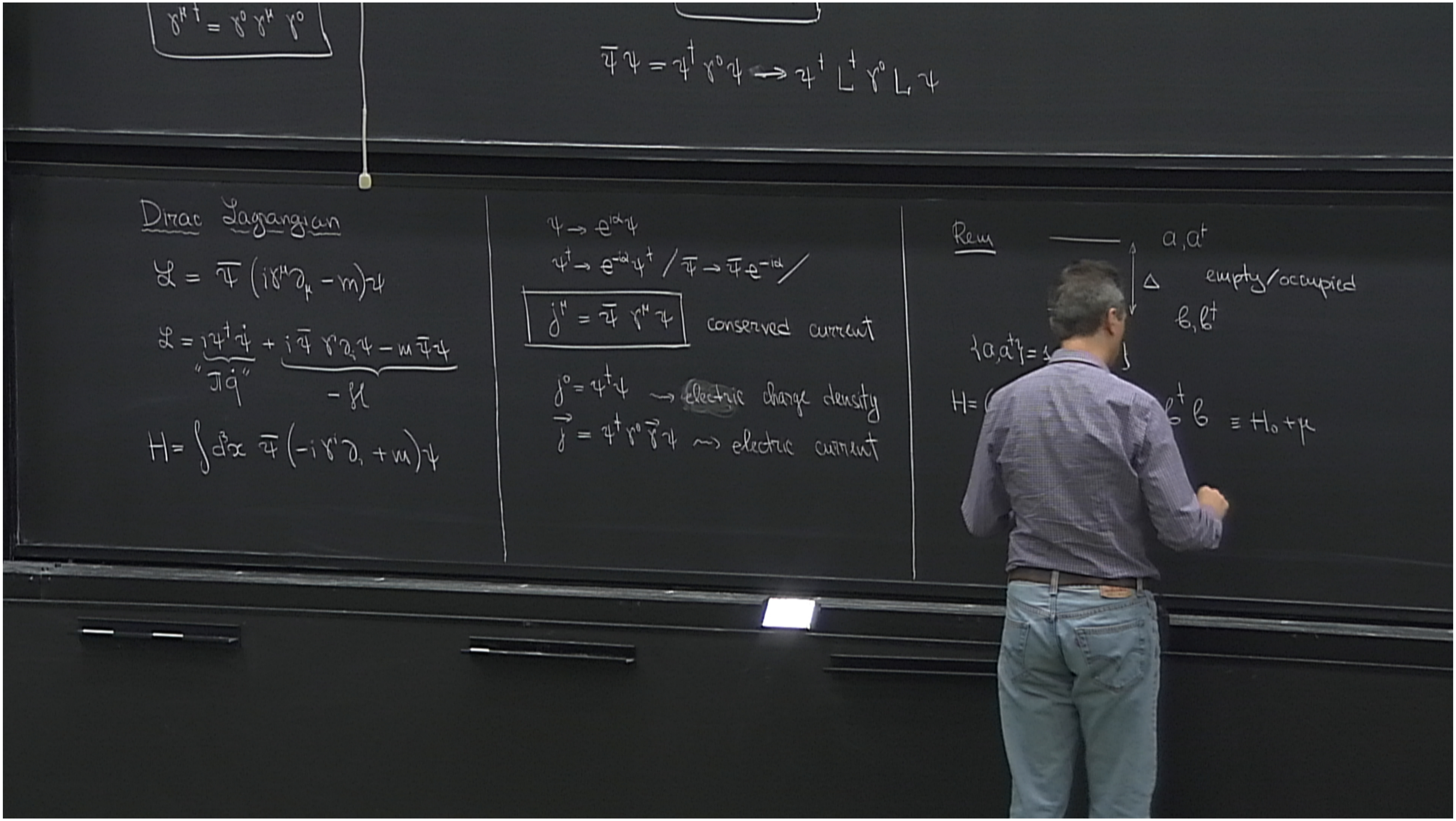
Title: Quantum Field Theory I - Lecture 5b

Date: Oct 07, 2011 10:00 AM

URL: http://pirsa.org/11100093

Abstract:


Ψ → Ψ e^{-iα}
conserved current
electric charge density
~> electric current


$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0$
$\bar\psi \psi = \psi^\dagger \gamma^0 \psi \rightarrow \psi^\dagger L^\dagger \gamma^0 L \psi$
Dirac Lagrangian
$\mathcal{L} = \bar\psi (i\gamma^\mu \partial_\mu - m)\psi$
$\mathcal{L} = \underbrace{i\psi^\dagger \dot\psi}_{\text{"}\pi \dot q\text{"}} + \underbrace{i\bar\psi \gamma^i \partial_i \psi - m\bar\psi\psi}_{-\mathcal{H}}$
$H = \int d^3x \, \bar\psi(-i\gamma^i \partial_i + m)\psi$
$\psi \to e^{i\alpha}\psi$
$\psi^\dagger \to e^{-i\alpha}\psi^\dagger$ / $\bar\psi \to \bar\psi e^{-i\alpha}$
$\boxed{j^\mu = \bar\psi \gamma^\mu \psi}$ conserved current
$j^0 = \psi^\dagger\psi \rightsquigarrow$ electric charge density
$\vec{j} = \psi^\dagger \gamma^0 \vec\gamma \psi \rightsquigarrow$ electric current
Rem
$a, a^\dagger$
Δ empty/occupied
$b, b^\dagger$
$\{a, a^\dagger\} = 1$
$H =$
$b^\dagger b \equiv H_0 + \mu$

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0$$

$$\bar\psi\psi = \psi^\dagger \gamma^0 \psi \rightarrow \psi^\dagger L^\dagger \gamma^0 L \psi$$

Dirac Lagrangian

$$\mathcal{L} = \bar\psi (i\gamma^\mu \partial_\mu - m)\psi$$

$$\mathcal{L} = \underbrace{i\psi^\dagger \dot\psi}_{\text{"}\pi\dot q\text{"}} + \underbrace{i\bar\psi \gamma^i \partial_i \psi - m\bar\psi\psi}_{-\mathcal{H}}$$

$$H = \int d^3x\, \bar\psi(-i\gamma^i\partial_i + m)\psi$$

$$\psi \to e^{i\alpha}\psi$$

$$\psi^\dagger \to e^{-i\alpha}\psi^\dagger \ / \ \bar\psi \to \bar\psi e^{-i\alpha} /$$

$$\boxed{j^\mu = \bar\psi \gamma^\mu \psi}$$ conserved current

$j^0 = \psi^\dagger\psi \rightsquigarrow$ electric charge density

$\vec{j} = \psi^\dagger \gamma^0 \vec\gamma \psi \rightsquigarrow$ electric current

Rem

$a, a^\dagger$

Δ empty/occupied

$b, b^\dagger$

$$\{a,a^\dagger\} = 1 = \{b,b^\dagger\}$$

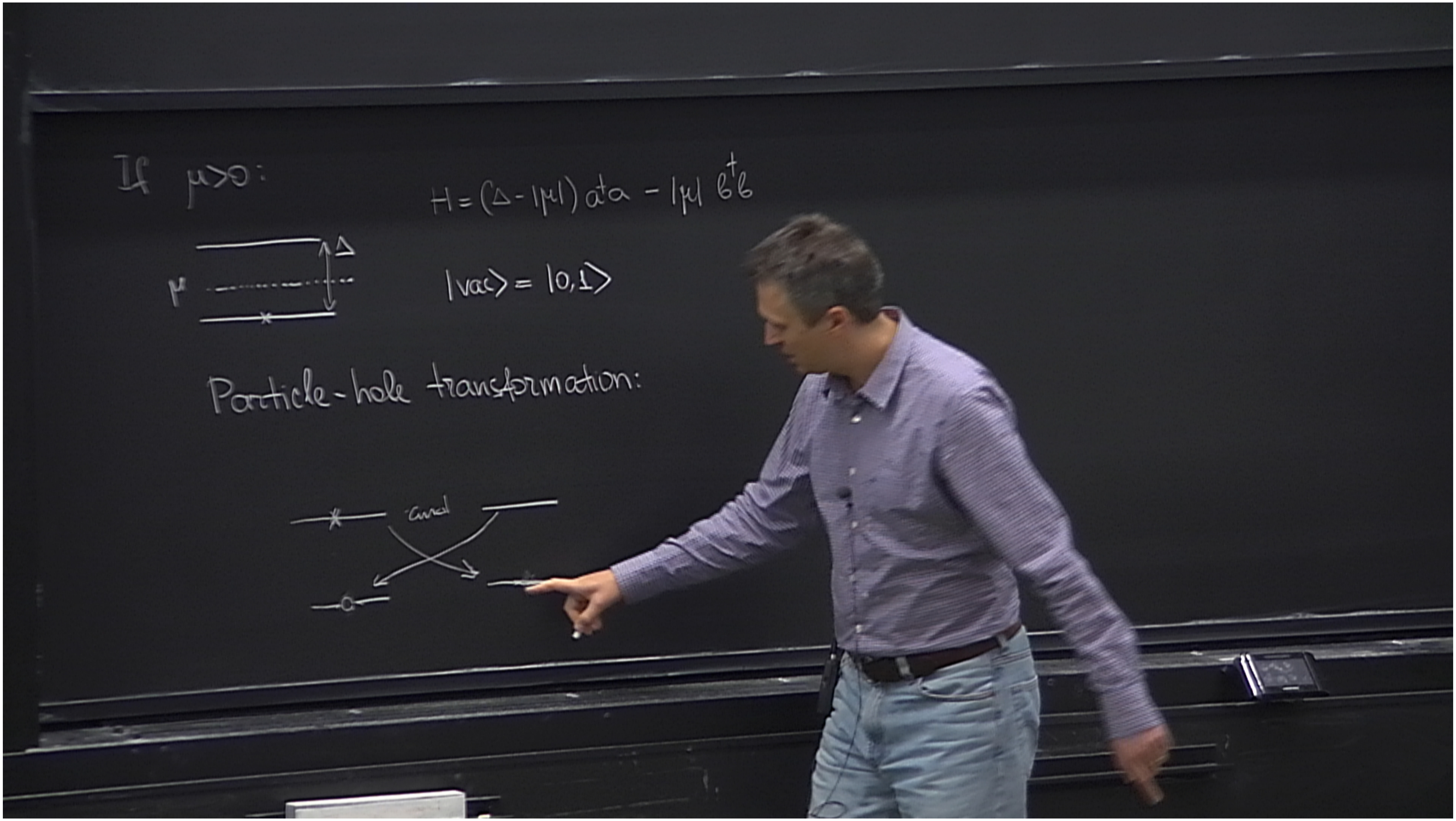
$$H = (\Delta+\mu)a^\dagger a + \mu b^\dagger b \equiv H_0 + \mu N$$

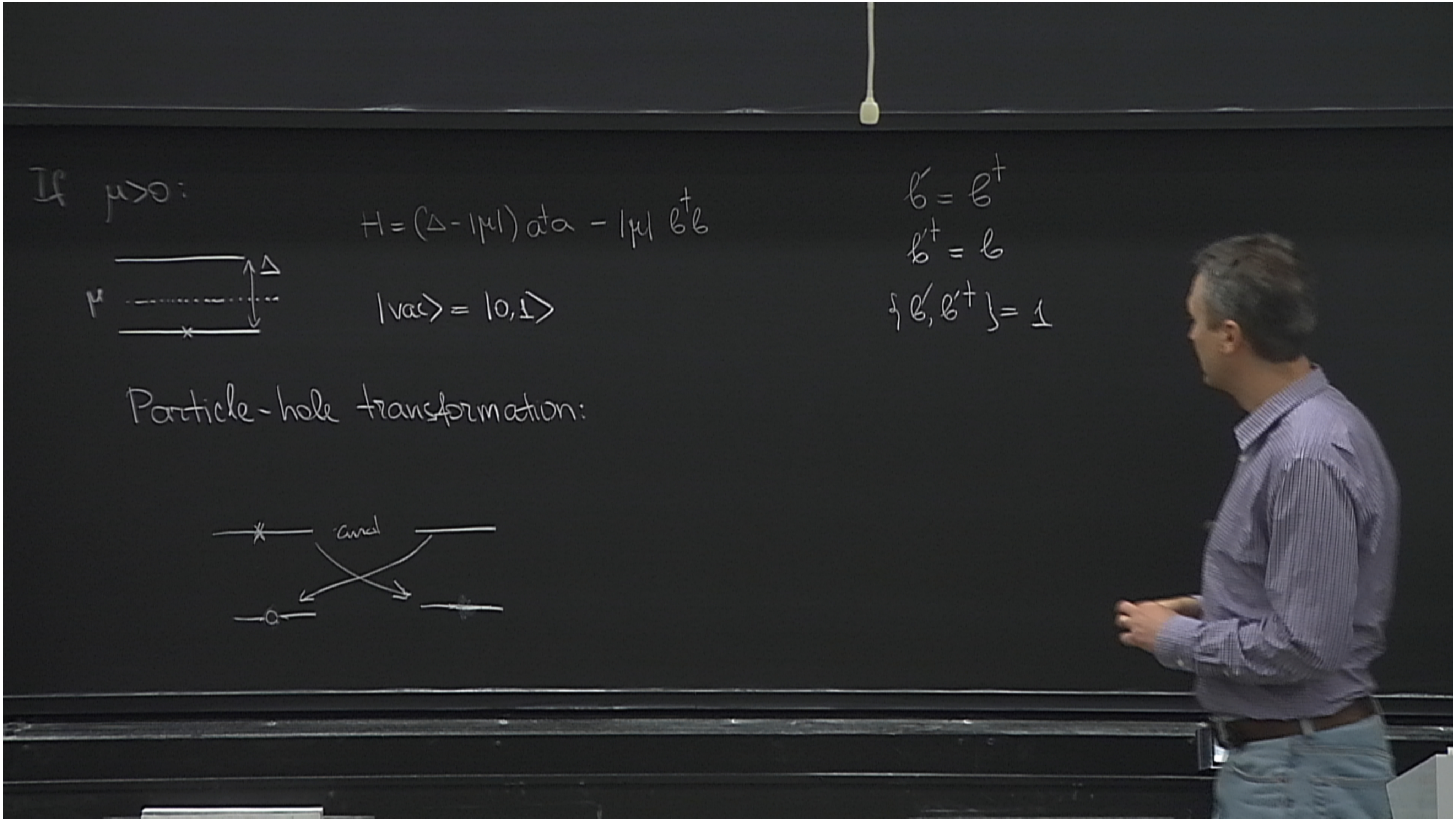
If $\mu < 0$:

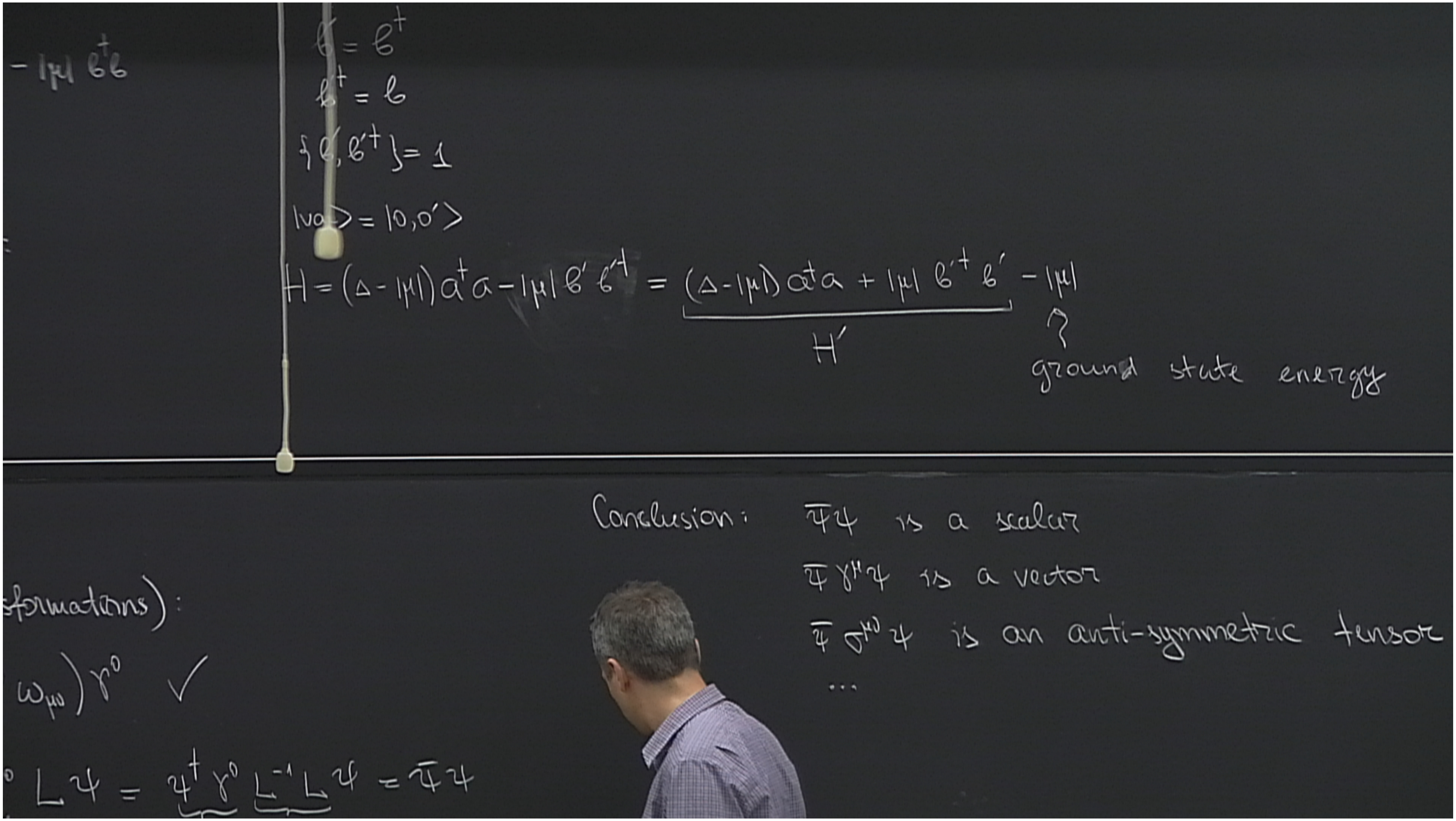
Δ

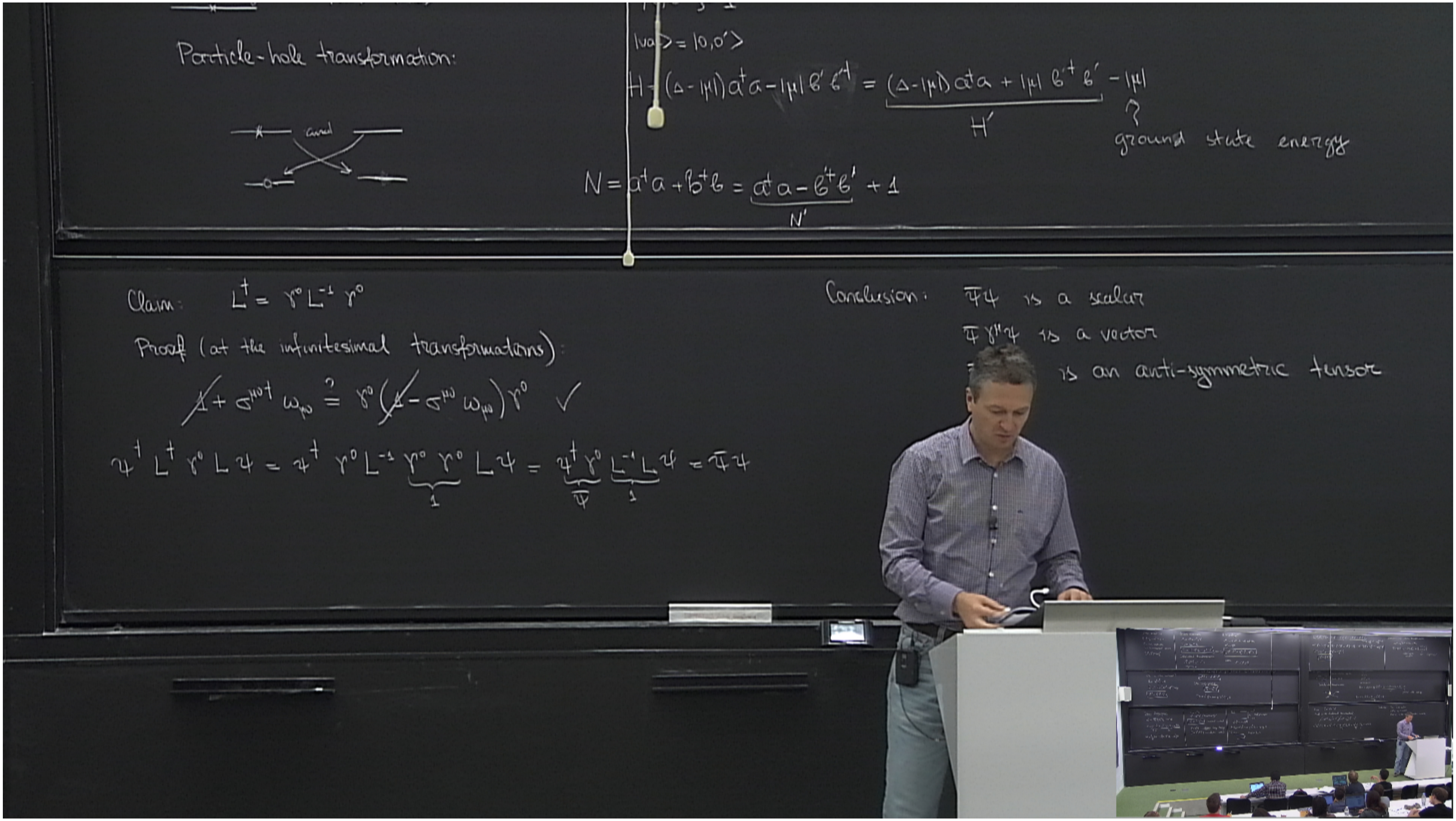
μ

$$|vac\rangle = |0,0\rangle$$


If $\mu > 0$:
$H = (\Delta - |\mu|)\, a^\dagger a - |\mu|\, b^\dagger b$
Δ
μ
$|vac\rangle = |0,1\rangle$
Particle-hole transformation:
and


If $\mu > 0$:
$H = (\Delta - |\mu|)\, a^\dagger a - |\mu|\, b^\dagger b$
Δ
μ
$|vac\rangle = |0,1\rangle$
$b' = b^\dagger$
$b'^\dagger = b$
$\{b', b'^\dagger\} = 1$
Particle-hole transformation:
and


$- |\mu| b^\dagger b$
$b' = b^\dagger$
$b'^\dagger = b$
$\{b', b'^\dagger\} = 1$
$|vac\rangle = |0,0'\rangle$
$H = (\Delta - |\mu|) a^\dagger a - |\mu| b' b'^\dagger = \underbrace{(\Delta - |\mu|) a^\dagger a + |\mu| b'^\dagger b'}_{H'} - |\mu|$
ground state energy
Conclusion: $\bar\psi\psi$ is a scalar
$\bar\psi\gamma^\mu\psi$ is a vector
$\bar\psi\sigma^{\mu\nu}\psi$ is an anti-symmetric tensor
...
sformations):
$\omega_{\mu\nu}) \gamma^0$
$L\psi = \psi^\dagger\gamma^0 L^{-1} L \psi = \bar\psi\psi$


Particle-hole transformation:
cancel
$|va\rangle = |0,0'\rangle$
$H = (\Delta - |\mu|)a^\dagger a - |\mu| b' b'^\dagger = \underbrace{(\Delta - |\mu|)a^\dagger a + |\mu| b'^\dagger b'}_{H'} - |\mu|$
ground state energy
$N = a^\dagger a + b^\dagger b = \underbrace{a^\dagger a - b'^\dagger b'}_{N'} + 1$
Claim: $L^\dagger = \gamma^0 L^{-1} \gamma^0$
Proof (at the infinitesimal transformations):
$\cancel{1} + \sigma^{\mu\nu\dagger}\omega_{\mu\nu} \overset{?}{=} \gamma^0(\cancel{1} - \sigma^{\mu\nu}\omega_{\mu\nu})\gamma^0$ ✓
$\psi^\dagger L^\dagger \gamma^0 L \psi = \psi^\dagger \gamma^0 L^{-1} \underbrace{\gamma^0 \gamma^0}_{1} L \psi = \underbrace{\psi^\dagger \gamma^0}_{\bar\psi} \underbrace{L^{-1} L}_{1} \psi = \bar\psi\psi$
Conclusion: $\bar\psi\psi$ is a scalar
$\bar\psi\gamma^\mu\psi$ is a vector
is an anti-symmetric tensor