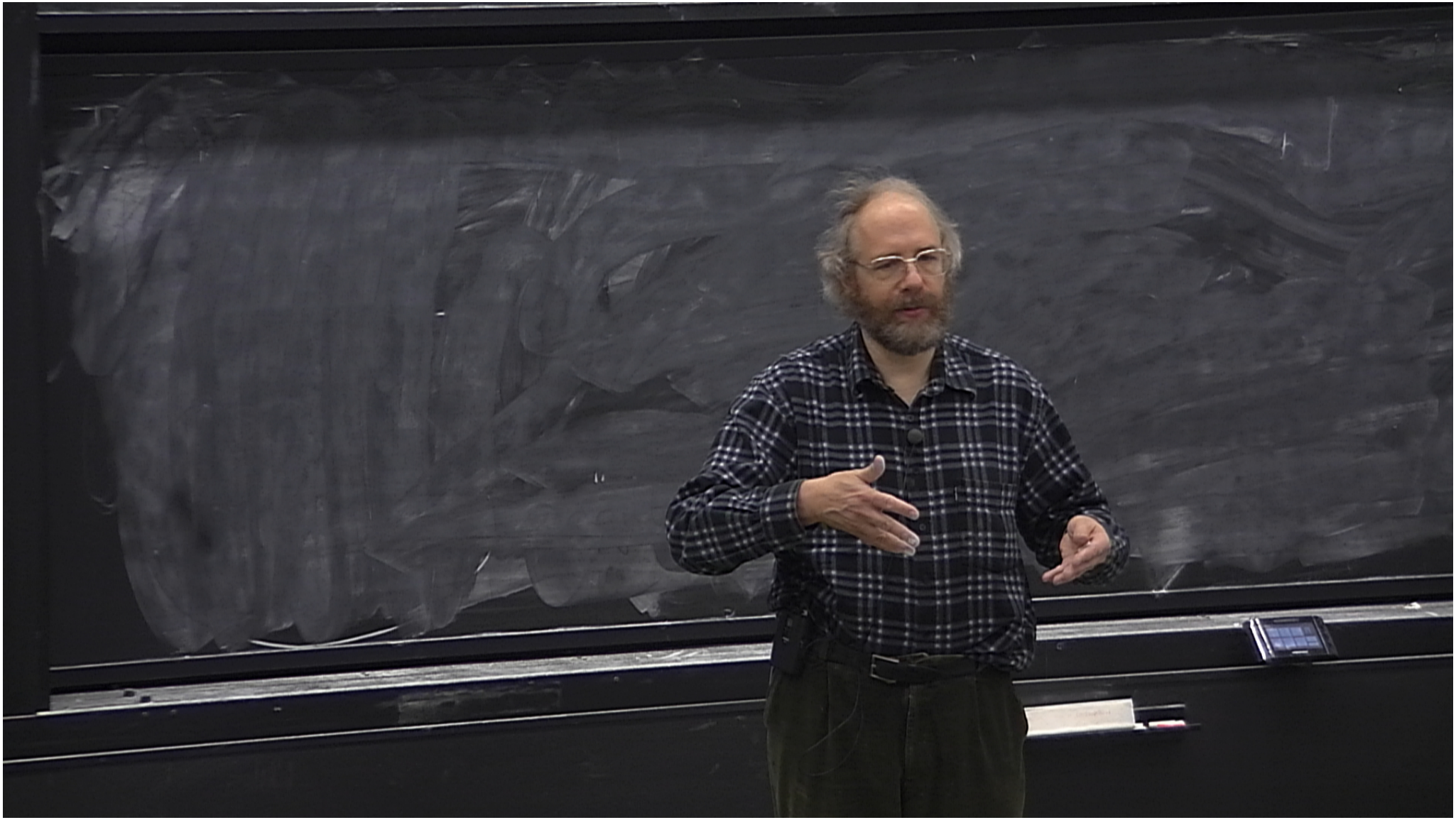
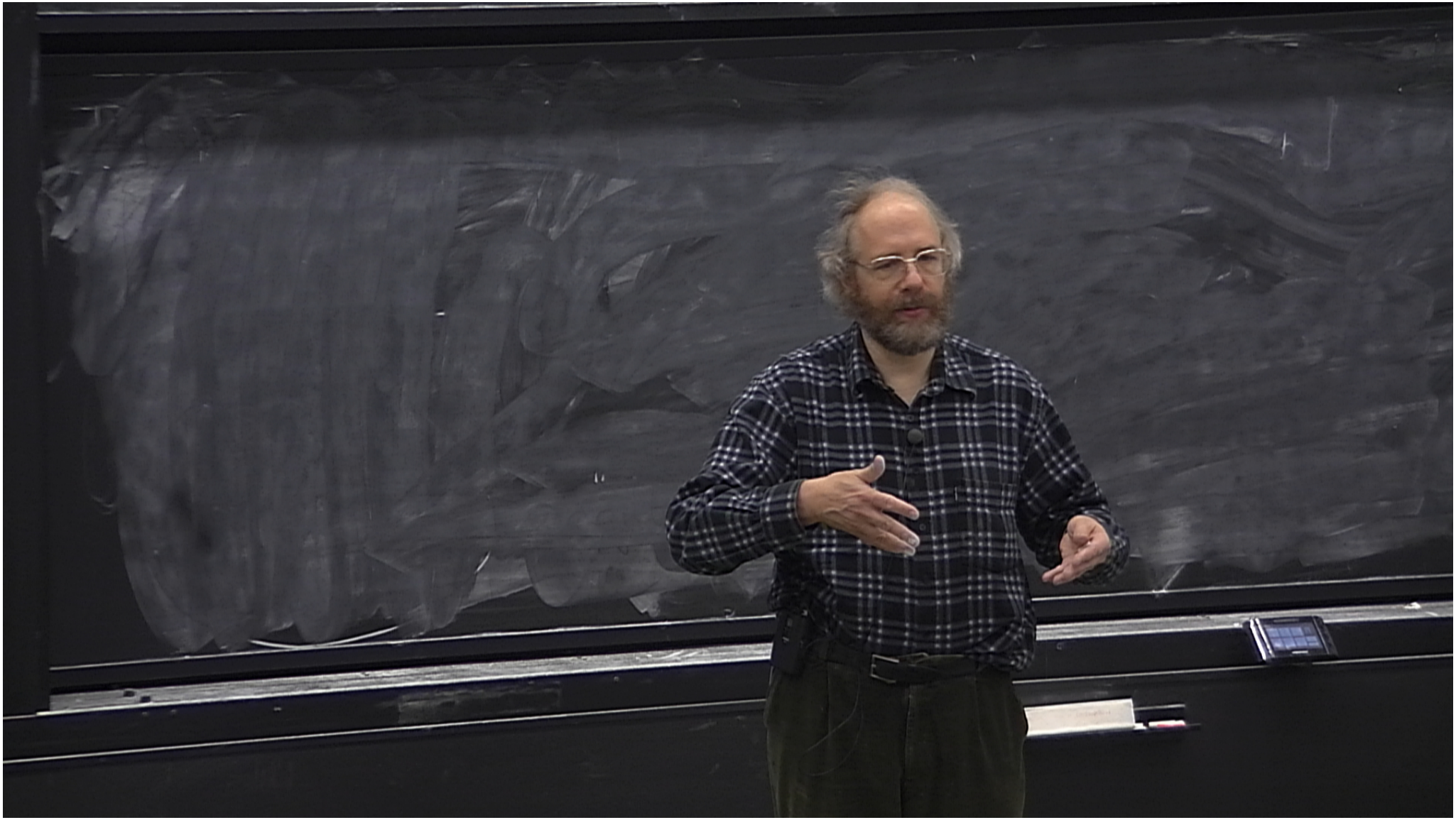
Title: Quantum Field Theory II - Lecture 3

Date: Nov 02, 2011 09:00 AM

URL: http://pirsa.org/11110005

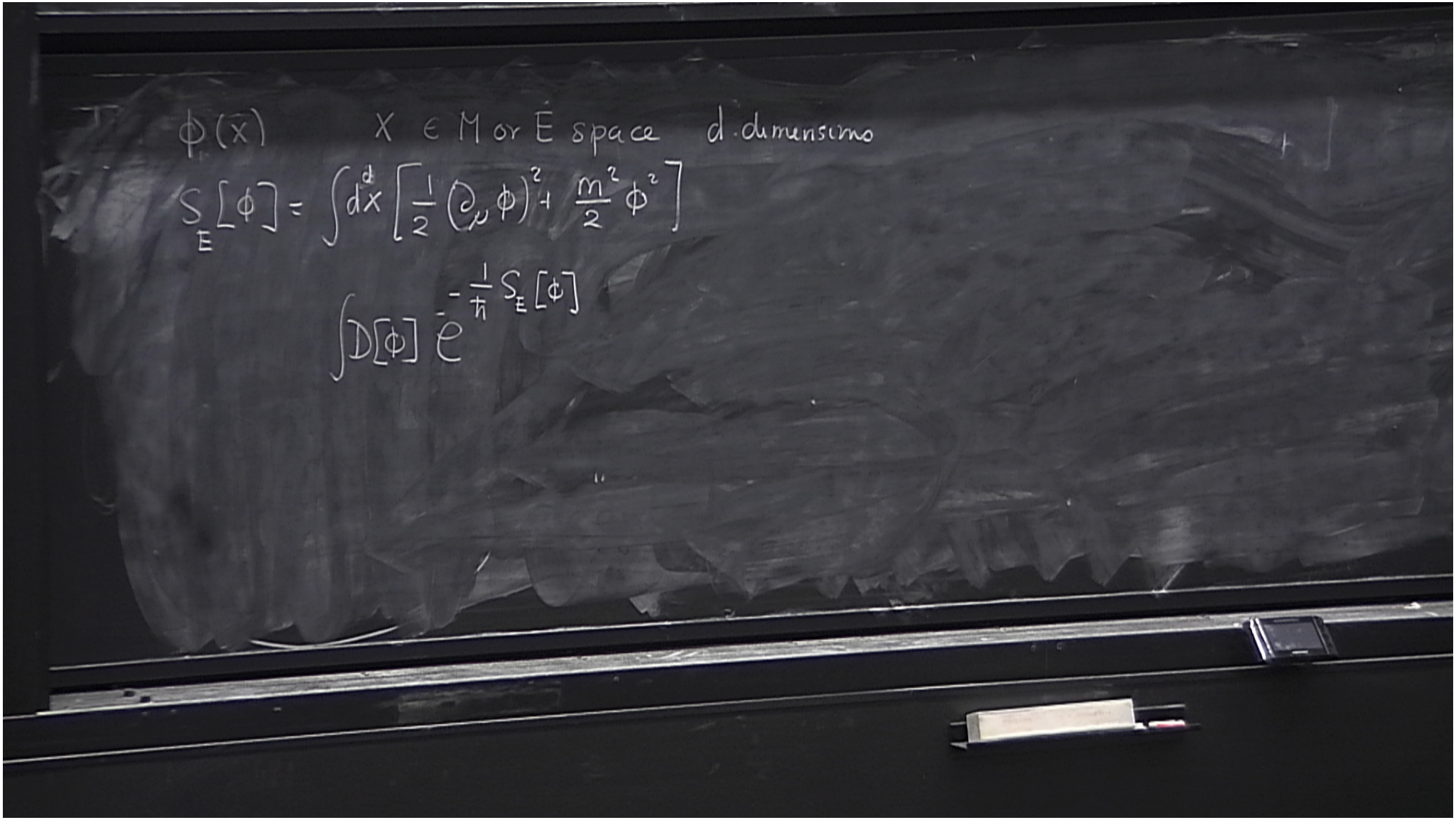
Abstract:

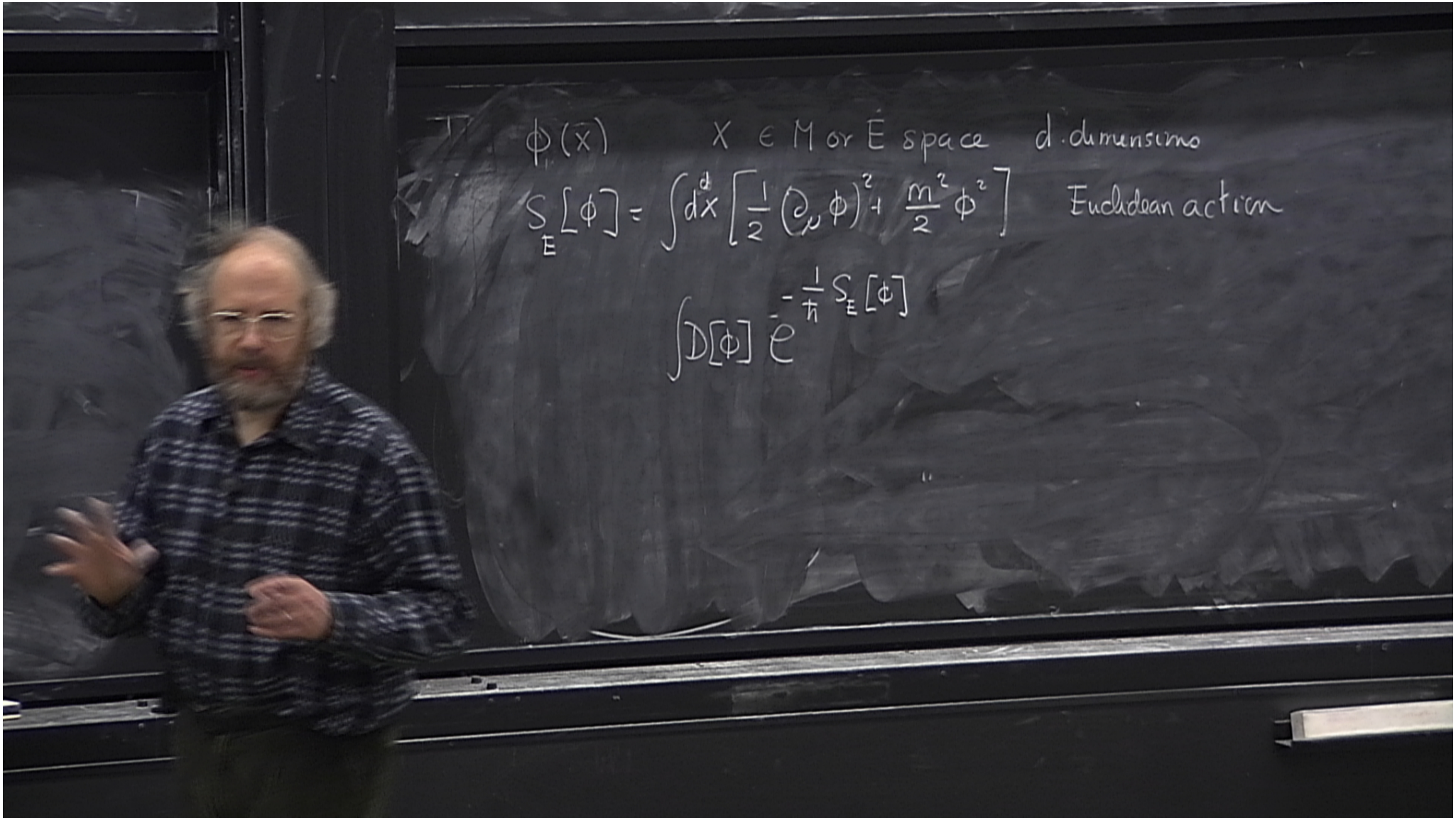


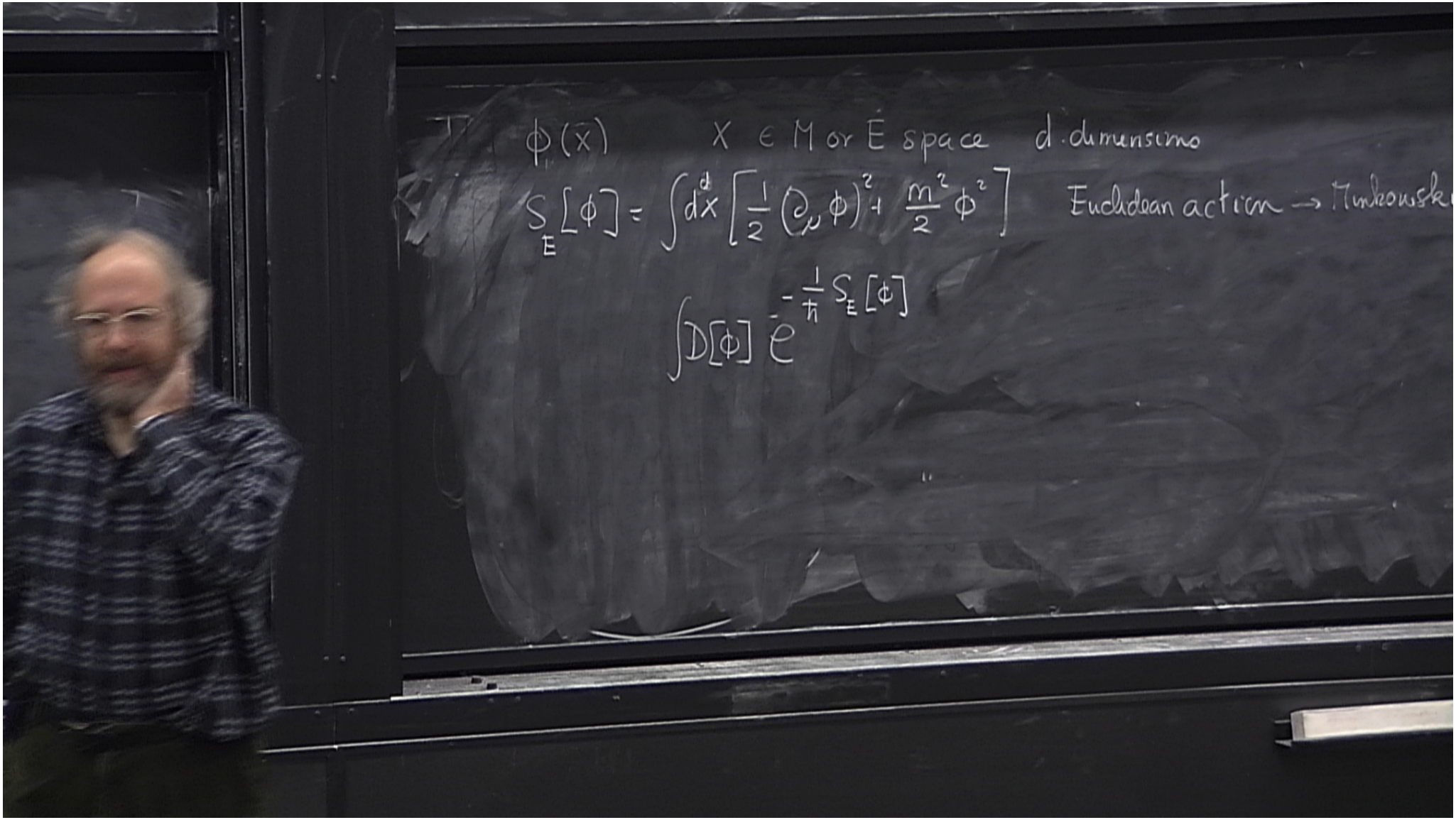


$\phi(\vec{x})$ $\quad x \in M$ or E space $\quad d$ dimensions

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 \right]$$


$\phi(x)$ $x \in$ Minkowski Espace d-dimensions
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$
$\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}$


$\phi(x)$ $x \in M$ or E space d-dimensional
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$ Euclidean action
$\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}$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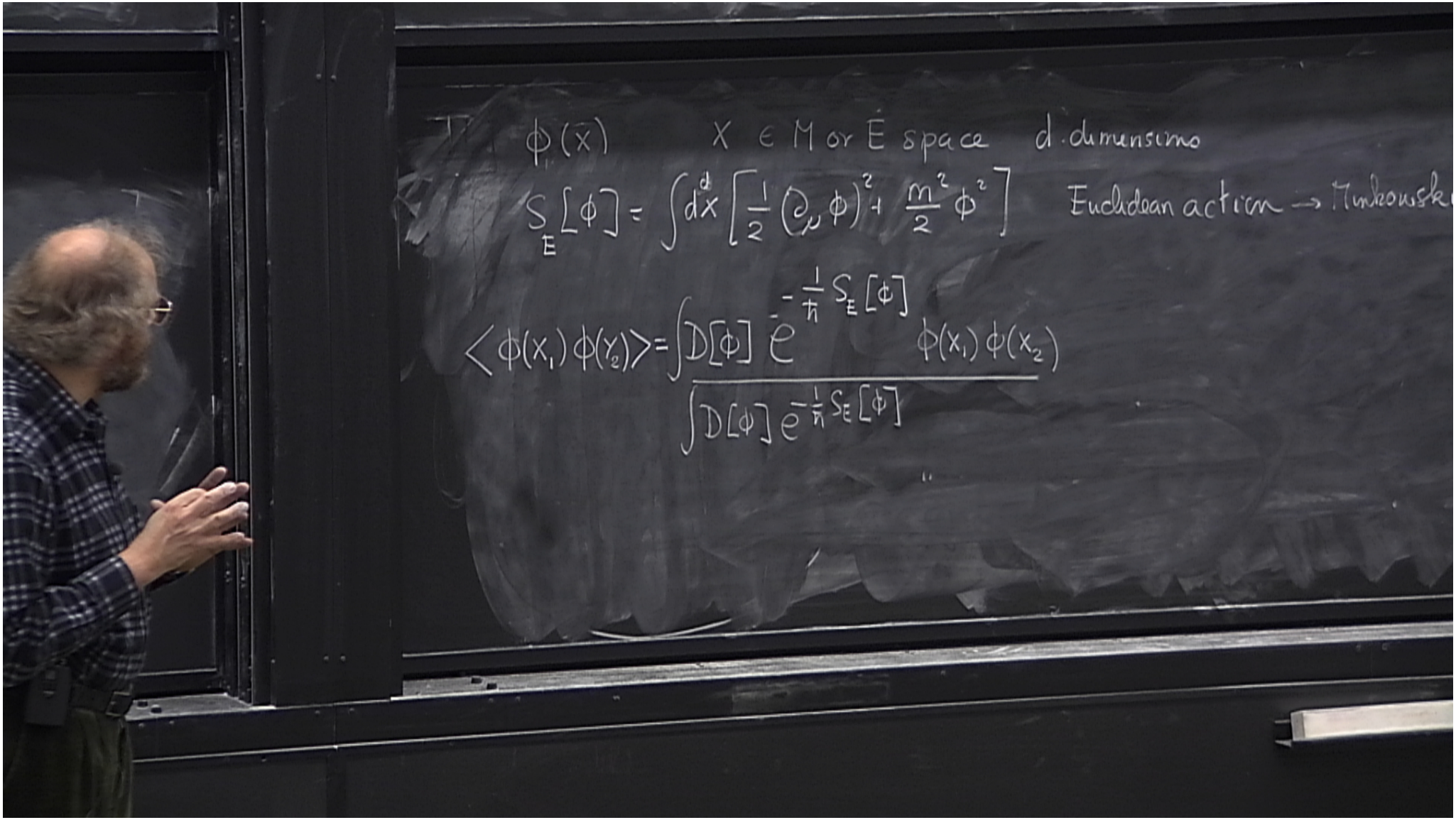
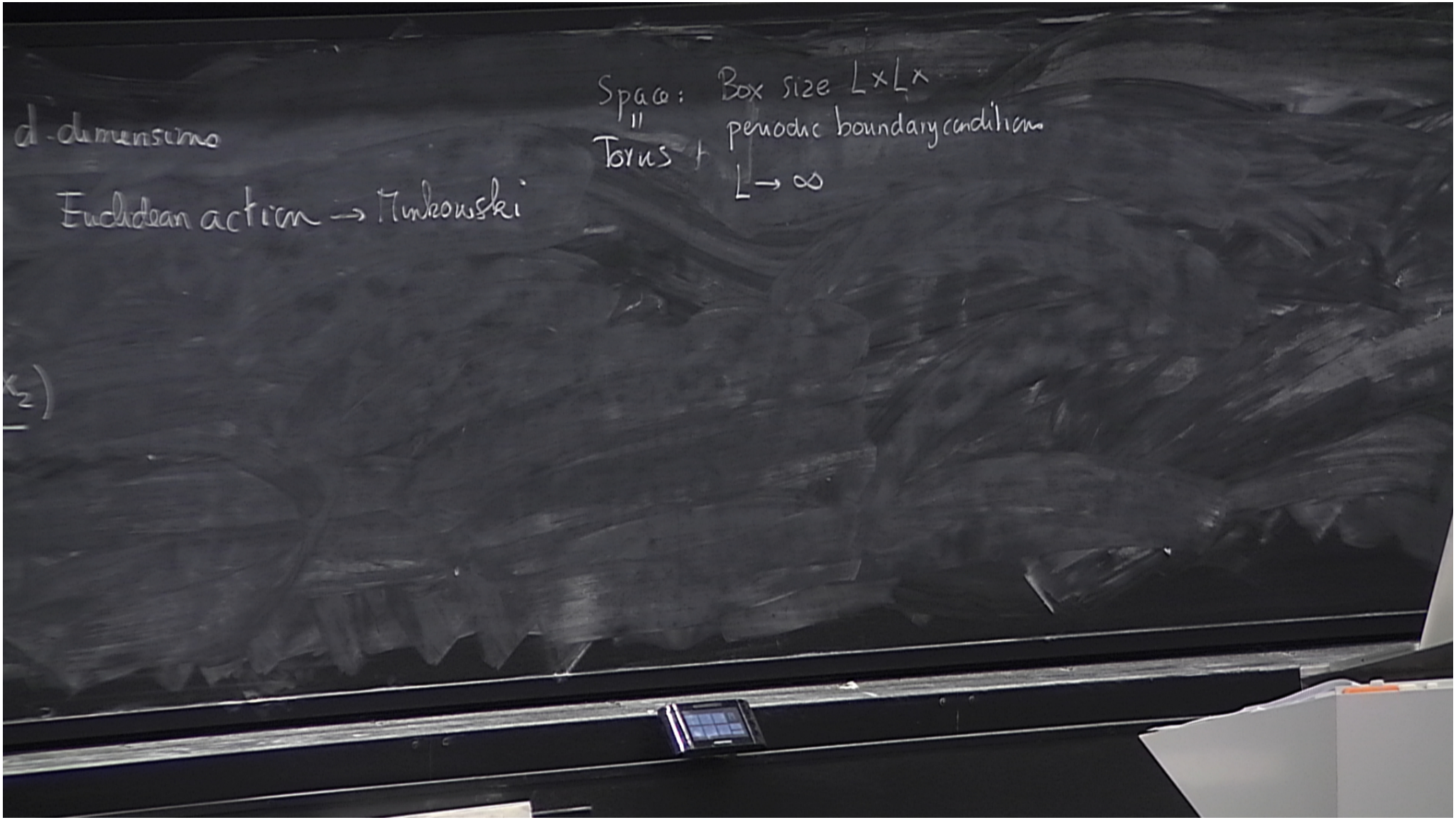
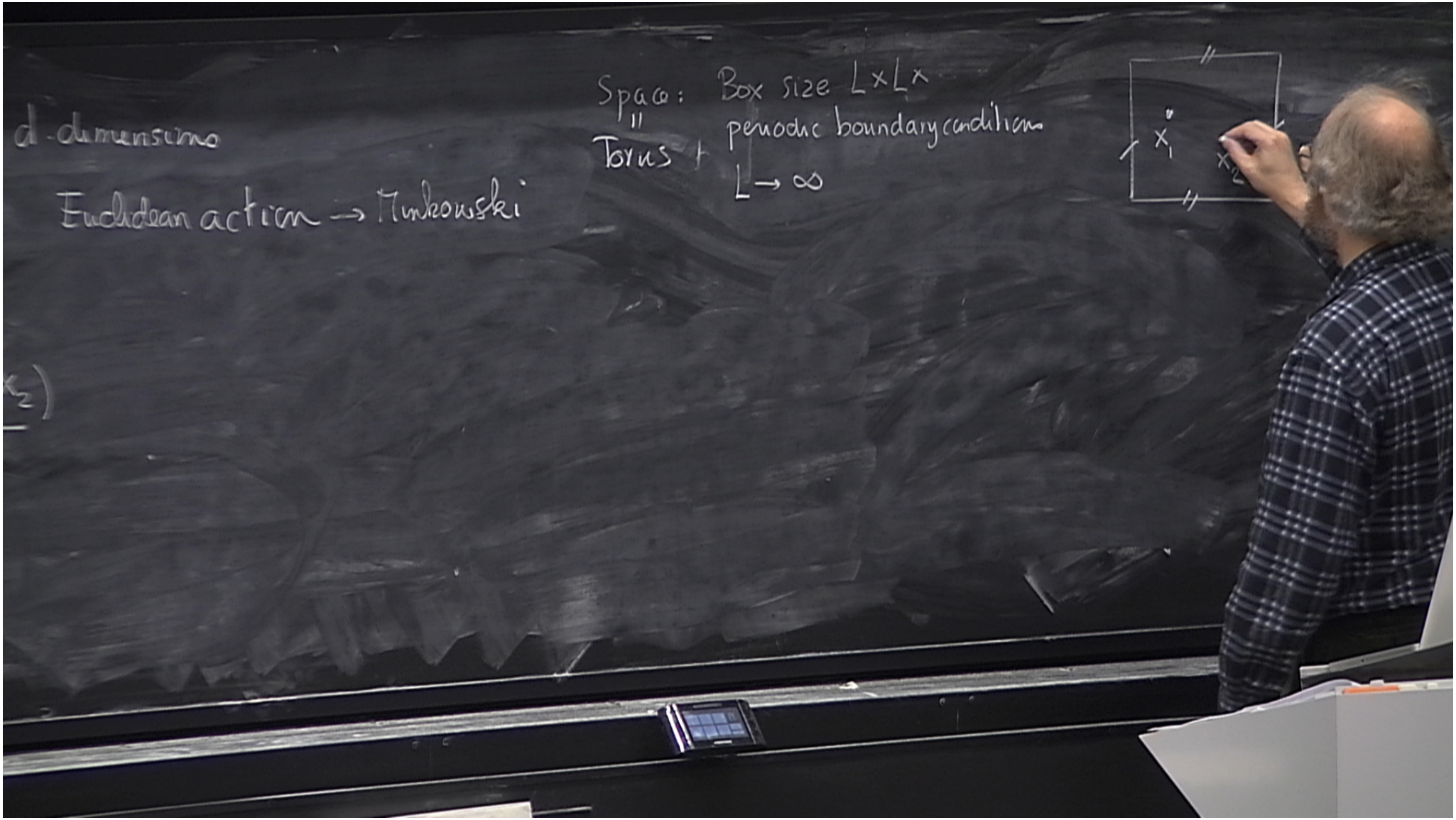
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$ Euclidean action → Minkowski
$\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}$


$\phi(x)$ $x \in M$ or E space d-dimensional
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$ Euclidean action → Minkowski
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}\, \phi(x_1)\phi(x_2)}{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}}$

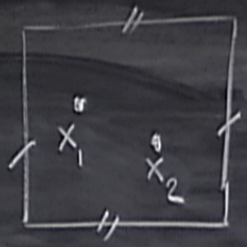

d-dimensions
Euclidean action → Minkowski
Space: Box size L×L×
Torus + periodic boundary condition
L → ∞


d-dimensions
Euclidean action → Minkowski
Space: Box size L×L×
Torus + periodic boundary condition
L → ∞
x1
x2

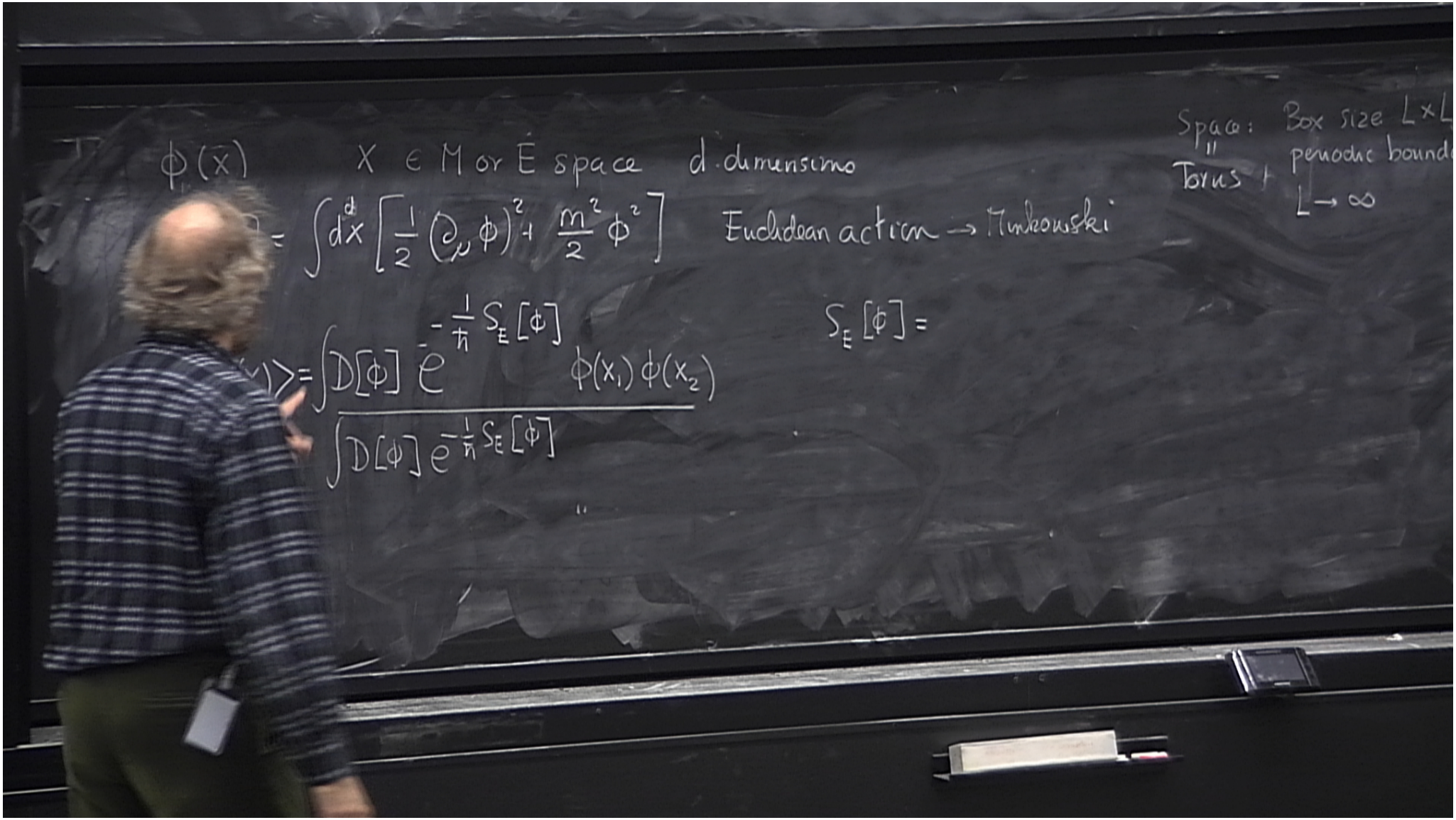
d-dimensions

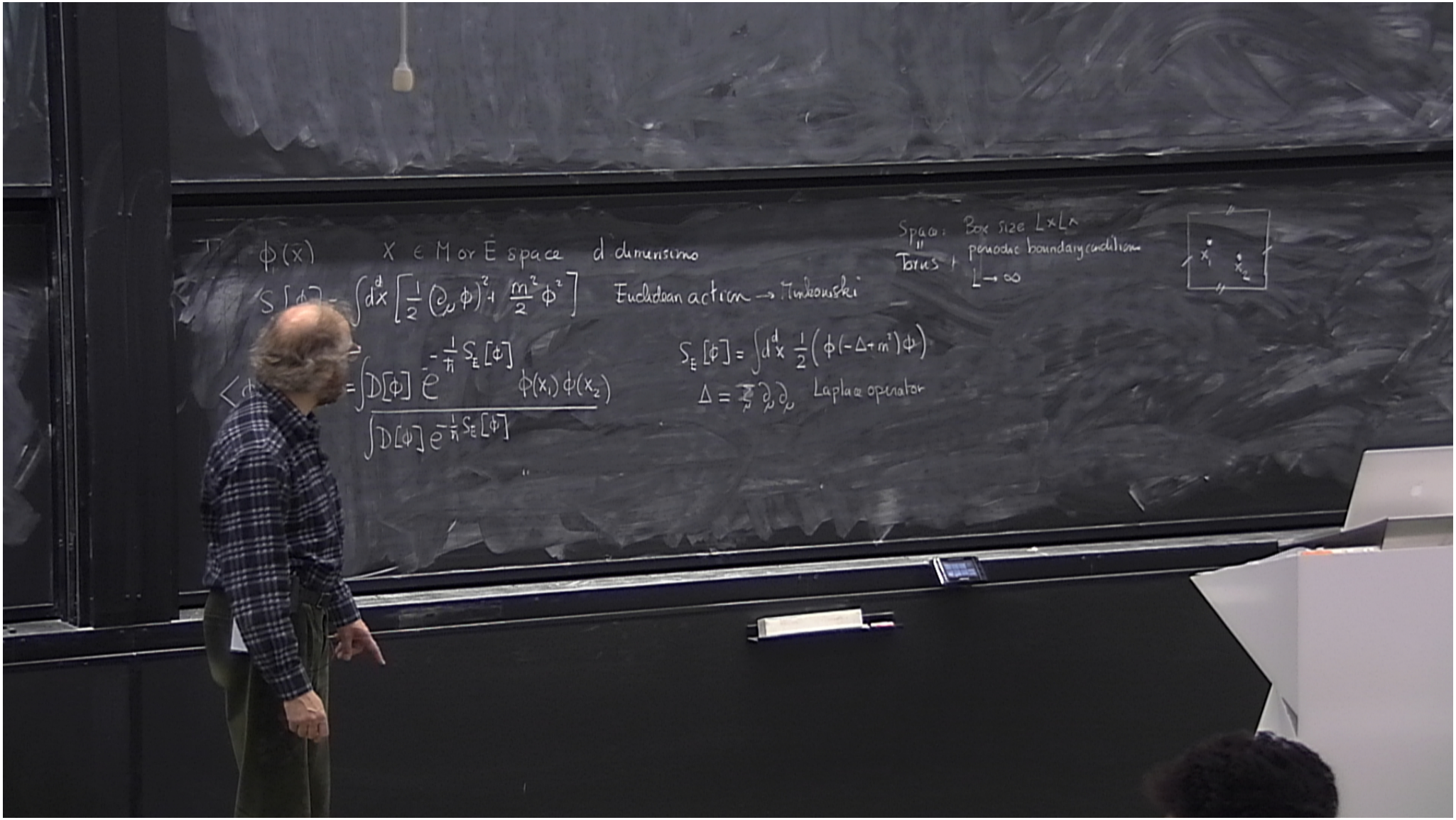
Euclidean action → Minkowski

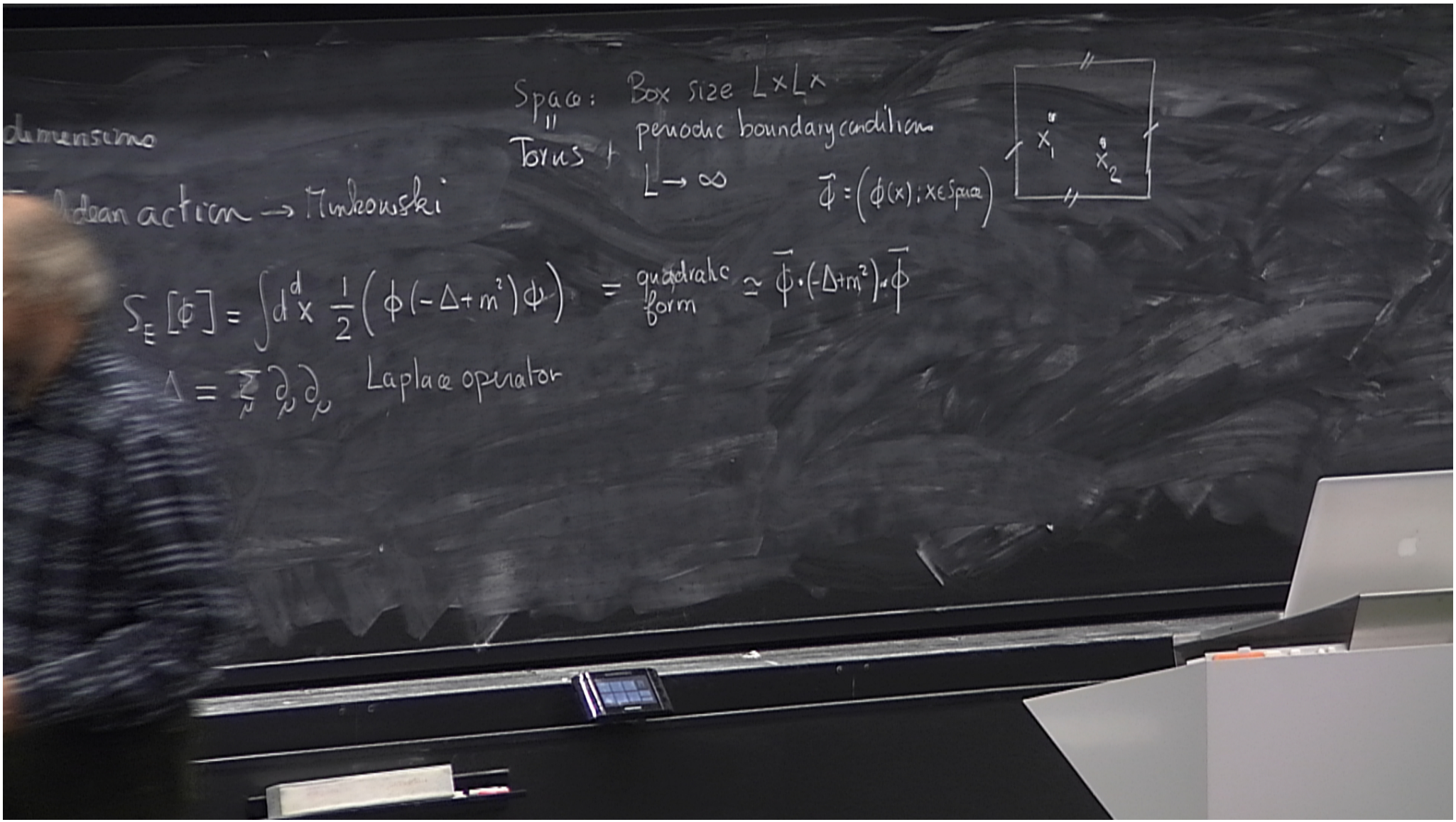
Space: Box size $L \times L \times$
=
Torus + periodic boundary condition
$L \to \infty$

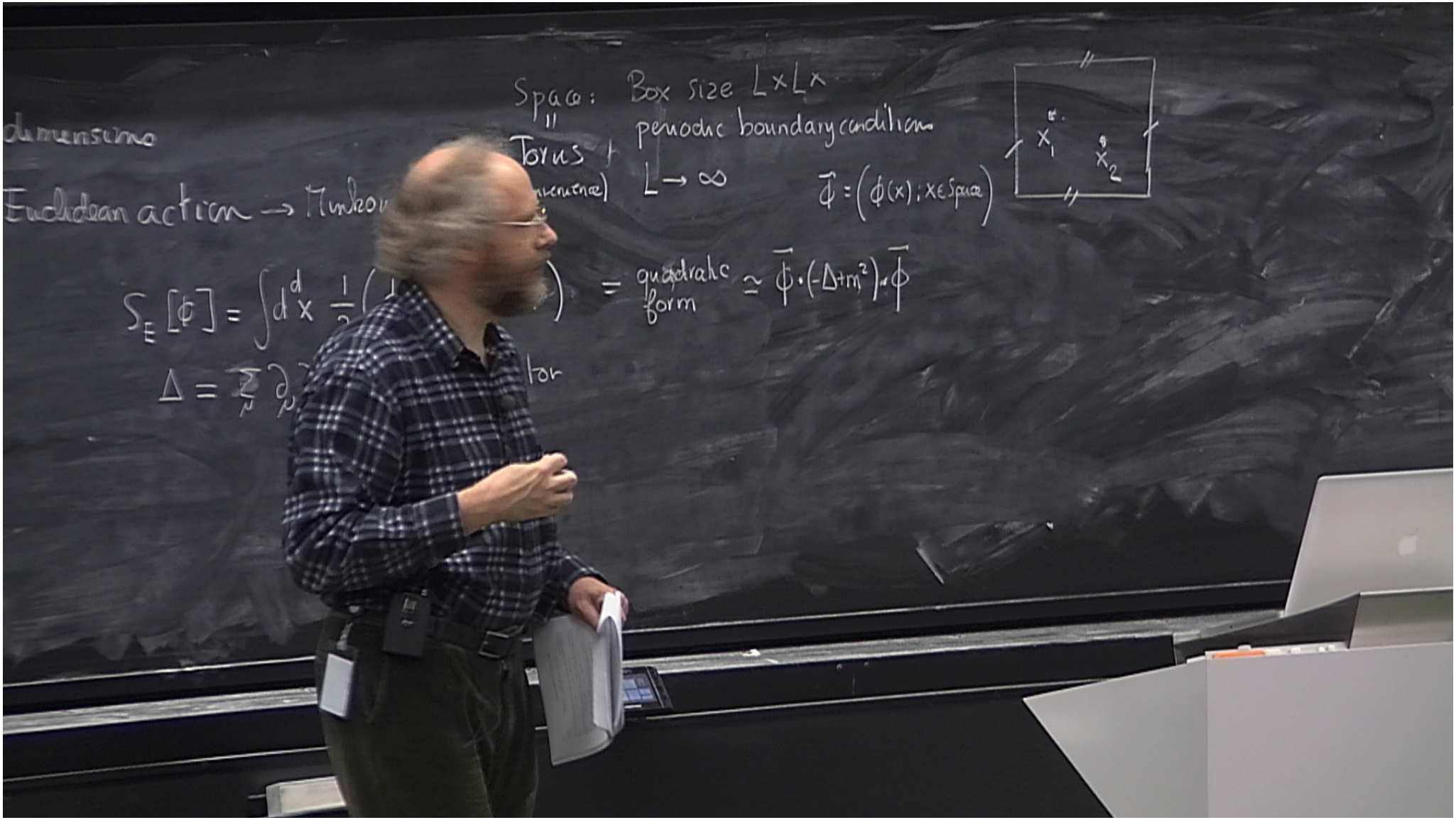


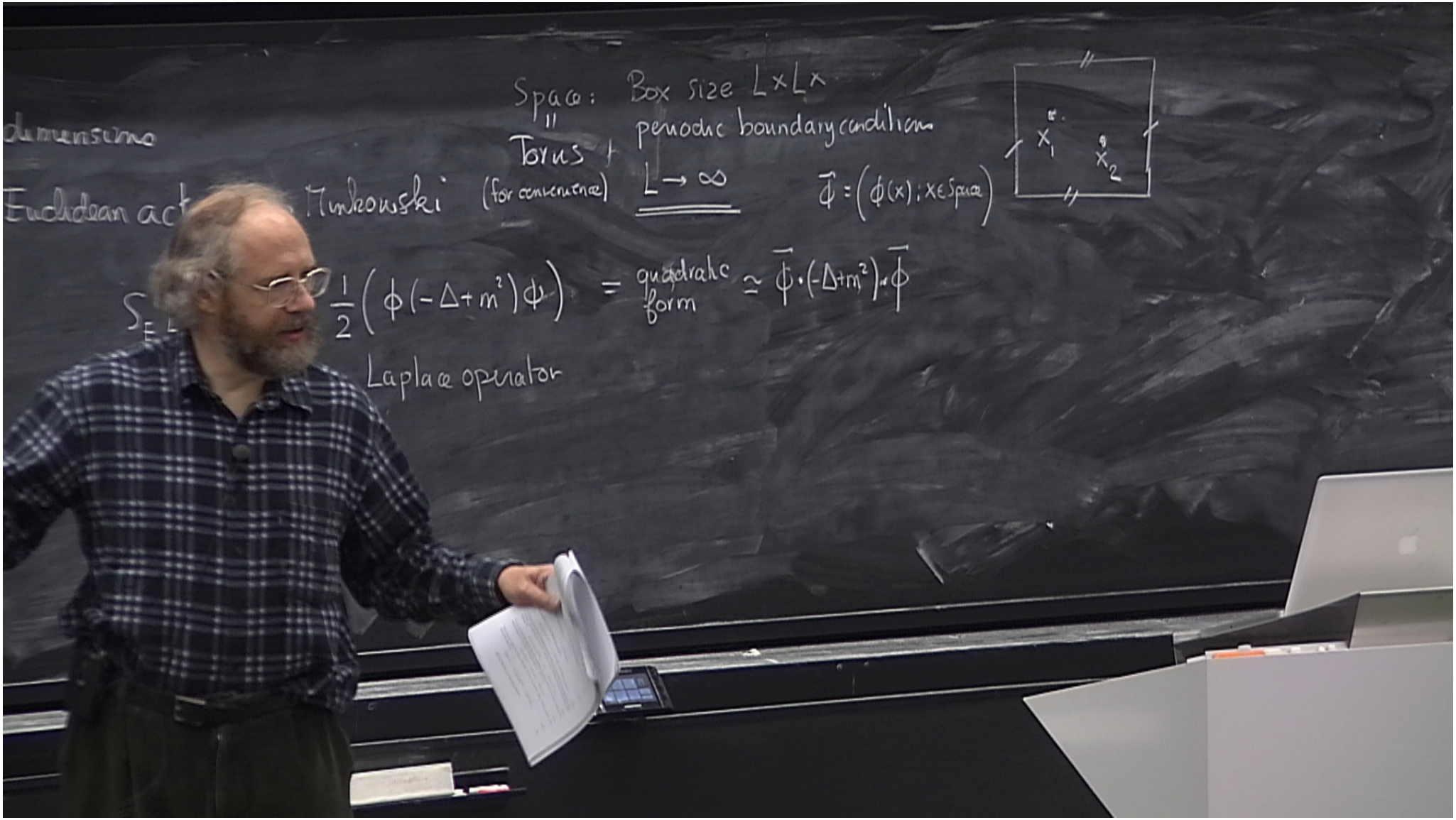
x_2)

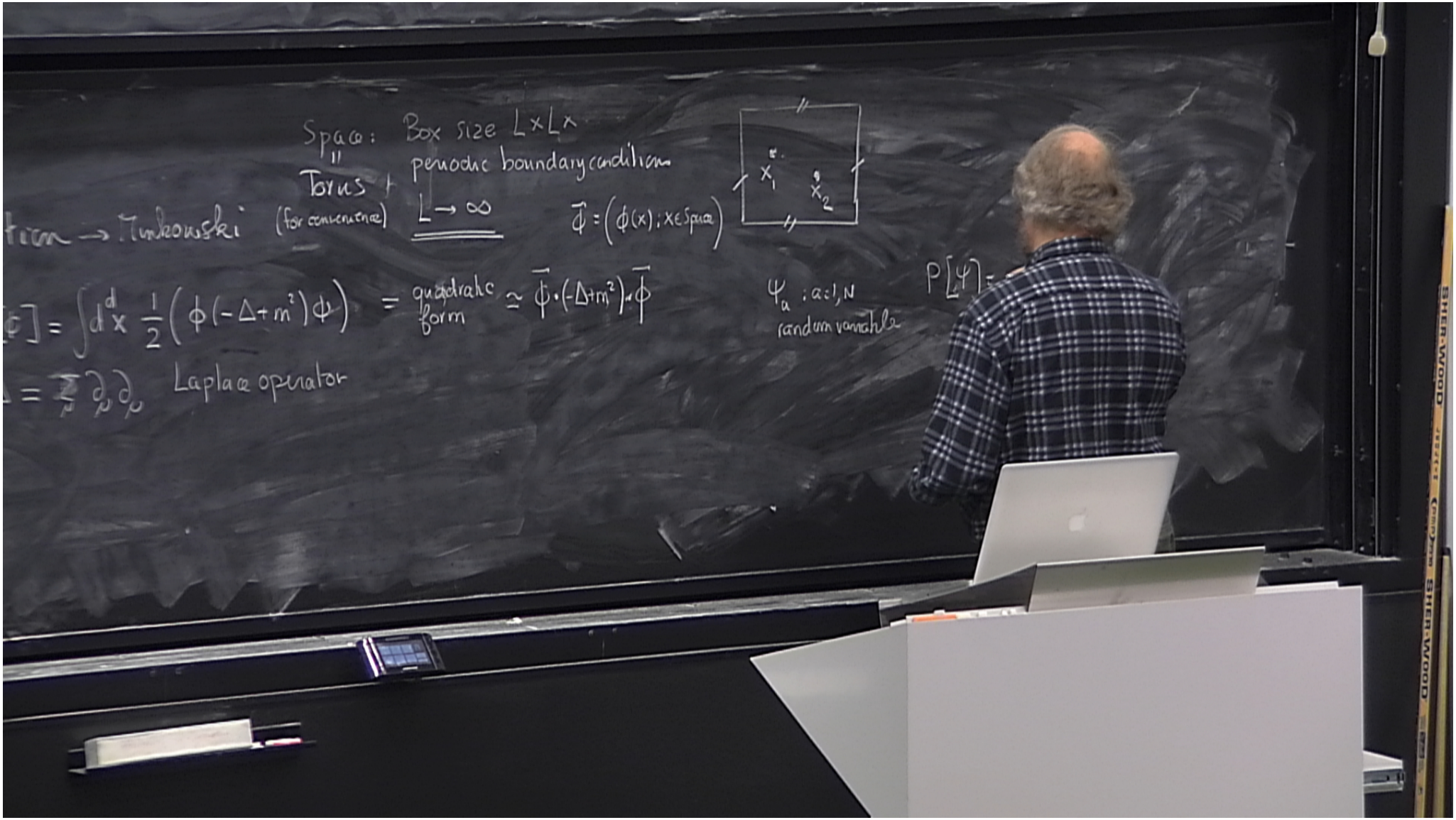

$\phi(\vec{x})$
$X \in M$ or E space
d dimensions
$= \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 \right]$
Euclidean action → Minkowski
$S_E[\phi] =$
$\langle \phi(x_1) \rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}\, \phi(x_1)\phi(x_2)}{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}}$
Space: Box size $L \times L$
Torus + periodic bound
$L \to \infty$

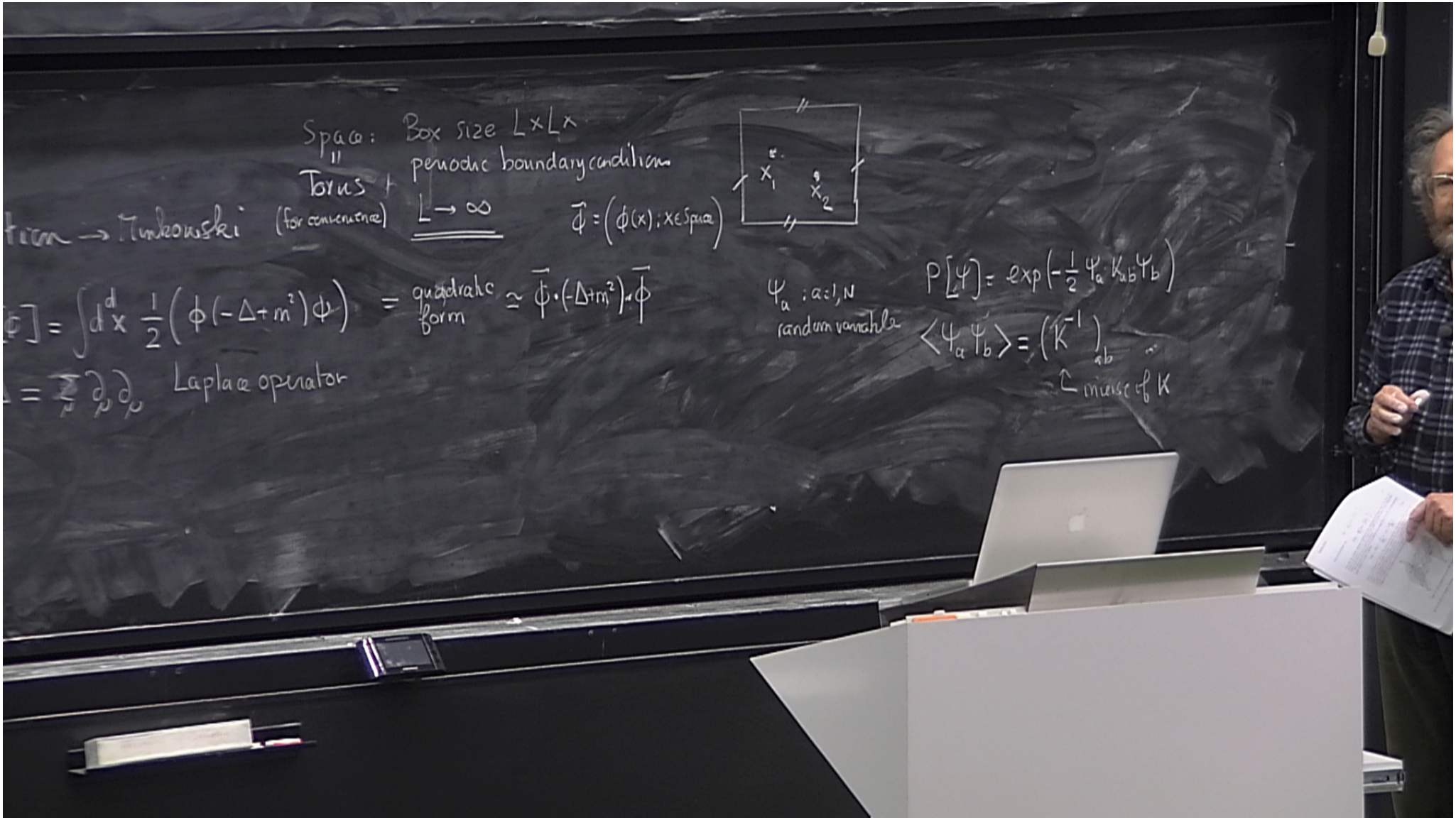

$\phi(x)$ $x \in M$ or E space d dimensions
$S[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$ Euclidean action → Minkowski
$\langle \phi \rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}\, \phi(x_1)\phi(x_2)}{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}}$
$S_E[\phi] = \int d^d x \, \frac{1}{2}\left(\phi(-\Delta + m^2)\phi\right)$
$\Delta = \sum_\mu \partial_\mu \partial_\mu$ Laplace operator
Space: Box size $L \times L$
" Torus + periodic boundary conditions
$L \to \infty$
x_1
x_2

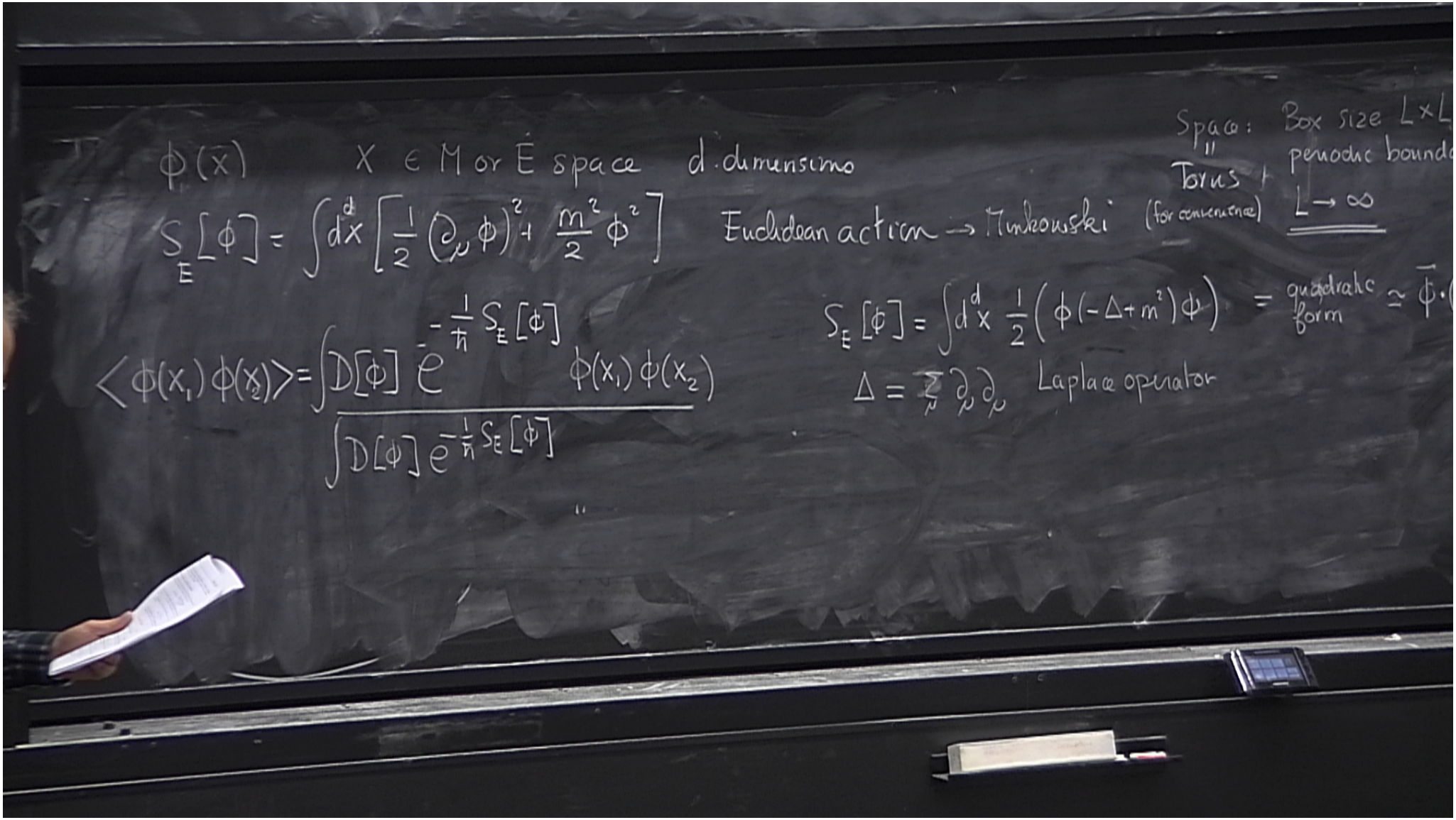

dimensions
Space: Box size L×L×
Torus + periodic boundary condition
L → ∞
$\vec{\phi} = (\phi(x); x \in \text{Space})$
x_1
x_2
Euclidean action → Minkowski
$S_E[\phi] = \int d^d x \, \frac{1}{2}\left(\phi(-\Delta + m^2)\phi\right)$ = quadratic form $\simeq \vec{\phi}\cdot(-\Delta+m^2)\cdot\vec{\phi}$
$\Delta = \sum_\mu \partial_\mu \partial_\mu$ Laplace operator


dimensions
Euclidean action → Minkow
Space: Box size L×L×
Torus = periodic boundary condition
L → ∞
$\vec{\phi} = (\phi(x); x \in \text{Space})$
$S_E[\phi] = \int d^d x$
= quadratic form $\simeq \vec{\phi} \cdot (-\Delta + m^2) \cdot \vec{\phi}$
$\Delta = \sum_\mu \partial_\mu$
x_1
x_2


dimensions
Euclidean act
Minkowski
Space: Box size L×L×
Torus = periodic boundary condition
(for convenience) L → ∞
$\vec{\phi} = (\phi(x); x \in \text{Space})$
x_1
x_2
S_E
$\frac{1}{2}(\phi(-\Delta+m^2)\phi)$ = quadratic form $\simeq \vec{\phi}\cdot(-\Delta+m^2)\cdot\vec{\phi}$
Laplace operator


Space: Box size L×L×
Torus + periodic boundary condition
(for convenience) L → ∞
tion → Minkowski
quadratic form
Laplace operator
random variable


Space: Box size L×L×
Torus + periodic boundary condition
(for convenience) L → ∞
$\vec{\phi} = (\phi(x); x \in \text{Space})$
x_1
x_2
tion → Minkowski
$S[\phi] = \int d^d x \, \frac{1}{2}(\phi(-\Delta + m^2)\phi) = \text{quadratic form} \simeq \vec{\phi} \cdot (-\Delta + m^2) \cdot \vec{\phi}$
$\Delta = \sum_\mu \partial_\mu \partial_\mu$ Laplace operator
ψ_a ; a = 1, N
random variable
$P[\psi] = \exp(-\frac{1}{2} \psi_a K_{ab} \psi_b)$
$\langle \psi_a \psi_b \rangle = (K^{-1})_{ab}$
inverse of K


$\phi(\vec{x})$ $x \in M$ or E space d. dimensions
$S_E[\phi] = \int d^dx \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$
Euclidean action → Minkowski
Space: Box size $L \times L$
Torus
periodic bound
(for convenience)
$L \to \infty$
$S_E[\phi] = \int d^dx \, \frac{1}{2}\left(\phi(-\Delta + m^2)\phi\right)$ = quadratic form $\simeq \vec{\phi}\cdot($
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}\, \phi(x_1)\phi(x_2)}{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}}$
$\Delta = \sum_\mu \partial_\mu \partial_\mu$ Laplace operator

$\phi(\vec{x})$ $\quad x \in M$ or E space $\quad$ d-dimensions

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 \right]$$

Euclidean action $\to$ Minkowski

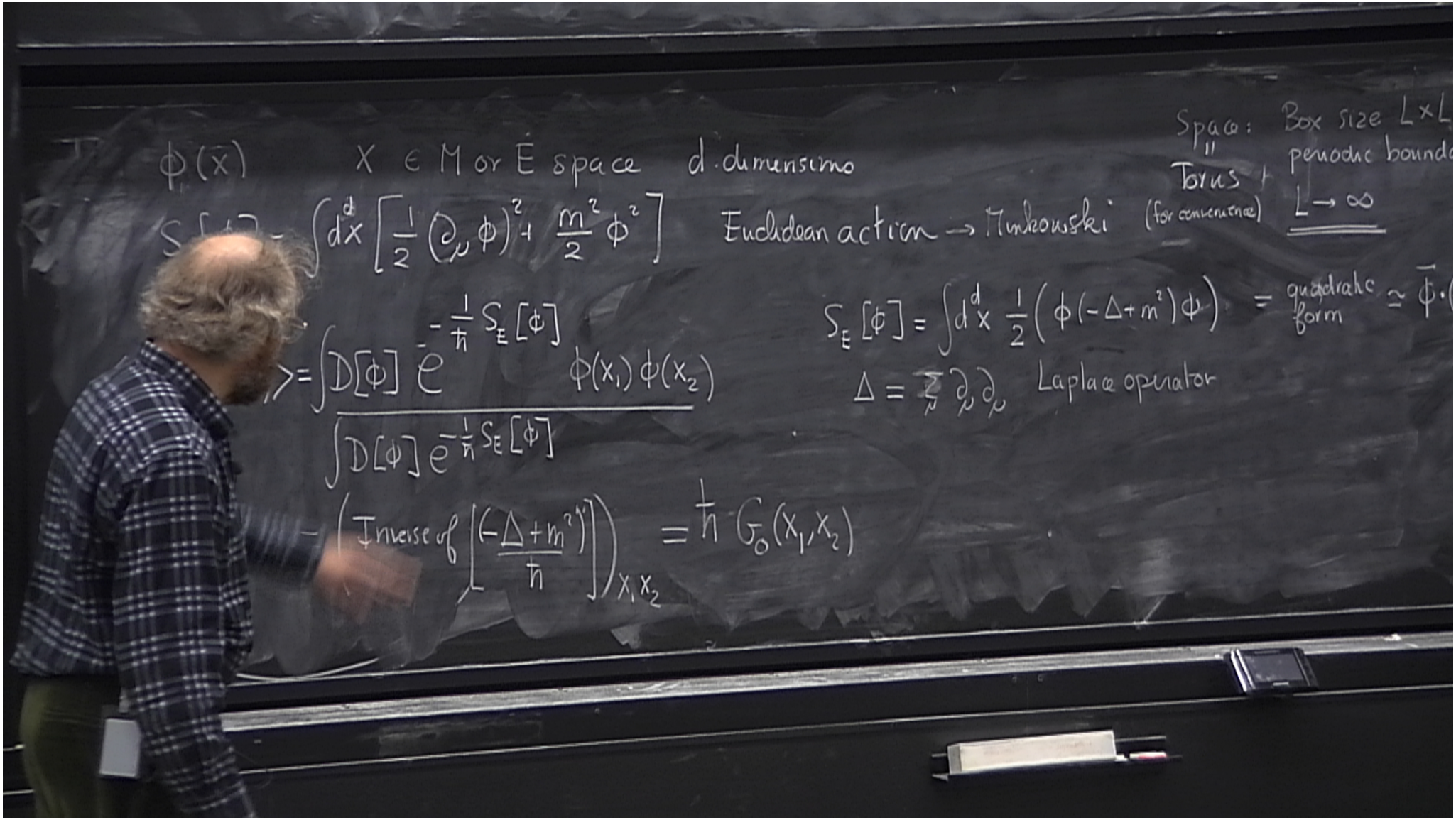
Space: Box size $L \times L$
$\parallel$
Torus, periodic bound
(for convenience) $\underline{\underline{L \to \infty}}$

$$S_E[\phi] = \int d^d x \, \frac{1}{2} \left(\phi (-\Delta + m^2) \phi \right) = \text{quadratic form} \simeq \vec{\phi} \cdot ($$

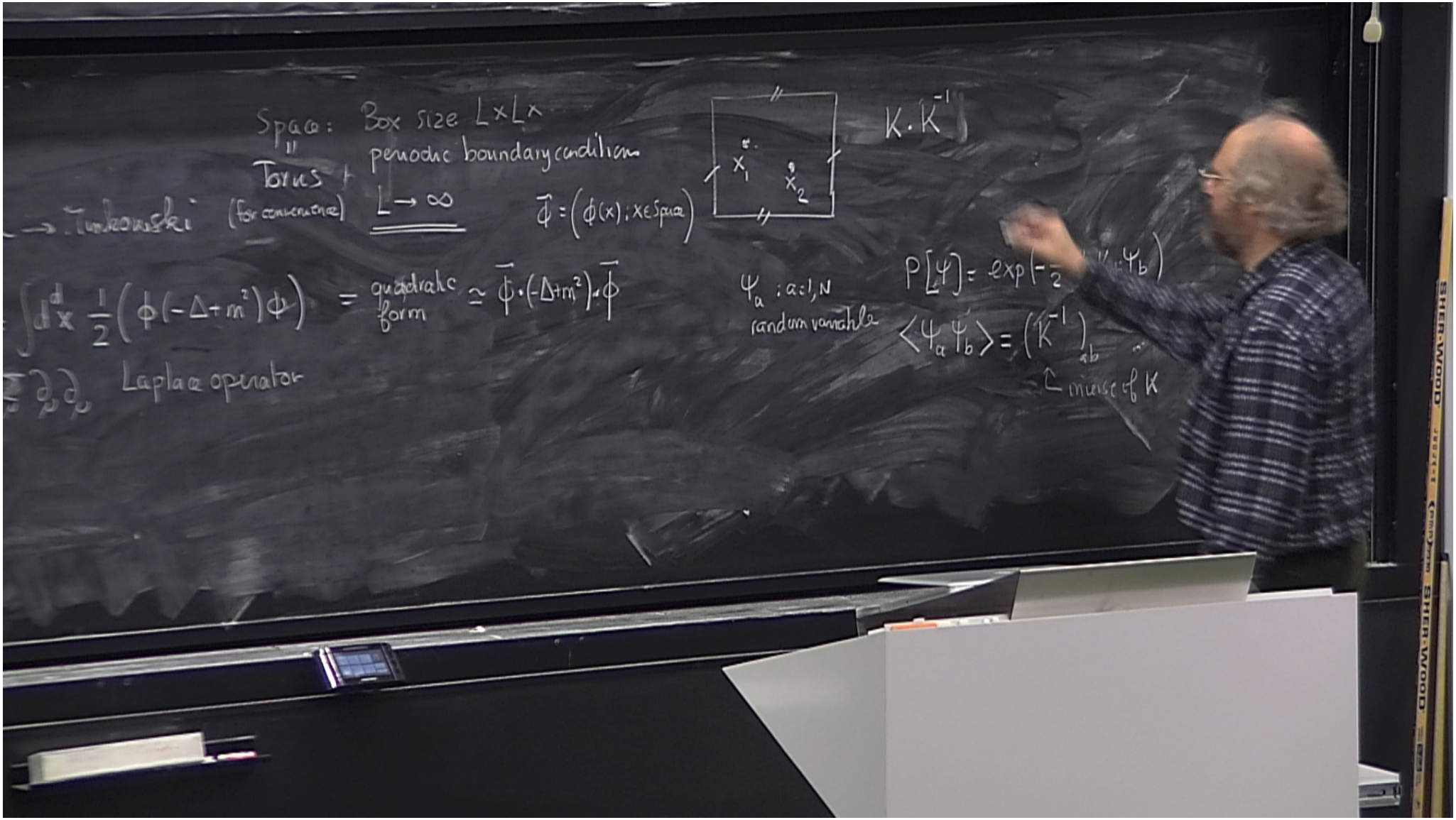
$$\Delta = \sum_\mu \partial_\mu \partial_\mu \quad \text{Laplace operator}$$

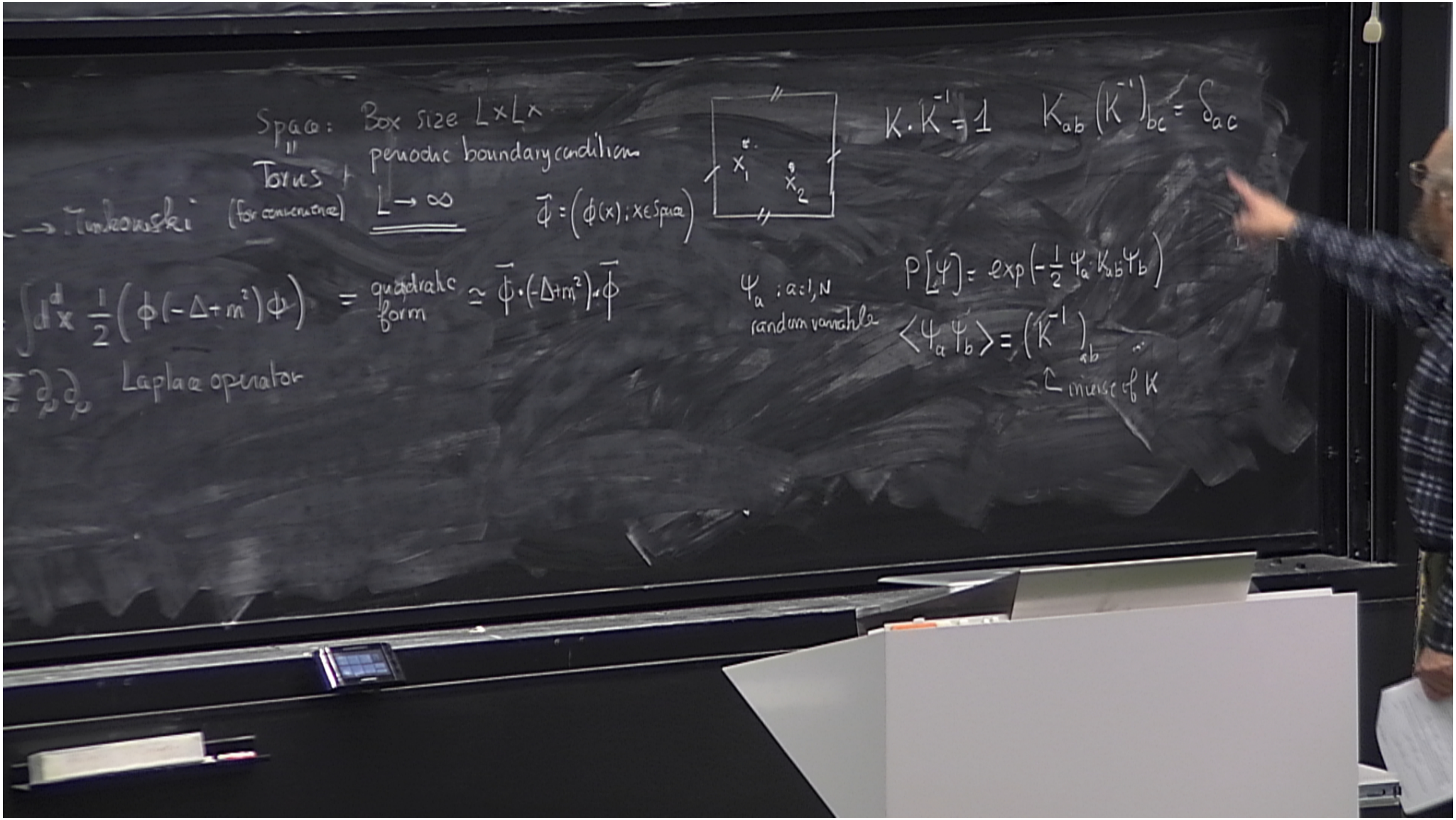
$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{\int D[\phi] \, e^{-\frac{1}{\hbar} S_E[\phi]} \, \phi(x_1) \phi(x_2)}{\int D[\phi] \, e^{-\frac{1}{\hbar} S_E[\phi]}}$$

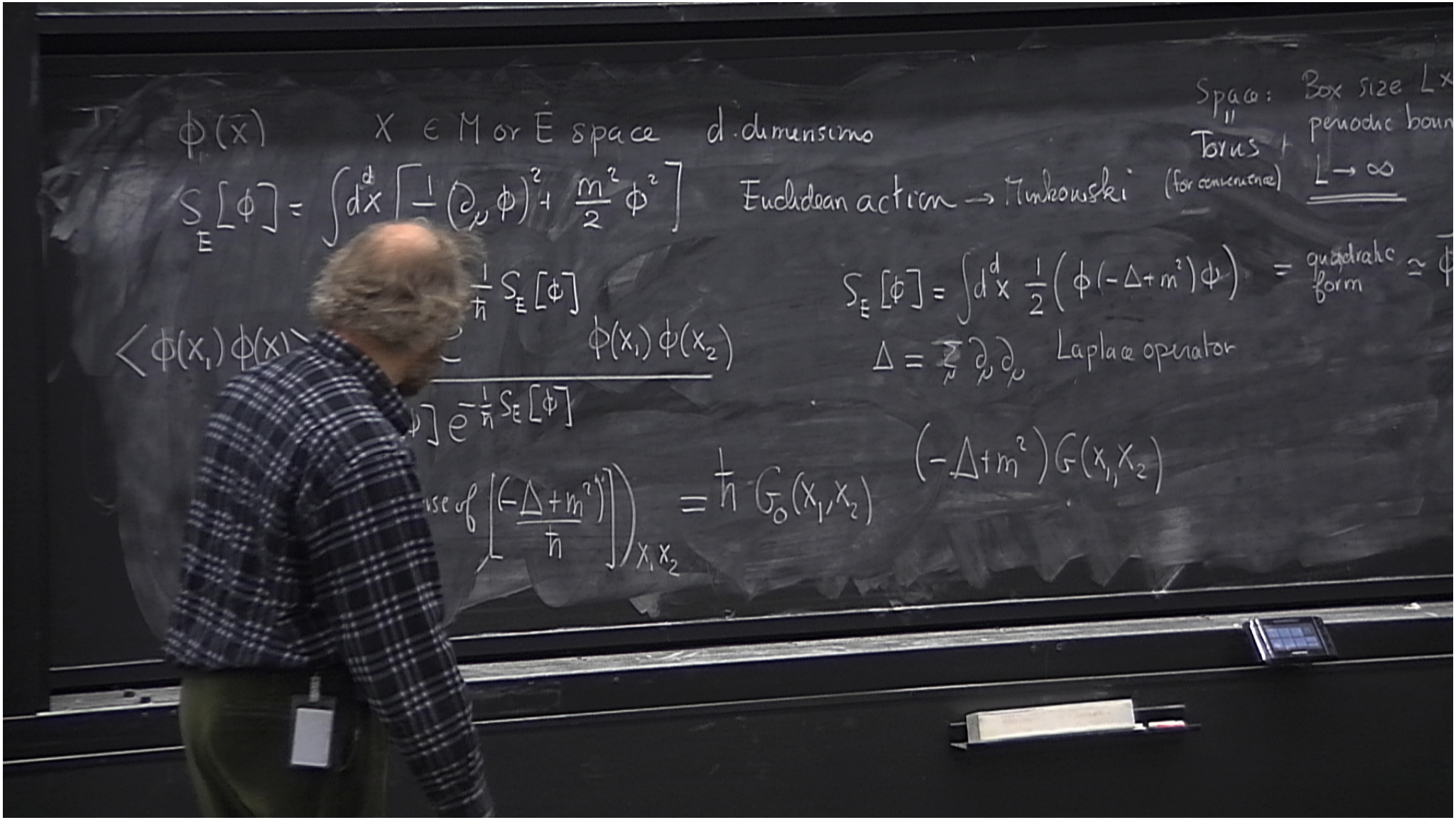
$$= \left(\text{Inverse of} \left[\frac{(-\Delta + m^2)}{\hbar} \right] \right)_{x_1 x_2}$$



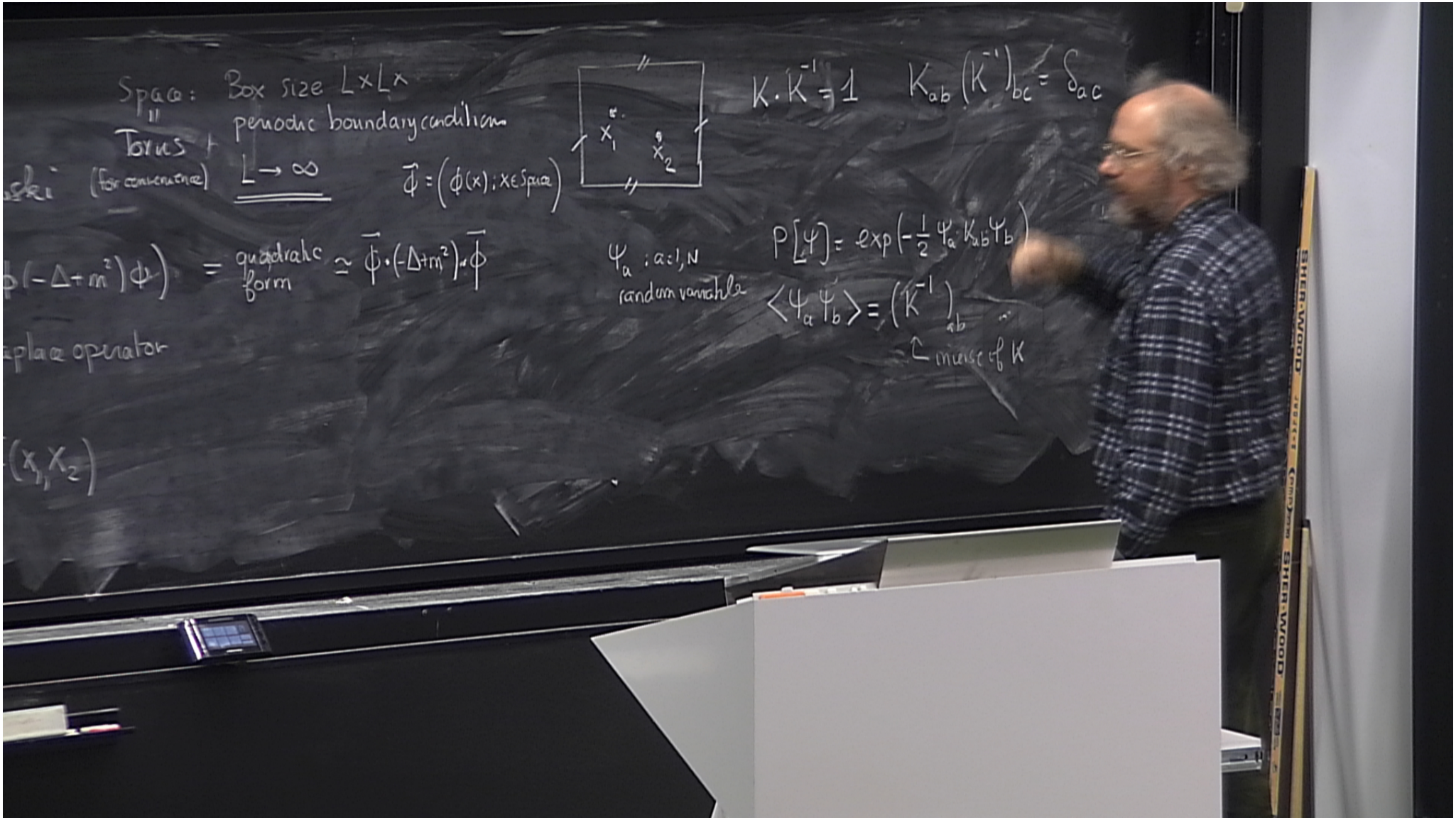
$\phi(\vec{x})$ $x \in M$ or E space d-dimensions
$S[\phi] = \int d^dx \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$
Euclidean action → Minkowski
Space: Box size $L \times L$
Torus
periodic bound
(for convenience)
$L \to \infty$
$S_E[\phi] = \int d^dx \, \frac{1}{2}\left(\phi(-\Delta + m^2)\phi\right)$ = quadratic form
$\Delta = \sum_\mu \partial_\mu \partial_\mu$ Laplace operator
$\langle \phi(x_1)\phi(x_2) \rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}\, \phi(x_1)\phi(x_2)}{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}}$
$\left(\text{Inverse of } \left[\frac{(-\Delta + m^2)}{\hbar}\right]\right)_{x_1 x_2} = \hbar\, G_0(x_1, x_2)$


Space: Box size L×L×
periodic boundary condition
Torus
(for convenience)
L → ∞
Minkowski
quadratic form
Laplace operator
random variable
inverse of K
K·K⁻¹


Space: Box size L×L×
Torus = periodic boundary condition
(for convenience) L → ∞
→ Minkowski
$\vec{\phi} = (\phi(x); x \in \text{Space})$
x_1
x_2
$K \cdot K^{-1} = 1$
$K_{ab} (K^{-1})_{bc} = \delta_{ac}$
$\int d^d x \frac{1}{2} \left(\phi (-\Delta + m^2) \phi \right) = \text{quadratic form} \simeq \vec{\phi} \cdot (-\Delta + m^2) \cdot \vec{\phi}$
Ψ_a : a : 1, N
random variable
$P[\Psi] = \exp\left(-\frac{1}{2} \Psi_a K_{ab} \Psi_b\right)$
$\langle \Psi_a \Psi_b \rangle = (K^{-1})_{ab}$
$\sum \partial_\mu \partial_\mu$ Laplace operator
↳ inverse of K



$\phi(\bar{x})$
$x \in M$ or E space
d dimensions
$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 \right]$
Euclidean action → Minkowski
Space: Box size $L \times$
Torus + periodic boun
(for convenience)
$L \to \infty$
$S_E[\phi] = \int d^d x \, \frac{1}{2} \left(\phi (-\Delta + m^2) \phi \right)$
= quadratic form
$\Delta = \sum_\mu \partial_\mu \partial_\mu$
Laplace operator
$\langle \phi(x_1) \phi(x_2) \rangle$
$\phi(x_1) \phi(x_2)$
$e^{-\frac{1}{\hbar} S_E[\phi]}$
$(-\Delta + m^2) G(x_1, x_2)$
$= \hbar \, G_0(x_1, x_2)$


Space: Box size L×L×
= Torus + periodic boundary condition
(for convenience) L → ∞
$\vec{\phi} = (\phi(x); x \in \text{Space})$
x_1
x_2
$K \cdot K^{-1} = 1$
$K_{ab} (K^{-1})_{bc} = \delta_{ac}$
$(-\Delta + m^2)\phi)$ = quadratic form $\simeq \vec{\phi} \cdot (-\Delta + m^2) \cdot \vec{\phi}$
ψ_a ; a: 1, N
random variable
$P[\psi] = \exp(-\frac{1}{2} \psi_a K_{ab} \psi_b)$
$\langle \psi_a \psi_b \rangle = (K^{-1})_{ab}$
↑ inverse of K
place operator
(x_1, x_2)

$\phi(x)$ $x \in$ Minkowski or Euclidean space d dimensions

Space: Torus (for convenience) Box size $L \times L$ periodic boundary $L \to \infty$

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 \right]$$ Euclidean action $\to$ Minkowski

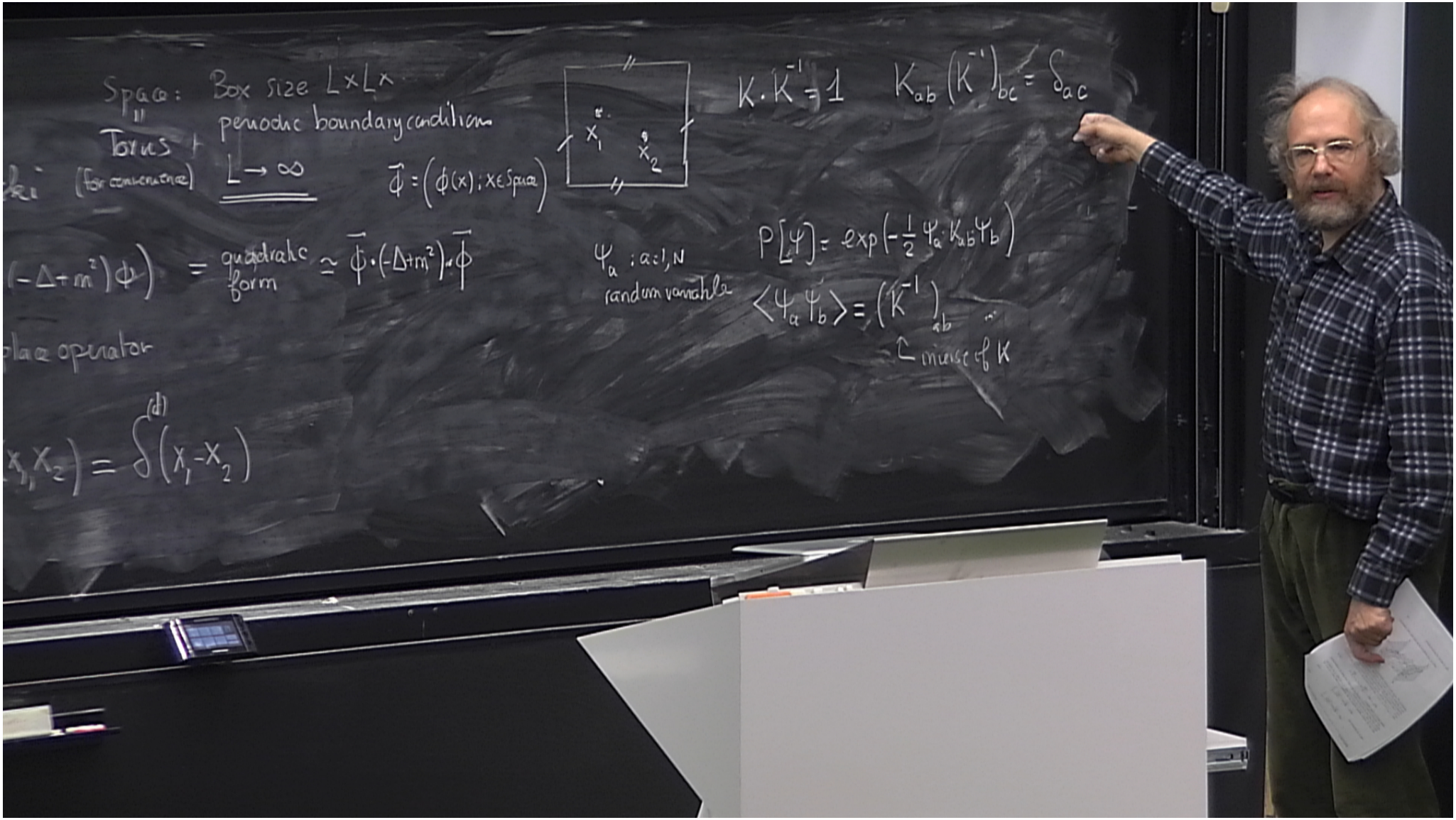
$$S_E[\phi] = \int d^d x \, \frac{1}{2} \left(\phi (-\Delta + m^2) \phi \right)$$ = quadratic form $\simeq \vec{\phi} \cdot$

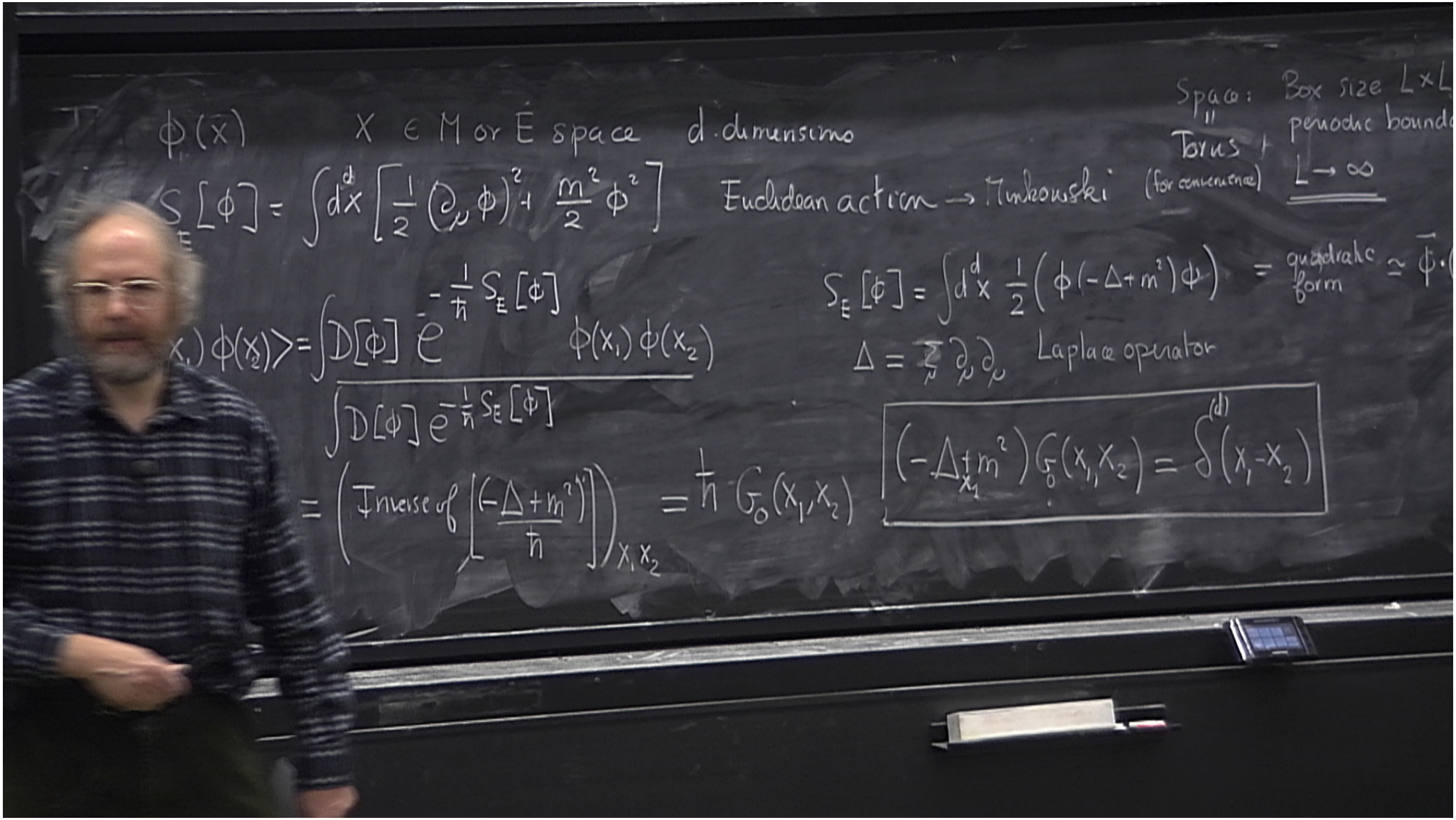
$$\Delta = \sum_\mu \partial_\mu \partial_\mu$$ Laplace operator

$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{\int D[\phi] \, e^{-\frac{1}{\hbar} S_E[\phi]} \, \phi(x_1) \phi(x_2)}{\int D[\phi] \, e^{-\frac{1}{\hbar} S_E[\phi]}}$$

$$= \left(\text{Inverse of } \left[\frac{(-\Delta + m^2)}{\hbar} \right] \right)_{x_1 x_2} = \hbar \, G_0(x_1, x_2)$$

$$(-\Delta_{x_1} + m^2) G(x_1, x_2) =$$


Space: Box size L×L×
Torus = periodic boundary condition
L → ∞
$\vec{\phi} = (\phi(x); x \in \text{Space})$
$K \cdot K^{-1} = 1$
$K_{ab} (K^{-1})_{bc} = \delta_{ac}$
$(-\Delta + m^2)\phi)$ = quadratic form $\simeq \vec{\phi} \cdot (-\Delta + m^2) \cdot \vec{\phi}$
ψ_a ; a = 1, N
random variable
$P[\psi] = \exp(-\frac{1}{2} \psi_a \cdot K_{ab} \psi_b)$
$\langle \psi_a \psi_b \rangle = (K^{-1})_{ab}$
↑ inverse of K
$\delta(x_1 - x_2)$


$\phi(x)$ $x \in M$ or E space d dimensions
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$
Euclidean action → Minkowski
Space: Torus (for convenience)
Box size L×L periodic bound
$L \to \infty$
$S_E[\phi] = \int d^d x \frac{1}{2}\left(\phi(-\Delta + m^2)\phi\right)$ = quadratic form
$\Delta = \sum_\mu \partial_\mu \partial_\mu$ Laplace operator
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi] e^{-\frac{1}{\hbar}S_E[\phi]} \phi(x_1)\phi(x_2)}{\int D[\phi] e^{-\frac{1}{\hbar}S_E[\phi]}}$
$= \left(\text{Inverse of } \left[\frac{(-\Delta + m^2)}{\hbar}\right]\right)_{x_1 x_2} = \hbar\, G_0(x_1, x_2)$
$(-\Delta_{x_1} + m^2) G_0(x_1, x_2) = \delta^{(d)}(x_1 - x_2)$

$\phi(\vec{x})$ $\quad x \in M$ or E space $\quad$ d-dimensions

Space: Box size $L \times L$
$\parallel$ periodic bound
Torus (for convenience) $\quad L \to \infty$

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 \right]$$

Euclidean action $\to$ Minkowski

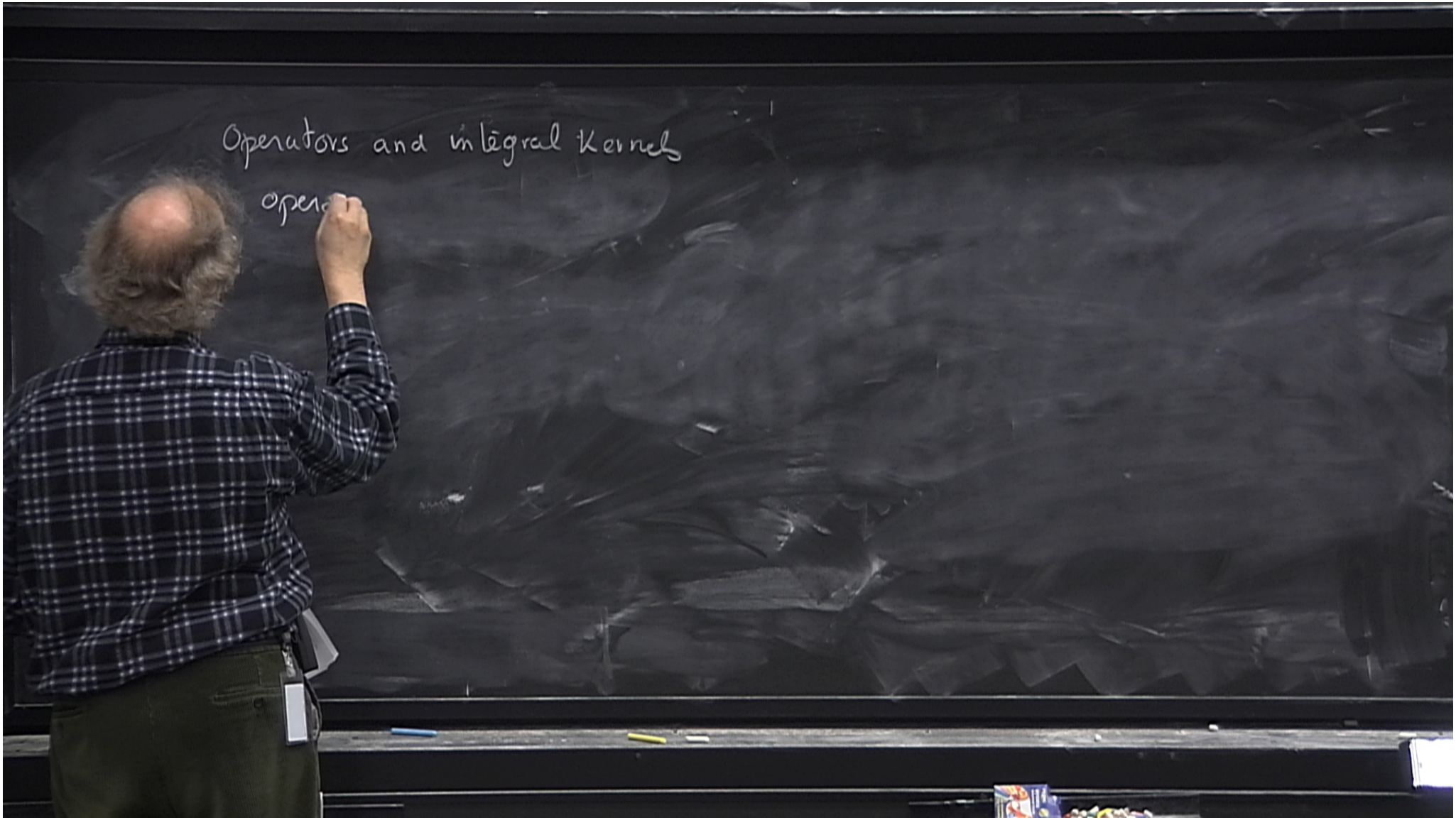
$$S_E[\phi] = \int d^d x \, \frac{1}{2} \left(\phi (-\Delta + m^2) \phi \right) = \text{quadratic form} \simeq \vec{\phi} \cdot ($$

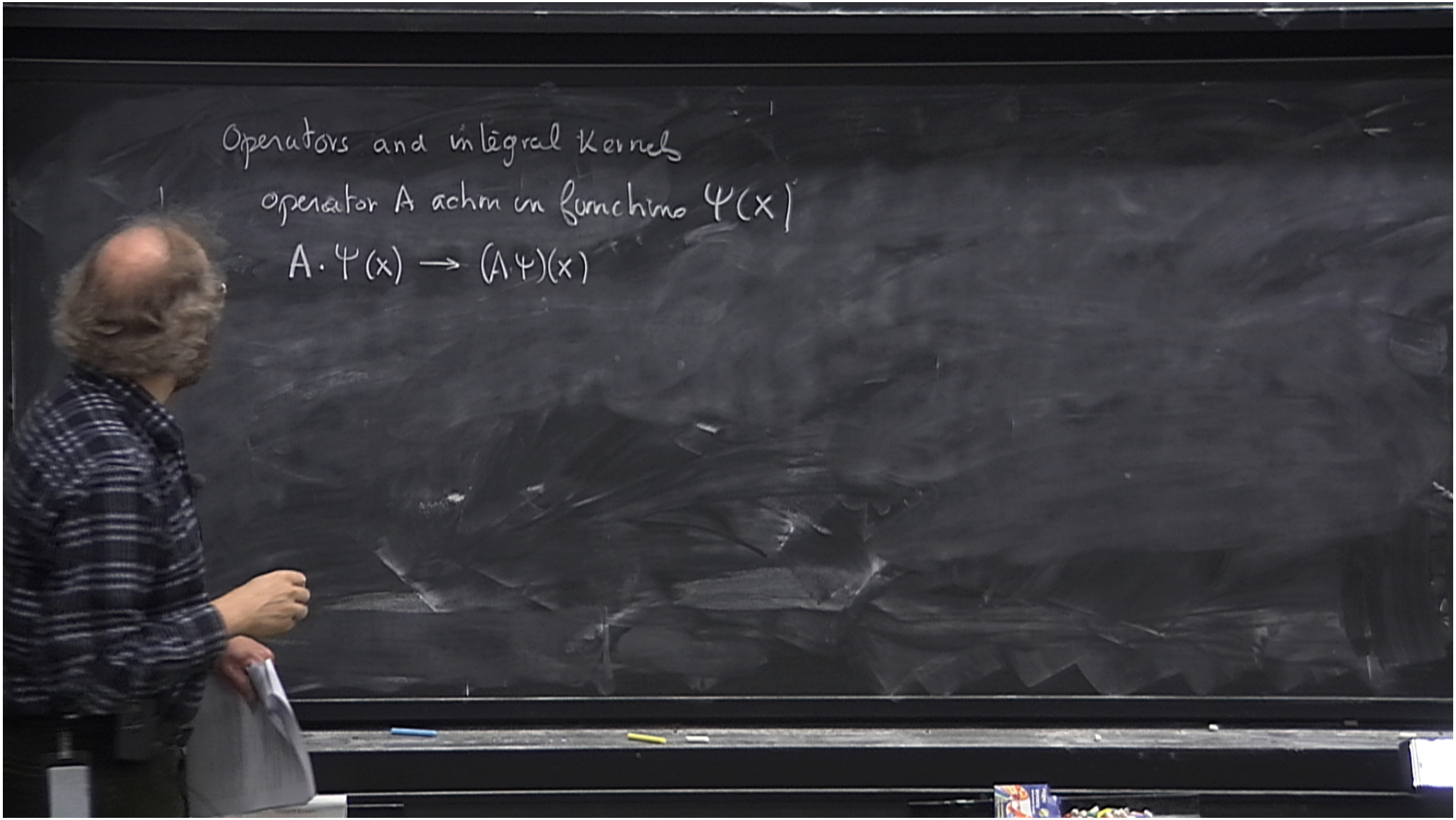
$$\Delta = \sum_\mu \partial_\mu \partial_\mu \quad \text{Laplace operator}$$

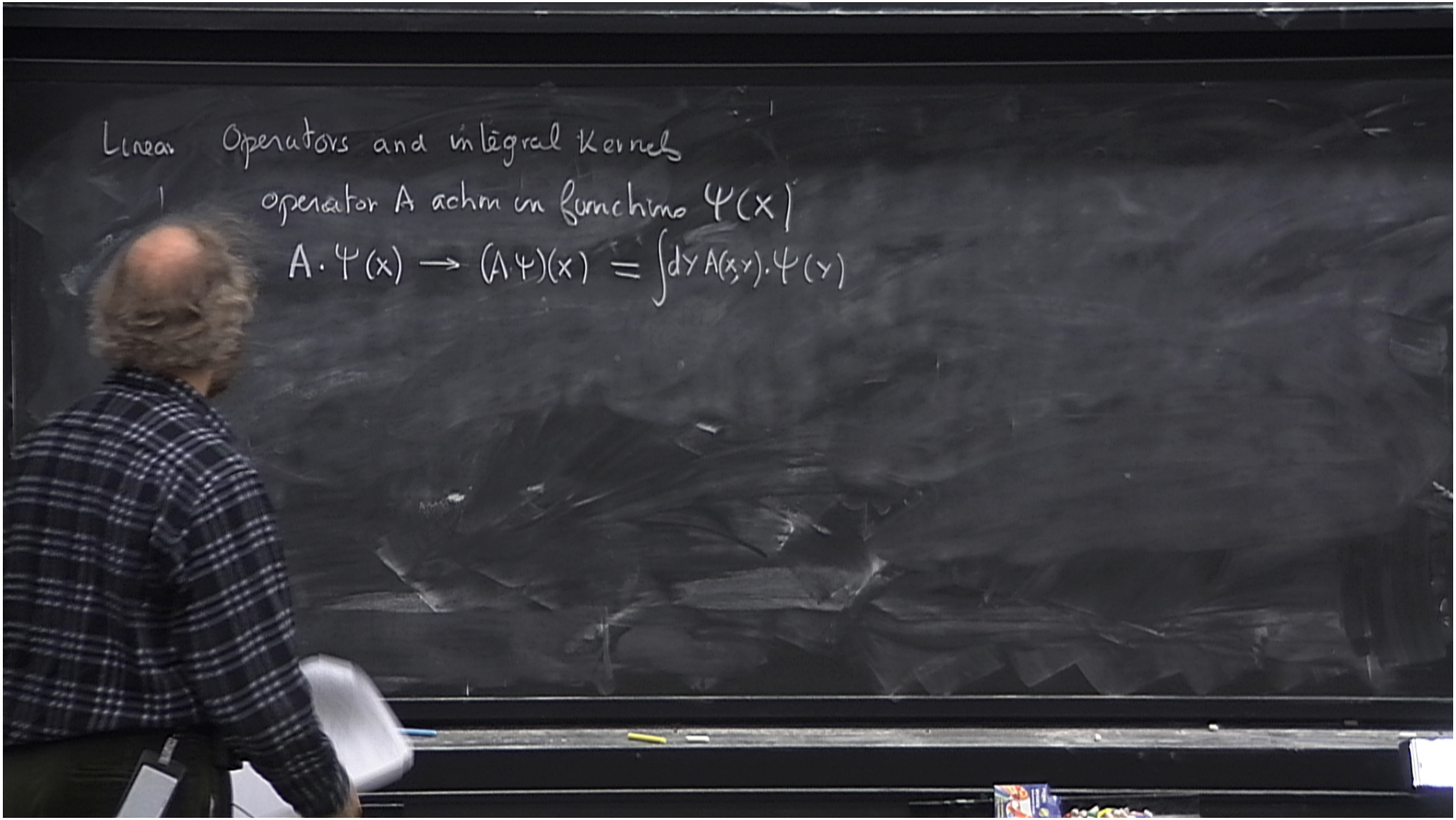
$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{\int D[\phi] \, e^{-\frac{1}{\hbar} S_E[\phi]} \phi(x_1) \phi(x_2)}{\int D[\phi] \, e^{-\frac{1}{\hbar} S_E[\phi]}}$$

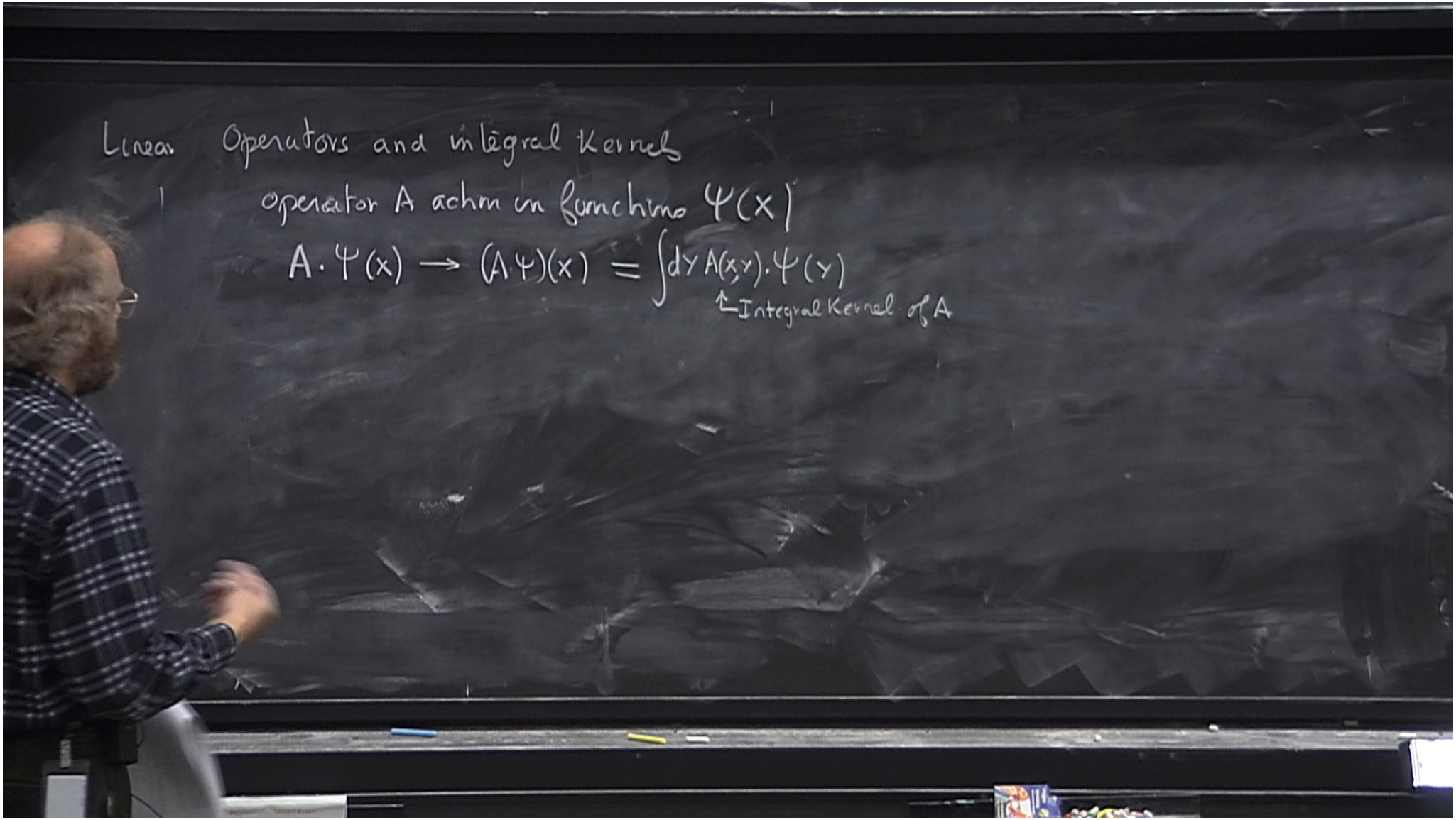
$$= \left(\text{Inverse of} \left[\frac{(-\Delta + m^2)}{\hbar} \right] \right)_{x_1 x_2} = \hbar \, G_0(x_1, x_2)$$

$$\boxed{(-\Delta_{x_1} + m^2) G_0(x_1, x_2) = \delta^{(d)}(x_1 - x_2)}$$


Operators and integral kernels


Operators and integral kernels
operator A acting on function $\Psi(x)$
$A \cdot \Psi(x) \rightarrow (A\Psi)(x)$


Linear Operators and integral Kernels
operator A acting on function $\Psi(x)$
$A \cdot \Psi(x) \rightarrow (A\Psi)(x) = \int dy\, A(x,y) \cdot \Psi(y)$


Linear Operators and integral Kernels
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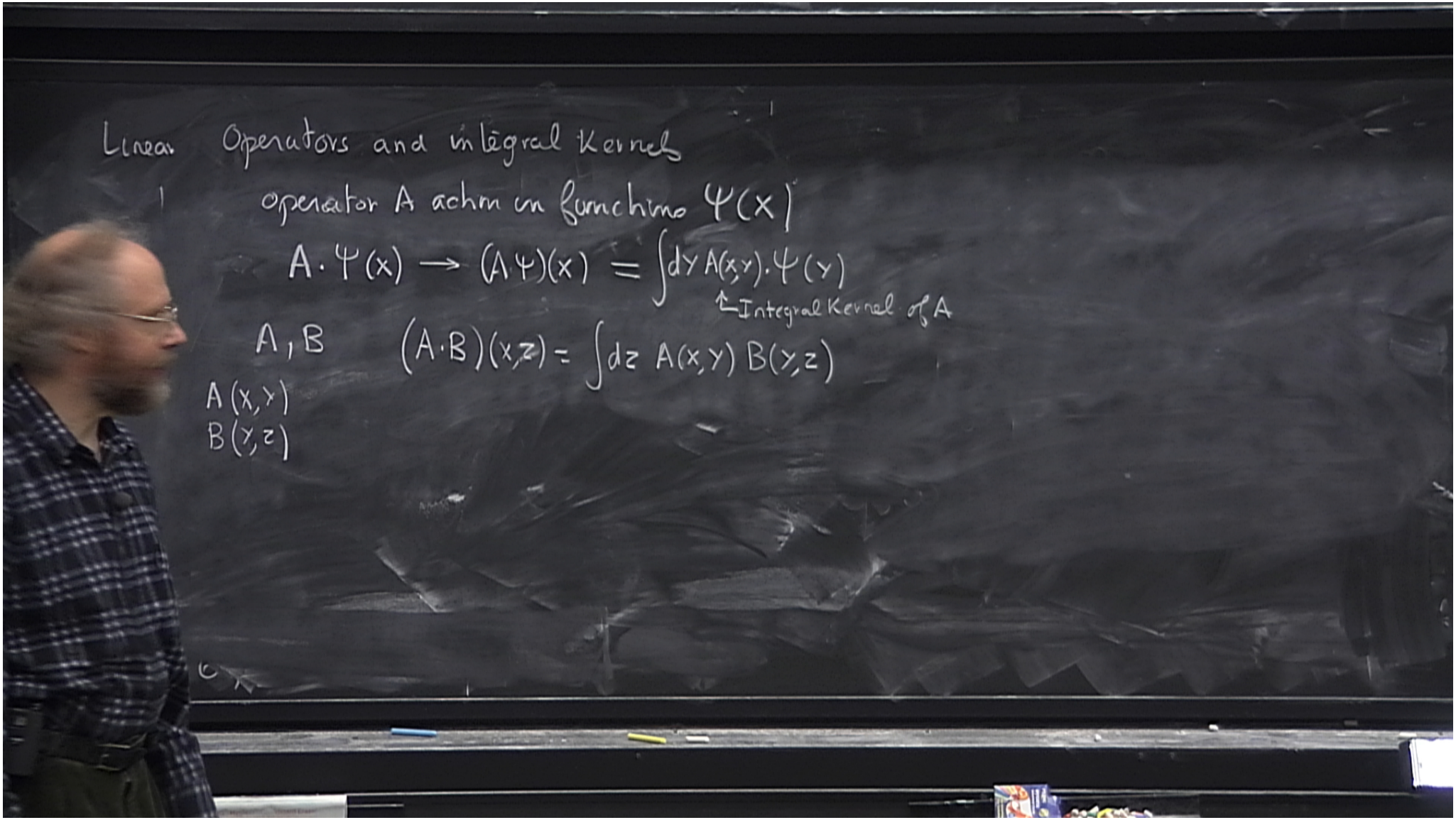
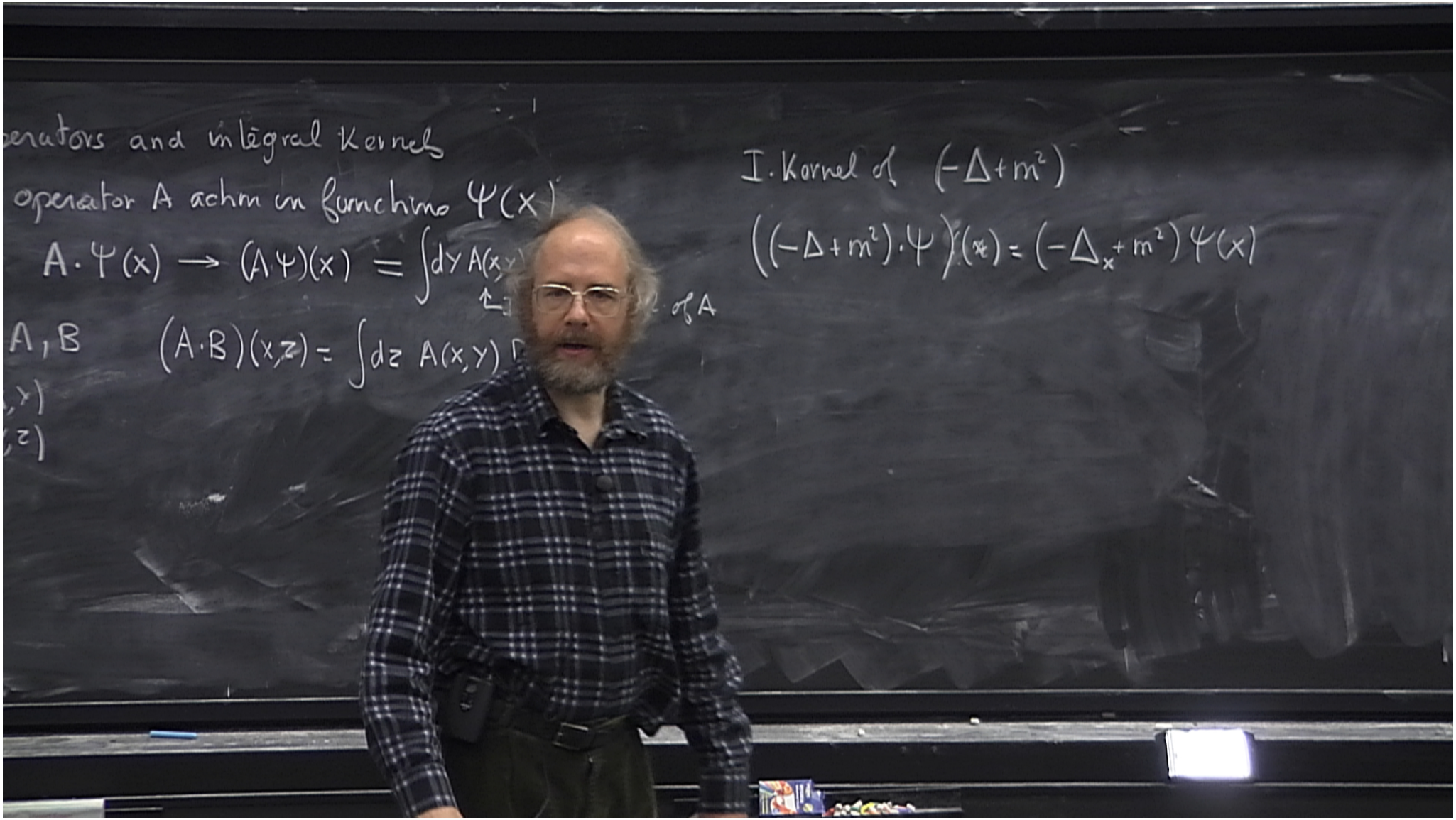
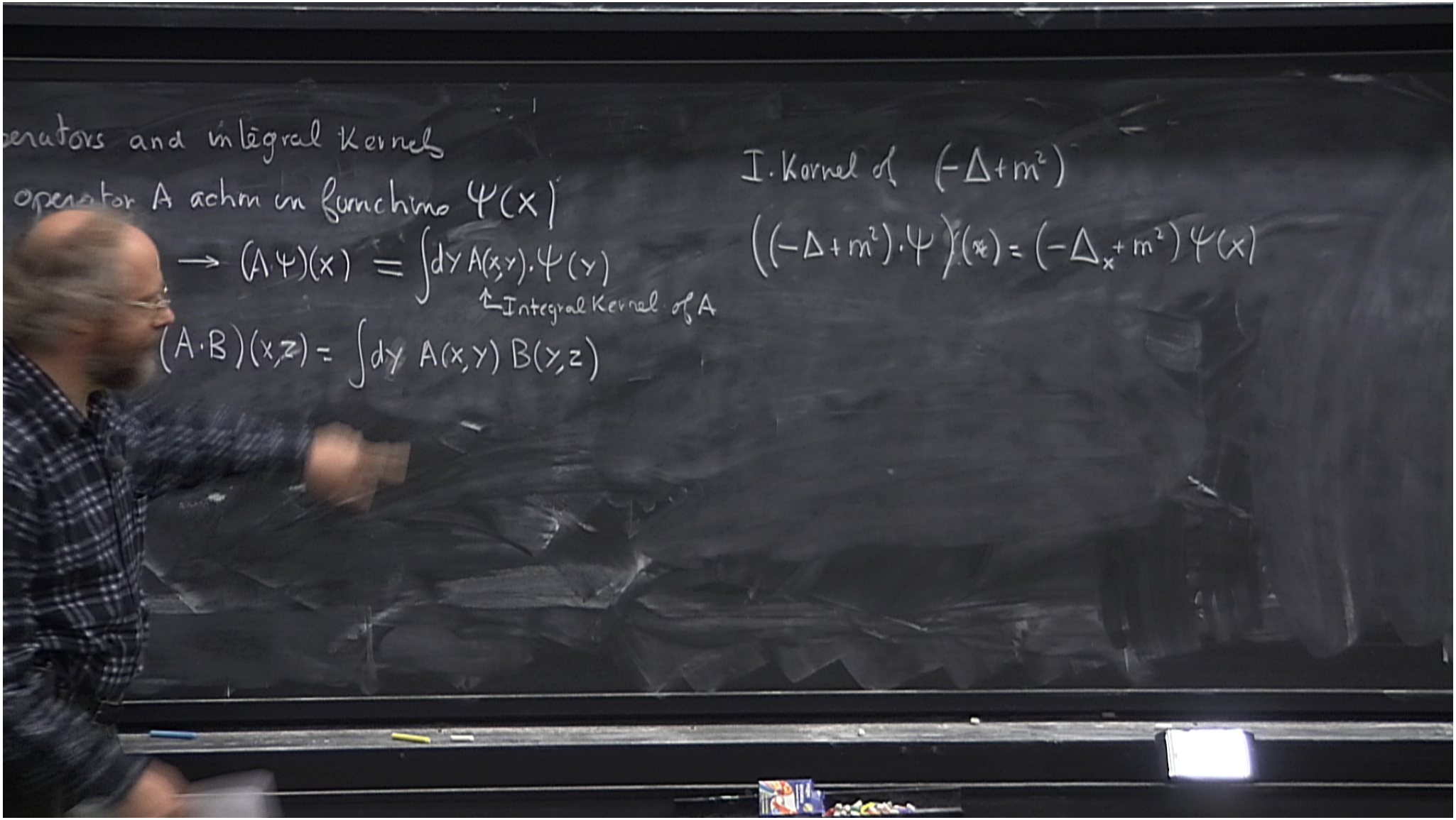
↑ Integral Kernel of A


Linear Operators and integral Kernels
operator A acting on function $\Psi(x)$
$A \cdot \Psi(x) \rightarrow (A\Psi)(x) = \int dy\, A(x,y) \cdot \Psi(y)$
↑ Integral Kernel of A
$A, B \quad (A \cdot B)(x,z) = \int dz\, A(x,y)\, B(y,z)$
$A(x,y)$
$B(y,z)$


erators and integral Kernels
operator A acting on function $\Psi(x)$
$A \cdot \Psi(x) \longrightarrow (A\Psi)(x) = \int dy\, A(x,y)$
of A
A, B
$(A \cdot B)(x,z) = \int dz\, A(x,y)$
I. Kernel of $(-\Delta + m^2)$
$\left((-\Delta + m^2) \cdot \Psi\right)(x) = (-\Delta_x + m^2)\,\Psi(x)$


erators and integral Kernels
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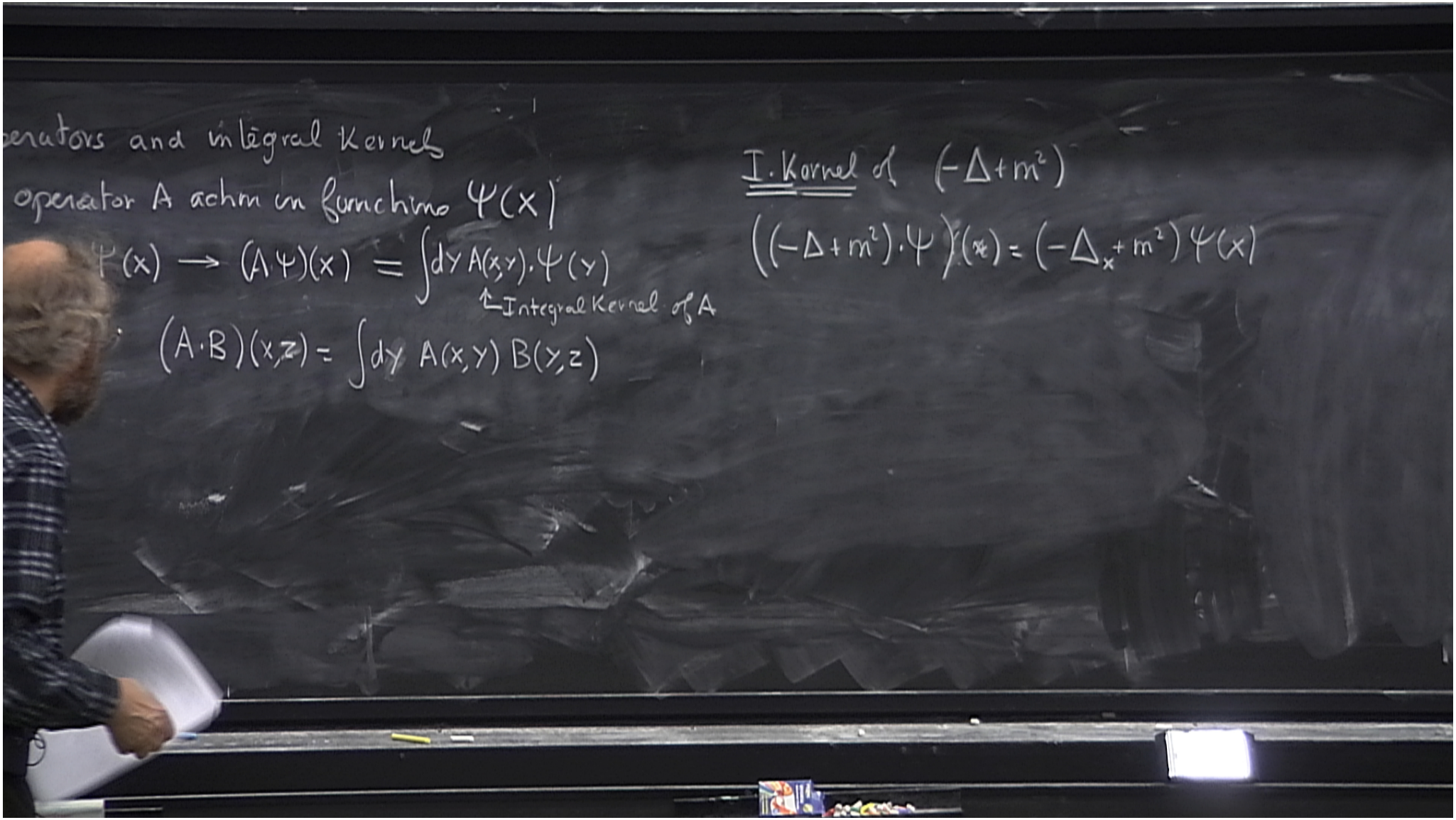
erators and integral Kernels
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$\Psi(x) \to (A\Psi)(x) = \int dy\, A(x,y)\cdot\Psi(y)$
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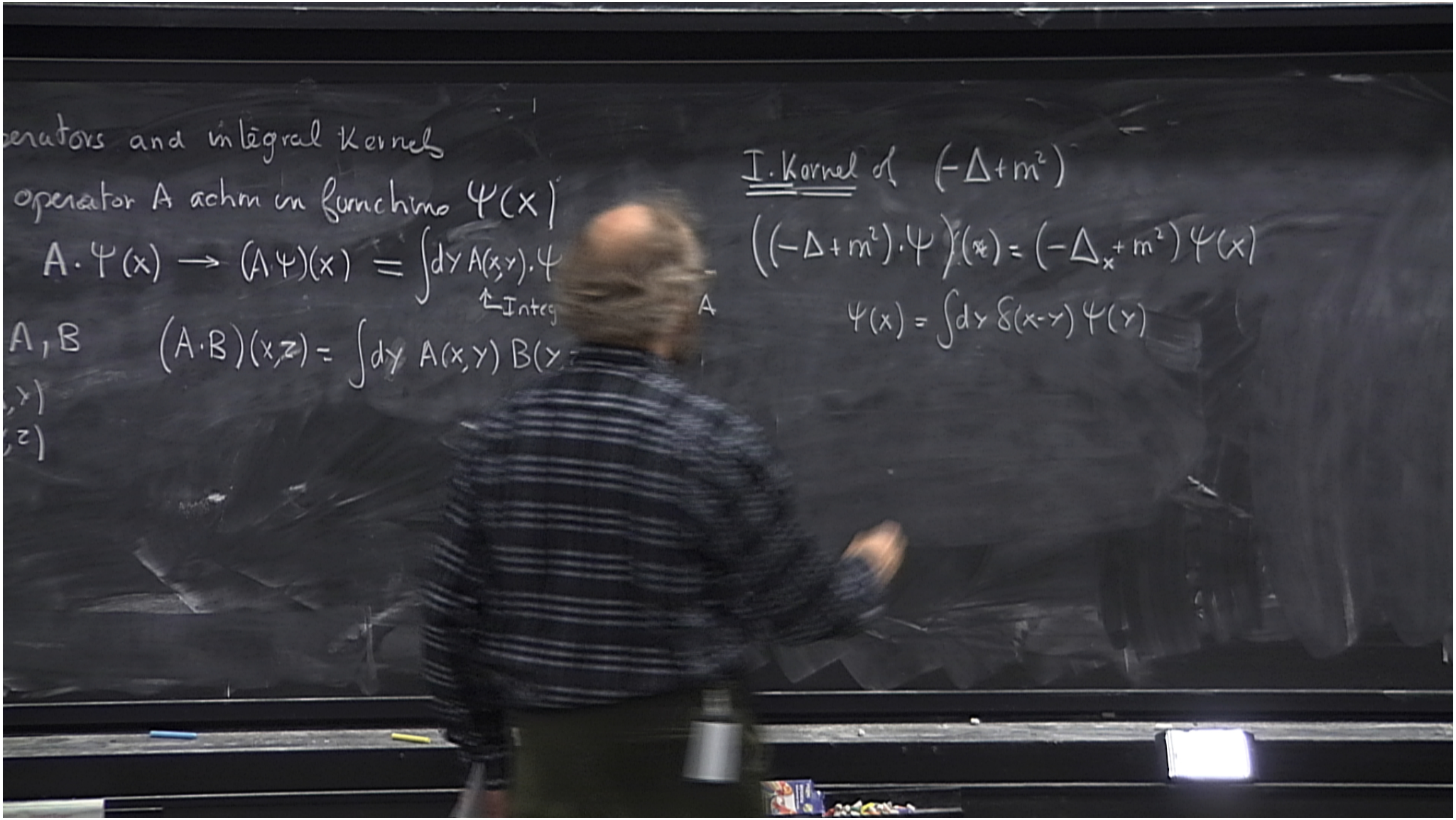
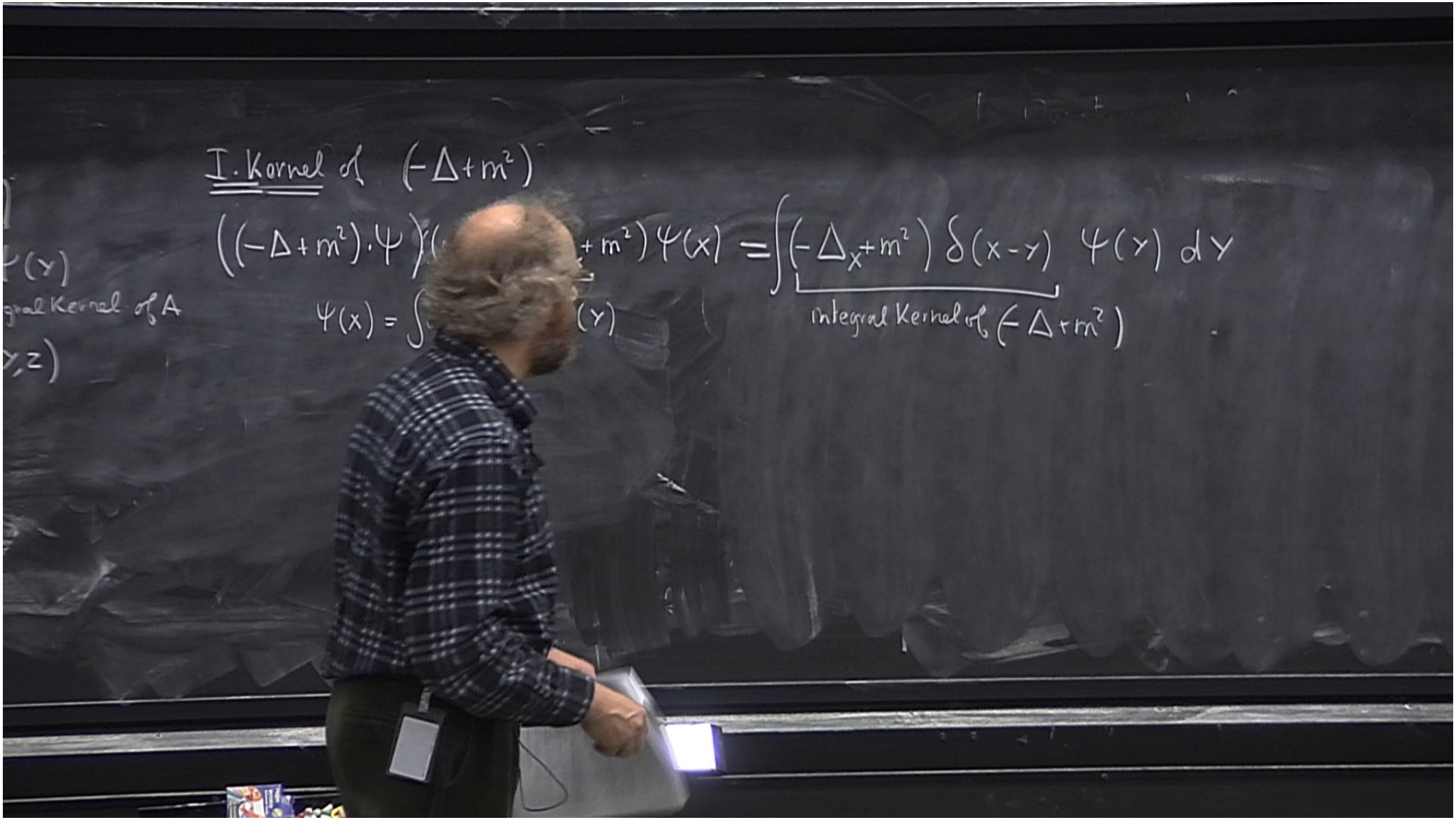
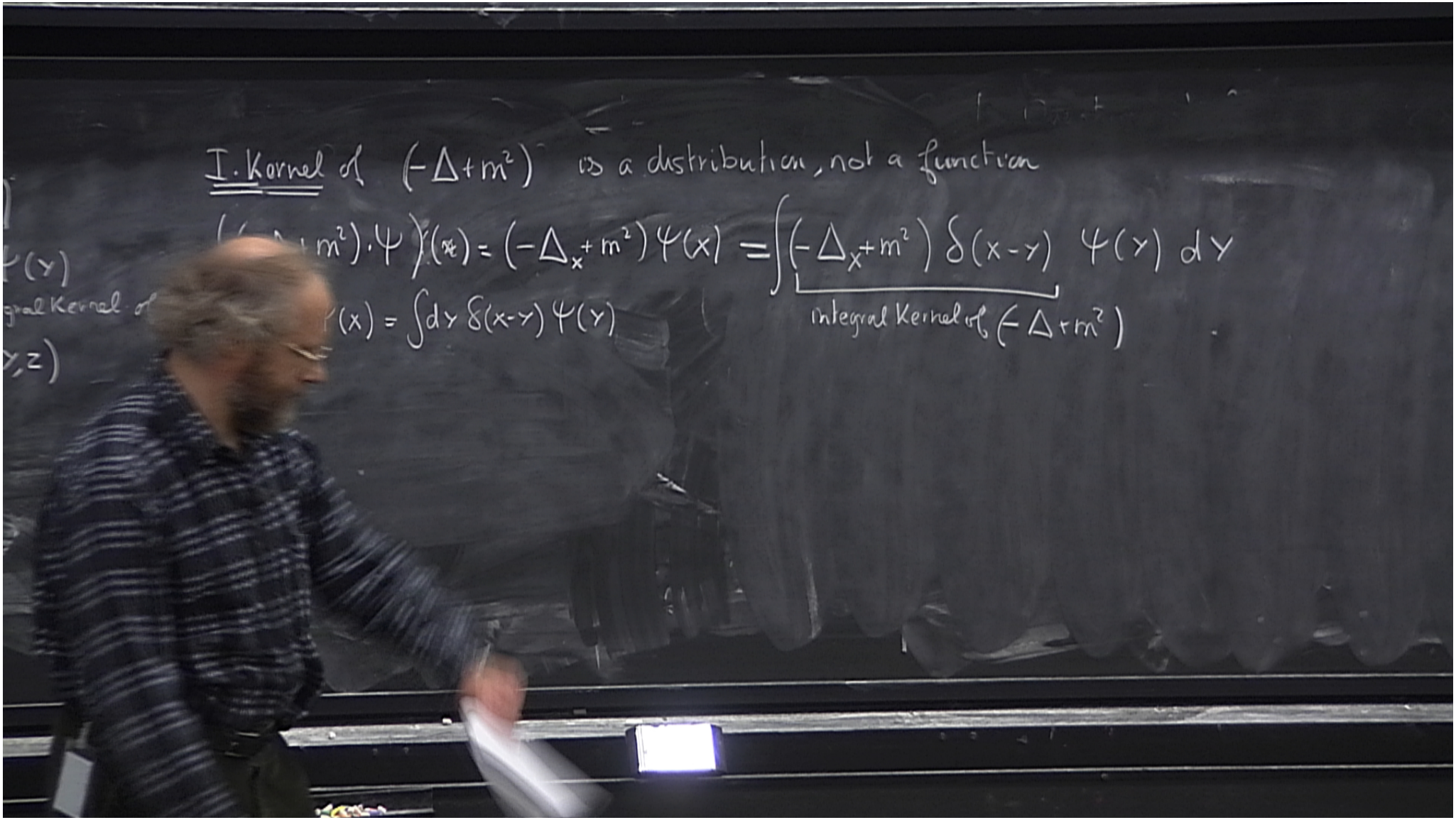
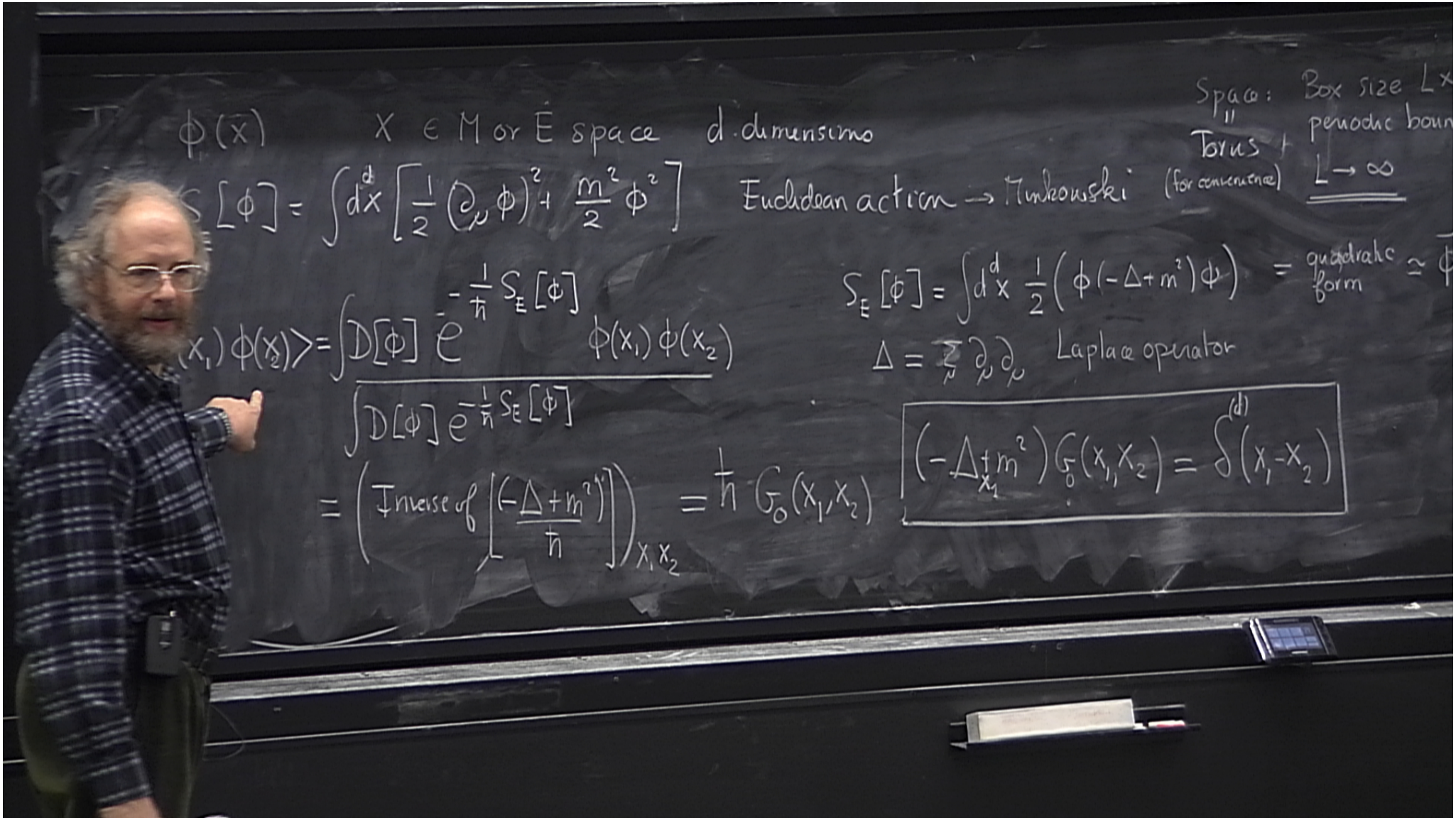
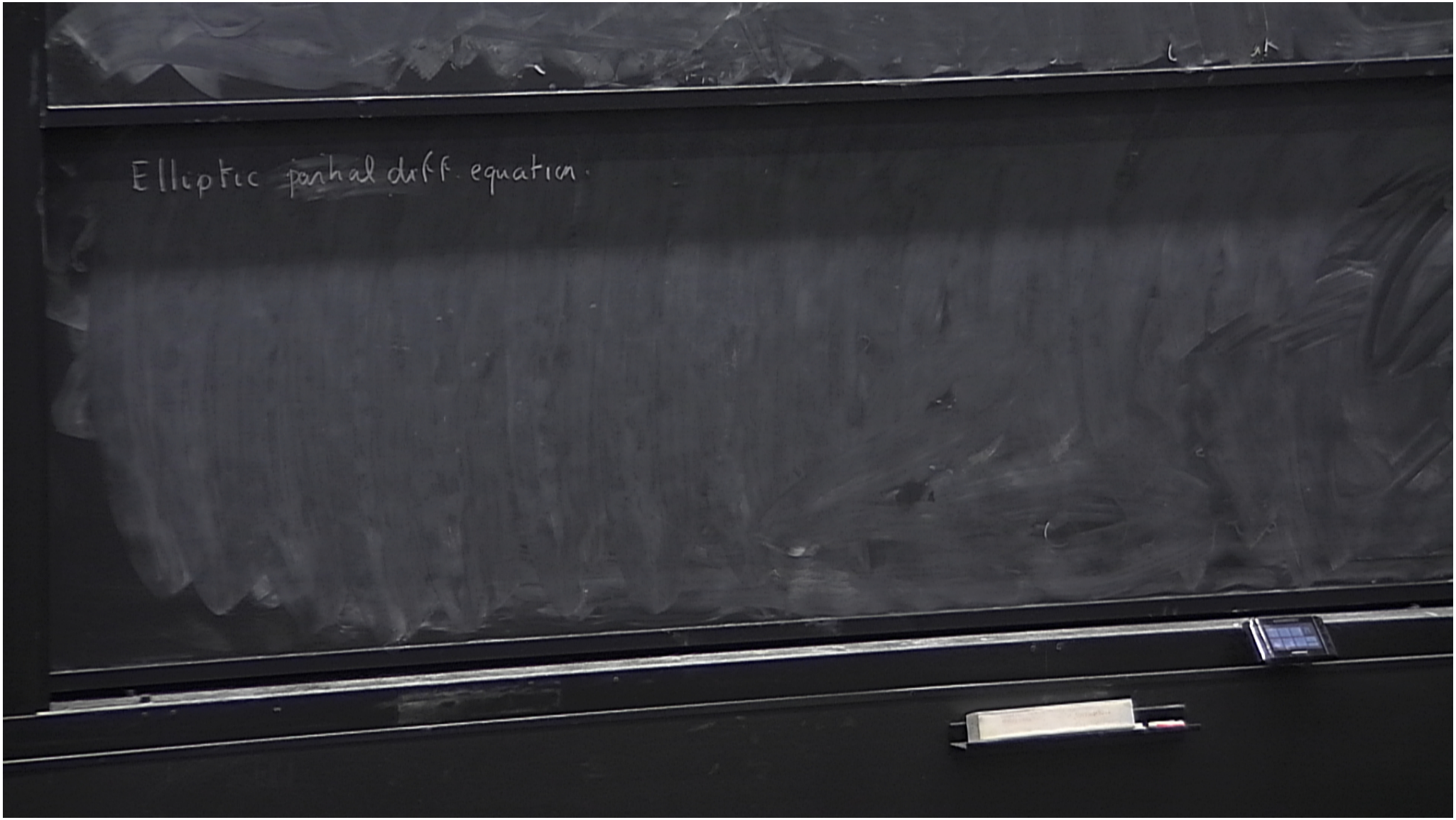
$((-\Delta+m^2)\cdot\Psi)(x) = (-\Delta_x+m^2)\,\Psi(x)$

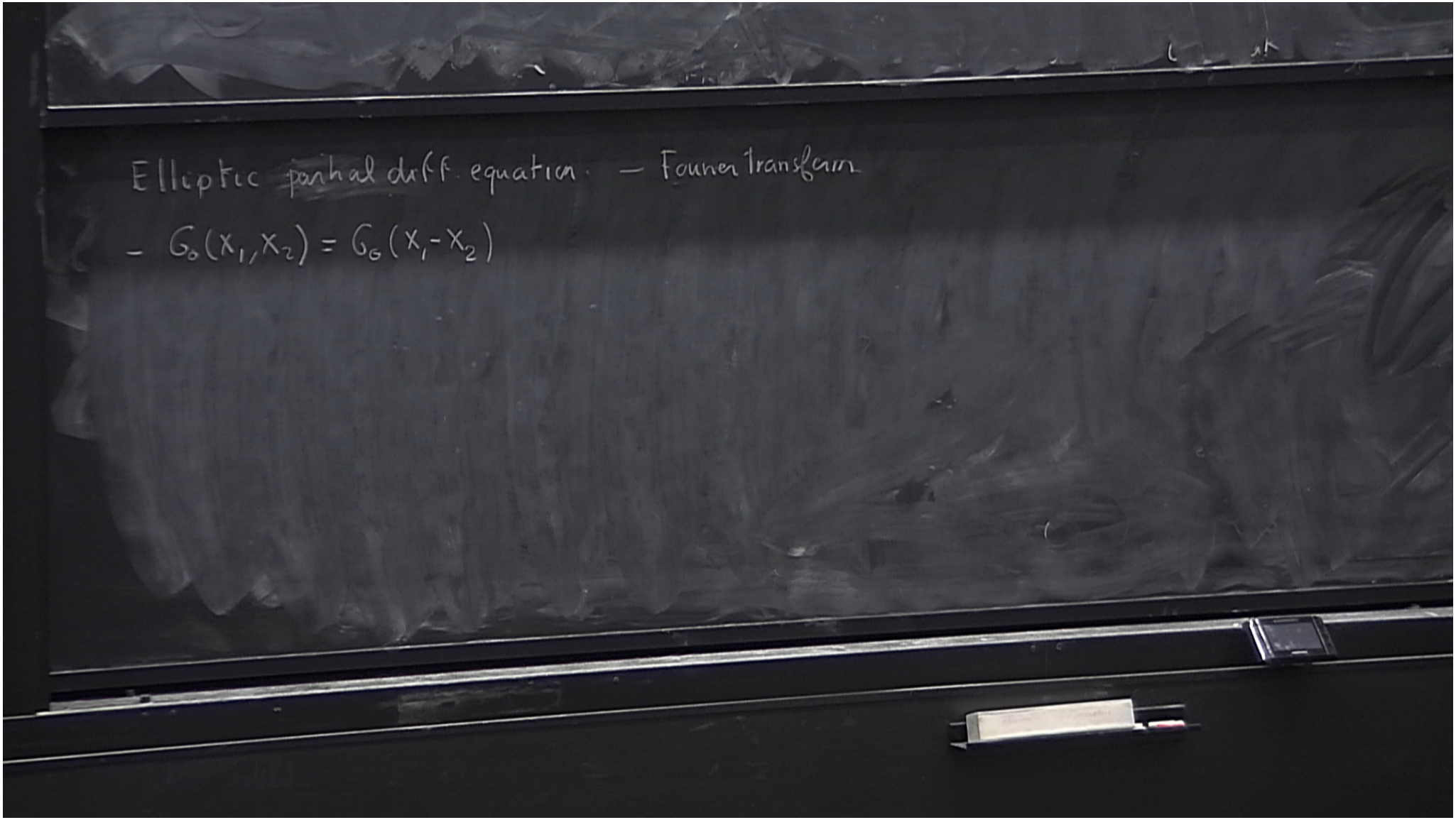

erators and integral Kernels
operator A acting on function $\Psi(x)$
$A \cdot \Psi(x) \longrightarrow (A\Psi)(x) = \int dy\, A(x,y) \cdot \Psi$
↑Integ
A
$A, B \quad (A \cdot B)(x,z) = \int dy\, A(x,y)\, B(y$
I. Kernel of $(-\Delta + m^2)$
$\left((-\Delta + m^2) \cdot \Psi\right)(x) = (-\Delta_x + m^2)\, \Psi(x)$
$\Psi(x) = \int dy\, \delta(x-y)\, \Psi(y)$

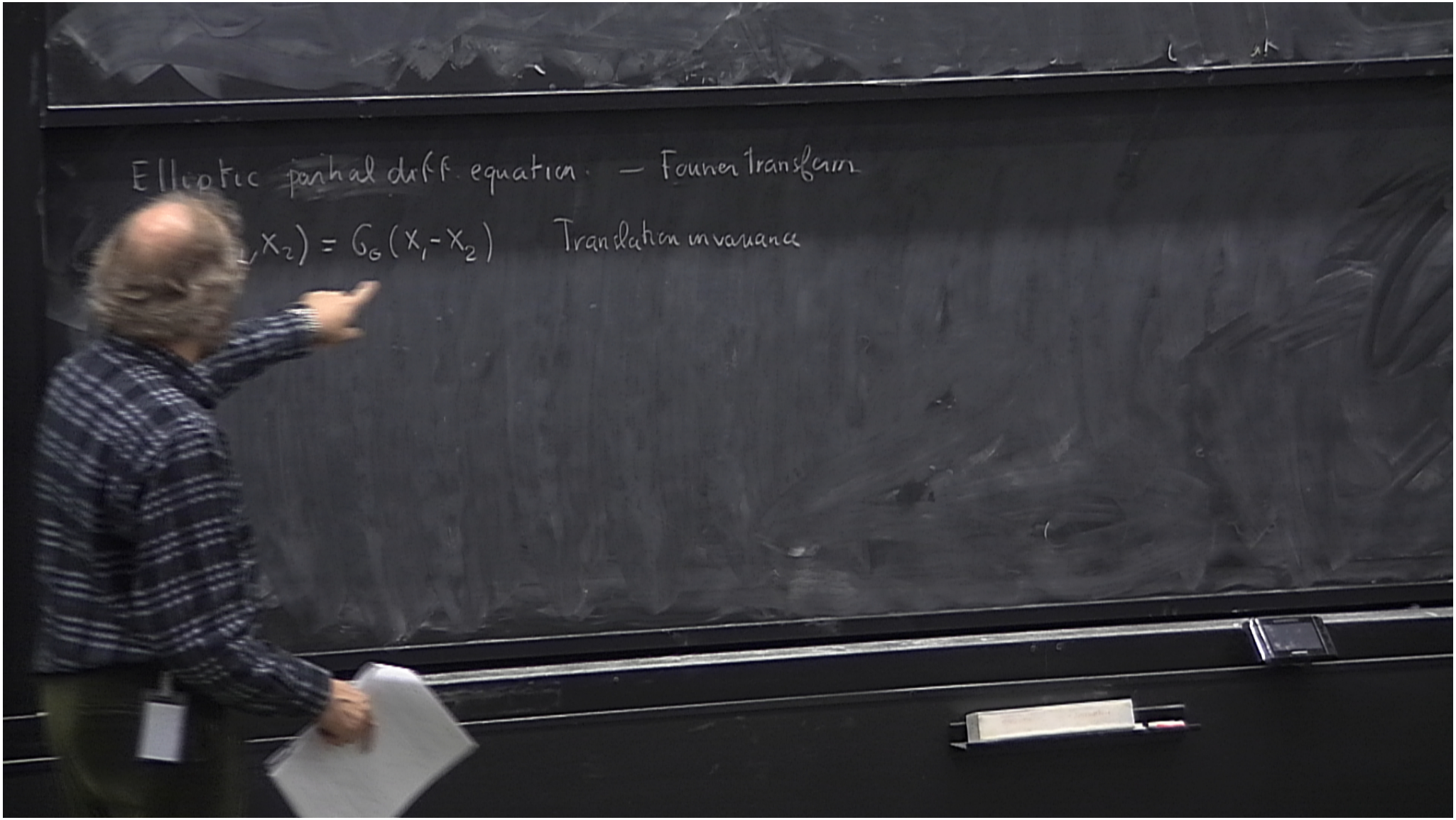

I. Kernel of $(-\Delta+m^2)$
$\left((-\Delta+m^2)\cdot\psi\right)($ $+m^2)\psi(x) = \int \underbrace{(-\Delta_x+m^2)\,\delta(x-y)}_{\text{integral Kernel of } (-\Delta+m^2)}\ \psi(y)\,dy$
$\psi(x) = \int$ (y)
$\psi(y)$
gral Kernel of A
$y,z)$


I. Kernel of $(-\Delta+m^2)$ is a distribution, not a function
$(-\Delta+m^2)\cdot\psi)(x) = (-\Delta_x+m^2)\psi(x) = \int (-\Delta_x+m^2)\,\delta(x-y)\,\psi(y)\,dy$
integral Kernel of $(-\Delta+m^2)$
$(x) = \int dy\,\delta(x-y)\,\psi(y)$
$\psi(y)$
gral Kernel of
y,z)


$\phi(\vec{x})$
$x \in M$ or E space
d dimensions
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2\right]$
Euclidean action → Minkowski
Space: Box size $L \times$
Torus + periodic boun
(for convenience) $L \to \infty$
$S_E[\phi] = \int d^d x \, \frac{1}{2}\left(\phi(-\Delta + m^2)\phi\right)$ = quadratic form
$\Delta = \sum_\mu \partial_\mu \partial_\mu$ Laplace operator
$\langle \phi(x_1)\phi(x_2)\rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}\, \phi(x_1)\phi(x_2)}{\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]}}$
$= \left(\text{Inverse of } \left[\frac{(-\Delta + m^2)}{\hbar}\right]\right)_{x_1 x_2} = \hbar\, G_0(x_1, x_2)$
$(-\Delta_{x_1} + m^2)\, G_0(x_1, x_2) = \delta^{(d)}(x_1 - x_2)$


Elliptic partial diff. equation.


Elliptic partial diff. equation — Fourier transform
- $G_\sigma(x_1, x_2) = G_\sigma(x_1 - x_2)$


Elliptic partial diff. equation — Fourier transform
$G(x_1, x_2) = G_0(x_1 - x_2)$ Translation invariance

Elliptic partial diff. equation — Fourier transform

- $G_0(x_1, x_2) = G_0(x_1 - x_2)$ Translation invariance

$$(-\Delta_x + m^2)\, G_0(x) = \delta^{(d)}(x)$$

Elliptic partial diff. equation — Fourier transform

$-G_0(x_1, x_2) = G_0(x_1 - x_2)$ Translation invariance

$(-\Delta_x + m^2) G_0(x) = \delta^{(d)}(x)$

$\hat{G}_0(k) = \int d^d x\, e^{-ik\cdot x} G_0(x)$

Elliptic partial diff. equation — Fourier transform

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$\hat{G}_0(k) = \int d^d x\, e^{-ik\cdot x}\, G_0(x)$

$(k^2 + m^2) \cdot \hat{G}_0(k) = 1$

Elliptic partial diff. equation — Fourier transform

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$$\frac{\partial}{\partial x_\mu} \to -i k_\mu \qquad \Delta \to -k^2 = -\sum_\mu k_\mu k_\mu$$

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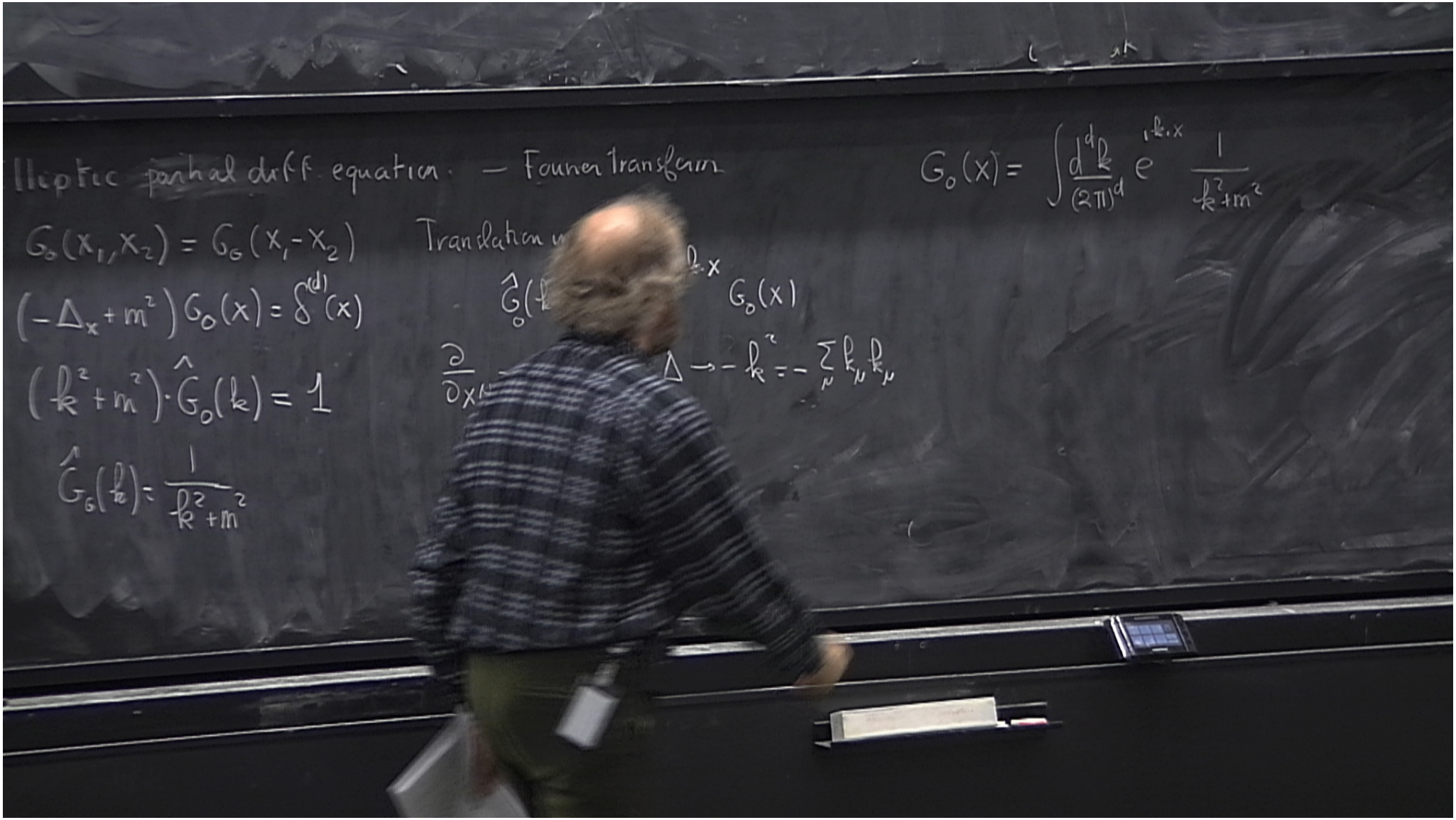
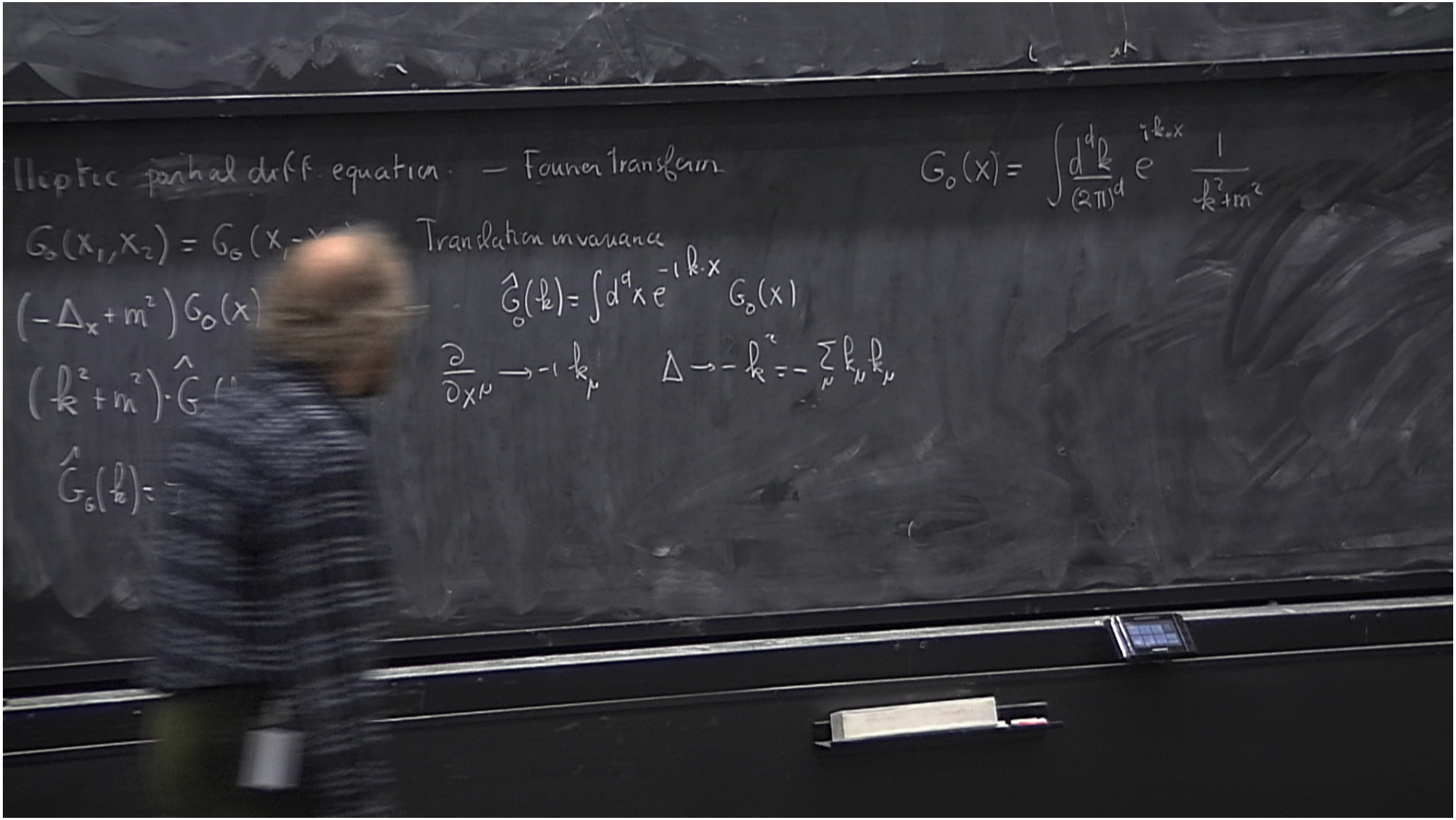
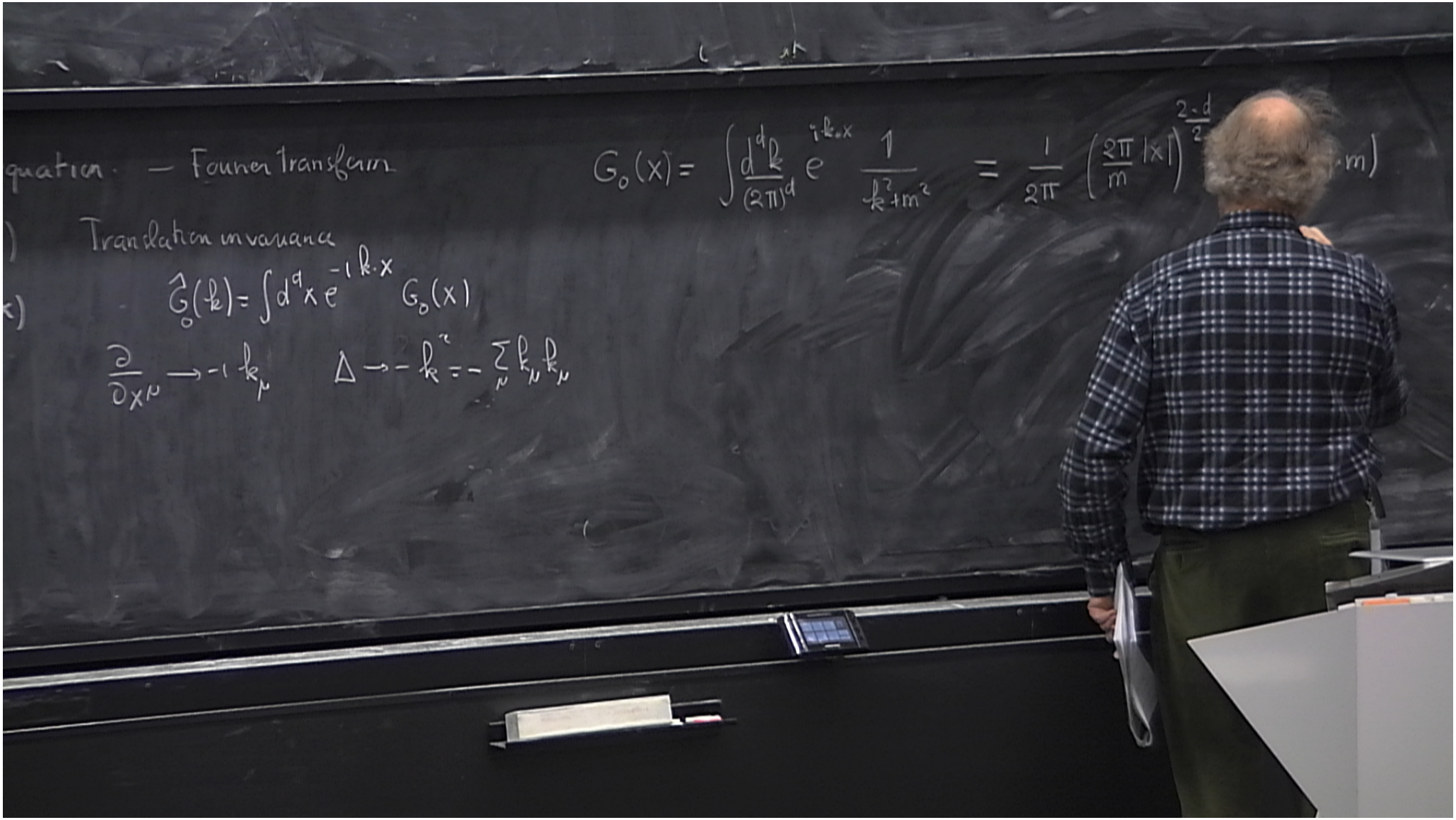
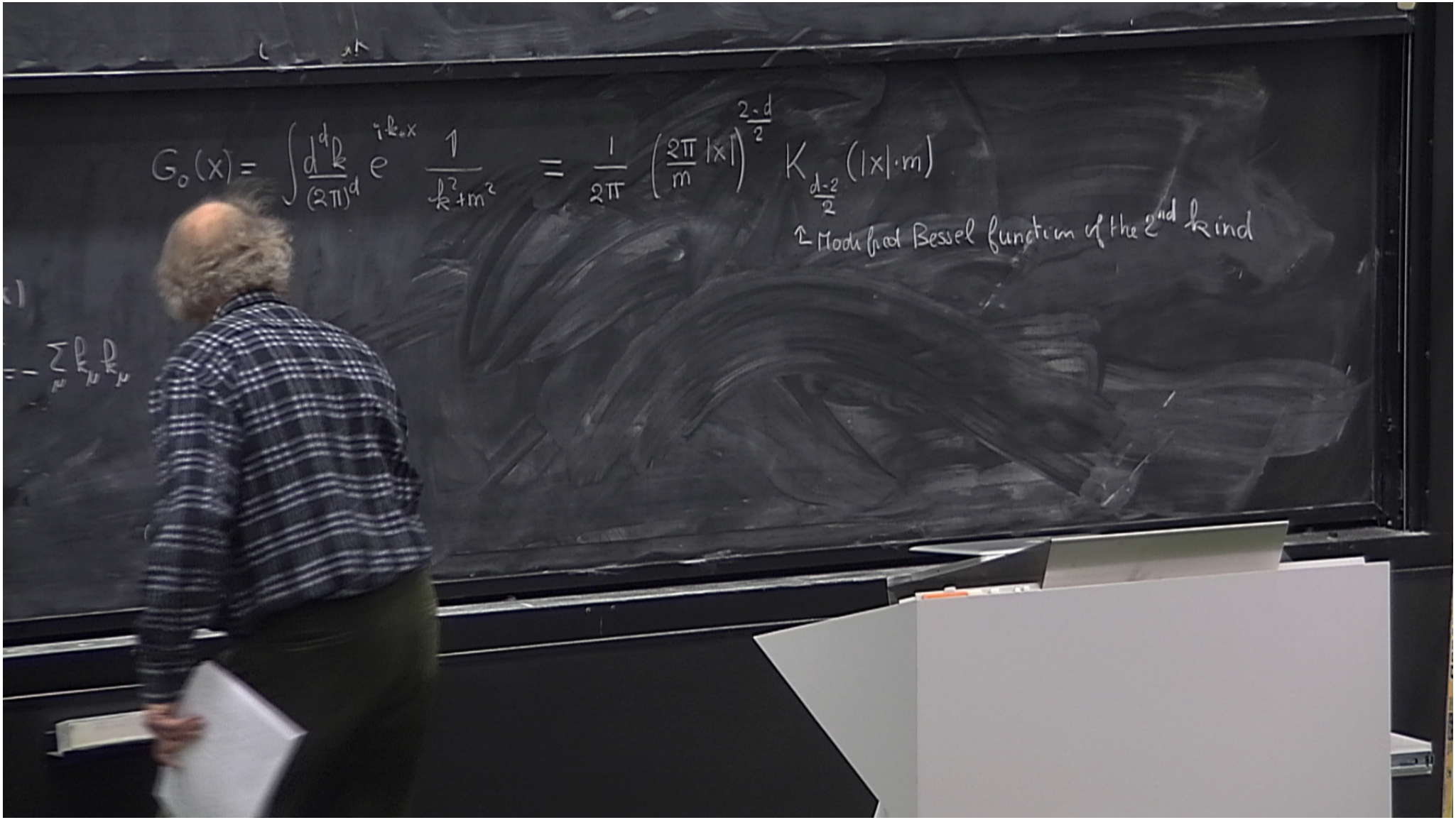
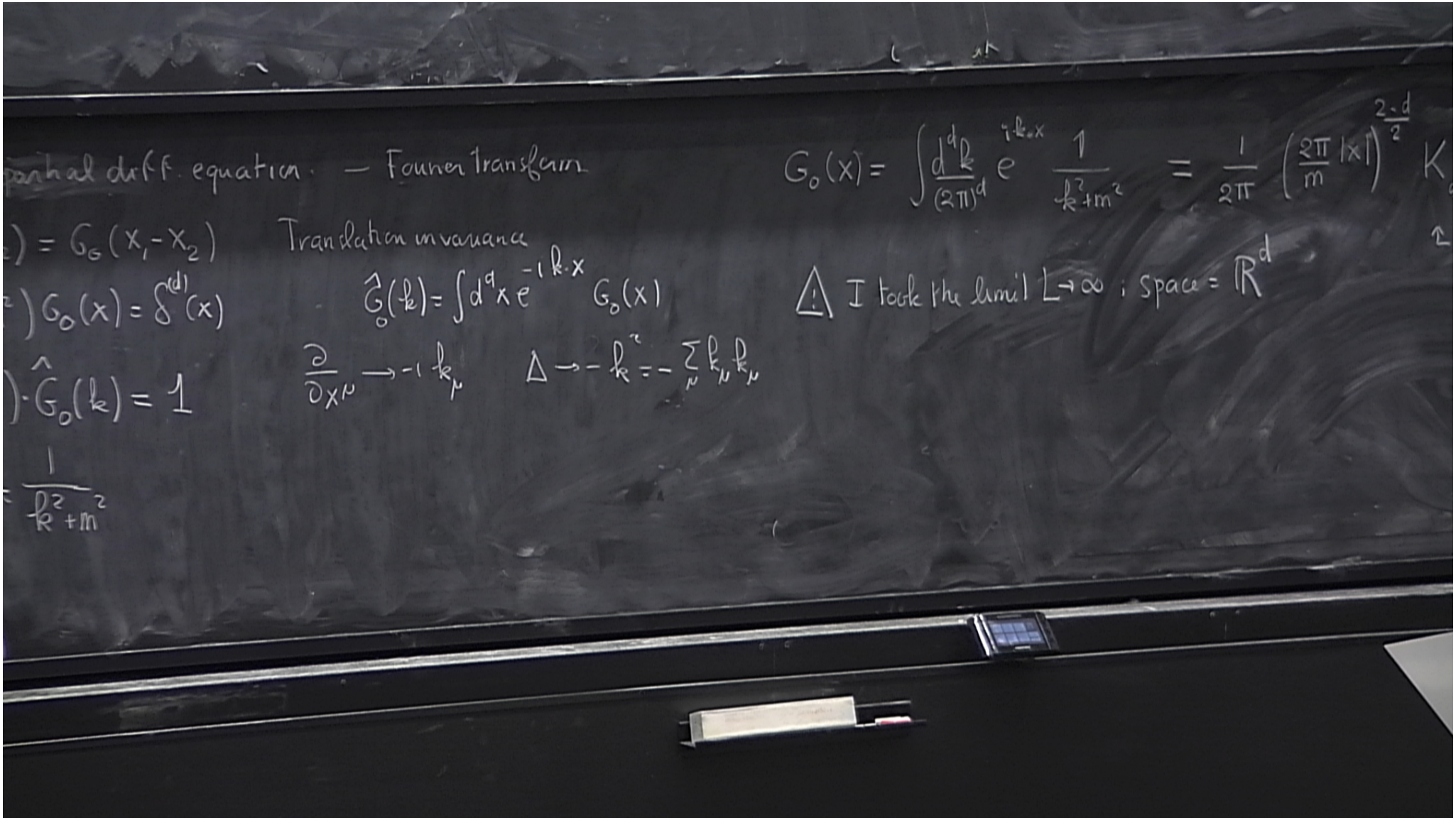
$\hat{G}_0(k) = \frac{1}{k^2 + m^2}$

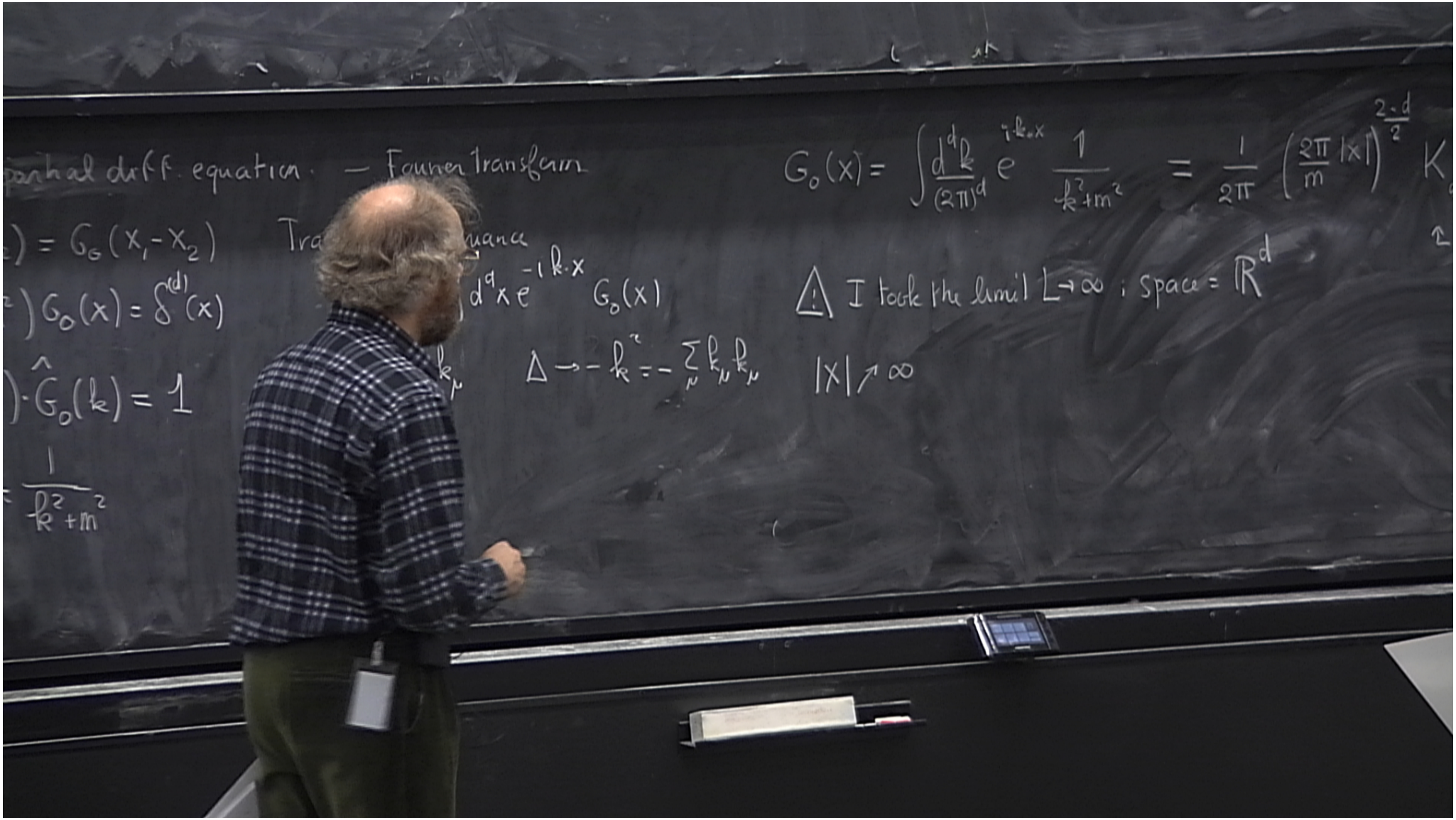

Elliptic partial diff. equation — Fourier Transform
$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{ik\cdot x} \frac{1}{k^2+m^2}$
$G_0(x_1, x_2) = G_0(x_1 - x_2)$
Translation
$(-\Delta_x + m^2) G_0(x) = \delta^{(d)}(x)$
$\hat{G}_0(k)$ $G_0(x)$
$(k^2 + m^2) \cdot \hat{G}_0(k) = 1$
$\frac{\partial}{\partial x_\mu}$ $\Delta \to -k^2 = -\sum_\mu k_\mu k_\mu$
$\hat{G}_0(k) = \frac{1}{k^2+m^2}$

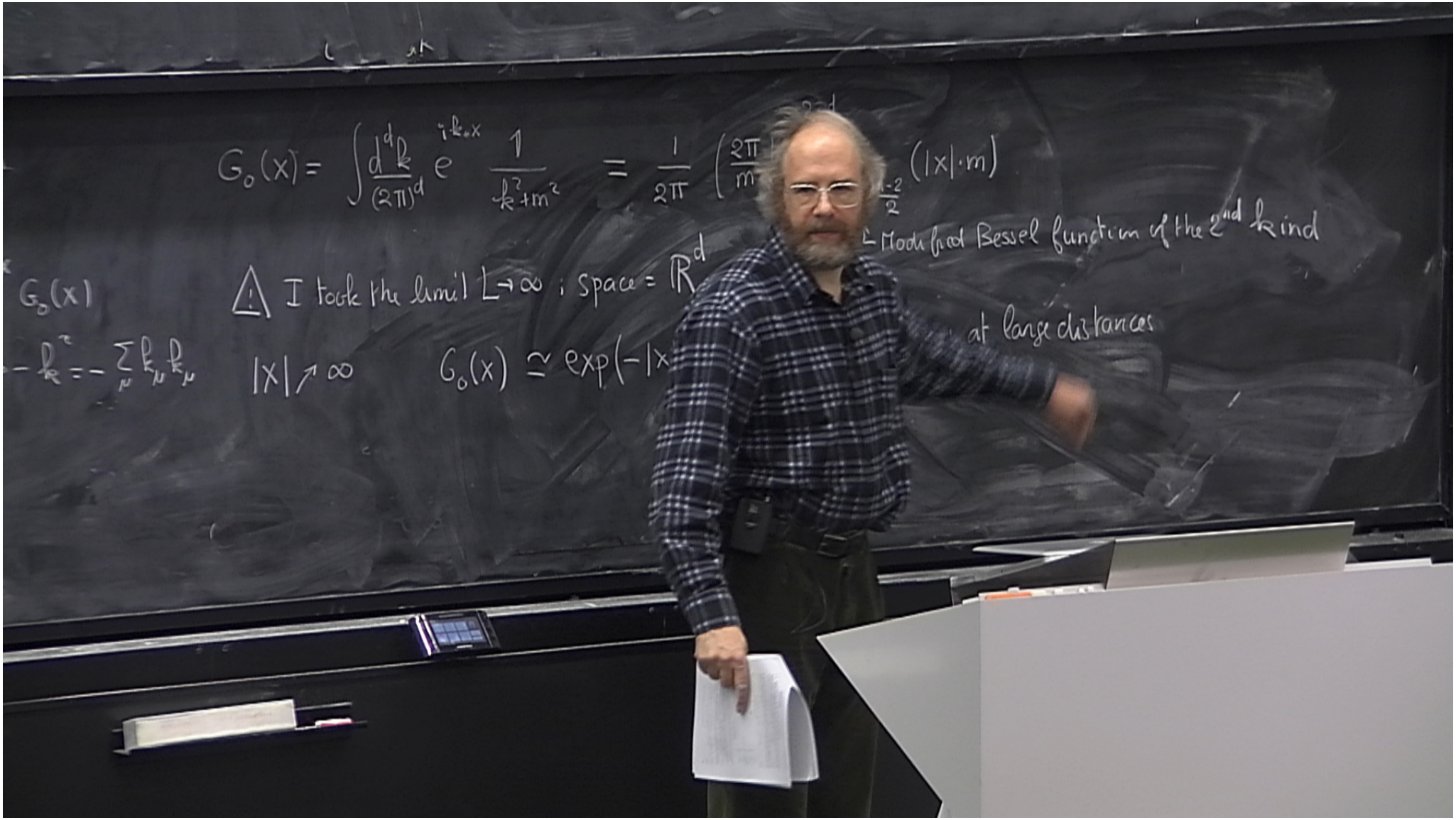

Elliptic partial diff. equation — Fourier Transform
$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{ik\cdot x} \frac{1}{k^2+m^2}$
$G_0(x_1, x_2) = G_0(x_1 - x_2)$ Translation invariance
$(-\Delta_x + m^2) G_0(x)$
$\hat{G}_0(k) = \int d^d x\, e^{-ik\cdot x} G_0(x)$
$(k^2 + m^2) \cdot \hat{G}$
$\frac{\partial}{\partial x_\mu} \to -i k_\mu \qquad \Delta \to -k^2 = -\sum_\mu k_\mu k_\mu$
$\hat{G}_0(k) =$

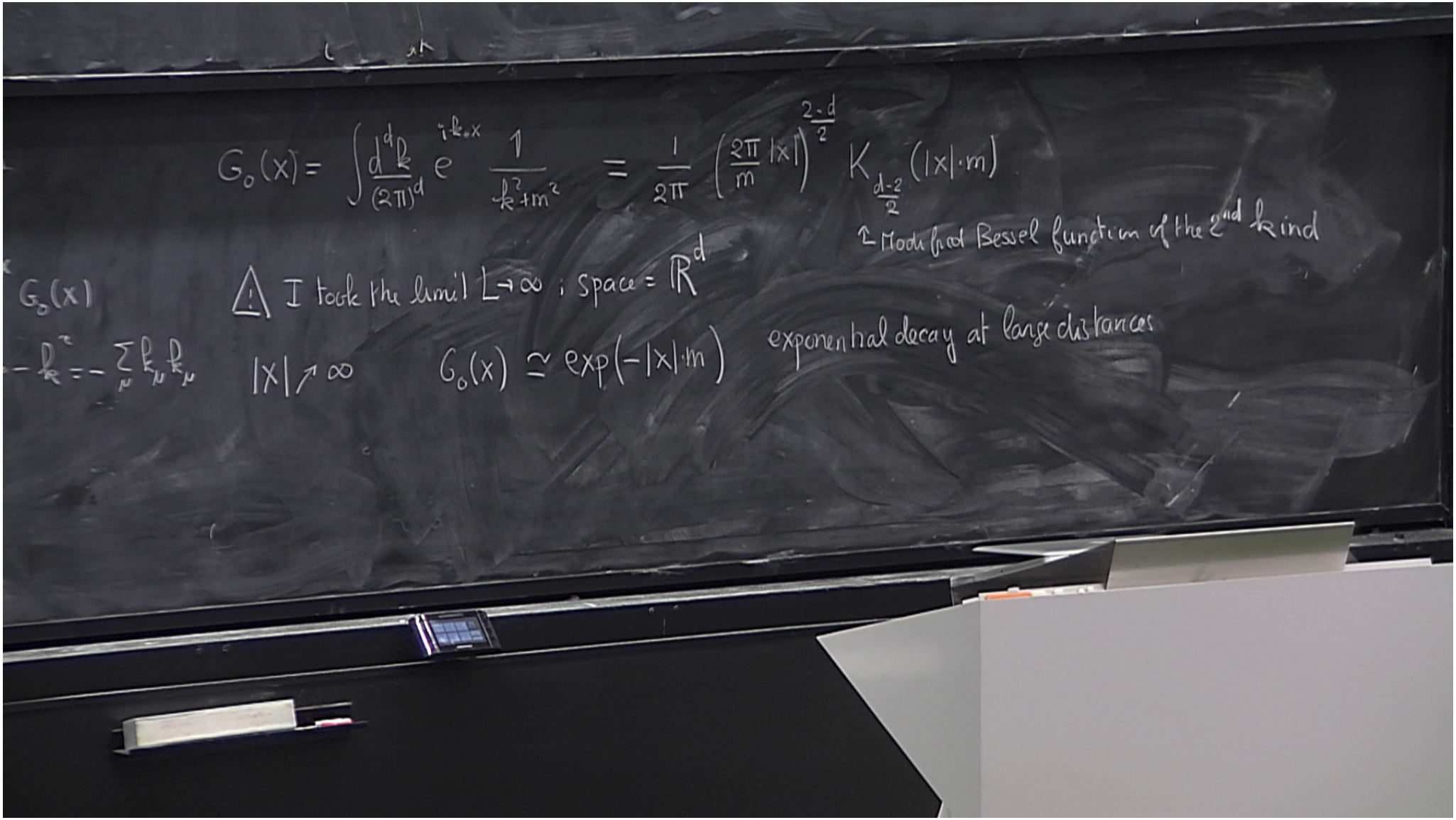

quation — Fourier transform
Translation invariance
$\tilde{G}_0(k) = \int d^d x\, e^{-ik\cdot x} G_0(x)$
$\frac{\partial}{\partial x_\mu} \to -i k_\mu \qquad \Delta \to -k^2 = -\sum_\mu k_\mu k_\mu$
$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{ik\cdot x} \frac{1}{k^2+m^2} = \frac{1}{2\pi} \left(\frac{2\pi}{m}|x|\right)^{\frac{2-d}{2}}$
m)

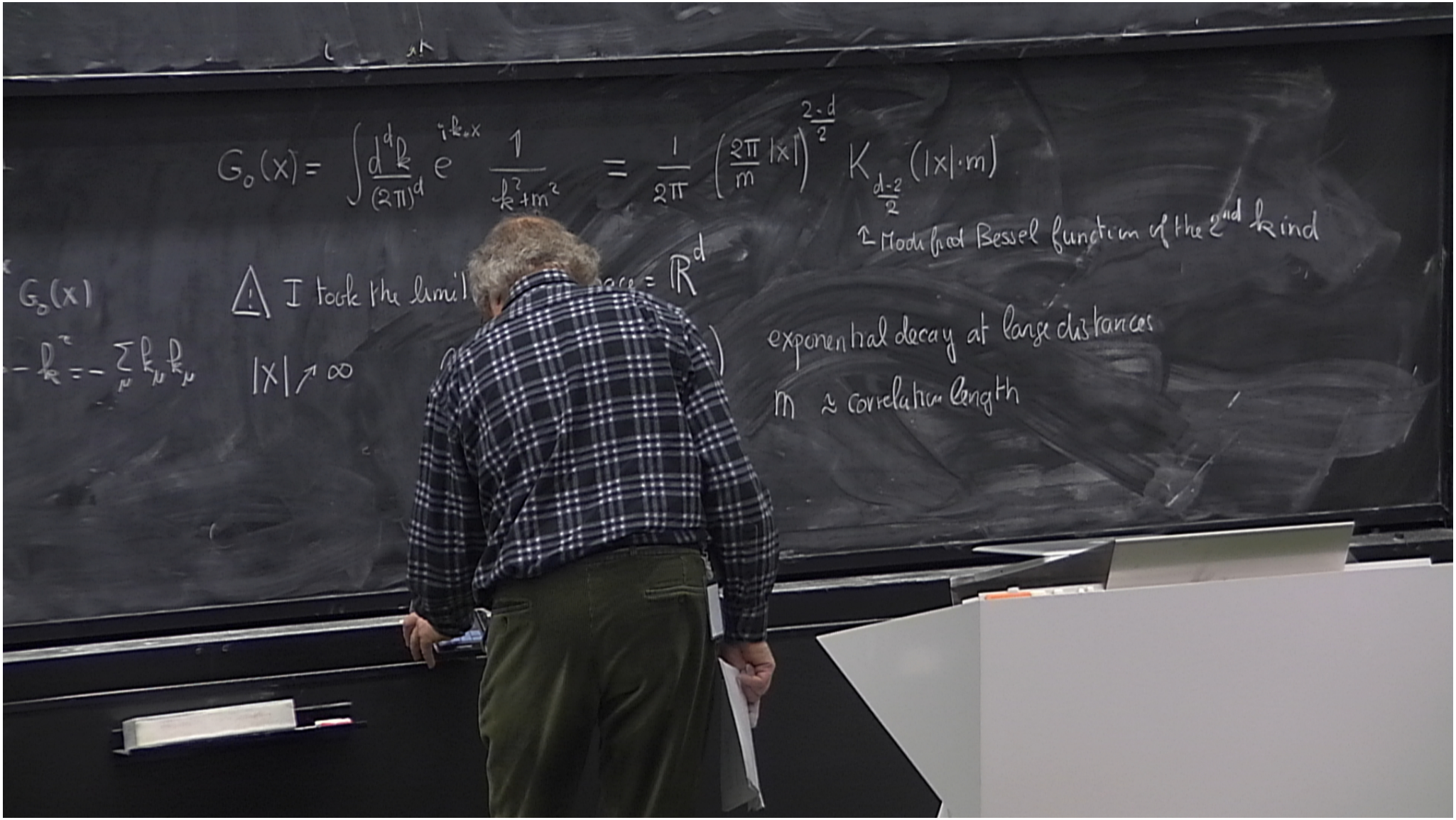

$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{i k \cdot x} \frac{1}{k^2 + m^2} = \frac{1}{2\pi} \left(\frac{2\pi}{m}|x|\right)^{\frac{2-d}{2}} K_{\frac{d-2}{2}}(|x| \cdot m)$
↑ Modified Bessel function of the 2nd kind

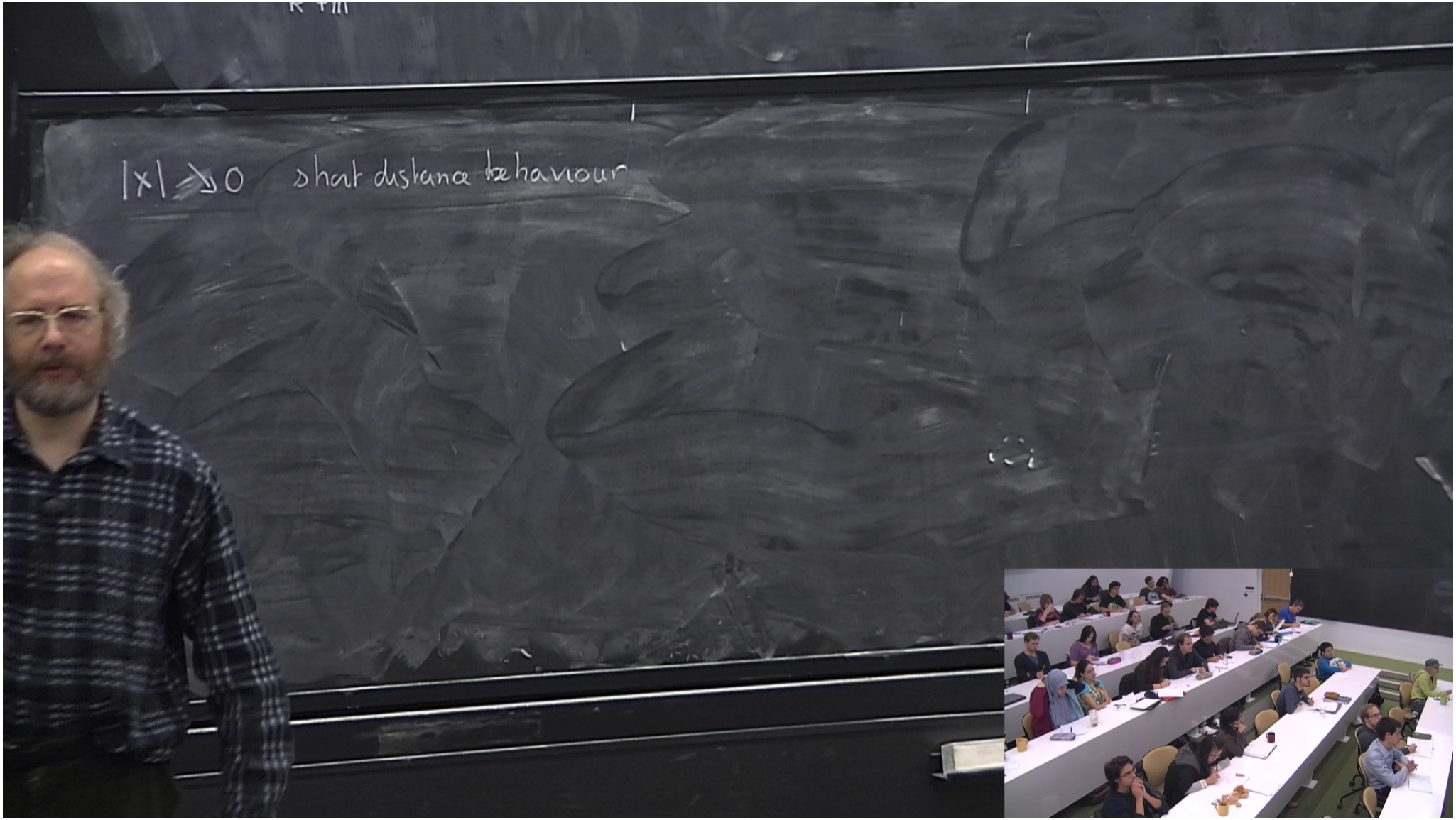

onhal diff. equation. — Fourier Transform
$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{ik\cdot x} \frac{1}{k^2+m^2} = \frac{1}{2\pi}\left(\frac{2\pi}{m}|x|\right)^{\frac{2-d}{2}} K$
$) = G_0(x_1 - x_2)$ Translation invariance
$\hat{G}_0(k) = \int d^d x\, e^{-ik\cdot x} G_0(x)$
$) G_0(x) = \delta^{(d)}(x)$
I took the limit $L \to \infty$; space = $\mathbb{R}^d$
$\frac{\partial}{\partial x^\mu} \to -i k_\mu$ $\Delta \to -k^2 = -\sum_\mu k_\mu k_\mu$
$) \cdot \hat{G}_0(k) = 1$
$= \frac{1}{k^2+m^2}$

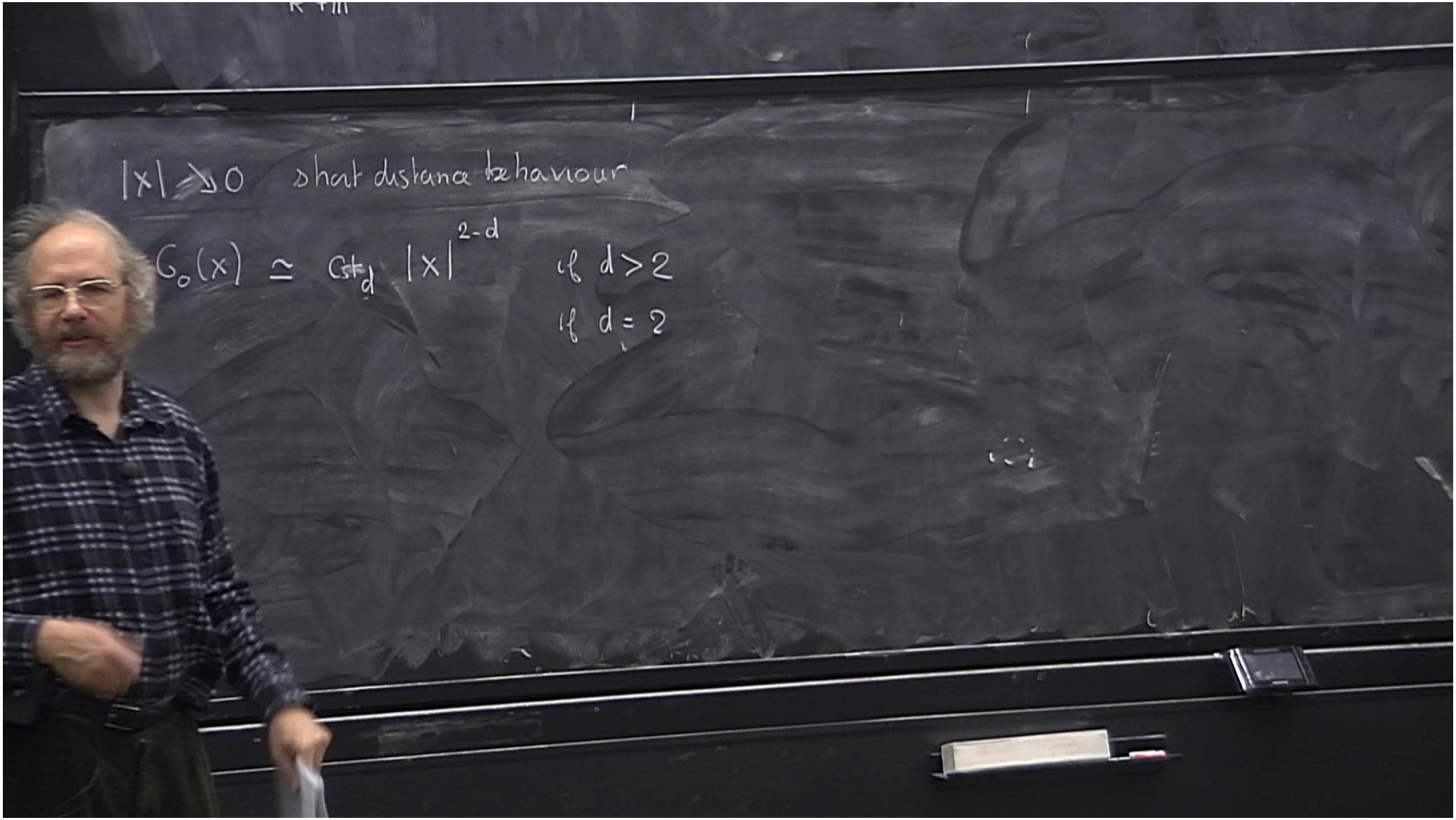

$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{i k \cdot x} \frac{1}{k^2 + m^2} = \frac{1}{2\pi} \left(\frac{2\pi}{m}|x|\right)^{\frac{2-d}{2}} K$
$= G_0(x_1 - x_2)$
$G_0(x) = \delta^{(d)}(x)$
$\cdot \hat{G}_0(k) = 1$
$\frac{1}{k^2 + m^2}$
$\int d^d x e^{-ik \cdot x} G_0(x)$
k_μ
$\Delta \to -k^2 = -\sum_\mu k_\mu k_\mu$
I took the limit $L \to \infty$; space = $\mathbb{R}^d$
$|x| \nearrow \infty$

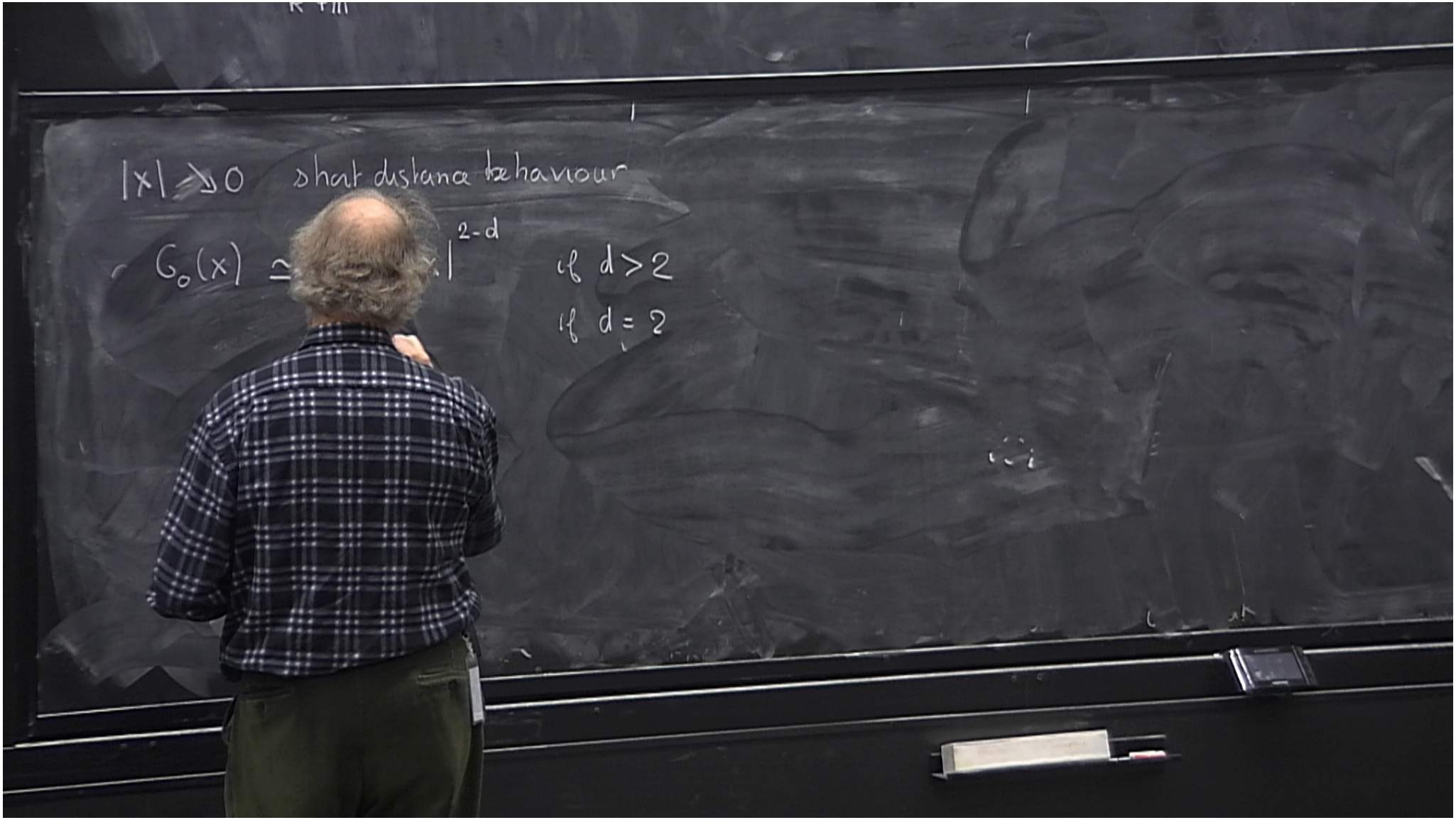

$G_0(x) = \int \frac{d^q k}{(2\pi)^d} e^{i k \cdot x} \frac{1}{k^2 + m^2} = \frac{1}{2\pi} \left(\frac{2\pi}{m} \right)^{\frac{d-2}{2}} (|x| \cdot m)$
Modified Bessel function of the 2nd kind
$G_0(x)$
I took the limit $L \to \infty$; space = $\mathbb{R}^d$
$-k^2 = -\sum_\mu k_\mu k_\mu$
$|x| \nearrow \infty$
$G_0(x) \simeq \exp(-|x$
at large distances


$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{i k \cdot x} \frac{1}{k^2 + m^2} = \frac{1}{2\pi} \left(\frac{2\pi}{m}|x|\right)^{\frac{2-d}{2}} K_{\frac{d-2}{2}}(|x| \cdot m)$
↑ Modified Bessel function of the 2nd kind
$G_0(x)$
⚠ I took the limit $L \to \infty$; Space = $\mathbb{R}^d$
$-k^2 = -\sum_\mu k_\mu k_\mu$
$|x| \nearrow \infty$ $G_0(x) \simeq \exp(-|x| \cdot m)$ exponential decay at large distances


$G_0(x) = \int \frac{d^d k}{(2\pi)^d} e^{i k \cdot x} \frac{1}{k^2 + m^2} = \frac{1}{2\pi} \left(\frac{2\pi}{m}|x|\right)^{\frac{2-d}{2}} K_{\frac{d-2}{2}}(|x| \cdot m)$
$K_{\frac{d-2}{2}}$ Modified Bessel function of the 2nd kind
$G_0(x)$
$-k^2 = -\sum_\mu k_\mu k_\mu$
I took the limit
$|x| \nearrow \infty$
$= \mathbb{R}^d$
exponential decay at large distances
$m \simeq$ correlation length


|x| → 0 what distance behaviour


$|x| \to 0$ short distance behaviour
$G_0(x) \simeq C_d |x|^{2-d}$ if $d > 2$
if $d = 2$


|x| → 0 short distance behaviour
$G_0(x) \simeq |x|^{2-d}$ if $d > 2$
if $d = 2$

$|x| \to 0$ short distance behaviour : singularity

$$G_0(x) \simeq C_d |x|^{2-d} \quad \text{if } d > 2$$

$$-\frac{1}{2\pi} \log |x| \quad \text{if } d = 2$$

$|x| \to 0$ short distance behaviour : singularity

$$G_0(x) \simeq C_d\, |x|^{2-d} \quad \text{if } d > 2$$

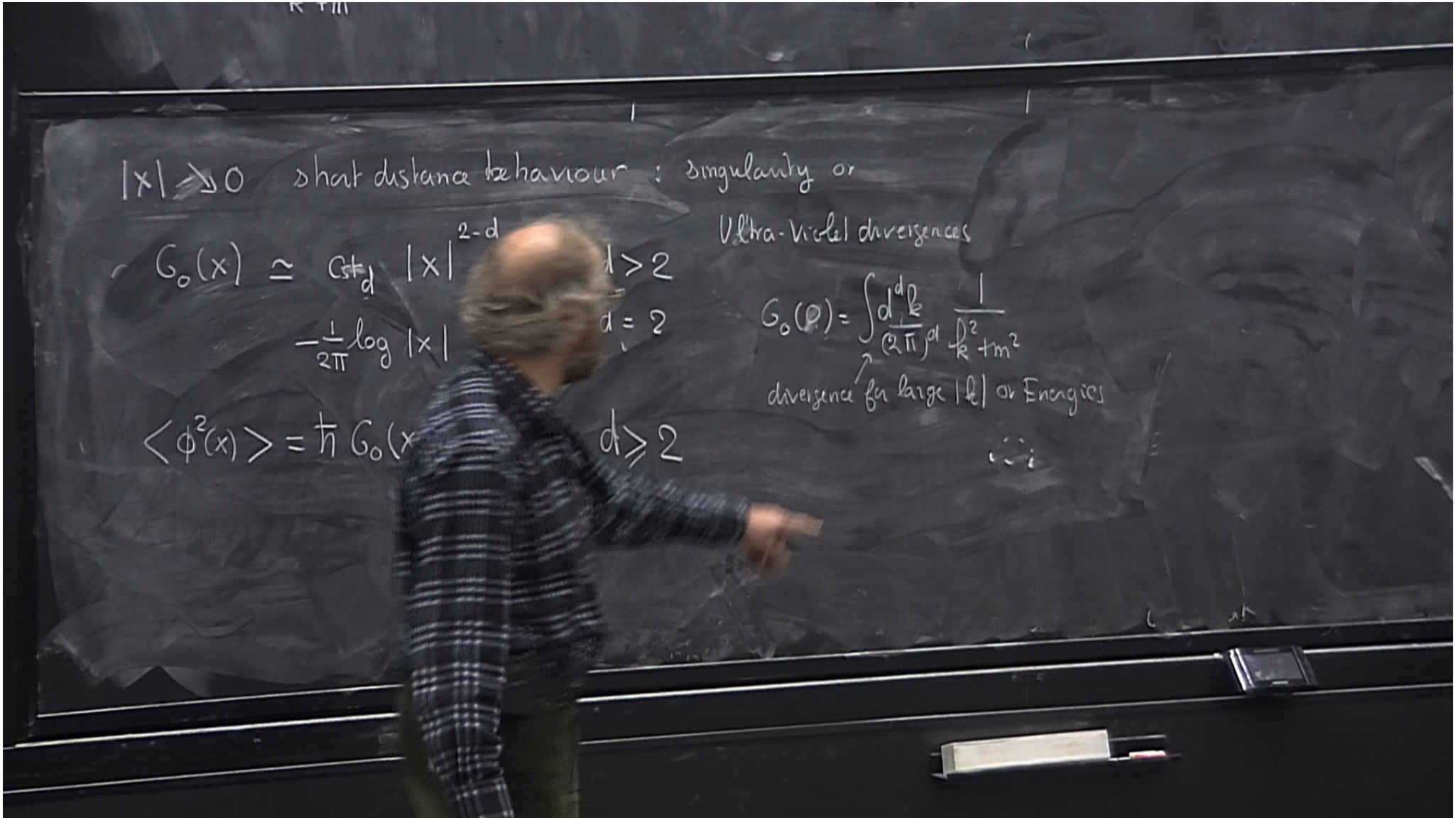
$$-\frac{1}{2\pi} \log |x| \quad \text{if } d = 2$$

$$\langle \phi^2(x) \rangle = \hbar\, G_0(x,x) = \infty \quad d \geq 2$$

$|x| \to 0$ short distance behaviour : singularity or Ultra-Violet divergences

$$G_0(x) \simeq \begin{cases} C_d\, |x|^{2-d} & \text{if } d > 2 \\ -\frac{1}{2\pi} \log |x| & \text{if } d = 2 \end{cases}$$

$$\langle \phi^2(x) \rangle = \hbar\, G_0(x,x) = \infty \quad d \geq 2$$


|x| → 0 short distance behaviour : singularity or
Ultra-Violet divergences
$G_0(x) \simeq C_d |x|^{2-d}$ $d > 2$
$-\frac{1}{2\pi} \log |x|$ $d = 2$
$\langle \phi^2(x) \rangle = \hbar G_0(x$ $d \geq 2$
$G_0(0) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + m^2}$
divergence for large |k| or Energies

$|x| \to 0$ short distance behaviour : singularity or Ultra-Violet divergences

$$G_0(x) \simeq C_d\, |x|^{2-d} \quad \text{if } d>2$$

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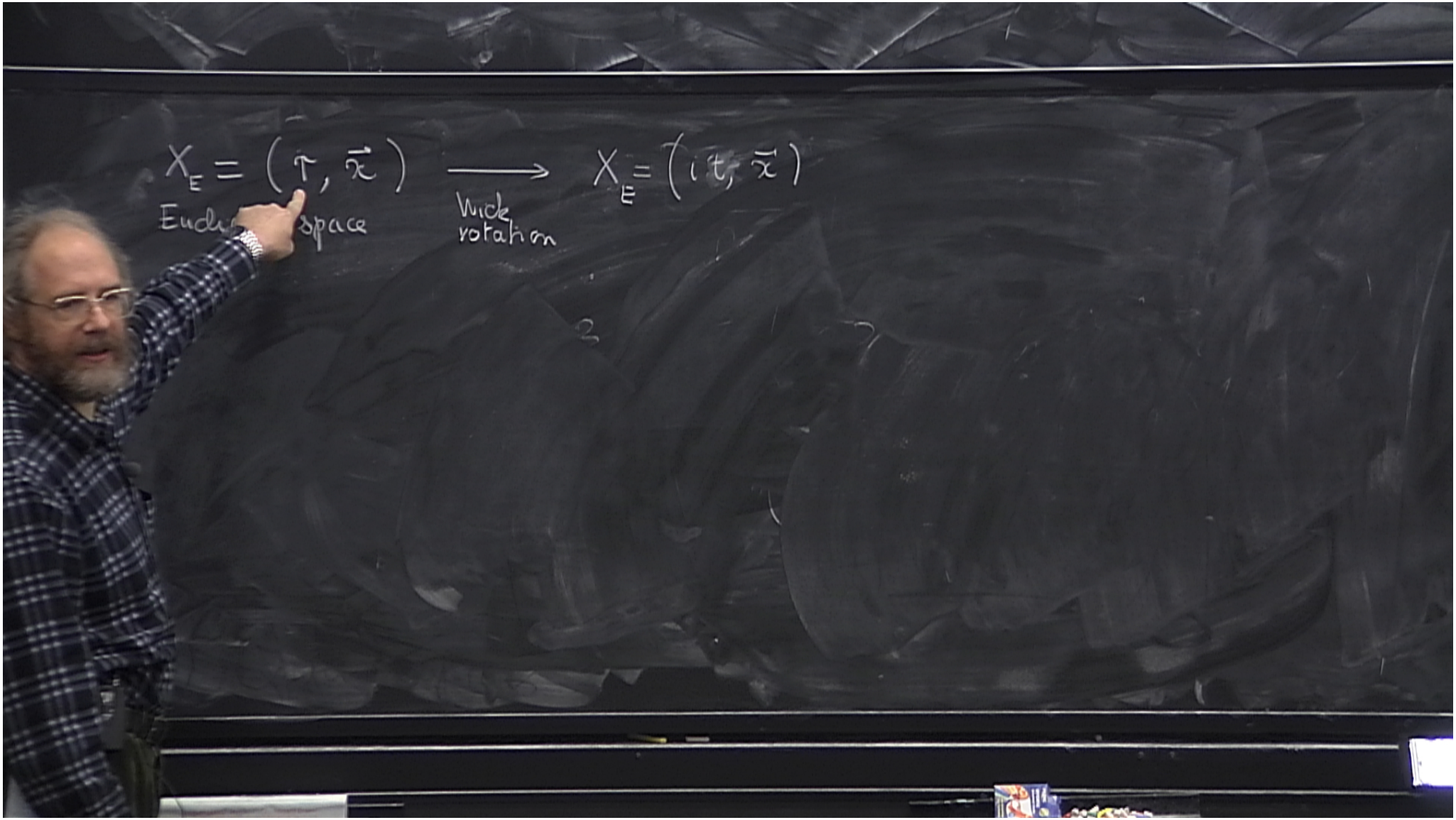
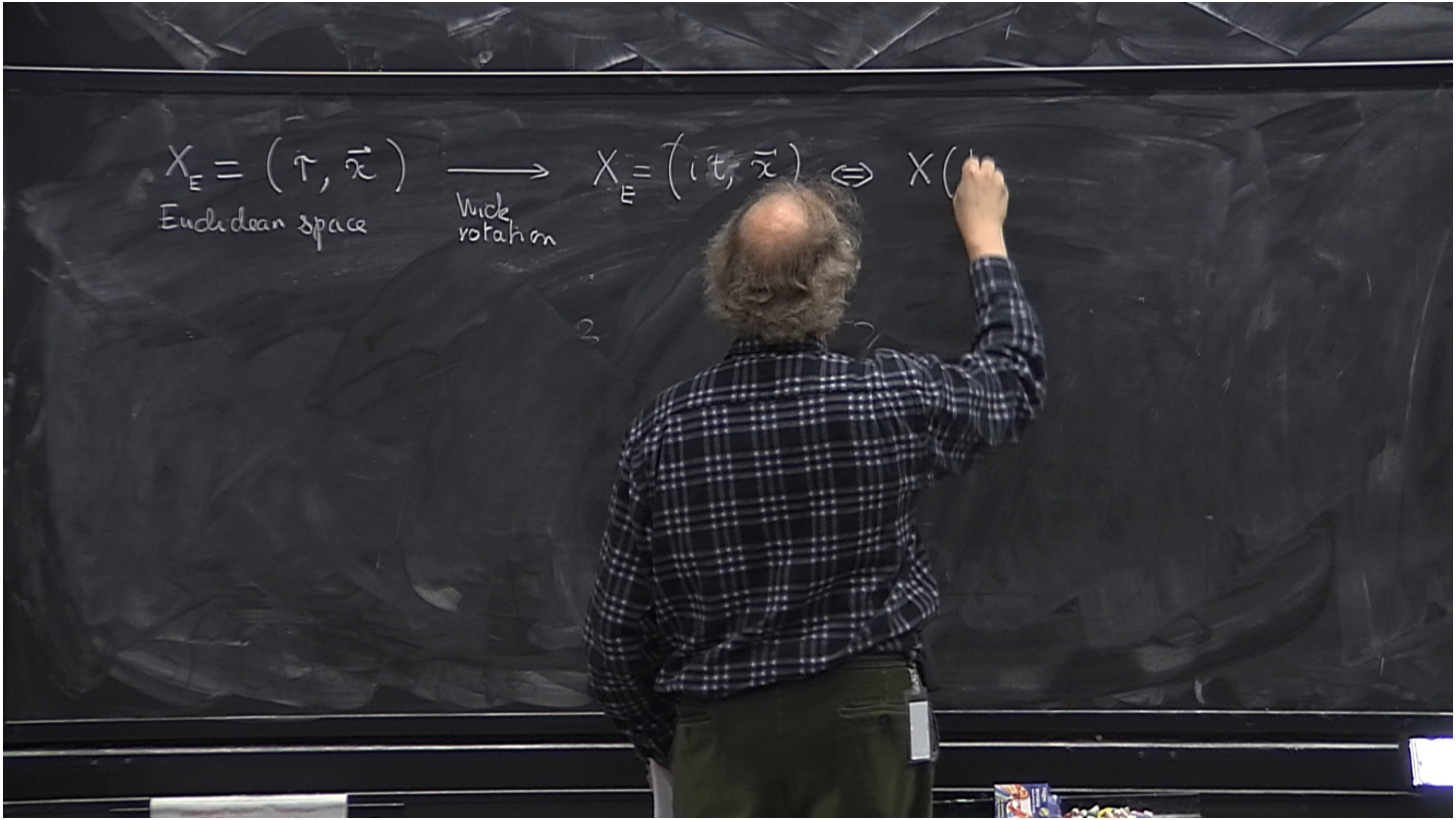
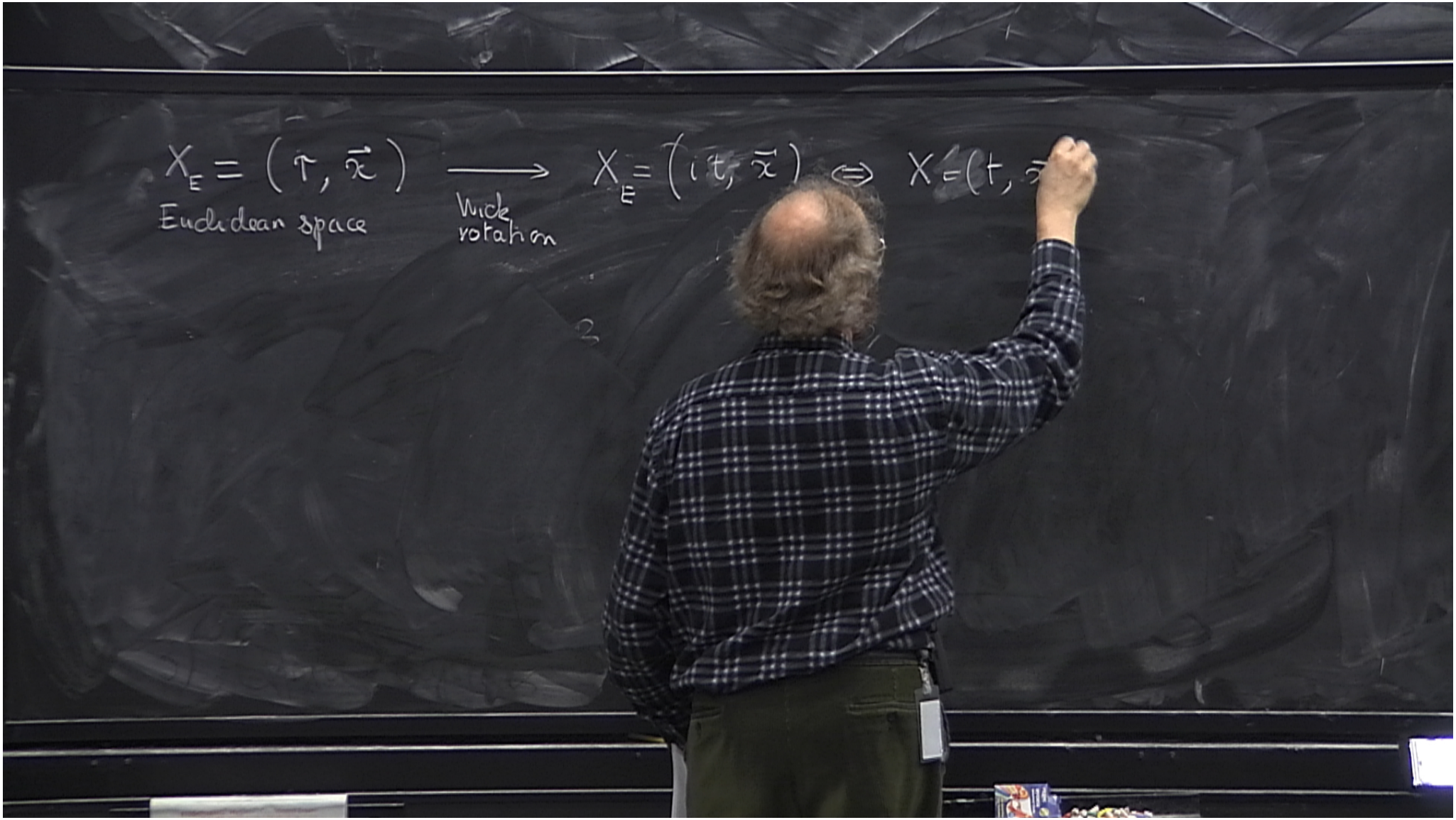
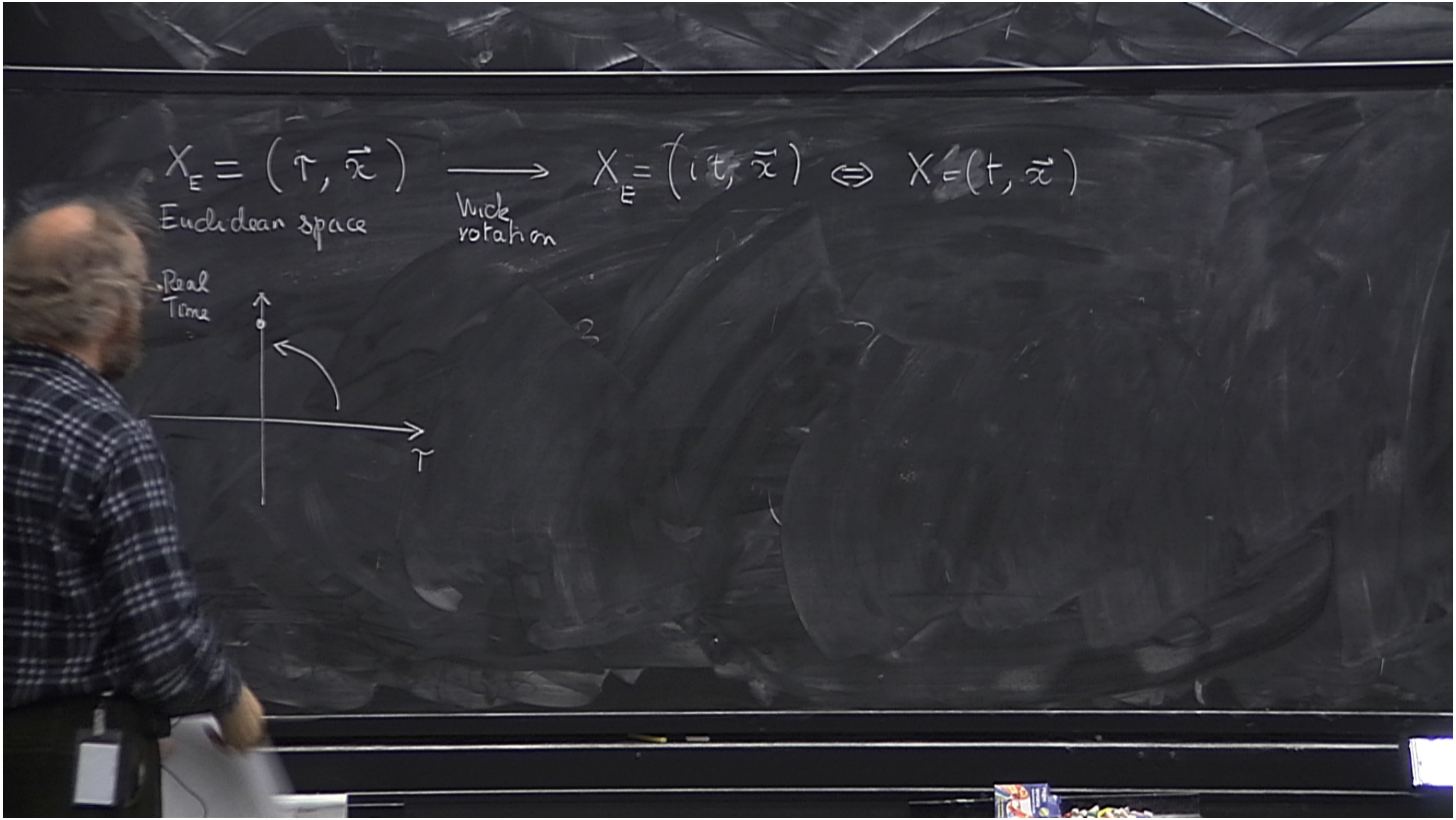
$$G_0(0) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2+m^2}$$

divergence for large $|k|$ or Energies


$X_E = (\tau, \vec{x})$
Wick rotation
$X_E = (it, \vec{x})$


$X_E = (\tau, \vec{x})$
Euclidean space
Wick rotation
$X_E = (it, \vec{x}) \Leftrightarrow X($

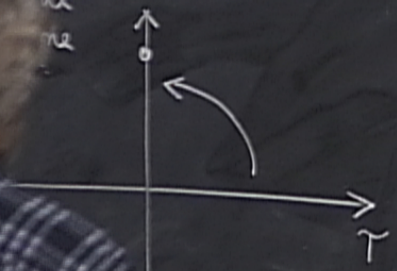

$X_E = (\tau, \vec{x})$
Euclidean space
Wick rotation
$X_E = (it, \vec{x})$
$\Leftrightarrow$
$X = (t, \vec{x})$


$X_E = (\tau, \vec{x})$
Euclidean space
Wick rotation
$X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$
Real Time
τ

$X_E = (\tau, \vec{x})$ ⟶ $X_E = (it, \vec{x})$ ⇔ $X = (t, \vec{x})$

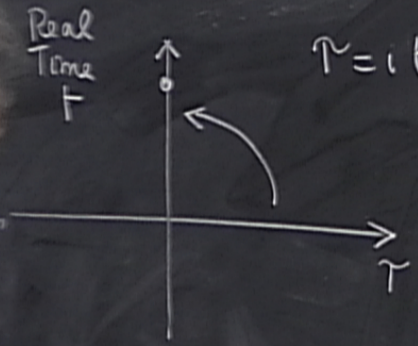
Euclidean space

Wick rotation



$X_E = (\tau, \vec{x})$ ⟶ $X_E = (i t, \vec{x})$ ⇔ $X = (t, \vec{x})$

Euclidean space — Wick rotation

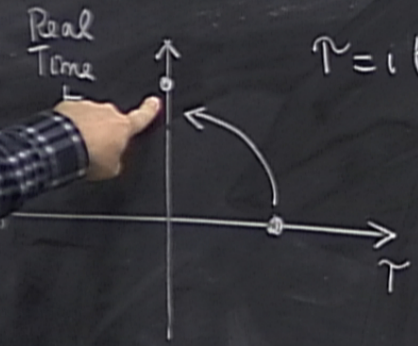


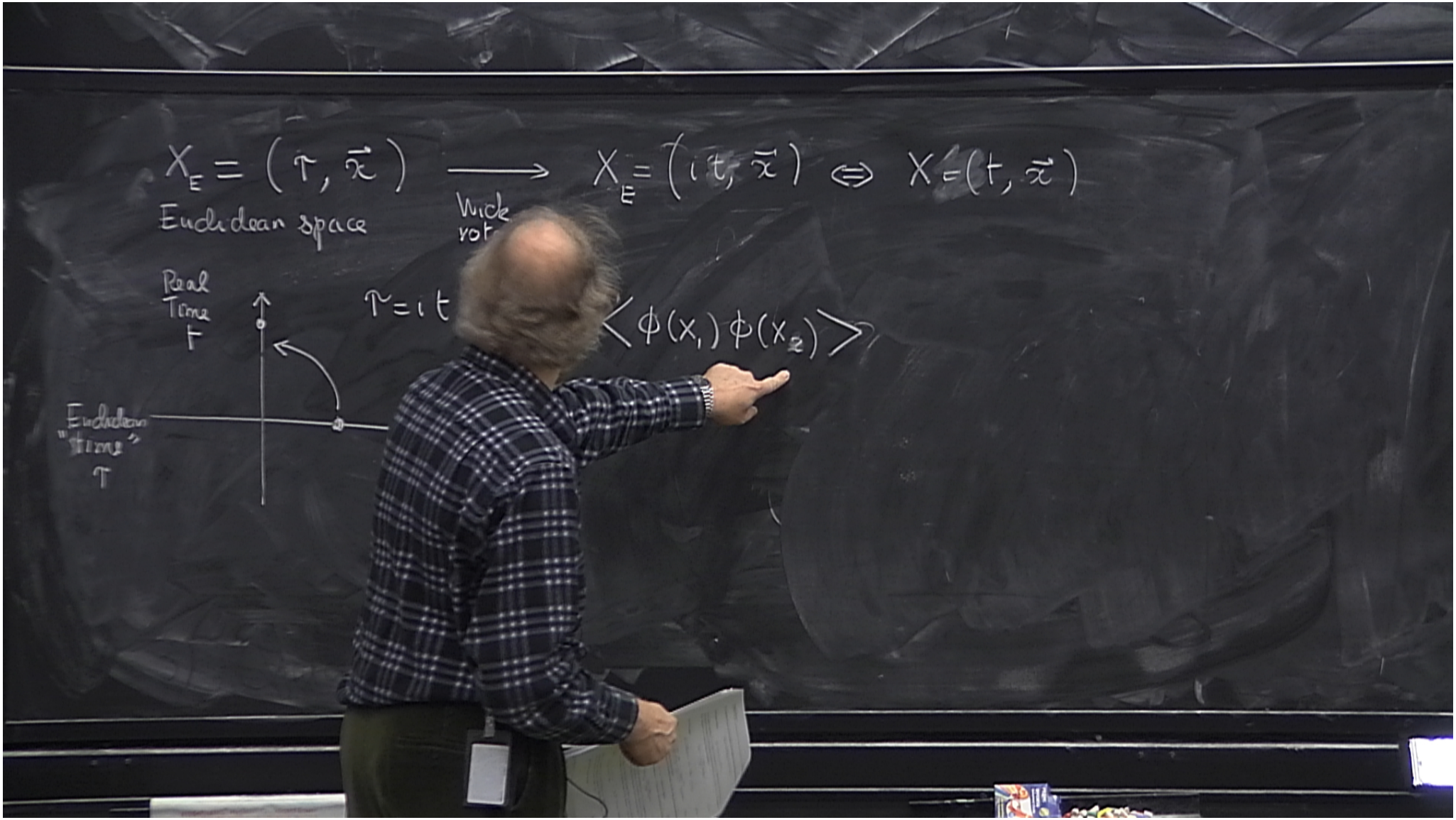
$\tau = i t$

$X_E = (\tau, \vec{x}) \longrightarrow X_E = (i t, \vec{x}) \Leftrightarrow X = (t, \vec{x})$

Euclidean space

Wick rotation

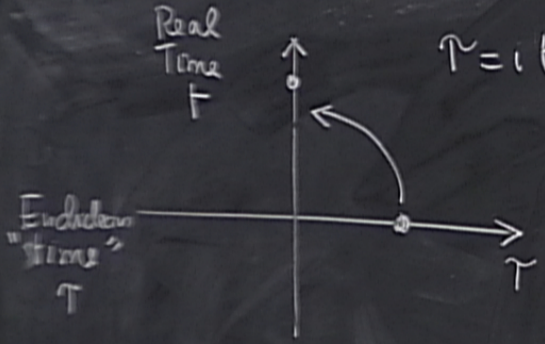



$X_E = (\tau, \vec{x})$
Euclidean space
Wick rot
$X_E = (it, \vec{x})$
$X = (t, \vec{x})$
Real Time t
$\tau = it$
Euclidean "time" τ
$\langle \phi(x_1)\phi(x_2) \rangle$

$X_E = (\tau, \vec{x})$ ⟶ $X_E = (i t, \vec{x})$ ⇔ $X = (t, \vec{x})$

Euclidean space

Wick rotation



$\tau = i t$

$2 < \phi(x_1) \phi$

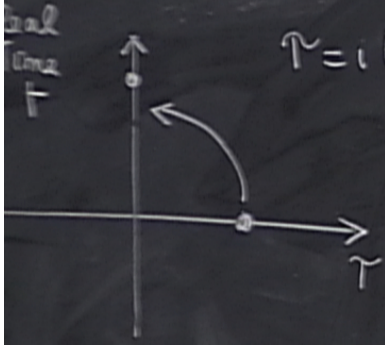
Wick Rotation ⟶ $< 0 | T(\phi(x_1)\phi(x_2) | 0 >$

dean

$X_E = (\tau, \vec{x}) \longrightarrow X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$

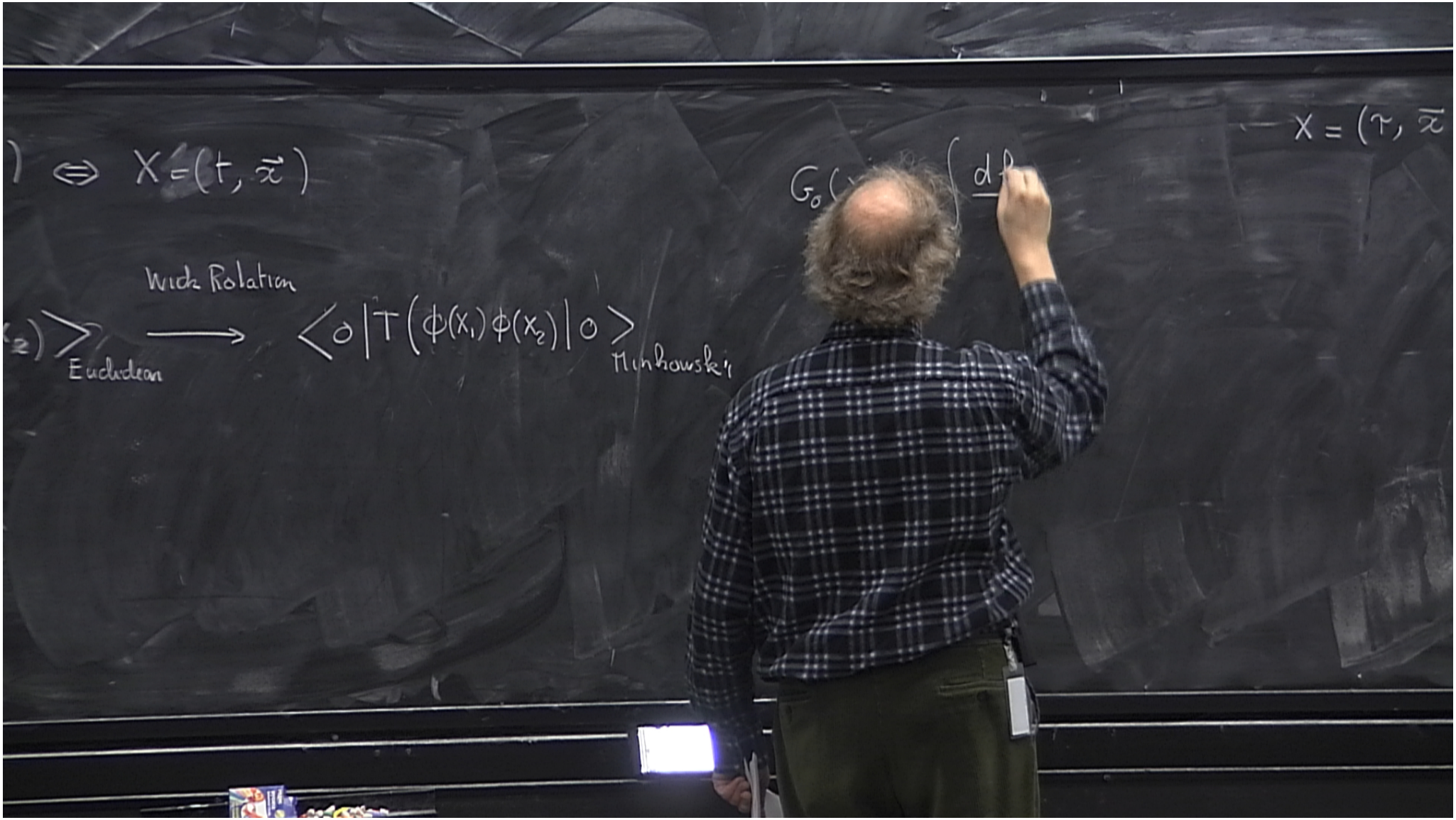
Euclidean space

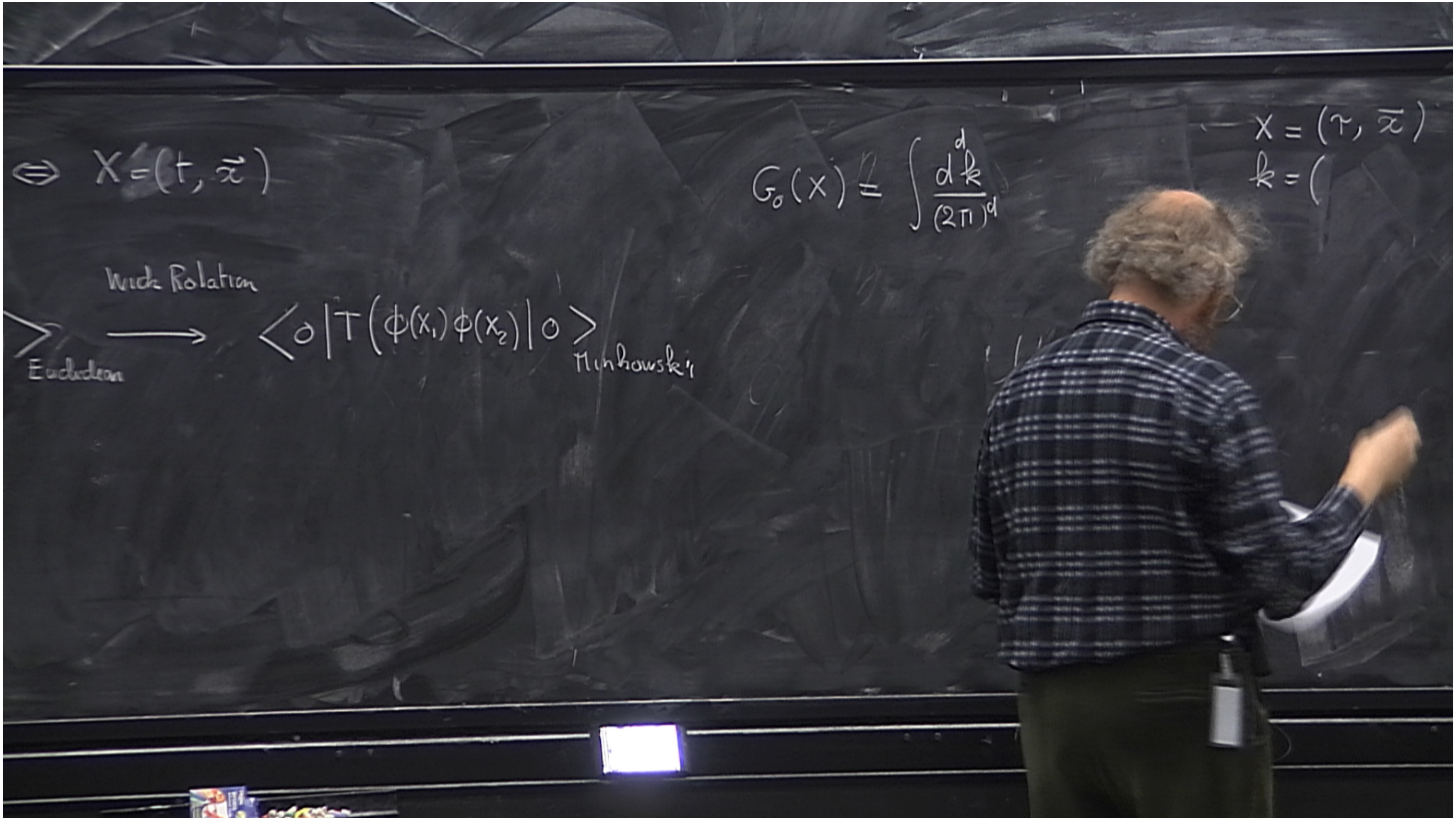
Wick rotation

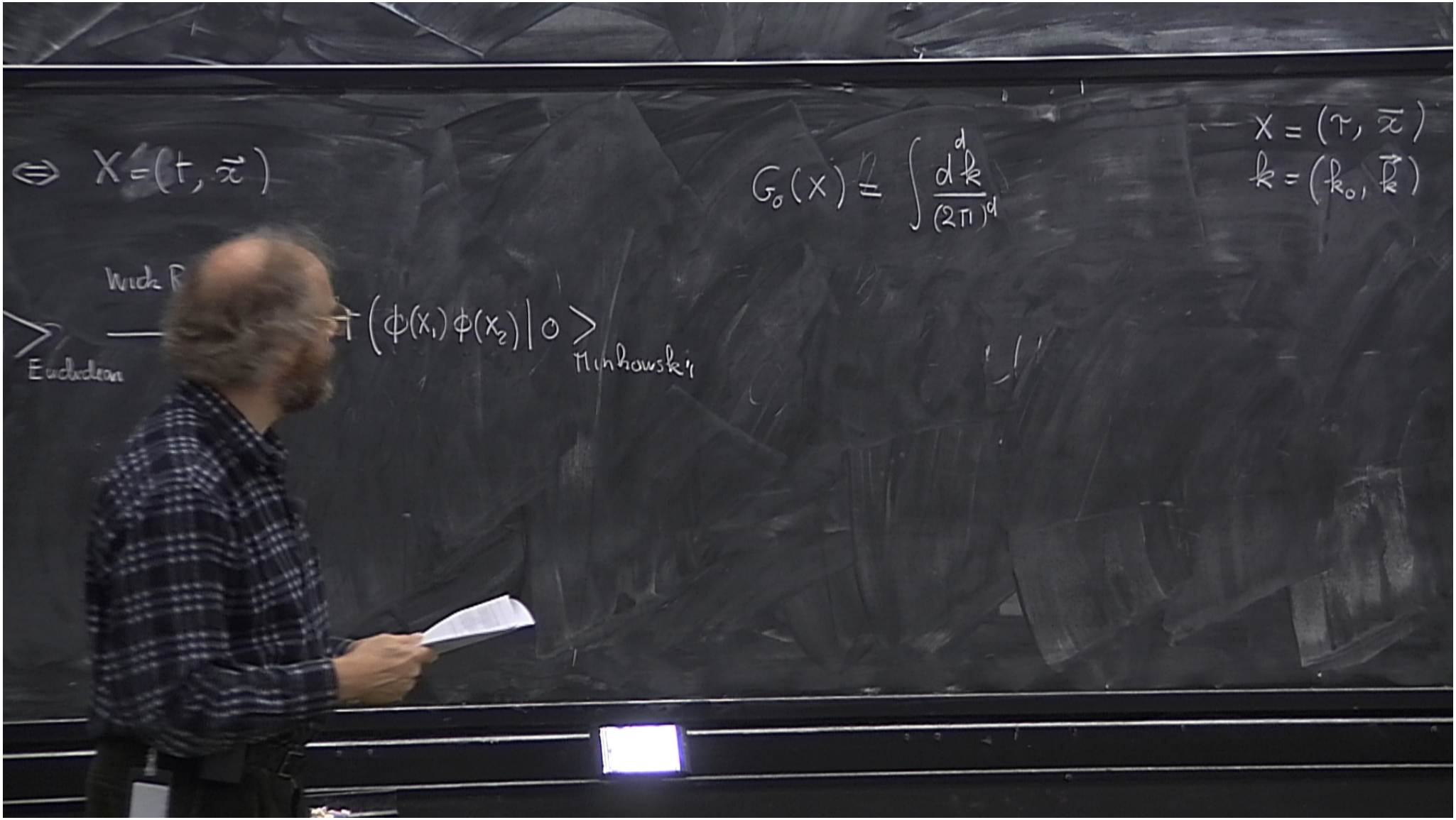


$\tau = it$

$\langle \phi(x_1)\phi(x_2) \rangle$ Euclidean $\xrightarrow{\text{Wick Rotation}}$ $\langle 0 | T(\phi(x_1)\phi(x_2)) | 0 \rangle$ Minkowski


$\Leftrightarrow X=(t,\vec{x})$
$X=(\tau,\vec{x}$
Wick Rotation
$\langle 0|T(\phi(x_1)\phi(x_2)|0\rangle$
Euclidean
Minkowski


$\Leftrightarrow X=(t,\vec{x})$
$G_0(X) = \int \frac{d^d k}{(2\pi)^d}$
$X=(\tau,\vec{x})$
$k=($
Wick Rotation
Euclidean
$\langle 0|T(\phi(x_1)\phi(x_2)|0\rangle$
Minkowski


$\Leftrightarrow X=(t,\vec{x})$
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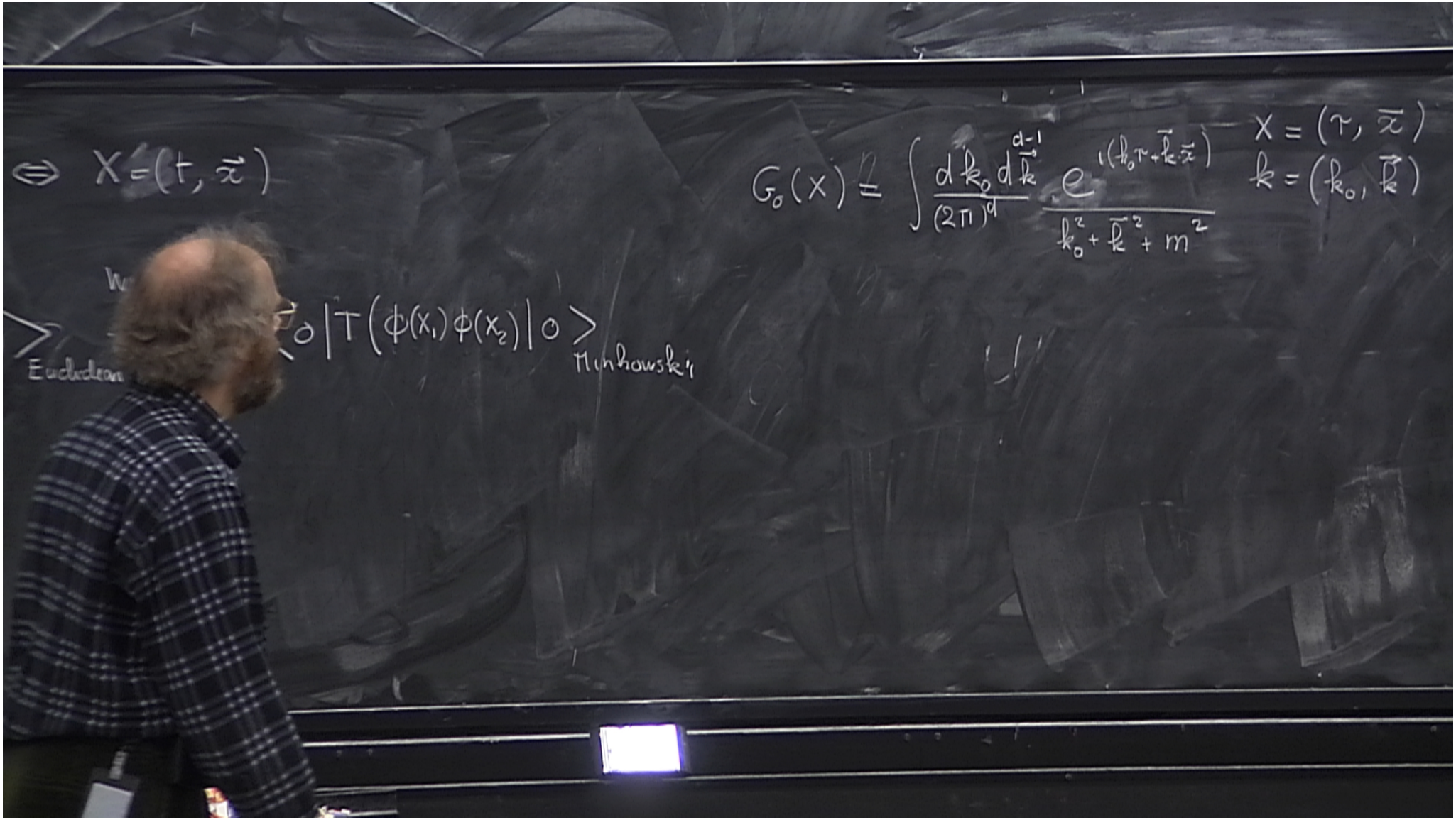
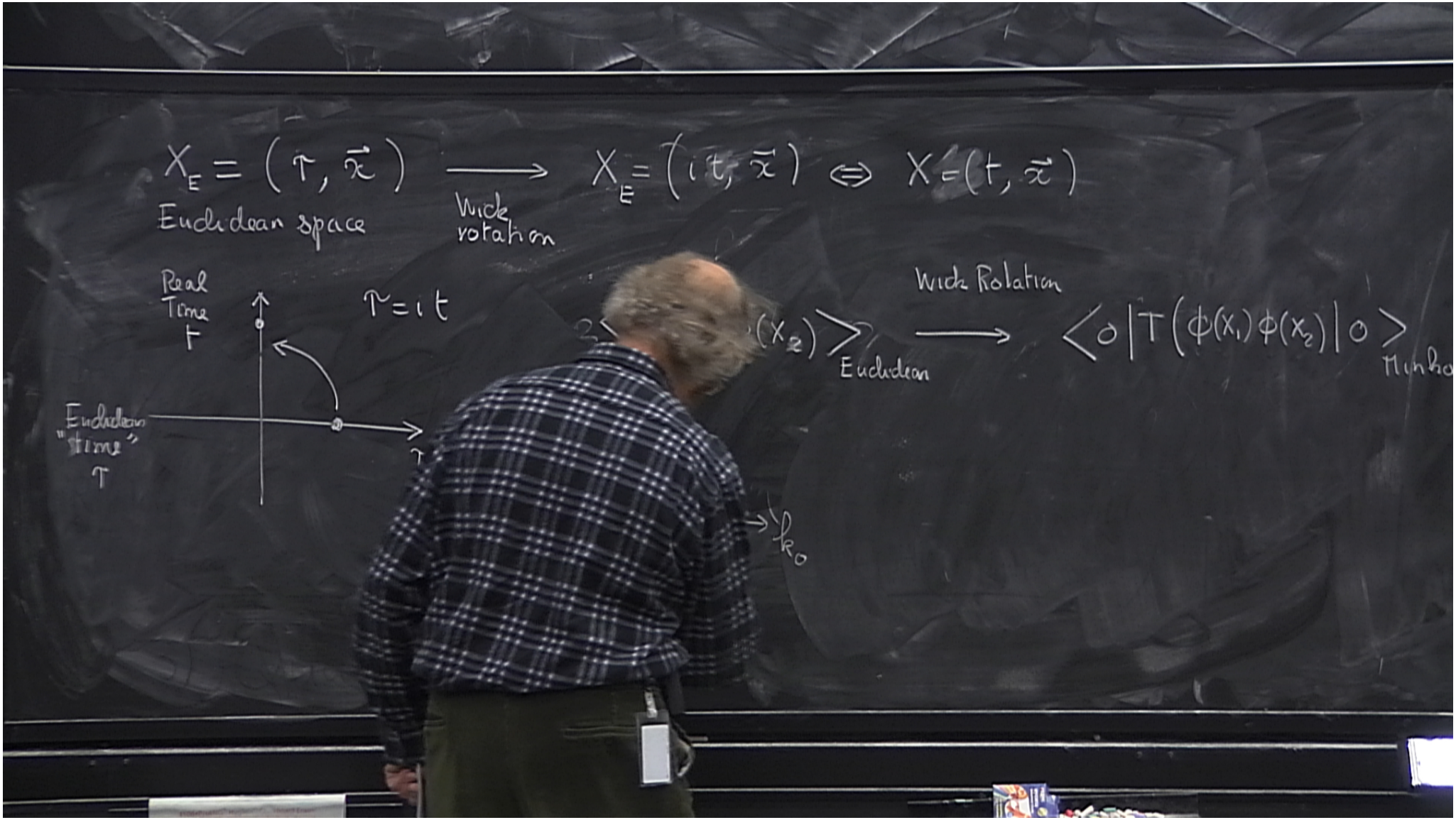
$X=(\tau,\vec{x})$
$k=(k_0,\vec{k})$
Euclidean
Minkowski

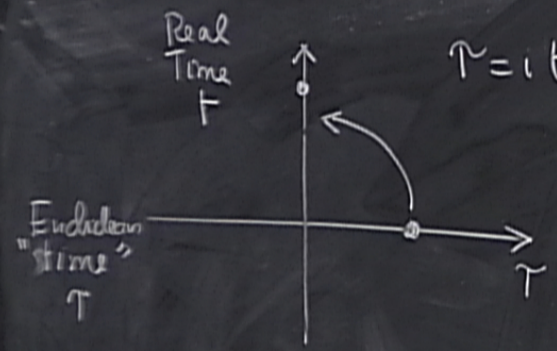

$\Leftrightarrow X=(t,\vec{x})$
$G_0(X) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau+\vec{k}\cdot\vec{x})}}{k_0^2+\vec{k}^2+m^2}$
$X=(\tau,\vec{x})$
$k=(k_0,\vec{k})$
$\langle 0|T(\phi(x_1)\phi(x_2)|0\rangle$
Euclidean
Minkowski


$X_E = (\tau, \vec{x})$
Euclidean space
Wick rotation
$X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$
Real Time t
$\tau = it$
Euclidean "time" τ
Wick Rotation
Euclidean
$\langle 0 | T(\phi(x_1)\phi(x_2)|0 \rangle$
k_0

$X_E = (\tau, \vec{x})$ ⟶ $X_E = (it, \vec{x})$ ⇔ $X = (t, \vec{x})$

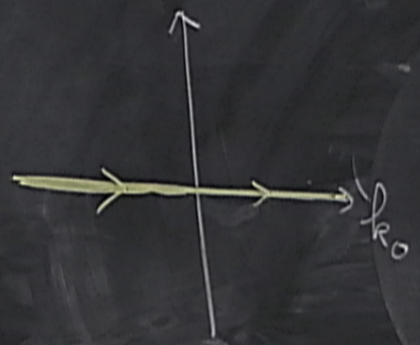
Euclidean space

Wick rotation



Wick Rotation

$2\langle \phi(x_1)\phi(x_2) \rangle$ ⟶ (Euclidean) $\langle 0 | T(\phi(x_1)\phi(x_2)) | 0 \rangle$ (Minkowski)



$= (\tau, \vec{x}) \longrightarrow X_E = (i\tau, \vec{x}) \Leftrightarrow X = (t, \vec{x})$

Euclidean space

Wick rotation

$G_0(X$

$\tau = it$

Wick Rotation

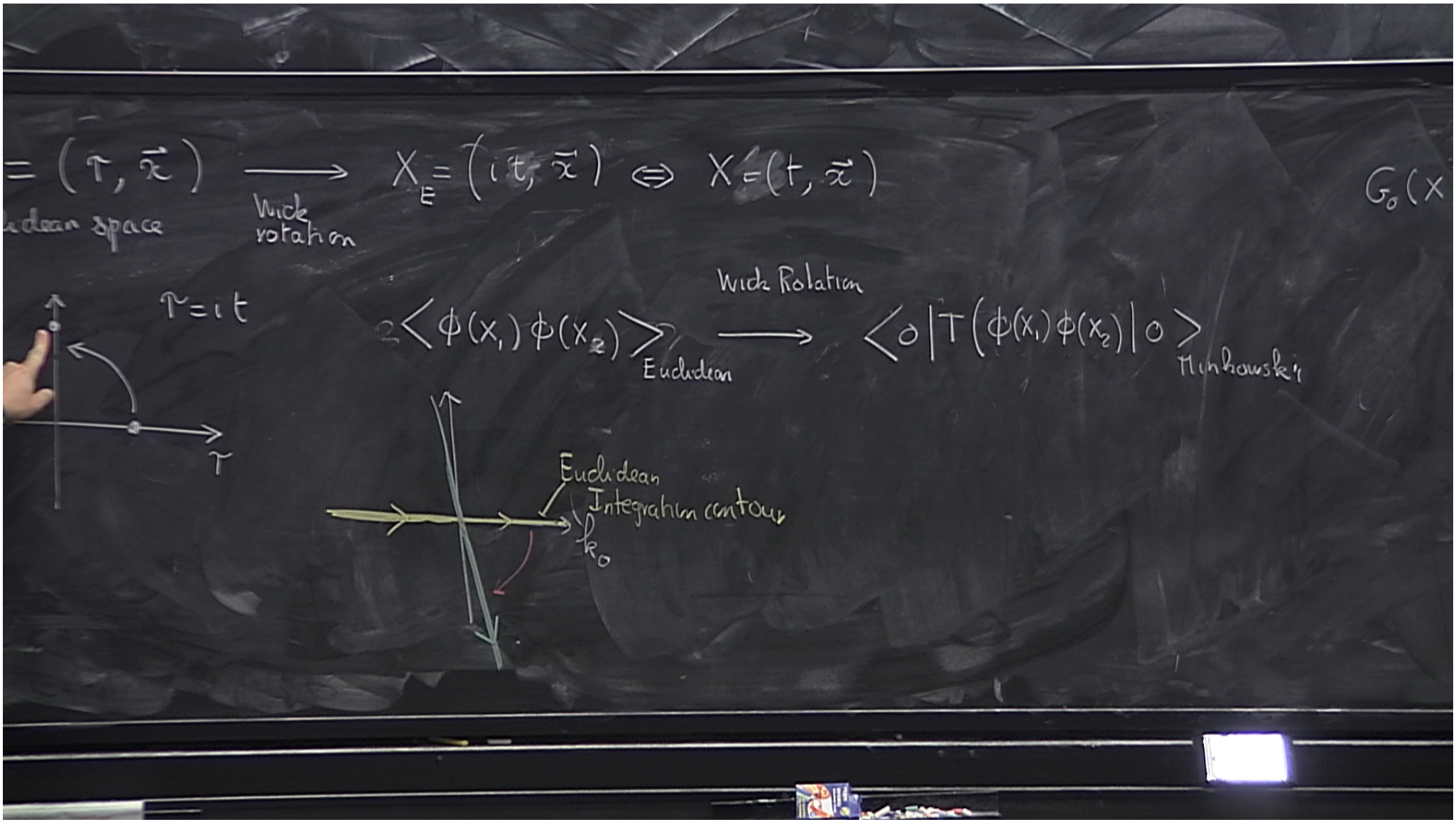
$2\langle \phi(x_1)\phi(x_2) \rangle \longrightarrow \langle 0|T(\phi(x_1)\phi(x_2)|0\rangle$

Euclidean

Minkowski

τ

k_0


= (τ, x⃗) ⟶ X_E = (it, x⃗) ⇔ X = (t, x⃗)
idean space
Wick rotation
G_0(X
τ = it
τ
Wick Rotation
⟨φ(x_1) φ(x_2)⟩ ⟶ ⟨0|T(φ(x_1) φ(x_2)|0⟩
Euclidean
Minkowski
Euclidean Integration contour
k_0

$$G_0(X) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2}$$

$X = (\tau, \vec{x})$

$k = (k_0, \vec{k})$

$\tau = i\,t$

$k_0 = -i\,\omega$

$$G_0(X) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\vec{x})}}{k_0^2 + \vec{k}^2 + m^2}$$

$X = (\tau, \vec{x})$

$k = (k_0, \vec{k})$

$\tau = i\,t$ Real time

$k_0 = -i\,\omega$ Real "energy"

$$= \int \frac{d\omega\, d^{d-1}\vec{k}}{(2\pi)^d} (-i)$$

Minkowski

$$G_0(X) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\vec{x})}}{k_0^2 + \vec{k}^2 + m^2} \qquad X = (\tau, \vec{x}) \qquad k = (k_0, \vec{k})$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega\, d^{d-1}\vec{k}}{(2\pi)^d} (-i)$$

$\tau = i\,t$ Real time

$k_0 = -i\,\omega$ Real "energy"

Minkowski

$$G_0(x) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2} \qquad x = (\tau, \vec{x}) \qquad k = (k_0, \vec{k})$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{(-i)\, e^{i(\omega t + \vec{k}\cdot\vec{x})}}{-\omega^2 + \vec{k}^2 + m^2}$$

$\tau = i t$ Real time

$k_0 = -i\,\omega$ Real "energy"

Minkowski

$$G_0(X) \equiv \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2} \qquad X = (\tau, \vec{x}) \qquad k = (k_0, \vec{k})$$

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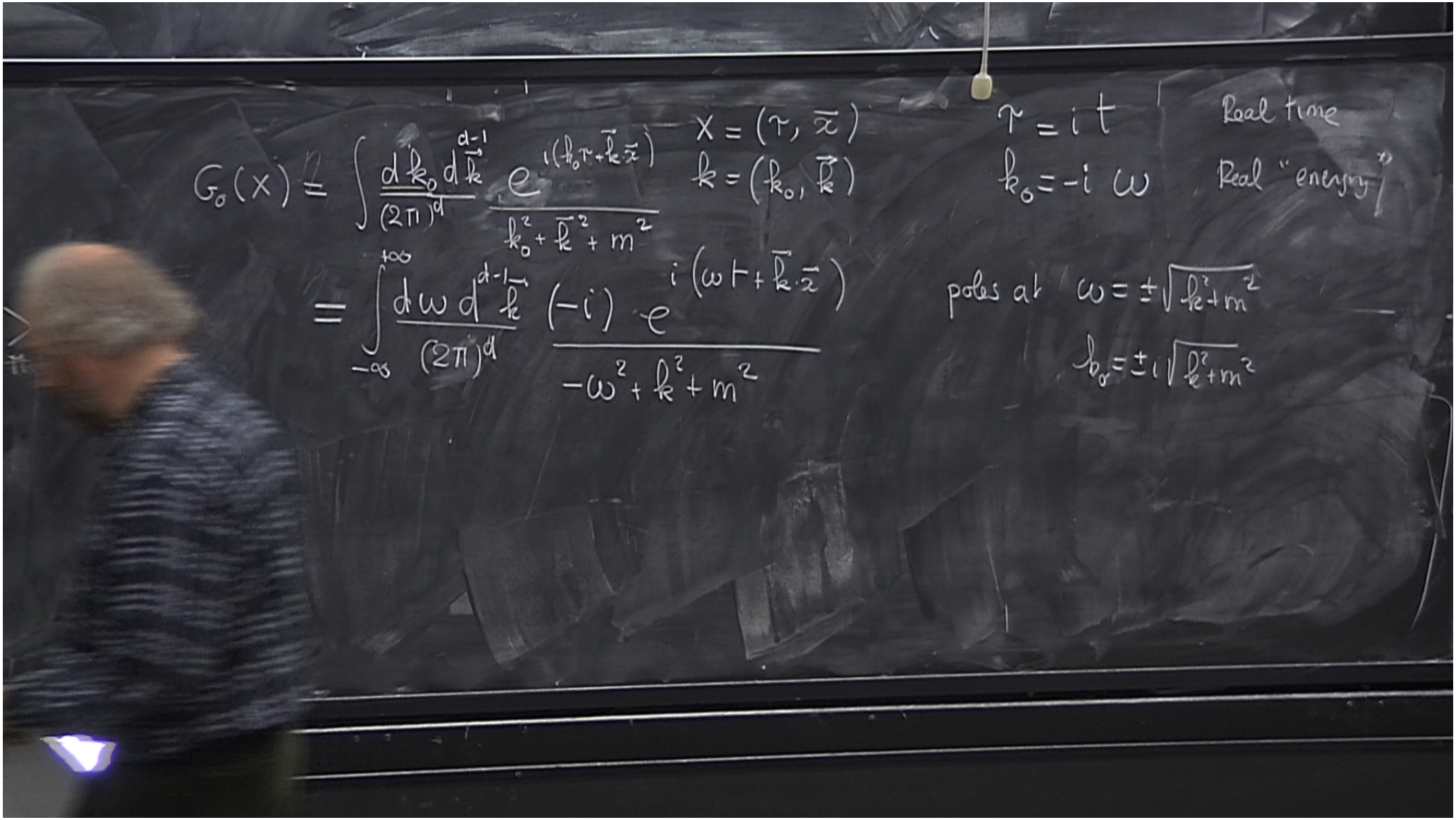
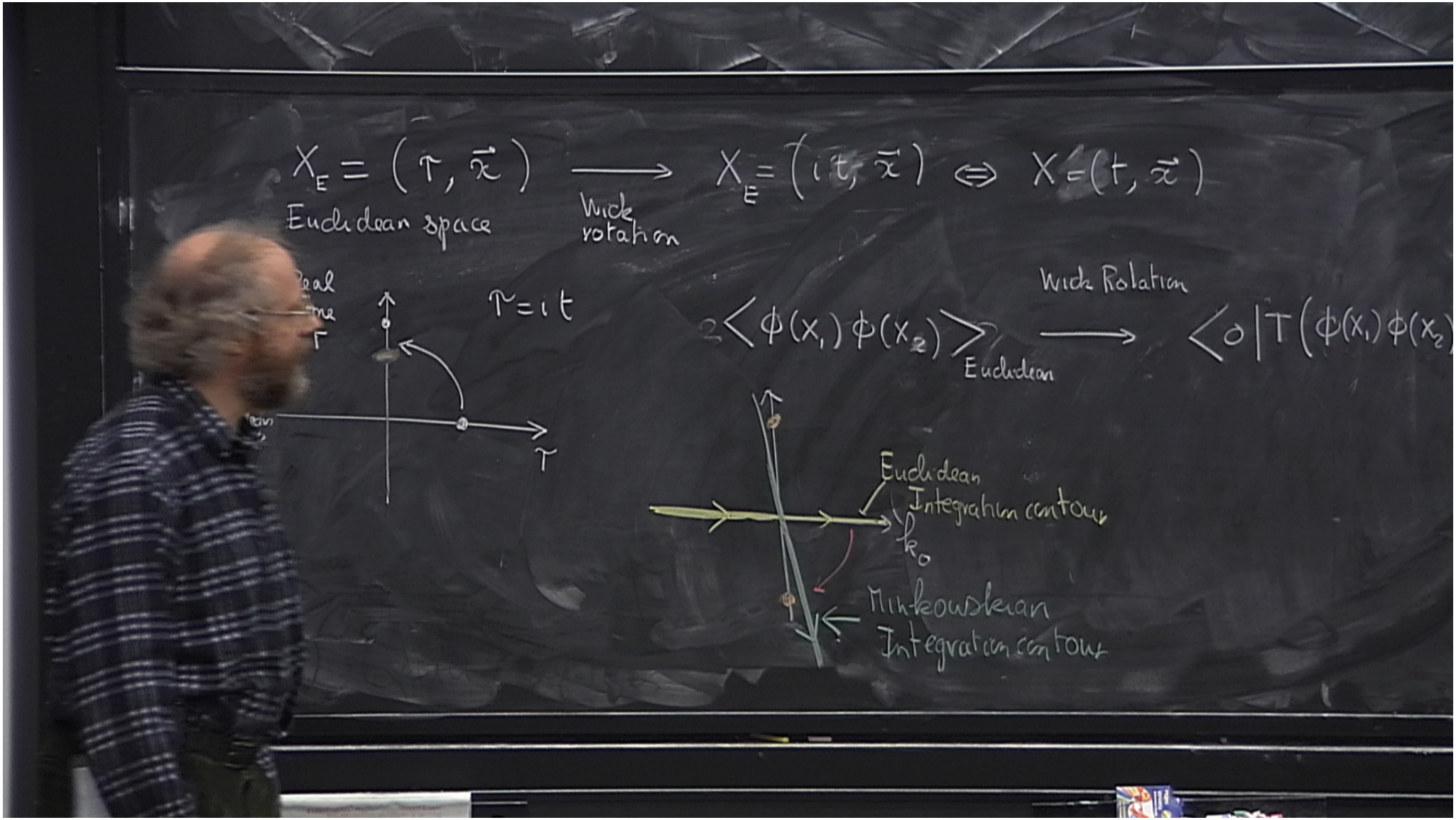
$\tau = it$ Real time

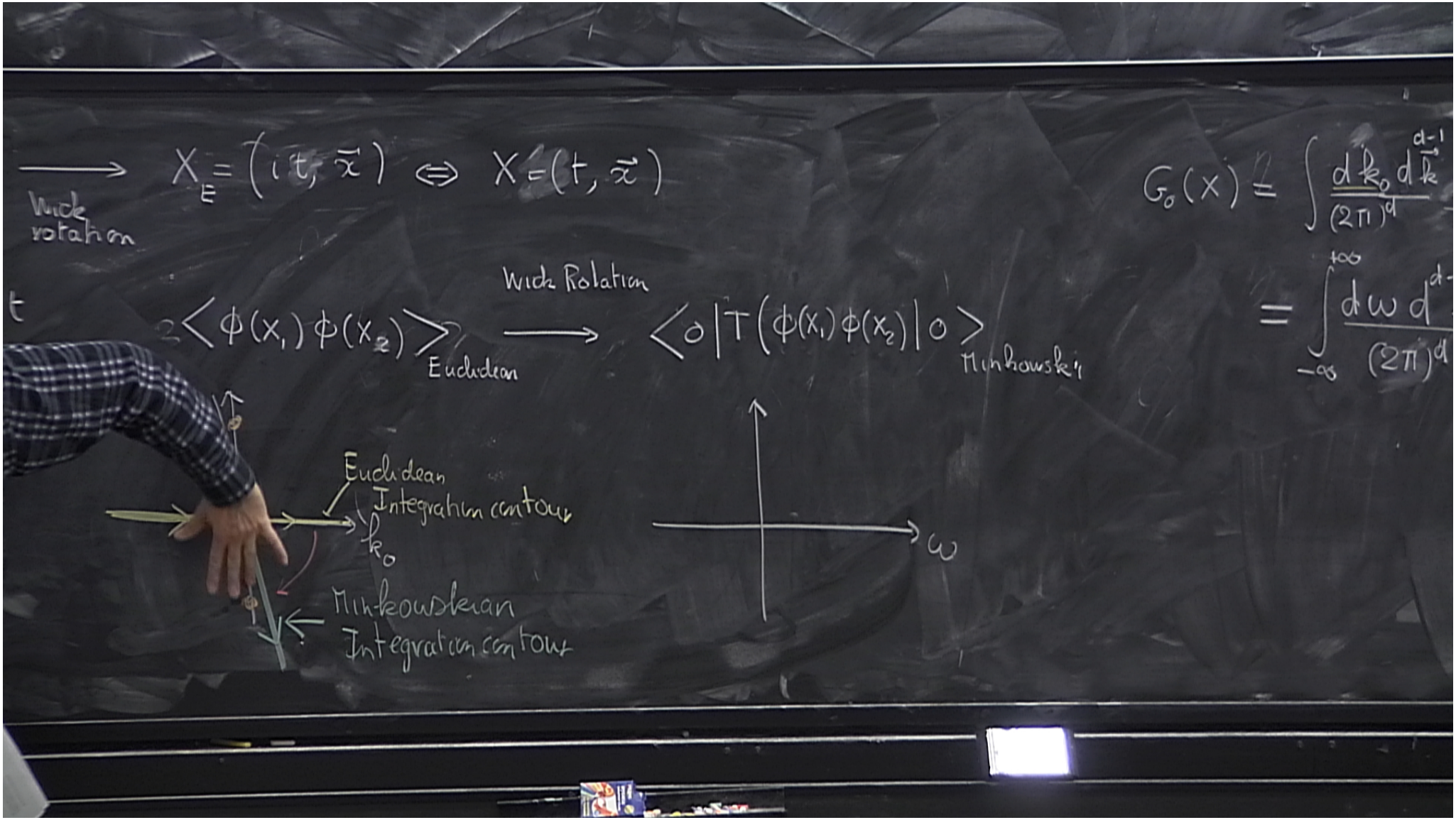
$k_0 = -i\omega$ Real "energy"

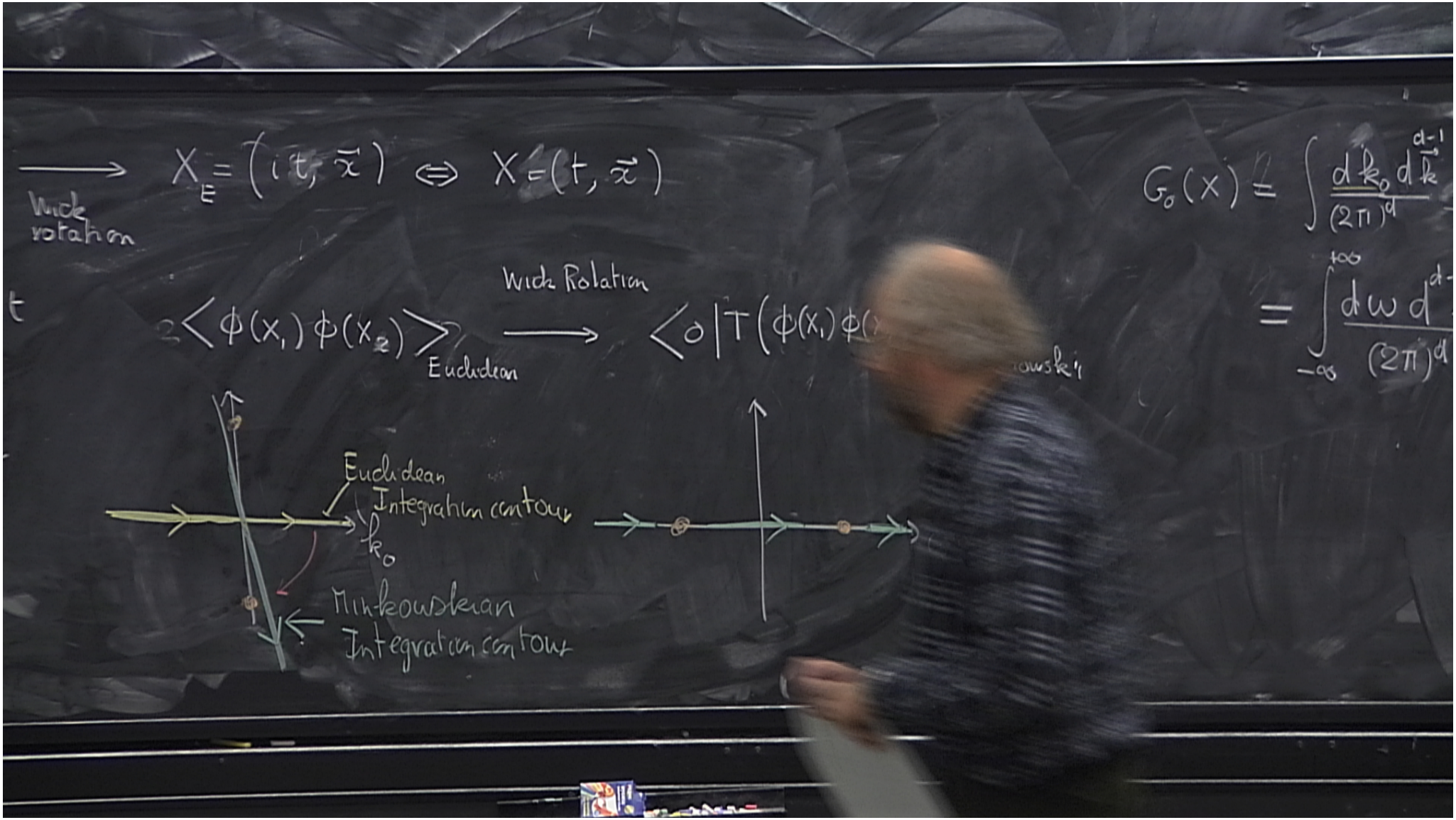
poles at $\omega = \pm\sqrt{\vec{k}^2 + m^2}$

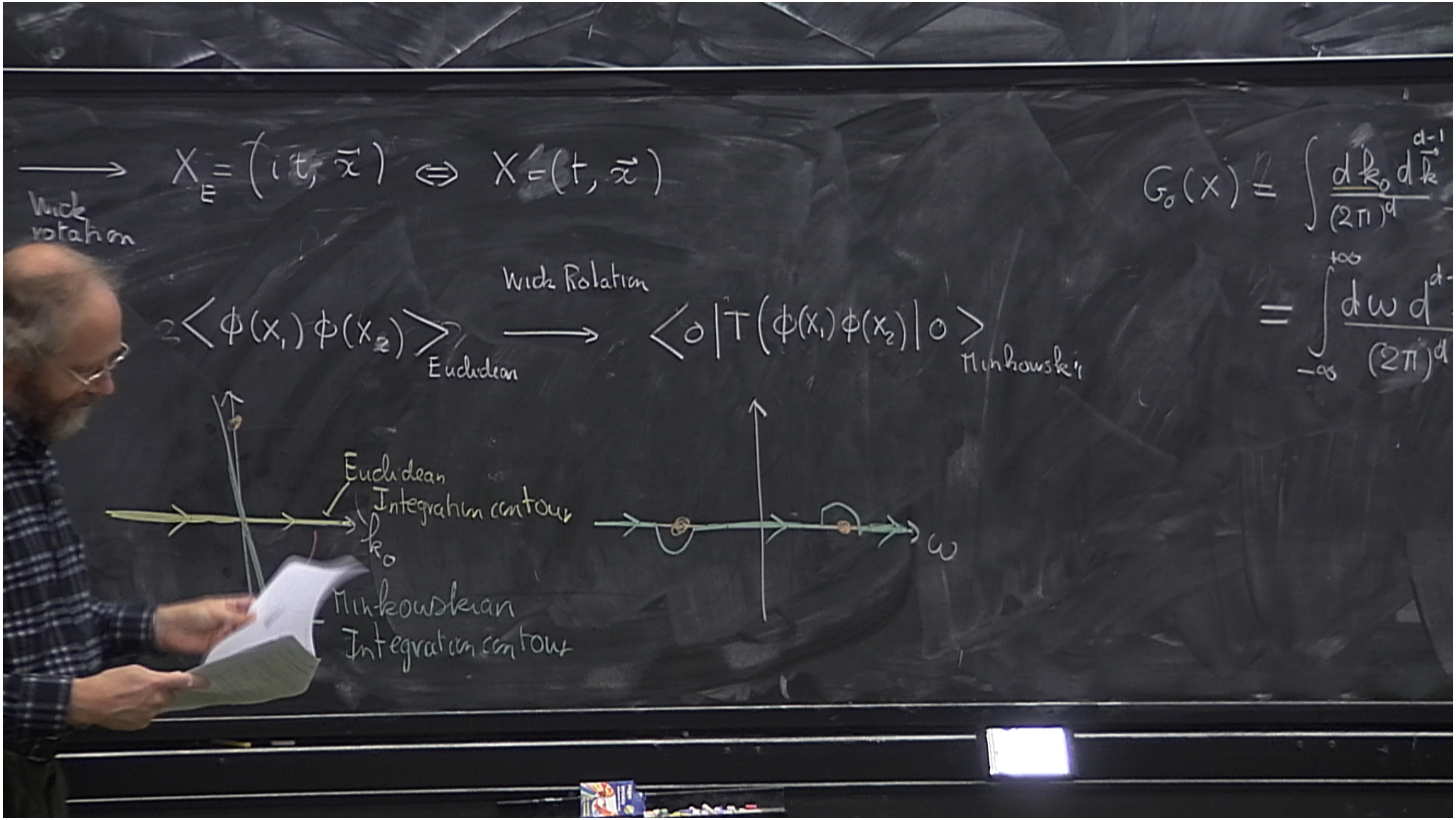
Minkowski


$G_0(X) = \int \frac{dk_0 \, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2}$
$X = (\tau, \vec{x})$
$k = (k_0, \vec{k})$
$\tau = i\,t$
Real time
$k_0 = -i\,\omega$
Real "energy"
$= \int_{-\infty}^{+\infty} \frac{d\omega \, d^{d-1}\vec{k}}{(2\pi)^d} \frac{(-i)\, e^{i(\omega t + \vec{k}\cdot\vec{x})}}{-\omega^2 + \vec{k}^2 + m^2}$
poles at $\omega = \pm\sqrt{\vec{k}^2 + m^2}$
$k_0 = \pm i\sqrt{\vec{k}^2 + m^2}$


$X_E = (\tau, \vec{x})$
Euclidean space
Wick rotation
$X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$
$\tau = it$
τ
Wick Rotation
Euclidean
Euclidean Integration contour
k_0
Minkowskian Integration contour


Wick rotation
$X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$
$G_0(x) = \int \frac{dk_0 d^{d-1}\vec{k}}{(2\pi)^d}$
Wick Rotation
$i\,\langle \phi(x_1)\phi(x_2) \rangle$ Euclidean
$\langle 0|T(\phi(x_1)\phi(x_2)|0\rangle$ Minkowski
$= \int_{-\infty}^{+\infty} \frac{d\omega\, d^{d-}}{(2\pi)^d}$
Euclidean Integration contour
k_0
Minkowskian Integration contour
ω


Wick rotation
$X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$
$G_0(x) = \int \frac{dk_0 d\vec{k}}{(2\pi)^d}$
Wick Rotation
$\langle \phi(x_1)\phi(x_2) \rangle$
Euclidean
$\langle 0 | T(\phi(x_1)\phi($
$= \int_{-\infty}^{+\infty} \frac{dw\, d$
Euclidean Integration contour
k_0
Minkowskian Integration contour


Wick rotation
$X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$
$G_0(X) = \int \frac{dk_0 d^{d-1}\vec{k}}{(2\pi)^d}$
Wick Rotation
$2\langle \phi(x_1)\phi(x_2) \rangle$
Euclidean
$\langle 0|T(\phi(x_1)\phi(x_2)|0\rangle$
Minkowski
$= \int_{-\infty}^{+\infty} \frac{d\omega d^{d-1}}{(2\pi)^d}$
Euclidean Integration contour
k_0
Minkowskian Integration contour
ω

$$G_0(x) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2} \qquad x = (\tau, \vec{x}) \qquad k = (k_0, \vec{k})$$

$$\tau = i\,t \quad \text{Real time} \qquad k_0 = -i\,\omega \quad \text{Real "energy"}$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{(-i)\, e^{i(\omega t + \vec{k}\cdot\vec{x})}}{-\omega^2 + \vec{k}^2 + m^2}$$

poles at $\omega = \pm\sqrt{\vec{k}^2 + m^2}$

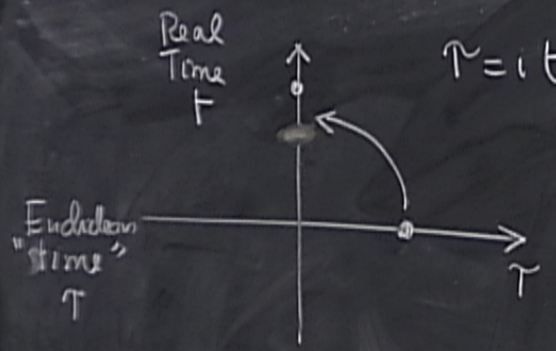
$k_0 = \pm i\sqrt{\vec{k}^2 + m^2}$

Feynman integration prescription contour $\longrightarrow$ Feynman propagator

$X_E = (\tau, \vec{x})$ Euclidean space $\longrightarrow$ Wick rotation $X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$

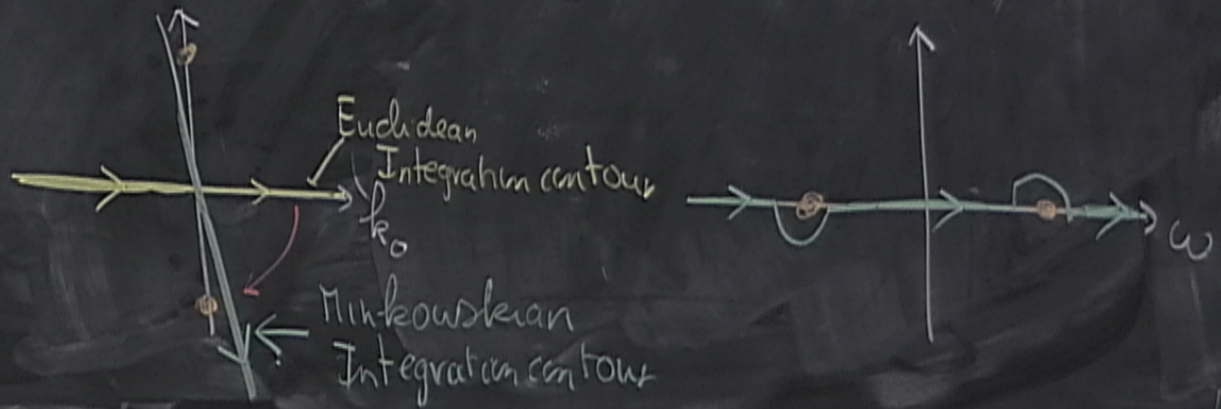
$X = (X^\mu)$

$k = (k_\mu)$



Wick Rotation

$2\langle \phi(x_1)\phi(x_2) \rangle_{\text{Euclidean}} \longrightarrow \langle 0 | T(\phi(x_1)\phi(x_2)|0 \rangle_{\text{Min}}$



$X = (X^\mu)$ contra v.

$k = (k_\mu)$ covariant

$$G_0(X) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2}$$

$X = (\tau, \vec{x})$

$k = (k_0, \vec{k})$

$\tau = i\,t$

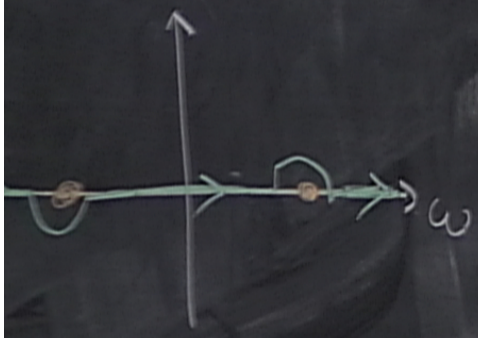
$k_0 = -i\,\omega$

$$= \int_{-\infty}^{+\infty} \frac{d\omega\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{(-i)\, e^{i(\omega t + \vec{k}\cdot\vec{x})}}{-\omega^2 + \vec{k}^2 + m^2}$$

poles at $\omega =$

$k_0 =$

$\langle 0 | T(\phi(x_1)\phi(x_2)) | 0 \rangle$ Minkowski



Feynman integration prescription contour ⟶ Feynman propag

$X = (X^\mu)$ contra v.

$k = (k_\mu)$ covariant

$$G_0(X) = \int \frac{dk_0\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0\tau + \vec{k}\cdot\vec{x})}}{k_0^2 + \vec{k}^2 + m^2}$$

$X = (\tau, \vec{x})$

$k = (k_0, \vec{k})$

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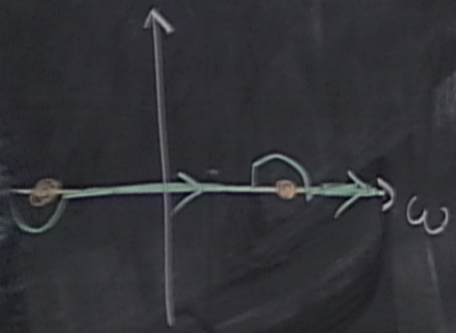

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Minkowski

$$= \int_{-\infty}^{+\infty} \frac{d\omega\, d^{d-1}\vec{k}}{(2\pi)^d} \frac{(-i)\, e^{i(\omega t + \vec{k}\cdot\vec{x})}}{-\omega^2 + \vec{k}^2 + m^2}$$

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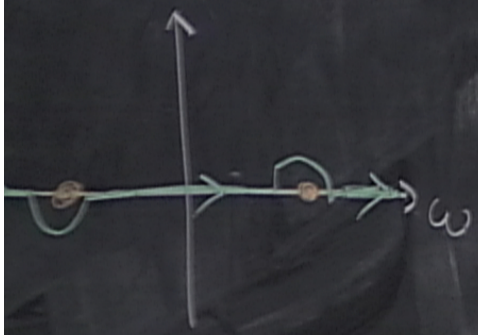

Feynman integration prescription contour ⟶ Feynman propag

$X_E = (it, \vec{x}) \Leftrightarrow X = (t, \vec{x})$

$X = (X^\mu)$ contra v.

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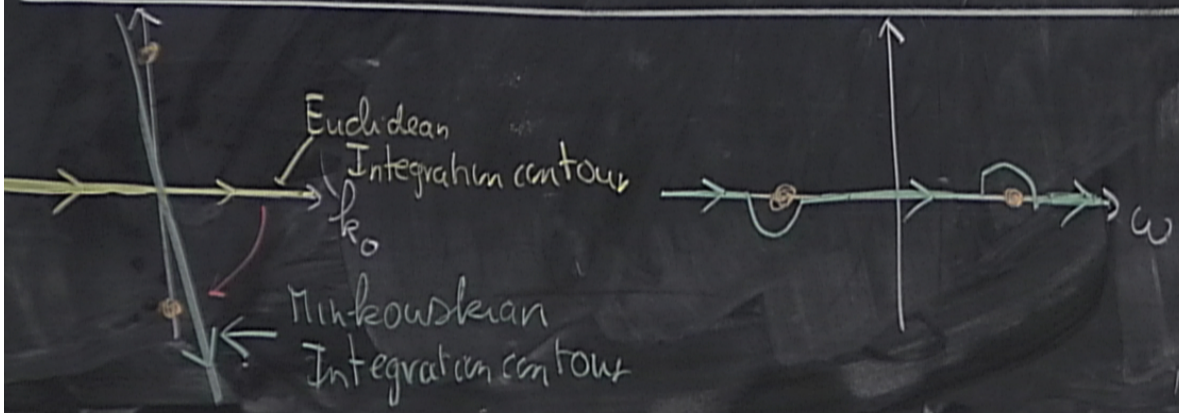
$$G_0(X) = \int \frac{dk_0 \, d^{d-1}\vec{k}}{(2\pi)^d} \frac{e^{i(k_0 x^0 + \vec{k}}}{k_0^2 + \vec{k}^2 + m}$$

Wick Rotation

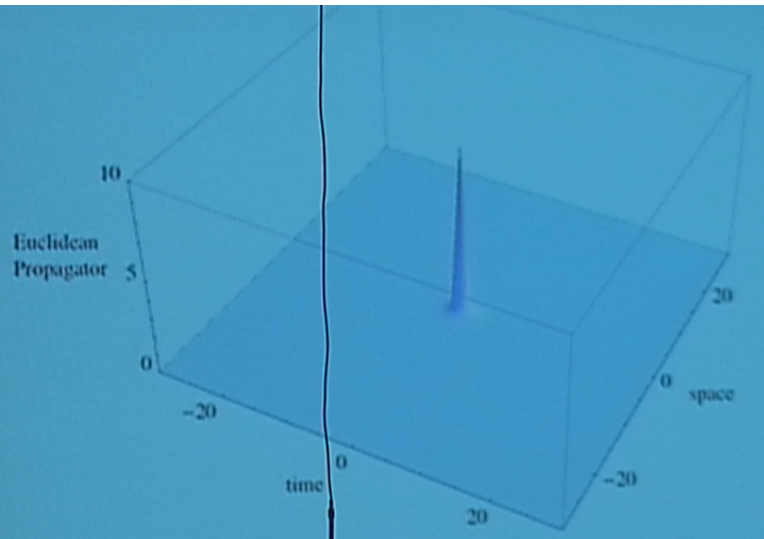
$$\langle \phi(x_1)\phi(x_2) \rangle_{\text{Euclidean}} \longrightarrow \langle 0 | T(\phi(x_1)\phi(x_2)) | 0 \rangle_{\text{Minkowski}} = G_{\text{Feynman}}$$

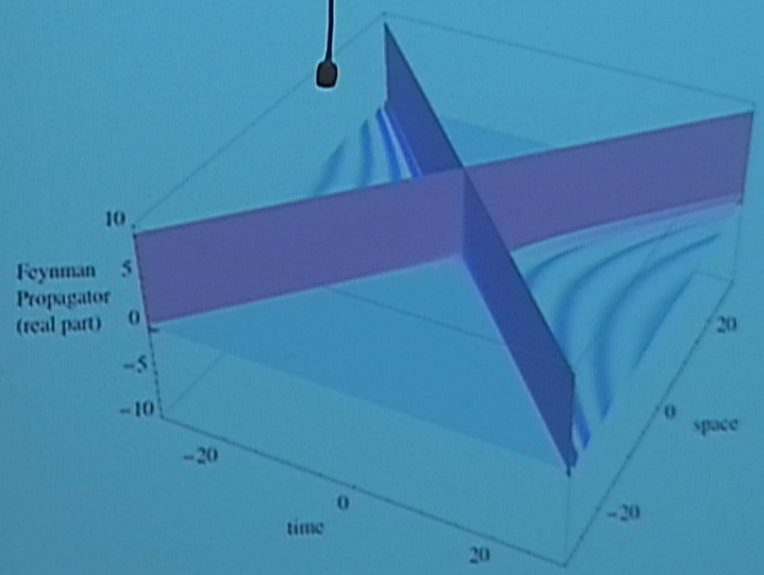
$$= \int_{-\infty}^{+\infty} \frac{d\omega \, d^{d-1}\vec{k}}{(2\pi)^d} \frac{(-i)}{-\omega^2}$$

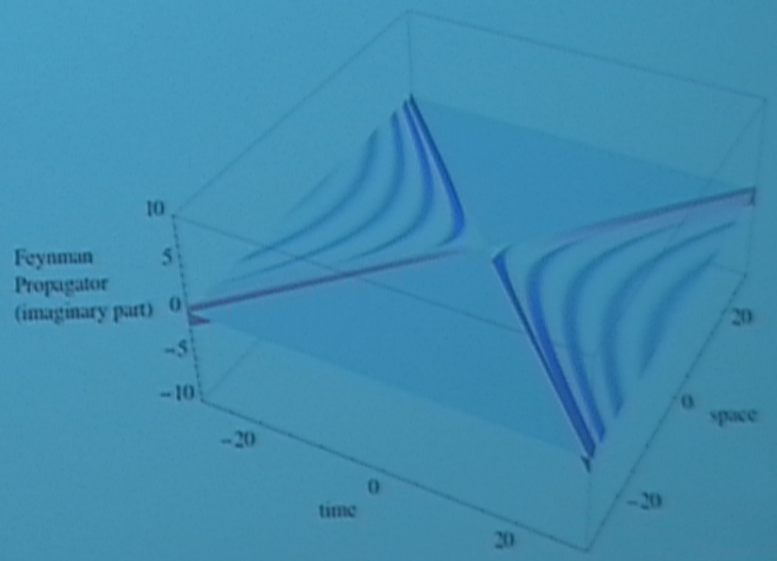
Feynman integration pre



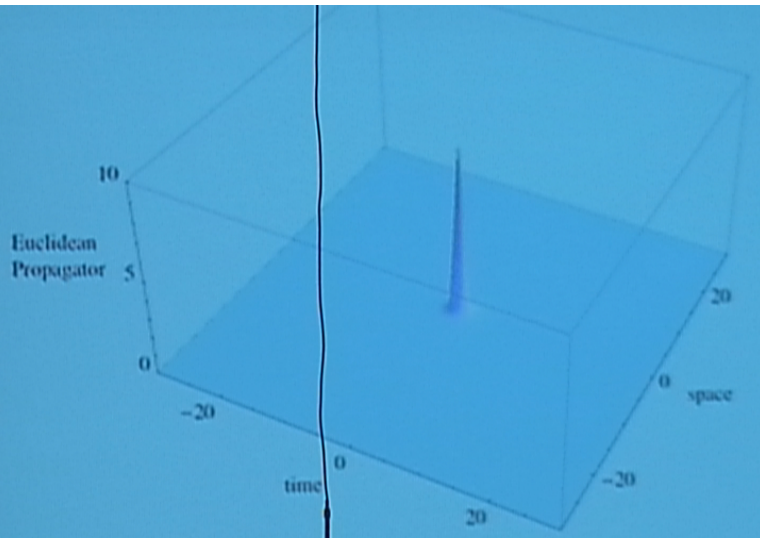
Massive Propagators in d=2

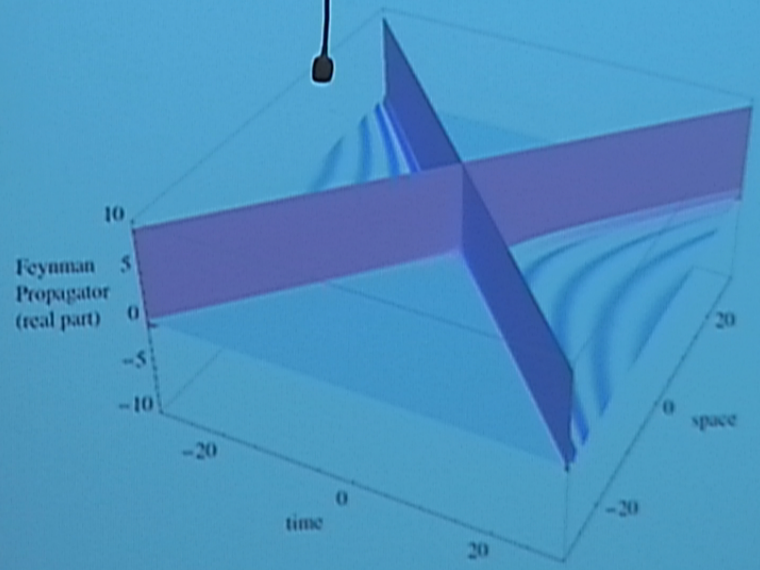


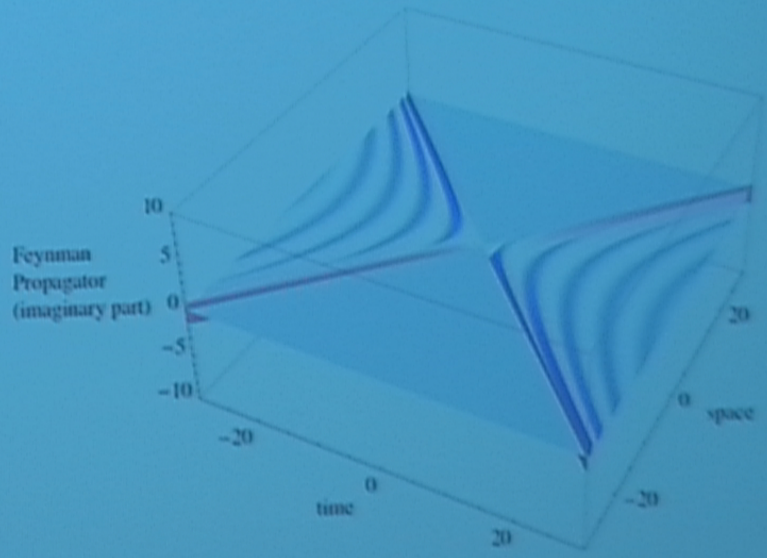


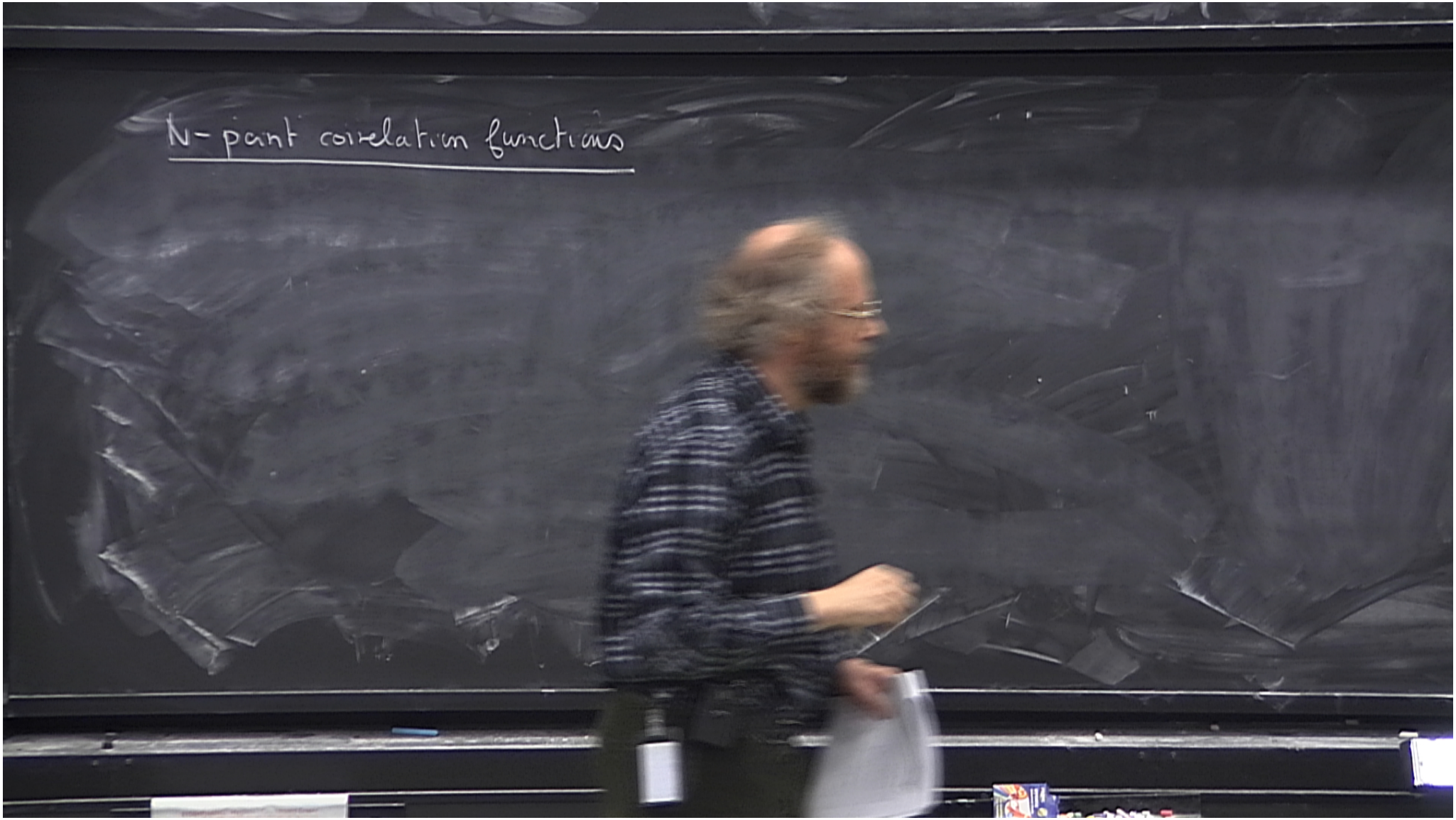


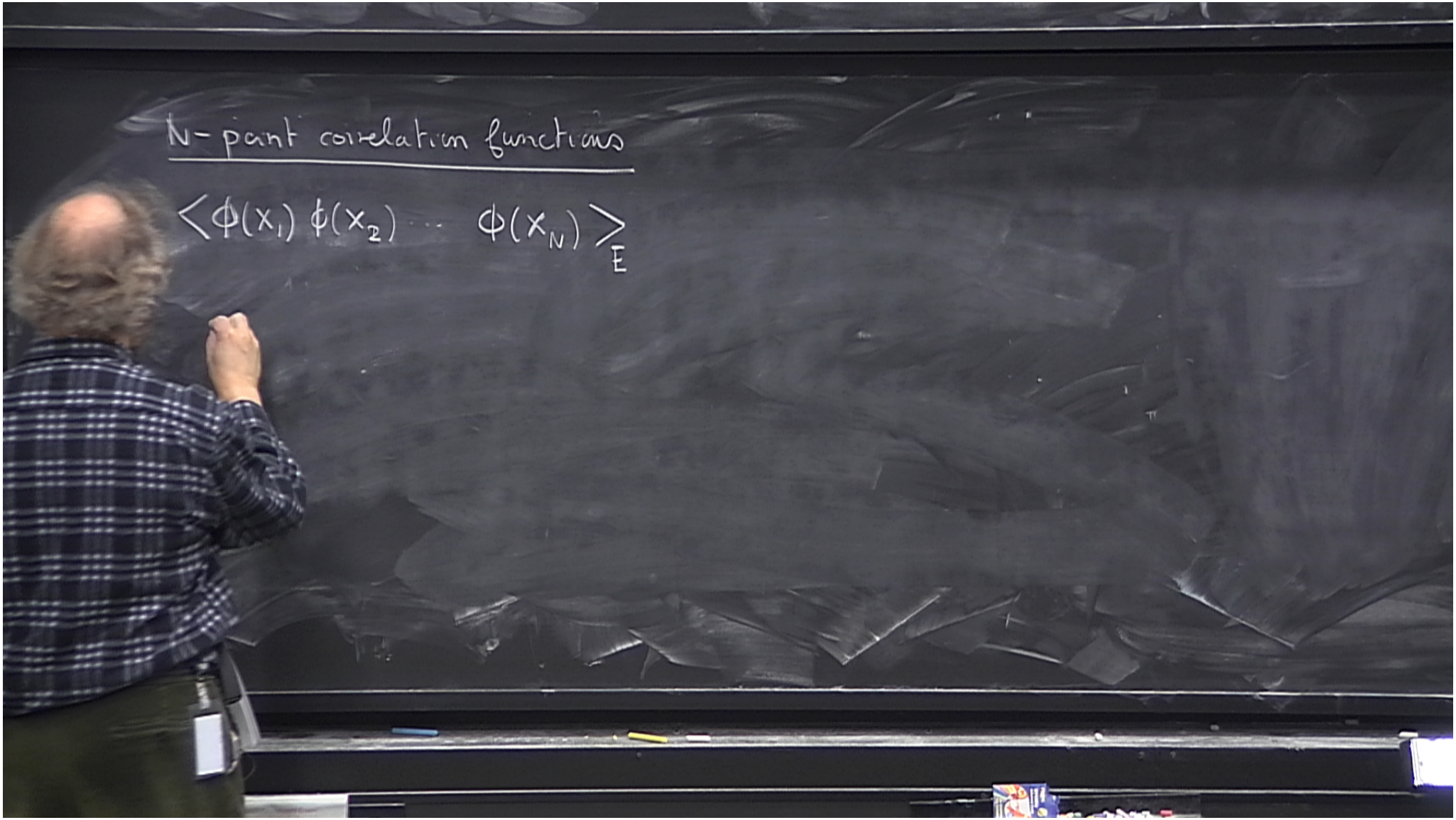
Massive Propagators in d=2

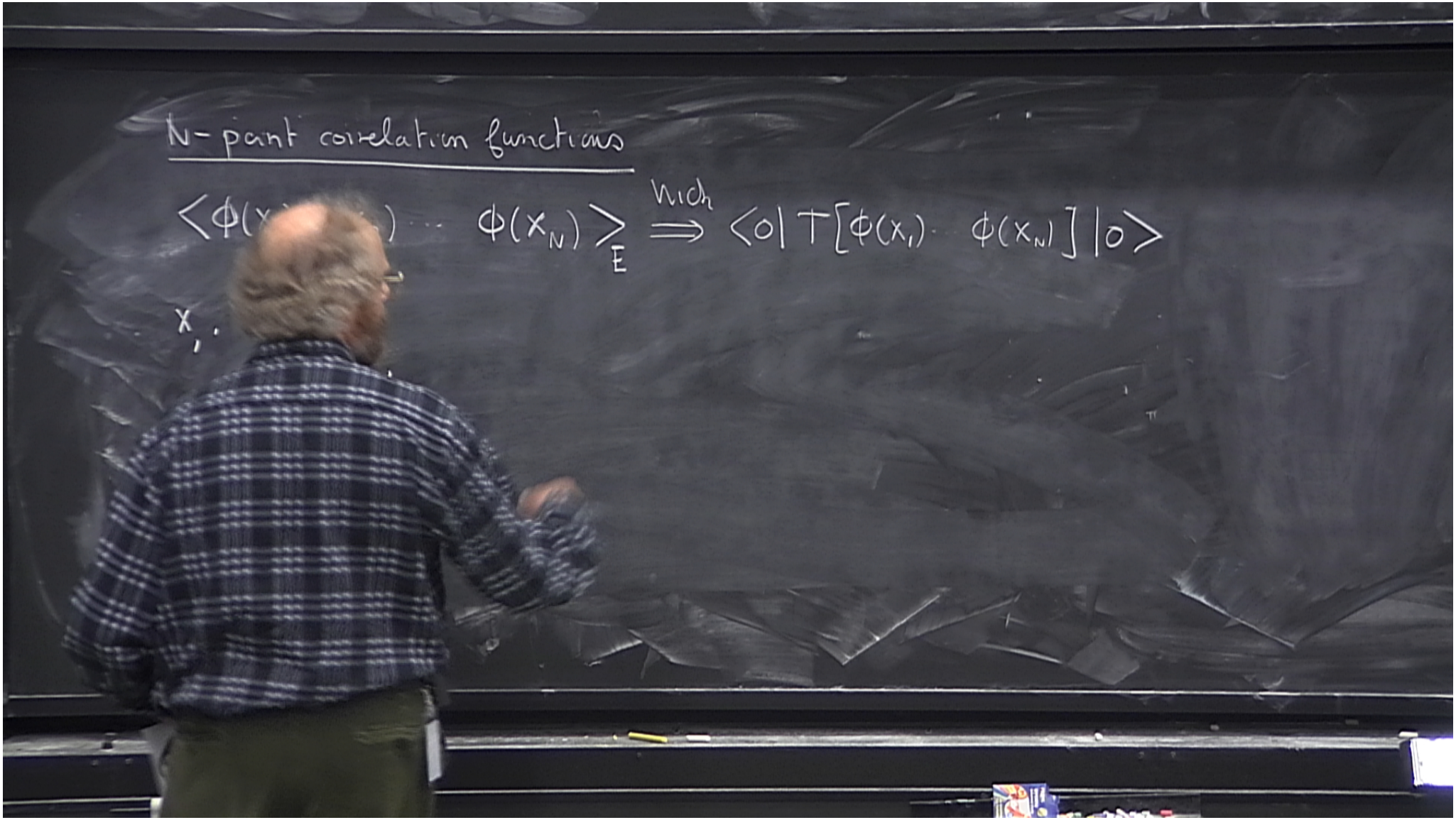


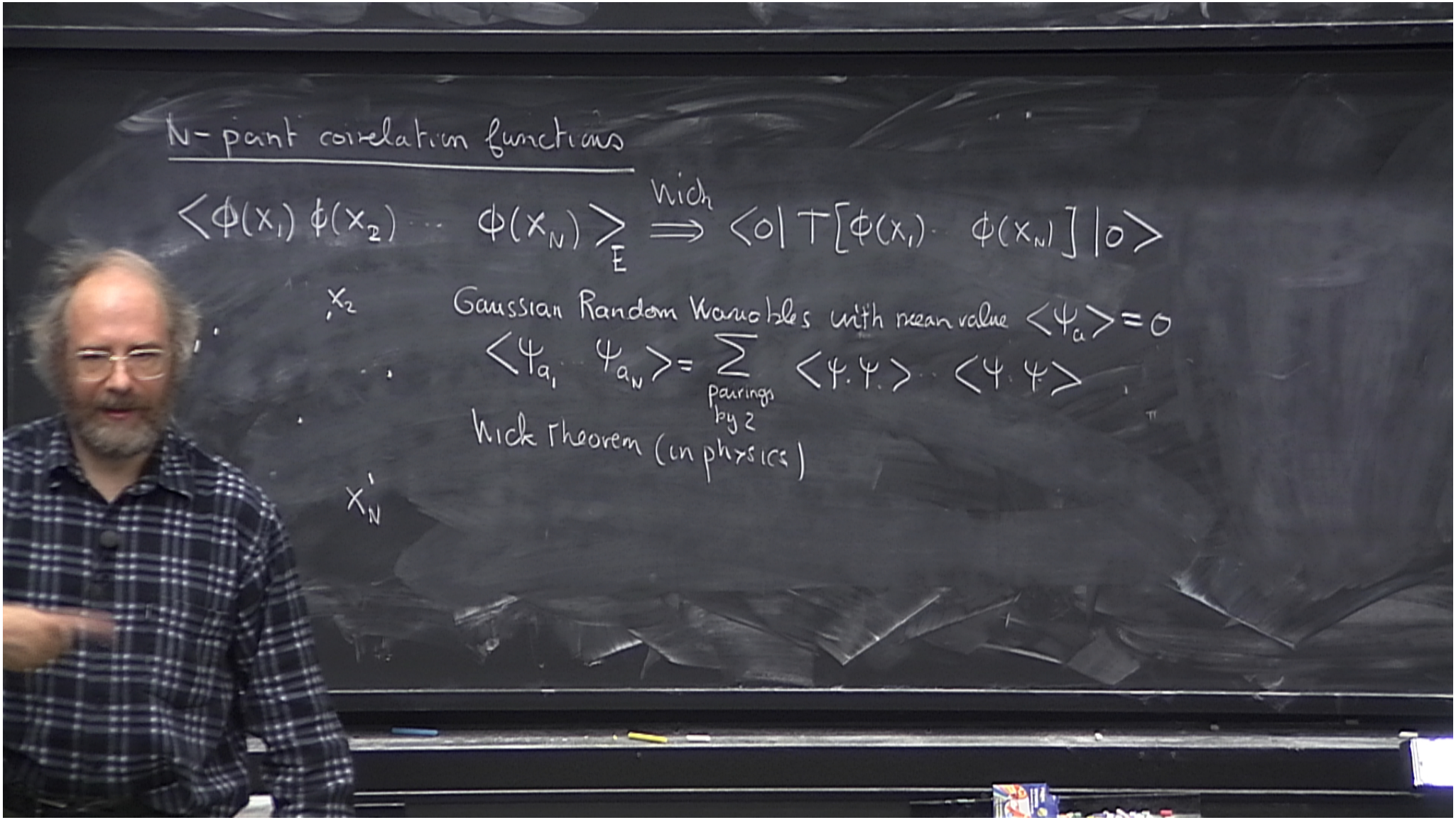


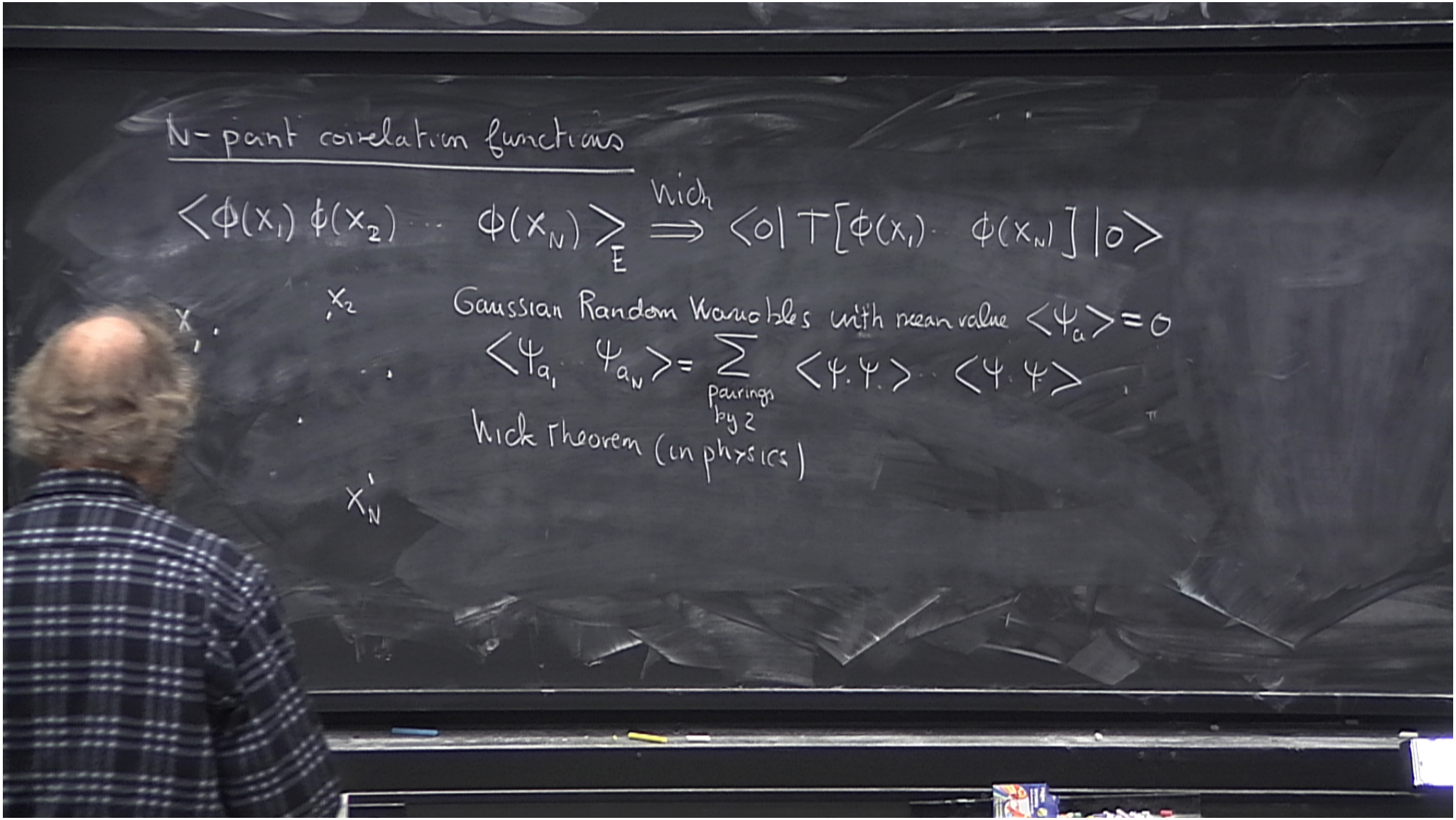



N-point correlation functions


N-point correlation functions
$\langle \phi(x_1)\, \phi(x_2) \cdots \phi(x_N) \rangle_E$


N-point correlation functions
$\langle \phi(x_1) \cdots \phi(x_N) \rangle_E \xrightarrow{\text{Wick}} \langle 0 | T[\phi(x_1) \cdots \phi(x_N)] | 0 \rangle$
x_1,


N-point correlation functions
$\langle \phi(x_1)\,\phi(x_2) \cdots \phi(x_N) \rangle_E \overset{\text{Wick}}{\Longrightarrow} \langle 0|\, T[\phi(x_1) \cdots \phi(x_N)]\,|0\rangle$
x_2
x_N'
Gaussian Random Variables with mean value $\langle \psi_a \rangle = 0$
$\langle \psi_{a_1} \cdots \psi_{a_N} \rangle = \sum_{\substack{\text{pairings} \\ \text{by 2}}} \langle \psi \cdot \psi \rangle \cdots \langle \psi \cdot \psi \rangle$
Wick Theorem (in physics)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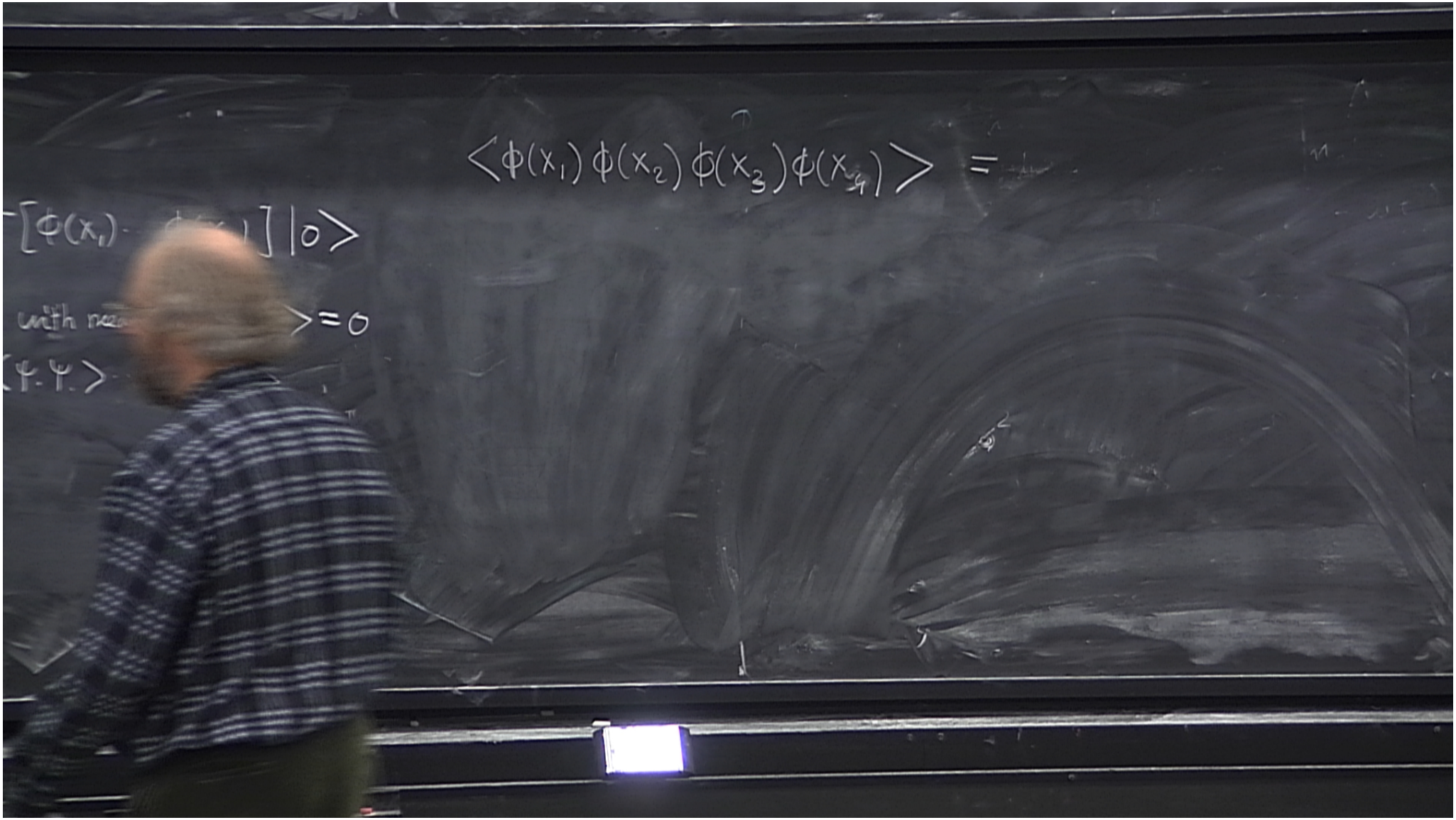
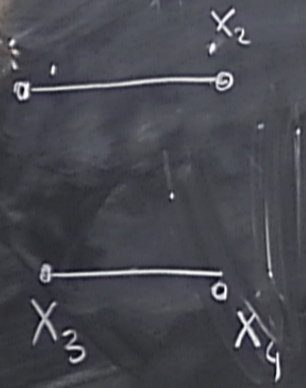

$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle =$
$[\phi(x_1) \cdots] |0\rangle$
with
$\rangle = 0$
$\langle \psi \cdot \psi \rangle$

N-point correlation functions

$$\langle \phi(x_1)\,\phi(x_2)\cdots\phi(x_N)\rangle_E \overset{\text{Wick}}{\Longrightarrow} \langle 0|\,T[\phi(x_1)\cdots\phi(x_N)]\,|0\rangle$$



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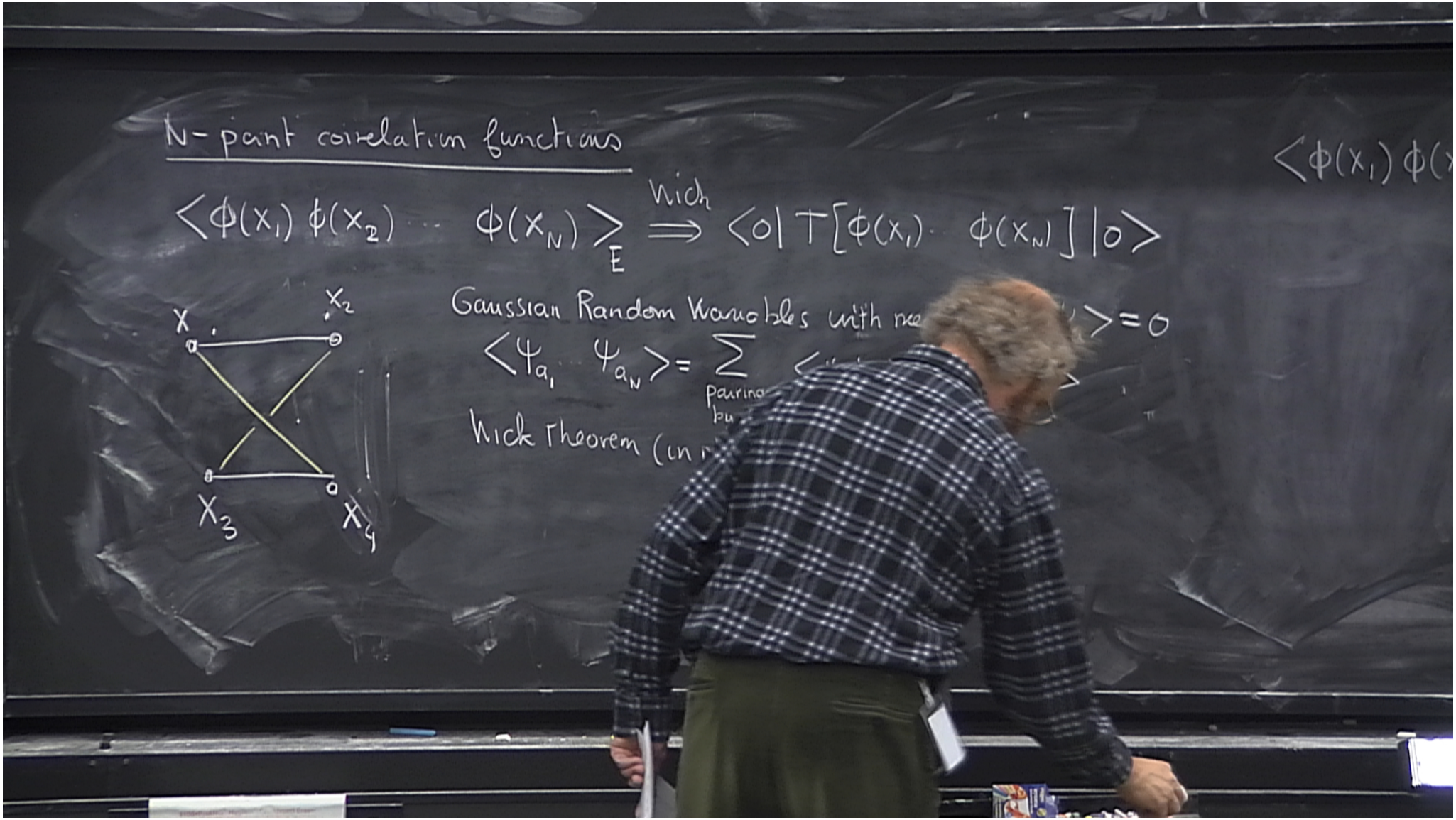
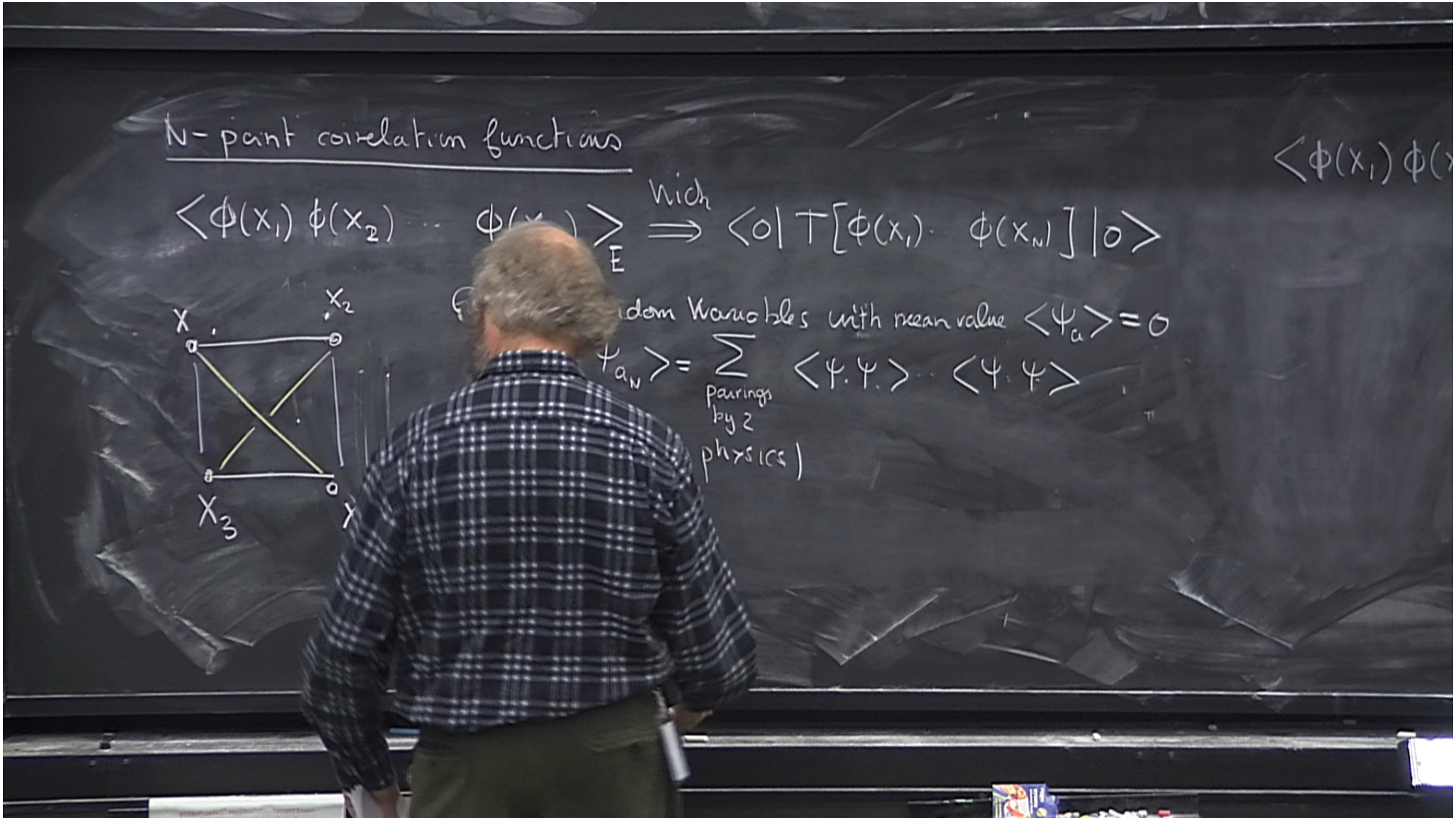
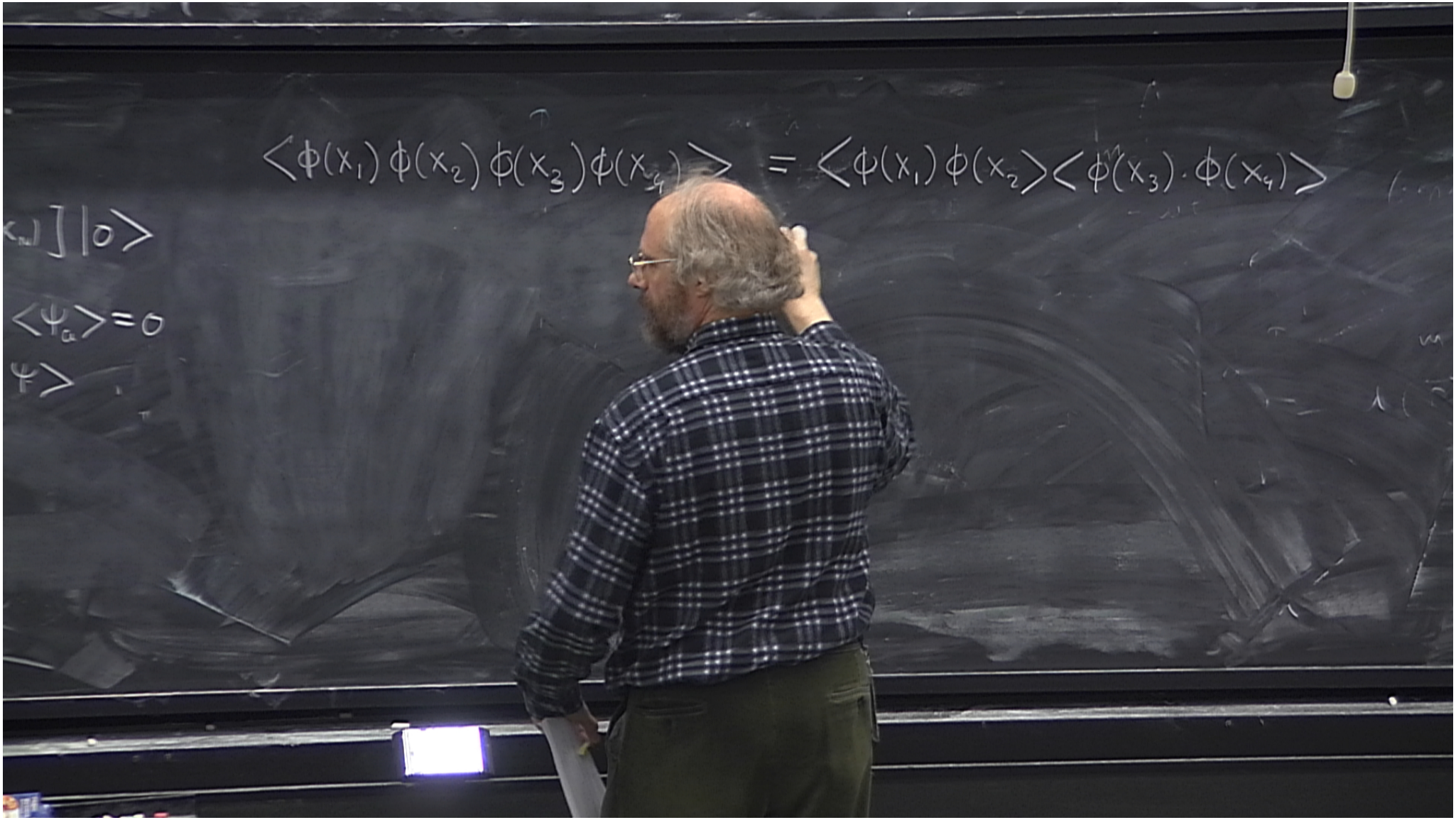
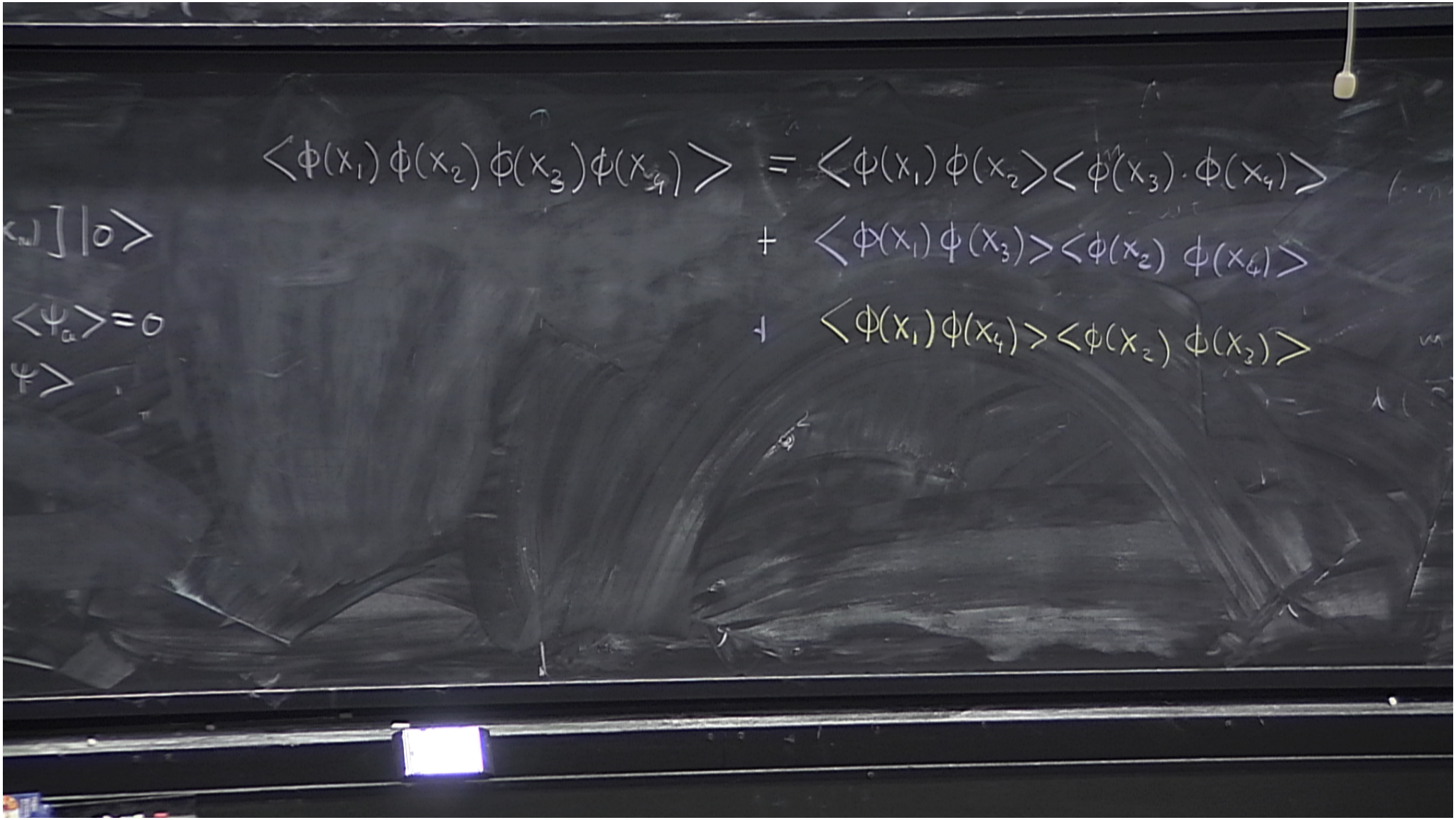
Wick Theorem (in physics)

$\langle \phi(x_1)\,\phi($


N-point correlation functions
$\langle \phi(x_1)\, \phi(x_2) \cdots \phi(x_N) \rangle_E \overset{\text{Wick}}{\Longrightarrow} \langle 0 | T[\phi(x_1) \cdots \phi(x_N)] | 0 \rangle$
$\langle \phi(x_1) \phi($
x_1
x_2
x_3
x_4
Gaussian Random Variables with zero
$\langle \psi_{a_1} \cdots \psi_{a_N} \rangle = \sum_{\text{pairing}}$
Wick Theorem (in


N-point correlation functions
$\langle \phi(x_1)\,\phi(x_2) \cdots \phi(x_N) \rangle_E \Rightarrow$ Wick $\Rightarrow \langle 0|\,T[\phi(x_1) \cdots \phi(x_N)]\,|0\rangle$
$\langle \phi(x_1)\,\phi($
dom variables with mean value $\langle \psi_a \rangle = 0$
$\psi_{a_N} \rangle = \sum_{\text{pairings by 2}} \langle \psi \cdot \psi \rangle \cdots \langle \psi \cdot \psi \rangle$
physics)
x_1
x_2
x_3


$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle = \langle \phi(x_1)\phi(x_2) \rangle \langle \phi(x_3)\cdot\phi(x_4) \rangle$
$\langle \Psi_a \rangle = 0$


$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle\phi(x_1)\phi(x_2)\rangle\langle\phi(x_3)\cdot\phi(x_4)\rangle$
$+ \langle\phi(x_1)\phi(x_3)\rangle\langle\phi(x_2)\phi(x_4)\rangle$
$+ \langle\phi(x_1)\phi(x_4)\rangle\langle\phi(x_2)\phi(x_3)\rangle$
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$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle \phi(x_1)\phi(x_2)\rangle\langle \phi(x_3)\cdot\phi(x_4)\rangle$$
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$$+ \langle \phi(x_1)\phi(x_4)\rangle\langle \phi(x_2)\,\phi(x_3)\rangle$$

Feynman Diagrams

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle \phi(x_1)\phi(x_2)\rangle\langle \phi(x_3)\cdot\phi(x_4)\rangle$$
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Feynman Diagrams

$\langle \phi(x)\phi(y)\rangle =$ $= \hbar\, G_0(x_1 - x_2)$

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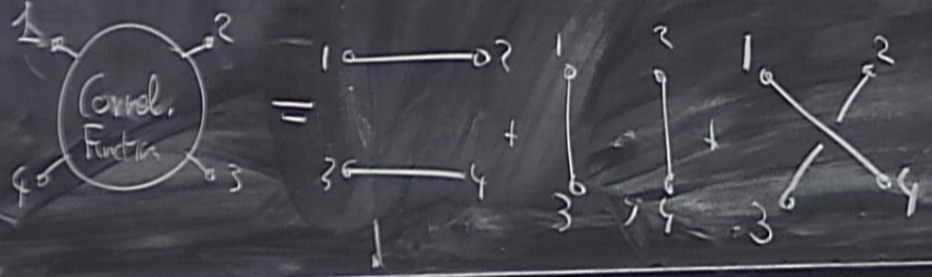
Feynman Diagrams

$$\langle \phi(x)\,\phi(y)\rangle = \quad x_1 \quad x_2 \quad = \hbar\, G_0(x_1 - x_2)$$



$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle \phi(x_1)\phi(x_2)\rangle\langle \phi(x_3)\cdot\phi(x_4)\rangle$$
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Feynman Diagrams

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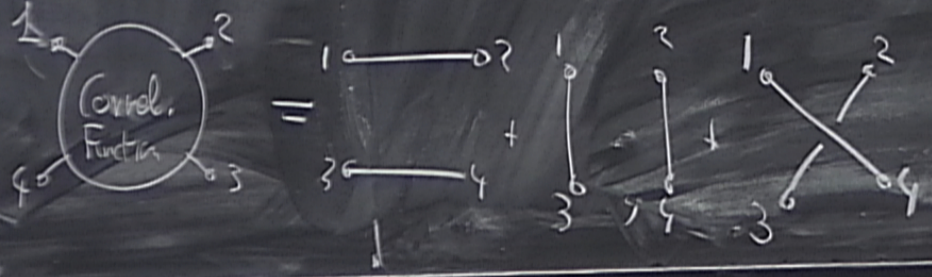
Generalize to any $N = 2M$ even



$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle\phi(x_1)\phi(x_2)\rangle\langle\phi(x_3)\cdot\phi(x_4)\rangle$$
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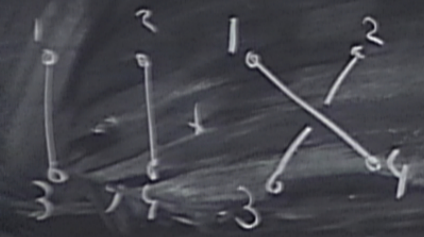
$$+ \langle\phi(x_1)\phi(x_4)\rangle\langle\phi(x_2)\phi(x_3)\rangle$$

x_1 ——— x_2 $= \hbar\, G_0(x_1 - x_2)$

Generalize to any $N = 2M$ even

$N = 2M+1 \Rightarrow 0$ $\quad \phi(x) \to -\phi(x)$



$\phi(x_1)\rangle = \langle\phi(x_1)\phi(x_2)\rangle\langle\phi(x_3)\cdot\phi(x_4)\rangle$

$+ \langle\phi(x_1)\phi(x_3)\rangle\langle\phi(x_2)\phi(x_4)\rangle$

$+ \langle\phi(x_1)\phi(x_4)\rangle\langle\phi(x_2)\phi(x_3)\rangle$

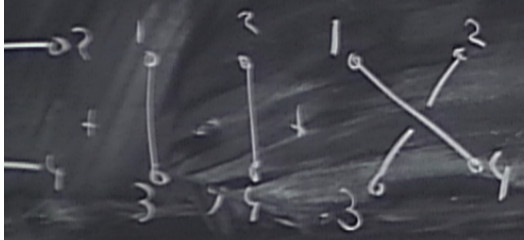
$= \hbar G_0(x_1 - x_2)$

Generalize to any $N = 2M$ even

$N = 2M+1 \Rightarrow 0$

Symmetry of $S[\phi]$

$\phi(x) \to -\phi(x)$



$|x| \to 0$ short distance behaviour : singularity or

Ultra-Violet divergences

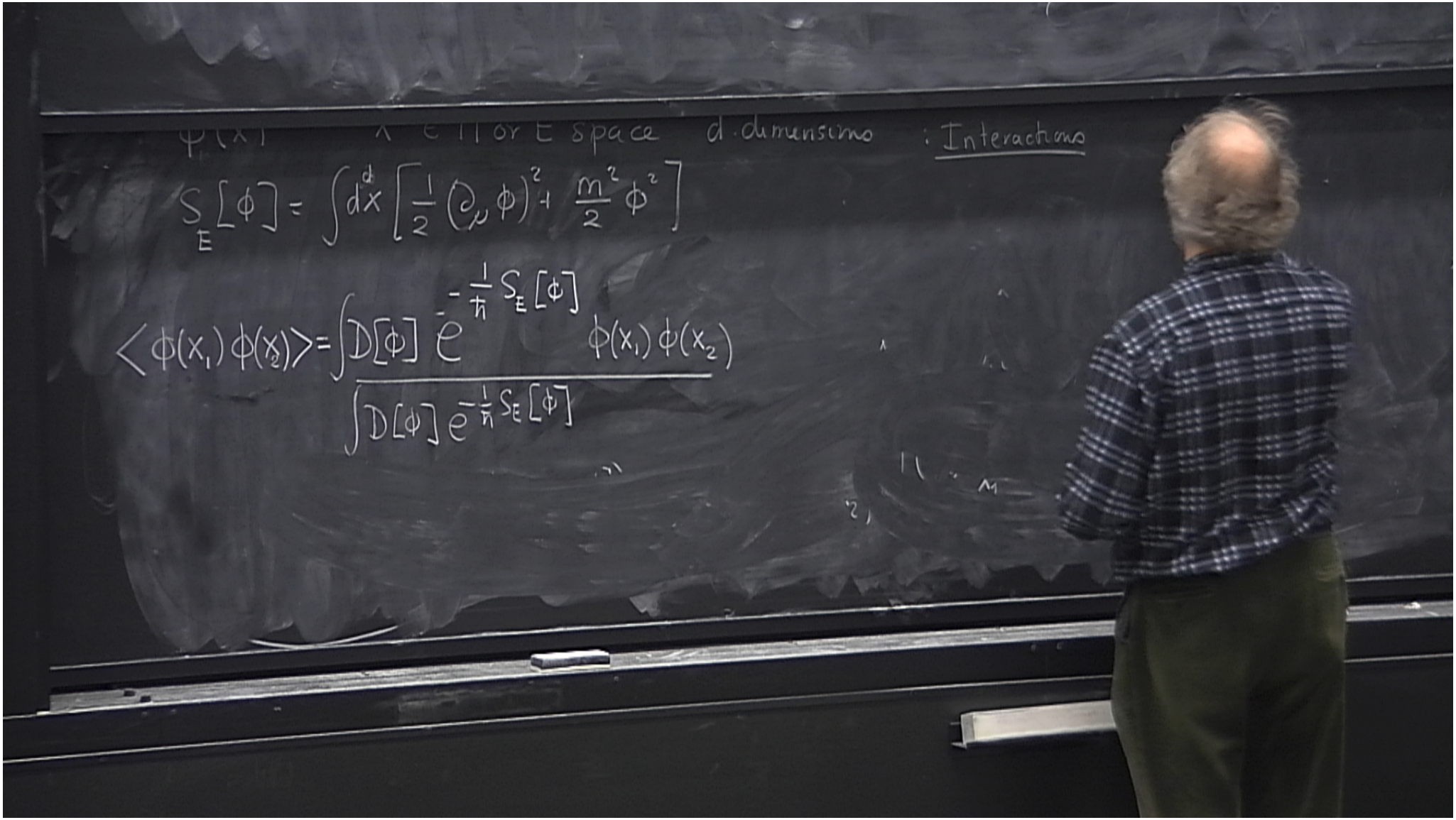
$$G_0(x) \simeq C_d\, |x|^{2-d} \quad \text{if } d > 2$$

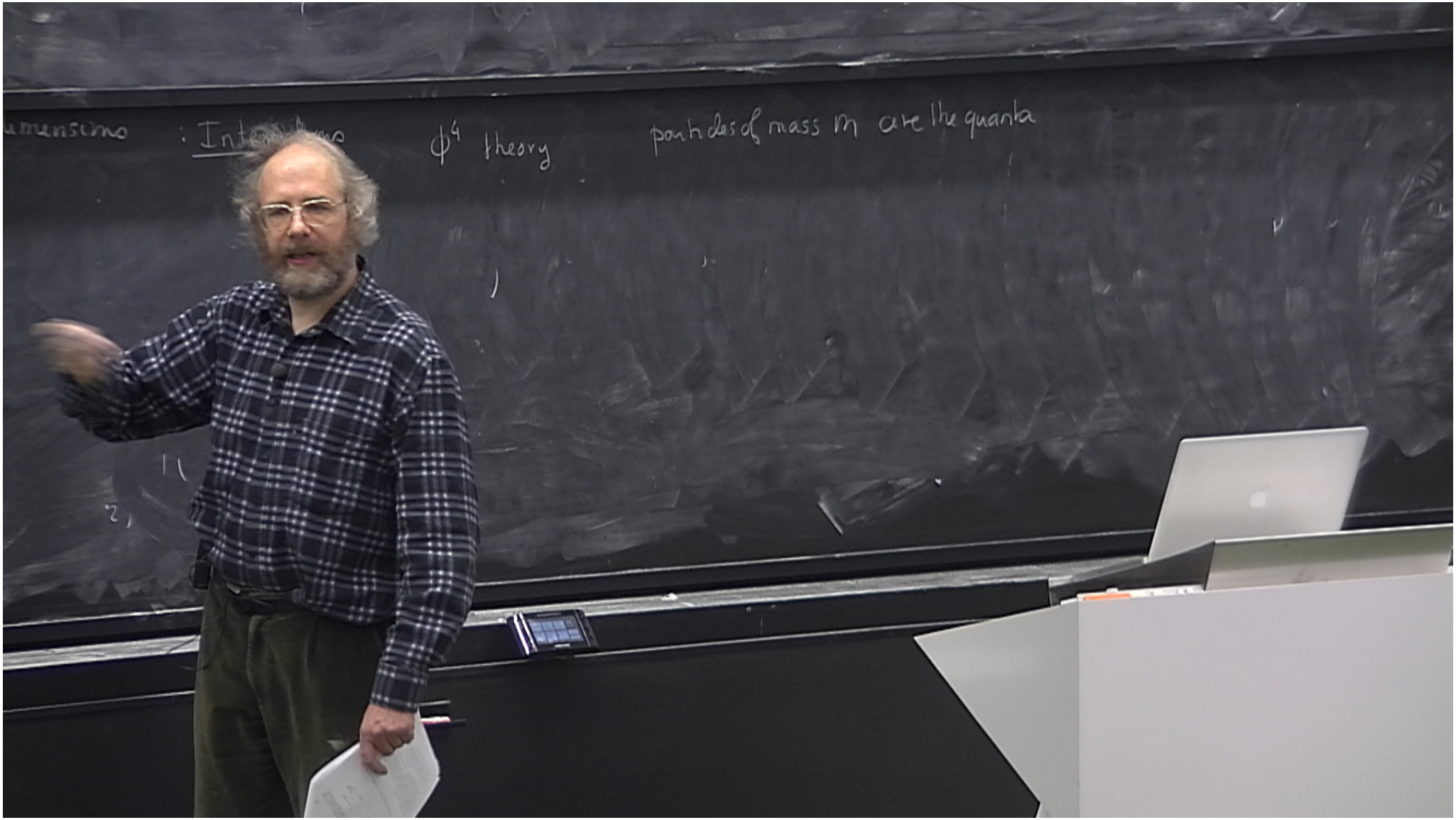
$$-\frac{1}{2\pi} \log |x| \quad \text{if } d = 2$$

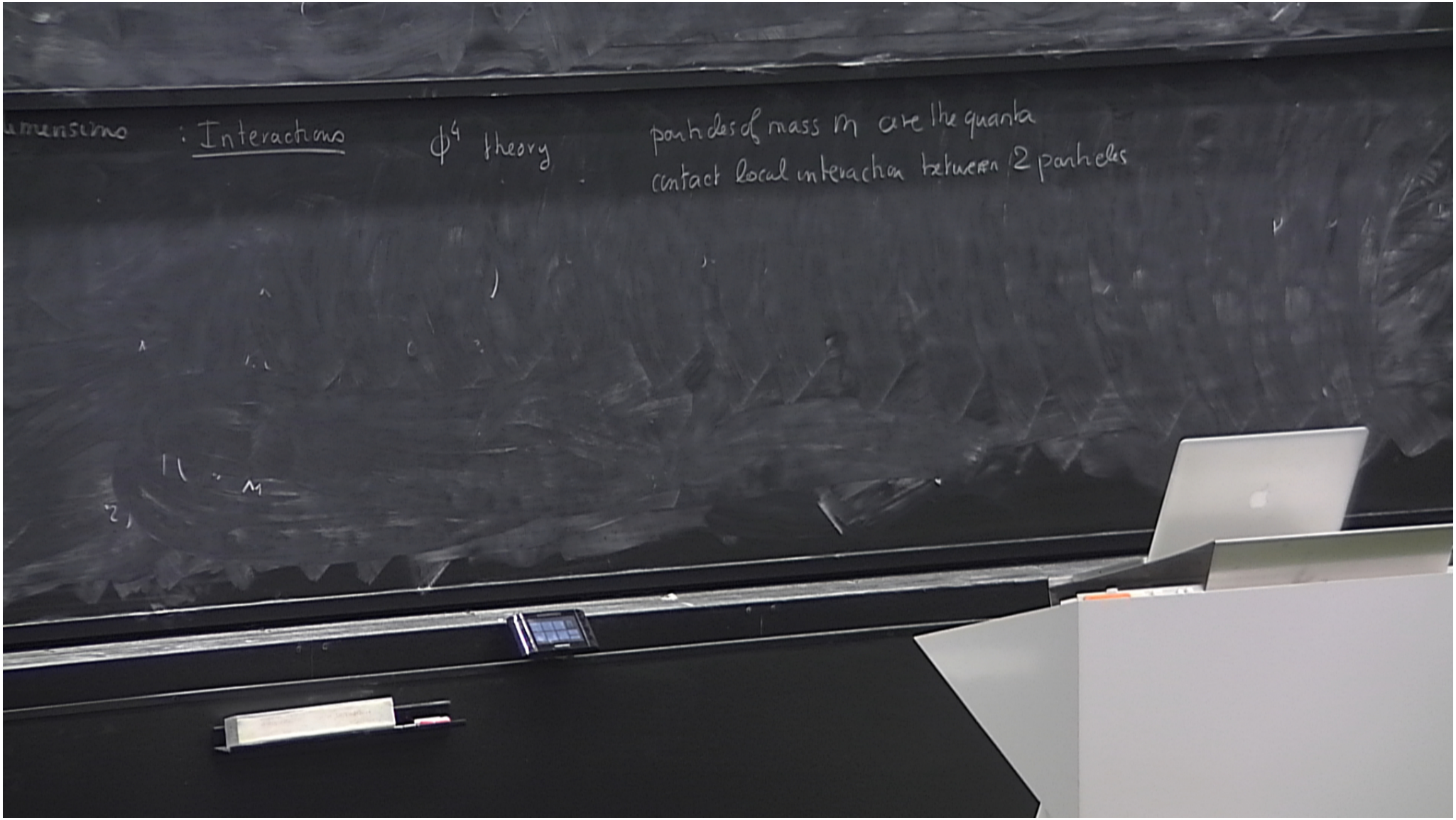
$$G_0(0) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 + m^2}$$

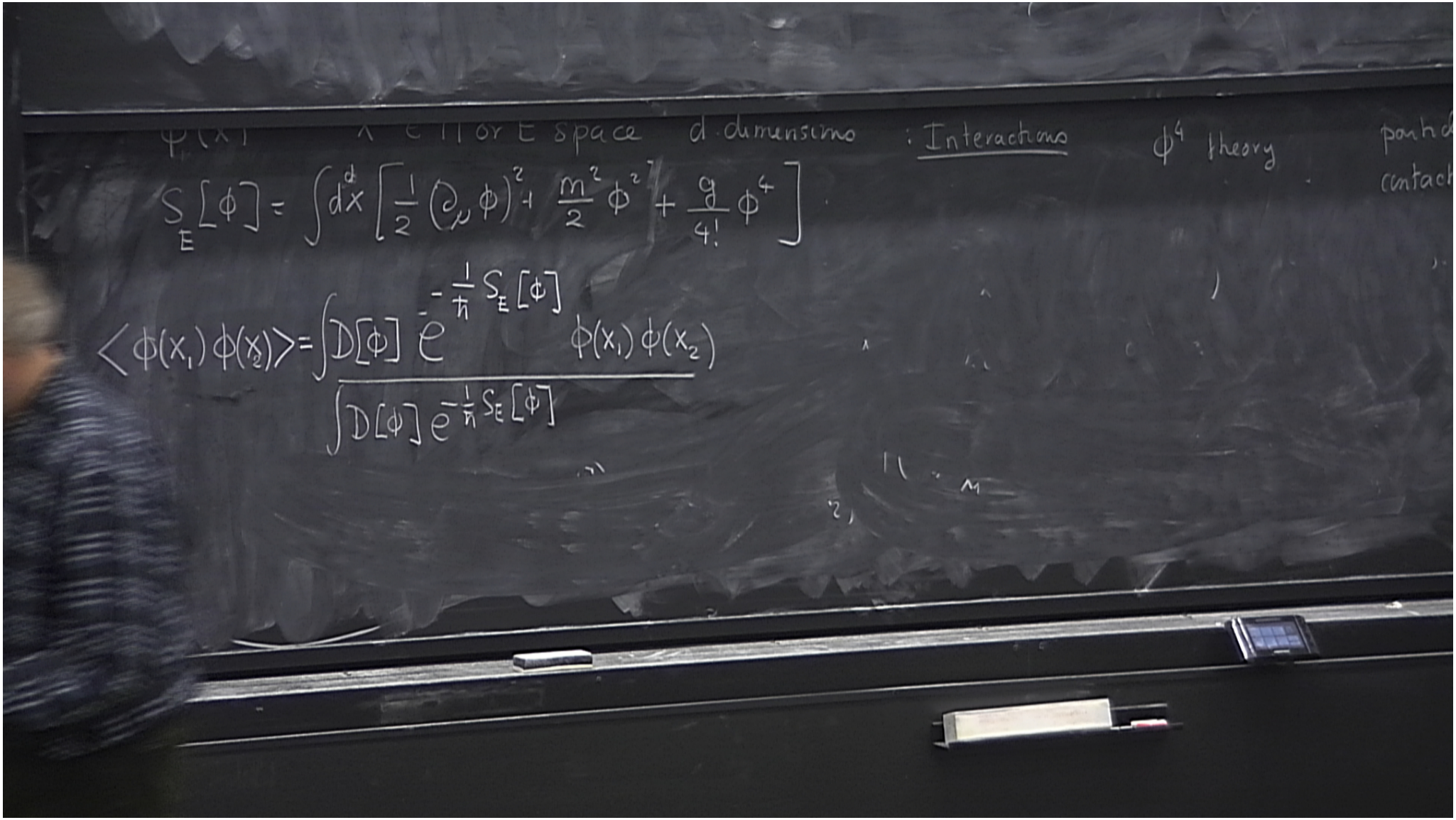
divergence for large $|k|$ or Energies

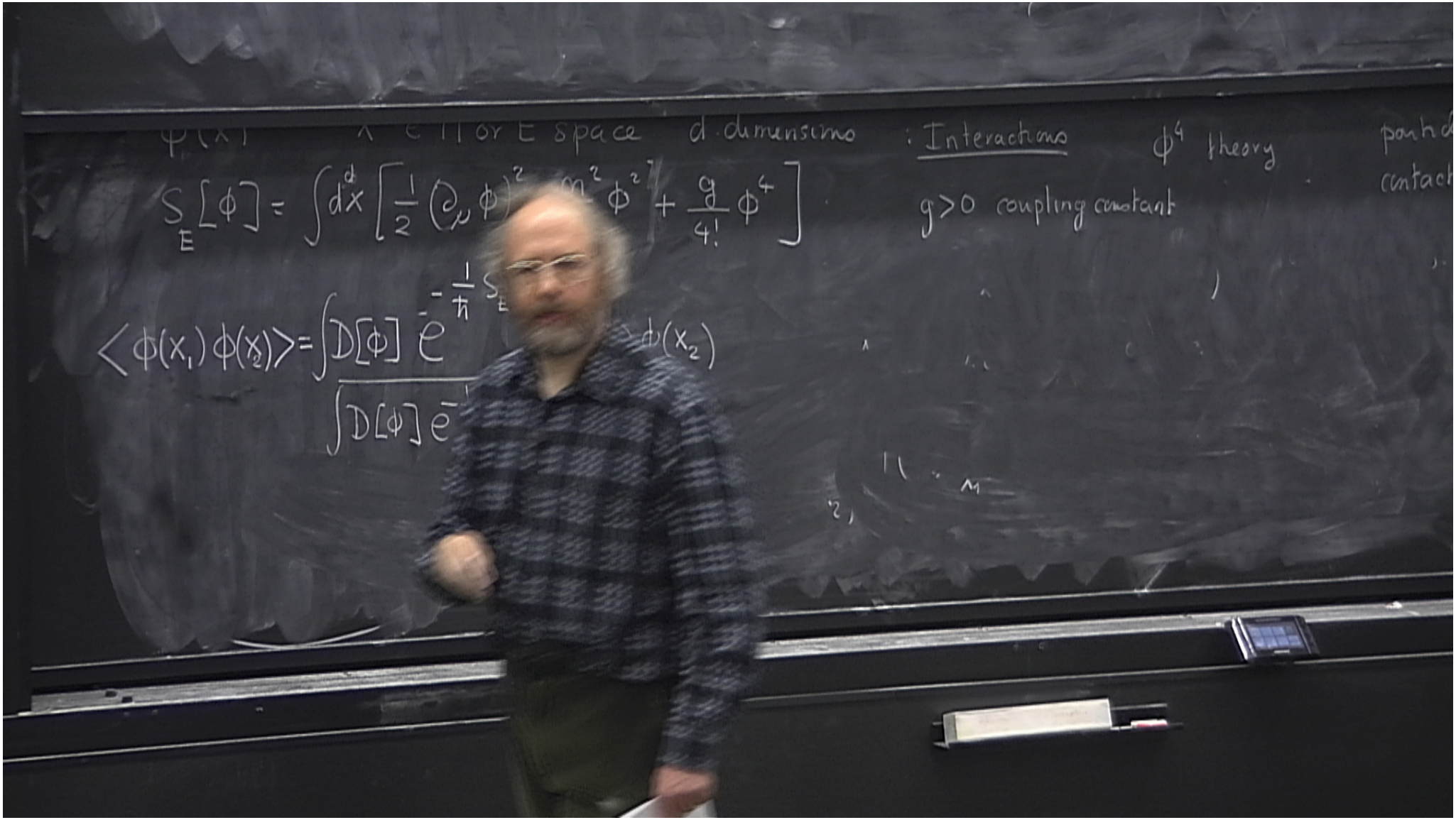
$$\langle \phi^2(x) \rangle = \hbar\, G_0(x,x) = \infty \quad d \geq 2$$

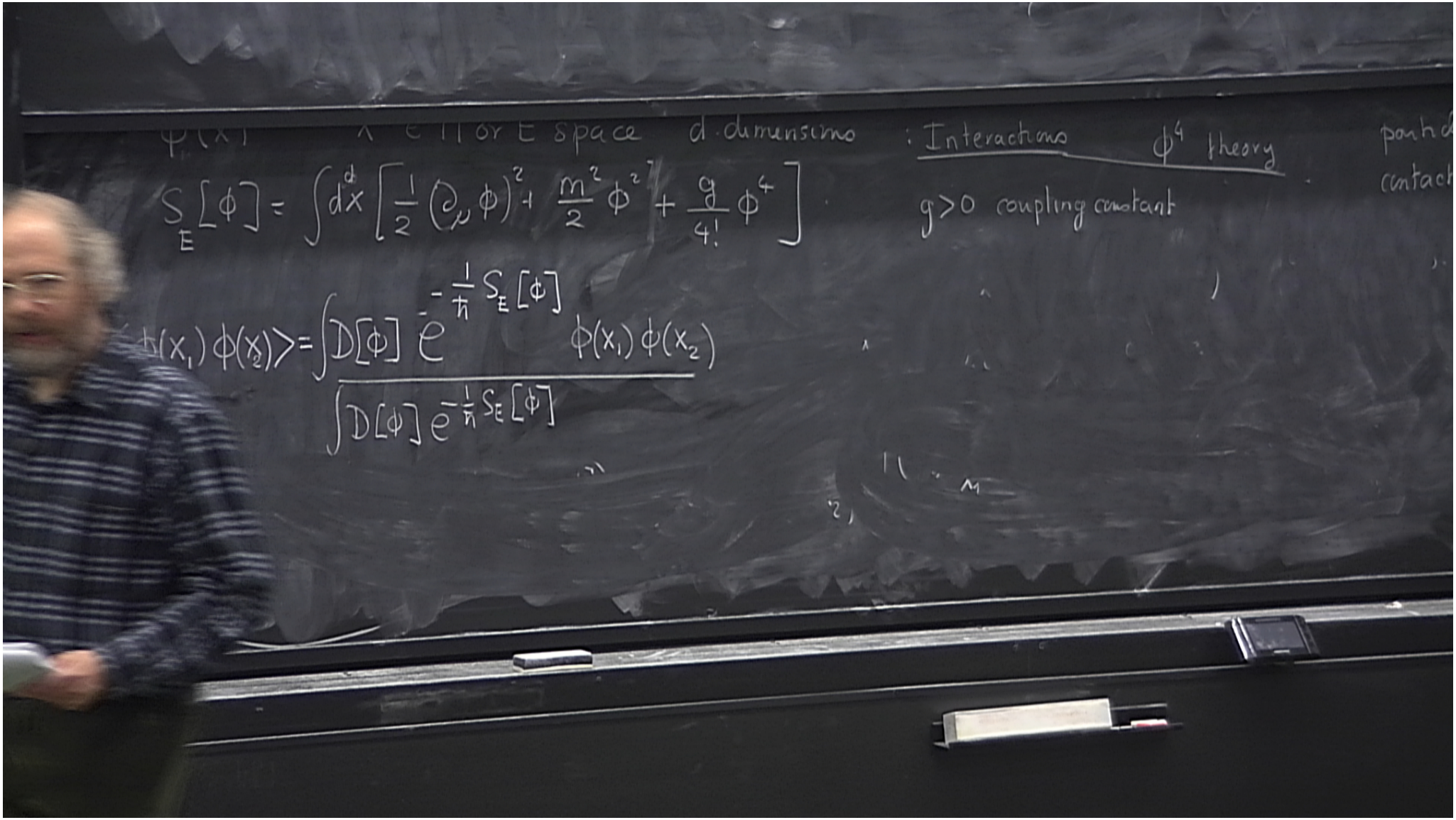

Euclidean space d dimensions : Interactions
$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 \right]$
$\langle \phi(x_1) \phi(x_2) \rangle = \frac{\int D[\phi] e^{-\frac{1}{\hbar} S_E[\phi]} \phi(x_1) \phi(x_2)}{\int D[\phi] e^{-\frac{1}{\hbar} S_E[\phi]}}$


: Int
ϕ^4 theory
particles of mass m are the quanta


mensions : Interactions ϕ^4 theory
particles of mass m are the quanta
contact local interaction between 2 particles


$\varphi(x)$ $x \in$ Euclidean space d dimensions : Interactions ϕ^4 theory
$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$
$\langle \phi(x_1) \phi(x_2) \rangle = \frac{\int D[\phi] e^{-\frac{1}{\hbar} S_E[\phi]} \phi(x_1) \phi(x_2)}{\int D[\phi] e^{-\frac{1}{\hbar} S_E[\phi]}}$


Interactions
ϕ^4 theory
$g>0$ coupling constant
$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 \; m^2 \phi^2 + \frac{g}{4!} \phi^4 \right]$
$\langle \phi(x_1) \phi(x_2) \rangle = \int D[\phi] \, e^{-\frac{1}{\hbar} S_E} \, \phi(x_1) \phi(x_2)$
$\int D[\phi] \, e^{-}$


$\varphi(x)$ $x \in$ Euclidean space d dimensions : Interactions ϕ^4 theory
$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$
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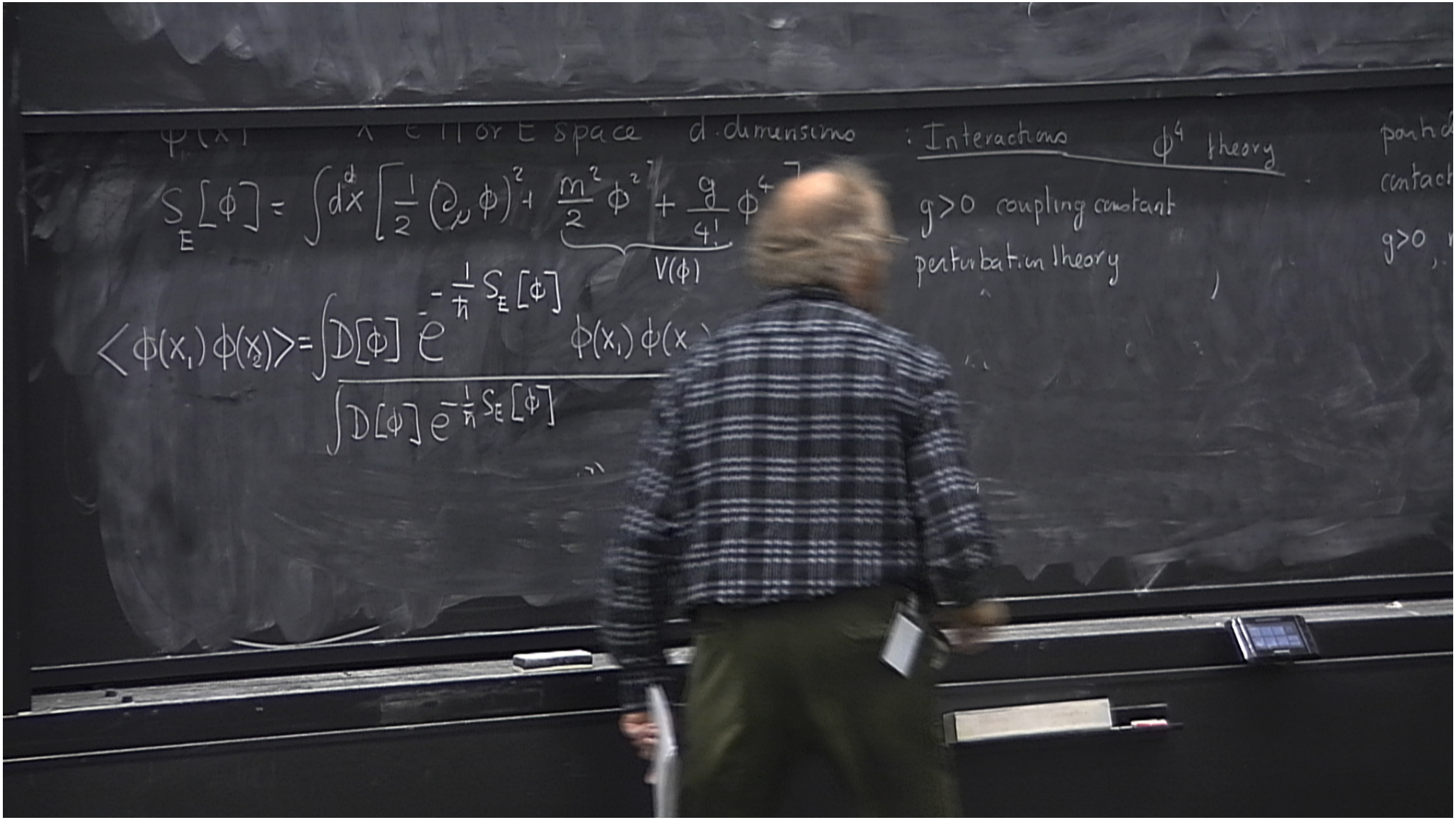
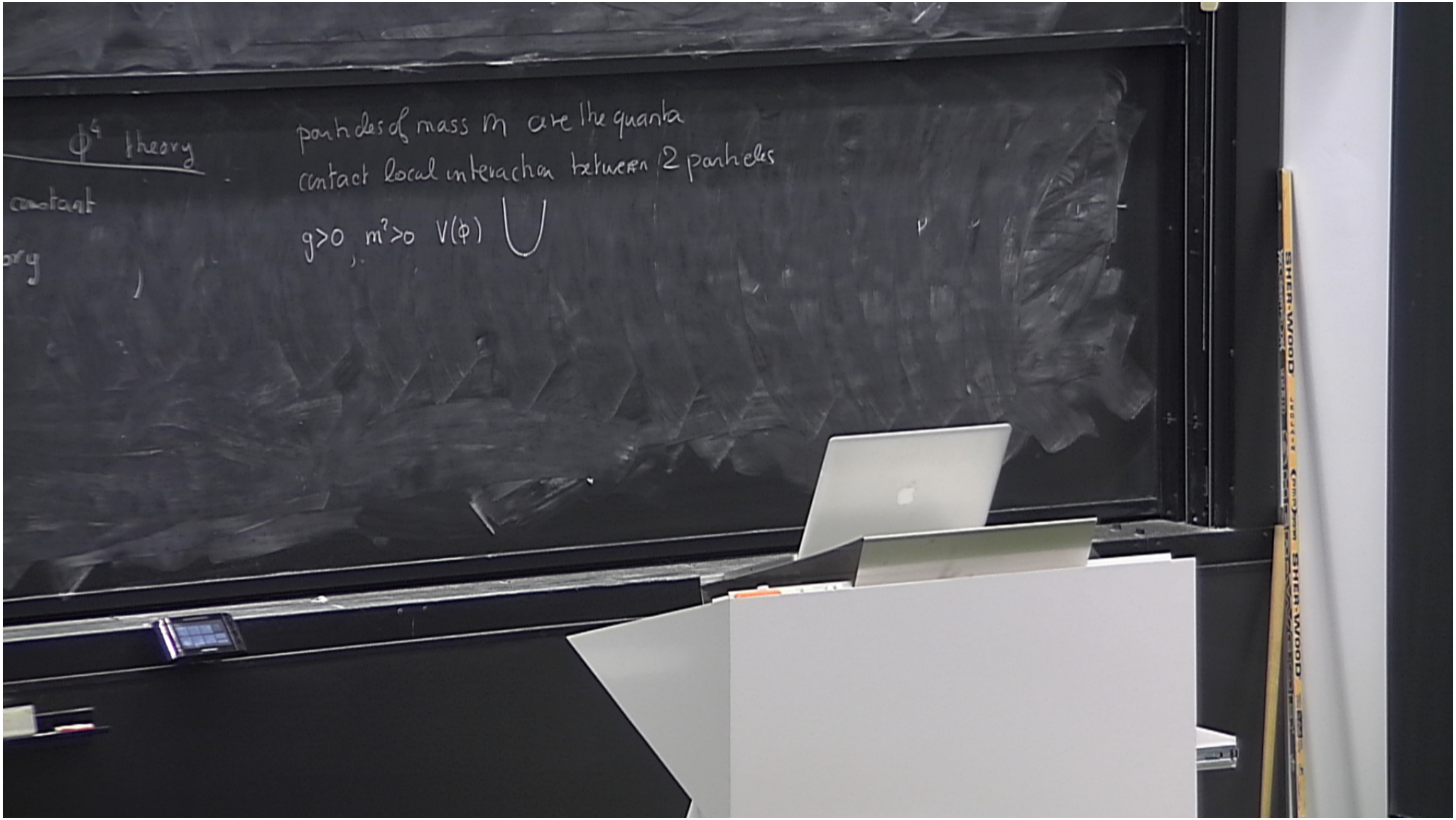
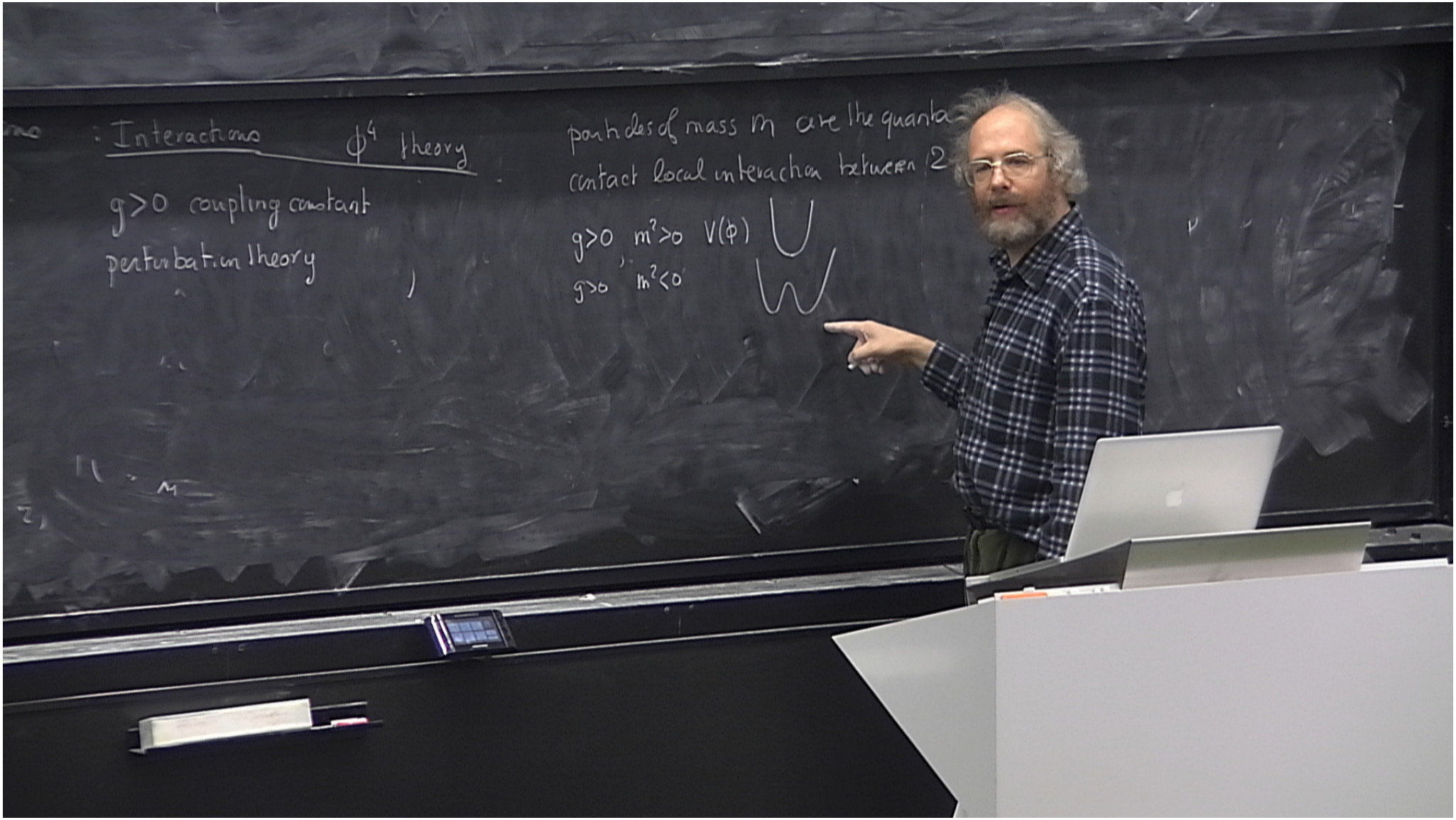
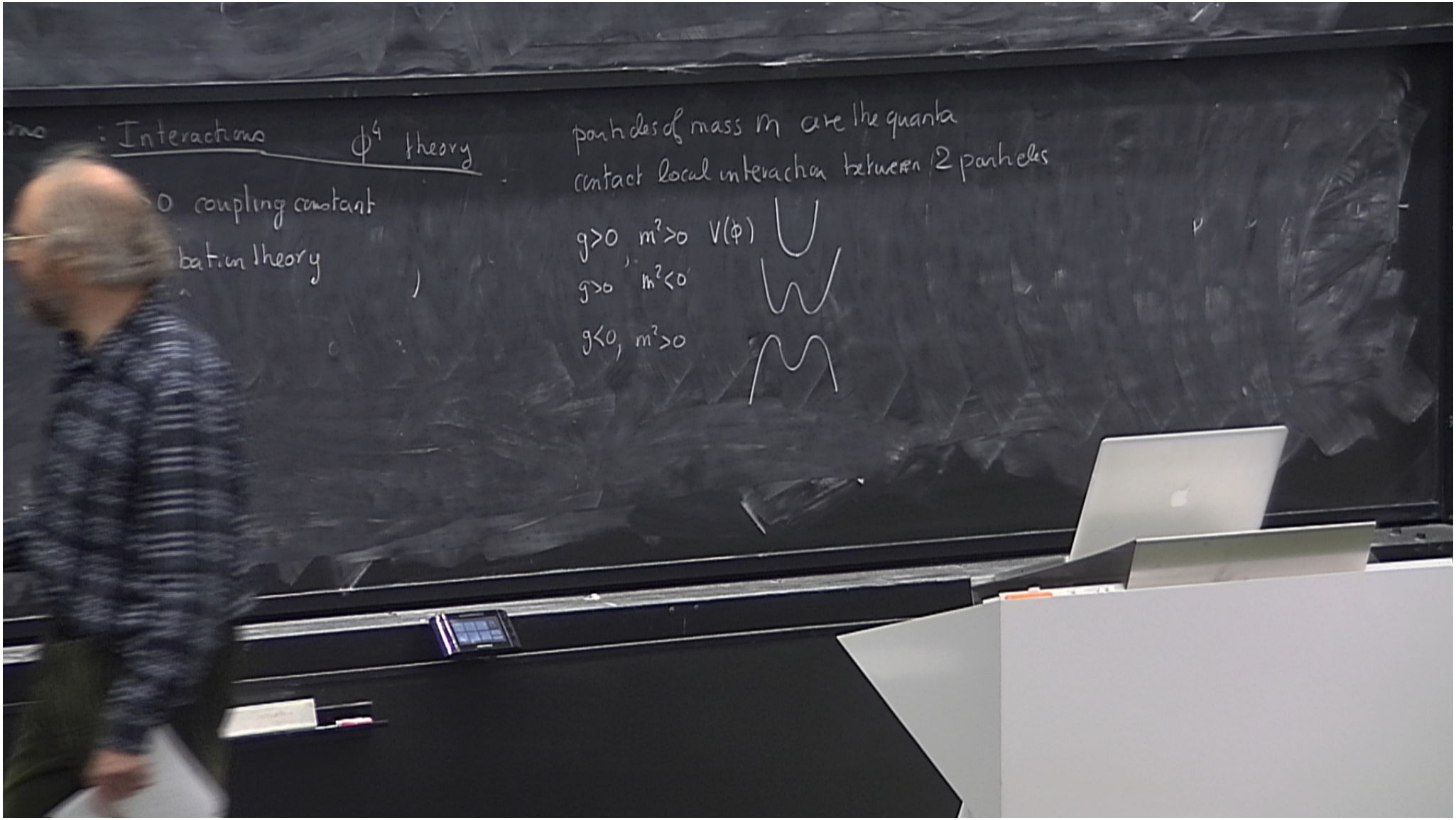
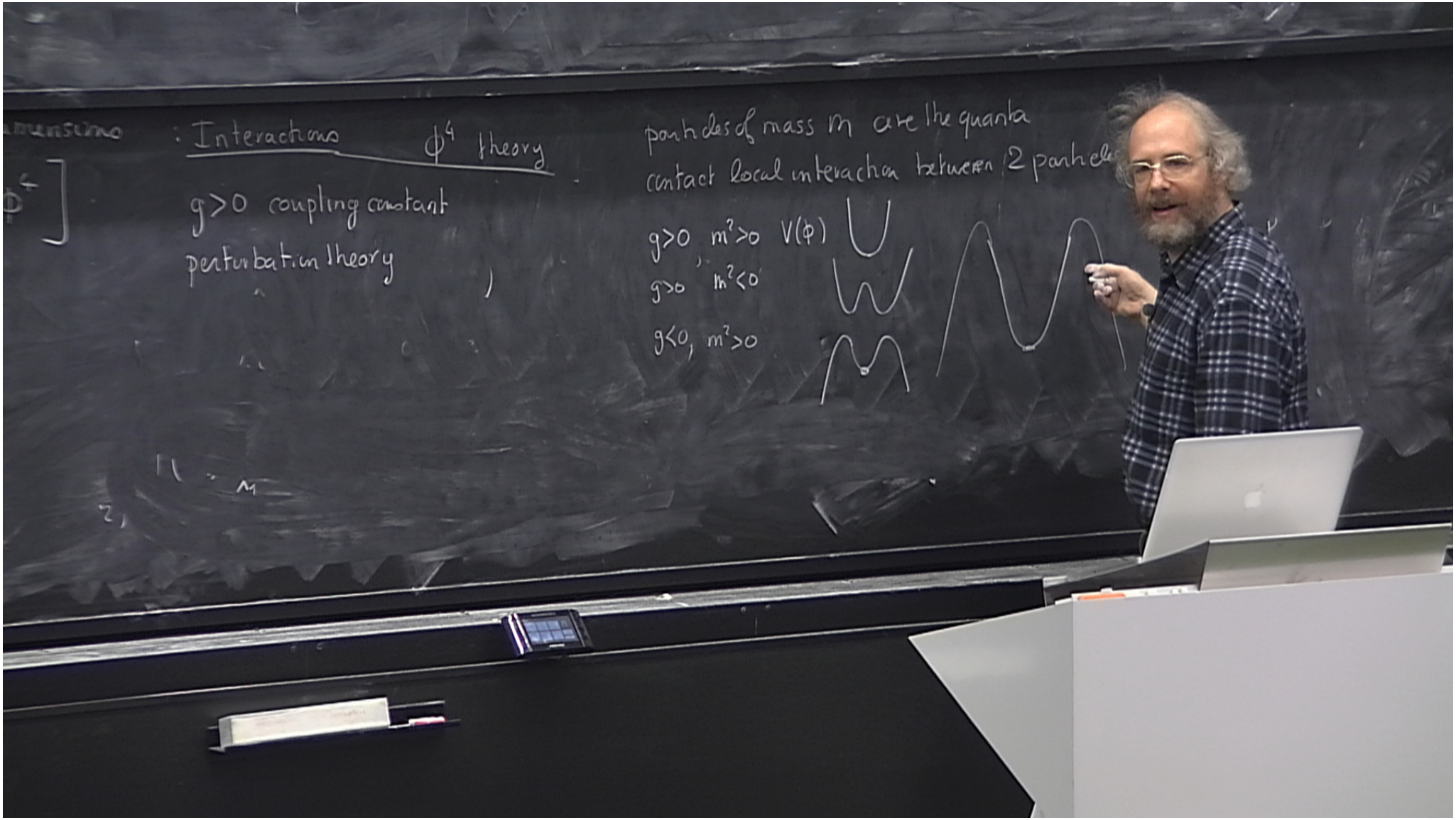

$\varphi(x)$ $x \in$ Euclidean space d-dimensions
: Interactions ϕ^4 theory
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$
$V(\phi)$
$g > 0$ coupling constant
perturbation theory
$\langle \phi(x_1)\phi(x_2) \rangle = \frac{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}\, \phi(x_1)\phi(x}{\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]}}$


ϕ^4 theory
particles of mass m are the quanta
contact local interaction between 2 particles
$g>0$, $m^2>0$ $V(\phi)$
SHERWOOD


: Interactions ϕ^4 theory
$g>0$ coupling constant
perturbation theory
particles of mass m are the quanta
contact local interaction between 2
$g>0$, $m^2>0$ $V(\phi)$
$g>0$ $m^2<0$


: Interactions ϕ^4 theory
particles of mass m are the quanta
contact local interaction between 2 particles
O coupling constant
bat. in theory
$g>0$, $m^2>0$ $V(\phi)$
$g>0$ $m^2<0$
$g<0$, $m^2>0$


: Interactions ϕ^4 theory
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$g>0$, $m^2>0$ $V(\phi)$
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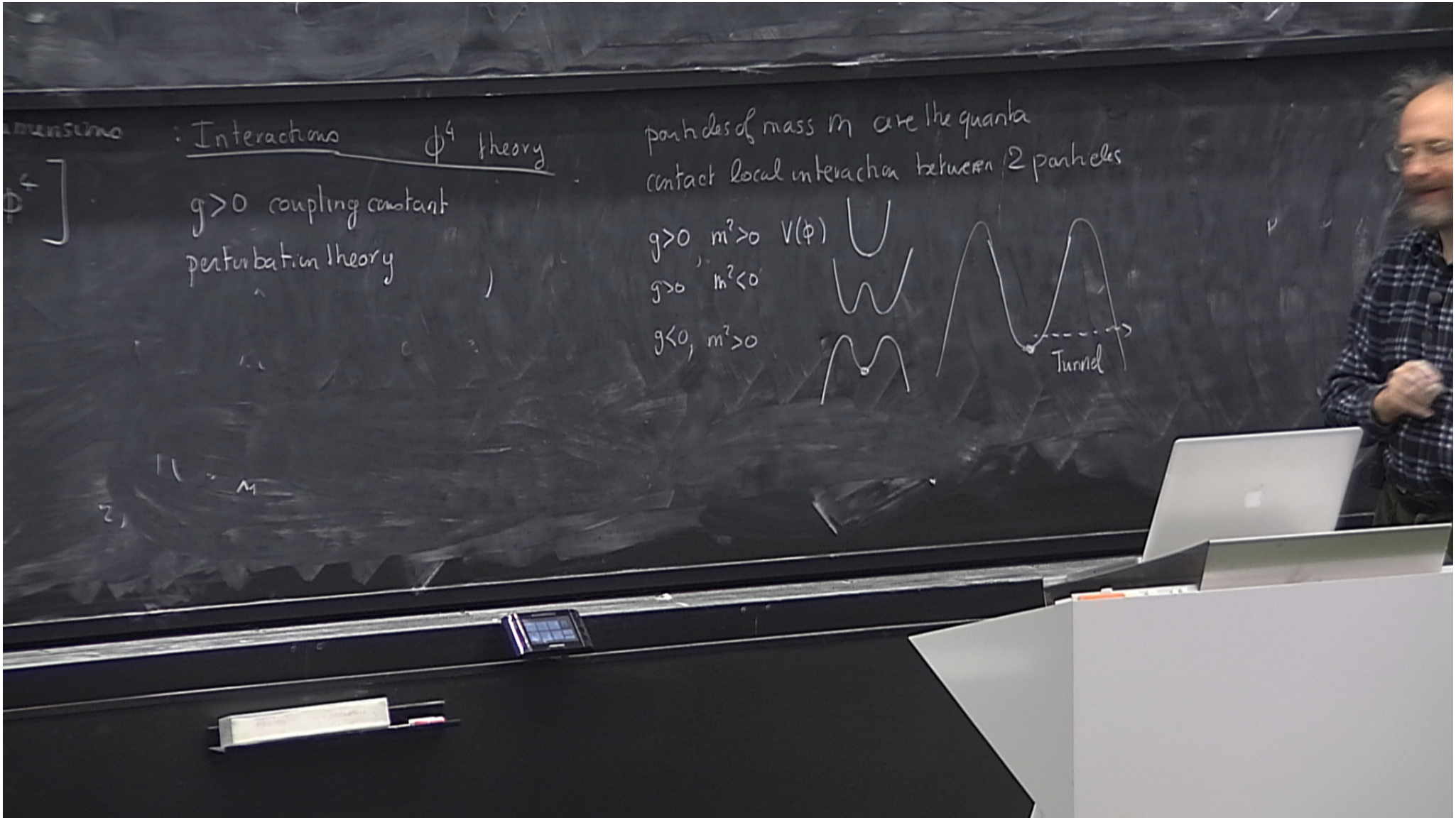
: Interactions ϕ^4 theory
$g>0$ coupling constant
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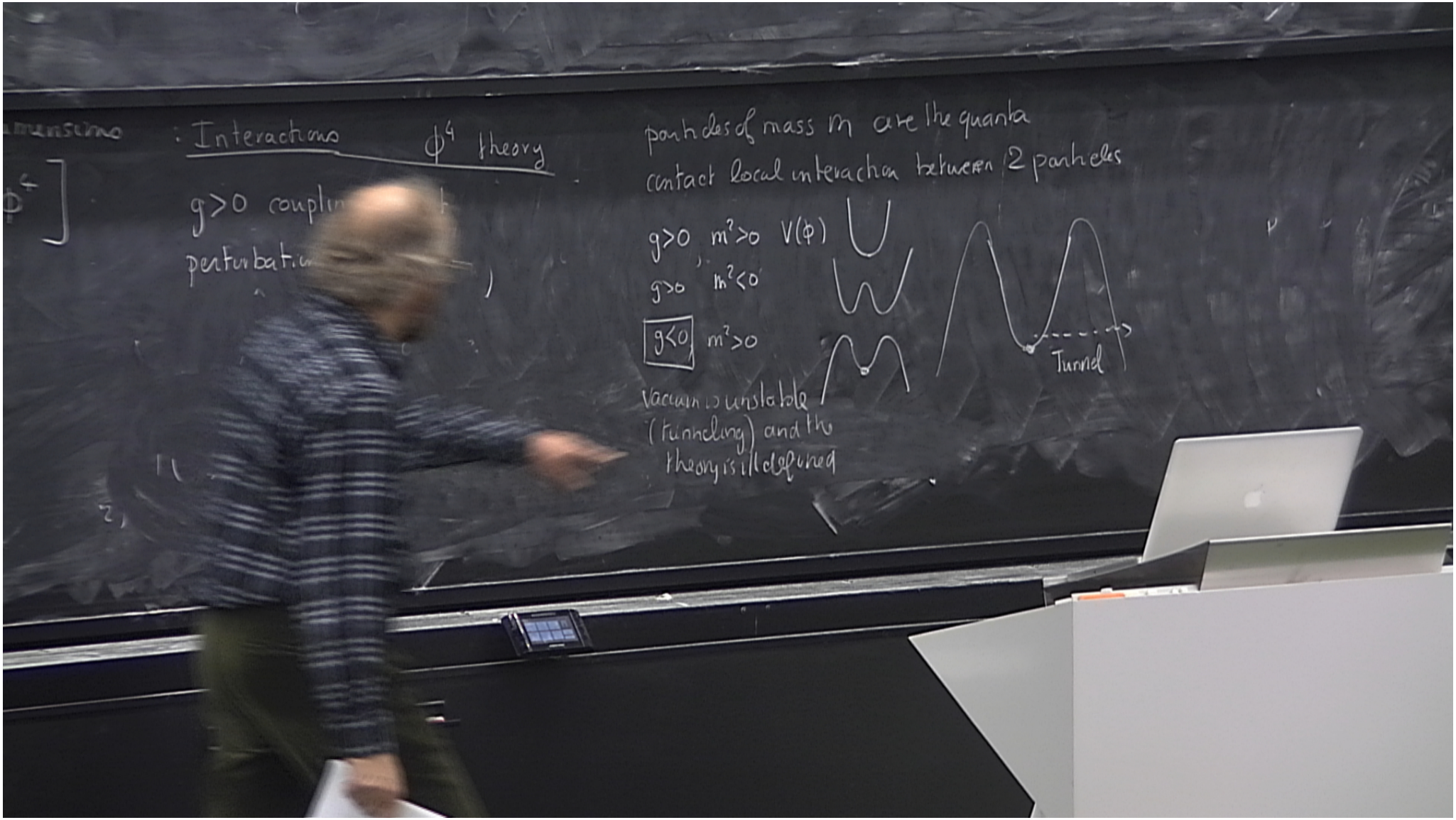
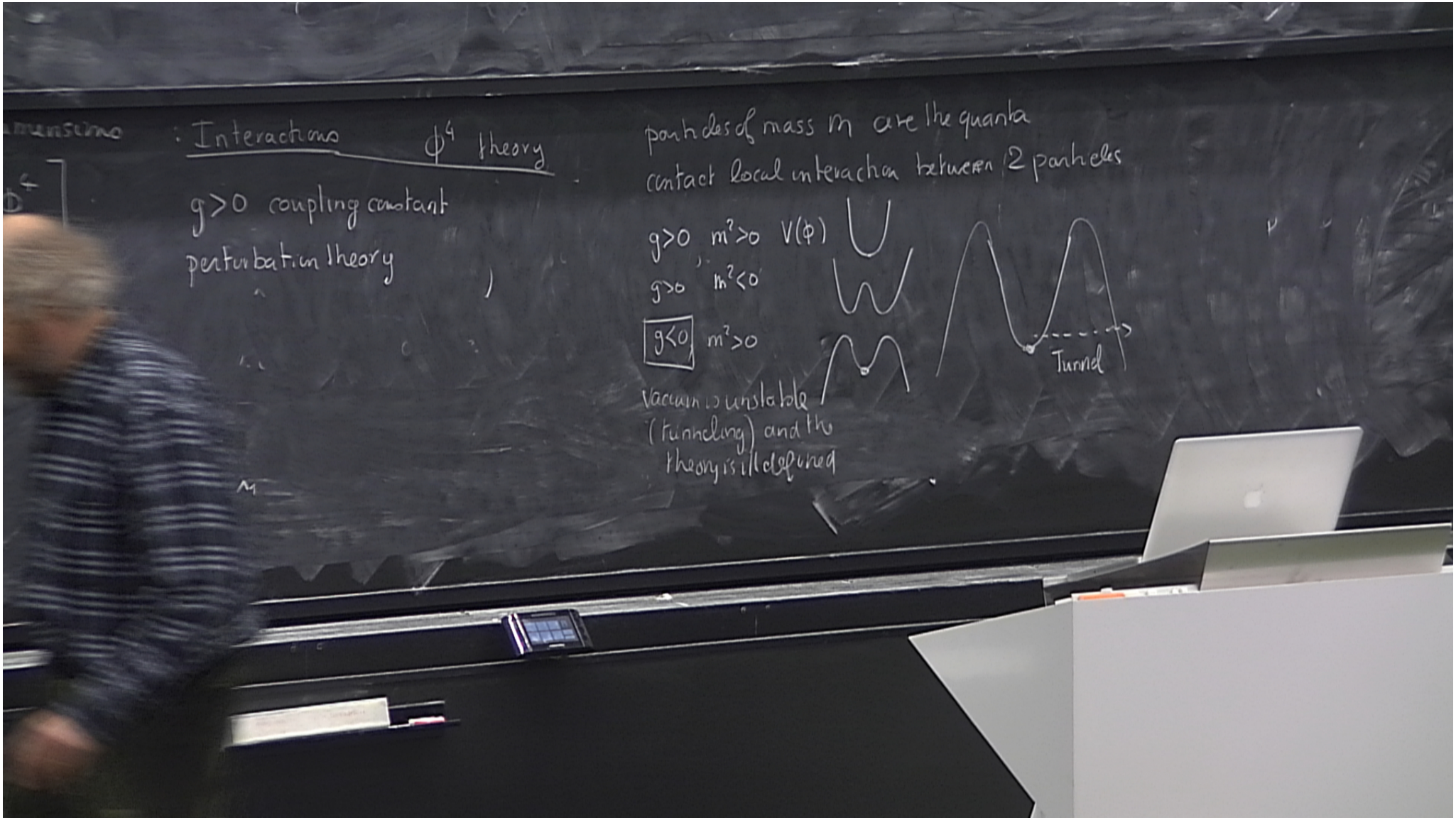
$g>0$, $m^2>0$ $V(\phi)$
$g>0$ $m^2<0$
$g<0$, $m^2>0$
Tunnel


: Interactions ϕ^4 theory
particles of mass m are the quanta
contact local interaction between 2 particles
$g>0$ $m^2>0$ $V(\phi)$
$g>0$ $m^2<0$
$g<0$ $m^2>0$
vacuum is unstable (tunneling) and the theory is ill defined
Tunnel


: Interactions ϕ^4 theory
$g>0$ coupling constant
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particles of mass m are the quanta
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$g>0$ $m^2>0$ $V(\phi)$
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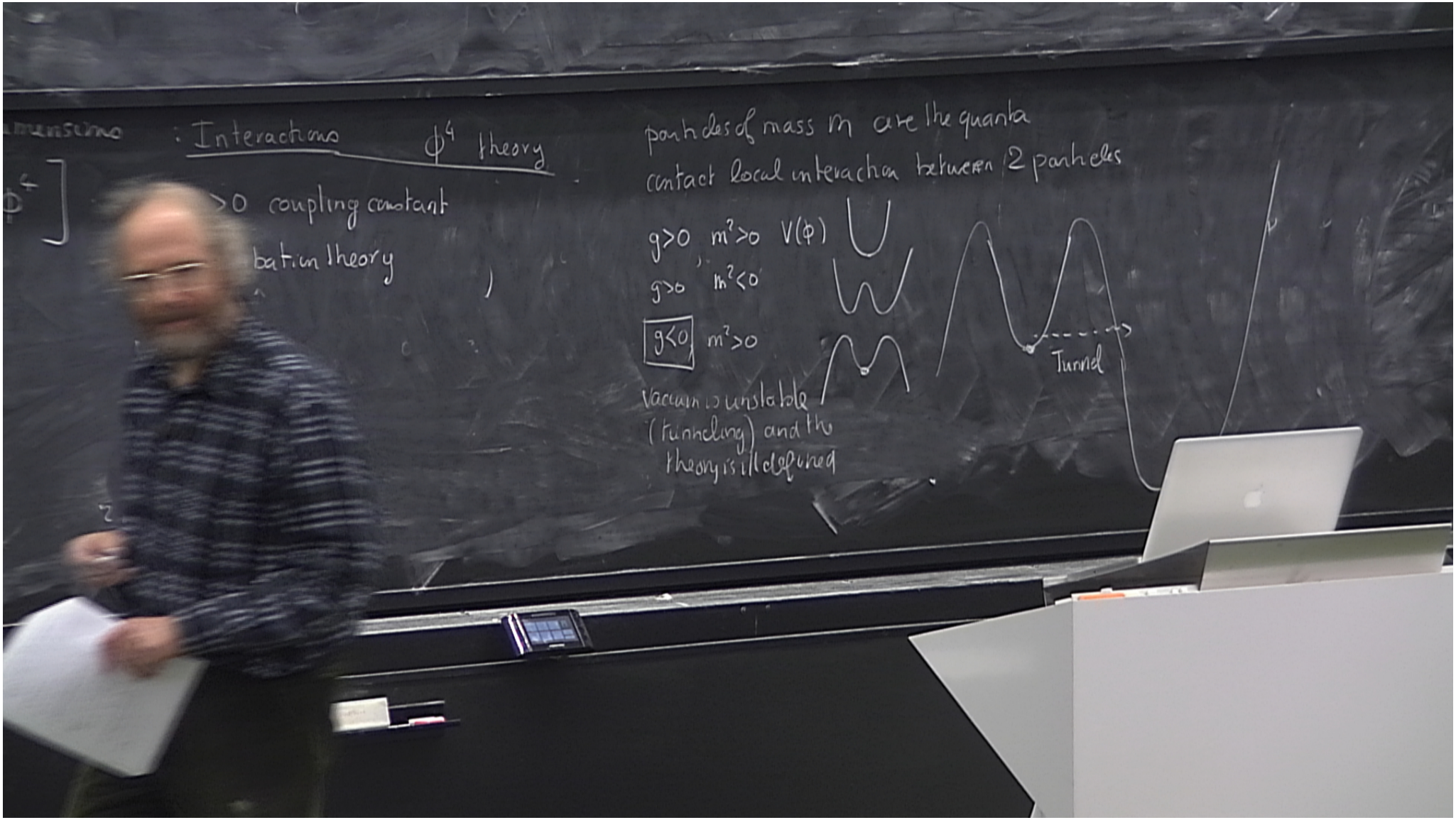
: Interactions
ϕ^4 theory
>0 coupling constant
bation theory
particles of mass m are the quanta
contact local interaction between 2 particles
g>0 m²>0 V(φ)
g>0 m²<0
g<0 m²>0
vacuum is unstable
(tunneling) and the
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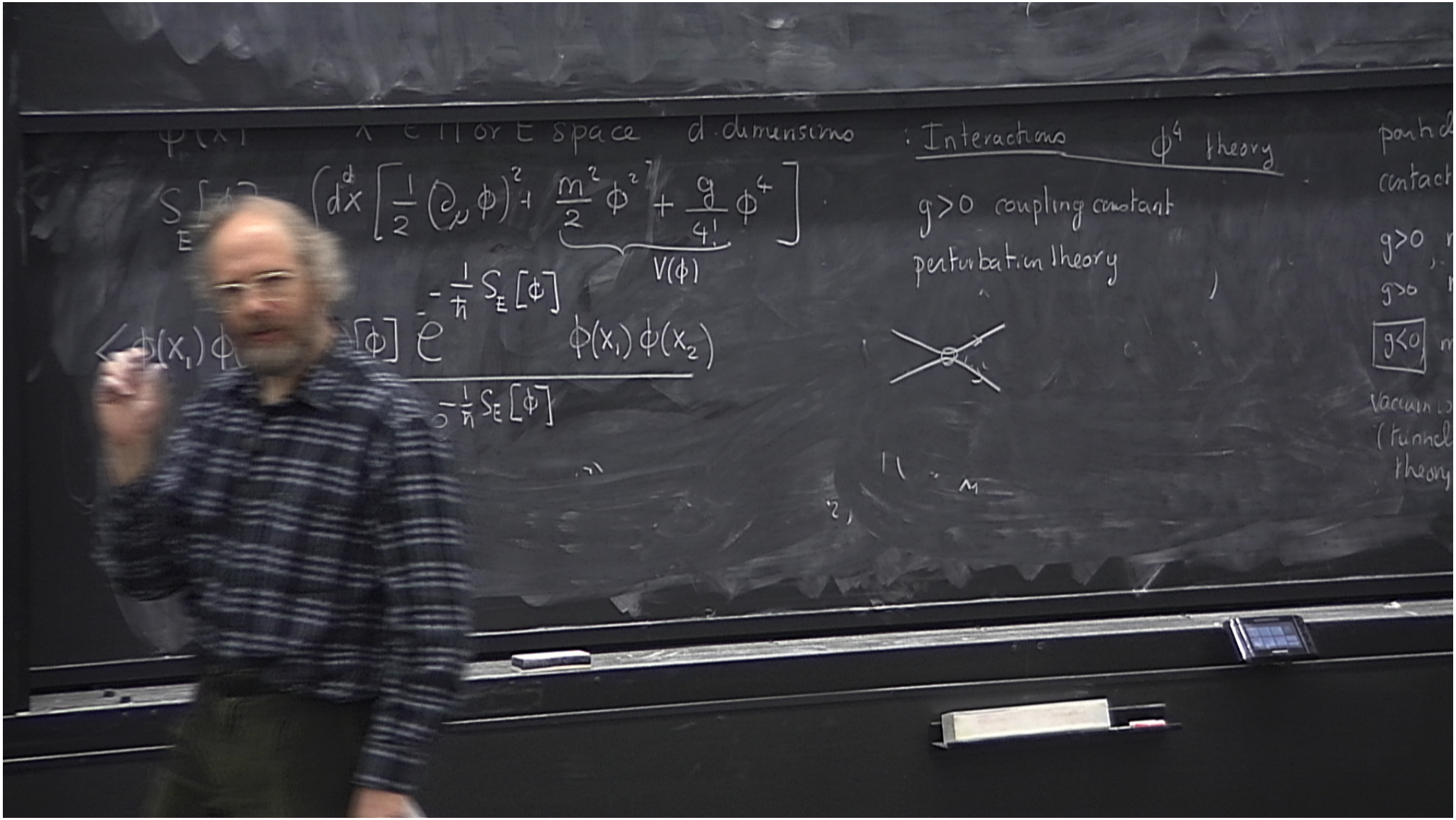
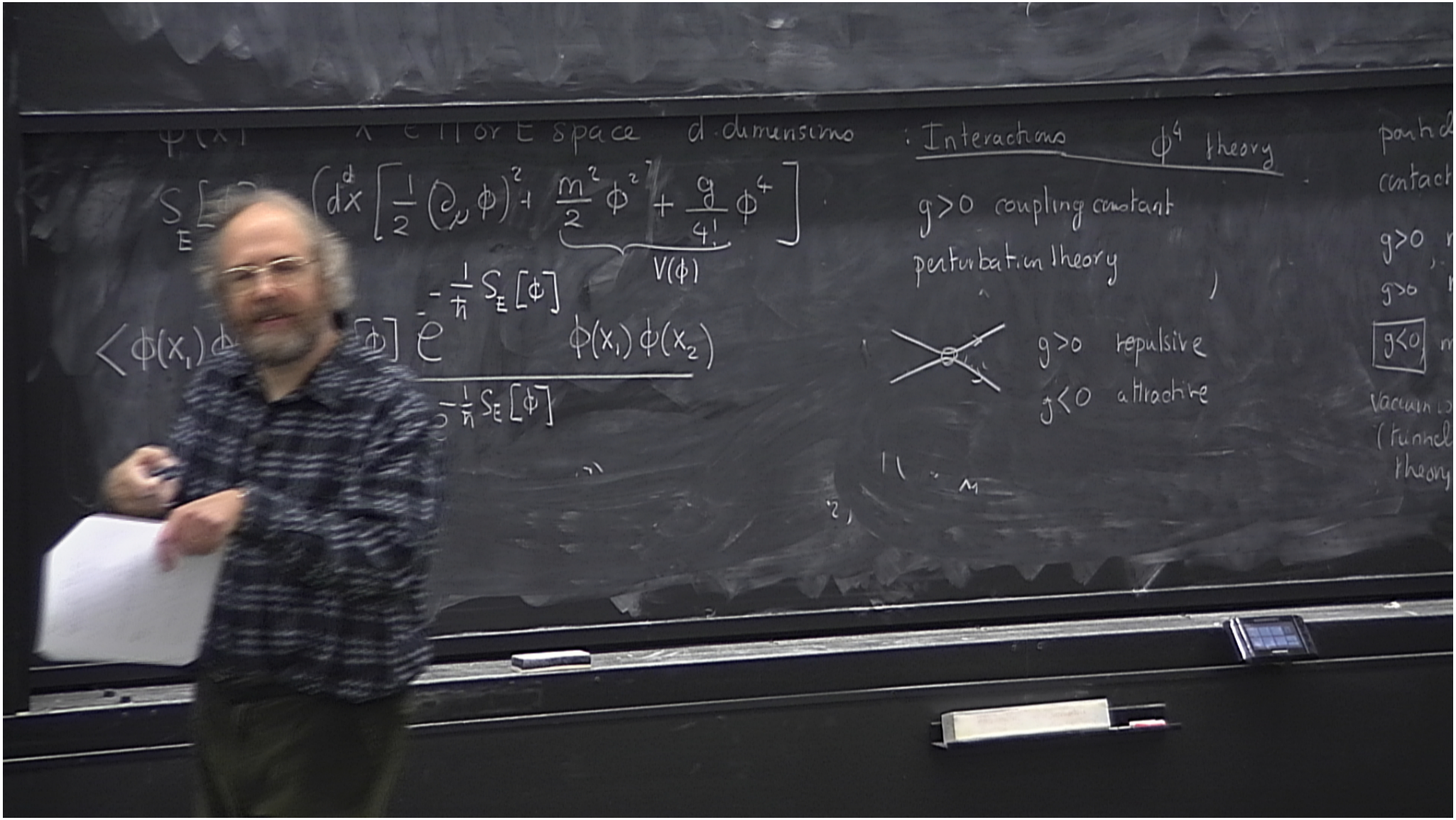
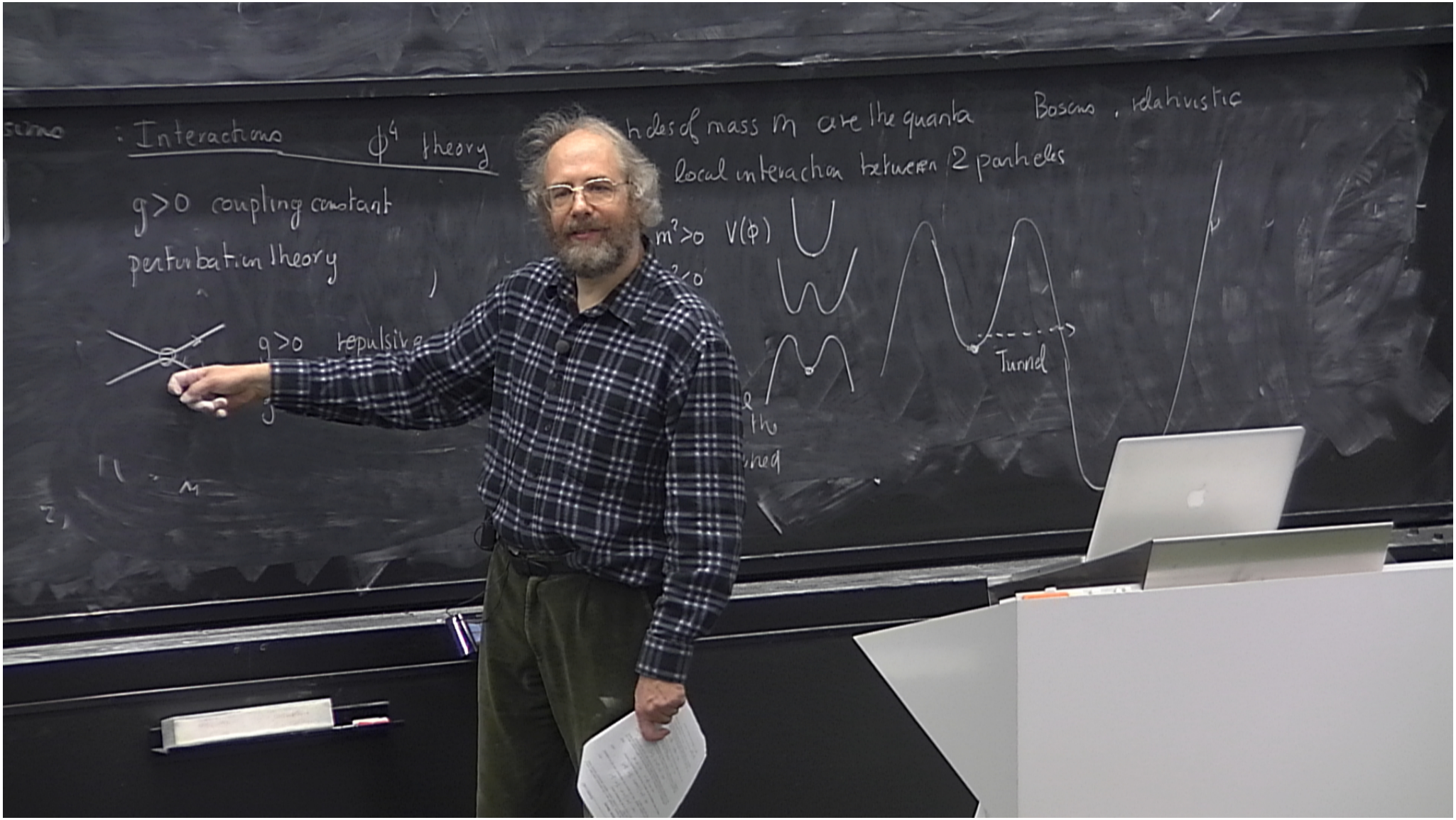
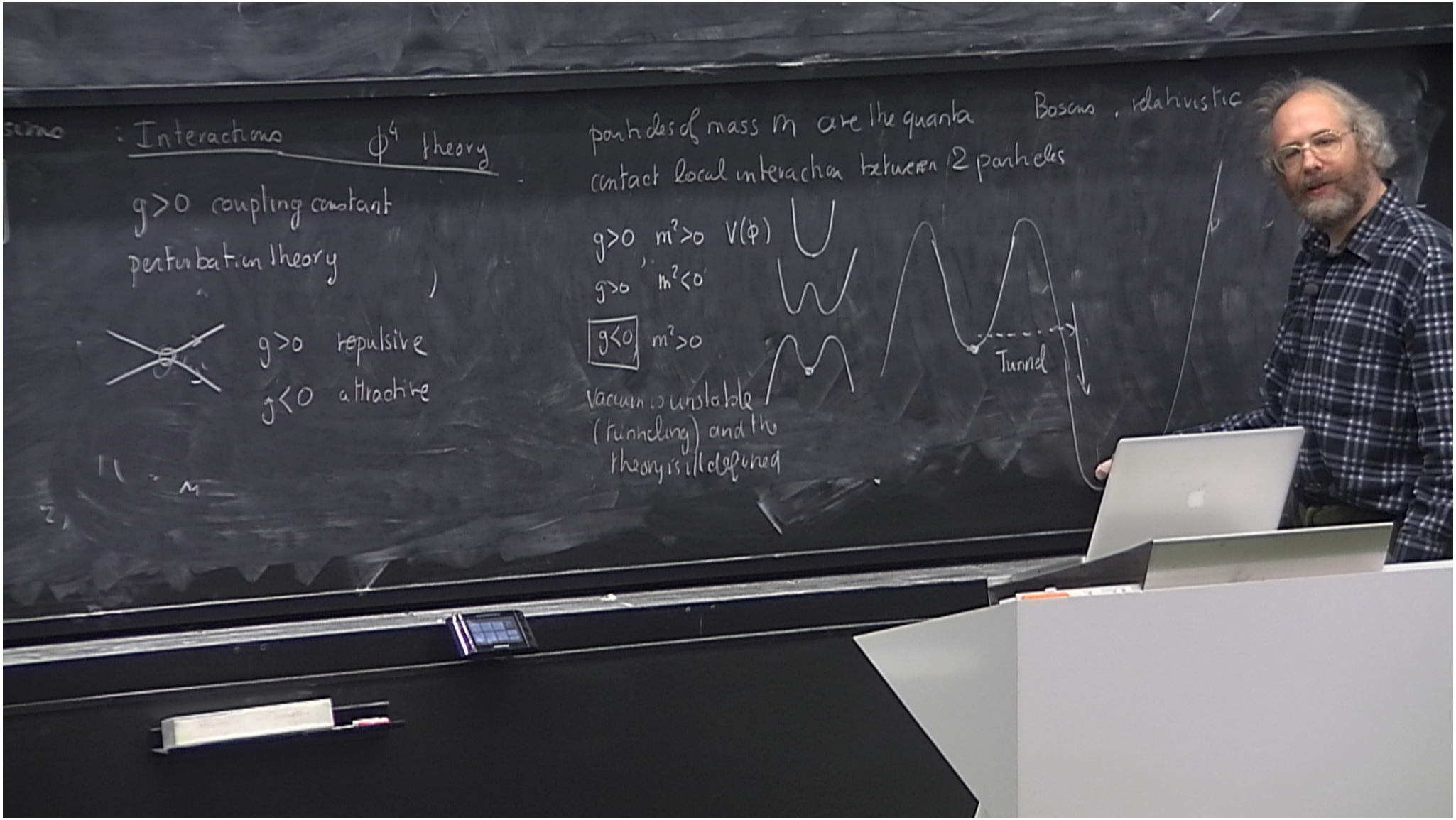
Tunnel


Euclidean space d dimensions
Interactions ϕ^4 theory
$g>0$ coupling constant
perturbation theory
$V(\phi)$
$g<0$


Interactions
ϕ^4 theory
$g>0$ coupling constant
perturbation theory
$g>0$ repulsive
$g<0$ attractive
$V(\phi)$


: Interactions
ϕ^4 theory
g>0 coupling constant
perturbation theory
g>0 repulsive
Bosons, relativistic
local interaction between 2 particles
$m^2>0$ V(φ)
Tunnel


: Interactions ϕ^4 theory
$g>0$ coupling constant
perturbation theory
$g>0$ repulsive
$g<0$ attractive
particles of mass m are the quanta Bosons, relativistic
contact local interaction between 2 particles
$g>0$, $m^2>0$ $V(\phi)$
$g>0$ $m^2<0$
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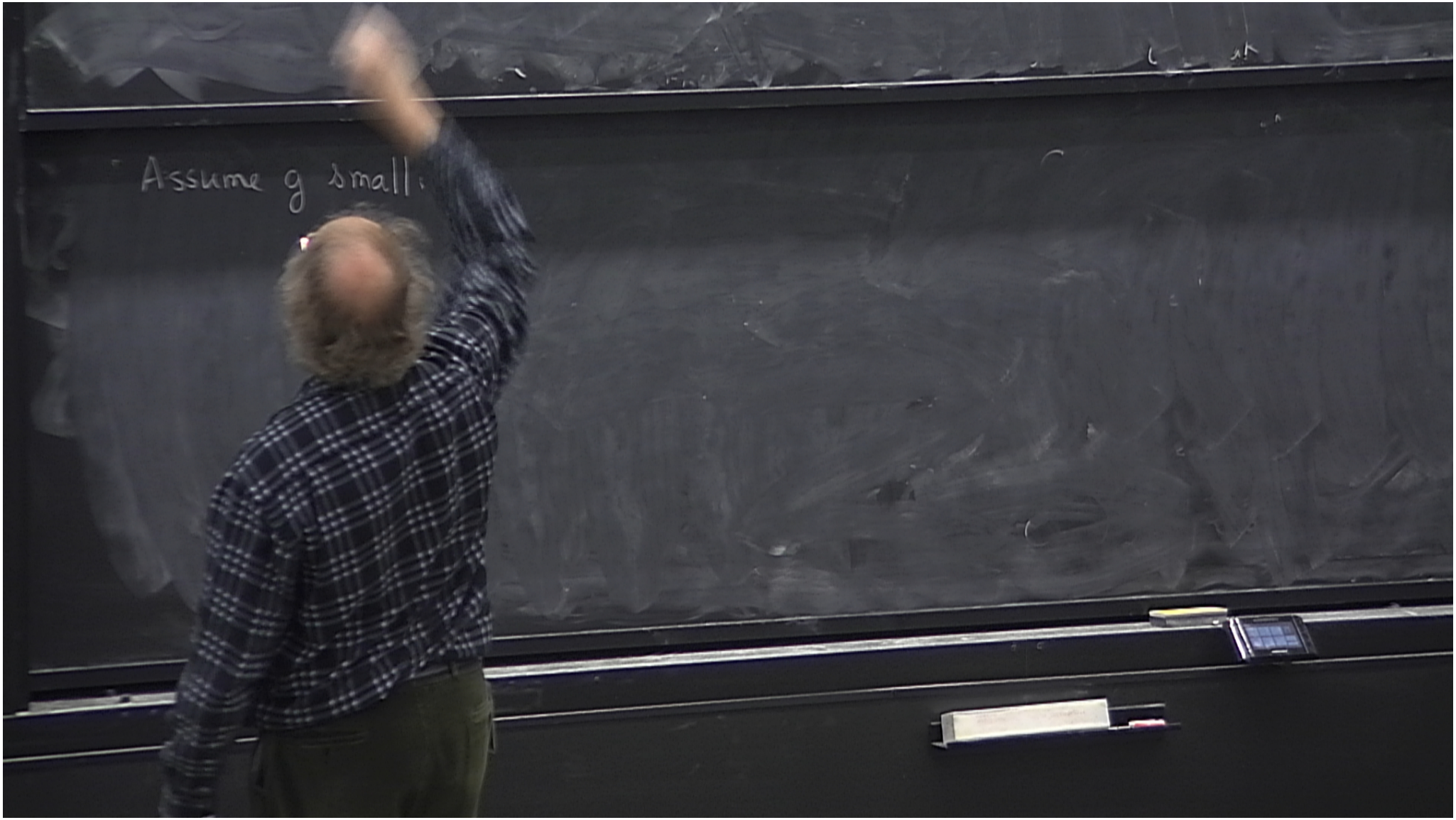
vacuum is unstable (tunneling) and the theory is ill-defined
Tunnel


Assume g small:

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$$e^{-\frac{1}{\hbar}\int d^4x \frac{g}{4!}\phi^4(x)}$$

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$$e^{-\frac{1}{\hbar}\int d^4x \frac{g}{4!}\phi^4(x)} = \sum_{k=0}^{\infty} \left(\frac{-g}{4!\hbar}\right)^k \frac{1}{k!} \int d^dx_1 \cdots d^dx_k \; \phi^4(x_1) \cdots \phi^4(x_k)$$

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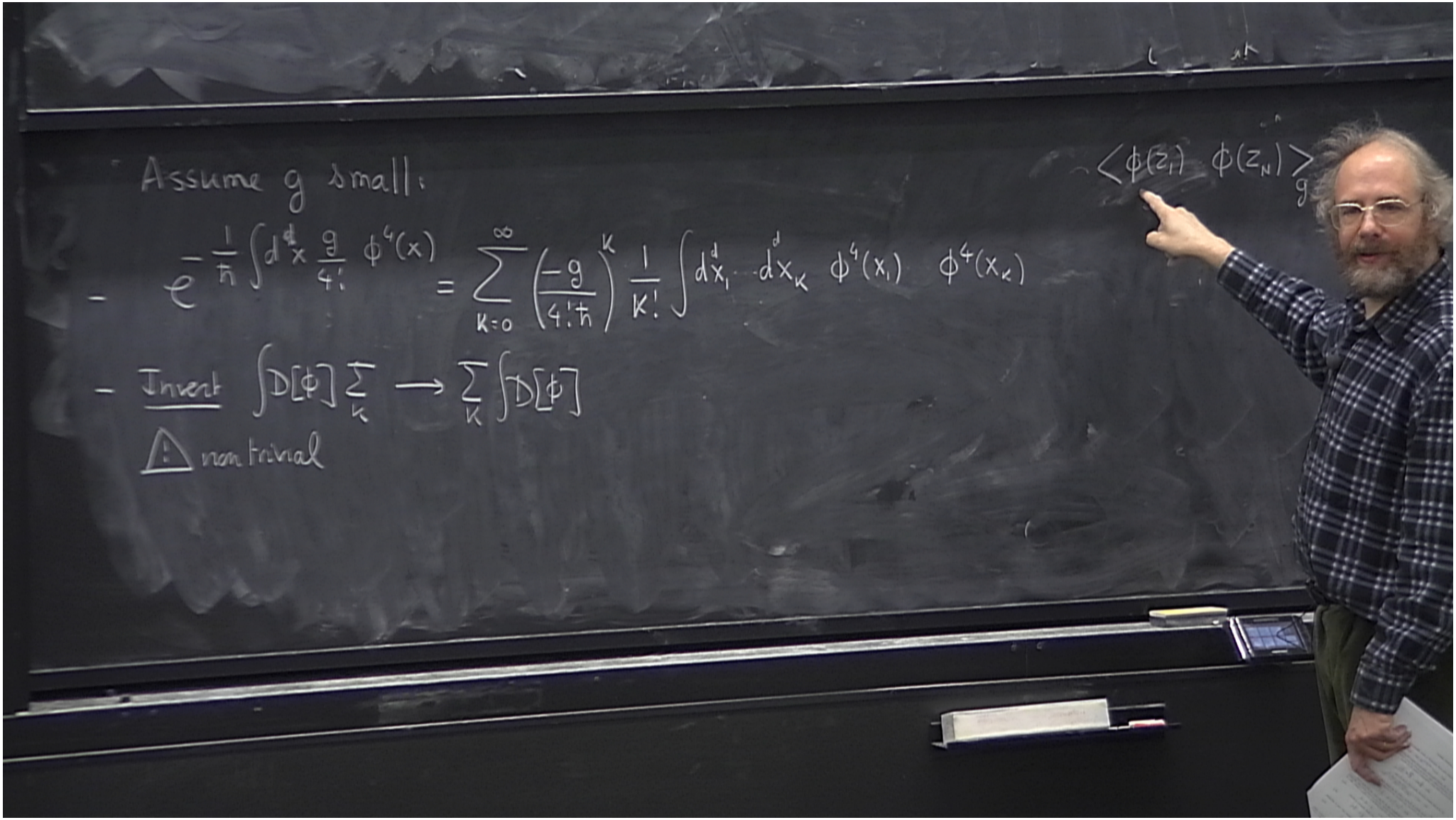
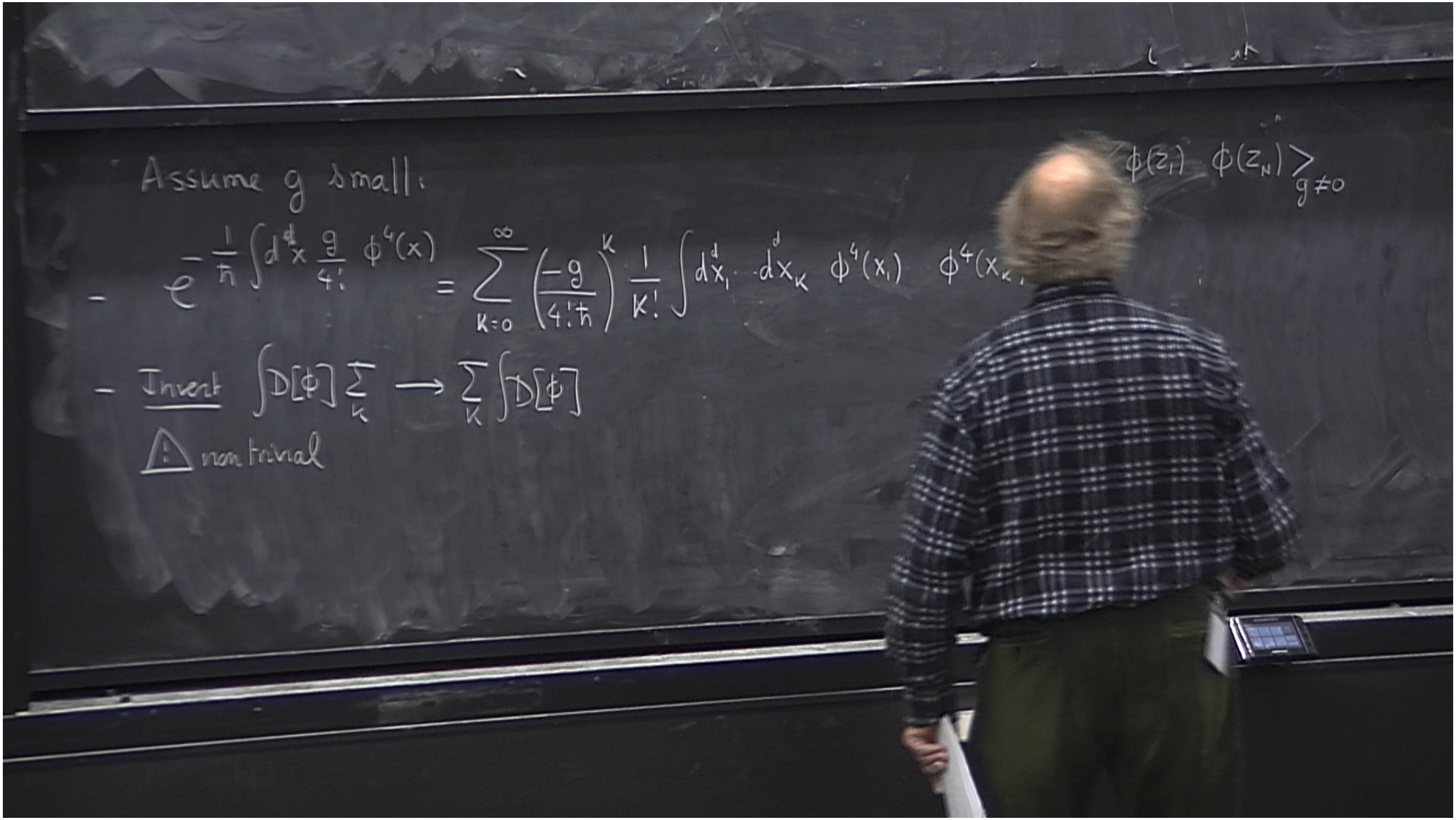
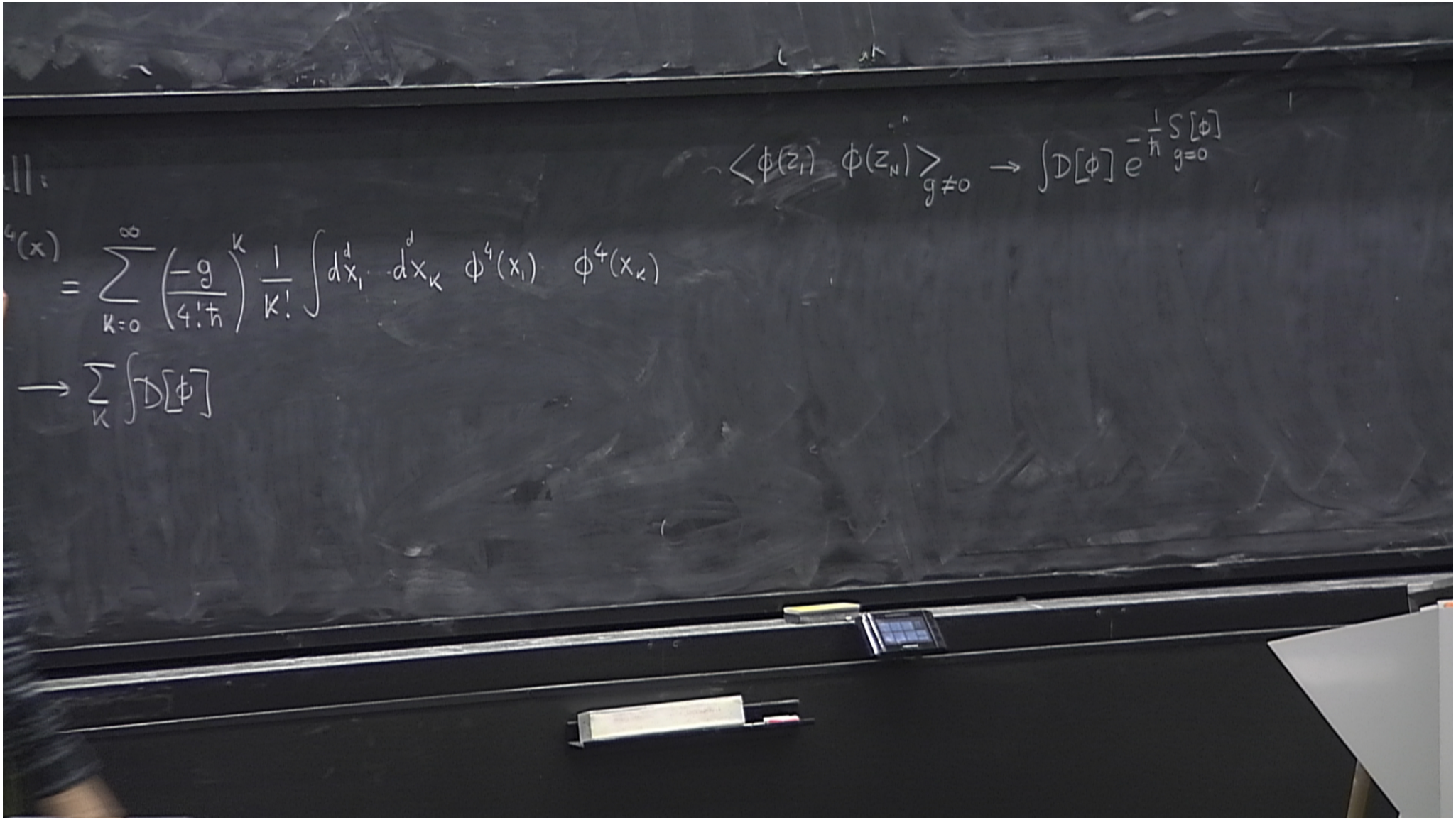
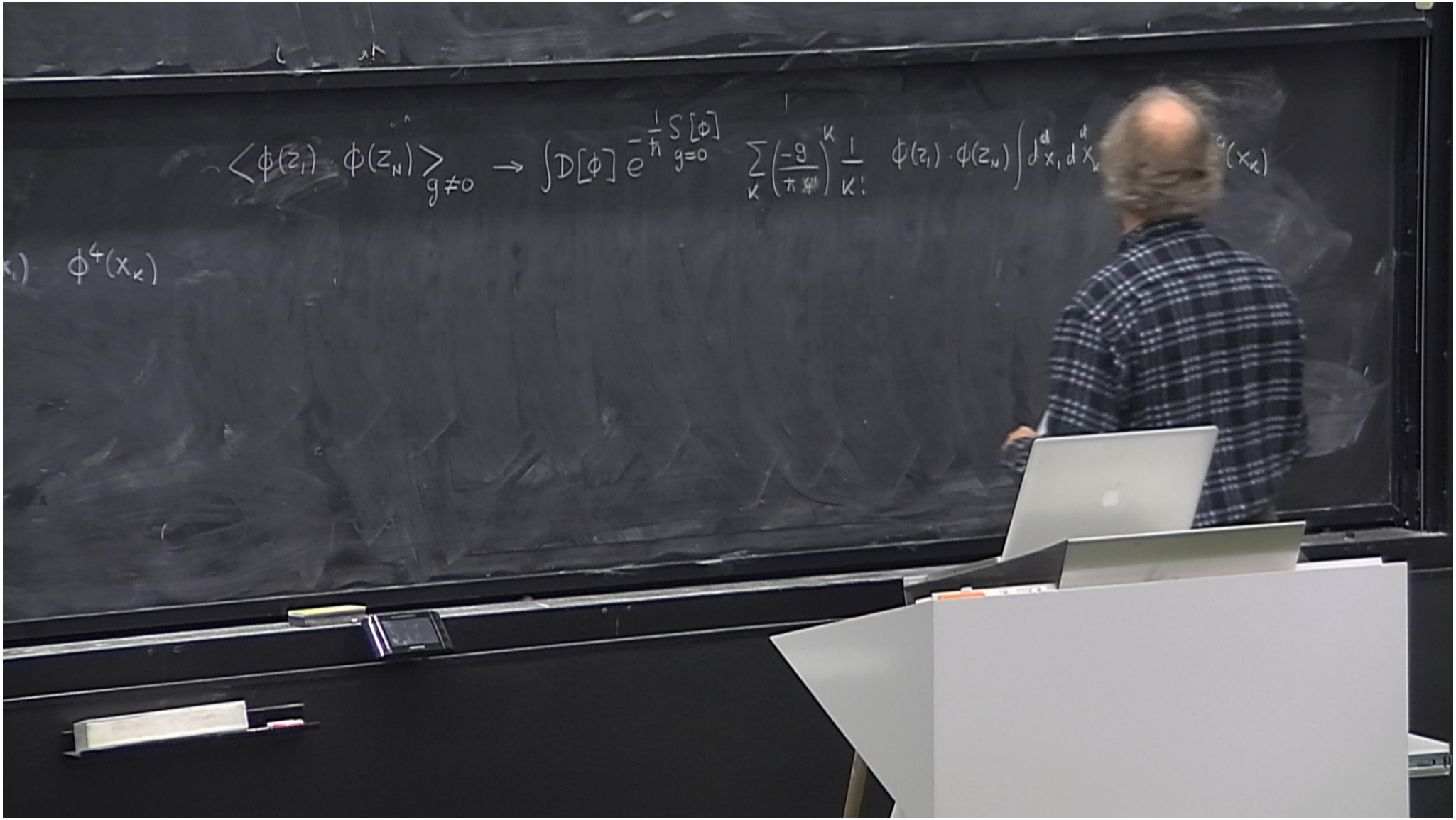
- Invert $\int D[\phi] \sum_k \rightarrow \sum_k \int D[\phi]$

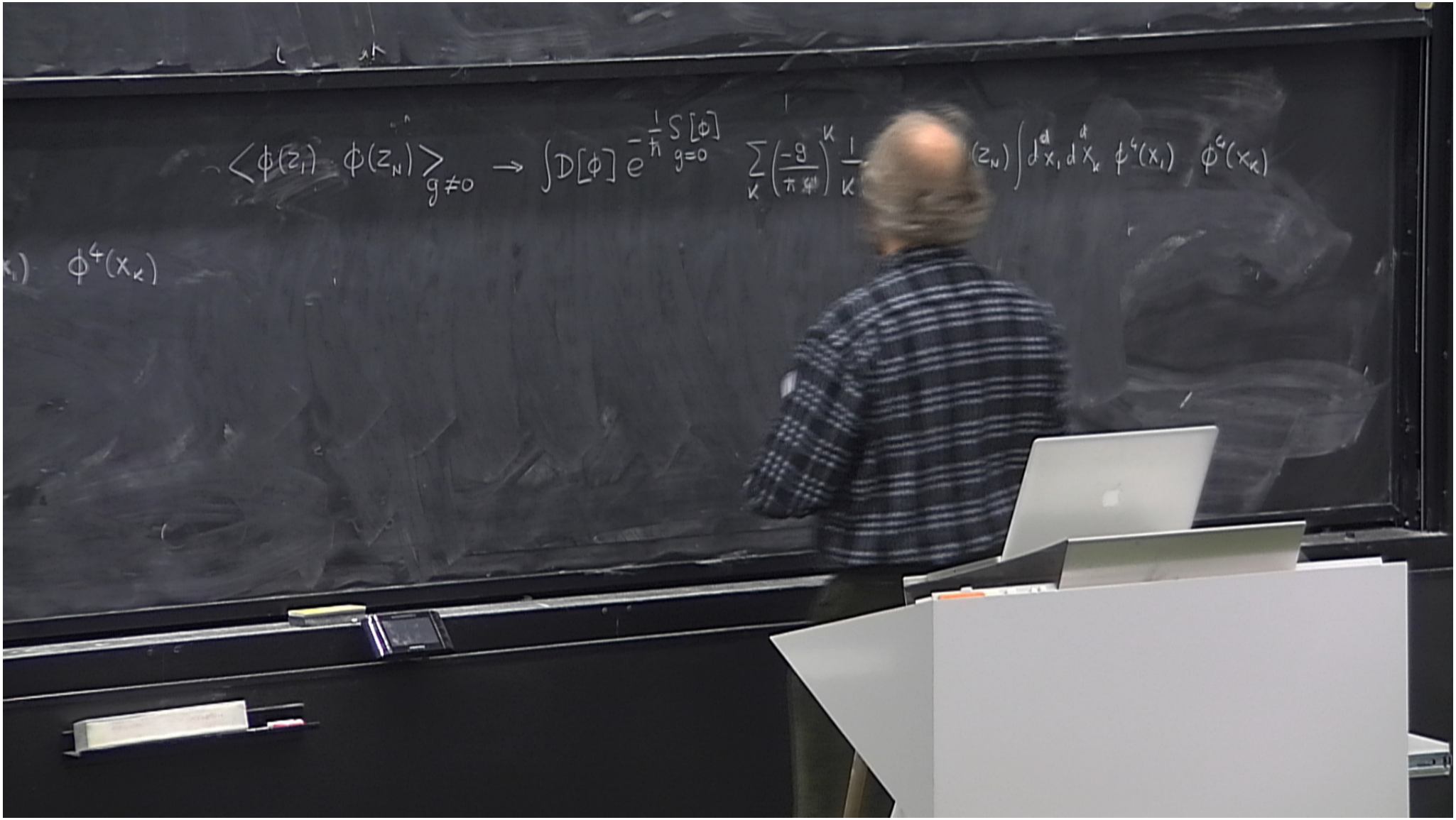
⚠ non trivial

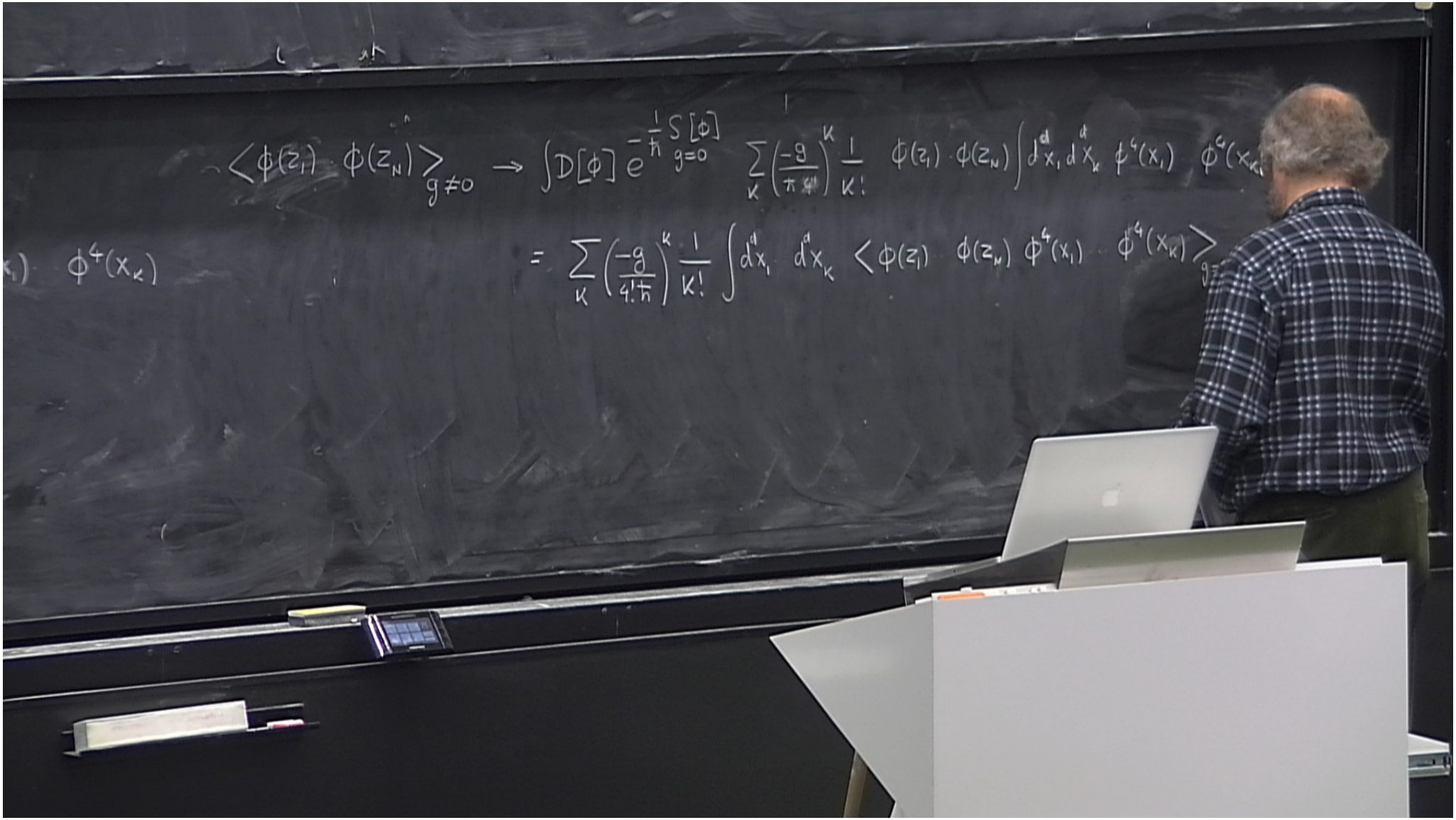

Assume g small:
$\langle \phi(z_1) \ldots \phi(z_N) \rangle_g$
$e^{-\frac{1}{\hbar}\int d^4x \frac{g}{4!}\phi^4(x)} = \sum_{k=0}^{\infty} \left(\frac{-g}{4!\hbar}\right)^k \frac{1}{k!} \int d^dx_1 \ldots d^dx_k \; \phi^4(x_1) \ldots \phi^4(x_k)$
- Invert $\int D[\phi] \sum_k \to \sum_k \int D[\phi]$
⚠ non trivial

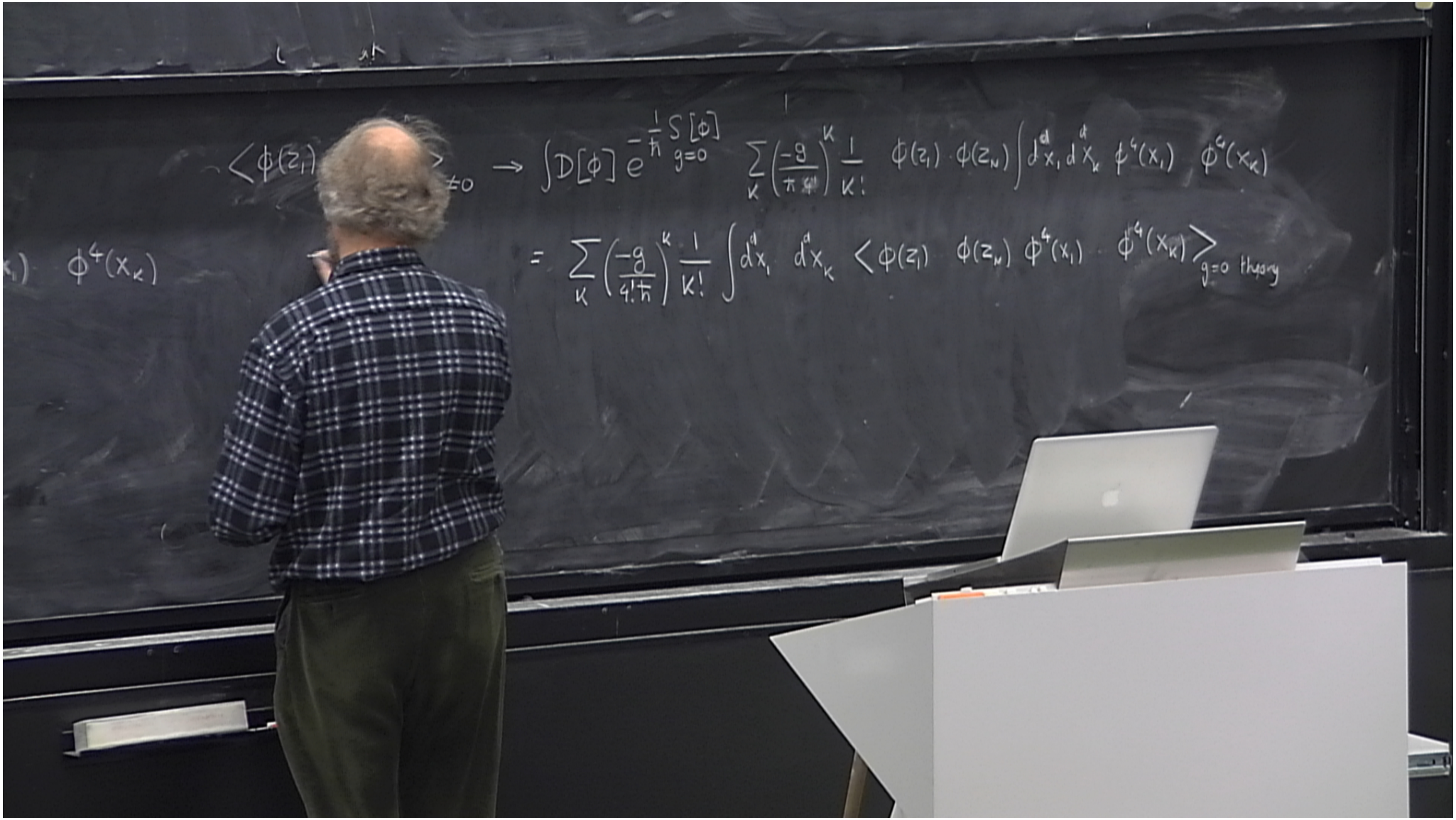

Assume g small:
$\phi(z_1) \ldots \phi(z_N)\rangle_{g\neq 0}$
$e^{-\frac{1}{\hbar}\int d^4x \frac{g}{4!}\phi^4(x)} = \sum_{k=0}^{\infty} \left(\frac{-g}{4!\hbar}\right)^k \frac{1}{k!} \int d^4x_1 \ldots d^4x_k \; \phi^4(x_1) \ldots \phi^4(x_k)$
- Invert $\int \mathcal{D}[\phi] \sum_k \rightarrow \sum_k \int \mathcal{D}[\phi]$
⚠ non trivial

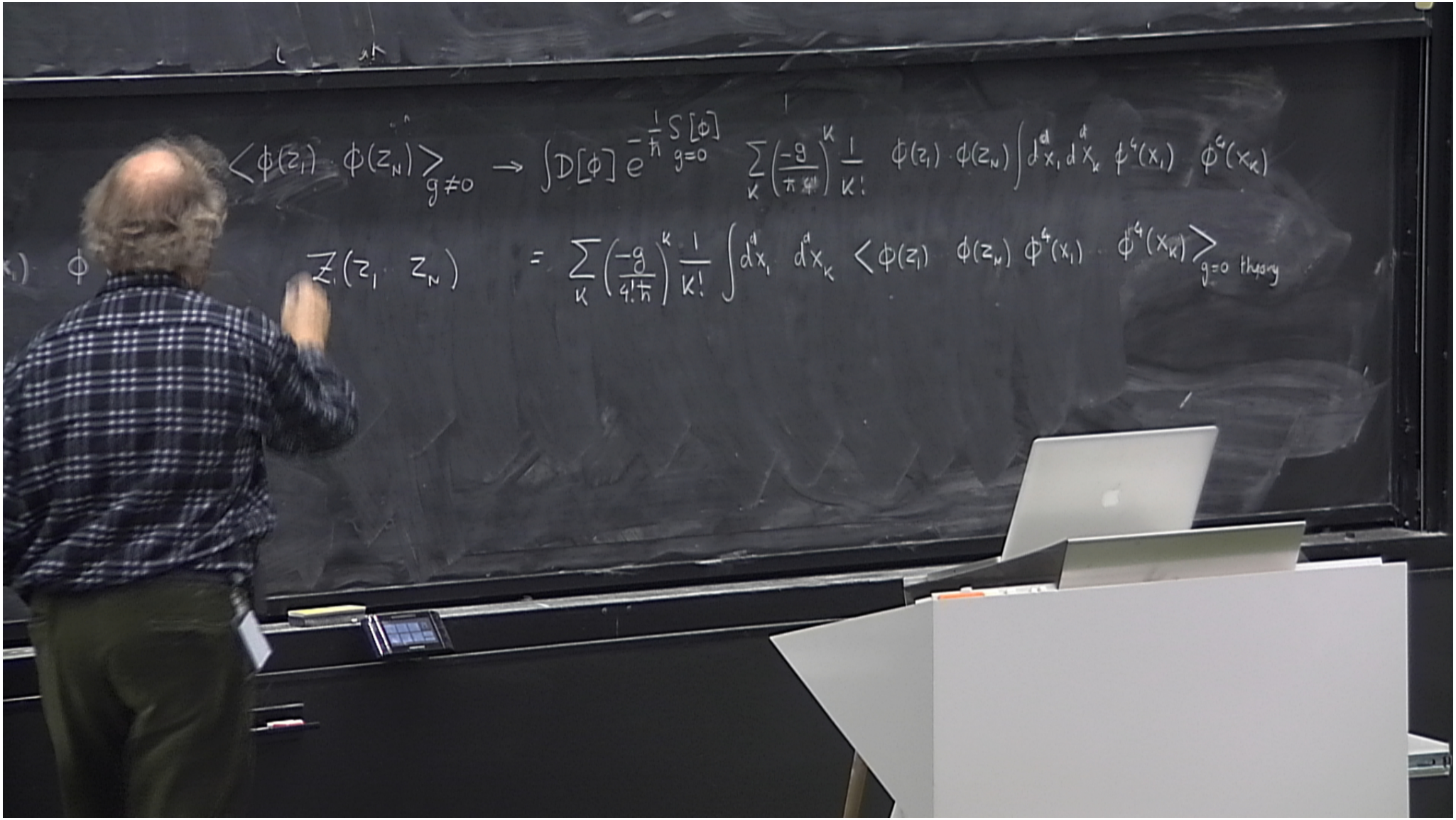

$\langle \phi(z_1) \ldots \phi(z_N) \rangle_{g \neq 0} \to \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}}$
$\phi^4(x) = \sum_{K=0}^{\infty} \left(\frac{-g}{4!\hbar}\right)^K \frac{1}{K!} \int d^d x_1 \ldots d^d x_K \; \phi^4(x_1) \ldots \phi^4(x_K)$
$\to \sum_K \int D[\phi]$

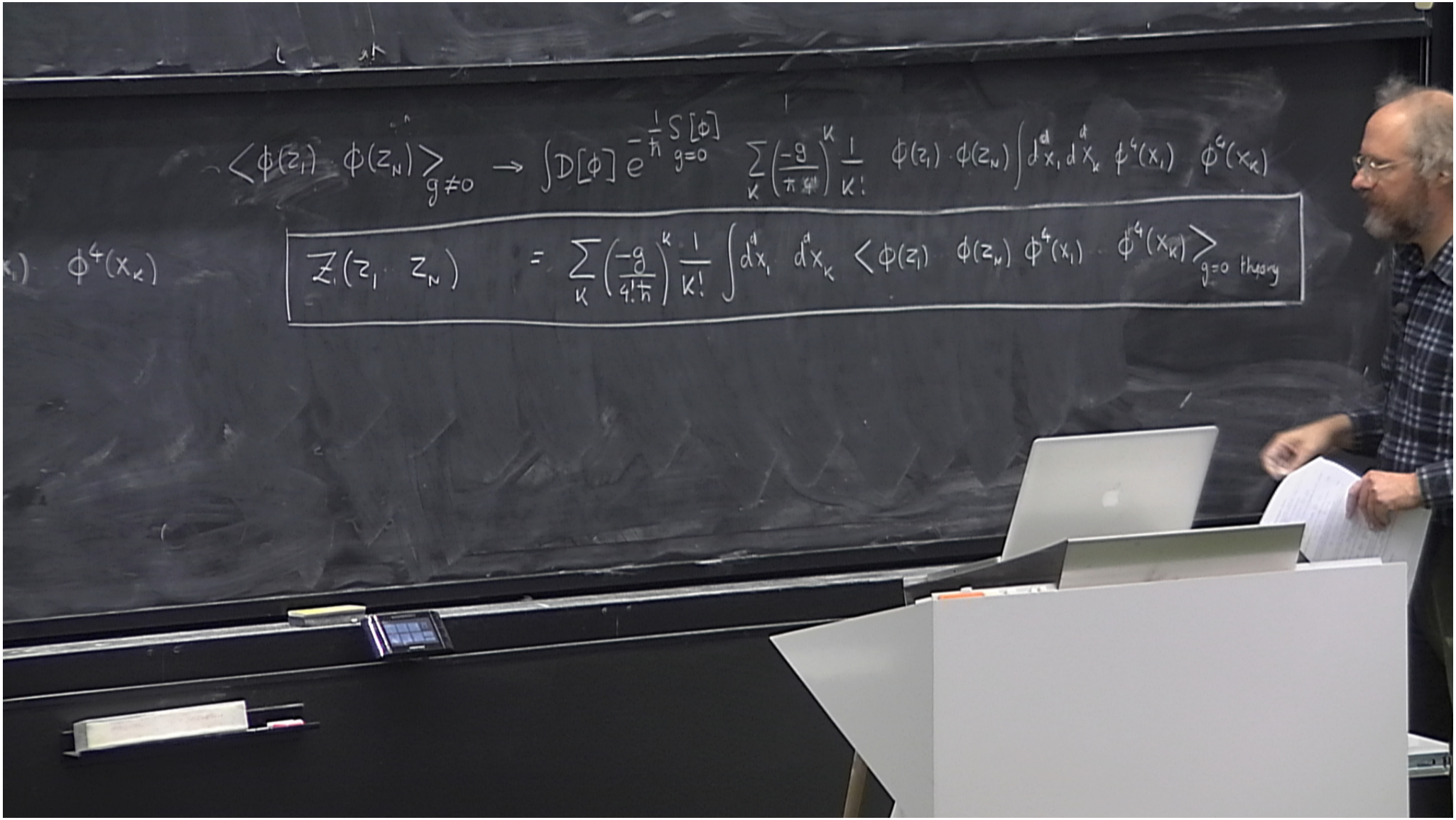

$\langle \phi(z_1) \cdots \phi(z_N) \rangle_{g \neq 0} \to \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_K \left(\frac{-g}{\pi \hbar}\right)^K \frac{1}{K!} \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \, d^d x_K$
$\phi^4(x_K)$


$\langle \phi(z_1) \ldots \phi(z_N) \rangle_{g \neq 0} \to \int \mathcal{D}[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_k \left(\frac{-g}{\pi \ldots}\right)^k \frac{1}{k!} \ldots (z_N) \int d^d x_1 \ldots d^d x_k \ldots \phi^4(x_1) \ldots \phi^4(x_k)$
$\ldots) \ldots \phi^4(x_k)$


$\langle \phi(z_1) \cdots \phi(z_N) \rangle_{g \neq 0} \to \int \mathcal{D}[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_K \left(\frac{-g}{\pi \hbar}\right)^K \frac{1}{K!} \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \cdots d^d x_K \, \phi^4(x_1) \cdots \phi^4(x_K)$
$= \sum_K \left(\frac{-g}{4! \hbar}\right)^K \frac{1}{K!} \int d^d x_1 \cdots d^d x_K \, \langle \phi(z_1) \cdots \phi(z_N) \, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_{g=}$


$\langle \phi(z_1) \quad \neq 0 \rightarrow \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_K \left(\frac{-g}{\pi\, 4!}\right)^K \frac{1}{K!}\, \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \cdots d^d x_K\, \phi^4(x_1) \cdots \phi^4(x_K)$
$\phi^4(x_K) \quad = \sum_K \left(\frac{-g}{4!\hbar}\right)^K \frac{1}{K!} \int d^d x_1 \cdots d^d x_K \langle \phi(z_1) \cdots \phi(z_N)\, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_{g=0 \text{ theory}}$


$\langle \phi(z_1) \cdots \phi(z_N) \rangle_{g \neq 0} \to \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_k \left(\frac{-g}{\pi \cdots}\right)^k \frac{1}{k!} \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \cdots d^d x_k \, \phi^4(x_1) \cdots \phi^4(x_k)$
$Z(z_1 \cdots z_N) = \sum_k \left(\frac{-g}{4! \hbar}\right)^k \frac{1}{k!} \int d^d x_1 \cdots d^d x_k \langle \phi(z_1) \cdots \phi(z_N) \phi^4(x_1) \cdots \phi^4(x_k) \rangle_{g=0}$ theory


$\langle \phi(z_1) \cdots \phi(z_N) \rangle_{g \neq 0} \to \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_K \left(\frac{-g}{\pi \hbar}\right)^K \frac{1}{K!} \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \cdots d^d x_K\, \phi^4(x_1) \cdots \phi^4(x_K)$
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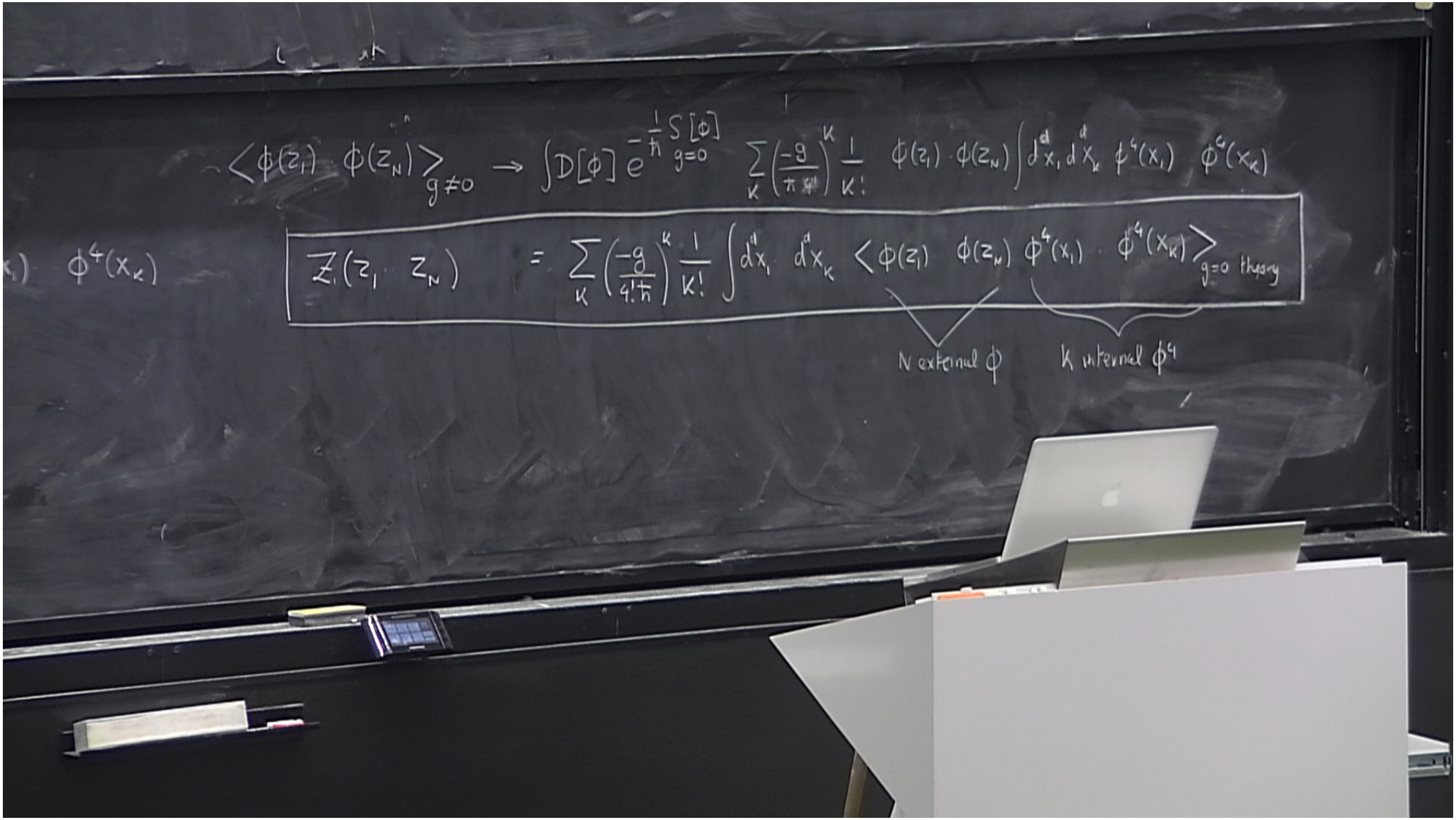
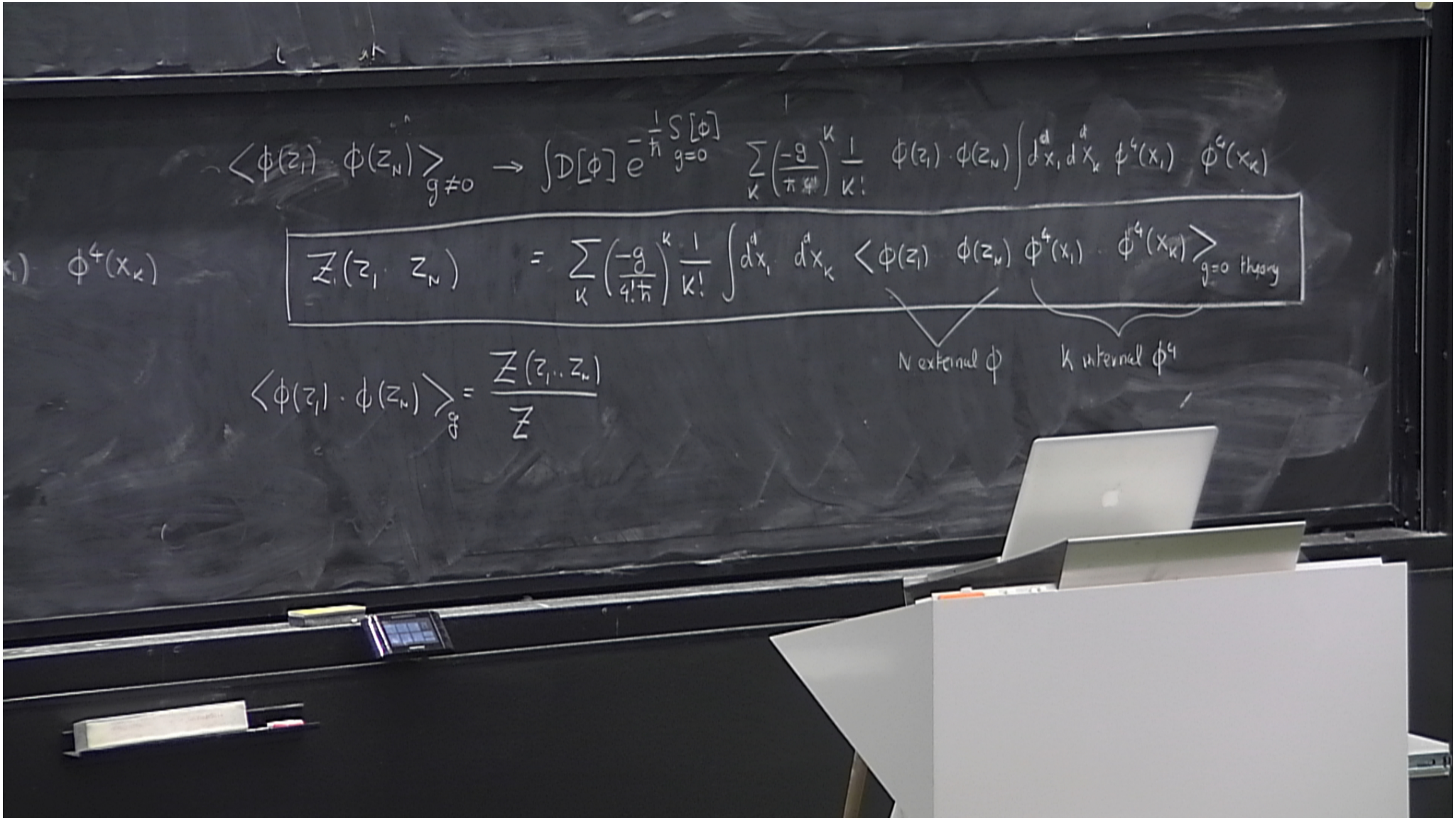
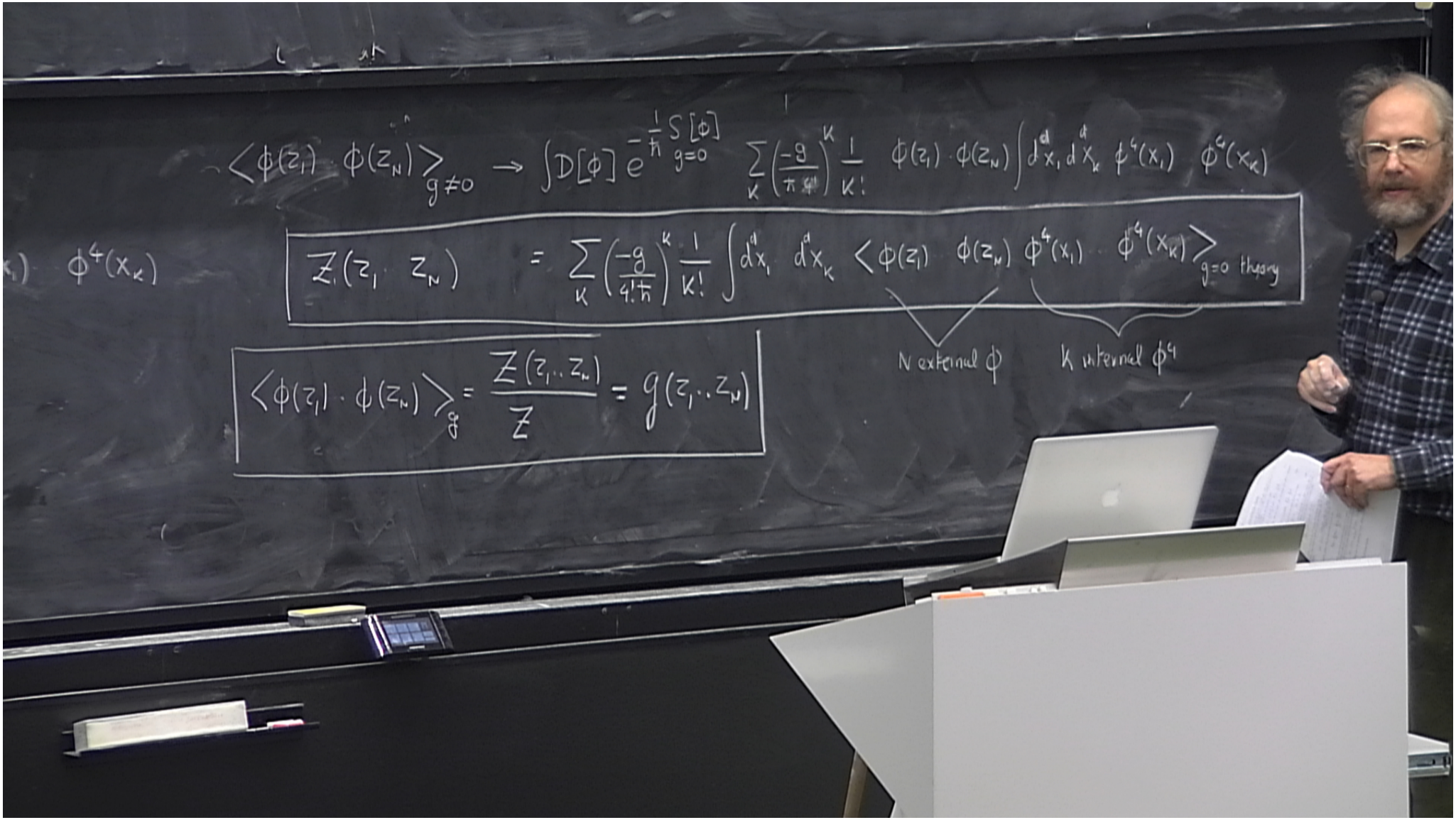
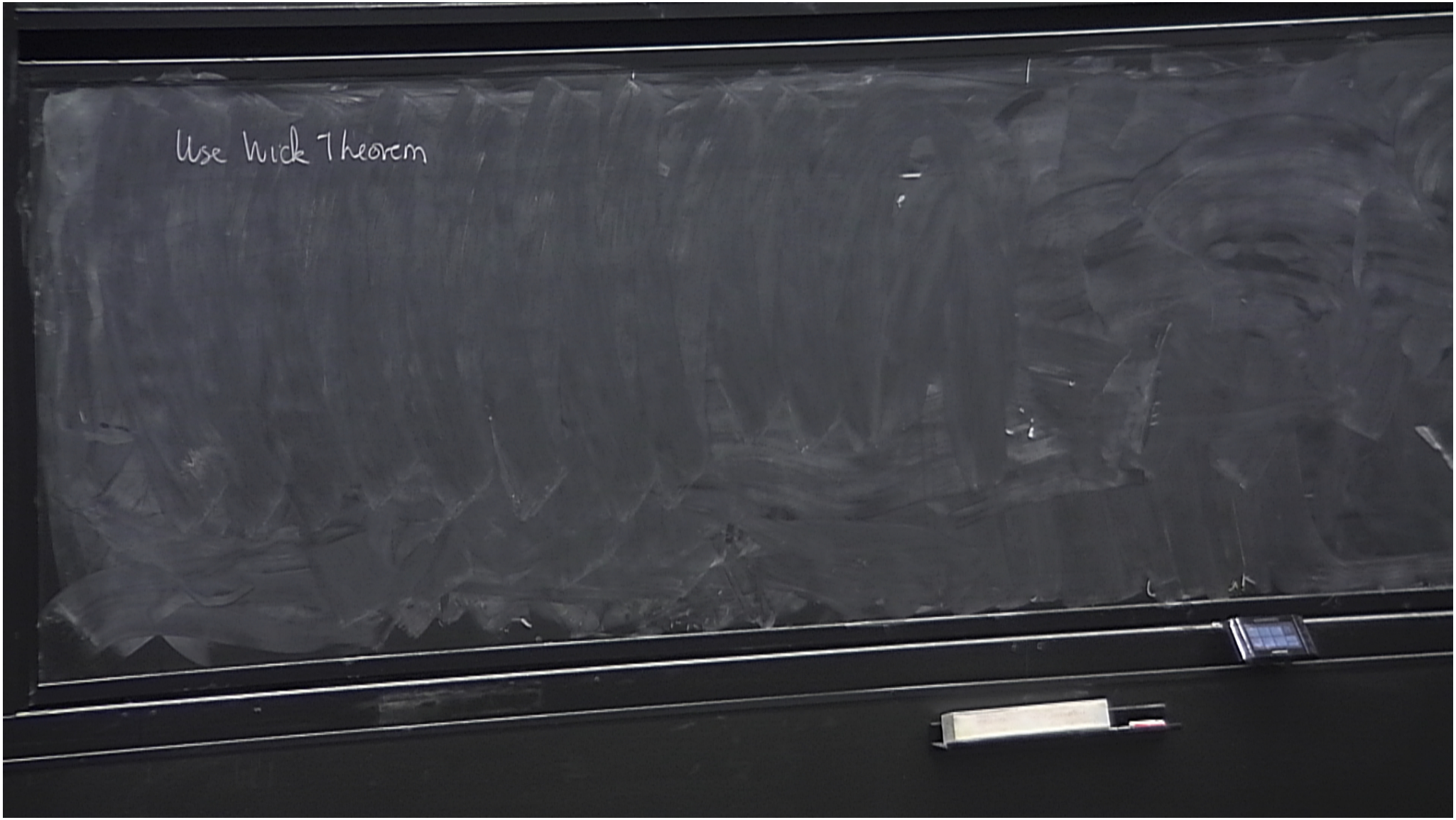
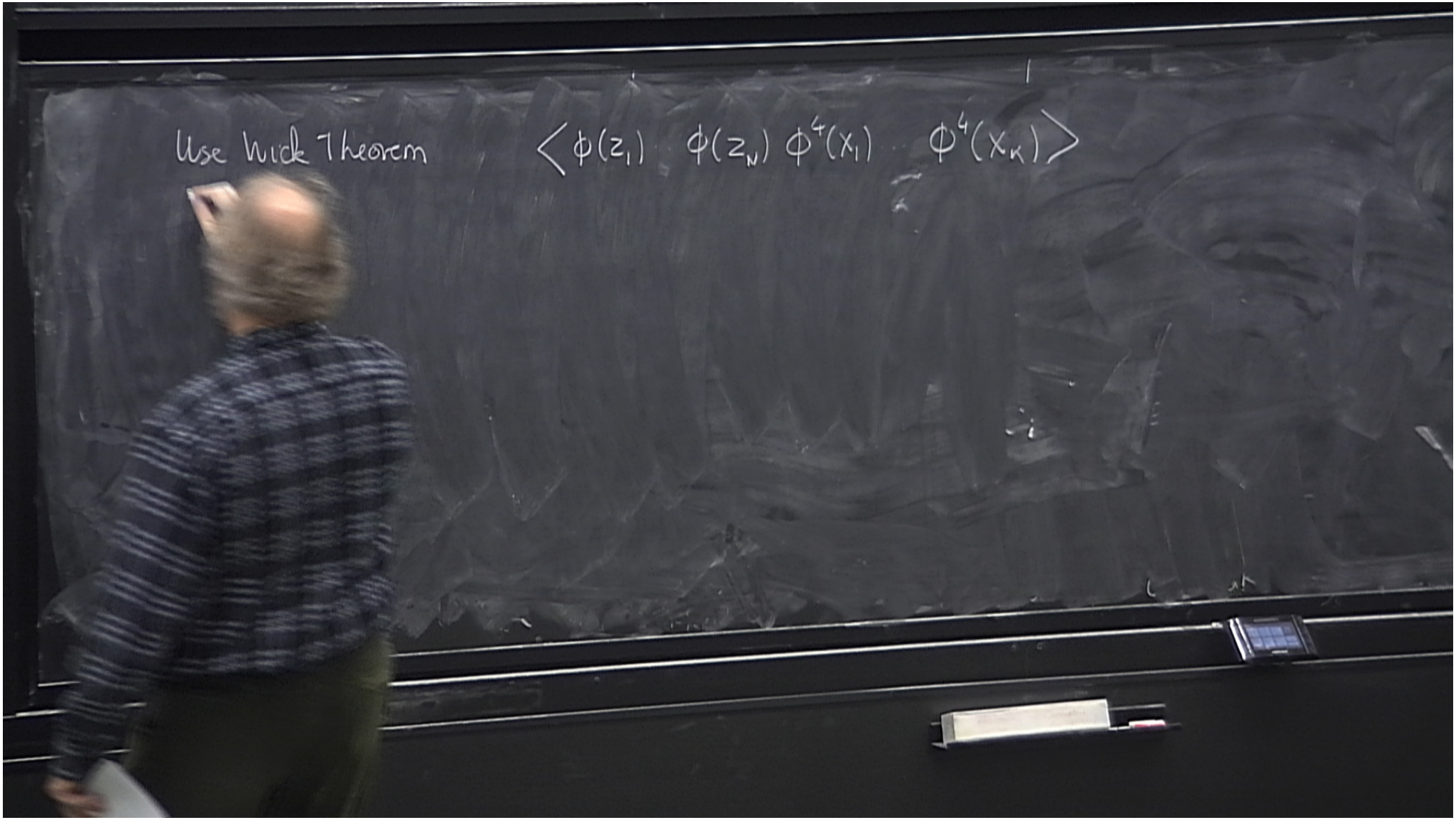

$\langle \phi(z_1) \cdots \phi(z_N) \rangle_{g \neq 0} \to \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_K \left(\frac{-g}{\pi \cdots}\right)^K \frac{1}{K!} \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \cdots d^d x_K \, \phi^4(x_1) \cdots \phi^4(x_K)$
$Z(z_1 \cdots z_N) = \sum_K \left(\frac{-g}{4! \hbar}\right)^K \frac{1}{K!} \int d^d x_1 \cdots d^d x_K \langle \phi(z_1) \cdots \phi(z_N) \, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_{g=0 \text{ theory}}$
N external ϕ
K internal ϕ^4


$\langle \phi(z_1) \cdots \phi(z_N) \rangle_{g \neq 0} \to \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_K \left(\frac{-g}{\pi \cdots}\right)^K \frac{1}{K!} \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \cdots d^d x_K \, \phi^4(x_1) \cdots \phi^4(x_K)$
$Z(z_1 \cdots z_N) = \sum_K \left(\frac{-g}{4! \hbar}\right)^K \frac{1}{K!} \int d^d x_1 \cdots d^d x_K \langle \phi(z_1) \cdots \phi(z_N) \, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_{g=0 \text{ theory}}$
N external ϕ
K internal ϕ^4
$\langle \phi(z_1) \cdots \phi(z_N) \rangle_g = \frac{Z(z_1 \cdots z_N)}{Z}$


$\langle \phi(z_1) \cdots \phi(z_N) \rangle_{g \neq 0} \to \int D[\phi]\, e^{-\frac{1}{\hbar} S[\phi]_{g=0}} \sum_K \left(\frac{-g}{\pi \hbar}\right)^K \frac{1}{K!} \phi(z_1) \cdots \phi(z_N) \int d^d x_1 \cdots d^d x_K\, \phi^4(x_1) \cdots \phi^4(x_K)$
$Z(z_1 \cdots z_N) = \sum_K \left(\frac{-g}{4! \hbar}\right)^K \frac{1}{K!} \int d^d x_1 \cdots d^d x_K \langle \phi(z_1) \cdots \phi(z_N)\, \phi^4(x_1) \cdots \phi^4(x_K) \rangle_{g=0 \text{ theory}}$
N external ϕ
K internal ϕ^4
$\langle \phi(z_1) \cdots \phi(z_N) \rangle_g = \frac{Z(z_1 \cdots z_N)}{Z} = g(z_1 \cdots z_N)$


Use Wick Theorem


Use Wick Theorem
$\langle \phi(z_1) \cdots \phi(z_N)\, \phi^4(x_1) \cdots \phi^4(x_K) \rangle$

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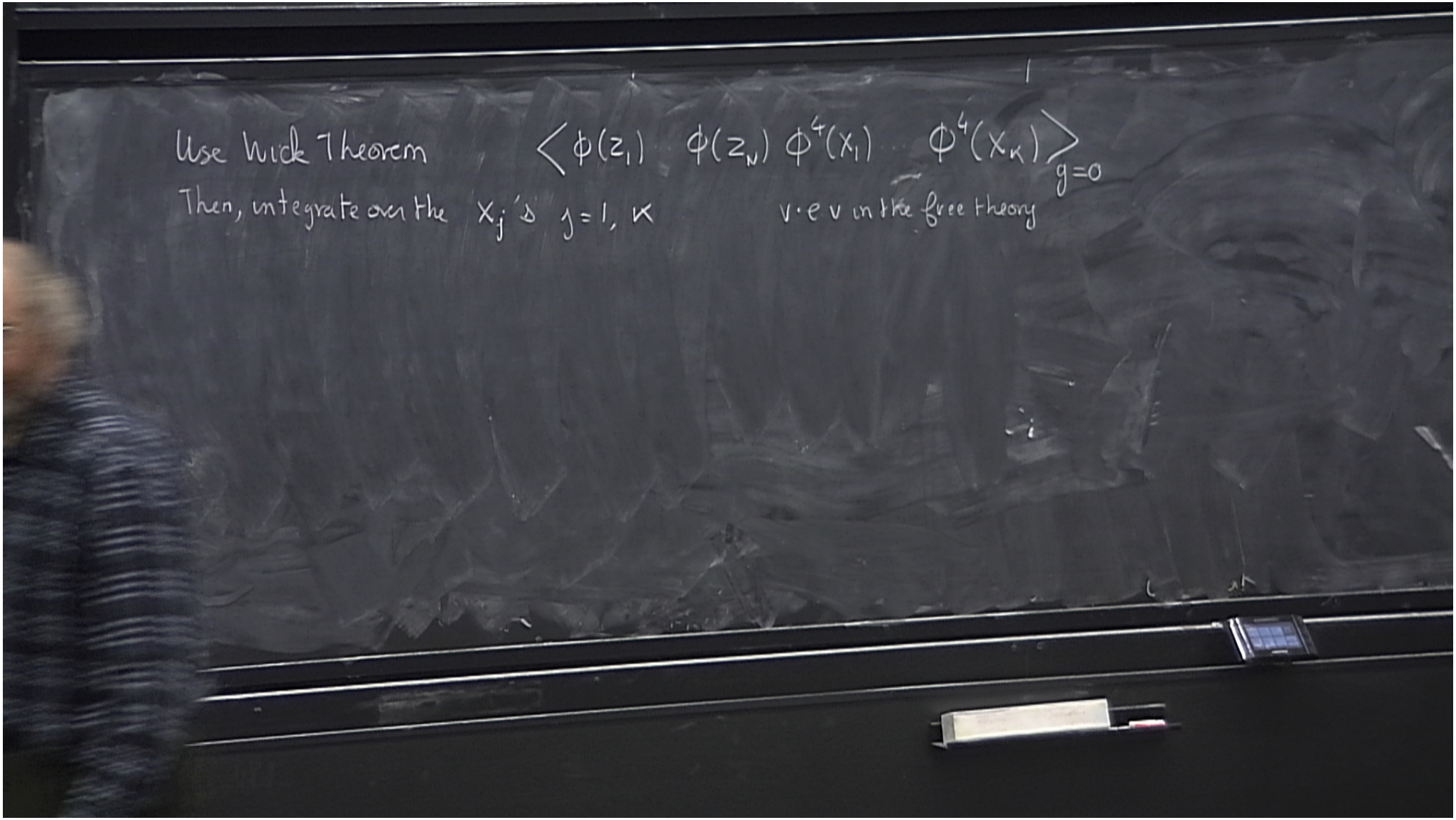
Then, integrate over the x_j's $j = 1, K$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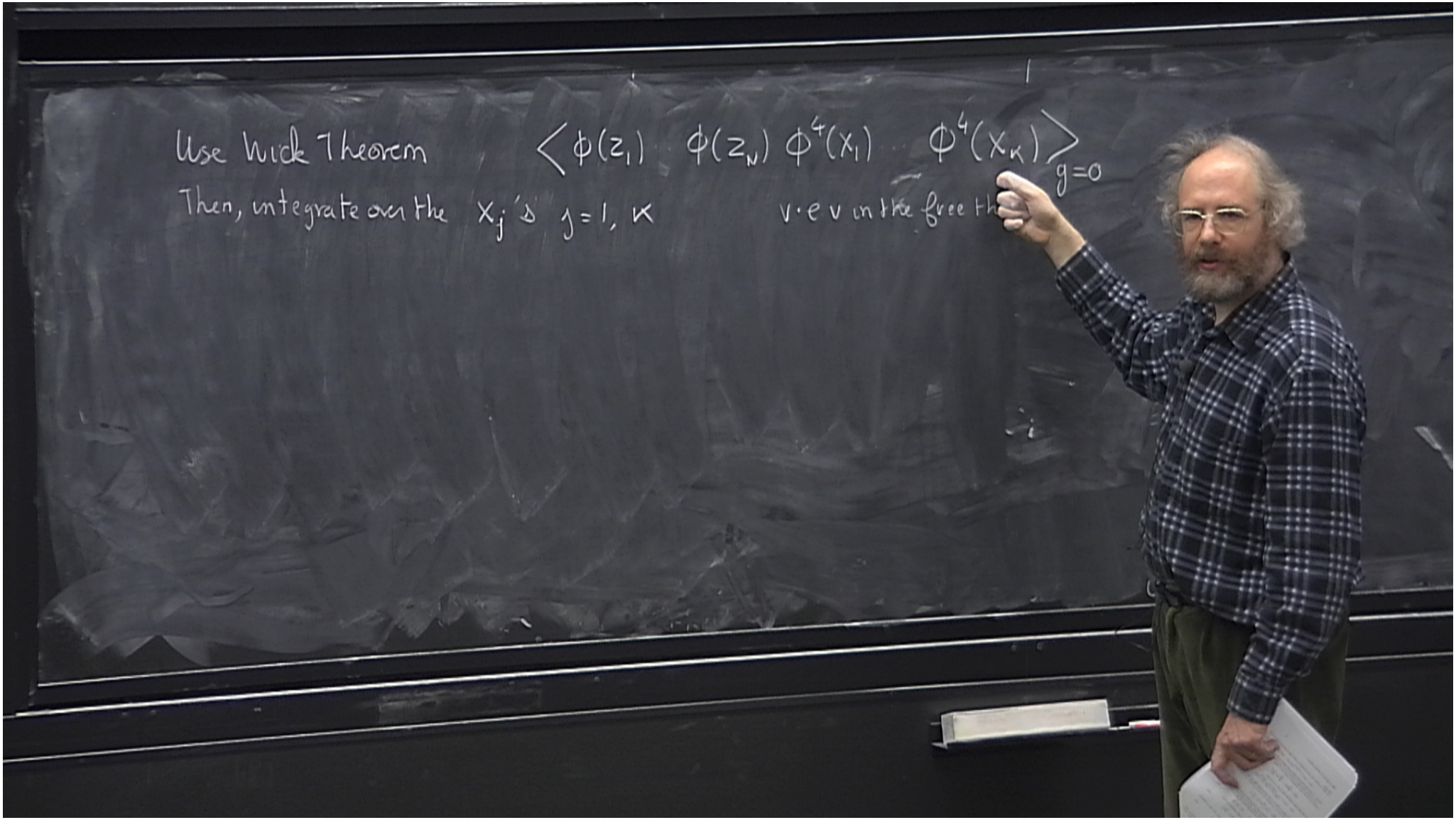
Then, integrate over the x_j's $j=1,K$
v.e.v in the free theory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Then, integrate over the x_j's $j = 1, K$ v.e.v in the free theory

Diagrams with "internal loops"