Title: First Principle Construction of Holographic Duals

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Abstract: Topological Quantum field theories(TQFTs) are a special class of QFTs. Their actions do not depend on the metric of the background space-time manifold. Thus, it is very natural to define TQFTs on an arbitrary triangulation of the space-time manifold and they are independent on the triangulation. More importantly, TQFTs defined on triangulations are always a finite theory associated with a well defined cut-off. A well known example is the Turaev-Viro states sum invariants. Essentially, the Turaev-Viro constructions are (local) tensor network representations of a special class of 1+2D TQFTs. In this talk, I will show a new class of TQFTs that can be derived based on the (local) tensor network representations in arbitrary dimensions. They can be regarded as the discrete analogy of topological Berry phase terms of (discrete) non-linear sigma models. The edge theory of such a new class of TQFTs can be regarded as the discrete analogy of WZW terms. This new class of TQFTs naturally classify (bosonic) symmetry protected topological orders in arbitrary dimensions. Finally, I will also discuss new classes of fermionic TQFTs based on the Grassmann tensor network representations and possible new route towards Quantum Gravity(QG).

Strongly coupled QFT is hard, but

- There are theories that have weak coupling descriptions in terms of dual variables
  - Original `particles' remain strongly coupled and have short life time, yet they are organized into long-lived (weakly coupled) collective excitations
  - Duality provides new windows into strong coupling physics
  - Dual variable may carry new (sometimes fractional) quantum numbers : fractionalization
  - Dual variable may live in different space : holography

Slide-particle approach to fractionalized phases

$$\vec{S}_r = f^\dagger_{r\alpha}\vec{\sigma}_{\alpha\beta}f_{r\beta}$$ Spinon : EM charge 0, spin 1/2

Gauge redundancy : $f_{r\alpha} \rightarrow f_{r\alpha}e^{i\theta_r}$

- An exact change of variable; applicable to any system
- Phase redundancy : gauge theory
- Gauge field introduced as Lagrangian multiplier of constraint
- No bare kinetic term : bare gauge coupling is infinite

- Large 'N'
  - Deconfinement
  - Emergent gauge field
  - Quantum order
  - Emergent internal space
- Small `N'
  - Confinement
  - No emergent gauge field
  - No quantum order
  - No emergent internal space

Quantum order in fractionalized phase

[Wen]

- `Order' in the pattern of long range entanglement
- Provide `explanation' for why there exist gapless modes whose robustness is not from any microscopic symmetry
- Can be used to classify phases of matter beyond the symmetry breaking scheme
  - In particular, phases with different quantum order form different universality classes
- Associated with the suppression of topological defects

$$Z = \sum e^{-S} \quad \rightarrow \quad Z = \sum e^{-S}$$

Gauge-string duality

[Maldacena; Gubser, Klebanov, Polyakov; Witten]

$$Z[J(x)] = \int D\phi(x) e^{-S_{field\ theory}[\phi]}$$ D-dimensional gauge theory

$$= \int D \text{ ``}J(x,z)\text{''} \, e^{-S'[J(x,z)]}\Big|_{J(x,0)=J(x)}$$ (D+1)-dimension string theory

- Best understood in the maximally supersymmetric gauge theory in 4D
  - Weak coupling description for strongly coupled QFT
  - Non-perturbative definition of string theory (quantum gravity)
- Believed to be a general framework for a large class of QFT's

[Das, Jevicki; Gopakumar; Heemskerk, Penedones, Polchinski; Lee; Faulkner, Liu, Rangamani; Douglas, Mazzucato, Razamat,…]

Gauged matrix model

$$S[U] = NM^2 \sum_{<i,j>} \mathrm{tr}(U_{ij}^\dagger U_{ij}) + N^2 V[W_C/N]$$

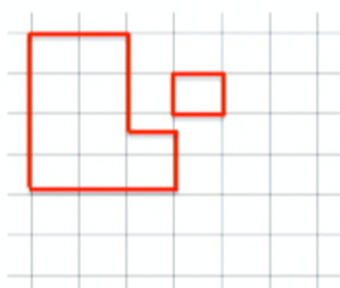
$$V = -\sum_{n=1}^{\infty} N^{-n} \sum_{\{C_1,..,C_n\}} J_{\{C_1,..,C_n\}} \prod_{k=1}^{n} W_{C_k}$$

U_{ij} : N × N complex matrices

$$W_C = \mathrm{tr} \prod_{<i,j>\in C} U_{ij}$$

: Wilson loop

U(N) gauge symmetry : $U_{ij} \to V_i^\dagger U_{ij} V_j$



D-dimensional Euclidean lattice

$\mathcal{J}_C$: Sources for single-trace operators

$\mathcal{J}_{C_1,C_2}$: Sources for double-trace operators

⋮

$\mathcal{J}_{\{C_1,..,C_n\}}$: Sources for general multi-trace operators

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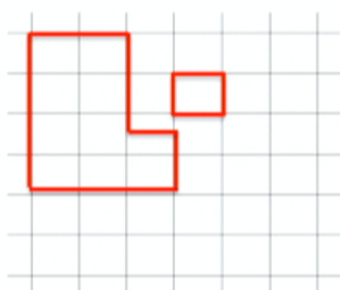

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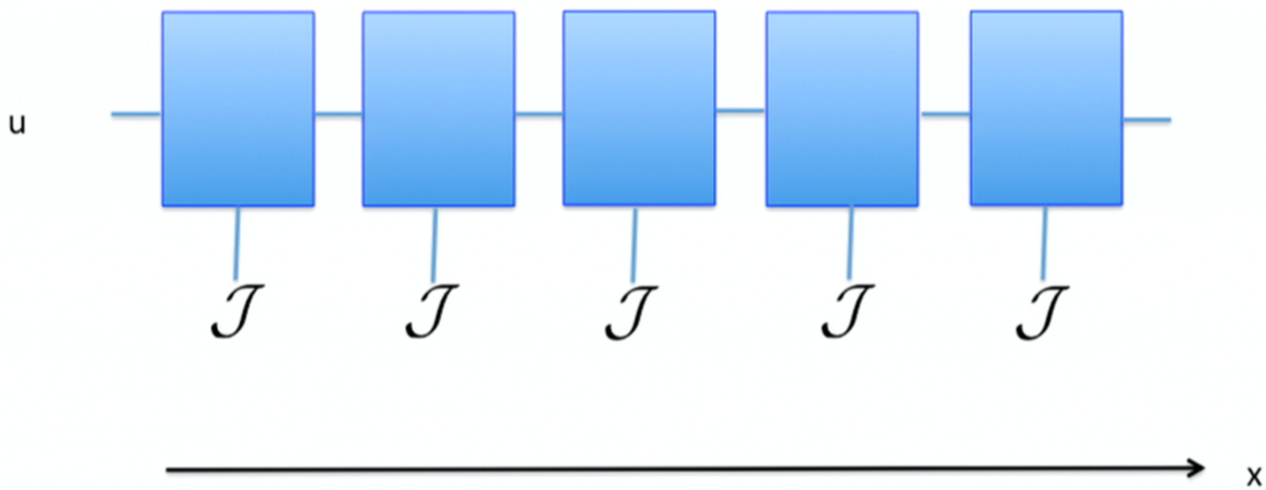
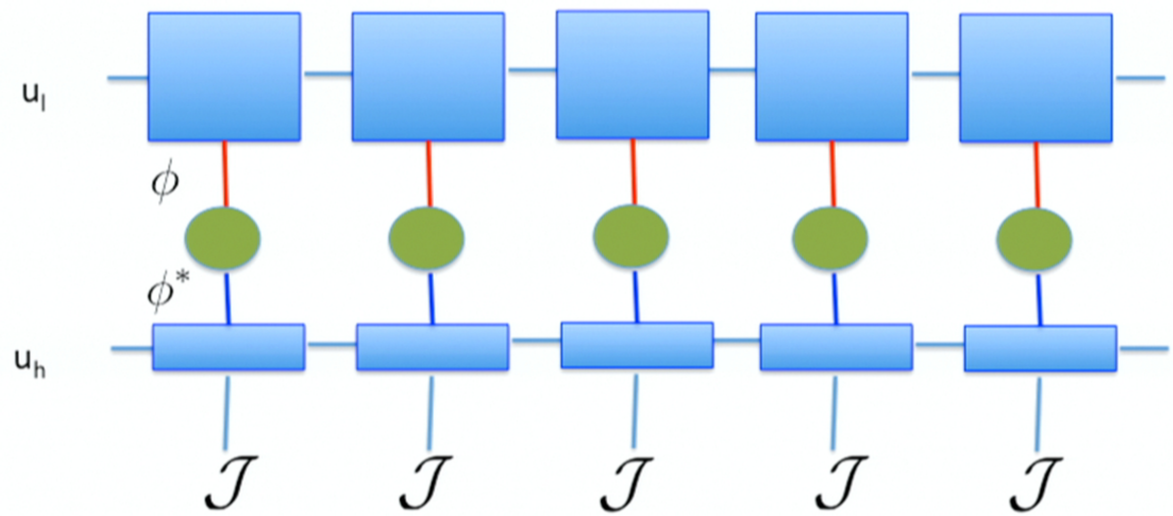
Construction of holographic theory

Partition function can be viewed as contractions of an D-dimensional array of tensors which depend on external sources



Construction of holographic theory

- High energy fields can be viewed as fluctuating sources for the low energy fields



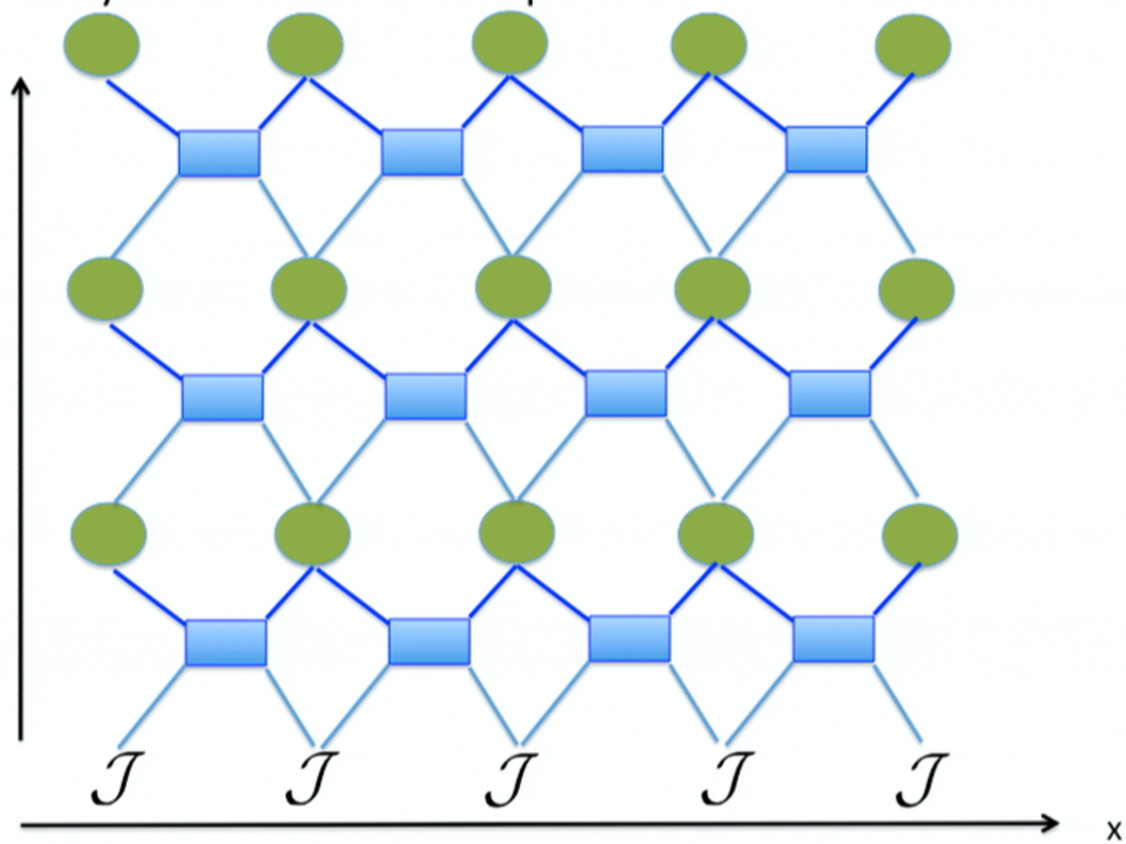
ϕ : an auxiliary field that plays the role of fluctuating source for low energy field
ϕ^* : a Lagrangian multiplier than impose the constraint between ϕ and J

Construction of holographic theory

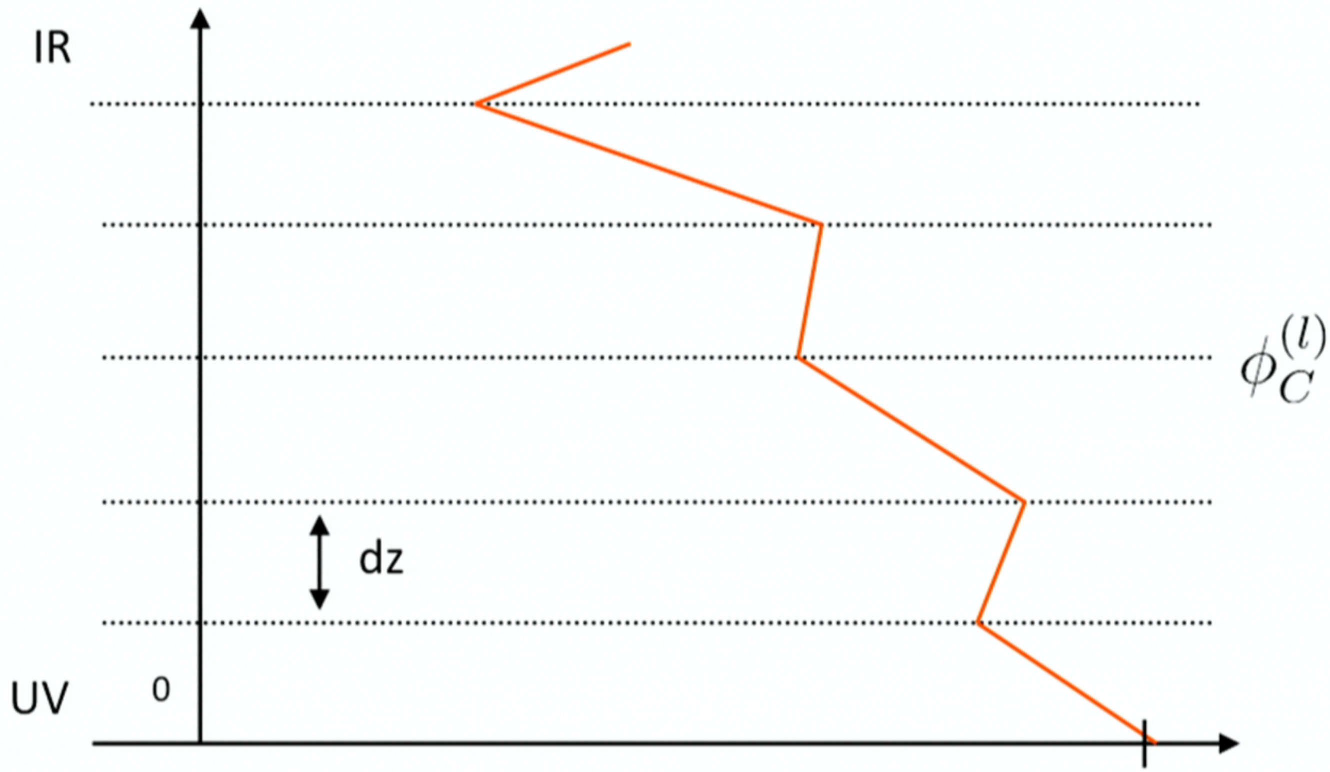
Repetition of these step leads to contractions of (D+1)-dimensional array of matrices for the partition function

Extra dimension, z : length scale in RG [J. de Boer, Verlinde, Verlinde]

Cf. MERA [Vidal]



Extra dimension as a length scale



(D+1)-dimensional field theory of closed loops

$$Z = \int D\phi_C D\phi_C^* \;\; e^{-\left(S_{bulk}[\phi_C^*(z),\phi_C(z)] + N^2\phi_C^*(0)\phi_C(0) + N^2 V[\phi_C^*(0)]\right)}$$

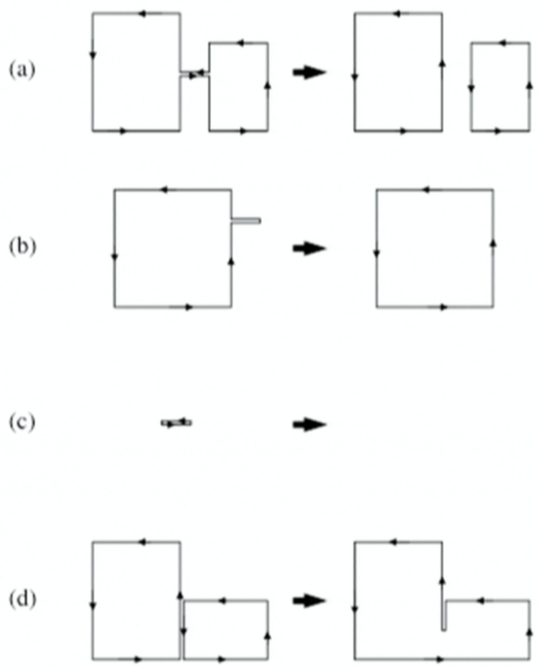
$$S_{bulk} = N^2 \int_0^\infty dz \Big[\phi_C^* \partial_z \phi_C + \alpha L_C \phi_C^* \phi_C$$
$$- \frac{\alpha}{M^2} \Big(F_{ij}[C_1, C_2] \phi_{C_1}^* \phi_{C_2}^* \phi_{[C_1+C_2]_{ij}} + G_{ij}[C_1, C_2] \phi_{(C_1+C_2)_{ij}}^* \phi_{C_1} \phi_{C_2} \Big) \Big]$$

- $\phi_C(z), \phi_C^*(z)$: coherent fields for annihilation/creation operators of loop
- S_{bulk} : action for closed loop fields in (D+1)-dimensions
- V : J_C dependent action for the UV(z=0) boundary fields
  - Single trace potential : the standard Dirichlet B.C.
  - Multi-trace potential : mixed B.C.

Loop Hamiltonian in the bulk

$$Z = \lim_{\beta \to \infty} < \Psi_f | e^{-\beta H} | \Psi_i >$$

$$H = \alpha L_C a_C^\dagger a_C - \frac{\alpha}{NM^2} \left(F_{ij}[C_1, C_2] a_{C_1}^\dagger a_{C_2}^\dagger a_{[C_1+C_2]_{ij}} + G_{ij}[C_1, C_2] a_{(C_1+C_2)_{ij}}^\dagger a_{C_1} a_{C_2} \right)$$

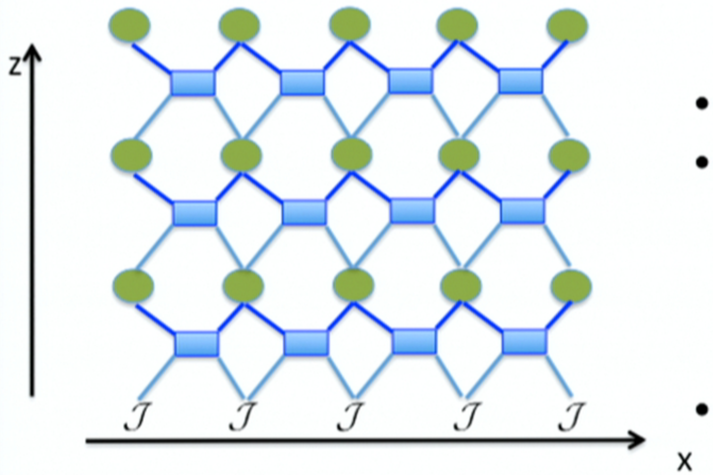


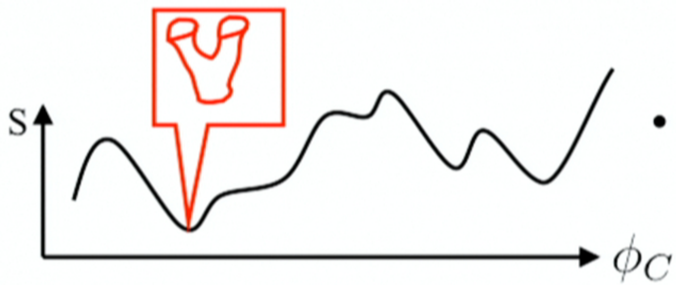
– Tension

– Joining/splitting

Similar to the loop Hamiltonian studied by Kawai & Ishibashi; Jevicki & Rodrigues, ...

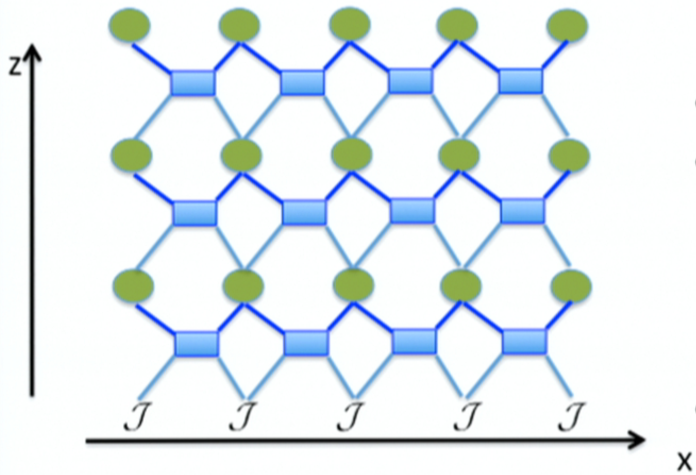
Saddle point and beyond

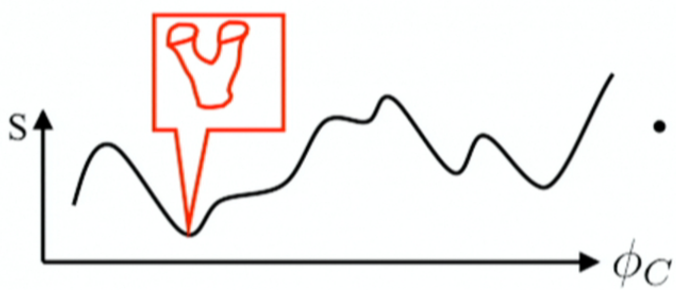




- S ~ N^2 (…)
- Fluctuations of loop fields around a saddle point describe weakly interacting closed strings in (D+1)-dimensional space for a large N
- The background (metric and the two-form gauge field) for closed strings are determined by the saddle point solution
- Key question : When is the saddle point solution stable ?

Saddle point and beyond





- $S \sim N^2$ (…)
- Fluctuations of loop fields around a saddle point describe weakly interacting closed strings in (D+1)-dimensional space for a large N
- The background (metric and the two-form gauge field) for closed strings are determined by the saddle point solution
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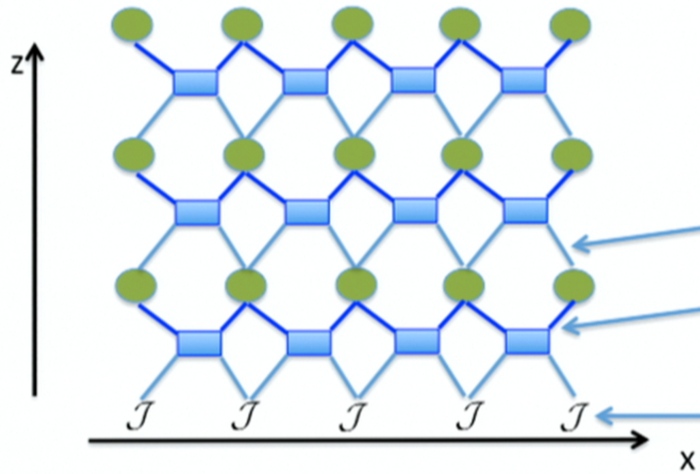
Gauge symmetry

- Relative phases between sources and operators are ill-defined $J_n \to e^{i\theta_n} J_n, \quad O_n \to e^{-i\theta_n} O_n$

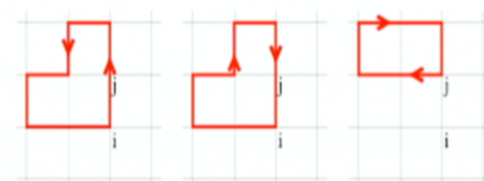


$$\phi_C(z) \to \; e^{i\theta_\mu(i,z) Q_{ii+\mu}[C]} \; \phi_C(z),$$

$$\phi^*_C(z) \to \; e^{-i\theta_\mu(i,z) Q_{ii+\mu}[C]} \; \phi^*_C(z),$$

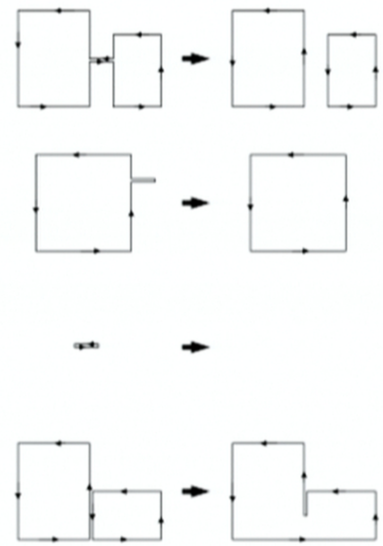
$$J_C \to \; e^{i\theta_\mu(i,0) Q_{ii+\mu}[C]} \; J_C$$

External sources J_C explicitly break the gauge symmetry at the UV boundary.



$Q_{ij}[C] =$ +1 -1 0

Consequences of gauge symmetry

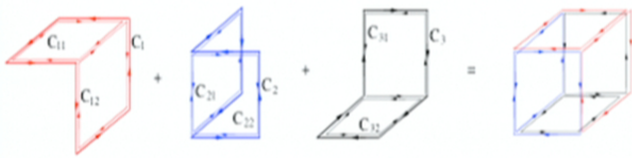


- No-quadratic hopping : flux conservation
- The cubic interactions between loops generate the kinetic term for strings

$$-\frac{\alpha < \phi_C >}{M^2} a^\dagger_{C+C'} a_{C'}$$

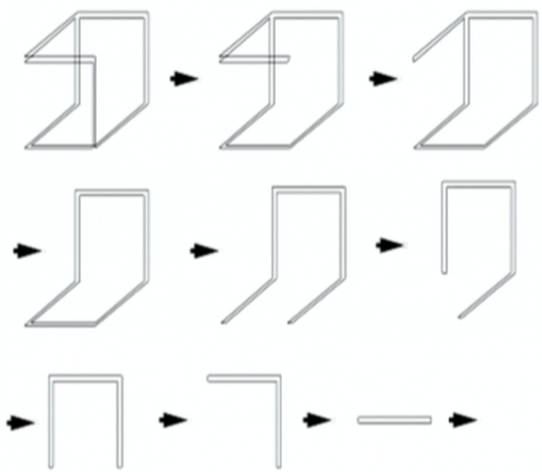
- The phase of the background loop field provides a Berry phase for strings that moves in space $\phi_C = |\phi_C| e^{ib_C}$
- $b_C = \int_{A_C} B$ ← compact two-form gauge field $b_C \sim b_C + 2\pi$

Quantum fluctuations generate kinetic energy for the two-form gauge field



(a)

- Integrating out heavy (long) loops generate the kinetic energy for the two-form gauge field

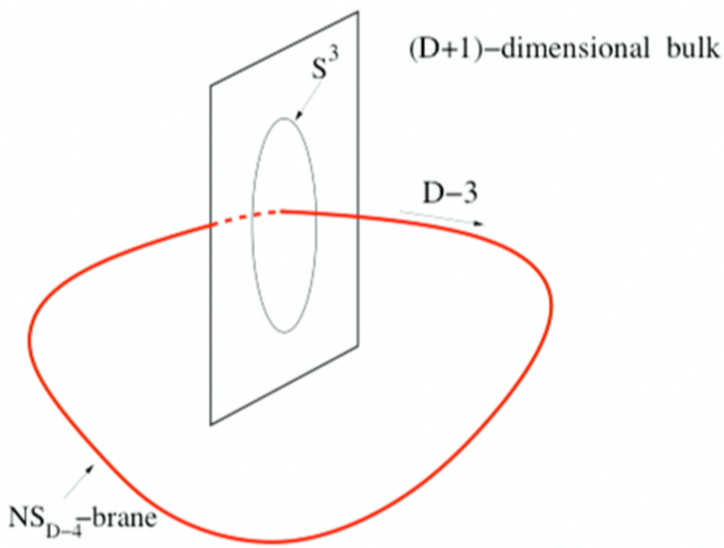


(b)

$$S_{eff} = \frac{1}{g_{KR}^2} \int dz \Big(\sum_{\square} (\partial_z B_{\mu\nu})^2 - \sum_{\text{cubes}} \cos\left[a^3(\Delta_\mu B_{\nu\lambda} + \Delta_\nu B_{\lambda\mu} + \Delta_\lambda B_{\mu\nu})\right] \Big)$$

$$g_{KR}^2 \sim 1/(|\phi_\square|^6 N^2)$$

Topological defect for the compact two-form gauge field



$$H = dB$$

$$\int_{S^3} H = 2\pi$$

- Tension of the brane ~ N^2

NS-brane determines the fate of string

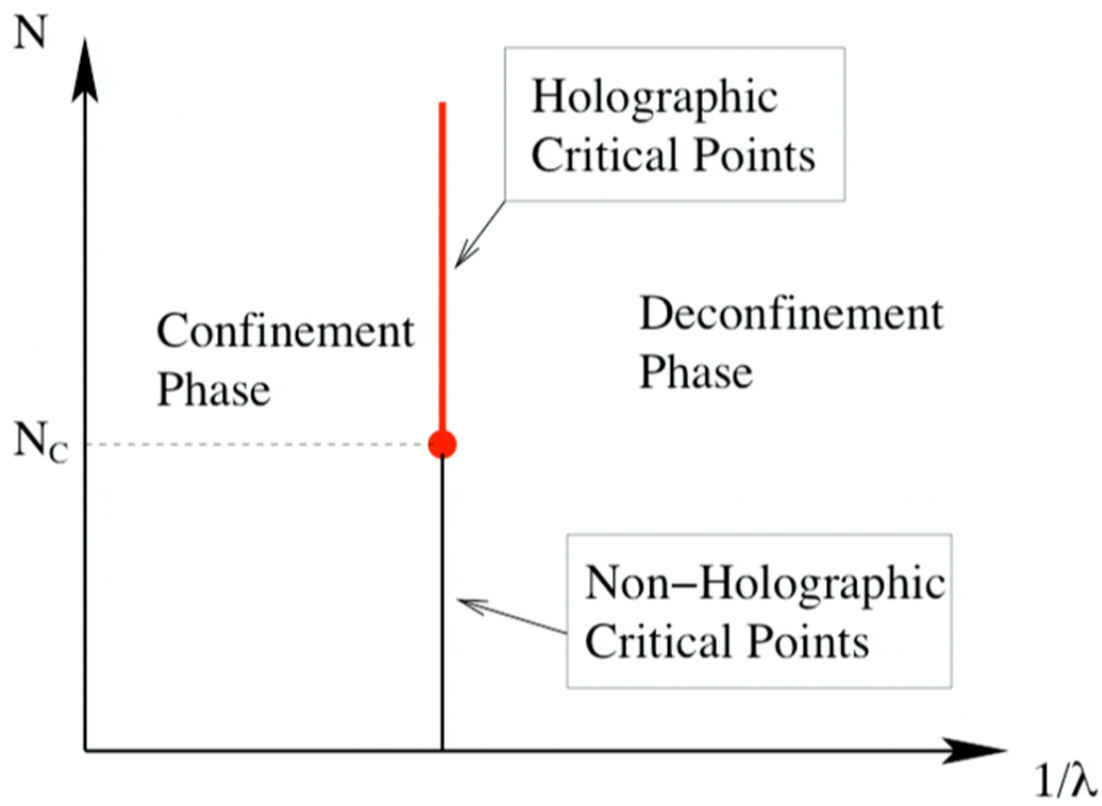
Gapped NS-brane

- Emergent Bianchi identity dH=0 at long distances
- Strings are deconfined
- Emergent space
- Two-form gauge field remains light even at strong coupling
- Non-trivial quantum order!

Condensed NS-brane

- Bianchi identity is violated at all distance scales
- Strings are confined
- No emergent space
- No light propagating mode deep inside the bulk
- No quantum order

A proposed phase diagram for a pure bosonic gauged matrix model in D>4



Summary

- General D-dimensional gauged matrix model can be mapped into (D+1)-dimensional string field theory which include compact two-form gauge field
- Those phases that admit holographic description have a distinct quantum order
  - Emergent space
  - Deconfined string
  - Protected scaling dimension