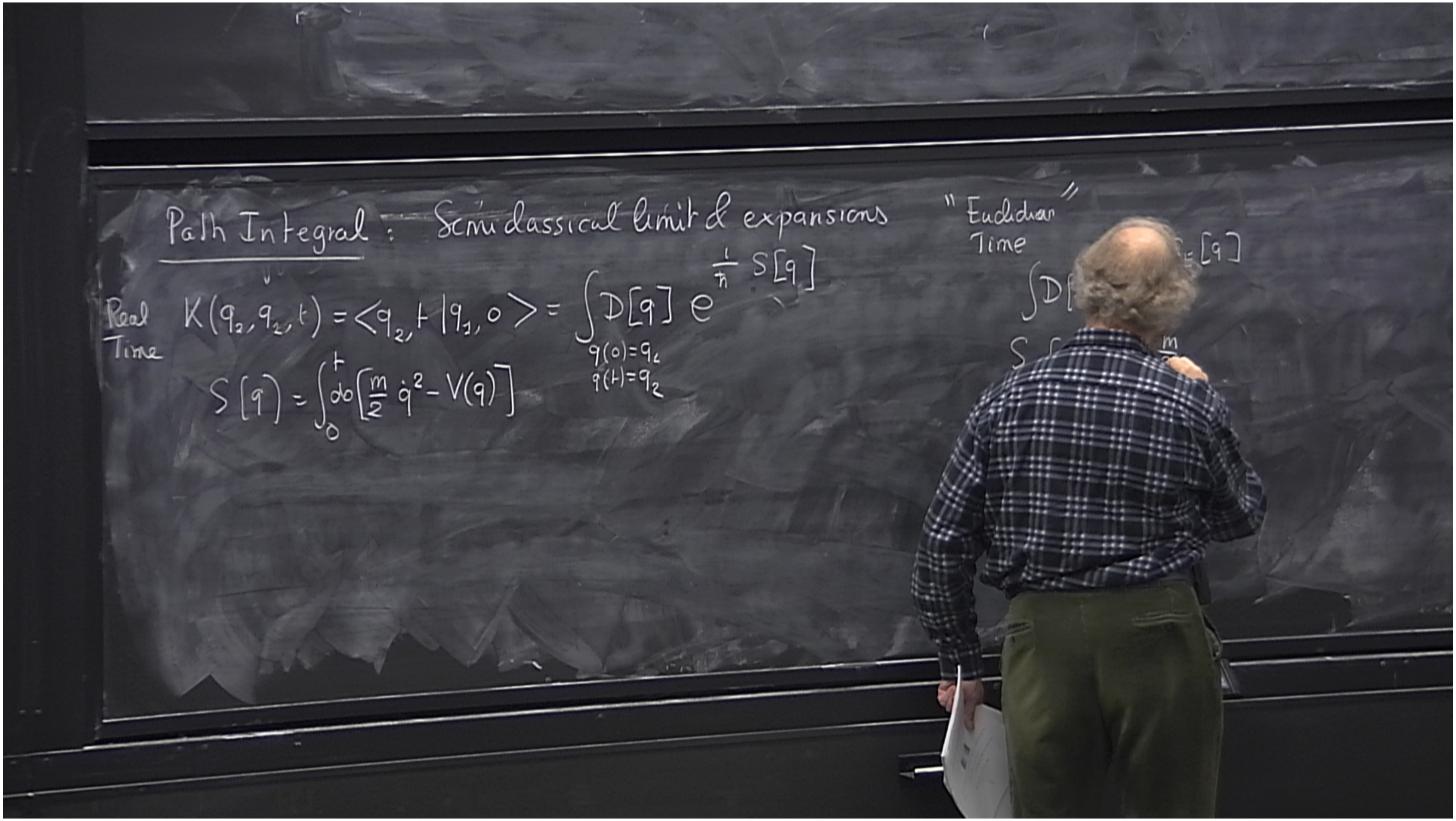
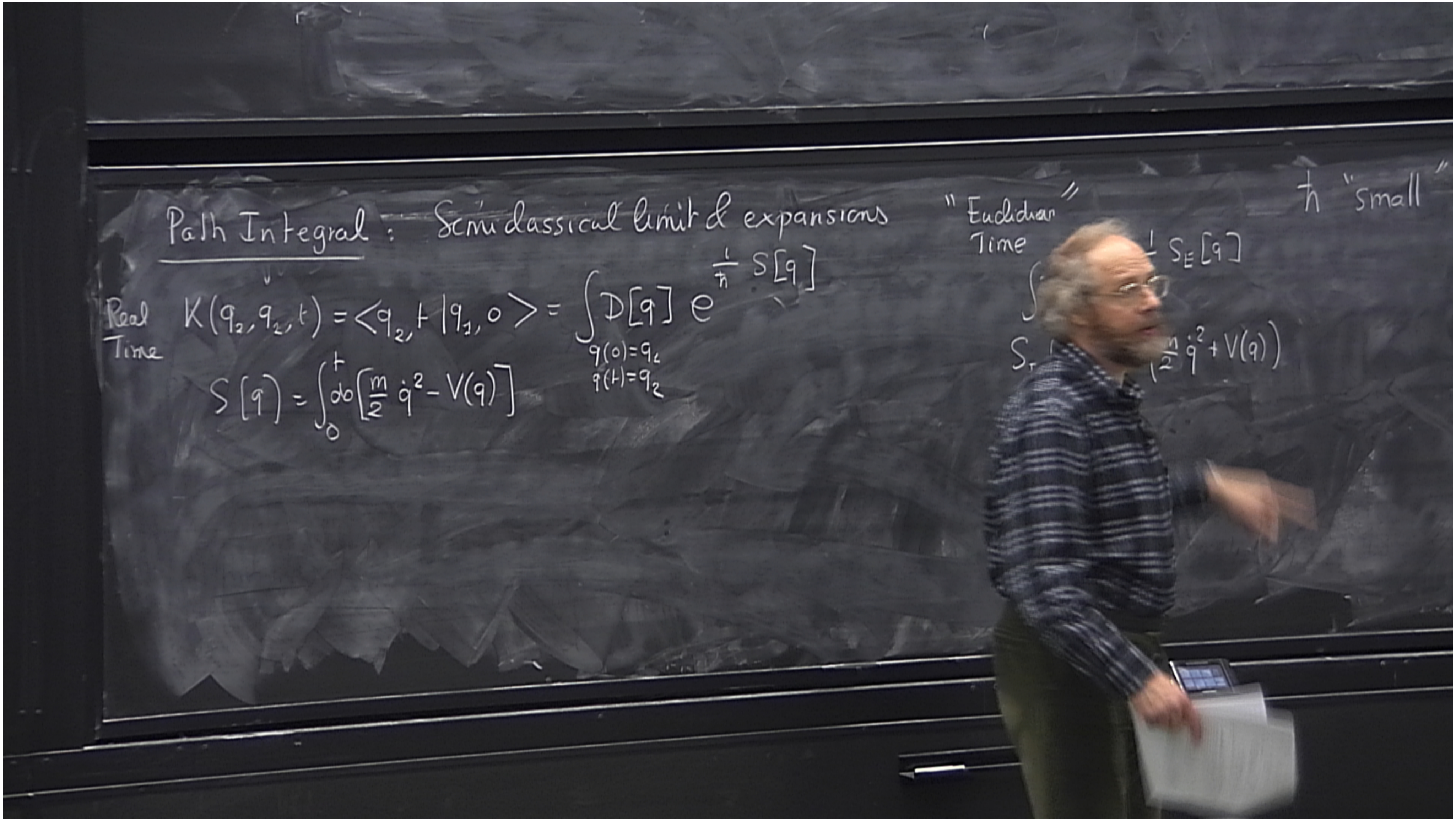
Title: Quantum Field Theory II - Lecture 2

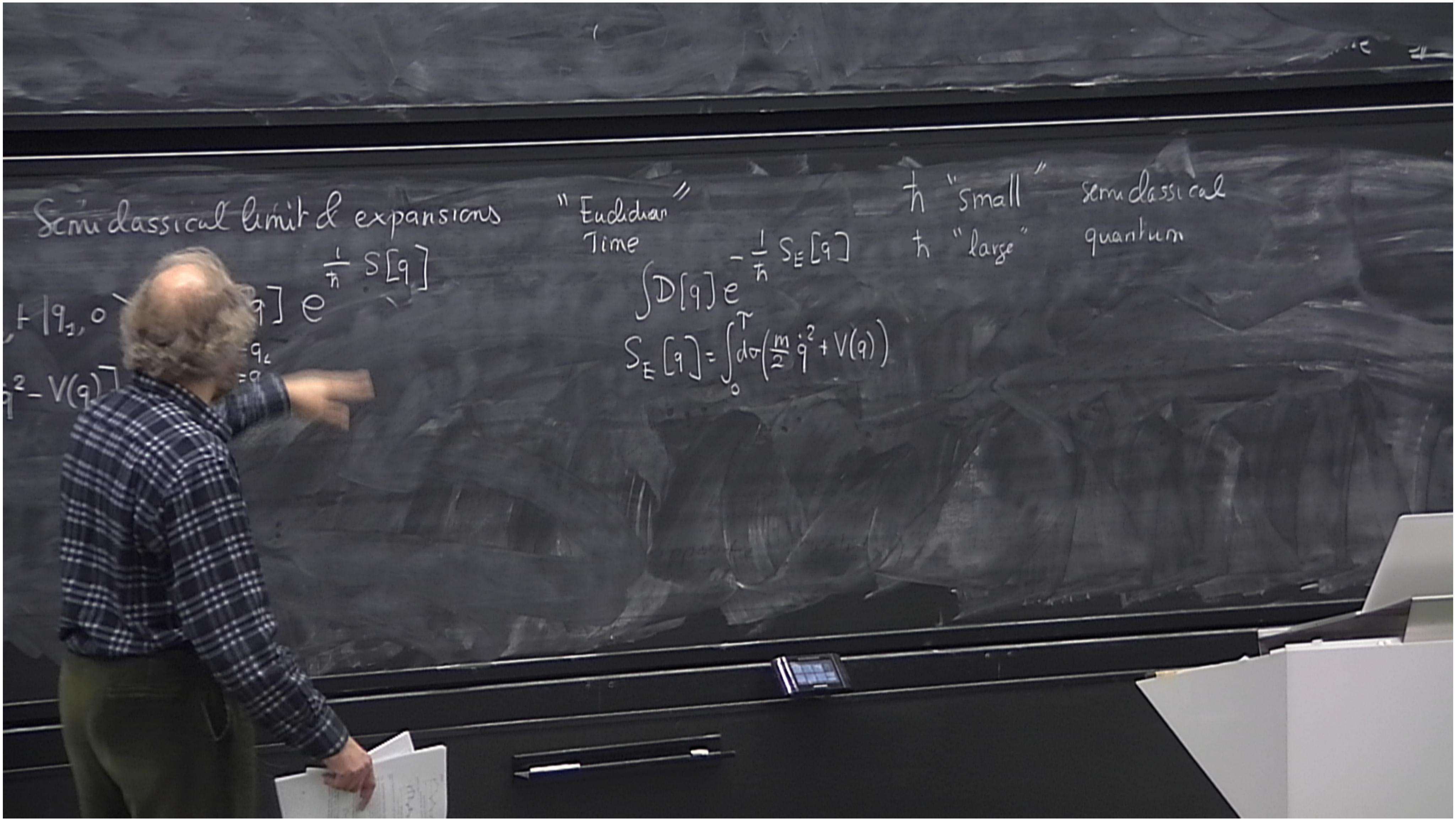
Date: Nov 01, 2011 09:00 AM

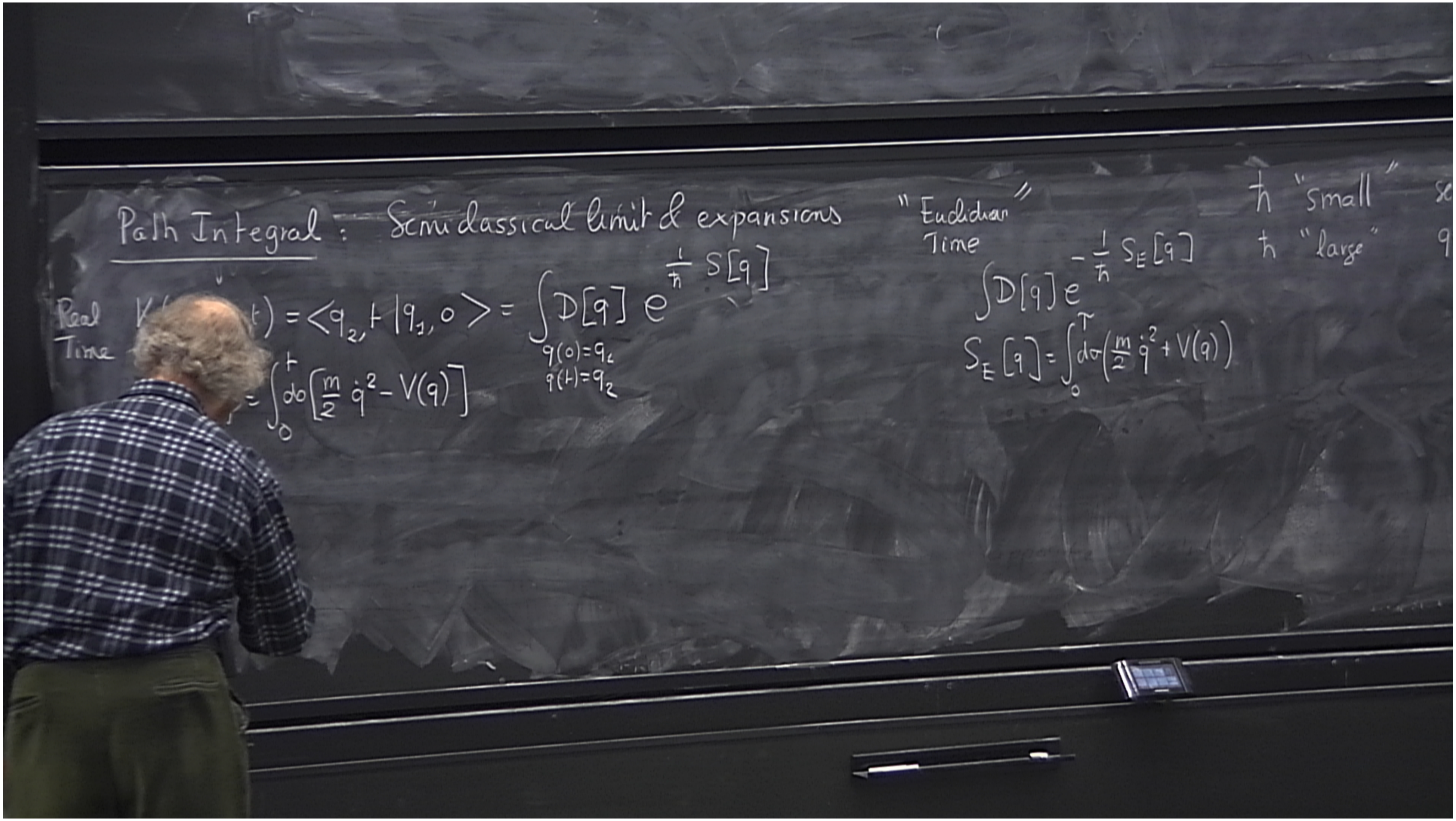
URL: http://pirsa.org/11110004

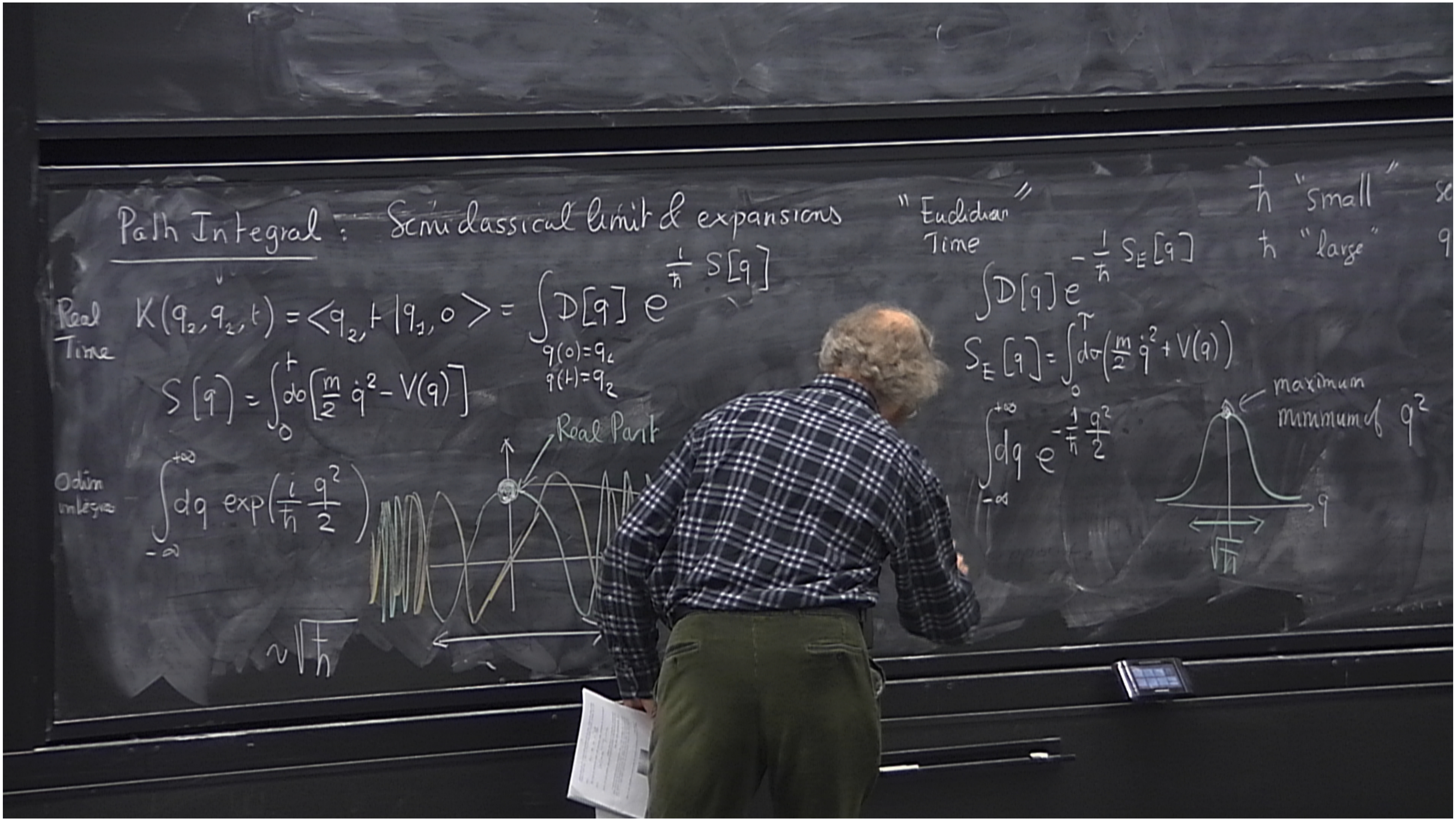
Abstract:

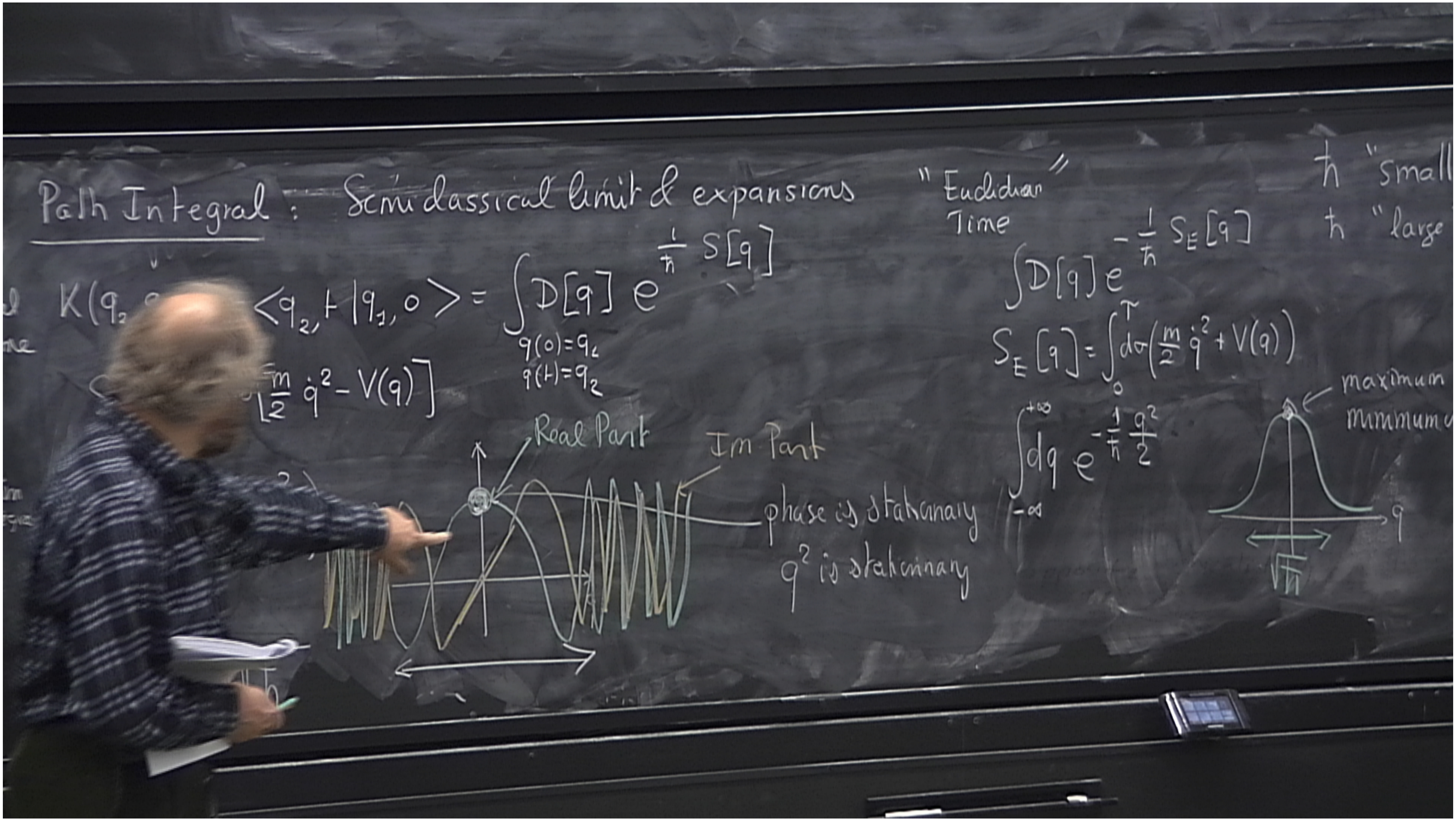

Path Integral : Semiclassical limit & expansions
Real Time
$K(q_2, q_2, t) = \langle q_2, t | q_1, 0 \rangle = \int D[q]\, e^{\frac{i}{\hbar} S[q]}$
$q(0) = q_1$
$q(t) = q_2$
$S[q] = \int_0^t d\sigma \left[\frac{m}{2} \dot{q}^2 - V(q)\right]$
"Euclidean"
Time


Path Integral : Semiclassical limit & expansions
"Euclidean" Time
$\hbar$ "small"
Real Time
$K(q_2, q_2, t) = \langle q_2, t | q_1, 0 \rangle = \int_{\substack{q(0)=q_1 \\ q(t)=q_2}} D[q]\, e^{\frac{i}{\hbar} S[q]}$
$S[q] = \int_0^t d\sigma \left[\frac{m}{2} \dot{q}^2 - V(q) \right]$

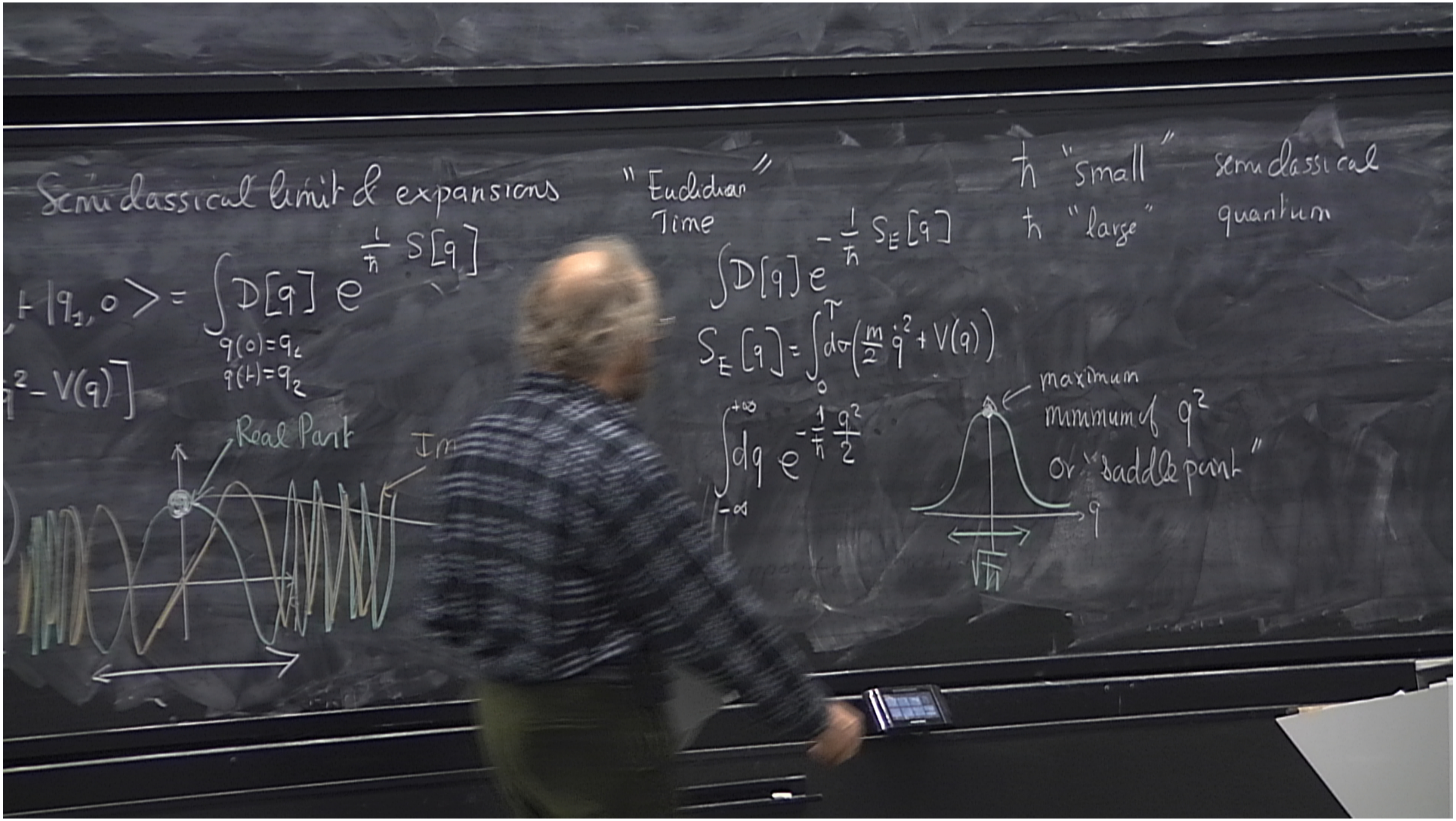

Semiclassical limit & expansions
"Euclidean" Time
ħ "small" semiclassical
ħ "large" quantum
∫D[q] e^{-1/ħ S_E[q]}
S_E[q] = ∫_0^T dσ (m/2 q̇² + V(q))

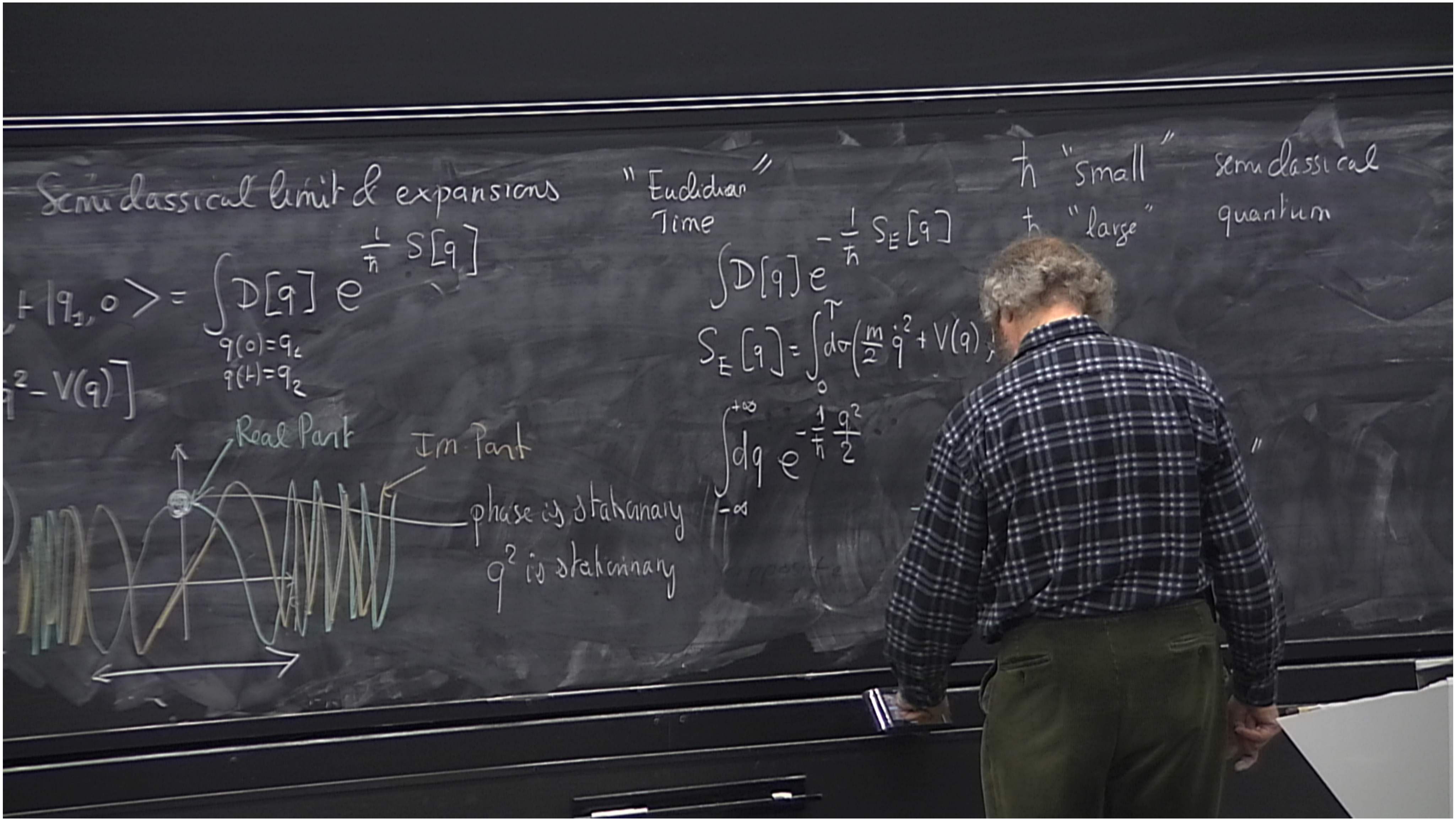

Path Integral: Semiclassical limit & expansions
Real Time
$= \langle q_2, t | q_1, 0 \rangle = \int D[q]\, e^{\frac{i}{\hbar} S[q]}$
$q(0)=q_1$
$q(t)=q_2$
$= \int_0^t d\sigma \left[\frac{m}{2}\dot{q}^2 - V(q)\right]$
"Euclidean" Time
$\int D[q]\, e^{-\frac{1}{\hbar} S_E[q]}$
$S_E[q] = \int_0^T d\sigma \left(\frac{m}{2}\dot{q}^2 + V(q)\right)$
$\hbar$ "small"
$\hbar$ "large"


Path Integral: Semiclassical limit & expansions
"Euclidean" Time
ħ "small"
ħ "large"
Real Time
$K(q_2, q_2, t) = \langle q_2, t | q_1, 0 \rangle = \int D[q]\, e^{\frac{i}{\hbar} S[q]}$
$q(0) = q_1$
$q(t) = q_2$
$S[q] = \int_0^t d\sigma \left[\frac{m}{2}\dot{q}^2 - V(q)\right]$
$\int D[q]\, e^{-\frac{1}{\hbar} S_E[q]}$
$S_E[q] = \int_0^T d\sigma \left(\frac{m}{2}\dot{q}^2 + V(q)\right)$
0 dim integral
$\int_{-\infty}^{+\infty} dq\, \exp\left(\frac{i}{\hbar}\frac{q^2}{2}\right)$
Real Part
$\sim \sqrt{\hbar}$
$\int_{-\infty}^{+\infty} dq\, e^{-\frac{1}{\hbar}\frac{q^2}{2}}$
maximum
minimum of q^2
q
$\sqrt{\hbar}$



Path Integral : Semiclassical limit & expansions
"Euclidian" Time
$\hbar$ "small
$\hbar$ "large
$\langle q_2, t \mid q_1, 0 \rangle = \int D[q]\, e^{\frac{i}{\hbar} S[q]}$
$q(0)=q_1$
$q(t)=q_2$
$\frac{m}{2}\dot{q}^2 - V(q)\Big]$
$\int D[q]\, e^{-\frac{1}{\hbar} S_E[q]}$
$S_E[q] = \int_0^T d\sigma \left(\frac{m}{2}\dot{q}^2 + V(q)\right)$
$\int_{-\infty}^{+\infty} dq\, e^{-\frac{1}{\hbar}\frac{q^2}{2}}$
Real Part
Im Part
phase is stationary
q^2 is stationary
maximum
minimum
$\sqrt{\hbar}$


Semiclassical limit & expansions
"Euclidean" Time
ħ "small" semiclassical
ħ "large" quantum
Real Part
maximum
minimum of q²
or "saddle point"

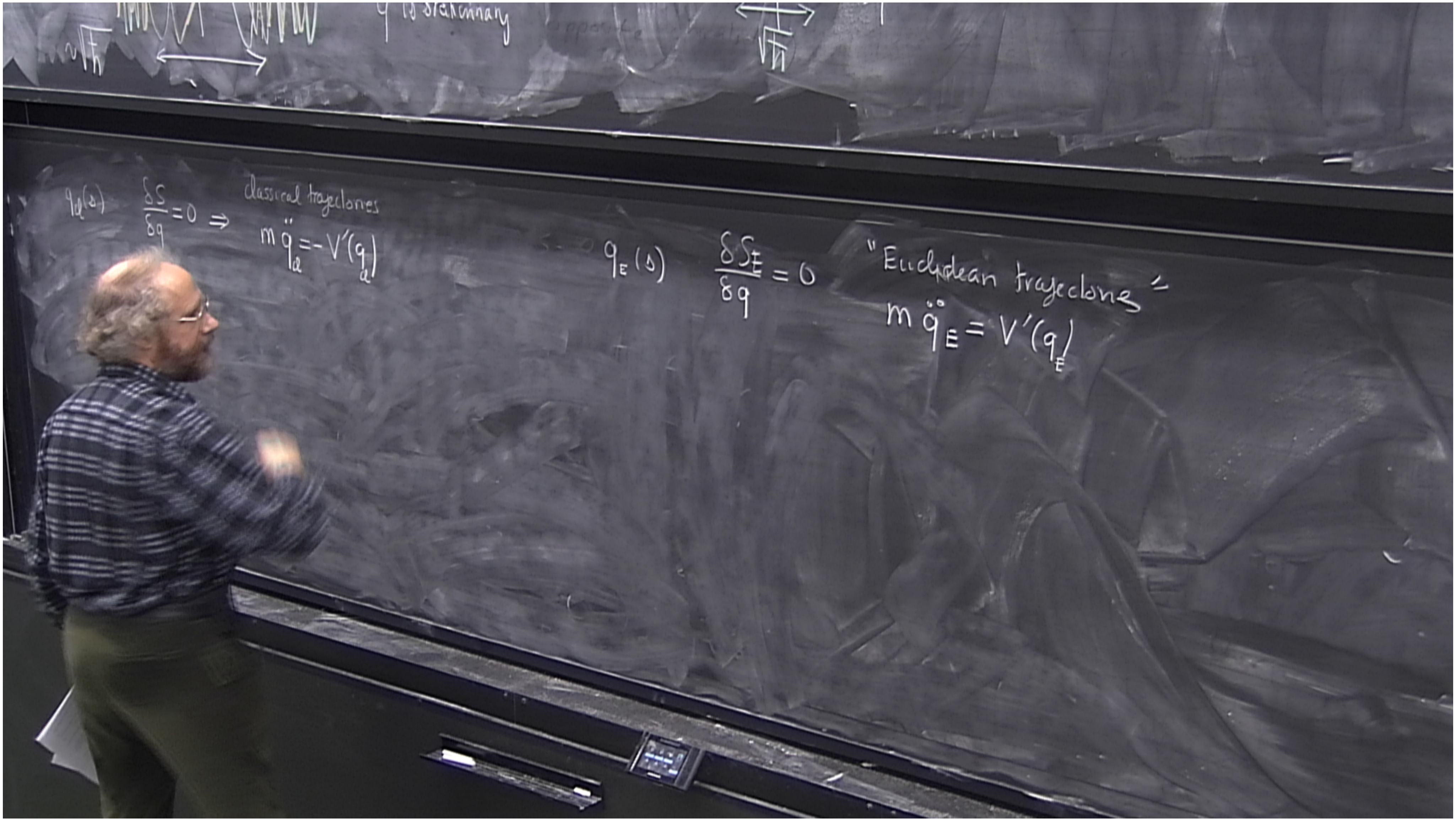


Semiclassical limit & expansions
"Euclidean" Time
ħ "small" semiclassical
ħ "large" quantum
$|q_1, 0\rangle = \int D[q]\, e^{\frac{i}{\hbar} S[q]}$
$q(0)=q_1$
$q(t)=q_2$
$\dot{q}^2 - V(q)]$
$\int D[q]\, e^{-\frac{1}{\hbar} S_E[q]}$
$S_E[q] = \int_0^T d\sigma \left(\frac{m}{2}\dot{q}^2 + V(q)\right)$
$\int_{-\infty}^{+\infty} dq\, e^{-\frac{1}{\hbar}\frac{q^2}{2}}$
Real Part
Im Part
phase is stationary
q^2 is stationary

classical trajectories

$q_{cl}(s)$ $\qquad \dfrac{\delta S}{\delta q} = 0 \Rightarrow m\ddot{q} = -V'(q)$

$q_E(s)$ $\qquad \dfrac{\delta S_E}{\delta q} = ($



$q_{cl}(s)$ $\frac{\delta S}{\delta q} = 0 \Rightarrow$ classical trajectories
$m\ddot{q}_{cl} = -V'(q_{cl})$
$q_E(s)$ $\frac{\delta S_E}{\delta q} = 0$
"Euclidean trajectories"
$m\ddot{q}_E = V'(q_E)$

classical trajectories

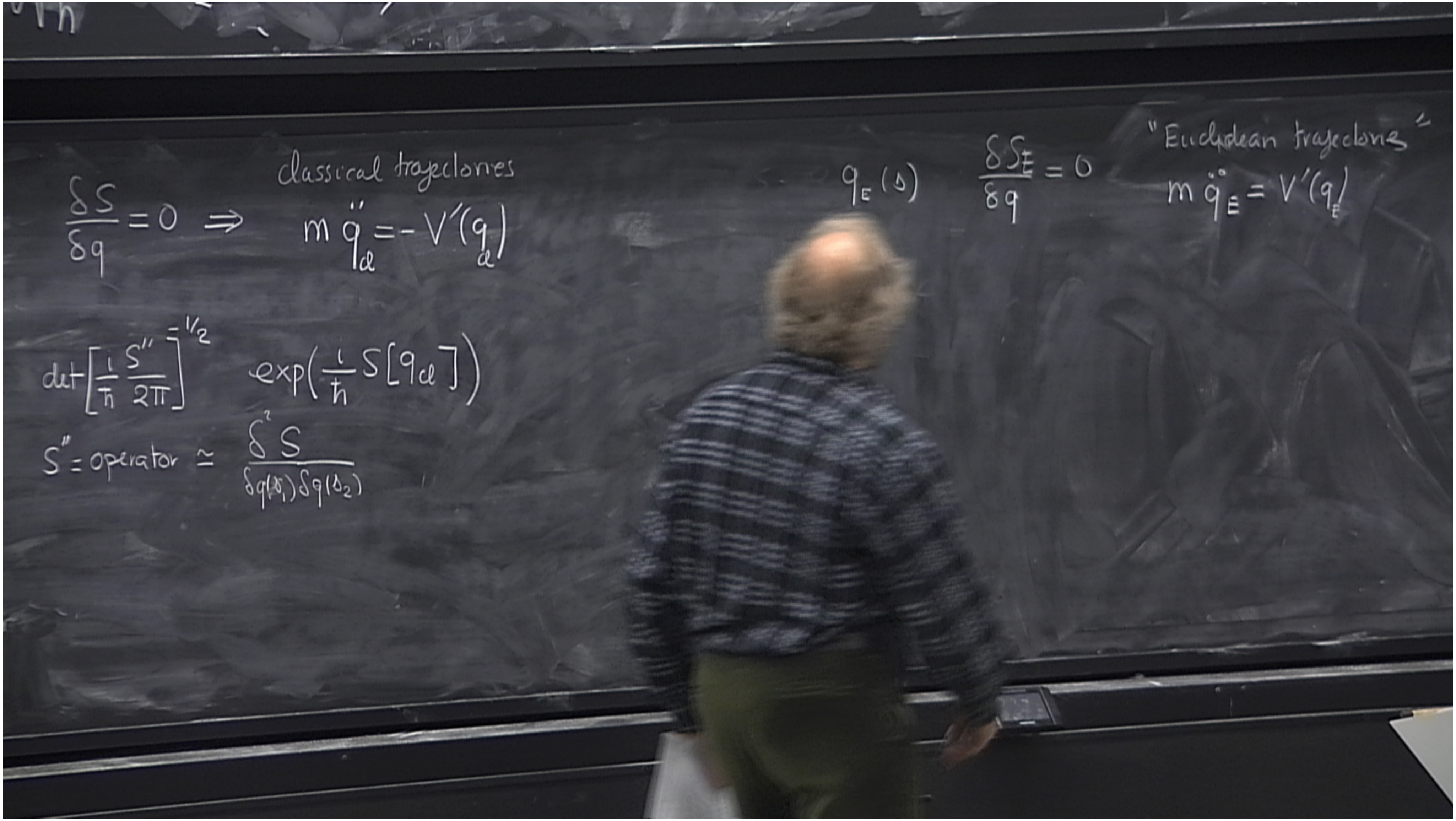
$$\frac{\delta S}{\delta q} = 0 \Rightarrow m\ddot{q}_{cl} = -V'(q_{cl})$$

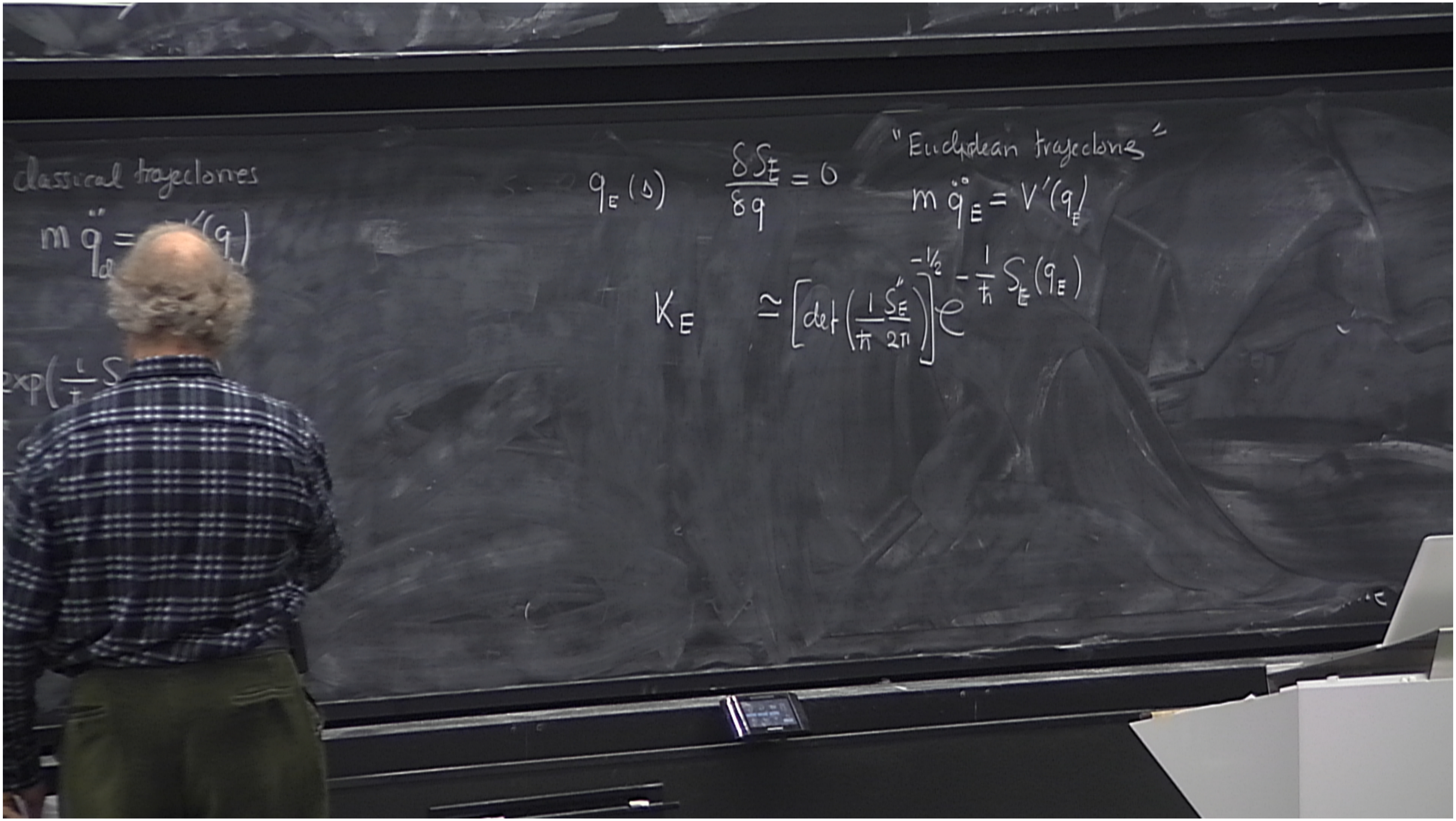
$$\exp\left(\frac{i}{\hbar} S[q_{cl}]\right)$$

$$q_E(s) \qquad \frac{\delta S_E}{\delta q} = 0$$

"Euclidean trajectories"

$$m\ddot{q}_E = V'(q_E)$$


classical trajectories
$\frac{\delta S}{\delta q} = 0 \Rightarrow m\ddot{q}_{cl} = -V'(q_{cl})$
$q_E(s)$
$\frac{\delta S_E}{\delta q} = 0$
"Euclidean trajectories"
$m\ddot{q}_E = V'(q_E)$
$\det\left[\frac{i}{\hbar}\frac{S''}{2\pi}\right]^{-1/2} \exp\left(\frac{i}{\hbar}S[q_{cl}]\right)$
$S'' =$ operator $\simeq \frac{\delta^2 S}{\delta q(s_1)\delta q(s_2)}$


classical trajectories
"Euclidean trajectories"
$q_E(s)$
$\frac{\delta S_E}{\delta q} = 0$
$m\ddot{q}_E = V'(q_E)$
$K_E \simeq \left[\det\left(\frac{1}{\hbar}\frac{S_E''}{2\pi}\right)\right]^{-1/2} e^{-\frac{1}{\hbar}S_E(q_E)}$

$\sim\sqrt{\hbar}$

classical trajectories

$q_{cl}(s)$ $\quad \frac{\delta S}{\delta q} = 0 \Rightarrow m\ddot{q}_{cl} = -V'(q_{cl})$

$q_E(s)$ $\quad \frac{\delta S_E}{\delta q} = 0$

$$K \simeq \left[\det\left(\frac{1}{\hbar}\frac{S''}{2\pi}\right)\right]^{-1/2} \exp\left(\frac{i}{\hbar}S[q_{cl}]\right)$$

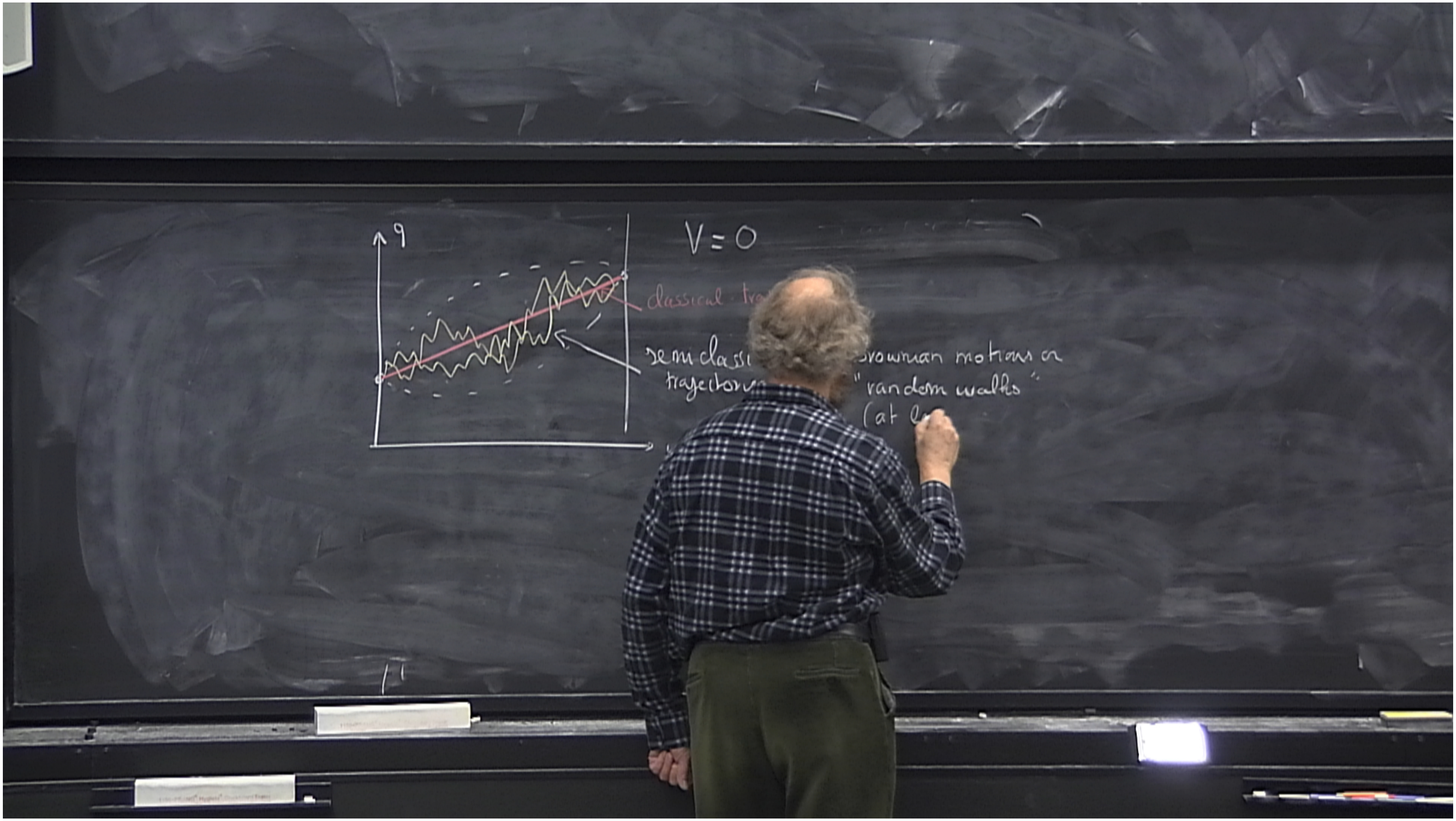
classical trajectory

S'' = operator $\simeq \frac{\delta^2 S}{\delta q(s_1)\delta q(s_2)}$

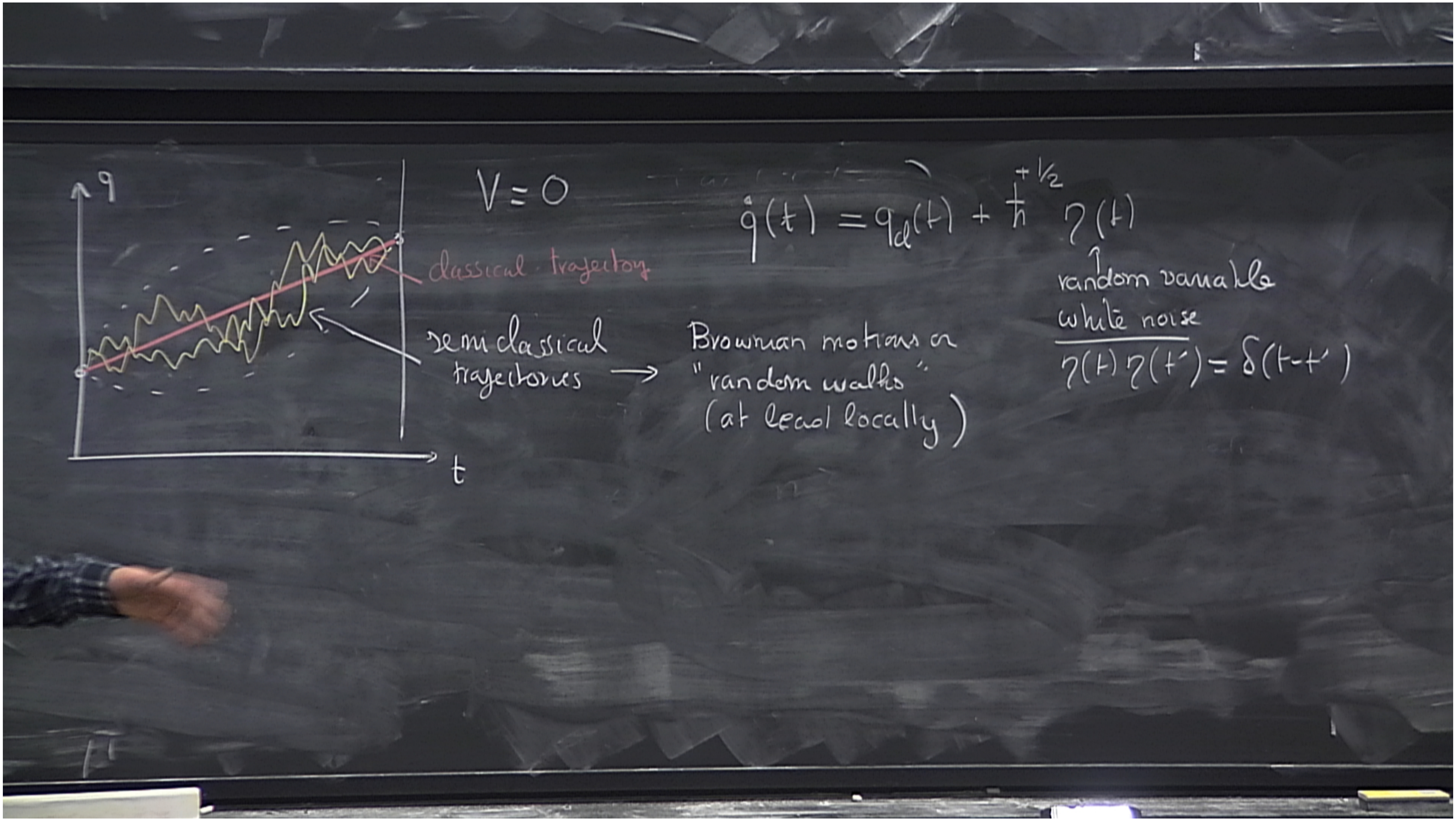
non classical but close to classical trajectories

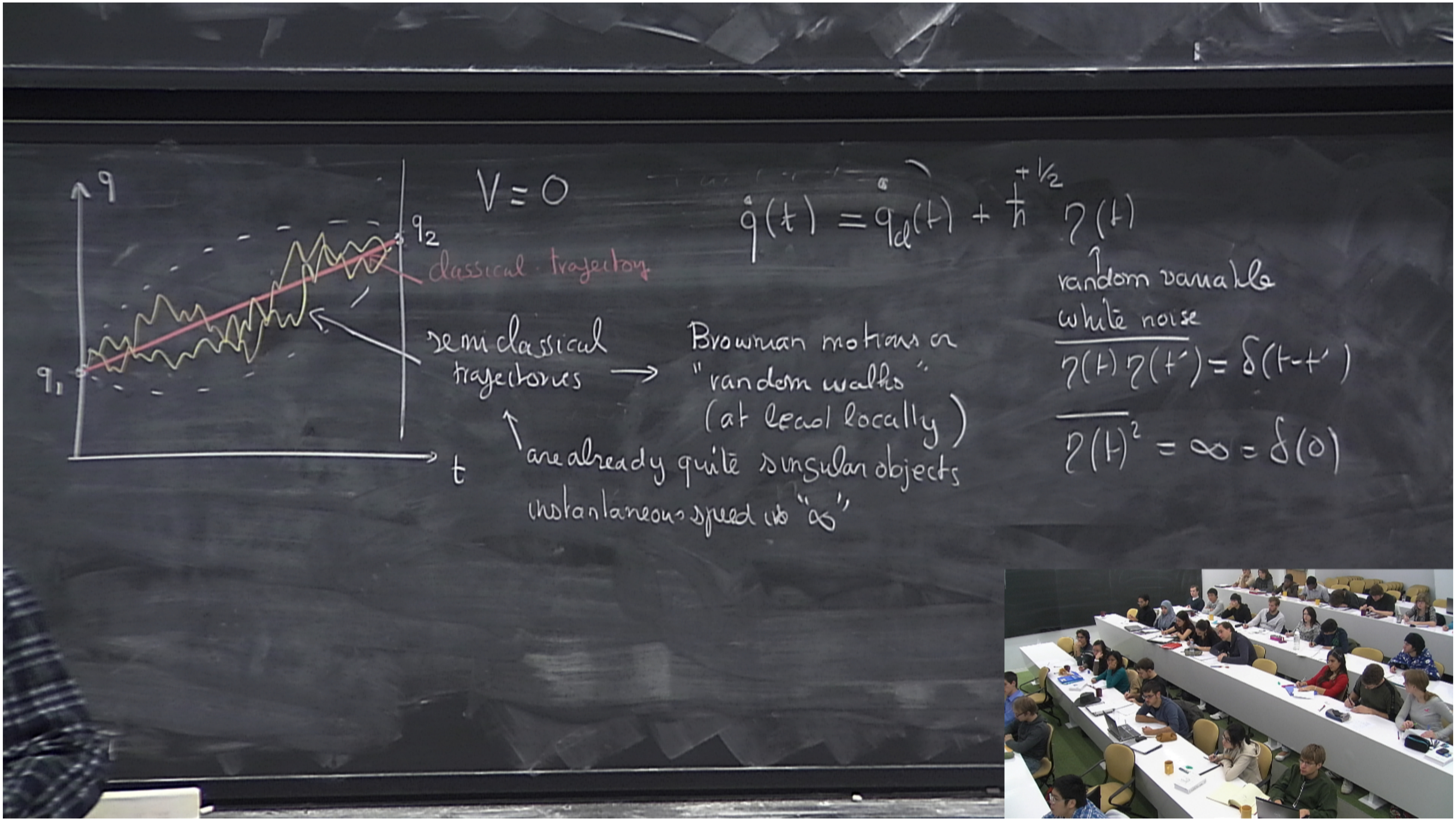
what do they look like

$K_E \simeq \left[\det\left(\frac{1}{\hbar}\frac{S_E''}{2\pi}\right.\right.$

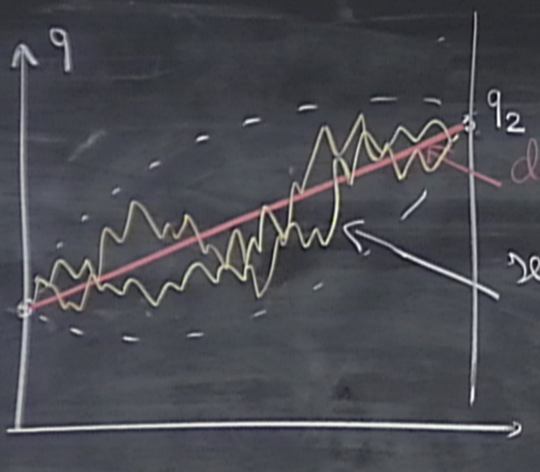


q
V = 0
classical tra
semiclassi
trajectori
brownian motions or
"random walks"
(at l


q
t
V = 0
$\dot{q}(t) = q_{cl}(t) + \hbar^{+1/2}\,\eta(t)$
classical trajectory
semiclassical trajectories
Brownian motions or "random walks" (at least locally)
random variable
white noise
$\overline{\eta(t)\,\eta(t')} = \delta(t-t')$



q
q_2
q_1
t
V = 0
$\dot{q}(t) = \dot{q}_{cl}(t) + \hbar^{+1/2}\, \eta(t)$
classical trajectory
semiclassical trajectories
Brownian motions or "random walks" (at least locally)
are already quite singular objects
instantaneous speed is "∞"
random variable
white noise
$\overline{\eta(t)\eta(t')} = \delta(t-t')$
$\overline{\eta(t)^2} = \infty = \delta(0)$



$V = 0$

semiclassical trajectories $\longrightarrow$ Brownian motions or "random walks" (at least locally)

are already quite singular objects

instantaneous speed is "∞"

$$\dot q(t) = \dot q_{cl}(t) + \hbar^{+1/2}\,\eta(t)$$

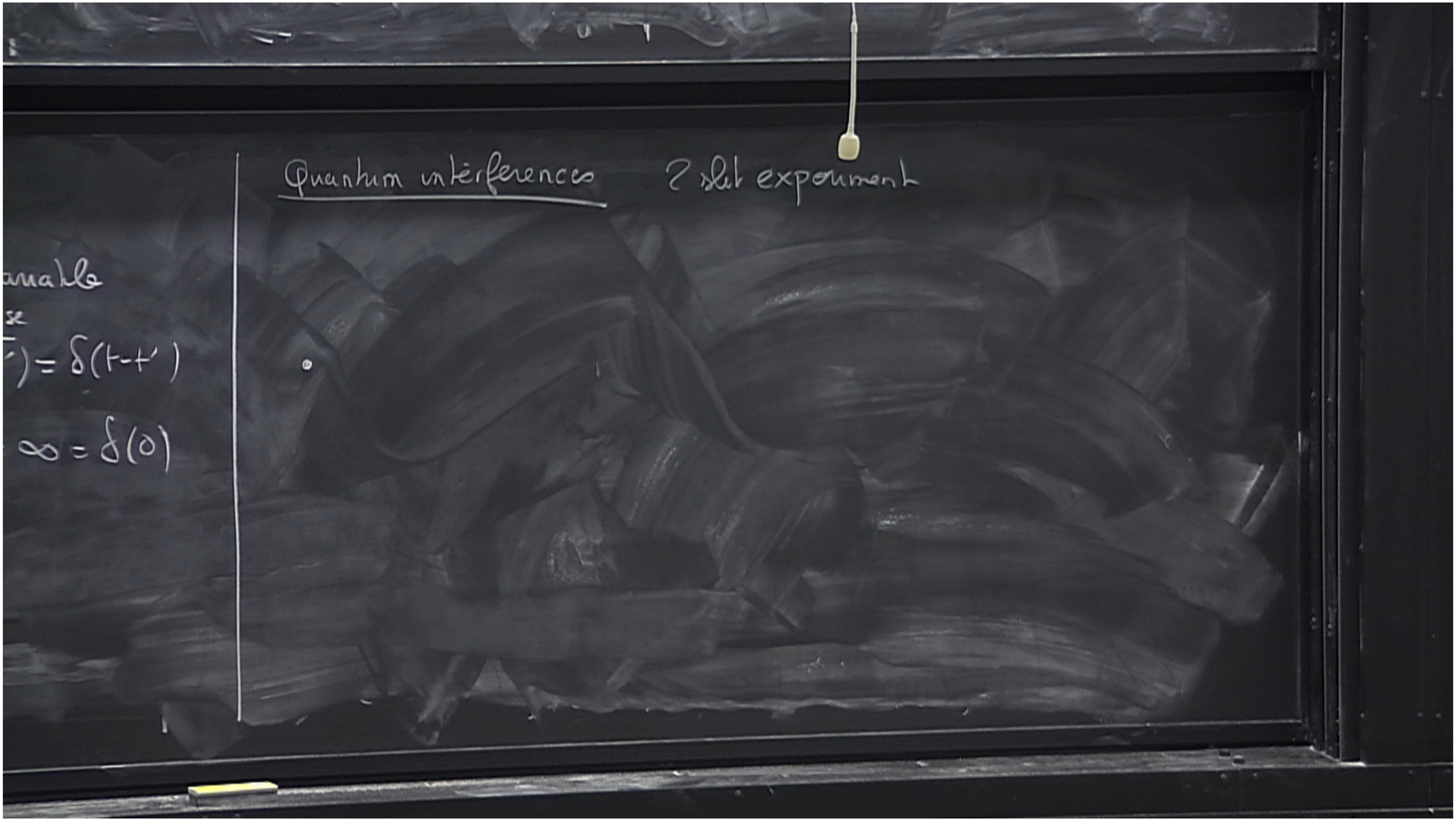
random variable

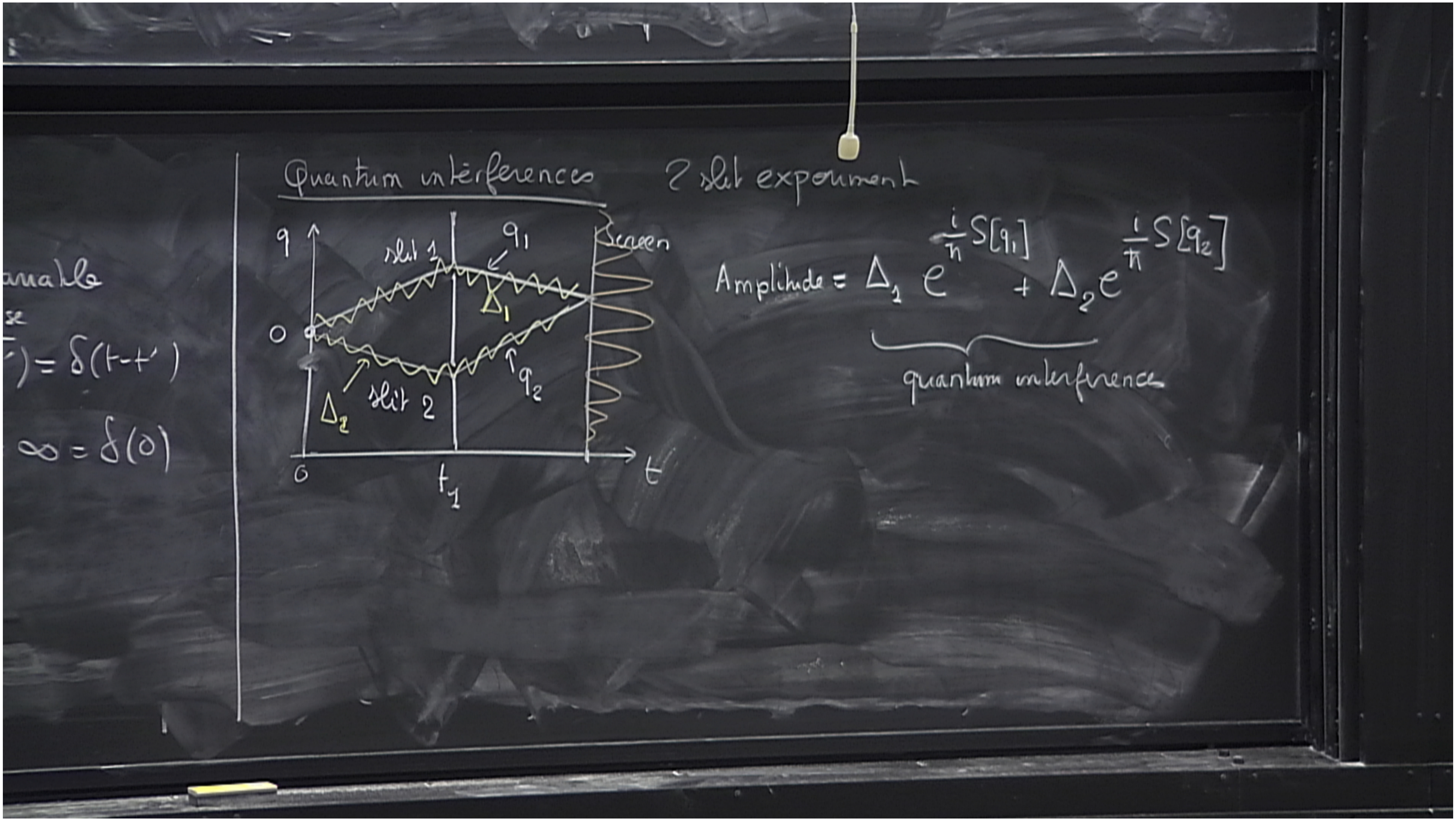
white noise

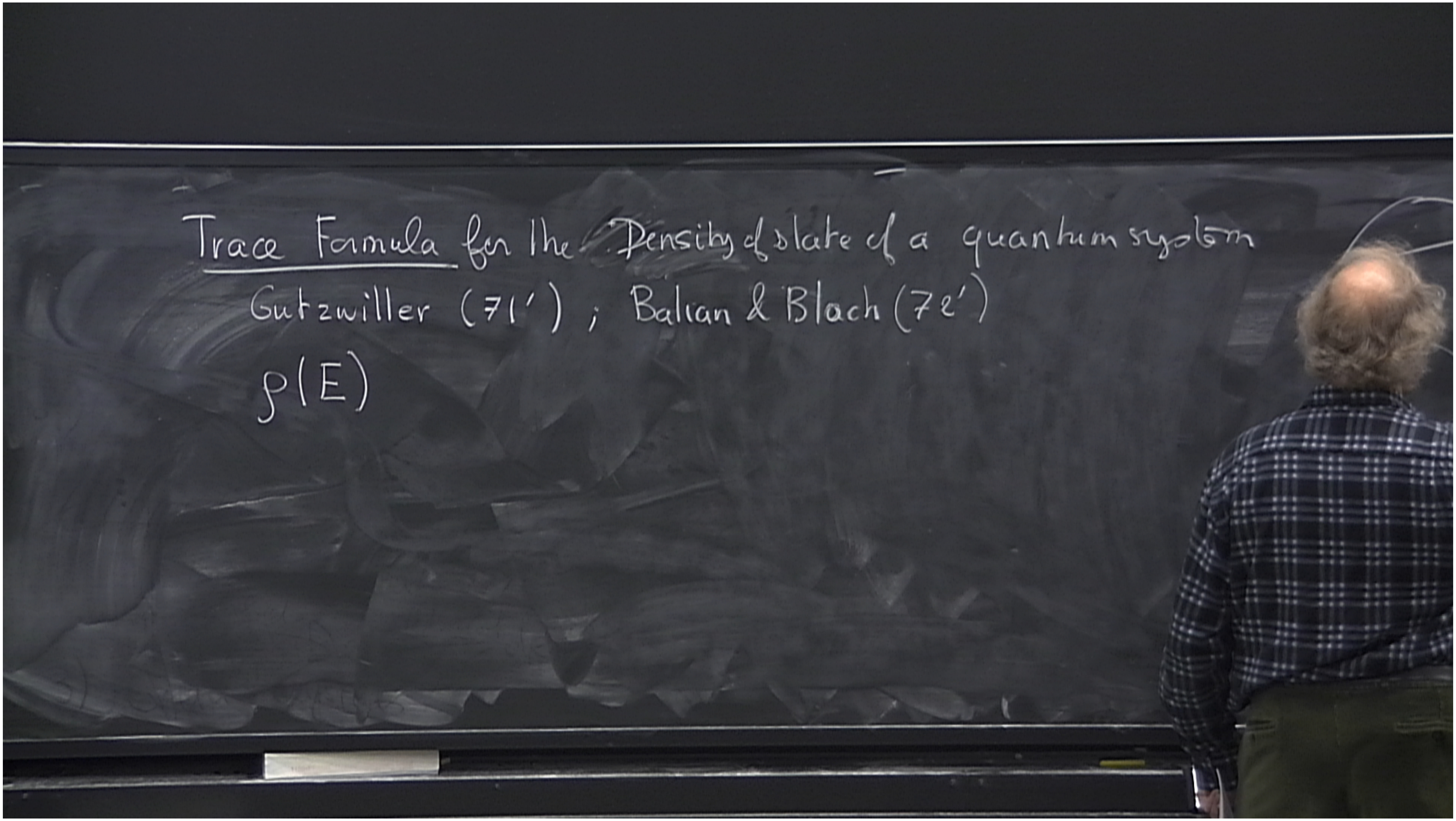
$$\overline{\eta(t)\,\eta(t')} = \delta(t-t')$$

$$\overline{\eta(t)^2} = \infty = \delta(0)$$

Real & Euclidean Time


Quantum interferences
2 slit experiment
ariable
se
) = δ(t-t')
∞ = δ(0)

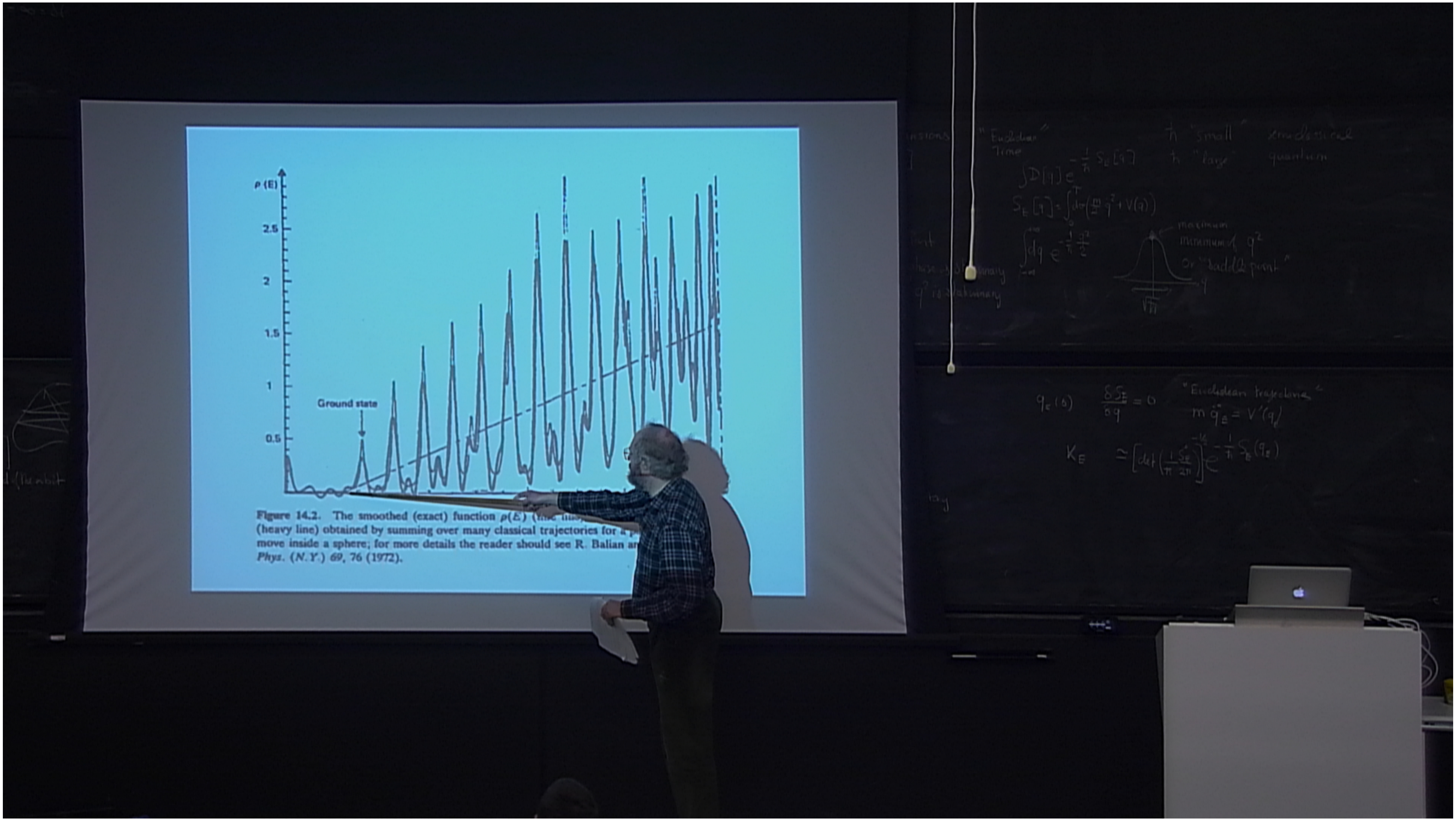

Quantum interferences
2 slit experiment
Amplitude = $\Delta_1 e^{\frac{i}{\hbar}S[q_1]} + \Delta_2 e^{\frac{i}{\hbar}S[q_2]}$
quantum interference
q
slit 1
q_1
Screen
Δ_1
Δ_2
slit 2
q_2
t_1
t
$) = \delta(t-t')$
$\infty = \delta(0)$

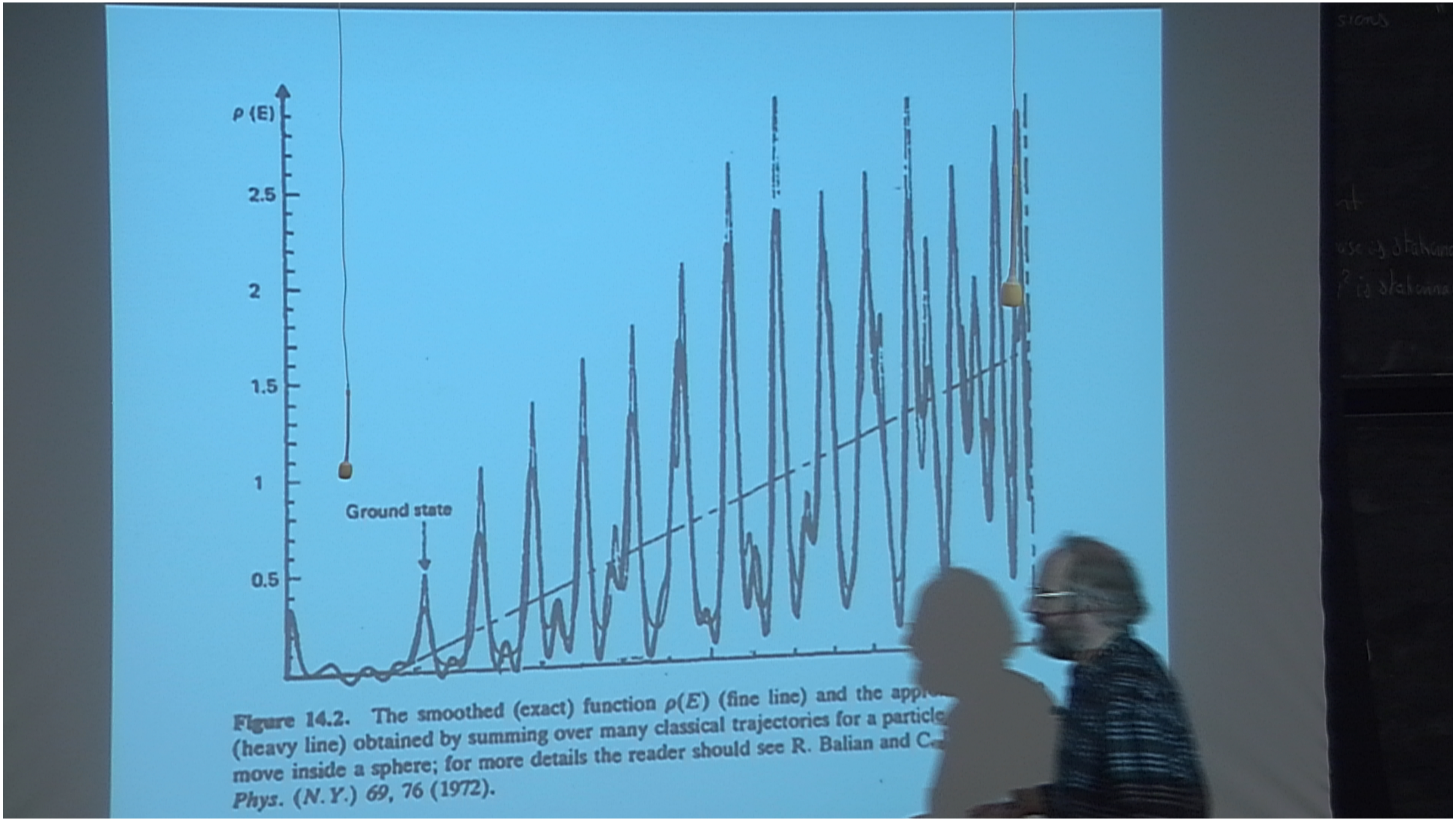

Trace Formula for the Density of state of a quantum system
Gutzwiller (71') ; Balian & Bloch (72')
ρ(E)

Trace Formula for the Density of state of a quantum system

Gutzwiller (71') ; Balian & Bloch (72') $\hbar \to 0$

$$\rho(E) = \rho_{classical}(E) + \sum_{\substack{\text{periodic orbits} \\ \text{of energy } E}} \mathrm{Re}\Big[\underbrace{\Delta_{orbit}(E)}_{\text{contribution of the fluctuations}}\, e^{\frac{i}{\hbar}\left(S_{orbit} + \underbrace{T_{orbit}}_{\text{period of the orbit}} E\right)}\Big]$$


ρ (E)
2.5
2
1.5
1
0.5
Ground state
Figure 14.2. The smoothed (exact) function ρ(E)
(heavy line) obtained by summing over many classical trajectories for a
move inside a sphere; for more details the reader should see R. Balian
Phys. (N.Y.) 69, 76 (1972).


ρ (E)
2.5
2
1.5
1
0.5
Ground state
Figure 14.2. The smoothed (exact) function ρ(E) (fine line) and the
(heavy line) obtained by summing over many classical trajectories for a
move inside a sphere; for more details the reader should see R. Balian and
Phys. (N.Y.) 69, 76 (1972).
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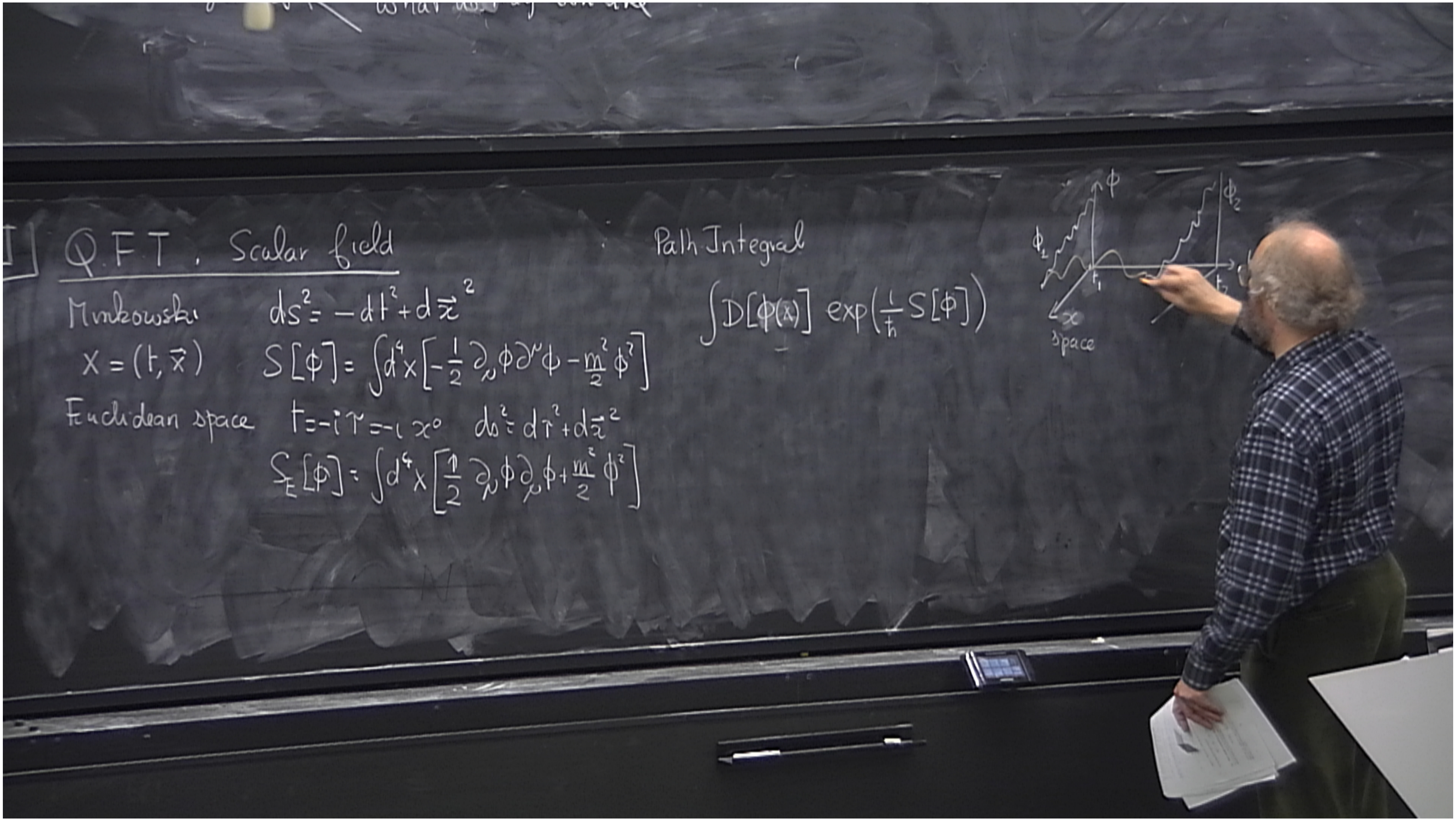
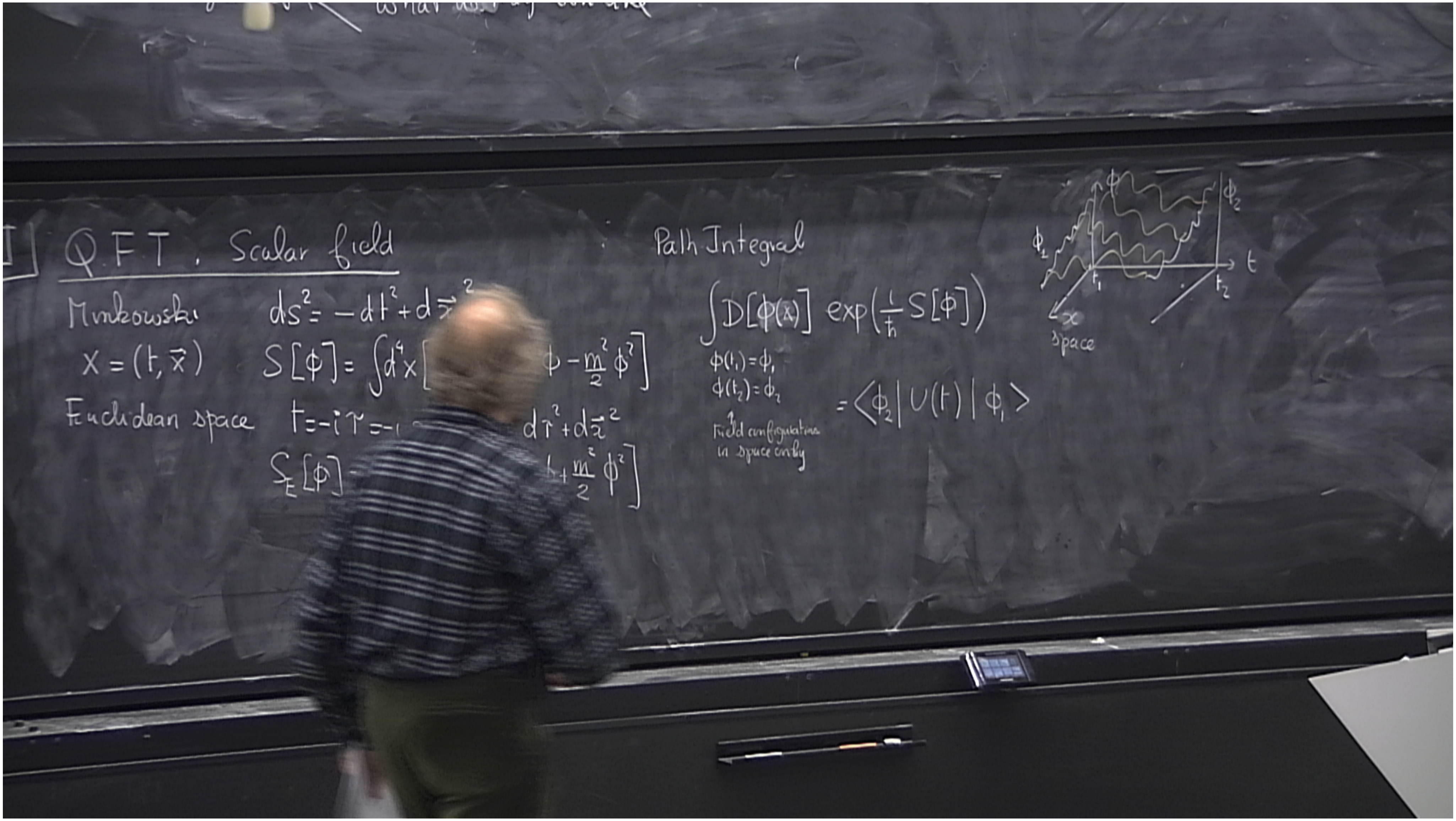
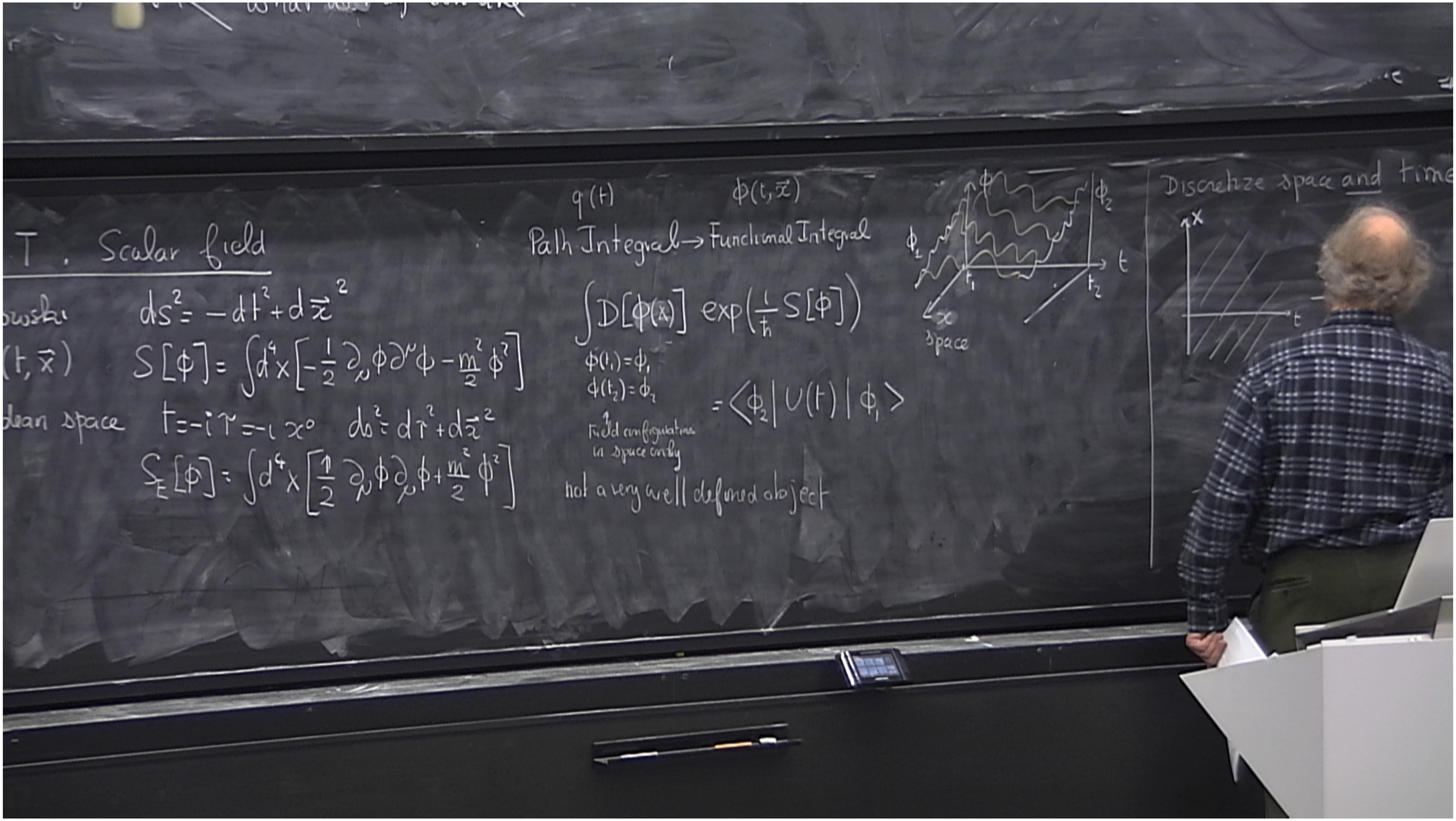
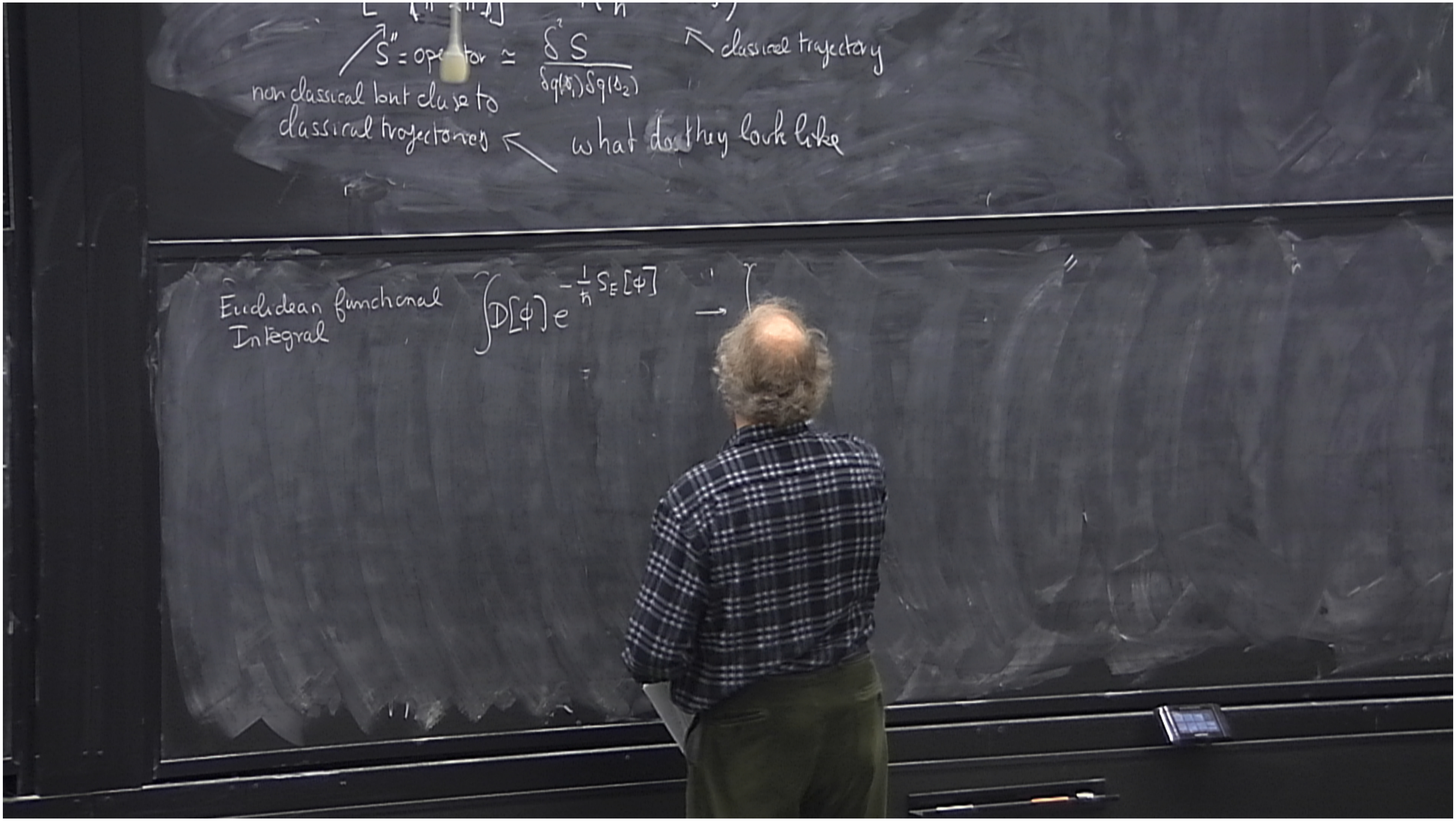
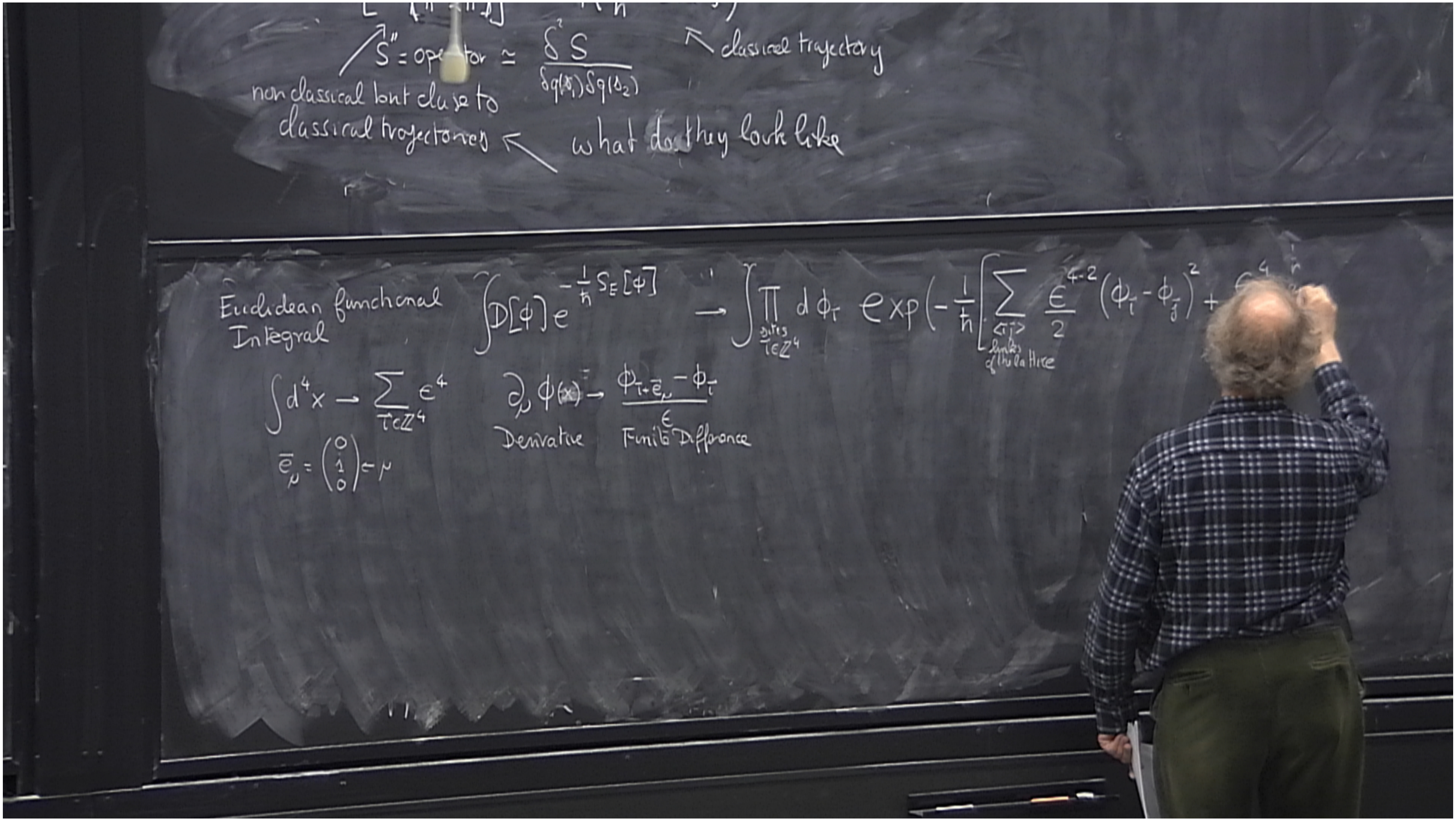
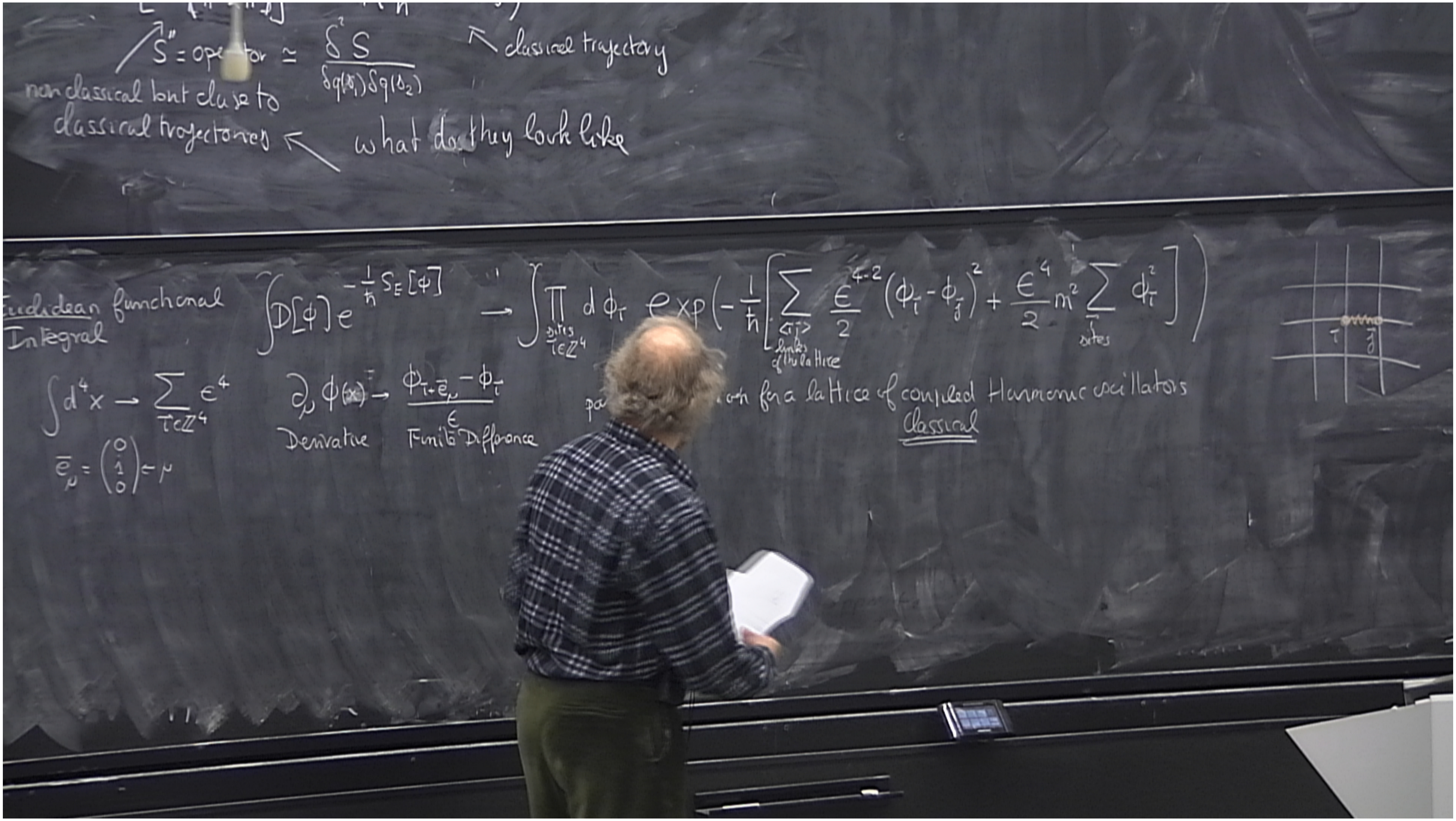

Q.F.T. , Scalar field
Minkowski $ds^2 = -dt^2 + d\vec{x}^2$
$x = (t, \vec{x})$ $S[\phi] = \int d^4x \left[-\frac{1}{2} \partial_\nu \phi \partial^\nu \phi - \frac{m^2}{2} \phi^2 \right]$
Euclidean space $t = -i\tau = -i x^0$ $ds^2 = d\tau^2 + d\vec{x}^2$
$S_E[\phi] = \int d^4x \left[\frac{1}{2} \partial_\nu \phi \partial_\nu \phi + \frac{m^2}{2} \phi^2 \right]$
Path Integral
$\int D[\phi(x)] \exp\left(\frac{i}{\hbar} S[\phi]\right)$
ϕ
ϕ_2
t_1
t_2
x
space

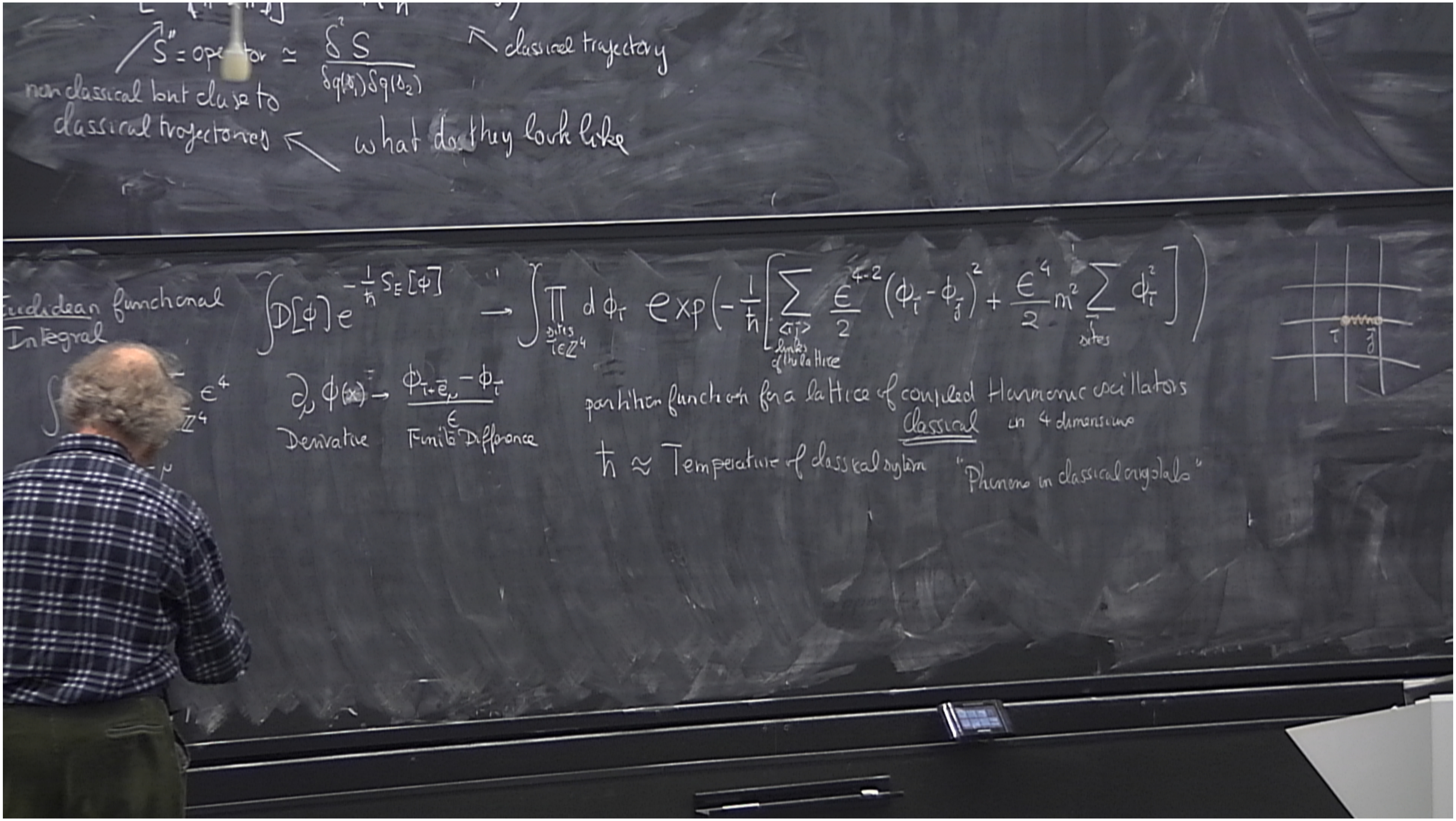

QFT, Scalar field
Minkowski
$ds^2 = -dt^2 + d\vec{x}^2$
$X = (t, \vec{x})$
$S[\phi] = \int d^4x$
Euclidean space
$S_E[\phi]$
Path Integral
$\int D[\phi(\vec{x})] \exp\left(\frac{i}{\hbar} S[\phi]\right)$
$\phi(t_1) = \phi_1$
$\phi(t_2) = \phi_2$
$= \langle \phi_2 | U(t) | \phi_1 \rangle$
field configurations in space only
space
t


T . Scalar field
$ds^2 = -dt^2 + d\vec{x}^2$
$S[\phi] = \int d^4x \left[-\frac{1}{2} \partial_\nu \phi \partial^\nu \phi - \frac{m^2}{2} \phi^2 \right]$
$t = -i\tau = -i x^0 \quad ds^2 = d\tau^2 + d\vec{x}^2$
$S_E[\phi] = \int d^4x \left[\frac{1}{2} \partial_\nu \phi \partial_\nu \phi + \frac{m^2}{2} \phi^2 \right]$
$q(t)$
$\phi(t, \vec{x})$
Path Integral → Functional Integral
$\int D[\phi(x)] \exp\left(\frac{i}{\hbar} S[\phi]\right)$
$\phi(t_1) = \phi_1$
$\phi(t_2) = \phi_2$
field configuration in space only
$= \langle \phi_2 | U(t) | \phi_1 \rangle$
not a very well defined object
space
Discretize space and time

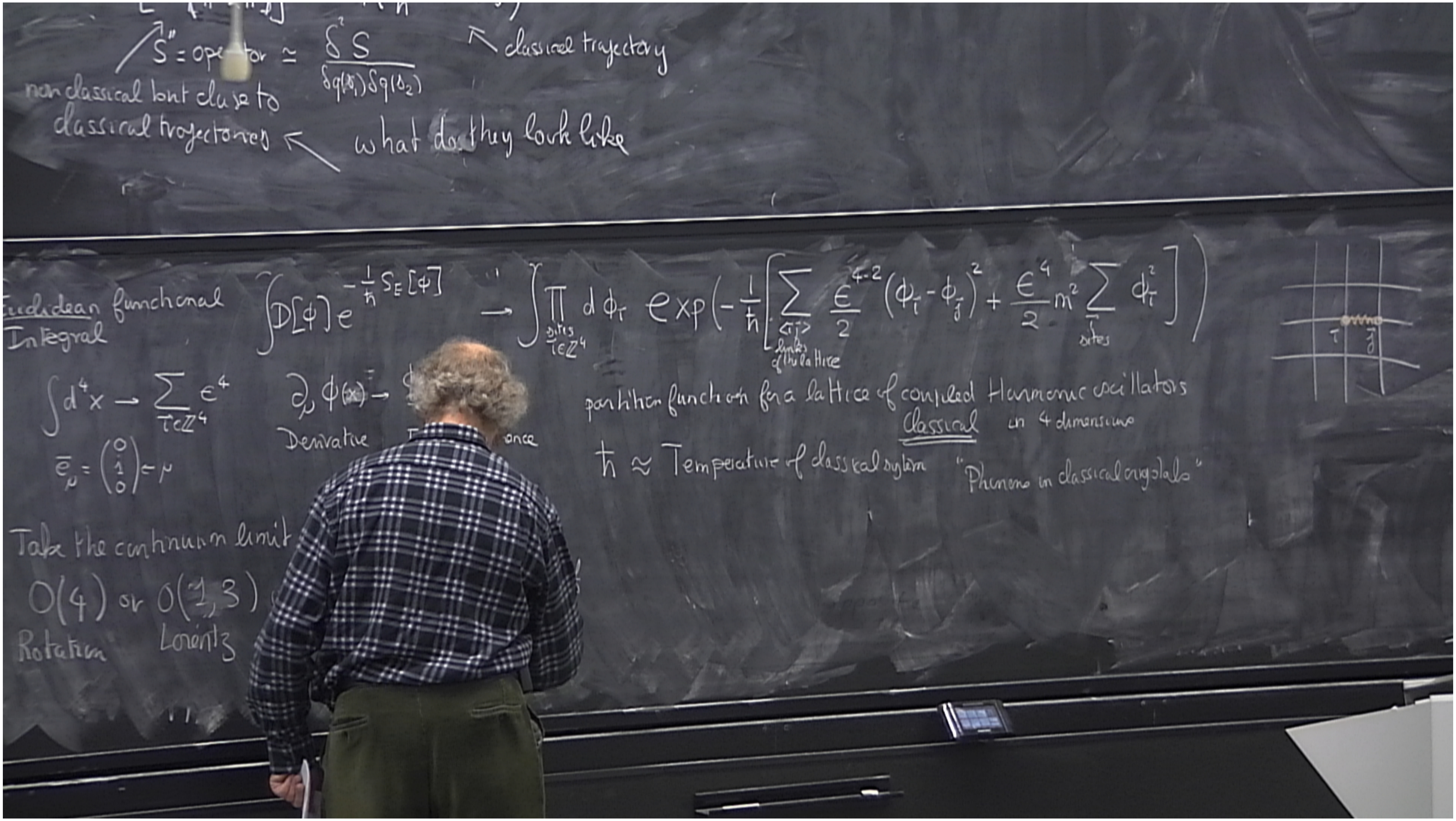

classical trajectory
non classical but close to
classical trajectories
what do they look like
Euclidean functional
Integral
$\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]} \rightarrow$


"S''" = operator ≃ $\frac{\delta^2 S}{\delta q(s_1)\delta q(s_2)}$
classical trajectory
non classical but close to classical trajectories
what do they look like
Euclidean functional Integral
$\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]} \longrightarrow \int \prod_{\substack{\text{sites} \\ i \in \mathbb{Z}^4}} d\phi_i \, \exp\left(-\frac{1}{\hbar}\left[\sum_{\langle ij \rangle} \frac{\epsilon^{4-2}}{2}(\phi_i - \phi_j)^2 + \epsilon^4\right.\right.$
links of the lattice
$\int d^4x \to \sum_{i \in \mathbb{Z}^4} \epsilon^4$
$\partial_\mu \phi(x) \to \frac{\phi_{i+\bar{e}_\mu} - \phi_i}{\epsilon}$
Derivative
Finite Difference
$\bar{e}_\mu = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \leftarrow \mu$


$S'' = \text{operator} \simeq \frac{\delta^2 S}{\delta q(t_1)\delta q(t_2)}$
classical trajectory
non classical but close to classical trajectories
what do they look like
Euclidean functional Integral
$\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]} \longrightarrow \int \prod_{\substack{\text{sites}\\ i\in\mathbb{Z}^4}} d\phi_i \, \exp\left(-\frac{1}{\hbar}\left[\sum_{\langle ij\rangle} \frac{\epsilon^{4-2}}{2}(\phi_i-\phi_j)^2 + \frac{\epsilon^4}{2} m^2 \sum_{i} \phi_i^2\right]\right)$
links of the lattice
sites
$\int d^4x \to \sum_{i\in\mathbb{Z}^4} \epsilon^4$
$\partial_\mu \phi(x) \to \frac{\phi_{i+\bar{e}_\mu} - \phi_i}{\epsilon}$
Derivative
Finite Difference
$\bar{e}_\mu = \begin{pmatrix}0\\ \vdots\\ 1\\ 0\end{pmatrix} \leftarrow \mu$
for a lattice of coupled Harmonic oscillators
Classical



$S'' =$ operator $\simeq \frac{\delta^2 S}{\delta q(s_1)\,\delta q(s_2)}$
classical trajectory
non classical but close to classical trajectories
what do they look like
Euclidean functional Integral
$\int D[\phi]\, e^{-\frac{1}{\hbar} S_E[\phi]} \longrightarrow \int \prod_{\substack{\text{sites}\\ i\in\mathbb{Z}^4}} d\phi_i \, \exp\left(-\frac{1}{\hbar}\left[\sum_{\langle ij\rangle} \frac{\epsilon^{4-2}}{2}(\phi_i-\phi_j)^2 + \frac{\epsilon^4}{2} m^2 \sum_{i} \phi_i^2\right]\right)$
links of the lattice
sites
$\partial_\mu \phi(x) \to \frac{\phi_{i+\vec{e}_\mu} - \phi_i}{\epsilon}$
Derivative
Finite Difference
partition function for a lattice of coupled Harmonic oscillators
Classical in 4 dimensions
$\hbar \approx$ Temperature of classical system
"Phonons in classical crystals"

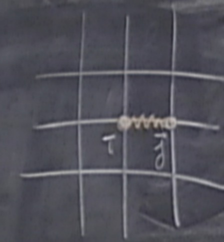

classical trajectory
non classical but close to classical trajectories
what do they look like
Euclidean functional Integral
sites
links of the lattice
Derivative
partition function for a lattice of coupled Harmonic oscillators
Classical in 4 dimensions
Temperature of classical system
"Phonons in classical crystals"
Take the continuum limit
O(4) or O(1,3)
Rotation
Lorentz

$\vec{S}'' =$ operator $\simeq \dfrac{\delta^2 S}{\delta q(t_1)\,\delta q(t_2)}$ ↖ classical trajectory

non classical but close to classical trajectories ← what do they look like

Euclidean functional Integral

$$\int D[\phi]\, e^{-\frac{1}{\hbar}S_E[\phi]} \longrightarrow \int \prod_{\substack{\text{sites}\\ \vec{i}\in\mathbb{Z}^4}} d\phi_{\vec{i}}\, \exp\left(-\frac{1}{\hbar}\left[\sum_{\substack{\langle \vec{i}\,\vec{j}\rangle\\ \text{links of the lattice}}} \frac{\epsilon^{4-2}}{2}(\phi_{\vec{i}}-\phi_{\vec{j}})^2 + \frac{\epsilon^4}{2} m^2 \sum_{\substack{\vec{i}\\ \text{sites}}} \phi_{\vec{i}}^2\right]\right)$$



$\int d^4x \to \sum_{\vec{i}\in\mathbb{Z}^4}\epsilon^4$

$\partial_\mu \phi(x) \to \dfrac{\phi_{\vec{i}+\vec{e}_\mu}-\phi_{\vec{i}}}{\epsilon}$

Derivative — Finite Difference

$\vec{e}_\mu = \begin{pmatrix}0\\1\\0\end{pmatrix} \leftarrow \mu$

partition function for a lattice of coupled Harmonic oscillators <u>Classical</u> in 4 dimensions

$\hbar \approx$ Temperature of classical system "Phonons in classical crystals"

Take the continuum limit $\epsilon \to 0$, recover Lorentz & Poincaré invariance (or Euclidean invariance)

$O(4)$ or $O(1,3)$ is explicitly broken — Symmetries of the hypercubic lattice

Rotation — Lorentz

Trace Formula for the Density of state of a quantum system

Gutzwiller (71') ; Balian & Bloch (72') $\hbar \to 0$

$$\rho(E) = \rho_{classical}(E) + \sum_{\substack{\text{periodic orbits}\\ \text{of energy } E}} \mathrm{Re}\left[\Delta_{orbit}(E)\, e^{\frac{i}{\hbar}\left(S_{orbit} + T_{orbit} E\right)}\right]$$

period of the orbit

contribution of the fluctuations

D = 1

$q(t)$

Random Walk

Functions

$\phi_{cl}(t,x) = 0$ $\phi_{typical}(t,x)$ not a function but a distributions

NEXT TIME: Correlation function of the Field

$$\langle \phi(x)\phi(y)\rangle \longmapsto \langle 0|\, T\left[\phi(x)\phi(y)\right] |0\rangle = \text{Feynman Propagator}$$

Euclidean Time

Real Time

Time ordered Product.

vaccum expectation value

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vaccum expectation value

$$\simeq |x-y|^{-\frac{d-2}{2}}$$

$|x-y| \to 0$

d = dimension of space time $\quad d > 2$