Title: More is Different in the Quantum World, in its Own Way

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Abstract: In his article `More is Different', P W Anderson (1972) sensitized the physics community about importance of emergence, by using concepts such as broken symmetry, emergent hierarchical structures, constructionists converse of reductionism etc. The manifestation of complexity and hierarchy in the quantum many particle systems go beyond broken symmetries. In certain quantum systems we have the opposite - emergence of new local gauge symmetries. This idea was introduced, for example, for Mott insulators, by Anderson and us in 1988. Properties such as entanglement, different possible statistics of identical particles, multi particle wave interference, vastness of Hilbert space etc;, which are unique to the world of quantum, leads to a wonderful richness in physics and mathematics, including emergent gauge symmetries, gauge fields, quantum order, holographic correspondences and continuing surprises.

More is Different
in Quantum World
in its Own Way

Symmetry Enhancement
as opposed to
Symmetry Reduction

Emergence & Effective Field Theories
October 26-28, 2011

The Institute of Mathematical Sciences

G Baskaran

More is Different
Science (1972)

P W Anderson
July 2011

Introduction

More is different ...

symmetry breaking, emergence of hierarchy,
information bearing symmetry breaking, complexity, depth ...

Different in different ways in the World of Quantum Matter

Strong Correlation & Symmetry enhancement

Emergence of gauge symmetries, gauge particles, quantum order,
non Abelion anyons, topological protection, entanglement, holography ...

Mott Insulator & Quantum Spin Liquid

as examples

More Is Different

SCIENCE

4 August 1972, Volume 177, Number 4047

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

at each level of complexity entirely new properties appear

X	Y
solid state or many-body physics	elementary particle physics
chemistry	many-body physics
molecular biology	chemistry
cell biology	molecular biology
•	•
•	•
•	•
psychology	physiology
social sciences	psychology

But this hierarchy does not imply that science X is "just applied Y."

we have begun to formulate a general theory of just how this shift from quantitative to qualitative differentiation takes place. This formulation, called the theory of "broken symmetry," may be of help in making more generally clear the breakdown of the constructionist converse of reductionism.

World of Quantum Matter

Matter, including light

Natural	& Man Made
	2 dimensional electron gas (heterostructure) & large B
Solids at Low T (phonons) Magnets, Fe, Ni, Gd, … Superconductors, Hg, Pb, Nb .. Insulators Diamond, Si, GaAs, ..	Doped La_2CuO_4 Superconductor Sr_2RuO_4 p + ip superconductor
Oxides, sulfides, polymers. … Minerals ….	$SrTiO_3$ Quantum paraelectric Heavy fermions, Kondo insulators
Biological Matter Photosynthesis, electron transfer proton tunneling Bird navigation	Superfluid & supersolid ^4He Quantum dots, Cold atom gas Photonic crystals …

Quantum Mechanics

Schrodinger Equation

$$-i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

Hilbert Space
formed by physical States
Linear Vector Space
$|i\rangle, \dots; |j\rangle \dots$

ψ – complex
PROBABILITY
AMPLITUDE

$|\psi\rangle \equiv u|i\rangle + v|j\rangle$ is a physically meaningful state

⇓

NON CLASSICAL CORRELATION & ENTANGLEMENT

Identical particles

BOSONS

$$\psi(1,2) = \psi(2,1)$$

FERMIONS

$$\psi(1,2) = -\psi(2,1)$$

BEC

state

FERMI SEA

Classical Mechanics: 6N dimensional phase space

VASTNESS OF HILBERT SPACE

$$\psi(\vec{r}_1\sigma_1, \vec{r}_2\sigma_2, \ldots\ldots \vec{r}_N\sigma_N)$$

In many problems,
Direct product of finite dimensional Hilbert space:

N Qubits: 2^N dim. Hilbert Space

Evolution in the World of Quantum Matter

Effectively Non interacting
Non Entangled Ground States

Photon (Einstein)

Crystals Phonon (Einstein, Debye)
Ferro, Antiferromagnets Magnon -spin waves
Superconductor, Superfluids
Charge density wave, spin density wave …

Fermi sea
Band insulators, Topological insulators
Holes, electrons, excitons, plasmons, polaritons

Strongly Interacting Quantum Matter

Mott Insulators, Quantum Spin liquids
Quantum Hall States

Luttinger liquids, Kondo Lattice system
Heavy Fermions

Superfluid ^4He, Supersolid ^4He
High Tc Superconductors (Cuprates, Fe pnictides)
P-wave superconductors (Sr_2RuO_4)

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Hilbert Space Restriction is a basic ingredient for interaction effects to become important and appearance of unexpected properties of matter

Free electron gas → tight binding models
(periodic potential produces bands, narrow bands, gaps …)
$LaCuO_4$ (narrow band), Quantum Spin Liquids
High Tc Superconductivity

Free electron gas → highly degenerate Landau Levels

A rich Quantum Hall Physics

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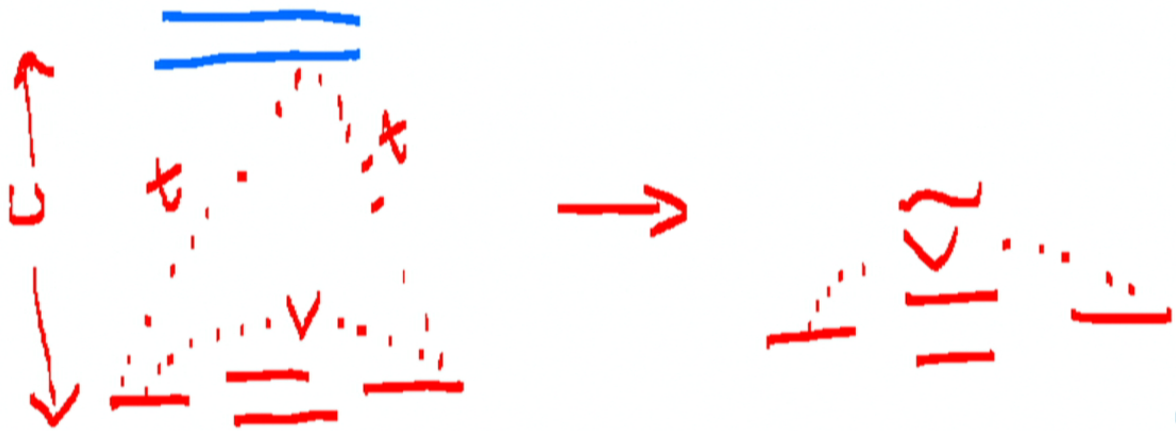
A rich Quantum Hall Physics

Finding the relevant Hilbert space and Focusing our attention there is a process of renormalization

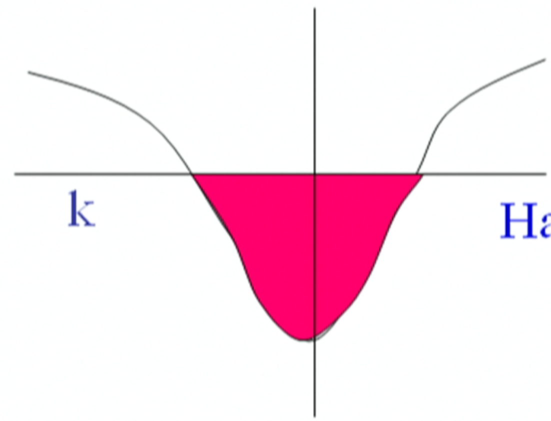
Often it is a finite number of discrete step

Elimination of high energy state above a gap



$$\tilde{V} = V + \frac{t^2}{U}$$

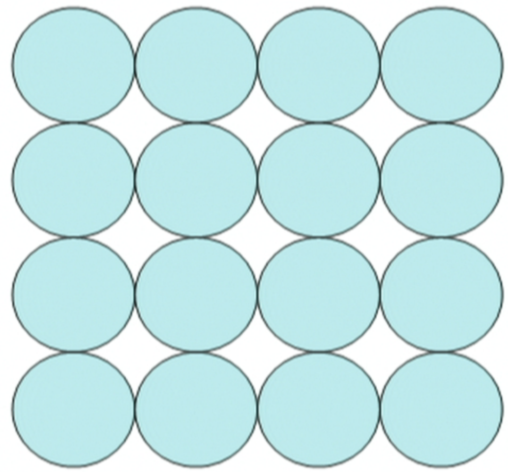
A collection of hydrogen atoms forming a hypothetical 3D lattice



Half filled band

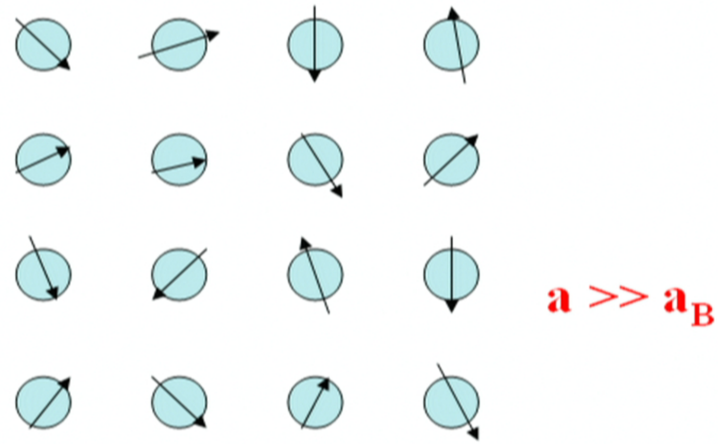
1s states of individual hydrogen atoms strongly overlap and form a tight binding half filled band

It is a metal



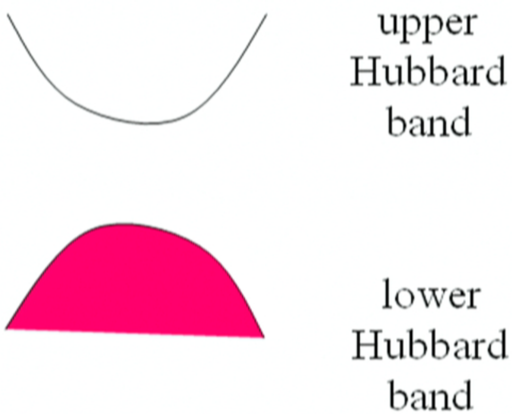
$a \sim a_B$

Let us expand the lattice

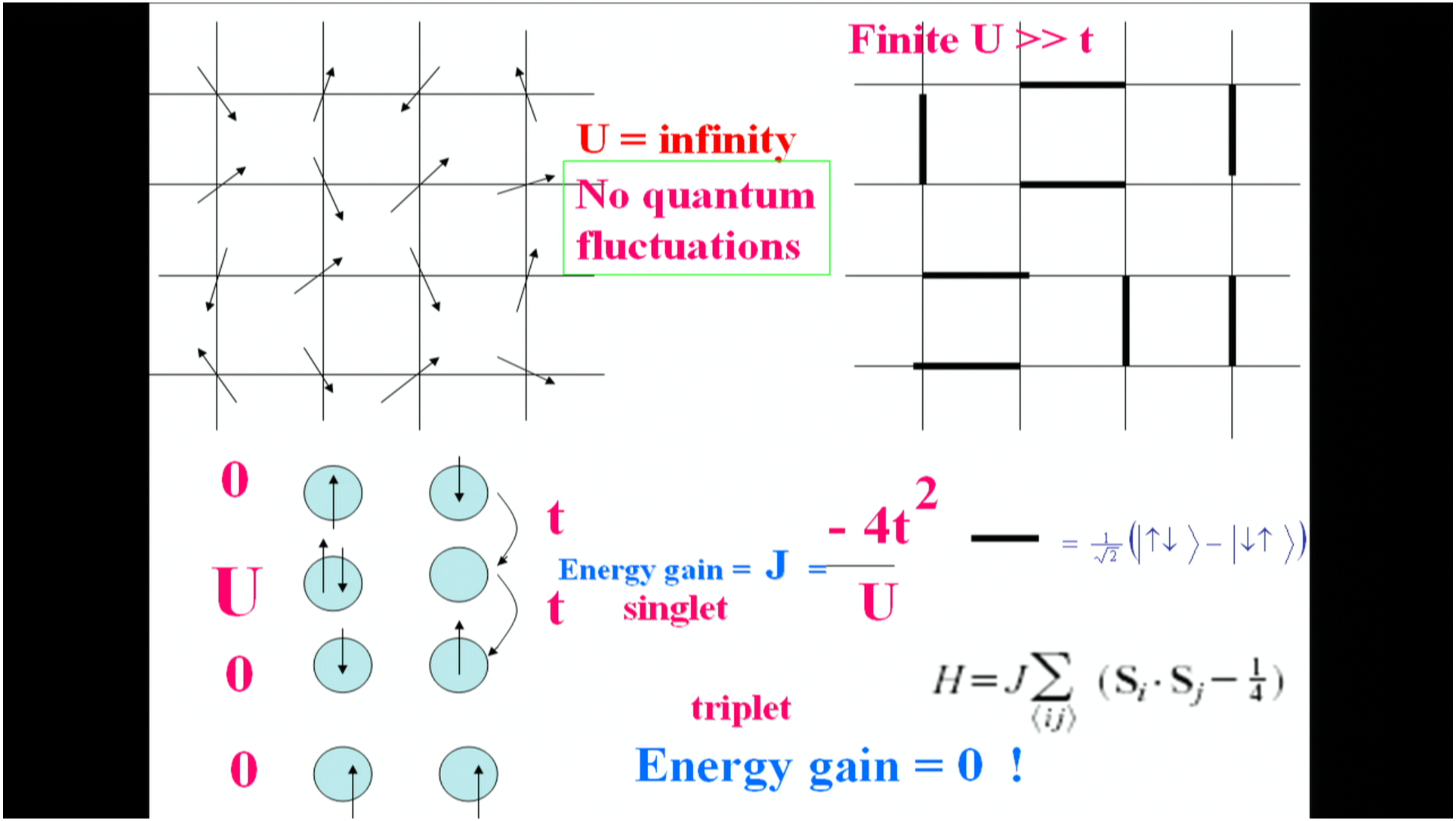


For $a \gg a_B$

we get a Mott insulator



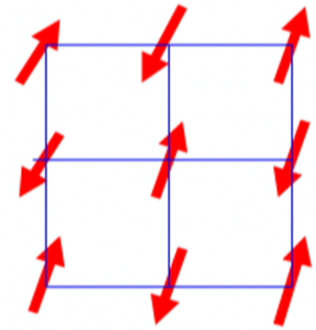
Spins are soft degrees of freedom
while charges are frozen


Finite U >> t
U = infinity
No quantum fluctuations
0
U
0
0
t
t
Energy gain = J = - 4t^2 / U
singlet
= 1/√2 (|↑↓ ⟩ − |↓↑ ⟩)
triplet
H = J Σ⟨ij⟩ (Si · Sj − 1/4)
Energy gain = 0 !

Resonating Valence Bond (RVB) States

Pauling
Anderson 1973
Fazekas

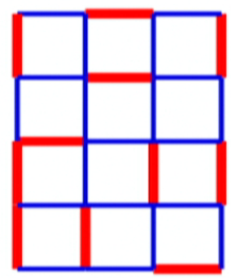
Quantum Heisenberg Antiferromagnets (spin half)



Quantum fluctuations
Encouraged by lattice frustration and lower dimensionality
may destroy long range AFM order (spin crystal)

Resulting in a Quantum Spin Liquid, a RVB state

$$\left|RVB\right\rangle = \sum_{C}\left|C\right\rangle$$



$$|RVB;\phi\rangle \equiv P_G(\sum_{ij}\phi_{ij}b^{\dagger}_{ij})^{\frac{N}{2}}|0\rangle$$

Short range spin correlations

$$\text{—} = \frac{1}{\sqrt{2}}[\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle]$$

$$\langle S_{iz}S_{jz}\rangle \approx e^{\frac{-|i-j|}{\xi}} \quad or \quad \frac{1}{|i-j|^{\alpha}}$$

How do we get pseudo Fermi surface from theory ?

RVB Meanfield Theory (GB, Zou, Anderson 1987)

$$J(S_i.S_j - \frac{1}{4}n_i n_j) \equiv -J \quad b_{ij}^{+} b_{ij}$$

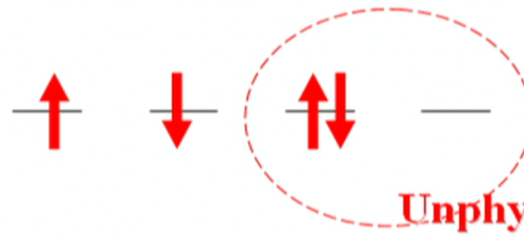
$$\vec{S}_i \equiv C_{i\alpha}^{+} \vec{\sigma}_{\alpha\beta} C_{i\beta} \qquad n_{i\uparrow} + n_{i\downarrow} = 1$$

$$b_{ij}^{+} = \frac{1}{\sqrt{2}}(C_{i\uparrow}^{+}C_{j\downarrow}^{+} - C_{i\downarrow}^{+}C_{i\uparrow}^{+})$$

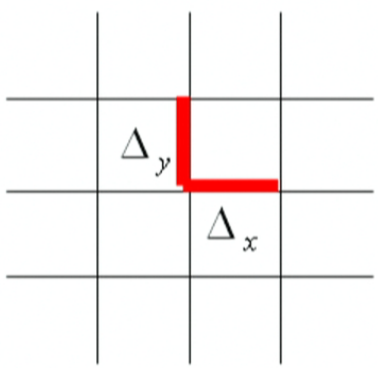
Two complex fermion Hilbert space $\quad 2^2 = 4$

$$b_{ij}^{+}b_{ij} \rightarrow \langle b_{ij}^{+} \rangle b_{ij} + b_{ij}^{+} \langle b_{ij} \rangle$$

Δ_{ij}^{*}



Unphysical Hilbert space



Extended - S $\longrightarrow \Delta_x = \Delta_y$ **BZA**

Flux Phase $\longrightarrow \Delta_x = -\Delta_y$ **Kotliar, Affleck, Marston**

Extended - S $\longrightarrow \Delta_x = \Delta_y$

$$H_s = J\sum_{\langle ij\rangle}(S_i \cdot S_j - \frac{1}{4}) = -J\sum_{\langle ij\rangle} b_{ij}^{\dagger} b_{ij}$$

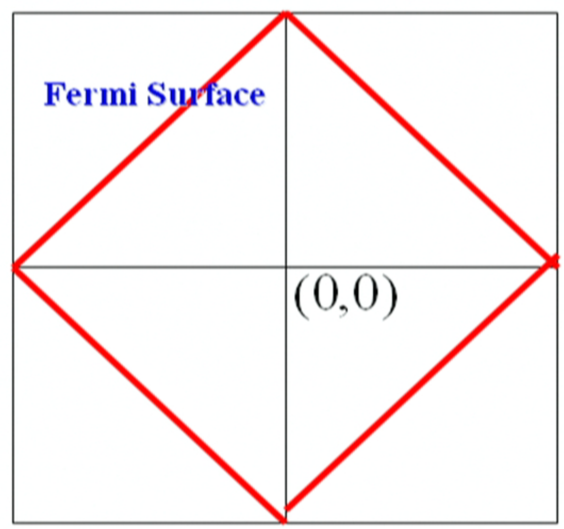
$$b_{ij}^{+} b_{ij} \rightarrow \langle b_{ij}^{+}\rangle b_{ij} + b_{ij}^{+}\langle b_{ij}\rangle$$

$$H_{pair} = -J\sum_{k,k'} \gamma(\mathbf{k}-\mathbf{k}') c_{-k'\downarrow}^{\dagger} c_{k'\uparrow}^{\dagger} c_{k\uparrow} c_{-k\downarrow}$$

$$H_{mF} \sim J\sum_{k\alpha} |\cos k_x + \cos k_y| \; \alpha_{k\sigma}^{\dagger}\alpha_{k\sigma}$$

$$|2D\,RVB\rangle = P_G \prod_k (u_k + v_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger})|0\rangle$$

(π, π)



GB, Zou, Anderson 1987

Dynamically generated Gauge Fields

GB, Anderson 1987

$$2^N \rightarrow 4^N$$

Spin waves (Goldstone modes) are elementary excitations
In magnetically ordered systems

Hilbert space enlargement helped us to see
presence of dynamically generated gauge field degree of freedom
in addition to topological excitations such as a spinon

Local U(1) gauge symmetry

$$H_s = J\sum_{\langle ij\rangle}(\mathrm{S}_i\cdot\mathrm{S}_j-\frac{1}{4}) = -J\sum_{\langle ij\rangle} b_{ij}^{\dagger}b_{ij}$$

$$C_{i\alpha}^{+}\rightarrow e^{i\theta_i}C_{i\alpha}^{+} \qquad b_{ij}^{+}\rightarrow e^{i\theta_i}b_{ij}^{+}e^{i\theta_j} \qquad \Delta_{ij}^{*}\rightarrow e^{i\theta_i}\Delta_{ij}^{*}e^{i\theta_j}$$

GB, Anderson

U(1) RVB magnetic field $\Re\ e^{i\oint \mathbf{A}(\mathbf{r}).d\mathbf{l}} \sim \mathbf{S}_i\times(\mathbf{S}_j\times\mathbf{S}_k)$

Wen Wilczek Zee

Local SU(2) symmetry $c_{i\uparrow}\rightarrow u_i c_{i\uparrow}+v_i c_{i\downarrow}^{\dagger}$ $\quad |u_i|^2+|v_i|^2=1$

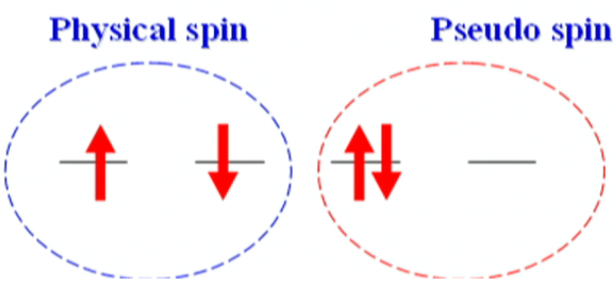
Affleck Anderson Zou Hsu

Two complex fermion

Hilbert space $2^2 = 4$

$$2^{2N} = 2^N\times\ldots\ldots\times 2^N$$

$$\leftarrow 2^N\, times \rightarrow$$



When a physical spin and a Pseudo spin get identified the above 2^N sectors become gauge copies of a Z_2 gauge theory

Quantum Order Topological Order

Kivelson Rokhsar

Haldane
Rezayi

Wen

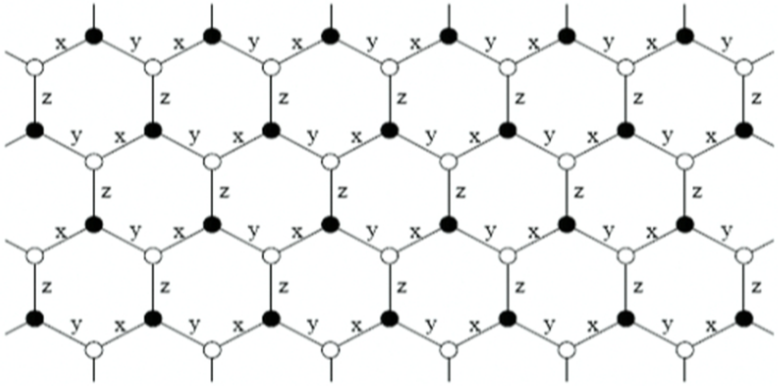
GB, Anderson

$e^{i\oint \vec{A}\cdot d\vec{l}} = \pm 1$

4 FOLD DEGENERACY

$\oint \vec{A}\cdot d\vec{l}$

dynamically generated RVB flux

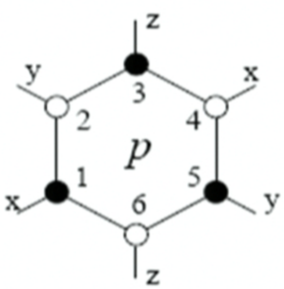


Kitaev Model
it is a highly Frustrated Quantum Spin System

exactly solvable

$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z$$

Local Conserved Quantities



$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

$W_p^2 = 1$

Eigen values of W_p = 1, -1

Dirac or complex Fermion

$\{\psi_i, \psi_j^\dagger\} = \delta_{ij}$

$\psi = \frac{1}{2}(\zeta + i\eta)$

$\{\zeta_i, \zeta_j\} = \delta_{ij}$ $\eta^2 = \zeta^2 = 1$

$|0\rangle, |1\rangle$

Majorana or Real fermions

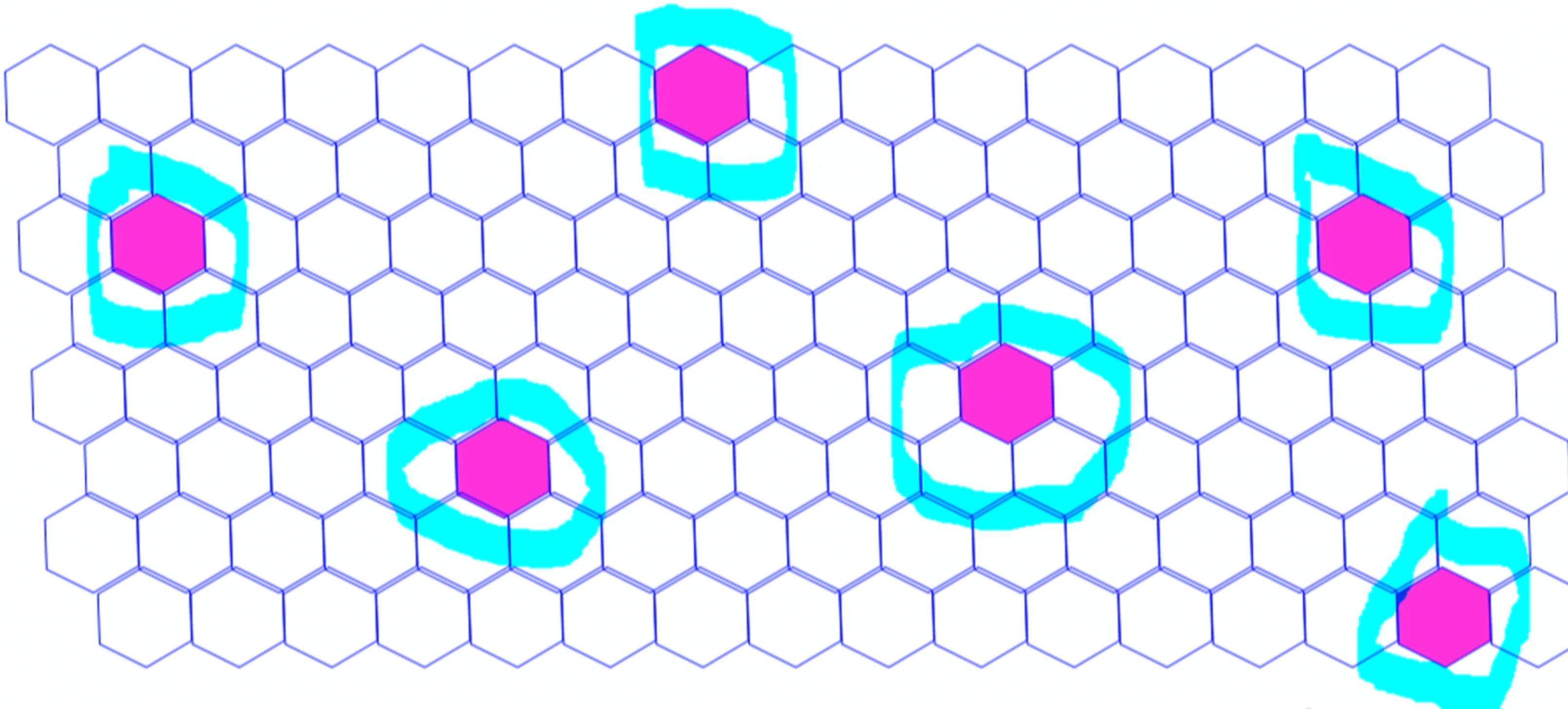
Hilbert space dimensions

$D_\zeta = \sqrt{2}$

$D_\xi = \sqrt{2}$

$D_F = D_\zeta \times D_\xi = \sqrt{2} \times \sqrt{2} = 2$

M well separated vortices Ground state degeneracy is $(\sqrt{2})^M$



6 well separated vortices Ground state degeneracy is $(\sqrt{2})^6 = 8$