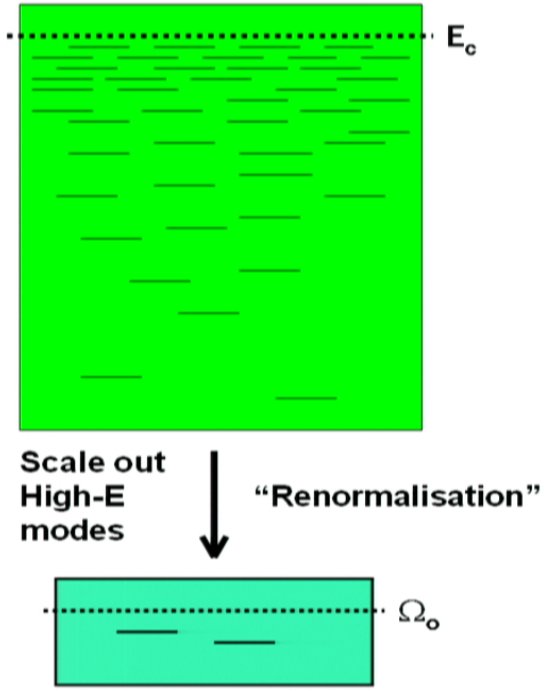
Title: Decoherence and Effective Field Theories

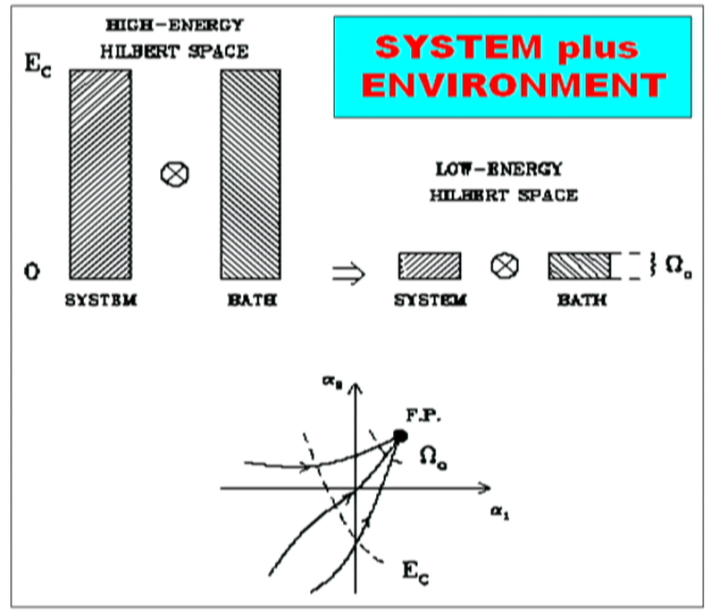
Date: Oct 28, 2011 09:30 AM

URL: http://pirsa.org/11100121

Abstract: Effective field theories, underpinned by the resnormalization framework, are a central feature of condensed matter physics and relativistic field theory. However the phenomenon of decoherence is not so easily subsumed under this framework. Ordinary environmental decoherence may lead to very unusual effective theories, and recent ideas about intrinsic decoherence in Nature (eg., Penrose's ideas aobut gravitational decoherence) do not obviously lead to any effective field theory. I will review our ideas aobut environmental decoherence, with some examples from condensed matter physics, highlighting some of the peculiar features of these. I will then discuss what we know of intrinsic decoherence (which in some cases amounts to a breakdonw of quantum mechanics, focussing on a new path integral formulation of Penrose's ideas.

Orthodox view of $\mathcal{H}_{\text{eff}}$



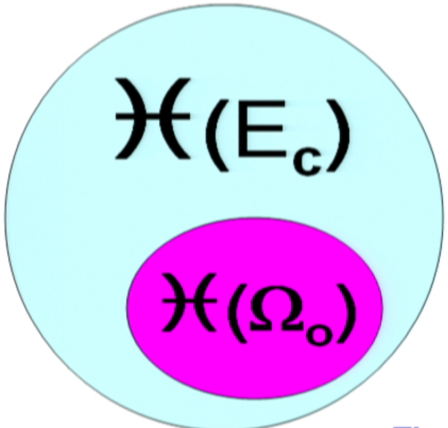


$\mathcal{H}_{\text{eff}}(E_c) \rightarrow \mathcal{H}_{\text{eff}}(\Omega_o)$

$|\psi_i\rangle\ H_{ij}(E_c)\ \langle\psi_j| \quad \rightarrow \quad |\phi_\alpha\rangle\ \mathcal{H}_{\alpha\beta}(\Omega_o)\ \langle\phi_\beta|$

Flow of Hamiltonian & Hilbert space with UV cutoff

The RG mantra is: RG flow
fixed points
low-energy $\mathcal{H}_{\text{eff}}$
universality classes

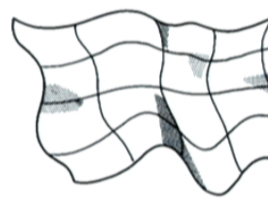


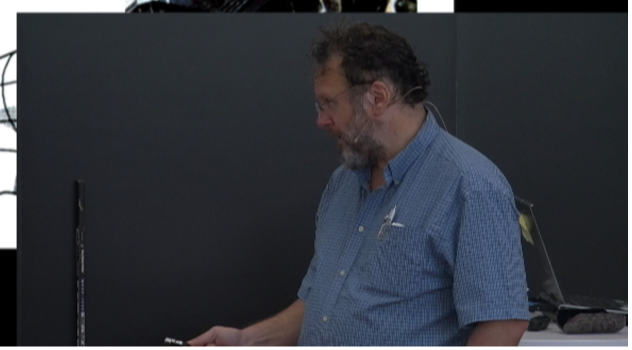
MORE ORTHODOXY

One supposes that for a given system, there will be a sequence of Hilbert spaces, over which the effective Hamiltonian and all the other relevant physical operators (**effective operators**) are defined.

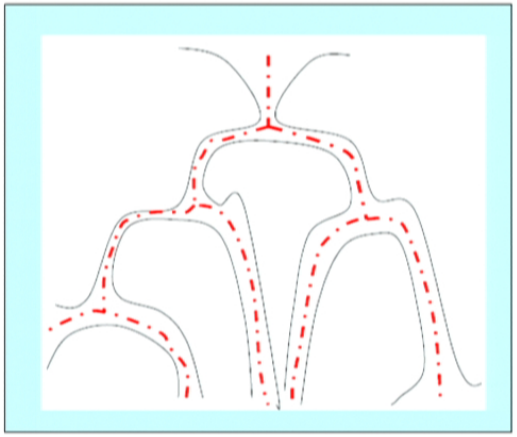
Then, we suppose, as one goes to low energies we approach the '*real vacuum*'; the approach to the fixed point tells us about the excitations about this vacuum. This is a little simplistic- the effective vacuum and the excitations change with the energy scale (often discontinuously, at phase transitions); & the effective Hamiltonian never one describes the full N-particle states.

Nevertheless, most believe that the basic structure is correct - that the effective Hamiltonian (***& note that ALL Hamiltonians or Actions are effective***) *captures all the basic physics*





RG PHILOSOPHY vs QCP PHILOSOPHY; T.O.E.'s



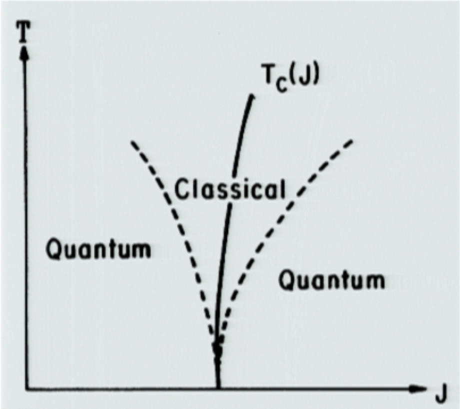
Two different views of the RG flow in a typical condensed matter system.

LEFT: a 'hierarchy' of fixed points, cascading down to ever lower energies. One determines a succession of effective Hamiltonians & field theories by integrating out high energy modes.

Reality check: In any complex system like a glass (or indeed any real solid) this cascade continues to extremely low energies – *ad infinitum* in the thermodynamic limit (if there is one!).

RIGHT: the 'Quantum critical point' philosophy – the structure of effective field theories is determined from BELOW by a few zero-energy fixed points. Some even argue that QCP framework allows classification of all low-E states → low-energy "Theory of Everything" (cf., eg., Preskill).

Reality check: The information in all the excited states vastly exceeds that in ground state; ground state does not determine excited states.



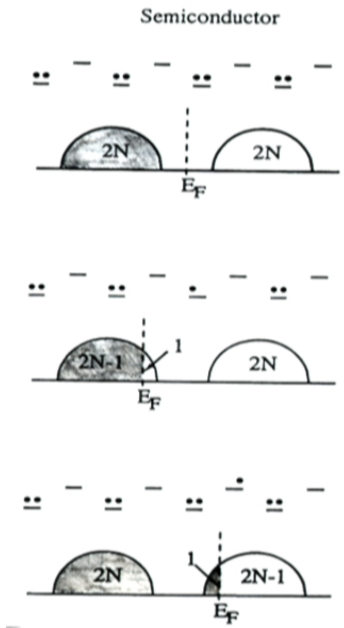
So much for the THEORY/PHILOSOPHY: now let's look at the

REAL WORLD

Only wimps specialize in the general case. Real scientists pursue examples.

MV Berry: Ann NY Acad Sci 755, 303 (1995)

1ST CONUNDRUM- the HUBBARD MODEL

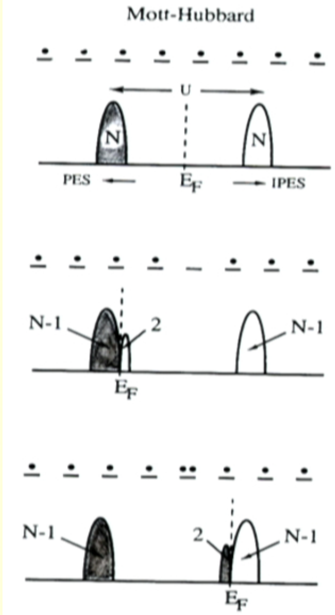


The 'standard model' of condensed matter physics for a lattice system is the 'Hubbard model', having effective Hamiltonian at electronic energy scales given by

$$H = -t \sum_{\langle i,j \rangle} \left(c^{\dagger}_{i\sigma} c_{j\sigma} + h.c. \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

This simple Hamiltonian has very bizarre properties. Suppose we try to find a low energy effective Hamiltonian, valid near the Fermi energy- eg., when the system is near "half-filling". Assume a UV Cutoff **<< U** (splitting between Mott-Hubbard sub-bands - assume that **U > t**).

Seems to be impossible!



1) **No well-defined low-E Hilbert space – no low-E effective $\mathcal{H}$**

2) **Spectral weight transfer → analogue of "UV / IR mixing"**

2ND CONUNDRUM: REAL Solids at low T

The canonical high-E Hamiltonian:

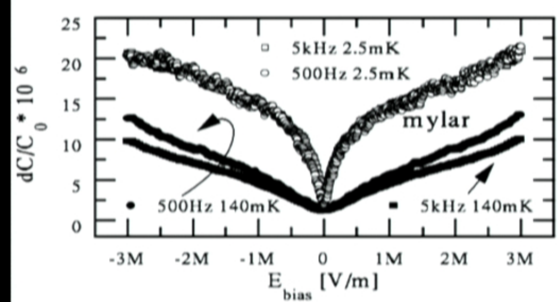
$$H^{sp}_{env} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V^{\alpha\beta}_{kk'} \sigma^{\alpha}_{k} \sigma^{\beta}_{k'}$$

Frustrating interactions and/or residual long-range interactions, & boundaries give:

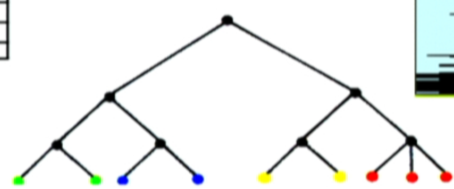
1) hierarchy of states → 'ultrametric' picture
2) Infinite Hierarchy of relaxation times
3) Infinite hierarchy of effective Hamiltonians

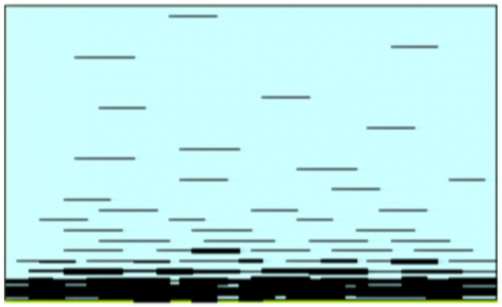
-- No ground state
-- effective H changes ad infinitum

"The deepest and most interesting unsolved problem in solid state theory is probably the theory of the nature of glass and the glass transition. This could be the next breakthrough in the coming decade. The solution of the problem of spin glass in the late 1970s had broad implications in unexpected fields like neural networks, computer algorithms, evolution, and computational complexity. The solution of the more important and puzzling glass problem may also have a substantial intellectual spin-off. Whether it will help make better glass is questionable."—P. W. Anderson [*Science* **1995**, *267*, 1615]



Capacitance in pure SiO_2





structure of low-energy eigenstates for interacting TLS model, before relaxation

ATTEMPTS at a LOW-T EFFECTIVE HAMILTONIAN

ONE ATTEMPT:

M Schechter, PCE Stamp: J Phys Cond Matt 20, 244136 (2008)
EuroPhys Lett 88, 66002 (2009)
/condmat: 0910.1283 v2

We get: $H_{\rm eff} = -\sum_j [D_j \hat{S}_j^x + \Delta_j \hat{\tau}_j^x] + V_{\rm eff}$ **where** $\mathbf{V_{eff} = V_{S\tau} + V_{RF}}$

Inter-defect interactions: $H_{S\tau} = \sum_{ij} [J_{ij}^{SS} S_i^z S_j^z + J_{ij}^{S\tau} S_i^z \tau_j^z + J_{ij}^{\tau\tau} \tau_i^z \tau_j^z]$

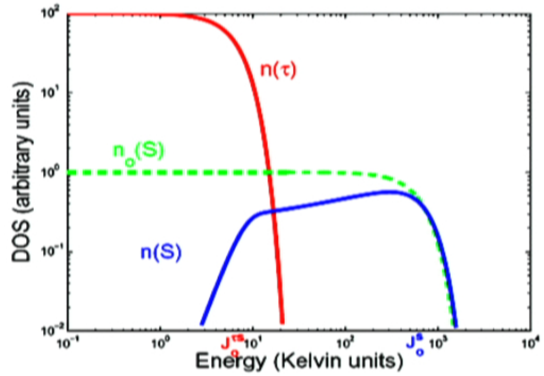
Random fields: $H_{RF} = \sum_j [h_j^S S_j^z + h_j^\tau \tau_j^z]$

where: $J_o^{\tau\tau} \sim g J_o^{S\tau} \sim g^2 J_o^{SS}$

$J_o \equiv J_o^{SS}$ **with** $J_o = \gamma_s^2 / \rho c^2 R_0^3 \sim 300\ K$

and $h_j^S \sim J_o$, $h_j^\tau \sim g J_o$

in which $g = \bar{\gamma}_w / \gamma_s = E_\phi / E_C$ **~ 0.01-0.03**



OTHER ATTEMPTS

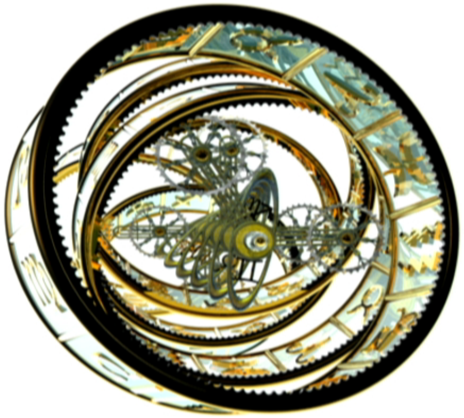
One such attempt tries to scale out 'blocks' of system to get low-T Hamiltonian – the hope is these excitations look a little more like collective phonons:

DC Vural, AJ Leggett, /arXiv 1103.5530

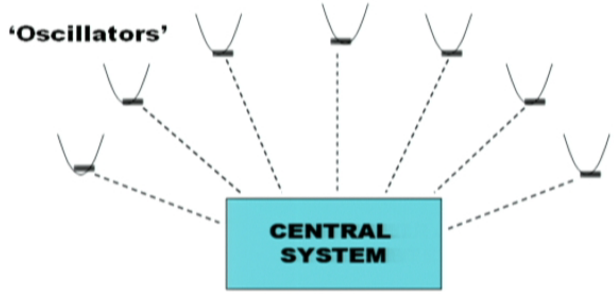
A quite different theory:

Lubchenko, V. & Wolynes, P. G. Intrinsic quantum excitations of low temperature glasses. *Phys. Rev. Lett.* **87**, 195901 (2001).

EFFECTIVE FIELD THEORIES: SUMMARY



CURRENT MODELS of ENVIRONMENTAL DECOHERENCE



$$H_{\rm eff}^{\rm osc} = H_0 + H_{\rm int} + H_{\rm env}^{\rm osc}$$

Bath: $$H_{osc} = \sum_{q=1}^{N_o} \left(\frac{p_q^2}{m_q} + m_q \omega_q^2 x_q^2 \right)$$

Int: $$H_{\rm int}^{\rm osc} = \sum_{q=1}^{N} [F_q(Q) x_q + G_q(P) p_q]$$

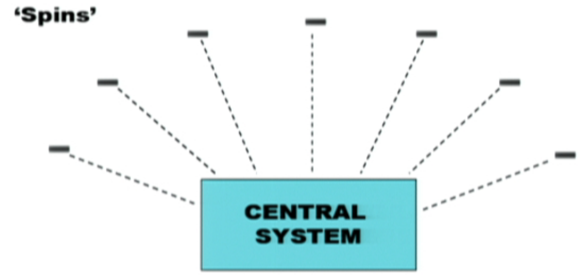
Very SMALL (~ O(1/N$^{1/2}$))

Phonons, photons, magnons, spinons, Holons, Electron-hole pairs, gravitons,..

Feynman & Vernon, Ann. Phys. 24, 118 (1963)

Caldeira & Leggett, Ann. Phys. 149, 374 (1983)
AJ Leggett et al, Rev Mod Phys 59, 1 (1987

DELOCALIZED BATH MODES
↓
OSCILLATOR BATH



$$H_{\rm eff}^{\rm sp}(\Omega_0) = H_0 + H_{\rm int}^{\rm sp} + H_{\rm env}^{\rm sp}$$

Bath: $$H_{\rm env}^{\rm sp} = \sum_k^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^\alpha \sigma_{k'}^\beta$$

Interaction: $$H_{\rm int}^{\rm sp} = \sum_k^{N_s} \boldsymbol{F}_k(P,Q) \cdot \boldsymbol{\sigma}_k$$

NOT SMALL !

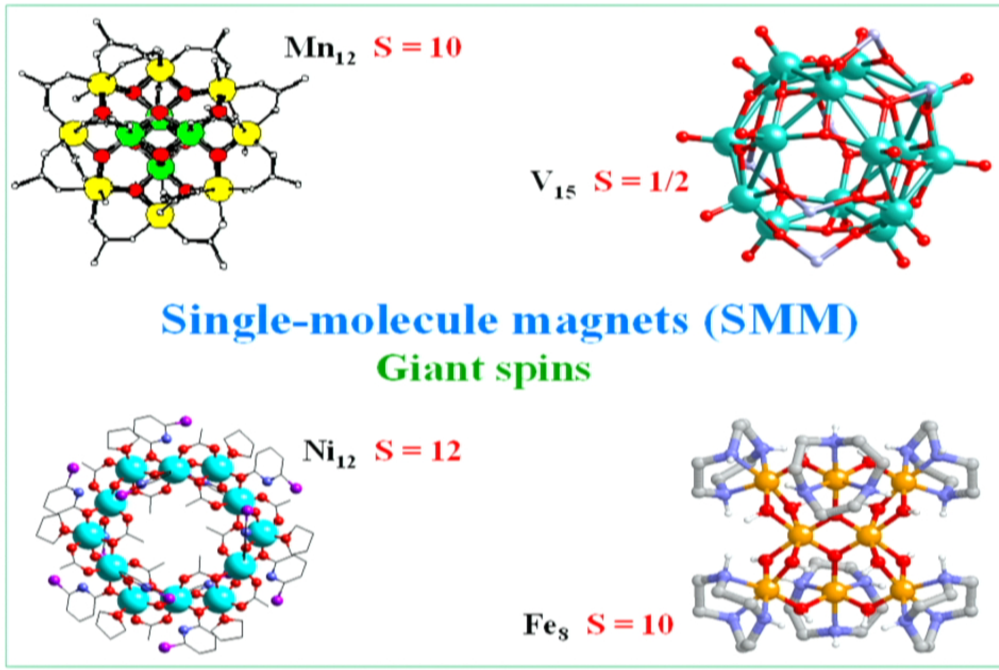
Defects, dislocation modes, vibrons, Localized electrons, spin impurities, nuclear spins, ...

LOCALIZED BATH MODES
↓
SPIN BATH

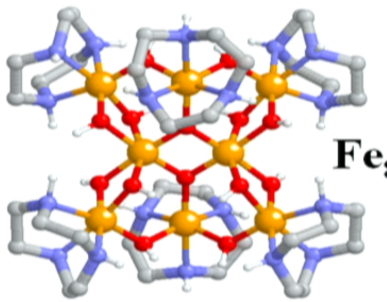
(1) P.C.E. Stamp, PRL 61, 2905 (1988)
(2) NV Prokof'ev, PCE Stamp, J Phys CM5, L663 (1993)
(3) NV Prokof'ev, PCE Stamp, Rep Prog Phys 63, 669 (2000)

Example: Experiments on Magnetic Molecules

These molecules behave as a set of 2-state qubits, with the spin bath provided by nuclear spins, and the oscillator bath by phonons. There are thousands of high-spin molecules which behave like this – here are some well-studied examples:



A Typical tunneling molecule: the Fe-8 MOLECULE



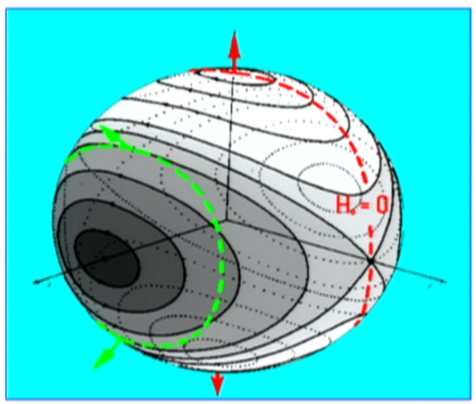
Fe_8 **S = 10**

Low-T Quantum regime- effective Hamiltonian
(T < 0.36 K): $\mathcal{H}_o(\hat{\tau}) = (\Delta_o \hat{\tau}_x + \epsilon_o \hat{\tau}_z)$

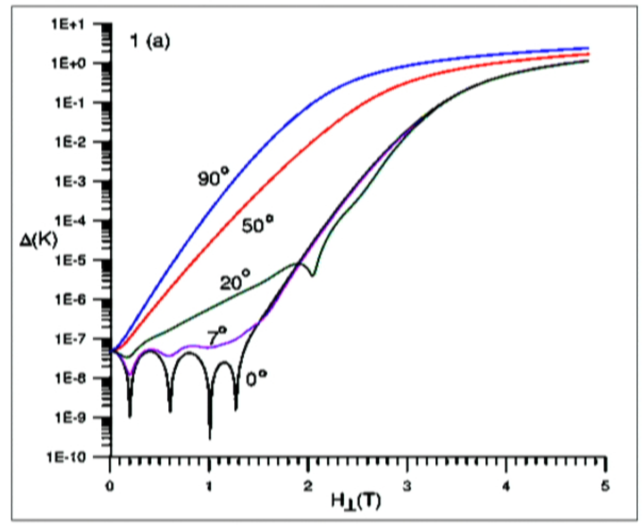
Longitudinal bias: $\epsilon_o = g\mu_B S_z H_o^z$

Eigenstates: $|\pm\rangle = [|\uparrow\rangle \pm |\downarrow\rangle]/\sqrt{2}$

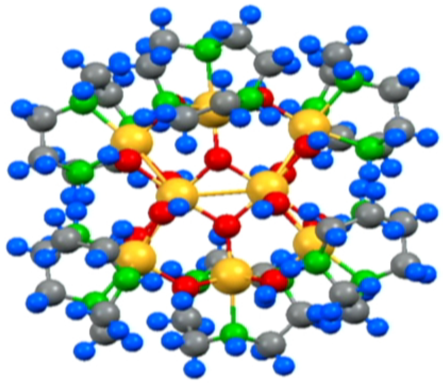
Which also defines orthonormal states: $|\uparrow\rangle, |\downarrow\rangle$



Feynman Paths on the spin sphere for a biaxial potential. Application of a field pulls the paths towards the field

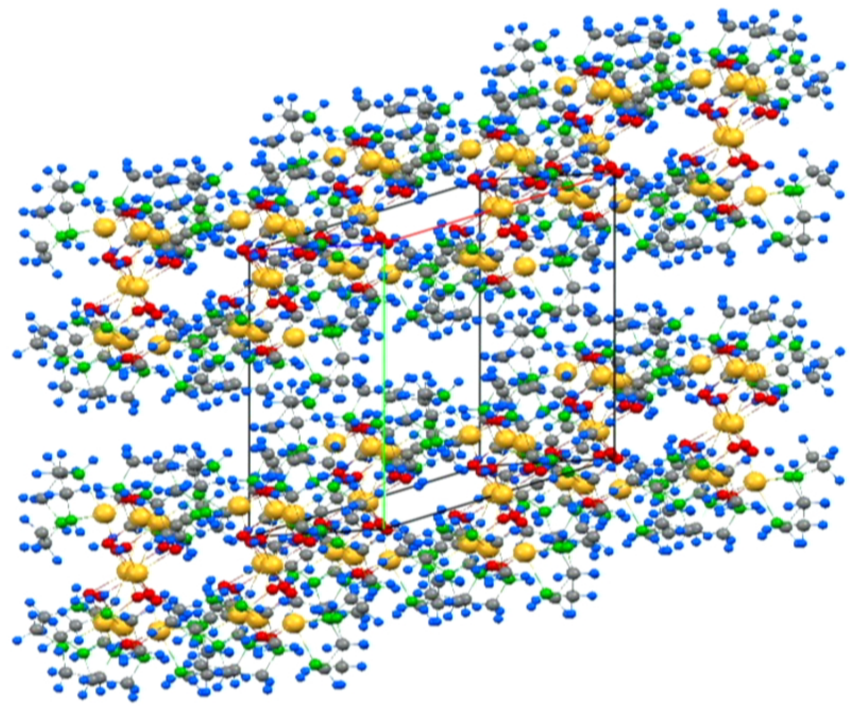


GEOMETRICAL ARRANGEMENT



Single molecule

Crystal lattice (triclinic symmetry)



THEORETICAL DESCRIPTION of INTERACTING SPIN QUBITS

1) EFFECTIVE HAMILTONIAN

Interqubit dipolar interactions

$$\mathbf{H} = \Sigma_j (\Delta_j \tau_j^x + \varepsilon_j \tau_j^z) + V_{dip}(\{\tau_j\})$$
$$\mathbf{H}_{spin}(\{\sigma_k\}) + \mathbf{H}_{osc}(\{x_q\}) + U_{int}(\tau_j, \{\sigma_k\}, \{x_\alpha\})$$

$$H_{\rm env}^{\rm sp} = \sum_{k}^{N_s} \mathbf{h}_k \cdot \boldsymbol{\sigma}_k + \sum_{k,k'}^{N_s} V_{kk'}^{\alpha\beta} \sigma_k^\alpha \sigma_{k'}^\beta$$

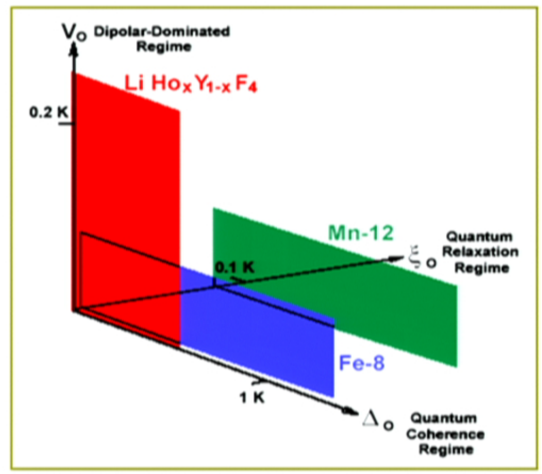
$$H_{osc} = \sum_{q=1}^{N_o} (\frac{p_q^2}{m_q} + m_q \omega_q^2 x_q^2)$$

Hyperfine interactions &
Spin-phonon interactions

2) QUANTUM vs CLASSICAL REGIMES

At low T, we are in the quantum regime, where the behaviour can be dominated by any of the 3 interactions (see right).

However if we raise T, we cross over to classical activated dynamics.



***QUANTUM COHERENCE REGIME:* here quantitative predictions were made long before any experiments were done.**

DECOHERENCE IN Fe-8 SYSTEM

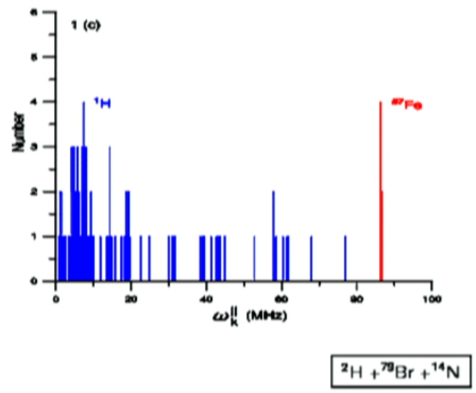
(A) Nuclear Spin Bath

Hyperfine couplings of all 213 nuclear spins are well known

$$H_{eff}^{CS} = [\Delta_o \hat{\tau}_+ e^{-i\sum_k \boldsymbol{\alpha}_k \cdot \boldsymbol{\sigma}_k} + H.c.] + \hat{\tau}^z(\epsilon_o + \sum_k \boldsymbol{\omega}_k \cdot \boldsymbol{\sigma}_k) + H_{env}^{sp}([\boldsymbol{\sigma}_k])$$

Nuclear spin decoherence rate

$$\gamma_\phi^{\rm NS} = E_0^2/2\Delta_0^2 \qquad \textbf{where} \qquad E_o^2 = \sum_k \frac{I_k+1}{3I_k}(\omega_k^{\parallel} I_k)^2$$



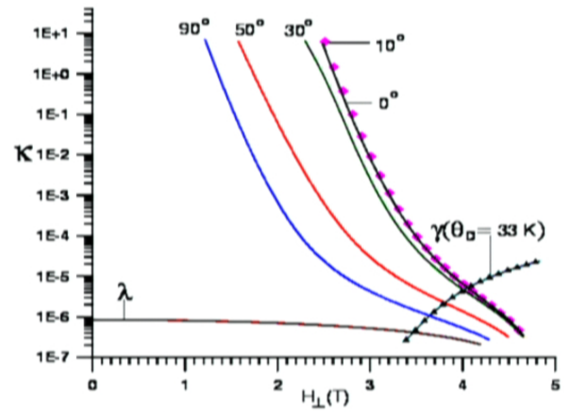
(b) Phonon Bath

Phonon spectrum and spin-phonon couplings are known. Phonon decoherence rate is:

$$\gamma_\phi^{\rm ph} = \frac{\mathcal{M}_{\mathcal{AS}}^2 \Delta_0^2}{\pi \rho c_s^5 \hbar^3} \coth\left(\frac{\Delta_0}{k_B T}\right)$$

$$\mathcal{M}_{\mathcal{AS}}^2(H_y) \approx \tfrac{4}{3} D^2 |\langle \mathcal{A}| S_y S_z + S_z S_y |\mathcal{S}\rangle|^2$$

Total SINGLE QUBIT decoherence rate shown in Figure at right:



(c) Dipolar Decoherence

This is an example of "correlated errors" caused by inter-qubit interactions. It turns out to be very serious.

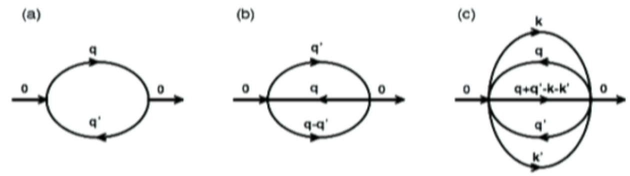
The high-T (van Vleck) limiting form is $$(\gamma_\phi^{vV})^2 \approx \left[1 - \tanh^2\left(\frac{\Delta_0}{k_B T}\right)\right]\sum_{i\neq j}\left(\frac{\mathcal{A}^{ij}_{yy}}{\Delta_0}\right)^2,$$

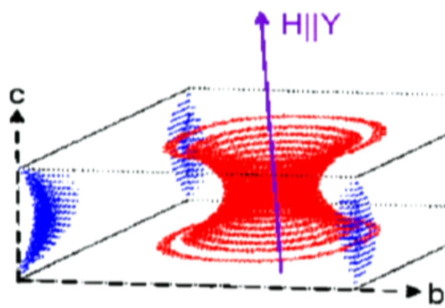
$$\mathcal{A}^{ij}_{yy} = \frac{U_d}{(2g_e S)^2}[(2\tilde{g}_y^2 + \tilde{g}_z^2)\mathcal{R}^{ij}_{yy} - (\tilde{g}_x^2 - \tilde{g}_z^2)\mathcal{R}^{ij}_{xx}],$$

$$\mathcal{R}^{ij}_{\mu\nu} = \mathcal{V}_c(|\mathbf{r}^{ij}|^2\delta_{\mu\nu} - 3r^{ij}_\mu r^{ij}_\nu)/|\mathbf{r}^{ij}|^5$$

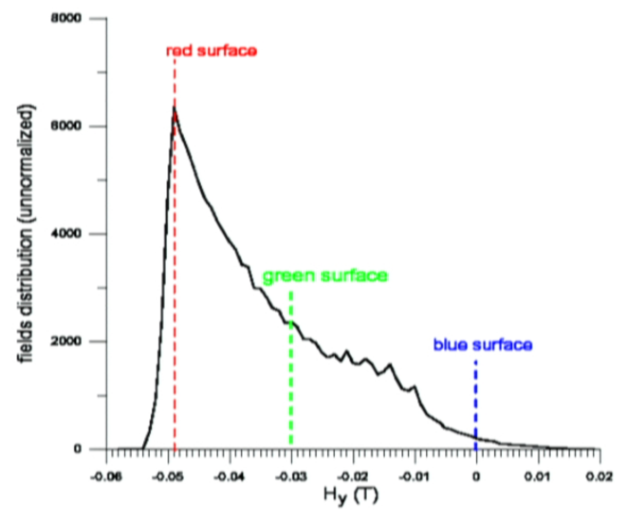
At low T one gets a quite different form

$$\gamma_\phi^m = \frac{2\pi}{\hbar\Lambda_c}\sum|\Gamma^{(4)}_{\mathbf{q}\mathbf{q}'}|^2\mathcal{F}[\bar{n}_\mathbf{q}]\delta(\omega_0 + \omega_\mathbf{q} - \omega_{\mathbf{q}'} - \omega_{\mathbf{q}-\mathbf{q}'}).$$





RESONANT SURFACES

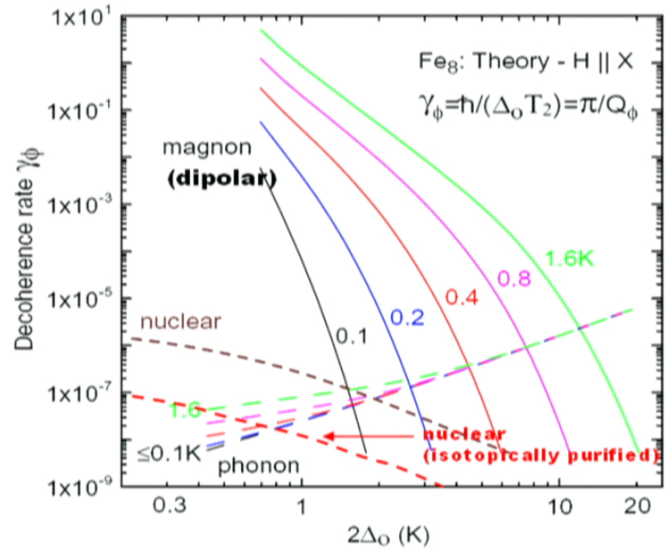


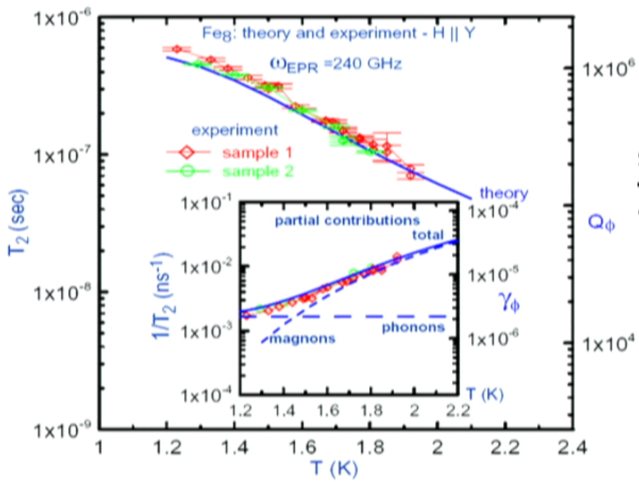
EXPERIMENTAL VERIFICATION: the Fe-8 SYSTEM

Advantage of using Fe-8: it can be made very pure, with few defects in a crystal. To raise the 'Q-factor' of this system it is very useful to go to high fields.

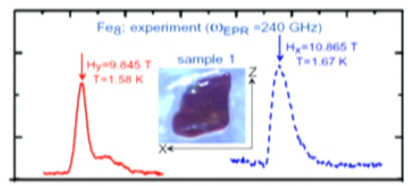
WE GET STRIKING CONFIRMATION of the PREDICTION of LARGE-SCALE COHERENCE ACROSS THE SAMPLE

S. Takahashi + al., Nature 476, 76 (2011)



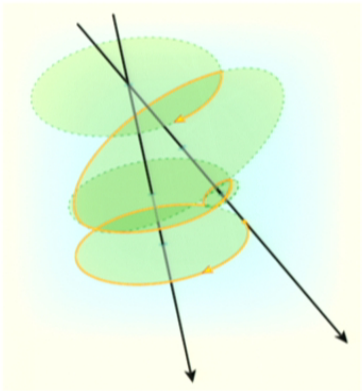


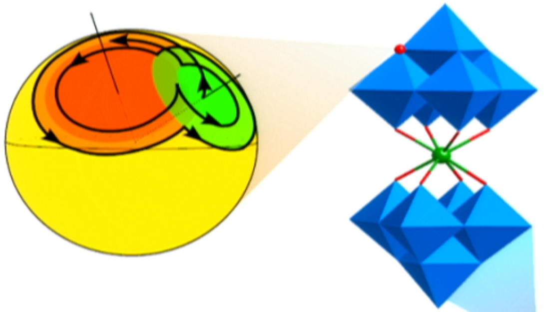
Using 'Hahn echo' ESR experiments, get good agreement with theory; no evidence for extrinsic decoherence sources.
This is first ever agreement with theoretical predictions for a non-trivial solid-state system.

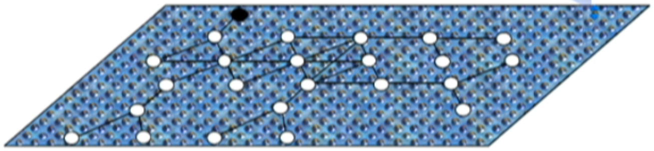


These experiments need to be pursued further.

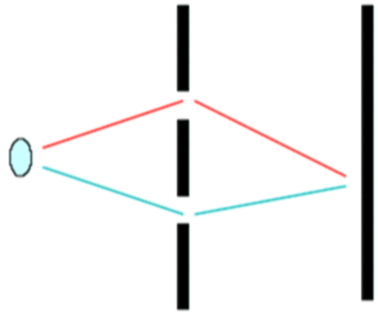
ENVIRONMENTAL DECOHERENCE: SUMMARY





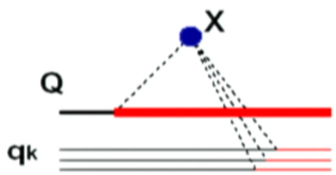


3^{rd} PARTY DECOHERENCE:



This is decoherence in the dynamics of a system A (with coordinate Q) caused by *indirect* entanglement with an environment E- the entanglement is achieved via a 3^{rd} party B (coordinate X).

Ex: Buckyball decoherence



Consider the 2-slit expt with buckyballs. The COM coordinate Q of the buckyball does not couple directly to the vibrational modes {qk } of the buckyball - by definition.

However BOTH couple to the slits in the system, in a distinguishable way.

Note: the state of the 2 slits, described by a coordinate X, is irrelevant- it does not need to change at all. We can think of it as a scattering potential, caused by a system with infinite mass. It is a PASSIVE 3^{rd} party. We can also have ACTIVE 3^{rd} parties

PCE Stamp, Stud. Hist Phil Mod Phys 37, 467 (2006)

See also PCE Stamp, WG Unruh, in preparation

There is a problem however – consider the system moving in some parabolic well (to make the calculations simple). Then we have a correction of form:

$$\int \mathcal{D}\underline{r}(t) \int \mathcal{D}\underline{r}'(t) \; \exp\left\{ \frac{i}{\hbar} \int dt \; \pi G m \frac{1}{|r(t) - r'(t)|} \left[(\dot{\underline{r}}^2 + \dot{\underline{r}}'^2) - \omega_0^2 (r^2 + r^3) \right] \right\}$$

But this contains unavoidable and very severe UV divergences, as we see by writing

$$\underline{R} = \underline{r} + \underline{r}' \qquad \underline{q}(t) = \sum_{n=1}^{N(\Lambda)} q_n \sin \frac{n\pi t}{t_2 - t_1}$$

$$\underline{q} = \underline{r} - \underline{r}'$$

and then letting the UV cutoff increase. Very close paths dominate the sum. Making the calculation fully covariant would not help – indeed it makes it worse. Short distances correspond to higher energy scales, and the energy-momentum tensor is cut-off dependent.

There is another problem. The correction kernel is the exponential of a pure phase. It will not cause ecoherence in the conventional sense, but only a geometric phase shift, which will depend very sensitively on initial and final states, etc.

However, one can still do some useful things....

There is a problem however – consider the system moving in some parabolic well (to make the calculations simple). Then we have a correction of form:

$$\int \mathcal{D}\underline{r}(t) \int \mathcal{D}\underline{r}'(t) \; \exp\left\{ \frac{i}{\hbar} \int dt \; \pi G m \frac{1}{|r(t) - r'(t)|} \left[(\dot{\underline{r}}^2 + \dot{\underline{r}}'^2) - \omega_0^2 (r^2 + r'^2) \right] \right\}$$

But this contains unavoidable and very severe UV divergences, as we see by writing

$$\underline{R} = \underline{r} + \underline{r}' \qquad \underline{q}(t) = \sum_{n=1}^{N(\Lambda)} q_n \sin \frac{n\pi t}{t_2 - t_1}$$

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There is another problem. The correction kernel is the exponential of a pure phase. It will not cause ecoherence in the conventional sense, but only a geometric phase shift, which will depend very sensitively on initial and final states, etc.

However, one can still do some useful things....

Let's separate out the centre of mass $\{r_i(t)\} = R(t), \{x_q(t)\}$

Total Lagrangian $\mathcal{L}(R, \{x_q\}) = L_o(\underline{R}) + L_\phi(\{x_q\}) + L_{int}(R, \{x_q\})$

Total Hamiltonian $\mathcal{H}(R, \{x_q\}) = H_o(\underline{R}) + \mathcal{H}_\phi(\{x_q\}) + V_{int}(R, \{x_q\})$

Effective Lagrangian & Hamiltonian for the secondary 'fast' variables:

$$L_f(\underline{R}, \{x_q\}) = L_\phi(\{x_q\}) + L_{int}(\underline{R}, \{x_q\})$$

$$\mathcal{H}_f(R, \{x_q\}) = \mathcal{H}_\phi(\{x_q\}) + \mathcal{H}_{int}(R, \{x_q\})$$

such that: $\mathcal{H}_f(\underline{R}, \{x_q\}) \,|\, n_q(R)\rangle = \mathcal{H}_f(R, \{x_q\}) \,|\, n_q(R)\rangle$

QM gives us: $\mathcal{G}_o(2,1) \equiv \mathcal{G}_o(\underline{R}_2, \underline{R}_1; \{x_q(2), x_q(1)\}; t_2 t_1)$

$$= \int_1^2 \mathcal{D}\underline{R}\; e^{i/\hbar \int dt\, L_o(\underline{R})}\, G_f^o(\{x_q(2), x_q(1)\}; t_2 t_1 \,|\, [R(t)])$$

But here we get a correction kernel: $\bar{K}_{mn}(\underline{R}, R'; t) \sim e^{i\, \bar{\Phi}_{mn}[R, R']}$

With the phase $\Phi_{mn}[R, R'] = \frac{1}{\hbar} \int dt \prod_q \dot{\underline{R}} \cdot \langle m_q(R) | \nabla_R | n_q(R) \rangle \frac{4\pi G}{|R - R'|} \langle m_q(\underline{R}') | \nabla_{R'} | n_q(R') \rangle$

$$= \hbar \int dt \prod_q \dot{\underline{R}} \cdot A^q_{mn}(\underline{R}) \frac{4\pi G}{|R(t) - R'(t)|} A^q_{nm}(\underline{R}') \cdot \dot{\underline{R}}'$$

INTRINSIC DECOHERENCE: SUMMARY

CONCLUSION: WHAT HAVE WE LEARNED?