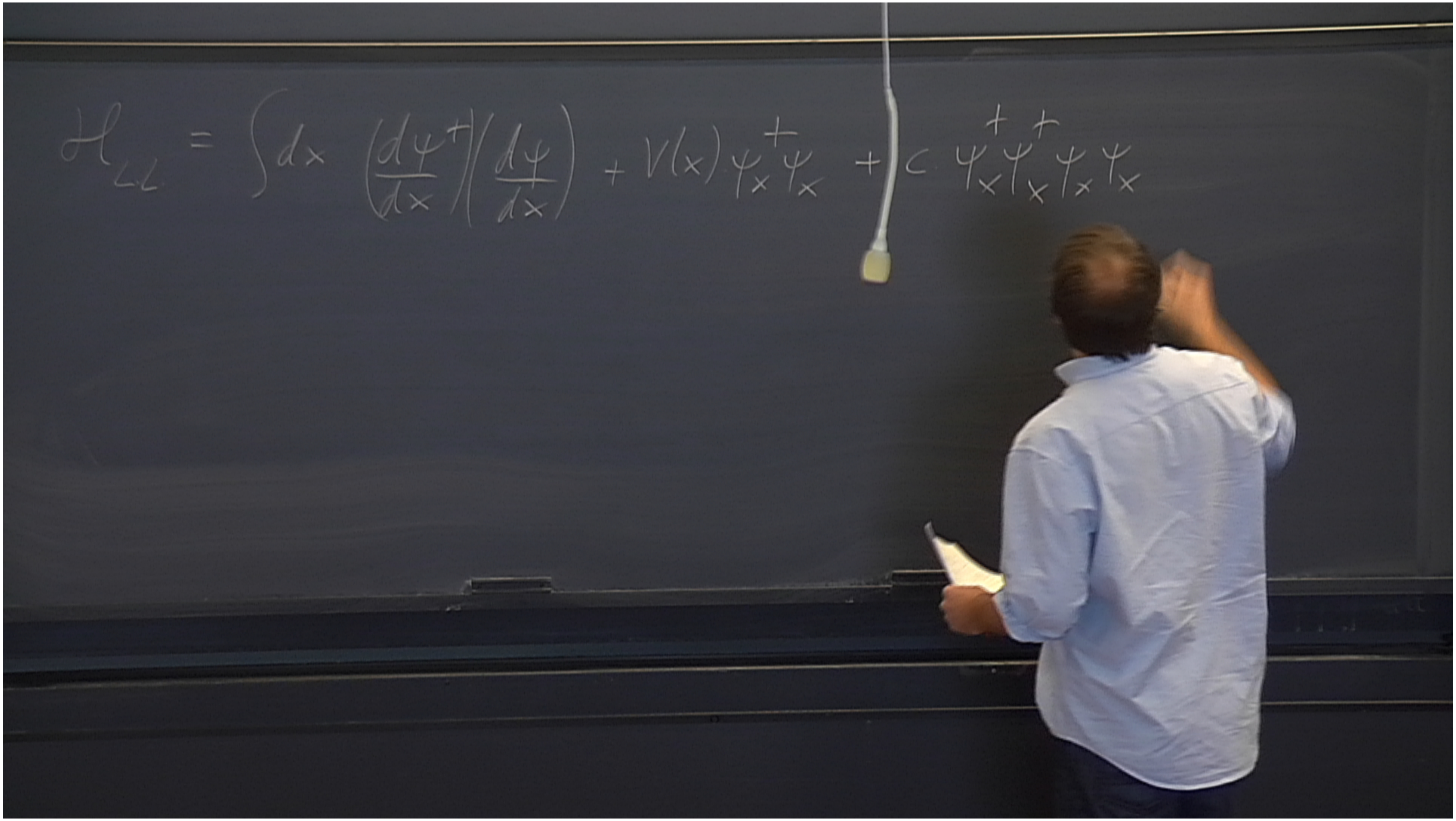
Title: MPS for QFTs

Date: Oct 24, 2011 02:00 PM

URL: http://pirsa.org/11100091

Abstract: I will talk about matrix product states and their suitability for simulating quantum many-body systems in the continuum.

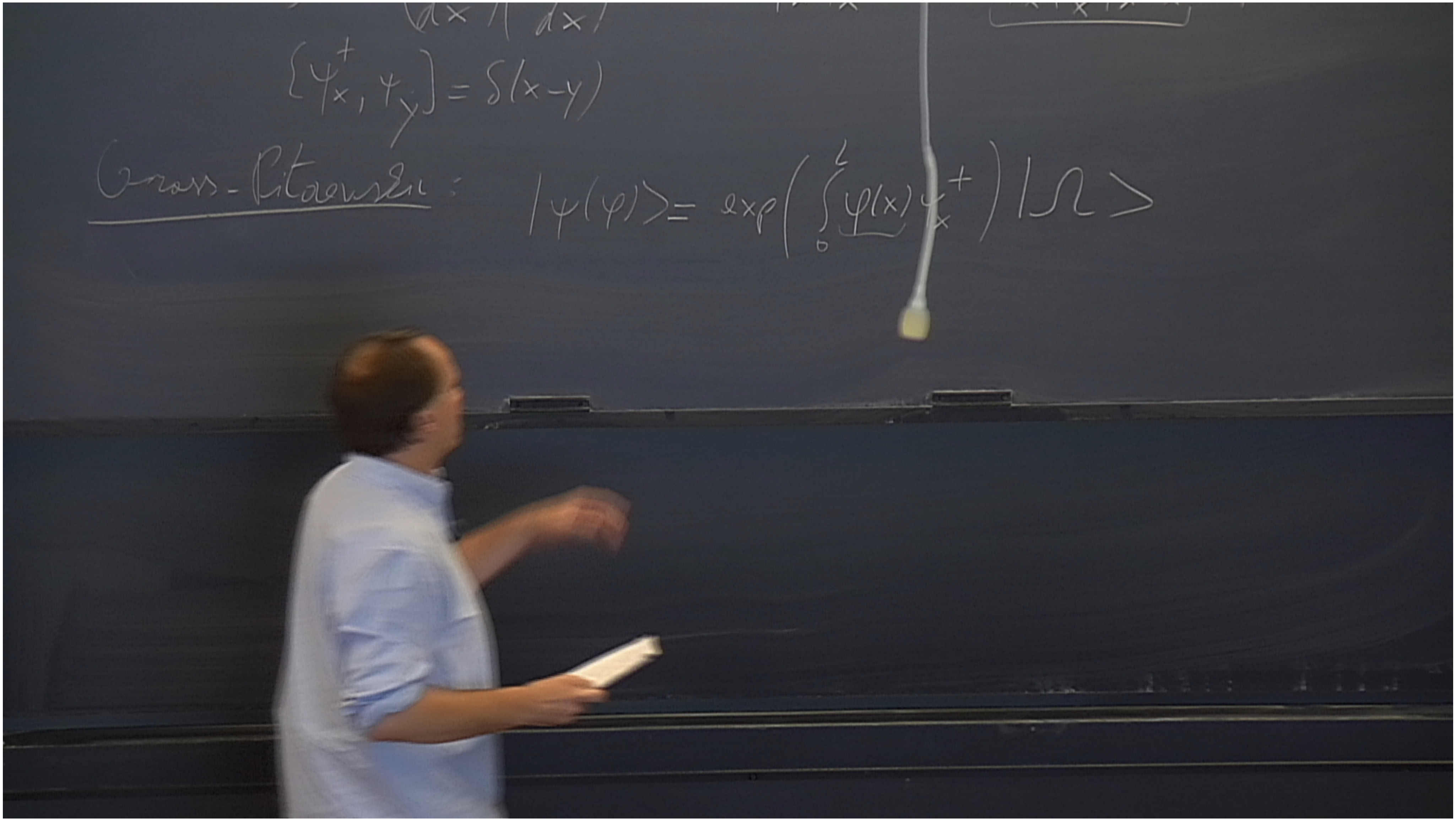

$\mathcal{H}_{LL} = \int dx \left(\frac{d\psi^{+}}{dx}\right)\left(\frac{d\psi}{dx}\right) + V(x)\,\psi^{+}_{x}\psi_{x} + c\,\psi^{+}_{x}\psi^{+}_{x}\psi_{x}\psi_{x}$

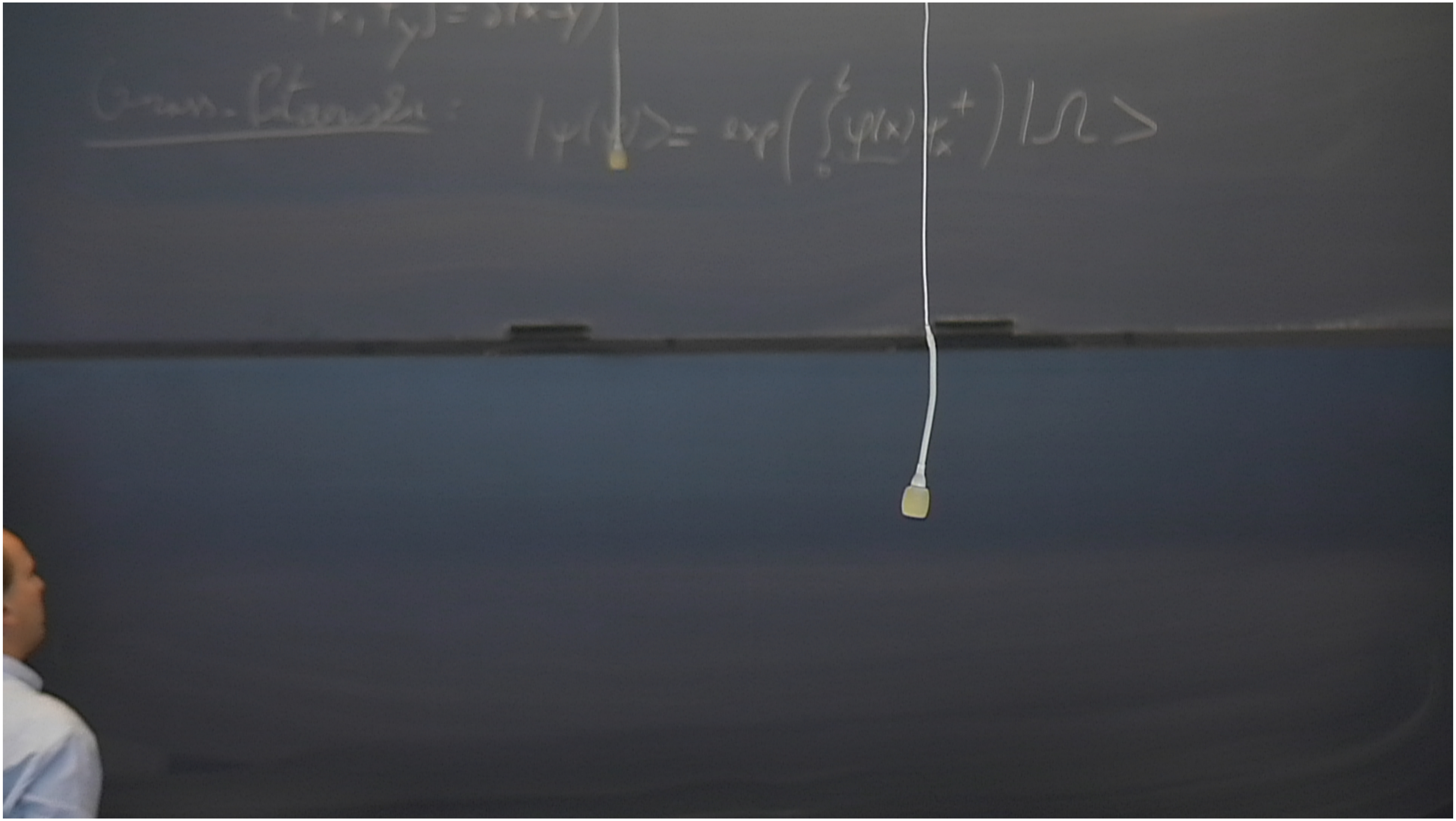
$$\mathcal{H}_{LL} = \int dx \left(\frac{d\psi_x^{\dagger}}{dx}\right)\left(\frac{d\psi_x}{dx}\right) + V(x)\,\psi_x^{\dagger}\psi_x + c\,\underbrace{\psi_x^{\dagger}\psi_x^{\dagger}\psi_x\psi_x} - \mu\,\psi_x^{\dagger}\psi_x$$

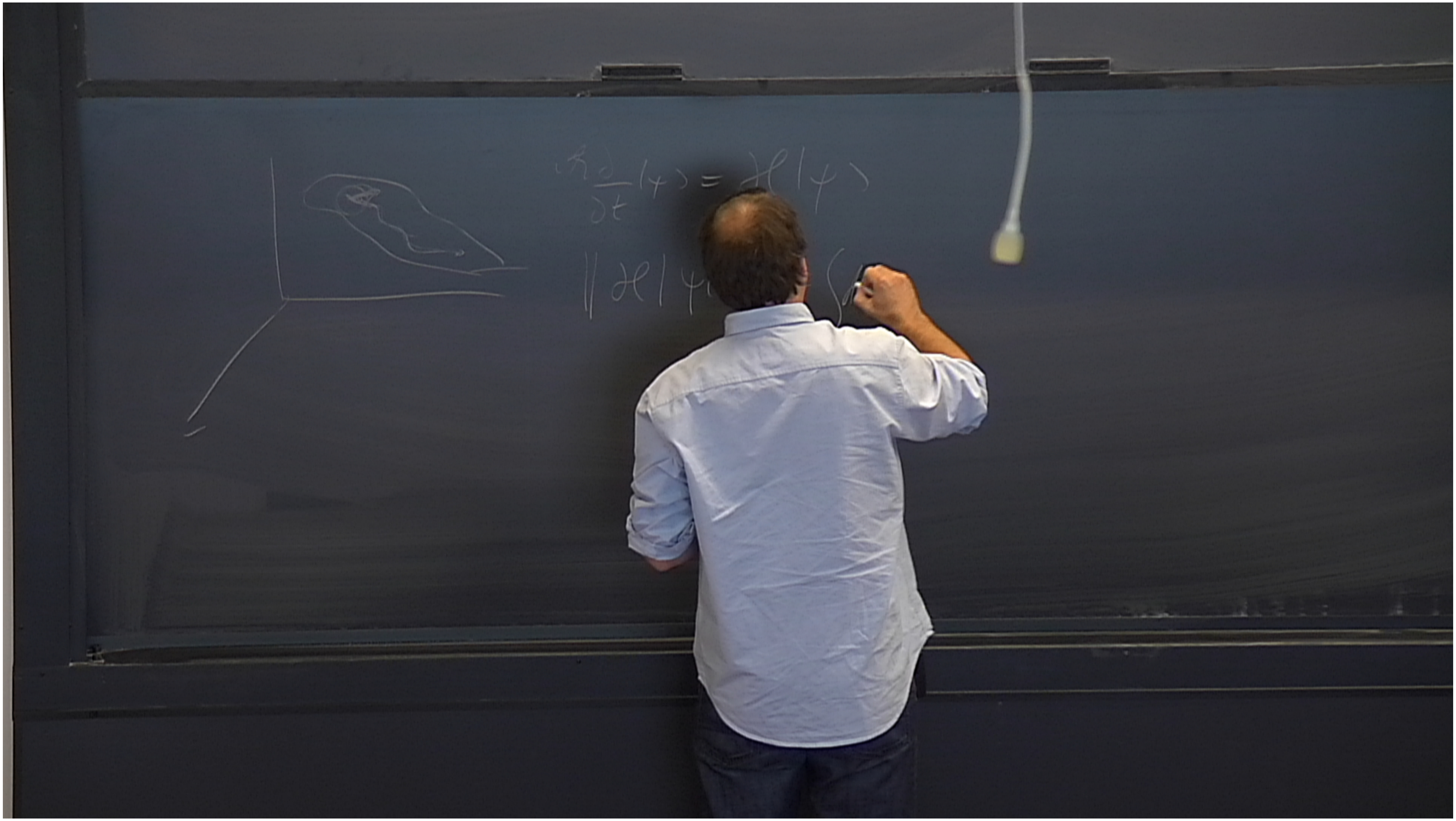
$$\{\psi_x^{\dagger}, \psi_y] = \delta(x-y)$$

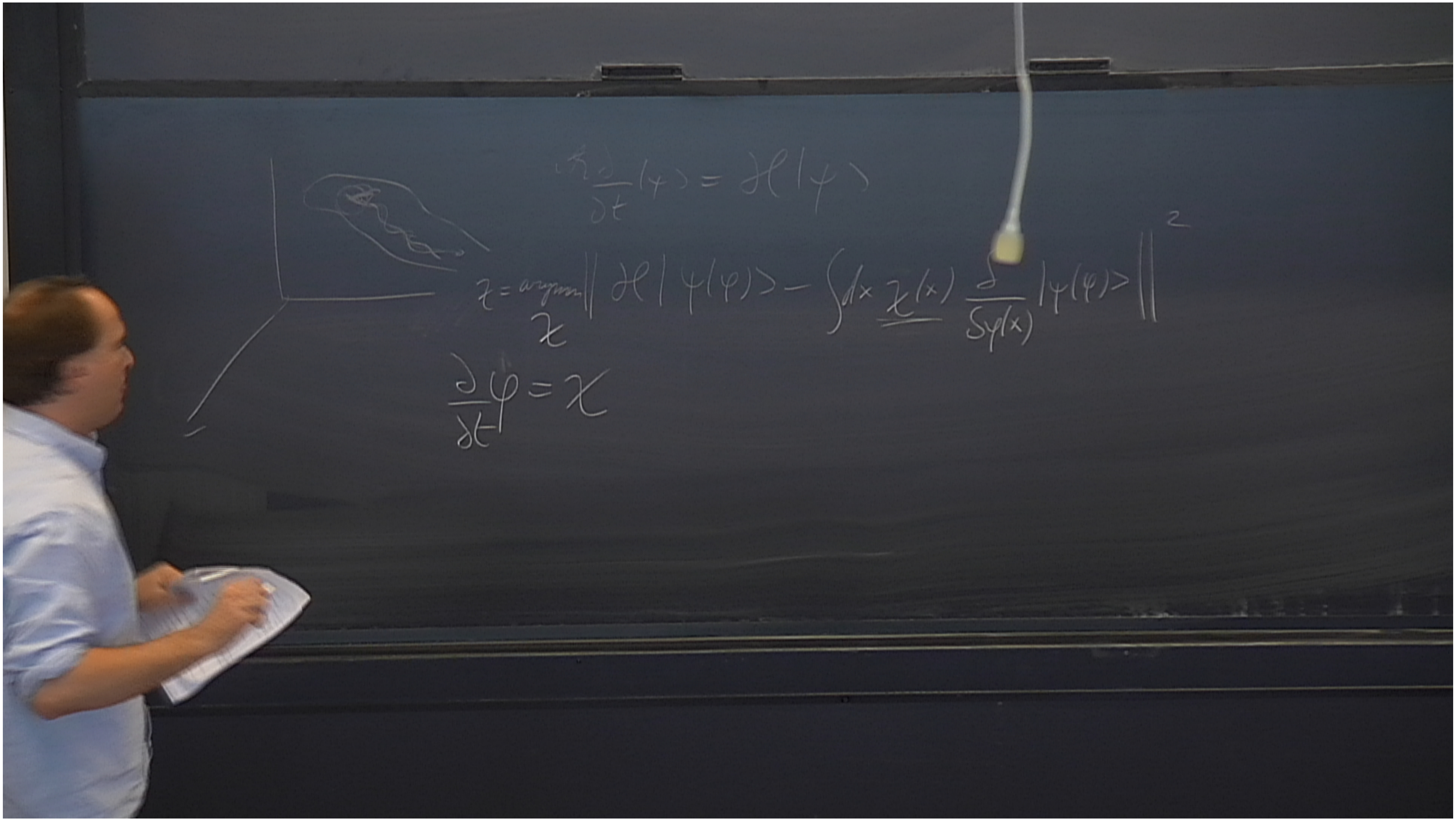
$$\mathcal{H}_{LL} = \int dx \left(\frac{d\psi_x^\dagger}{dx}\right)\left(\frac{d\psi_x}{dx}\right) + V(x)\,\psi_x^\dagger\psi_x + c\,\underbrace{\psi_x^\dagger\psi_x^\dagger\psi_x\psi_x} - \mu\,\psi_x^\dagger\psi_x$$

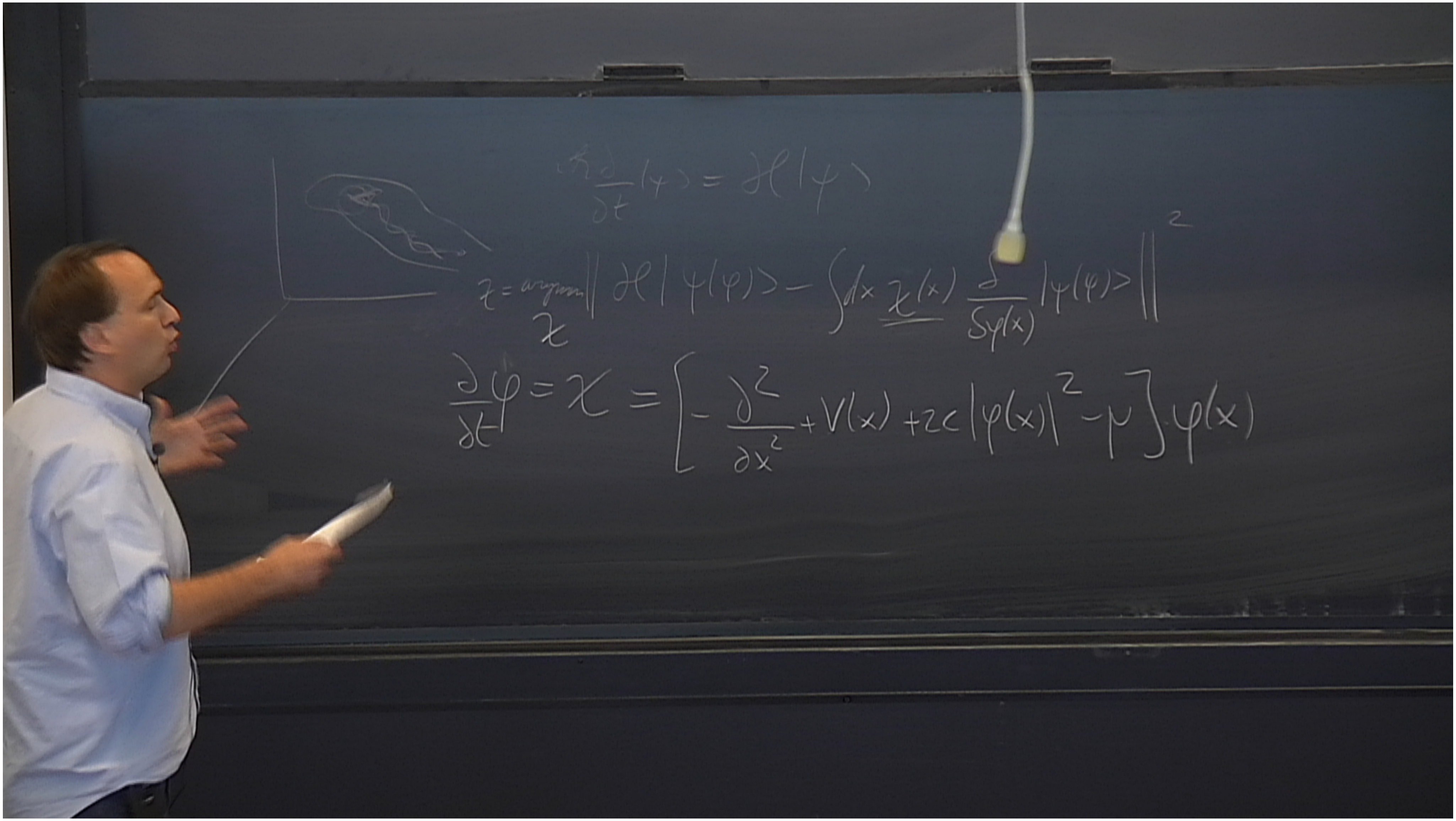
$$\{\psi_x^\dagger, \psi_y] = \delta(x-y)$$

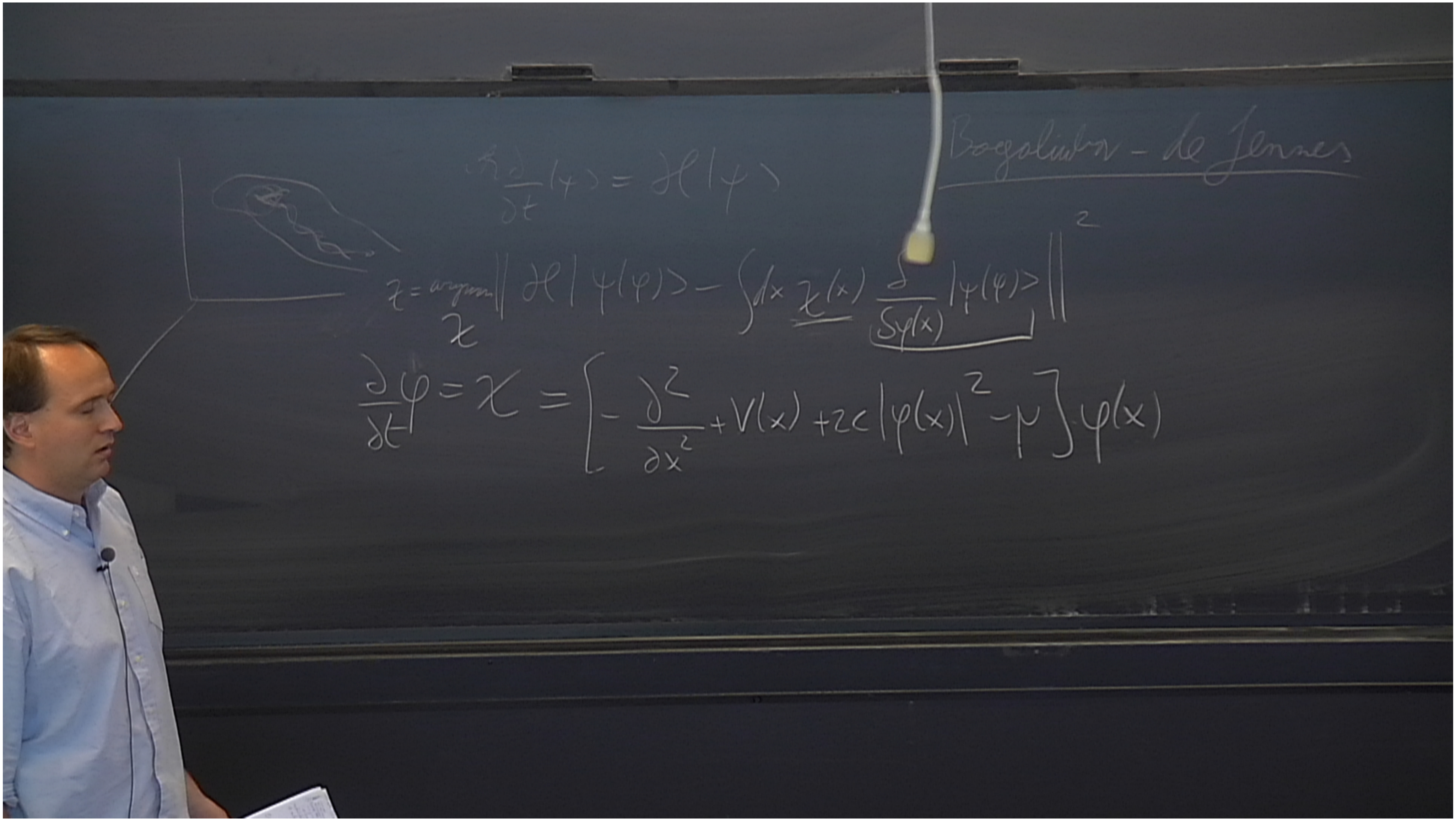

{ψ⁺ₓ, ψ_y} = δ(x−y)
Gross-Pitaevskii:
|ψ(φ)> = exp(∫₀ᴸ φ(x) ψ⁺ₓ) |Ω>




$i\hbar \frac{\partial}{\partial t}|\psi\rangle = \mathcal{H}|\psi\rangle$


$i\hbar \frac{\partial}{\partial t}|\psi\rangle = \mathcal{H}|\psi\rangle$
$\chi = \underset{\chi}{\operatorname{argmin}} \left\| \mathcal{H}|\psi(\varphi)\rangle - \int dx\, \underline{\chi(x)} \frac{\delta}{\delta\varphi(x)}|\psi(\varphi)\rangle \right\|^2$
$\frac{\partial}{\partial t}\varphi = \chi$


$i\hbar \frac{\partial}{\partial t}|\psi\rangle = \mathcal{H}|\psi\rangle$
$\chi = \operatorname{argmin}_{\chi} \left\| \mathcal{H}|\psi(\varphi)\rangle - \int dx\, \chi(x) \frac{\delta}{\delta\varphi(x)} |\psi(\varphi)\rangle \right\|^2$
$\frac{\partial}{\partial t}\varphi = \chi = \left[-\frac{\partial^2}{\partial x^2} + V(x) + 2c|\varphi(x)|^2 - \mu\right]\varphi(x)$


Bogoliubov - de Gennes
$i\hbar \frac{\partial}{\partial t}|\psi\rangle = \mathcal{H}|\psi\rangle$
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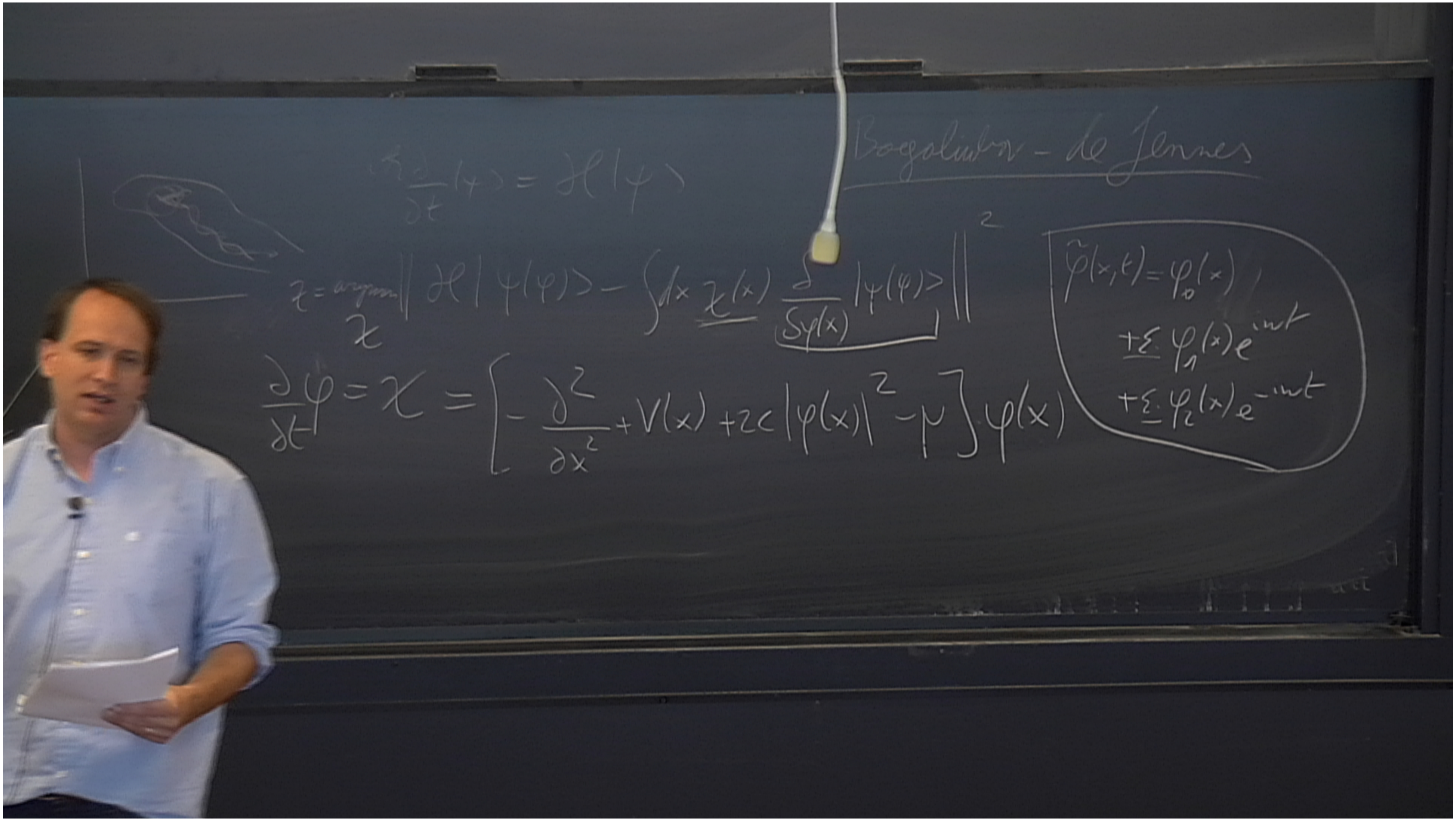
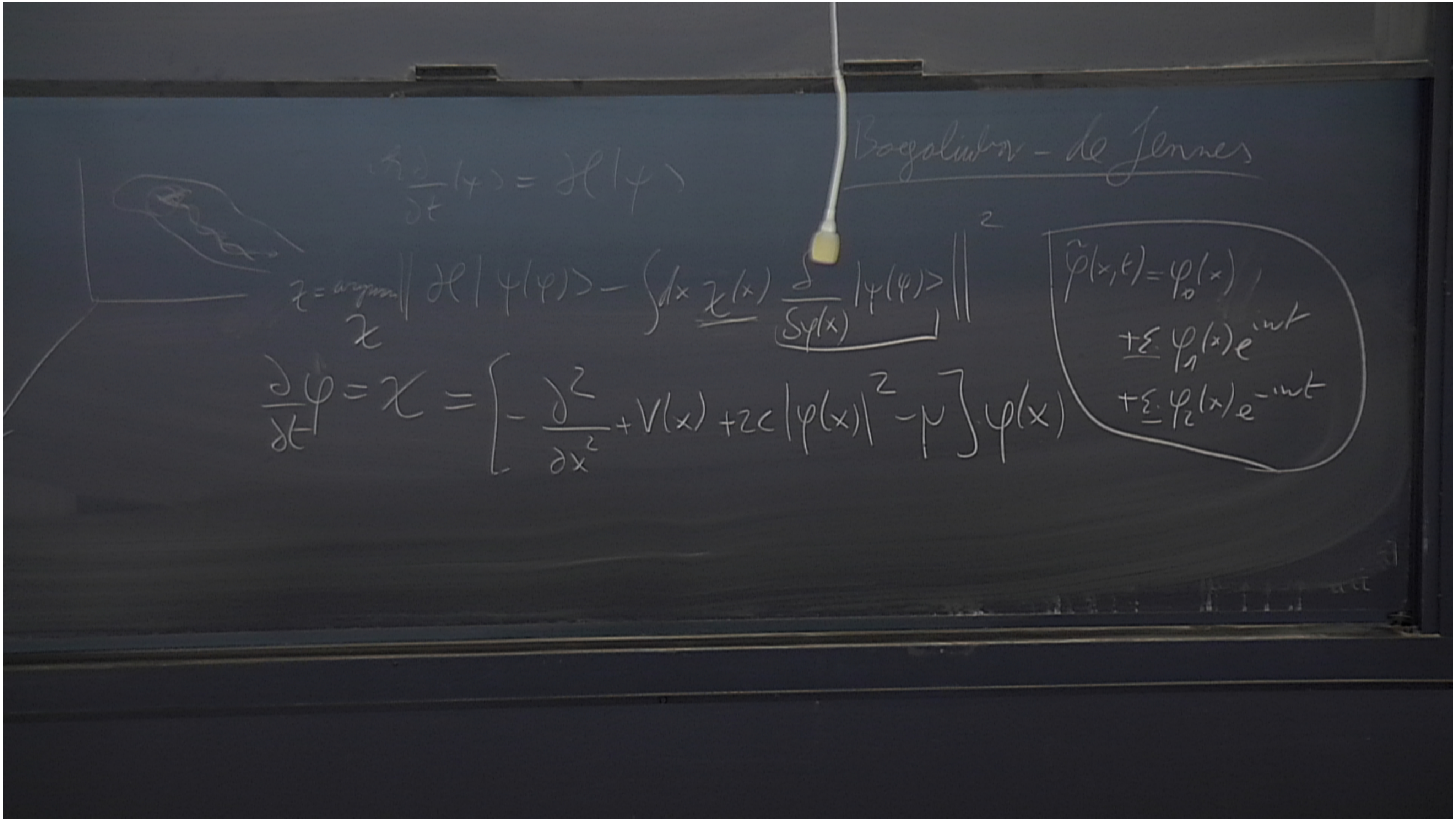
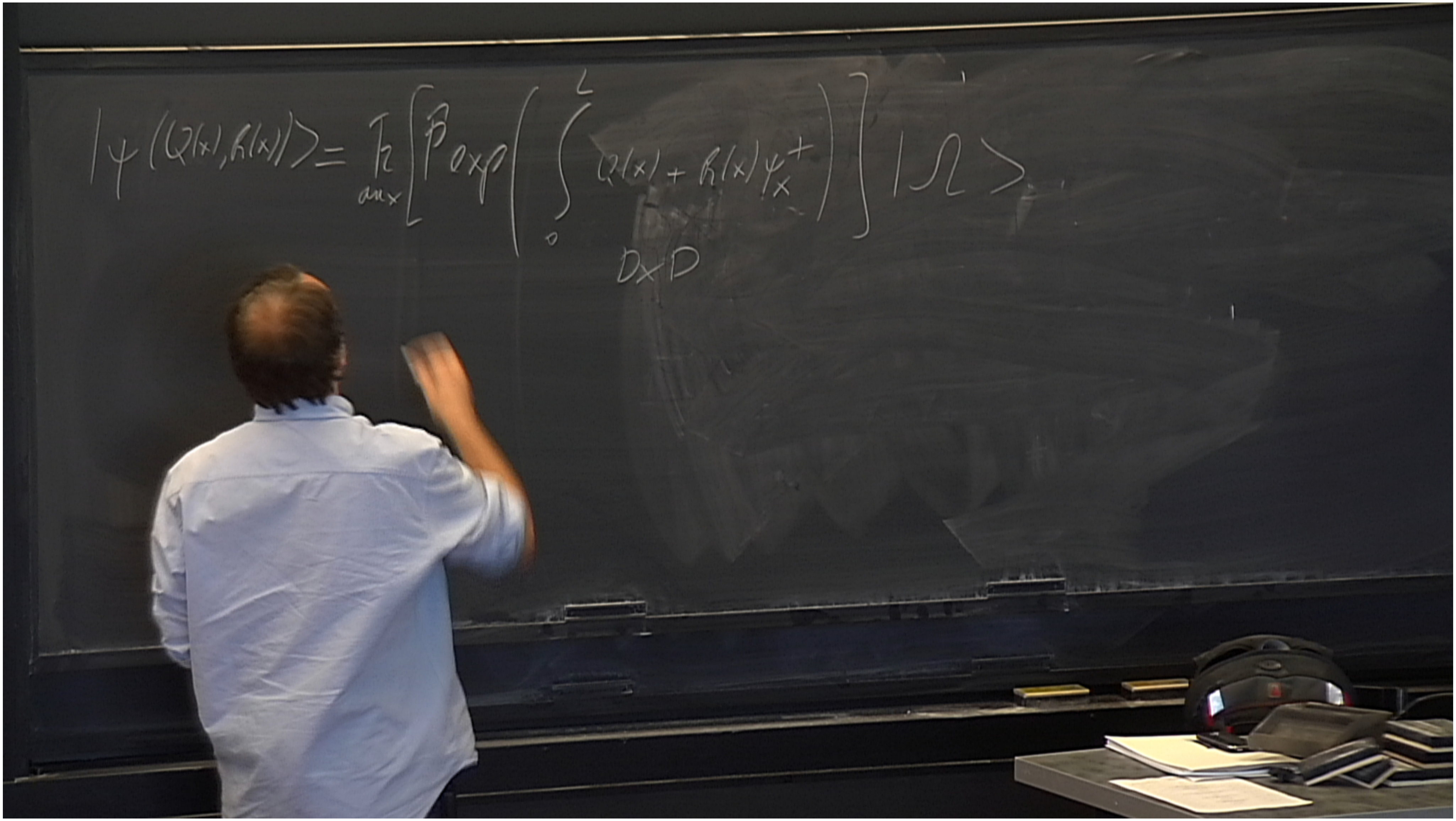
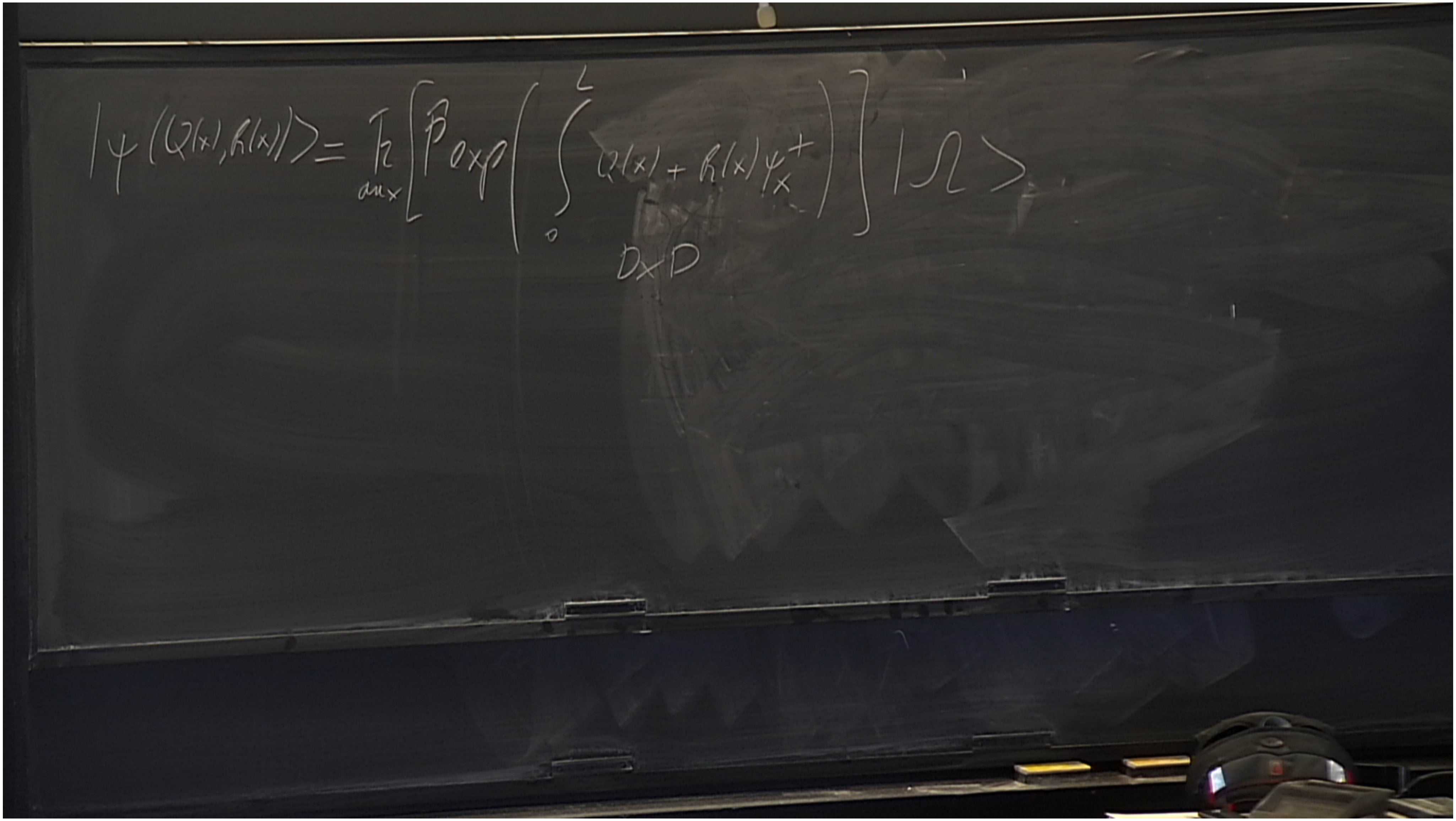
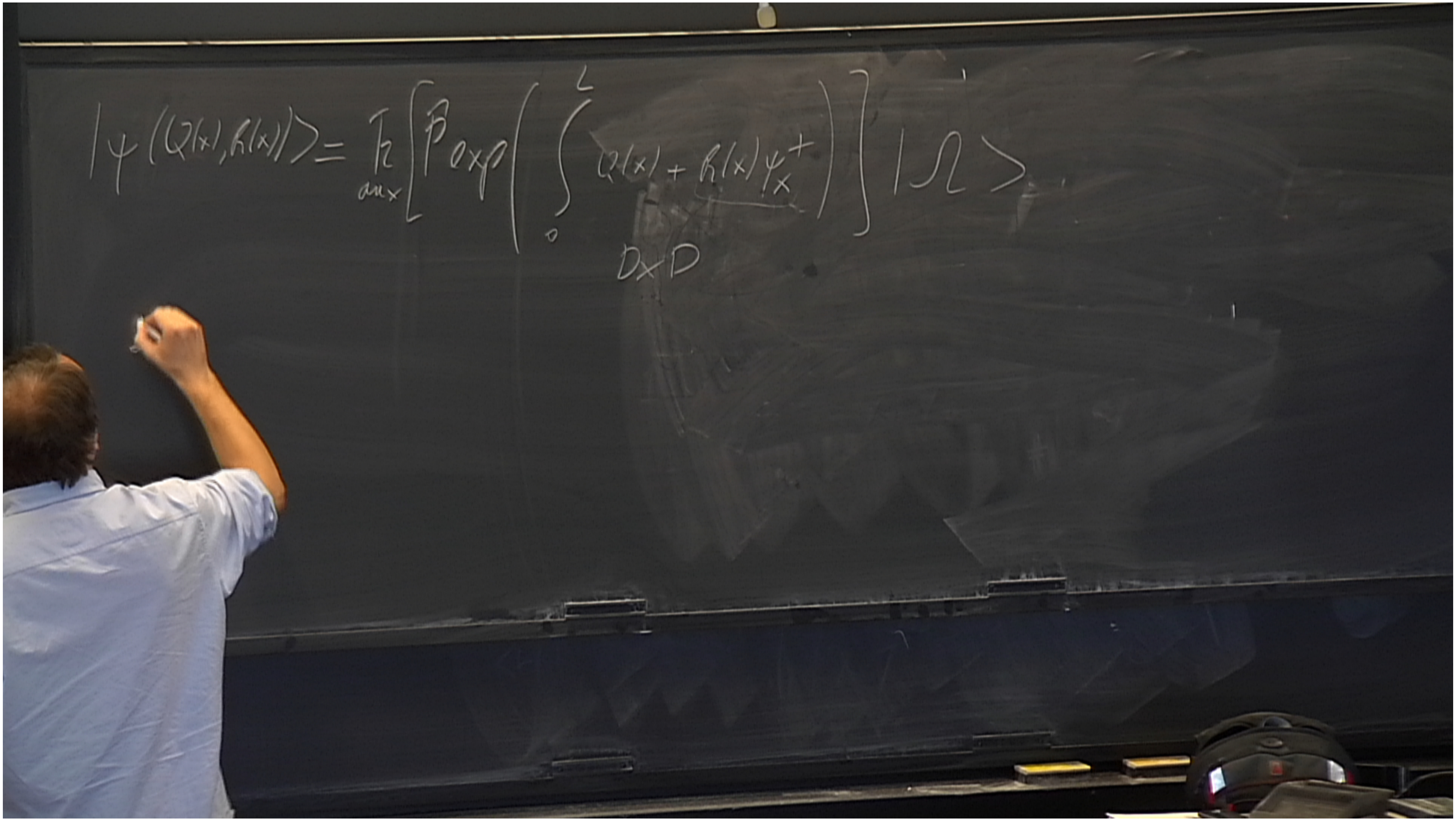
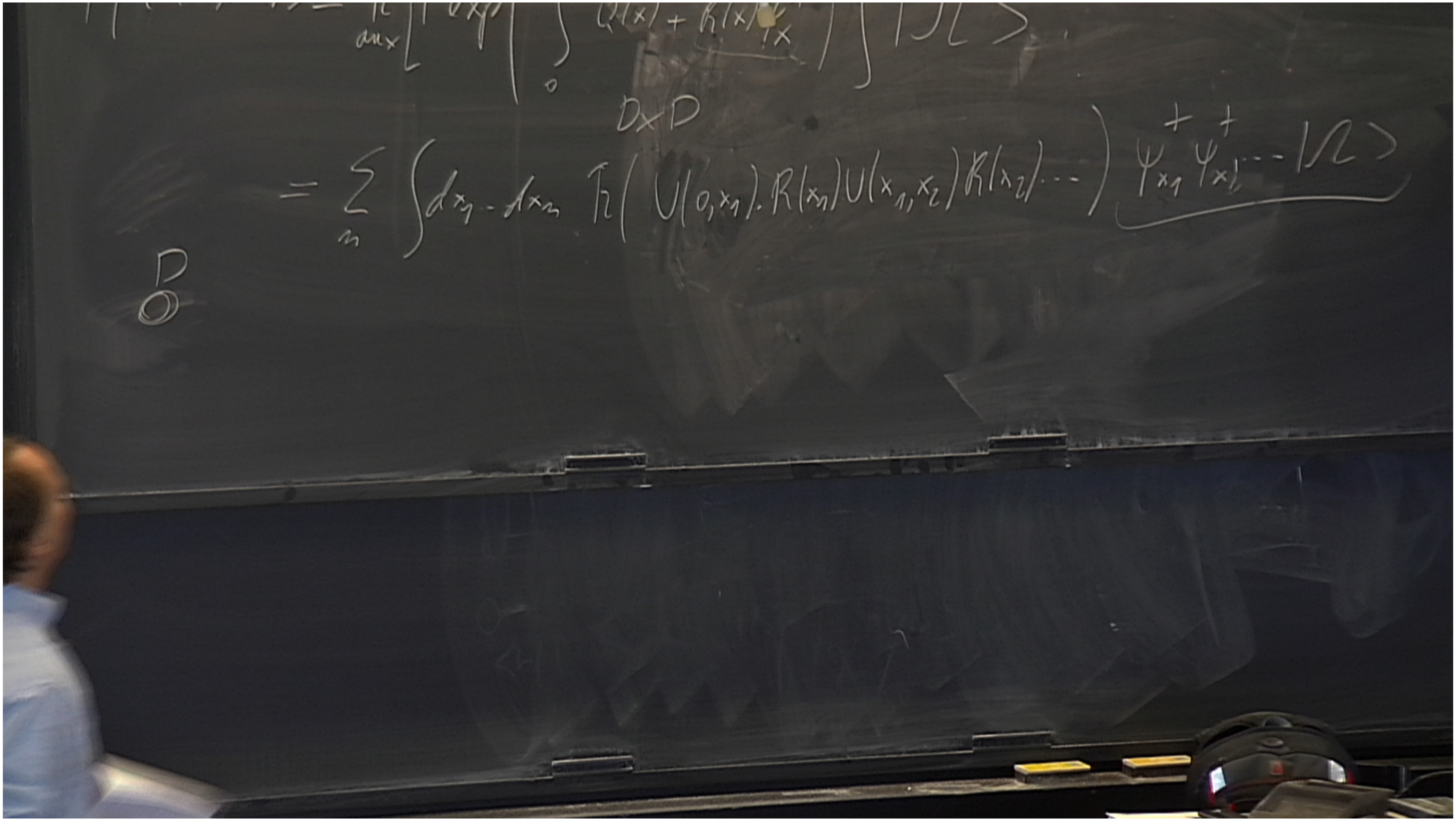

Bogoliubov - de Gennes

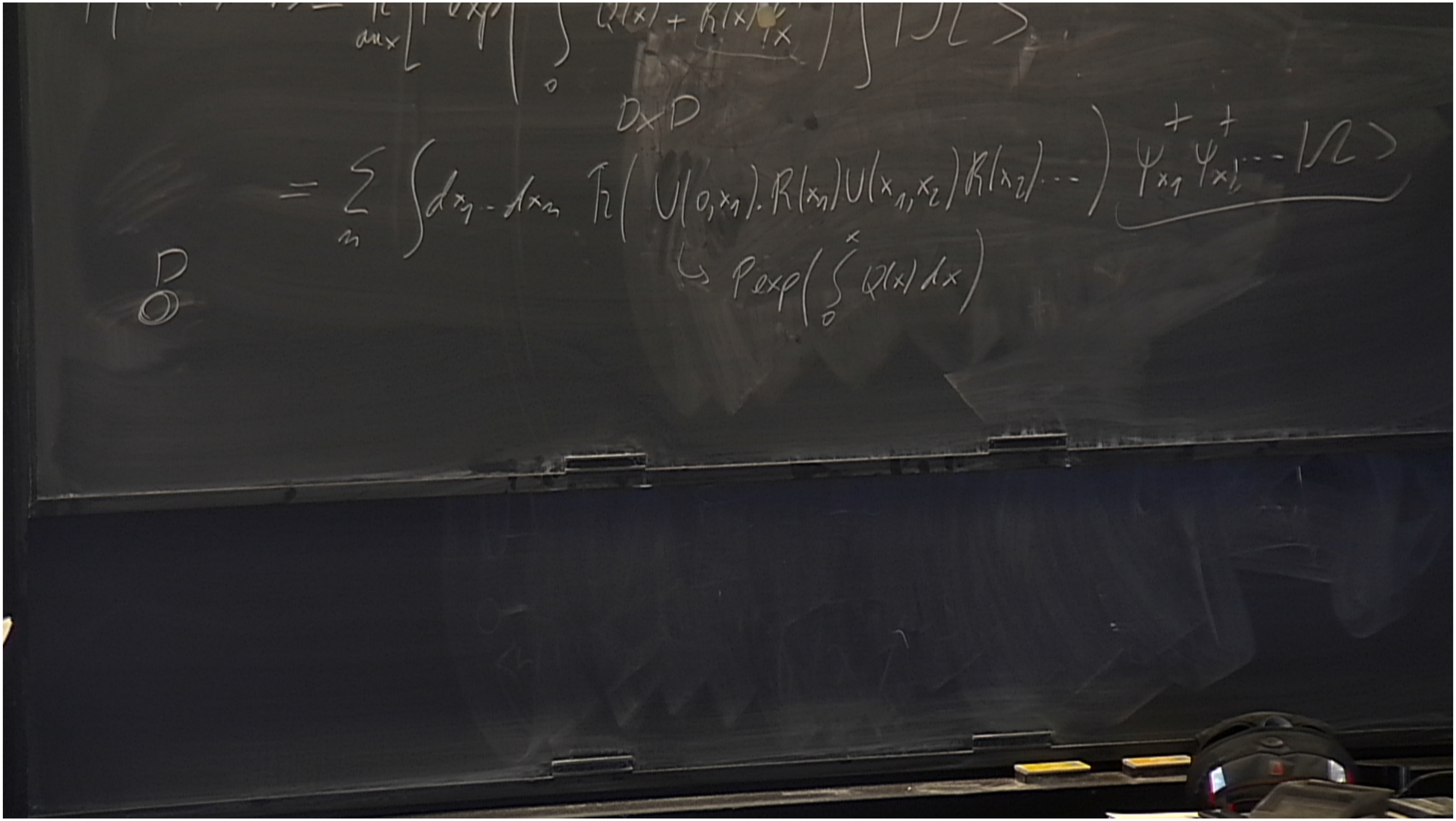

Bogoliubov - de Gennes
$i\hbar \frac{\partial}{\partial t}|\psi\rangle = \mathcal{H}|\psi\rangle$
$\chi = \mathrm{argmin}_{\chi} \left\| \mathcal{H}|\psi(\varphi)\rangle - \int dx\, \chi(x) \frac{\delta}{\delta\varphi(x)}|\psi(\varphi)\rangle \right\|^2$
$\frac{\partial}{\partial t}\varphi = \chi = \left[-\frac{\partial^2}{\partial x^2} + V(x) + 2c|\varphi(x)|^2 - \mu \right]\varphi(x)$
$\tilde{\varphi}(x,t) = \varphi_0(x) + \varepsilon\, \varphi_1(x) e^{i\omega t} + \varepsilon\, \varphi_2(x) e^{-i\omega t}$

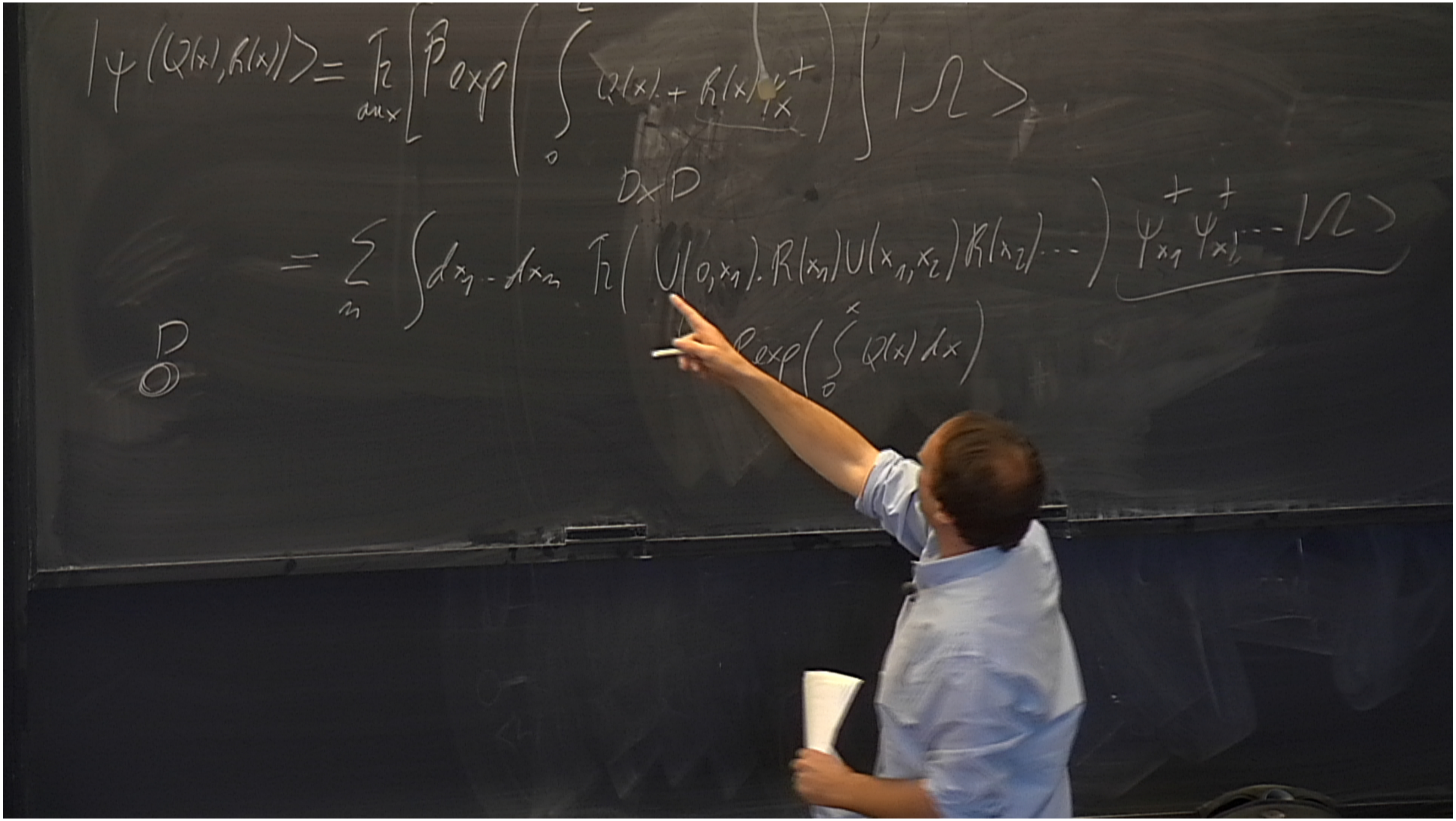

|ψ(Q(x), R(x))> = Tr_aux [P exp(∫_0^L Q(x) + R(x) ψ_x^+)] |Ω>
D×D

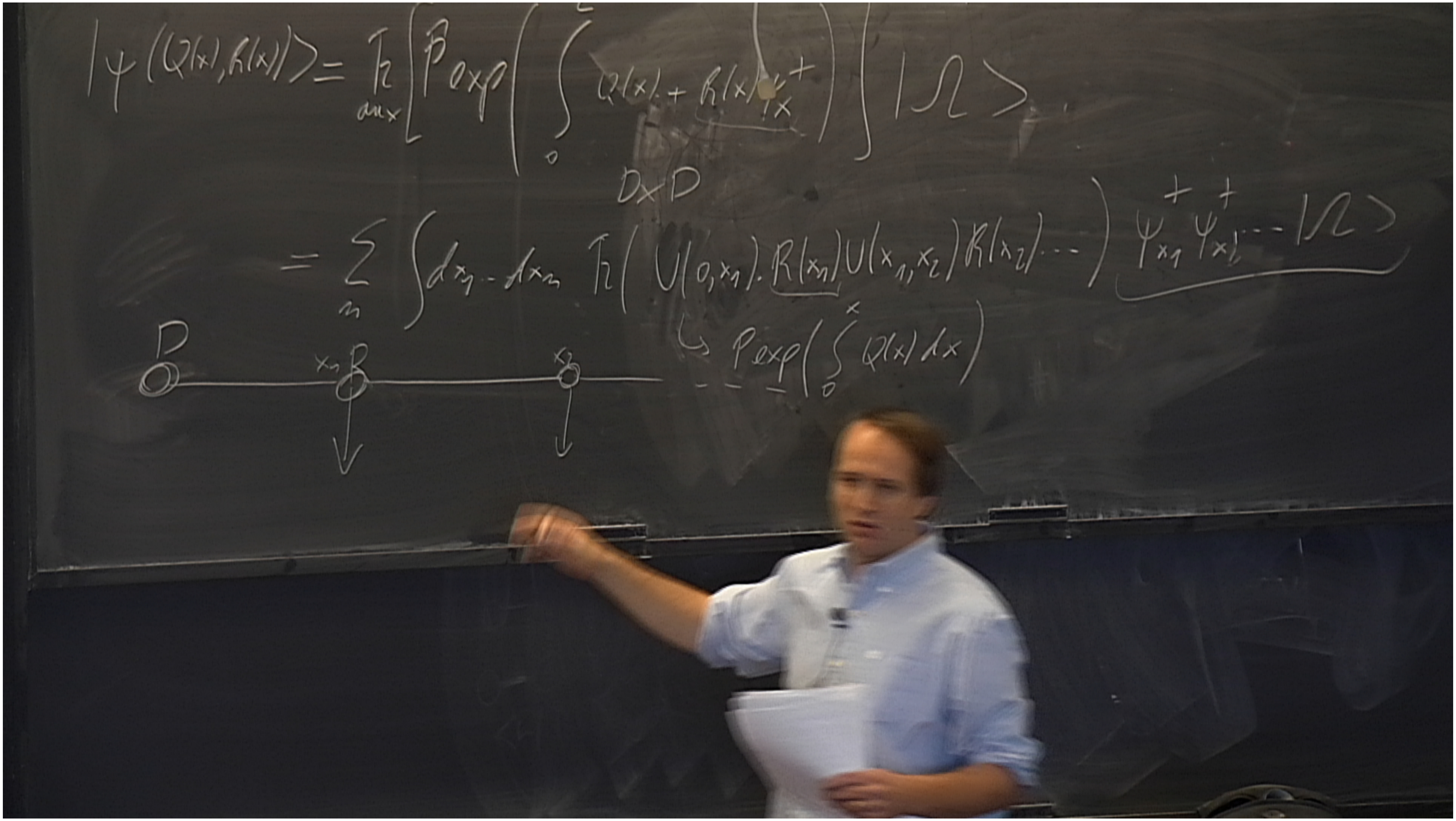

$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[\mathcal{P}\exp\left(\int_0^L Q(x) + R(x)\psi_x^\dagger\right)\right]|\Omega\rangle$
$D\times D$


$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[\hat{P}\exp\left(\int_0^L Q(x) + R(x)\psi_x^\dagger\right)\right]|\Omega\rangle$
D×D

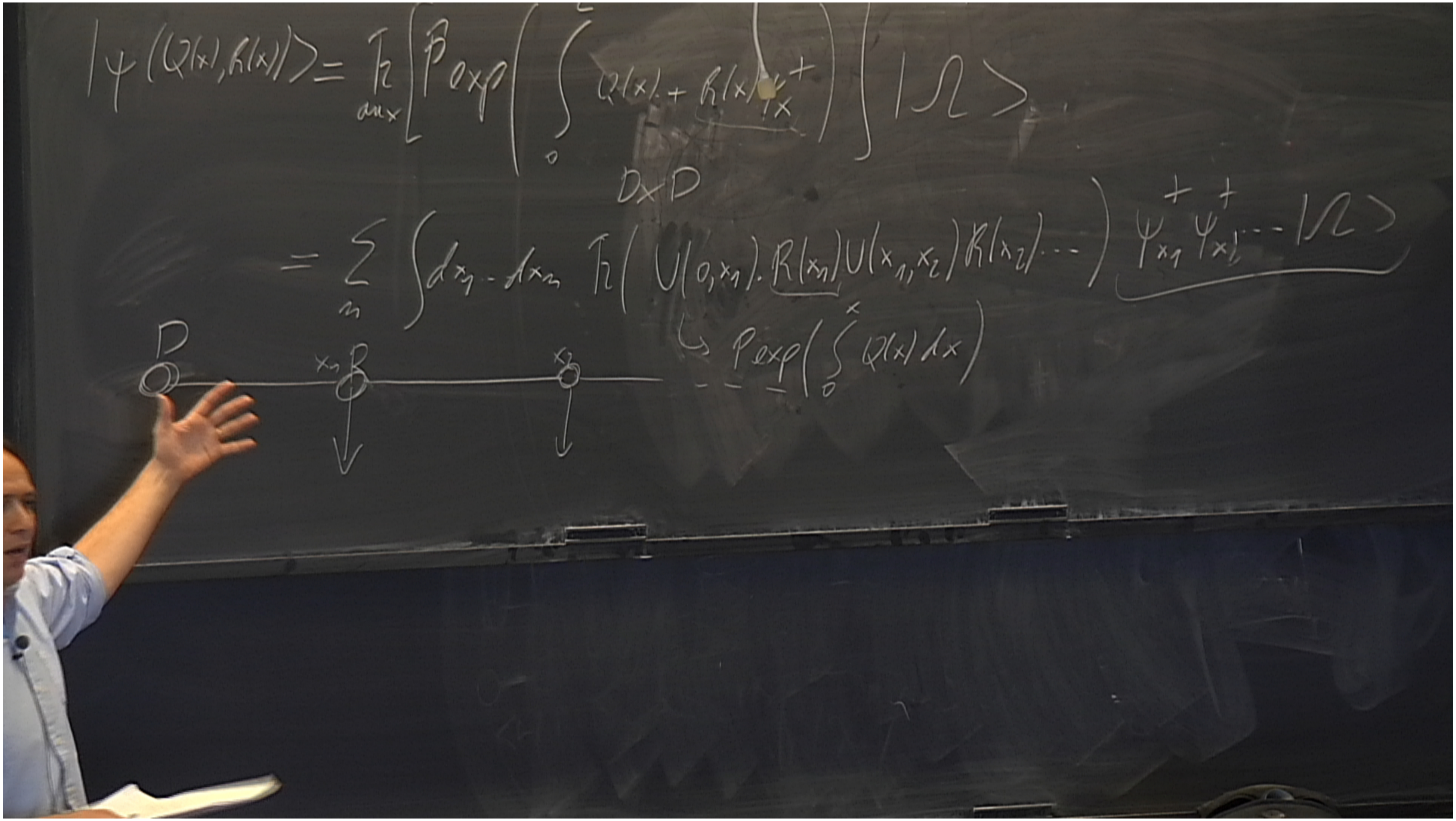

D×D
= Σ ∫dx1 .. dxn Tr(U(0,x1). R(x1) U(x1,x2) R(x2) ...) ψ+x1 ψ+x2 ... |Ω>


D×D
= Σ_n ∫dx_1 .. dx_n Tr(U(0,x_1). R(x_1) U(x_1,x_2) R(x_2) ...) ψ†_{x_1} ψ†_{x_2} ... |Ω>
↳ P exp(∫_0^x Q(x) dx)


$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[P\exp\left(\int_0 Q(x) + R(x)\psi_x^+ \right)\right] |\Omega\rangle$
$D\times D$
$= \sum_n \int dx_1 .. dx_n \, \mathrm{Tr}\left(U(0,x_1) R(x_1) U(x_1,x_2) R(x_2) \dots \right) \psi_{x_1}^+ \psi_{x_2}^+ \dots |\Omega\rangle$
$P\exp\left(\int_0^x Q(x)\, dx \right)$



$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[P\exp\left(\int_0^L Q(x) + R(x)\psi_x^+\right)\right]|\Omega\rangle$
$D\times D$
$= \sum_n \int dx_1 \ldots dx_n \, \mathrm{Tr}\left(U(0,x_1).R(x_1)U(x_1,x_2)R(x_2)\ldots\right)\psi_{x_1}^+\psi_{x_2}^+\ldots|\Omega\rangle$
$P\exp\left(\int_0^x Q(x)\,dx\right)$
D
x_1
x_2



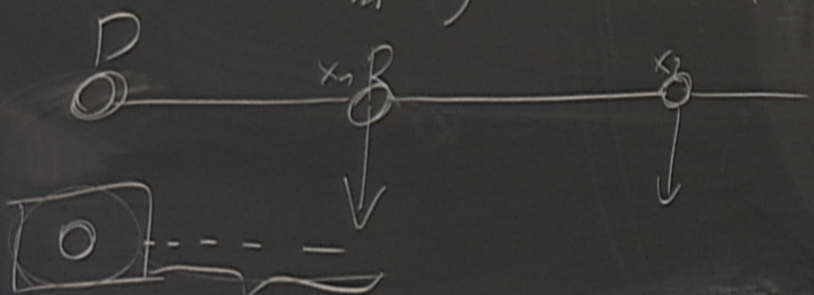
|ψ(Q(x), R(x))> = Tr_aux [P exp(∫ Q(x) + R(x) ψ+_x)] |Ω>
D×D
= Σ_n ∫dx_1 .. dx_n Tr(U(0,x_1). R(x_1) U(x_1,x_2) R(x_2) ...) ψ+_x1 ψ+_x2 ... |Ω>
P exp(∫_0^x Q(x) dx)
D
x_1
x_2

$$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[P\exp\left(\int_0^L Q(x) + R(x)\,\psi_x^{+}\right)\right]|\Omega\rangle$$

$D \times D$

$$= \sum_n \int dx_1 \ldots dx_n \, \mathrm{Tr}\left(U(0,x_1)\, R(x_1)\, U(x_1,x_2)\, R(x_2) \cdots \right) \psi^{+}_{x_1}\psi^{+}_{x_2}\cdots|\Omega\rangle$$

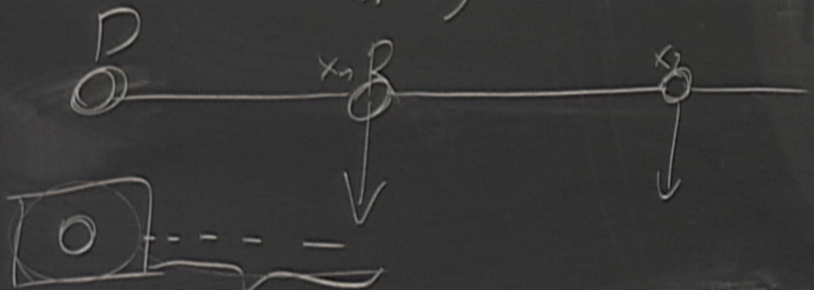
$\hookrightarrow P\exp\left(\int_0^x Q(x)\,dx\right)$

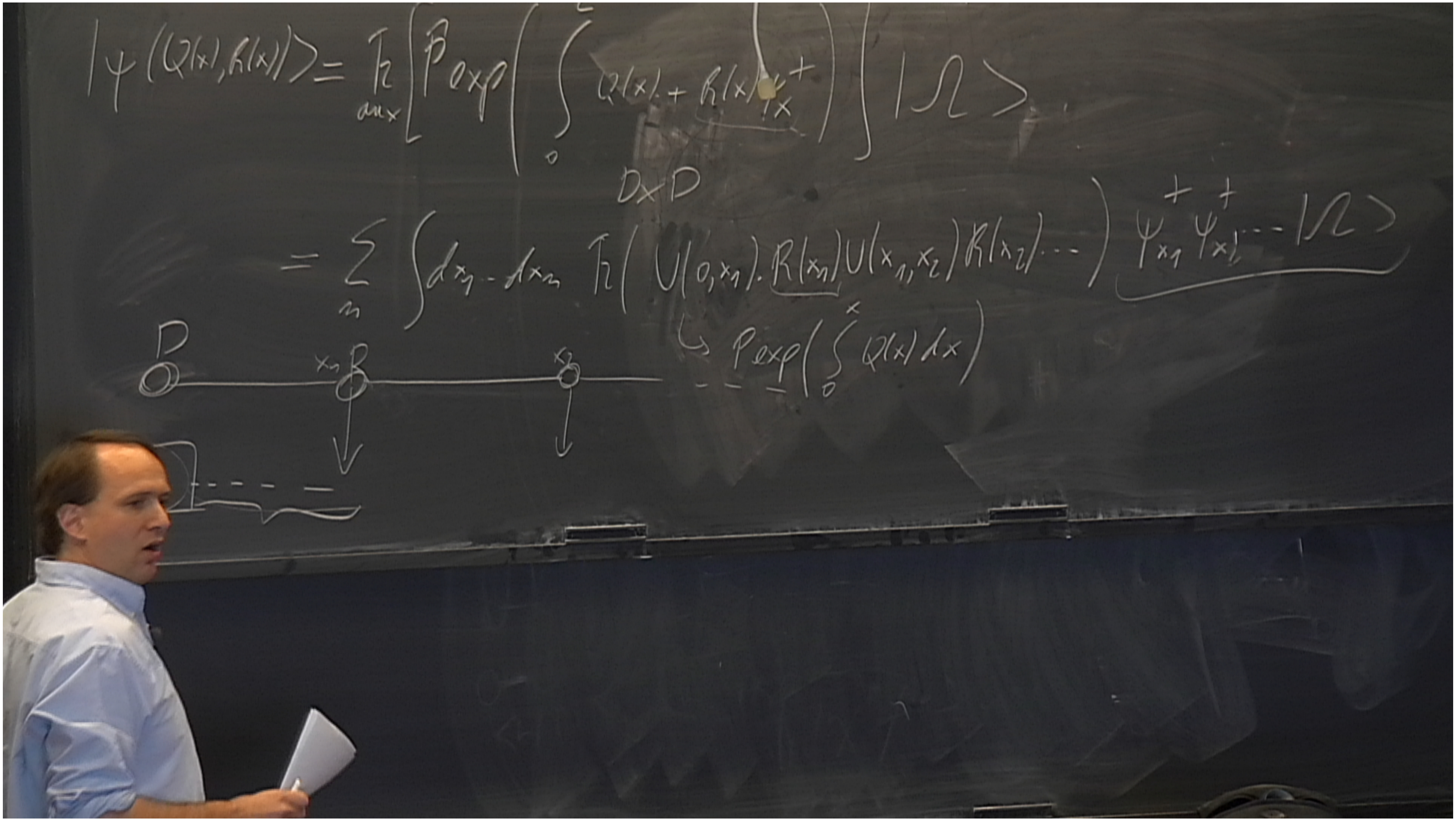


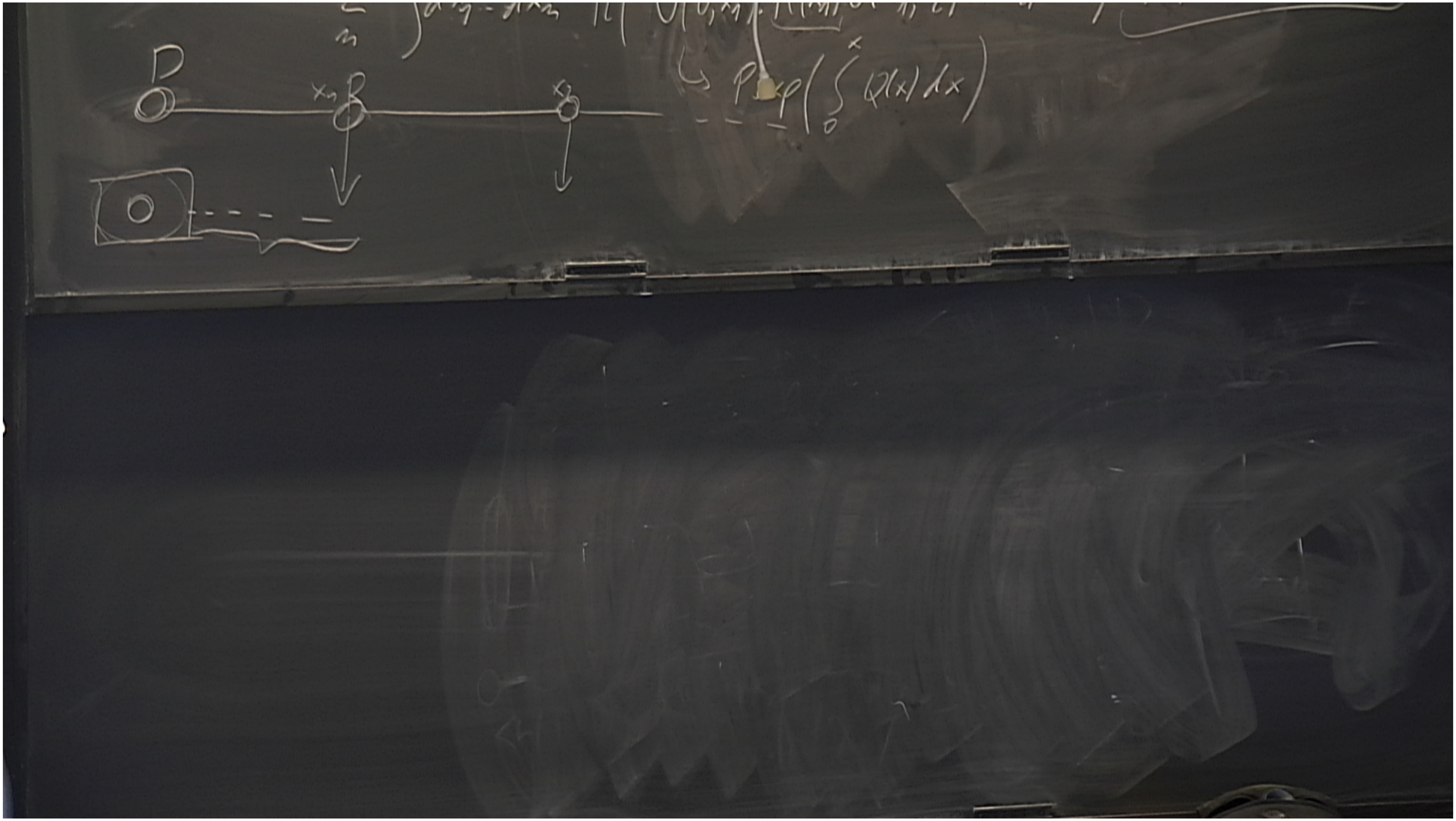
$$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[\mathcal{P}\exp\left(\int_0^L Q(x) + \underbrace{R(x)\psi_x^\dagger}_{D\times D}\right)\right]|\Omega\rangle$$

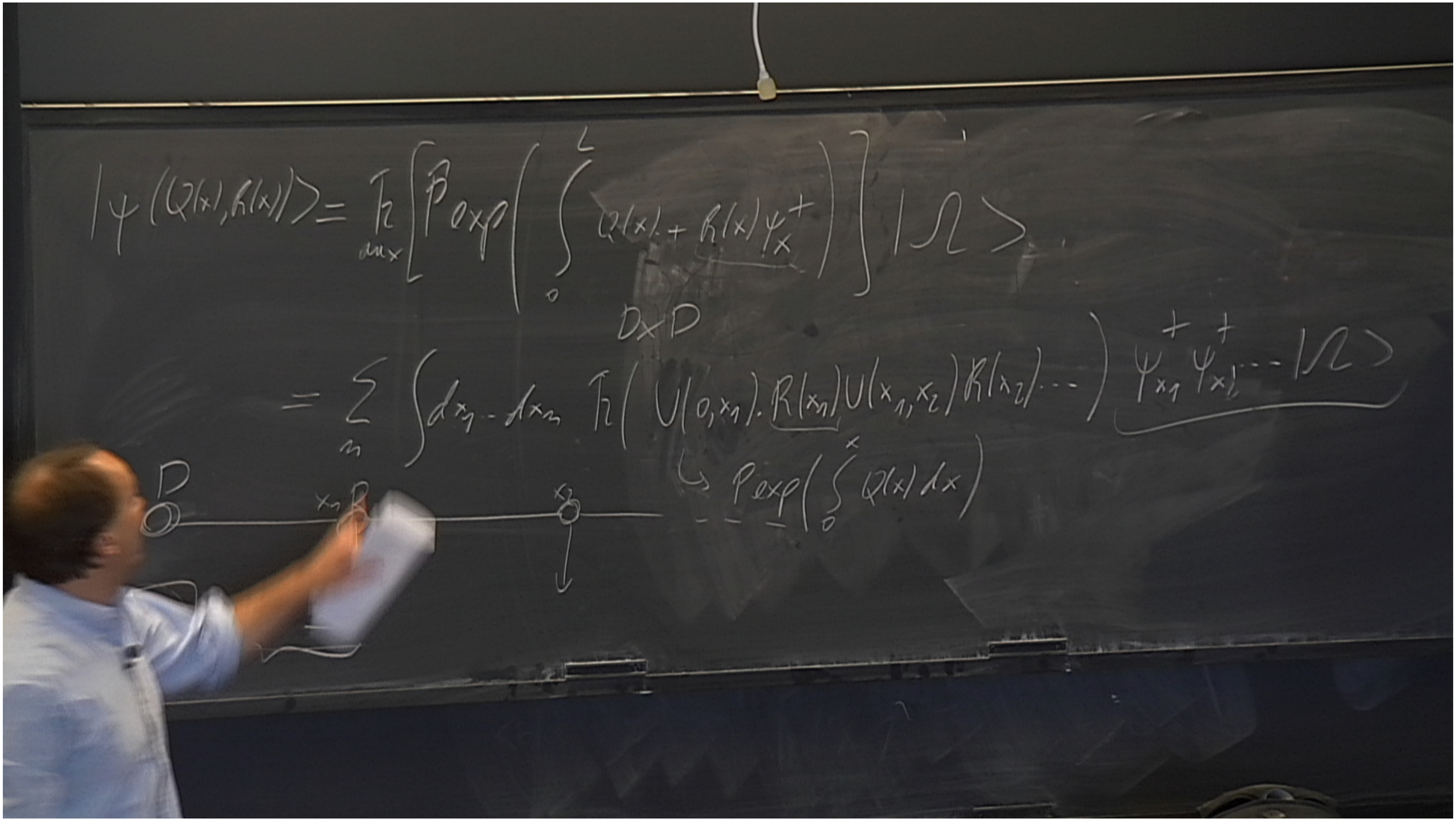
$$= \sum_n \int dx_1 \ldots dx_n \, \mathrm{Tr}\left(U(0,x_1)\,\underline{R(x_1)}\,U(x_1,x_2)\,R(x_2)\cdots\right)\underline{\psi_{x_1}^\dagger \psi_{x_2}^\dagger \cdots |\Omega\rangle}$$

$$\hookrightarrow \mathcal{P}\exp\left(\int_0^x Q(x)\,dx\right)$$

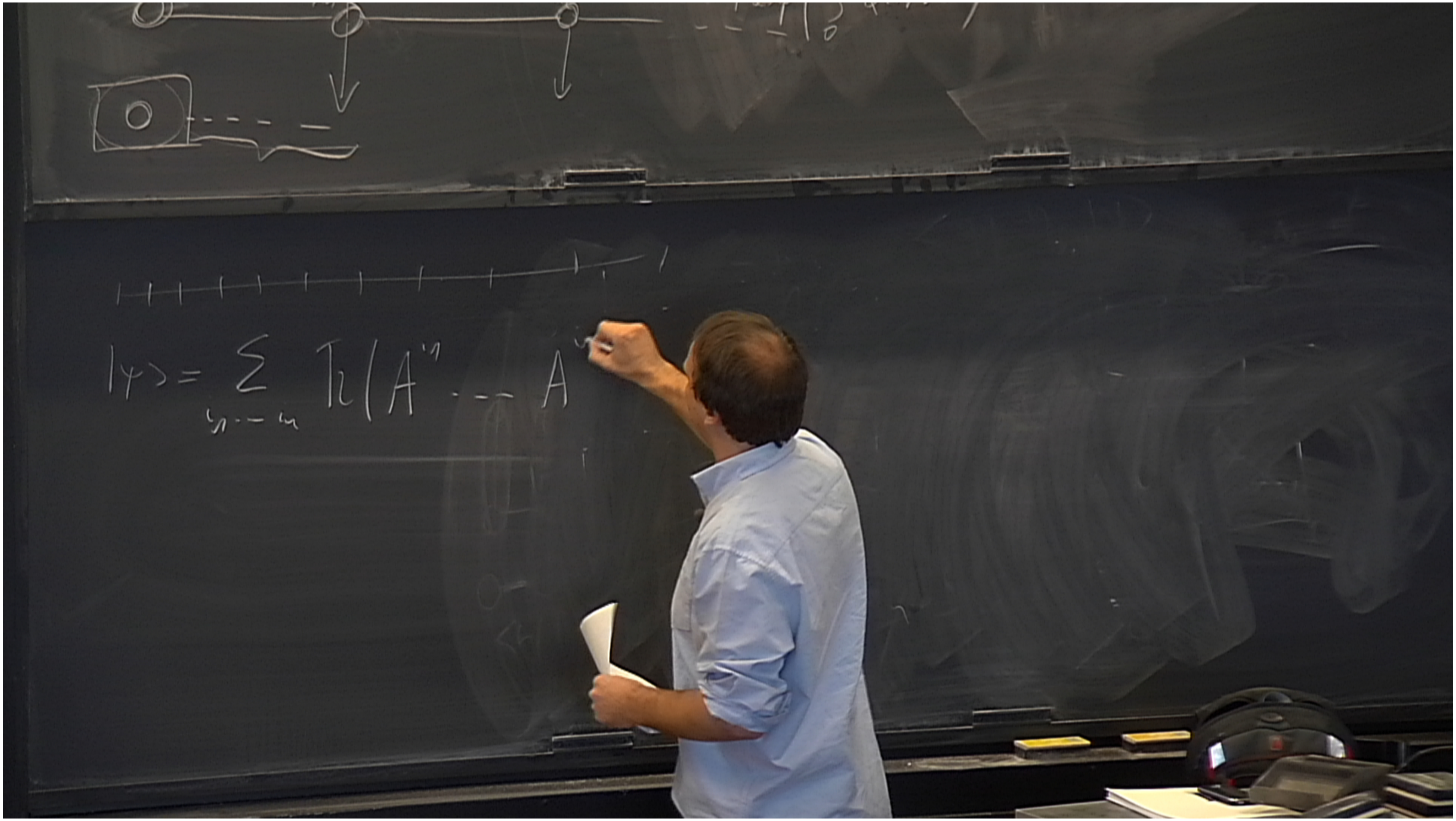


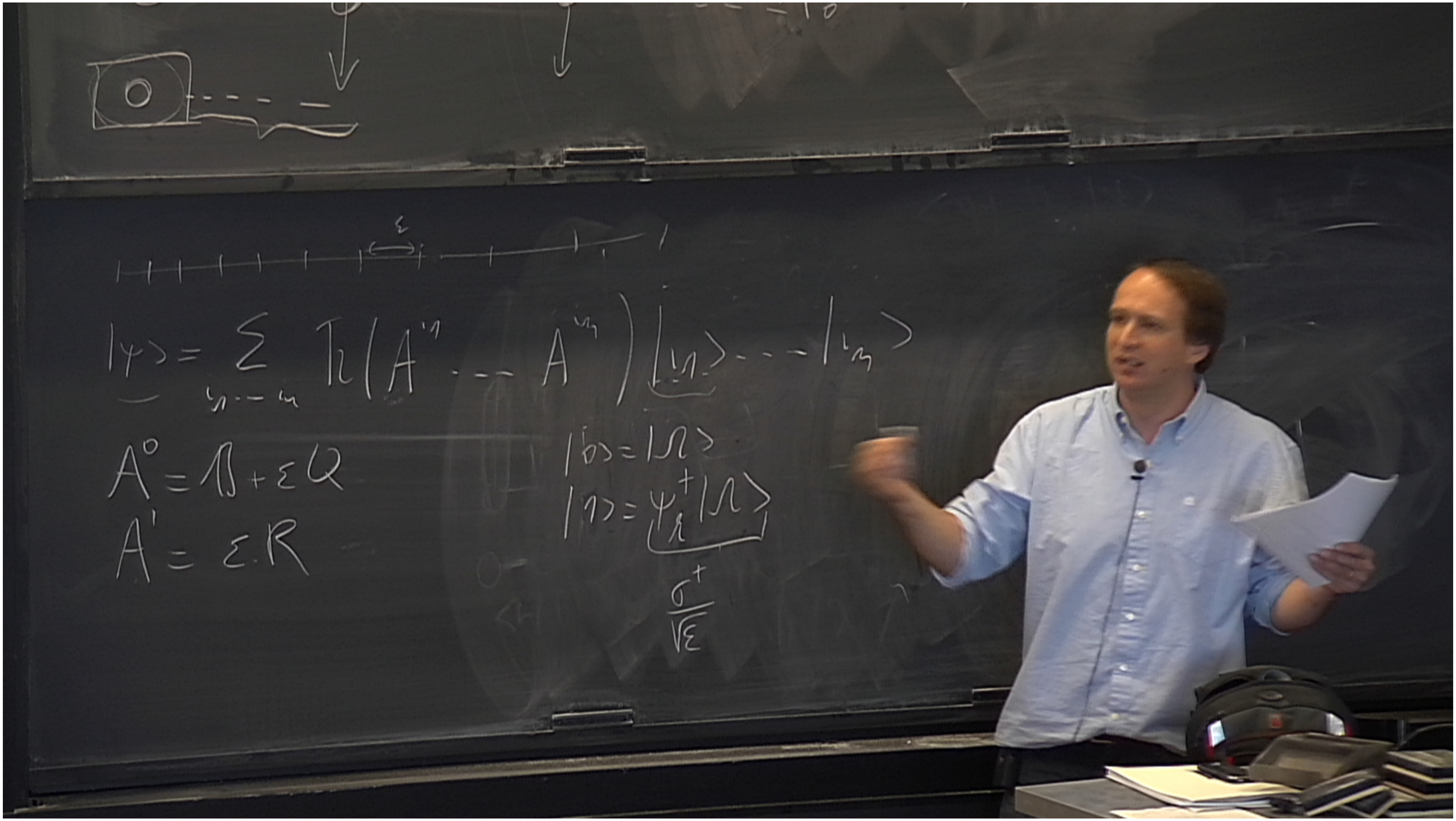

$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[P\exp\left(\int_0^L Q(x) + R(x)\psi_x^+ \right)\right] |\Omega\rangle$
$D \times D$
$= \sum_n \int dx_1 \ldots dx_n \; \mathrm{Tr}\left(U(0,x_1) R(x_1) U(x_1,x_2) R(x_2) \ldots \right) \psi_{x_1}^+ \psi_{x_2}^+ \ldots |\Omega\rangle$
$P\exp\left(\int_0^x Q(x)\,dx \right)$



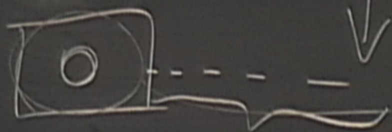


$|\psi(Q(x),R(x))\rangle = \mathrm{Tr}_{aux}\left[P\exp\left(\int_0^L Q(x) + R(x)\psi_x^+\right)\right]|\Omega\rangle$
$D\times D$
$= \sum_n \int dx_1 \dots dx_n \, \mathrm{Tr}\left(U(0,x_1).R(x_1)U(x_1,x_2)R(x_2)\dots\right)\psi_{x_1}^+\psi_{x_2}^+\dots|\Omega\rangle$
$P\exp\left(\int_0^x Q(x)\,dx\right)$


$|\psi\rangle = \sum_{i_1 \ldots i_n} \mathrm{Tr}(A^{i_1} \ldots A$



$|\psi\rangle = \sum_{i_1 \ldots i_n} \mathrm{Tr}(A^{i_1} \ldots A^{i_n}) |i_1\rangle \ldots |i_n\rangle$
$A^0 = 1\!\!1 + \varepsilon Q$
$A^1 = \varepsilon . R$
$|0\rangle = |\Omega\rangle$
$|1\rangle = \psi_x^\dagger |\Omega\rangle$
$\frac{\sigma^+}{\sqrt{\varepsilon}}$



$$|\psi\rangle = \sum_{i_1 \dots i_n} \mathrm{Tr}\left(A^{i_1} \dots A^{i_n}\right) |i_1\rangle \dots |i_n\rangle$$

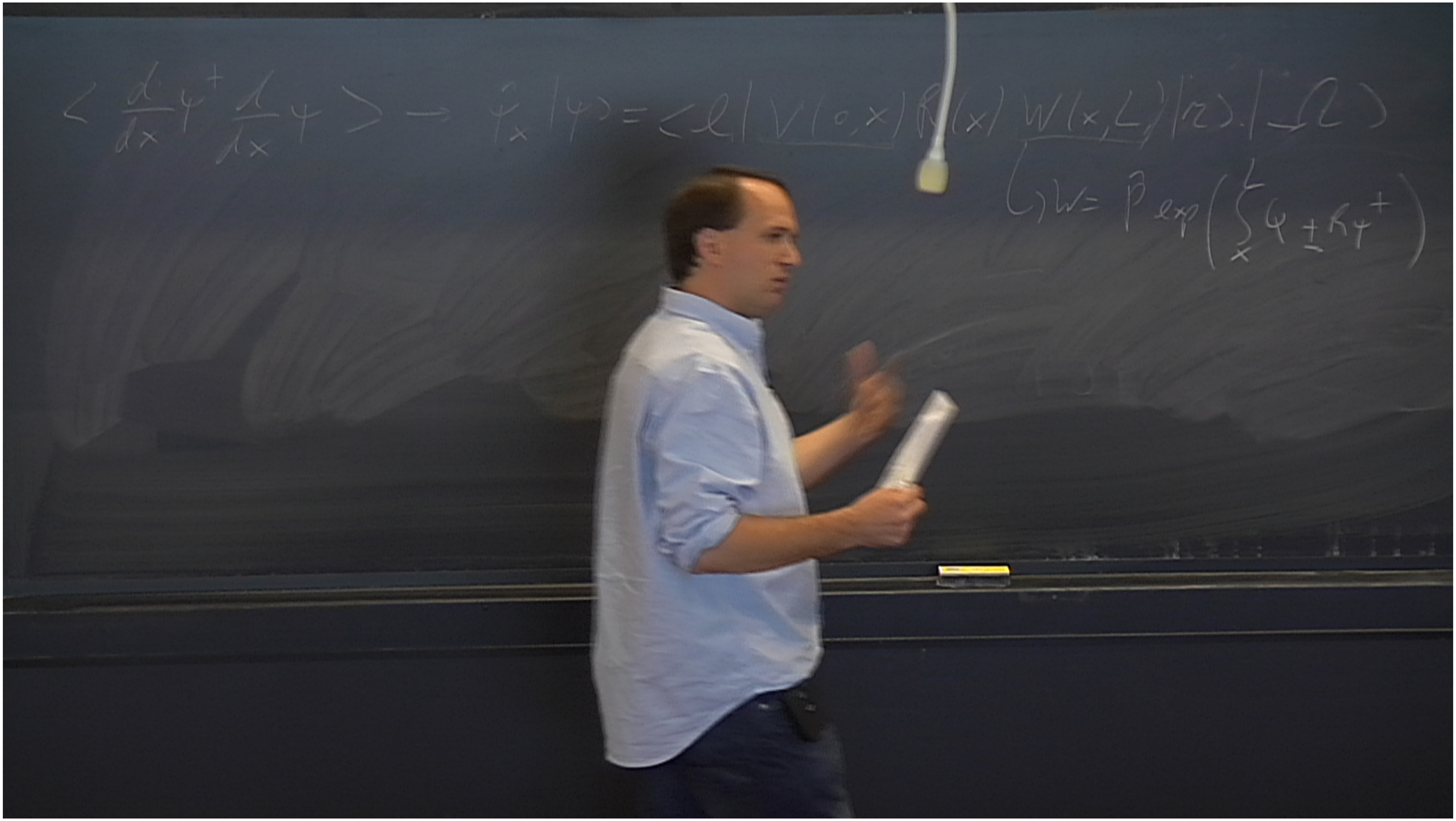
$$A^0 = \mathbb{1} + \varepsilon Q$$

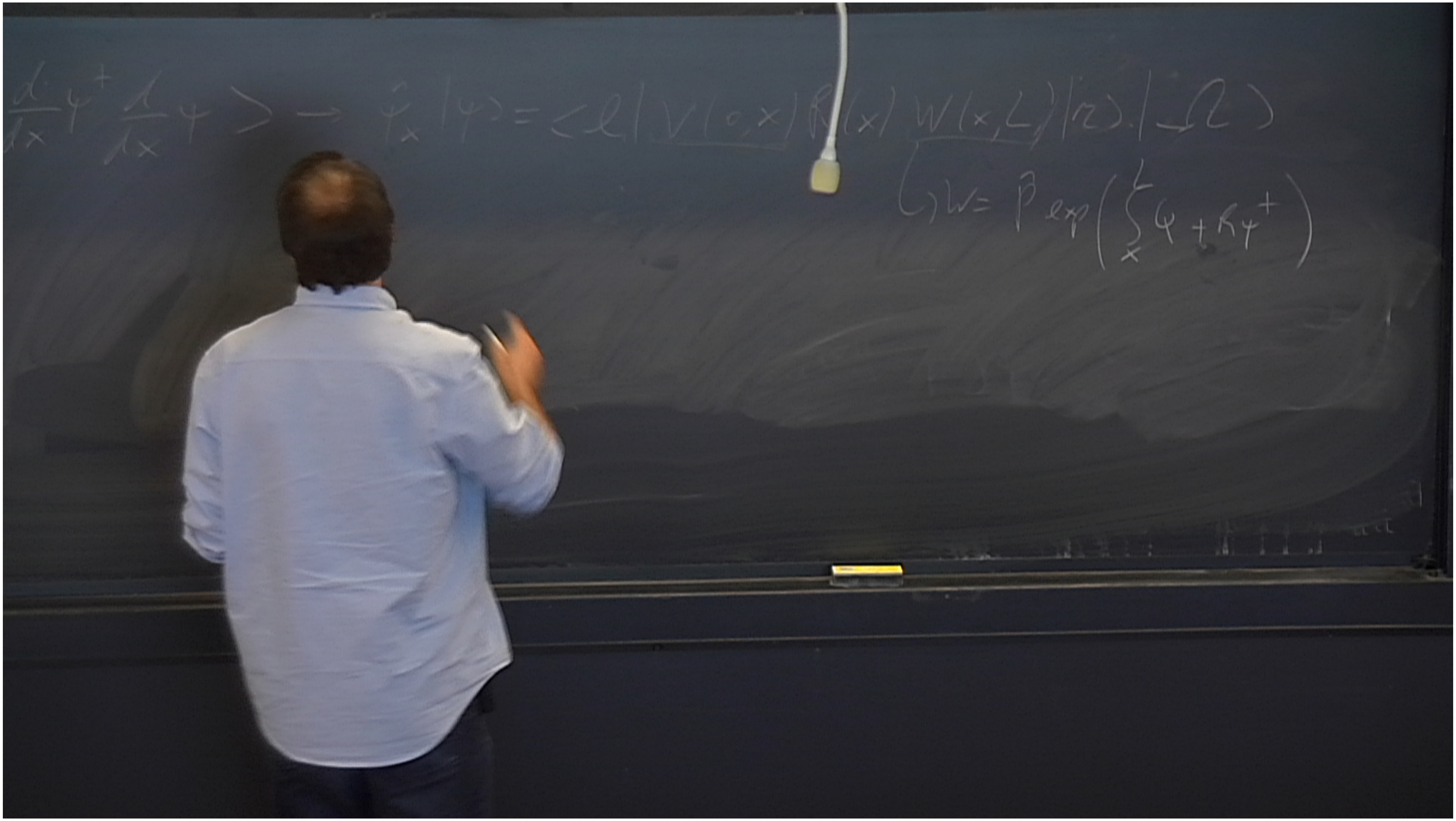
$$A^1 = \varepsilon R$$

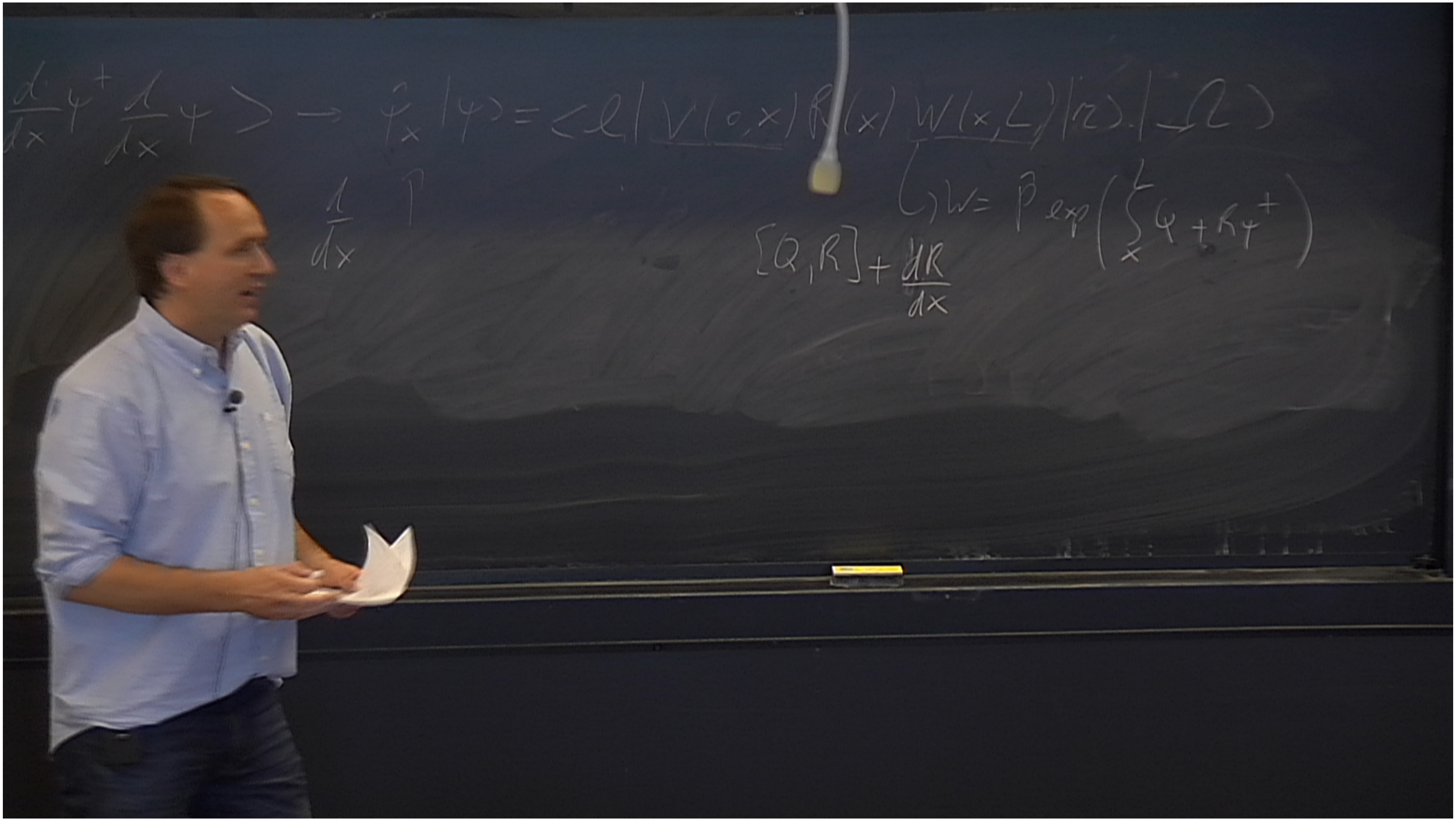
$$|0\rangle = |\Omega\rangle$$

$$|1\rangle = \psi_x^\dagger |\Omega\rangle$$

$$\frac{\sigma^+}{\sqrt{\varepsilon}} = \psi_x^\dagger$$







$$\frac{d}{dx}\psi^+ \frac{d}{dx}\psi \rangle \to \psi_x |\psi\rangle = \langle \ell | V(0,x) R(x) W(x,L) | r\rangle \, | \Omega \rangle$$

$$\frac{d}{dx} \quad P$$

$$\hookrightarrow W = P \exp\left(\int_x^L Q + R\psi^+ \right)$$

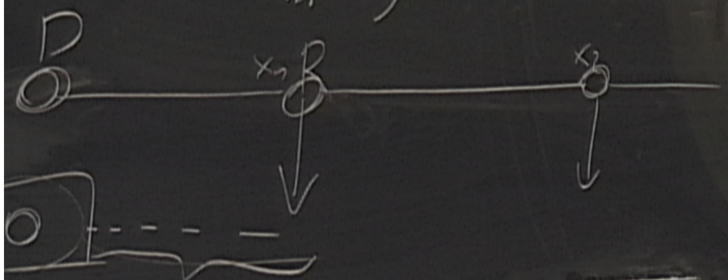
$$[Q,R] + \frac{dR}{dx}$$

$$+ (-1)^s R^2 - R^2$$

$$|\chi(Q(x),R(x))\rangle = \mathrm{Tr}_{\mathrm{aux}}\left[\mathcal{P}\exp\left(\int_0^L \underbrace{Q(x)\otimes 1 + \sum_\alpha R_\alpha(x)\otimes\psi^{\dagger}_{\alpha,x}}_{D\times D}\right)\right]|\Omega\rangle$$

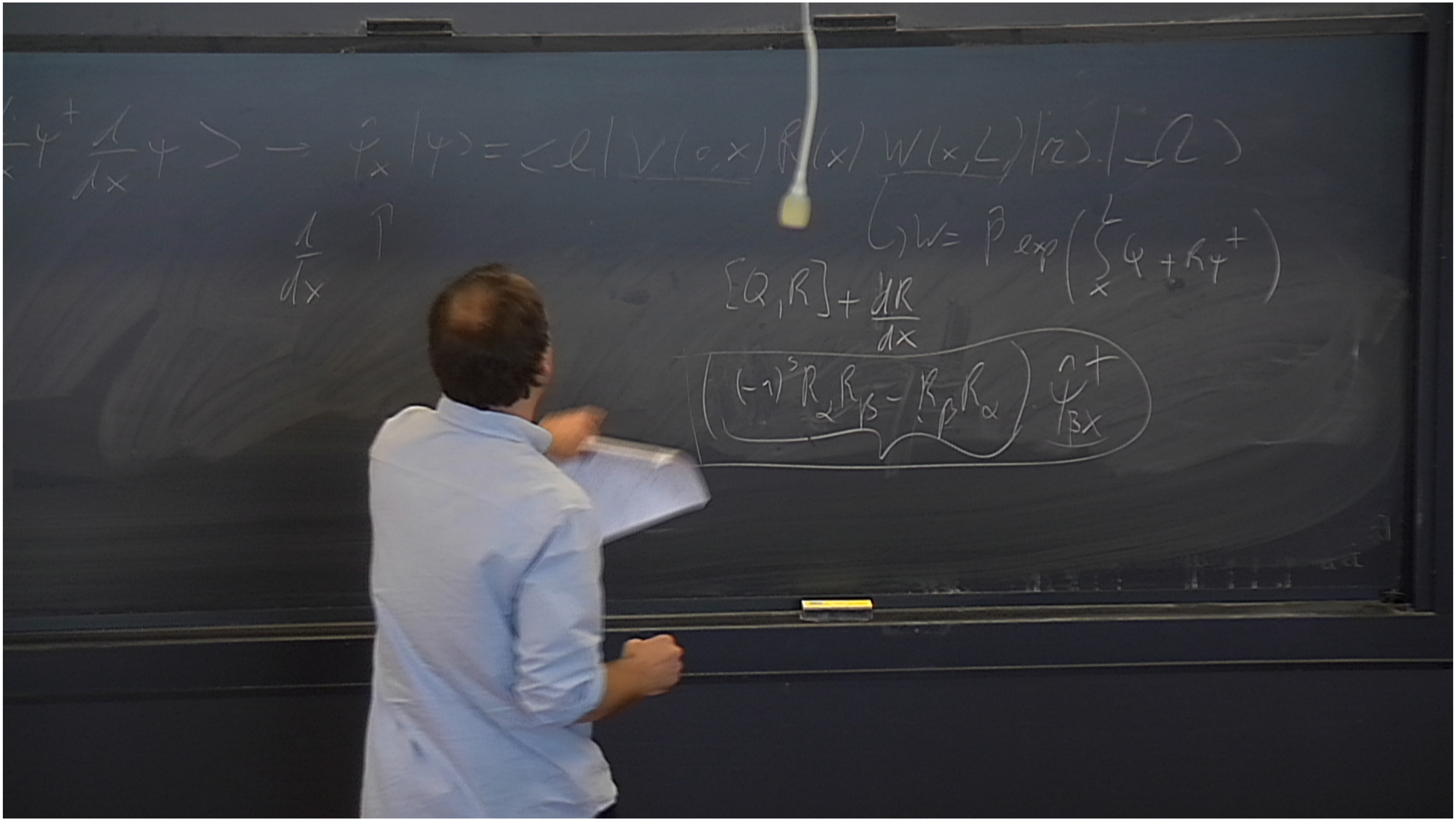
$$= \sum_n \int dx_1 \ldots dx_n \, \mathrm{Tr}\left(U(0,x_1)\, R(x_1)\, U(x_1,x_2)\, R(x_2)\ldots\right) \psi^{\dagger}_{x_1}\psi^{\dagger}_{x_2}\ldots|\Omega\rangle$$

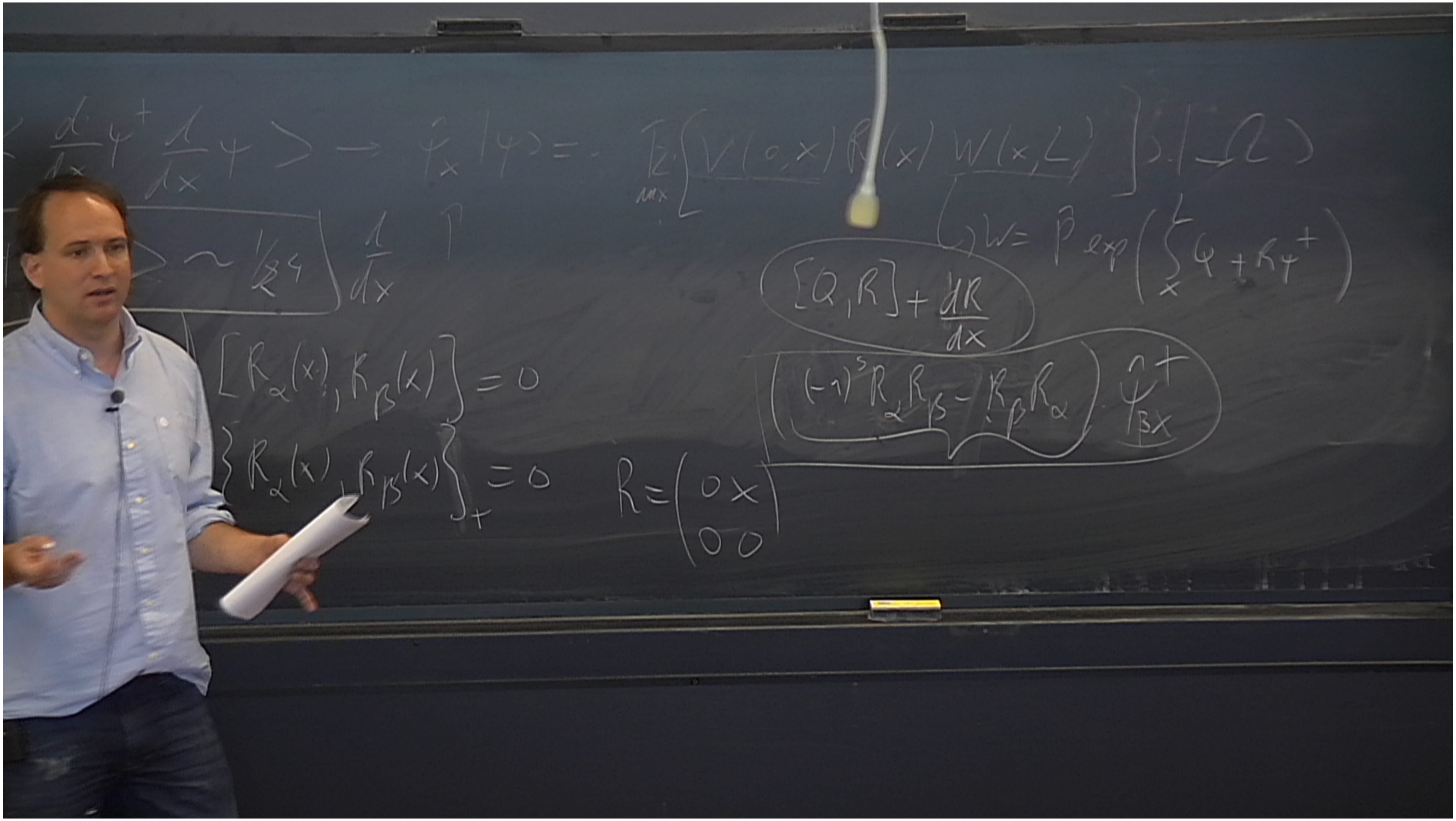
$$U(0,x_1) \hookrightarrow \mathcal{P}\exp\left(\int_0^x Q(x)\,dx\right)$$

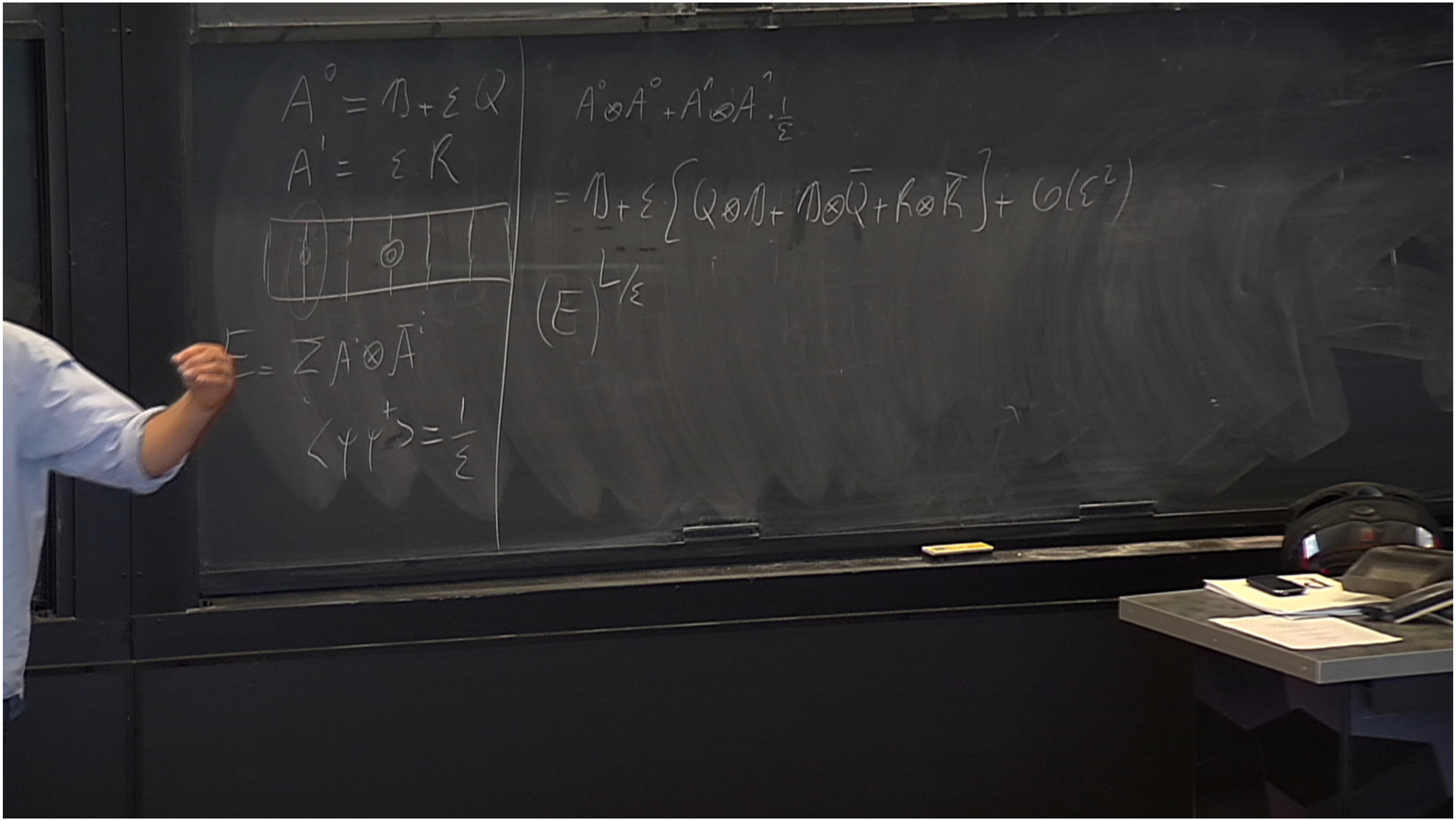


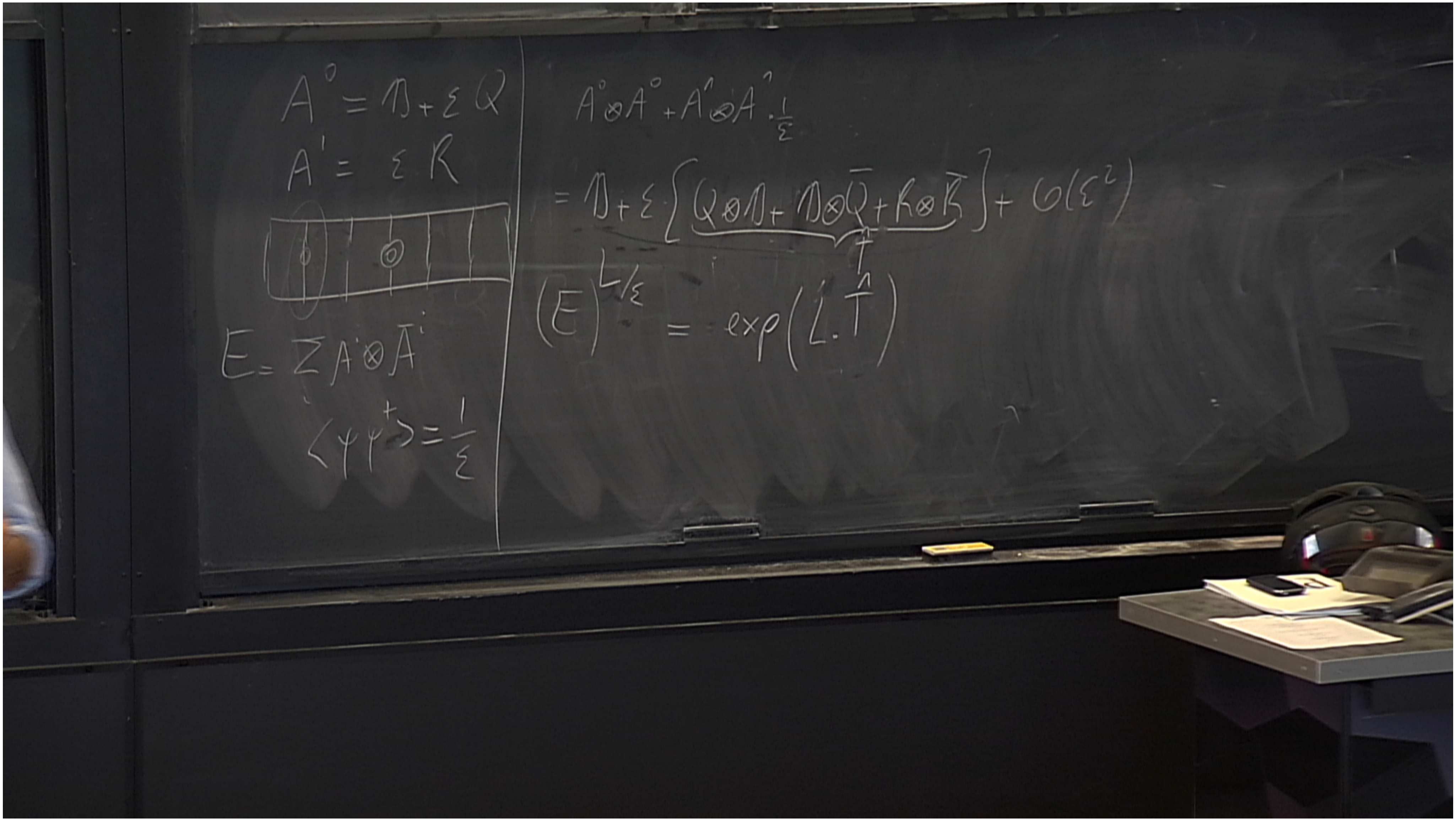
$$H = \sum K$$

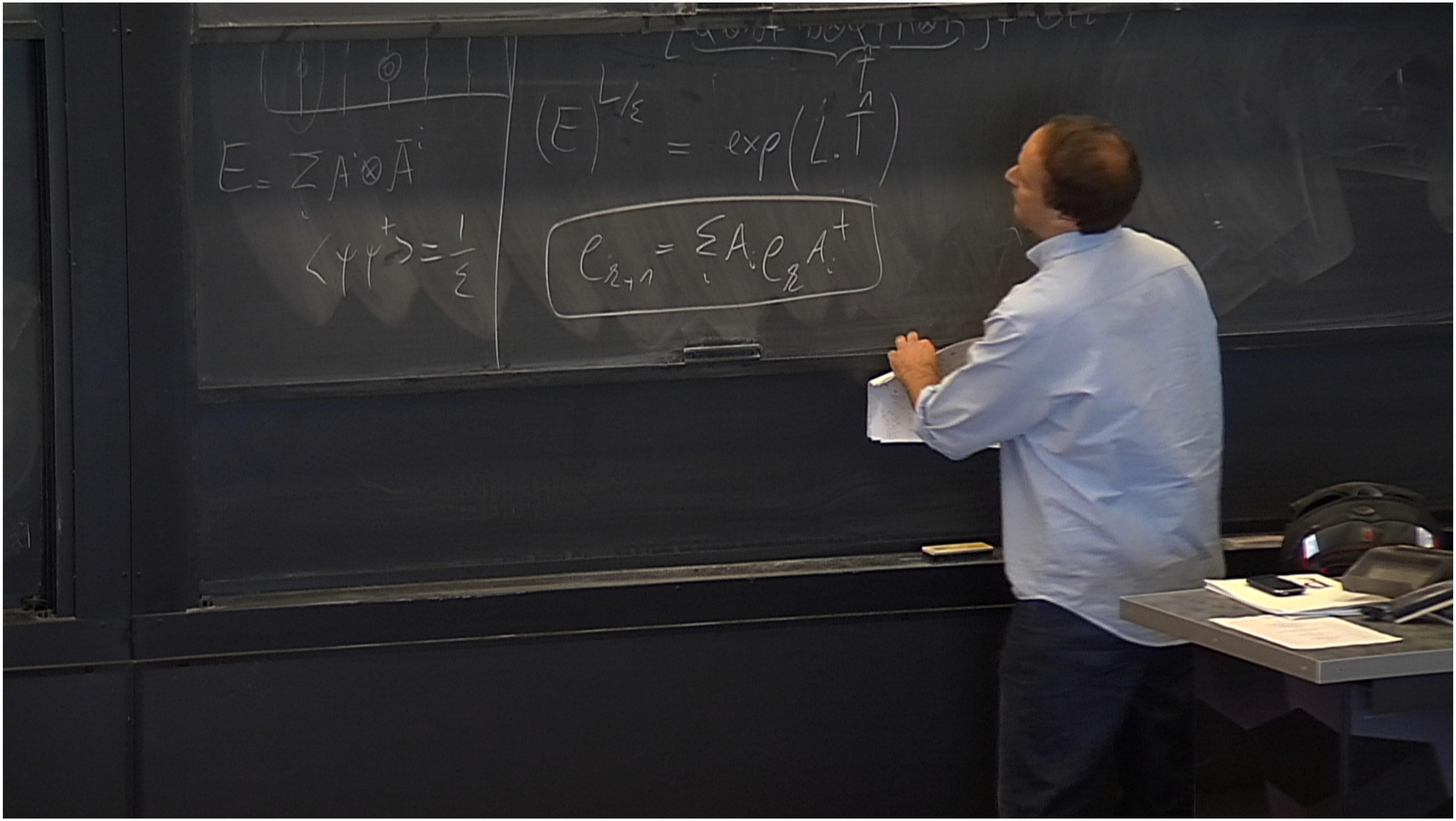
$$\frac{\sigma^{+}}{\sqrt{\varepsilon}} = \psi^{\dagger}_x$$

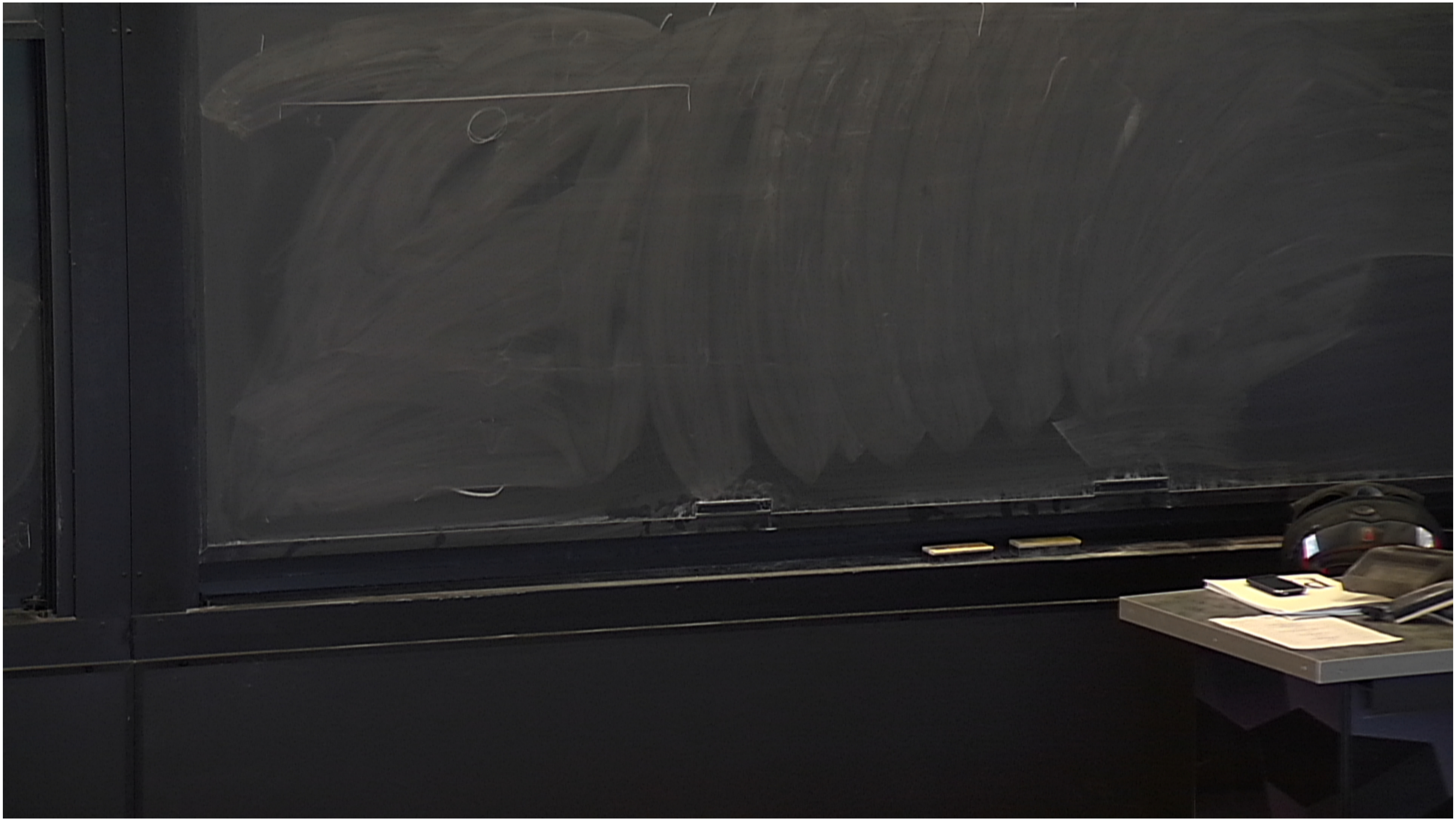


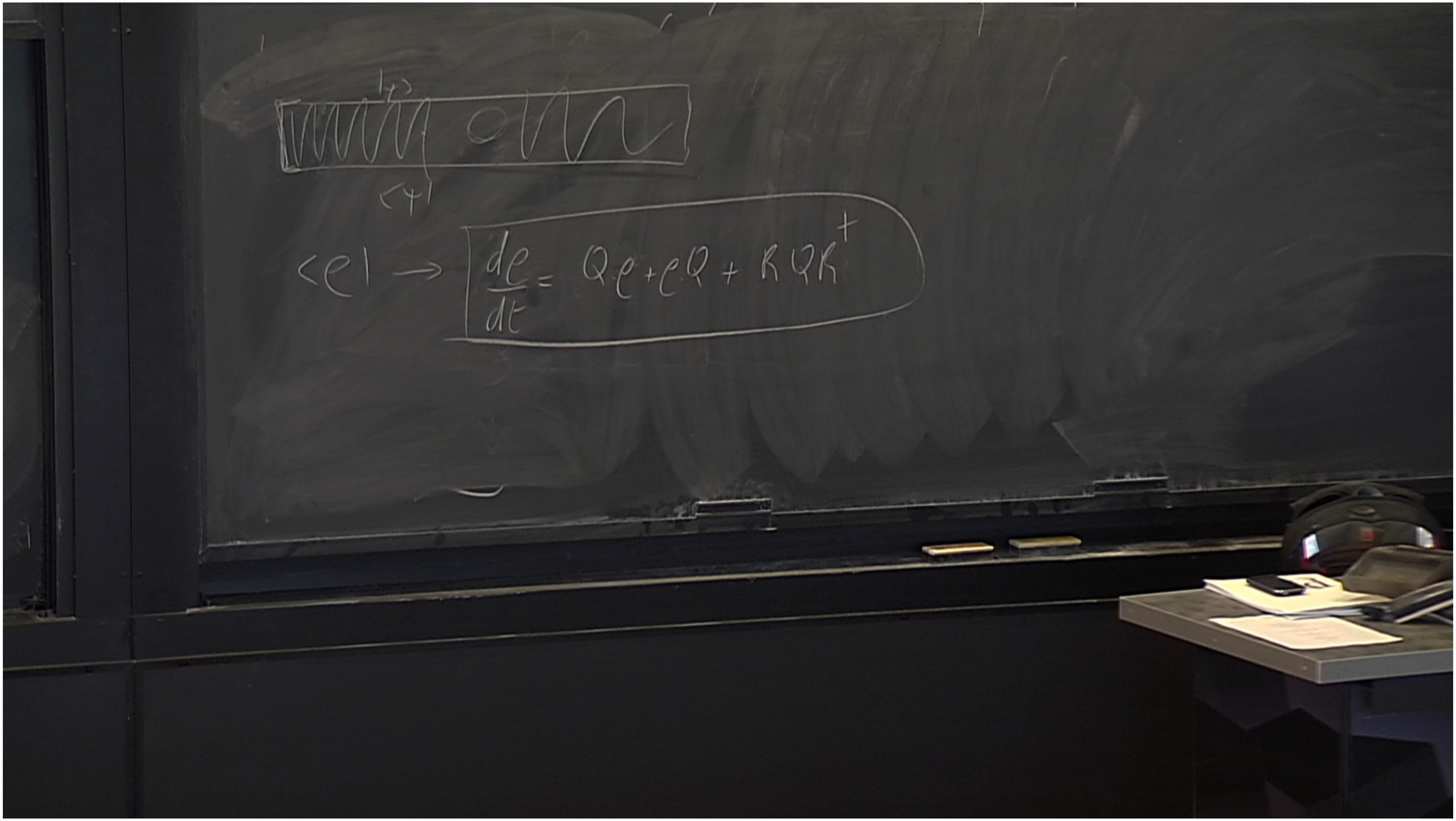



A^0 = 1 + ε Q
A^1 = ε R
E = Σ A^i ⊗ Ā^i
⟨ψ ψ^+⟩ = 1/ε
A^0 ⊗ A^0 + A^1 ⊗ A^1 · 1/ε
= 1 + ε {Q ⊗ 1 + 1 ⊗ Q̄ + R ⊗ R̄} + O(ε^2)
(E)^{L/ε}


$A^0 = \mathbb{1} + \varepsilon Q$
$A^1 = \varepsilon R$
$E = \sum A^i \otimes \bar{A}^i$
$\langle \psi \psi^\dagger \rangle = \frac{1}{\varepsilon}$
$A^0 \otimes A^0 + A^1 \otimes A^1 \cdot \frac{1}{\varepsilon}$
$= \mathbb{1} + \varepsilon \{ \underbrace{Q \otimes \mathbb{1} + \mathbb{1} \otimes \bar{Q} + R \otimes \bar{R}}_{\hat{T}} \} + O(\varepsilon^2)$
$(E)^{L/\varepsilon} = \exp(L \cdot \hat{T})$


$E = \sum_i A^i \otimes \bar{A}^i$
$\langle \psi \psi^\dagger \rangle = \frac{1}{\varepsilon}$
$(E)^{L/\varepsilon} = \exp(L \cdot \hat{T})$
$\rho_{k+1} = \sum_i A_i \rho_k A_i^\dagger$




<ψ|
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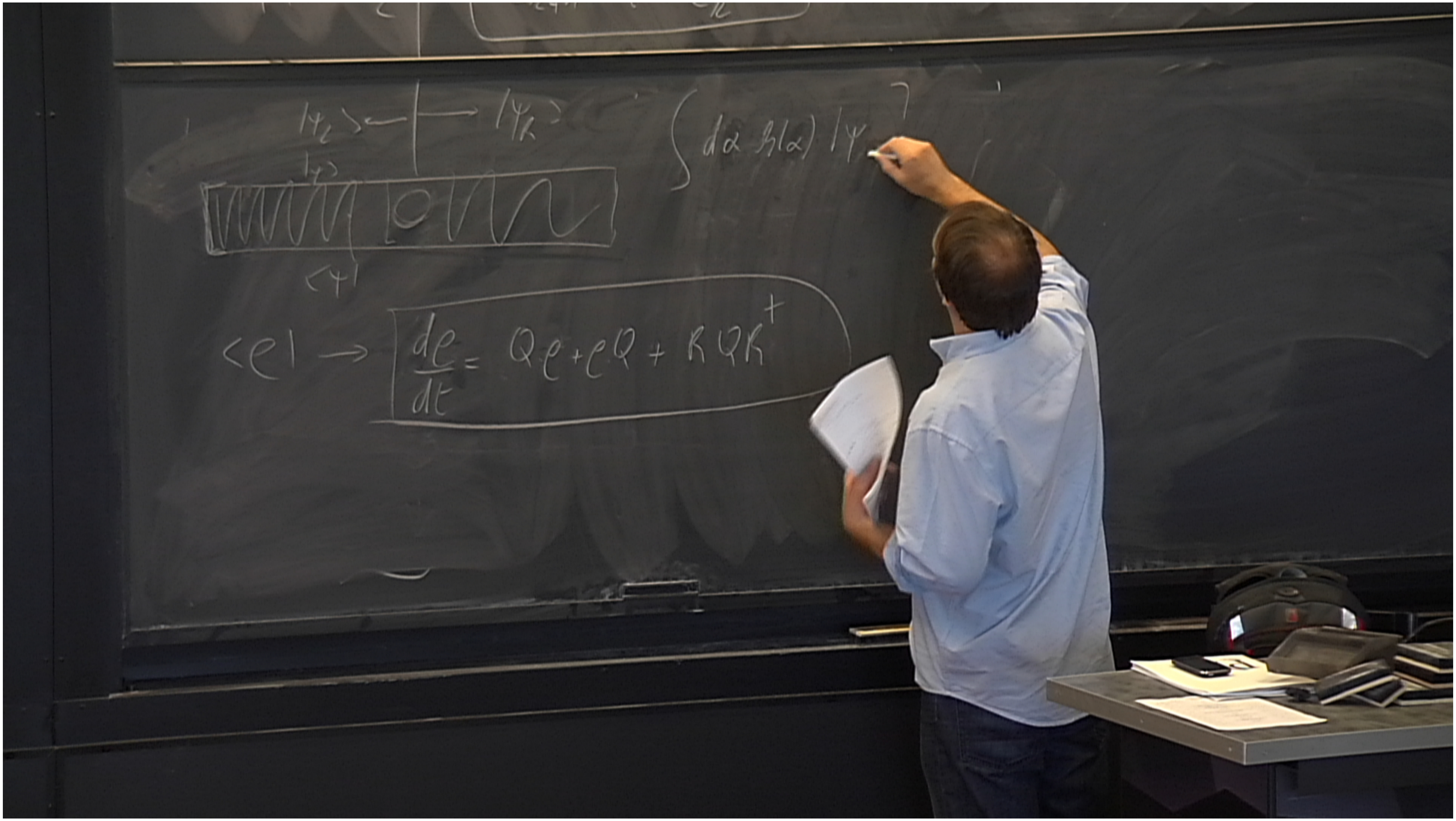

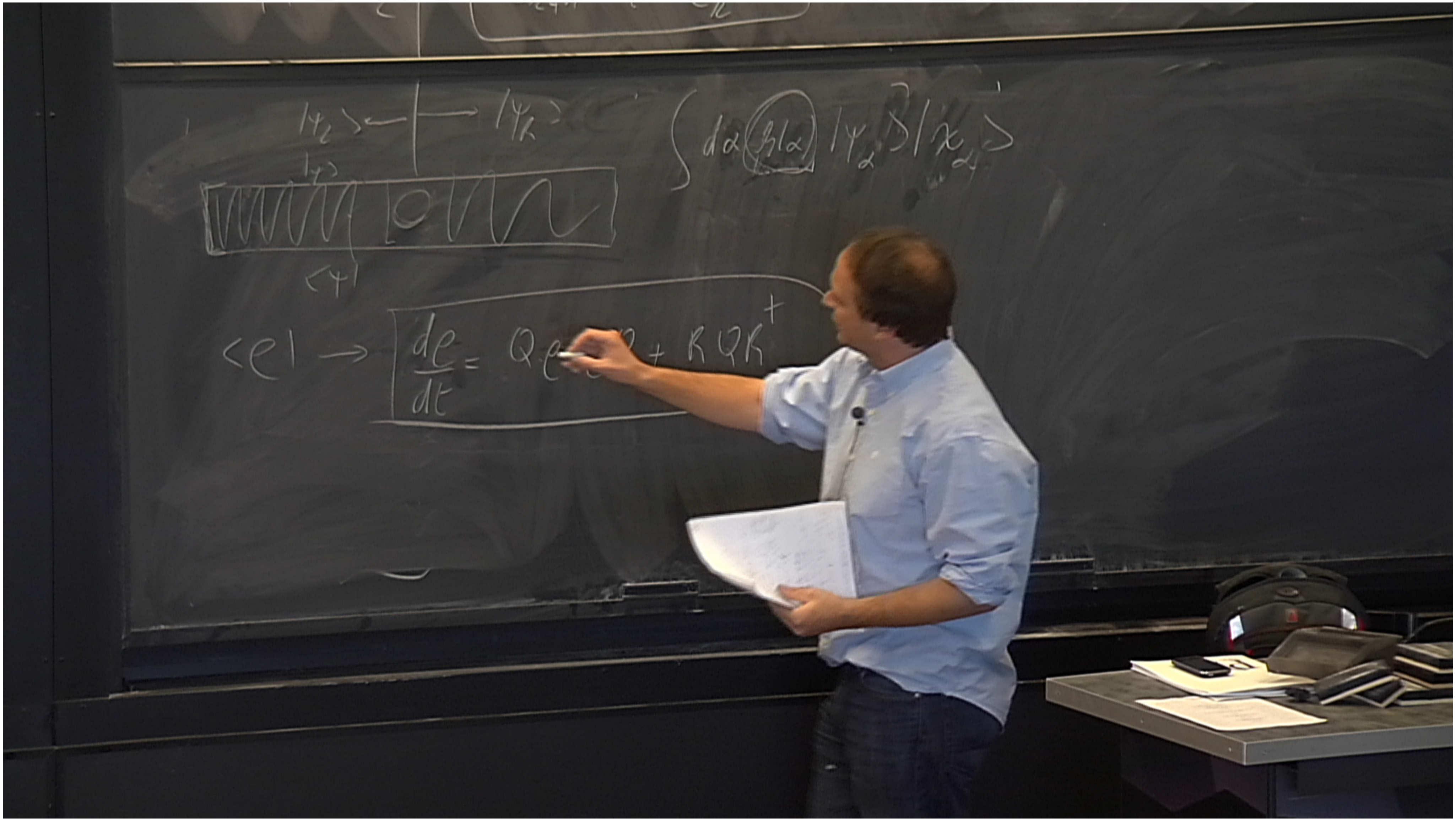

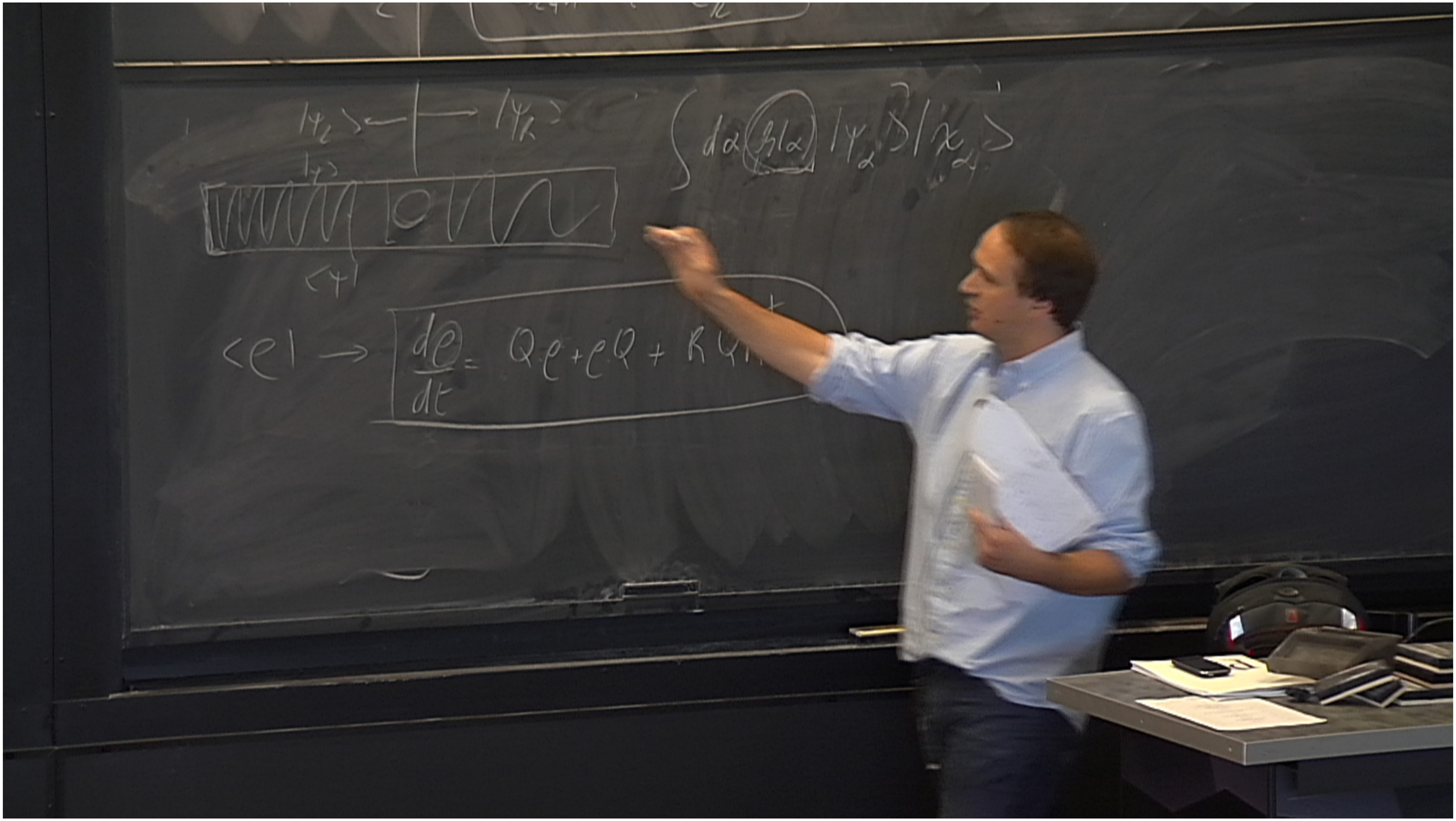

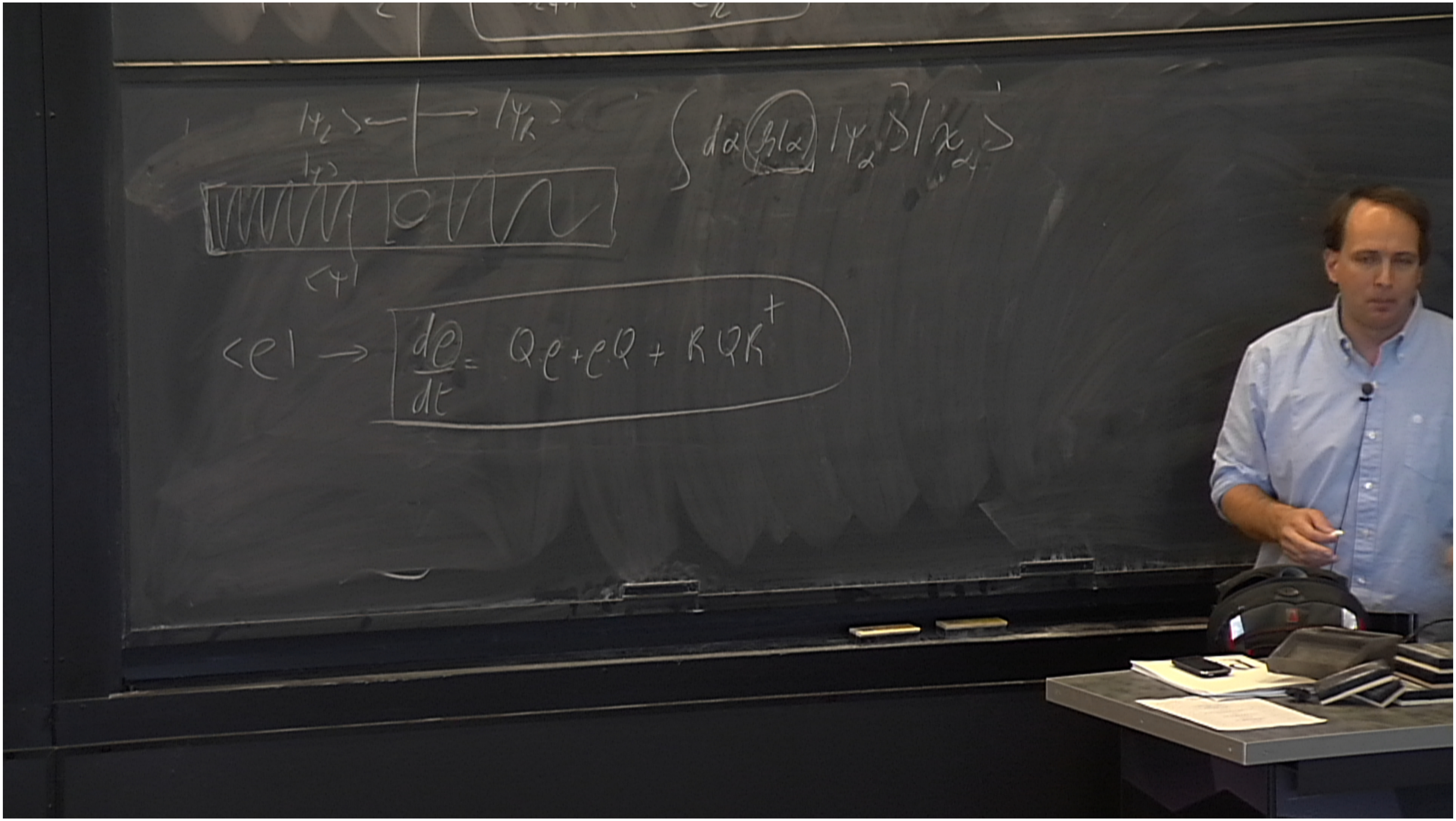

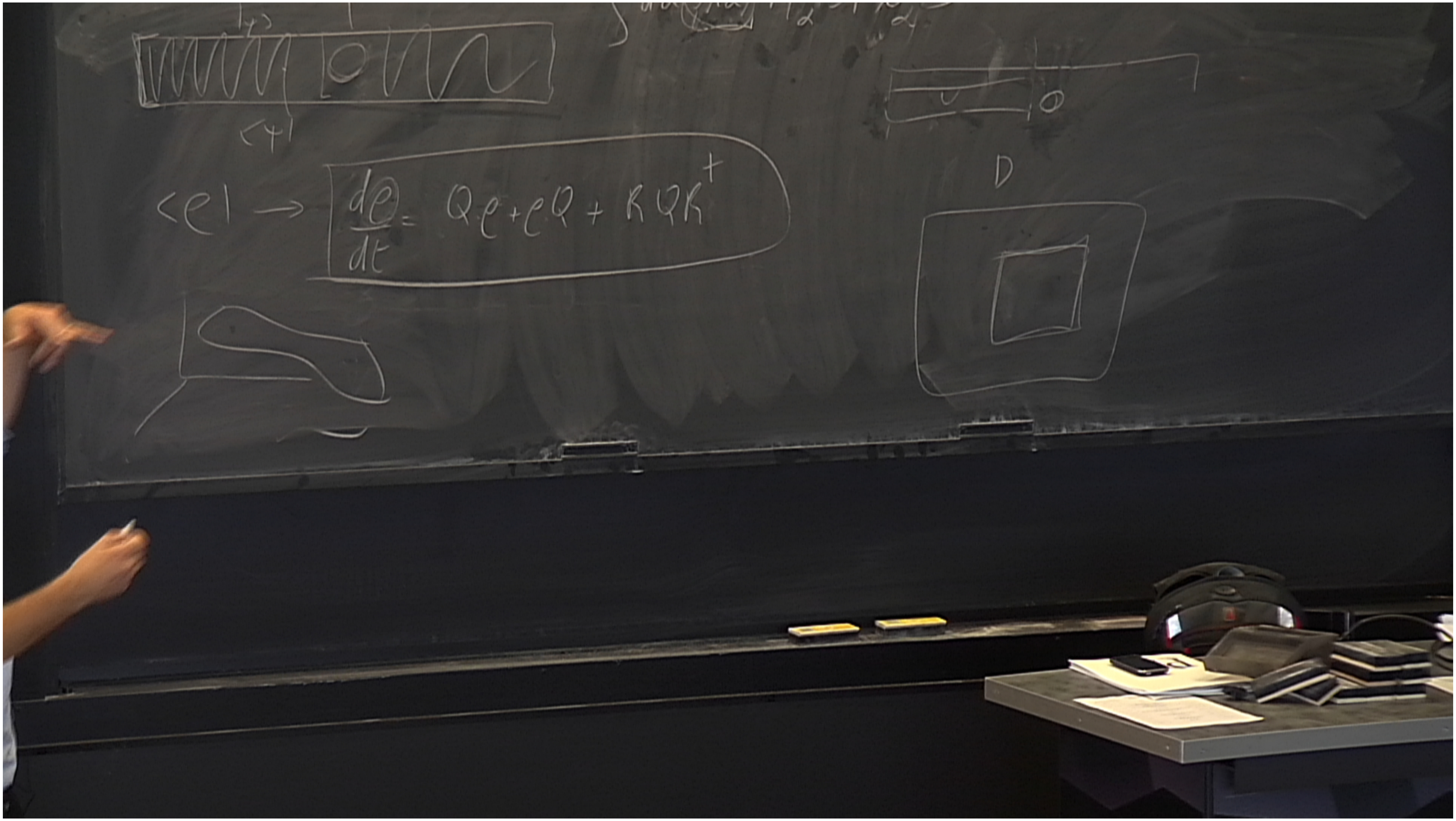
$\langle \rho | \to \frac{d\rho}{dt} = Q\rho + \rho Q + RQR^{\dagger}$
D