Title: Is the renormalization Group Really that Ugly?

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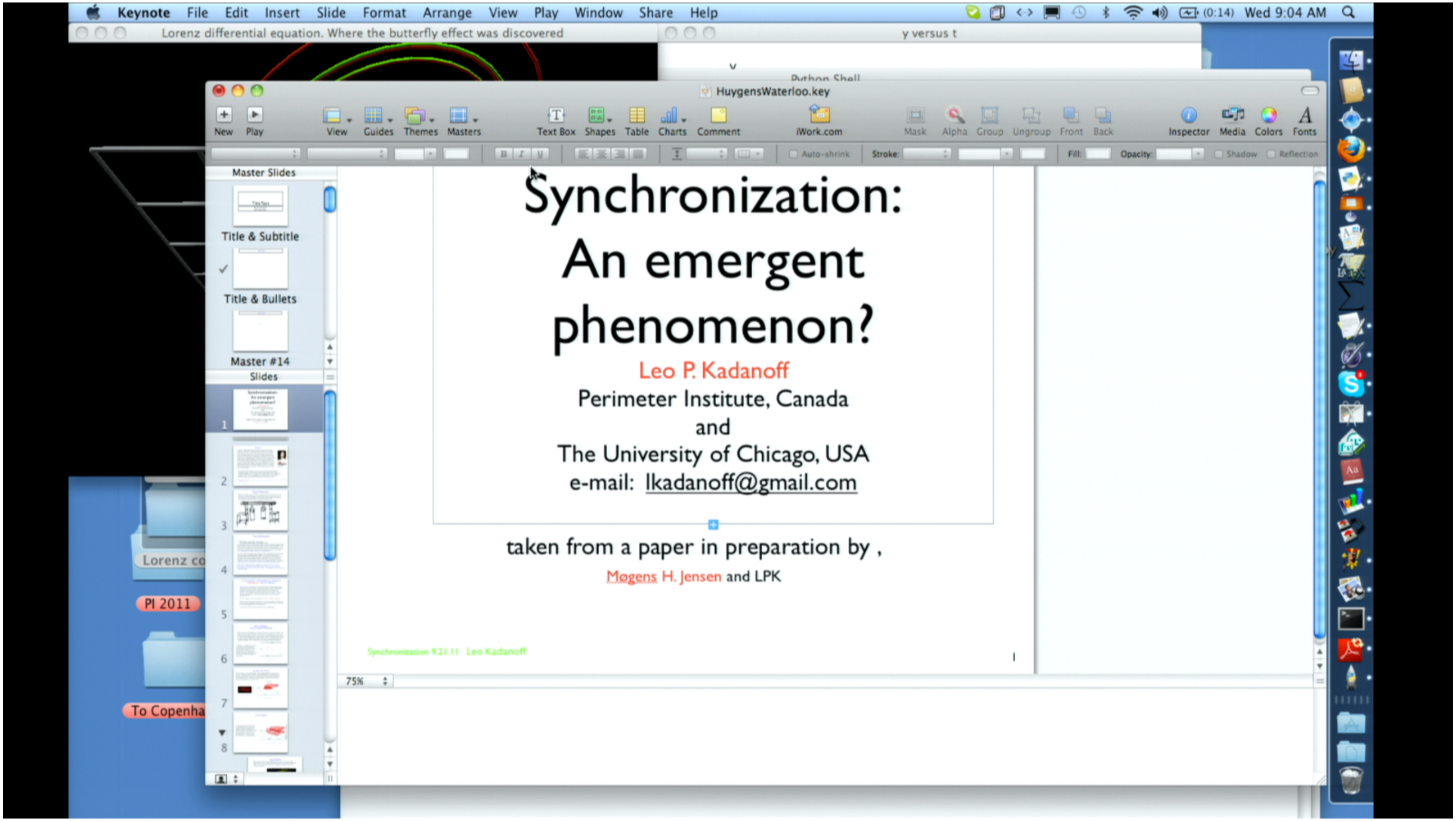
Abstract: In 1665, the clockmaker Christiaan Huygens noticed that two pendulum clocks hanging on a wall tend to synchronize the motion of their pendulums. A similar scenario occurs with two metronomes placed on a piano: they interact through vibrations in the wood and will eventually coordinate their motion. These effects are stable against small perturbations. Such stability is not predicted by either Hamiltonian mechanics or by few-body quantum theory. Nonetheless they can be seen as occurring within a simple model introduced by Kolmogorov. Surprisingly, this model leads to a very complex phase diagram. In turn, the complexities of this phase diagram have been observed within experimental observations of fluid flow, solid state devices, and non-linear electrical circuits. It is reflective of the structure of number theory and of the relation between rational and irrational numbers. Of course, the synchronization arises from friction, an effect often neglected in fundamental theories. Should we then regard synchronization, and its deeply mathematical explanation, as an example of an emergent phenomenon? What does emergence mean? Is is just something that surprises us? How are emergent phenomena connected with the fundamentals of our physical theories?


Synchronization:
An emergent
phenomenon?
Leo P. Kadanoff
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taken from a paper in preparation by ,
Møgens H. Jensen and LPK
HuygensWaterloo.key
Lorenz differential equation. Where the butterfly effect was discovered
y versus t

abstract

In 1665, the clockmaker Christiaan Huygens noticed that two clocks hanging on a wall tend to synchronize the motion of their pendulums. A similar scenario occurs with two metronomes placed on a piano: they interact through vibrations in the wood and will eventually coordinate their motion. These effects are stable against small perturbations. Such stability is not predicted by either Hamiltonian mechanics or by few-body quantum theory. Nonetheless they can be seen as occurring within a simple model introduced by Kolmogorov. Surprisingly, this model leads to a very complex phase diagram. In turn, the complexities of this phase diagram have been observed within experimental observations of fluid flow, solid state devices, and non-linear electrical circuits. It is reflective of the structure of number theory and of the relation between rational and irrational numbers.

Christiaan Huygens by Bernard Vaillant, Museum Hofwijck, Voorburg

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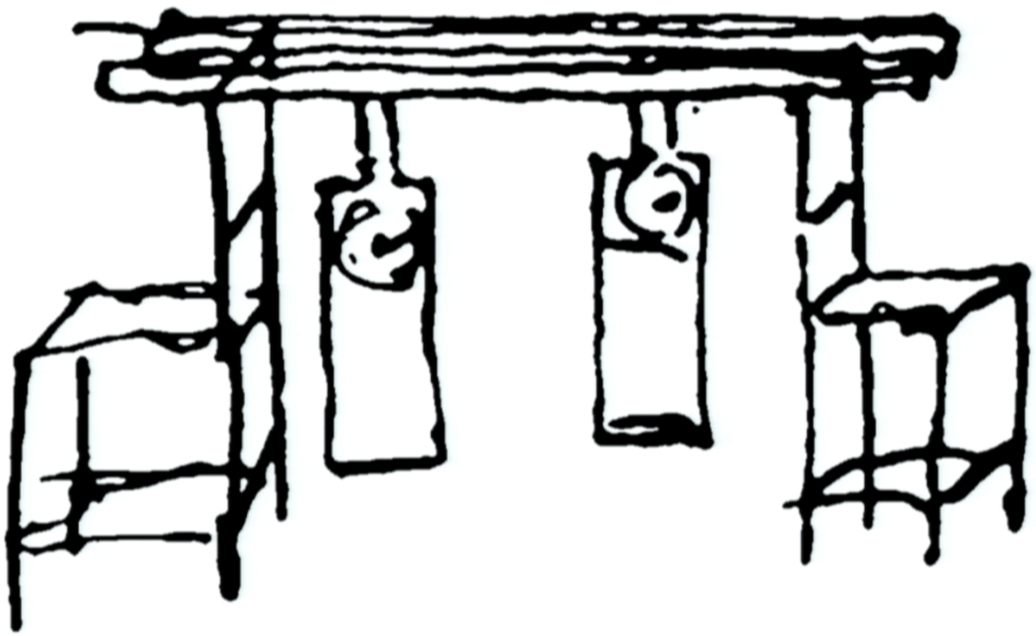
Huygens wrote the first book on probability theory, De ratiociniis in ludo aleae ("On Reasoning in Games of Chance"), which he had published in 1657.

Huygen's Experiment

After his surprising observation of the synchronization between the motions of the pendulums of two clocks hanging on his wall, Huygen's set an experiment to get a reproducible and reportable effect. His experiment is depicted here. The observed synchronization was duly reported to the Royal Society in London.



Surprising Emergence

The synchronization is surprising for several reasons:

The interaction between clocks or metronomes is weak:

To our surprise we see that repeated weak interactions may have large effects, especially when the interactions start out almost synchronous with one another. This special behavior is well-known in classical mechanics, and is described as the effect of a "secular perturbation". This kind of effect is not built in any obvious way into the structure of classical mechanics, but only arises after some careful analysis. Do the words "emergent phenomenon" then describe surprising things that are only exposed after some careful analysis?

However, in addition, the stability of the locking contradicts results built into the structure of few-body classical or quantum mechanics. Stability is not a generic result of classical mechanics. With care, two pendulums coupled by a spring can be synchronized, but small perturbations will produce oscillations about the synchronized state. That is not what Huygens saw. He saw a stable locking, a result of friction, that is impossible in the simple Hamiltonian mechanics of few bodies. Do the words "emergent phenomenon" then describe surprising things that contradict the simple form of ones theoretical understanding?

Both of these readings are possible. I suggest a third reading: The words "emergent phenomenon" then describe surprising things that turn out to be far richer and more interesting than we would have expected at first sight. I further suggest that the physical world is full of such things.

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Friction and thus synchronization are examples of P.W. Anderson's "More IS Different"

Physics statement: In comparison to many body theory, few body quantum theory and few body classical mechanics are pretty dull subjects. Friction, synchronization, entropy, phase transitions, chaos, black holes, temperature, pressure... are all examples of things that happen in large *N* systems but not in small *N*. Math statement: All the above are effects that results from limit processes, mostly the limit as *N* goes to infinity. In theoretical analysis, they are best seen after the passage to the limit.

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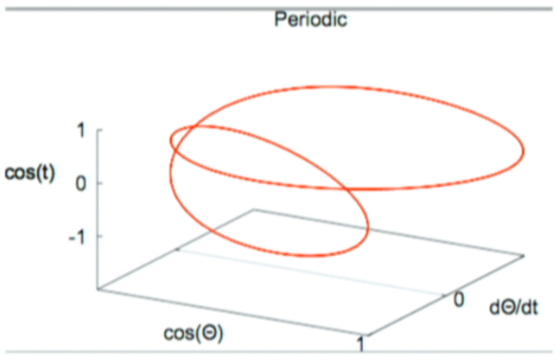
Back to Huygens: Synchronization to Rationals

Each clock has its own eigenfrequency. We may call the two frequencies Ω_1 and Ω_2. The tendency to lock was first observed to arise when the two frequencies are close to one another. They then "pull" each other and arrive at a common frequency, ω, close to Ω_1 and Ω_2. However, this pulling happens in other situations: when Ω_1 is approximately twice Ω_2 or indeed whenever the two eigenfrequencies are in a rational relation with one another, in symbols: $P\,\Omega_1$ is close to $Q\,\Omega_2$, with P and Q being small integers.

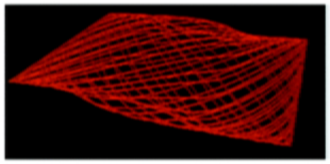
I show this sort of behavior in a picture of the motion of a forced damped pendulum, with forcing close to twice the pendulum's natural period. The pendulum cycles periodically, with a period close to twice the period of the forcing and produces the trajectory shown here.



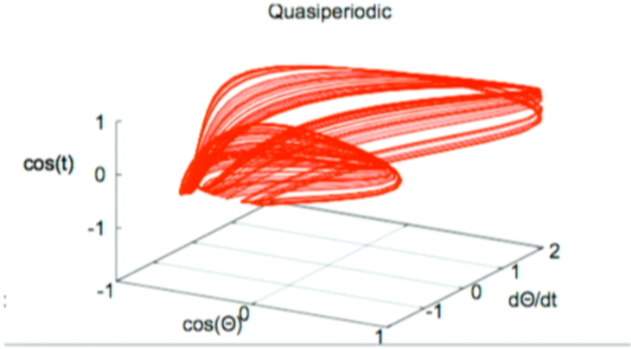
Simulation by Mogens Jensen

Quasiperiodic Motion

That is however, not the only possibility. If the coupling between the two periodic motions is sufficiently weak and if Ω_1/Ω_2 is sufficiently far from any simple ratio of rationals, the system may persist in showing two incommensurate frequencies and a frequency ratio, ω that is an irrational number. In that case, a plot of the trajectory will show motion around a torus.



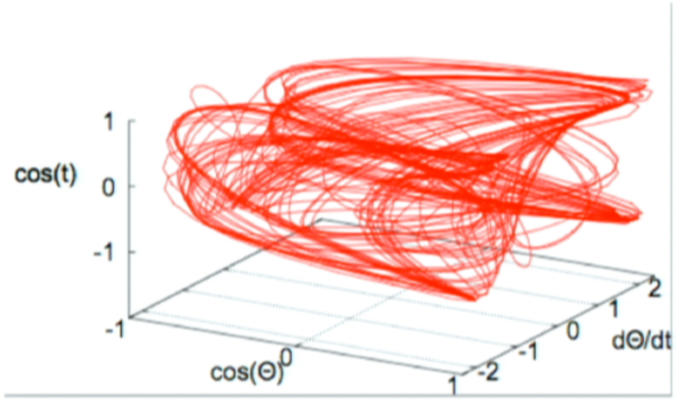
Forced pendulum Simulation by LPK



Simulation by Mogens Jensen

Chaotic Motion

A third possibility arises when the coupling between the two kinds of motion is really strong. Then the system may enter a situation in which the motion never comes close to repeating itself. It does one thing, then another, in a complex pattern.

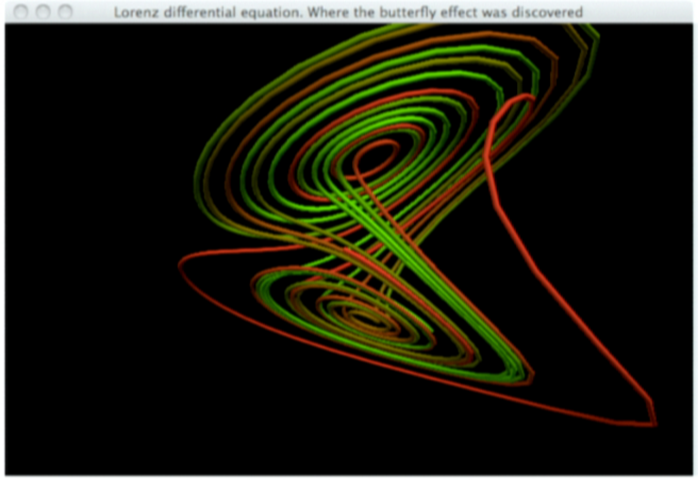


Chaotic Forced pendulum Simulation by Mogens Jensen

Butterfly effect

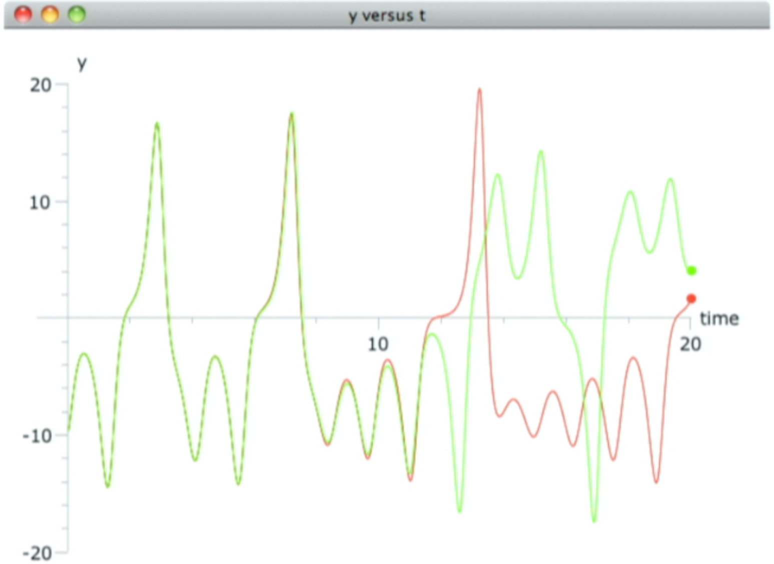
Edward Lorenz, "Deterministic Nonperiodic Flow". Journal of the Atmospheric Sciences 20 (2): 130–141 (1963). One of the most important scientific papers in the 1960s. Prior to this people believed that the behavior of physical systems was essentially predictable.

Two orbits:
after Ed. Lorenz



Two chaotic orbits in Lorenz model simulation LPK

Lorenz created a three-equation model of weather. I show two trajectories from that model.



Two trajectories. Here I show *y* versus time. They start from initial data different by 0.1 %.
The orbits, respectively plotted in red and green, diverge exponentially, and after a time are entirely different. The exponential divergence of orbits is called the butterfly effect, and is rhetorically described as "the flapping of a butterfly in Brazil can be expected to modify weather in Texas"

Predictability

Synchrony and quasiperiodicity can produce rather predictable forms of motion. In the synchronized cases, a small perturbation will generically give a small effect. In the quasiperiodic case, a small perturbation can give rise to a change that grows no more than algebraically in time. In contrast, the chaotic situation in one of exponential separation of orbits. In this situation, we find that the difficulty of making predictions grows exponential with time.

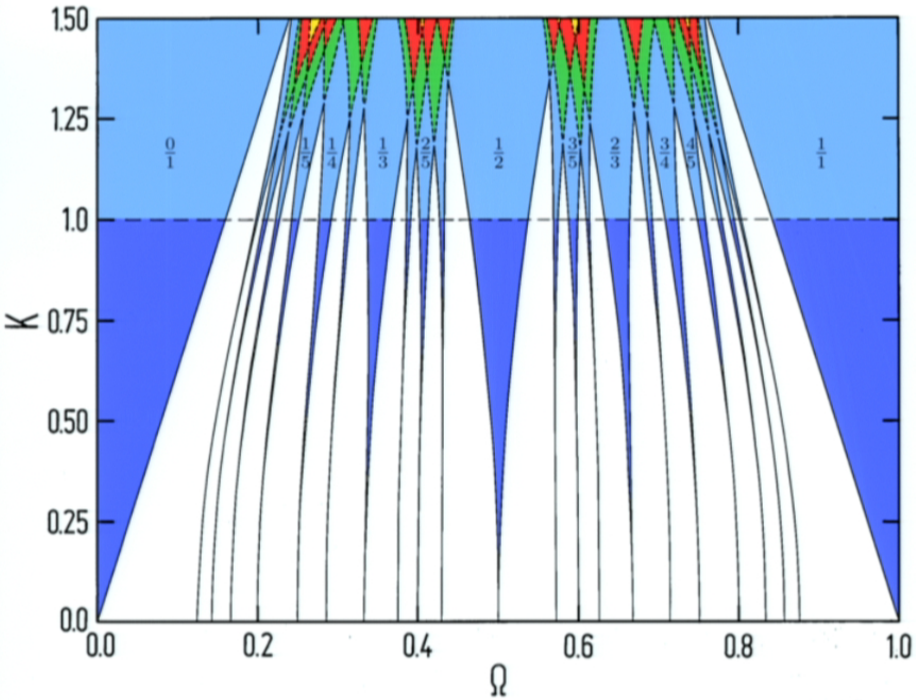
These possibilities have proven to be rather important in our qualitative view of the physical world. Any belief that important events are either entirely predictable (Karl Marx and Leo Tolstoy) or completely unpredictable (the traditional conservative view of Edmund Burke or Donald Rumsfeld) has been eroded away. We thus are left with both options as possibilities, depending upon the situation.

A model

The relation among the different kinds of behaviors was put into a framework by the circle model of A.N. Kolmogorov:

$$x_{j+1} = x_j + \Omega + [K/(2\pi)] \sin(2\pi x_j)$$

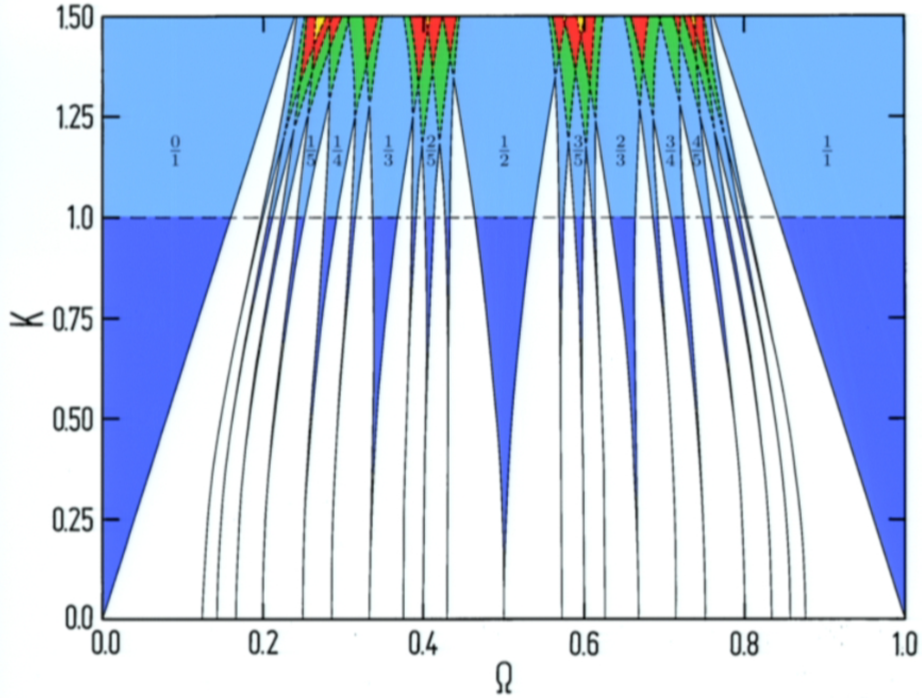
The two frequencies are Ω and unity, the natural inverse period of the sine function. This model was extensively analyzed by V. I. Arnold. The phase diagram is shown on the next slide



blue= locked regions
red= chaos
white=mixed quasiperiodic and locked

numbers are P/Q

Below K=1, ω values increase as Ω increases. Above K=1, this simplicity disappears.



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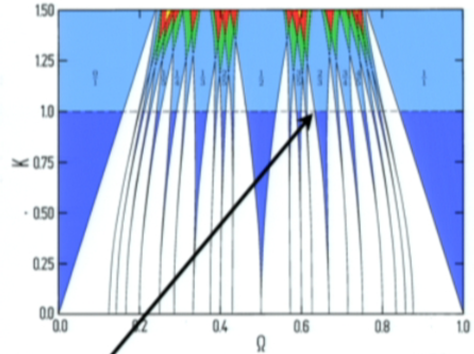
Scientists have observed in detail effects within this phase diagram

Blue region synchronization: Huygens, others e.g., Martha McClintock "Menstrual synchrony and suppression", Nature **229**, 244-245 (1971).

white regions: mostly quasiperiodic behavior. stability of quasiperiodicity studied by D.Ruelle, F. Takens Comm. Math. Phys. **20**, 167 (1971). Experiment J.P. Gollub and S.V. Benson, J. Fluid. Mech. **100**, 449 (1980)

red regions: mixtures of different kinds of orbits, chaos: E. Lorenz, S. Smale



K=1 line: fractal area occupied by quasiperiodic orbits. theory: Bak, Jensen, Bohr d=0.87 Phys. Rev. Lett. 50 1637, 1639 (1983) experiments e.g.: E. G. Gwinn and R. M. Westervelt, Frequency Locking, quasiperiodicity, and Chaos in Extrinsic Ge Phys. Rev. Lett. {\bf 57} 1060-1063 (1986).

M.H. Jensen, L.P. Kadanoff, A. Libchaber, I. Procaccia and J. Stavans, Onset of chaos for quasiperiodic orbit: Global Universality at the Onset of Chaos: Results of a Forced Rayleigh-Bénard Experiment, Physica D 5, 370-386, 1982.

"Feigenbaum, M. J.; Kadanoff, L. P.; and Shenker, S. J"Quasiperiodicity in Dissipative Systems: A Renormalization Group Analysis.. Phys. Rev. Letts. 55 2798 (1985).

Synchronization 9.21.11 Leo Kadanoff

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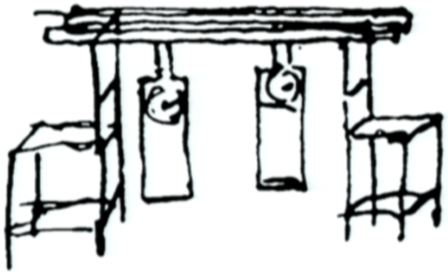
In contrast with statements by Nancy Cartwright who argues that physical models have a very small range of applicability in the real world,

I suggest that the effects of physical models are observable everywhere, if one but has the eyes to look. No fancy apparatus is required.



To see the effect of models one does not require trips to the far ends of the universe. Just look up to see the monthly period of the moon, incommensurate with the year. However, the moon's daily rotation is synchronized with that of the earth, so we always see the man in the moon.

To see the effects of model behavior one need only look at the chaos around us

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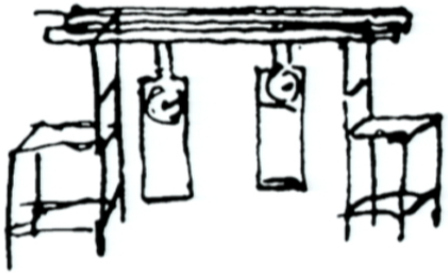

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Indeed models are important. Just reflect about how the present financial difficulties around the world were caused first by greed, but second by weak analysis of faulty economic models.

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