

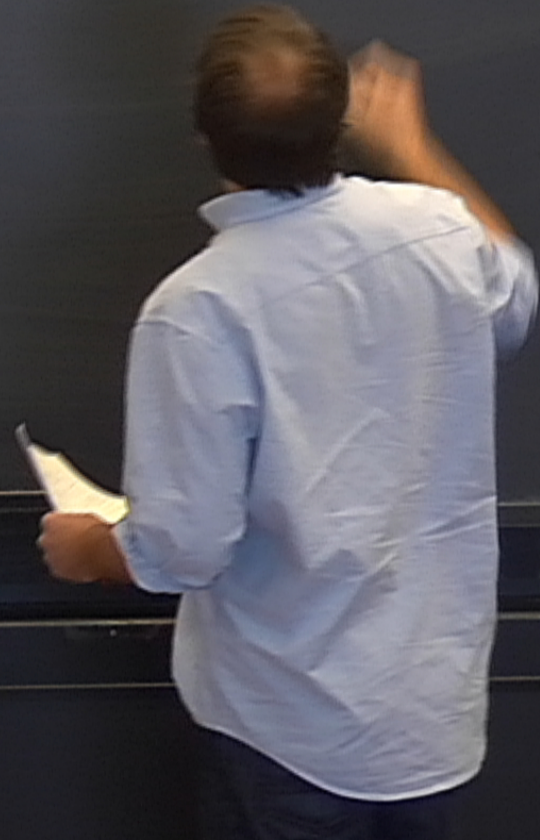
Title: MPS for QFTs

Date: Oct 24, 2011 02:00 PM

URL: <http://pirsa.org/11100091>

Abstract: I will talk about matrix product states and their suitability for simulating quantum many-body systems in the continuum.

$$\mathcal{H}_{LL} = \int dx \left(\frac{d\psi^+}{dx} \frac{d\psi}{dx} + V(x) \psi^+ \psi + c \psi^+ \psi^+ \psi \psi \right)$$



$$\mathcal{H}_{LL} = \int dx \left(\frac{d\psi^+}{dx} \right) \left(\frac{d\psi}{dx} \right) + V(x) \psi^+ \psi + c \left(\psi^+ \psi^+ \psi \psi - \psi \psi^+ \psi \right)$$

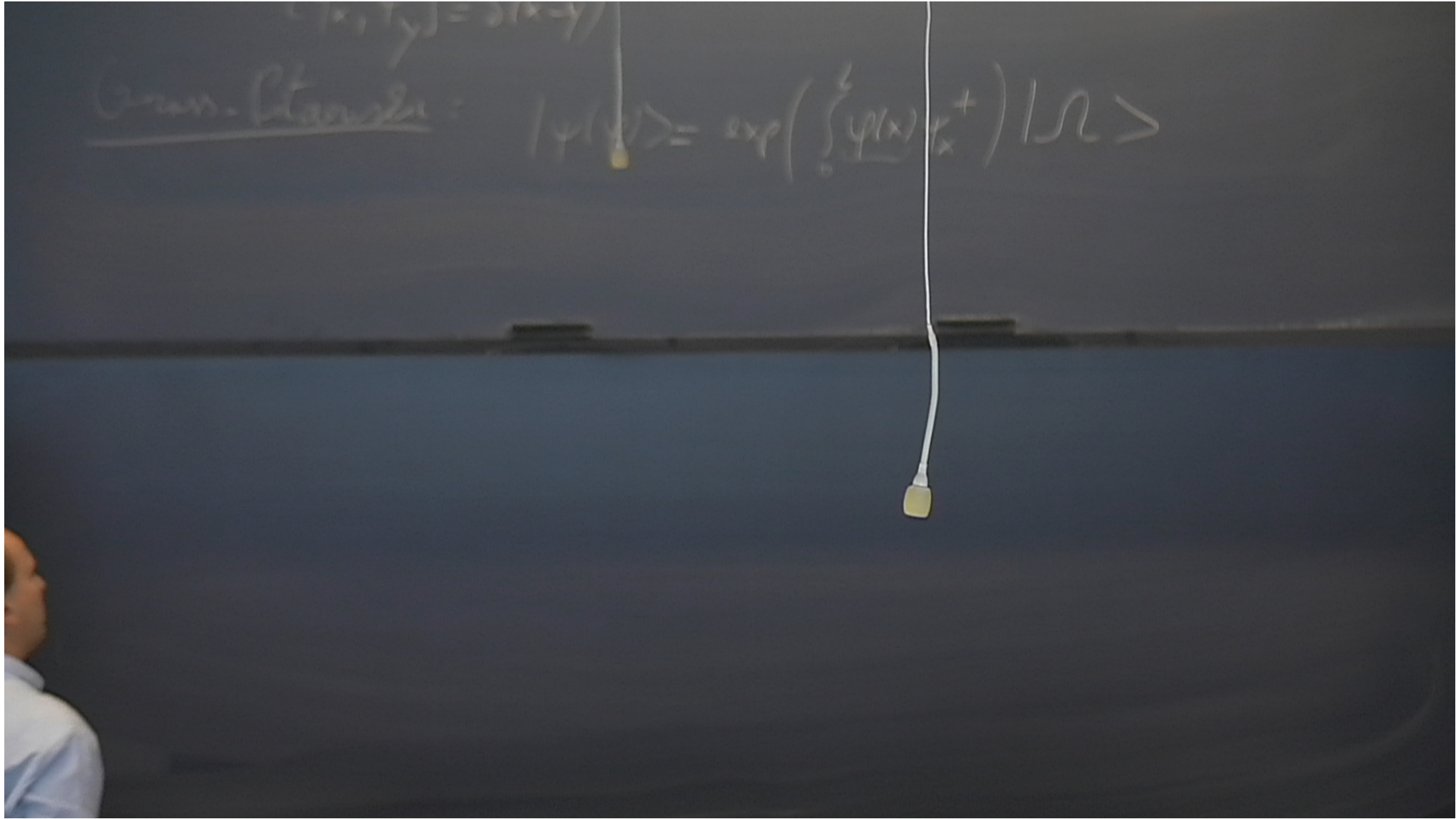
$$\{\psi_x^+, \psi_y\} = \delta(x-y)$$

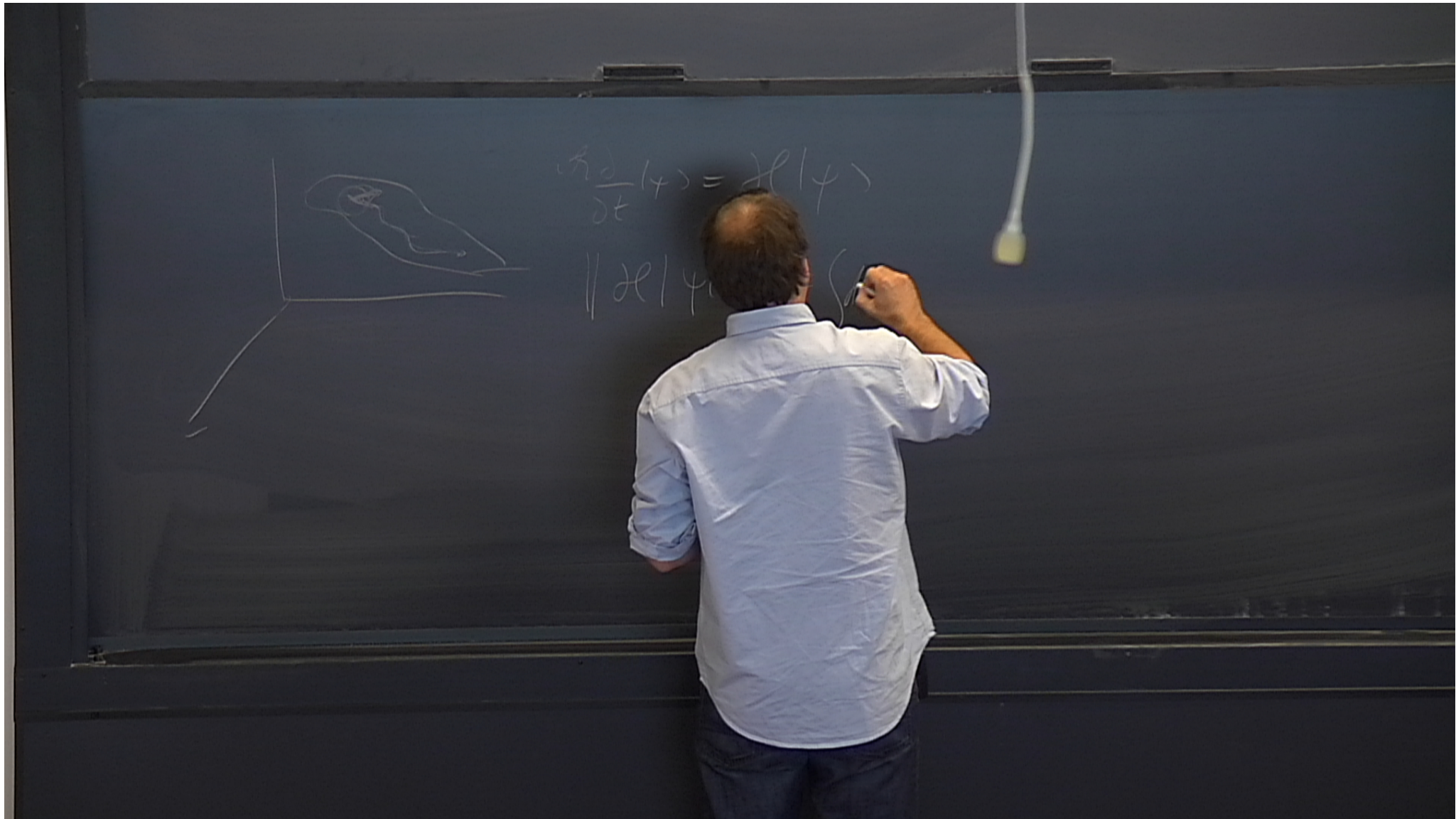
$$\mathcal{H}_{LL} = \int dx \left(\frac{d\psi_x^+}{dx} \right) \left(\frac{d\psi_x}{dx} \right) + V(x) \psi_x^+ \psi_x + c \left(\psi_x^+ \psi_x^+ \psi_x \psi_x - \mu \psi_x^+ \psi_x \right)$$

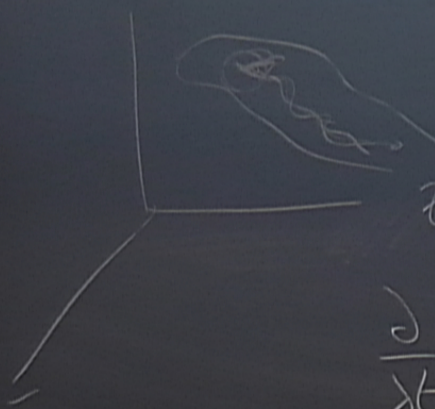
$$\{\psi_x^+, \psi_y\} = \delta(x-y)$$

$$(\mathcal{H} \times 1) (\mathcal{H} \times 1)$$
$$\{\psi_x^+, \psi_y\} = \delta(x-y)$$

Gross-Pitaevskii: $|\psi(\varphi)\rangle = \exp\left(\int_0^L \varphi(x) \psi_x^+\right) |\Omega\rangle$



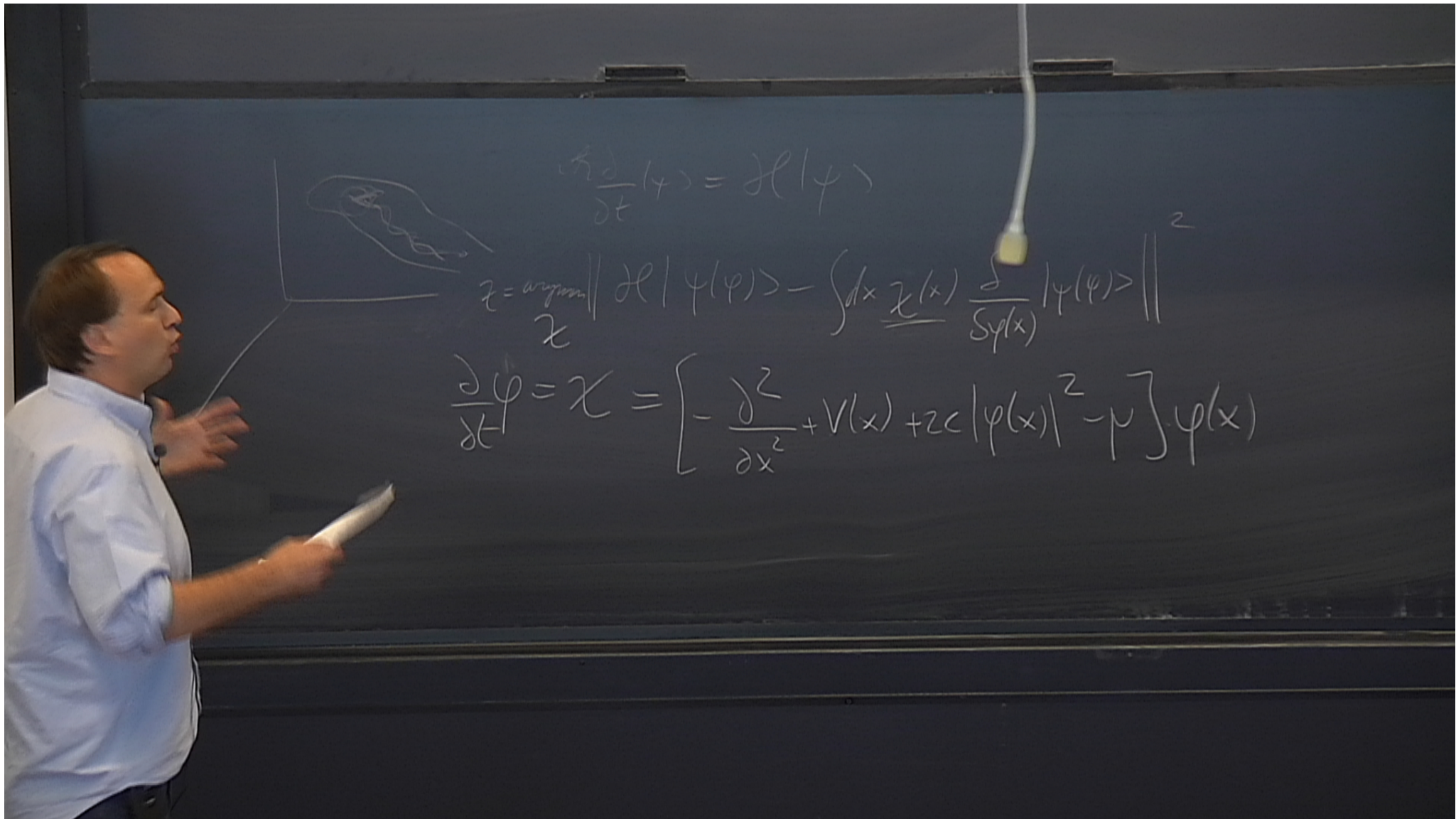




$$\hbar \frac{\partial}{\partial t} |\psi\rangle = \mathcal{H} |\psi\rangle$$

$$\chi = \underset{\chi}{\text{argmin}} \left\| \mathcal{H} |\psi(\varphi)\rangle - \int dx \chi(x) \frac{\delta}{\delta \varphi(x)} |\psi(\varphi)\rangle \right\|^2$$

$$\frac{\partial \varphi}{\partial t} = \chi$$



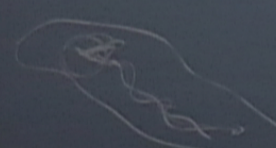
$$i\hbar \frac{\partial}{\partial t} \langle \psi | \psi \rangle = \langle \psi | \hat{H} | \psi \rangle$$

$$\lambda = \text{argmin}_{\psi} \langle \psi | \hat{H} | \psi \rangle = \int dx \lambda(x) \frac{\delta}{\delta \psi(x)} \langle \psi | \hat{H} | \psi \rangle$$

$$\frac{\partial \psi}{\partial t} = \chi = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + 2c |\psi(x)|^2 - \mu \right] \psi(x)$$

Bogoliubov - de Gennes

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \mathcal{H} |\psi\rangle$$



$$\chi = \text{argmin}_{\chi} \left\| \mathcal{H} |\psi(\varphi)\rangle - \int dx \chi(x) \frac{\delta}{\delta \varphi(x)} |\psi(\varphi)\rangle \right\|^2$$

$$\frac{\partial \varphi}{\partial t} = \chi = \left[-\frac{\partial^2}{\partial x^2} + V(x) + 2c |\varphi(x)|^2 - \mu \right] \varphi(x)$$

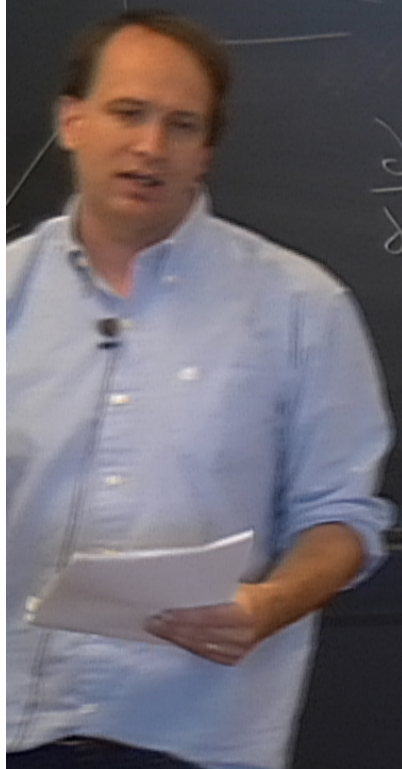
Bogoliubov - de Gennes

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \mathcal{H} |\psi\rangle$$

$$\chi = \text{argmin}_{\chi} \left\| \mathcal{H} |\psi(\chi)\rangle - \int dx \chi(x) \frac{\delta}{\delta \psi(x)} |\psi(\chi)\rangle \right\|^2$$

$$\frac{\partial \chi}{\partial t} = \chi = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) + 2c |\psi(x)|^2 - \mu \right] \psi(x)$$

$$\tilde{\psi}(x,t) = \psi_0(x) + \epsilon \psi_1(x) e^{i\omega t} + \epsilon \psi_2(x) e^{-i\omega t}$$



Bogoliubov - de Gennes

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \mathcal{H} |\psi\rangle$$

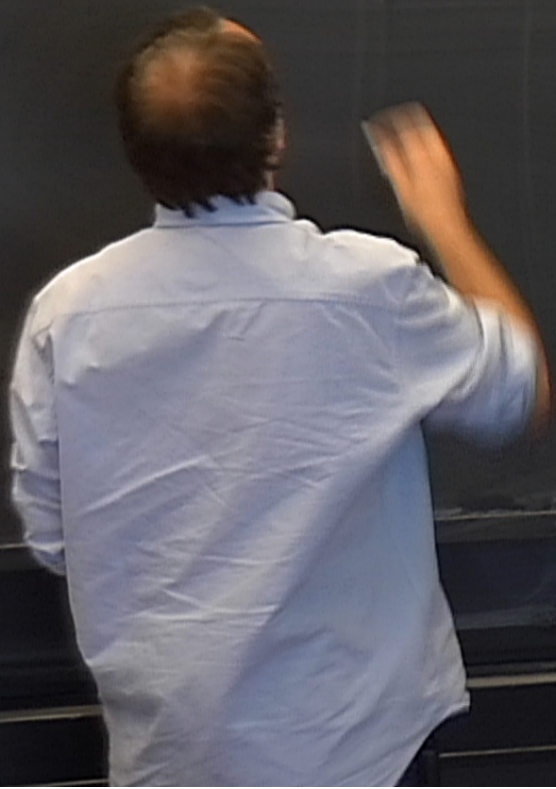
$$\chi = \frac{\text{argmin}}{\chi} \left\| \mathcal{H} |\psi(\chi)\rangle - \int dx \chi(x) \frac{\delta}{\delta \psi(x)} |\psi(\chi)\rangle \right\|^2$$

$$\frac{\partial \psi}{\partial t} = \chi = \left[-\frac{\partial^2}{\partial x^2} + V(x) + 2c |\psi(x)|^2 - \mu \right] \psi(x)$$

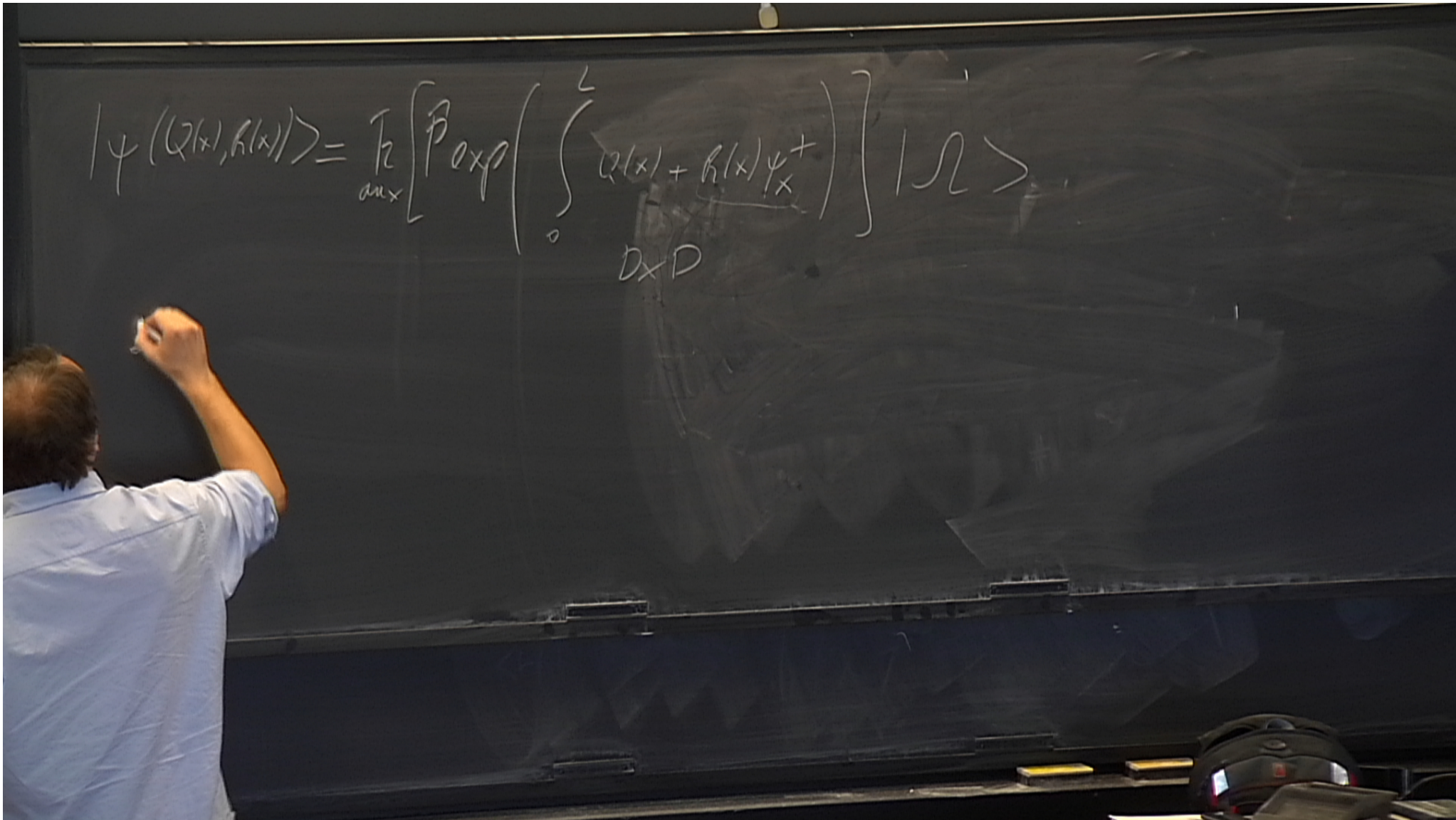
$$\tilde{\psi}(x,t) = \psi_0(x) + \epsilon \psi_1(x) e^{i\omega t} + \epsilon \psi_2(x) e^{-i\omega t}$$

$$|\psi(Q(x), R(x))\rangle = \int_{\mathcal{D}} \mathcal{P} \exp\left(\int_0^L (Q(x) + R(x)\psi_x^+)\right) |\Omega\rangle$$

$\mathcal{D} \times \mathcal{D}$



$$|\psi(Q(x), R(x))\rangle = \int_{\mathcal{D} \times \mathcal{D}} \mathcal{P} \exp\left(\int_0^L Q(x) + R(x) \psi_x^\dagger\right) |\Omega\rangle$$



$$\begin{aligned}
 & \text{aux} \left[\begin{array}{c} \dots \\ \dots \\ \dots \end{array} \right] \left(\begin{array}{c} Q(x) + R(x) \\ \dots \end{array} \right) \int |U\rangle \\
 & \quad \quad \quad D \times D \\
 & = \sum_n \int dx_1 \dots dx_n \text{Tr} \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \underbrace{\Psi_{x_1}^+ \Psi_{x_1}^+ \dots |R\rangle}
 \end{aligned}$$

0

\mathbb{D}
 \mathbb{O}

$$= \sum_n \int_{x_{n-1}}^{x_n} \text{Tr} \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \underbrace{\Psi_{x_1}^+ \Psi_{x_2}^+ \dots}_{\text{---}} | \mathcal{L} \rangle$$

$\hookrightarrow P \exp \left(\int_0^x Q(x) dx \right)$

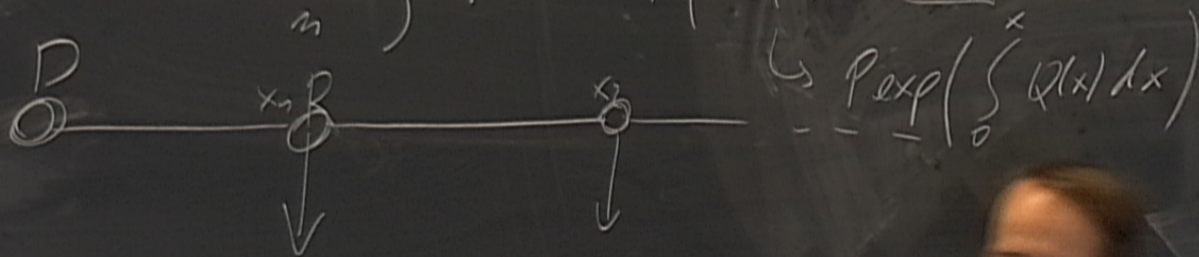
$$\begin{aligned}
 |\Psi(Q(x), R(x))\rangle &= \int_{\mathcal{D}} \mathcal{P} \exp\left(\int_0^+ Q(x) + R(x) \frac{d}{dx}\right) |\Omega\rangle \\
 &= \sum_n \int dx_1 \dots dx_n \int_{\mathcal{D}} \mathcal{P} \exp\left(\int_0^+ Q(x) dx\right) \text{Tr}\left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots\right) \underbrace{\Psi_{x_1}^+ \Psi_{x_2}^+ \dots}_{|\Omega\rangle}
 \end{aligned}$$

\mathcal{D}
0

$\mathcal{P} \exp\left(\int_0^+ Q(x) dx\right)$

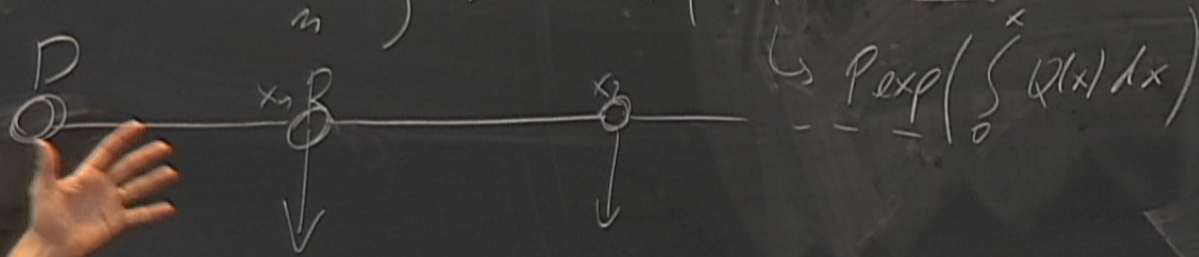
$$|\psi(Q(x), R(x))\rangle = \int_{\mathcal{D}} \mathcal{P} \exp \left(\int_0^x (Q(x) + R(x)) \frac{d}{dx} \right) |\Omega\rangle$$

$$= \sum_n \int dx_1 \dots dx_n \mathcal{P} \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \underbrace{\psi_{x_1}^+ \psi_{x_2}^+ \dots}_{|\Omega\rangle}$$



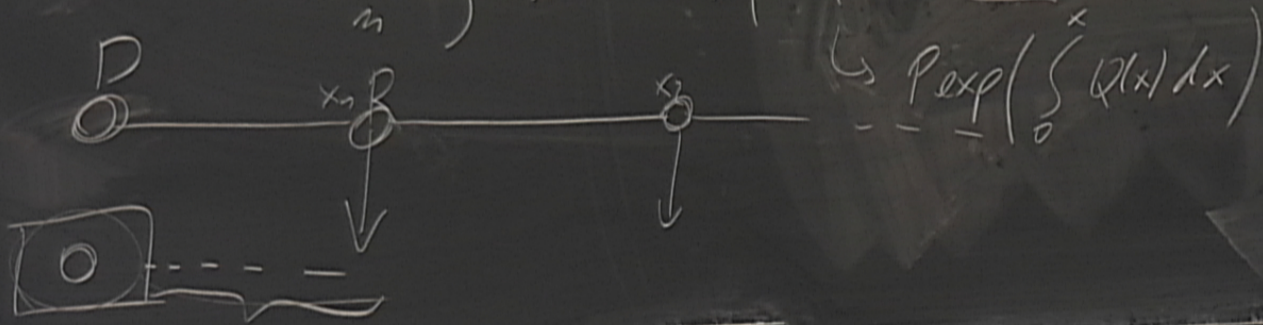
$$|\psi(Q(x), R(x))\rangle = \int_{\mathcal{D}} \mathcal{P} \exp\left(\int_0^{\beta} (Q(x) + R(x)) \frac{d}{dx}\right) |\Omega\rangle$$

$$= \sum_n \int dx_1 \dots dx_n \text{Tr}\left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots\right) \underbrace{\psi_{x_1}^+ \psi_{x_2}^+ \dots}_{|\Omega\rangle}$$



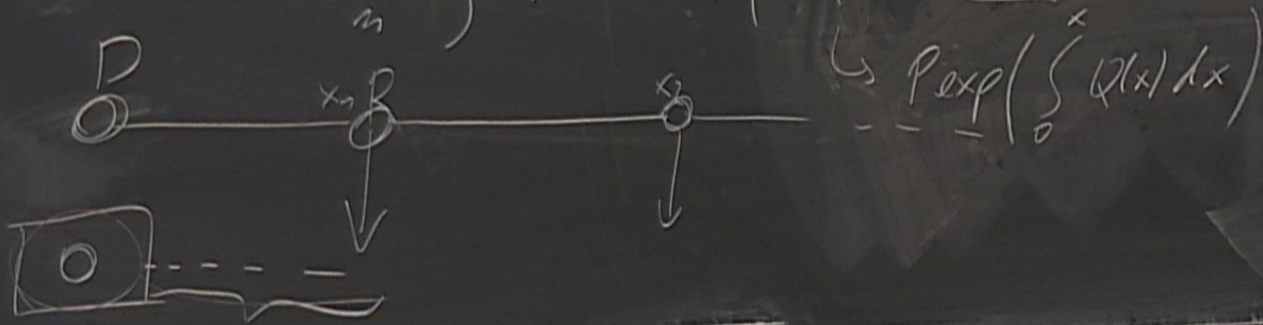
$$|\psi(Q(x), R(x))\rangle = \int_{\mathcal{D}} \mathcal{P} \exp\left(\int_0^x Q(x) + R(x) \frac{d}{dx}\right) |\Omega\rangle$$

$$= \sum_n \int dx_1 \dots dx_n \mathcal{T} \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \underbrace{\psi_{x_1}^+ \psi_{x_2}^+ \dots}_{|\Omega\rangle}$$



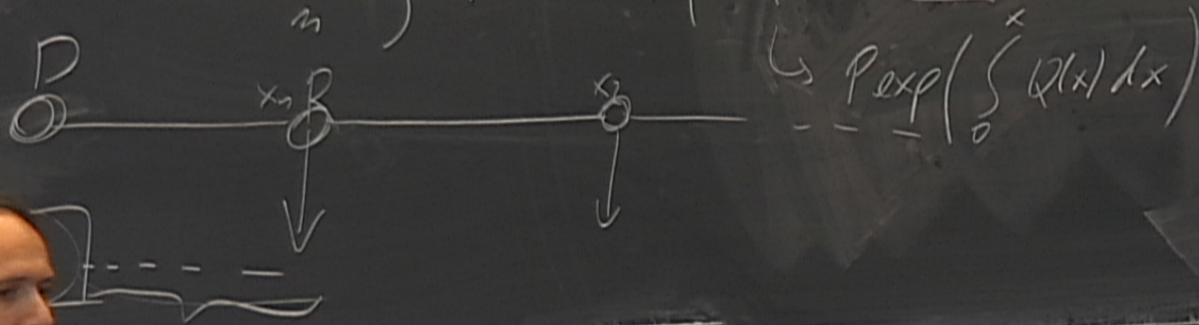
$$|\psi(Q(x), R(x))\rangle = \int_{\mathcal{D}} \mathcal{P} \exp\left(\int_0^x Q(x) + R(x) \frac{d}{dx}\right) |\Omega\rangle$$

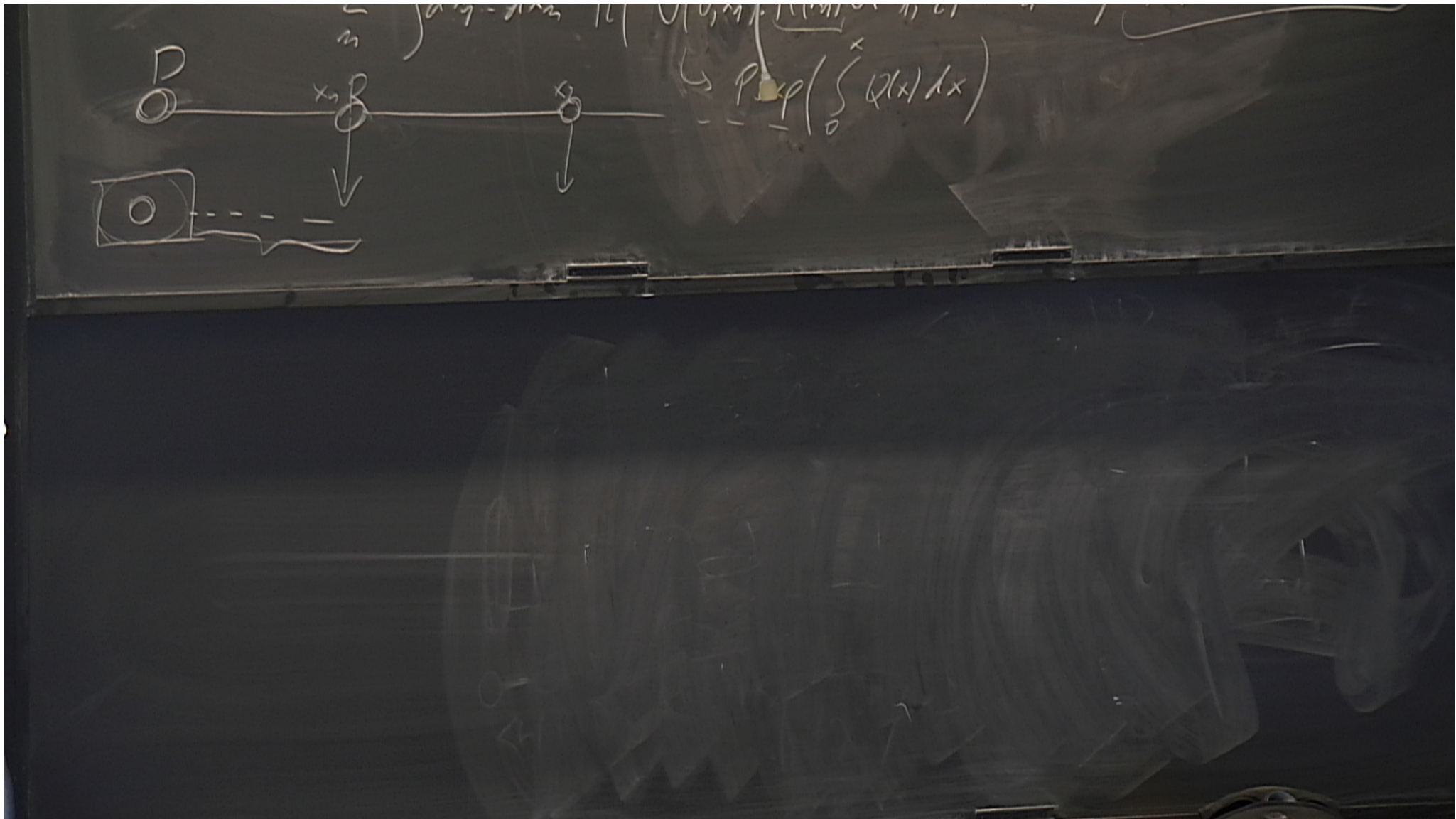
$$= \sum_n \int dx_1 \dots dx_n \mathcal{T} \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \underbrace{\psi_{x_1}^+ \psi_{x_2}^+ \dots}_{|\Omega\rangle}$$



$$|\psi(Q(x), R(x))\rangle = \int_{\mathcal{D}} \mathcal{P} \exp\left(\int_0^x Q(x) + R(x) \frac{d}{dx}\right) |\Omega\rangle$$

$$= \sum_n \int dx_1 \dots dx_n \mathcal{P} \exp\left(\int_0^x Q(x) dx\right) \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \psi_{x_1}^+ \psi_{x_2}^+ \dots |\Omega\rangle$$

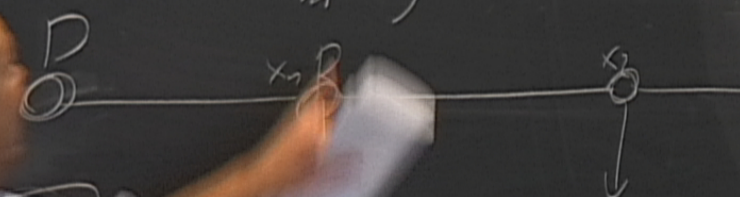


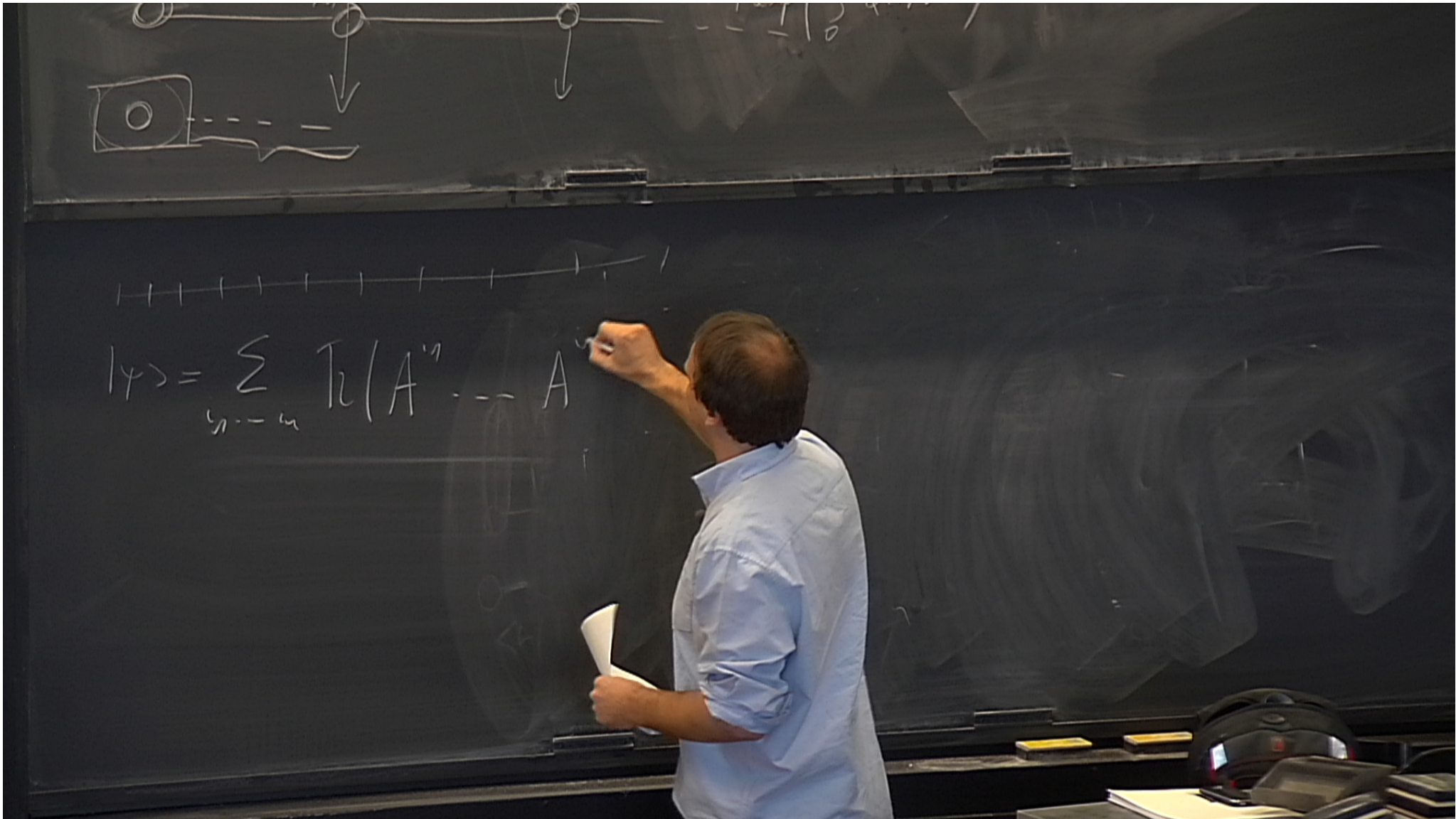


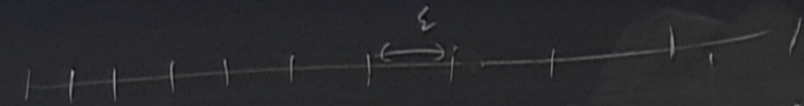
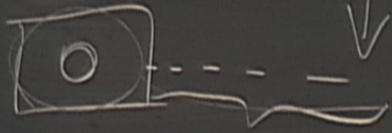
$$|\psi(Q(x), R(x))\rangle = \mathcal{T}_x \left[\mathcal{P} \exp \left(\int_0^L (Q(x) + R(x) \psi_x^+) \right) \right] |\Omega\rangle$$

$$= \sum_n \int_{x_1}^{x_2} dx_1 \dots dx_n \mathcal{T}_x \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \underbrace{\psi_{x_1}^+ \psi_{x_2}^+ \dots}_{|\Omega\rangle}$$

$$\hookrightarrow \mathcal{P} \exp \left(\int_0^L Q(x) dx \right)$$







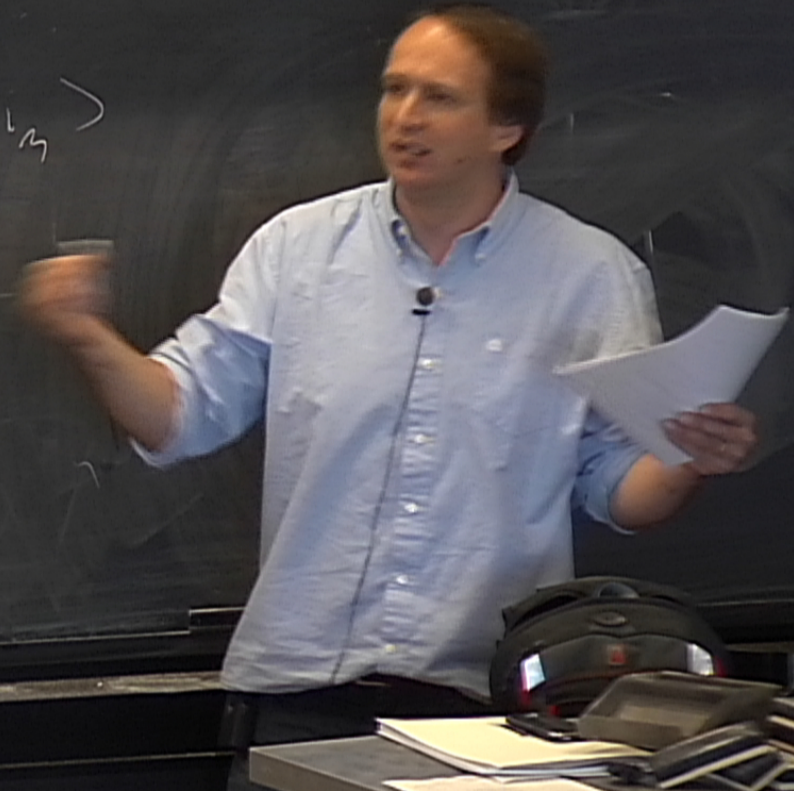
$$|\psi\rangle = \sum_{n=0}^{\infty} T_n(A^{(1)} \dots A^{(n)}) |v_n\rangle \dots |v_n\rangle$$

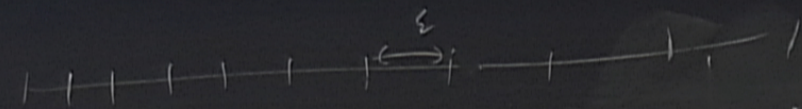
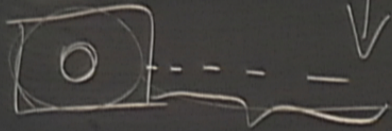
$$A^0 = \mathbb{1} + \epsilon Q$$

$$A^1 = \epsilon R$$

$$|0\rangle = |\Omega\rangle$$

$$|1\rangle = \frac{\psi^\dagger |\Omega\rangle}{\frac{\sigma^\dagger}{\sqrt{E}}}$$





$$|\psi\rangle = \sum_{n=0}^{\infty} T_n(A^{(0)} - A^{(n)}) |u_n\rangle \dots |u_n\rangle$$

$$A^{(0)} = \mathbb{1} + \epsilon Q$$

$$A^{(1)} = \epsilon R$$

$$|0\rangle = |\Omega\rangle$$

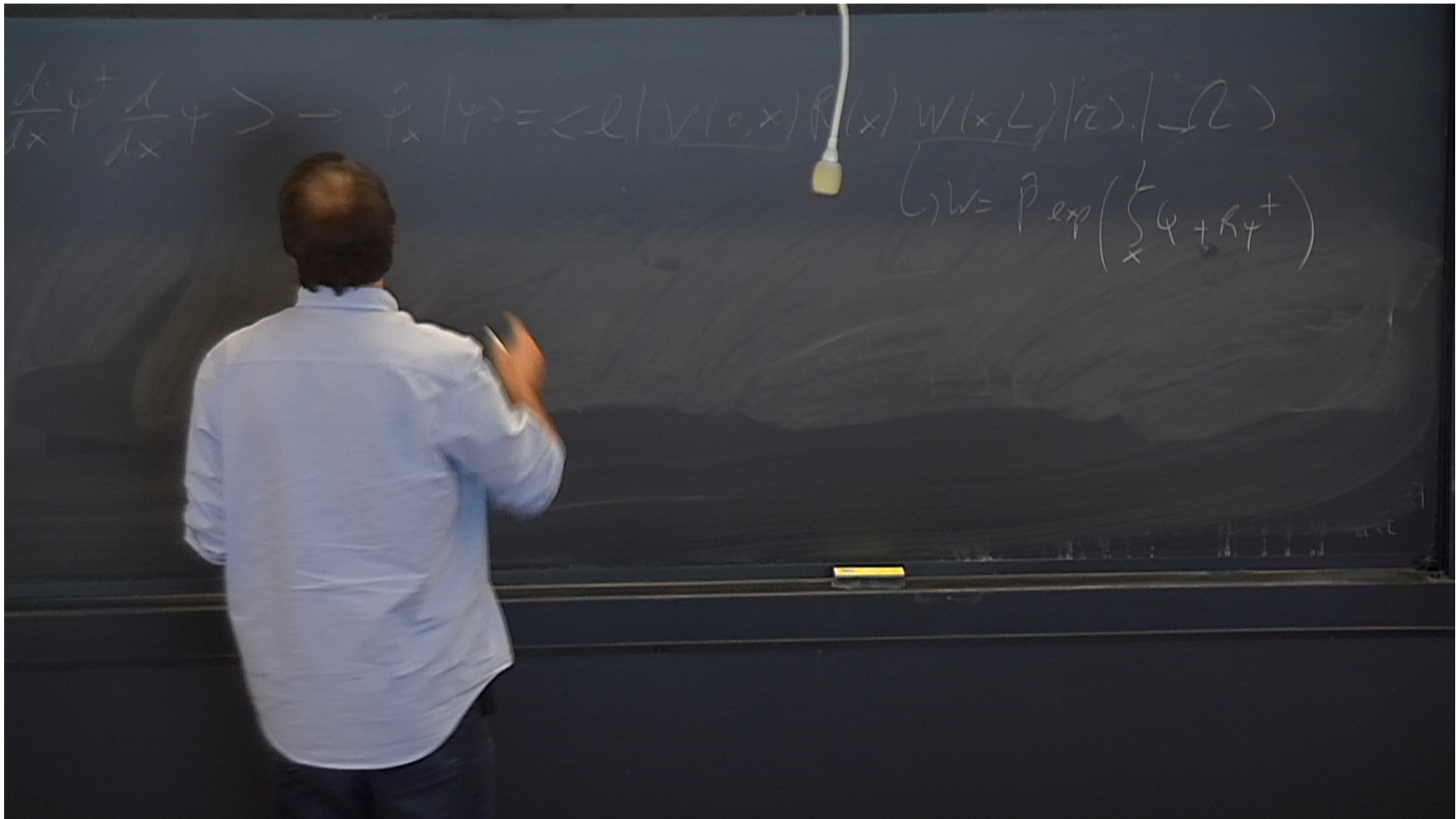
$$|1\rangle = \frac{\sigma^+}{\sqrt{\epsilon}} |\Omega\rangle$$

$$\frac{\sigma^+}{\sqrt{\epsilon}} \equiv \psi_x^+$$

$$\left\langle \frac{d\psi^+}{dx} \frac{d\psi}{dx} \right\rangle \rightarrow \langle \psi_x | \psi \rangle = \langle \ell | \frac{V(\sigma, x) R(x) W(x, L)}{i\hbar} | r \rangle \cdot \int_{\Omega} \psi$$

$$\hookrightarrow W = \beta \exp\left(\int_x^L \psi \pm R \psi^+\right)$$



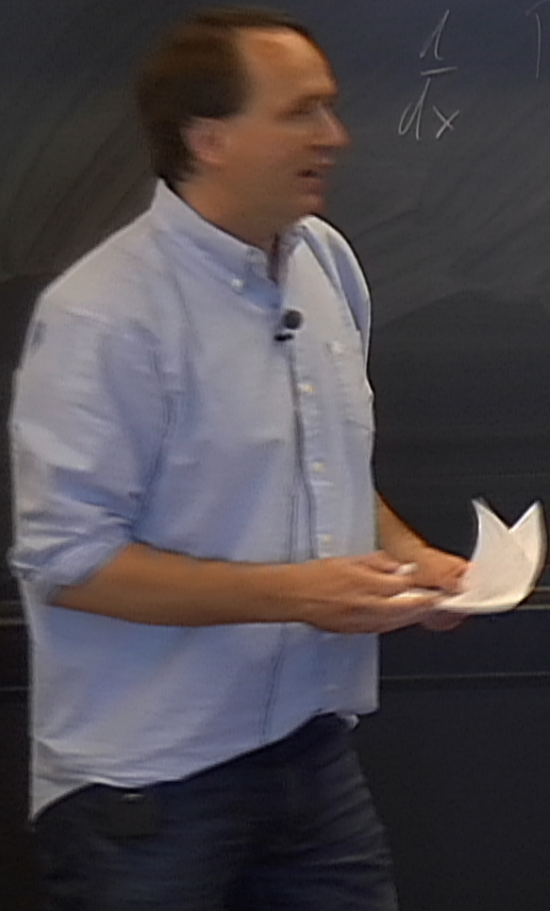


$$\frac{d}{dx} \psi + \frac{1}{\psi} \psi' \rightarrow \langle \psi | \psi \rangle = \langle \psi | V(x) R(x) W(x, L) | \psi \rangle$$

$$\frac{1}{\psi} \psi'$$

$$W = \beta \exp\left(\int_x^L \psi + R \psi'\right)$$

$$[Q, R] + \frac{dR}{dx}$$



$$\frac{d}{dx} \psi^+ \frac{d}{dx} \psi \rightarrow \hat{p}_x |\psi\rangle = \langle \psi | \frac{d}{dx} V(x) R(x) W(x, L) | \psi \rangle \int_{-\infty}^{\infty} dx$$

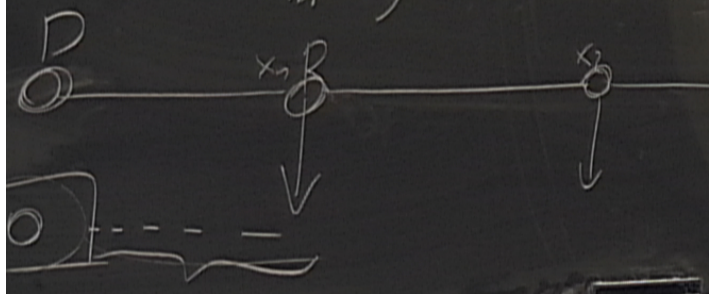
$$\frac{d}{dx} \uparrow$$

$$W = \beta \exp\left(\int_x^L Q + R_T^+\right)$$

$$[Q, R] + \frac{dR}{dx} + (-1)^s R^2 - R^2$$

$$\langle Q(x), R(x) \rangle = \text{Tr} \left[\mathcal{P} \exp \left(\int_0^L \underbrace{Q(x) + \sum_x R(x) \psi_x^\dagger}_{D \times D} \right) \right] | \Omega \rangle$$

$$= \sum_n \int dx_1 \dots dx_n \text{Tr} \left(U(0, x_1) R(x_1) U(x_1, x_2) R(x_2) \dots \right) \underbrace{\psi_{x_1}^\dagger \psi_{x_1}^\dagger \dots | \Omega \rangle}_{\mathcal{P} \exp \left(\int_0^L Q(x) dx \right)}$$



$$T = \sum K$$

$$\frac{\sigma^\dagger}{\sqrt{\epsilon}} \equiv \psi_x^\dagger$$

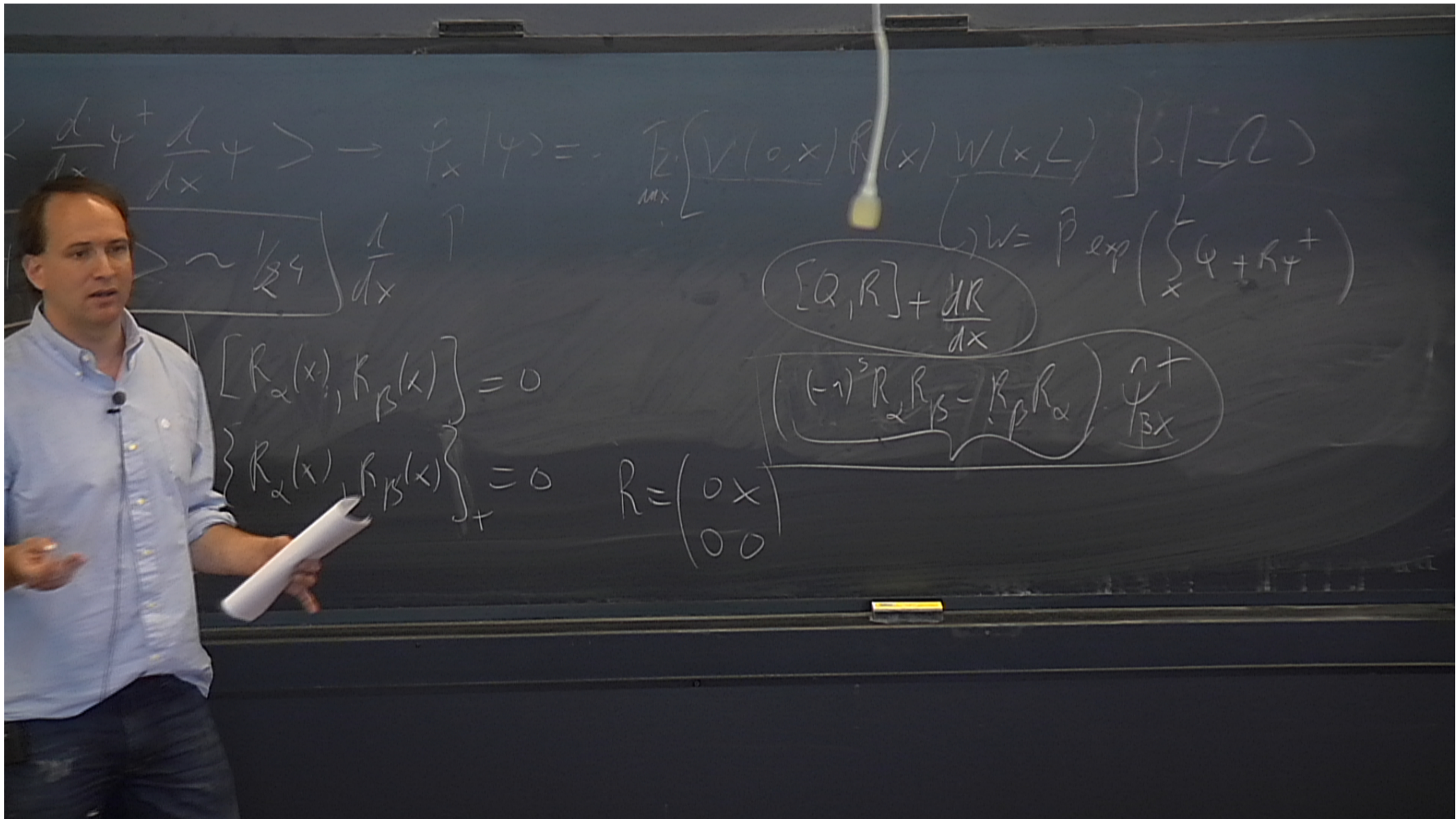
$$\langle \psi^+ | \frac{d}{dx} \psi \rangle \rightarrow \langle \psi_x | \psi \rangle = \langle \psi | \frac{d}{dx} \psi \rangle = \langle \psi | V(x) R(x) W(x, L) | \psi \rangle \quad (\Omega)$$

$$\frac{d}{dx} \uparrow$$

$$W = \rho \exp\left(\int_x^L \psi + R \psi^+\right)$$

$$[Q, R] + \frac{dR}{dx}$$

$$\left((-1)^s R_\alpha R_\beta - R_\beta R_\alpha \right) \begin{matrix} \psi^+ \\ \psi \\ \psi_x \end{matrix}$$



$$\left\langle \frac{d}{dx} \psi^+ + \frac{1}{2} \psi \right\rangle \rightarrow \hat{f}_x |\psi\rangle =$$

$$\int_{\text{max}} \left[\frac{V(x) R(x) W(x, L)}{L} \right] \psi(x) dx$$

$$W = \beta \exp\left(\int_x^L \psi + R \psi^+\right)$$

$$\left[R_\alpha(x), R_\beta(x) \right] = 0$$

$$\left. \begin{matrix} R_\alpha(x) \\ R_\beta(x) \end{matrix} \right\} = 0$$

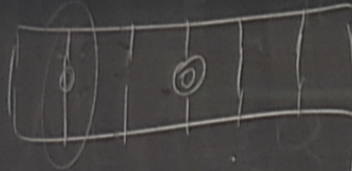
$$R = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}$$

$$\left[Q, R \right] + \frac{dR}{dx}$$

$$\left((-1)^s R_\alpha R_\beta - R_\beta R_\alpha \right) \begin{matrix} \uparrow \\ \psi \\ \downarrow \\ \beta x \end{matrix}$$

$$A^0 = \mathbb{1} + \varepsilon Q$$

$$A^1 = \varepsilon R$$



$$E = \sum A^i \otimes \bar{A}^i$$

$$\langle \psi \psi^+ \rangle = \frac{1}{\varepsilon}$$

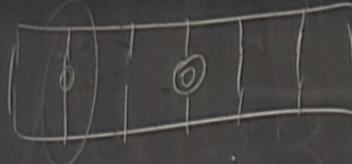
$$A^0 \otimes A^0 + A^1 \otimes A^1 \cdot \frac{1}{\varepsilon}$$

$$= \mathbb{1} + \varepsilon \left\{ Q \otimes \mathbb{1} + \mathbb{1} \otimes \bar{Q} + R \otimes \bar{R} \right\} + \mathcal{O}(\varepsilon^2)$$

$$(E)^{L/\varepsilon}$$

$$A^0 = \mathbb{1} + \varepsilon Q$$

$$A^1 = \varepsilon R$$



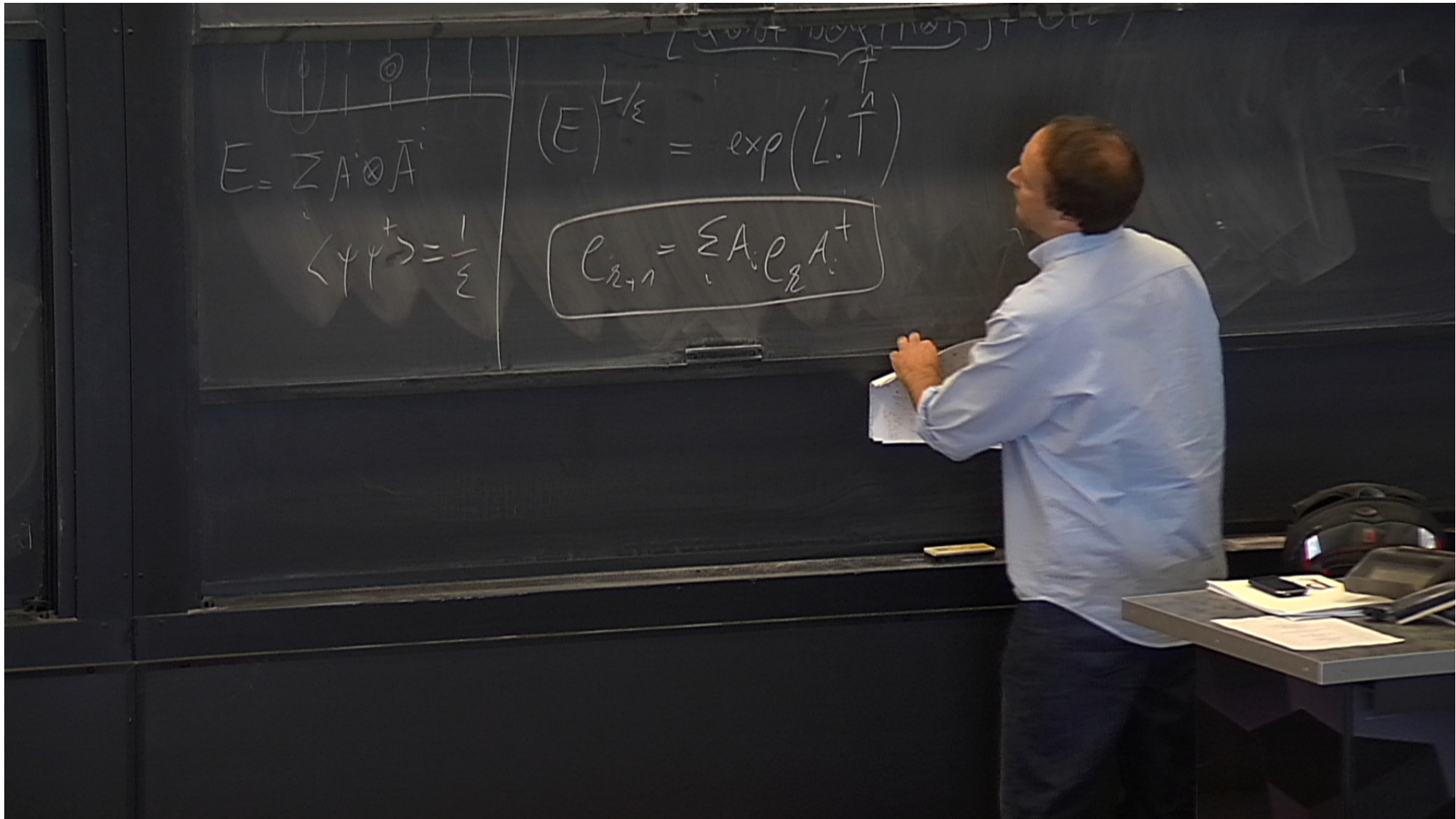
$$E = \sum A^i \otimes \bar{A}^i$$

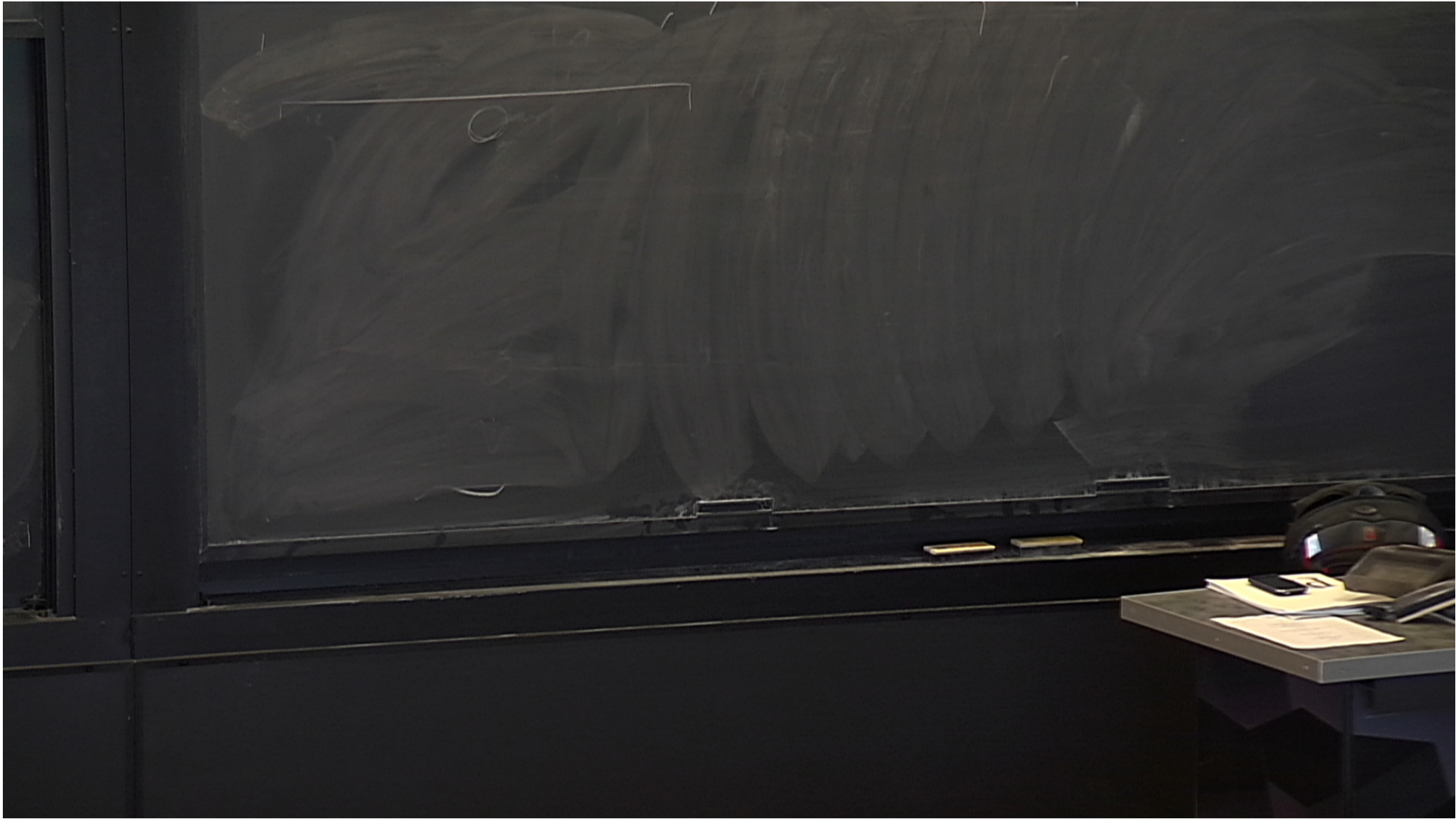
$$\langle \psi \psi^\dagger \rangle = \frac{1}{\varepsilon}$$

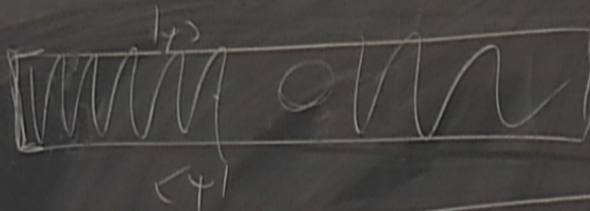
$$A^0 \otimes A^0 + A^1 \otimes A^1 \cdot \frac{1}{\varepsilon}$$

$$= \mathbb{1} + \varepsilon \left[\underbrace{Q \otimes \mathbb{1} + \mathbb{1} \otimes \bar{Q}}_{\uparrow} + R \otimes \bar{R} \right] + \mathcal{O}(\varepsilon^2)$$

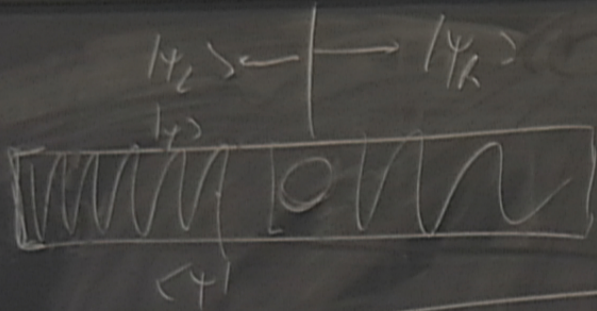
$$(E)^{L/\varepsilon} = \exp(L \cdot \hat{T})$$





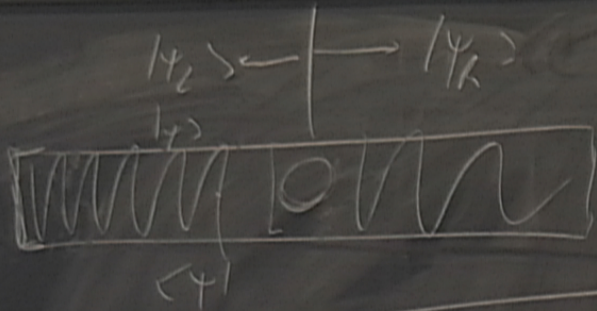


$$v(t) \rightarrow \left[\frac{de}{dt} = RQ + eQ + RQK^+ \right]$$



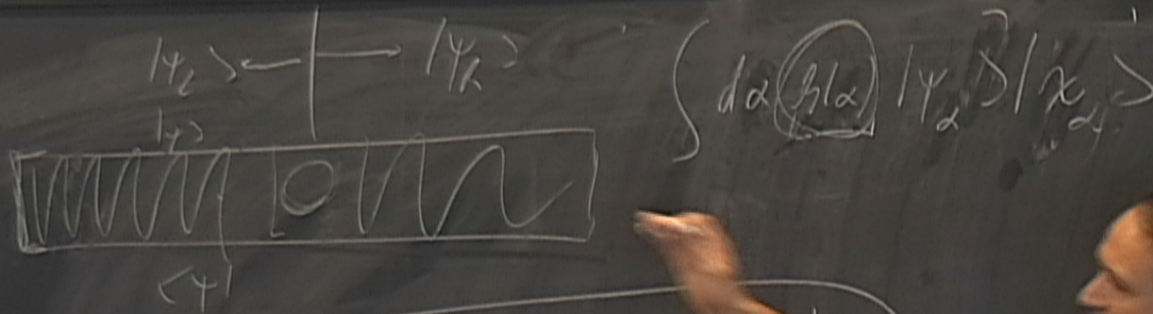
$$\left(d\alpha h(\alpha) |\psi \right.$$

$$\langle e| \rightarrow \left[\frac{de}{dt} = Qe + eQ + RQR^\dagger \right]$$



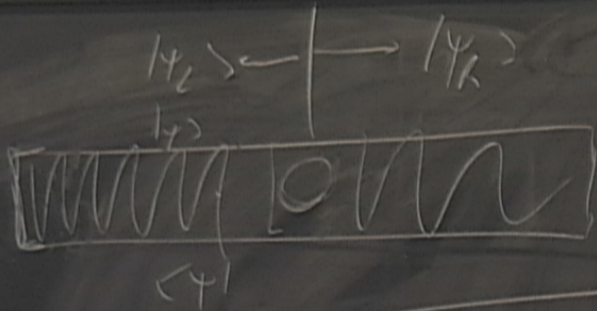
$$\left(d\alpha \frac{h}{2\pi} \int \psi \frac{\delta}{\delta \psi} \right)$$

$$\langle e | \rightarrow \frac{de}{dt} = Qe + RQR^+$$



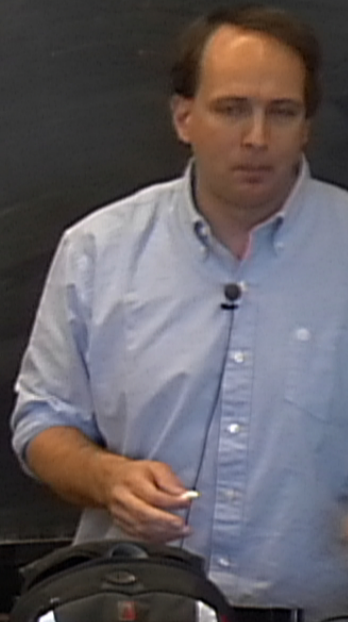
$$\left(d\alpha \frac{h\alpha}{2\pi} |y_i\rangle \right)$$

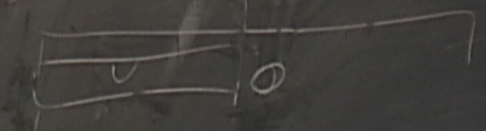
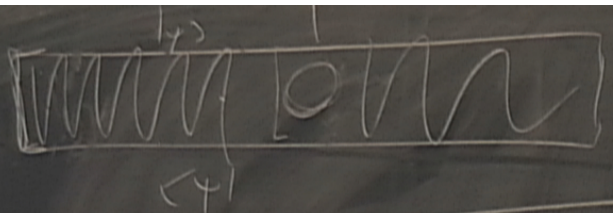
$$\langle e | \rightarrow \left[\frac{de}{dt} = Qe + eQ + RQ \right]$$



$$\left(d\alpha \frac{h}{2\pi} \right) |\psi_\alpha\rangle \langle \psi_\alpha|$$

$$\langle e | \rightarrow \left[\frac{de}{dt} = Qe + eQ + RQR^\dagger \right]$$





$$c_1 \rightarrow \left[\frac{de}{dt} = Qe + eQ + RQR^+ \right]$$

