Title: MERA for QFTs

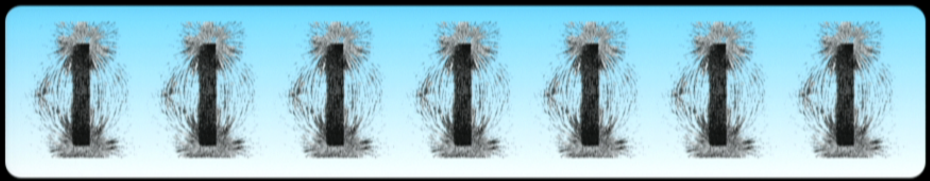
Date: Oct 25, 2011 02:00 PM

URL: http://pirsa.ca/11100090

Abstract: In this talk I will describe how to generalize the multiscale entanglement renormalization ansatz to quantum fields. The resulting variational class of wavefunctions, cMERA, arising from this RG flow are translation invariant and exhibit an entropy-area law. I'll illustrate the construction for some example fields, and describe how to cover the case of interacting theories.

Part 1: quantum spin systems

Physical systems, in this part, are 1D quantum spin systems, which are collections of *n* quantum spins:

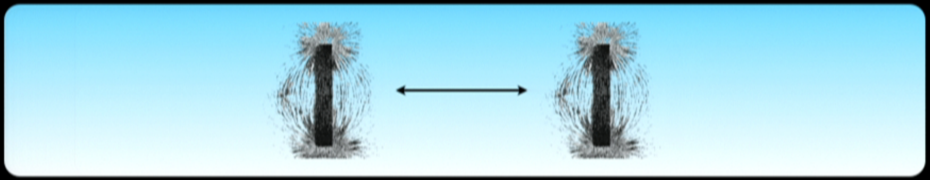


n quantum spins with global hilbert space:

$$(\mathbb{C}^d)^{\otimes n}$$

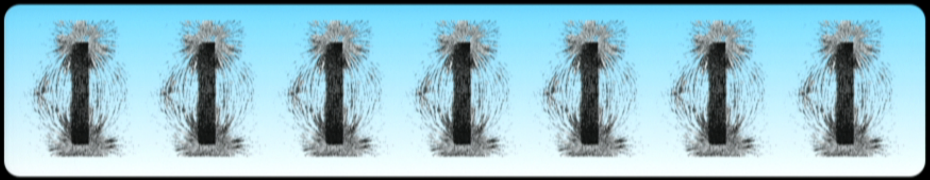
Interactions

The way our spins interact is via their nearest-neighbours:



$$h = \begin{pmatrix} \alpha_{11} & \dots & \alpha_{1d^2} \\ \vdots & \ddots & \vdots \\ \alpha_{d^2 1} & \dots & \alpha_{d^2 d^2} \end{pmatrix}$$

Hamiltonian



$$H = \sum_{j=1}^{n-1} h_j$$

where

$$h_j = \mathbb{I}_{1\cdots j-1} \otimes h \otimes \mathbb{I}_{j+2\cdots n}$$

The multiscale entanglement renormalisation ansatz

G. Vidal, Phys. Rev. Lett. **99**, 220405 (2007)

- Stage 0: *initialisation* to "all 0"s state $|\mathbf{0}\rangle$
- Stage 1: *local interaction* U_1
- Stage 2: transform *scale* by factor of 2
- Stage 3: *new uncorrelated* spins via $\mathcal{R}$
- Stage 4: repeat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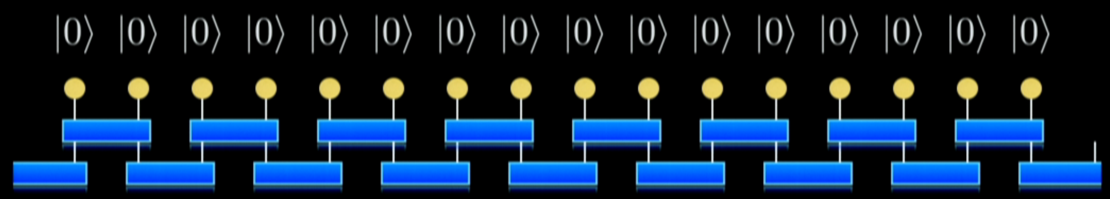
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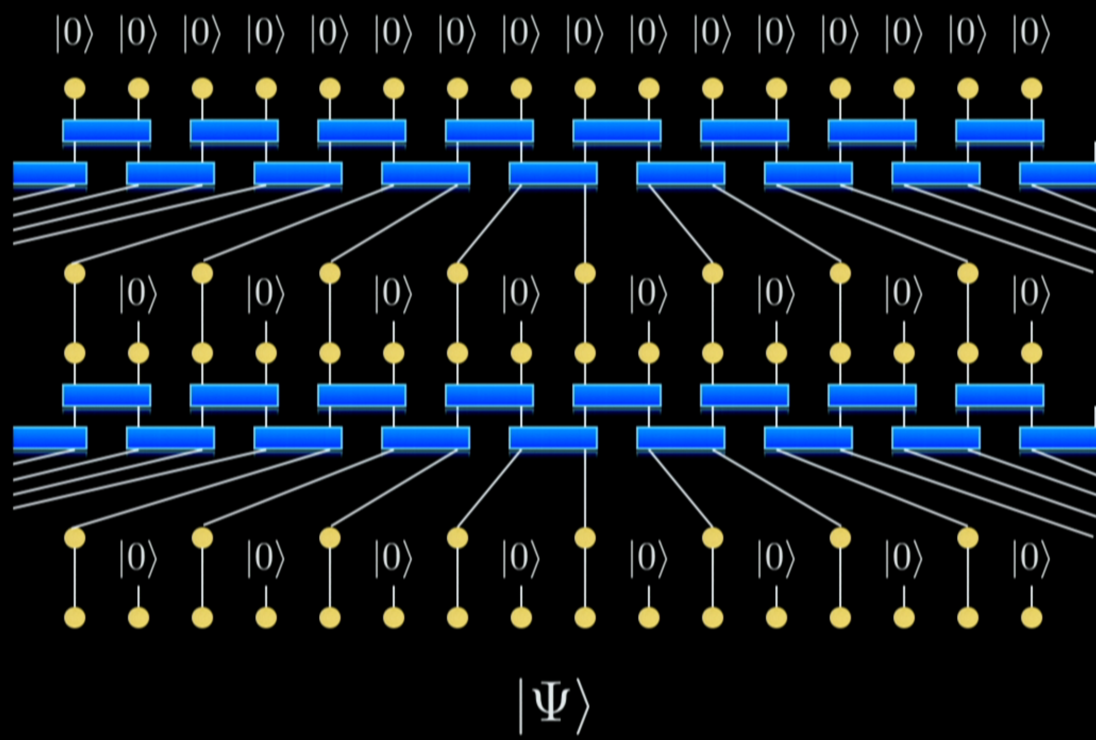
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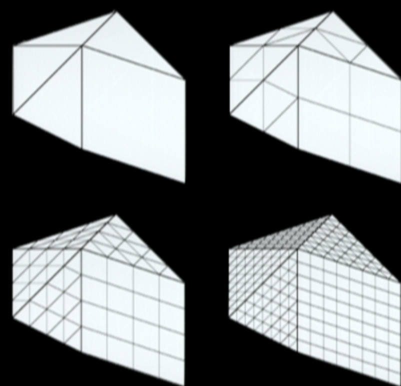
$|\Psi\rangle$

$$|\Psi_{\mathrm{MERA}}\rangle = U_m \mathcal{R} U_{m-1} \mathcal{R} \cdots \mathcal{R} U_1 |\mathbf{0}\rangle$$

MERA = dilation +
local interaction

The passage to the continuum

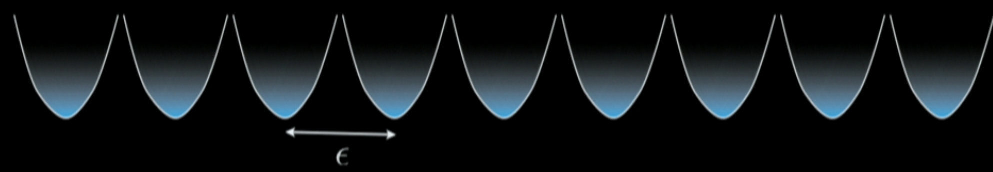


$$H = \int \frac{d\Psi^\dagger}{dx}\frac{d\Psi}{dx} + \int V(x-y)\psi^\dagger(x)\psi(x)\psi^\dagger(y)\psi(y)\, dy\, dx$$

Quantum fields

$$H = \int \pi^2 + (\nabla\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\, d^d x$$

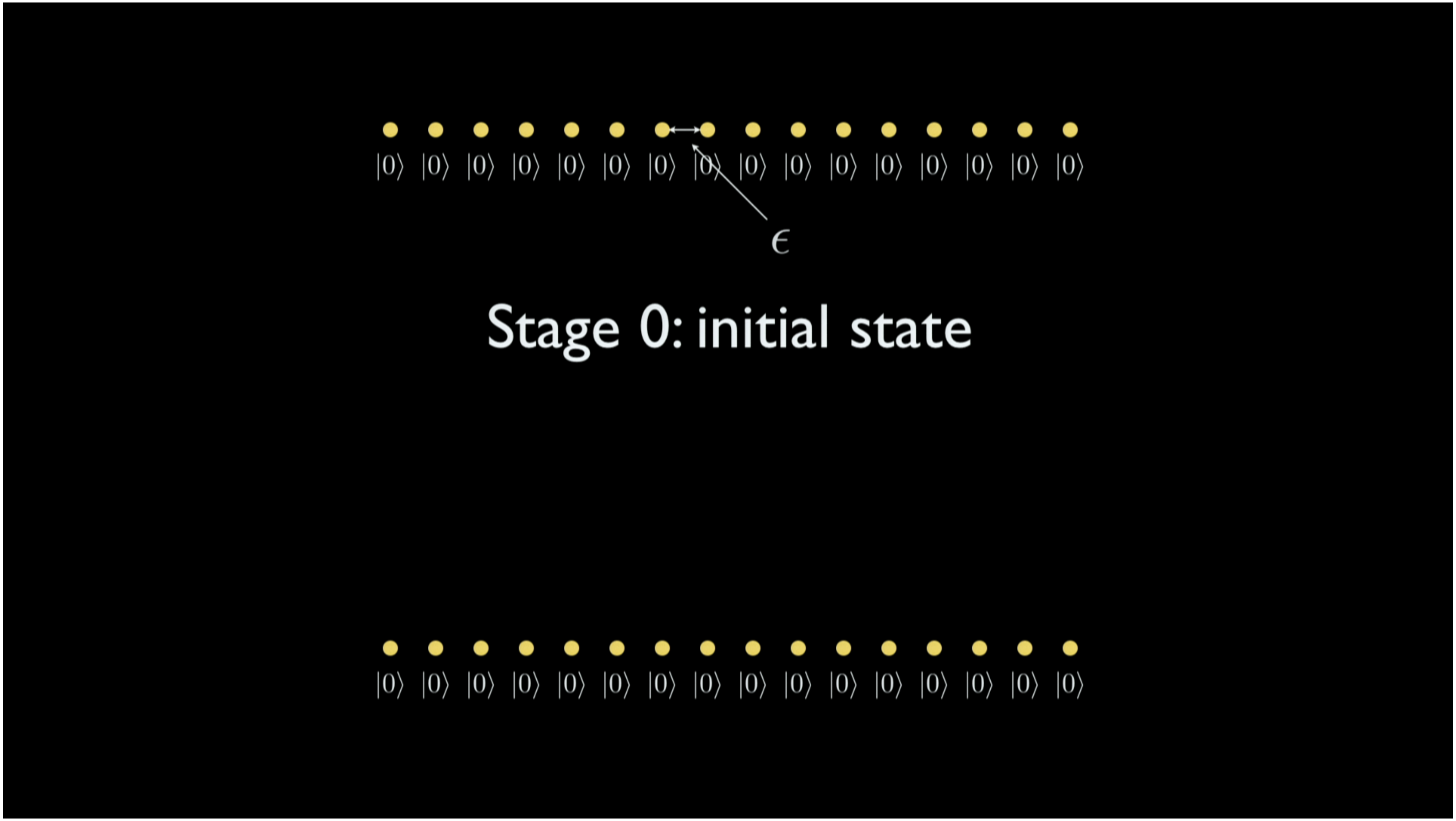
$$H = \int \left[-i\psi^\dagger\alpha\frac{d}{dx}\psi + m\psi^\dagger\beta\psi\right] - g^2 : (\psi^\dagger\psi)^2 :\, dx$$



$$\psi_j = a_j/\sqrt{\epsilon}$$

$$[a_j, a_k^\dagger] = \delta_{jk}$$

cMERA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ϵ
Stage 0: initial state
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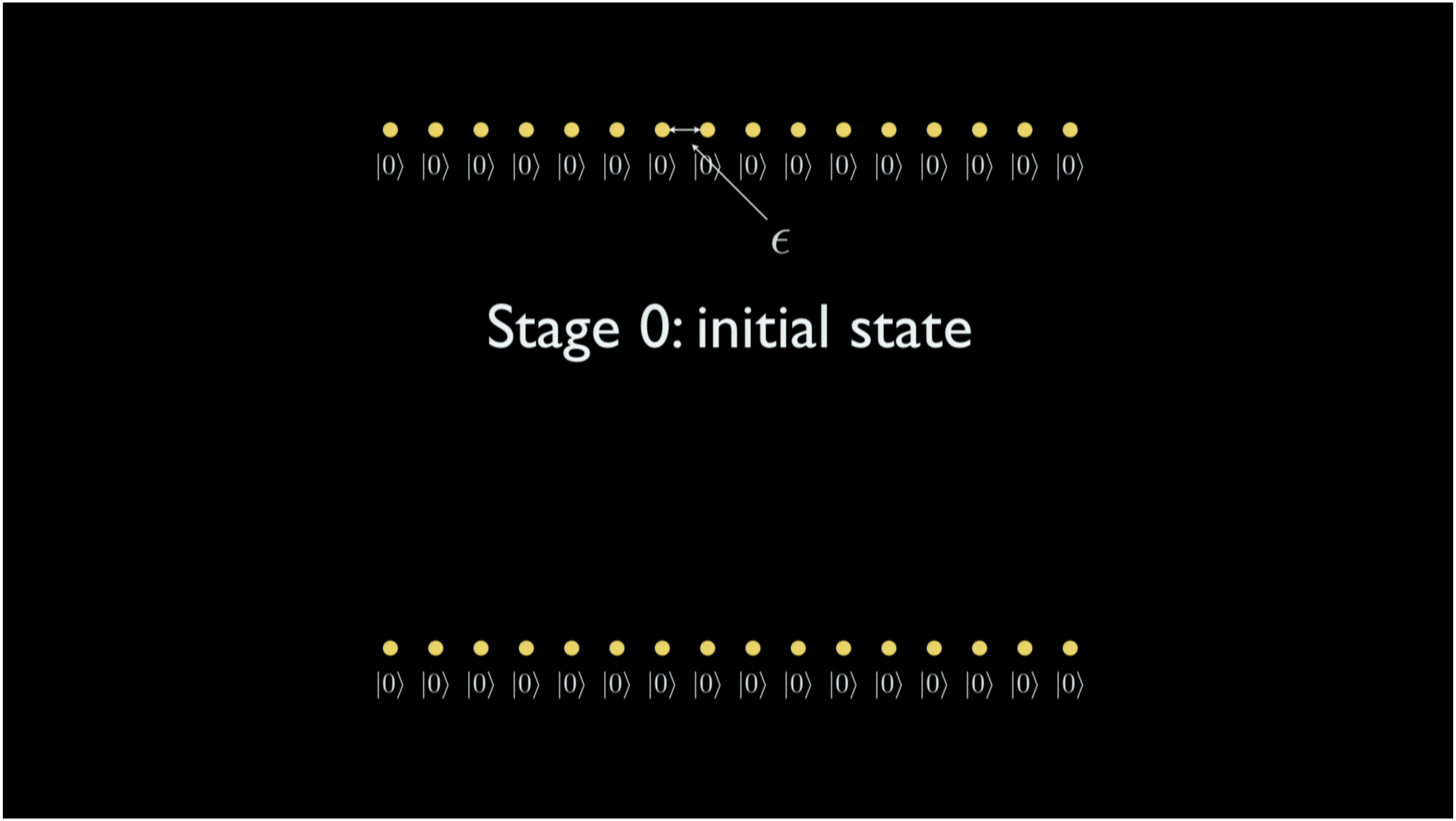
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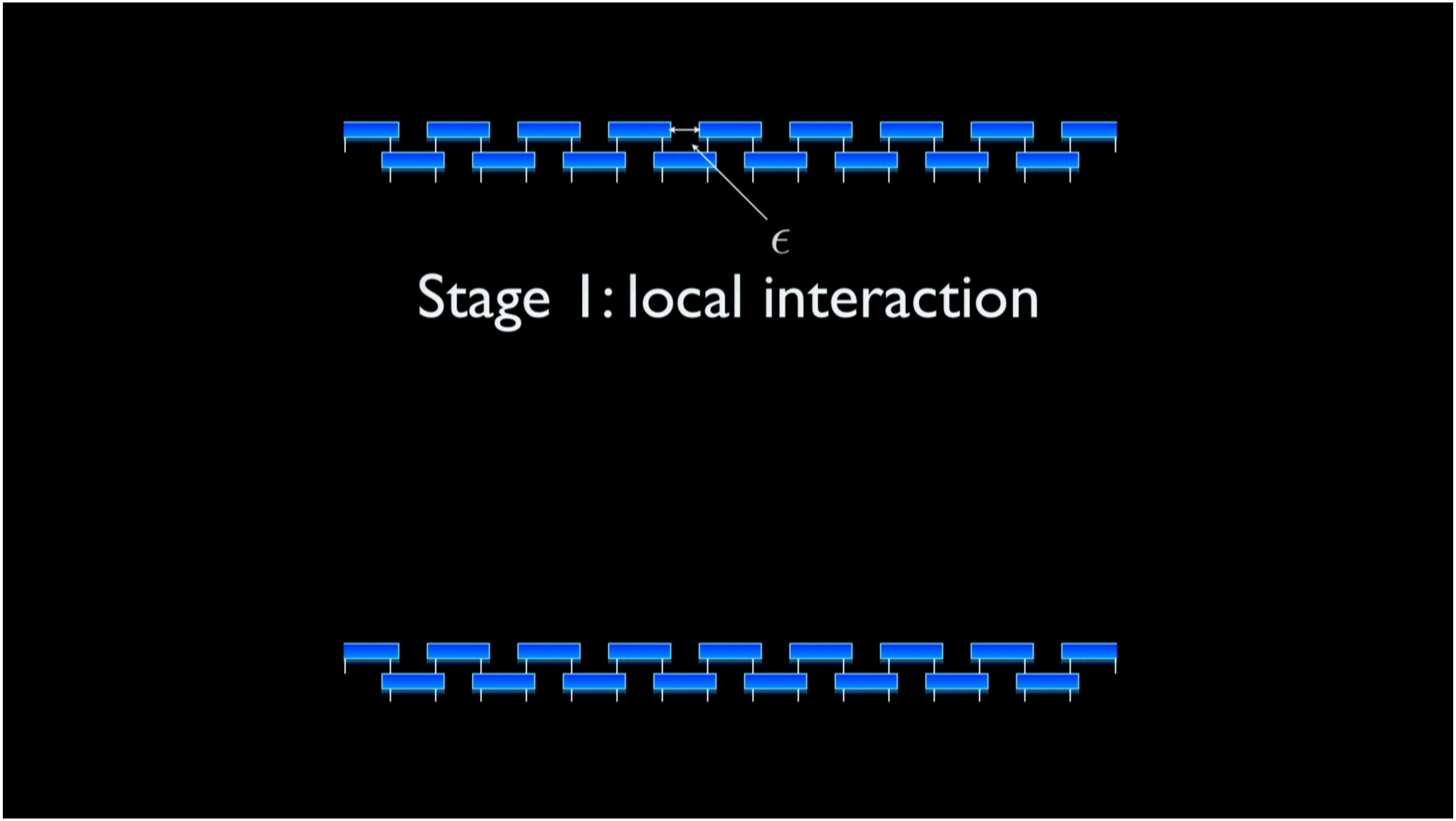
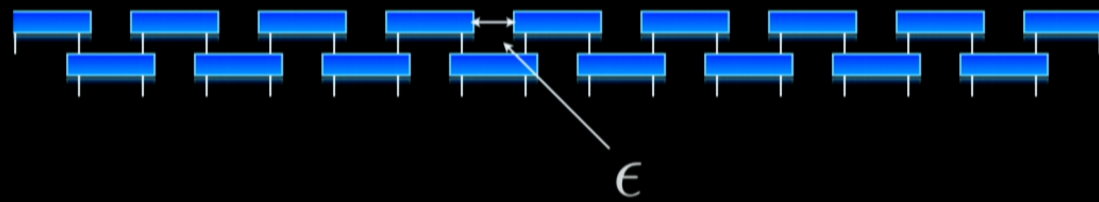
ϵ
Stage 0: initial state
|0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩ |0⟩


ϵ
Stage 1: local interaction



Stage 1: local interaction

$$K = \int dx\, k(x)$$

$$U_1 = e^{-i\delta K}$$

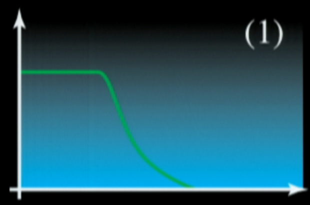
Stage 2: scale transform

Generator:

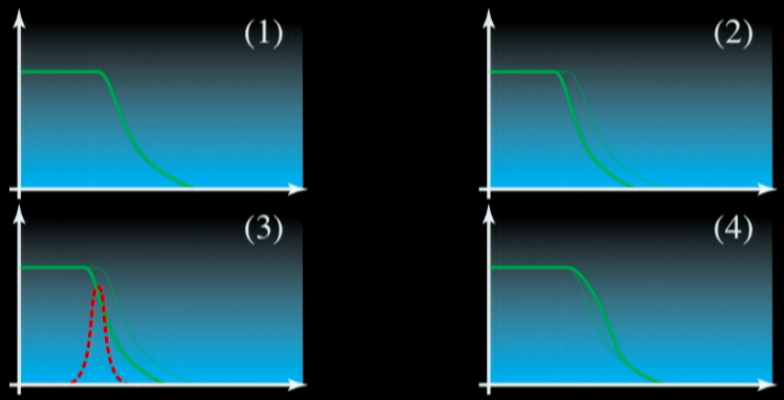
$$L = -\frac{i}{2}\int \psi^\dagger(x) x \frac{d\psi(x)}{dx} - x\frac{d\psi^\dagger(x)}{dx}\psi(x)\, dx$$

$$\mathcal{R} \sim e^{-i\delta L}$$

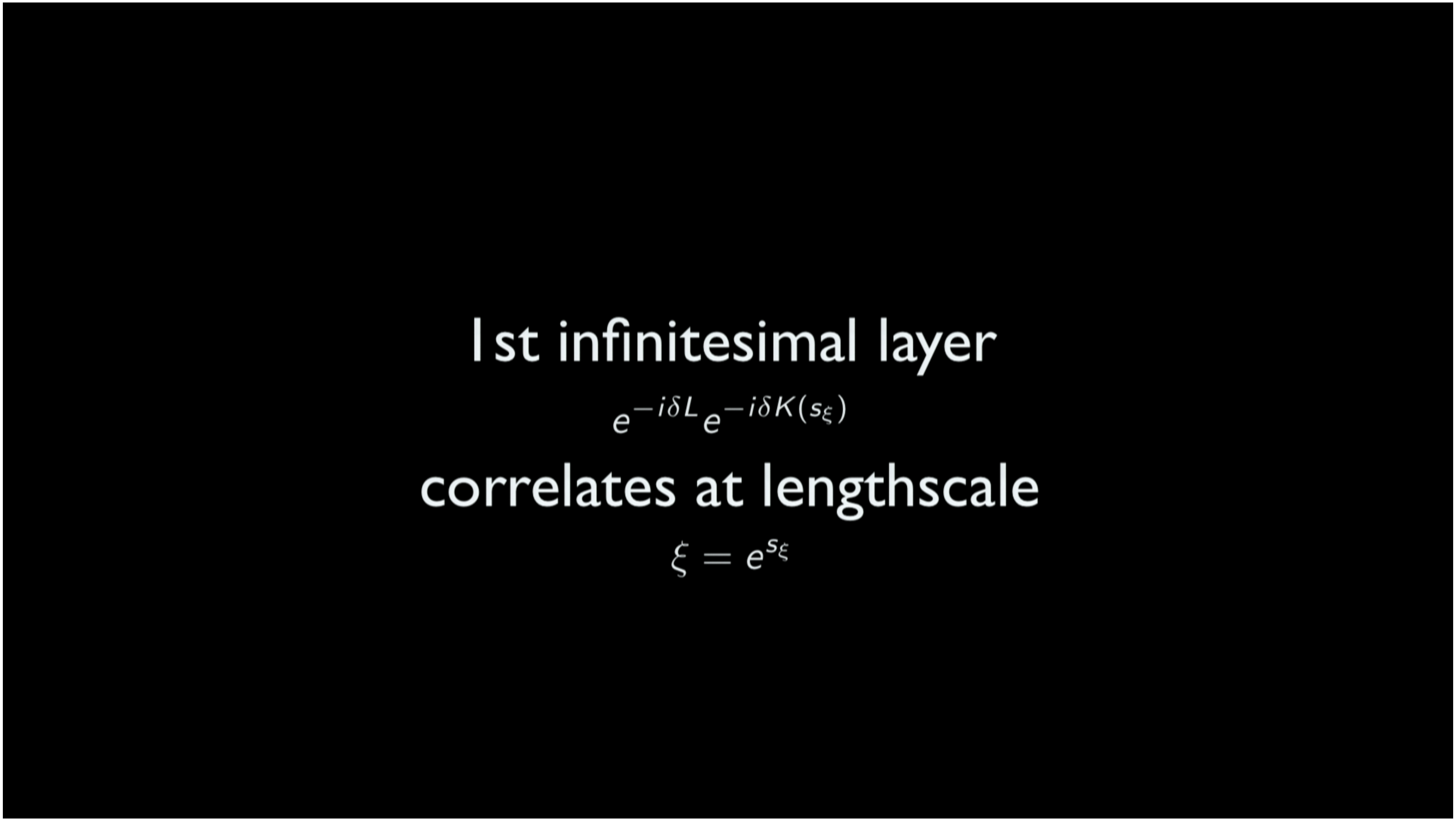
Stage 3: new degrees of freedom?

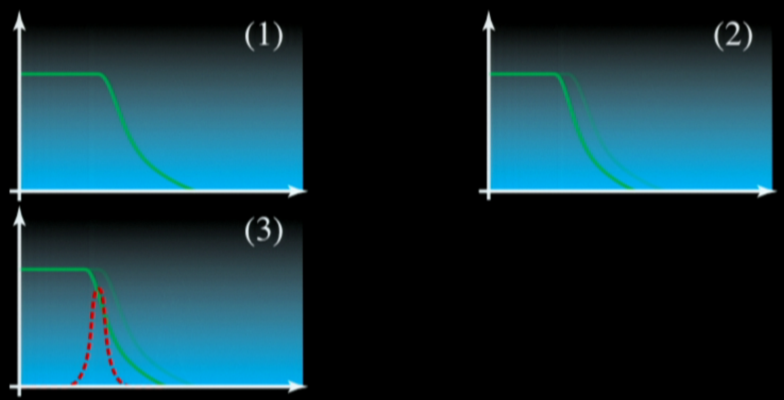


cMERA steps in momentum space



cMERA steps in momentum space


1st infinitesimal layer
$e^{-i\delta L} e^{-i\delta K(s_\xi)}$
correlates at lengthscale
$\xi = e^{s_\xi}$

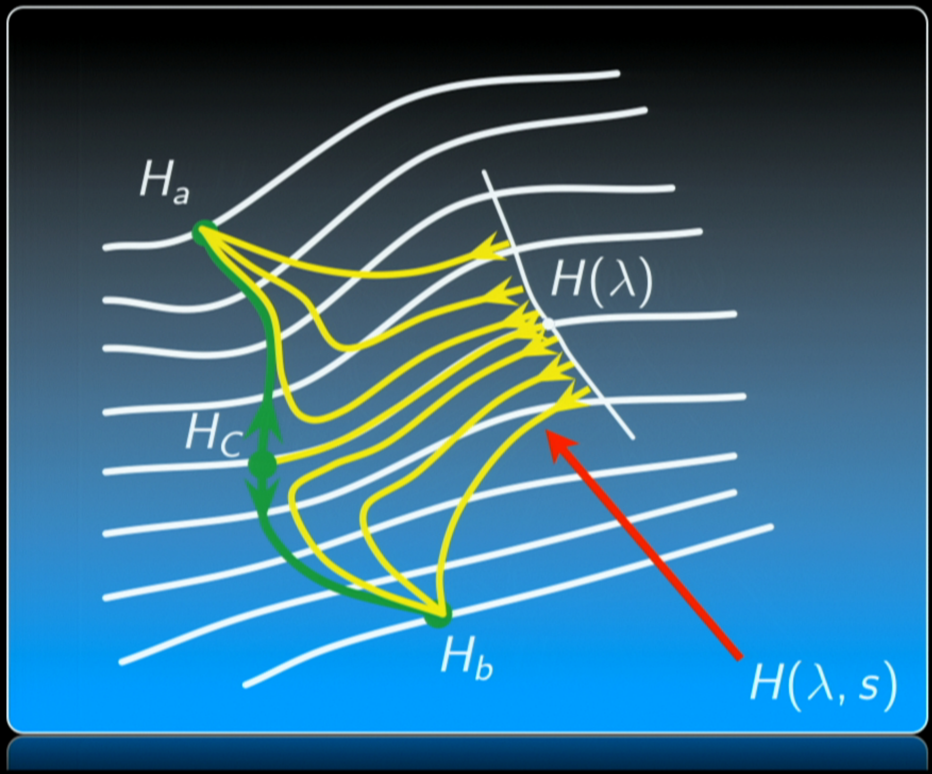


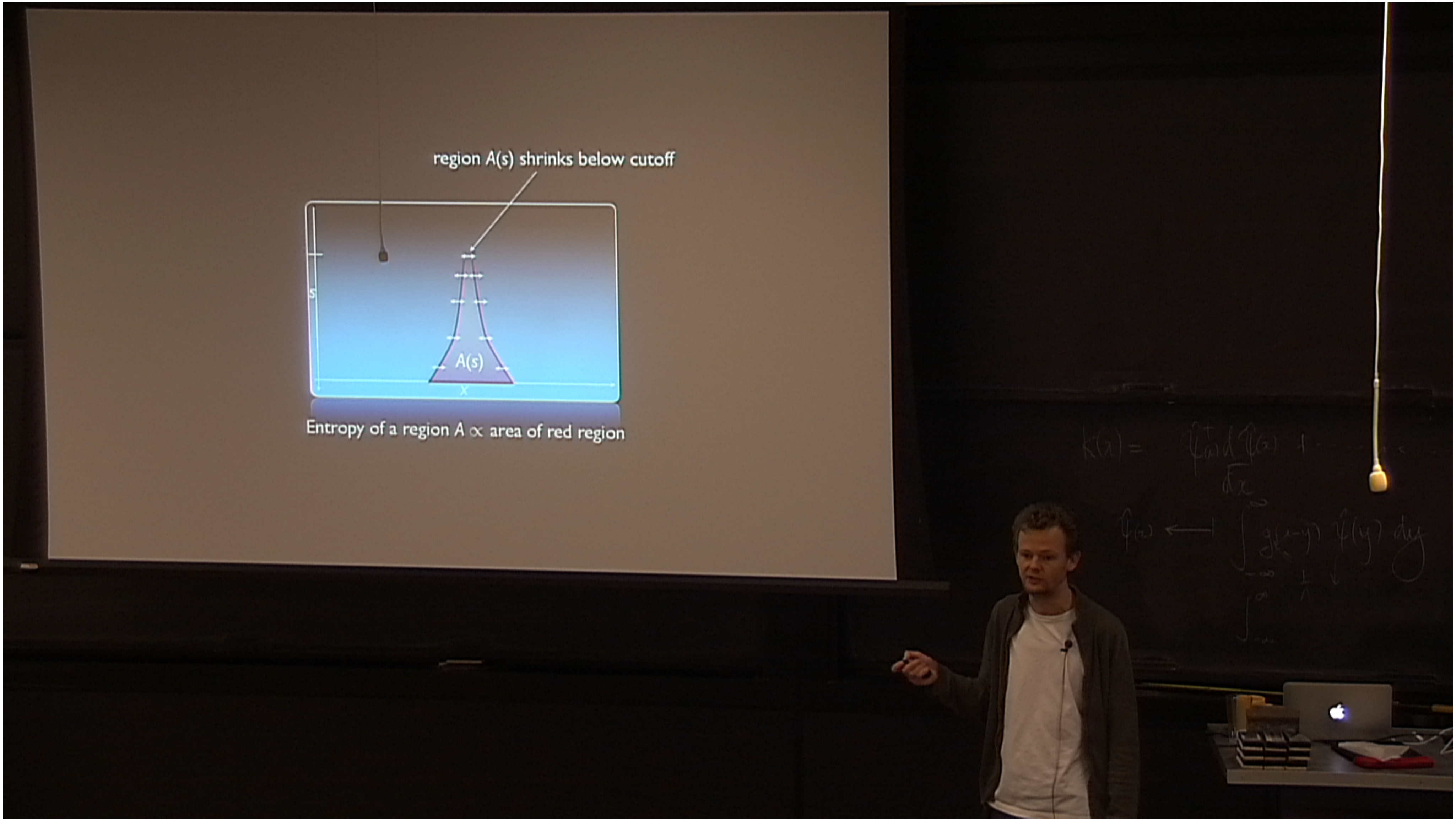
cMERA steps in momentum space

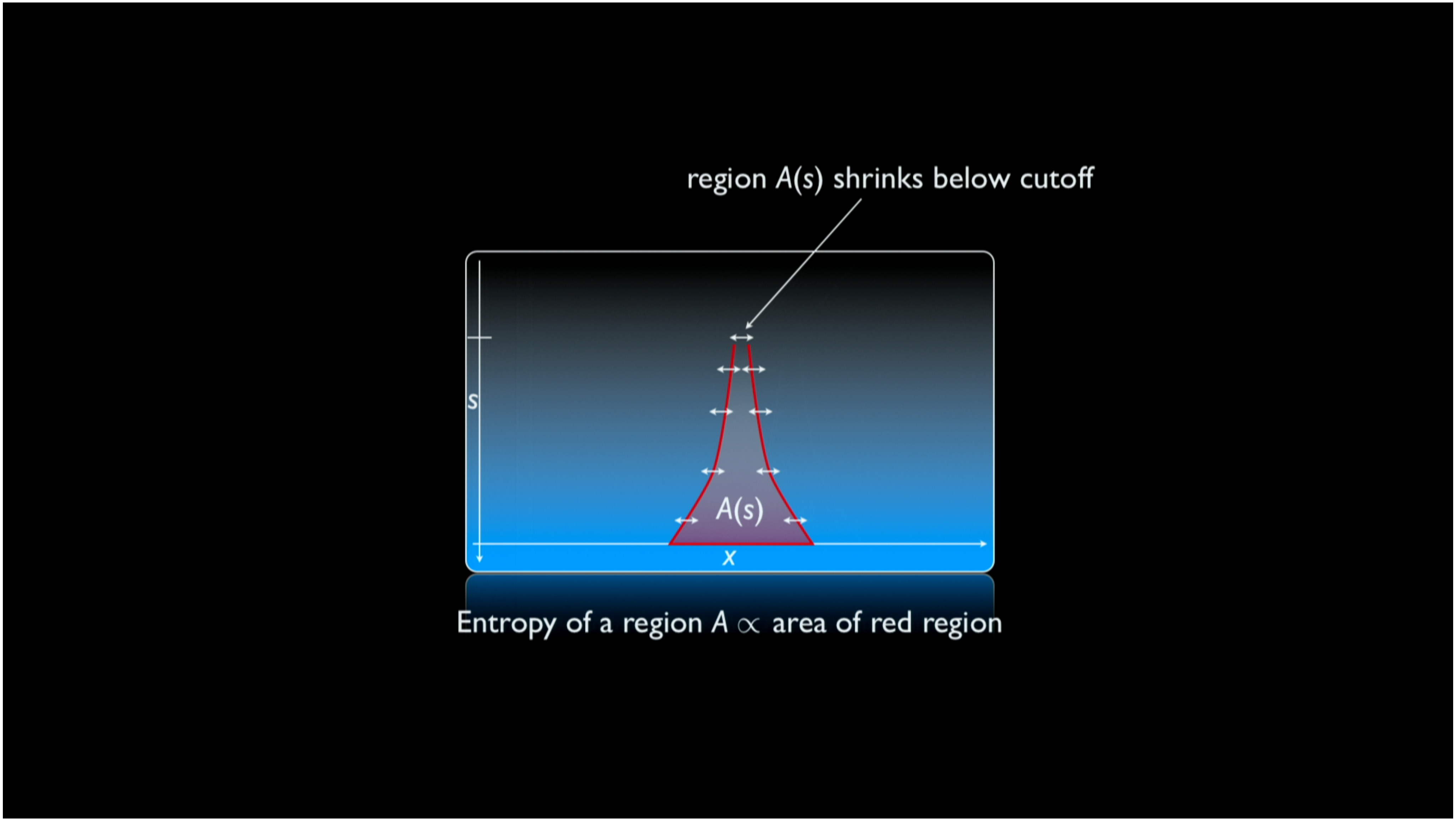
Renormalized operator

$$A_R(s) \equiv U^\dagger(s_\epsilon, s) A U(s_\epsilon, s)$$

$$\frac{dA_R(s)}{ds} = i[K(s) + L, A_R(s)]$$


H_a
$H(\lambda)$
H_C
H_b
$H(\lambda, s)$


region A(s) shrinks below cutoff
s
A(s)
x
Entropy of a region A ∝ area of red region


region $A(s)$ shrinks below cutoff
s
$A(s)$
x
Entropy of a region $A \propto$ area of red region

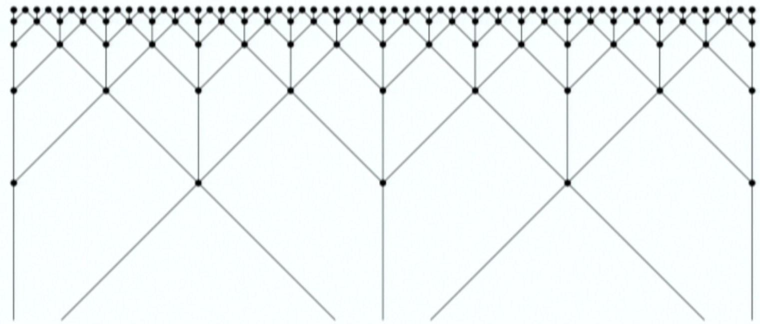
Entanglement between A and rest of field generated by K:

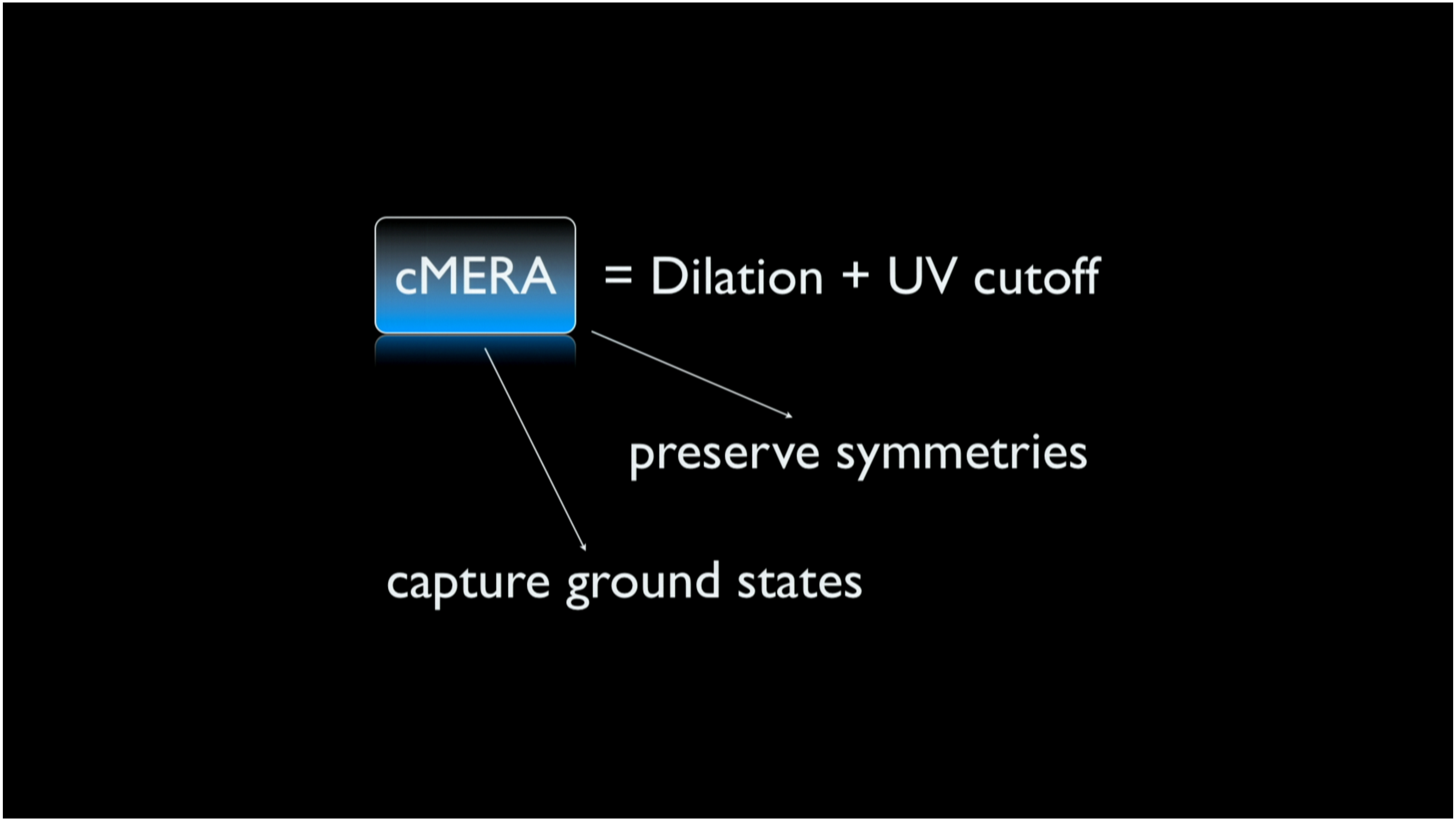
$$\frac{dS_A(t)}{dt} \leq c|\partial A|$$

Non-rel. bosonic ground state:

$$H = \int \left[\frac{d\psi^\dagger}{dx} \frac{d\psi}{dx} + \mu \psi^\dagger \psi - \nu ({\psi^\dagger}^2 + \psi^2) \right] dx$$

MERA as causal set




cMERA
= Dilation + UV cutoff
preserve symmetries
capture ground states