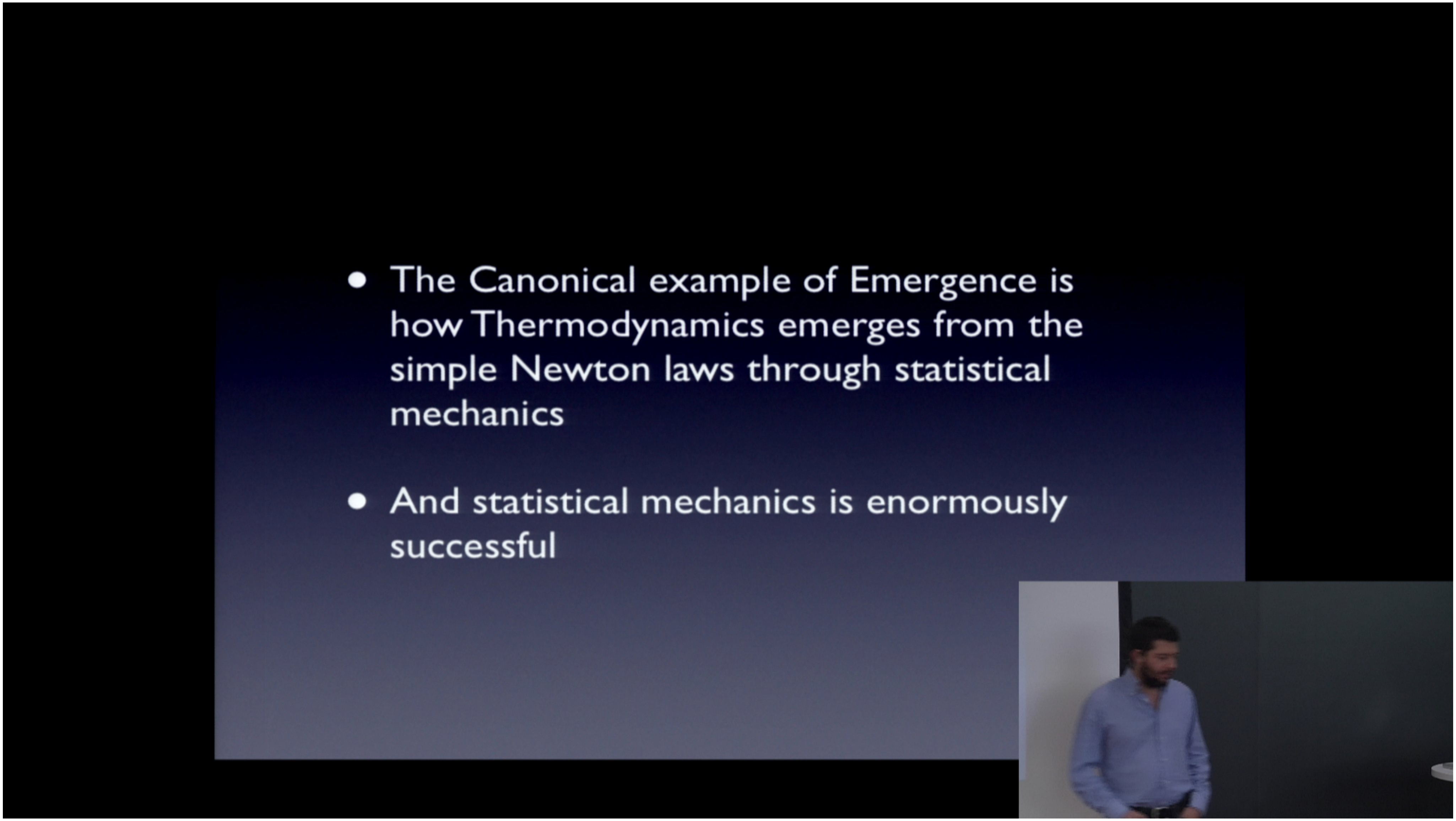
Title: Entanglement and the Emergence of Thermalization

Date: Oct 28, 2011 09:00 AM

URL: http://www.pirsa.org/11100115

Abstract: The canonical example of emergence is how thermodynamics emerges from microscopic laws through statistical mechanics. One of the vexing questions in the foundations of statistical mechanics is though how is it possible to justify thermalization in a closed system. In quantum statistical mechanics, entanglement can give the key to answer this question, provided that they are typically very entangled. Fortunately, most states in the Hilbert space are maximally entangled. Unfortunately, most states in the Hilbert space of a quantum many body system are not physically accessible. We show that the typical entanglement in physical ensembles of states is still very high.


• The Canonical example of Emergence is how Thermodynamics emerges from the simple Newton laws through statistical mechanics
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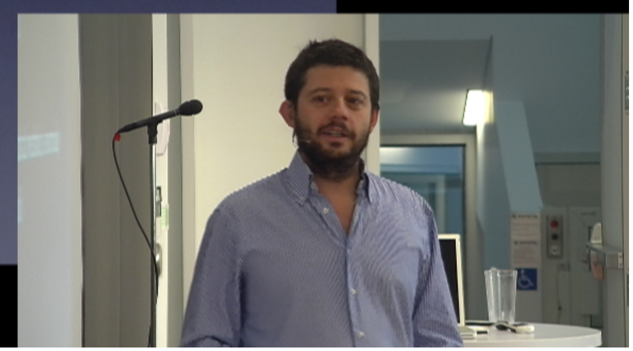
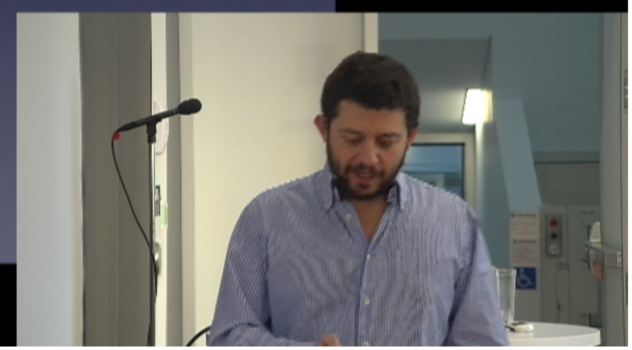
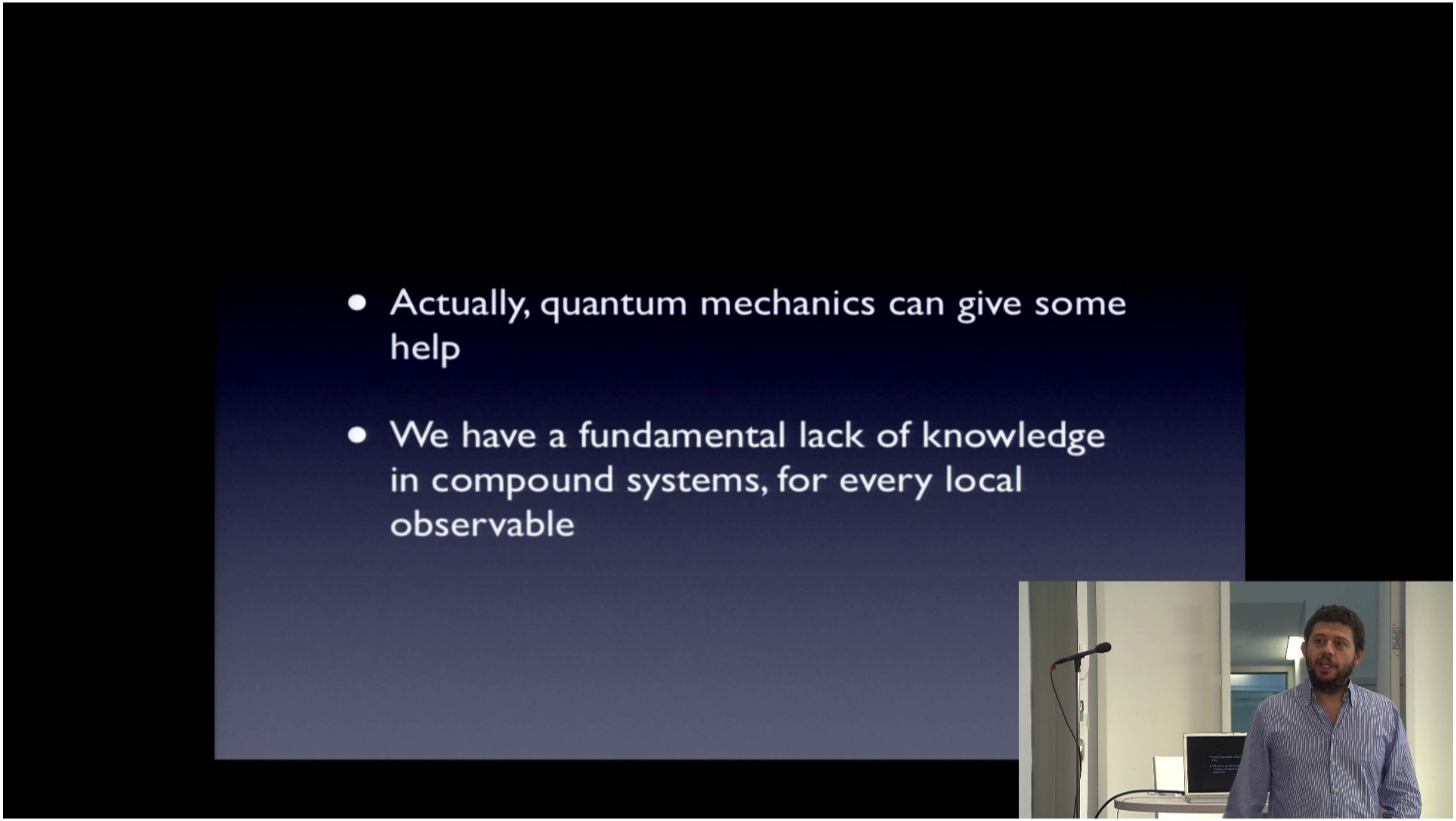


- Shank Keng Ma: "Statistical Mechanics is Established on a Daring Assumption" $S = \log \Gamma$



Thermalization in a Closed Quantum System

- In a closed quantum system evolution is unitary
- The state is always pure and its entropy is thus zero
- How can it thermalize?
- The H theorem is always trouble, even more so in Quantum stat mec!


• Actually, quantum mechanics can give some help
• We have a fundamental lack of knowledge in compound systems, for every local observable

- The state of the first spin is completely mixed, it is a thermal state at infinite temperature

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

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$$\mathcal{E}_R = \frac{\mathbb{I}_R}{d_R}$$

its restriction to S is $\Omega_S = \mathrm{Tr}_E \mathcal{E}_R$

now pick a random state ψ in $\mathcal{H}_R$ and look at it locally

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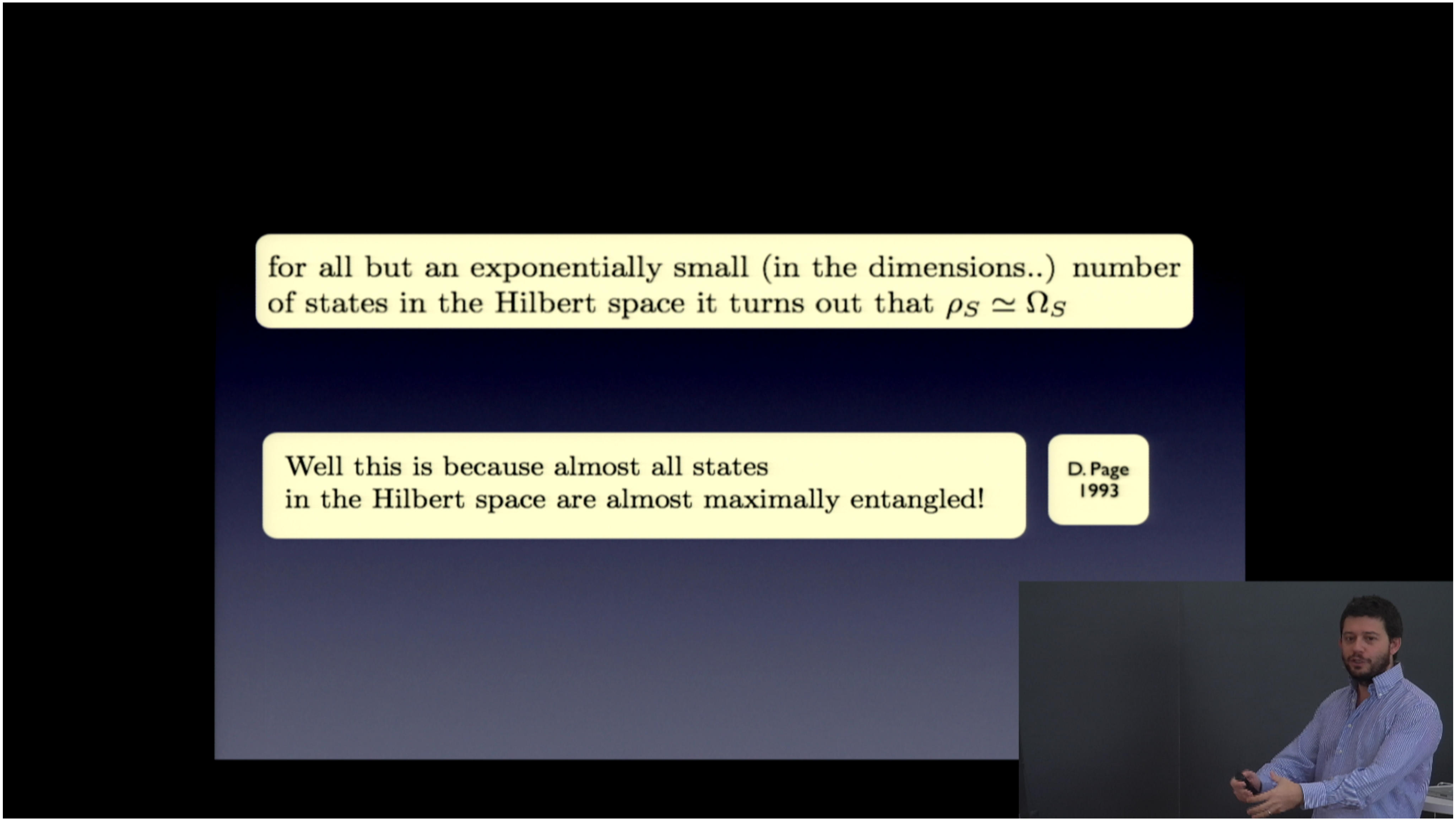

for all but an exponentially small (in the dimensions..) number
of states in the Hilbert space it turns out that $\rho_S \simeq \Omega_S$
Well this is because almost all states
in the Hilbert space are almost maximally entangled!
D. Page
1993

if R is an energetic restriction the canonical state is $\Omega_s \propto \exp\left(-\frac{H_S}{k_B T}\right)$

So because typical states in a large Hilbert space are very entangled, typically subsystems appear completely mixed (or in the thermal state if there are restrictions)

Popescu, Short, Winter, Nature Phys 2, 754 (2006)
Bocchieri, Loinger, Lloyd, Gemmer, Michel, Mahler, Tasaki...

The Problem of Physical States

We have taken random states in $\mathcal{H}_R$ with the Haar measure... but how physical are these states?

The states that can be obtained by evolution with a local Hamiltonian in polynomial time are an exponentially small fraction of the Hilbert space

Poulin, Qarry, Somma, Versraete (2011)

$$H(t) = \sum_X H_X(t)$$

$$U_t \simeq \prod_{j=1}^{n} \prod_X \exp\{-iH_X(j\Delta t)\Delta t\}$$

Typicality of Entanglement in Ensembles of Physical States

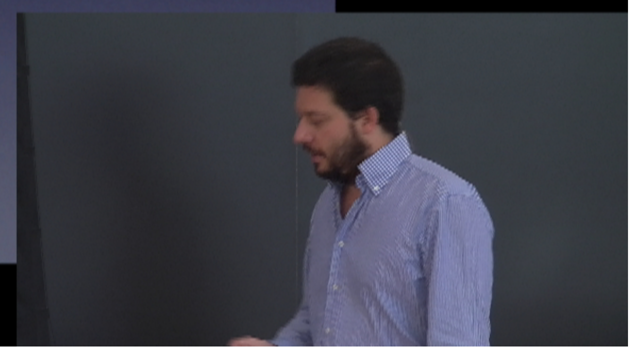
A.H. S, Santra, P. Zanardi, arXiv:1109.4391

$\mathcal{H} = \otimes_{x \in V} \mathcal{H}_x$

local Hilbert spaces of qudits

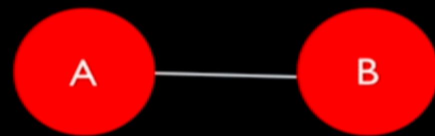
a completely factorized state is $|\Phi\rangle = \otimes_{x \in V} |\phi_x\rangle$

$\omega = |\Phi\rangle\langle\Phi|$



WARM UP EXAMPLE, SINGLE EDGE MODEL

$\mathcal{H}_V = \mathcal{H}_A \otimes \mathcal{H}_B$



here $p(V) = 1$ and the $d\mu$ is the Haar measure over $\mathcal{U}(\mathcal{H}_V)$

$$\overline{P}^U = \int dU \mathrm{Tr}[\rho^{\otimes 2} T_A] = \mathrm{Tr}[\omega^{\otimes 2} \int dU (U^\dagger)^{\otimes 2} T_A U^{\otimes 2}]$$

performing the integration we get

$$\overline{P}^U = 2d/(d^2+1) \equiv 2N_d$$

For large k we find $\bar{P}_k \simeq 2\left[N_d/(1-N_d)\right]^k$

The average 2−Renyi entropy is

$$\bar{S}_2 \geq k \log\left(\frac{1-N_d}{N_d}\right) - \log 2 \simeq k \log d - \log 2$$

The average is also typical in view of the Markov inequality

For $k \to \infty$ the fixed point is the completely mixed state

Conclusions and Perspectives

- Typical physical states have an area law if they are obtained with O(1) evolution
- They show Volume Law if the time scales with the size of the system
- We would like to show area and volume law on 2D and 3D lattices
- How do we implement energy constraints in order to obtain the thermal states at finite temperature?