Title: Quantum Field Theory I - Lecture 1b

Date: Oct 03, 2011 10:00 AM

URL: http://pirsa.org/11100081

Abstract:

$$H = \int d^3x \, \frac{\hbar^2}{2m} \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi + \frac{1}{2}\int d^3x \, d^3y \; U(x-y) \; :\rho(x)\,\rho(y):$$

Ex $\quad U(x) = g\,\delta(x)$

$$H = \int d^3x \; \frac{\hbar^2}{2m} \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi + \frac{1}{2}\int d^3x \, d^3y \; U(x-y) \; :\rho(x)\,\rho(y):$$

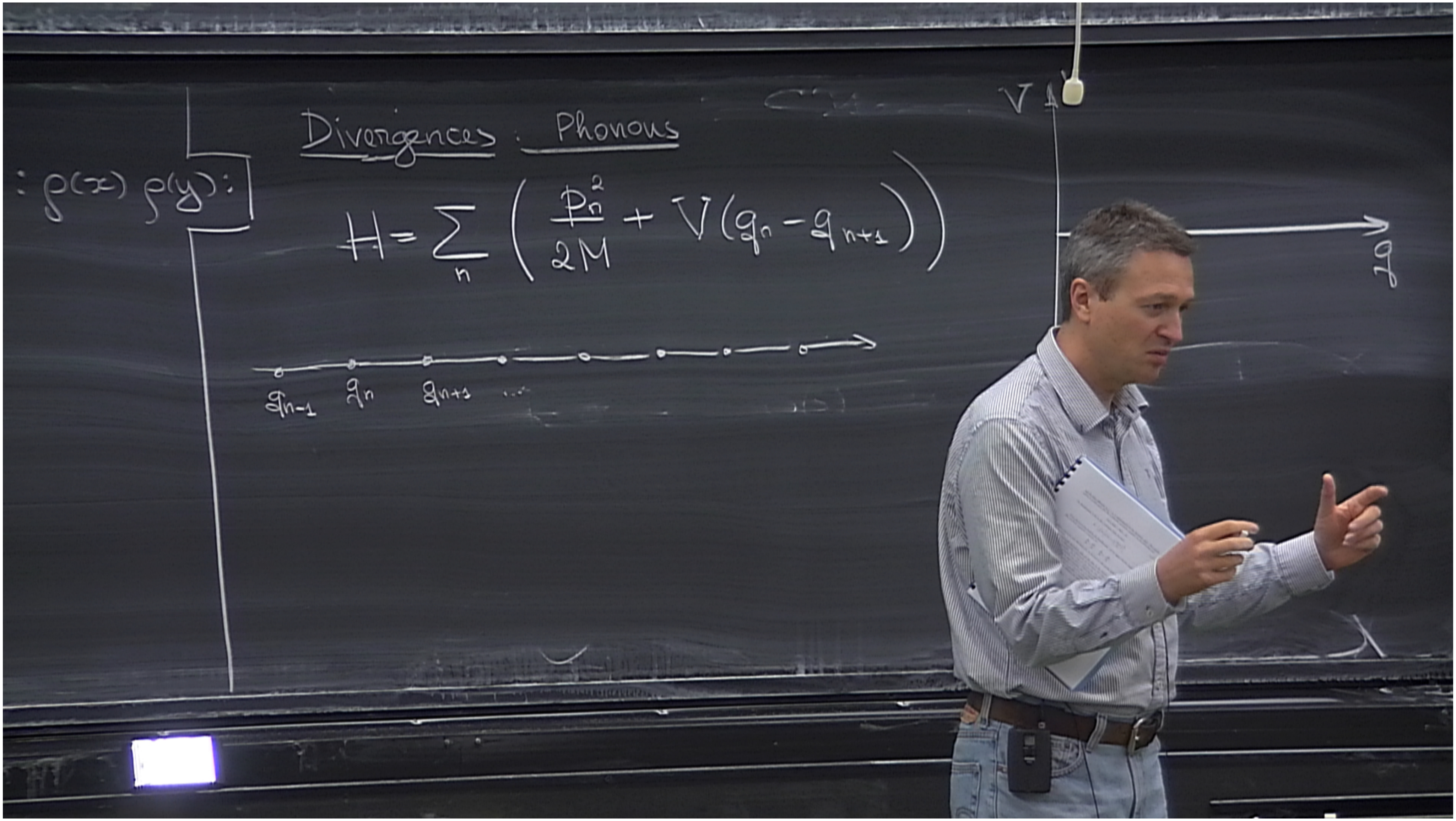
Ex $\quad U(x) = g\,\delta(x)$

$$H = \int d^3x \left[\frac{\hbar^2}{2m} \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi + \frac{g}{2} :(\psi^\dagger \psi)^2: \right]$$

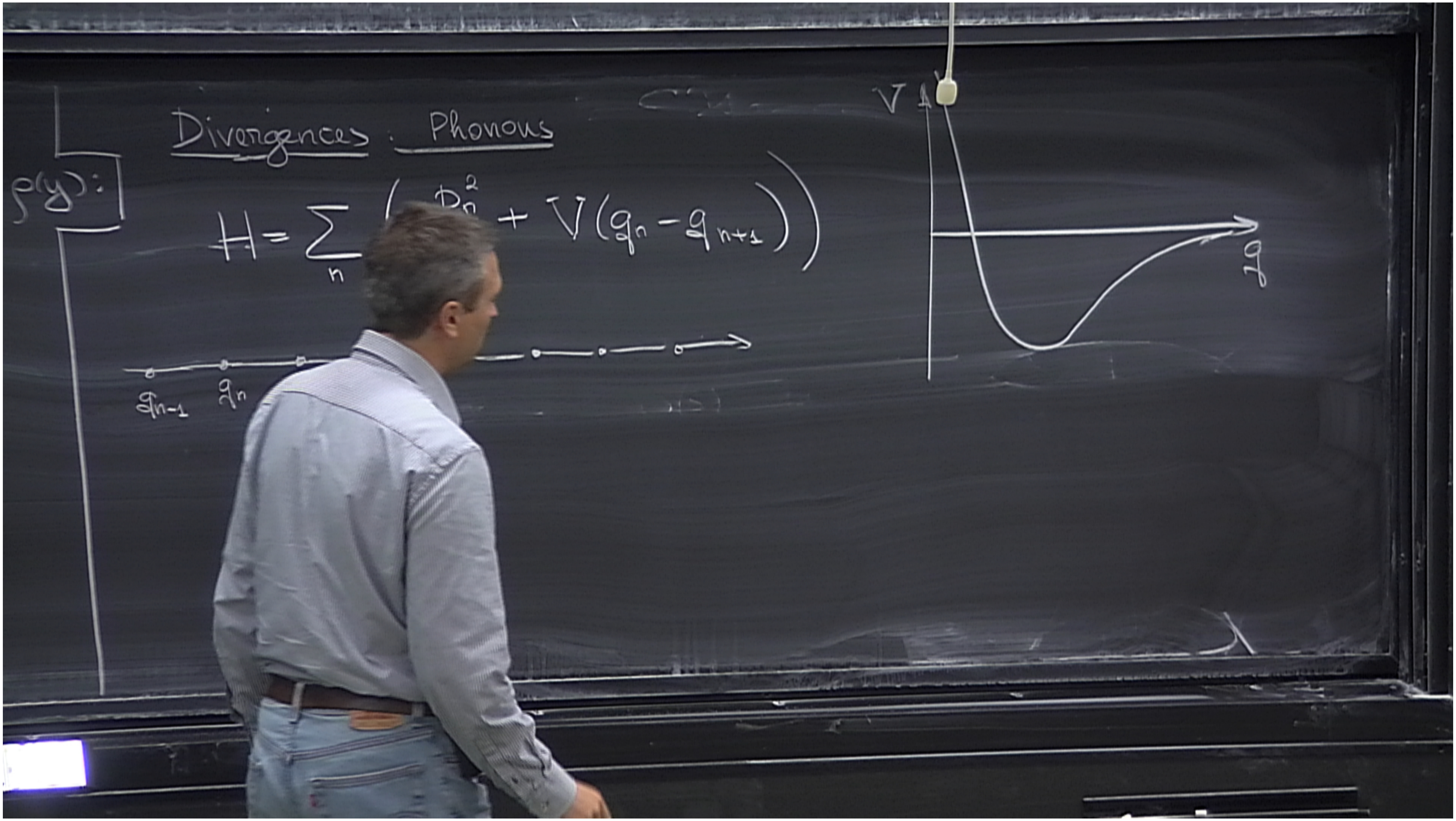
$$H = \int d^3x \, \frac{\hbar^2}{2m} \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi + \frac{1}{2}\int d^3x \, d^3y \; U(x-y) \; :\rho(x)\,\rho(y):$$

$\underline{\text{Ex}}$ $\quad U(x) = g\,\delta(x)$

$$H = \int d^3x \underbrace{\left[\frac{\hbar^2}{2m} \vec{\nabla}\psi^\dagger \cdot \vec{\nabla}\psi + \frac{g}{2} :(\psi^\dagger\psi)^2: \right]}_{\text{energy density}}$$



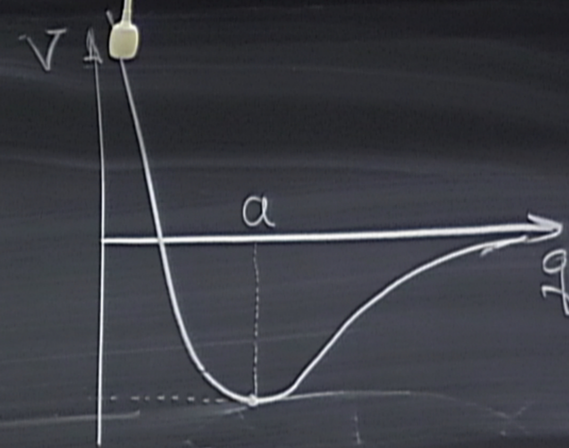
: ρ(x) ρ(y) :
Divergences : Phonons
$H = \sum_n \left(\frac{p_n^2}{2M} + V(q_n - q_{n+1}) \right)$
q_{n-1} q_n q_{n+1} ...
V
q


Divergences : Phonons
H = Σ_n (p_n^2 + V(q_n - q_{n+1}))
ρ(y):
V
q
q_{n-1}
q_n

$\rho(y)$:

Divergences: Phonons

$$H = \sum_n \left(\frac{p_n^2}{2M} + V(q_n - q_{n+1}) \right)$$

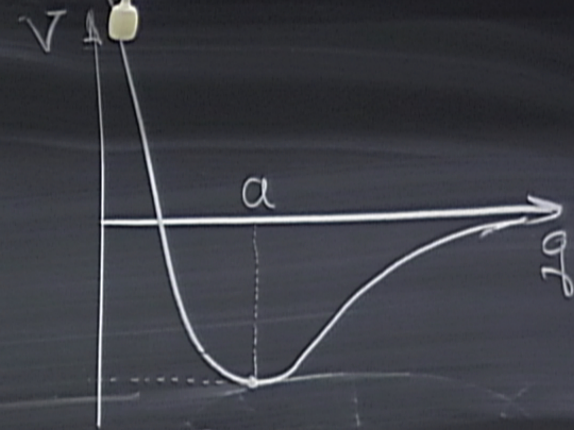


In equilibrium: $q_n = na$

$\rho(y)$:

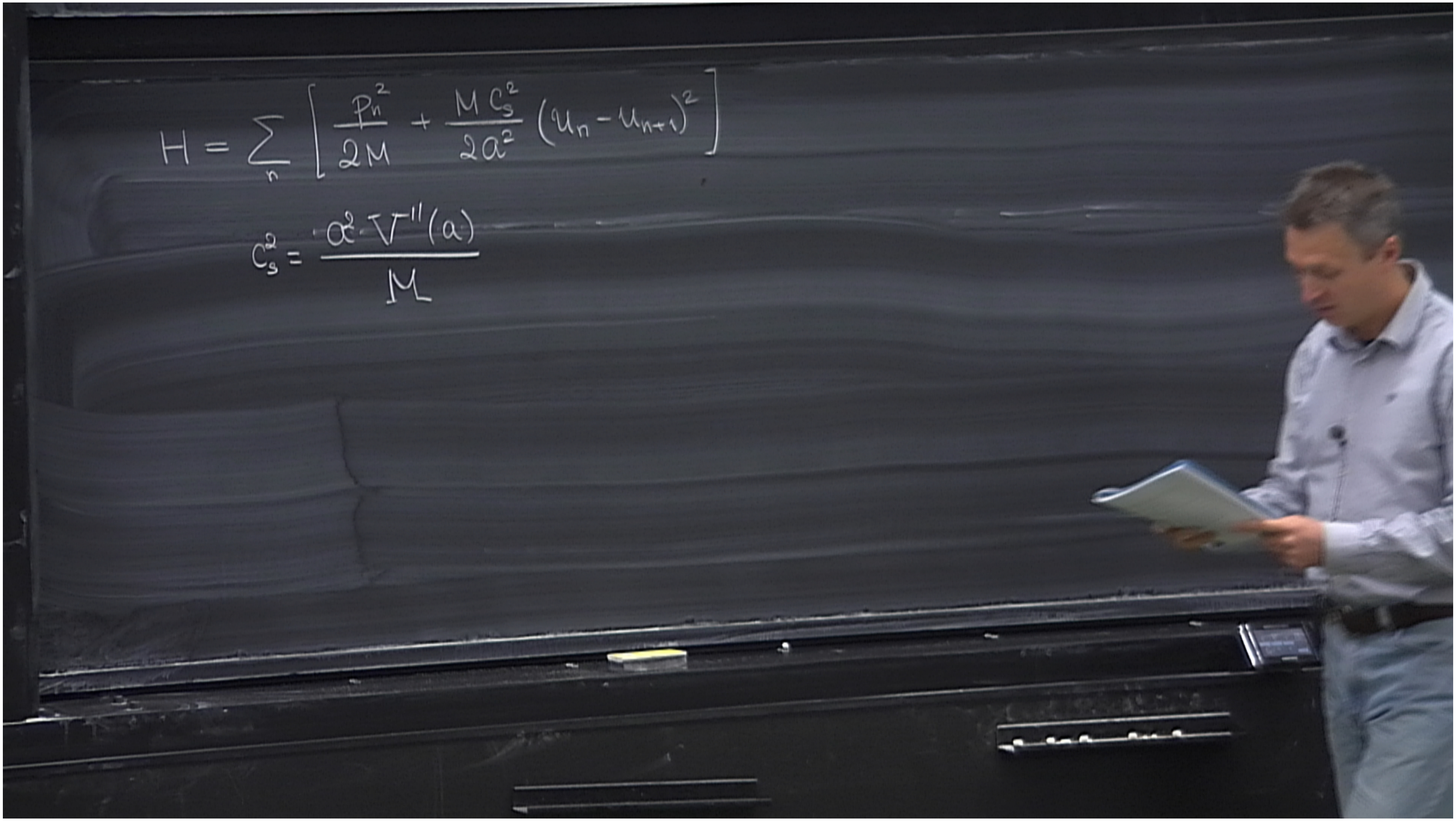
Divergences: Phonons

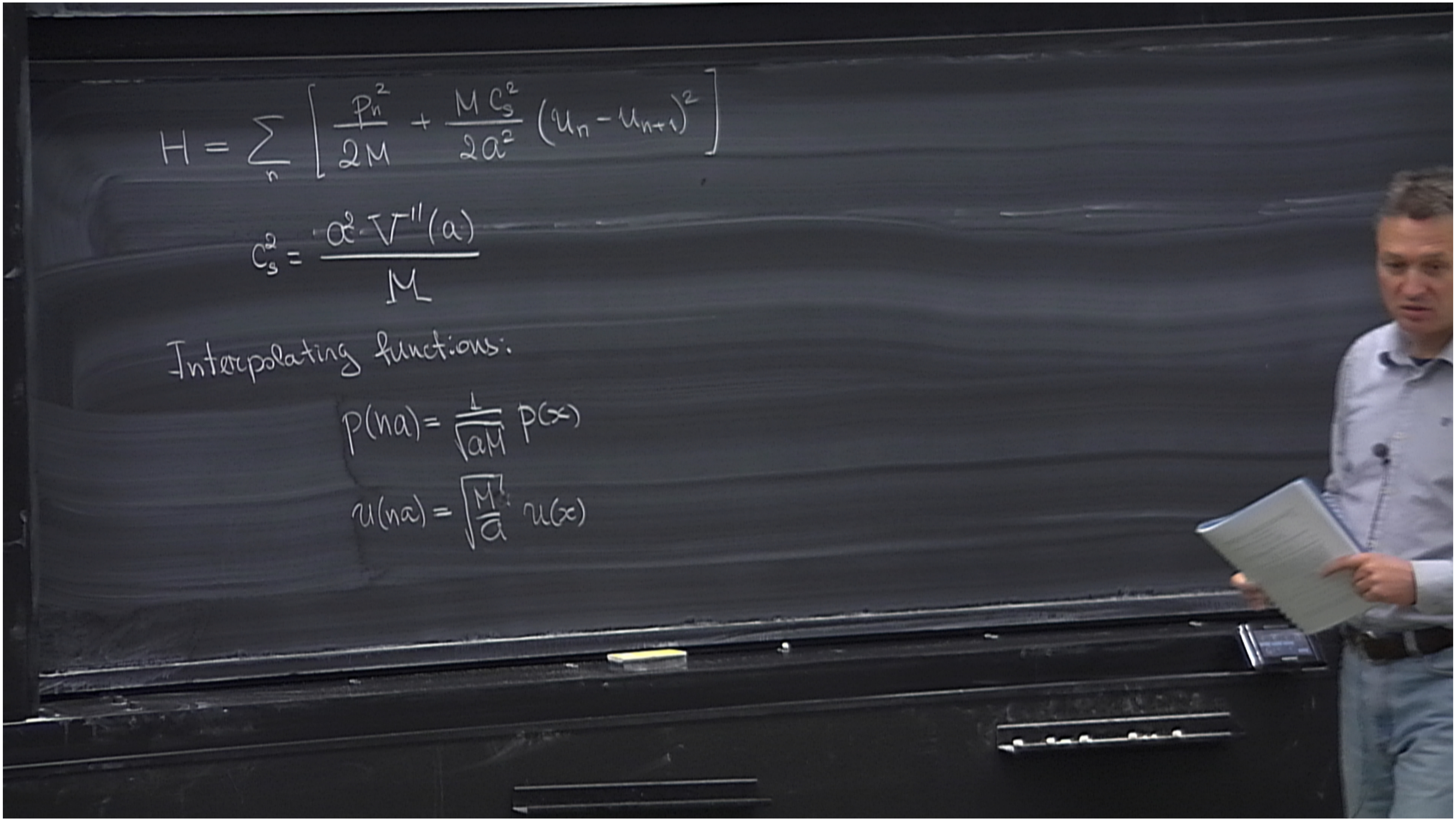
$$H = \sum_n \left(\frac{p_n^2}{2M} + V(q_n - q_{n+1}) \right)$$



In equilibrium: $q_n = na$

Deviations from equilibrium: $q_n = na + u_n$; $|u_n| \ll a$


$H = \sum_n \left[\frac{p_n^2}{2M} + \frac{M c_s^2}{2a^2} (u_n - u_{n+1})^2 \right]$
$c_s^2 = \frac{\alpha^2 \cdot V''(a)}{M}$


$H = \sum_n \left[\frac{p_n^2}{2M} + \frac{M c_s^2}{2a^2} (u_n - u_{n+1})^2 \right]$
$c_s^2 = \frac{a^2 V''(a)}{M}$
Interpolating functions:
$p(na) = \frac{1}{\sqrt{aM}} p(x)$
$u(na) = \sqrt{\frac{M}{a}} u(x)$

$$H = \sum_n \left[\frac{p_n^2}{2M} + \frac{M c_s^2}{2a^2} \left(u_n - u_{n+1} \right)^2 \right]$$

$$c_s^2 = \frac{a^2 V''(a)}{M}$$

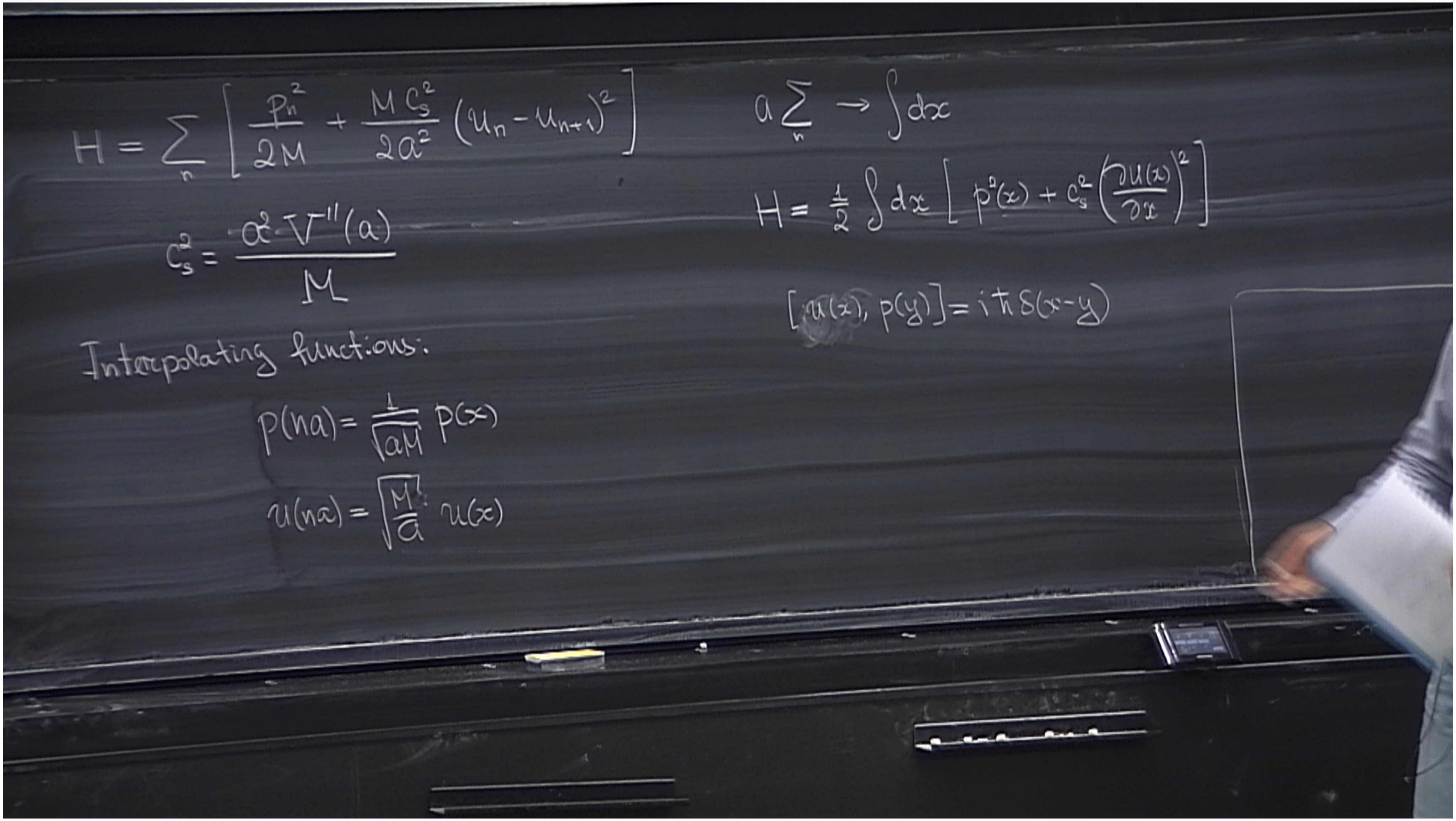
Interpolating functions:

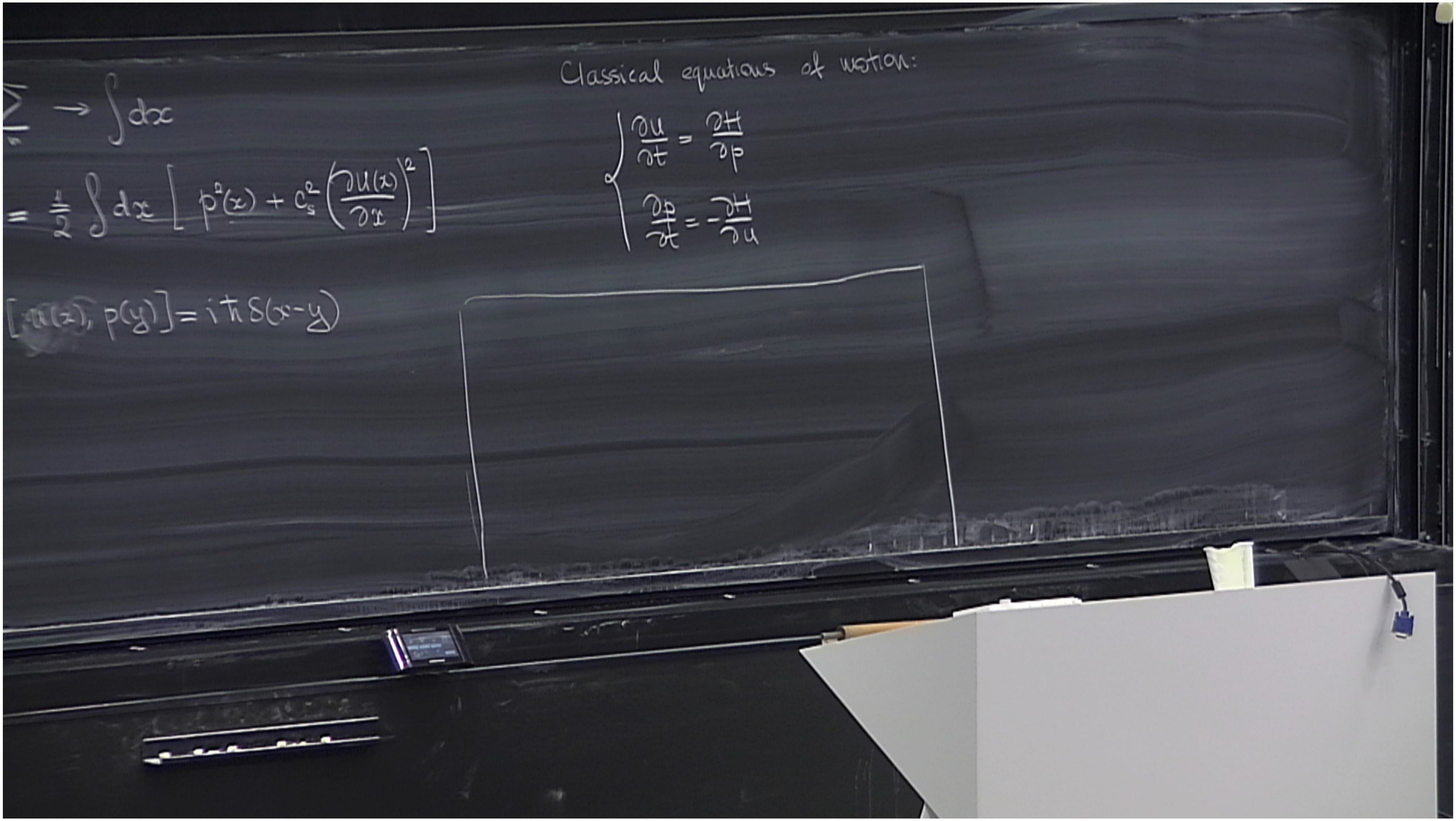
$$p(na) = \frac{1}{\sqrt{aM}} p(x)$$

$$u(na) = \sqrt{\frac{M}{a}}\, u(x)$$

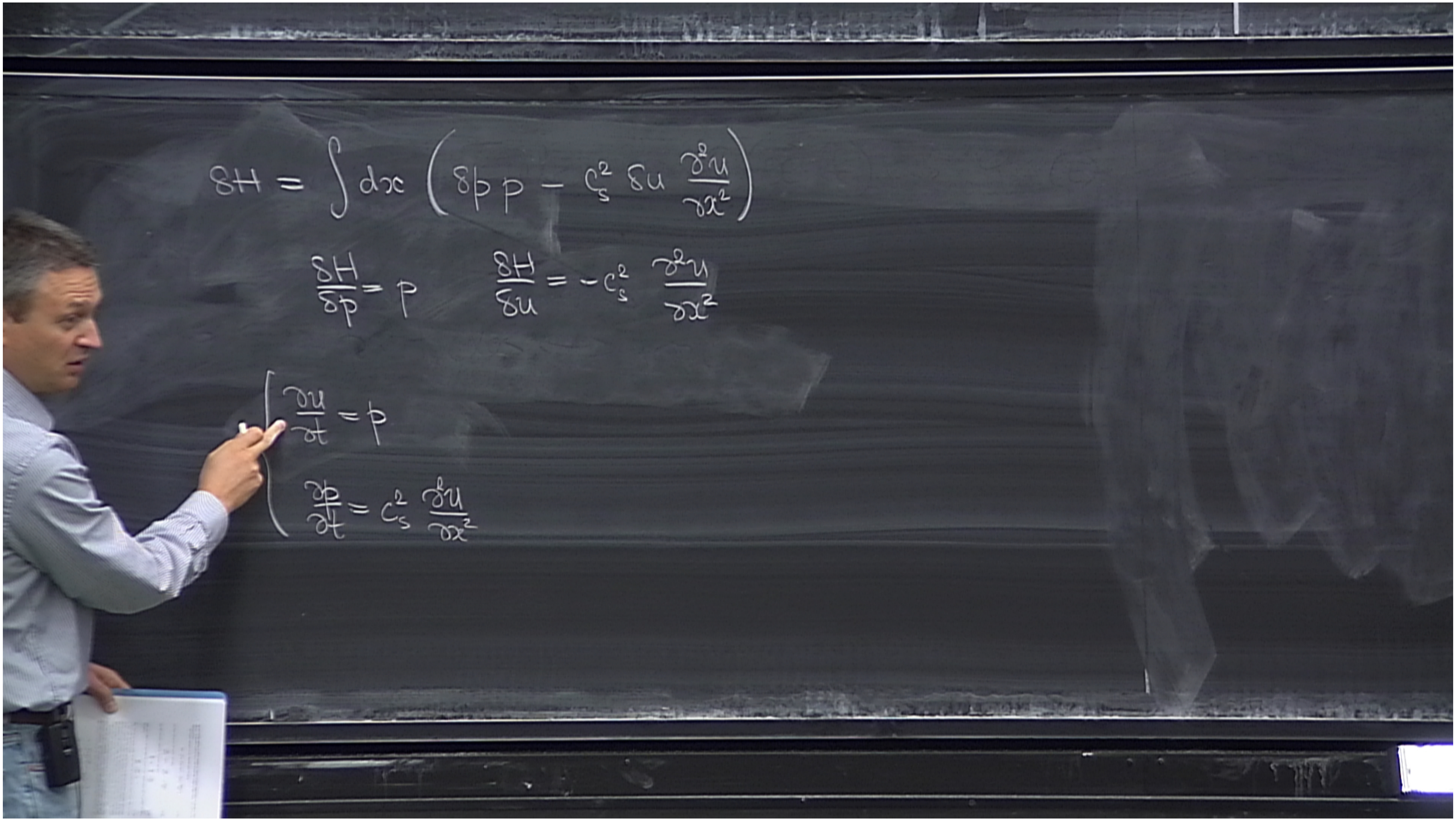
$$a \sum_n \rightarrow \int dx$$

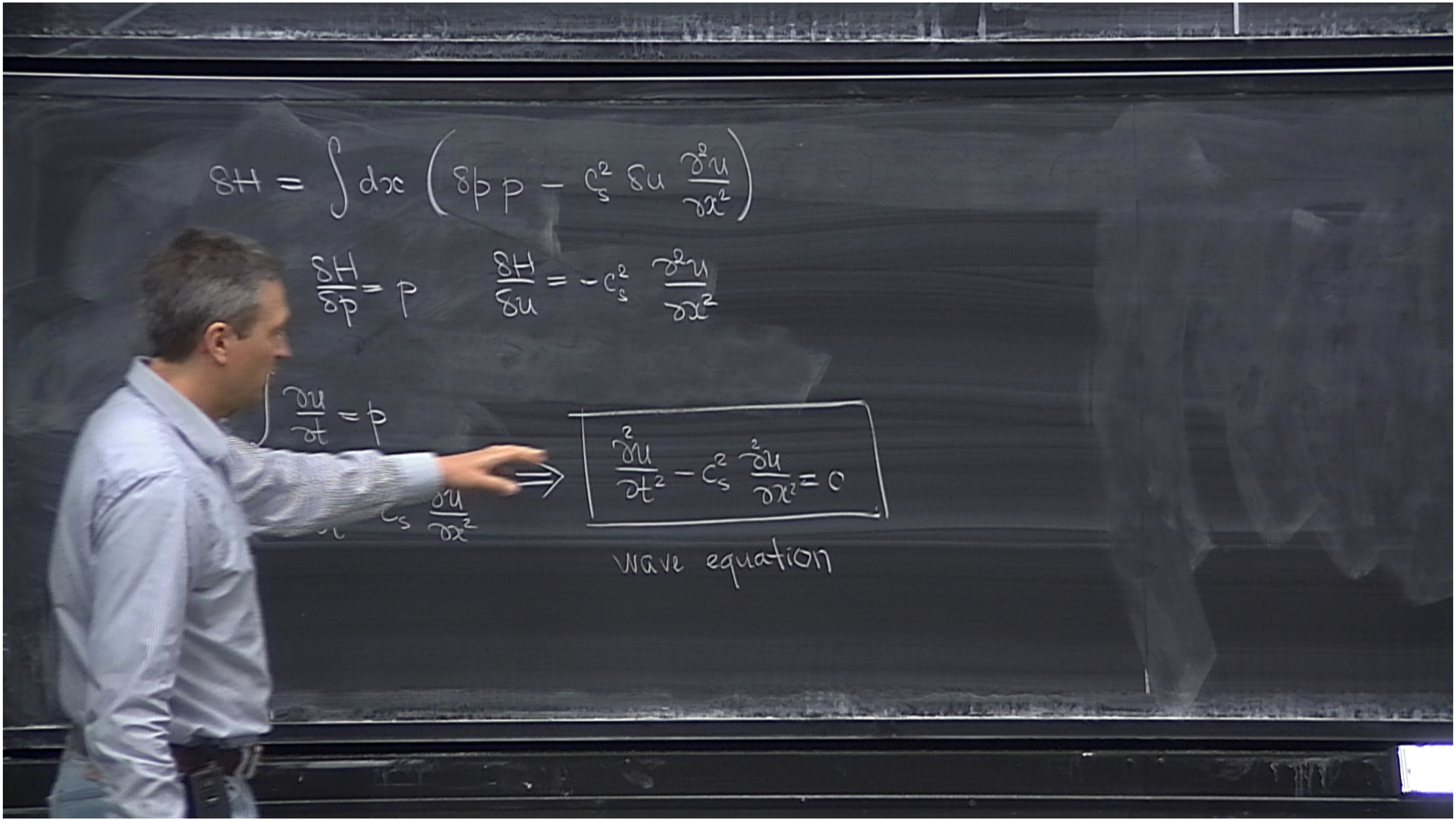
$$H = \frac{1}{2} \int dx \left[p^2(x) + c_s^2 \left(\frac{\partial u(x)}{\partial x} \right)^2 \right]$$


$H = \sum_n \left[\frac{p_n^2}{2M} + \frac{M c_s^2}{2a^2} (u_n - u_{n+1})^2 \right]$
$c_s^2 = \frac{a^2 V''(a)}{M}$
Interpolating functions:
$p(na) = \frac{1}{\sqrt{aM}} p(x)$
$u(na) = \sqrt{\frac{M}{a}} u(x)$
$a \sum_n \to \int dx$
$H = \frac{1}{2} \int dx \left[p^2(x) + c_s^2 \left(\frac{\partial u(x)}{\partial x} \right)^2 \right]$
$[u(x), p(y)] = i\hbar \delta(x-y)$



$\sum_n \to \int dx$
$= \frac{1}{2} \int dx \left[p^2(x) + c_s^2 \left(\frac{\partial u(x)}{\partial x} \right)^2 \right]$
$[u(x), p(y)] = i\hbar\,\delta(x-y)$
Classical equations of motion:
$\frac{\partial u}{\partial t} = \frac{\partial H}{\partial p}$
$\frac{\partial p}{\partial t} = -\frac{\partial H}{\partial u}$


$\delta H = \int dx \left(\delta p \, p - c_s^2 \, \delta u \, \frac{\partial^2 u}{\partial x^2} \right)$
$\frac{\delta H}{\delta p} = p \qquad \frac{\delta H}{\delta u} = -c_s^2 \frac{\partial^2 u}{\partial x^2}$
$\begin{cases} \frac{\partial u}{\partial t} = p \\ \frac{\partial p}{\partial t} = c_s^2 \frac{\partial^2 u}{\partial x^2} \end{cases}$


$\delta H = \int dx \left(\delta p \, p - c_s^2 \, \delta u \, \frac{\partial^2 u}{\partial x^2} \right)$
$\frac{\delta H}{\delta p} = p$
$\frac{\delta H}{\delta u} = -c_s^2 \frac{\partial^2 u}{\partial x^2}$
$\frac{\partial u}{\partial t} = p$
$\Longrightarrow \boxed{\frac{\partial^2 u}{\partial t^2} - c_s^2 \frac{\partial^2 u}{\partial x^2} = 0}$
wave equation

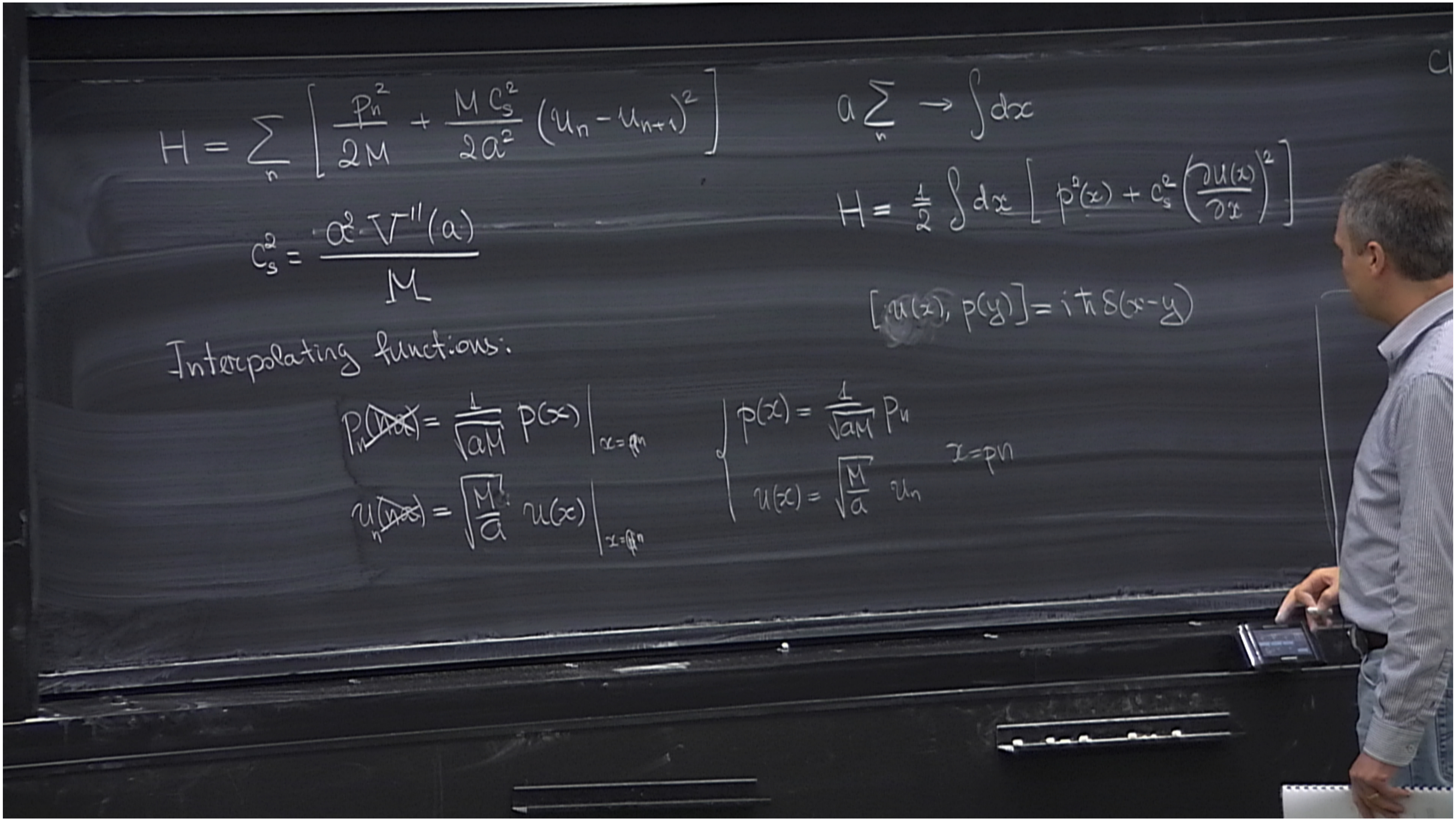
$$\delta H = \int dx \left(\delta p \, p - c_s^2 \, \delta u \, \frac{\partial^2 u}{\partial x^2} \right)$$

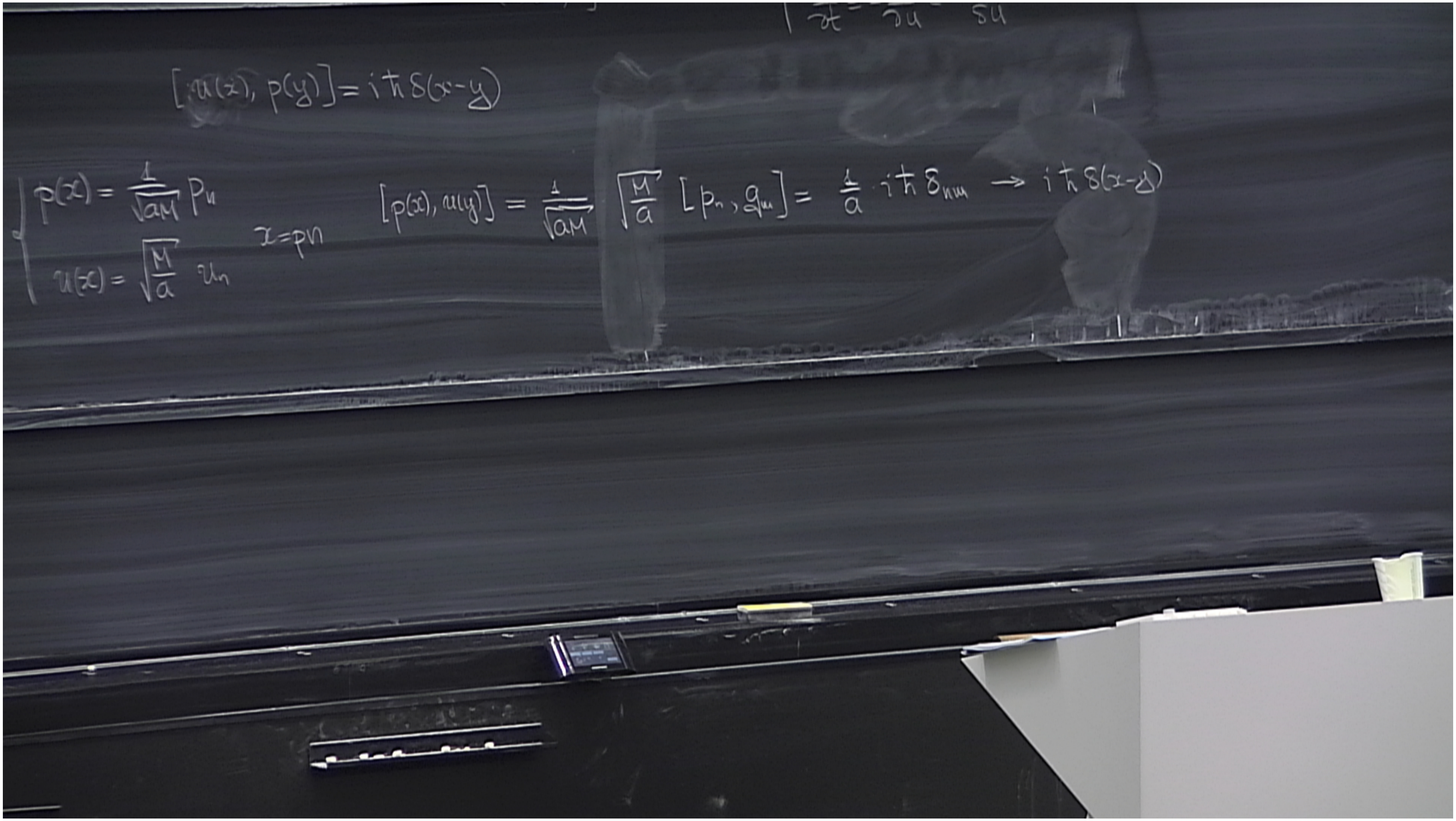
c_s - speed of sound

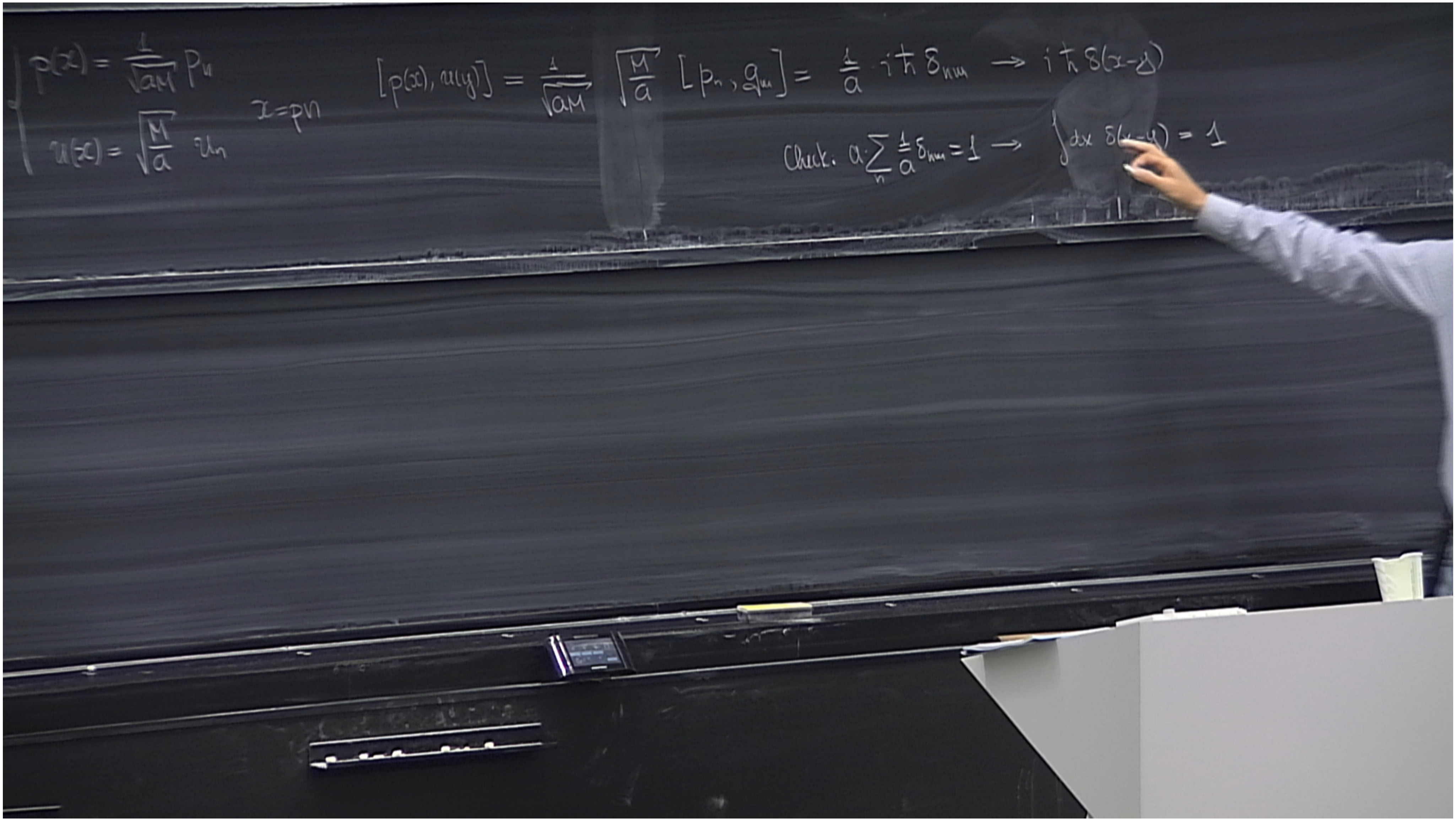
$$\frac{\delta H}{\delta p} = p \qquad \frac{\delta H}{\delta u} = -c_s^2 \frac{\partial^2 u}{\partial x^2}$$

$$\begin{cases} \dfrac{\partial u}{\partial t} = p \\[2ex] \dfrac{\partial p}{\partial t} = c_s^2 \dfrac{\partial^2 u}{\partial x^2} \end{cases} \Rightarrow \boxed{\frac{\partial^2 u}{\partial t^2} - c_s^2 \frac{\partial^2 u}{\partial x^2} = 0}$$

wave equation


$H = \sum_n \left[\frac{p_n^2}{2M} + \frac{M c_s^2}{2a^2} (u_n - u_{n+1})^2 \right]$
$c_s^2 = \frac{a^2 V''(a)}{M}$
Interpolating functions:
$p_n = \frac{1}{\sqrt{aM}} p(x) \Big|_{x=an}$
$u_n = \sqrt{\frac{M}{a}} u(x) \Big|_{x=an}$
$a \sum_n \to \int dx$
$H = \frac{1}{2} \int dx \left[p^2(x) + c_s^2 \left(\frac{\partial u(x)}{\partial x} \right)^2 \right]$
$[u(x), p(y)] = i\hbar \delta(x-y)$
$p(x) = \frac{1}{\sqrt{aM}} p_n$
$u(x) = \sqrt{\frac{M}{a}} u_n$
$x = an$


$[u(x), p(y)] = i\hbar\delta(x-y)$
$p(x) = \frac{1}{\sqrt{aM}} p_n$
$u(x) = \sqrt{\frac{M}{a}} u_n$
$x = pn$
$[p(x), u(y)] = \frac{1}{\sqrt{aM}} \sqrt{\frac{M}{a}} [p_n, q_m] = \frac{1}{a} i\hbar\delta_{nm} \rightarrow i\hbar\delta(x-y)$



$p(x) = \frac{1}{\sqrt{aM}} p_n$
$u(x) = \sqrt{\frac{M}{a}}\, u_n$
$x = pn$
$[p(x), u(y)] = \frac{1}{\sqrt{aM}} \sqrt{\frac{M}{a}} [p_n, q_m] = \frac{1}{a} i\hbar \delta_{nm} \longrightarrow i\hbar\delta(x-y)$
Check. $a \cdot \sum_n \frac{1}{a}\delta_{nm} = 1 \longrightarrow \int dx\, \delta(x-y) = 1$