

Title: A Macroscopic-scale Wave-particle Duality : the Role of a Wave Mediated Path Memory

Date: Oct 26, 2011 02:00 PM

URL: <http://pirsa.org/11100119>

Abstract: It is usually assumed that the quantum wave-particle duality can have no counterpart in classical physics. We were driven into revisiting this question when we found that a droplet bouncing on a vibrated bath could couple to the surface wave it excites. It thus becomes a self-propelled &quot;walker&quot;, a symbiotic object formed by the droplet and its associated wave.

Through several experiments, we addressed one central question. How can a continuous and spatially extended wave have a common dynamics with a localized and discrete droplet? Surprisingly, quantum-like behaviors emerge; both a form of uncertainty and a form of quantization are observed. This is interesting because the probabilistic aspects of quantum mechanics are often said to be intrinsic and to have no possible relation with underlying unresolved dynamical phenomena. In our experiment we find probabilistic behaviors and they do have a relation with chaotic individual trajectories. These quantum like properties are related in our system to the non-locality of a walker that we called its &quot;wave mediated path memory&quot;. The relation of this experiment with the pilot wave model proposed for quantum mechanics by de Broglie will be discussed.

**A macroscopic-scale wave-particle duality :  
the role of a wave mediated path memory**

**With :**

**Emmanuel Fort,  
Suzie Protière, Antonin Eddi,  
Julien Moukhtar, Eric Sultan  
Arezki Boudaoud, Charles Henri Gautier,  
Frédéric Moisy and Maurice Rossi, ,**

**Matières et Systèmes Complexes (Université Paris–Diderot)**

Can any of the phenomena characteristic of the quantum wave-particle duality be observed in a non quantum system?

**We were drawn into investigating this question by the, almost accidental, finding of a wave-particle association at macroscopic scale.**

## **Introduction**

**A massive particle driven by the wave it generates**

The bouncing droplet and its coupling to surface waves.

**Part I : Walking straight**

The wave-field structure and its “path-memory”: a non locality in time.

**Part II: Walking in circles**

The orbits of walkers when submitted to a transverse force

**Part III : Walking when confined**

(a) In corrals

(b) Through slits (diffraction and interference)

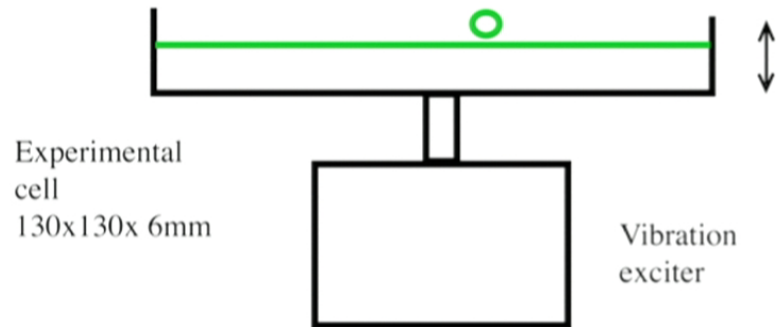
(c) By a single barrier: a tunnel effect of a kind

**Discussion**

The relation to de Broglie’s pilot waves

Trajectories and probability of visit

## The basic experimental set-up



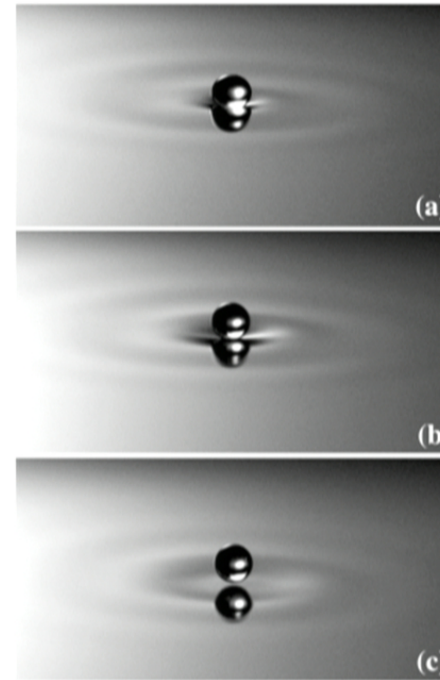
Silicon oil of viscosity  
 $\mu = 50 \cdot 10^{-3} \text{ Pa}\cdot\text{s}$

Vertical acceleration

$$\gamma = \gamma_m \cos(\omega t)$$

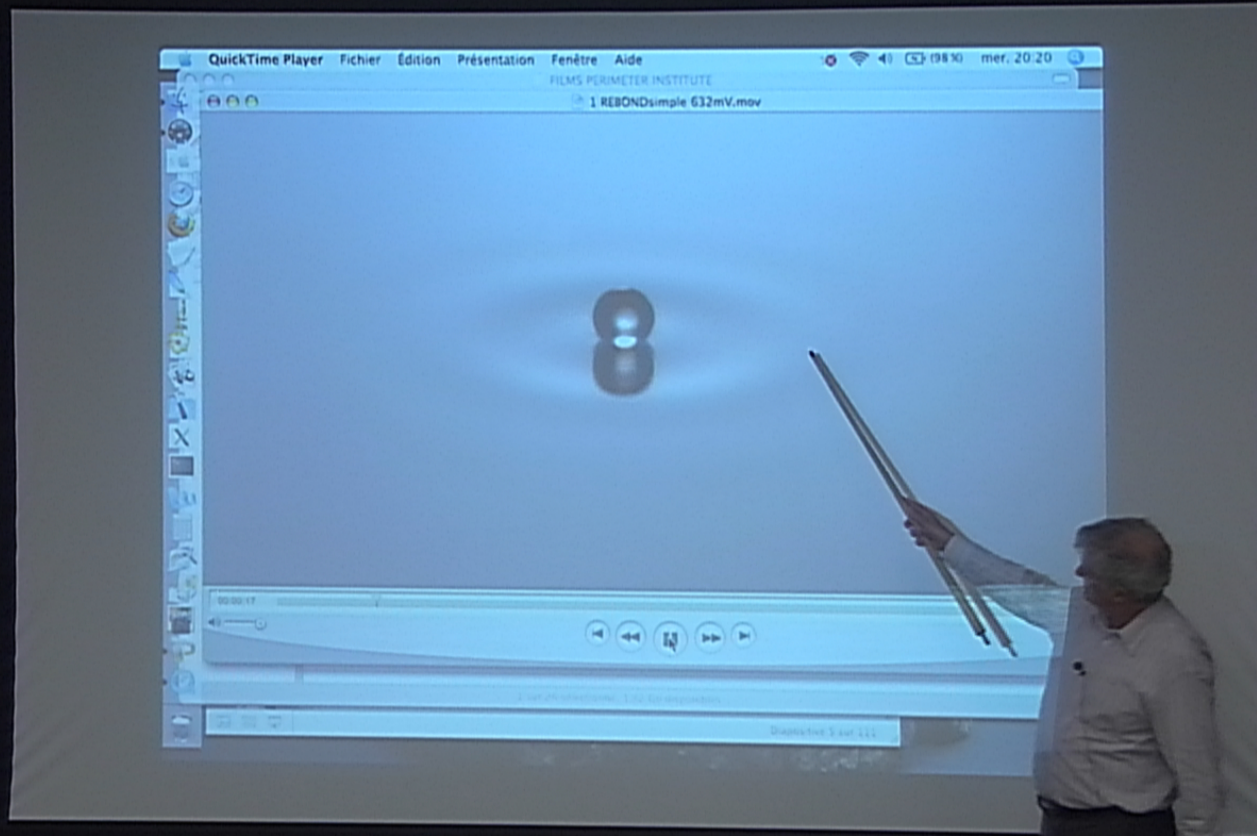
with  $\omega/2\pi = 80 \text{ Hz}$

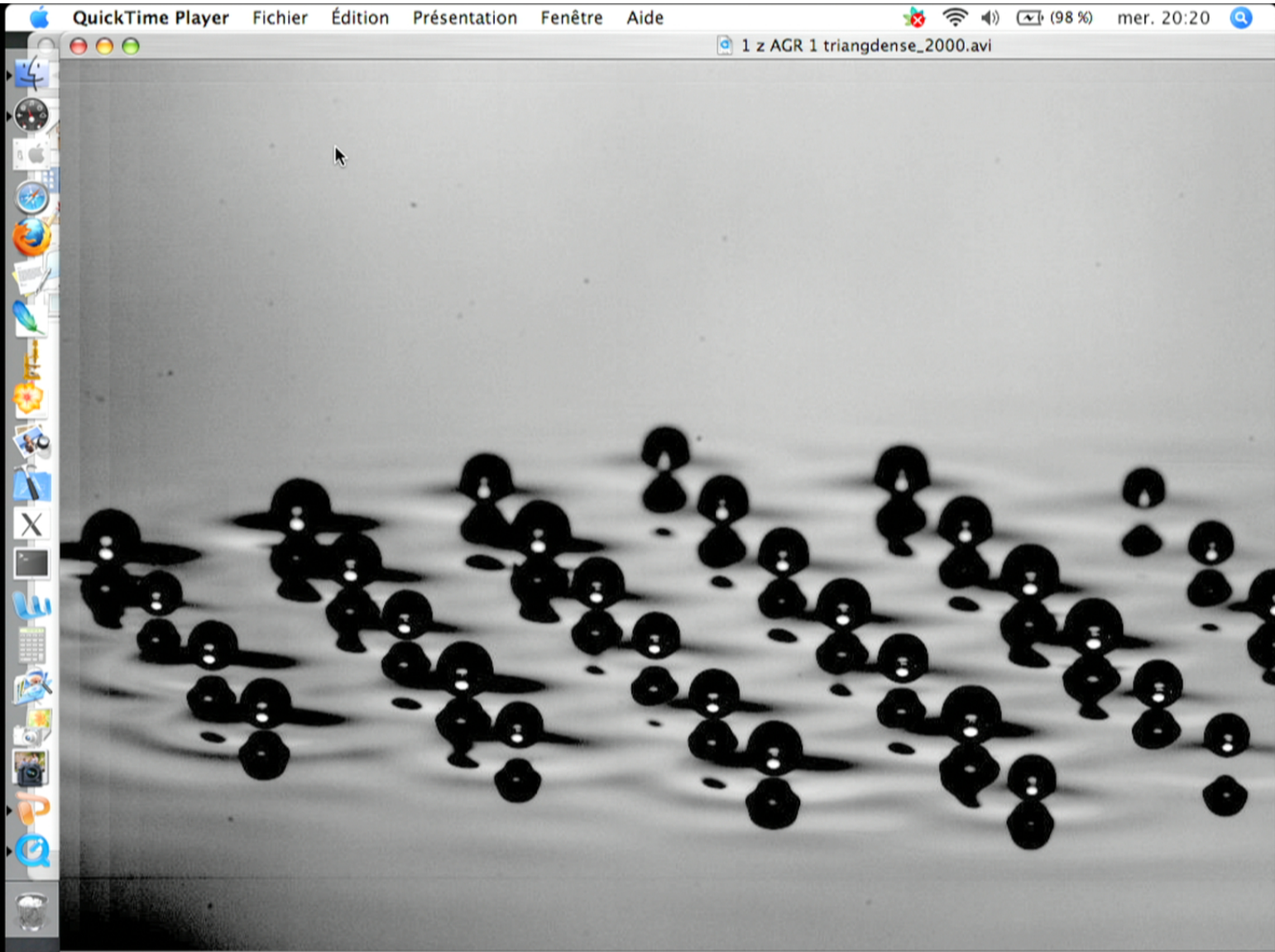
and  $0 < \gamma_m < 5$

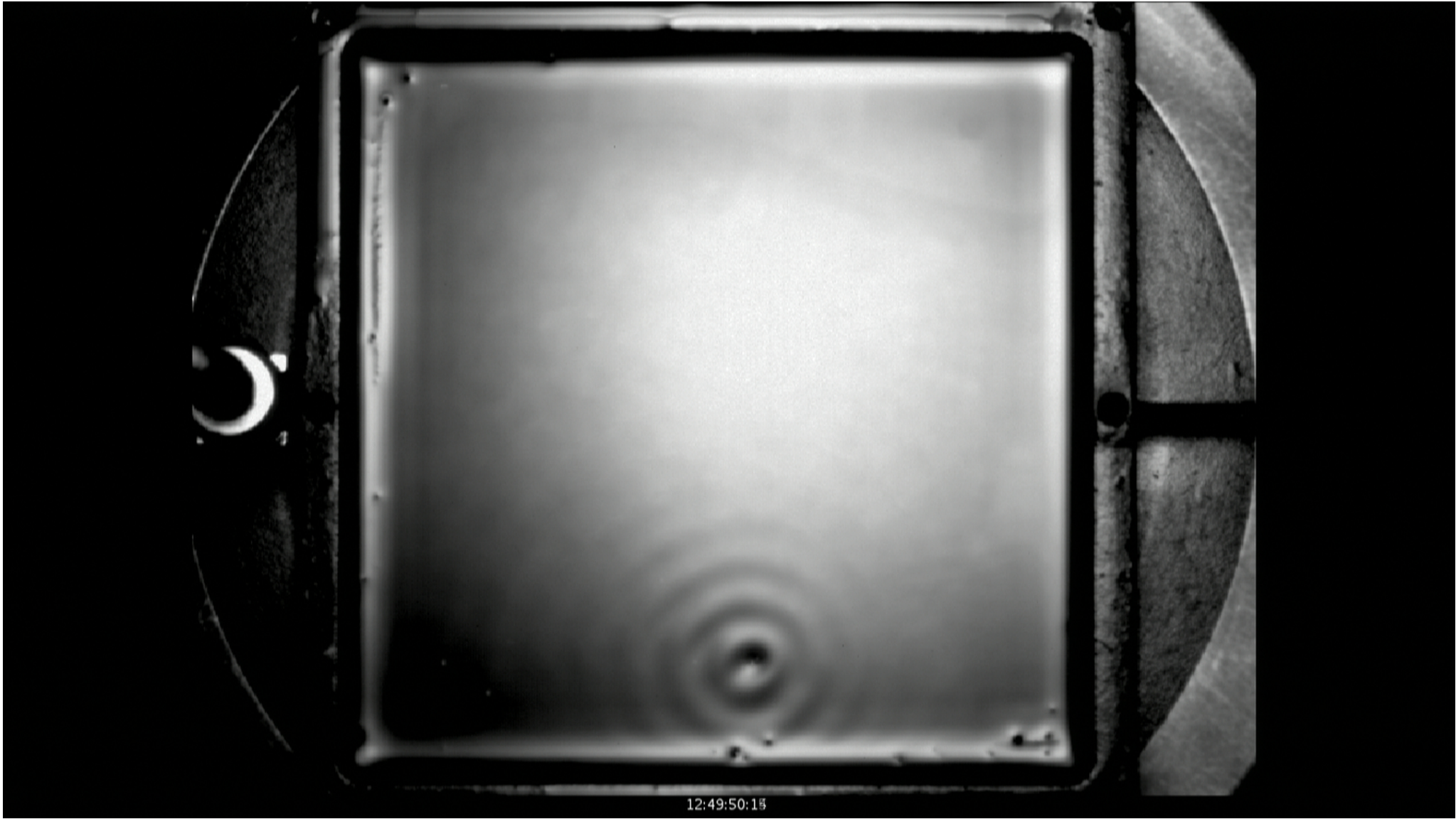


**There is always an air film between the drop and the substrate**

*The drop can bounce for days!*

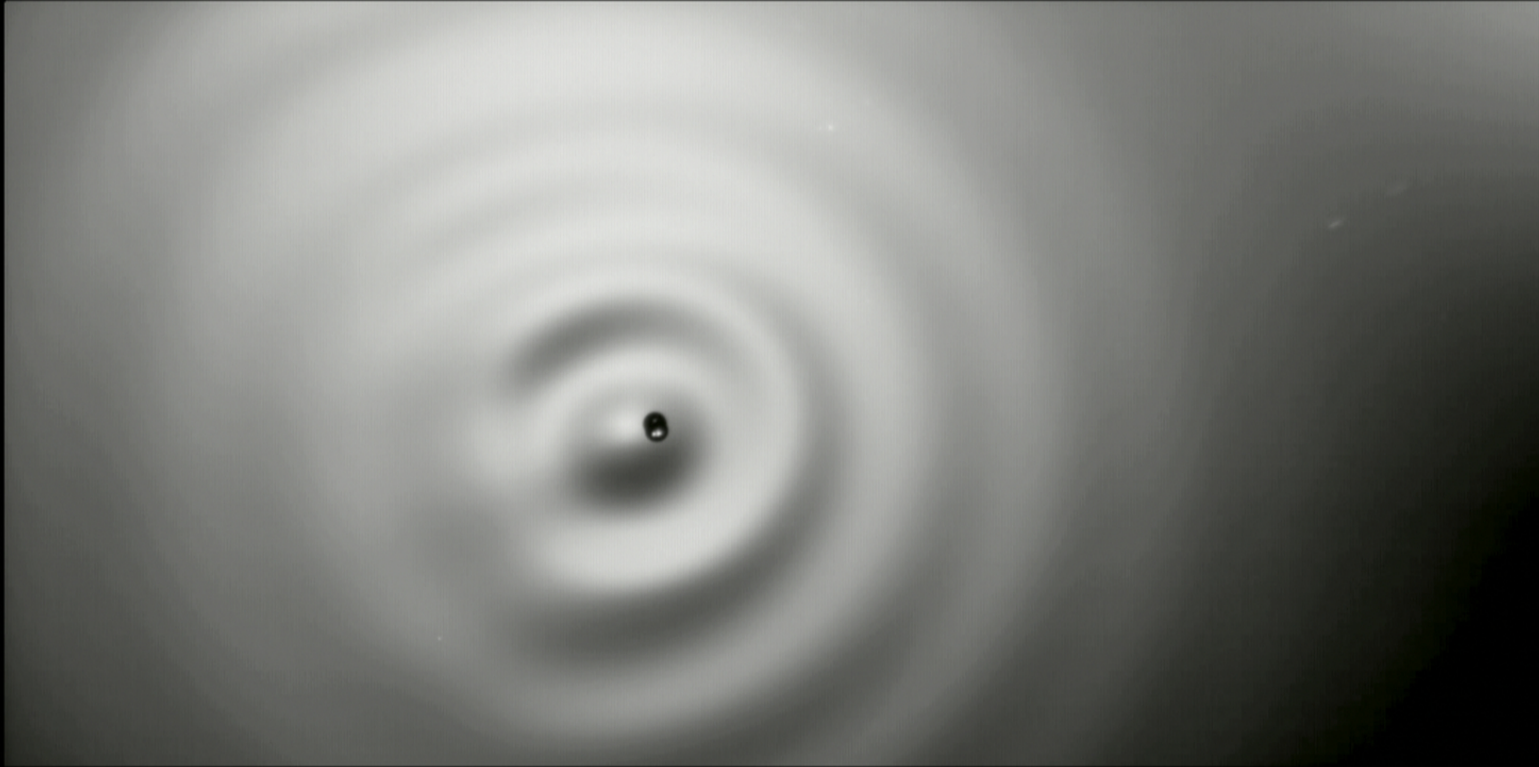






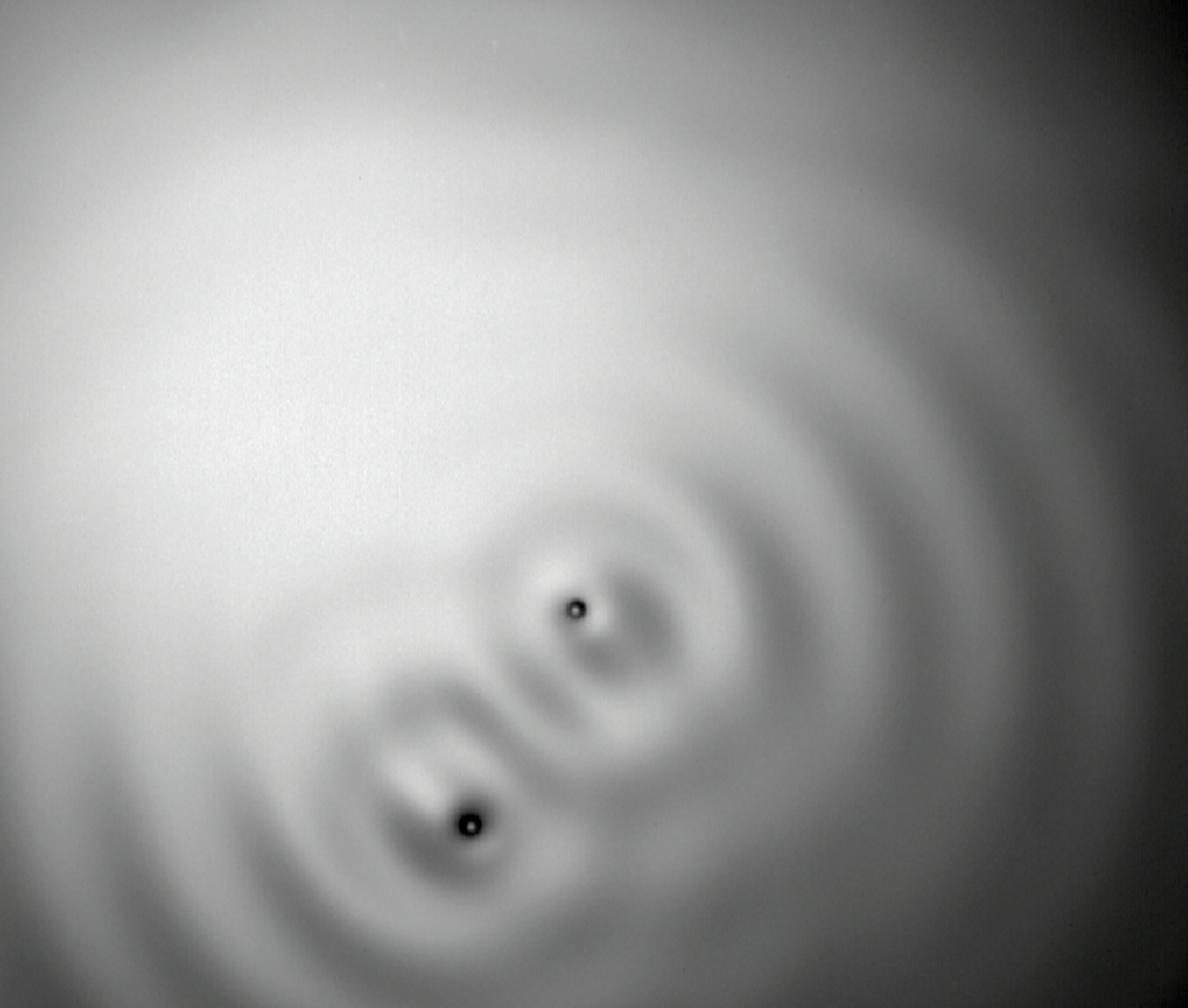








Finder



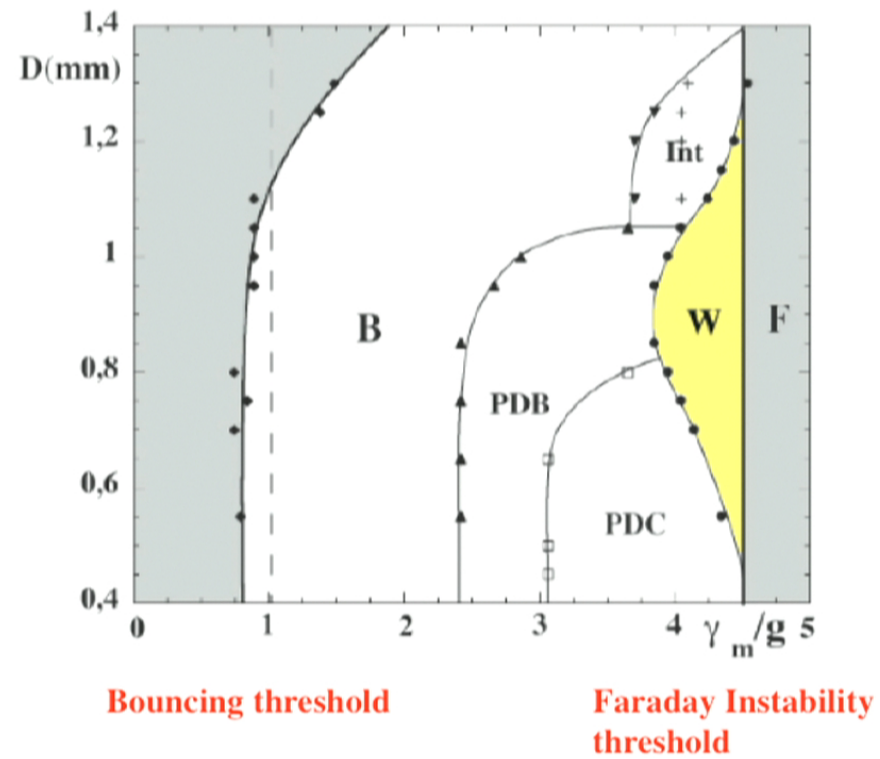
14:44:25:04

## Phase diagram of the different types of bouncing

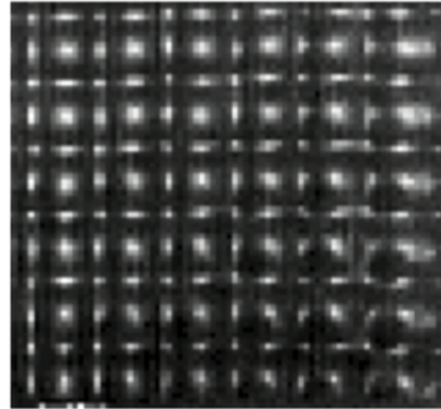
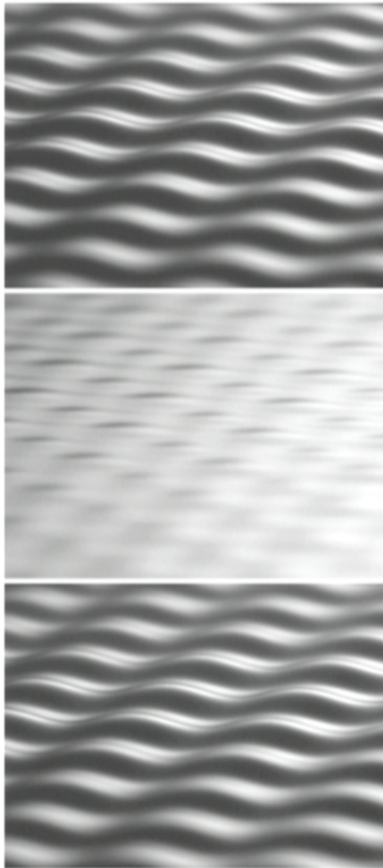
(for  $\mu=50 \cdot 10^{-3} \text{ Pa s}$  and  $\omega/2\pi=80 \text{ Hz}$ )

as a function of the droplet size and the amplitude of the forcing

- B : simple bouncing at the forcing frequency
- PDB : Period doubling
- PDC: Temporal chaos
- W : “walkers“
- F : Faraday instability



*The Faraday instability of a vertically vibrated liquid surface*



**Vertical oscillations**

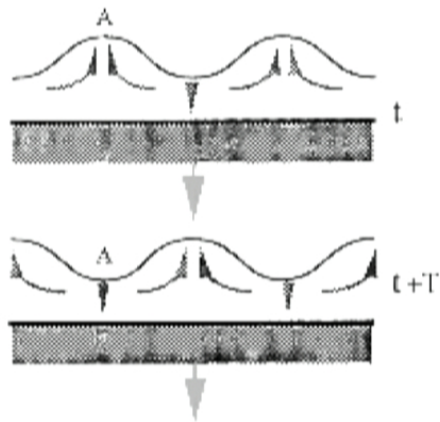
$$\gamma = \gamma_m \cos(\omega t)$$

**When  $\gamma_m \geq \gamma_m^{Far}$   
the surface becomes covered  
with standing waves of frequency  $\omega/2$**

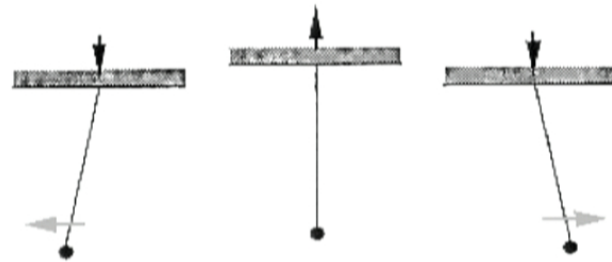
**In the present experiment**

$$\omega/2\pi = 80Hz \quad \text{and} \quad \gamma_m^{Far} = 4.5 g$$

*The Faraday instability results from the parametric forcing of the surface waves*  
cf e.g. Stéphane Douady, *Thesis* (1989)



*Analogous to the parametric forcing of a rigid pendulum*



**The motion is given by Mathieu's equation:**

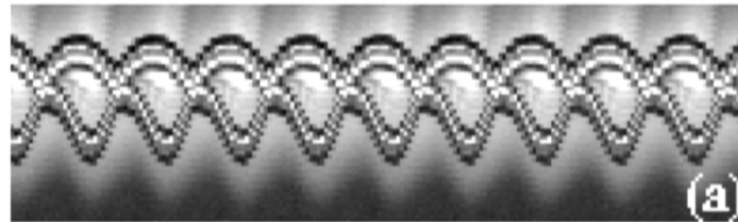
$$\frac{\partial^2 a}{\partial t^2} + 2f \frac{\partial a}{\partial t} + \omega_0^2 (1 + 2\varepsilon \cos \omega t) = 0$$

**and its frequency is half that of the forcing**

The drop's bouncing: (for  $0.6 < D < 1.1\text{mm}$ )  
Spatio-temporal diagrams of the vertical motion

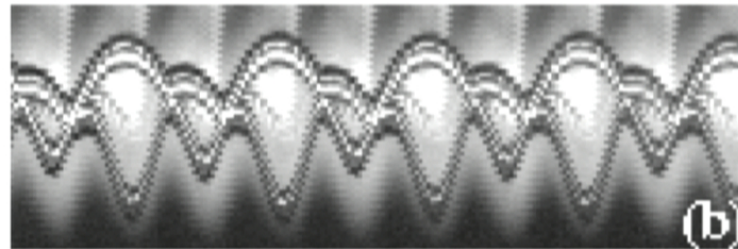
$\gamma_m = 1.5g$

Bouncing at the  
forcing frequency



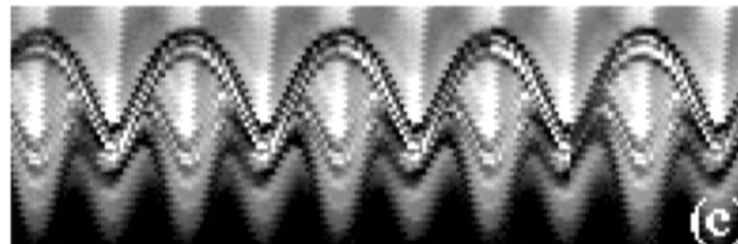
$\gamma_m = 3g$

Period doubling



$\gamma_m = 4g$

no chaos and  
complete period  
doubling



Time

## Part I (b)

# A simple model of the walking bifurcation

*(Arezki Boudaoud)*

Protière S., Boudaoud A. & Couder Y. *J. Fluid Mech.* 554, 85-108, (2006)



## The first model for the walking transition

(Arezki Boudaoud)

Newton's equation, the fast vertical motion being averaged over one period

$$m \frac{d^2 x}{dt^2} = F^b \sin\left(2\pi \frac{dx/dt}{V_F^\varphi}\right) - f^v dx/dt$$

-  $m \sim 10^{-6}$  kg mass of the droplet

-  $F^b$  effective force due to the bouncing on an inclined surface

$$F^b \approx m \gamma_b \frac{A_w}{\lambda} \left(\frac{\tau}{T_F}\right) \approx 10^{-6} \text{ N}$$

$\gamma_b$  Vertical acceleration,  
 $A_w/\lambda$  slope of the surface  
 $\tau$  duration of the collision

-  $f^v$  : damping due to the shearing of the air film

$$f^v \approx \frac{\mu_a S}{b} \left(\frac{\tau}{T_F}\right) \approx 10^{-6} \text{ N}$$

## The “walking“ bifurcation

$$m\ddot{x} = F^b \sin\left(\frac{2\pi \dot{x}}{V_F^\varphi}\right) - f^v \dot{x}$$

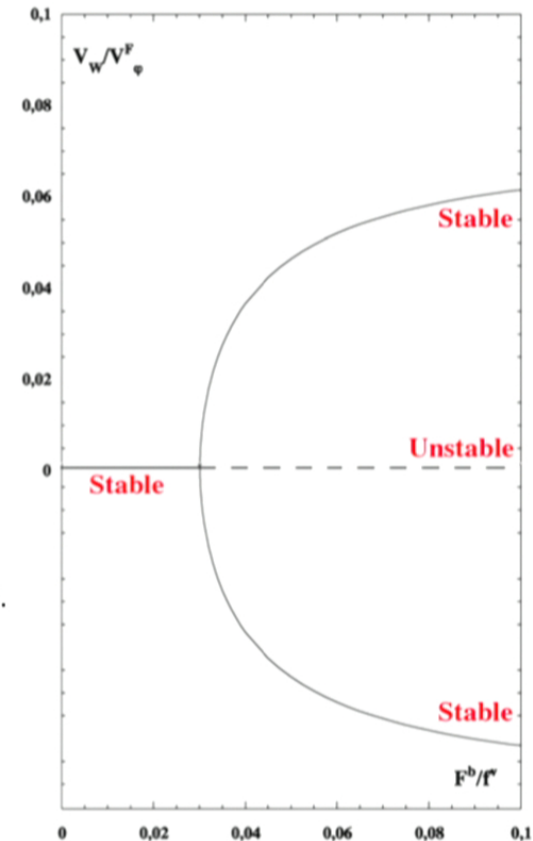
**Seeking steady solutions**  
(and in the limit of small velocities)

$$f^v \dot{x} = F^b \left[ \frac{2\pi \dot{x}}{V_F^\varphi} - \frac{1}{6} \left( \frac{2\pi \dot{x}}{V_F^\varphi} \right)^3 \right]$$

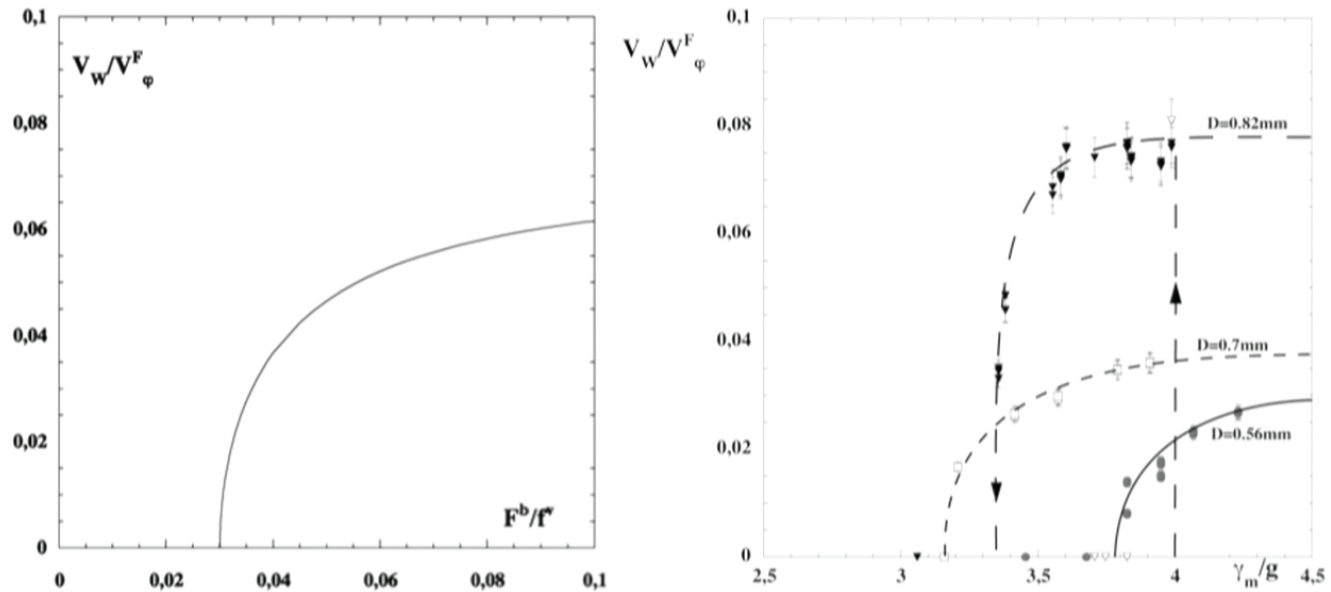
for small values of  $F^b$  the only solution is  $\dot{x} = 0$

Above a threshold, the motionless solution becomes unstable  
And two self propagative solutions of opposite velocities appear.

$$\dot{x}/V_F^\varphi = \pm \left( \sqrt{6}/2\pi \right) \sqrt{\left( F^b - F_c^b \right) / F^b}$$



## The computed and observed bifurcation



*A more complete model based on the same principle has been developed recently by Jan Molacek and John Bush*

## The energy balance

The system is dissipative : viscous friction damps the droplet motion and the wave

However steady regimes are obtained because energy is provided by the forcing to both the droplet and the wave.

- The droplet is kicked up at each of its collision with the interface.  
( similar to the escapement mechanism of mechanical clocks)
- The wave being a Faraday wave is almost sustained by parametric forcing  
(in the vicinity of the instability threshold).

**The main limitation of this experiment is that the forcing imposes a fixed frequency: the energy is fixed**

*A walker is formed of:*

*- A spread-out and continuous wave*

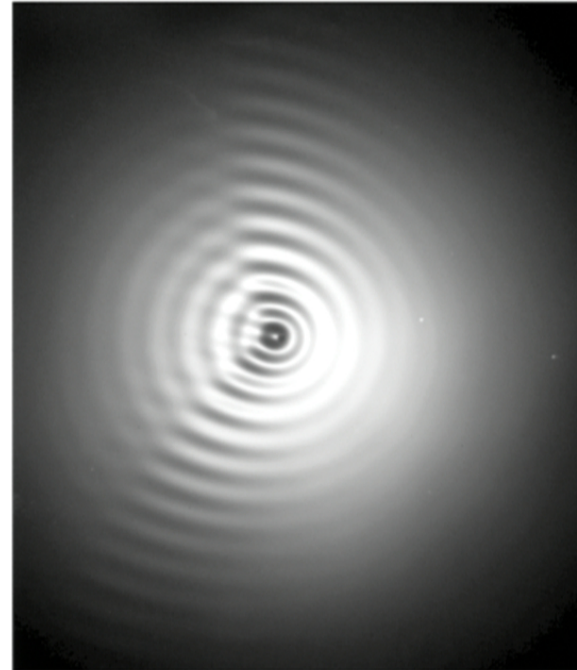
*- A discrete and localized droplet*

*How can they have a common dynamics?*

**I Walking straight**

**II Walking in circles**

**III Walking when confined**



**Part I**  
**Walking straight**

**The wave field structure and its “path-memory”,**

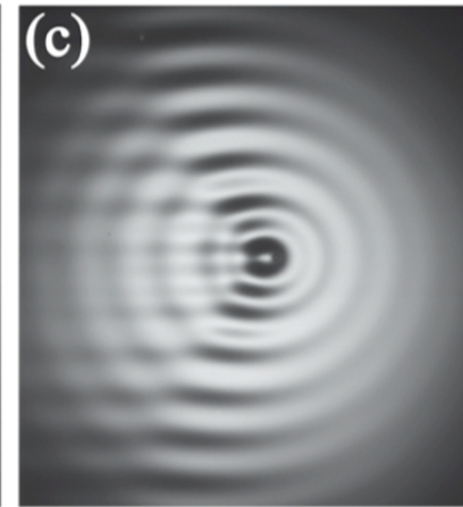
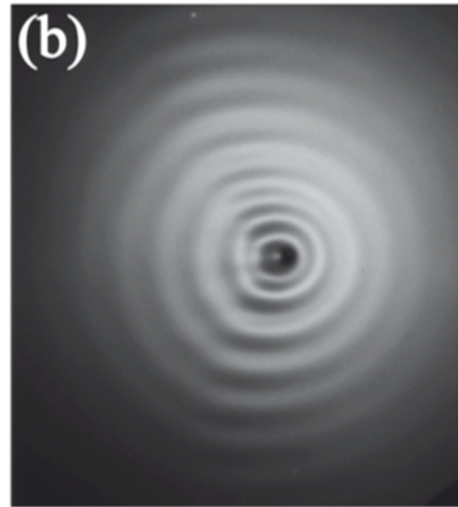
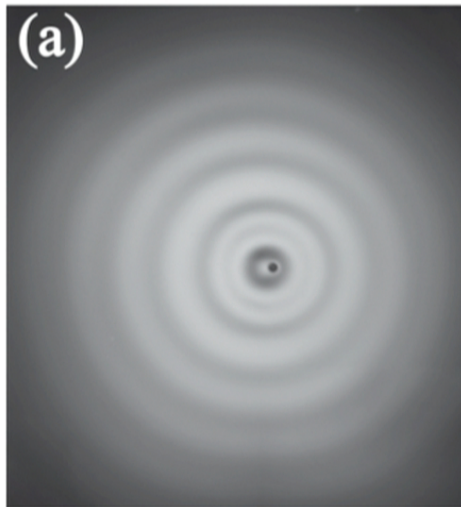
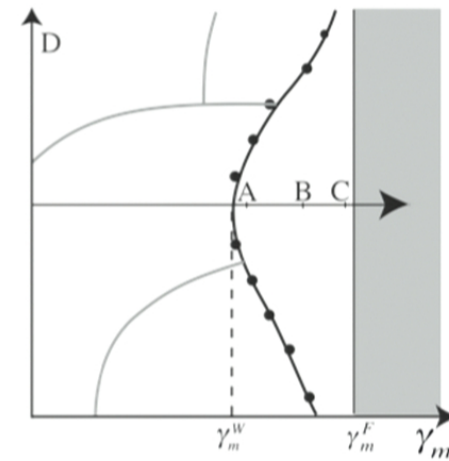
**A. Eddi, E. Sultan, J. Moukhtar, E. Fort, M. Rossi, and Y. Couder,**  
*J. Fluid Mech.*, 674, 433- 464, (2011).

## Evolution of the wave field as a function of the distance to the Faraday instability threshold

$\Gamma$  the non-dimensional distance to threshold tends to zero

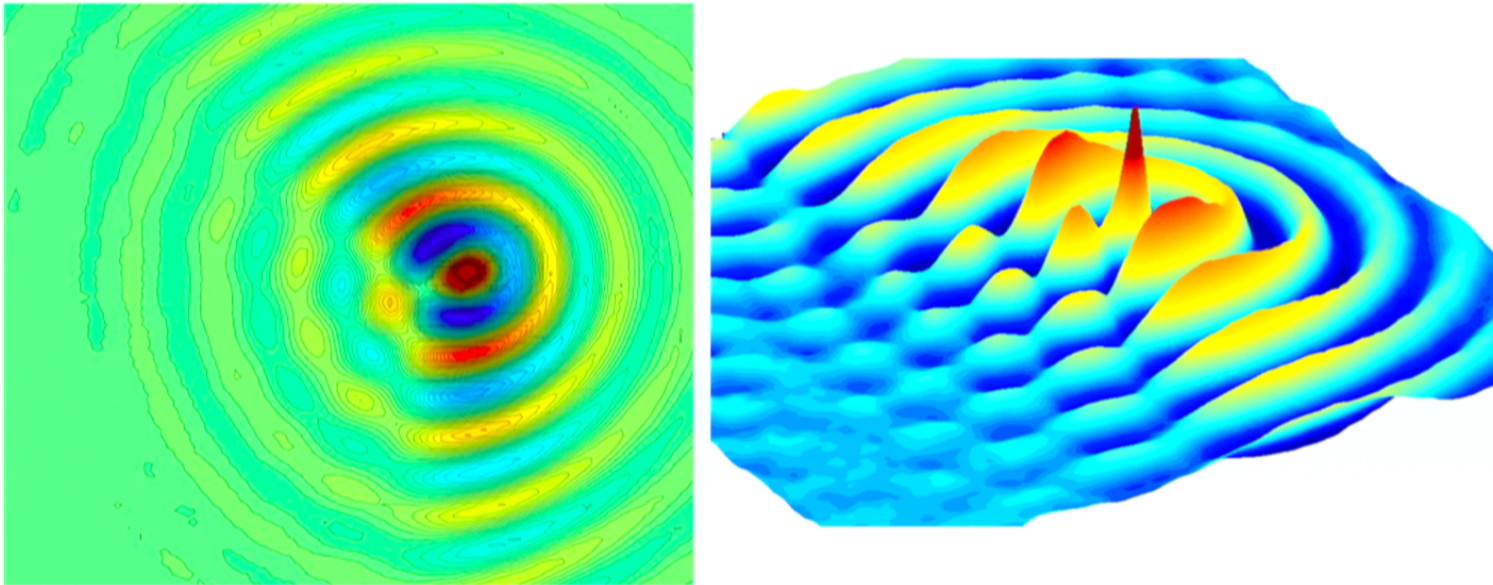
$$\Gamma = (\gamma_m^F - \gamma_m) / \gamma_m^F$$

A detail of the phase diagram



## The measured wave field

Obtained by an adaptation of a particle image velocimetry technique (PIV) to measure the shape of the interface, a technique due to **Frédéric Moisy and Marc Rabaud (FAST Orsay)**





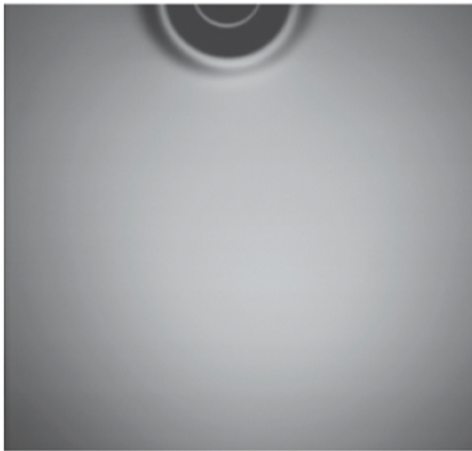
**The interface is disturbed by the repeated impacts of the droplet**

**What type of wave is generated by one single collision?**

**The wave-field produced  
by one single collision**

**(a 1 mm steel ball dropped in the  
bath)**

**t=10 ms**



**t=300 ms**



**The Faraday waves decay  
with a characteristic time:**

$$\tau \propto |\gamma_m - \gamma_m^F|^{-1}$$

**Without periodic  
forcing**

**With a periodic forcing near  
the Faraday instability  
threshold**

QuickTime Player Fichier Édition Présentation Fenêtre Aide

PERIMETER OCT 2011.ppt

6 pivPloufSansExcitation.mov

0.03  
0.02  
0.01  
0  
-0.01  
-0.02  
-0.03

h (mm)

10 0 -10 -20 -30 10 20 30 40

y (mm) x (mm)

00:00:00

PERIMETER OCT 2011.ppt

The wave-field by one single  
(a 1 mm steel ball bath)  
t=10 ms

FILMS PERIMETER INSTITUTE

	Date de modification	Taille	Type
mov	28 juin 2008, 11:01	60,9 Mo	Séque...kTime
00.avi	6 janvier 2011, 00:29	19,1 Mo	Film AVI
	14 mai 2009, 08:25	149,7 Mo	Séque...kTime
	15 mai 2009, 08:24	16,5 Mo	Séque...kTime
	15 mai 2009, 08:29	18,9 Mo	Séque...kTime
	18 juin 2010, 18:12	22,8 Mo	Séque...kTime
	6 janvier 2011, 00:30	24,3 Mo	Séque...kTime
	4 février 2008, 09:06	95,7 Mo	Séque...kTime
	10 mai 2011, 19:10	1,7 Mo	Séque...kTime
	10 mai 2011, 19:02	2,9 Mo	Séque...kTime
	10 mai 2011, 18:44	3,4 Mo	Séque...kTime
	16 juillet 2010, 14:52	392,6 Mo	Séque...kTime
	14 mai 2009, 11:16	143 Mo	Séque...kTime
	14 mai 2009, 11:12	18,8 Mo	Séque...kTime
	12 mai 2011, 15:28	35,4 Mo	Séque...kTime
	14 mai 2009, 10:11	70,8 Mo	Séque...kTime
	3 mai 2011, 15:18	5,9 Mo	Séque...kTime
	12 mai 2011, 12:34	4,5 Mo	Séque...kTime
	26 mai 2009, 11:45	4,3 Mo	Séque...kTime
	26 mai 2009, 09:26	4,9 Mo	Séque...kTime

Music

Pictures

34

35

36

37

5 a collision répulsive.mov

5 b Collision attract n=2 .mov

6 pivPloufSansExcitation.mov

7 pivPlouf avec excitation .mov

8 z Piv marcheur.mov

9 Mort\_MarcheurV10-1000Hz.mov

10 LANDAU 7,1 V n=1\_court\_L.mov

11 LANDAU 13,8 V n=0,5.mov

12 billard ds cavite cour\_bis.mov

13 chaos ds cavite bis\_L.mov

14 FILM DIFFRACTION-tt.mov

15 INTERFERENCE DoubleSlit.mov

17 TUNNEL cycle -808mV.mov

18 TUNNEL court-784mV.mov

QuickTime Player Fichier Édition Présentation Fenêtre Aide

PERIMETER OCT 2011.ppt

7 pivPlouf avec excitation .mov

0.05  
h (mm)  
0  
-0.05  
10 0 -10 20 30 40  
y (mm) x (mm)

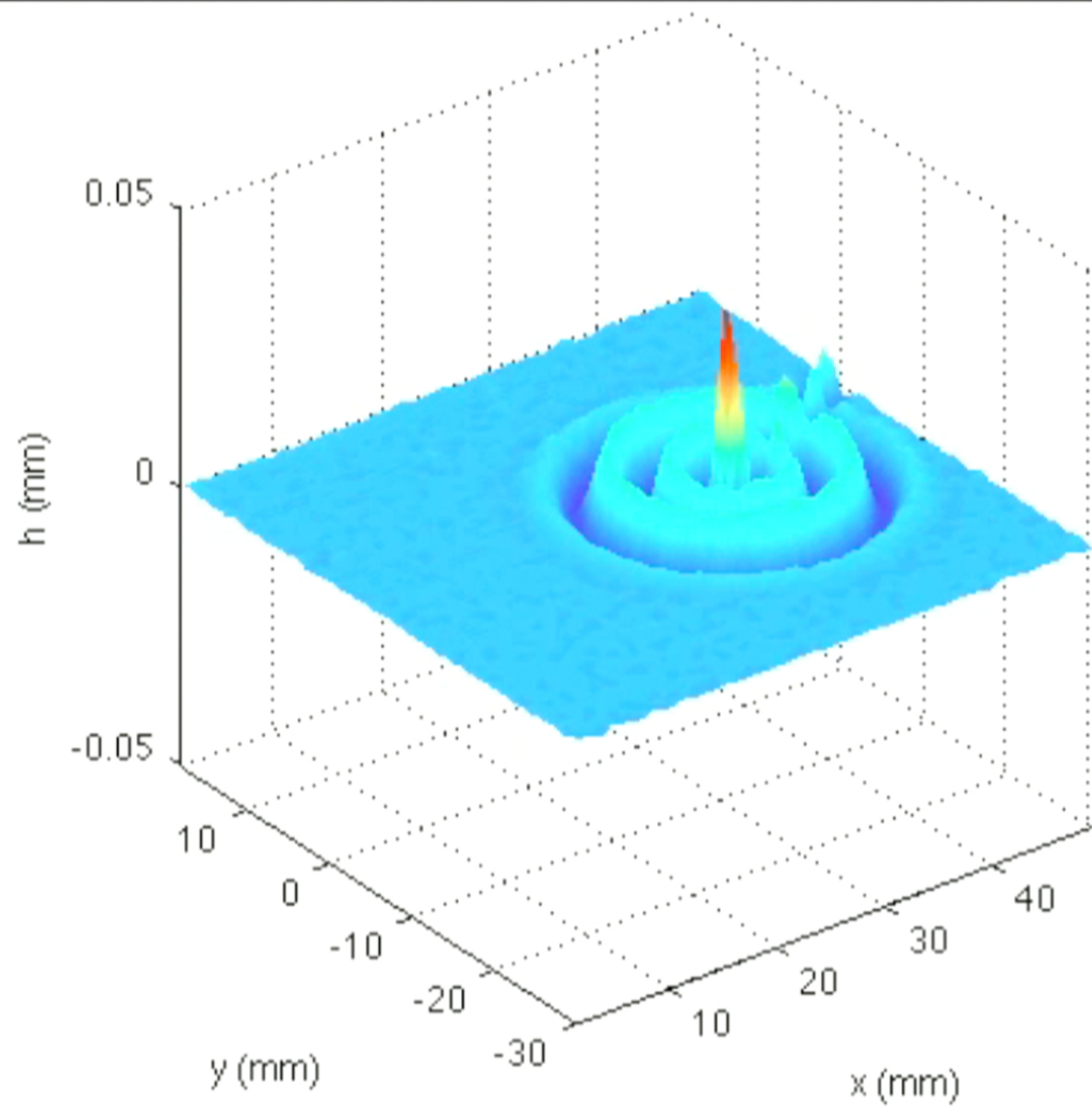
00:00:00

PERIMETER OCT 2011.ppt

The wave-field by one single (a 1 mm steel ball bath) t=10 ms

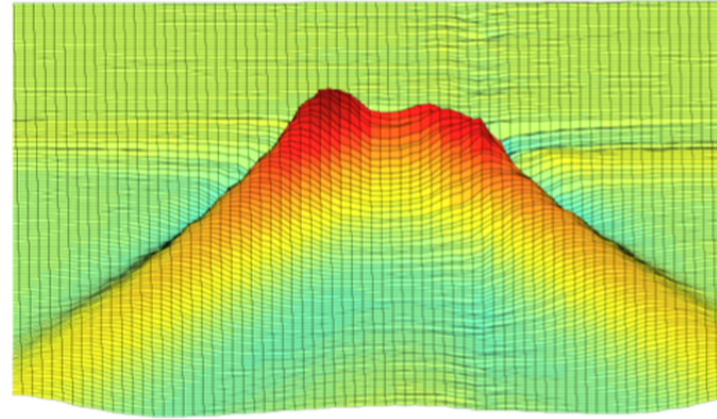
FILMS PERIMETER INSTITUTE

	Date de modification	Taille	Type
	28 juin 2008, 11:01	60,9 Mo	Séque...kTime
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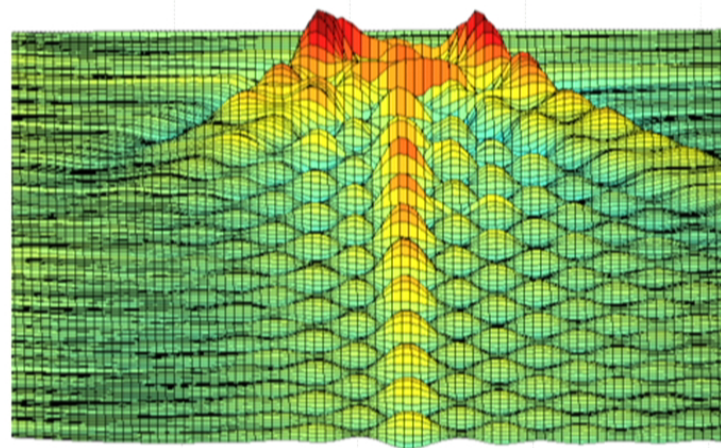


**Spatio-temporal** evolution of the radial profile of the wave emitted by one bounce

Without periodic forcing



With a periodic forcing near the Faraday instability threshold

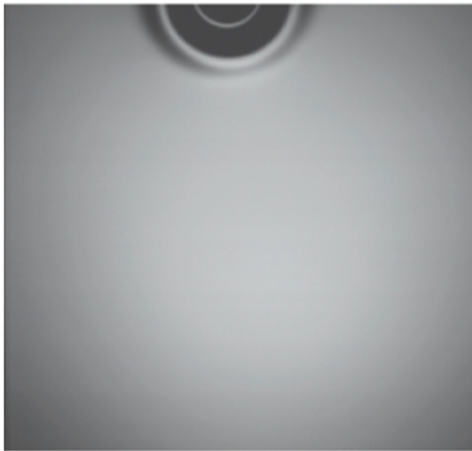


**Conclusion: near the Faraday threshold, a point which has been disturbed remains the centre of a localized state of almost sustained Faraday waves**

**The wave-field produced  
by one single collision**

**(a 1 mm steel ball dropped in the  
bath)**

**t=10 ms**



**t=300 ms**



**The Faraday waves decay  
with a characteristic time:**

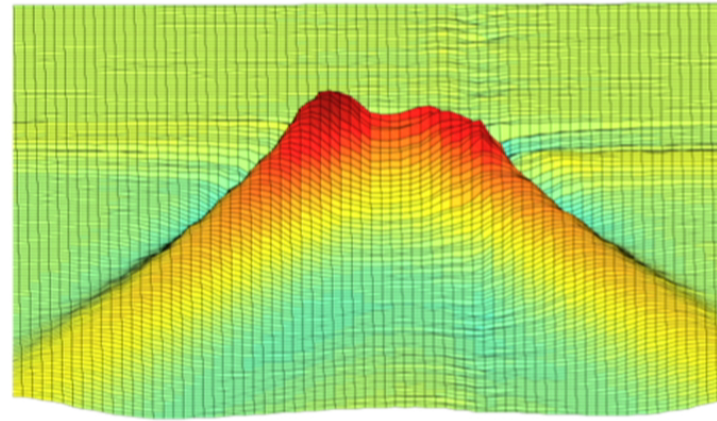
$$\tau \propto |\gamma_m - \gamma_m^F|^{-1}$$

**Without periodic  
forcing**

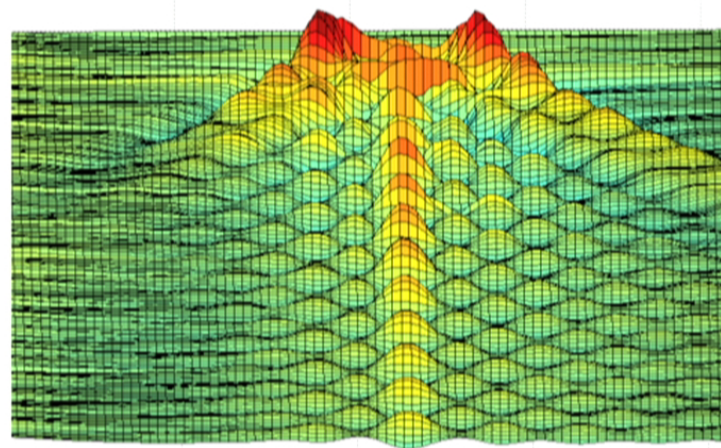
**With a periodic forcing near  
the Faraday instability  
threshold**

**Spatio-temporal** evolution of the radial profile of the wave emitted by one bounce

Without periodic forcing



With a periodic forcing near the Faraday instability threshold



**Conclusion: near the Faraday threshold, a point which has been disturbed remains the centre of a localized state of almost sustained Faraday waves**

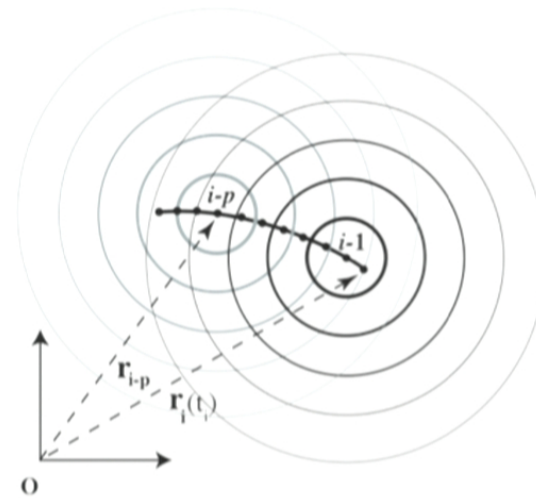


**The numerical model of walkers**  
*(Emmanuel Fort)*

## The numerical model of walkers (Emmanuel Fort)

### A/ Motion of the droplet :

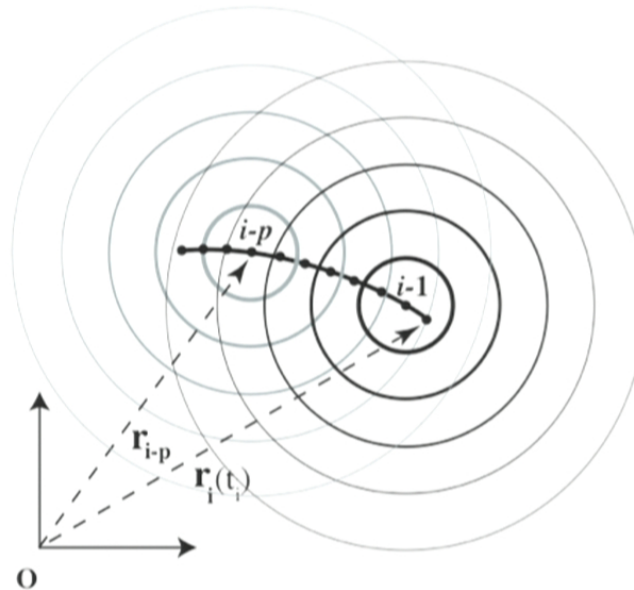
- (1) *Take-off and landing times are determined by the forcing oscillations only.*
- (2) *The walk result from successive displacements  $\delta r_n$  due to the kicks. The direction and modulus of  $\delta r_n$  are determined by the surface slope at the point of landing.*
- (3) *This slope results of the interfering waves due to the previous bounces*



### B/ Computation of the wave-field

- 1) *At each bounce, a circular localized mode of Faraday waves is generated.*
- 2) *The points of the surface visited by the droplet in the past remain the centres of such a localized mode.*
- 3) *The wave field results from the superposition of all these waves, and thus contains a memory of the path followed by the droplet*

## B/ The computation of the wave-field



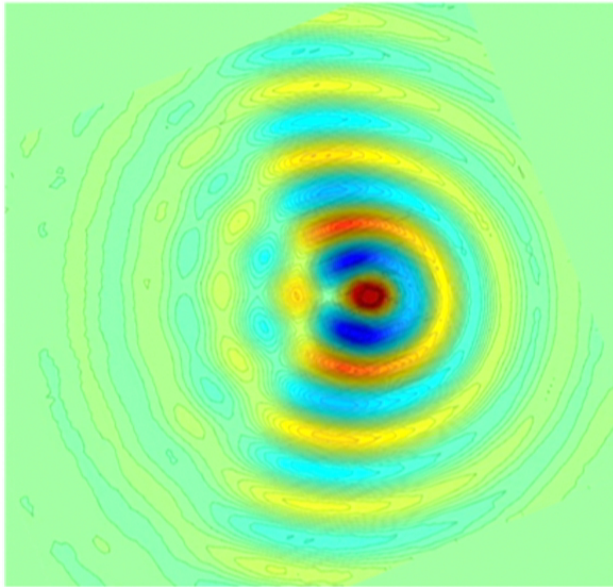
$$h(\mathbf{r}, t_i) = \sum_{p=i-1}^{-\infty} \operatorname{Re} \left[ \frac{A}{|\mathbf{r} - \mathbf{r}_p|^{1/2}} \exp\left(-\frac{t_i - t_p}{\tau}\right) \exp\left(-\frac{|\mathbf{r} - \mathbf{r}_p|}{\delta}\right) \exp i \left( \frac{2\pi |\mathbf{r} - \mathbf{r}_p|}{\lambda_F} + \varphi \right) \right]$$

$\mathbf{r}_p$  position of the droplet at time  $t_p = t_i - (i - p)T_F$

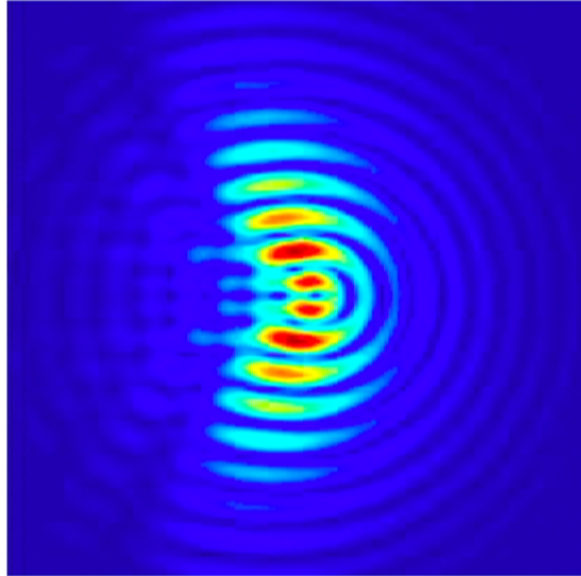
the damping time is related to the distance to Faraday instability onset:  $\tau \propto |\gamma_m - \gamma_m^F|^{-1}$

## First results of the numerical simulation

- (1) The walking bifurcation is recovered
- (2) A realistic structure of the wave field is obtained for a rectilinearly moving walker



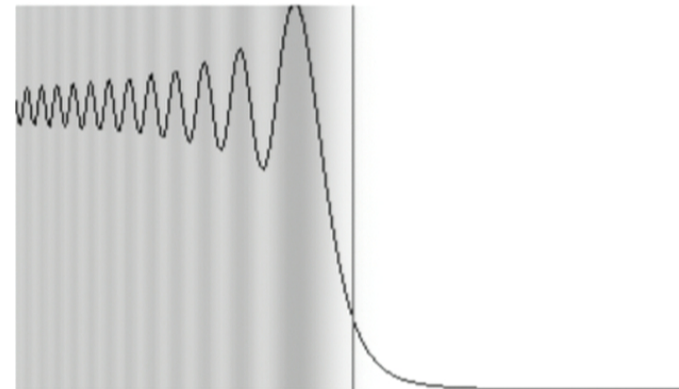
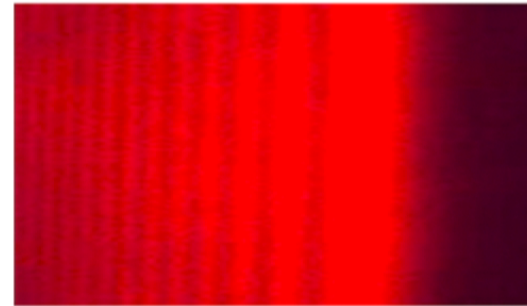
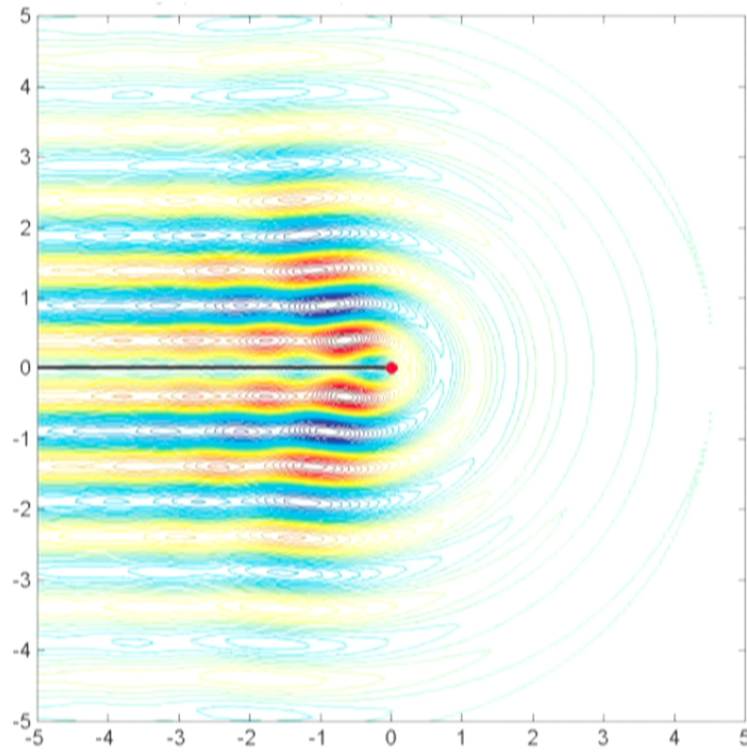
*The measured field*



*Its simulation*

*The structure of the wave field exhibits Fresnel interference fringes*

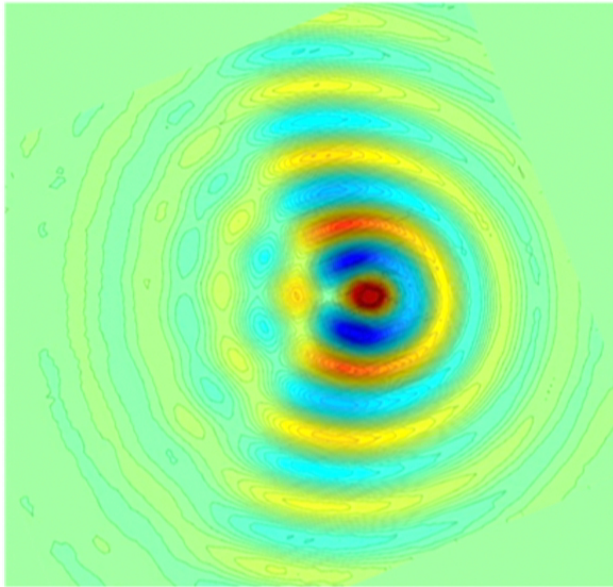
**In the limit of weak decay times:**  
*Fresnel fringes*



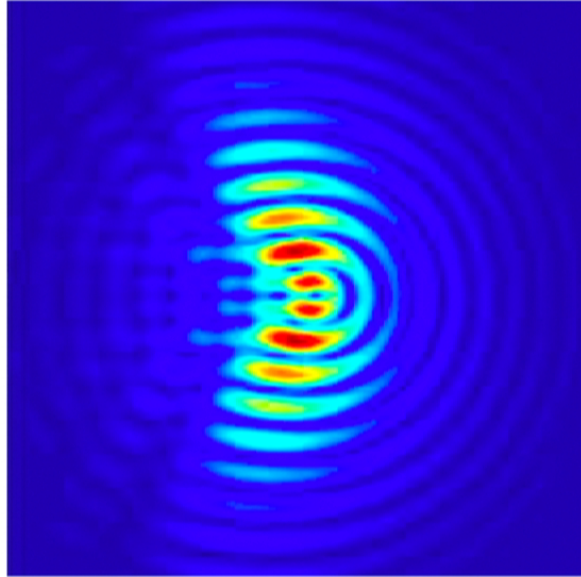
**Fresnel diffraction behind an edge**  
(simulation John Talbot 1997)

## First results of the numerical simulation

- (1) The walking bifurcation is recovered
- (2) A realistic structure of the wave field is obtained for a rectilinearly moving walker



*The measured field*



*Its simulation*

*The structure of the wave field exhibits Fresnel interference fringes*

**Part II**  
**Walking in circles**

**Orbiting due to an external force**

**E. Fort, A. Eddi, A. Boudaoud, J. Moukhtar, and Y. Couder,**  
*PNAS*, 107, 17515-17520, (2010)

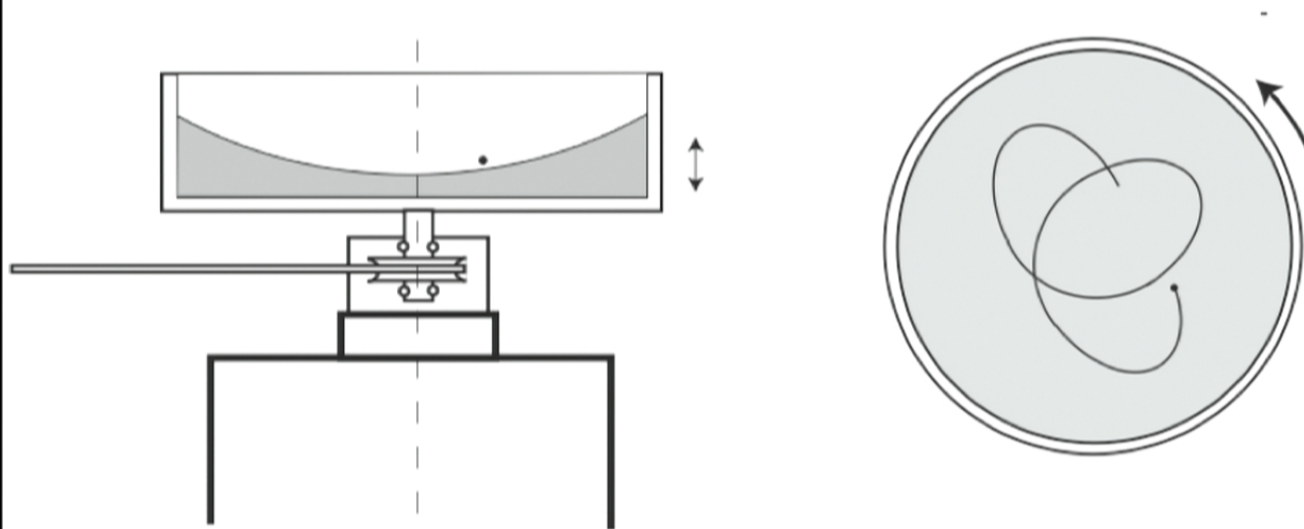
## How to obtain circular trajectories?

Use either a magnetic field or a rotating system,  
*(an analogy used by Michael Berry  
 to obtain a fluid mechanics analog of the Aharonov-Bohm effect)*

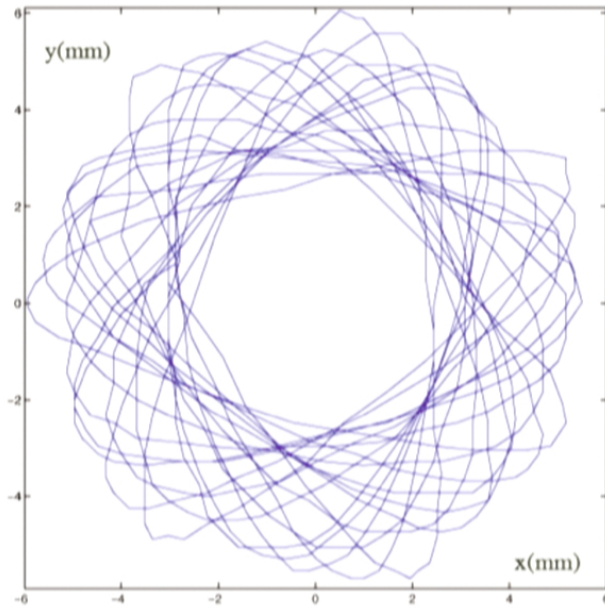
In a magnetic field $\mathbf{B}$  $\vec{F} = q(\vec{V} \wedge \vec{B})$	On a surface rotating with angular velocity $\Omega$  $\vec{F}_c = -2m(\vec{V} \wedge \vec{\Omega})$
Orbital motion	Orbital motion in the rotating frame
Larmor angular velocity  $\omega_L = qB/m$	Orbital motion angular velocity  $2\Omega$
Orbit radius  $\rho_L = V/\omega_L$	Orbit radius  $R = V/2\Omega$



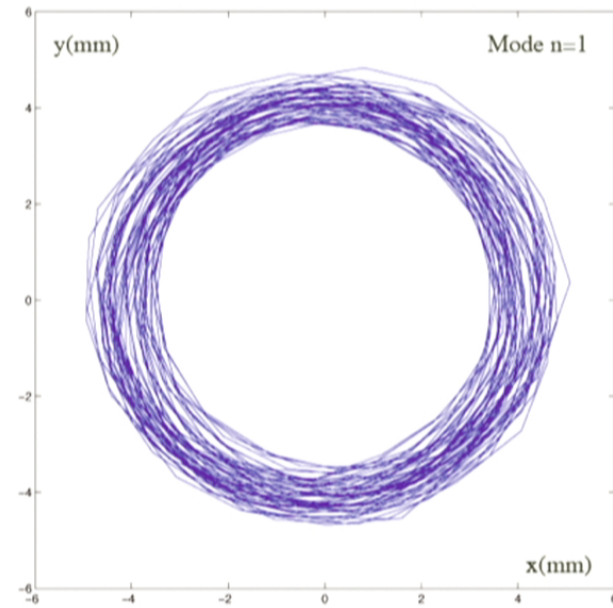
## The rotating Faraday experiment



## Measured trajectories



**Trajectory in the laboratory  
frame of reference**

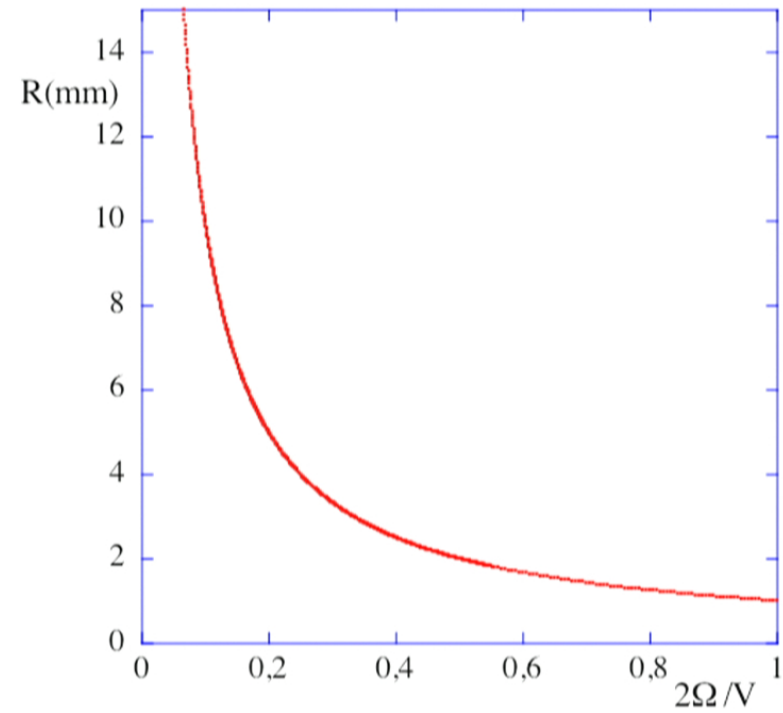


**Trajectory in the rotating  
frame of reference**

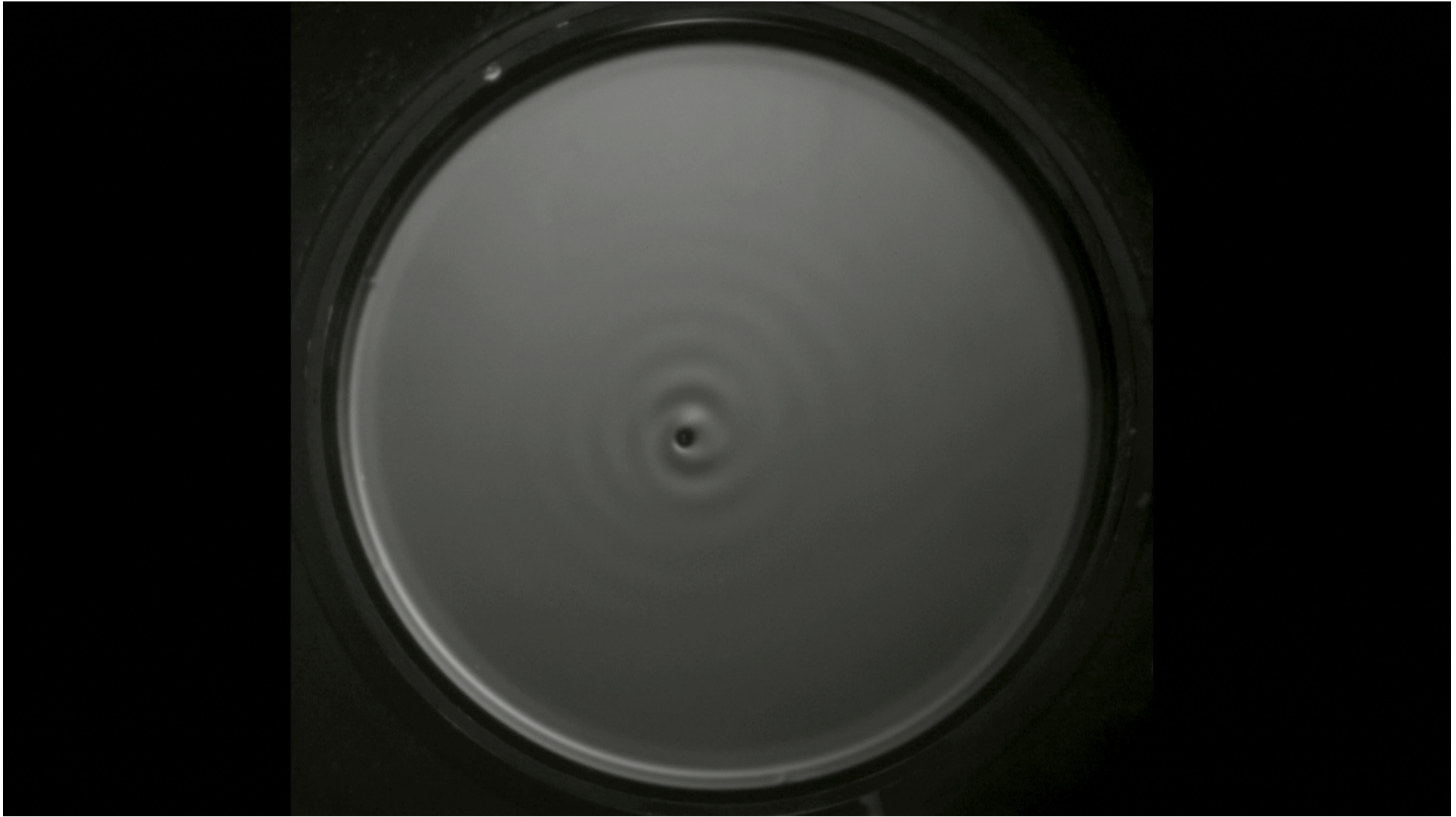
## Classical radius of the orbits

The “classical“ radius of the orbit observed in the rotating frame and due to Coriolis effect should be:

$$R = V/2\Omega$$







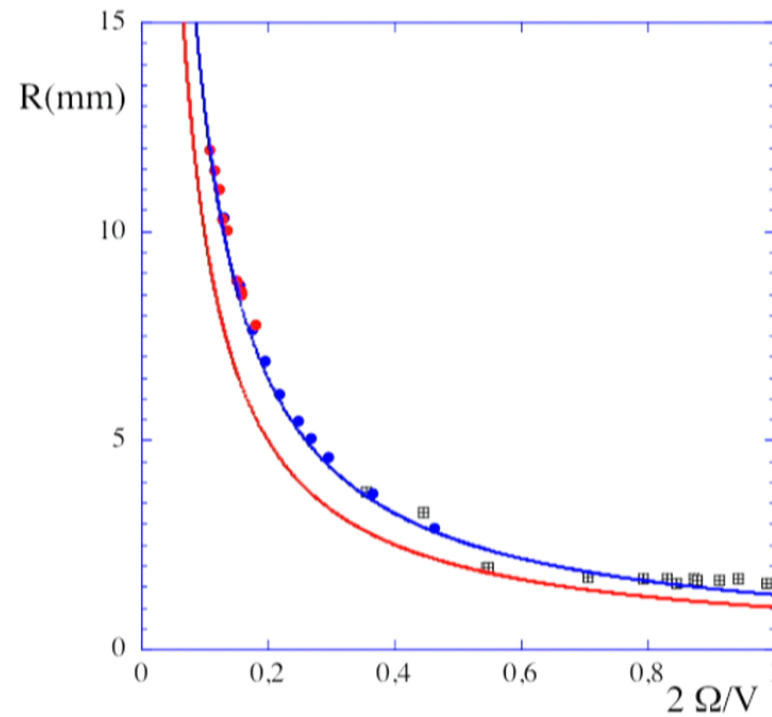
## Measured radius of the orbits for walkers with weak path memory (far from the Faraday threshold)

The radius of the orbit observed in the rotating frame has a “classical” dependence, but slightly shifted

$$R = a (V/2\Omega)$$

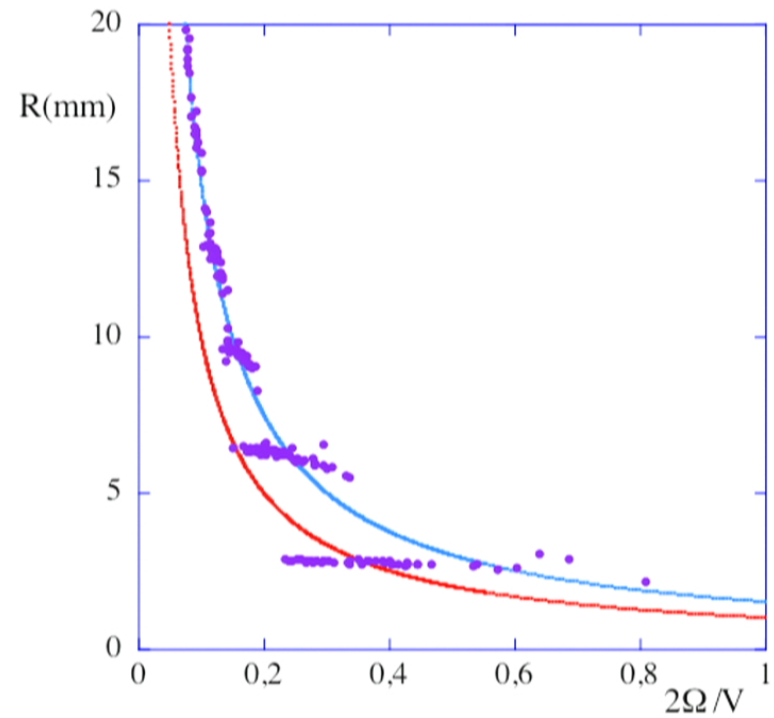
With

$$a \approx 1.3$$



## Measured radius of the orbits of walkers having long term path memory (near the Faraday threshold)

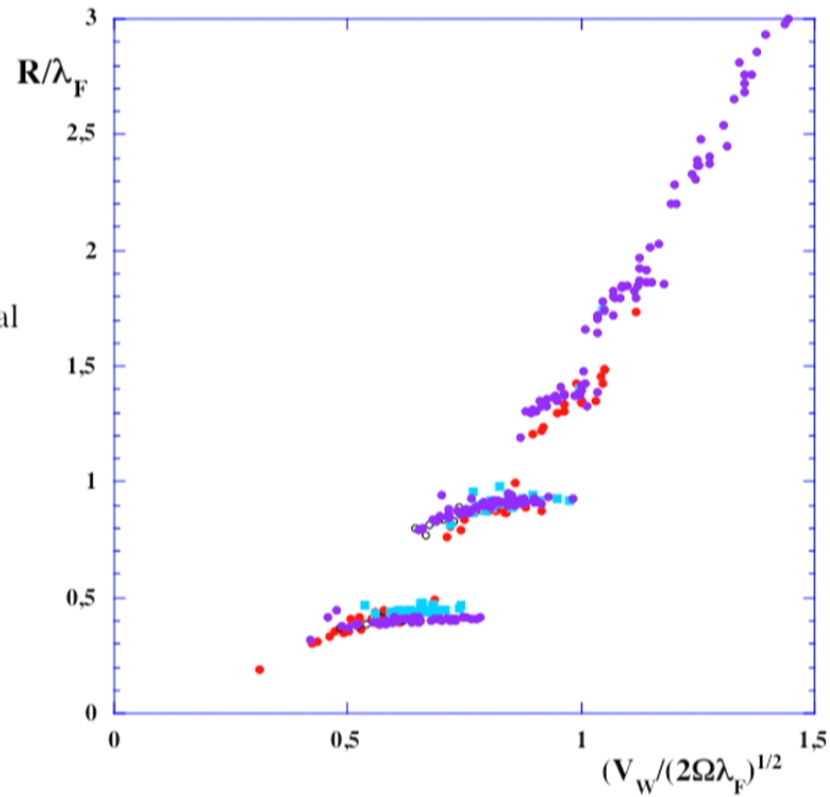
*Near the Faraday instability threshold, the radius of the orbit evolves by discrete jumps when  $\Omega$  is increased*



## Dimensionless radius of the orbits

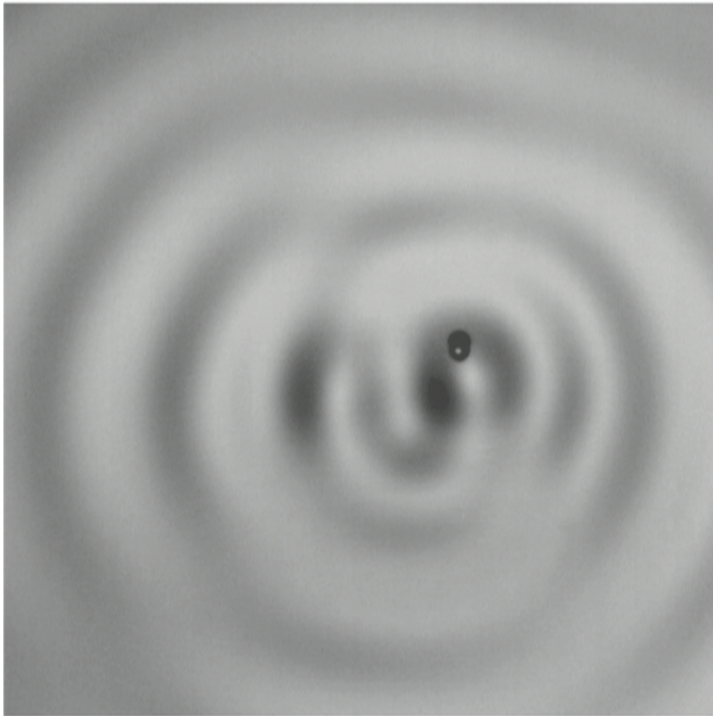
The radii of the orbits obtained at various frequencies and for walkers of various velocities are all rescaled by expressing:

$\frac{R_n}{\lambda_F}$  as a function of the non - dimensionnal parameter  $\sqrt{\frac{V_W}{2\Omega\lambda_F}}$

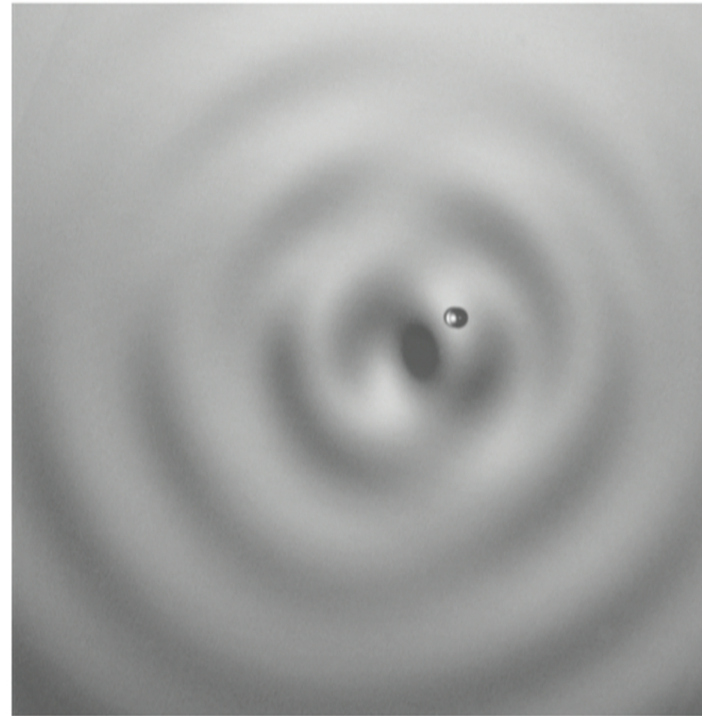




The first two modes



$$d_n^{orb} = 2 \lambda_F$$



$$d_n^{orb} = \lambda_F$$

## Landau levels:

the Bohr-Sommerfeld quantization in a magnetic field

**The Bohr Sommerfeld condition imposes:**

$$\oint (\mathbf{p} - e\mathbf{A}) \cdot d\mathbf{l} = (n + \gamma)h$$

**The Larmor radius can only take discrete values**

$$\rho_L^n = \frac{1}{\sqrt{\pi}} \sqrt{\frac{h}{qB} \left( n + \frac{1}{2} \right)}$$

## The radii of the Landau orbits in a magnetic field

The quantized Larmor radius:

$$\rho_L^n = \frac{1}{\sqrt{\pi}} \sqrt{\left(n + \frac{1}{2}\right) \frac{h}{qB}}$$

can be expressed as a function of the de Broglie wavelength

$$\lambda_{dB} = \frac{h}{mV}$$

$$\frac{\rho_n}{\lambda_{dB}} = \sqrt{1/\pi} \sqrt{\left(n + \frac{1}{2}\right) \frac{m}{qB} \frac{V}{\lambda_{dB}}}$$

$$\frac{m}{qB} \Leftrightarrow \frac{1}{2\Omega}$$
$$\lambda_{dB} \Leftrightarrow \lambda_F$$

The analogy suggests:

$$\frac{R_n}{\lambda_F} = \sqrt{1/\pi} \sqrt{\left(n + \frac{1}{2}\right) \frac{V_w}{2\Omega \lambda_F}}$$

## Dimensionless radius of the orbits

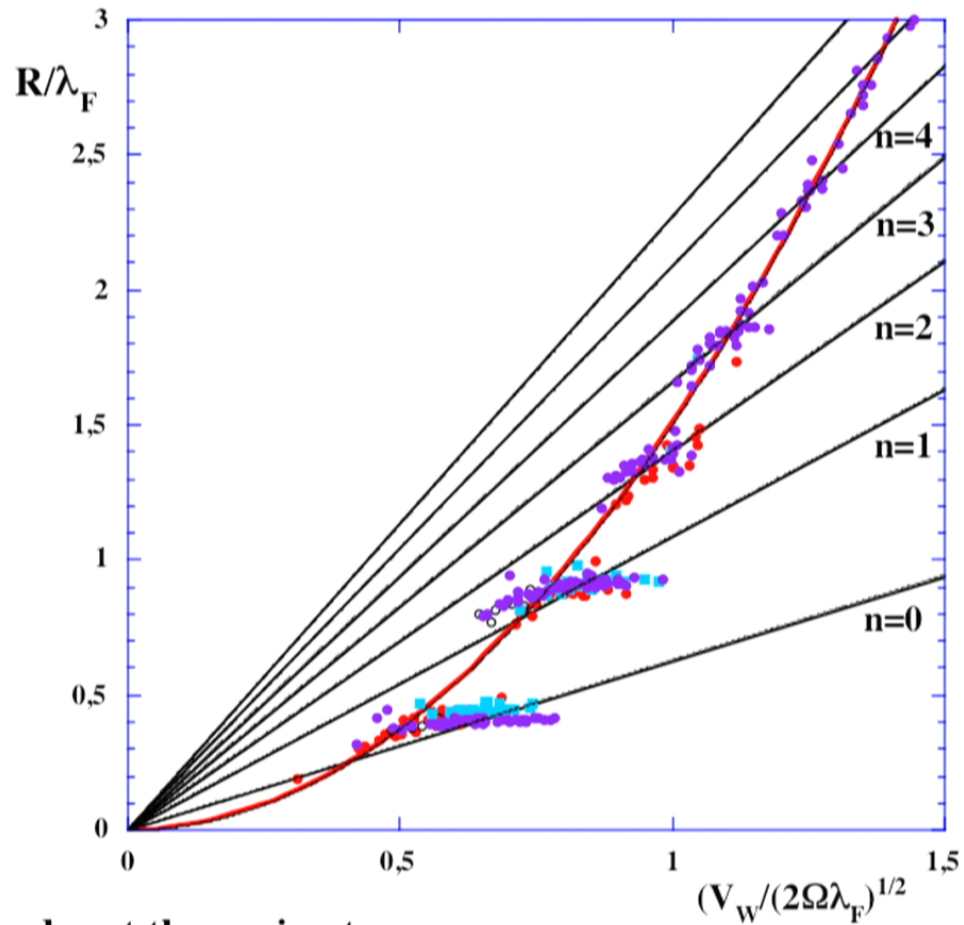
The analogy suggests:

$$\frac{R_n}{\lambda_F} = \sqrt{1/\pi} \sqrt{\left(n + \frac{1}{2}\right) \frac{V_W}{2\Omega \lambda_F}}$$

$$\sqrt{1/\pi} = 0.564$$

We find

$$\frac{R_n}{\lambda_F} = 0.89 \sqrt{\left(n + \frac{1}{2}\right) \frac{V_W}{2\Omega \lambda_F}}$$



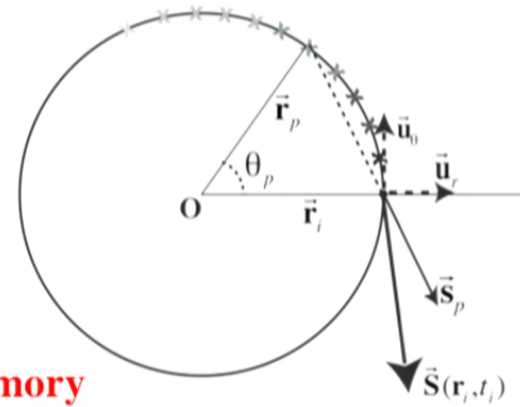
**The diameters are quantized, not the perimeters**

## The path memory effect in the case of circular trajectories

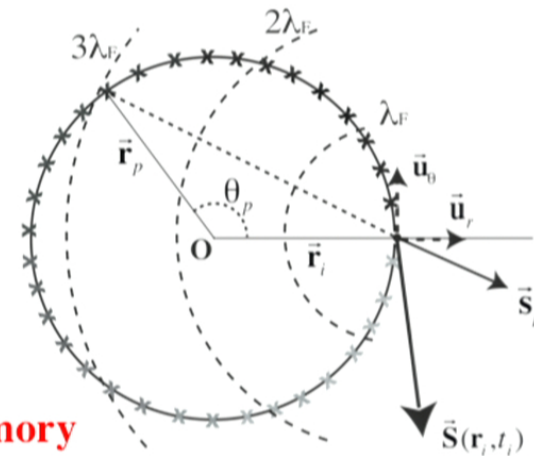
How to compute the local slope induced at the point of impact by the superposition of the waves due to “sources” distributed on a circular orbit

$$\vec{S}(\vec{r}_i, t_i) = \vec{\nabla} h(\vec{r}_i, t_i)$$

$$\vec{S}(\vec{r}_i, t_i) = \sum_{p=i-1}^{\infty} \vec{s}(\vec{r}_i - \vec{r}_p, t_i - t_p)$$

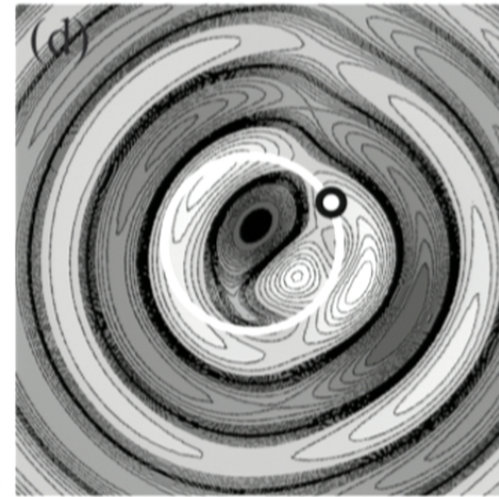
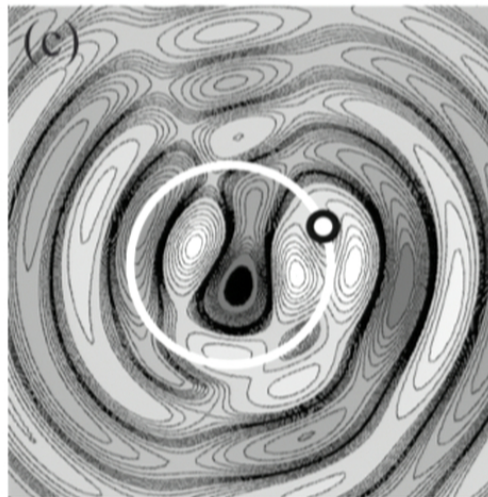
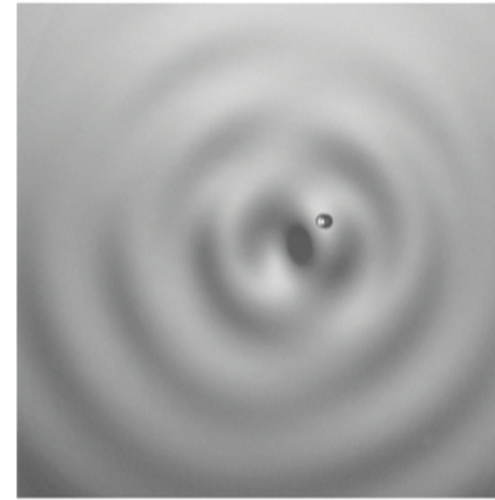
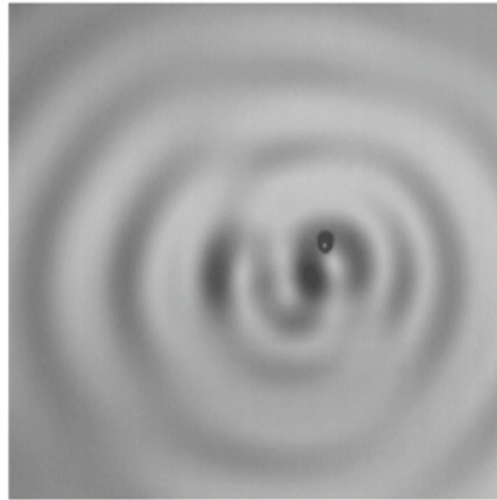


Short-term memory



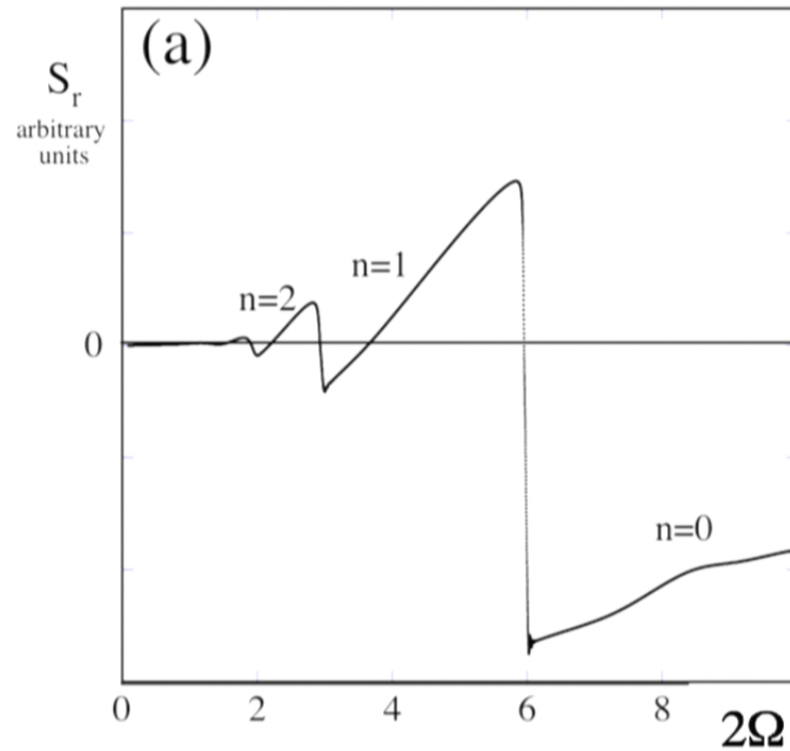
Long-term memory

The first two modes  
as observed and  
simulated

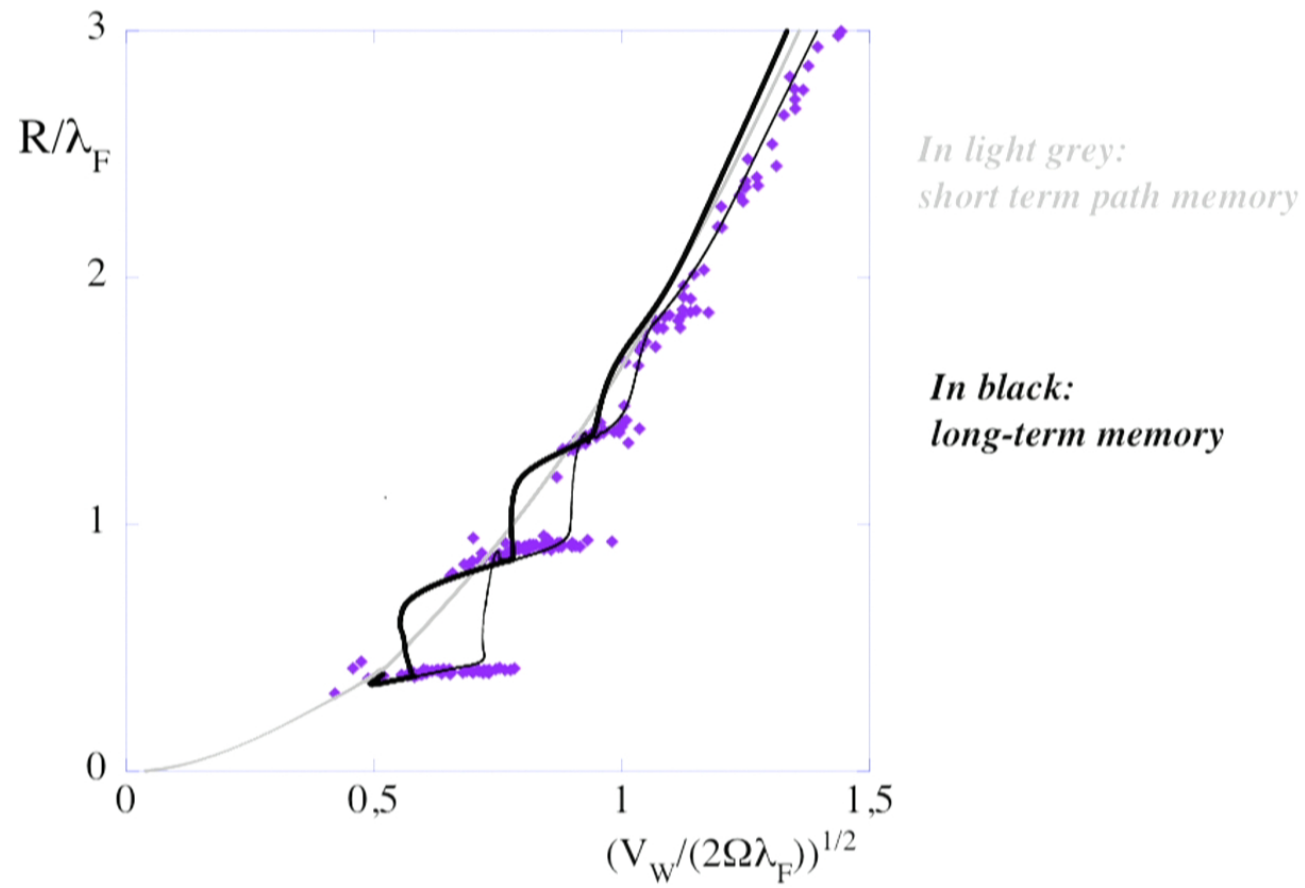


The radial slope  $S_r$  at the point of bouncing generates an additional centripetal (or centrifugal) force.

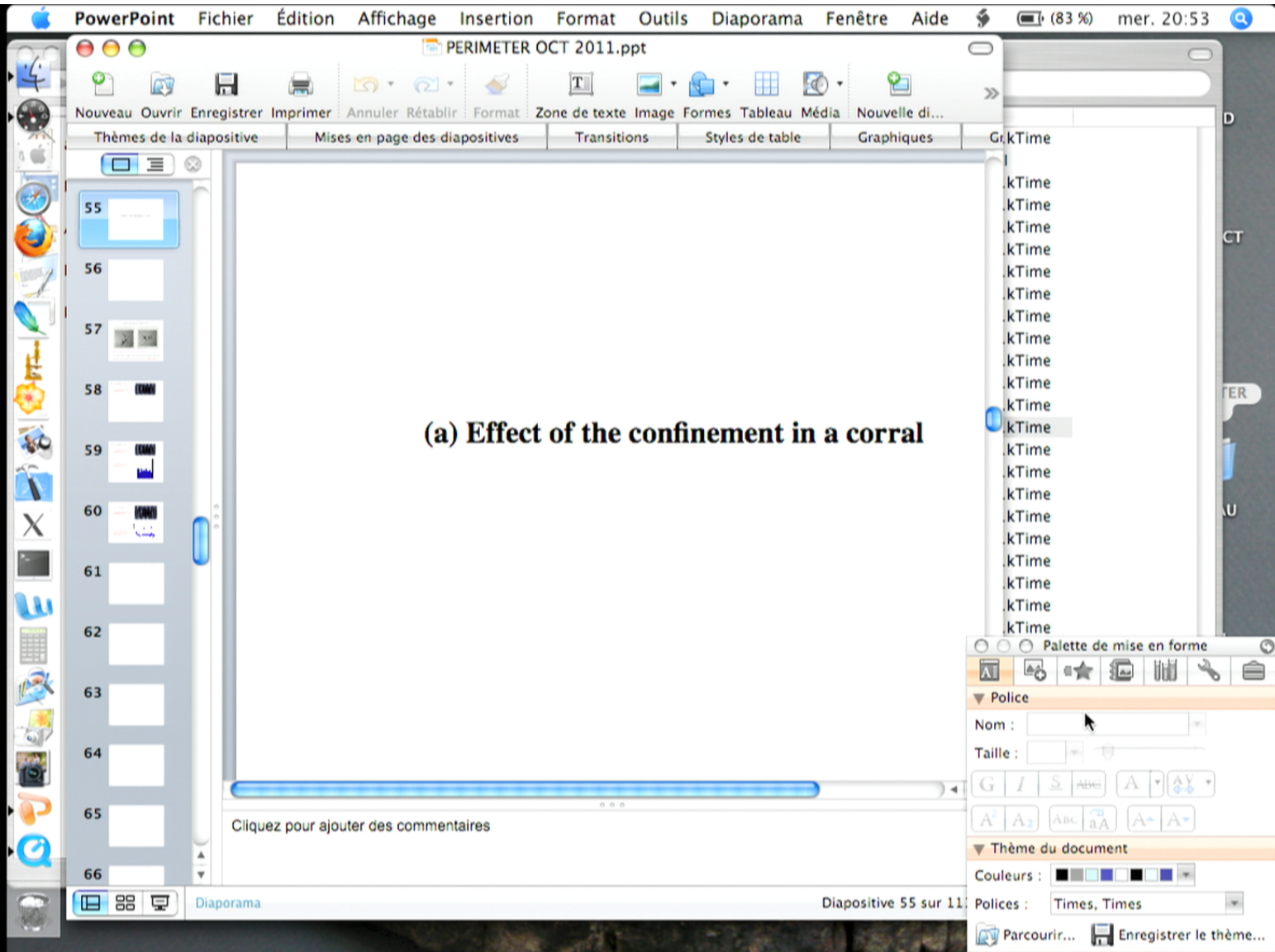
It is responsible for the formation of plateaus



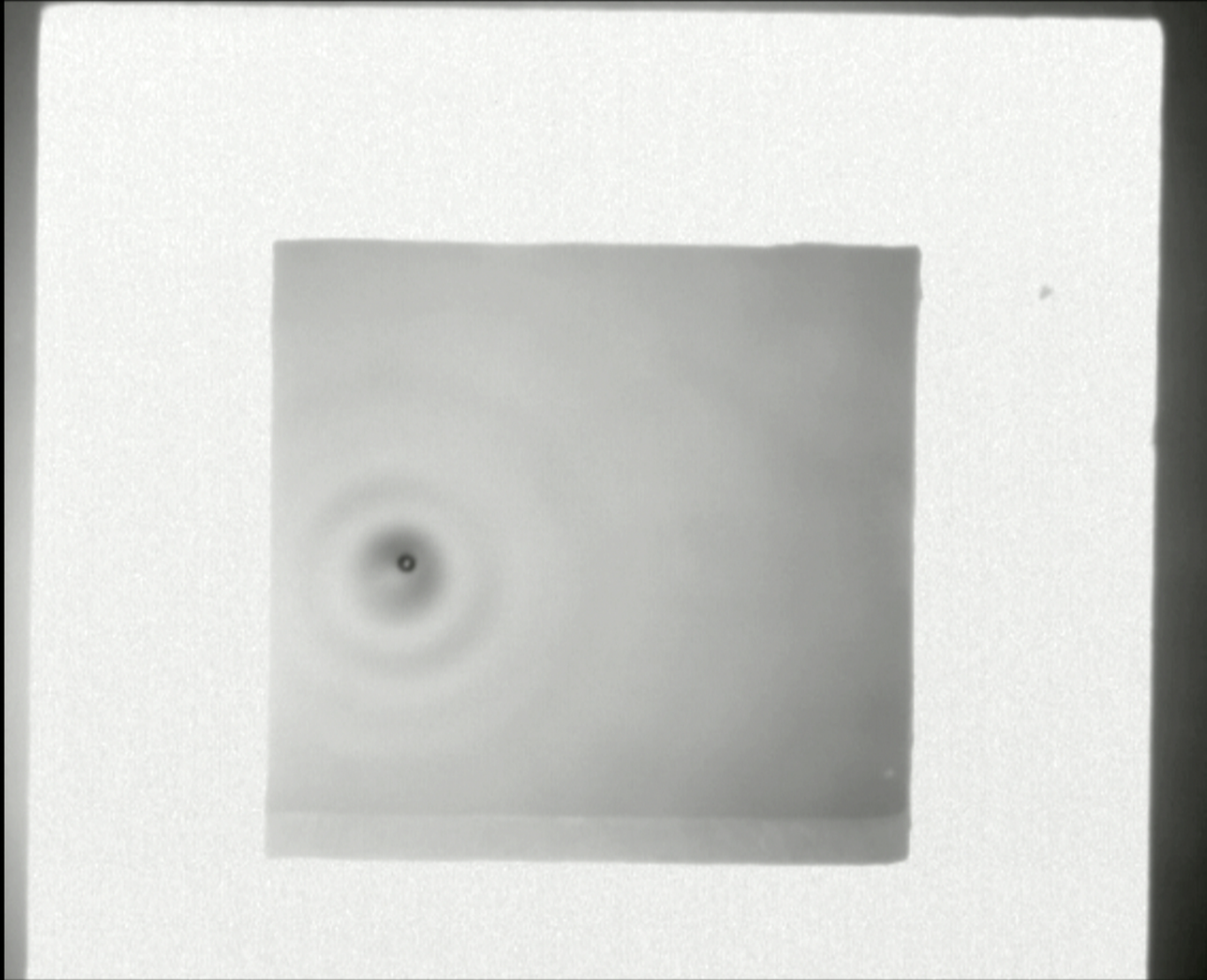
The results of numerical simulations (Emmanuel Fort's model)



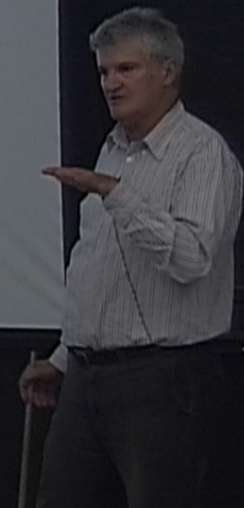
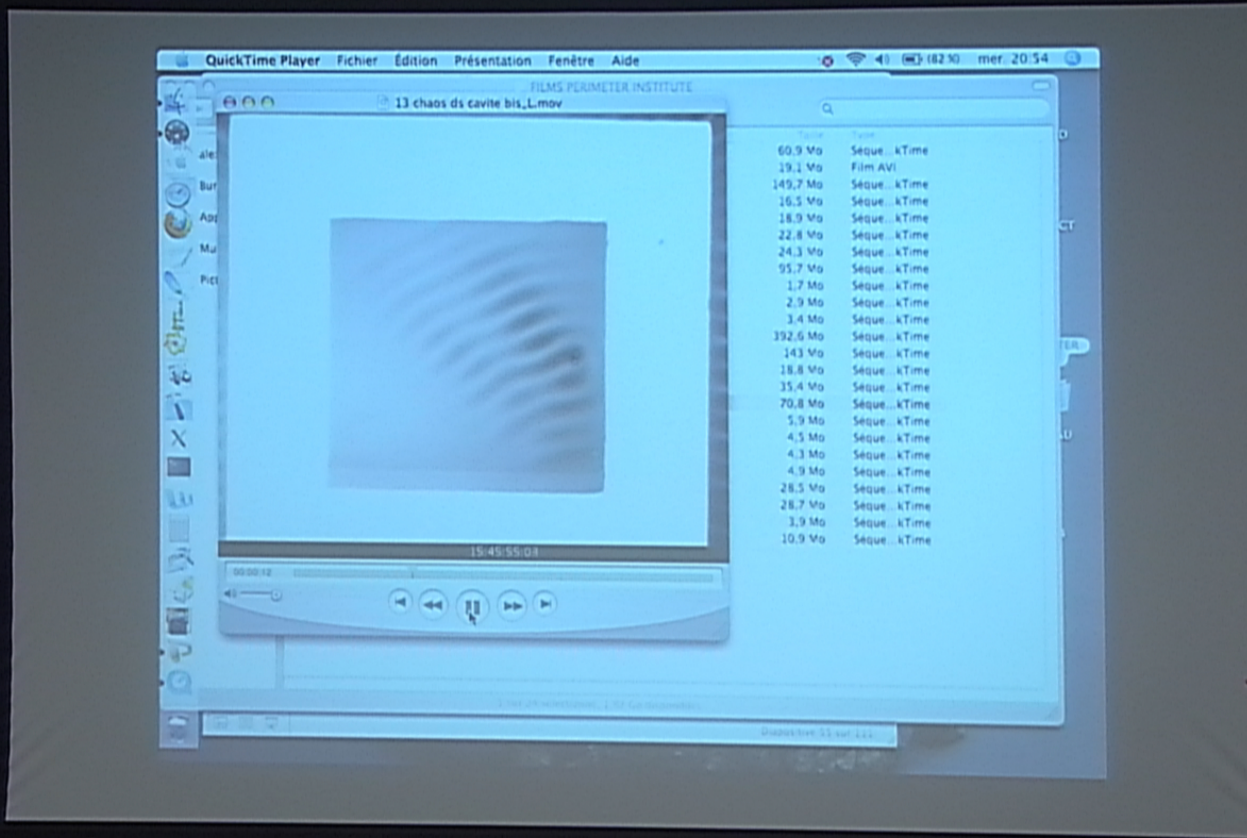




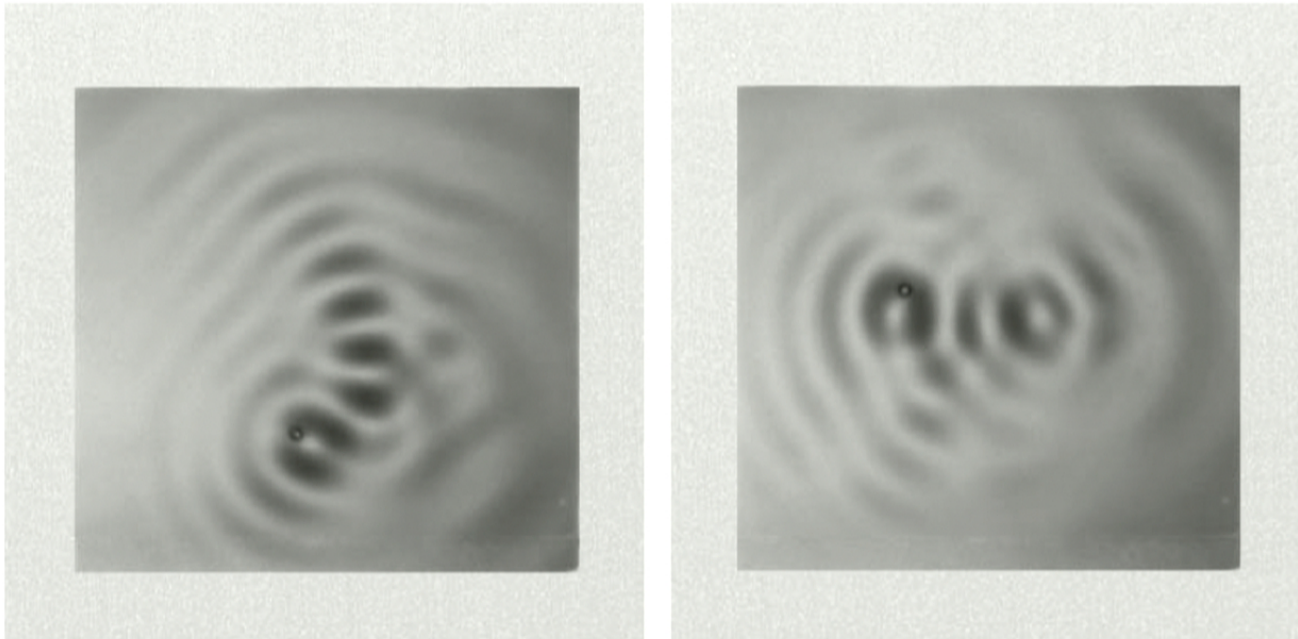
**(a) Effect of the confinement in a corral**



15:29:47:04



## Chaotic trajectories in a square corral



Has the probability of visit a relation to the eigenmodes of the cavity?

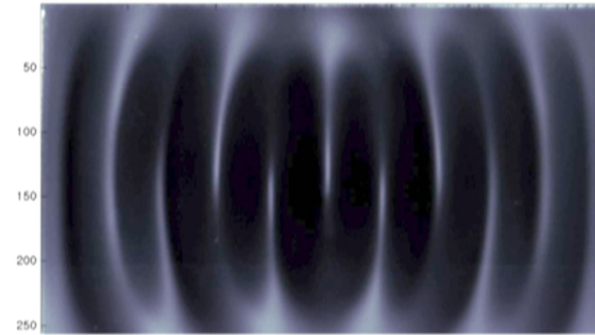
We chose to study rectangular cavities tuned with one dominant resonant mode,

Dan Harris and John Bush (MIT) chose circular cavities (**John's talk**)

In a rectangular cavity

with *Julien Moukhtar*

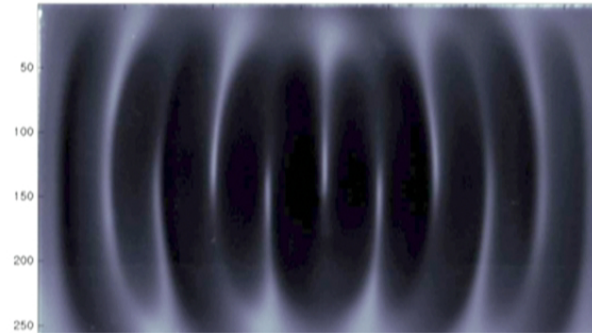
**The Faraday eigenmode of the  
rectangular cavity**



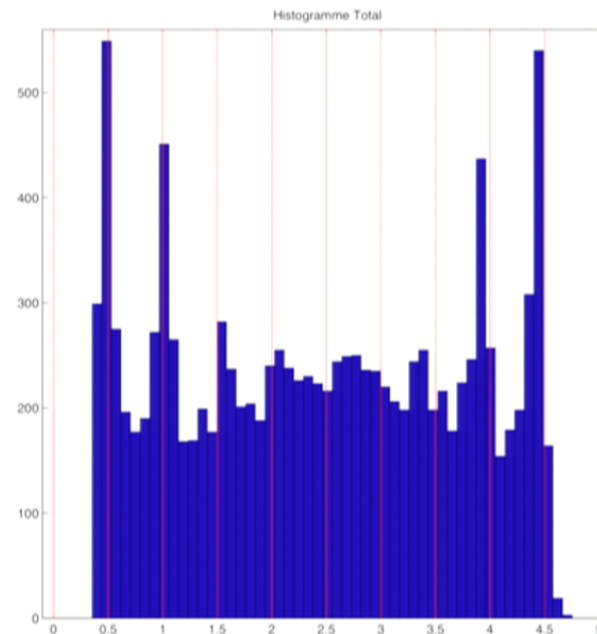
In a rectangular cavity

with *Julien Moukhtar*

**The Faraday eigenmode of the rectangular cavity**



**The probability of presence of a walker along the main axis of the cavity**



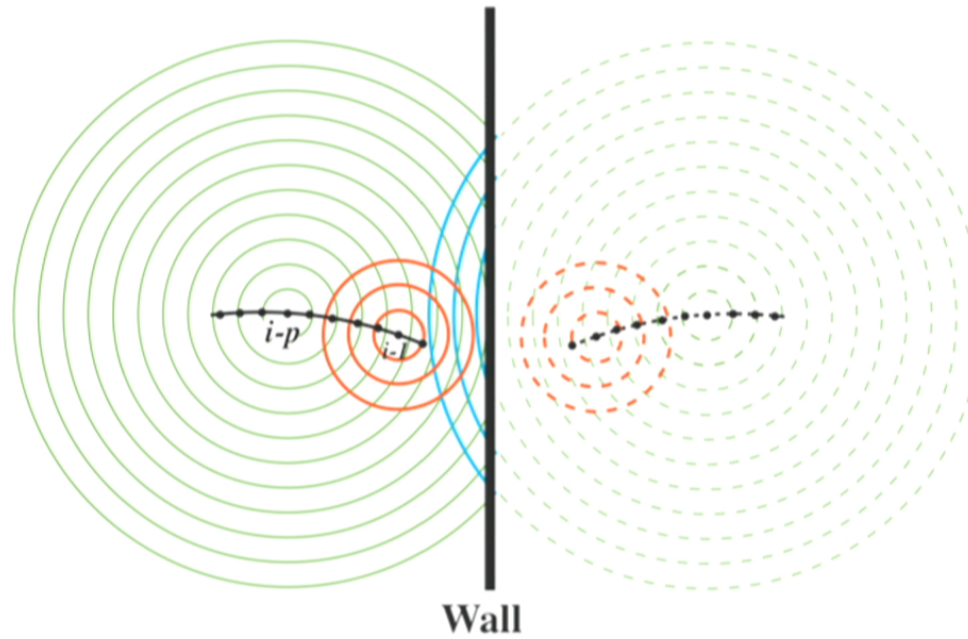
## Part III (b)

### Single particles diffraction and interference

*Couder Y. & Fort E. PRL., 97, 15101, (2006)*

## The numerical model of walkers in the vicinity of boundaries

**The echolocation of the walker: interaction with boundaries, reflected waves are also taken into account**





## **Part III (b)**

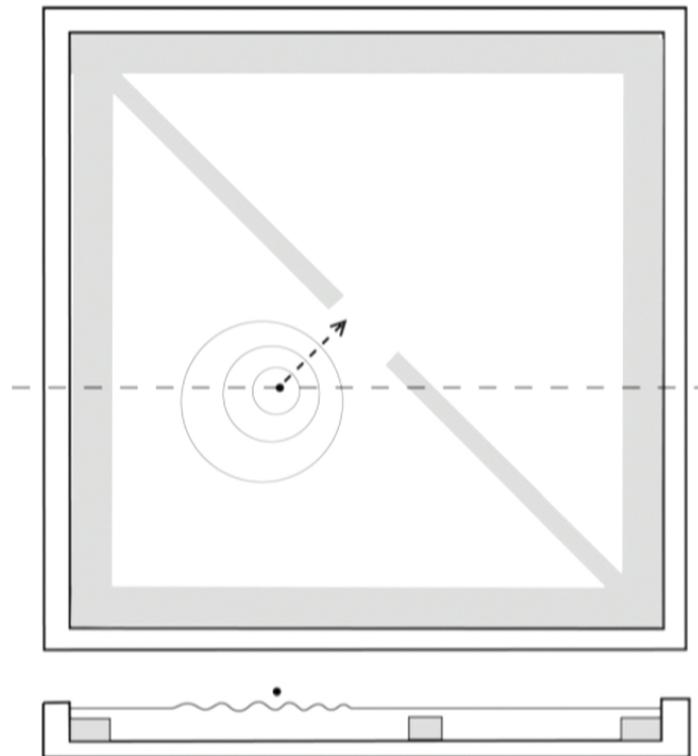
### **Single particles diffraction and interference**

*Couder Y. & Fort E. PRL., 97, 15101, (2006)*

## The experimental set up for diffraction and interference experiments

In the grey regions the fluid layer thickness is reduced to  $h_1=1\text{mm}$  ( $h_0=4\text{mm}$  elsewhere)

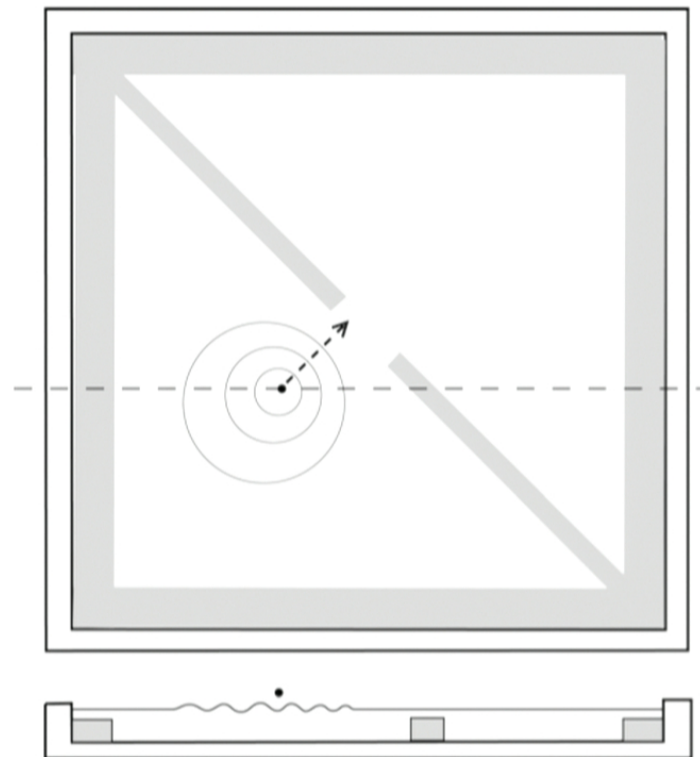
In these regions the Faraday threshold being shifted, the walkers do not propagate

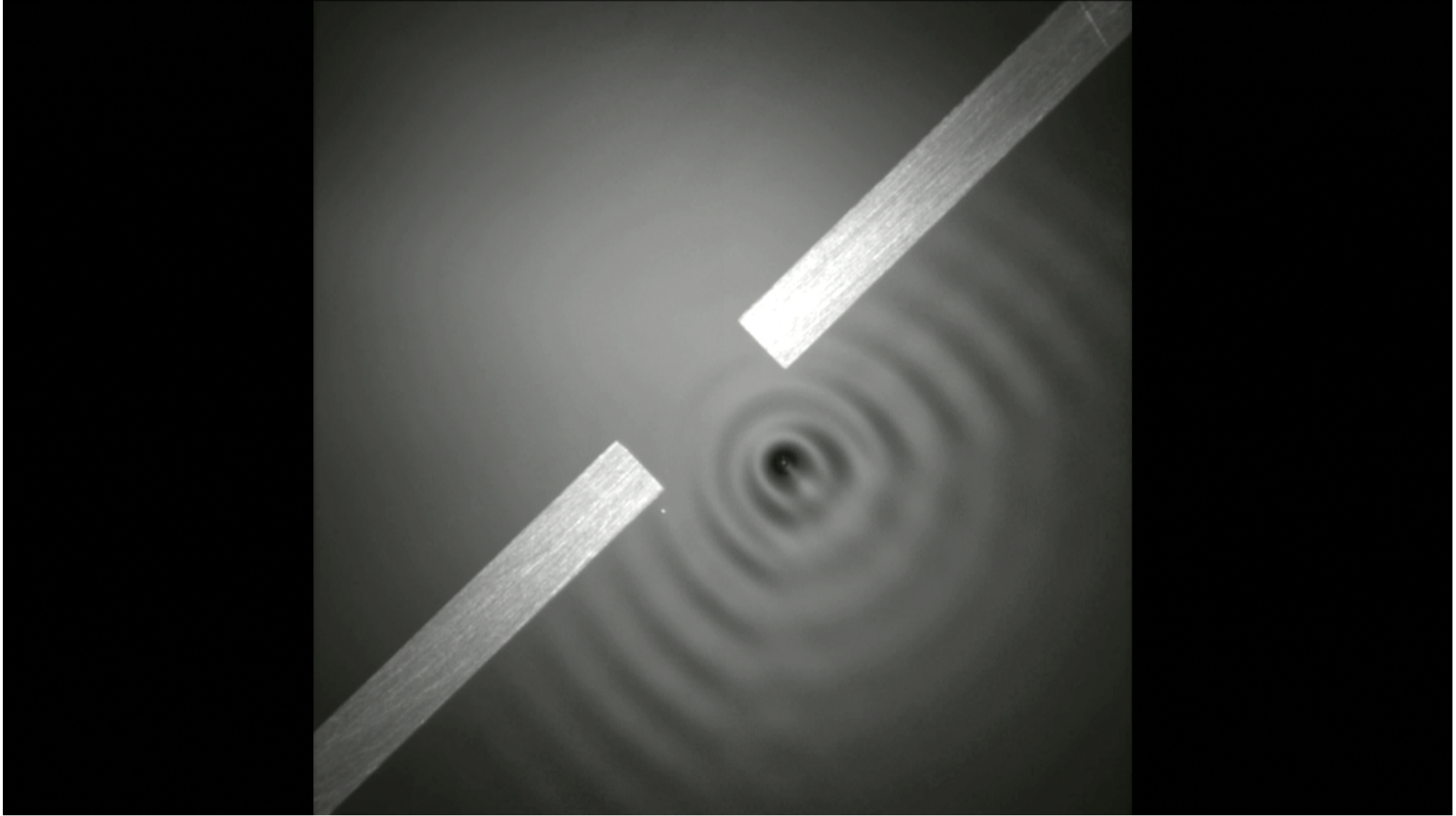


## The experimental set up for diffraction and interference experiments

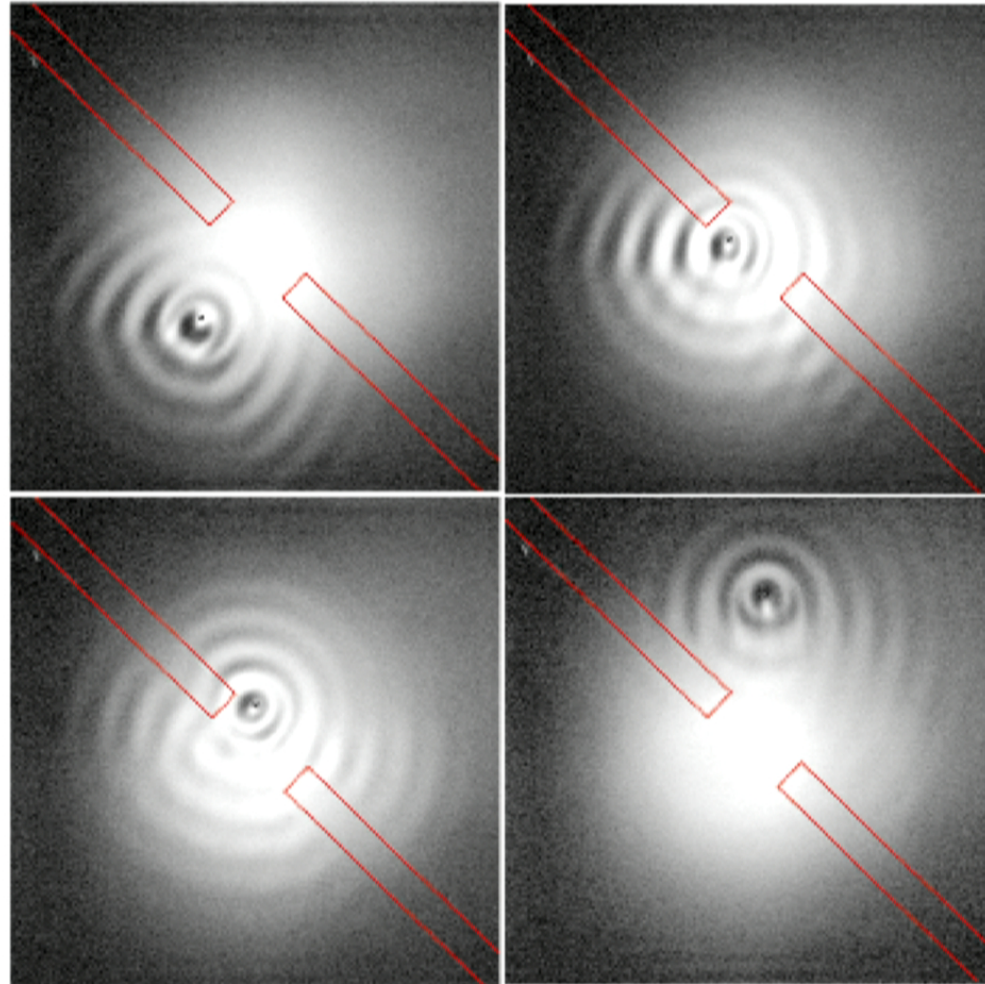
In the grey regions the fluid layer thickness is reduced to  $h_1=1\text{mm}$  ( $h_0=4\text{mm}$  elsewhere)

In these regions the Faraday threshold being shifted, the walkers do not propagate





Four  
photographs of  
the  
wave pattern  
during the  
diffraction  
of a walker



## Measurements on the droplet's trajectory

The relevant parameters:

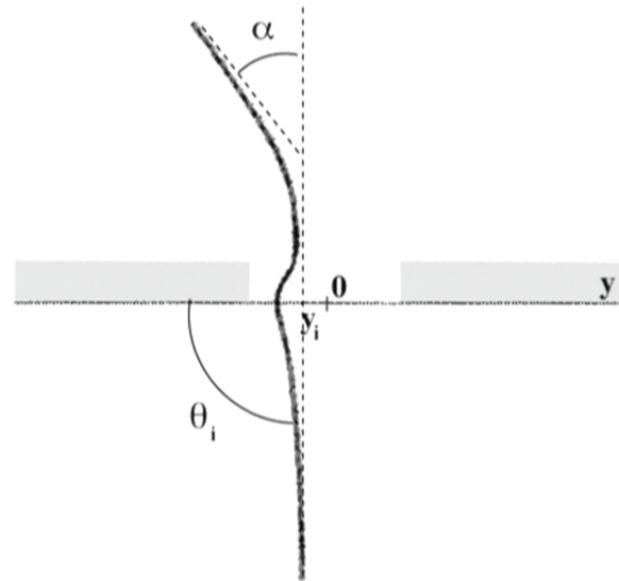
$L$  : the width of the slit,

$\theta_i$  : angle of incidence (chosen  $\theta_i = \pi/2$ ),

$\alpha$  : the angle of deviation

$Y_i = y_i/L$  : the impact parameter

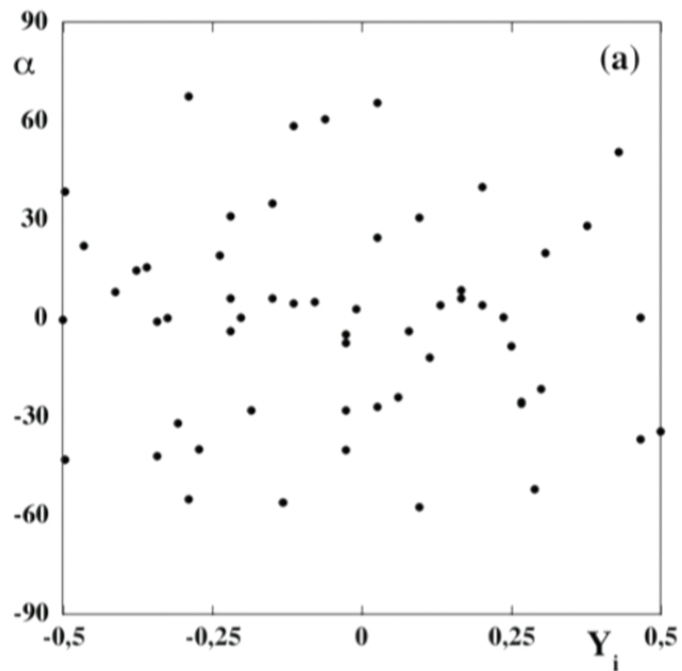
(With  $-0.5 < Y_i < 0.5$ )



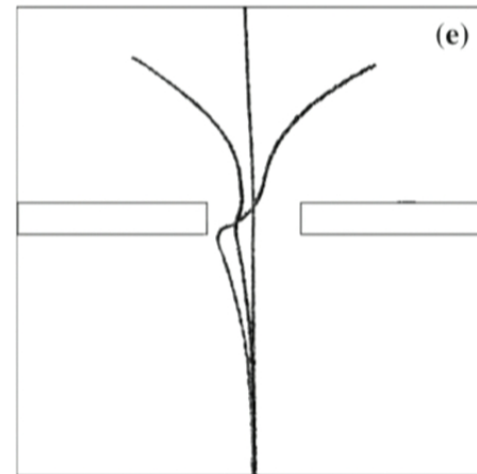
## Is there a link between the deviation $\alpha$ and the impact parameter $Y_i$ ?

*The measured deviation in experiments performed with the same walker, the same angle of incidence, but various impact parameters*

$L/\lambda_F=3.1$  ( $L=14.7\text{mm}$  and  $\lambda_F= 4.75 \text{ mm}$ ).

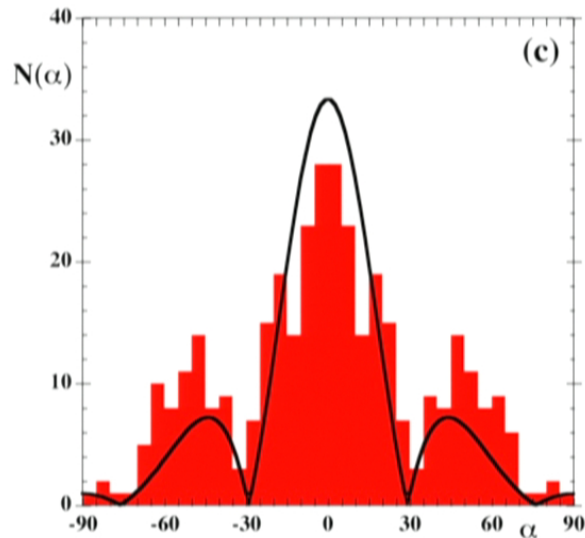


Three independent trajectories with the same initial conditions (within experimental accuracy)



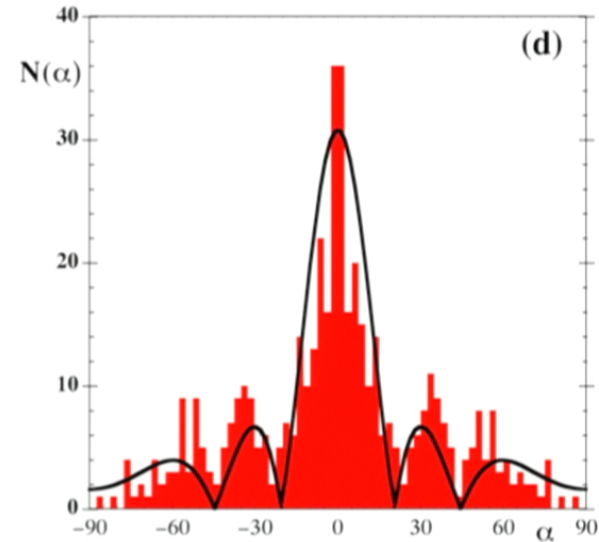
*What about statistical properties?*

## Cumulative histograms of the observed deviations in N independent crossings



$$L/\lambda_F = 2.1$$

(L=14.7mm,  $\lambda_F = 6.9$  mm)



$$L/\lambda_F = 3.1$$

L=14.7mm,  $\lambda_F = 4.7$  mm

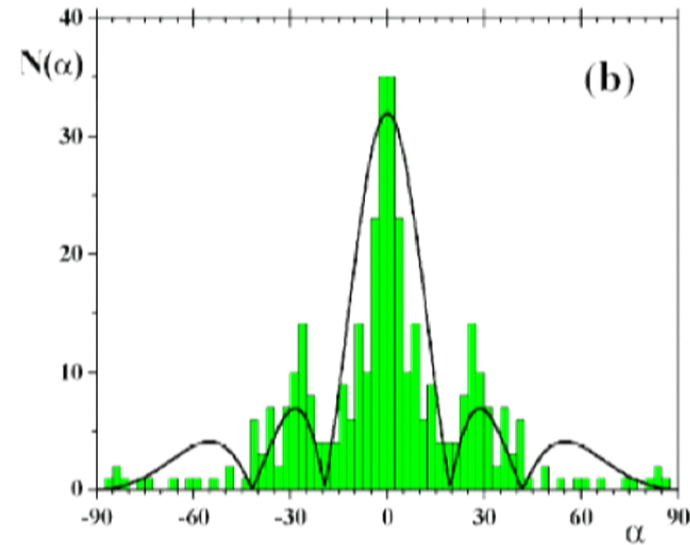
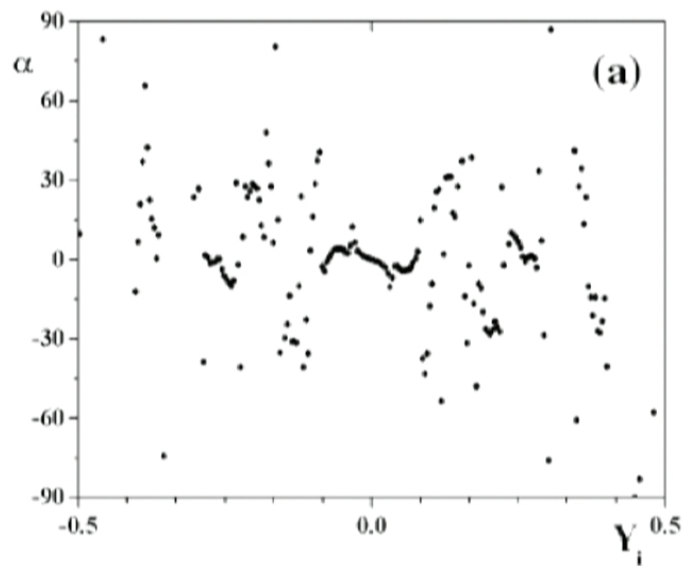
The curve is the modulus of the amplitude of a plane wave of the same wavelength diffracted through a slit of the same width

$$f(\alpha) = A \left| \frac{\sin(\pi L \sin \alpha / \lambda_F)}{\pi L \sin \alpha / \lambda_F} \right|$$



## The numerical simulation of the diffraction

In the presence of path memory the deviation becomes a complex function of the impact parameter  
And the pdfs of deviations are similar to those observed experimentally



$$L/\lambda_F = 3$$

## Diffraction of waves...

It is not a surprise that a wave passing through a slit is diffracted. This is the standard result from Fourier transform. The wave truncation results in its spreading in the transverse direction.

## ... or diffraction of particules?

*Here, however, we do not measure the wave-field but the trajectories of successive particles. Their individual deviations are unpredictable, exhibiting an uncertainty linked with the spreading of the wave:*

*The Fourier spreading of the wave generates an uncertainty for the direction of the velocity of the particle and thus for its momentum.*

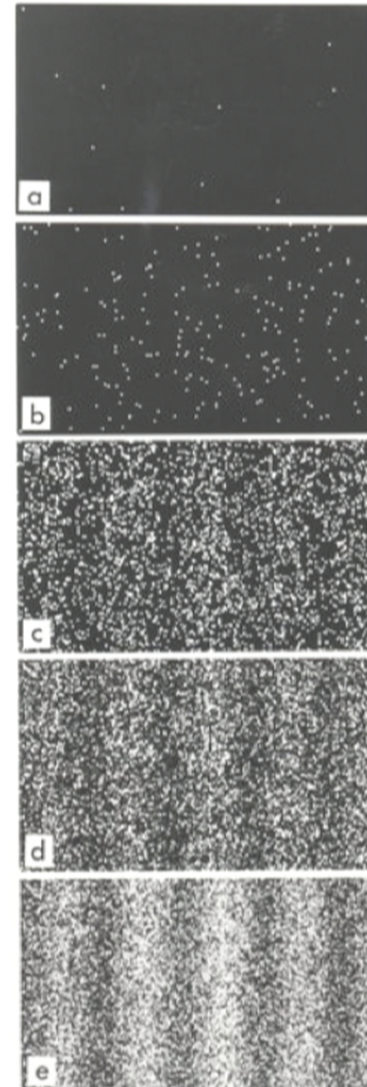
## The Young double-slit interference with single particles

*A phenomenon which is assumed to have no equivalent in classical physics*

*R. Feynman's, Lectures on Physics, vol. 3, Quantum Mechanics, (First chapter)*

*« ... In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way and which is at the heart of quantum mechanics. In reality it contains the only mystery. We cannot make the mystery go away by explaining how it works. We will just tell you how it works.... »*

The build-up of the interference pattern  
(with electrons, after Tonomura)



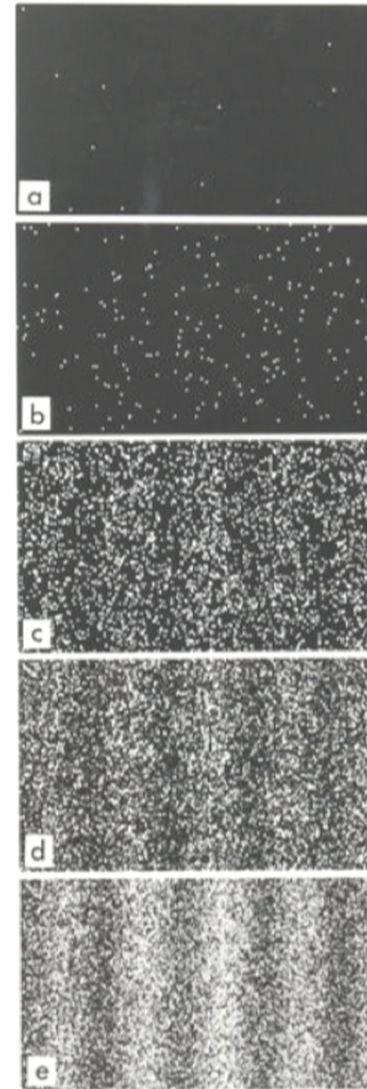
## The Young double-slit interference with single particles

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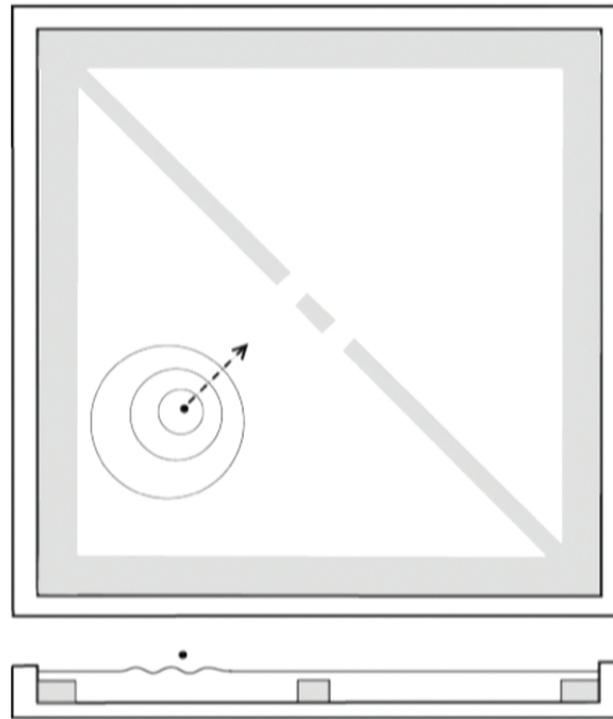
*R. Feynman's, Lectures on Physics, vol. 3, Quantum Mechanics, (First chapter)*

*« ... In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, absolutely impossible, to explain in any classical way and which is at the heart of quantum mechanics. In reality it contains the only mystery. We cannot make the mystery go away by explaining how it works. We will just tell you how it works.... »*

The build-up of the interference pattern  
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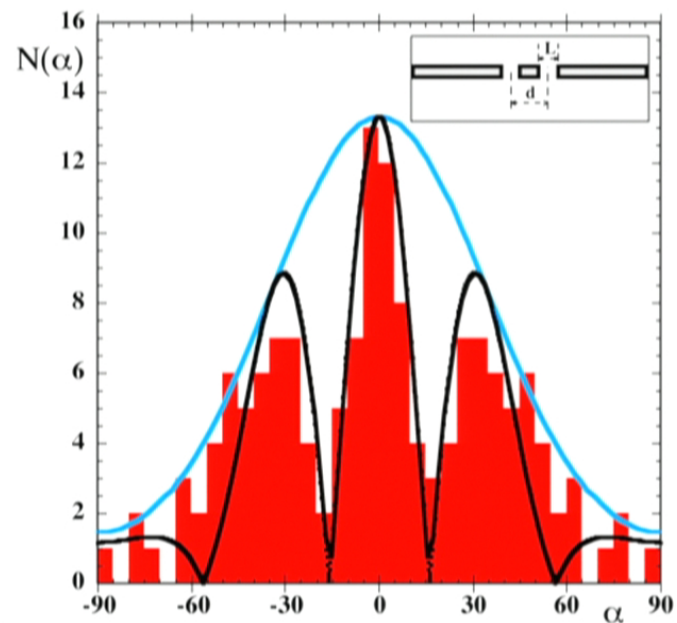
## Young's two slits experiment



## Interference: histogram for 75 realizations

The curve is the modulus of the  
amplitude of the interference of a  
plane wave through two slits with  
 $L/\lambda_F = 0.9$  and  $d/\lambda_F = 1.7$ .

$$f(\alpha) = A \left| \frac{\sin(\pi L \sin \alpha / \lambda_F)}{\pi L \sin \alpha / \lambda_F} \cos(\pi d \sin \alpha / \lambda_F) \right|$$

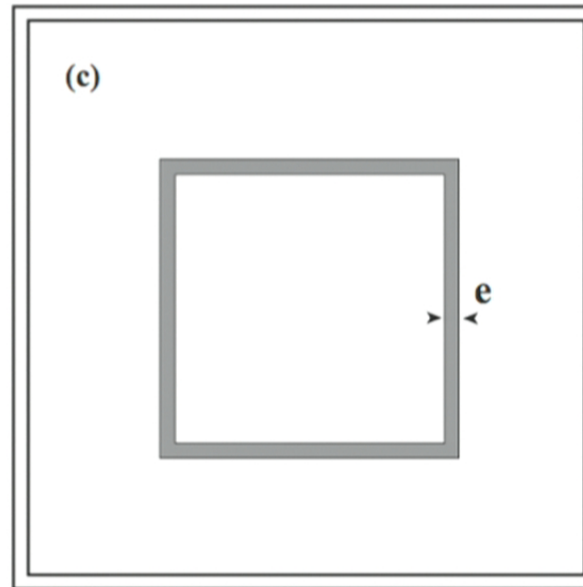


*Part II (b)*  
*Tunneling through a barrier*

*With Antonin Eddi*

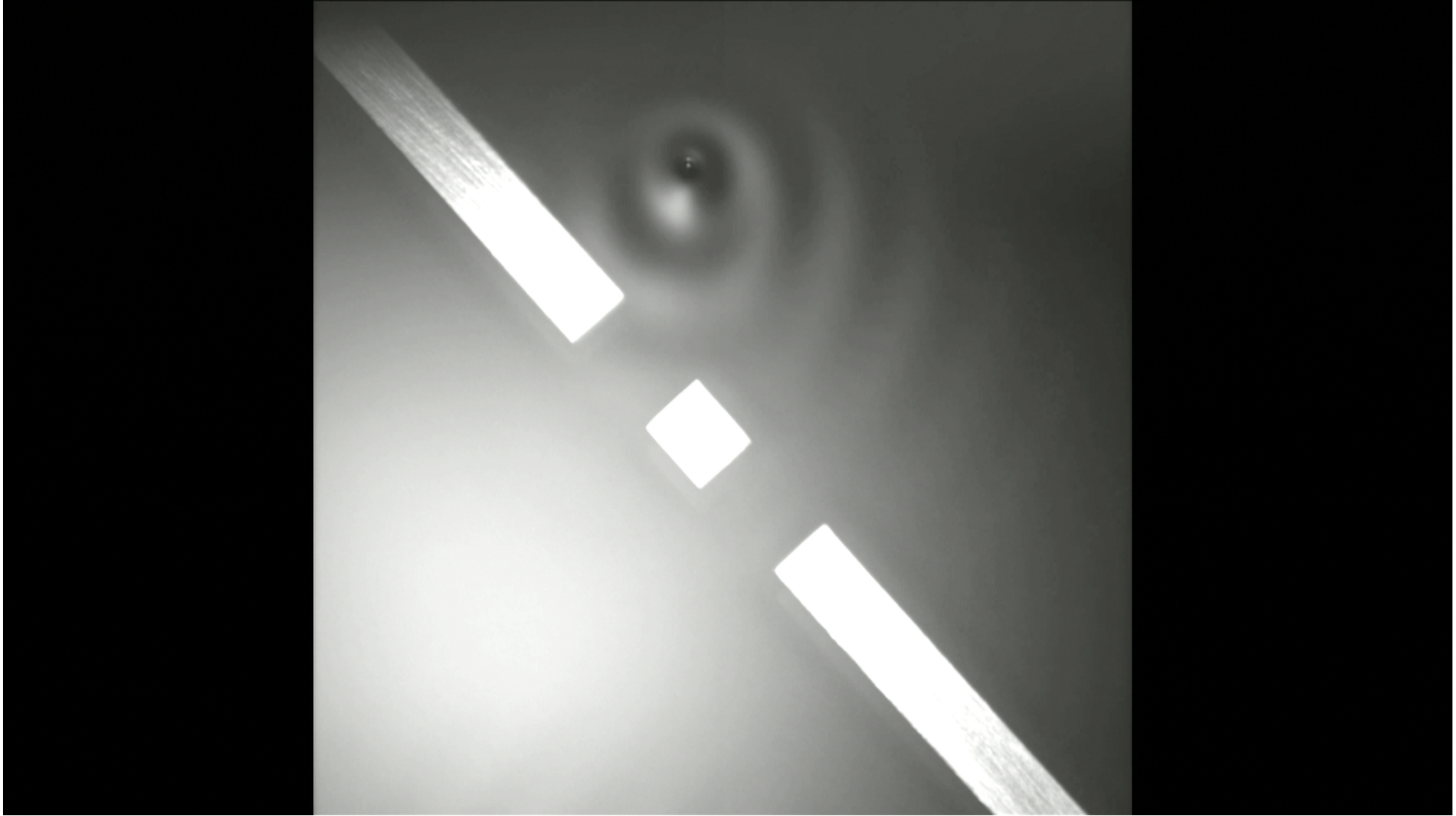
A.Eddi, F. Moisy, E. Fort & Y.Couder  
" Unpredictable tunneling of a classical wave-particle association"  
*Phys. Rev. Lett.* **102**, 240401, (2009)

## First experimental set-up



**Escape out of a closed cavity**





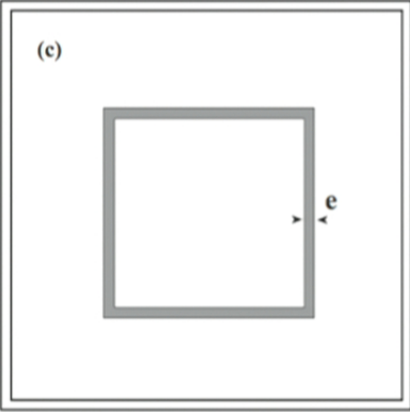
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Thèmes de la diapositive Mises en page des diapositives Transitions Styles de table Graphiques

### First experimental set-up



(e)

Escape out of a closed cavity

Cliquez pour ajouter des commentaires

Affichage normal Diapositive 76 sur 111

PaLETTE de mise en forme

Police

Nom :

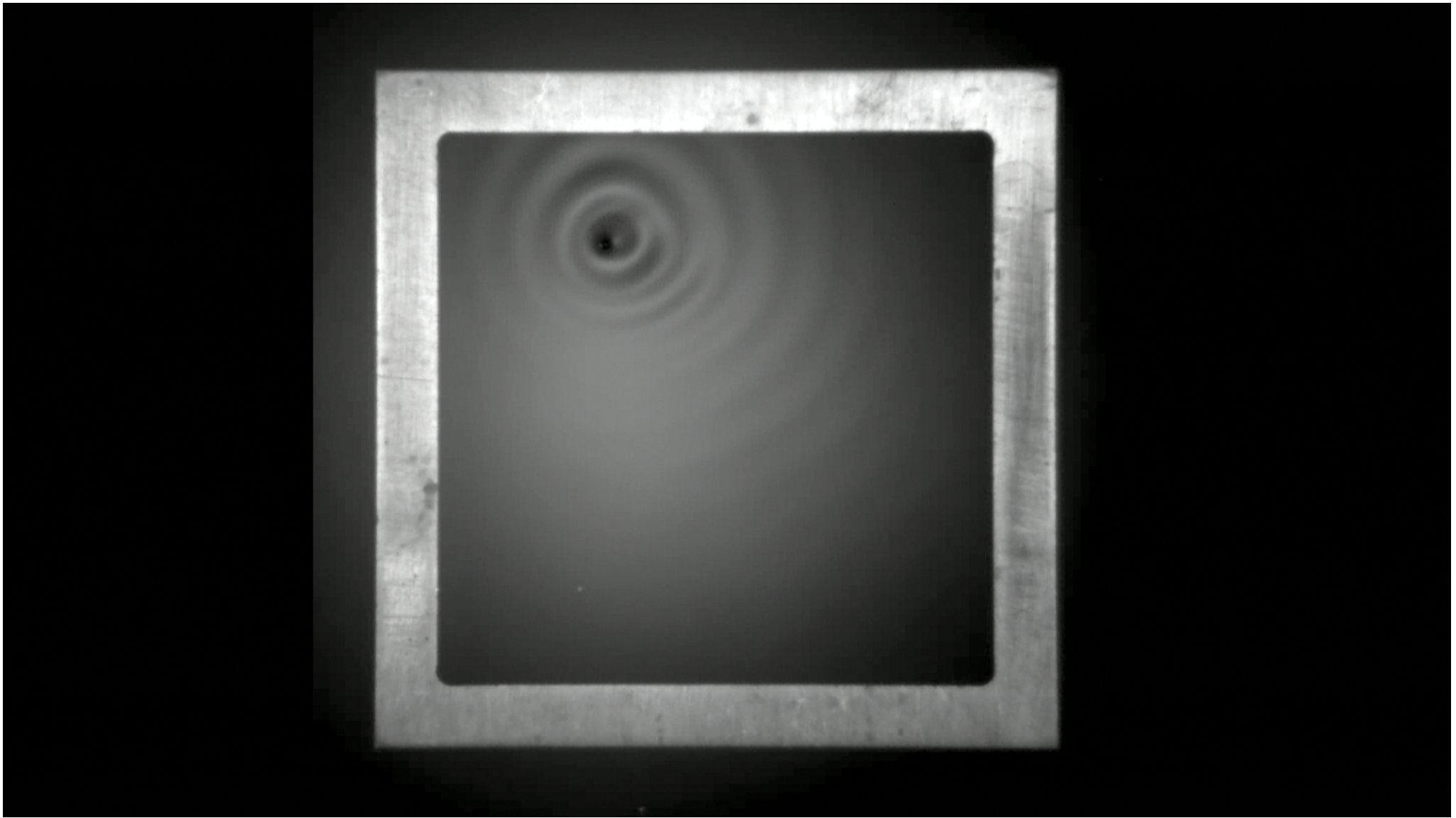
Taille :

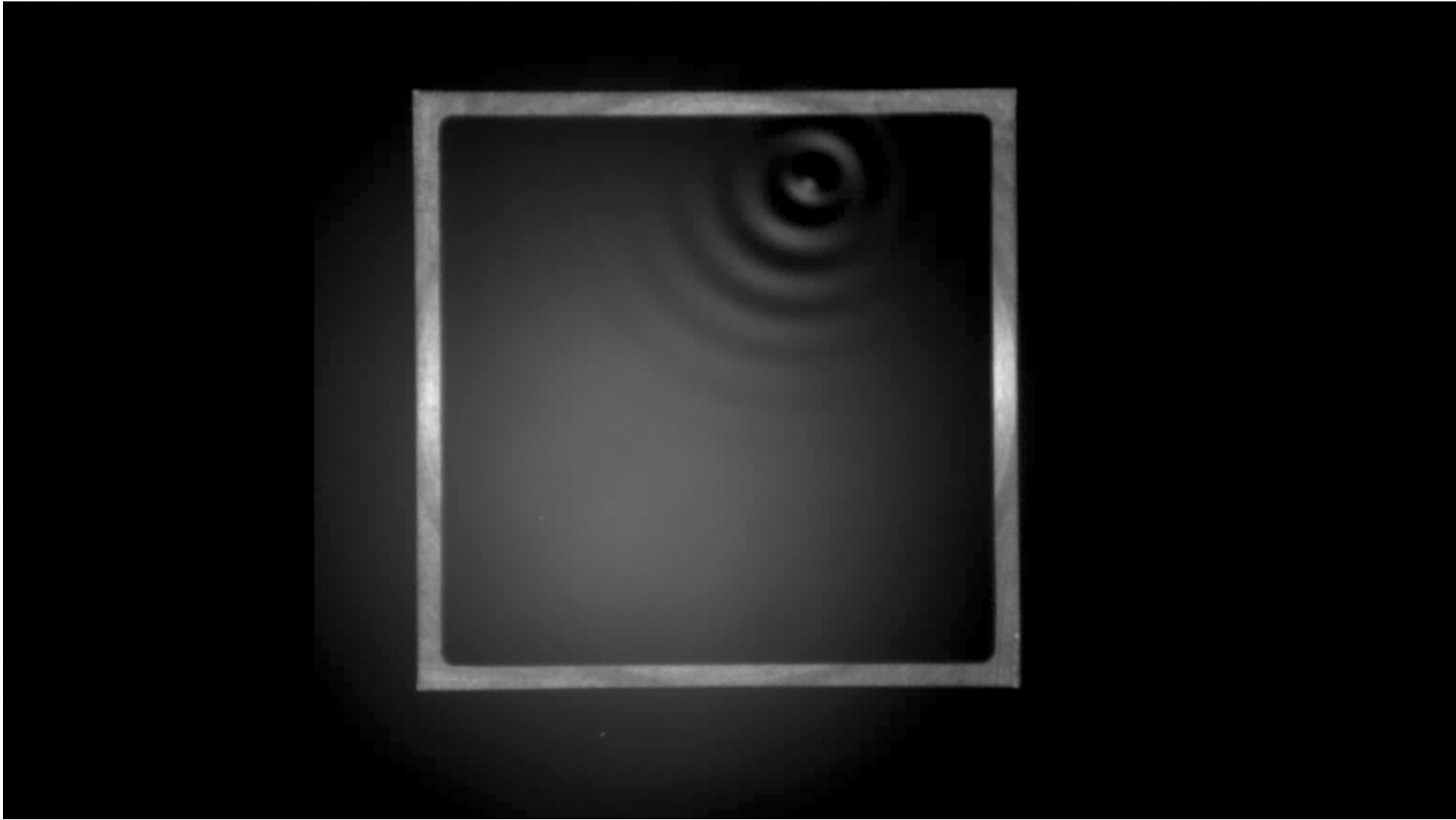
Thème du document

Couleurs :

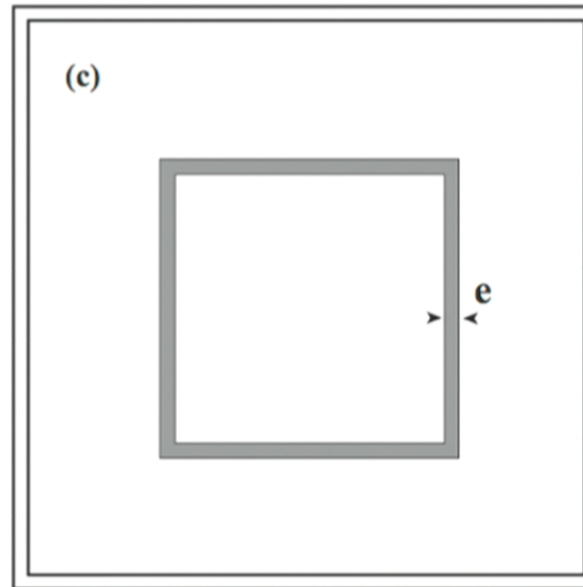
Polices : Times, Times

Parcourir... Enregistrer le thème...





## First experimental set-up

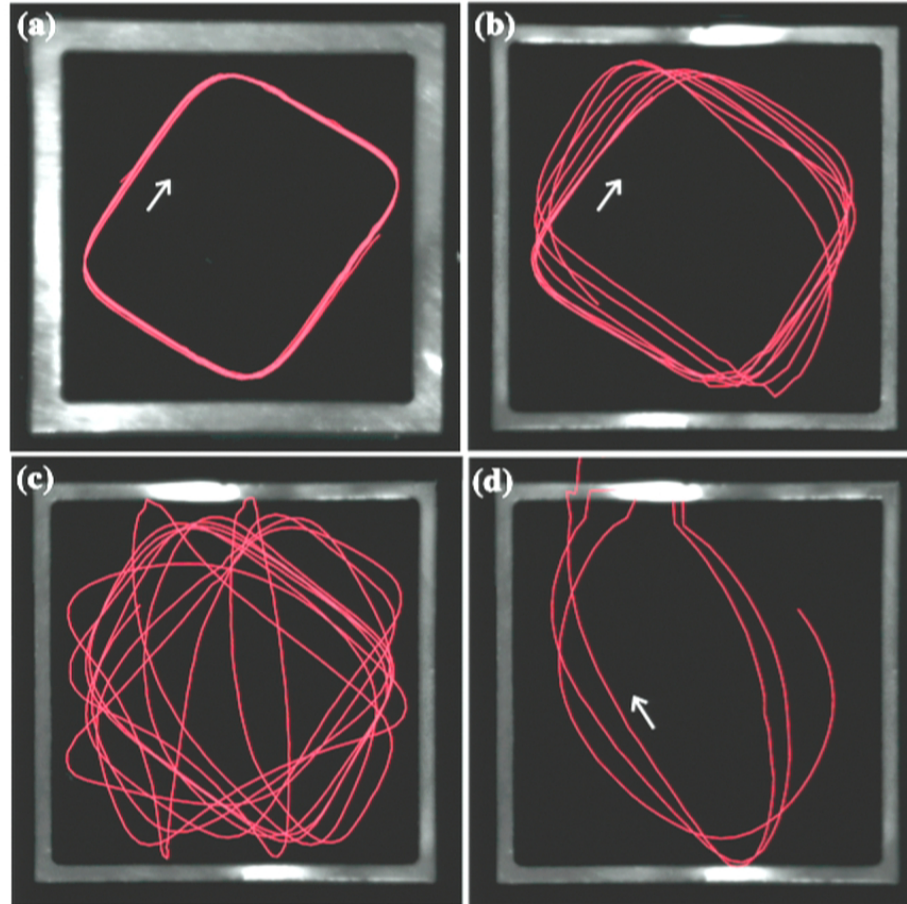


**Escape out of a closed cavity**

**The origin of the probabilistic behaviour:  
the trajectories inside  
the frame**

*For thick walls the walker  
reaches a stable limit cycle*

*With thin walls the reflections  
are imperfect, leading to  
different types of collisions  
with the wall and to a  
probability of escape*

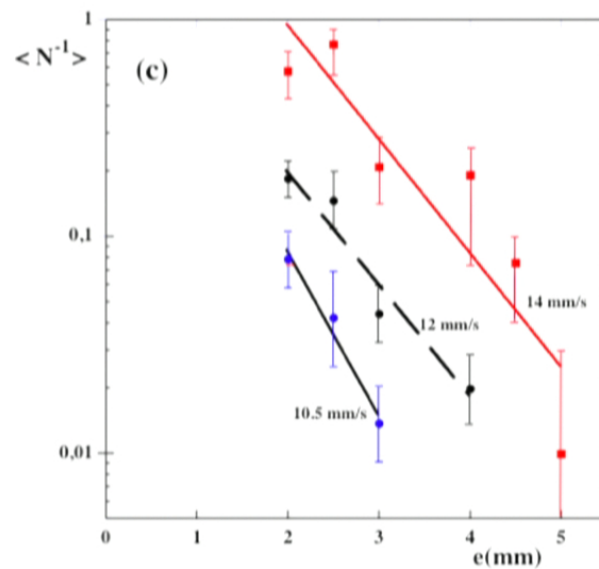


## A probability of escape can be measured

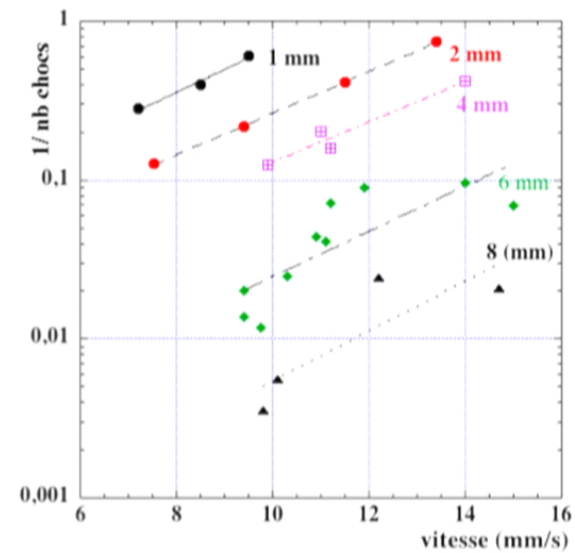
*The walker has a billiard motion inside the frame and escapes out of it once in a while.*

*Repeating the experiment, a probability of escape can be defined as the ratio of the number of escapes over the total number of collisions with the wall*

*It turns out to depend on the barrier thickness and the velocity of the walker*

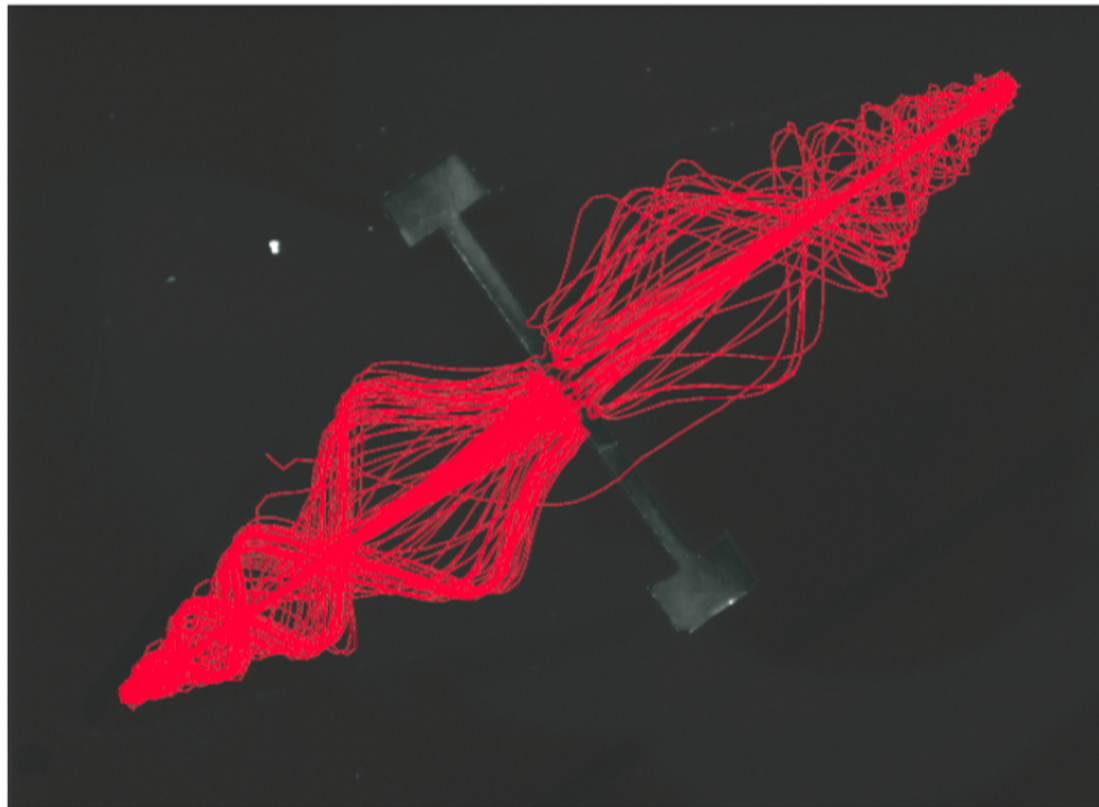


Semi-log plot of the probability of crossing as a function of the barrier's thickness



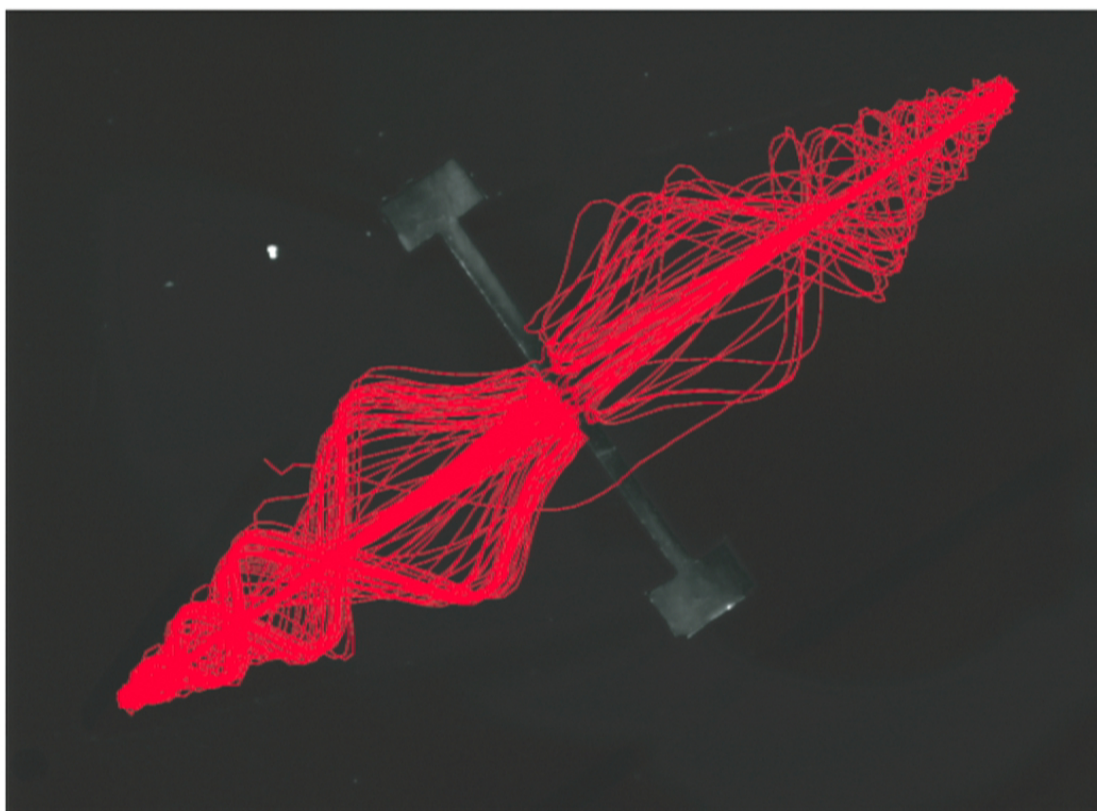
Semi-log plot of the probability of crossing as a function of the walker's velocity

## Accumulation of events

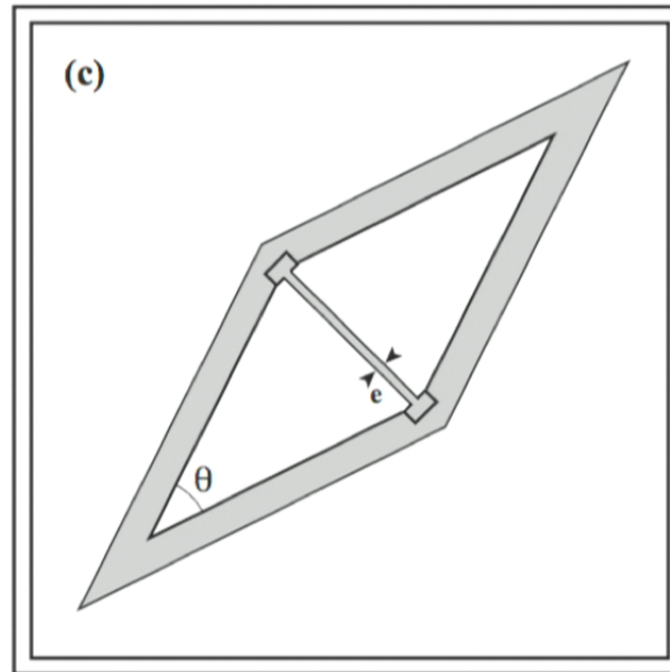




## Accumulation of events

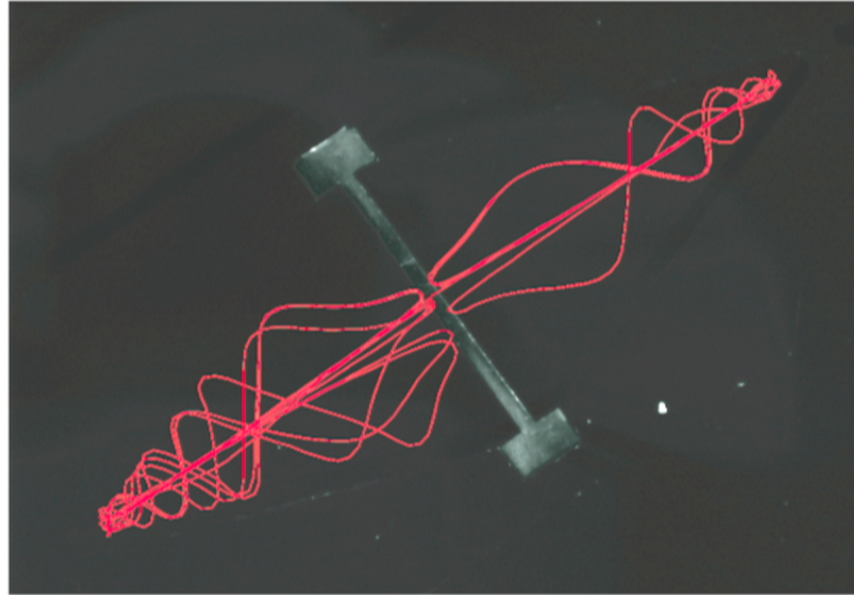


## Second experimental set-up



The particle is guided by the divergent walls so that it impinges perpendicularly on the barrier of thickness  $e$ .

## The relation to the incident trajectory



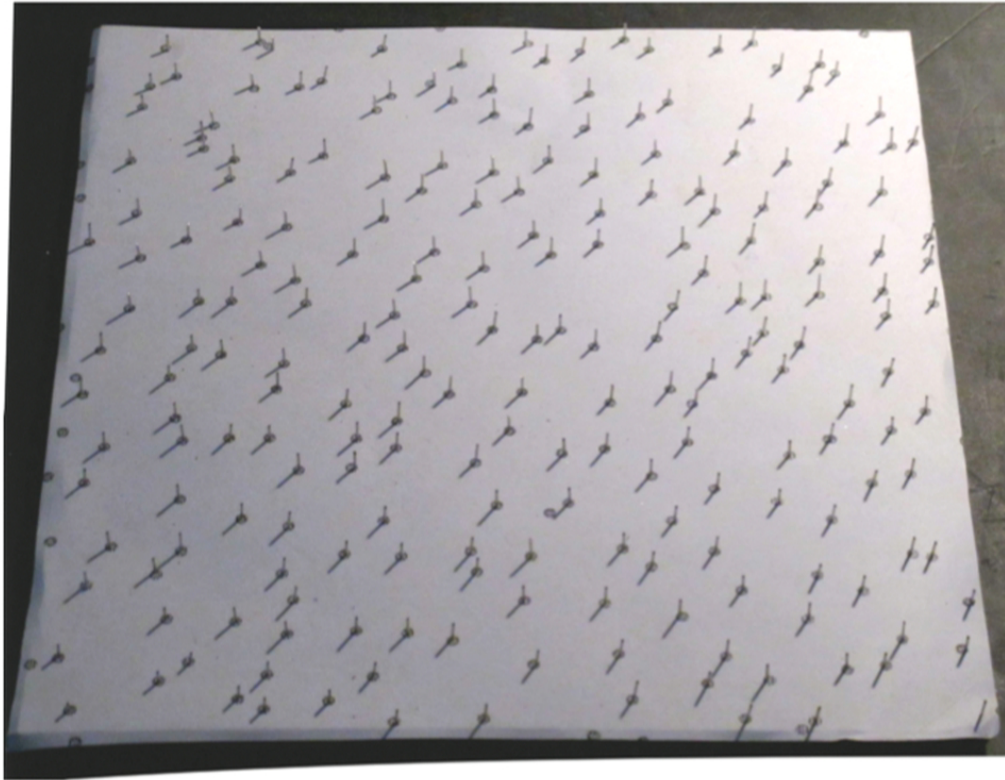
*Far from the barriers, all walkers have a normal incidence.*

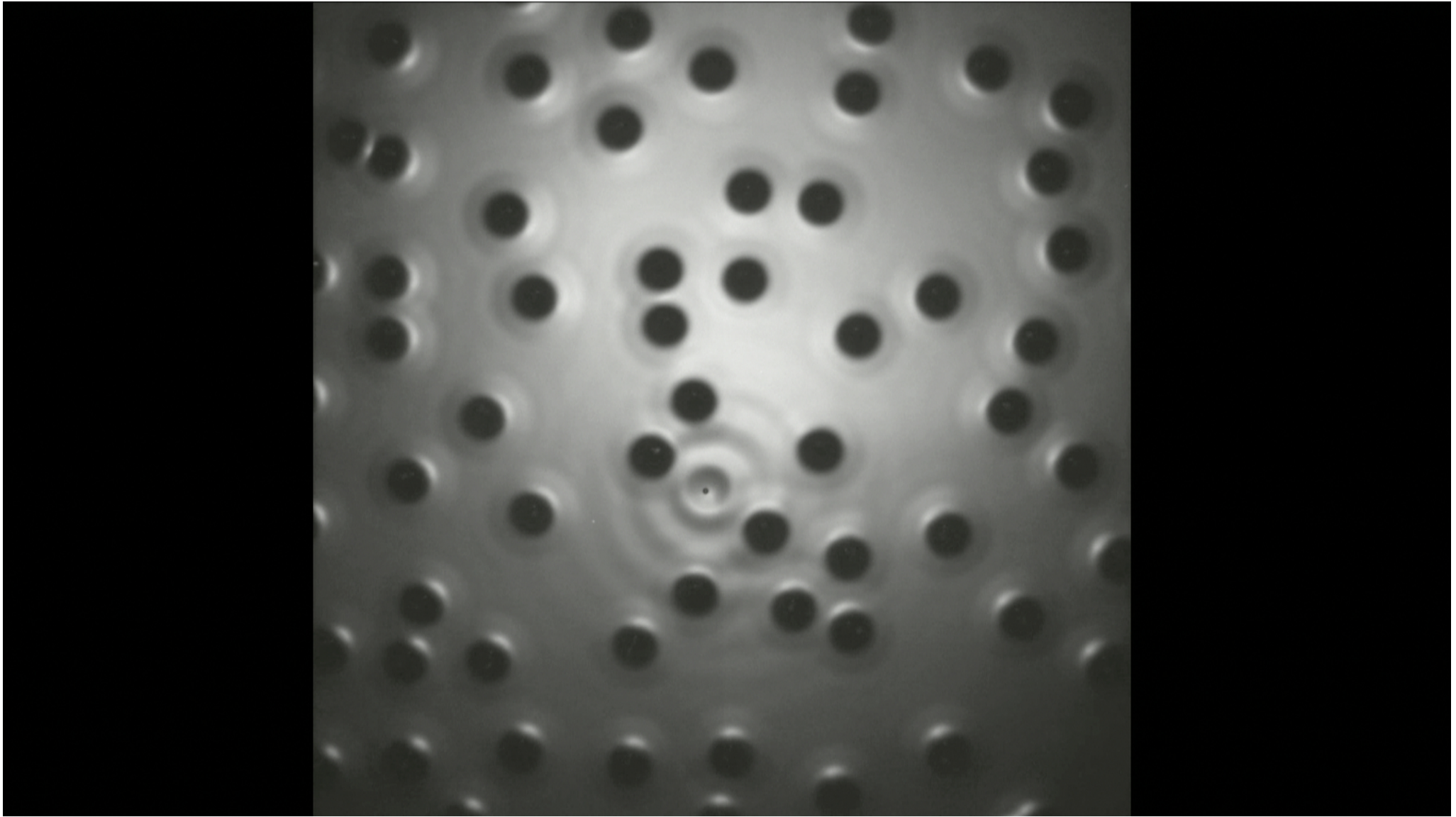
*They deviate because of the reflected waves*

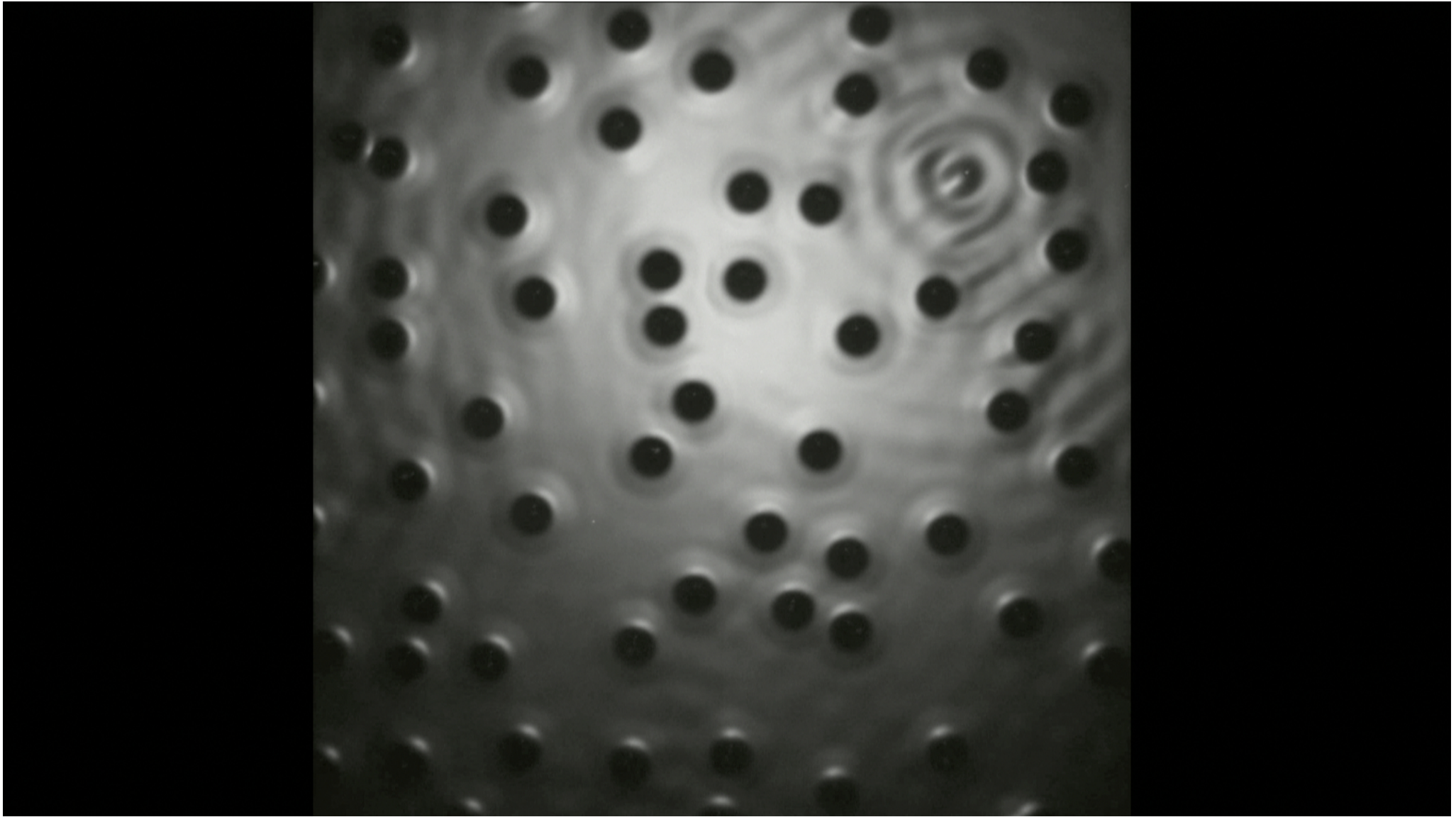
*Only those walkers which have had a weak deviation have a probability of crossing*

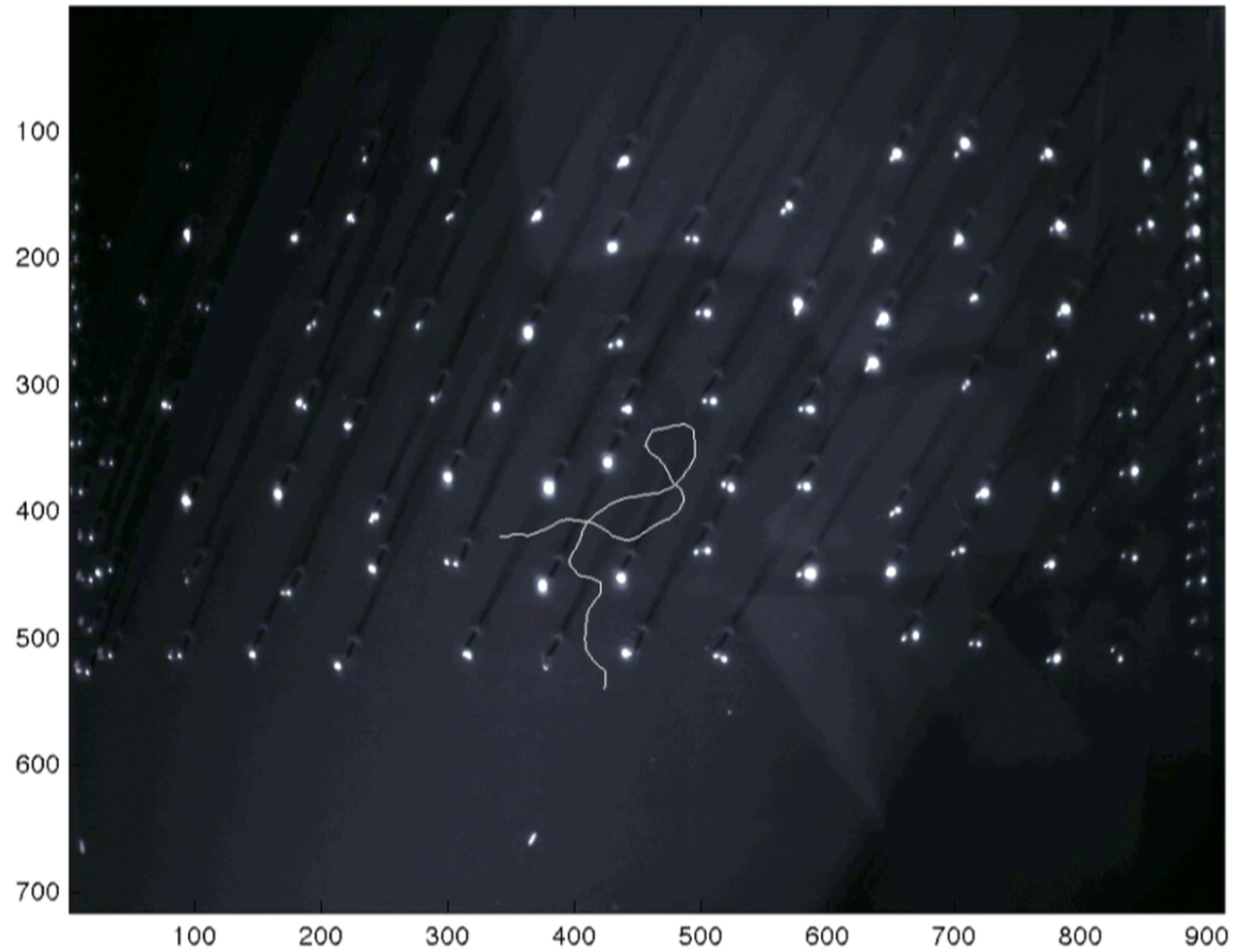
*The walker deviates have a weaker probability of being deviated when the reflected waves are weaker (thin barriers), hence a larger probability of crossing*

## Propagation in a random medium









## Discussion:

**is there a relation to quantum mechanics?**

*In our system we clearly have a particle driven by a wave it generates. It is therefore interesting to revisit the unorthodox “pilot wave” models of quantum mechanics.*



## The “pilot wave” models

The association of particles to waves was initially proposed by de Broglie

L. de Broglie, *Ondes et mouvements*, (Gautier Villars Paris) (1926).

In 1952 D. Bohm obtain trajectories by deriving an equation of motion out of the Schrödinger equation

D. Bohm, *Phys. Rev.* 85: 166-179, and 180-193, (1952),  
*Phys. Rev.* 89: 458-466 (1953).

**These two approaches are often identified to each other and called the de Broglie-Bohm pilot-wave models.**

**They are in fact very different from each other and should be dissociated.**

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**They are in fact very different from each other and should be dissociated.**

## D. Bohm uses the Madelung transformation of the Schrödinger equation

*revisited recently by John Bush*

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi$$

$$\Psi = \sqrt{\rho} e^{imS/\hbar}$$

Continuity :  $\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0$

Quantum Hamilton - Jacobi  $\frac{\partial S}{\partial t} + \frac{1}{2} \mathbf{u}^2 - \frac{\hbar^2}{2m^2} \frac{1}{\sqrt{\rho}} \nabla^2 \sqrt{\rho} + \frac{V}{m} = 0$

where :

$\rho = |\Psi|^2$  is the probability density

$\mathbf{u} = \nabla S$  is the quantum velocity of the probability

$\mathbf{j} = \rho \mathbf{u}$  is the quantum probability flux

**The quantum Hamilton-Jacobi can be written:**

$$\frac{\partial S}{\partial t} + \frac{1}{2} \mathbf{u}^2 = -\frac{Q}{m} - \frac{V}{m}$$

**Where Q is the quantum potential**

$$Q = -\frac{\hbar^2}{2m^2} \frac{1}{\sqrt{\rho}} \nabla^2 \sqrt{\rho}$$

**Taking the gradient**

$$m \frac{Du}{Dt} = -\nabla Q - \nabla V$$

**Equating the quantum velocity and the “particle” velocity**

$$m \ddot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

**Bohmian mechanics consists in solving Schrödinger equation for  $\Psi$ , from which Q is then computed, before solving the trajectory equation.**

## The limit of Bohmian mechanics

**The trajectory equation:**

$$m \dot{\mathbf{x}}_p = -\nabla Q - \nabla V$$

**does not define the trajectory of the particle but the trajectory of the probability density.**

**For this reason de Broglie wrote in 1953 :**

**“A year and a half ago, David Bohm took up the pilot-wave theory again. His work is very interesting in many ways (...) But since Bohm’s theory regards the wave  $\Psi$  as a physical reality, it seems to me to be unacceptable in its present form”.**

COLLECTION DE PHYSIQUE MATHÉMATIQUE

DIRECTEURS : ÉMILE BOREL ET MARCEL BRILLOUIN

# ONDES ET MOUVEMENTS

PAR

L. DE BROGLIE

Docteur en Sciences



PARIS

GAUTHIER-VILLARS ET C<sup>o</sup>. ÉDITEURS

LIBRAIRES DU BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE  
Quai des Grands-Augustins, 55

1926

de Broglie original model

L. de Broglie, *Ondes et Mouvements*,  
(Gauthier-Villars Paris) (1926).

## The pre-Shrödinger de Broglie model (1926)

de Broglie assumes that there are well defined particles that he considers as point sources.

This material point is considered as having an internal oscillation and emitting in the surrounding medium a wave of frequency :

$$\nu_0 = \frac{1}{h} m_0 c^2$$

**The phase of the particle oscillation and that of the wave are locked to each other**

The particle is surrounded by a stationary spherical wave, the superposition of a divergent and a convergent wave.

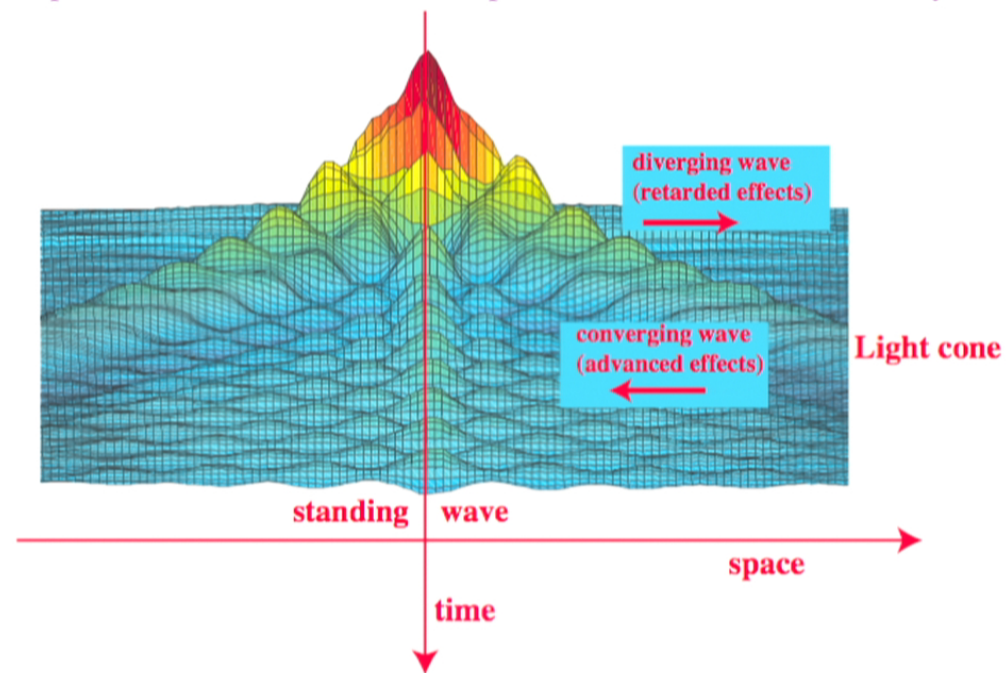
$$\varphi(r_0, t_0) = \frac{A}{2r_0} \left\{ \cos \left[ 2\pi\nu_0 \left( t_0 - \frac{r_0}{c} \right) + c_1 \right] - \cos \left[ 2\pi\nu_0 \left( t_0 + \frac{r_0}{c} \right) + c_2 \right] \right\}$$

He writes :

**« But there is also the convergent wave, the interpretation of which could raise interesting philosophical issues, but that appears necessary to insure the stability of the material point »**

In our system a standing wave is also associated to the particle.  
How is it generated?

Measured spatio-temporal evolution of the radial profile of the wave emitted by one bounce



A travelling droplet. Because of the excitability of the medium, it leaves behind a Faraday standing wave

*Each point of the wave front emits a wave moving backwards towards the source.*



## De Broglie proposed what he called a “double solution”

### *First solution*

*The particle has an oscillation at a frequency  $\nu_0 = m_0 c^2 / h$  and is surrounded by a standing wave with a singularity (or non linear region) at its core. This structure forms the individual particle and has well-defined trajectories in space-time.*

### *The second solution*

*A linear and smooth wave, solution of the Schrödinger equation that corresponds to the averaged behaviours.*

## De Broglie “double solution”

*The first solution can be written*

$$u = f e^{\frac{2\pi i}{h}\varphi}$$

*Where  $f$  has very large values in a singular region*

*The second solution can be written*

$$\psi = a e^{\frac{2\pi i}{h}\varphi}$$

*Where  $a$  and  $\varphi$  are continuous.*

*$\Psi$  is the solution of the Schrödinger equation*

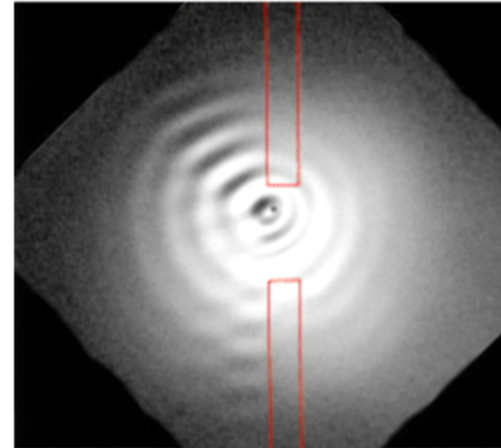
*The velocity of the particle is given by*

$$\vec{v} = -\frac{1}{m} \vec{\nabla} \varphi$$

*In our experiment we have a double solution situation of the type proposed by de Broglie.*

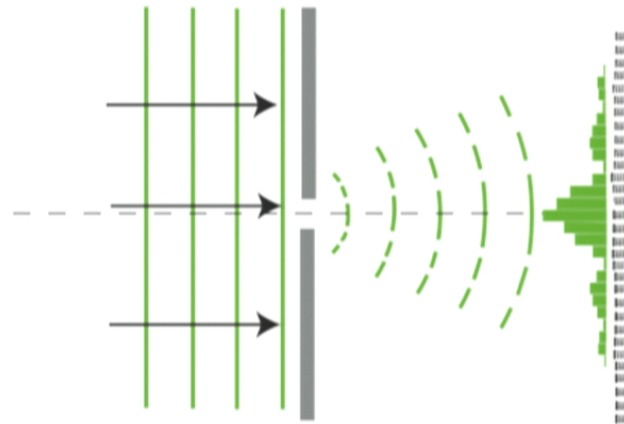
*There is a particle: the droplet. It is guided by a wave of wavelength  $\lambda_F$  but this wave is not a plane wave*

**Analogous to de Broglie's first solution**



*The probabilities of the various angles of deviation correspond to a diffracted plane wave of wavelength  $\lambda_F$*

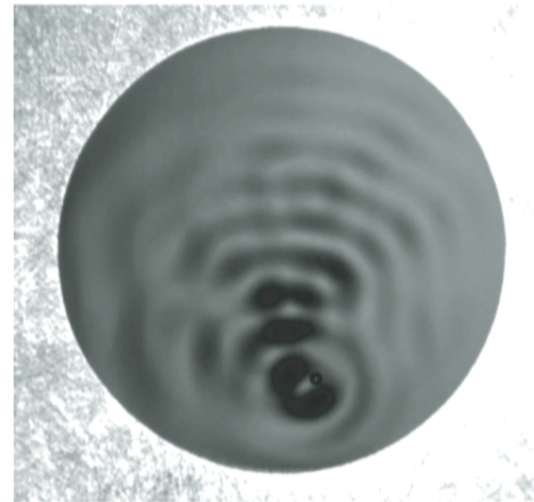
**Analogous to a Schrödinger wave, de Broglie's second solution**



## *The double solution in cavities*

### **First solution**

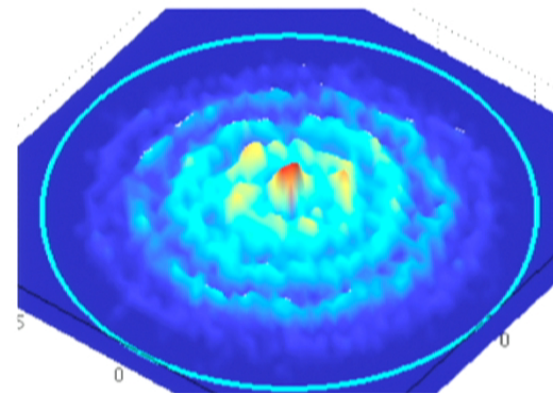
- One given walker is piloted by the wave it has generated and has an individual complex trajectory.
- The structure of the pilot wave is complex as it contains a memory of the past trajectory



**First solution**

### **Second solution**

- The probabilities are given by an underlying mean wave structure linked to the resonant modes of the cavity or more generally to the environment



**Second solution**

*Courtesy Dan Harris*

**Returning to our experiment.**  
**Its main drawback:**  
**it is very far from quantum mechanics**

- Macroscopic scale : no relation with Planck constant.
- The system is two-dimensional.
- The system is dissipative and sustained by external forcing.
- This forcing imposes a fixed frequency: the “energy” is fixed
- The waves live on a material medium: there is an “ether”.
-

**Its main interest:**  
**it is very far from quantum mechanics,**

-At quantum scale the Planck limitation imposes itself to all phenomena. It is not possible to do a non-intrusive measurement.

-Intrusive measurements generate a projection onto states, so that only the probabilities of those states can be measured.

*-Here we can do either intrusive or non intrusive measurements.*

- If we try to know the position of a walker by confining it in a cavity or by having it pass through slits we find probabilistic behaviours.

- The observation with light is non intrusive so that the undisturbed trajectory of the particle and the wave can be observed directly.

- **Non intrusive observations done during an intrusive measurement show that the latter generates chaotic trajectories that are responsible for the observed statistical properties.**

**All the observed quantum-like properties emerge from what we have called the “wave-mediated path-memory”.**

**This “path memory” generates a particular type of space and time non locality.**

**For this reason we believe the debate on hidden variables is not closed**