

Title: Quantum Information Science and Quantum Optics as Tools to Probe the Spacetime Structure

Date: Oct 31, 2011 04:00 PM

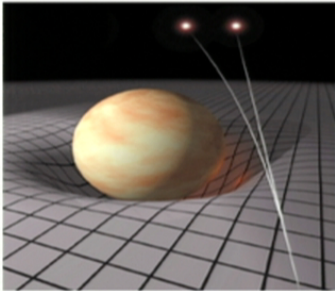
URL: <http://pirsa.org/11100109>

Abstract: Relativistic quantum information theory uses well-known tools coming from quantum information and quantum optics to study quantum effects provoked by gravity and to learn information about the spacetime. One can take advantage of our knowledge about quantum optics and quantum information theory to analyse from a new perspective the effects produced by the gravitational interaction. I will present some results and new ideas in this topic: from experimental proposals for detection of the Unruh and Hawking effects and cosmological implications of gravitationally generated entanglement to quantum simulation of general relativistic settings.

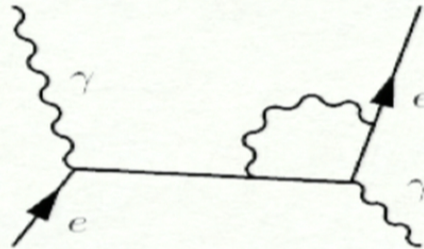
Quantum Information and Quantum Optics as tools to probe the Spacetime Structure

Eduardo Martín Martínez, QUINFOG (CSIC)
31st October, Perimeter Institute

Relativistic Quantum Information



General relativity

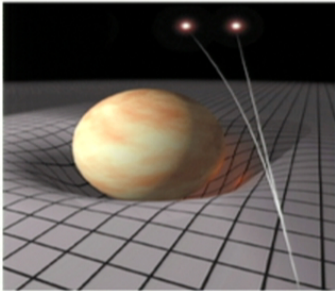


Quantum field theory

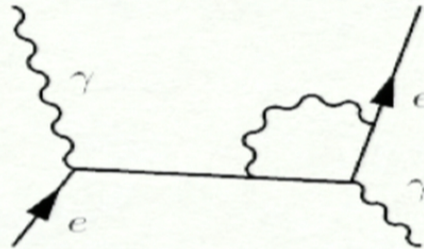


Quantum information

Relativistic Quantum Information



General relativity



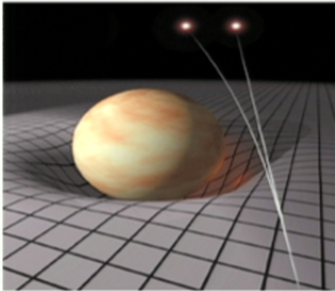
Quantum field theory



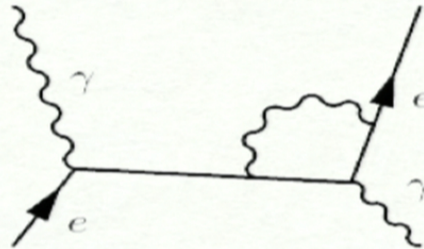
Quantum information

- Understanding quantum correlations behaviour and quantum information tasks in non-inertial settings

Relativistic Quantum Information



General relativity



Quantum field theory



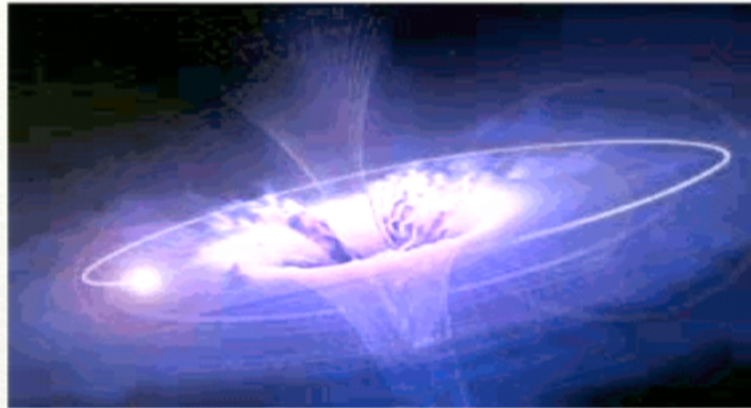
Quantum information

- Understanding quantum correlations behaviour and quantum information tasks in non-inertial settings
- Take advantage of gravity and acceleration as a feature to perform quantum information tasks

Outline

- Entanglement in a stellar collapse
- Entanglement generated by the expansion of the Universe
- Quantum optical tools to detect the Unruh effect (for real)
- Quantum simulation of general relativistic effects

Quantum Entanglement Created in a Gravitational Collapse



Entanglement generated in quantum fields due to the collapse

E. Martín-Martínez, L. J. Garay and J. León. Phys. Rev. D, 82, 064028 (2010)

Entanglement in a Stellar Collapse

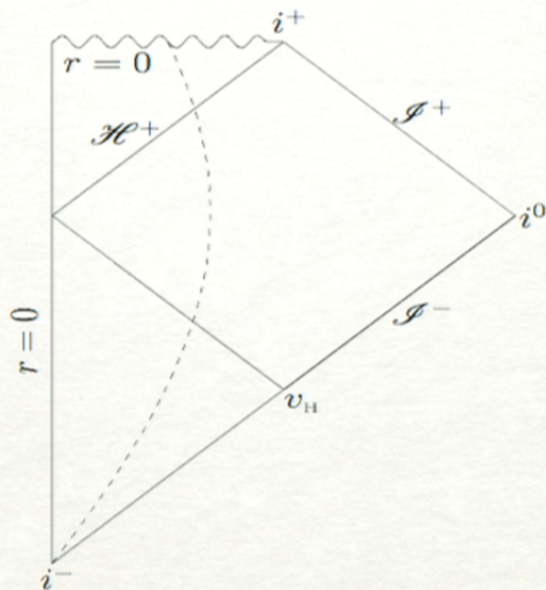
Analyse quantum fields in this background is not easy:

- Non-stationary spacetime: Problems with the field quantization.
- No unique notion of vacuum.
- Problems with the construction of a Fock space.

E. Martín-Martínez, L. J. Garay and J. León. *Phys. Rev. D*, 82, 064028 (2010)

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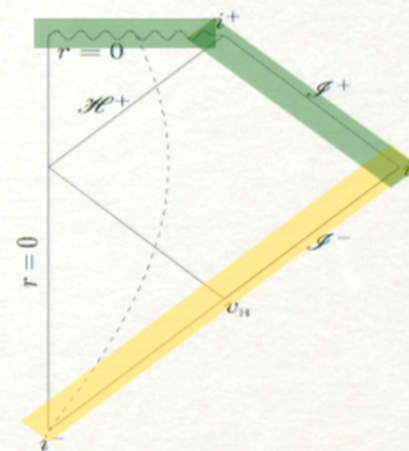
Fortunately, there are asymptotically stationary regions



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Entanglement in a Stellar Collapse

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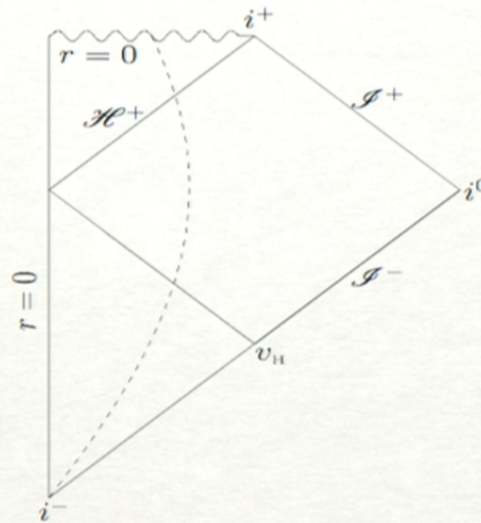


QFTs can be constructed in the asymptotic past and the asymptotic future

E. Martín-Martínez, L. J. Garay and J. León. Phys. Rev. D, 82, 064028 (2010)

Entanglement in a Stellar Collapse

We need to write the annihilation operators of field modes in the asymptotic past in terms of the corresponding creation and annihilation operators defined in terms of modes in the future:



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Entanglement in a Stellar Collapse

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The diagram is a Penrose diagram for a stellar collapse. The vertical axis is labeled $r=0$ and the horizontal axis is labeled v_H . The diagram shows a diamond-shaped region representing the spacetime. The top vertex is labeled i^+ , the bottom vertex is i^- , and the right vertex is i^0 . A vertical line on the left is labeled $r=0$. A dashed curve represents the event horizon, labeled \mathcal{H}^+ . Two diagonal lines are labeled \mathcal{I}^+ and \mathcal{I}^- . The region between \mathcal{I}^+ and \mathcal{I}^- is shaded. The region between \mathcal{I}^+ and \mathcal{H}^+ is also shaded. The region between \mathcal{I}^- and \mathcal{H}^+ is shaded. The region between \mathcal{I}^- and \mathcal{I}^+ is shaded. The region between \mathcal{I}^+ and \mathcal{I}^- is shaded. The region between \mathcal{I}^+ and \mathcal{I}^- is shaded.

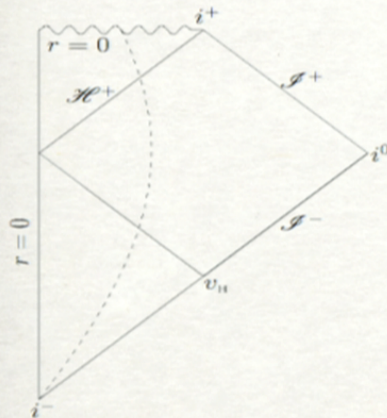
$$u_{\omega}^{\text{out}} \approx \frac{1}{4\pi r \sqrt{\omega}} e^{-i\omega(v_H - 4m \ln \frac{|v_H - v|}{4m})} \theta(v_H - v)$$

$$u_{\omega}^{\text{in}} \sim \frac{1}{4\pi r \sqrt{\omega}} e^{-i\omega v}$$

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$$u_{\omega}^{\text{hor}} \sim \frac{1}{4\pi r \sqrt{\omega}} e^{i\omega(v_{\text{H}} - 4m \ln \frac{|v_{\text{H}} - v|}{4m})} \theta(v - v_{\text{H}})$$

$$a_{\omega'}^{\text{in}} = \int d\omega \left[\alpha_{\omega\omega'}^* (a_{\omega}^{\text{out}} - \tanh r_{\omega} a_{\omega}^{\text{hor}\dagger}) + \alpha_{\omega\omega'} e^{i\varphi} (a_{\omega}^{\text{hor}} - \tanh r_{\omega} a_{\omega}^{\text{out}\dagger}) \right]$$

$$|0\rangle_{\text{in}} = \prod_{\omega} \frac{1}{\cosh r_{\omega}} \sum_{n=0}^{\infty} (\tanh r_{\omega})^n |n_{\omega}\rangle_{\text{hor}} |n_{\omega}\rangle_{\text{out}}$$

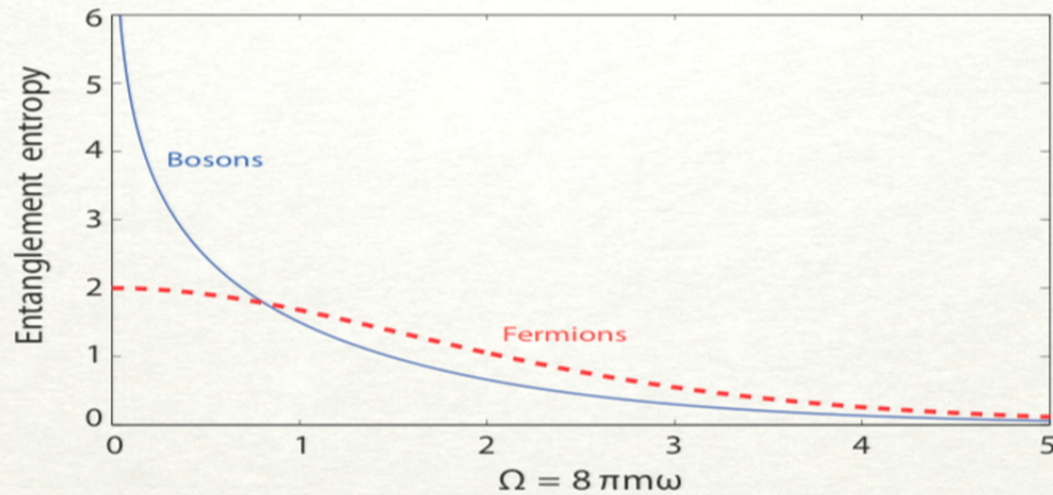
$$|0\rangle_{\text{in}} = \prod_{\omega} \left[(\cos \tilde{r}_{\omega})^2 |00\rangle_{\text{hor}} |00\rangle_{\text{out}} - \frac{\sin 2\tilde{r}_{\omega}}{2} (|01_{\omega}\rangle_{\text{hor}} |1_{\omega}0\rangle_{\text{out}} - |1_{\omega}0\rangle_{\text{hor}} |01_{\omega}\rangle_{\text{out}}) - (\sin \tilde{r}_{\omega})^2 |1_{\omega}1_{\omega}\rangle_{\text{hor}} |1_{\omega}1_{\omega}\rangle_{\text{out}} \right],$$

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Entanglement in a Stellar Collapse

Vacuum evolves to an entangled state of infalling and outgoing modes

$$\begin{aligned} \text{Scalar } S_{E,\omega} &= (\cosh r_\omega)^2 \log_2 (\cosh r_\omega)^2 - (\sinh r_\omega)^2 \log_2 (\sinh r_\omega)^2 \\ \text{Grassman } S_{E,\omega} &= 2 [(\cos \tilde{r}_\omega)^2 \log_2 (\cos \tilde{r}_\omega)^2 + (\sin \tilde{r}_\omega)^2 \log_2 (\sin \tilde{r}_\omega)^2] \end{aligned}$$

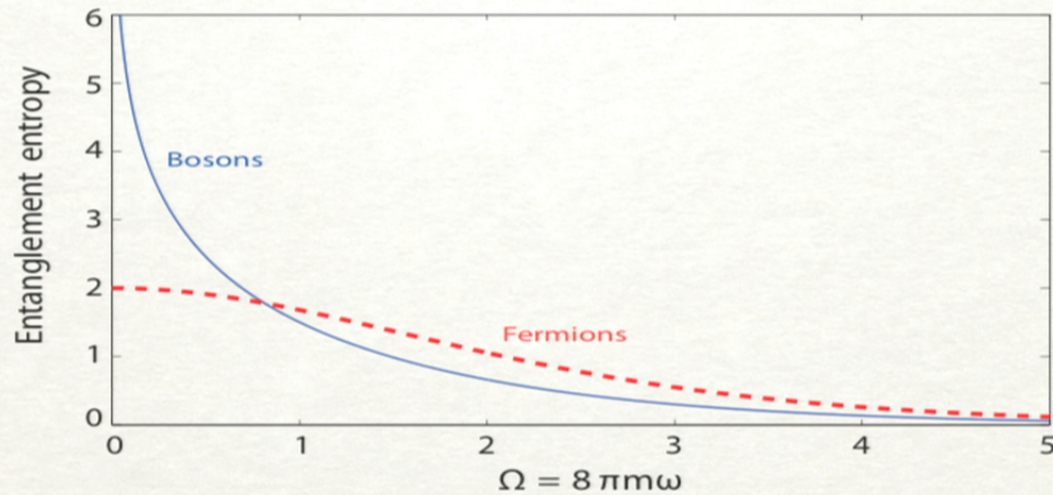


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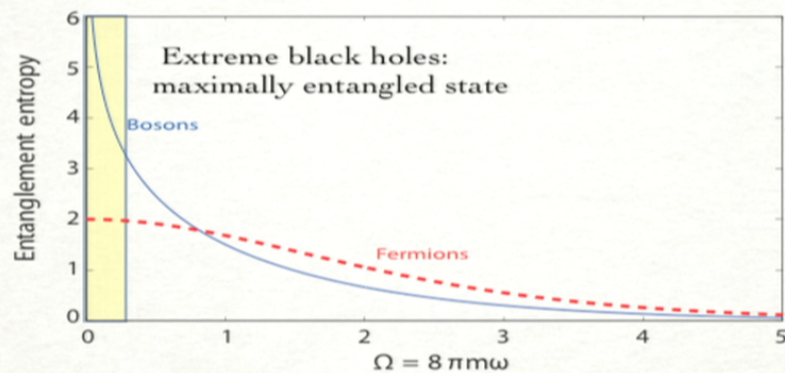
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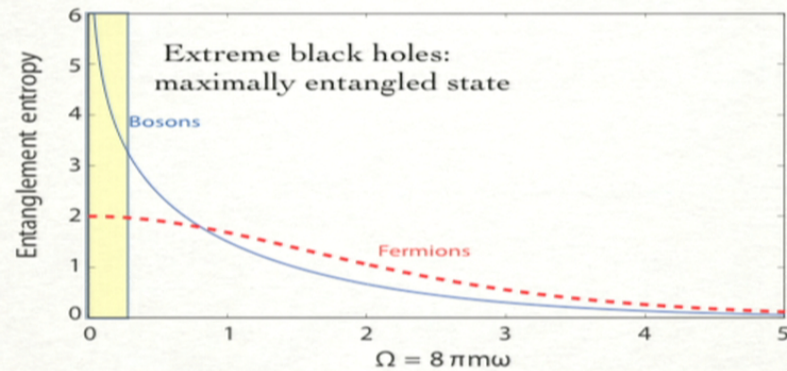


Analysing results I: Sci-Fi

- Micro-black holes, Final state of a black hole

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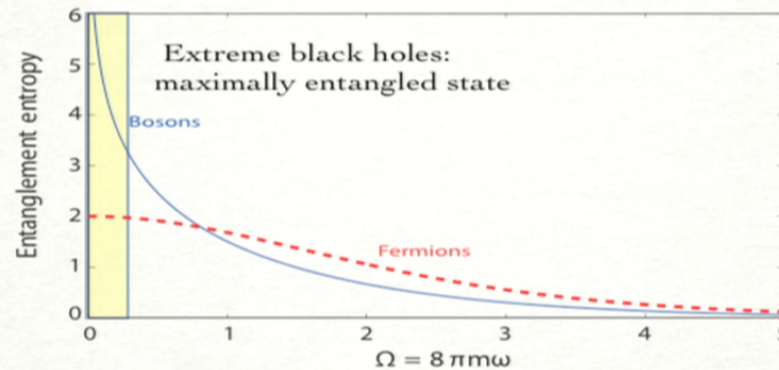


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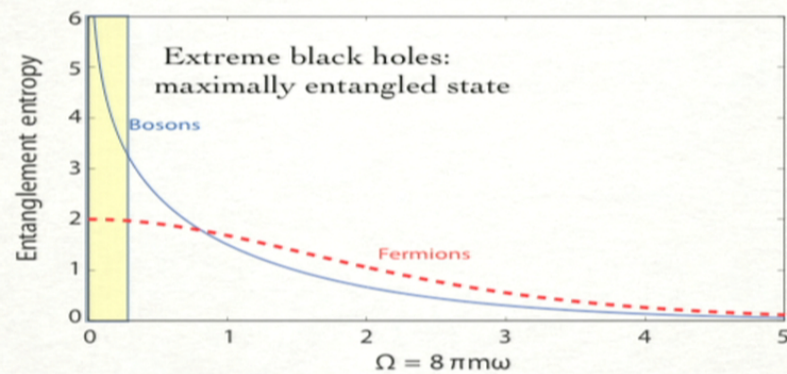


Analysing results I: Sci-Fi

- Micro-black holes, Final state of a black hole
- Hawking radiation maximally correlated with infalling radiation!
- e.g. high fidelity teleportation from the horizon to the asymptotic future

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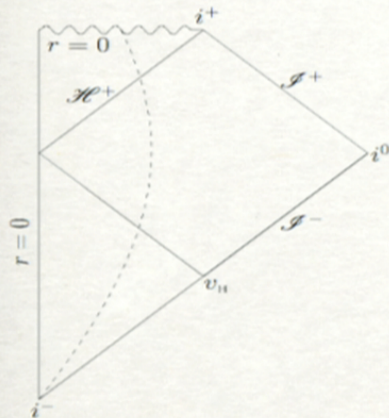


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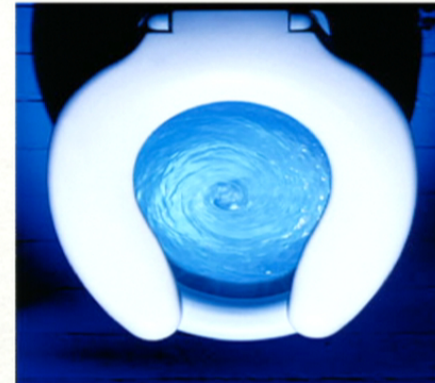
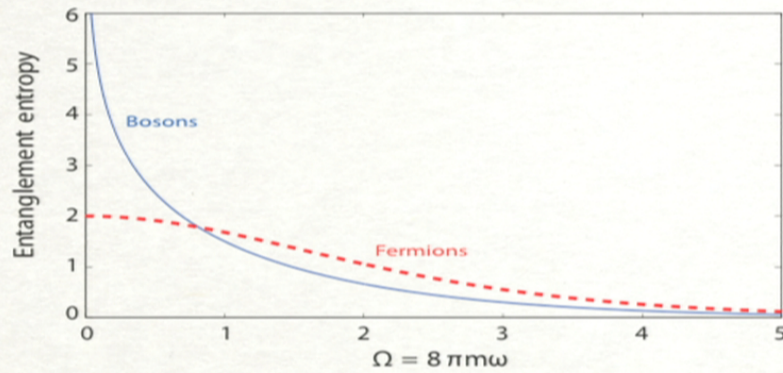
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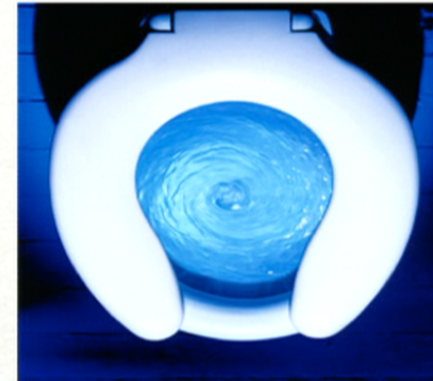
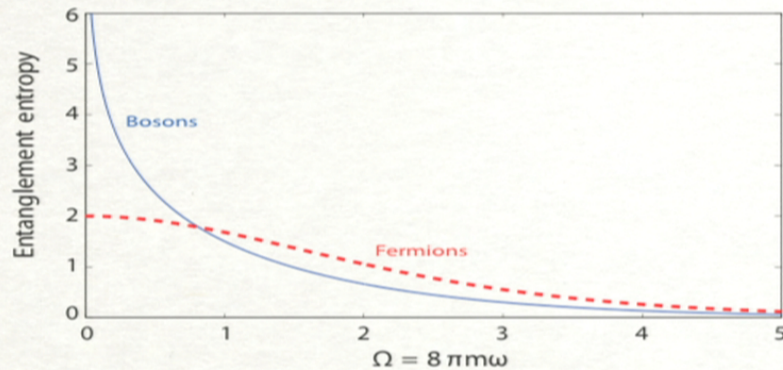


Analysing results II: non-Sci-Fi

- Analog-gravity experiments utility

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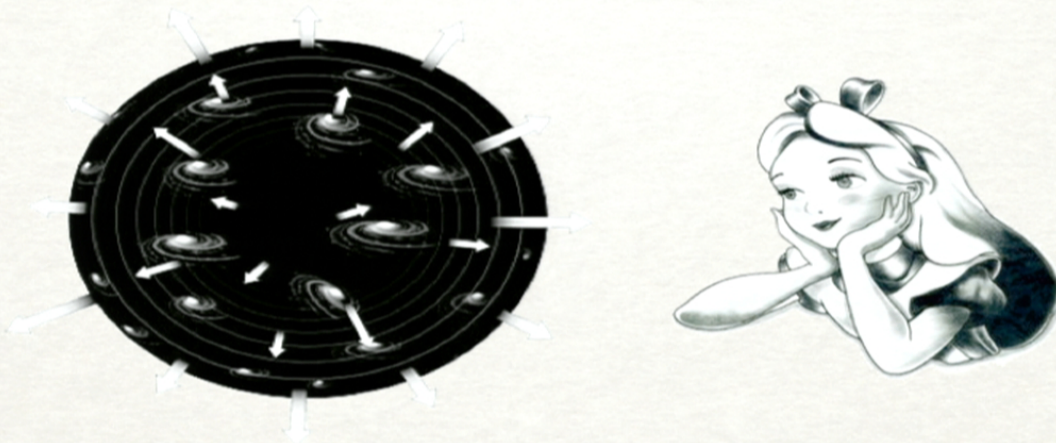


Analysing results II: non-Sci-Fi

- Analog-gravity experiments utility
 - Difference between classical induced thermal emission and Hawking effect
 - Entanglement has no classical analog: Use of entanglement to check the quantum nature of the Hawking radiation generation process
 - Fermions are more reliable for these experiments

E. Martín-Martínez, L. J. Garay and J. León. Phys. Rev. D, 82, 064028 (2010)

The expansion of the Universe as a source of quantum entanglement



I. Fuentes, R.B. Mann, E. Martín-Martínez and S. Moradi, Phys. Rev. D, 82, 045030 (2010)

The Setting

$$\text{FRW spacetime } ds^2 = C(\eta)(-d\eta^2 + dx_i dx^i)$$

Where $C(\eta)$ controls the expansion of the spacetime and η is the conformal time.

Regimes where there are asymptotically stationary regions, e.g.

$$C(+\infty) \rightarrow \text{const.} \quad C(-\infty) \rightarrow \text{const.}$$

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Positive & negative freq. modes can be identified in the far past and future where the spacetime admits the timelike Killing vector ∂_η

We can express the vacuum state in the past as a function of modes in the future

I. Fuentes, R.B. Mann, E. Martín-Martínez and S. Moradi, Phys. Rev. D, 82, 045030 (2010)

Cosmological Entanglement

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$$|0\rangle_{\text{in}} = \prod_k \frac{|0\rangle_{\text{out}} - \xi^{-*}(k)\chi(\bar{k})|1_k 1_{-k}\rangle_{\text{out}}}{\sqrt{1 + |\xi^-(k)\chi(\bar{k})|^2}}$$

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Massive fields get entangled by the expansion of the Universe

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Massive fields get entangled by the expansion of the Universe

Let us analyse as a toy model: the analytically solvable Duncan model

$$C(\eta) = (1 + \epsilon(1 + \tanh \rho\eta))^2$$

Where $\epsilon, \rho \in \mathbb{R}$ controls the total volume and rapidity of the expansion.

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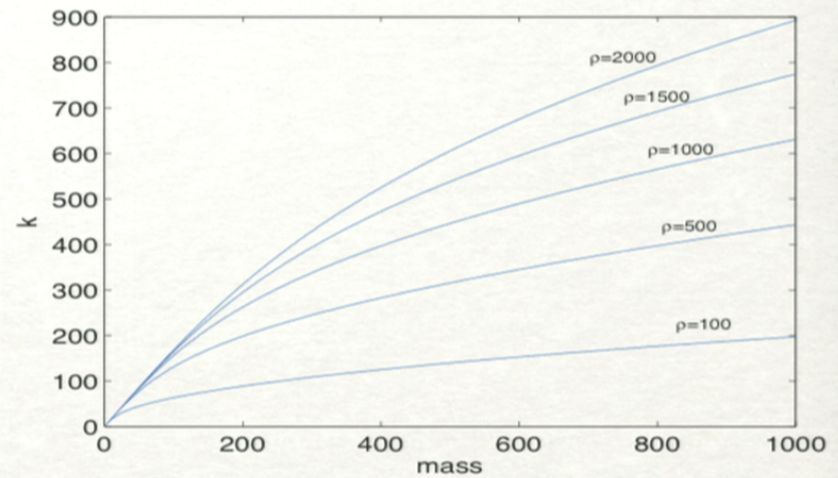
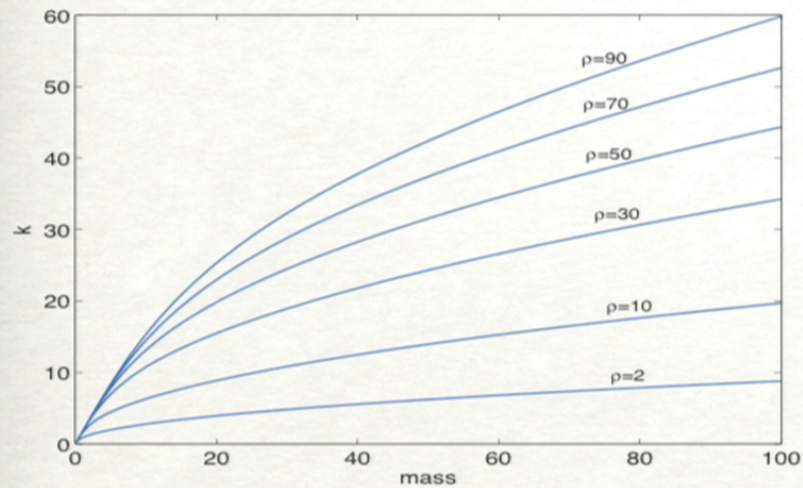
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Optimal Entanglement Curve

Mode of maximal entanglement creation for a given mass

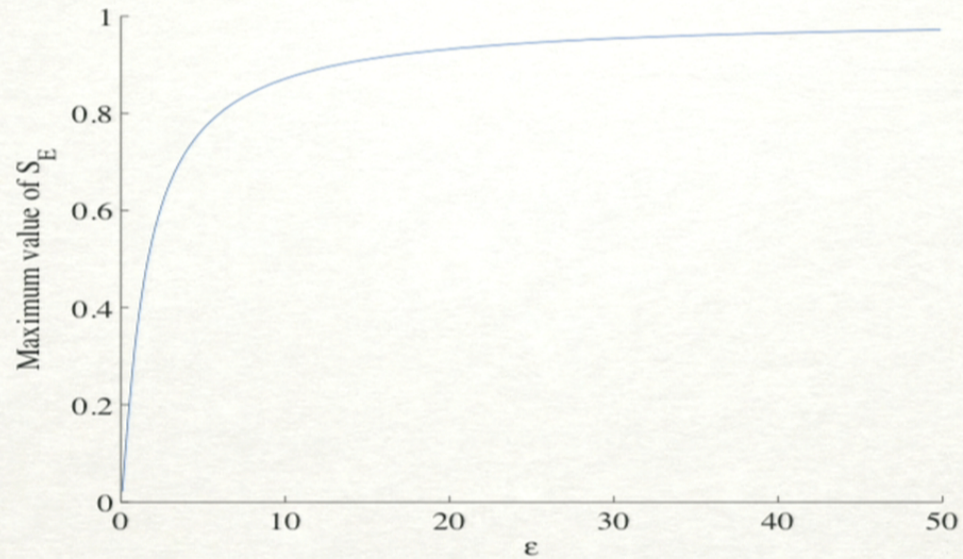


It 'sees' the rapidity but it is almost insensitive to ϵ

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Maximum Achievable Entanglement

Amount of entanglement in the optimal curve $S_E(m, k_{\text{opt}}(m))$



It 'sees' ϵ but it is insensitive to the rapidity

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Entanglement in Expanding Universes

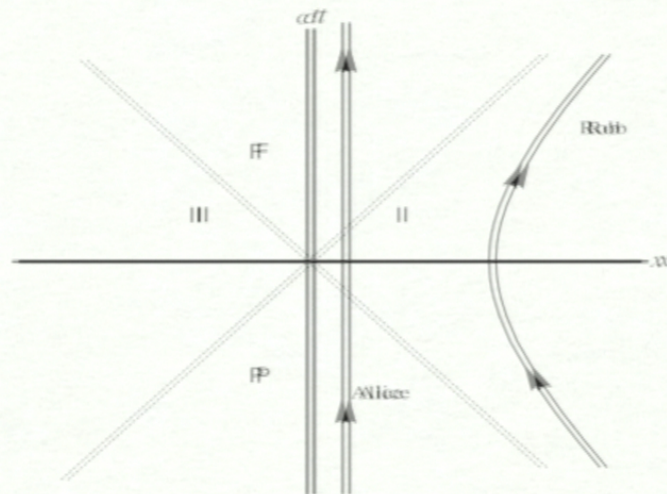
Analysing results

- Estimation of expansion parameters:
 - Maximally entangled mode gives the rapidity of the expansion
 - Amount of entanglement in this mode gives the total volume of expansion
- The expansion of the spacetime generates entanglement.
- Radical differences between fermionic and bosonic fields
- Being fermion is important: Information is better codified in fermionic fields.

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Field Quantization: Observer dependence

- Example: different observers of flat spacetime

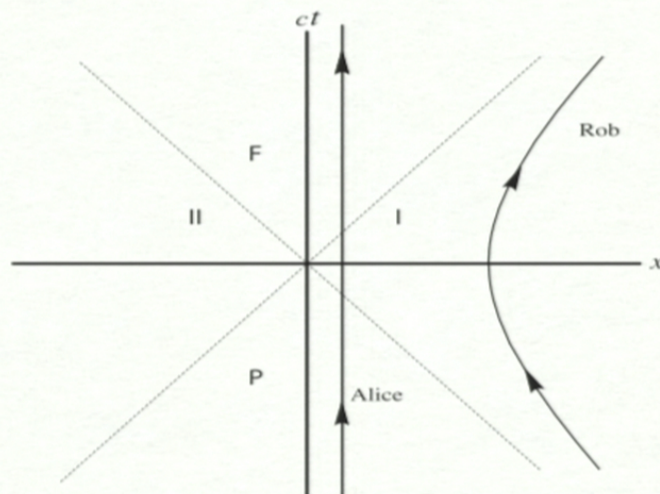


Basis $\{ (x, t) \rightarrow u_{\omega}^{\text{M}} \propto e^{-i\omega t}$

Positive freq. solutions $\nabla^{\mu} \nabla_{\mu} u = 0$

Field Quantization: Observer dependence

- Example: different observers of flat spacetime



$$ct = \xi \sinh\left(\frac{a\tau}{c}\right), \quad x = \xi \cosh\left(\frac{a\tau}{c}\right)$$

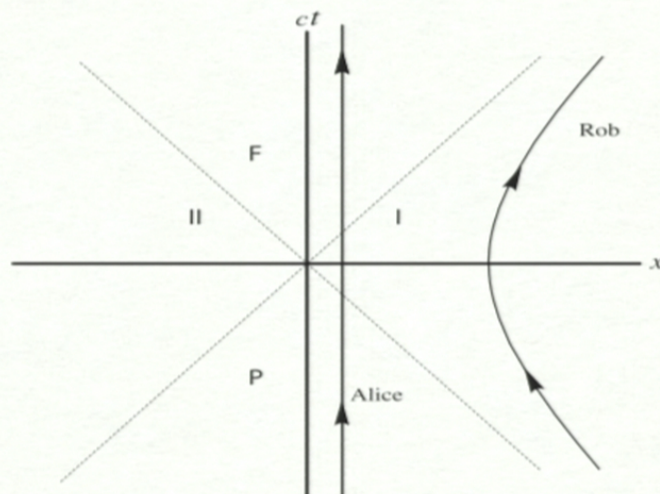
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$$(\xi, \tau) \rightarrow u_{\omega}^I \propto e^{-i\omega\tau}$$

Field Quantization: Observer dependence

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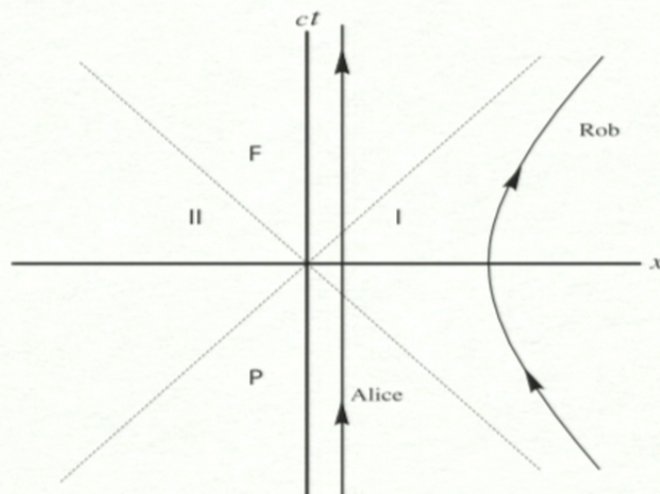
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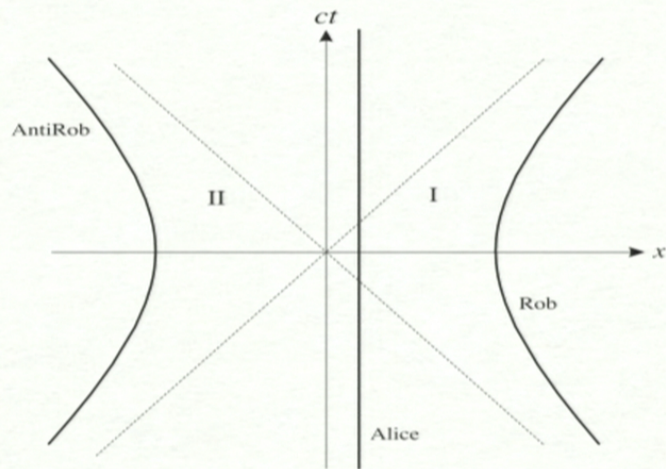
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Positive freq. solutions $\nabla^{\mu} \nabla_{\mu}$



Field Quantization: Observer dependence

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The field ϕ

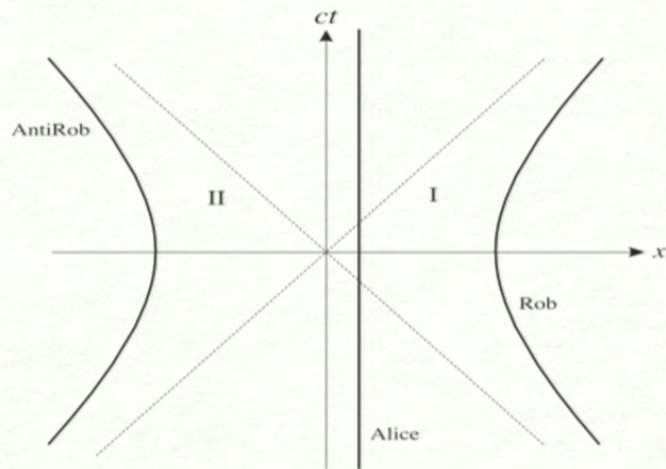
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Positive freq. solutions $\nabla^{\mu} \nabla_{\mu} u = 0$

Field Quantization: Observer dependence

- Example: different observers of flat spacetime



The field can be spanned in either basis

$$\phi = \sum_i \left(a_{\omega_i, M} u_{\omega_i}^M + a_{\omega_i, M}^\dagger u_{\omega_i}^{M*} \right)$$

or equi

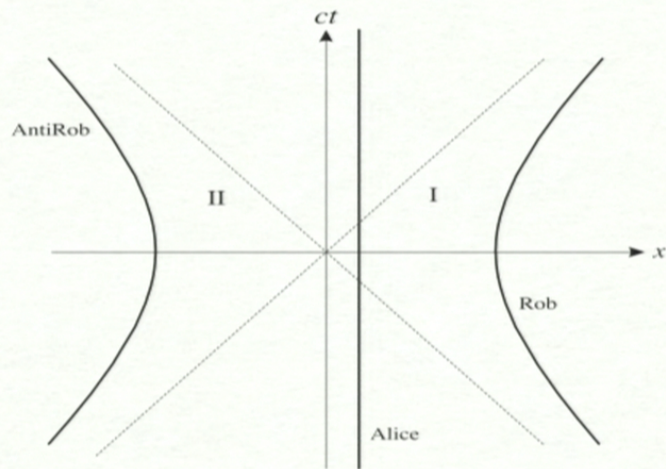
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Positive freq. solutions $\nabla^\mu \nabla_\mu u = 0$

Field Quantization: Observer dependence

- Example: different observers of flat spacetime



The field can be spanned in either basis

$$\phi = \sum_i \left(a_{\omega_i, M} u_{\omega_i}^M + a_{\omega_i, M}^\dagger u_{\omega_i}^{M*} \right)$$

or equivalently

$$\phi = \sum_i \left(a_{\omega_i, I} u_{\omega_i}^I + a_{\omega_i, I}^\dagger u_{\omega_i}^{I*} + a_{\omega_i, II} u_{\omega_i}^{II} + a_{\omega_i, II}^\dagger u_{\omega_i}^{II*} \right)$$

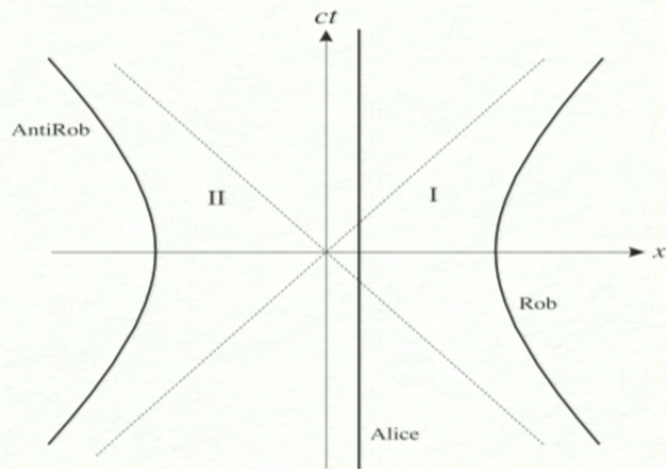
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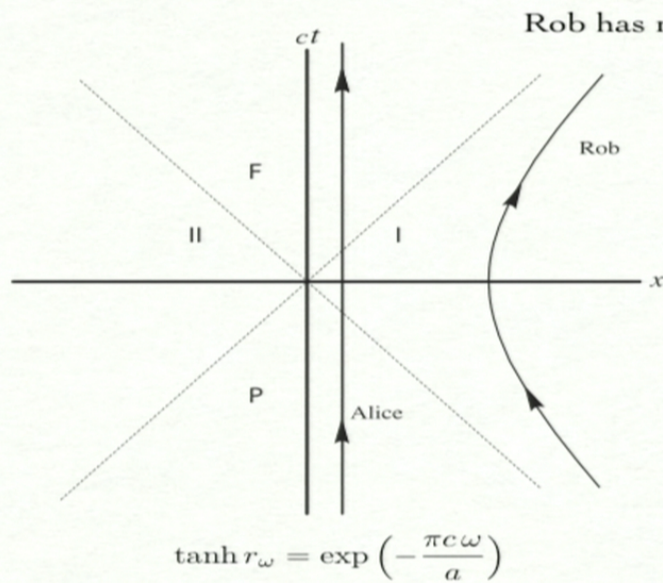
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Positive freq. solutions $\nabla^\mu \nabla_\mu u = 0$

The Importance of the Horizon



Example: Minkowskian vacuum. Rob's perspective

$$|0\rangle_M$$

First: change of Fock basis

$$|0\rangle_M = \bigotimes_{\omega} \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n r_\omega |n\rangle_I |n\rangle_{II}$$

Second: Trace out the disconnected region

$$\rho_{R,\omega} = \text{Tr}_{II} (|0_\omega\rangle\langle 0_\omega|) = \frac{1}{\cosh^2 r_\omega} \sum_n \tanh^{2n} r_\omega |n_\omega\rangle_I \langle n_\omega|_I$$

Result: thermal state

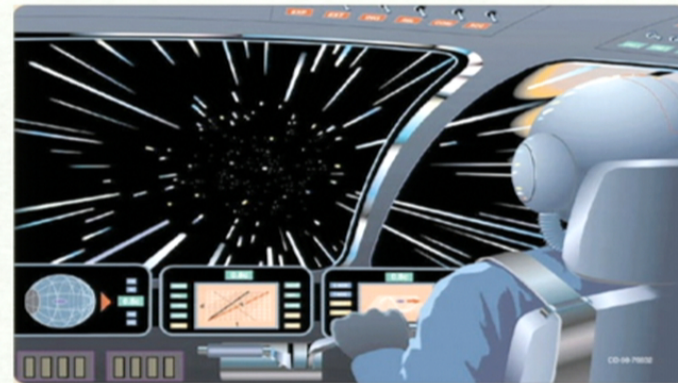
$$\langle N_{\omega,R} \rangle = \frac{1}{e^{2\pi c/\omega a} - 1} \quad T_U = \frac{\hbar a}{2\pi K_B}$$

THE UNRUH EFFECT

Inertial frame

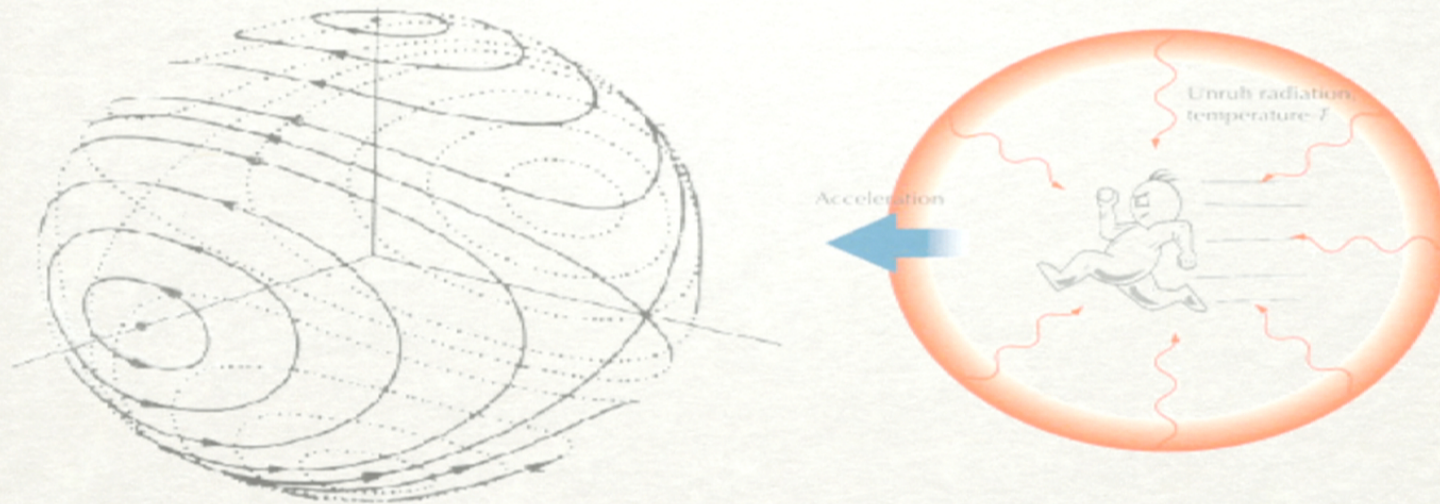


Accelerated frame



- Alice Observes the field vacuum.
- Rob observes a thermal bath of temperature $T_U \propto a$

Berry's Phase and the Detection of the Unruh Effect



E. Martín-Martínez, I. Fuentes and R.B. Mann. Physical Review Letters 107, 131301 (2011)

Geometrical Phases and adiabatic evolution

$$H(t) = H(R_1(t), \dots, R_k(t))$$

$R_1(t), \dots, R_k(t)$ Cyclicly and adiabatically varying parameters

$$|\psi\rangle \rightarrow e^{i\gamma} e^{i\phi} |\psi\rangle$$

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$$i\gamma = \oint_R \mathbf{A} \cdot d\mathbf{R} \quad \mathbf{A} = \begin{pmatrix} \langle \psi(t) | \partial_{R_1} | \psi(t) \rangle \\ \langle \psi(t) | \partial_{R_2} | \psi(t) \rangle \\ \vdots \\ \langle \psi(t) | \partial_{R_k} | \psi(t) \rangle \end{pmatrix}$$

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The Hamiltonian (Inertial)

Consider a single mode Unruh-Dewitt Hamiltonian

$$H_T = \Omega_a a^\dagger a + \Omega_b b^\dagger b + \lambda(b + b^\dagger)(a^\dagger e^{i\varphi} + a e^{-i\varphi})$$

For an inertial observer $\varphi = |\Omega_a|x/c - \Omega_a t$ Where (x, t) are Minkowskian coordinates

The movement of the detector in spacetime generates a change in the interaction Hamiltonian between the field and the atom.

E. Martín-Martínez, I. Fuentes and R.B. Mann. Physical Review Letters 107, 131301 (2011)

The Hamiltonian (Accelerated)

The Hamiltonian for the accelerated detector from the accelerated observer frame

$$\hat{H}_T = \Omega_a \hat{a}^\dagger \hat{a} + \Omega_b \hat{b}^\dagger \hat{b} + \lambda(\hat{b} + \hat{b}^\dagger)(\hat{a}^\dagger e^{i\varphi} + \hat{a} e^{-i\varphi})$$

On its own right, for an accelerated observer $\varphi = |\Omega_a|\xi - \Omega_a\tau$

Where (τ, ξ) are Rindler coordinates and \hat{a}, \hat{a}^\dagger are field operators in the Rindler basis

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Starting The Experiment

Inertial scenario

We consider the field and the detector initially in the bare ground state $|0_f 0_d\rangle$

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From the accelerated perspective, the initial state is $\rho_f \otimes |0_d\rangle\langle 0_d|$

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The interaction is suddenly switched on: small (but not negligible) excitation probability



Projective Measurement: the detector must be in the ground state

E. Martín-Martínez, I. Fuentes and R.B. Mann. Physical Review Letters 107, 131301 (2011)

Adiabatic Evolution

We require that if the detector starts in the ground state at time t_0
it evolves to the ground state at time t

$$|0_d\rangle \longrightarrow e^{i\gamma} e^{i\phi} |0_d\rangle$$

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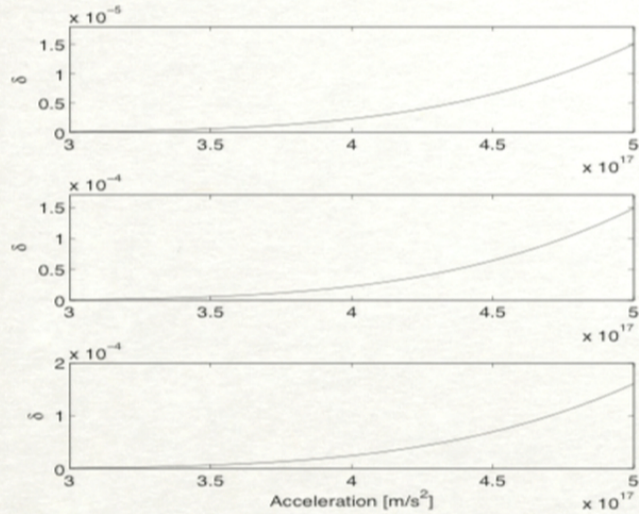
With realistic coupling regimes and the accelerations required this holds extremely well

$$P_{\text{excitation}} < 10^{-10}$$

We can use Berry's formalism to compute the phase acquired after a cycle of adiabatic evolution

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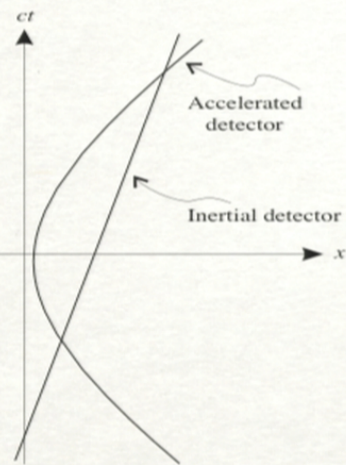
Phase Difference



- $\Omega_a \simeq 2.0$ GHz $\Omega_b \simeq 2.0$ GHz $\lambda \simeq 34$ Hz.
- $\Omega_a \simeq 2.0$ GHz $\Omega_b \simeq 2.0$ GHz $\lambda \simeq 0.10$ KHz.
- $\Omega_a \simeq 2.0$ GHz $\Omega_b \simeq 2.0$ GHz $\lambda \simeq 0.25$ KHz.

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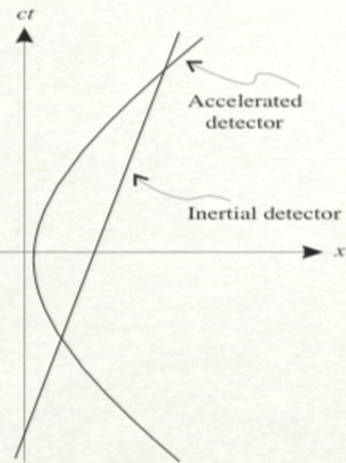
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Example: Atomic interferometry

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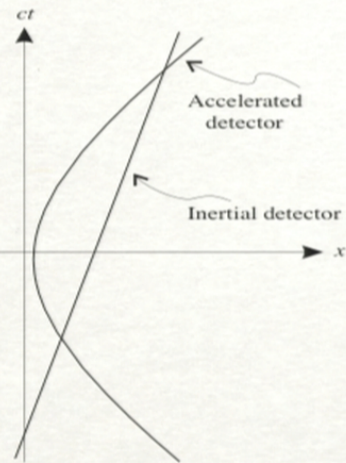


Example: Atomic interferometry

- Sustaining accelerations of $10^{16}g$ for times of nanoseconds
- Previous best result: $10^{25}g$ Phys. Rev. Lett. 83, 256 (1999)

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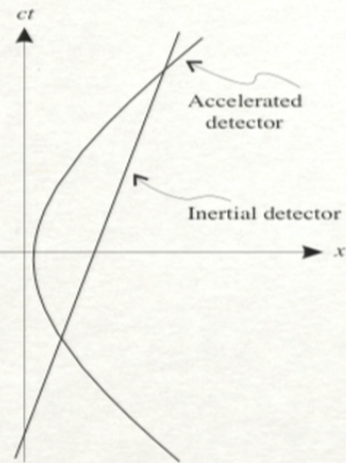


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- The difference of phase accumulates
 - We can wait! (microseconds for π dephase)
 - Independent of the acceleration sign.

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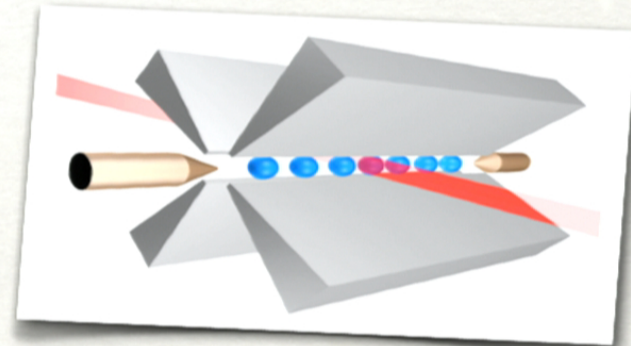
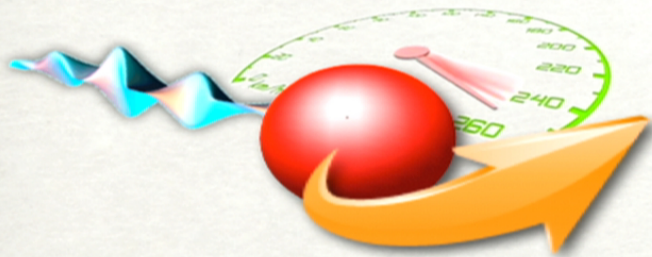


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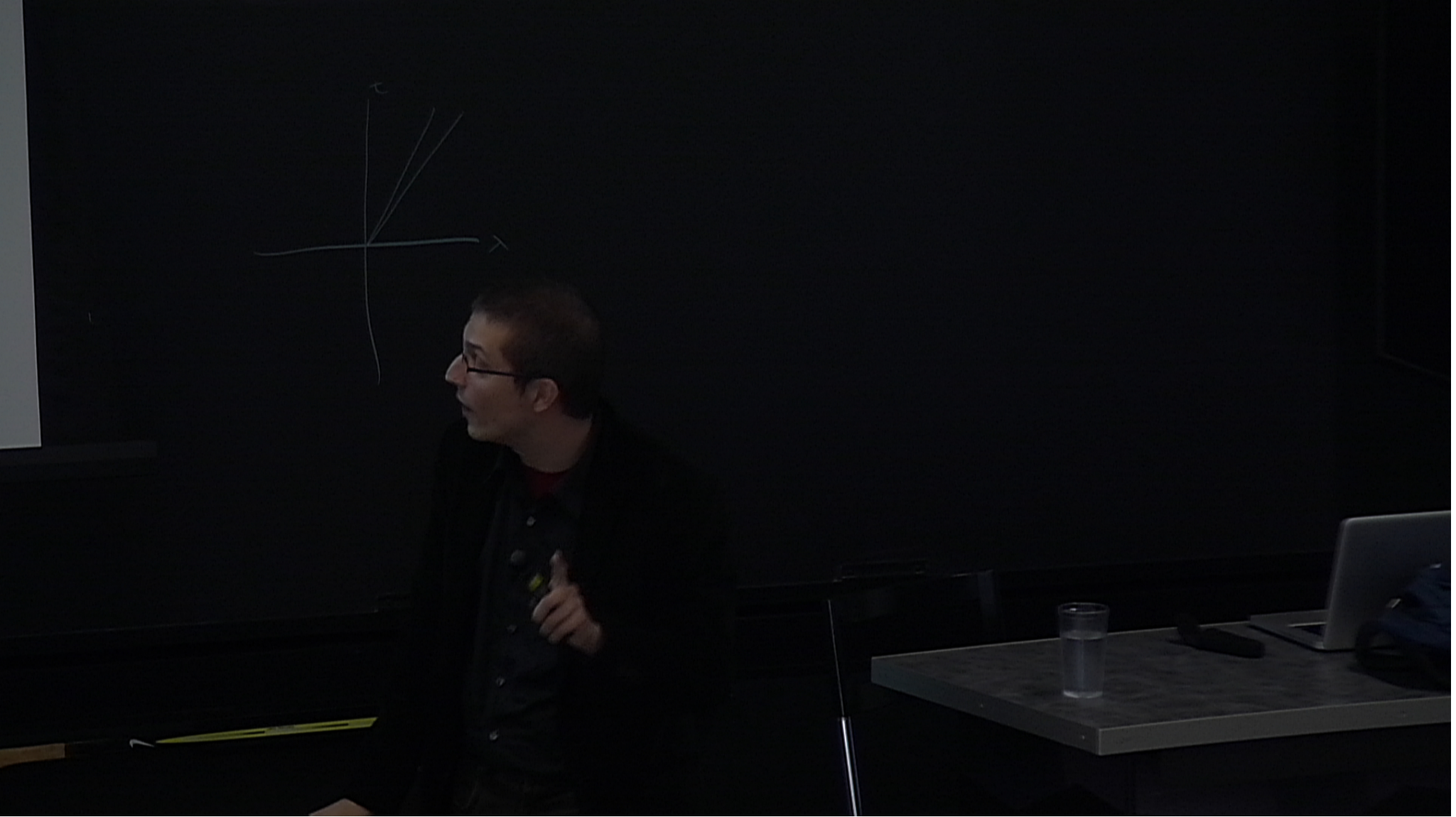
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Simulating accelerated atoms in quantum optical setups



M. del Rey, D. Porras and E. Martín-Martínez, ArXiv:1109.0209



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The movement of the detector in spacetime generates a change in the interaction Hamiltonian between the field and the atom.

The change is cyclic: After $\Delta t \sim \Omega_a^{-1}$ the parameter φ completes a 2π cycle.
Completing a closed trajectory in the parameter space

E. Martín-Martínez, I. Fuentes and R.B. Mann. Physical Review Letters 107, 131301 (2011)

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