Title: Lattice Study of a Technicolor Dark Matter Candidate

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Abstract: The simplest technicolor model contains would-be Goldstone bosons to provide masses for the observed W and Z particles, replacing the standard Higgs mechanism. Perhaps surprisingly, it also contains an additional Goldstone boson that is a natural dark matter candidate. A recent lattice simulation has confirmed the symmetry-breaking pattern, explored the mass spectrum of the lightest technihadrons, and established an effective field theory.

Lattice study of a technicolor dark matter candidate

arXiv:1109.3513

in collaboration with
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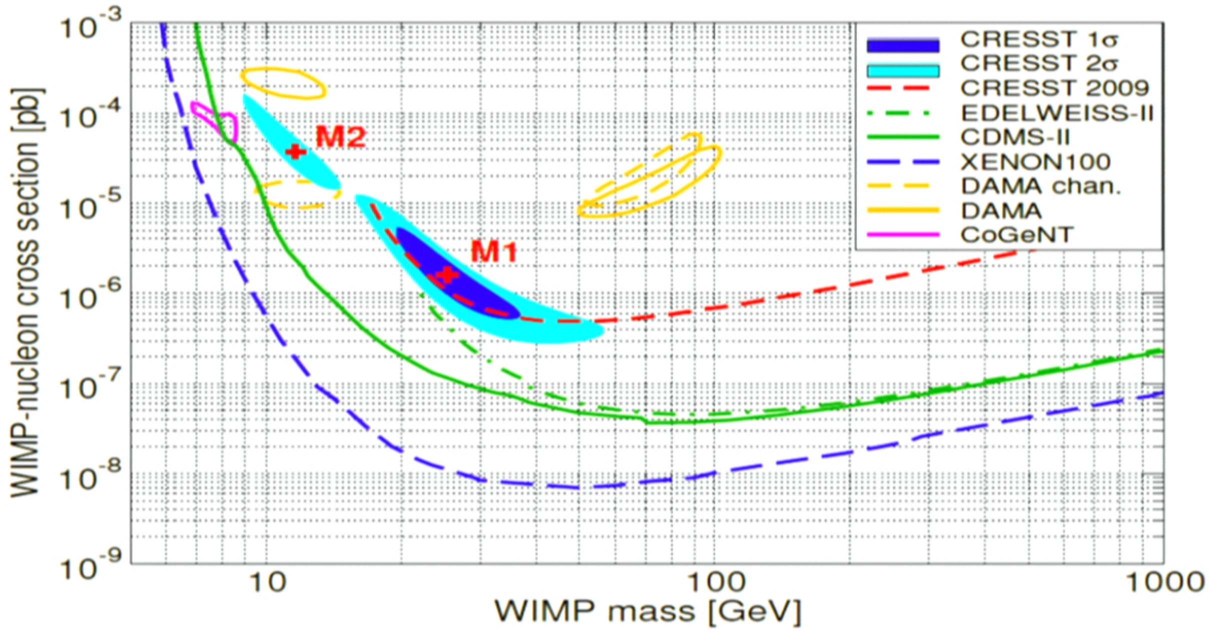
The experimental situation for WIMPs

(Figure 13 from CRESST-II, arXiv:1109.0702)



Standard model without Higgs

$SU(3)_C \times SU(2)_W \times U(1)_Y$

- Quarks and leptons are massless.
- The W and Z "eat" the pions to become massive.

Fig. 6.

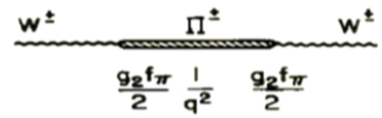


Fig. 7.

Fig. 8.

Figures from Farhi and Susskind, Physics Reports 74, 277 (1981).

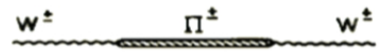
- $\frac{m_W}{m_Z}$ is protected by a custodial SU(2) global symmetry.
- m_W and m_Z are separately a factor of 2600 smaller than experiment.

Standard model without Higgs + simplest technicolor

$SU(3)_C \times SU(2)_W \times U(1)_Y \times SU(2)_{TC}$

- Quarks and leptons are massless.
- The W and Z "eat" the technipions to become massive.

Fig. 6.



$\frac{g_2 f_\pi}{2} \quad \frac{1}{q^2} \quad \frac{g_2 f_\pi}{2}$

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Figures from Farhi and Susskind, Physics Reports 74, 277 (1981).

- $\frac{m_W}{m_Z}$ is protected by a custodial SU(2) global symmetry.
- Choose f_Π such that m_W and m_Z agree with experiment.

Quantum numbers of the techniquarks

In the simplest technicolor model,

- U is a technicolor doublet (fundamental representation).
- D is a technicolor doublet (fundamental representation).

- U and D do not carry QCD color.

- U_L and D_L form a weak doublet.
- U_R and D_R are weak singlets.

- U has electric charge $+1/2$.
- D has electric charge $-1/2$.

There are many familiar extensions of this simple model.

The simple model is sufficient for our lattice study,
but extensions would be straightforward to implement.

Chiral symmetry breaking in QCD

$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$

- Neglecting u and d quark masses, the Lagrangian has global $SU(2)_L \times SU(2)_R$:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar{q}_L\gamma^\mu D_\mu q_L + i\bar{q}_R\gamma^\mu D_\mu q_R \\ q_L &= \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\ q_R &= \begin{pmatrix} u_R \\ d_R \end{pmatrix}\end{aligned}$$

- With degenerate u and d quark masses, the Lagrangian has global $SU(2)_V$:

$$\delta\mathcal{L} = -m\bar{q}_R q_L + \text{h.c.}$$

DYNAMICAL SYMMETRY BREAKING:

- Even without u and d quark masses, the hadron spectrum is $SU(2)_V$ multiplets, not $SU(2)_L \times SU(2)_R$ multiplets.

For example, $m_{\rho^\pm} \approx m_{\rho^0} \quad \neq \quad m_{a_1^\pm} \approx m_{a_1^0}$.

Chiral symmetry breaking in simplest technicolor

SU(4) $\longrightarrow$ Sp(4)

Appelquist, Rodriques da Silva & Sannino, PRD60, 116007 (1999)
Ryttov & Sannino, PRD78, 115010 (2008)

- Neglecting U and D quark masses, the Lagrangian has global SU(4):

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{Q}\gamma^\mu D_\mu Q$$

$$Q = \begin{pmatrix} U_L \\ D_L \\ -i\sigma^2 C\overline{U}_R^T \\ -i\sigma^2 C\overline{D}_R^T \end{pmatrix}$$

- With degenerate U and D quark masses, the Lagrangian has global Sp(4):

$$\delta\mathcal{L} = \frac{m}{2}Q^T(-i\sigma^2 C)EQ + \text{h.c.}, \qquad E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

HYPOTHESIS (dynamical symmetry breaking):

- Even without U and D quark masses, the hadron spectrum is Sp(4) multiplets, not SU(4) multiplets.

Seeing the SU(4) and Sp(4) symmetries

Act on the Lagrangian with an infinitesimal SU(4) transformation defined by

$$Q \to \left(1 + i \sum_{n=1}^{15} \alpha^n T^n\right) Q$$

- The kinetic terms are invariant *because the fundamental representation is real.* This is true for $N_{\rm TC} = 2$ but not for $N_{\rm TC} > 2$.

- The mass terms are not invariant under SU(4):

$$\mathcal{L} \to \mathcal{L} + \frac{im}{2} \sum_{n=1}^{15} \alpha^n Q^T(-i\sigma^2 C)\left(ET^n + T^{nT}E\right) Q + \text{h.c.}$$

Only 10 of the 15 generators leave $\mathcal{L}$ invariant: those that obey $ET^n + T^{nT}E = 0$. These 10 generators define an Sp(4) Lie algebra.

Recall: $E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$.

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Anticipating dynamical symmetry breaking

In QCD, the nonzero vacuum expectation value is $\langle \overline{u}u \rangle = \langle \overline{d}d \rangle \neq 0.$
This has the same form as the (degenerate) explicit mass terms.
Therefore dynamical breaking has the same structure as explicit breaking:

$$\mathrm{SU(2)_L \times SU(2)_R} \longrightarrow \mathrm{SU(2)_V}$$

The 3 broken generators require 3 Goldstone bosons: π^+, π^-, π^0.

In the simplest technicolor model, we expect $\langle \overline{U}U \rangle = \langle \overline{D}D \rangle \neq 0.$
This has the same form as the (degenerate) explicit mass terms.
Therefore dynamical breaking would have the same structure as explicit breaking:

$$\mathrm{SU(4)} \longrightarrow \mathrm{Sp(4)}$$

The 5 broken generators would require 5 Goldstone bosons.
We see these 5 Goldstone bosons in our lattice simulations.

Technihadron operators

local operators for technimesons:

$$\begin{aligned} \mathcal{O}^{(\Gamma)}_{\overline{U}D}(x) &= \overline{U}(x)\Gamma D(x) \\ \mathcal{O}^{(\Gamma)}_{\overline{D}U}(x) &= \overline{D}(x)\Gamma U(x) \\ \mathcal{O}^{(\Gamma)}_{\overline{U}U\pm\overline{D}D}(x) &= \frac{1}{\sqrt{2}}\Big(\overline{U}(x)\Gamma U(x) \pm \overline{D}(x)\Gamma D(x)\Big) \end{aligned}$$

local operators for technibaryons (techni-diquarks):

$$\begin{aligned} \mathcal{O}^{(\Gamma)}_{UD} &= U^T(x)(-i\sigma^2 C)\Gamma D(x) \\ \mathcal{O}^{(\Gamma)}_{DU} &= D^T(x)(-i\sigma^2 C)\Gamma U(x) \end{aligned}$$

(single-flavor baryon operators vanish)

Note: $\Gamma = 1$ or γ^5 or γ^μ or ... is any Dirac structure.

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Technihadron correlation functions

Put a creation operator at time t_x and an annihilation operator at time t_y.

Technimeson example:

$$\begin{aligned} C_{UD}^{(\Gamma)}(t_x - t_y) &= \sum_{\vec{x}} \sum_{\vec{y}} \mathcal{O}_{UD}^{(\Gamma)}(y) \left(\mathcal{O}_{UD}^{(\Gamma)}(x) \right)^\dagger \\ &= \sum_{\vec{x}} \sum_{\vec{y}} \mathrm{Tr}\left[\Gamma D(y) \overline{D}(x) \gamma^0 \Gamma^\dagger \gamma^0 U(x) \overline{U}(y) \right] \end{aligned}$$

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Therefore each technibaryon is mass-degenerate with a technimeson.

Parity partners and Goldstone bosons

The degenerate pairs have equal angular momentum but opposite parities:

$$J\left(\mathcal{O}^{(\Gamma)}_{UD}\right) = J\left(\mathcal{O}^{(\Gamma)}_{\overline{U}D}\right)$$
$$P\left(\mathcal{O}^{(\Gamma)}_{UD}\right) = -P\left(\mathcal{O}^{(\Gamma)}_{\overline{U}D}\right)$$

Because of this, we expect the 5 Goldstone bosons to be

- the pseudoscalars Π^+ and Π^-
- their degenerate scalars Π_{UD} and $\Pi_{\overline{UD}}$
- the neutral pseudoscalar Π^0

The 3 pseudoscalars will be eaten by W and Z.
The 2 scalars are our dark matter candidate and its antiparticle.

Choices for our lattice explorations

The Wilson action is used for technicolor:

$$S_W = \frac{\beta}{2}\sum_{x,\mu,\nu}\left(1-\frac{1}{2}\mathrm{ReTr}U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)\right)+(4+m_0)\sum_x\bar{\psi}(x)\psi(x)$$
$$-\frac{1}{2}\sum_{x,\mu}\left(\bar{\psi}(x)(1-\gamma_\mu)U_\mu(x)\psi(x+\hat{\mu})+\bar{\psi}(x+\hat{\mu})(1+\gamma_\mu)U_\mu^\dagger(x)\psi(x)\right)$$

Two choices for β gives two lattice spacings.

Six choices of m_0 give six techniquark masses per β.

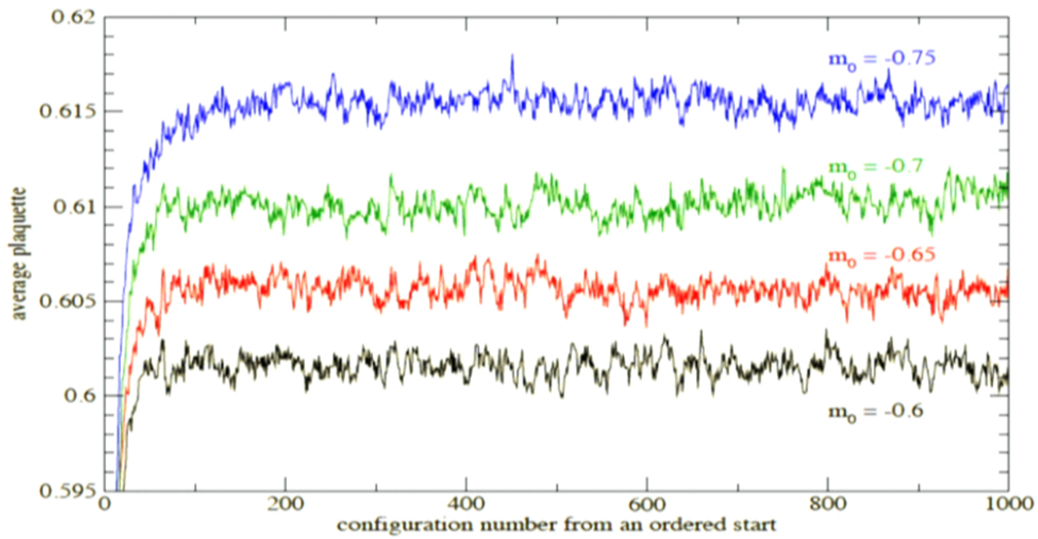
β	m_0
2.0	-0.85, -0.90, -0.94, -0.945, -0.947, -0.949
2.2	-0.60, -0.65, -0.68, -0.70, -0.72, -0.75

All lattices are $16^3\times 32$.

Electroweak interactions are perturbative, therefore neglected.

Creating ensembles of gauge fields

Raw data for $\beta = 2.2$.



We define each ensemble = {cfg320, cfg340, cfg360, . . . cfg1000}.

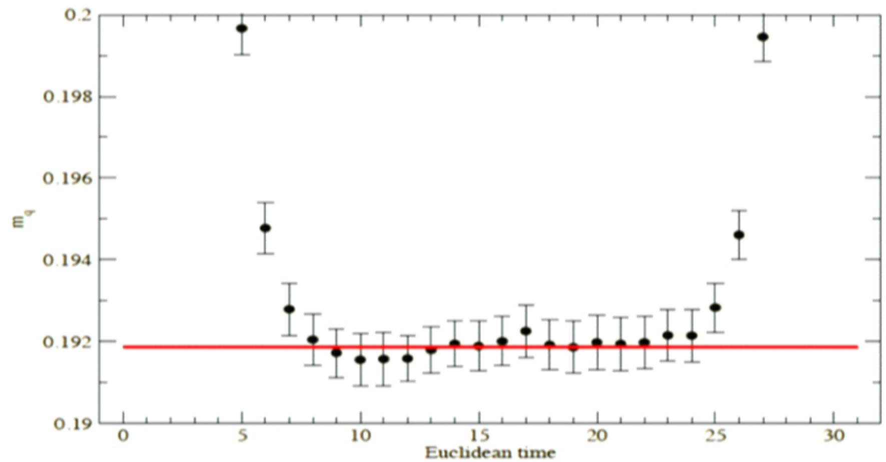
Techniquark propagators are random U(1) wall sources, averaged over each time step.

Determining a techniquark mass

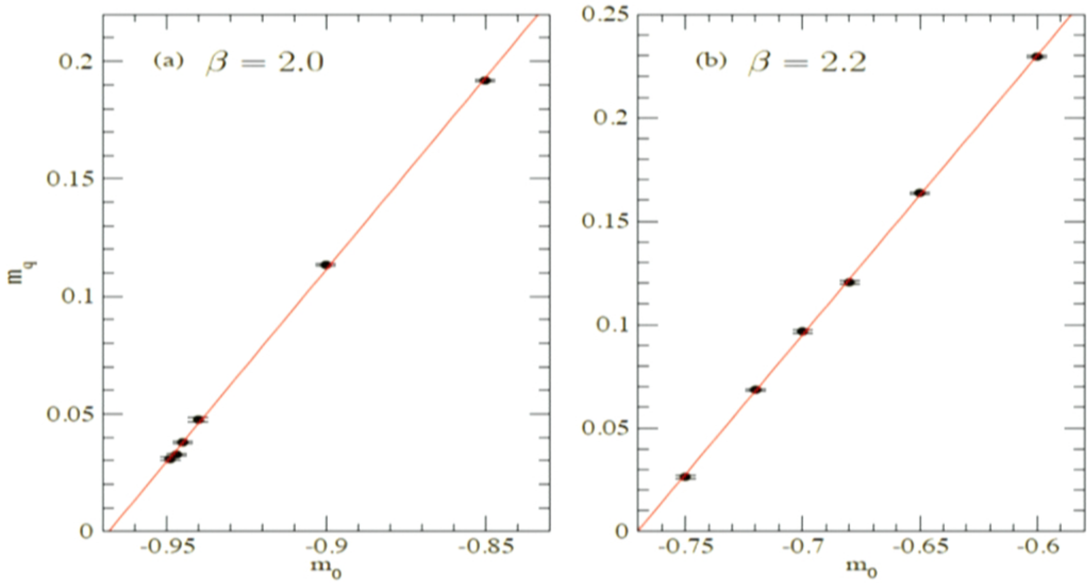
For lattice, a convenient definition of quark mass comes from PCAC:

$$m_q = \lim_{t\to\infty} \left(\frac{\langle A_4(t+1)P(0)\rangle - \langle A_4(t-1)P(0)\rangle}{4\langle P(t)P(0)\rangle} \right) ,$$

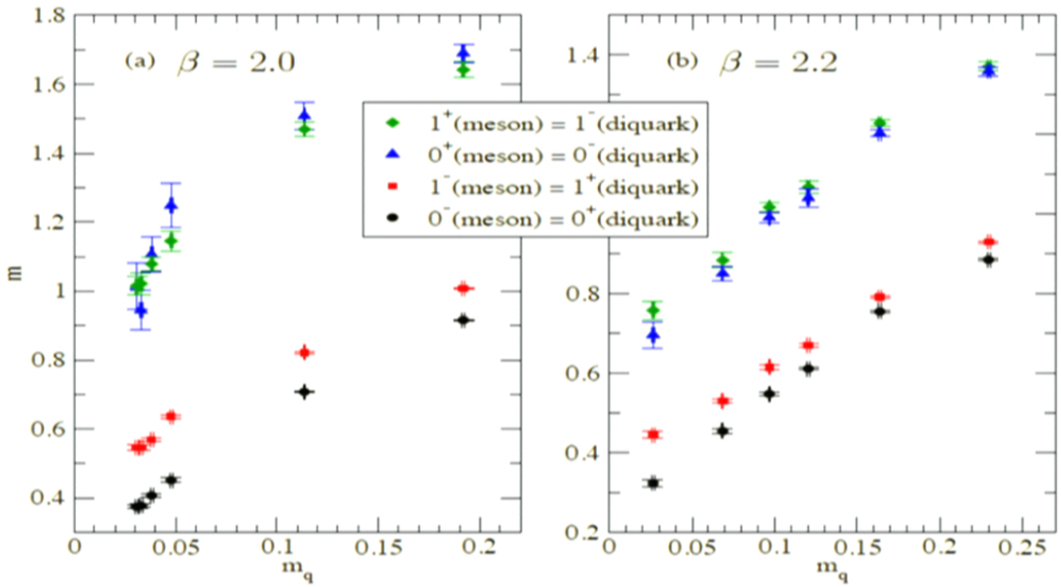
Here is the example of $\beta = 2.2$ and $m_0 = -0.75$:



The PCAC mass is linear in the bare mass

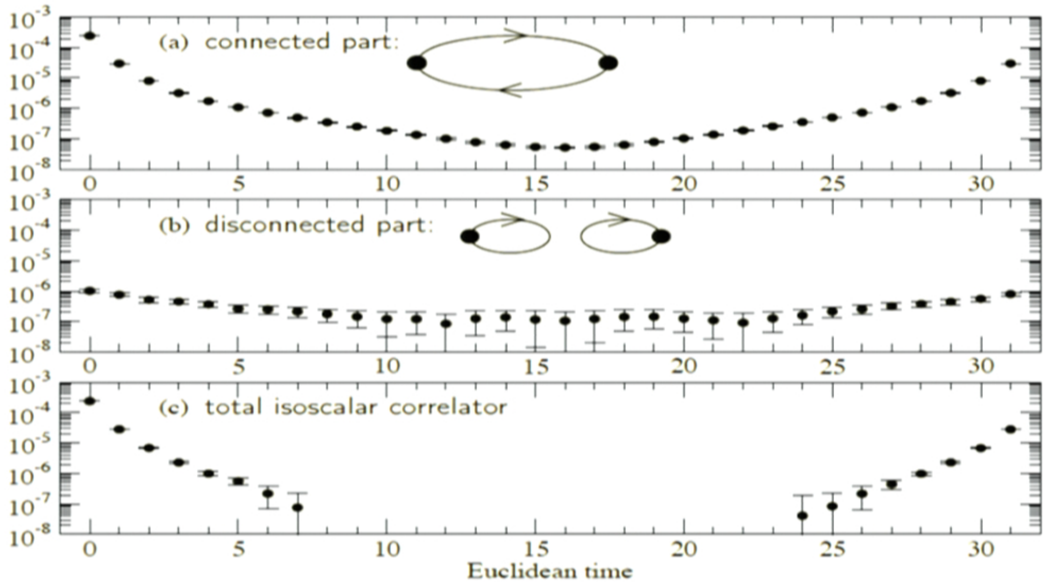


Exploring the spectrum of technihadron masses



Extrapolation to $m_q = 0$ gives nonzero masses to non-Goldstones.
For $\beta = 2.2$, all masses extrapolate to below the lattice cutoff, $m \sim 1$.

Isoscalar pseudoscalar meson



(a) shows our signal for the isovector Goldstone boson.

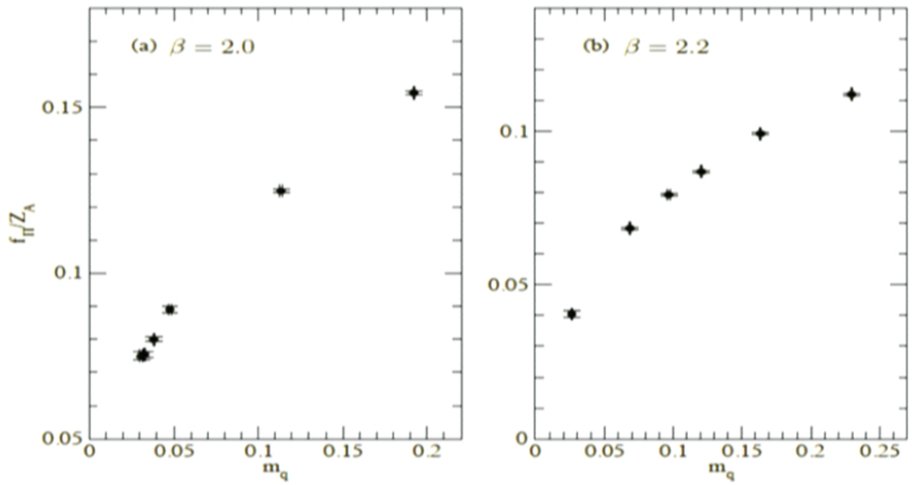
(c) shows no signal for any isoscalar Goldstone boson.

Goldstone decay constant (up to renormalization)

$$\langle A_4(t)A_4(0)\rangle = \sum_{j=1}^{n} \frac{2m_j}{\sqrt{L^3}} \left(\frac{f_j}{Z_A}\right)^2 \cosh\left(m_j\left(t-\frac{T}{2}\right)\right)$$

$$\langle A_4(t)P(0)\rangle = \sum_{j=1}^{n} \left(\frac{m_j}{2m_q Z_P}\right) \frac{2m_j}{\sqrt{L^3}} \left(\frac{f_j}{Z_A}\right)^2 \cosh\left(m_j\left(t-\frac{T}{2}\right)\right)$$

$$\langle P(t)P(0)\rangle = \sum_{j=1}^{n} \left(\frac{m_j}{2m_q Z_P}\right)^2 \frac{2m_j}{\sqrt{L^3}} \left(\frac{f_j}{Z_A}\right)^2 \cosh\left(m_j\left(t-\frac{T}{2}\right)\right)$$



An effective theory for the five Goldstones

Since $\frac{m_\rho}{(f_\Pi/Z_A)} = O(10)$, we can integrate out all but the five Goldstones.

The effective Lagrangian couple according to

$$\delta\mathcal{L}_{\mathcal{G}} = \sum_{n=1}^{5}\left(Q^T(-i\sigma^2 C)\gamma^5 T^n Q\right)\Pi^n$$

$$= Q^T(-i\sigma^2 C)\gamma^5 \mathcal{G} Q \quad \text{with} \quad \mathcal{G} = \frac{i}{2}\begin{pmatrix} 0 & \sqrt{2}\Pi_{UD} & \Pi^0 & \sqrt{2}\Pi^+ \\ -\sqrt{2}\Pi_{UD} & 0 & \sqrt{2}\Pi^- & -\Pi^0 \\ -\Pi^0 & -\sqrt{2}\Pi^- & 0 & -\sqrt{2}\Pi_{\overline{UD}} \\ -\sqrt{2}\Pi^+ & \Pi^0 & \sqrt{2}\Pi_{\overline{UD}} & 0 \end{pmatrix}$$

Similar to Ryttov&Sannino,PRD78(2008)115010, the resulting effective Lagrangian is

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left[D_\mu \mathcal{G} D^\mu \mathcal{G}^\dagger\right] + \cdots \quad \text{(linear form)}$$

$$\mathcal{L} = f_\Pi^2 \mathrm{Tr}\left[\omega_\mu^\perp \omega^{\perp\mu}\right] + \cdots \quad \text{(nonlinear form)}$$

where $\omega^\perp$ contains an exponential of the Goldstone fields as well as a covariant derivative.

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Summary

SU(2) technicolor with two techniquarks contains an extra Goldstone boson, which is a natural candidate for light asymmetric dark matter.

Our lattice exploration

- confirmed the symmetry-breaking pattern: SU(4)$\longrightarrow$Sp(4).
- explored the mass spectrum of the lightest technihadrons.
- established an effective field theory.

arXiv:1109.3513