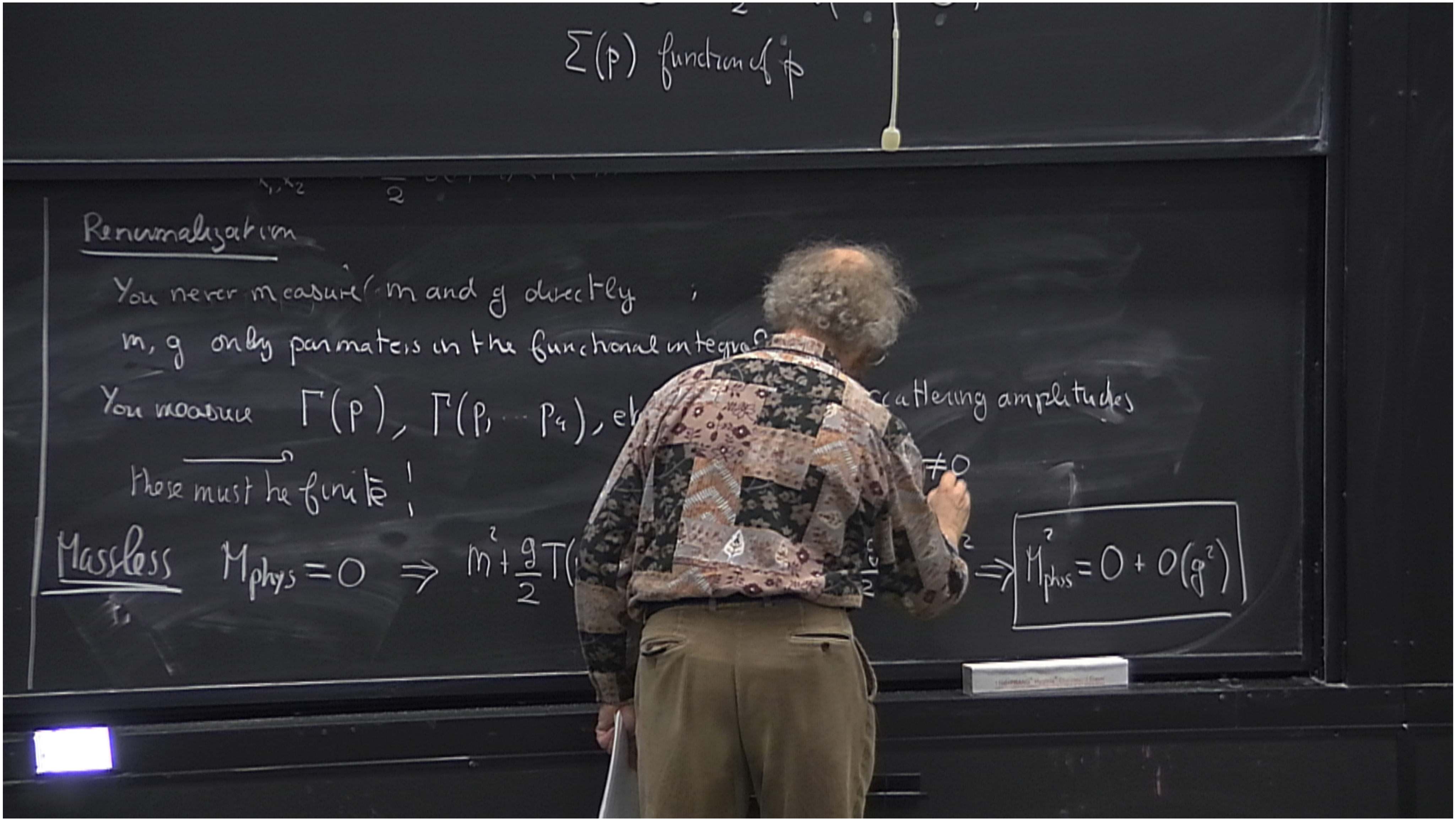
You measure Γ(p), Γ(p1 ··· p4), etc ⇒
amplitudes
These must be finite !
Massless   M_phys = 0 ⇒ m² + g/2 T(m)
M²_phys = 0 + O(g²)



Σ(p) function of p
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Massless
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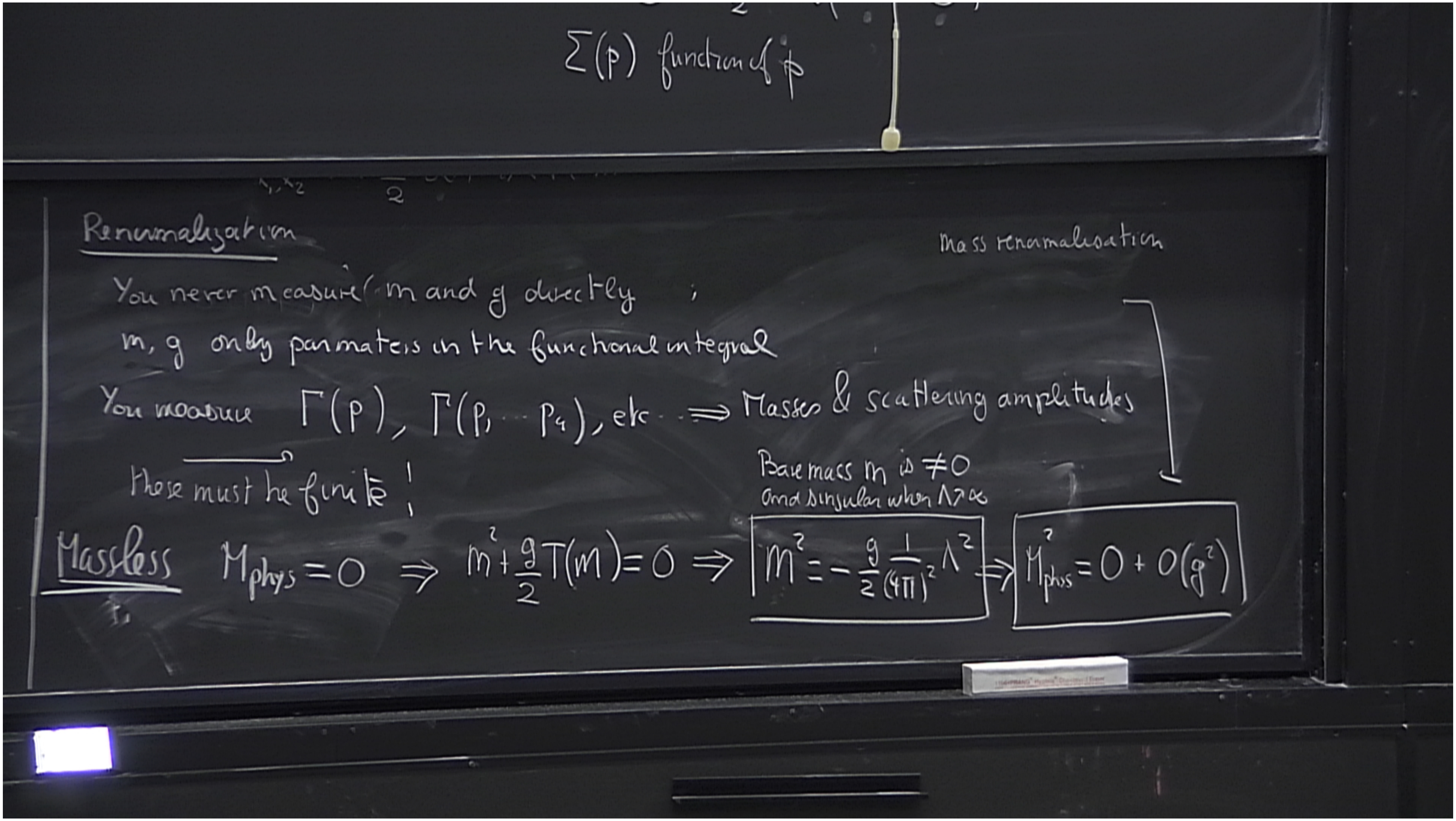
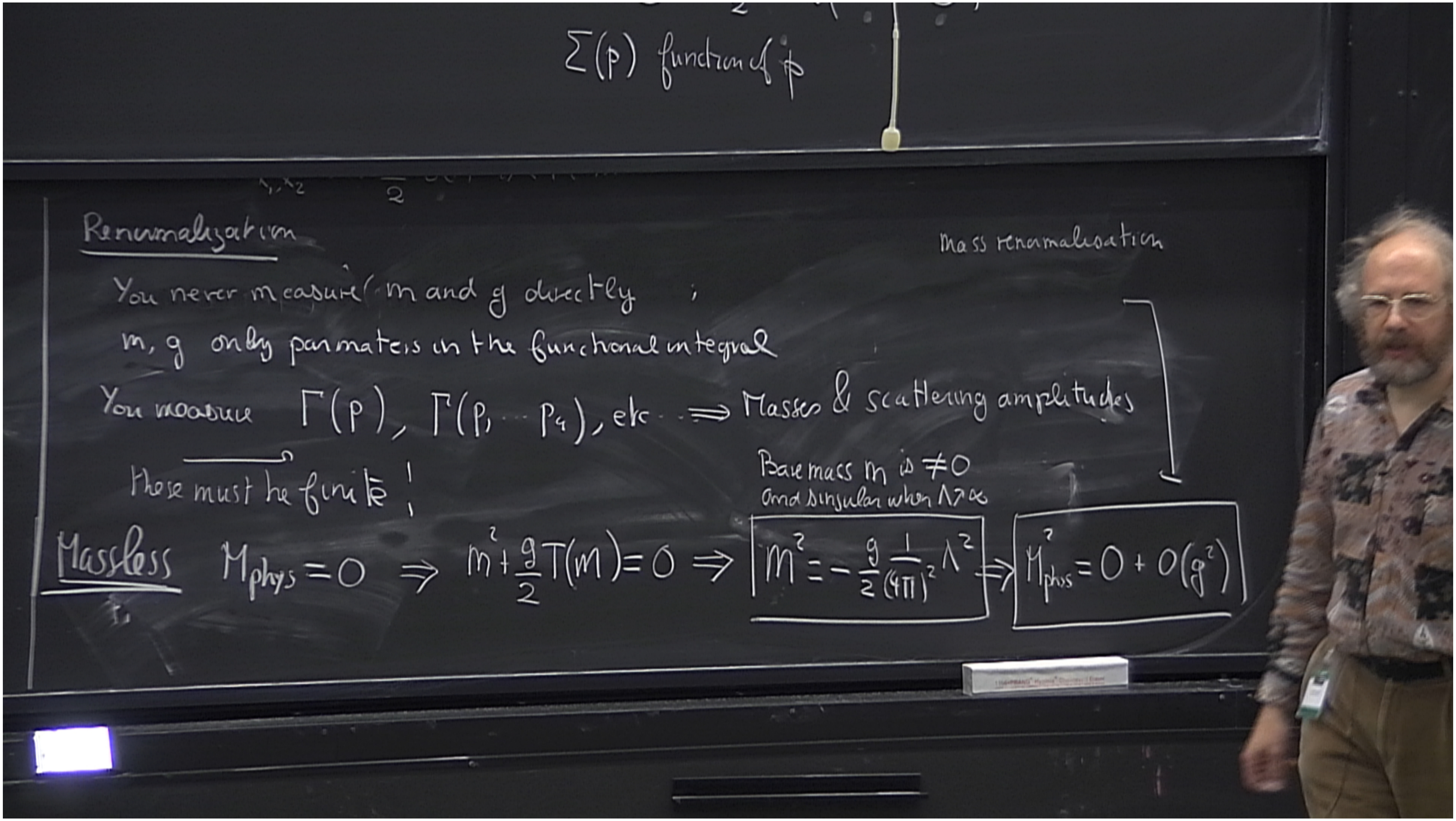

Σ(p) function of p
Renormalization
mass renormalisation
You never measure m and g directly ;
m, g only parameters in the functional integral
You measure Γ(p), Γ(p1 ··· p4), etc ⇒ Masses & scattering amplitudes
these must be finite !
Bare mass m is ≠ 0 and singular when Λ→∞
Massless Mphys = 0 ⇒ m² + g/2 T(m) = 0 ⇒ m² = − g/2 1/(4π)² Λ² ⇒ M²phys = 0 + O(g²)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