Title: MPS for Relativistic QFTs

Date: Oct 24, 2011 03:30 PM

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Abstract: In 1987, Feynman devoted one of his last lectures to highlighting three serious objections against the usefulness of the variational principle in the theory of relativistic quantum fields. In that same year, in a different branch of physics, Affleck, Kennedy, Lieb and Tasaki devised a quantum state that resulted in the development of a handful of different variational ansÃ¤tze for lattice models over the last two decennia. These quantum states are known as tensor network states and invalidate at least two of Feynman's arguments. They could thus be used in a variational study of relativistic quantum field theories on a lattice. However, two classes of tensor network states, namely the matrix product state and the multi-scale entanglement renormalization ansatz, have recently been ported to the continuous setting, so that we now have direct access to variational wave functions for quantum field theories and are no longer restricted to a lattice regularization.

Tensor Networks for Quantum Field Theories
October 24, 2011
Perimeter Institute, Waterloo

Matrix product states for relativistic quantum field theories

PRL 105, 251601 (2010)
arXiv:1006.2409

Jutho Haegeman, in collaboration with:
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Quantum Field Theory

- Non-relativistic ground state:
  collection of entangled particles

  $\rightarrow$ very dull and empty for
  distances $\ll$ interparticle distance $\rho^{-1/d}$

- Relativistic ground state:
  quantum fluctuations at all momentum scales due to Lorentz invariance

  $\Rightarrow$ divergences: need for regularization

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Variational Principle

- Advantages:
  - Non-perturbative effects
  - No sign problems
- (Successful) Examples:
  - Single-particle QM: e.g. quartic potential
  - Quantum chemistry: Hartree-Fock, DFT, ...
  - Quantum lattices: DMRG, tensor networks
  - Non-relativistic QFTs: cf. previous talk
- What about relativistic QFTs ?

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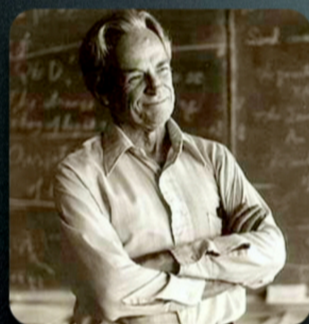
Variational Principle for relativistic QFTs



Lacking success of the variational method for relativistic QFTs was investigated by Feynman in 1987 *:

"It's no damn good at all!"

* R. P. Feynman in Proceedings of the International Workshop on Variational Calculations in Quantum Field Theory (L. Polley and D. E. L. Pottinger, eds.), World Scientific Publishing, Singapore, pp. 28-40 (1987).

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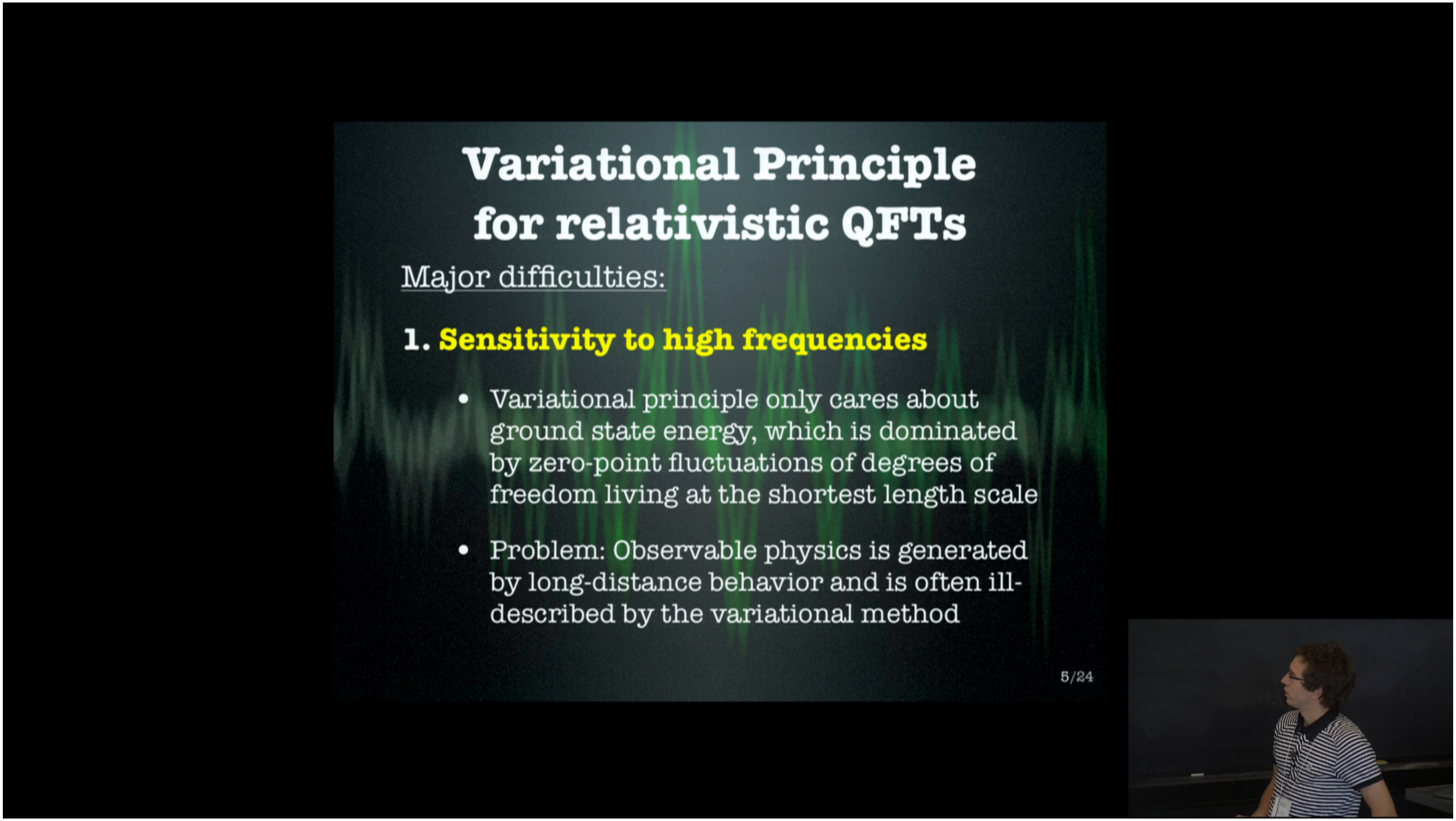
Variational Principle for relativistic QFTs
Major difficulties:
1. Sensitivity to high frequencies
• Variational principle only cares about ground state energy, which is dominated by zero-point fluctuations of degrees of freedom living at the shortest length scale
• Problem: Observable physics is generated by long-distance behavior and is often ill-described by the variational method
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Variational Principle for relativistic QFTs

Major difficulties:

2. Only Gaussians

- Typical scheme: (cf. configuration interaction)
  → Take ground state of free theory
  → Add 1,2,3,...-particle excitations
- Problem: all non-extensive states (n-particle excitations) do not contribute to the ground state
  ⇒ no improvement over Gaussian ansatz

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Variational Principle for relativistic QFTs

Second and third argument are generally applicable to **extended systems**:

↓

Lattice systems: DMRG → **tensor networks**
Non-relativistic field theories: **cMPS**

- extensive states
- efficient and accurate evaluation of expectation values

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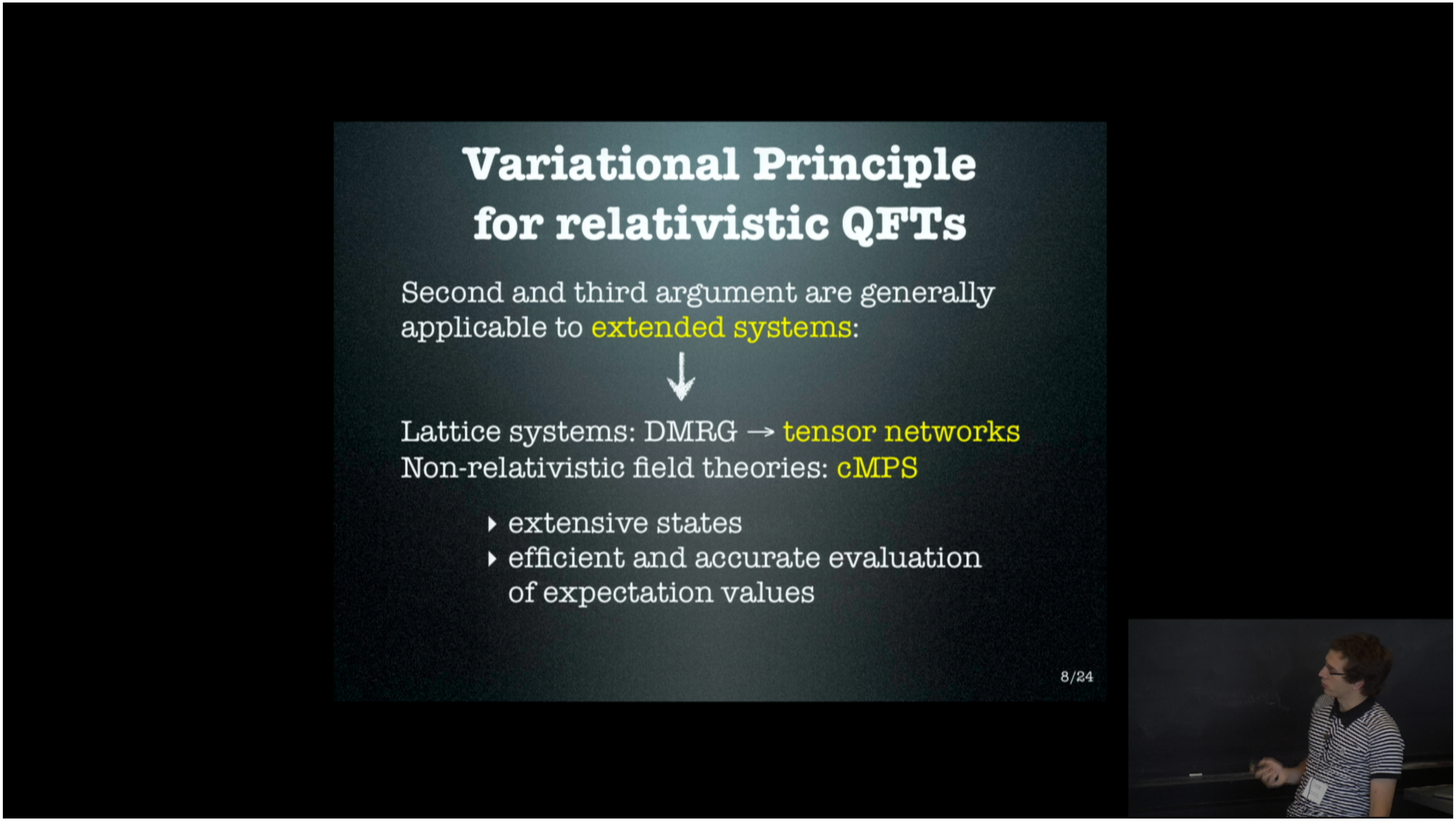
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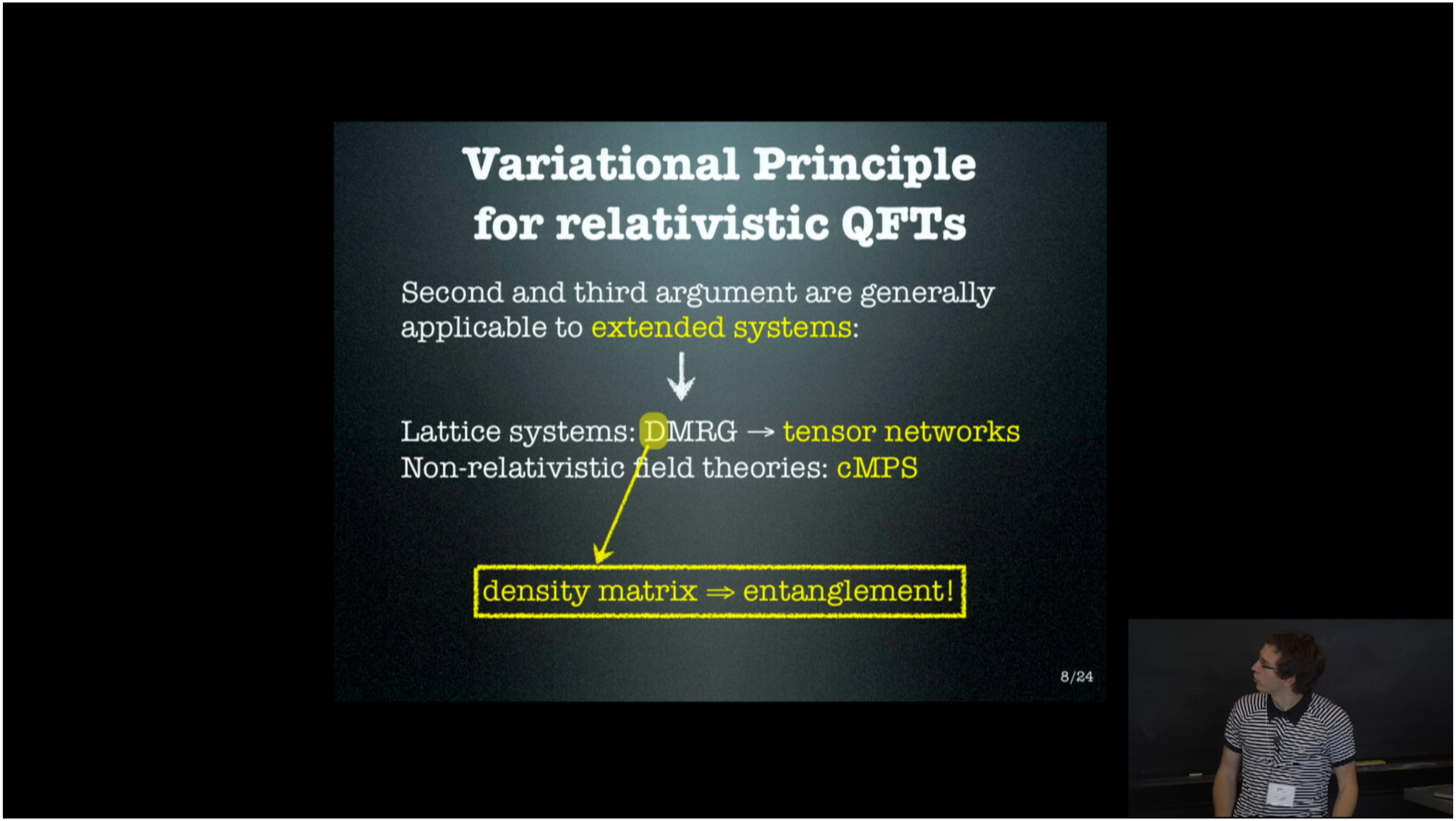
Variational Principle
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applicable to extended systems:
Lattice systems: DMRG → tensor networks
Non-relativistic field theories: cMPS
density matrix ⇒ entanglement!
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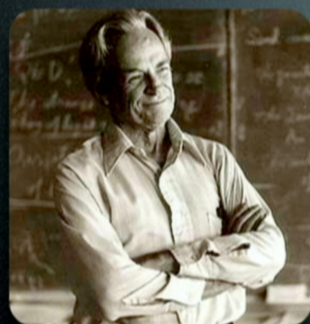
Variational Principle for relativistic QFTs



"... I think it should be possible some day to describe field theory in some other way than with the wave functions and amplitudes. It might be something like the density matrices where you concentrate on quantities in a given locality ..."

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Variational Principle for relativistic QFTs

Feynman's first concern:
applies to all theories with degrees of freedom over a wide range of length scales

0 — UV: $a = \Lambda^{-1}$ — IR: $\xi = M^{-1}$ — ∞

catastrophic when $\log(\xi / a) \to \infty$

- critical theories
- relativistic theories

$\Rightarrow$ (c)MERA (tomorrow)

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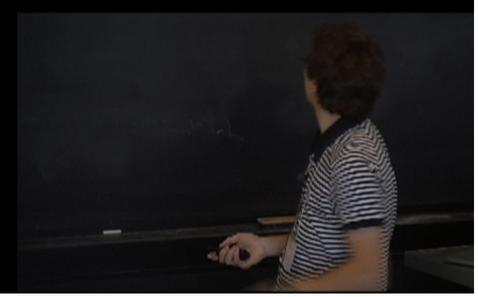


cMPS description

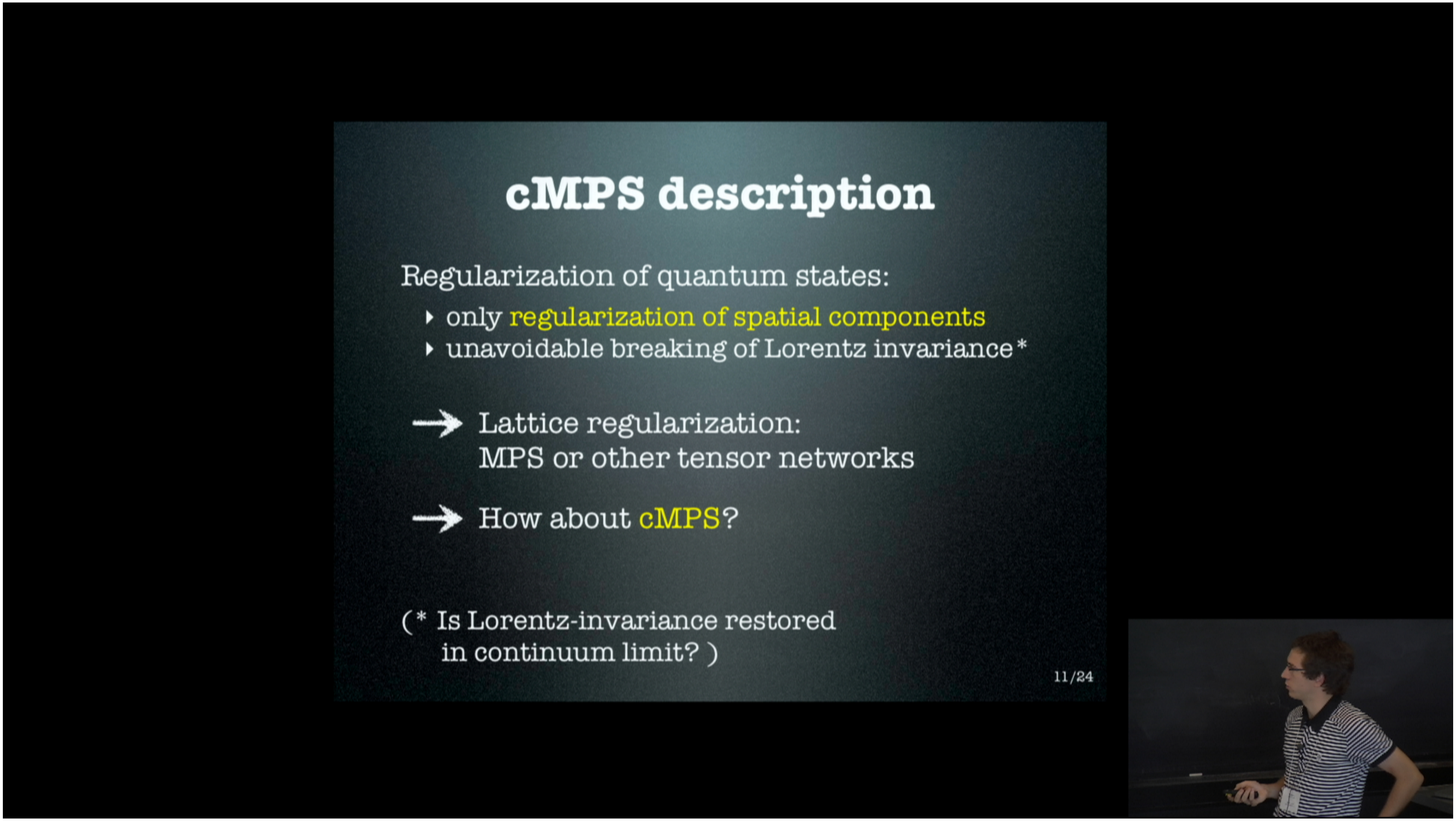
Regularization of quantum states:

- only regularization of spatial components
- unavoidable breaking of Lorentz invariance *

→ Lattice regularization:
MPS or other tensor networks

→ How about cMPS?

(* Is Lorentz-invariance restored
in continuum limit?)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cMPS description

Continuous Matrix Product State:

$$|\Psi\rangle = v_{\mathrm{L}}^{\dagger}\mathcal{P}\exp\left[\int_{-\infty}^{+\infty}\mathrm{d}x\,\left(Q\otimes\hat{\mathbb{1}}+\sum_{\alpha}R_{\alpha}\otimes\hat{\psi}_{\alpha}^{\dagger}(x)\right)\right]v_{\mathrm{R}}|\Omega\rangle$$

- matrices Q and R: bond dimension D
- thermodynamic limit
- translation invariant
- formulated using creation/annihilation operators:

$$[\hat{\psi}_{\alpha}^{\dagger}(x),\hat{\psi}_{\beta}(y)]_{\pm} = \delta_{\alpha,\beta}\delta(x-y) \quad (\alpha,\beta=1,2)$$

$\Rightarrow$ most straightforward for fermions

12/24

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- extensive: finite density
- non-gaussian
- efficient evaluation of expectation values
- regularized if $\{R_{\alpha},R_{\beta}\}=0$

$$\lim_{|p|\to\infty} n_{\alpha,\beta}(p)\sim\left(\frac{\Lambda}{p}\right)^{4}$$

with $\langle\Psi|\hat{\psi}_{\alpha}^{\dagger}(k')\hat{\psi}_{\beta}(k)|\Psi\rangle = 2\pi\delta(k'-k)n_{\alpha,\beta}(k)$

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cMPS description

Continuous Matrix Product State:

$$|\Psi\rangle = v_L^\dagger \mathcal{P}\exp\left[\int_{-\infty}^{+\infty} \mathrm{d}x\,\left(Q\otimes\hat{\mathbb{1}} + \sum_\alpha R_\alpha \otimes \hat{\psi}_\alpha^\dagger(x)\right)\right] v_R |\Omega\rangle$$

- scale transformation: $Q \mapsto cQ, R_\alpha \mapsto \sqrt{c}R_\alpha$

  $\Rightarrow n_{\alpha,\beta}(p) \mapsto n_{\alpha,\beta}(p/c) \Rightarrow \Lambda \mapsto c\Lambda$

  $\Rightarrow$ origin of the manifestation of Feynman's sensitivity to high frequencies for cMPS

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cMPS description

Sensitivity to high frequencies in cMPS:

- <u>real-space interpretation:</u>
  if $\langle \hat{t} \rangle < 0$, the (kinetic) energy can unboundedly be lowered through $c \to \infty$, since $\hat{t} \mapsto c^2 \hat{t}$ (together with other renormalizable terms)

- <u>momentum-space interpretation:</u>
  since $|\Omega\rangle$ does not contain the correct physics at large momenta, the cMPS can most quickly lower the energy by first fixing the degrees of freedom at large momentum

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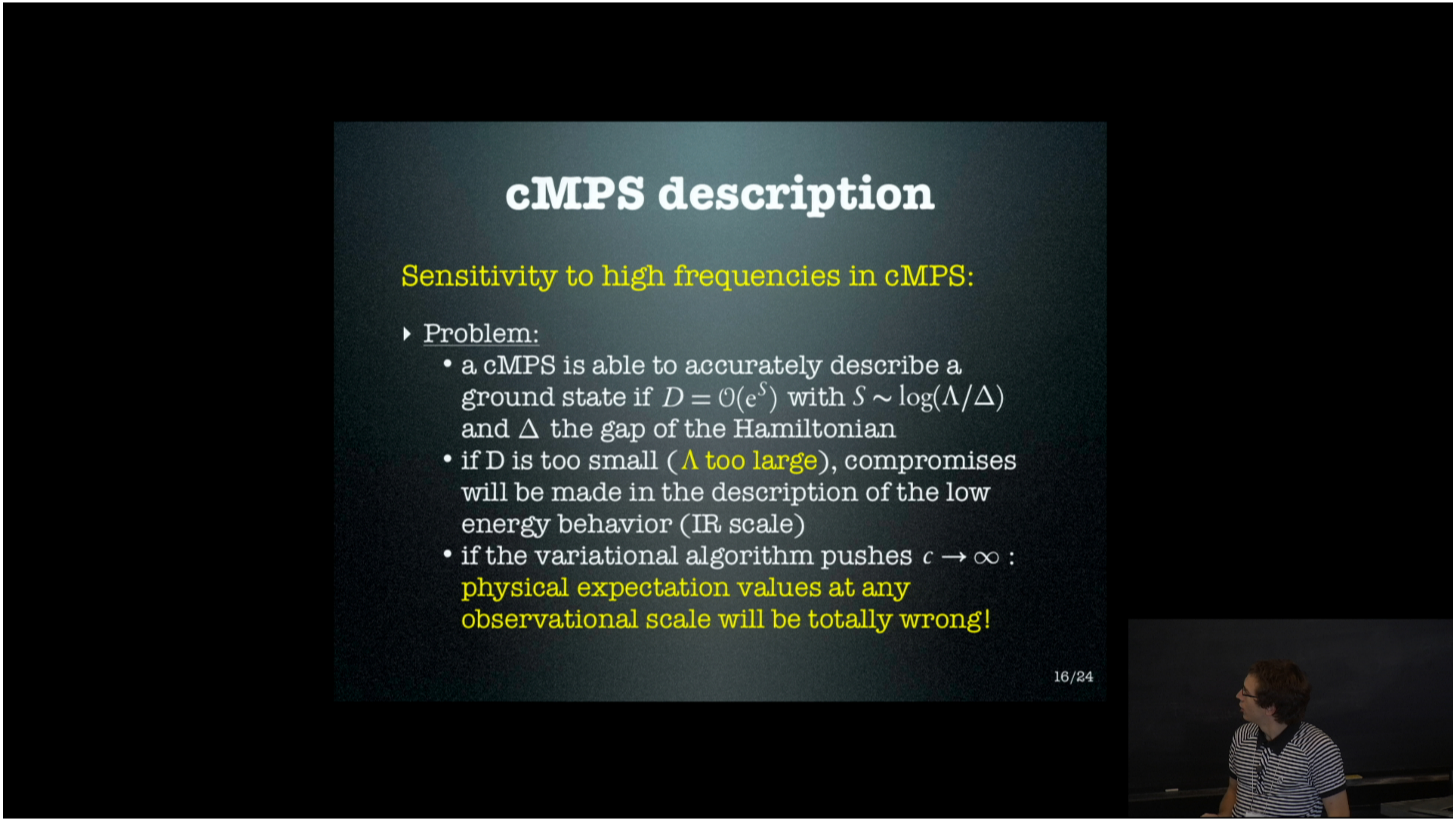
cMPS description
Sensitivity to high frequencies in cMPS:
Problem:
a cMPS is able to accurately describe a ground state if $D = \mathcal{O}(e^S)$ with $S \sim \log(\Lambda/\Delta)$ and Δ the gap of the Hamiltonian
if D is too small (Λ too large), compromises will be made in the description of the low energy behavior (IR scale)
if the variational algorithm pushes $c \to \infty$: physical expectation values at any observational scale will be totally wrong!
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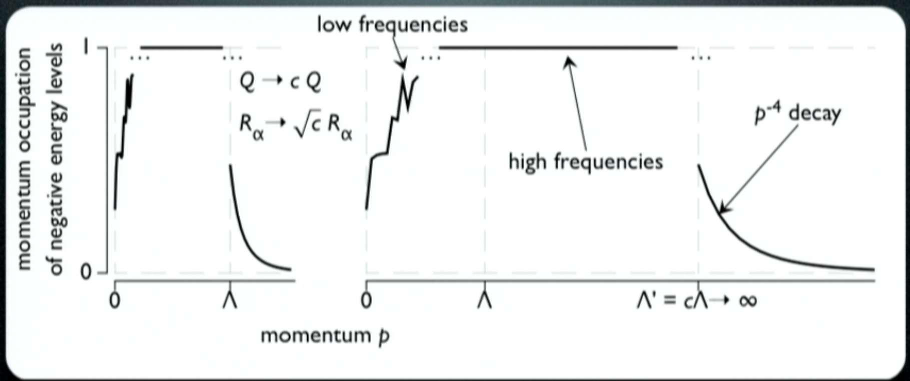
cMPS description

Sensitivity to high frequencies in cMPS:



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cMPS description

Sensitivity to high frequencies in cMPS:

- Solution:
  - prevent c from running to ∞
  - 'regularize' the field theory by modifying the Hamiltonian with an irrelevant term, such that
    - the low-energy dynamics are unchanged
    - the high-energy dynamics are such that the asymptotic solution for $|p| \to \infty$ is the empty vacuum $|\Omega\rangle$

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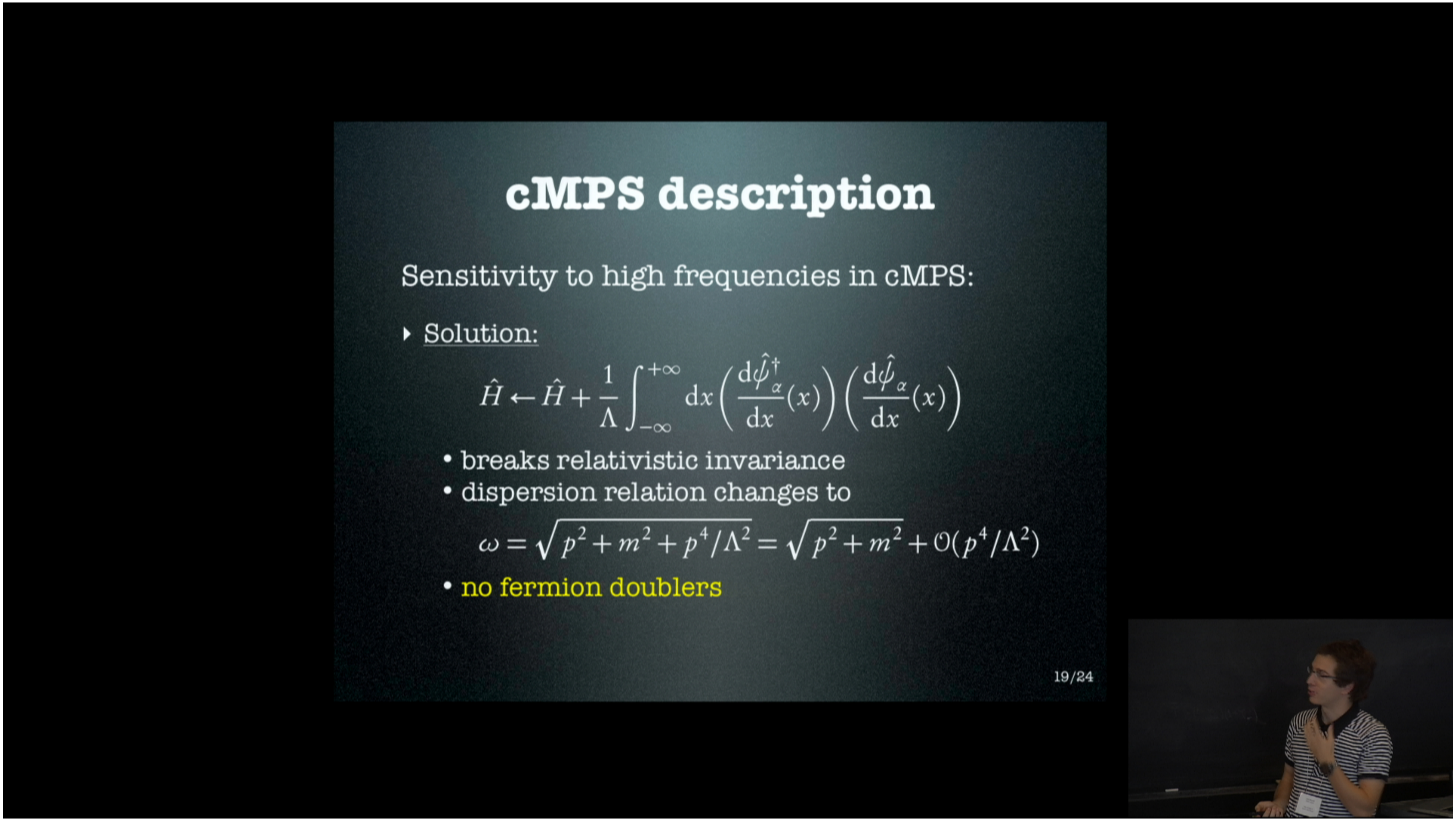
cMPS description
Sensitivity to high frequencies in cMPS:
Solution:
$\hat{H} \leftarrow \hat{H} + \frac{1}{\Lambda}\int_{-\infty}^{+\infty} \mathrm{d}x \left(\frac{\mathrm{d}\hat{\psi}^\dagger_\alpha}{\mathrm{d}x}(x)\right)\left(\frac{\mathrm{d}\hat{\psi}_\alpha}{\mathrm{d}x}(x)\right)$
• breaks relativistic invariance
• dispersion relation changes to
$\omega = \sqrt{p^2 + m^2 + p^4/\Lambda^2} = \sqrt{p^2+m^2} + \mathcal{O}(p^4/\Lambda^2)$
• no fermion doublers
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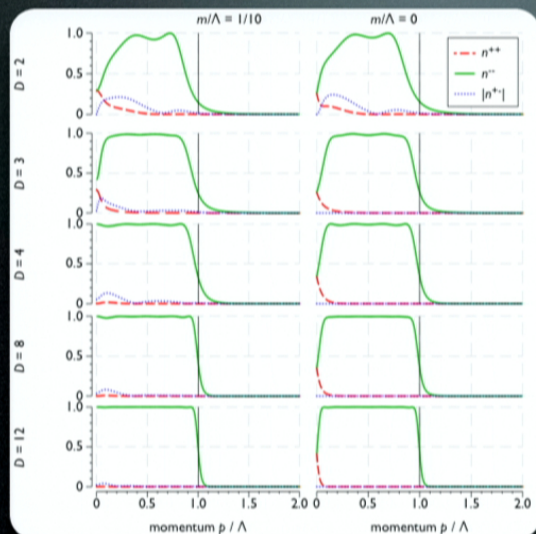
Examples

Relativistic free fermions



⇒ ground state energy is not interesting

⇒ for a **gapped system**, low energy behavior is very **well reproduced**

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Examples

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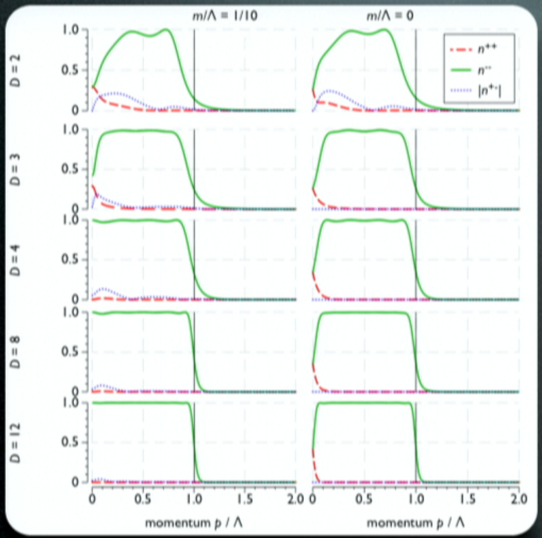

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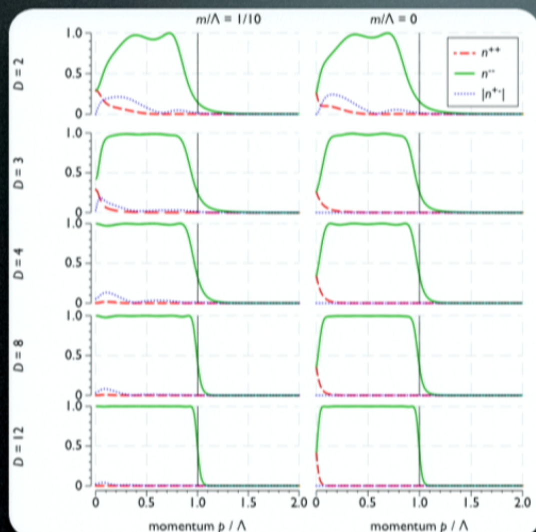

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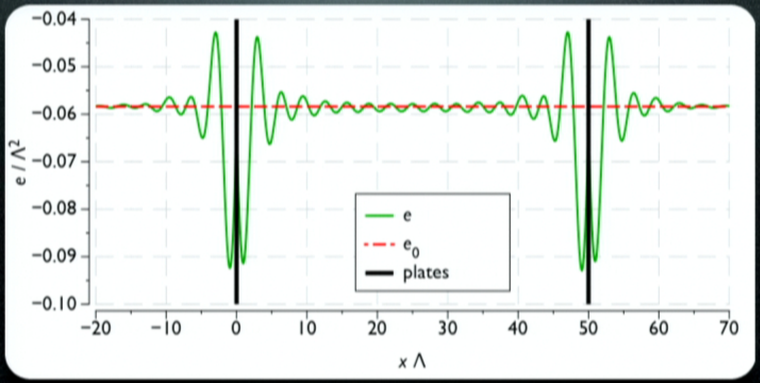
Examples

Relativistic free fermions

- Casimir: energy density between two plates



⇒ Friedel oscillations due to cutoff

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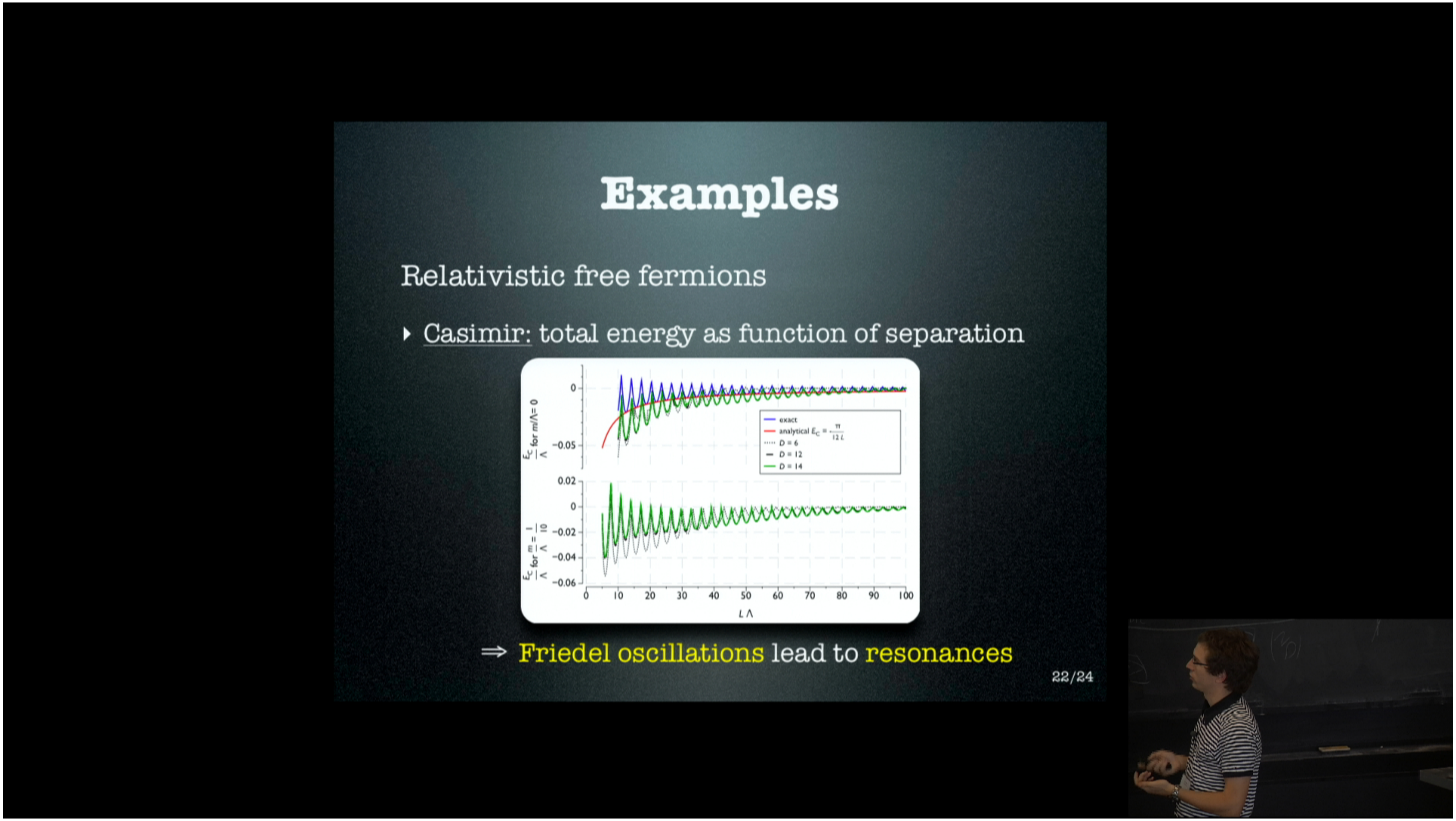
Examples
Relativistic free fermions
Casimir: total energy as function of separation
exact
analytical $E_C = -\frac{\pi}{12 L}$
$D = 6$
$D = 12$
$D = 14$
$\frac{E_C}{\Lambda}$ for $m/\Lambda = 0$
$\frac{E_C}{\Lambda}$ for $\frac{m}{\Lambda} = \frac{1}{10}$
$L\,\Lambda$
⇒ Friedel oscillations lead to resonances
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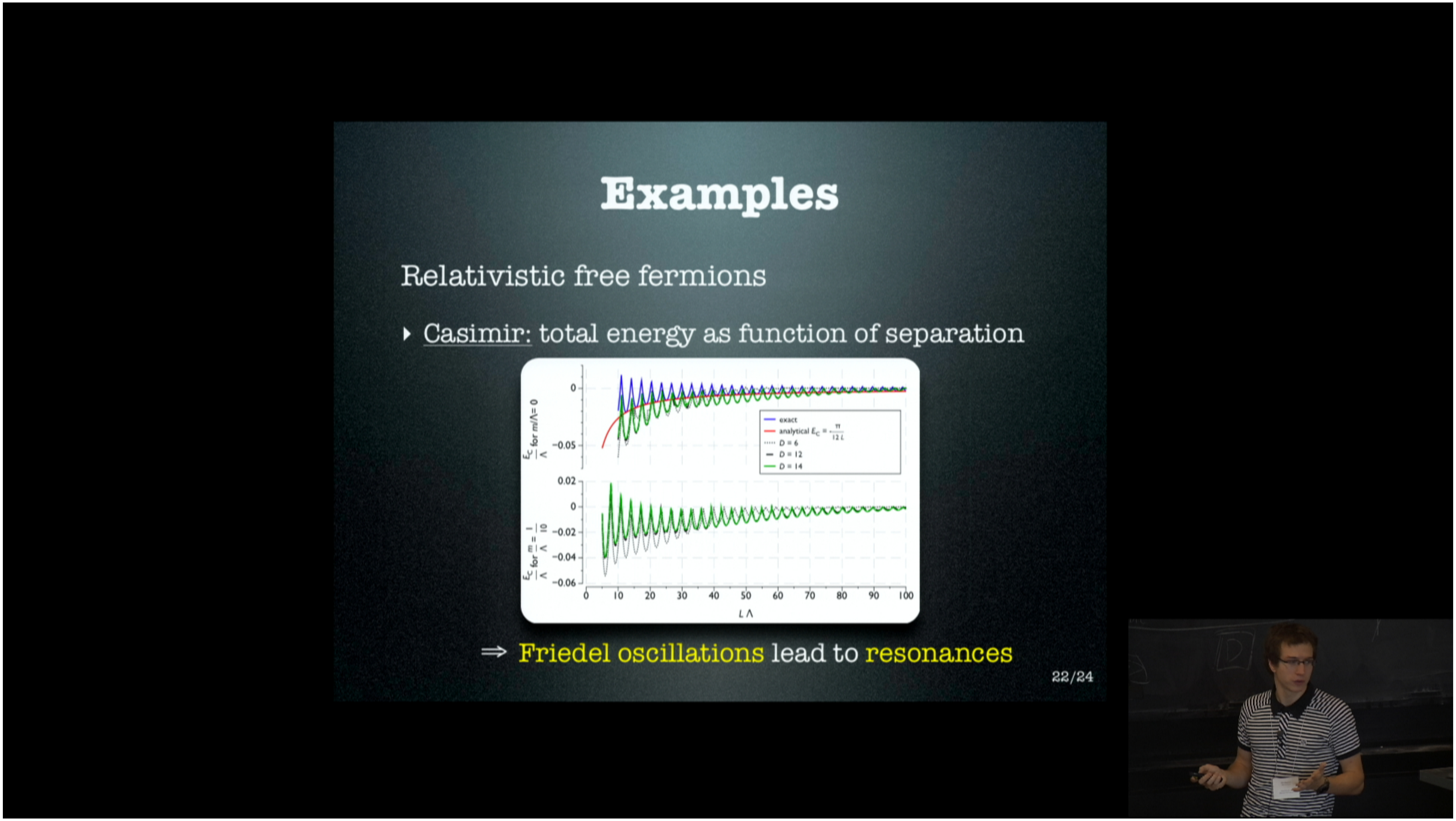
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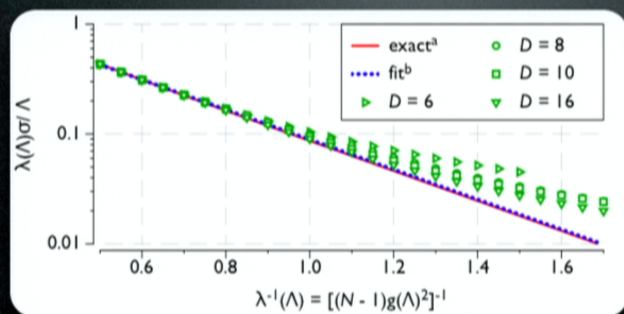
Examples

Gross-Neveu model:

- N flavors of massless fermions interacting with each other through a quartic potential
- dynamic mass generation related to spontaneous breaking of chiral symmetry



$$\sigma = \langle \hat{\psi}_1^\dagger \hat{\psi}_1 - \hat{\psi}_2^\dagger \hat{\psi}_2 \rangle$$

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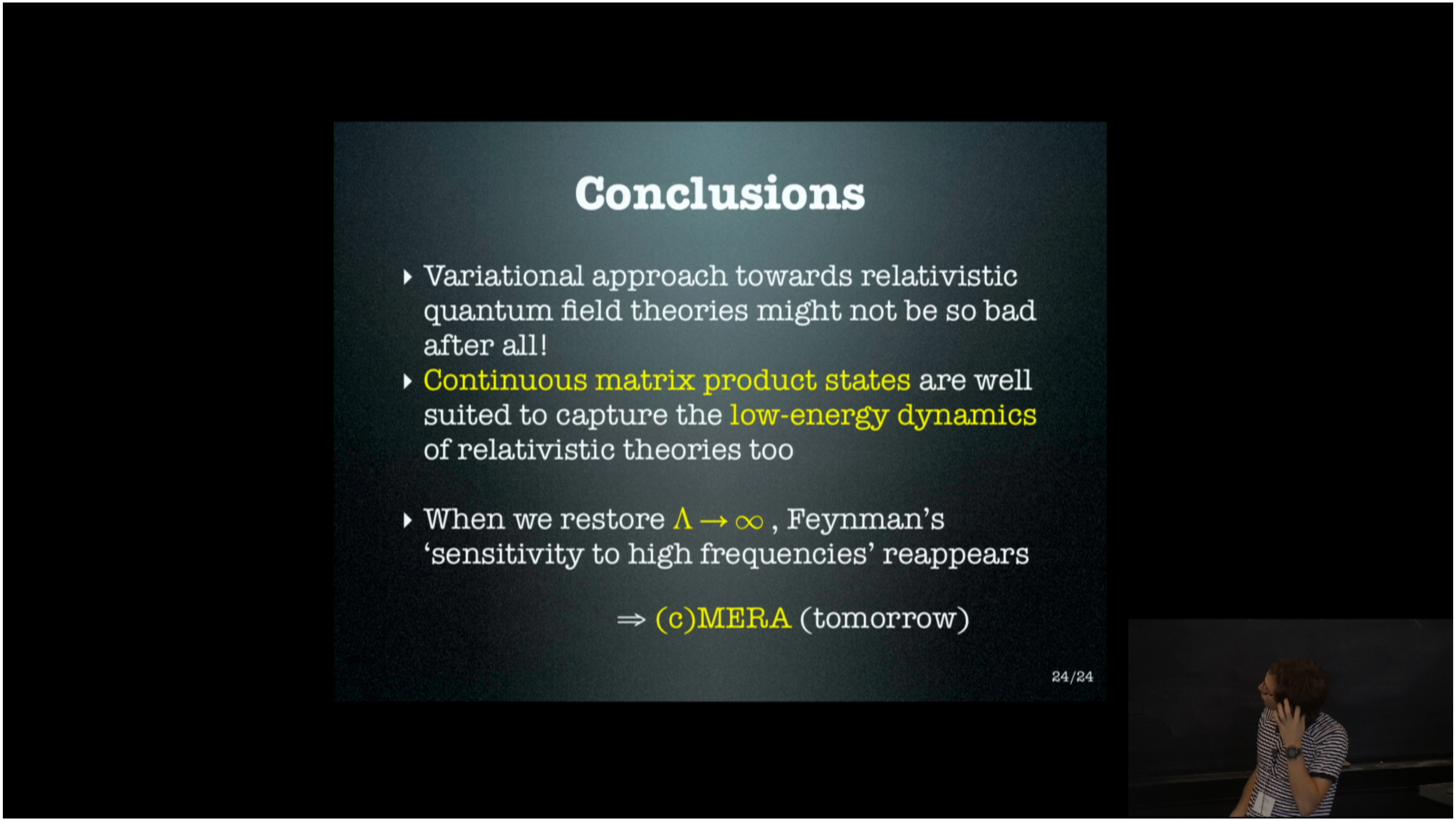
Conclusions
Variational approach towards relativistic quantum field theories might not be so bad after all!
Continuous matrix product states are well suited to capture the low-energy dynamics of relativistic theories too
When we restore $\Lambda \to \infty$, Feynman's 'sensitivity to high frequencies' reappears
$\Rightarrow$ (c)MERA (tomorrow)
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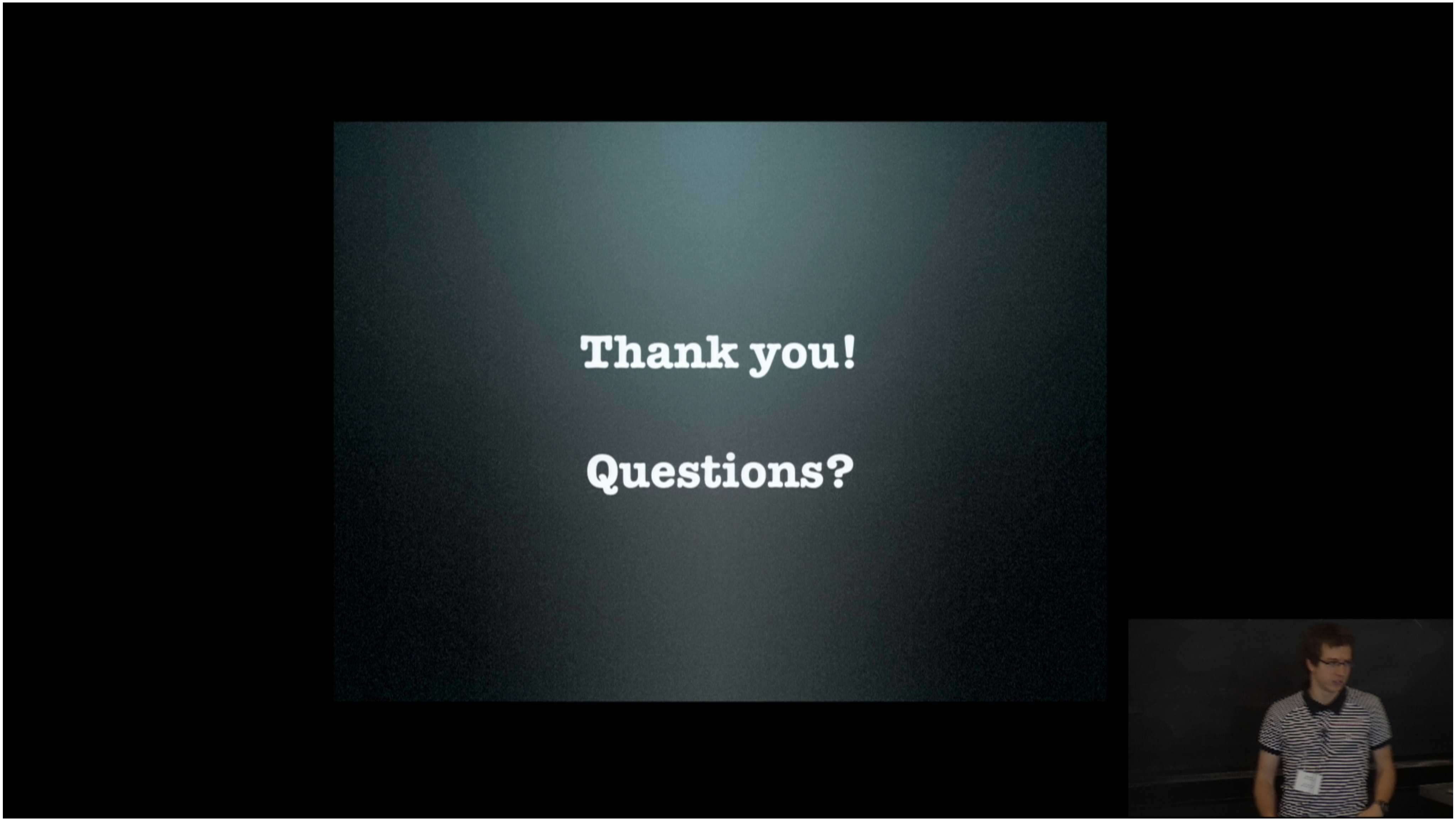
Thank you!
Questions?