Title: Topological Liquid Nucleation Induced by Vortex-vortex Interactions in Kitaev's Honeycomb Model

Date: Oct 26, 2011 04:00 PM

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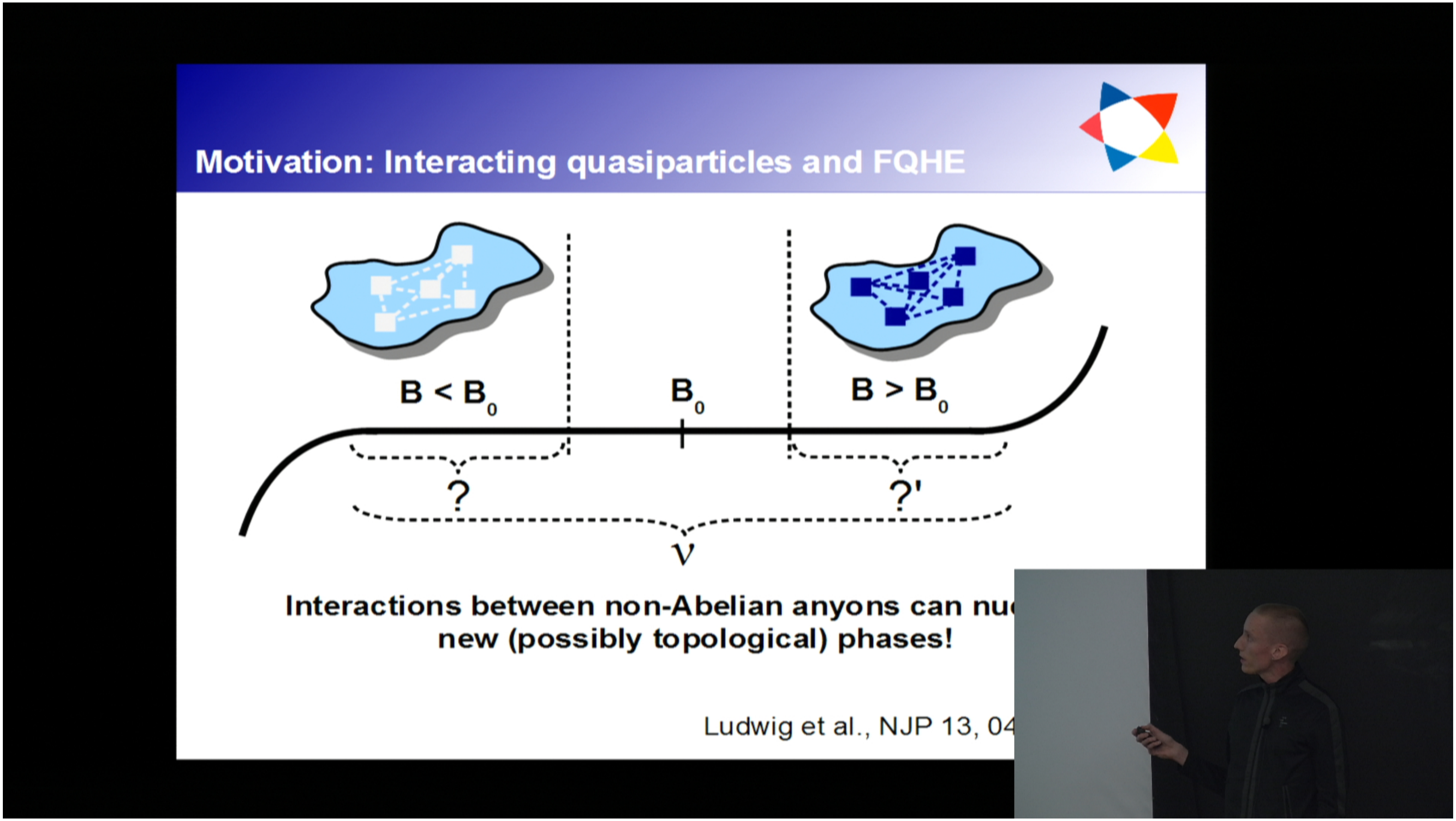
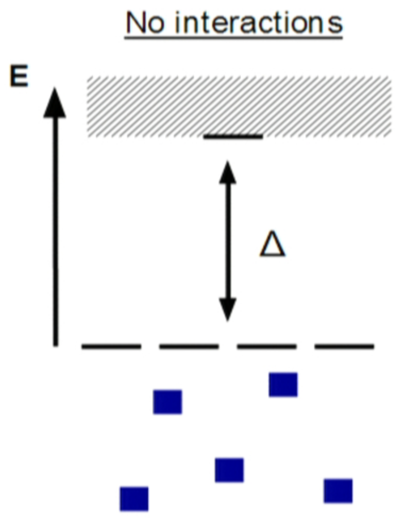
Abstract: We provide a microscopic understanding of the nucleation of topological quantum liquids that arise due to interactions between non-Abelian anyons. With the pairwise anyon interactions typically showing RKKY-type oscillations in sign, but decaying exponentially with distance, we show that the character of the nucleated phase is fully determined by anyon interactions beyond nearest neighbor exchange. We investigate this issue in the context of Kitaev's honeycomb lattice model. In the presence of vortex lattices, depending on microscopic parameters such as the vortex lattice spacing, we observe the nucleation of several distinct Abelian topological phases, that differ in their band structure and Chern number description. By employing an effective model of Majorana fermions, we show that these phases can be fully predicted from the vortex-vortex interactions. Corresponding microscopic results should hold for vortices forming an Abrikosov lattice in a p-wave superconductor or quasiholes forming a Wigner crystal in non-Abelian quantum Hall states.



Motivation: Interacting quasiparticles and FQHE
B < B0
B0
B > B0
?
?'
ν
Interactions between non-Abelian anyons can nu
new (possibly topological) phases!
Ludwig et al., NJP 13, 04

Motivation: Nucleation is destructive for TQC

- Topological quantum computing requires in general the manipulation of many (interacting) anyons in a finite system
- Nucleation can destroy any topological quantum computing scheme based on non-Abelian anyons even when the temperature is below the energy gap



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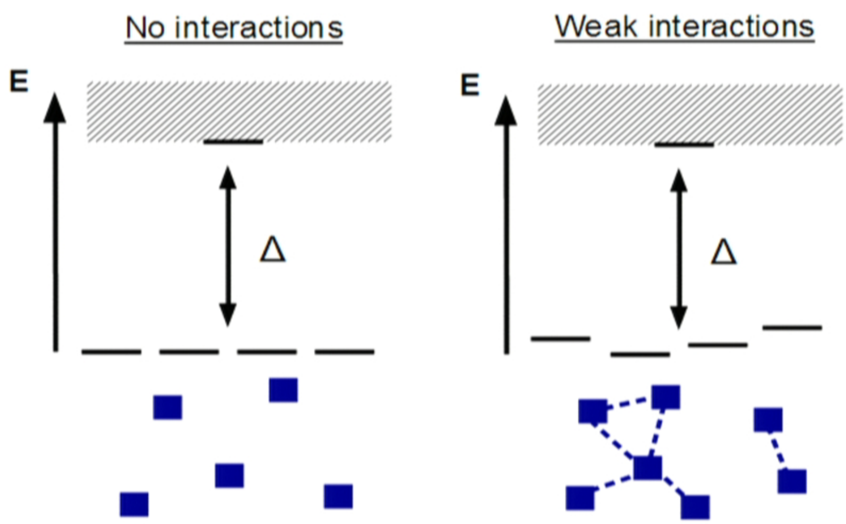

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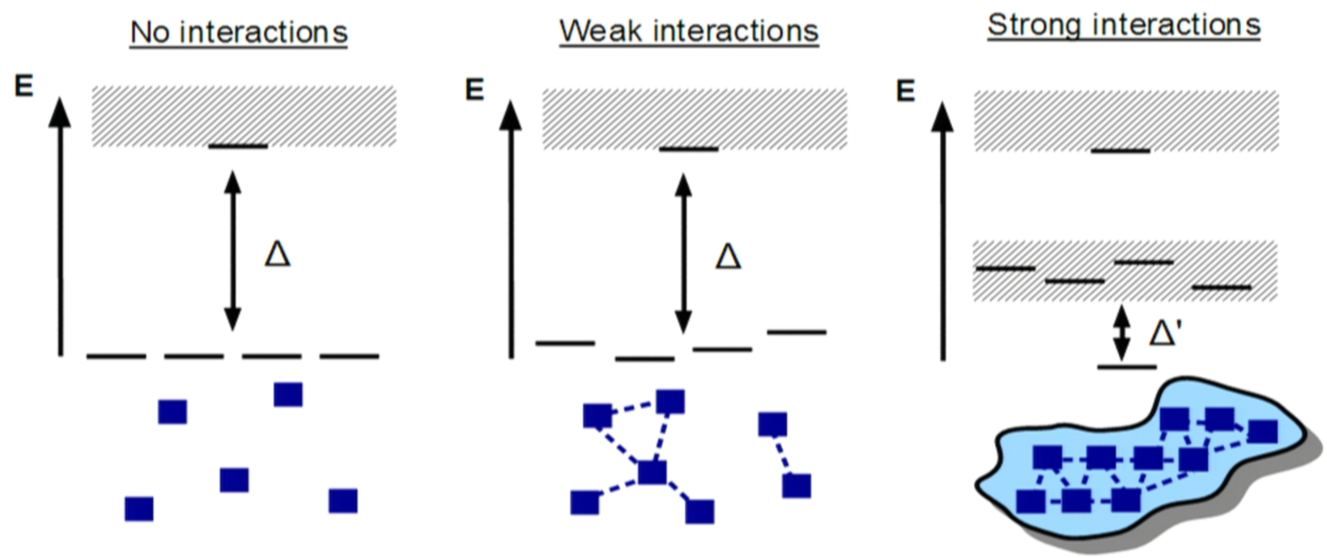
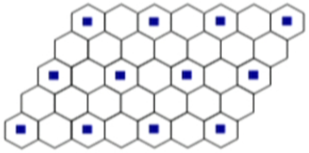


Outline

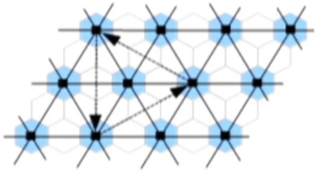
Laboratory: Kitaev's honeycomb lattice model

- New Abelian phases emerge in the presence of vortex lattices
- Band structure hybridization due to vortex-vortex interactions



Theory: Majorana model on the vortex lattice

- Include nearest and next to nearest neighbour hopping
- Relate the free parameters to the physical observables of the honeycomb model



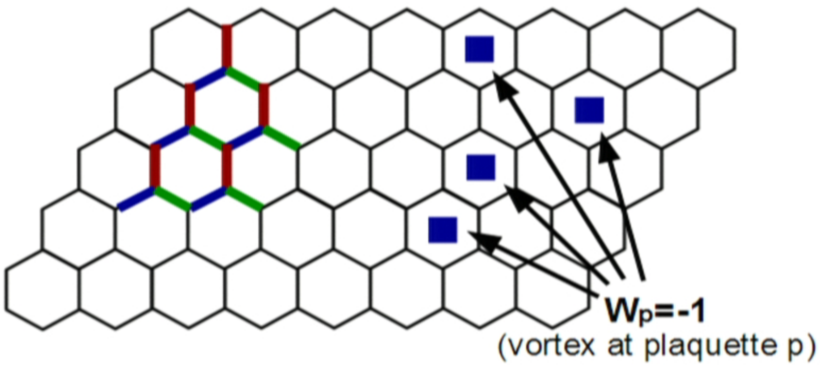
We show that the Majorana model fully predicts the emergent vortex lattice phases from the vortex-vortex interactions!

Laboratory: Kitaev's honeycomb lattice model

Spin ½-particles on the vertices of a honeycomb lattice interacting according to the Hamiltonian:

$$H(J_x, J_y, J_z, K, \{W_p\})$$

J_α: Nearest neighbour spin-spin interactions
K: TRS breaking three spin term
$\{W_p\}$: Local symmetry at every plaquette p



To solve the model:

(1) Fix vortex sector $\{W_p\}$

(2) Map the model into free Majorana fermions coupled to a Z_2 gauge field

(3) Diagonalize

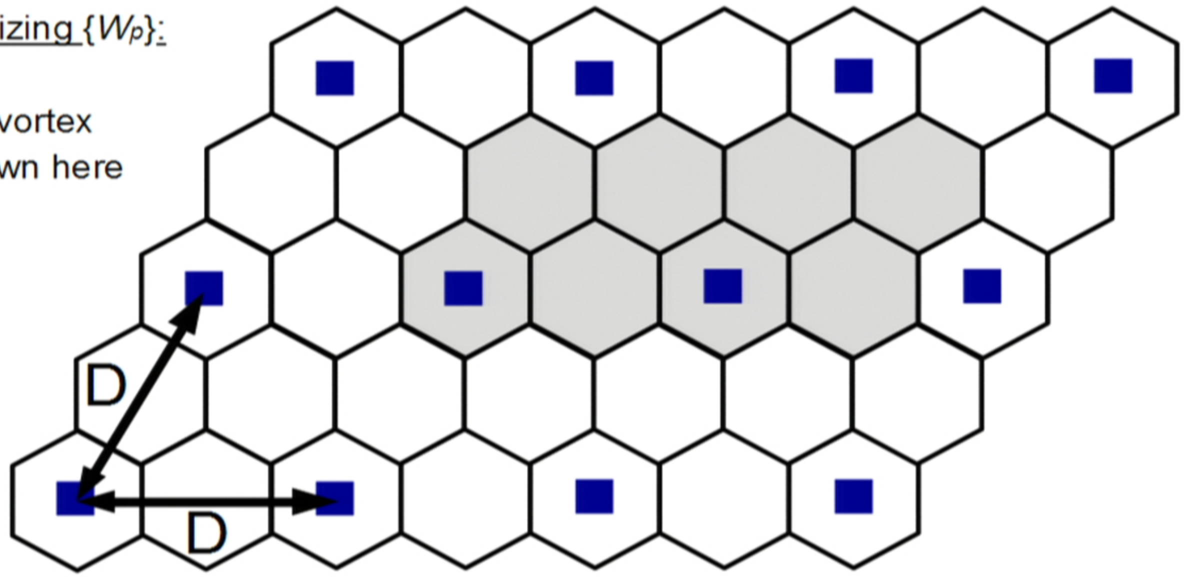
(4) Study how the spectrum and the Chern number depend on J_α, K and $\{W_p\}$

Kitaev, Ann. Phys. 321,2 (2006)

Laboratory: vortex lattices

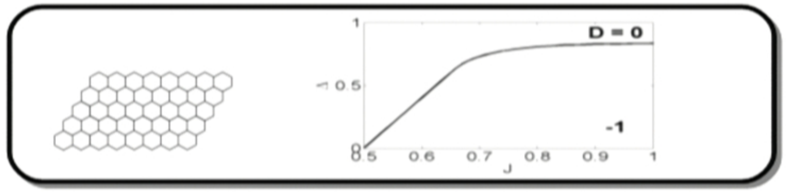
Parametrizing $\{W_p\}$:

D=1: full-vortex
D=2: shown here
....

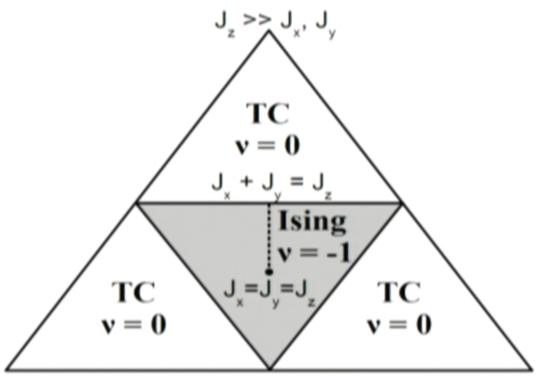


Vortex lattice of sparsity D $\leftrightarrow$ Bloch Hamiltonian of linear size $4D^2$

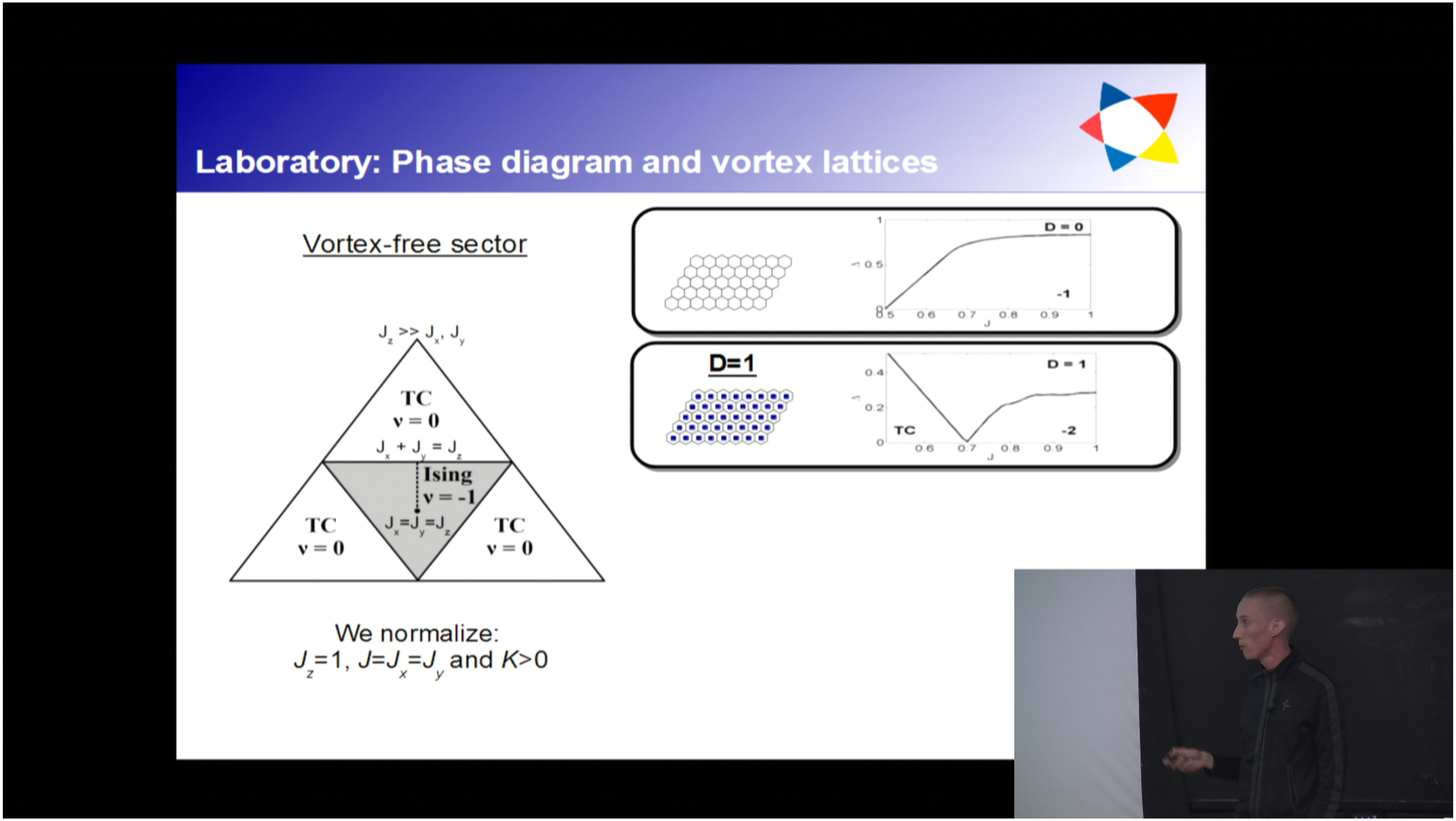
Laboratory: Phase diagram and vortex lattices



Vortex-free sector

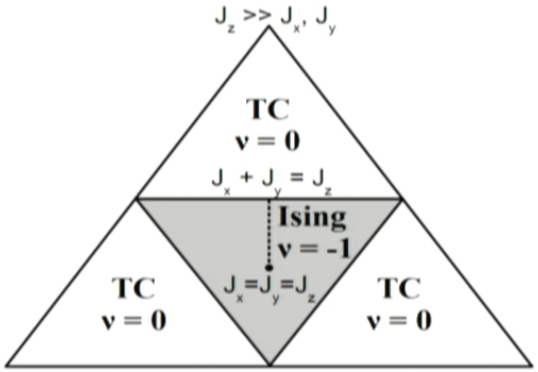


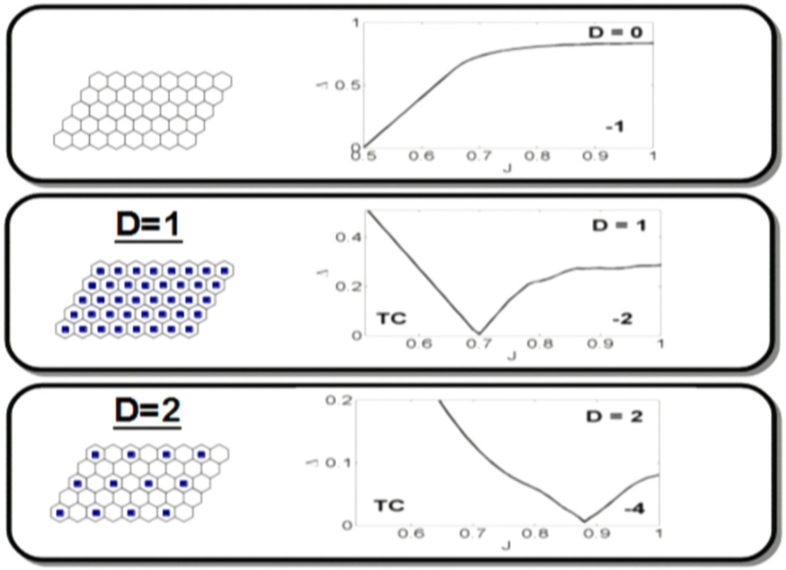
We normalize:
J_z=1, J=J_x=J_y and K>0


Laboratory: Phase diagram and vortex lattices
Vortex-free sector
J_z >> J_x, J_y
TC
ν = 0
J_x + J_y = J_z
Ising
ν = -1
J_x=J_y=J_z
TC
ν = 0
TC
ν = 0
We normalize:
J_z=1, J=J_x=J_y and K>0
D = 0
-1
D=1
D = 1
TC
-2

Laboratory: Phase diagram and vortex lattices

Vortex-free sector





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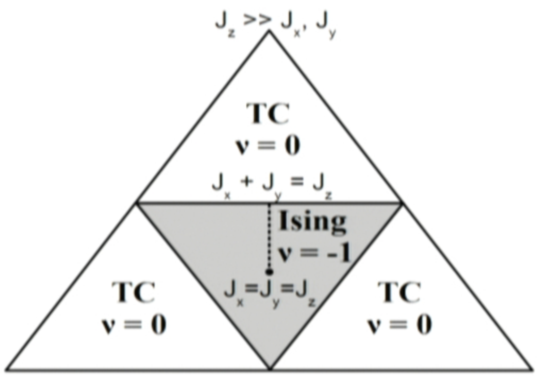
Laboratory: Phase diagram and vortex lattices

Vortex-free sector



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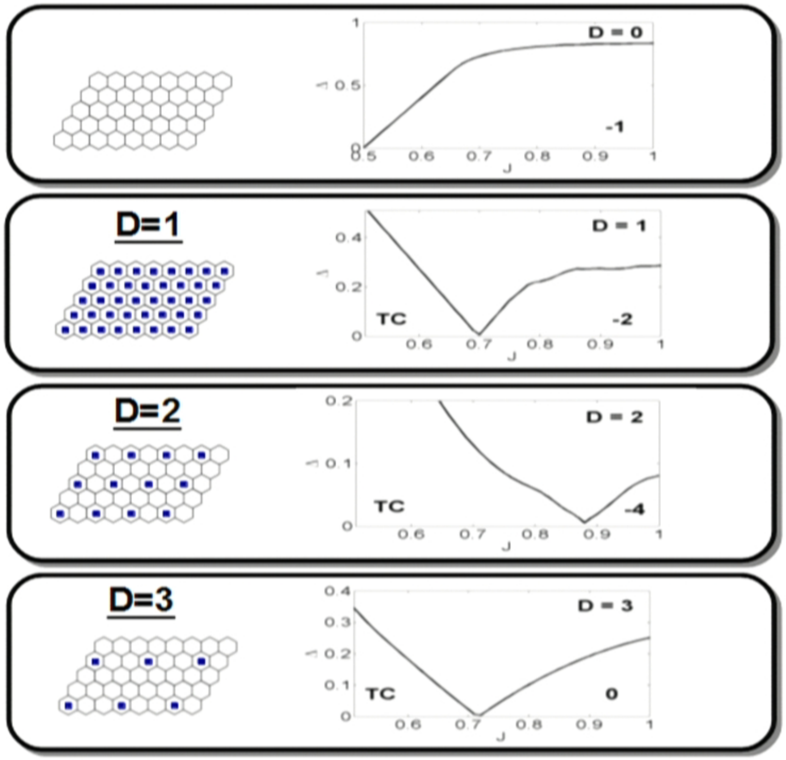
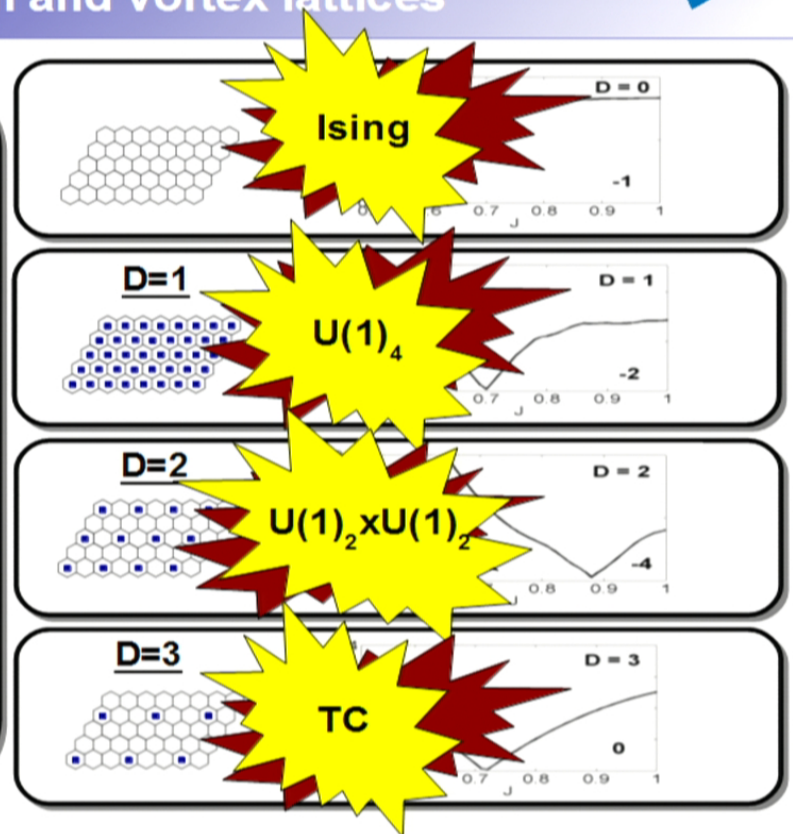
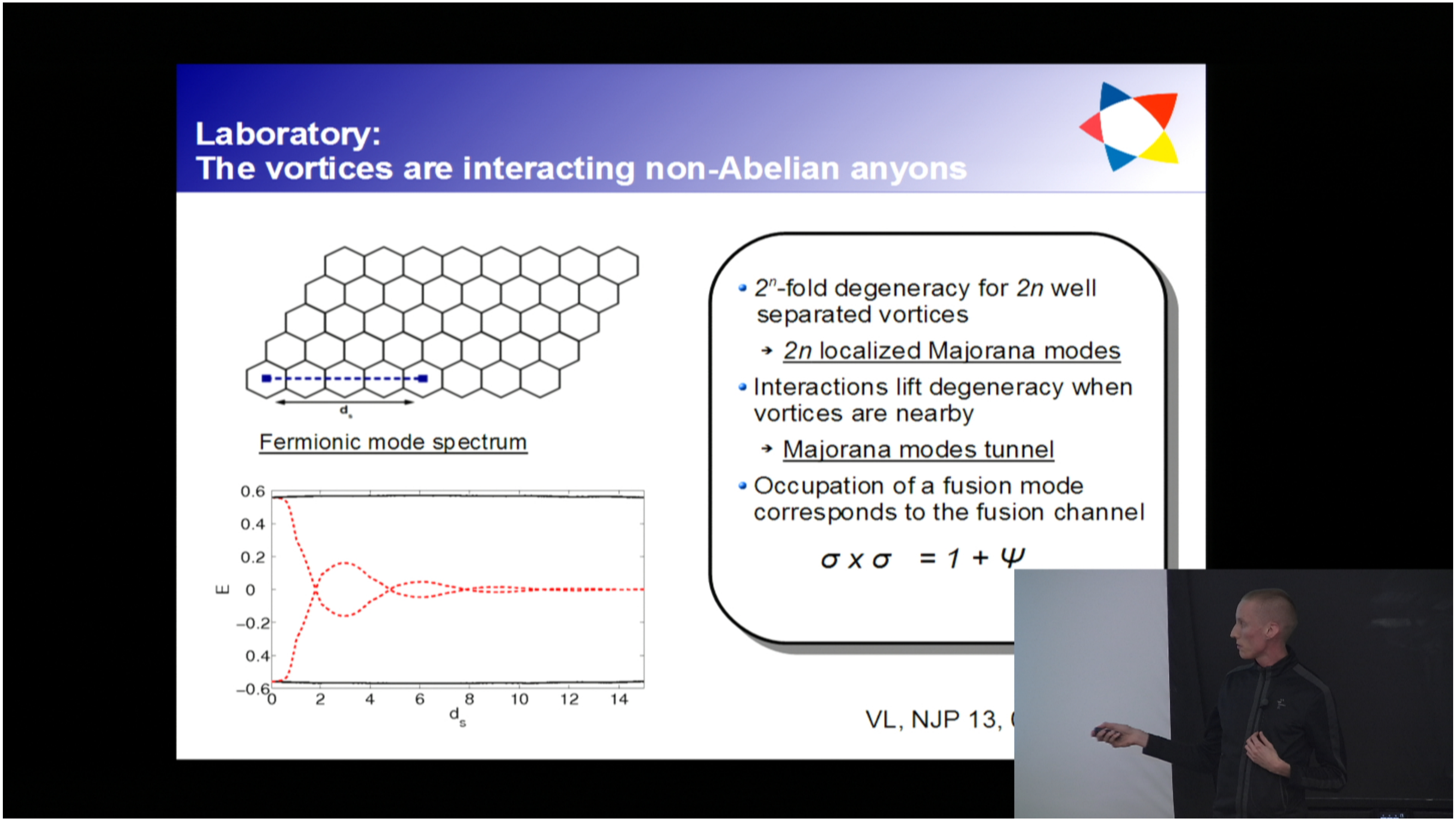


Laboratory: Phase diagram and vortex lattices

- Only Abelian phases emerge in the presence of triangular vortex lattices
- The theory of pinned interacting anyons by Ludwig et al.[1] predicts that the non-Abelian phase is lost and that either $U(1)_4$ or TC anyons should emerge.
- Are vortex-vortex interactions really responsible for the novel phases and if so, why does the $U(1)_2 x U(1)_2$ phase emerge?

[1] NJP 13, 045014 (2011)

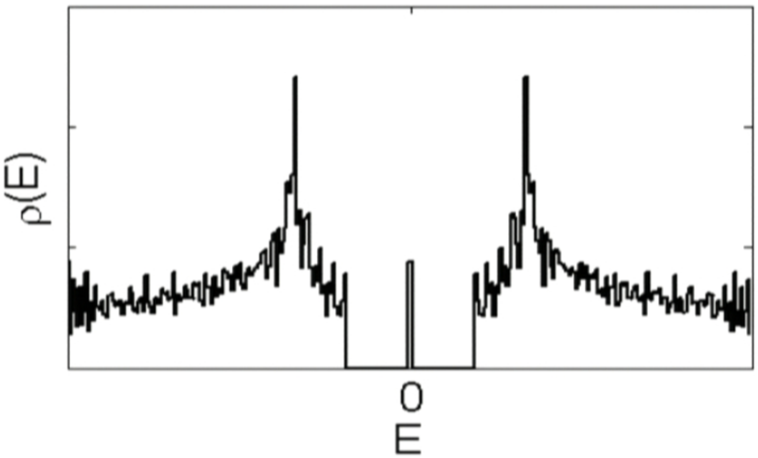


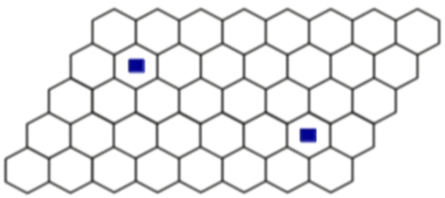


Laboratory:
The vortices are interacting non-Abelian anyons
d_s
Fermionic mode spectrum
0.6
0.4
0.2
E 0
−0.2
0.4
−0.6
0 2 4 6 8 10 12 14
d_s
2^n-fold degeneracy for 2n well separated vortices
→ 2n localized Majorana modes
Interactions lift degeneracy when vortices are nearby
→ Majorana modes tunnel
Occupation of a fusion mode corresponds to the fusion channel
σ x σ = 1 + Ψ
VL, NJP 13,

Laboratory:
Band hybridization due to the interactions

Evolution of the density of states as the vortex density is increased:





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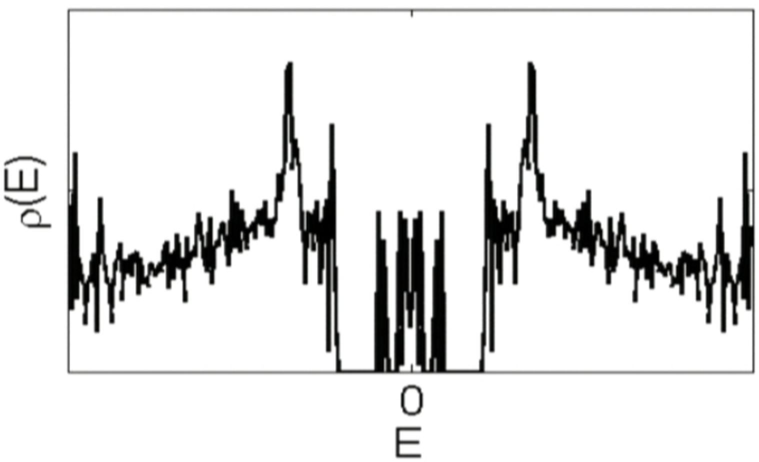

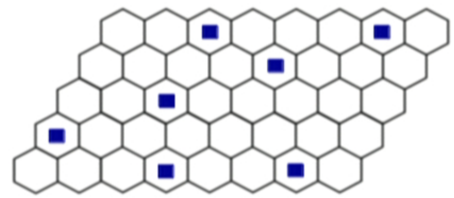

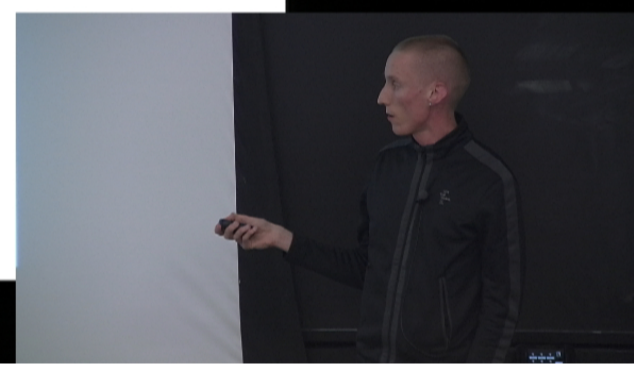

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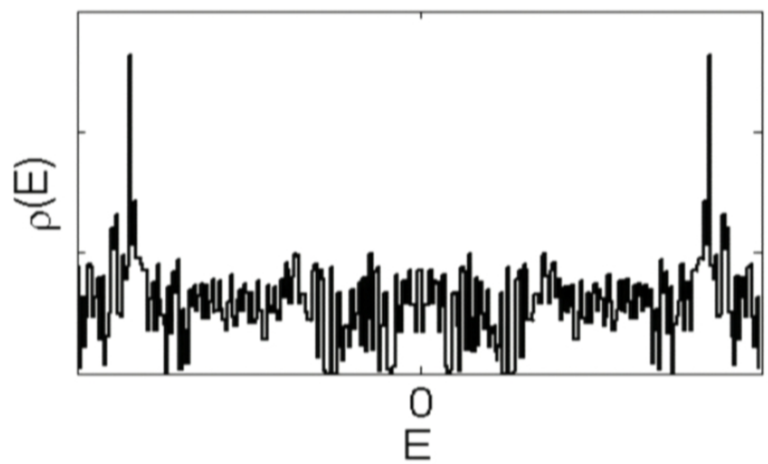

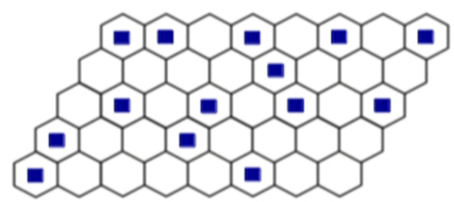

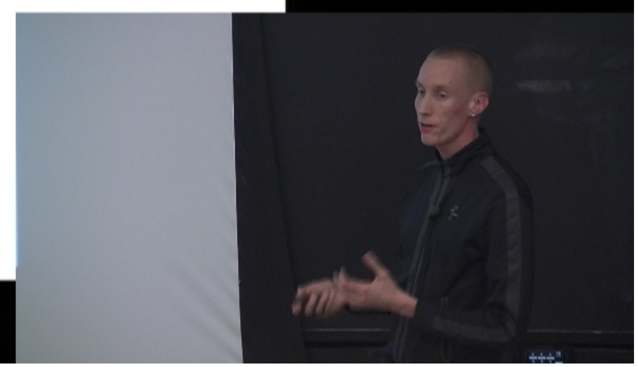

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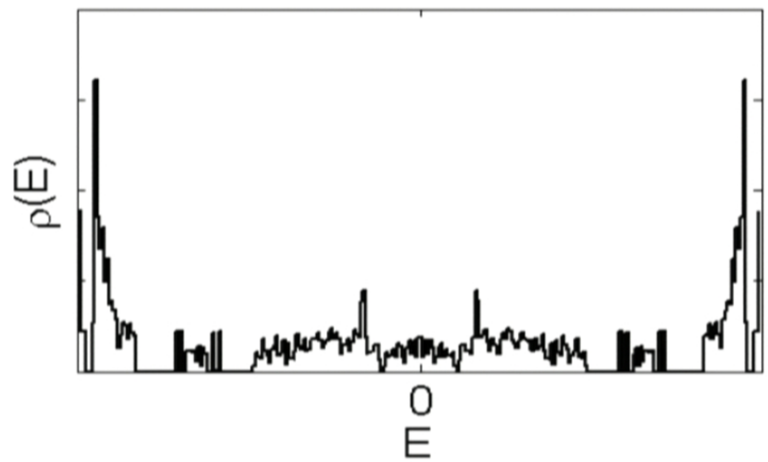

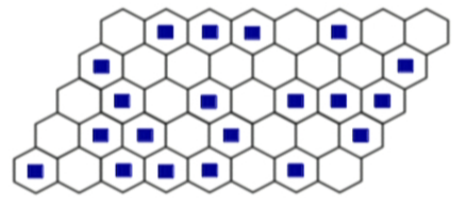

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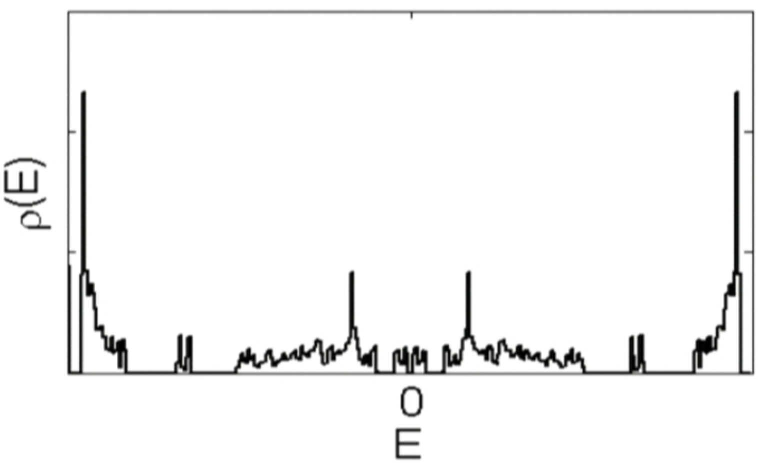

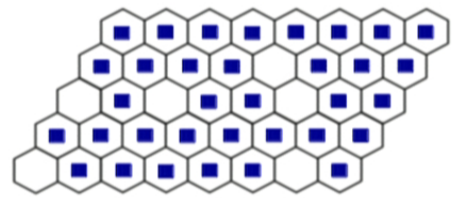

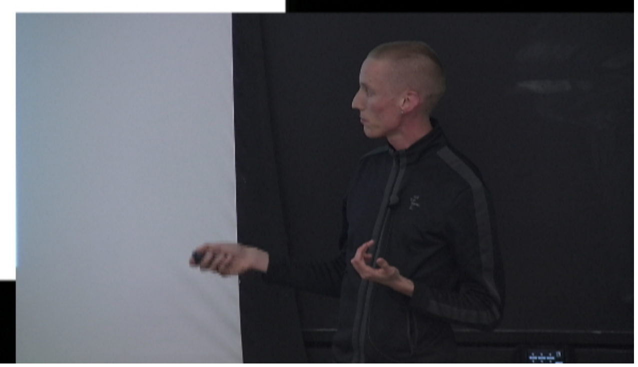

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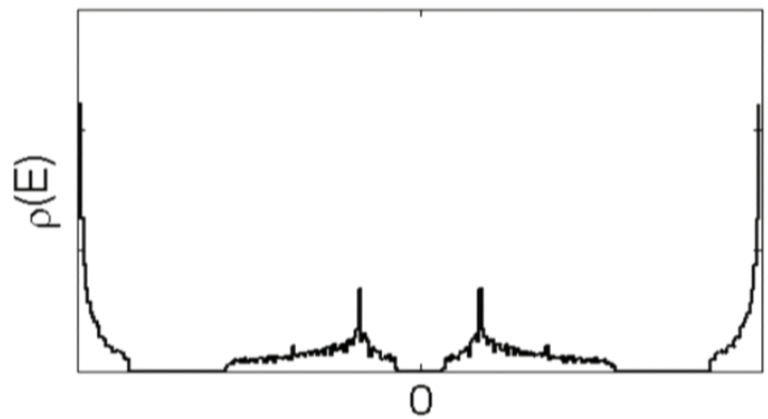
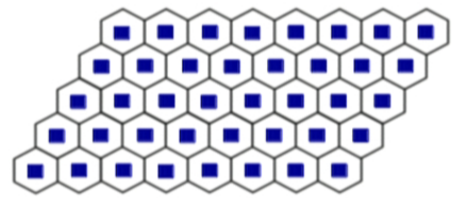
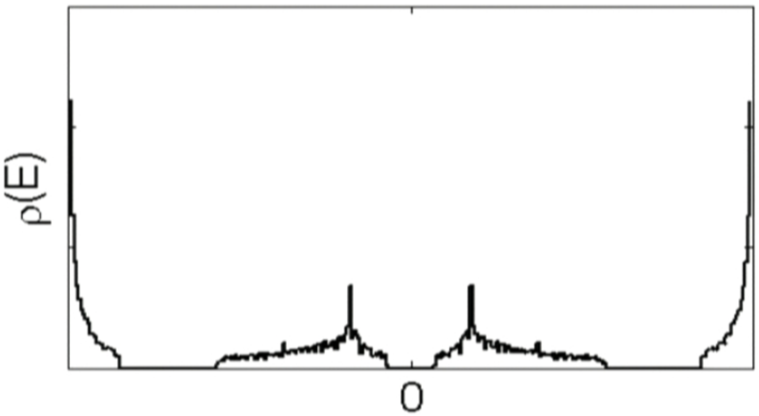


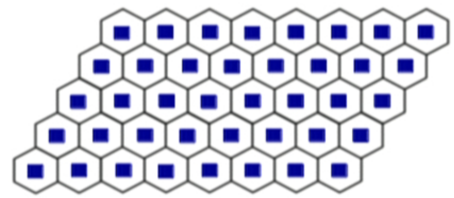


- Interactions hybridize new low-energy bands from the fusion mo
- General mechanism to all the vortex lattice phases

Laboratory: Band hybridization due to the interactions

Evolution of the density of states as the vortex density is increased:





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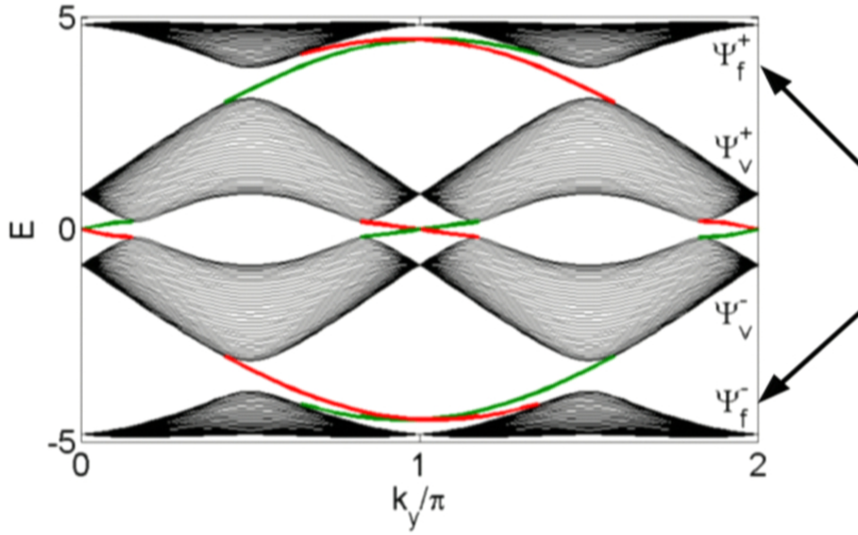
Laboratory:
Band hybridization due to the interactions

Full-vortex sector, $\nu = -2$



High-energy fermion bands

- Remnants of the free fermion bands in the vortex-free sector
- States have support across the whole honeycomb lattice

Laboratory:
Band hybridization due to the interactions

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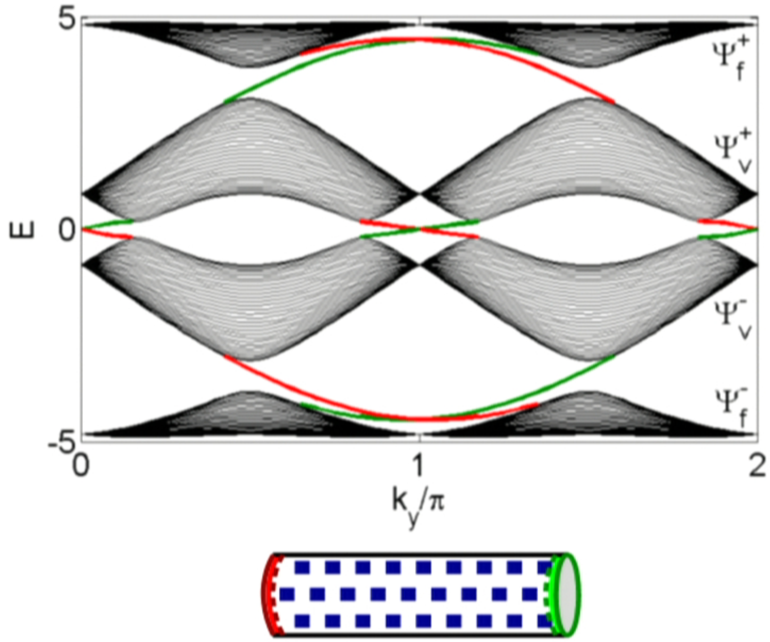


The Chern number in the vortex lattice phases

$$\nu = \nu_f + \nu_v$$

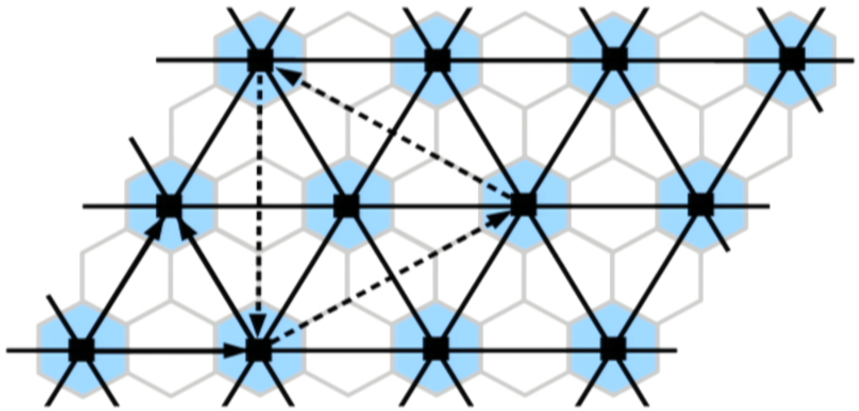
- ν_f = -1 always for K>0
- ν_v depends on the vortex lattice D

Can we find a model explaining how ν_v changes with D?

Theory: Majorana model on the vortex lattice

Hamiltonian with nearest t_1 (⟶) and next to nearest $t_{\sqrt{3}}$ (- - -►) hopping:

$$H_M = \sum_{l=1,\sqrt{3}} H_l, \qquad H_l = it_l \sum_{|i-j|=l} s^l_{ij}\gamma_i\gamma_j$$



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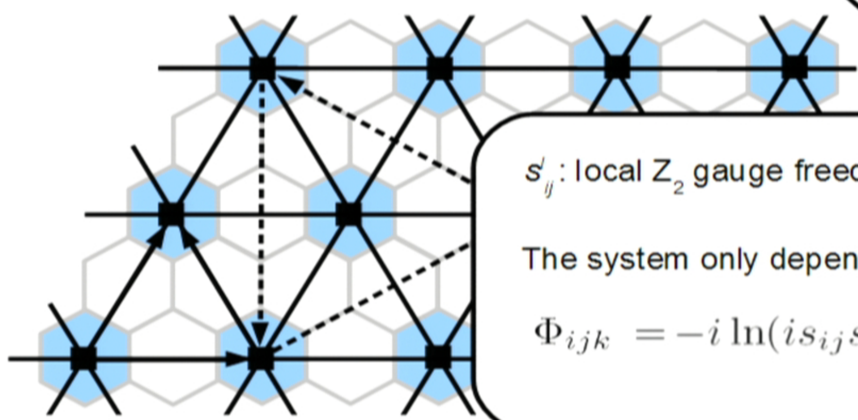

s^l_{ij}: local Z_2 gauge freedom on every link

The system only depends on the “flux”:

$$\Phi_{ijk} = -i\ln(is_{ij}s_{jk}s_{ki}) = \pm\frac{\pi}{2}$$

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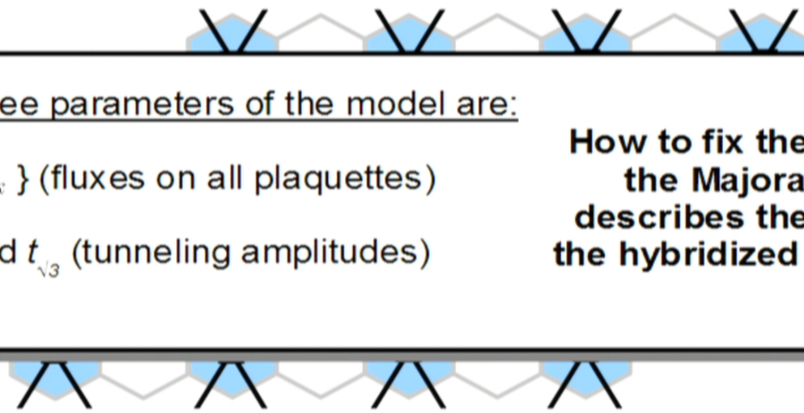


<u>The free parameters of the model are:</u>

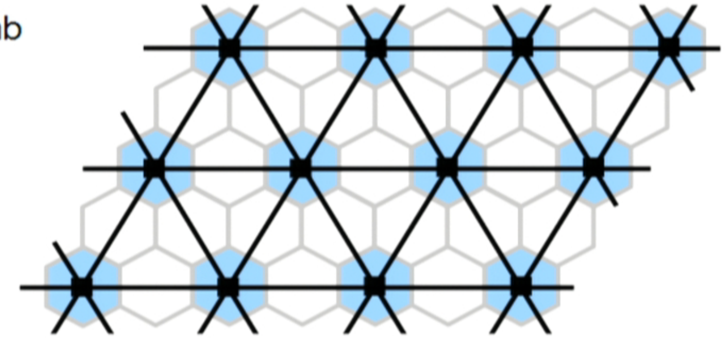
- { Φ_{ijk} } (fluxes on all plaquettes)
- t_1 and $t_{\sqrt{3}}$ (tunneling amplitudes)

How to fix these such that the Majorana model describes the behavior of the hybridized vortex band?

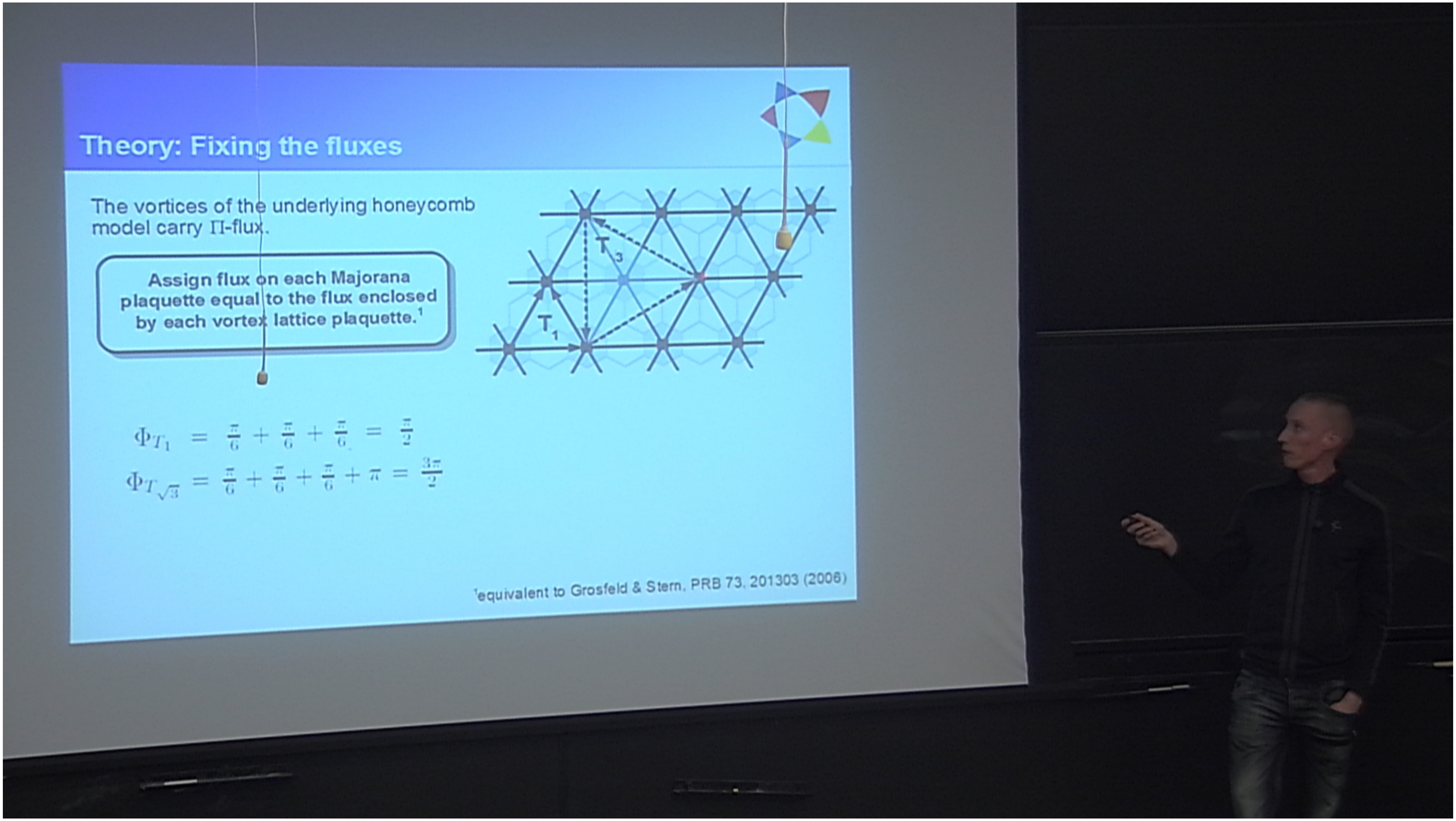
Theory: Fixing the fluxes

The vortices of the underlying honeycomb model carry Π-flux.

Assign flux on each Majorana plaquette equal to the flux enclosed by each vortex lattice plaquette.[1]



[1]equivalent to Grosfeld & Stern, PRB 73, 201303 (2006)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T_√3
T_1
$\Phi_{T_1} = \frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{2}$
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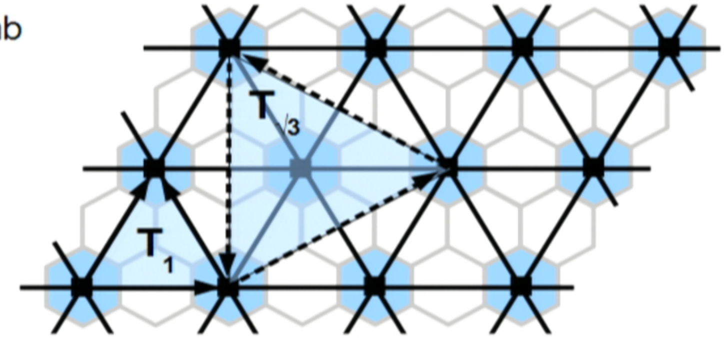

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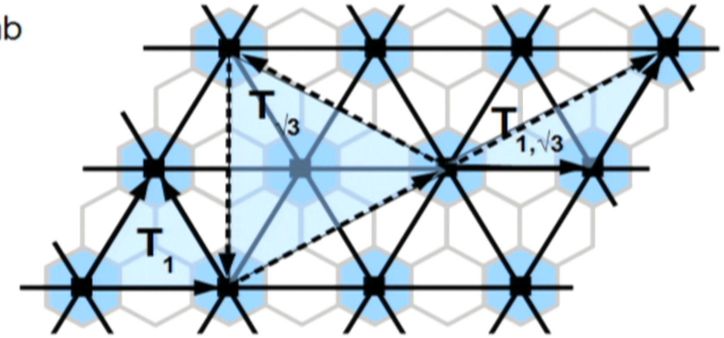

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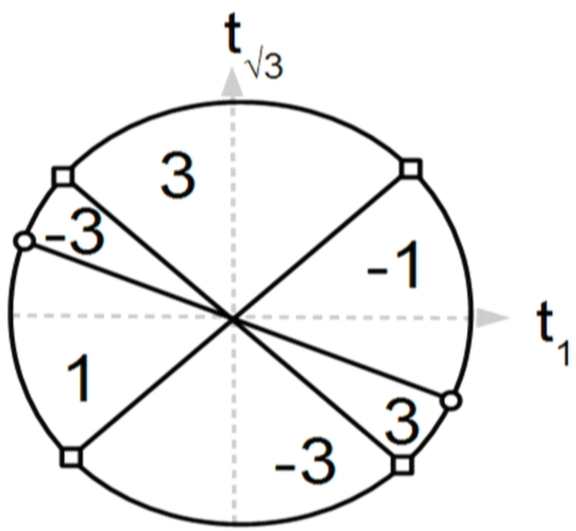
$$\Phi_{T_{1,\sqrt{3}}} = \frac{\pi}{12} + \frac{\pi}{12} + \frac{\pi}{3} = \frac{\pi}{2}$$

<u>We fix the Majorana model flux to:</u>

$$(\Phi_{T_1}, \Phi_{T_{\sqrt{3}}}, \Phi_{1,T_{\sqrt{3}}}) = (\frac{\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2})$$

[1] equivalent to Grosfeld & Stern, PRB 73, 201303 (2006)

Theory: Phase diagram of the Majorana model



Phase transitions:

- □ $|t_1| = |t_{\sqrt{3}}|$
- ○ $t_1 = -2t_{\sqrt{3}}$

Asymptotic behavior:

When t_l dominates, one always finds:

$$\nu_M = -\text{sign}(\Phi_{T_l})\text{sign}(t_l)l^2$$

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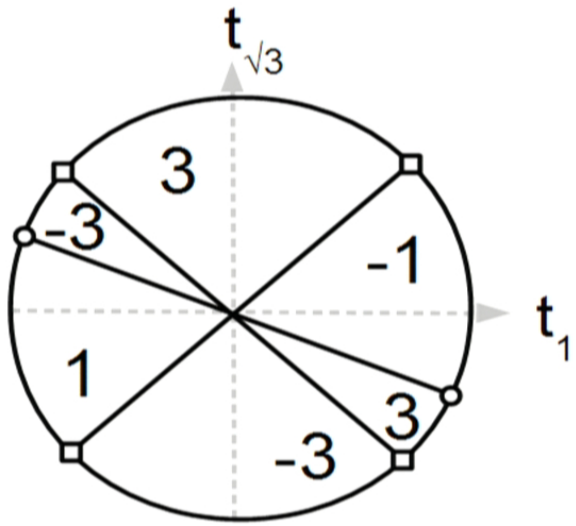

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Since the Majorana model describes only the vortex band, in the background of the ν_f = -1 fermion band our model can predict phases with:

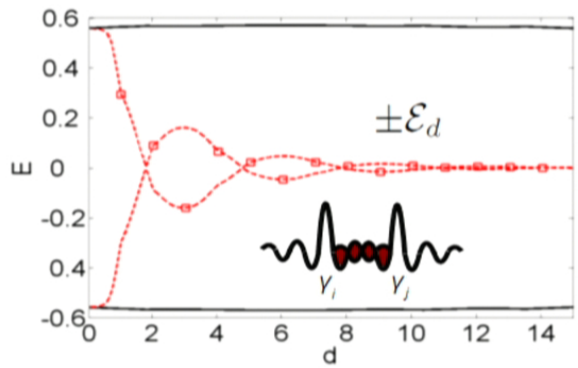
$$\nu = \nu_M - 1 = 0, \pm 2, -4$$

Promising!

Theory: Fixing the tunneling amplitudes

- Energy splitting $\mathcal{E}_d$ is proportional to the tunneling amplitude[1]
- Sign (favoured fusion channel) defined with respect to the fermionic parity P_d of the vortex sector

  $\mathcal{E}_d > 0 \rightarrow$ vacuum channel favoured
  $\mathcal{E}_d < 0 \rightarrow$ fermion channel favoured



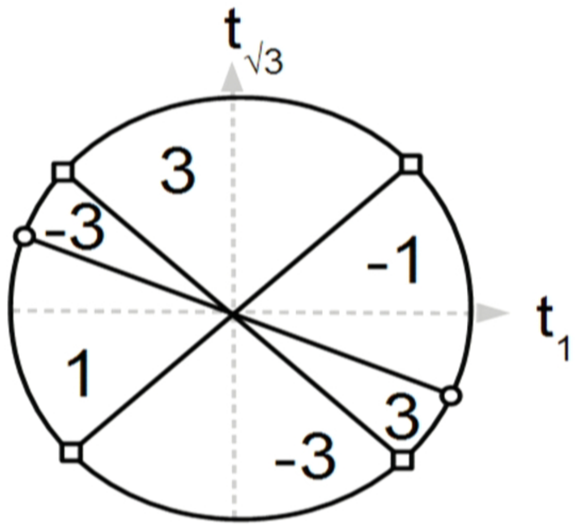
We assume the tunneling is bi-partite and use the ansatz:

$$t_l(K) = (-1)^{P_{lD}(K)} |\mathcal{E}_{lD}(K)|$$

Majorana model requires interactions to be isotropic → restrict to J=1 and vary K

[1] Cheng et al. PRL 103, 107001 (2009)

Theory: Phase diagram of the Majorana model



Phase transitions:

□ $|t_1| = |t_{\sqrt{3}}|$

○ $t_1 = -2t_{\sqrt{3}}$

Asymptotic behavior:

When t_l dominates, one always finds:

$$\nu_M = -\text{sign}(\Phi_{T_l})\text{sign}(t_l)l^2$$

Since the Majorana model describes only the vortex band, in the background of the ν_f = -1 fermion band our model can predict phases with:

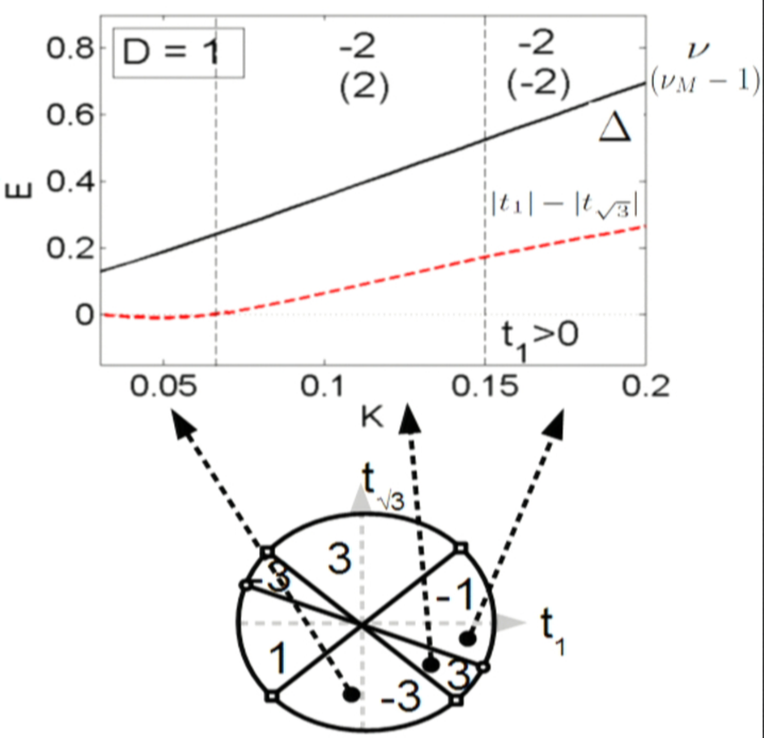
$$\nu = \nu_M - 1 = 0, \pm 2, -4$$

Promising!

Theory: Results

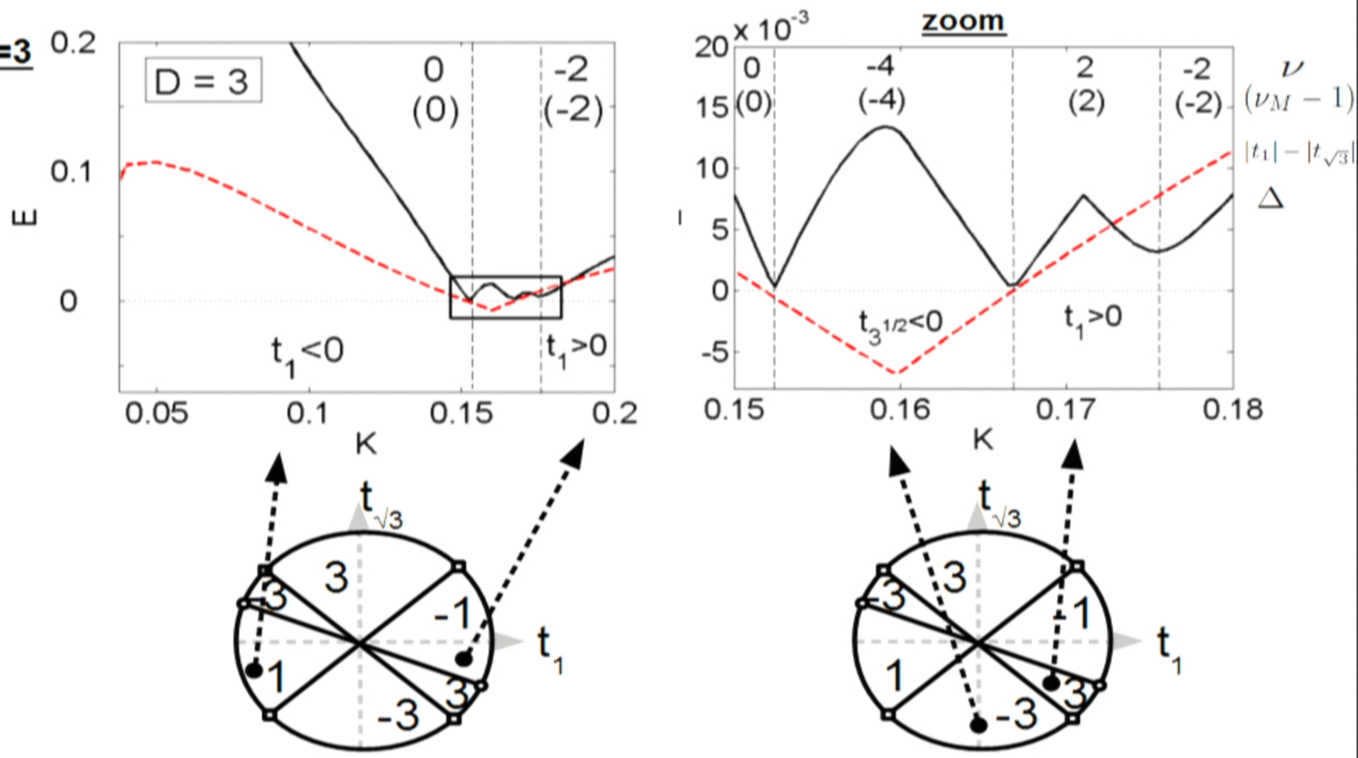
D=1 (full-vortex lattice)

- Agreement in the K > 0.15 region
- Disagreement for K < 0.15 due to our bi-partite ansatz overestimating $t_{\sqrt{3}}$
- In this region the coherence length $\zeta \sim K^{-1}$ is larger than D, which can lead to collective effects
- The prediction should be more accurate for larger K and larger D?



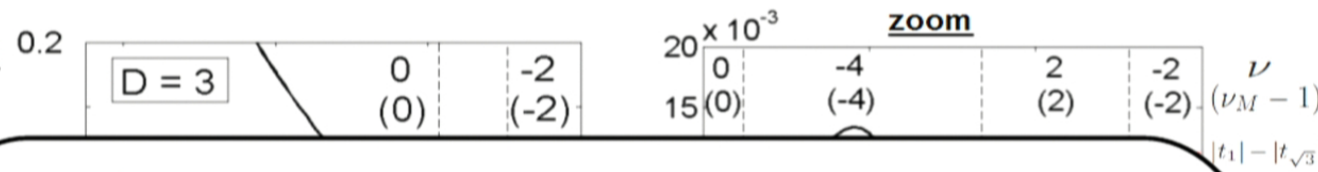
Theory: Results

D=3



Theory: Results

D=3



Perfect agreement!!!

- The Majorana model predicts correctly the emerging phases when the vortex separation in the vortex lattice is larger than the coherence length (the energy splitting captures directly the tunneling amplitudes)
- We have verified this for $D \leq 6$
- The existence of the weak, but robust $\nu = 2$ phase in the honeycomb model strong evidence for the correct flux sector in the Majorana model

Conclusions

- **First demonstration of the topological liquid nucleation in the context of a microscopic model**
- **The Majorana model provides full description directly from the interactions without fine-tuning or fitting parameters**
- **Microscopics important – oscillations can cause longer range interactions to determine the system's behavior**
- **Since the oscillations are ubiquitous, longer range interactions are likely to be relevant also in p-wave SCs and FQHE liquids**

Future work:

- Disordered vortex lattices and stability of the nucleated phases
- Can one protect the underlying non-Abelian phase by potentials?

References:

- This work appearing soon in arXiv near you
- The full-vortex lattice phase: **VL & JKP, PRB 81, 245132 (2010)**
- Interacting Ising anyons: **VL, NJP 13, 075009 (2011)**