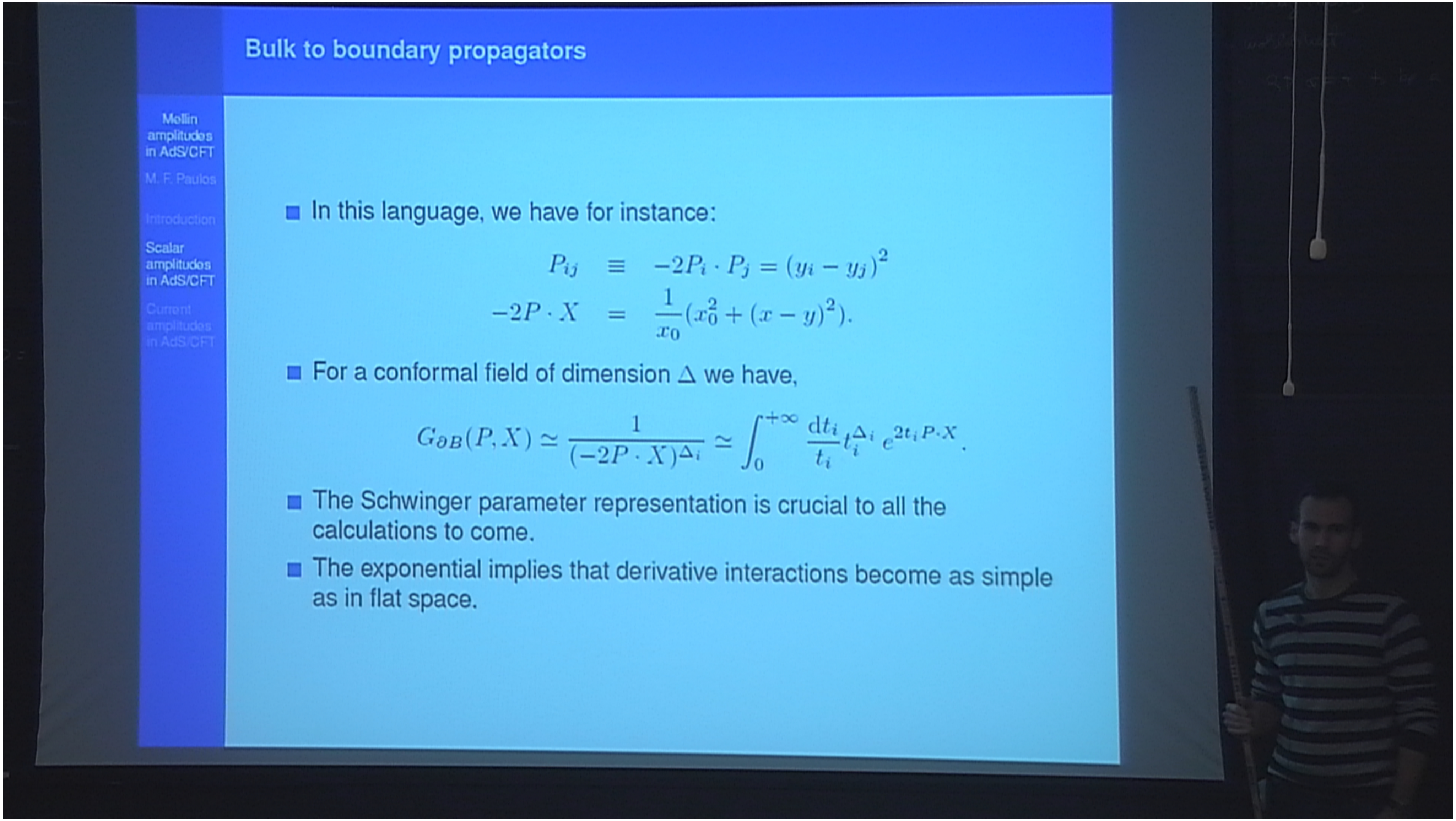
Title: Mellin Amplitudes in AdS/CFT

Date: Nov 22, 2011 02:00 PM

URL: http://pirsa.org/11110000

Abstract: We investigate the use of the embedding formalism and the Mellin transform in the calculation of tree-level conformal correlation functions in AdS/CFT.
We evaluate 5- and 6-point Mellin amplitudes in phi^3 theory and even a 12-pt diagram in phi^4 theory, enabling us to conjecture a set of Feynman rules for scalar Mellin amplitudes. We also show how to use the same combination of Mellin transform and embedding formalism for amplitudes involving fields with spin. The complicated tensor structures which usually arise can be written as certain operators acting as projectors on much simpler index structures - essentially the same ones appearing in a flat space amplitude. Using these methods we are able to evaluate a four-point current diagram with current exchange in Yang-Mills theory.


Bulk to boundary propagators
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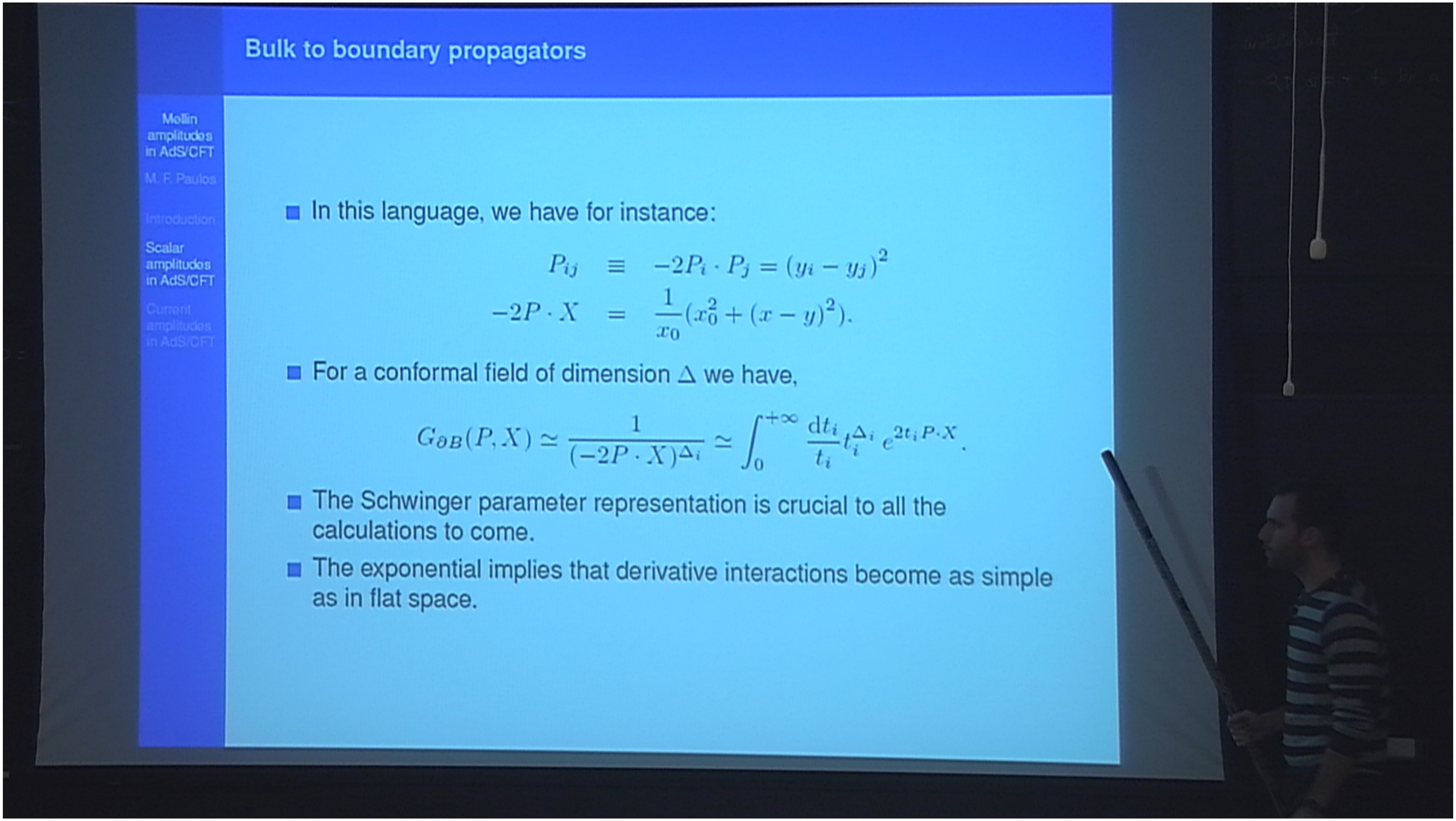
Scalar amplitudes in AdS/CFT
Current amplitudes in AdS/CFT
In this language, we have for instance:
$P_{ij} \equiv -2P_i \cdot P_j = (y_i - y_j)^2$
$-2P \cdot X = \frac{1}{x_0}(x_0^2 + (x - y)^2).$
For a conformal field of dimension Δ we have,
$G_{\partial B}(P, X) \simeq \frac{1}{(-2P \cdot X)^{\Delta_i}} \simeq \int_0^{+\infty} \frac{dt_i}{t_i} t_i^{\Delta_i} e^{2t_i P \cdot X}.$
The Schwinger parameter representation is crucial to all the calculations to come.
The exponential implies that derivative interactions become as simple as in flat space.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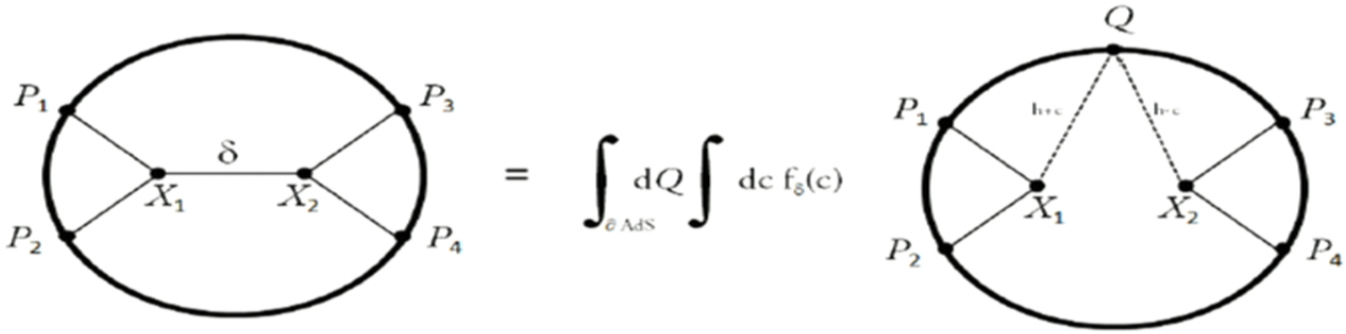
Bulk to bulk propagators

- The bulk-to-bulk propagator takes the remarkable form ($h \equiv d/2$):

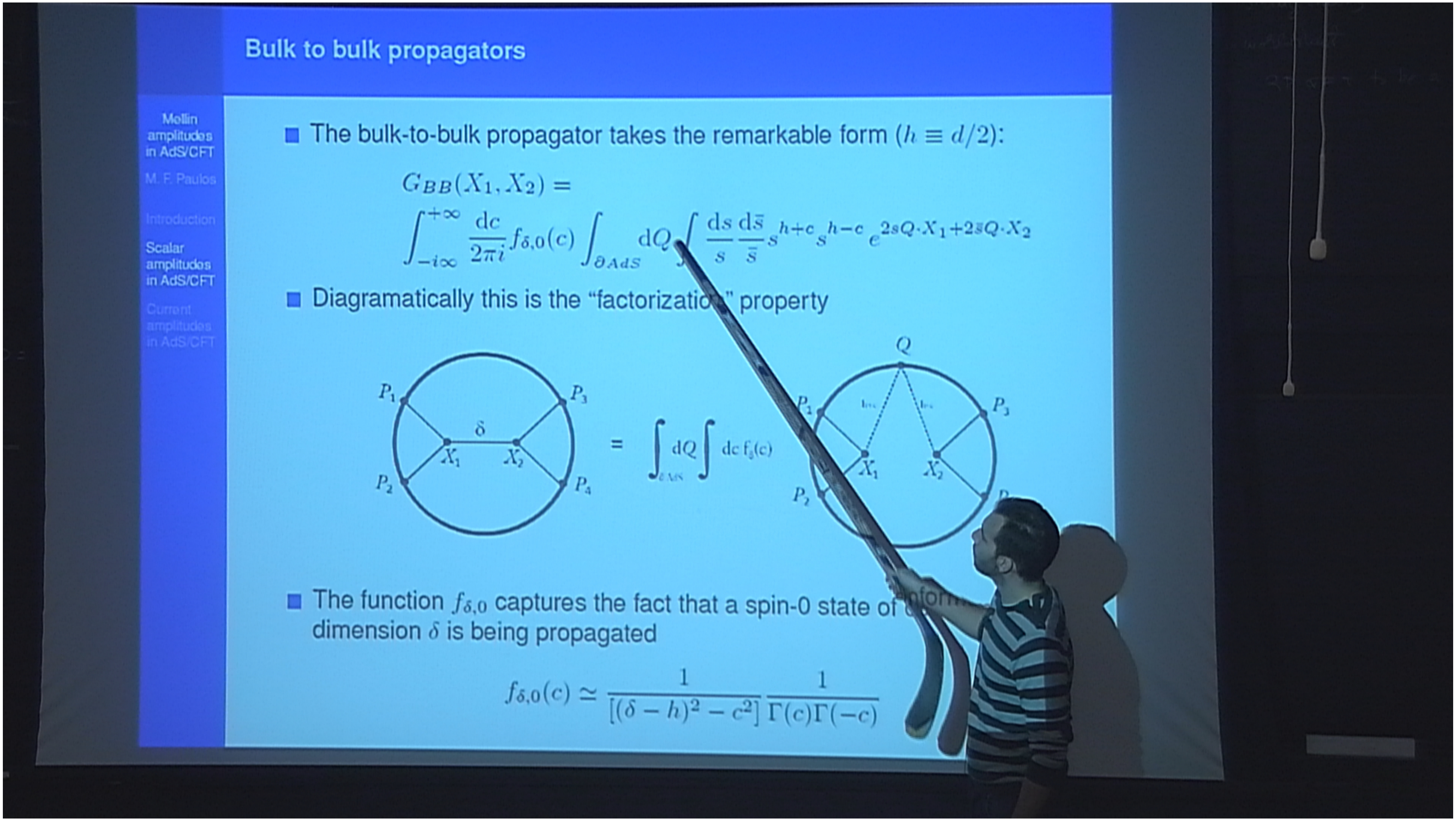
$$G_{BB}(X_1, X_2) = \int_{-i\infty}^{+\infty} \frac{\mathrm{d}c}{2\pi i} f_{\delta,0}(c) \int_{\partial AdS} \mathrm{d}Q \int \frac{\mathrm{d}s}{s} \frac{\mathrm{d}\bar{s}}{\bar{s}} s^{h+c} s^{h-c} e^{2sQ\cdot X_1 + 2\bar{s}Q\cdot X_2}$$

- Diagramatically this is the "factorization" property



- The function $f_{\delta,0}$ captures the fact that a spin-0 state of conformal dimension δ is being propagated

$$f_{\delta,0}(c) \simeq \frac{1}{[(\delta-h)^2 - c^2]} \frac{1}{\Gamma(c)\Gamma(-c)}$$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Diagramatically this is the "factorization" property
P_1
P_2
P_3
P_4
δ
X_1
X_2
$= \int_{\partial AdS} dQ \int dc\, f_\delta(c)$
Q
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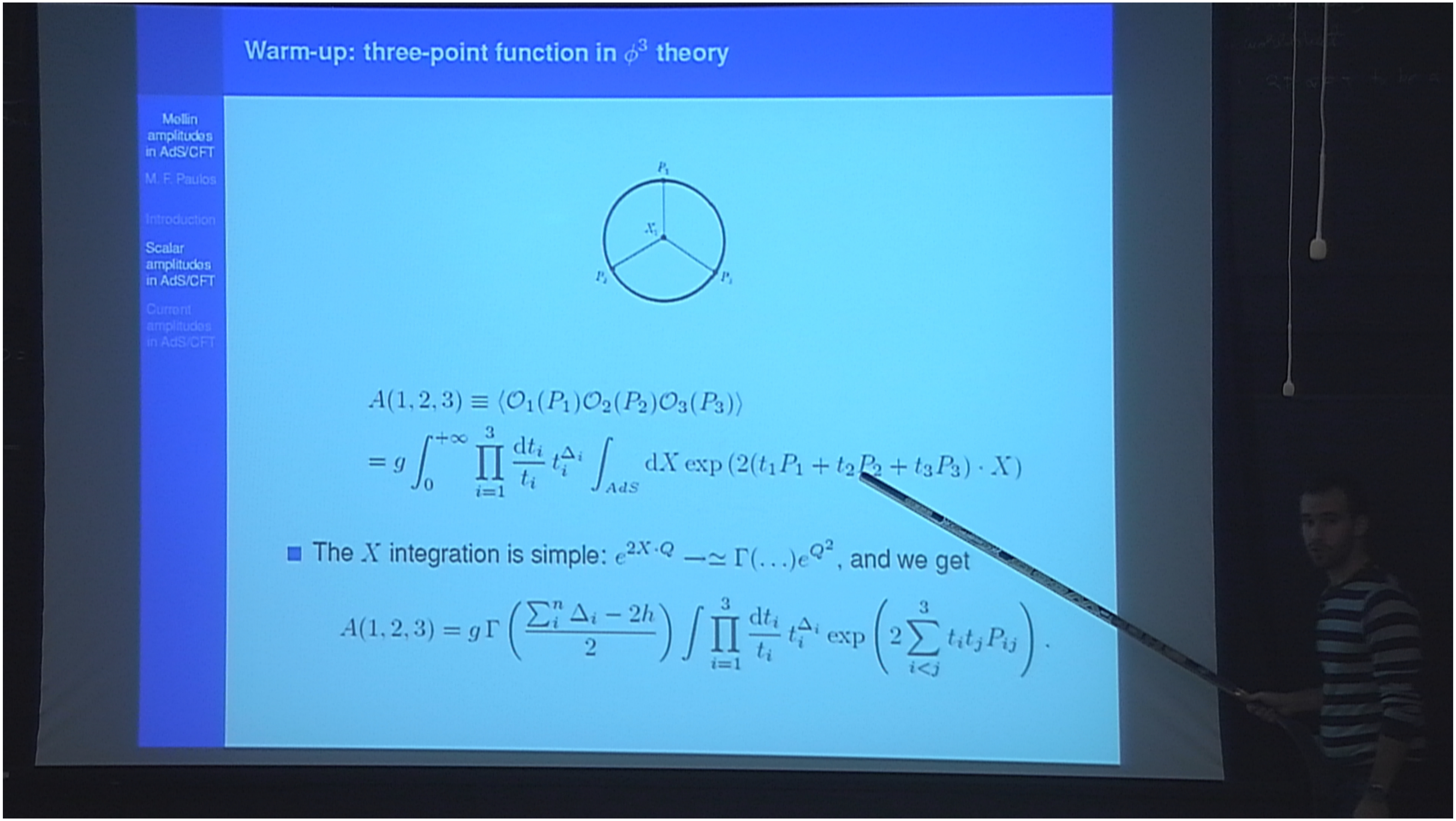
Warm-up: three-point function in ϕ^3 theory
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P_1
X_1
P_2
P_3
$A(1,2,3) \equiv \langle \mathcal{O}_1(P_1)\mathcal{O}_2(P_2)\mathcal{O}_3(P_3)\rangle$
$= g\int_0^{+\infty}\prod_{i=1}^{3}\frac{\mathrm{d}t_i}{t_i}\,t_i^{\Delta_i}\int_{AdS}\mathrm{d}X\exp\left(2(t_1P_1+t_2P_2+t_3P_3)\cdot X\right)$
The X integration is simple: $e^{2X\cdot Q} \to\simeq \Gamma(\ldots)e^{Q^2}$, and we get
$A(1,2,3) = g\,\Gamma\left(\frac{\sum_i^n \Delta_i - 2h}{2}\right)\int\prod_{i=1}^{3}\frac{\mathrm{d}t_i}{t_i}\,t_i^{\Delta_i}\exp\left(2\sum_{i<j}^{3}t_it_jP_{ij}\right).$

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$$A(1,2,3) = g\,\Gamma\left(\frac{\sum_i^n \Delta_i - 2h}{2}\right) \int \prod_{i=1}^{3} \frac{\mathrm{d}t_i}{t_i}\, t_i^{\Delta_i} \exp\left(2\sum_{i<j}^{3} t_i t_j P_{ij}\right).$$

Warm-up: three-point function in ϕ^3 theory

- Performing the integrals, and comparing with the Mellin form:

$$A(1,2,3) = g\,\Gamma\left(\frac{\sum_i^3 \Delta_i - 2h}{2}\right)\prod_{i<j}^{3}\Gamma(\delta_{ij})(x_i - x_j)^{-2\delta_{ij}}$$

$$A(x_1, x_2, \ldots, x_n) = \oint \mathrm{d}\delta_{ij}\; M(\delta_{ij})\prod_{i<j}^{n}\Gamma(\delta_{ij})(x_i - x_j)^{-2\delta_{ij}}.$$

we read off simply

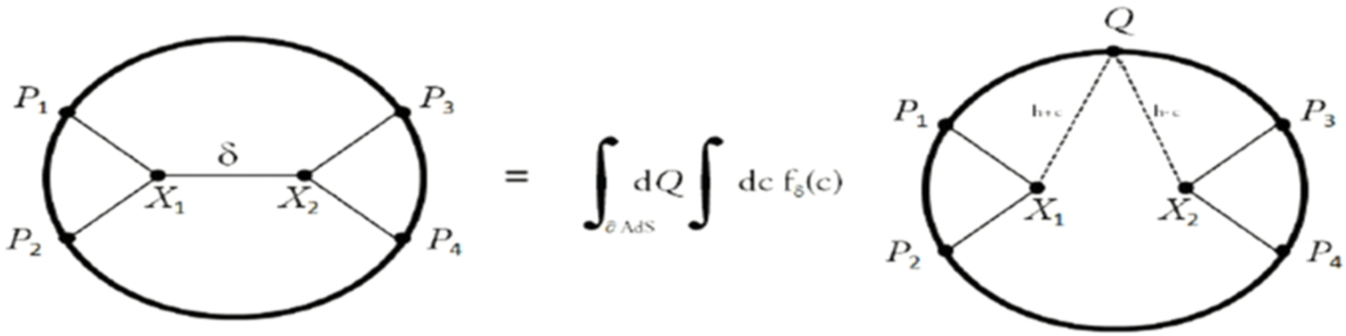
$$M_3 = V_{[0,0,0]}^{\Delta_1,\Delta_2,\Delta_3} \equiv g\,\Gamma\left(\frac{\sum_i^3 \Delta_i - 2h}{2}\right)$$

- Conformal symmetry (Mellin momentum conservation) completely fixes the parameters, say:

$$\delta_{12} = k_1 \cdot k_2 = \frac{\Delta_1 + \Delta_2 - \Delta_3}{2}$$

Comments on higher-point functions

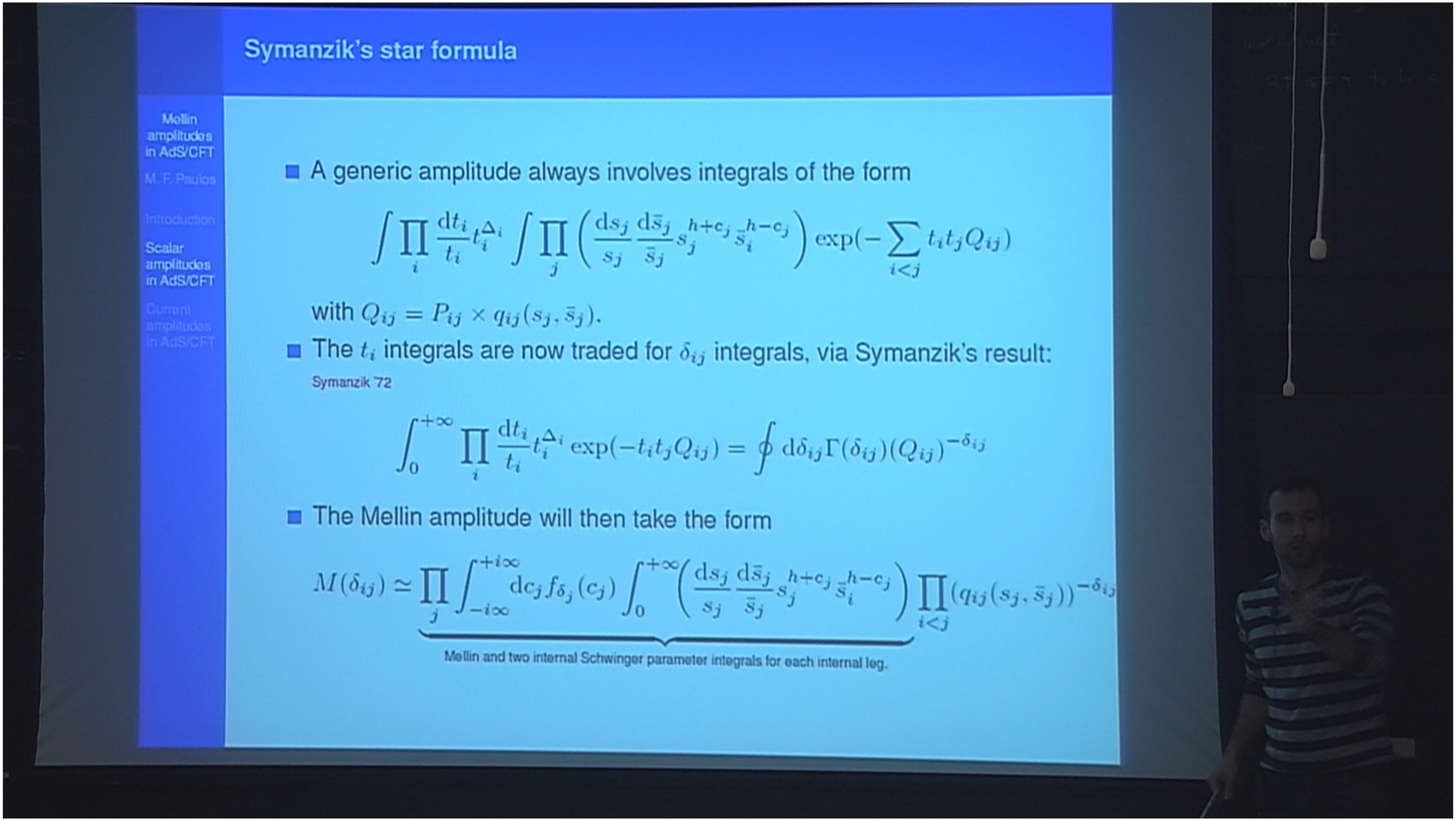
- Exchange diagrams are related to contact diagrams via the diagrammatic relation



- AdS integrations are trivial! Do them once for the 3-pt function, and that's it.
- Boundary integrations are also trivial in the embedding formalism, if we use Schwinger parameterisation:

$$\int \mathrm{d}Q e^{2P\cdot Q} \quad \rightarrow \quad e^{P^2}$$

- We are left with the Mellin integrals in c, and the Schwinger parameter integrals, one for each and every leg.


Symanzik's star formula
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A generic amplitude always involves integrals of the form
$$\int \prod_i \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} \int \prod_j \left(\frac{\mathrm{d}s_j}{s_j} \frac{\mathrm{d}\bar{s}_j}{\bar{s}_j} s_j^{h+c_j} \bar{s}_i^{h-c_j} \right) \exp(-\sum_{i<j} t_i t_j Q_{ij})$$
with $Q_{ij} = P_{ij} \times q_{ij}(s_j, \bar{s}_j)$.
The t_i integrals are now traded for δ_{ij} integrals, via Symanzik's result:
Symanzik '72
$$\int_0^{+\infty} \prod_i \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} \exp(-t_i t_j Q_{ij}) = \oint \mathrm{d}\delta_{ij} \Gamma(\delta_{ij}) (Q_{ij})^{-\delta_{ij}}$$
The Mellin amplitude will then take the form
$$M(\delta_{ij}) \simeq \underbrace{\prod_j \int_{-i\infty}^{+i\infty} \mathrm{d}c_j f_{\delta_j}(c_j) \int_0^{+\infty} \left(\frac{\mathrm{d}s_j}{s_j} \frac{\mathrm{d}\bar{s}_j}{\bar{s}_j} s_j^{h+c_j} \bar{s}_i^{h-c_j} \right)}_{\text{Mellin and two internal Schwinger parameter integrals for each internal leg.}} \prod_{i<j} (q_{ij}(s_j, \bar{s}_j))^{-\delta_{ij}}$$

Symanzik's star formula

- A generic amplitude always involves integrals of the form

$$\int \prod_i \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} \int \prod_j \left(\frac{\mathrm{d}s_j}{s_j} \frac{\mathrm{d}\bar{s}_j}{\bar{s}_j} s_j^{h+c_j} \bar{s}_i^{h-c_j} \right) \exp(-\sum_{i<j} t_i t_j Q_{ij})$$

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$$\int_0^{+\infty} \prod_i \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} \exp(-t_i t_j Q_{ij}) = \oint \mathrm{d}\delta_{ij} \Gamma(\delta_{ij})(Q_{ij})^{-\delta_{ij}}$$

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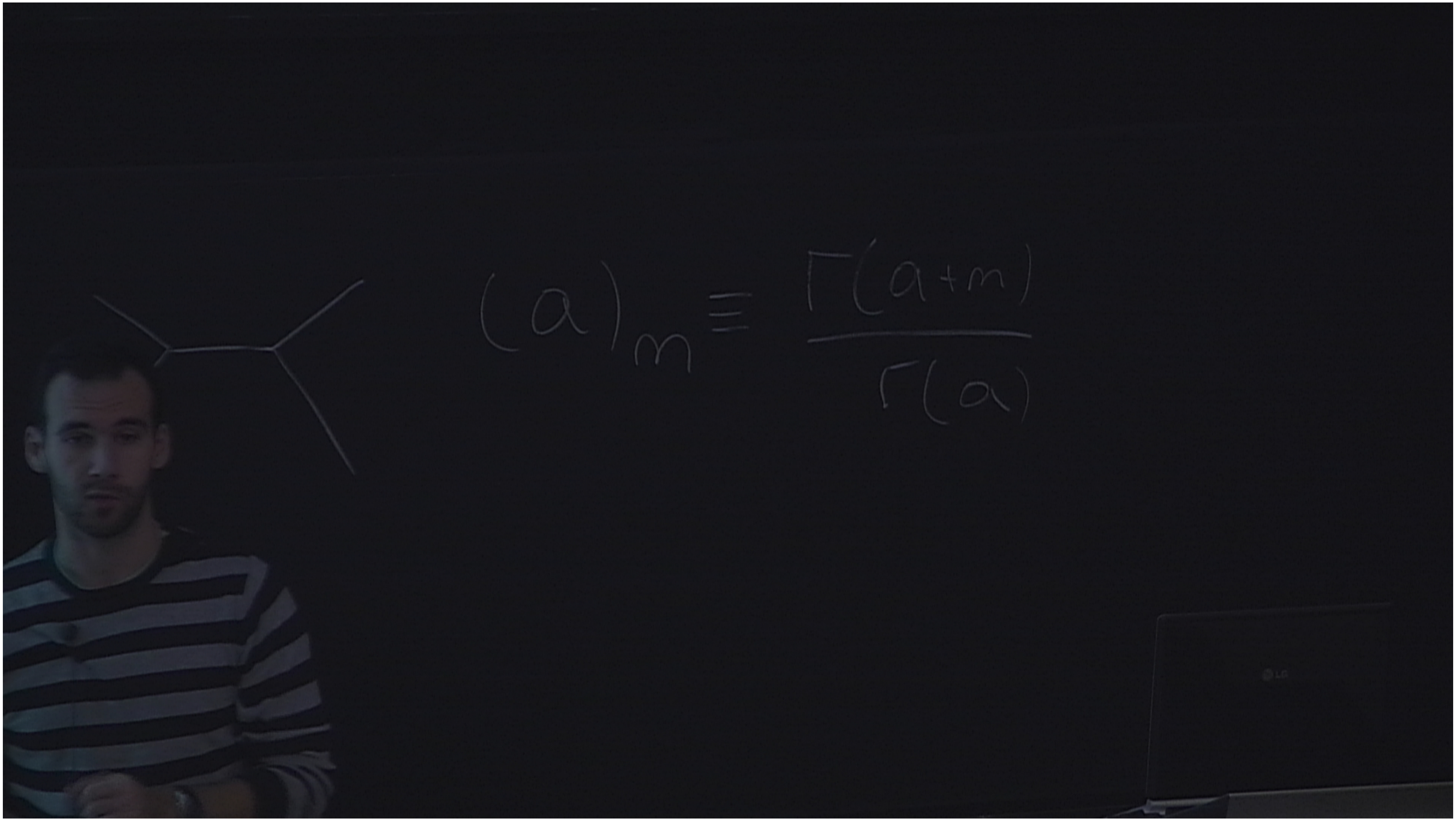
$(a)_m \equiv \frac{\Gamma(a+m)}{\Gamma(a)}$

Feynman rules?

- Test Feynman rules: compute higher n-point functions.
- This should also tell us what the vertices are.
- Computations are relatively straightforward - complicated part is integral over internal Schwinger parameters.

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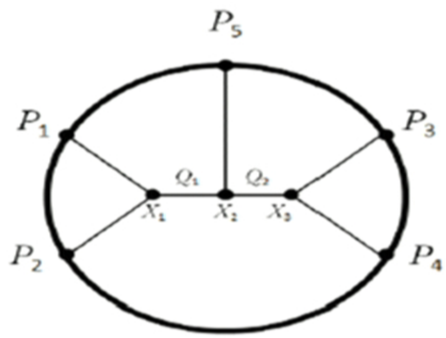
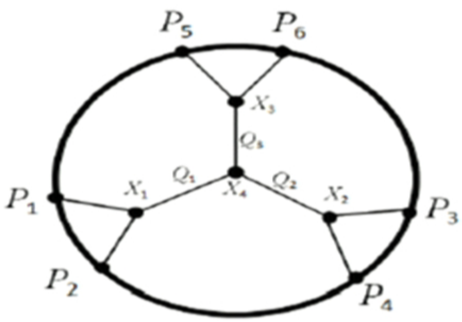




Computing the six-point amplitude

- After integrations over Q, t_i and half of the internal Schwinger parameters are performed, we obtain the Mellin amplitude.

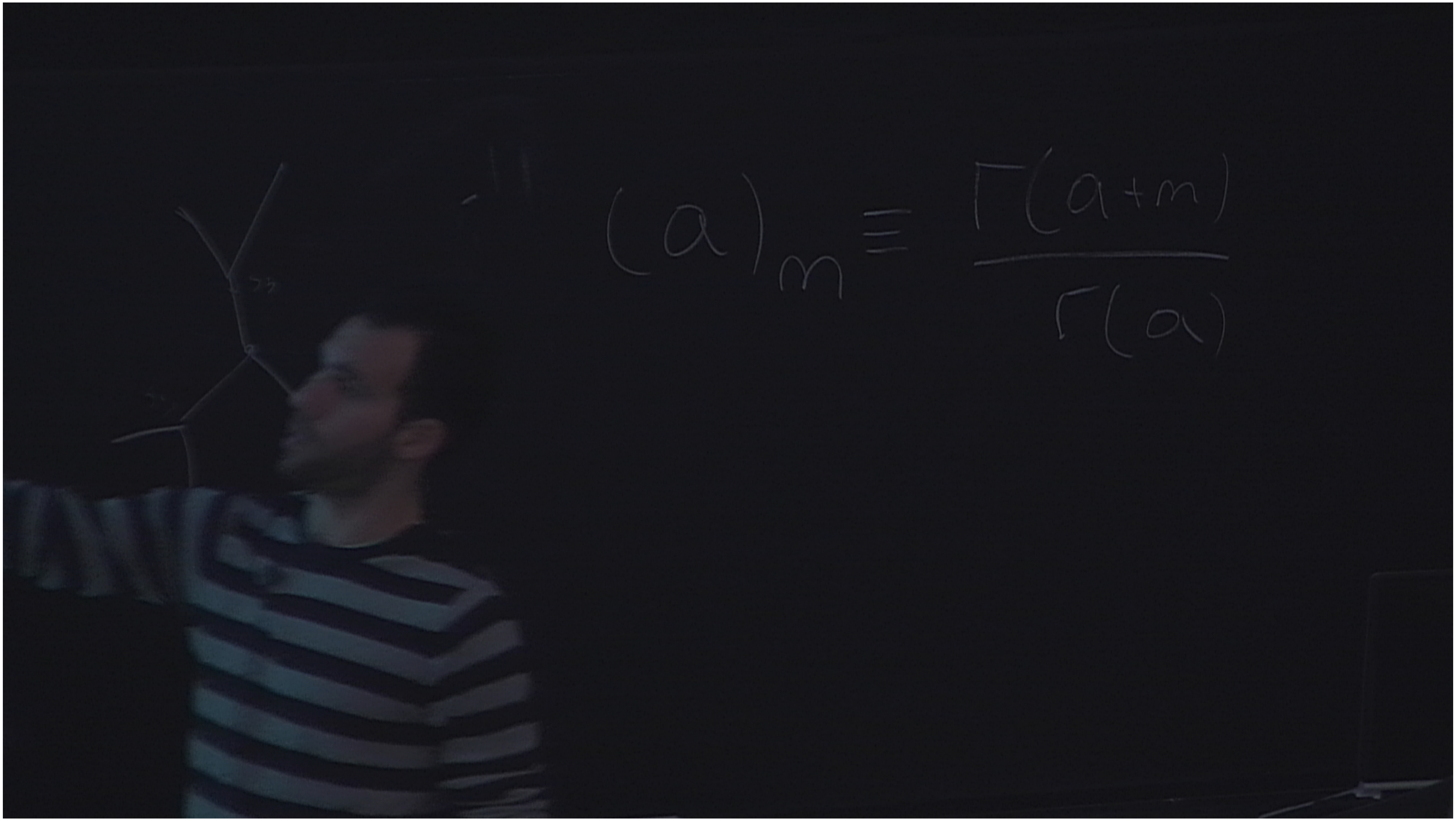
$$g^4 \int_{-i\infty}^{i\infty} \prod_{i=1}^{3} \left(\frac{\mathrm{d}c_k}{2\pi i} \frac{\Gamma\left(\frac{\Delta_{i,1}+\Delta_{i,2}+c_i-h}{2}\right)\Gamma\left(\frac{\Delta_{i,1}+\Delta_{i,2}-c_i-h}{2}\right)\Gamma\left(\frac{c_i+h-s_i}{2}\right)}{\Gamma\left(\frac{\Delta_{i,1}+\Delta_{i,2}-s_i}{2}\right)\Gamma(c_i)\Gamma(-c_i)\left[(\delta_i-h)^2-c_i^2\right]} \right)$$

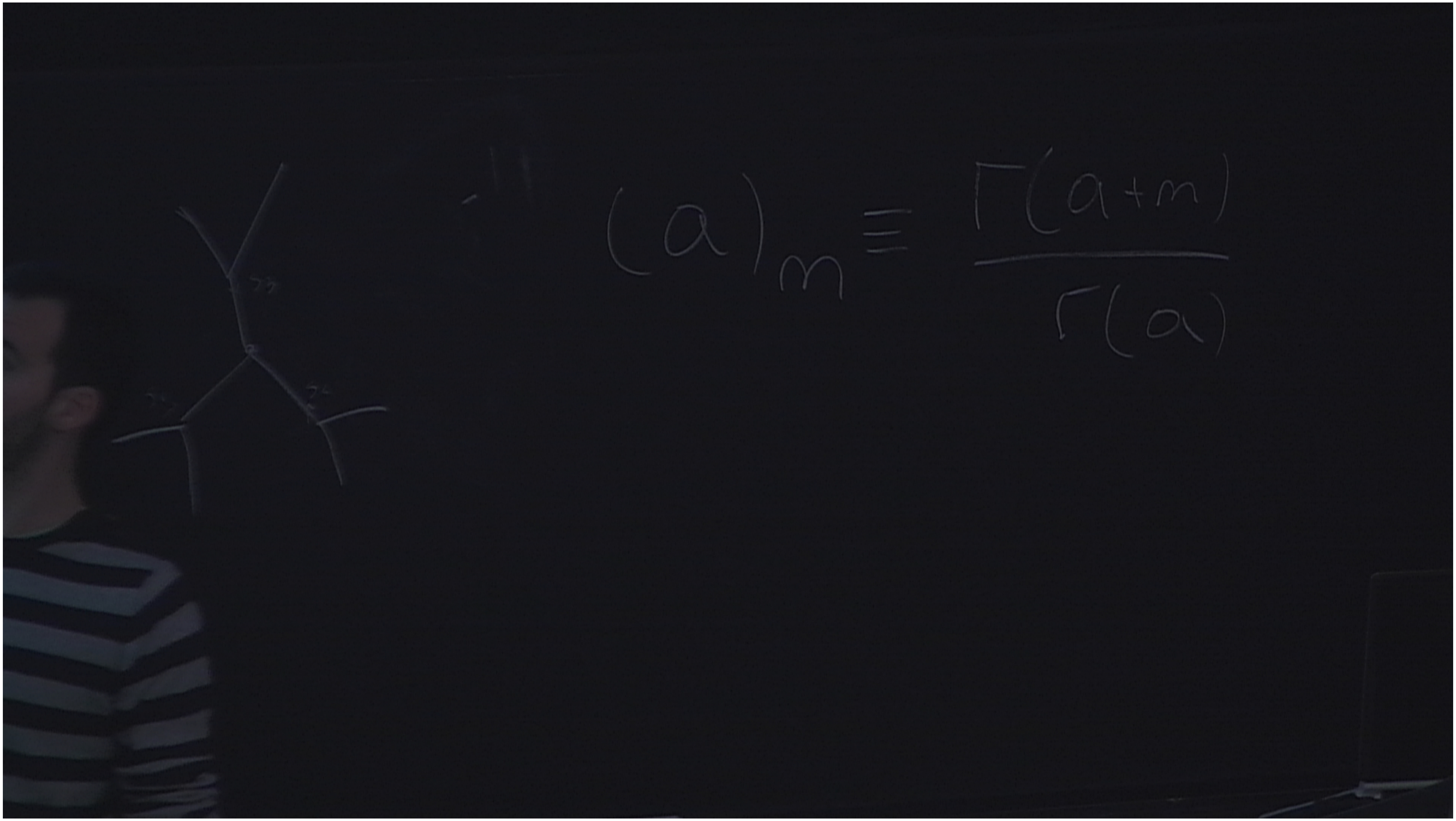
$$\Gamma\left(\frac{h-c_1-c_2-c_3}{2}\right)\int_0^{+\infty}\left(\prod_{i=1}^{3}\frac{\mathrm{d}x_i}{x_i}x_i^{a_i}(1+x_i)^{b_i}\right)\left(1+\sum_i^3 x_i\right)^c$$

with

$$a_i = \frac{-c_i+h-s_i}{2}, \quad b_i = \frac{-c_i-h-s_i}{2}, \quad c = \frac{c_1+c_2+c_3-h}{2}$$

- The c_i poles leads to poles in s_i at δ_i - conformal dimensions of exchanged states.
- Need to evaluate the triple integral!


$(a)_m \equiv \frac{\Gamma(a+m)}{\Gamma(a)}$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Mellin six-point amplitude

- Performing pole pinching we can finally write

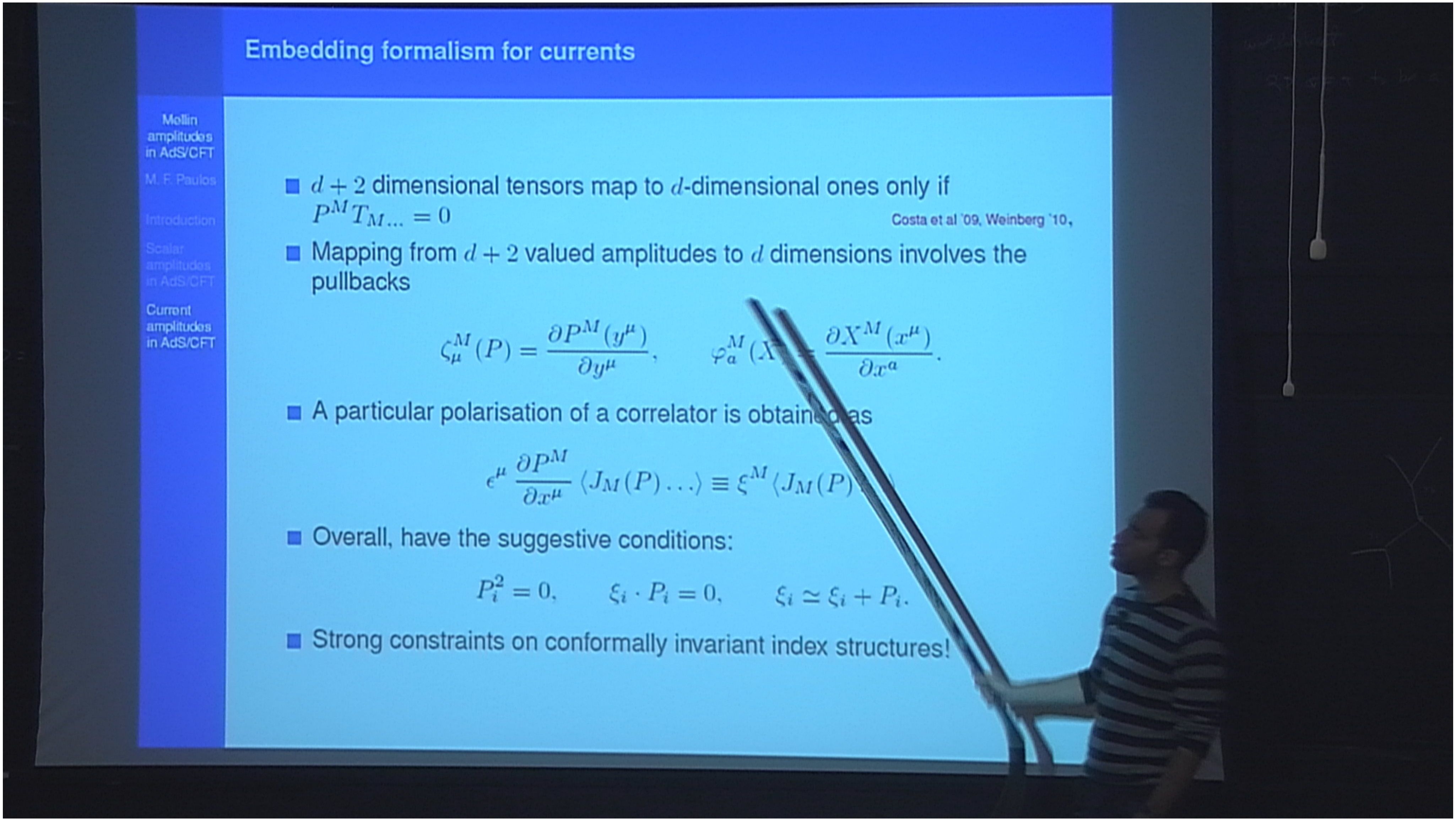
$$M_6 = \sum_{n_1,n_2,n_3=0}^{+\infty} \left(\prod_{i=1}^{3} \frac{P_{n_i}^{\delta_i}}{s_i - \delta_i - 2\,n_i} \right) V^{\Delta_1,\Delta_2,\delta_1}_{[0,0,n_1]} V^{\Delta_3,\Delta_4,\delta_2}_{[0,0,n_2]} V^{\Delta_5,\Delta_6,\delta_3}_{[0,0,n_3]} V^{\delta_1,\delta_2,\delta_3}_{[n_1,n_2,n_3]}$$

- Possible polynomial contributions not ruled out, but not expected.
- Feynman rules hold! General vertex is

$$V^{\Delta_1,\Delta_2,\Delta_3}_{[n_1,n_2,n_3]} = V^{\Delta_1,\Delta_2,\Delta_3}_{[0,0,0]} \, (1+\Delta_1-h)_{n_1} \, (1+\Delta_2-h)_{n_2} \, (1+\Delta_3-h)_{n_3}$$

$$F_A^{(3)} \left(\frac{\sum_i^3 \Delta_i - 2h}{2}, \{-n_1, -n_2, -n_3\}, \{1+\Delta_1-h, 1+\Delta_2-h, 1+\Delta_3-h\}; 1, 1, 1 \right).$$

- Series defining $F_A^{(3)}$ reduces to a finite sum for integer n_i.


Embedding formalism for currents
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$d+2$ dimensional tensors map to d-dimensional ones only if $P^M T_{M\dots} = 0$
Costa et al '09, Weinberg '10,
Mapping from $d+2$ valued amplitudes to d dimensions involves the pullbacks
$\zeta_\mu^M(P) = \frac{\partial P^M(y^\mu)}{\partial y^\mu}$,
$\varphi_a^M(X$
$\frac{\partial X^M(x^\mu)}{\partial x^a}$.
A particular polarisation of a correlator is obtain
as
$\epsilon^\mu \frac{\partial P^M}{\partial x^\mu} \langle J_M(P)\dots\rangle \equiv \xi^M \langle J_M(P)$
Overall, have the suggestive conditions:
$P_i^2 = 0, \qquad \xi_i \cdot P_i = 0, \qquad \xi_i \simeq \xi_i + P_i.$
Strong constraints on conformally invariant index structures!

Current propagators

- In the embedding formalism, current bulk to boundary propagator is

$$G_{\partial B}^{MA}(P,X) = \left(\eta^{MA} - \frac{P^A X^M}{P\cdot X}\right)\int_0^{+\infty}\frac{\mathrm{d}t_i}{t_i}t_i^{\Delta_i}\; e^{2t_i P\cdot X}.$$

- Satisfies

$$P^M G_{\partial B}^{MA} = G_{\partial B}^{MA} X^A = 0$$

- Equivalent to usual current propagator.
- The Schwinger parameterised form allows us to write

$$G_{\partial B}^{MA}(P,X) = D^{MA}\, G_{\partial B}(P,X)$$
$$D^{MA} \equiv \eta^{MA} + \frac{1}{\Delta}\, P^A \frac{\partial}{\partial P^M}$$

Current propagators

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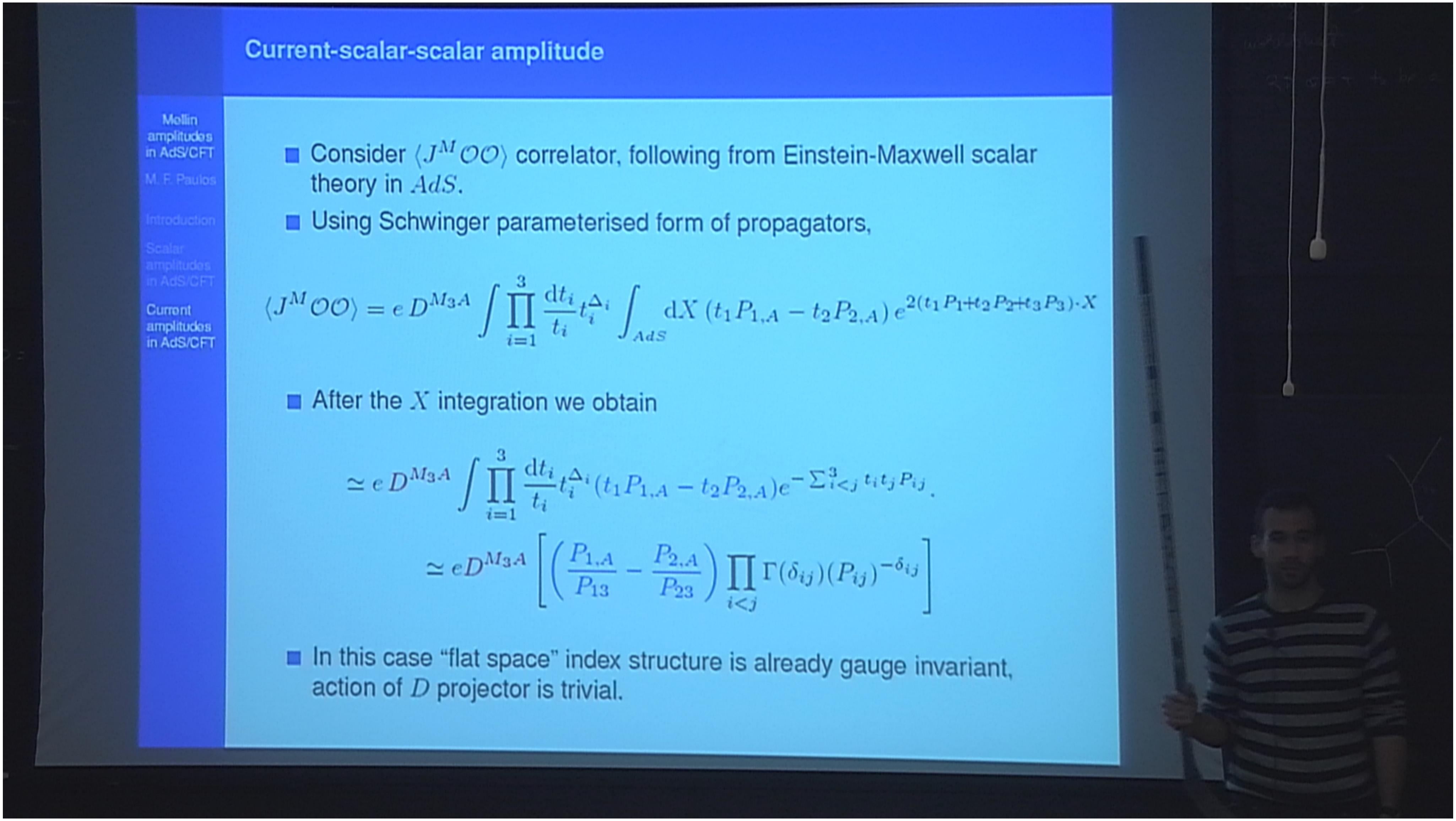
- The D projectors decouple from calculations, so that amplitudes satisfy

$$A_{M_1 \ldots M_N} = D^{M_1 A_1} \ldots D^{M_N A_N} \hat{A}_{A_1 \ldots A_N}$$

- Reduced amplitudes $\hat{A}$ have much simpler index structures - essentially those appearing in flat space.
- The D operators enforce correct behaviour under conformal transformations, e.g. dilatations:

$$P_1^{M_1} A_{M_1 \ldots} = 0 \Leftrightarrow P_1^{A_1} \left(1 + \frac{1}{\Delta} P_1 \cdot \frac{\partial}{\partial P_1}\right) \hat{A}_{A_1 \ldots}.$$

- Importantly, the same projectors exist for stress-tensors - index structure is dramatically simplified.


Current-scalar-scalar amplitude
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Consider $\langle J^M \mathcal{O}\mathcal{O}\rangle$ correlator, following from Einstein-Maxwell scalar theory in AdS.
Using Schwinger parameterised form of propagators,
$$\langle J^M \mathcal{O}\mathcal{O}\rangle = e\, D^{M_3 A} \int \prod_{i=1}^{3} \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} \int_{AdS} \mathrm{d}X\, (t_1 P_{1,A} - t_2 P_{2,A})\, e^{2(t_1 P_1 + t_2 P_2 + t_3 P_3)\cdot X}$$
After the X integration we obtain
$$\simeq e\, D^{M_3 A} \int \prod_{i=1}^{3} \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} (t_1 P_{1,A} - t_2 P_{2,A}) e^{-\sum_{i<j}^{3} t_i t_j P_{ij}}.$$
$$\simeq e D^{M_3 A} \left[\left(\frac{P_{1,A}}{P_{13}} - \frac{P_{2,A}}{P_{23}} \right) \prod_{i<j} \Gamma(\delta_{ij}) (P_{ij})^{-\delta_{ij}} \right]$$
In this case “flat space” index structure is already gauge invariant, action of D projector is trivial.

Current-scalar-scalar amplitude

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- After the X integration we obtain

$$\simeq e\, D^{M_3 A} \int \prod_{i=1}^{3} \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} (t_1 P_{1,A} - t_2 P_{2,A}) e^{-\sum_{i<j}^{3} t_i t_j P_{ij}}.$$

$$\simeq e D^{M_3 A} \left[\left(\frac{P_{1,A}}{P_{13}} - \frac{P_{2,A}}{P_{23}} \right) \prod_{i<j} \Gamma(\delta_{ij}) (P_{ij})^{-\delta_{ij}} \right]$$

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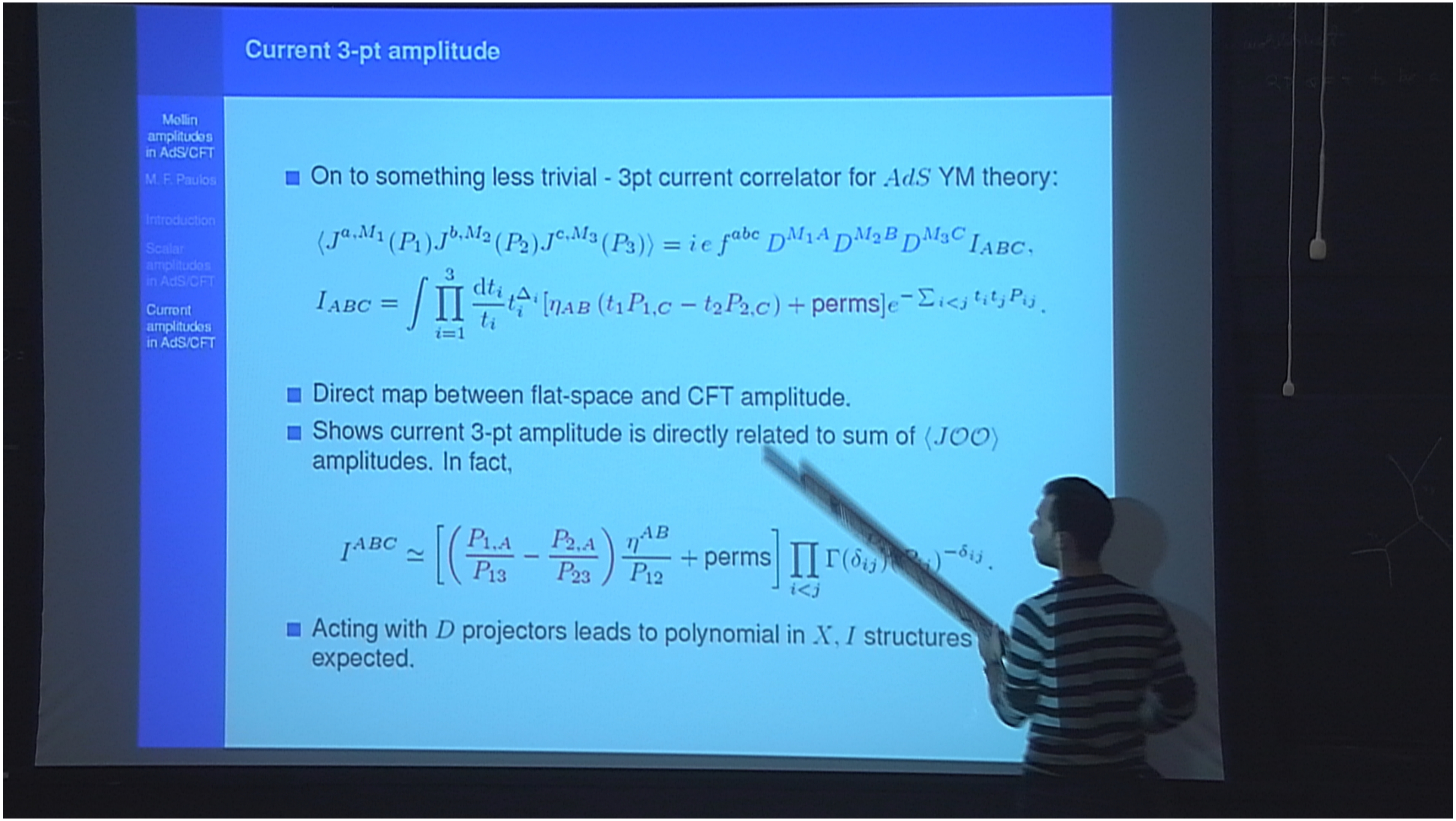
Current 3-pt amplitude
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On to something less trivial - 3pt current correlator for AdS YM theory:
$\langle J^{a,M_1}(P_1)J^{b,M_2}(P_2)J^{c,M_3}(P_3)\rangle = i\,e\,f^{abc}\,D^{M_1A}D^{M_2B}D^{M_3C}I_{ABC},$
$I_{ABC} = \int\prod_{i=1}^{3}\frac{\mathrm{d}t_i}{t_i}t_i^{\Delta_i}[\eta_{AB}(t_1P_{1,C}-t_2P_{2,C}) + \text{perms}]e^{-\sum_{i<j}t_it_jP_{ij}}.$
Direct map between flat-space and CFT amplitude.
Shows current 3-pt amplitude is directly related to sum of $\langle JOO\rangle$ amplitudes. In fact,
$I^{ABC} \simeq \left[\left(\frac{P_{1,A}}{P_{13}}-\frac{P_{2,A}}{P_{23}}\right)\frac{\eta^{AB}}{P_{12}} + \text{perms}\right]\prod_{i<j}\Gamma(\delta_{ij})(P_{ij})^{-\delta_{ij}}.$
Acting with D projectors leads to polynomial in X, I structures expected.

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Current 3-pt amplitude

- On to something less trivial - 3pt current correlator for AdS YM theory:

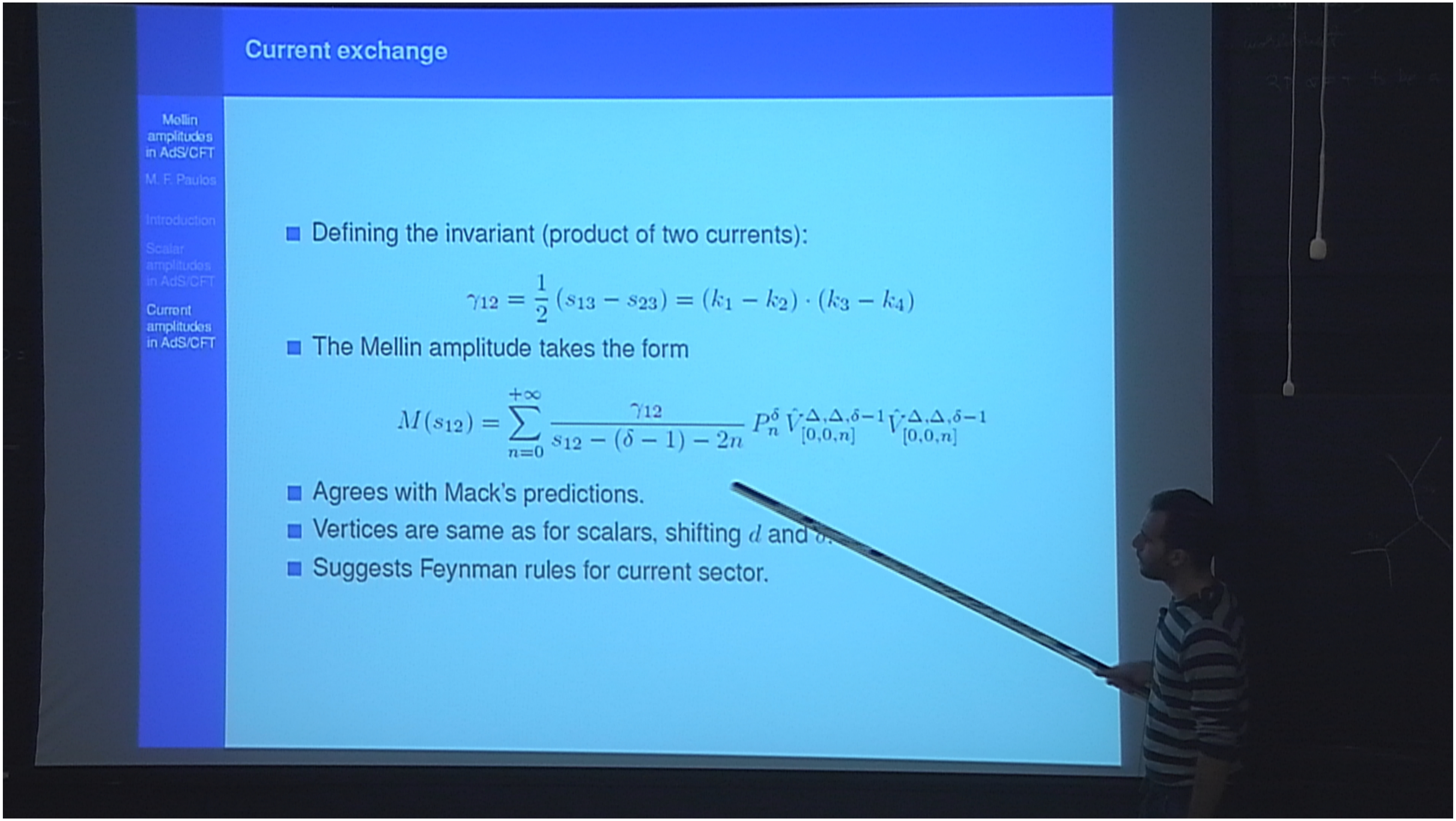
$$\langle J^{a,M_1}(P_1) J^{b,M_2}(P_2) J^{c,M_3}(P_3) \rangle = i\, e\, f^{abc}\, D^{M_1 A} D^{M_2 B} D^{M_3 C} I_{ABC},$$

$$I_{ABC} = \int \prod_{i=1}^{3} \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} [\eta_{AB} (t_1 P_{1,C} - t_2 P_{2,C}) + \text{perms}] e^{-\sum_{i<j} t_i t_j P_{ij}}.$$

- Direct map between flat-space and CFT amplitude.
- Shows current 3-pt amplitude is directly related to sum of $\langle J \mathcal{O} \mathcal{O} \rangle$ amplitudes. In fact,

$$I^{ABC} \simeq \left[\left(\frac{P_{1,A}}{P_{13}} - \frac{P_{2,A}}{P_{23}} \right) \frac{\eta^{AB}}{P_{12}} + \text{perms} \right] \prod_{i<j} \Gamma(\delta_{ij}) (P_{ij})^{-\delta_{ij}}.$$

- Acting with D projectors leads to polynomial in X, I structures as expected.


Current exchange
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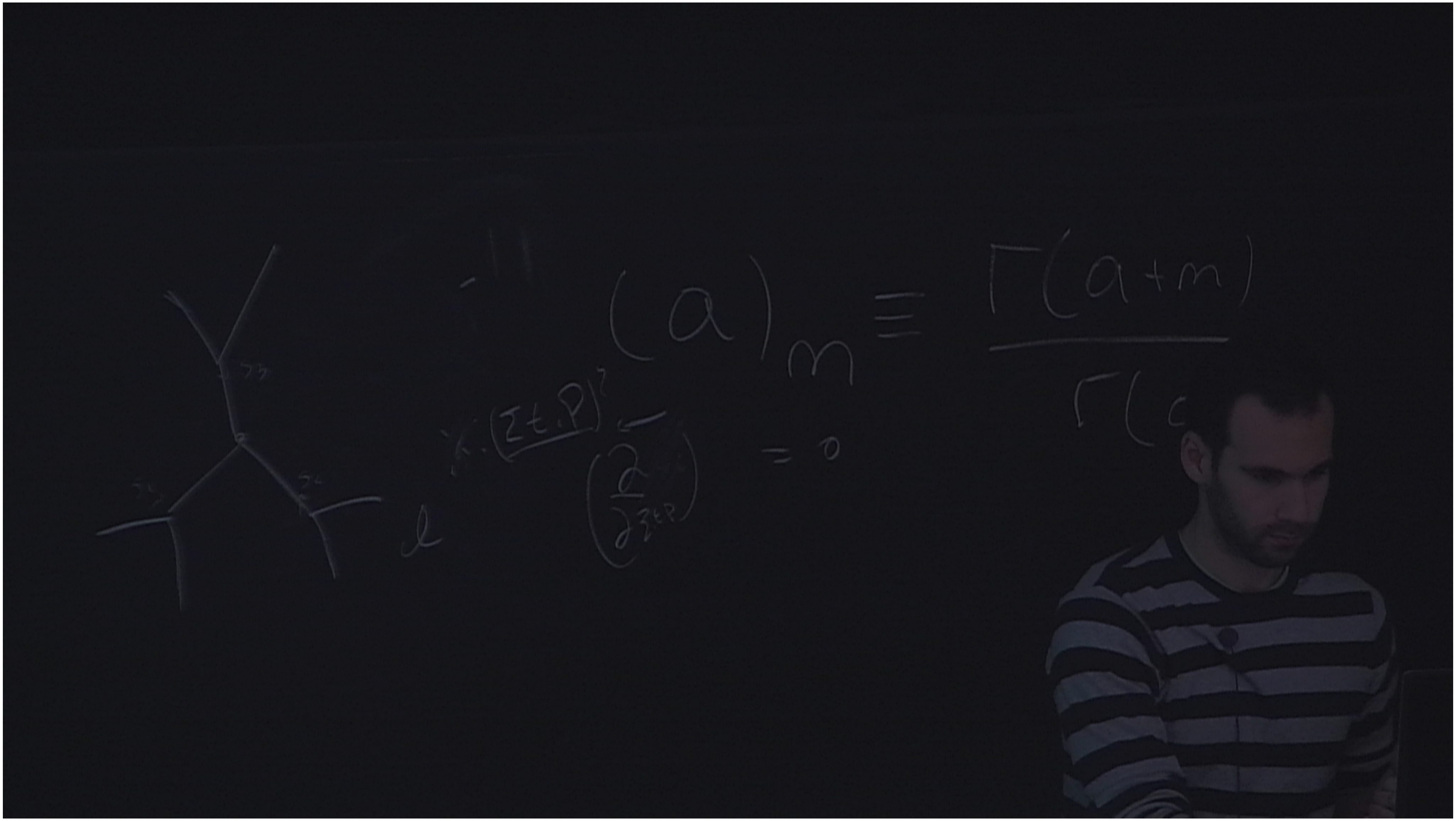
Current amplitudes in AdS/CFT
Defining the invariant (product of two currents):
$$\gamma_{12} = \frac{1}{2}(s_{13} - s_{23}) = (k_1 - k_2)\cdot(k_3 - k_4)$$
The Mellin amplitude takes the form
$$M(s_{12}) = \sum_{n=0}^{+\infty} \frac{\gamma_{12}}{s_{12} - (\delta - 1) - 2n} P_n^{\delta}\, \hat{V}^{\Delta,\Delta,\delta-1}_{[0,0,n]} \hat{V}^{\Delta,\Delta,\delta-1}_{[0,0,n]}$$
Agrees with Mack's predictions.
Vertices are same as for scalars, shifting d and
Suggests Feynman rules for current sector.


(a)m ≡ Γ(a+m)
= 0

Current four-point function

- Current four-point function calculation similar to current exchange.
- Life is made easier because of “momentum conservation” (!): originally we had $D^{MA}X_A = 0$, due to the transversality condition. After the X integrations are performed this means that

$$\int \left(\prod_i \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i}\right) D^{MA}\left(\sum t_i P_{i,A}\right) e^{-\sum t_i t_j P_{ij}} = 0.$$

- Upshot: under the integral sign, and action of D, there is “momentum” conservation at each vertex.
- Another piece of evidence that we’re missing some nice $(d+2)$ dimensional description of the physics.

Current 4pt exchange diagram

- Using momentum conservation, the index structure of the amplitude decouples from the exchange part.
- Calculation becomes almost exactly the same as in 4pt scalar current-exchange diagram:

$$\begin{aligned} M^{A_1\dots A_4} &= I^{A_1A_2A_3A_4}(s_{12},\gamma_{12}) \\ &\sum_{n=0}^{+\infty} \frac{P_n^{d-1}}{s_{12}-(d-2)-2n}\, \hat{V}^{d-1,d-1,d-2}_{[0,0,n]}\hat{V}^{d-1,d-1,d-2}_{[0,0,n]}. \end{aligned}$$

- For $d=4$ we get the remarkable result

$$M^{A_1\dots A_4} = I^{A_1A_2A_3A_4}(s_{12},\gamma_{12})\left(\frac{2}{s-2}+\frac{1}{s-4}\right)$$

- The index structure is fully known - there is a direct map from it to the flat space index structure.

Index structure of four-point function

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- To see how this how this map works, recall that due to the Schwinger parameterisation, P plays the role of momentum.
- At some point in the amplitude computation we have something like

$$A_4^J \simeq \prod_i^4 D^{M_i A_i} \int (\dots) \int \prod_{i=1}^4 \frac{\mathrm{d}t_i}{t_i} t_i^{\Delta_i} J_1^{A_1 A_2 B} J_2{}^{A_3 A_4}_{B}\, e^{-t_i t_j Q_{ij}},$$

- The currents are simply the three-point vertices of Yang-Mills theory with $k_i \to t_i P_i$. Their contraction gives the flat-space result.
- When going to Mellin space we have the simple rule

$$t_i t_j P_i^A P_j^B \to \delta_{ij} \frac{P_i^A P_j^B}{P_{ij}}$$

which determines $I^{M_1 M_2 M_3 M_4}$.

- The full amplitude index structure is obtained by acting with the D operators - straightforward but tedious (unnecessary?).

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Summary

- Mellin space is a powerful tool for representing CFT's with an (effectively) small number of primaries.
- Mellin amplitudes in AdS/CFT context seem to be described by a simple set of Feynman rules - possible solution of the scalar sector at tree-level.
- Embedding formalism simplifies description of conformal invariance dramatically, and seems to go hand in hand with the Mellin representation.
- Schwinger parameterisation+embedding formalism: more than a trick?