Title: MERA for Relativistic QFTs

Date: Oct 25, 2011 03:30 PM

URL: http://pirsa.org/11100086

Abstract: In this second presentation, we will revisit Feynman's first argument and discuss how it still strongly influences variational studies of relativistic field theories with MPS or cMPS. However, as we explain, this argument can be completely overcome by introducing different variational parameters for the different length scales in the system, a strategy that naturally results in the MERA for lattice systems, or its continuous version for field theories. We then illustrate how a cMERA representation for the ground state of free relativistic quantum field theories can be constructed and discuss the main properties of this representation.

Tensor Networks for Quantum Field Theories
October 25, 2011
Perimeter Institute, Waterloo

Entanglement renormalization for relativistic quantum field theories

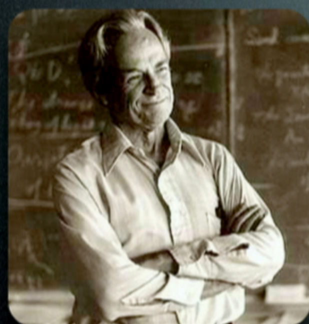
arXiv:1102.5524

Jutho Haegeman
in collaboration with:
Tobias J. Osborne,
Henri Verschelde, Frank Verstraete

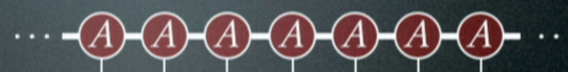
Outline

- Revisiting Feynman's first argument
- cMERA construction for free relativistic field theories:
  - Dirac fermions in (1+1) dimensions
  - Dirac fermions in (3+1) dimensions
  - Klein-Gordon bosons in (d+1) dimensions

Revisiting Feynman's first argument



Why do we have sensitivity to high frequencies with matrix product states?



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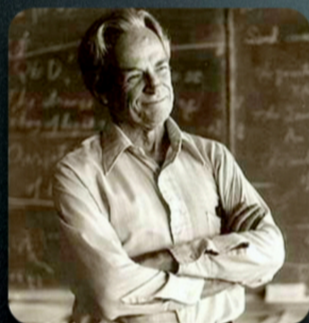
Revisiting Feynman's first argument



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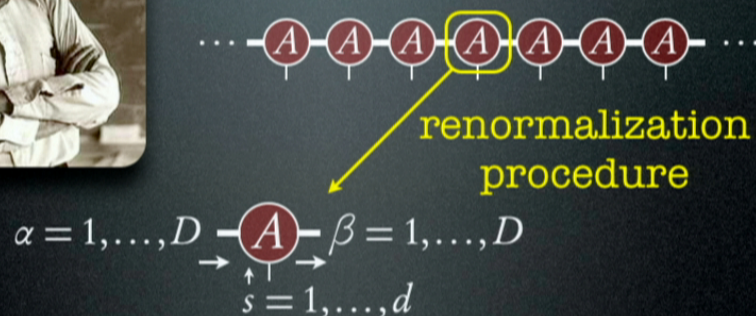

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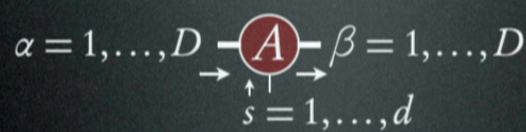
Revisiting Feynman's first argument

local degrees of freedom (in real space) → affect both **short and long** length scales



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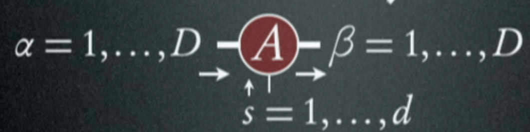
Revisiting Feynman's first argument

short length scales dominate in variational optimization ← affect both **short and long** length scales



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Revisiting Feynman's first argument

short length scales dominate in variational optimization → we need variational parameters that only affect the long length scales

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Multi-scale entanglement renormalization ansatz

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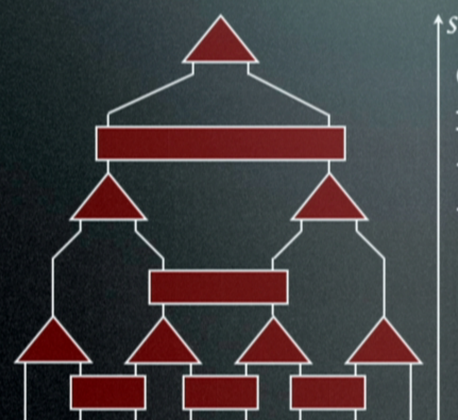
Revisiting Feynman's first argument

Multi-scale entanglement renormalization ansatz



degrees of freedom in a layer s act at:
- length scales $> ae^{s}$
- momentum scales $< \Lambda e^{-s}$

other degrees have already been integrated out

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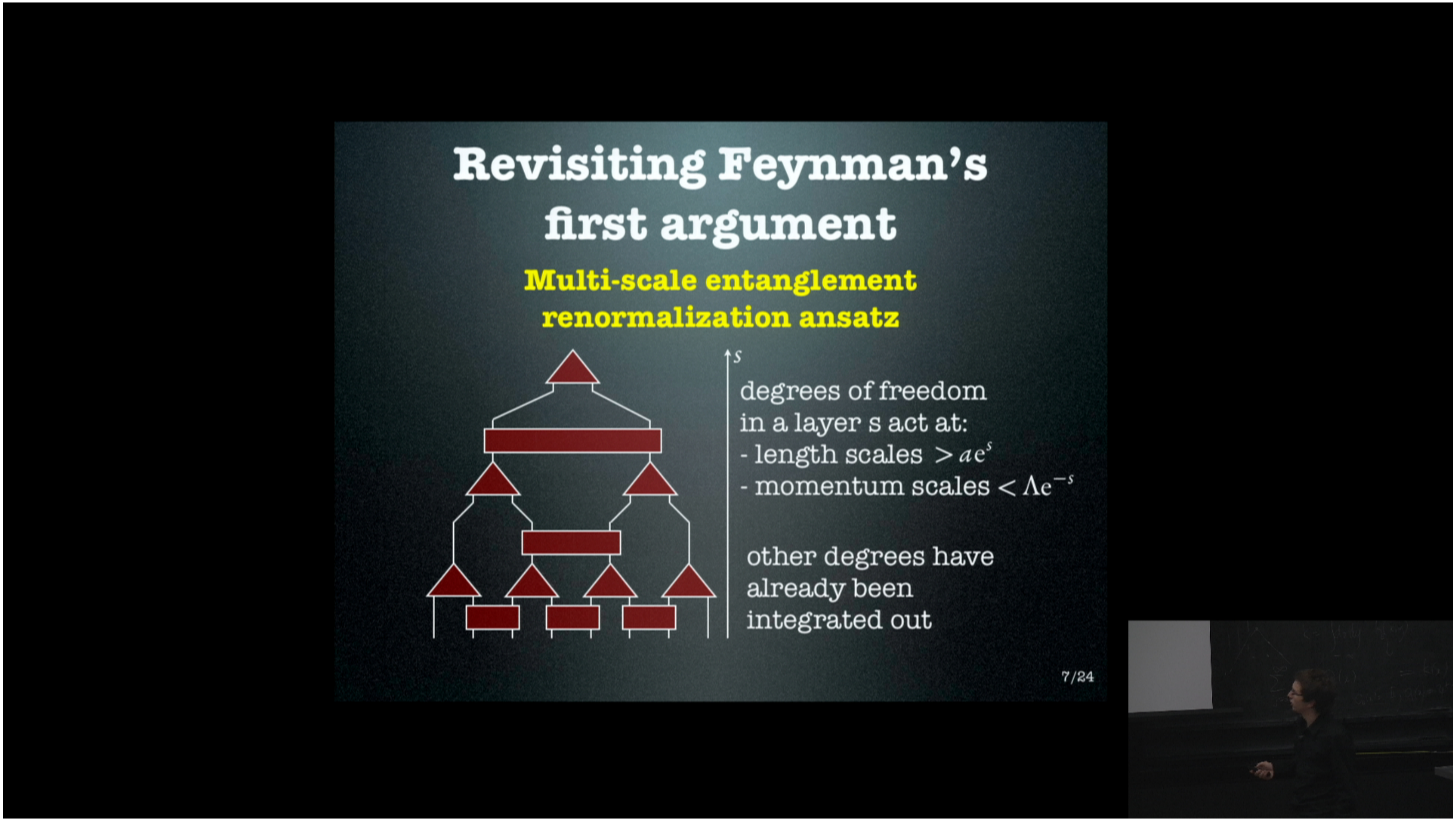
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cMERA constructions: generalities

cMERA: $|\Psi[\hat{K}]\rangle = \mathcal{S}\exp\underbrace{\left\{-\mathrm{i}\int_{s_\epsilon}^{s_\xi}\left[\hat{K}(s)+\hat{L}\right]\mathrm{d}s\right\}}_{\hat{U}(s_\epsilon,\, s_\xi)}|\Omega\rangle$

8/24

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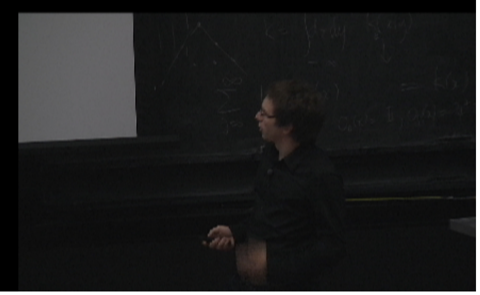
Renormalized operators:

$$\hat{O}_{\mathrm{R}}(s) = \hat{U}(s_\epsilon, s)^\dagger \hat{O}\,\hat{U}(s_\epsilon, s)$$

8/24



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where the underbraced part $\mathcal{S}\exp\left\{\dots\right\}$ is labelled $\hat{U}(s_\epsilon, s_\xi)$

↓

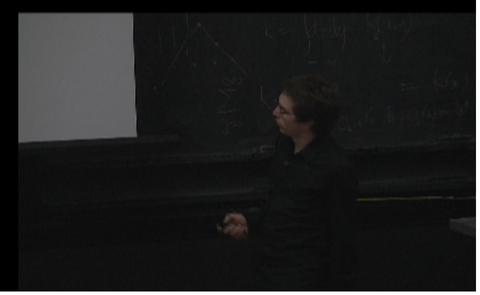
Renormalized operators:

$$\hat{O}_{\mathrm{R}}(s) = \hat{U}(s_\epsilon, s)^{\dagger}\hat{O}\,\hat{U}(s_\epsilon, s)$$

→ $\hat{O}_{\mathrm{R}}(s_\epsilon) = \hat{O}$

→ $\langle\Psi|\hat{O}|\Psi\rangle = \langle\Omega|\hat{O}_{\mathrm{R}}(s_\xi)|\Omega\rangle$

8/24



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RG equation: $\frac{\mathrm{d}}{\mathrm{d}s}\hat{O}_{\mathrm{R}}(s) = \mathrm{i}\left[\hat{K}(s)+\hat{L},\hat{O}_{\mathrm{R}}(s)\right]$

8/24

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$\underbrace{\qquad\qquad}_{\hat{U}(s_\epsilon,\, s_\xi)}$

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Renormalized operators:

RG equation: $\frac{\mathrm{d}}{\mathrm{d}s}\hat{O}_{\mathrm{R}}(s) = \mathrm{i}\left[\hat{K}(s)+\hat{L}, \hat{O}_{\mathrm{R}}(s)\right]$

$$\mathrm{i}[\hat{K}+\hat{L}, \hat{O}(vx)] = -\vec{x}\cdot\vec{\nabla}\hat{O}(\vec{x}) + \lambda\hat{O}(\vec{x})$$

scaling operator with scaling dimension λ

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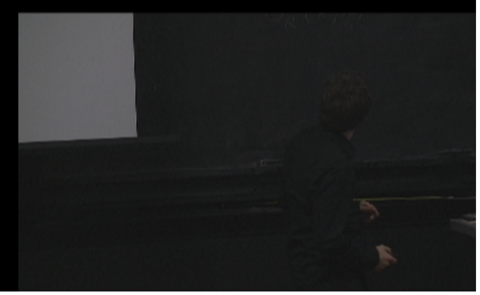
cMERA constructions: generalities

cMERA: $|\Psi[\hat{K}]\rangle = \mathcal{T}\exp\left\{-\mathrm{i}\int_{s_\xi}^{s_\epsilon}\left[\hat{K}(s)+\hat{L}\right]\mathrm{d}s\right\}|\Omega\rangle$

which $\hat{K}$? which $\hat{L}$? which $|\Omega\rangle$?

10/24



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which $\hat{K}$?

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↓

built using any set of operators; respect symmetries

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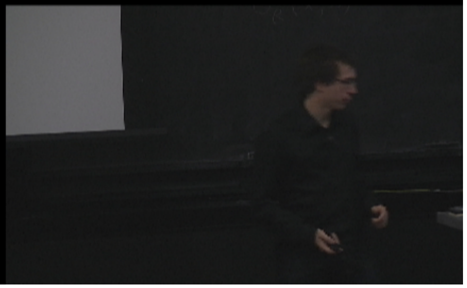
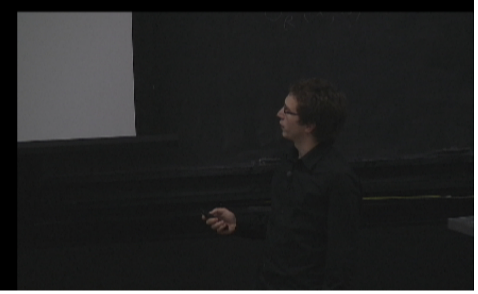


cMERA constructions: free fermions in (1+1)

$$\hat{H} = \int_{-\infty}^{+\infty} \left[-\frac{\mathrm{i}}{2} \hat{\psi}^{\dagger}(x) \alpha^{x} \frac{\mathrm{d}\hat{\psi}}{\mathrm{d}x}(x) + m \hat{\psi}^{\dagger}(x) \beta \hat{\psi}(x) \right] \mathrm{d}x$$

$$\alpha^{x} = \sigma^{y} \qquad \beta = \sigma^{z}$$

$$\gamma^{1} = \beta \alpha^{x} = -\mathrm{i}\sigma^{x} \qquad \gamma^{0} = \beta = \sigma^{z}$$



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$$\hat{K}(s) = \int \mathrm{d}p\, g(p;s)\left[\hat{\Psi}_1^\dagger(p)\hat{\Psi}_2^\dagger(p) + \hat{\Psi}_2(p)\hat{\Psi}_1(p)\right]$$

$$g(p;s) = \chi(s)\frac{p}{\Lambda}\Gamma(|p|/\Lambda)$$

$$\Gamma(x) = \exp(-x^2/2), \exp(-x), \theta(1-|x|), \ldots$$

(smooth cutoff at dimensionless scale 1)

$$\hat{L} = \frac{\mathrm{i}}{2}\int \mathrm{d}p \sum_{\alpha=1}^{2} p\left[\hat{\Psi}_\alpha^\dagger(p)\frac{\mathrm{d}\hat{\Psi}_\alpha}{\mathrm{d}p}(p) - \frac{\mathrm{d}\hat{\Psi}_\alpha^\dagger}{\mathrm{d}p}(p)\hat{\Psi}_\alpha(p)\right]$$

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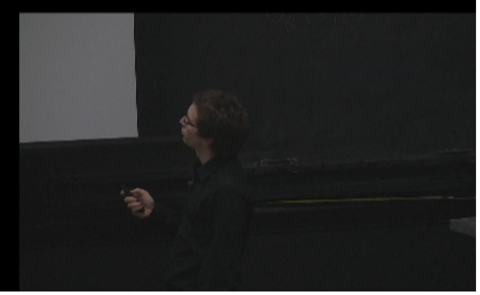

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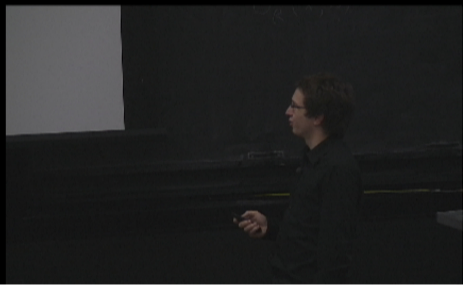

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12/24

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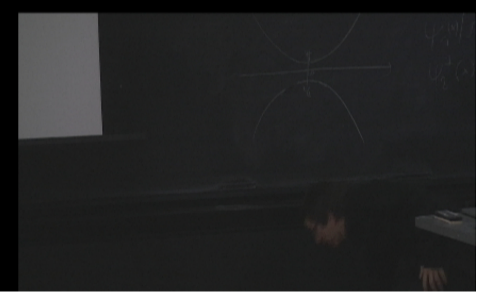

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cMERA constructions: free fermions in (1+1)

Choose $s_\epsilon = 0$:

$$\hat{\Psi}_{1,R}(p) = \cos\left(f(p;s)\right) e^{s/2} \hat{\Psi}_{1,R}(e^s p) - i \sin\left(f(p;s)\right) e^{s/2} \hat{\Psi}_{2,R}(e^s p)$$

$$\hat{\Psi}_{2,R}(p) = \cos\left(f(p;s)\right) e^{s/2} \hat{\Psi}_{2,R}(e^s p) - i \sin\left(f(p;s)\right) e^{s/2} \hat{\Psi}_{1,R}(e^s p)$$

with $f(p;s) = \int_{s_\epsilon}^{s} dw \, g(e^w p; w)$

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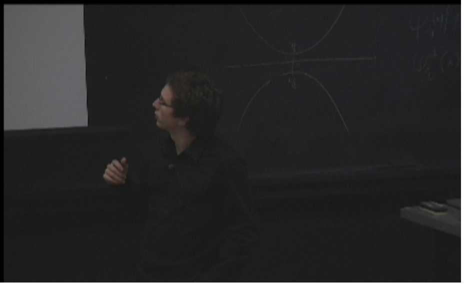

cMERA constructions: free fermions in (1+1)

$$
\begin{aligned}
\hat{H}_{\mathrm{R}}(s) = \mathrm{e}^{-s} \int \mathrm{d}p \, \Big[-p \sin\big(2f(\mathrm{e}^{-s} p; s)\big) + m\mathrm{e}^{s} \cos\big(2f(\mathrm{e}^{-s} p; s)\big) \Big] \\
\times \Big[\hat{\Psi}_1^\dagger(p)\hat{\Psi}_1(p) - \hat{\Psi}_2^\dagger(p)\hat{\Psi}_2(p) \Big] \\
-\mathrm{i} \Big[p \cos\big(2f(\mathrm{e}^{-s} p; s)\big) + m\mathrm{e}^{s} \sin\big(2f(\mathrm{e}^{-s} p; s)\big) \Big] \\
\times \Big[\hat{\Psi}_1^\dagger(p)\hat{\Psi}_2(p) - \hat{\Psi}_2^\dagger(p)\hat{\Psi}_1(p) \Big]
\end{aligned}
$$

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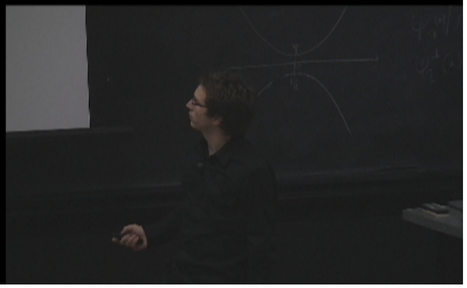

cMERA constructions: free fermions in (1+1)

$$\chi(s) = -\frac{1}{2}\arcsin\left[\frac{e^{-s}}{\sqrt{m^2/\Lambda^2 + e^{-2s}}}\right] + \frac{e^{-s}m/\Lambda}{2(m^2/\Lambda^2 + e^{-2s})}$$

$$-\delta(s)\frac{1}{2}\arcsin\left[\frac{1}{\sqrt{m^2/\Lambda^2 + 1}}\right]$$

Artefact of:
- sharp momentum cutoff
- freedom to have unbounded K
- non-regularized Hamiltonian

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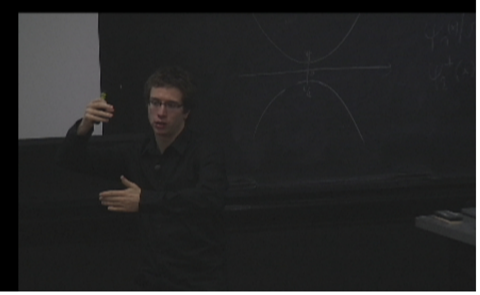

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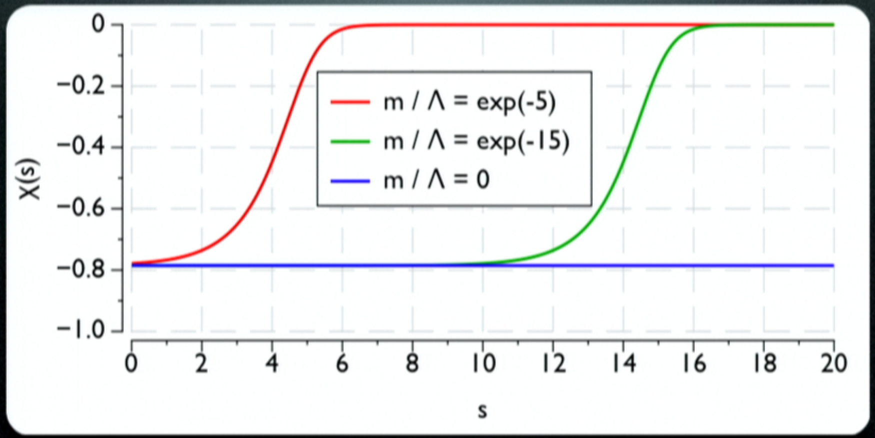

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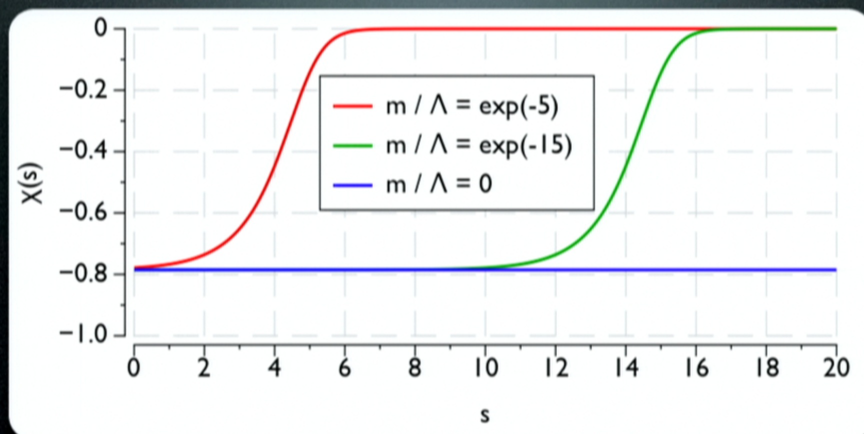

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cMERA constructions: free fermions in (1+1)

Fixed point Hamiltonians:

- critical case: $(m=0)$

$$\lim_{s\to\infty} \hat{H}_{\mathrm{R}}(s) = \mathrm{e}^{-s} \int \mathrm{d}p\, |p| \left[\hat{\Psi}_1^\dagger(p)\hat{\Psi}_1(p) - \hat{\Psi}_2^\dagger(p)\hat{\Psi}_2(p)\right]$$

- non-critical case: $(m \neq 0)$

$$\lim_{s\to\infty} \hat{H}_{\mathrm{R}}(s) = \int \mathrm{d}p\, m \left[\hat{\Psi}_1^\dagger(p)\hat{\Psi}_1(p) - \hat{\Psi}_2^\dagger(p)\hat{\Psi}_2(p)\right]$$

Note that $\hat{\Psi}_1(p)\,|\Omega\rangle = \hat{\Psi}_2^\dagger(p)\,|\Omega\rangle = 0, \forall p \in \mathbb{R}$

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cMERA constructions: free fermions in (3+1)

Symmetry for K:

$$\hat{K}(s) = \mathrm{i}\int \mathrm{d}p^1 \int \mathrm{d}p^2 \int \mathrm{d}p^3\, \chi(s)\Gamma\left(\frac{|p|}{\Lambda}\right)\frac{\vec{p}}{\Lambda}\cdot\left(\hat{\Psi}^\dagger(p)\vec{\gamma}\hat{\Psi}(p)\right)$$

$$= \frac{\mathrm{i}}{2}\int \mathrm{d}p^1 \int \mathrm{d}p^2 \int \mathrm{d}p^3\, \chi(s)\Gamma\left(\frac{|p|}{\Lambda}\right)\frac{p_i}{\Lambda}\hat{\bar{\Psi}}(p)\sigma^{0i}\hat{\Psi}(p)$$

- translation invariant
- rotation invariant
- Lorentz covariant: zero-component of a 4-vector

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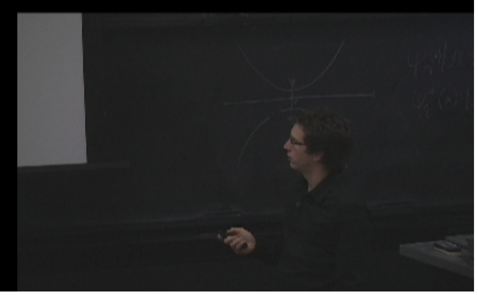

cMERA constructions: free bosons in (d+1)

Non-relativistic bosons:

$$\hat{\psi}(x), \hat{\psi}^\dagger(x) \longrightarrow \text{scaling dimensions: } d/2$$

Relativistic bosons:

$$\hat{\phi}(\vec{x}) \sim \hat{\psi}(\vec{x}) + \hat{\psi}^\dagger(\vec{x}) \quad \longrightarrow \quad (d-1)/2$$

$$\hat{\pi}(\vec{x}) \sim \hat{\psi}(\vec{x}) - \hat{\psi}^\dagger(\vec{x}) \quad \longrightarrow \quad (d+1)/2$$

which $\hat{L}$?

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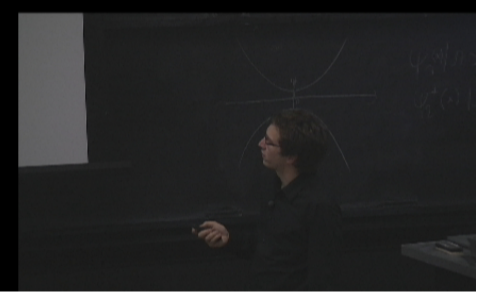

cMERA constructions: free bosons in (d+1)

Use non-relativistic L:

$$\hat{L} = -\frac{1}{2}\int \mathrm{d}^d x \left[\hat{\pi}(\vec{x})\vec{x}\cdot\vec{\nabla}\hat{\phi}(\vec{x}) + \vec{x}\cdot\vec{\nabla}\hat{\phi}(\vec{x})\hat{\pi}(\vec{x}) + \frac{\mathrm{d}}{2}\hat{\phi}(\vec{x})\hat{\pi}(\vec{x}) + \frac{d}{2}\hat{\pi}(\vec{x})\hat{\phi}(\vec{x})\right]$$

Use zero-component of 4-vector K:

$$\hat{K}(s) = \int \mathrm{d}^d p\, \frac{1}{2} g(\vec{p};s) \left[\hat{\Phi}(\vec{p})\hat{\Pi}(-\vec{p}) + \hat{\Pi}(-\vec{p})\hat{\Phi}(\vec{p})\right]$$

with $g(\vec{p};s) = \chi(s)\Gamma(||\vec{p}||/\Lambda)$

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cMERA constructions: free bosons in (d+1)

Solution:

$$\chi(s) = \frac{1}{2}\frac{e^{-2s}}{e^{-2s} + (m/\Lambda)^2}$$

⇒ restores relativistic scaling dimensions in the critical case m = 0

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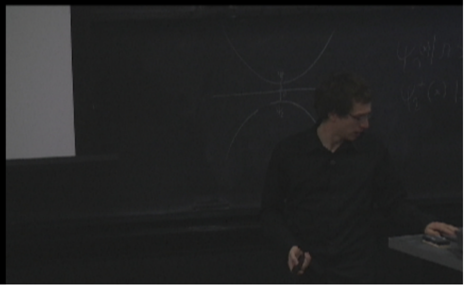

Conclusions

- cMERA are perfectly suited to study relativistic theories
- Range between IR and UV cutoff can be arbitrarily large: Feynman's sensitivity to high frequencies does not appear!
- Great flexibility and freedom
- Interacting theories is a priority !

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