

Title: Quantum Computational Matter

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Abstract: Low-temperature phases of strongly-interacting quantum many-body systems can exhibit a range of exotic quantum phenomena, from superconductivity to fractionalized particles. One exciting prospect is that the ground or low-temperature thermal state of an engineered quantum system can function as a quantum computer. The output of the computation can be viewed as a response, or 'susceptibility', to an applied input (say in the form of a magnetic field). For this idea to be sensible, the usefulness of a ground or low-temperature thermal state for quantum computation cannot be critically dependent on the details of the system's Hamiltonian; if so, engineering such systems would be difficult or even impossible. A much more powerful result would be the existence of a robust ordered phase which is characterised by its ability to perform quantum computation.

I'll discuss some recent results on the existence of such a quantum computational phase of matter. I'll outline some positive results on a phase of a toy model that allows for quantum computation, including a recent result that provides sufficient conditions for fault-tolerance. I'll also introduce a more realistic model of antiferromagnetic spins, and demonstrate the existence of a quantum computational phase in a two-dimensional system. Together, these results reveal that the characterisation of quantum computational matter has a rich and complex structure, with connections to renormalisation and recently-proposed concepts of 'symmetry-protected topological order'.

Quantum Computational Matter

Stephen Bartlett
Centre for Engineered Quantum Systems, School of Physics



eQus





eQuS

THE UNIVERSITY OF SYDNEY
EQUUS @ SYDNEY

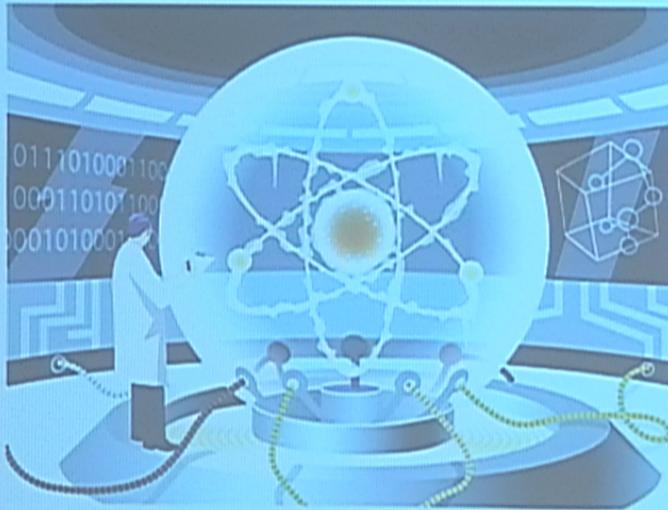
Quantum Computational Matter

Stephen Bartlett, Andrew Doherty

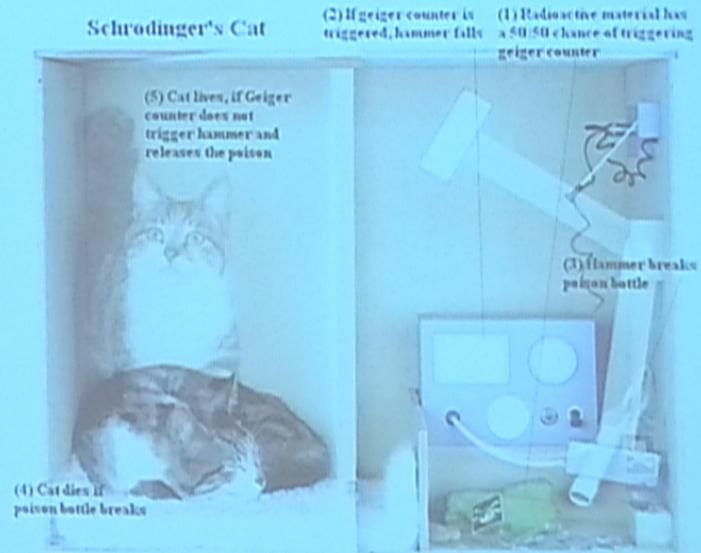
Students: Dominic Else (Hons), Andrew Darmawan (PhD)

Collaborators: Gavin Brennen, Akimasa Miyake, Joe Renes

What is a quantum computer?

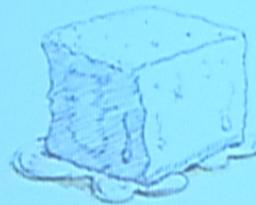
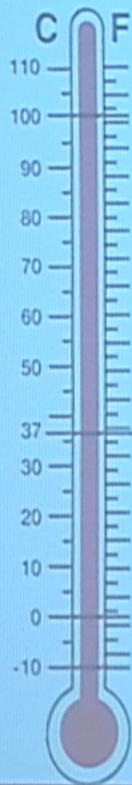


Schrodinger's Cat



Exotic quantum effects at equilibrium

Strongly-interacting quantum systems can lead to exotic quantum phenomena at equilibrium



SOLID



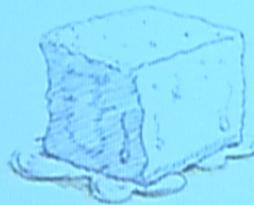
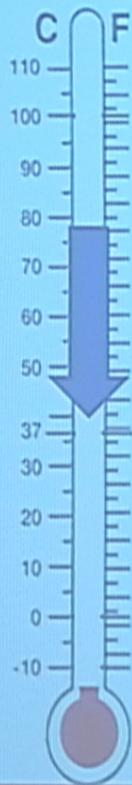
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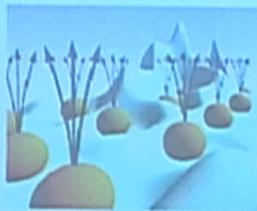
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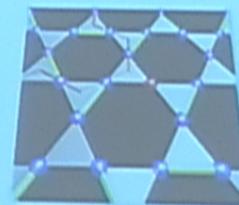
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Superconductivity



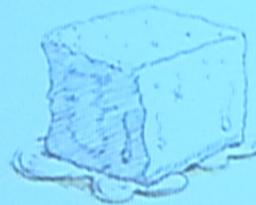
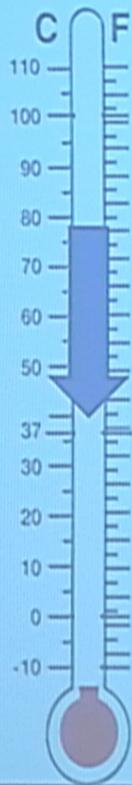
Fractionalised
Particles / Anyons



Quantum
Magnetism

Exotic quantum effects at equilibrium

Strongly-interacting quantum systems can lead to exotic quantum phenomena at equilibrium



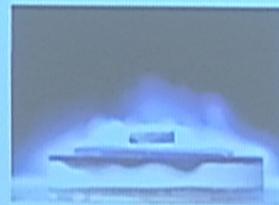
SOLID



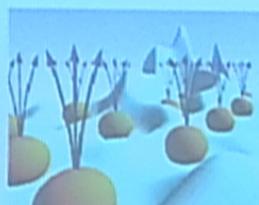
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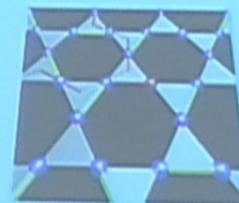
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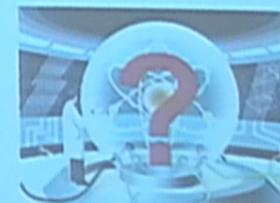
Superconductivity



Fractionalised
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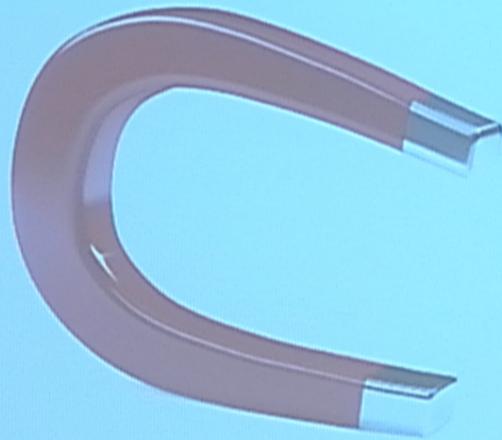
Quantum
Magnetism



Quantum
Computers?

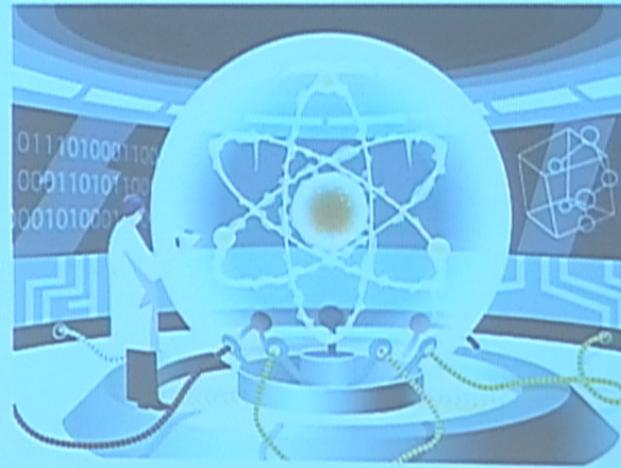
What would this mean?

Can characterise states of matter by their response to some **perturbation**



Magnetism:

- › Perturbation: applied magnetic field
- › Response: magnetic susceptibility

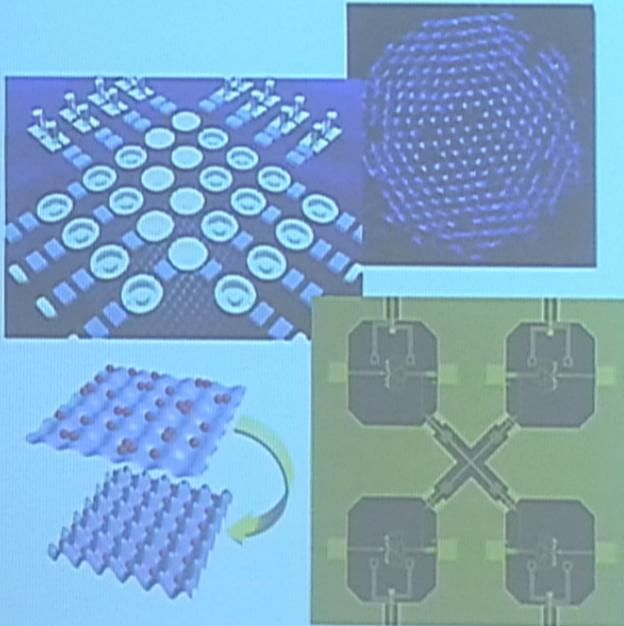


(Quantum) Computer:

- › Perturbation: input / question
- › Response: output of computation

Engineering 'synthetic quantum systems'

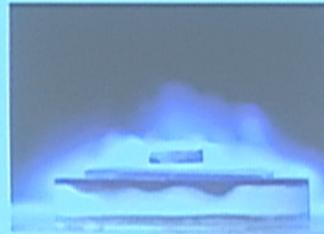
What is quantum computational matter made of?



- › FQH systems
- › Topological insulators
- › Coupled Josephson arrays
- › Ultracold neutral atoms or polar molecules
- › Frustrated magnets
- › Coupled optical cavities
- › Trapped ions in a crystal
- › Coupled electrons in semiconductor quantum dots
- › Interesting systems in 1- and 2-D

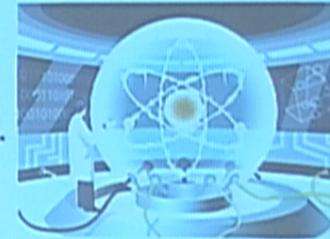
A robust quantum computational phase of matter?

- › What if the Hamiltonian was only “close” to the desired one?
- › Is the system **fragile** or **robust** to local perturbations, or finite temperature?
- › Is there a quantum computational phase of matter?



Superconductivity

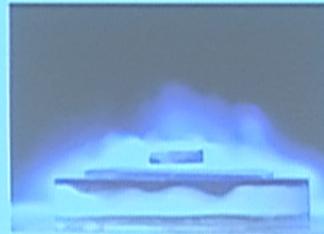
vs.



Quantum Computers

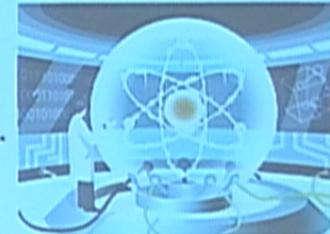
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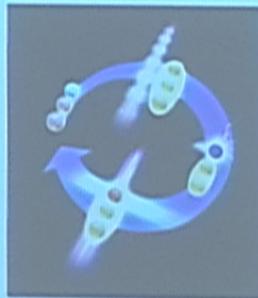
Superconductivity

vs.



Quantum Computers

What about quantum error correction?

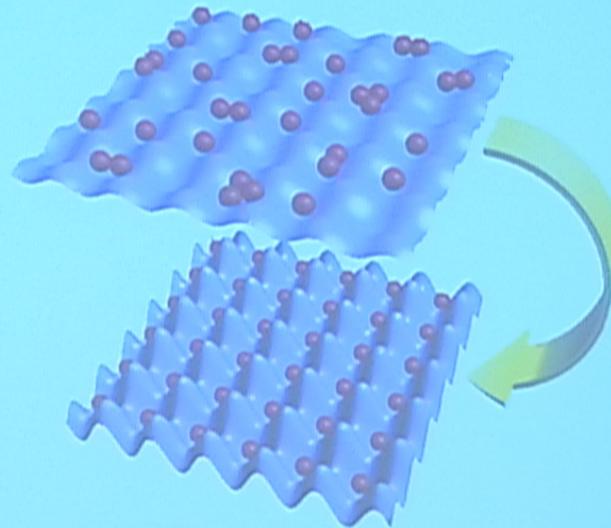


Error correction typically requires microscopic syndrome measurement and correction

Hamiltonian perturbations appear very different from the usual noise models: independent, Markovian, different notion of locality

- › Low temperature behavior
 - governed by ground-state properties
 - restrict to $T=0$ for simplicity

- 1. A toy model in 1D
- 2. Antiferromagnets in 1D
- 3. Antiferromagnets in 2D



A toy model for quantum computational matter

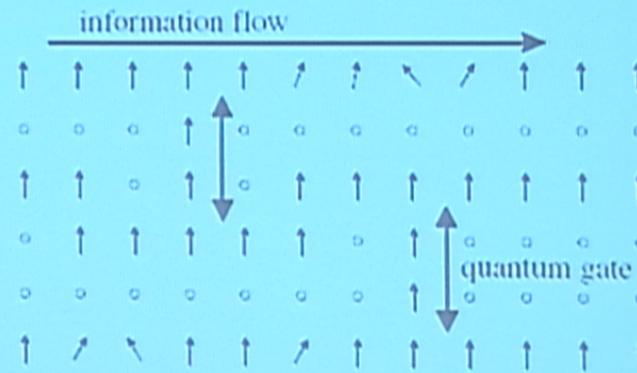
1

work with Dominic Else and Andrew Doherty



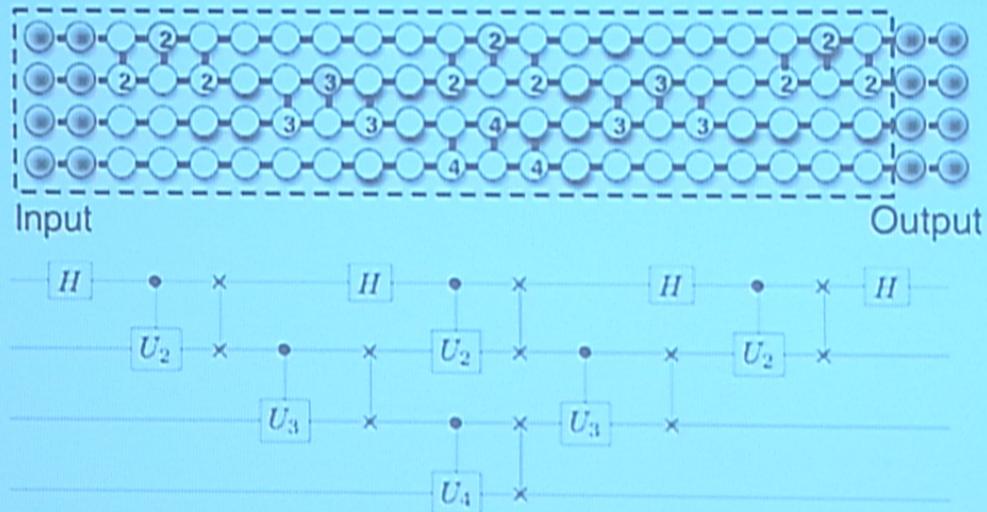
Cluster State Quantum Computing

- › Quantum computing can proceed through *measurements* rather than unitary evolution
- › Uses a resource state such as the *cluster state*: a universal circuit board
- › Resource states can be:
 - constructed with unitary gates
 - the ground state of a coupled quantum many-body system



Raussendorf and Briegel, PRL (2001)

Ground state quantum computing from the cluster state



Adiabatic quantum transistors

Bacon and Flammia, PRL (2010), PRA (2011), unpublished

- › Ground state of a designer many-body Hamiltonian
- › Apply a particular sequence of external magnetic fields
- › Adiabatically maps the state of the input to state of the output

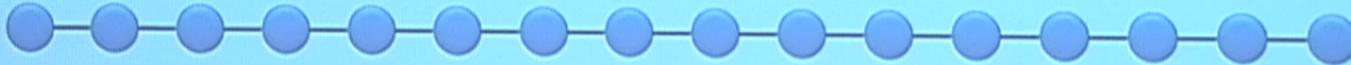
Qubit wires – 1D cluster model

Hamiltonian - gapped:

$$H = -J \sum_i Z_{i-1} X_i Z_{i+1}$$

"Frustration free" (all terms commute):

$$Z_{i-1} X_i Z_{i+1} |gs\rangle = |gs\rangle$$



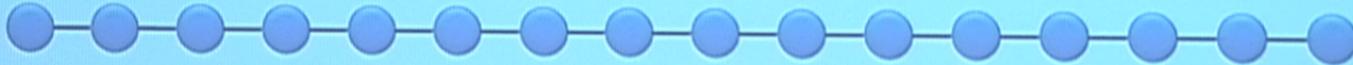
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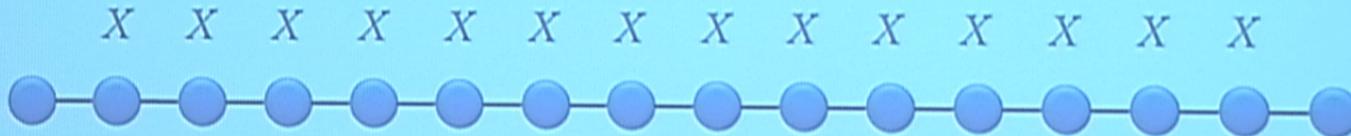
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Measure / apply local field



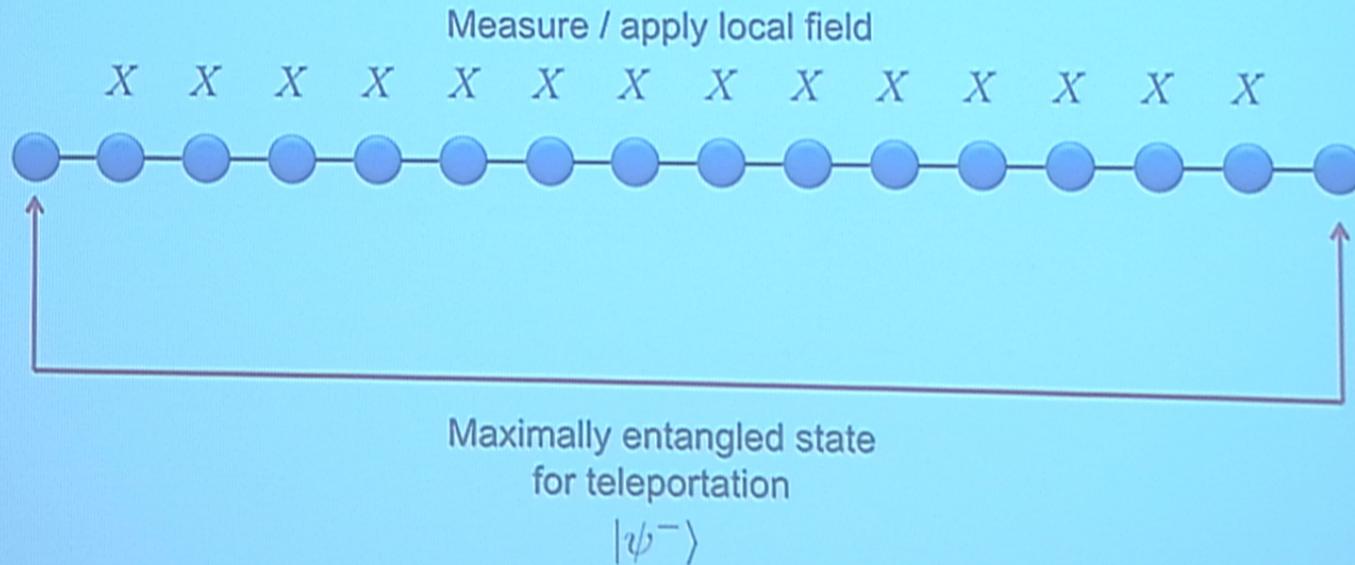
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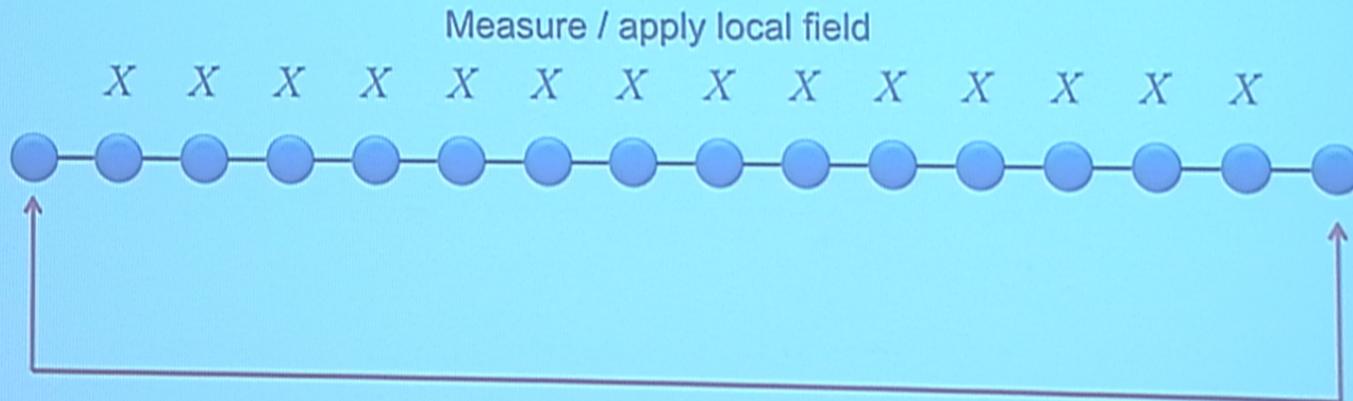
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Maximally entangled state
for teleportation

$$|\psi^-\rangle$$

Q: What are the
essential properties
of a qubit wire?

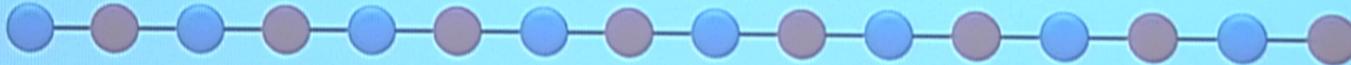
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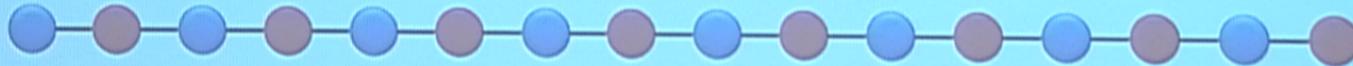
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Hamiltonian possesses
a symmetry: $Z_2 \times Z_2$

(0, 0)

i.e., 2 commuting
constants of motion

(0, 1)

(1, 0)

Four elements:

(1, 1)

Qubit wires – 1D cluster model

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constants of motion

Four elements:

- | | | |
|--------|---|------------|
| (0, 0) | ← | Do nothing |
| (0, 1) | | |
| (1, 0) | | |
| (1, 1) | | |

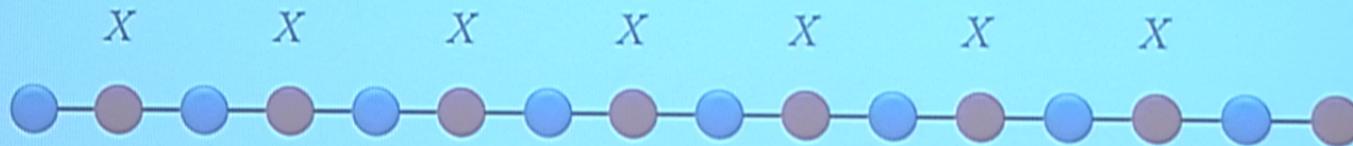
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Flip red spins

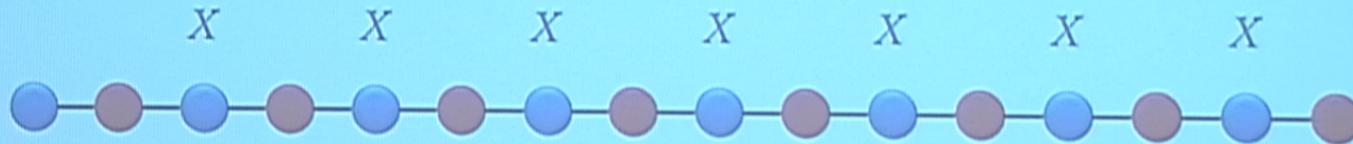
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(0, 0)

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Flip blue spins

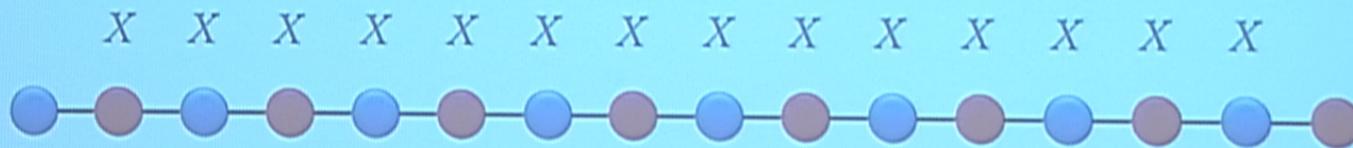
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i.e., 2 commuting
constants of motion

(0, 1)

(1, 0)

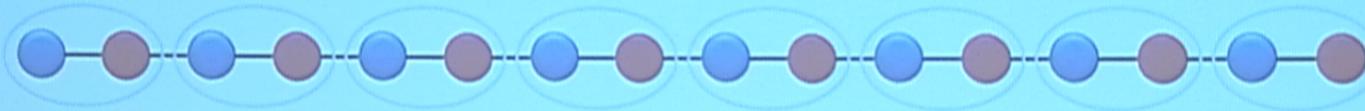
Four elements:

(1, 1)



Flip red and blue spins

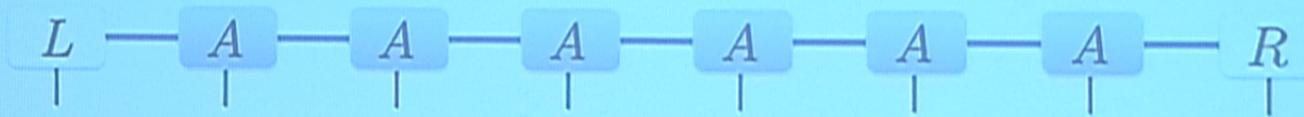
Ground state as a tensor network state



Ground state as a tensor network state



tensor network state (matrix product state)



$$\begin{array}{c}
 m \text{ --- } A_{mn}^{(i)} \text{ --- } n \\
 | \\
 i
 \end{array}$$

3 leg tensor

$$i = 1 \dots 4$$

index for basis of spin pairs

$$m, n = 1, 2$$

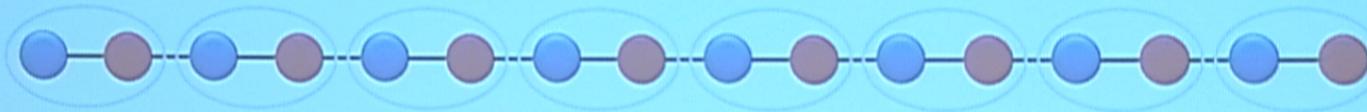
'virtual' index - contracted

Efficient representations of ground states of 1D gapped systems
 Natural language for ground-state quantum computation

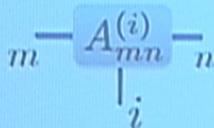
Gross, Eisert, Schuch, Perez-Garcia, PRA (2007)



Ground state as a tensor network state



tensor network state (matrix product state)



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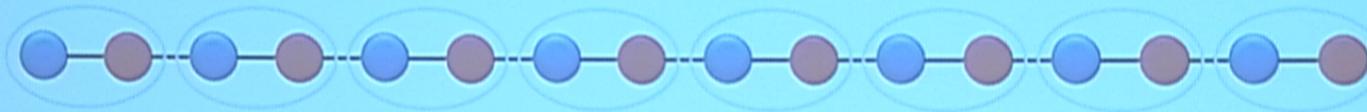
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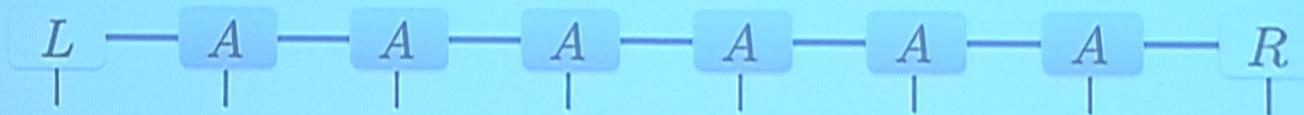
Goal:

Characterise properties of tensors, in terms of their symmetry, that make a good qubit wire

Ground state as a tensor network state



tensor network state (matrix product state)

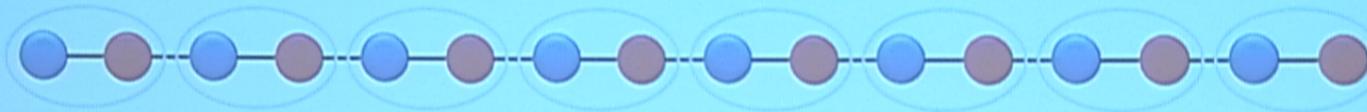


Cluster model possesses a symmetry: $Z_2 \times Z_2$

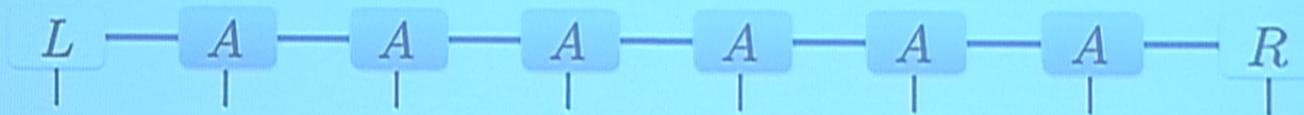
Tensors can carry a nontrivial *gauge* representation of this group

$$\begin{array}{c} \text{---} A \text{---} \\ | \\ U_g \end{array} = \begin{array}{c} \text{---} T_g \text{---} A \text{---} T_g^{-1} \text{---} \\ | \\ \end{array}$$

Ground state as a tensor network state



tensor network state (matrix product state)

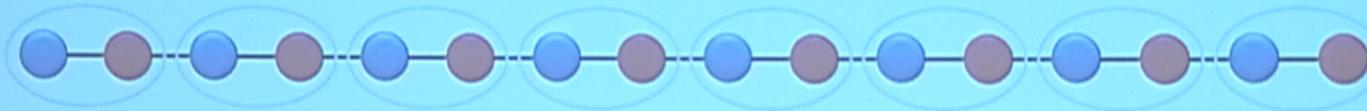


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$$\begin{array}{c}
 \text{---} A \text{---} \\
 | \\
 U_g \\
 \text{---}
 \end{array}
 =
 \begin{array}{c}
 \text{---} \tau_g \text{---} A \text{---} \tau_g^{-1} \text{---} \\
 | \\
 \text{---}
 \end{array}$$

Ground state as a tensor network state



tensor network state (matrix product state)



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Tensors can carry a nontrivial *gauge* representation of this group

For the cluster model, T_g is a projective representation: the Pauli group

$$\begin{array}{c} \text{---} A \text{---} \\ | \\ U_g \\ | \end{array} = \begin{array}{c} \text{---} T_g \text{---} A \text{---} T_g^{-1} \text{---} \\ | \end{array}$$

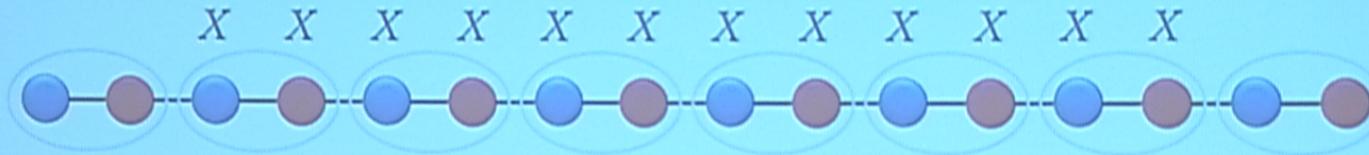
$$T_{(0,0)} = I$$

$$T_{(0,1)} = X$$

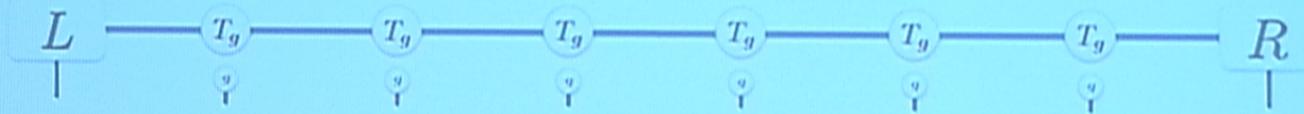
$$T_{(1,0)} = Z$$

$$T_{(1,1)} = Y$$

What makes a spin chain a good qubit wire?



tensor network state (matrix product state)



$$\begin{array}{c} \text{---} A \text{---} \\ | \\ \text{---} U_g \text{---} \\ | \end{array} = \begin{array}{c} \text{---} T_g \text{---} A \text{---} T_g^{-1} \text{---} \\ | \end{array}$$

Condition:

If T_g is a nontrivial projective representation of $Z_2 \times Z_2$ (Pauli representation), then the state is a good qubit wire.

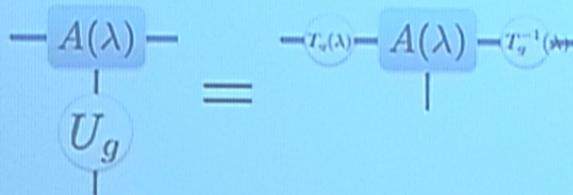
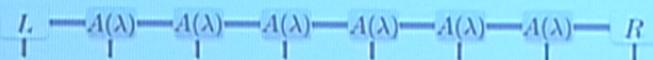
Quantum computing with perturbed ground states

$$H = H_0 + \lambda V$$

- › Symmetry-respecting perturbations V alter the ground state, but cannot change the type of representation^(*)

Chen, Gu, Wen, PRB (2011)

Ground state with Pauli gauge representation



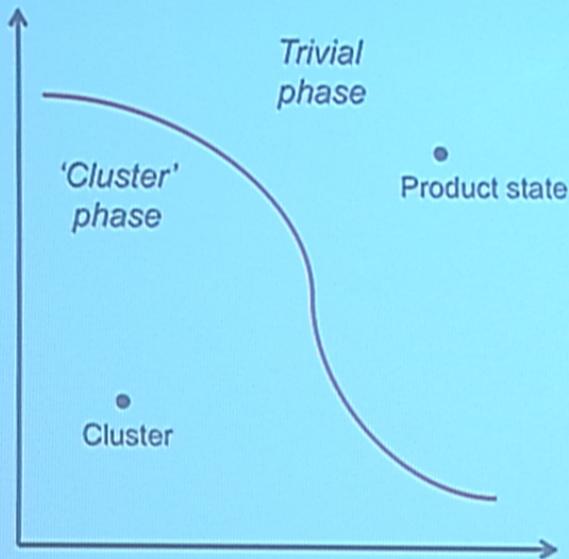
Phase transition

Ground state with trivial representation



^(*) technically, the second cohomology class
In this case, projective or true representation

1D cluster model in a nontrivial SP phase

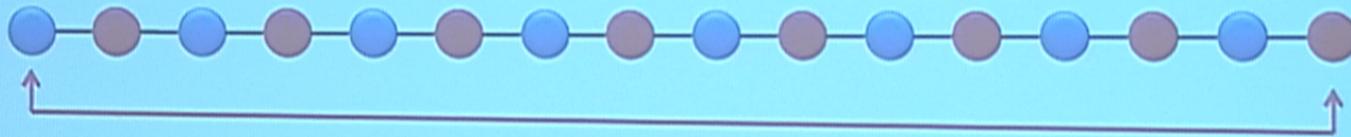


- › Ground states in the 'cluster' phase possess the long-range entanglement necessary for use as a qubit wire
- › Can we use states in this phase to do quantum logic?

Qubit wires – 1D cluster model

Measure / apply local field

X X X $X(\theta)X(\phi)$ X X X X $X(\theta)X(\phi)$ X X X



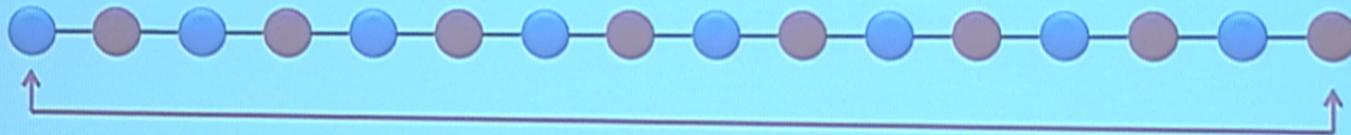
'Rotated' maximally entangled state for gate teleportation

$$I \otimes R(\theta, \phi) |\psi^-\rangle$$

Qubit wires – 1D cluster model

Measure / apply local field

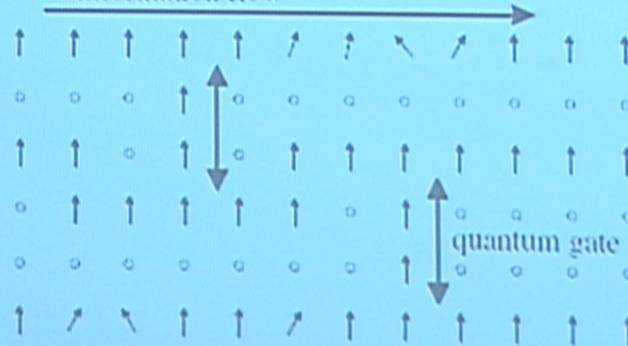
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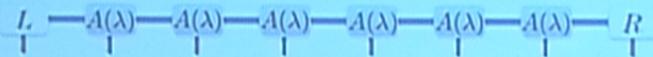
'Rotated' maximally entangled state for gate teleportation

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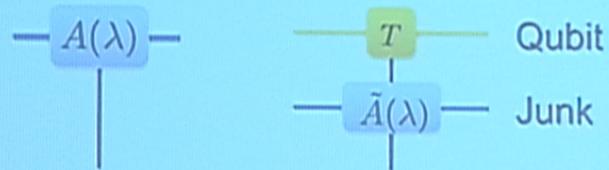
information flow



Equivalence to local Markovian error model



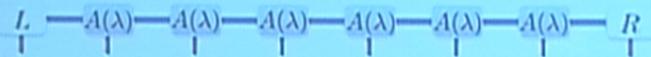
$$A(\lambda)^{(g)} = T_g \otimes \tilde{A}(\lambda)^{(g)}$$



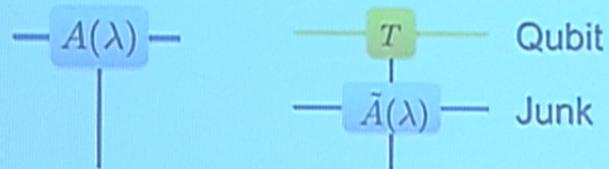
Tensor breaks up into a *structural part* (completely determined by representation) and a *'junk' part* affected by the perturbation

Singh, Pfeifer, Vidal, PRA (2010)

Equivalence to local Markovian error model



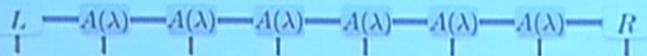
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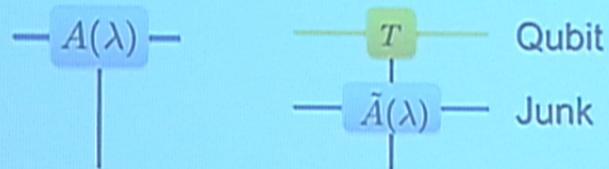
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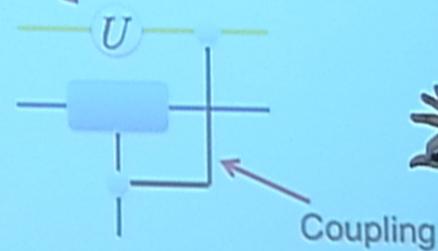


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Singh, Pfeifer, Vidal, PRA (2010)

Nontrivial gates lead to correlations between the qubit and the junk space

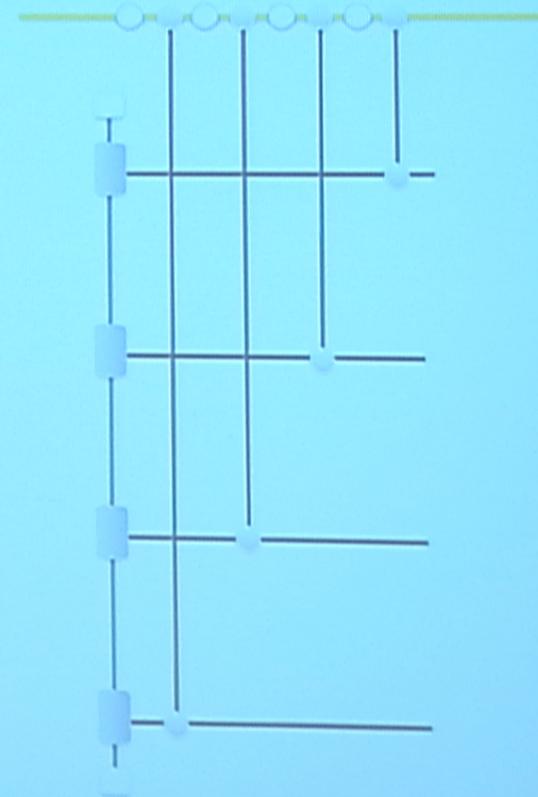
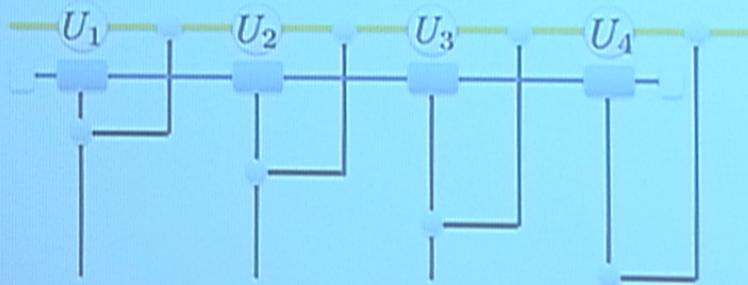
Quantum logic gate



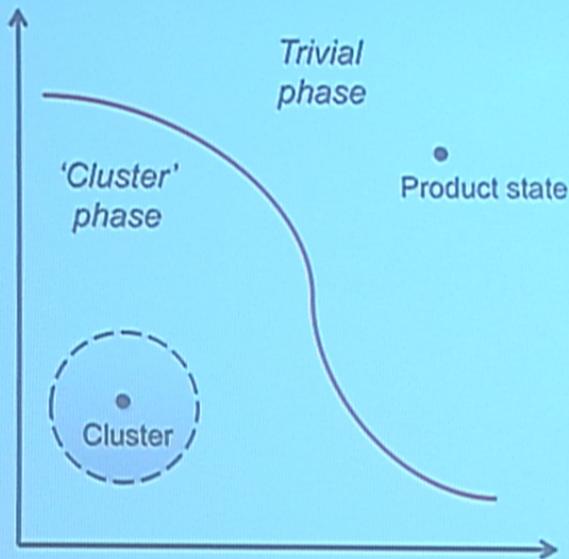
Equivalence to local Markovian error model

The 'junk space' state serves as an environment

- weak coupling for small perturbations
- noise is local in 'time', and independent
- space gates apart beyond correlation length:
Markovian noise model



1D cluster model in a nontrivial SP phase



- › Ground states in the 'cluster' phase possess the long-range entanglement necessary for use as a qubit wire
- › Quantum logic gates can be performed, with a local, independent, Markovian error model
- › Generalises to 2D cluster model
→ apply quantum error correction

Antiferromagnetic spin chains as quantum computational matter

2

work with Gavin Brennen, Akimasa Miyake, and Joe Renes

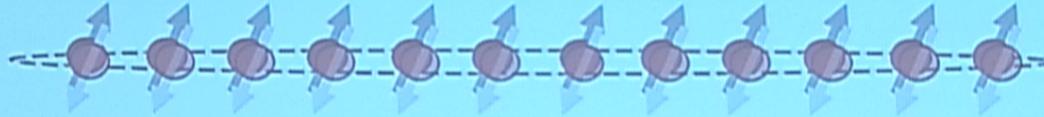


Affleck-Kennedy-Lieb-Tasaki (AKLT)



Rotationally-invariant spin chains

Affleck-Kennedy-Lieb-Tasaki (AKLT)



Rotationally-invariant spin chains

- **Haldane conjecture (1983):** ground state properties of Heisenberg spin chains depend on the spin. Integer spin antiferromagnets are *gapped*.

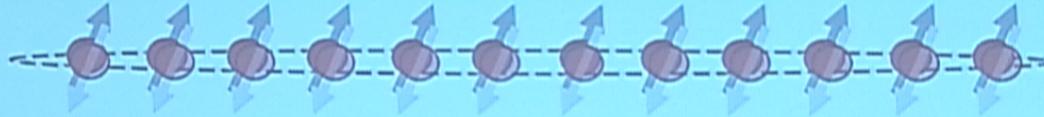


Rotationally-invariant spin chains

- Haldane conjecture (1983): ground state properties of Heisenberg spin chains depend on the spin. Integer spin antiferromagnets are *gapped*.
- AKLT state (1987): solution for a spin-1 Heisenberg chain with a gap.

$$H_{\text{AKLT}} = J \sum_j \left[\mathbf{S}_j \cdot \mathbf{S}_{j+1} + \frac{1}{3} (\mathbf{S}_j \cdot \mathbf{S}_{j+1})^2 \right]$$

Affleck-Kennedy-Lieb-Tasaki (AKLT)

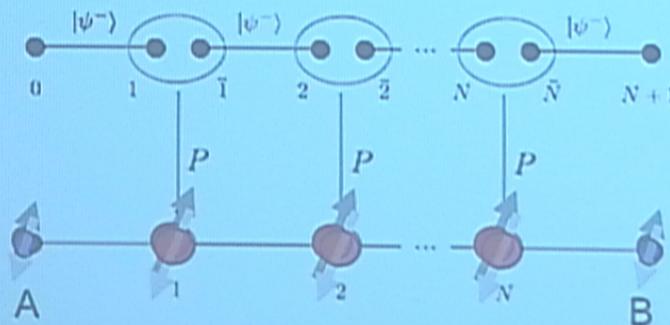


Rotationally-invariant spin chains

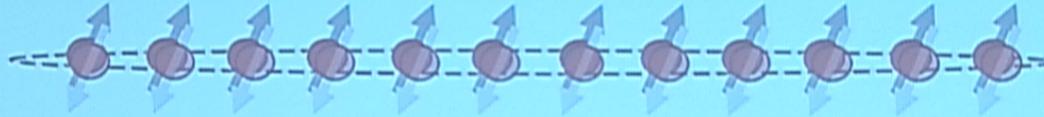
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- Ground state: valence bond solid – a 1D tensor network state



Affleck-Kennedy-Lieb-Tasaki (AKLT)

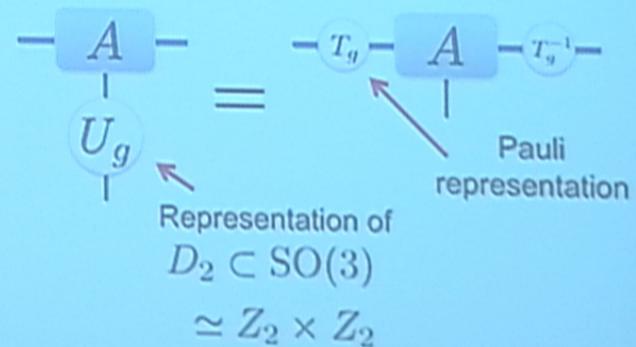
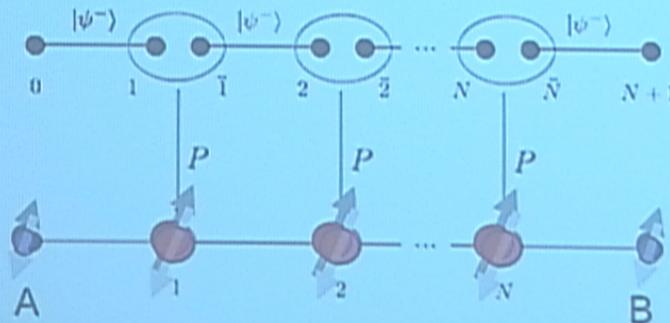


Rotationally-invariant spin chains

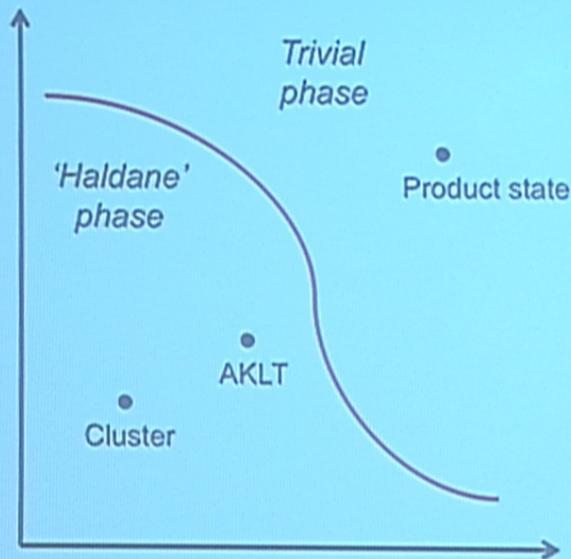
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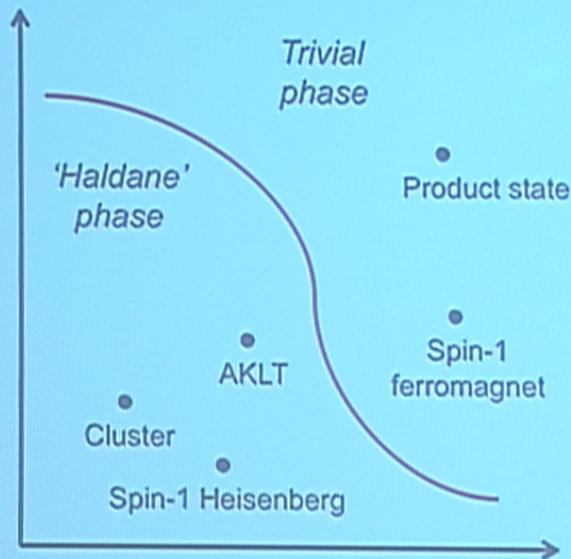


1D cluster and AKLT models are in the same SP phase



- › Our 'toy' cluster state model captures the essential 1D physics of antiferromagnetic spin-1 chains as qubit wires

1D cluster and AKLT models are in the same SP phase



› Our 'toy' cluster state model captures the essential 1D physics of antiferromagnetic spin-1 chains as qubit wires

› Quantum computing with spin-1 chains:

Gross, Eisert, Schuch, Perez-Garcia, PRA (2007)

Brennen and Miyake, PRL (2008)

› Computational renormalisation in Haldane phase:

Bartlett, Brennen, Miyake, Renes, PRL (2010)

› Holonomic quantum computing in spin-1 chains throughout the SP phase:

Renes, Miyake, Brennen, Bartlett, arXiv (2011)

Two-dimensional systems: Antiferromagnetic spin lattices

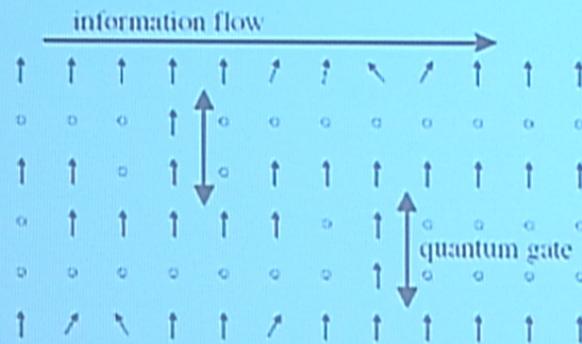
3

work with Andrew Darmawan and Gavin Brennen



Two-dimensional cluster models

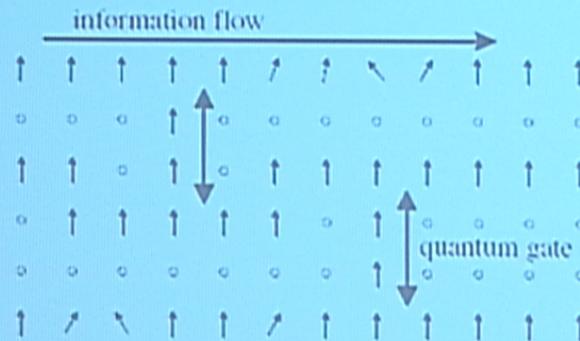
- › 2D cluster state allows for universal quantum computation
- › Ground state of a toy model



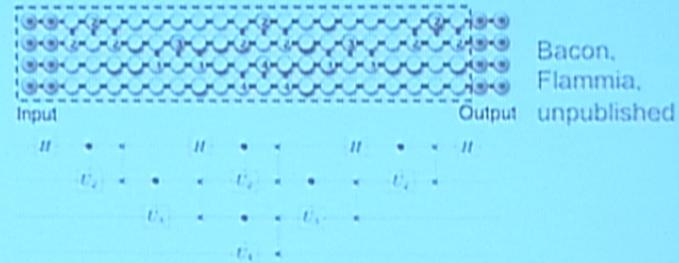
$$H = - \sum_{\text{sites}} Z \begin{array}{c} Z \\ | \\ X \\ | \\ Z \end{array} Z$$

Two-dimensional cluster models

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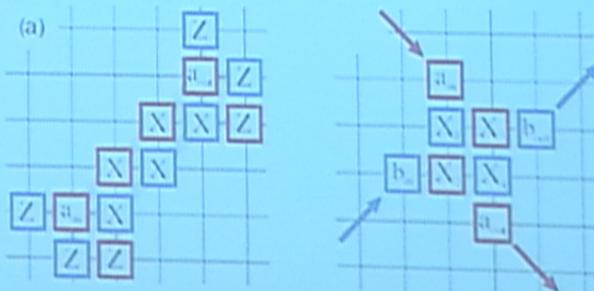
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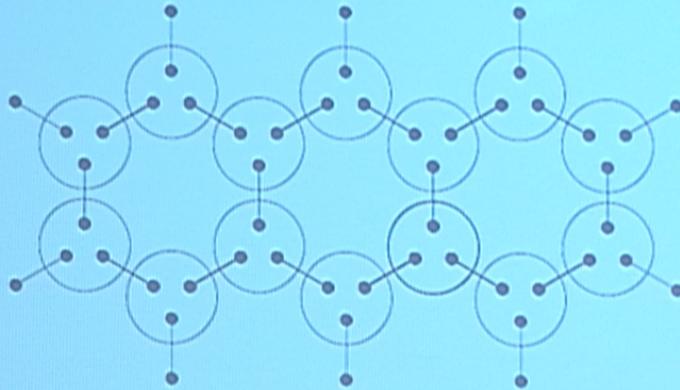
- › Is there a 2D 'cluster phase'?

Doherty, Bartlett, PRL (2009)

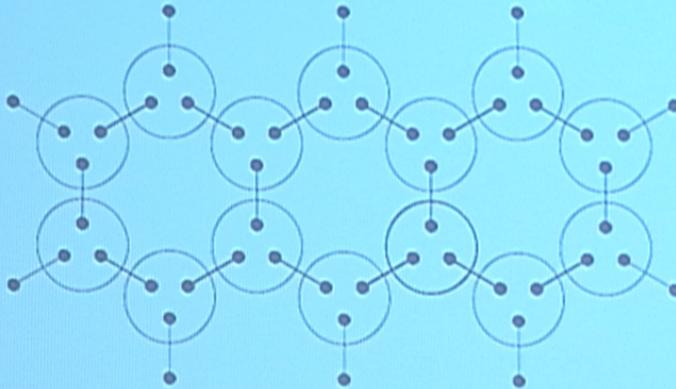
- › What are its properties? Related to 2D generalisations of SPTO?



› 2D AKLT state of spin-3/2



- › 2D AKLT state of spin-3/2



- › Unique ground state of Hamiltonian

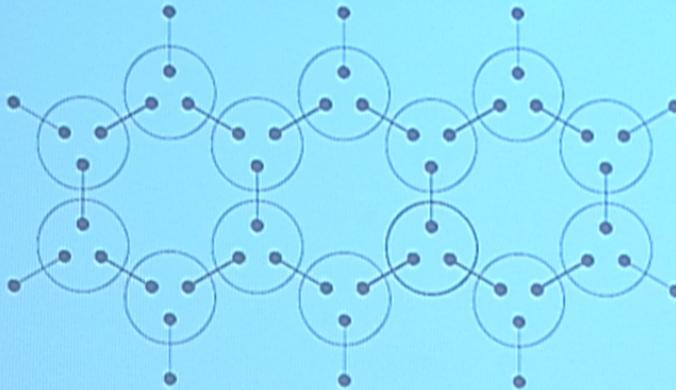
$$H = J \sum_{ij} \left[\frac{243}{1440} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{29}{360} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 + \frac{1}{90} (\mathbf{S}_i \cdot \mathbf{S}_j)^3 \right]$$

$$\propto J \sum_{ij} P_{ij}^{J=3}$$

- › Antiferromagnetic but not Néel ordered
- › Is it gapped?

2D AKLT antiferromagnet

› 2D AKLT state of spin-3/2



› Unique ground state of Hamiltonian

$$H = J \sum_{ij} \left[\frac{243}{1440} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{29}{360} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 + \frac{1}{90} (\mathbf{S}_i \cdot \mathbf{S}_j)^3 \right]$$

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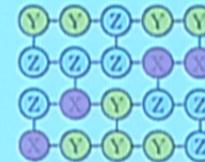
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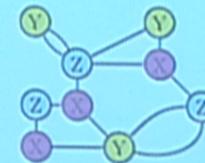
Wei, Affleck, Raussendorf, PRL (2011)

Miyake, Ann Phys (2011)

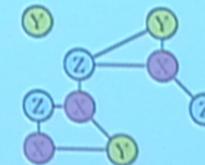
Filter:



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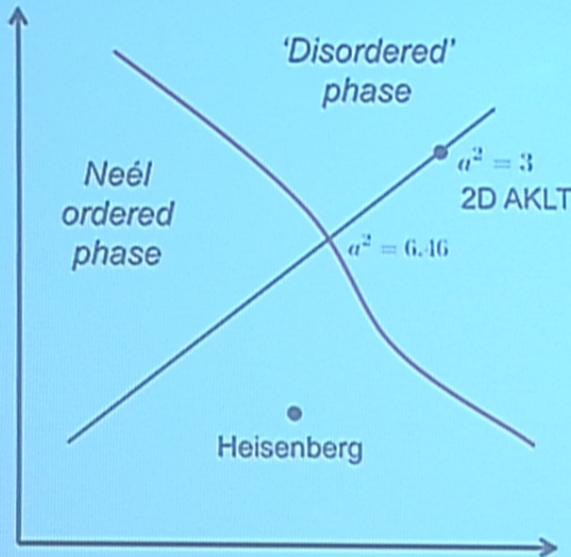


Random cluster state:



Large lattice yields a universal cluster state

A family of 2D antiferromagnet models

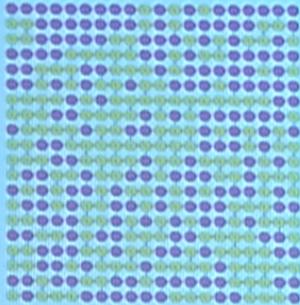


$$H(a) = J \sum_{ij} [D(a)_i D(a)_j] P_{ij}^{J=3} [D(a)_i D(a)_j]^\dagger$$

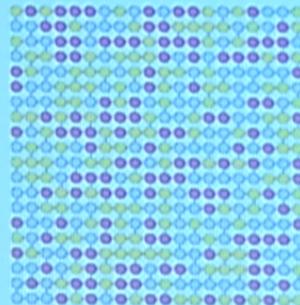
$$D(a) = \text{diag}(\sqrt{3}/a, 1, 1, \sqrt{3}/a)$$

Niggemann, Klumper, Zittartz, Z Phys B (1997)

Filtering for different values

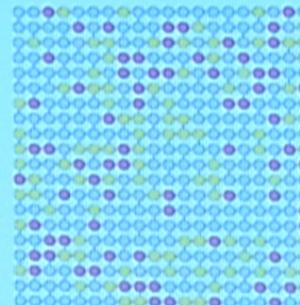


$$a^2 = 1$$



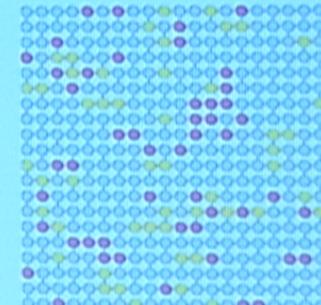
$$a^2 = 3$$

AKLT



$$a^2 = 5.70$$

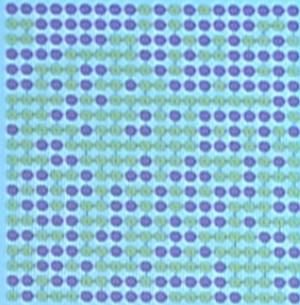
near transition



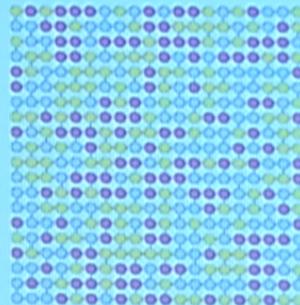
$$a^2 = 6.46$$

at transition

Filtering for different values

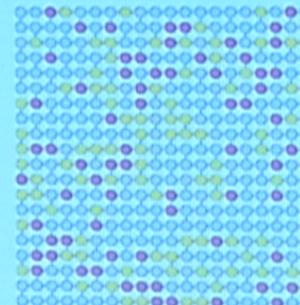


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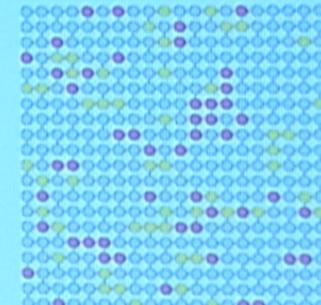
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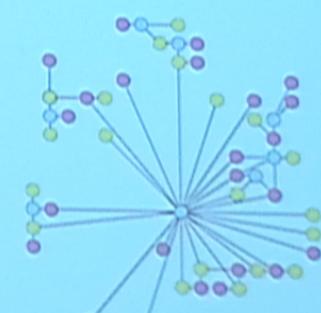
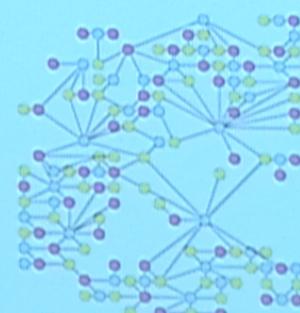
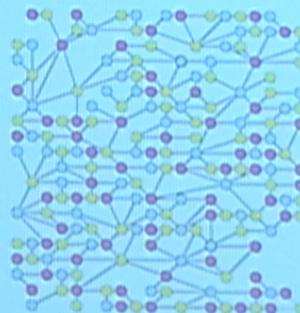
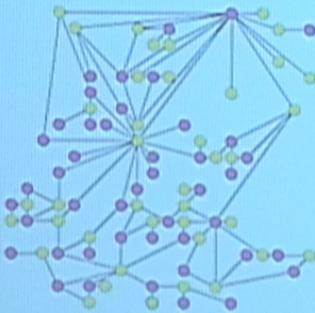
$$a^2 = 5.70$$

near transition



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at transition

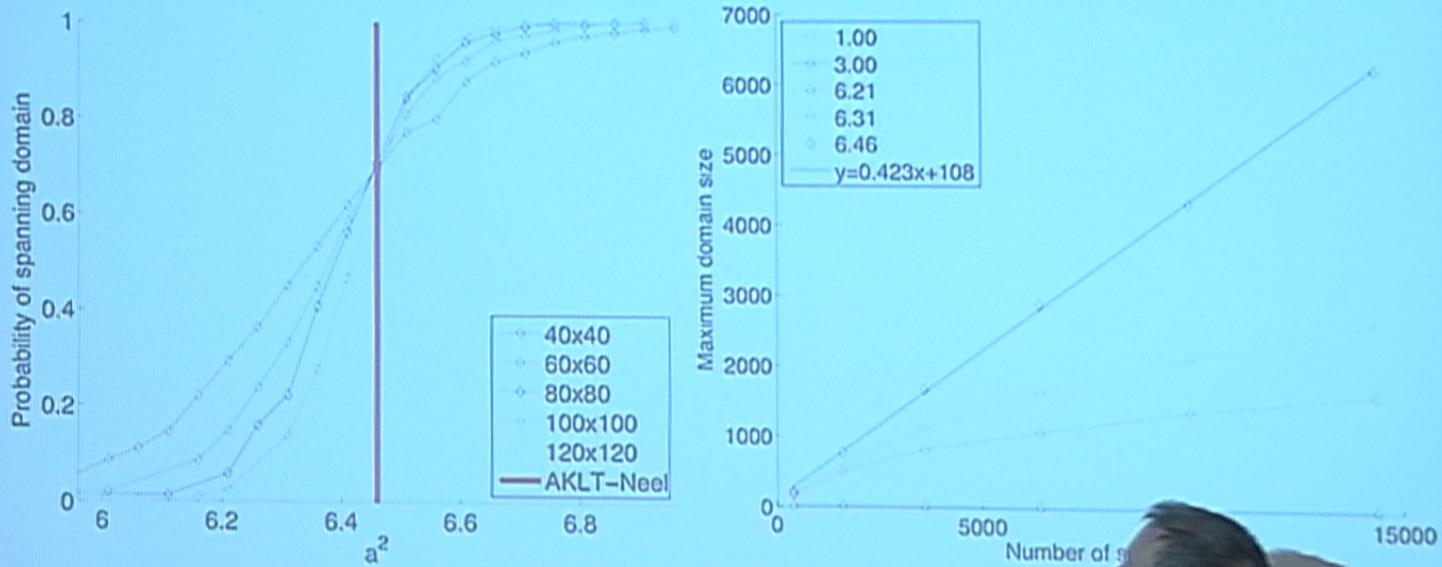


← genuine 2D structure

'star-like' graph

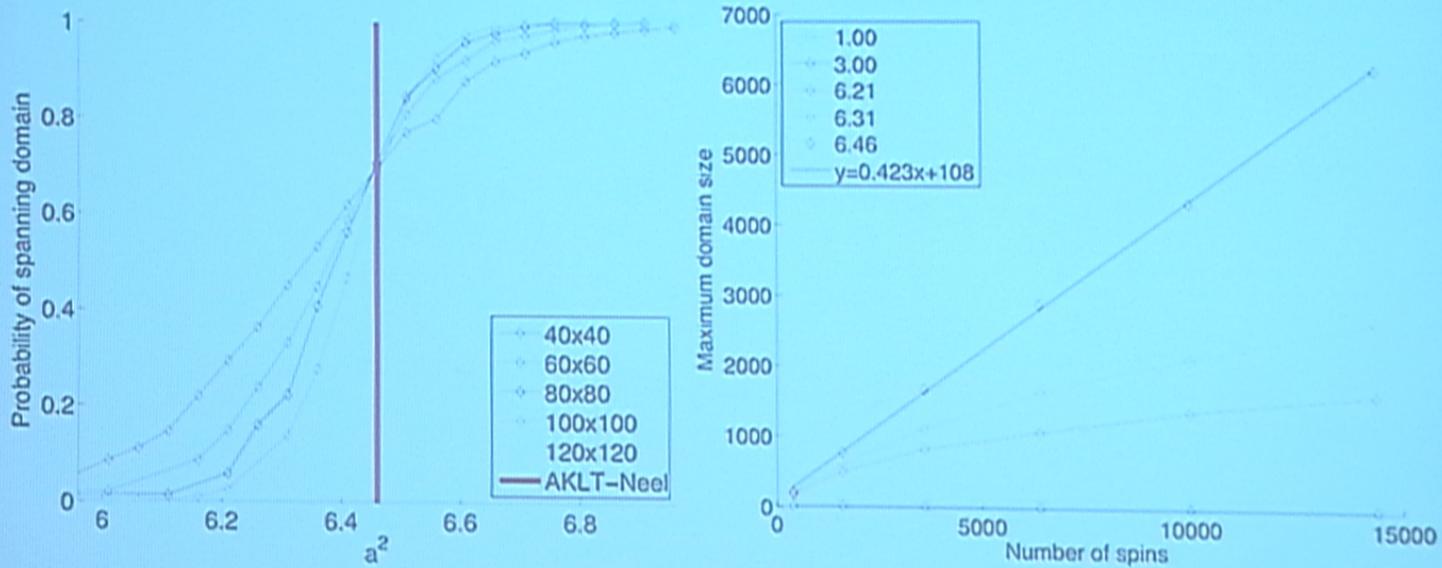
Darmawan, Brennen, Bartlett, arXiv (2011)

Quantum computational phase transition



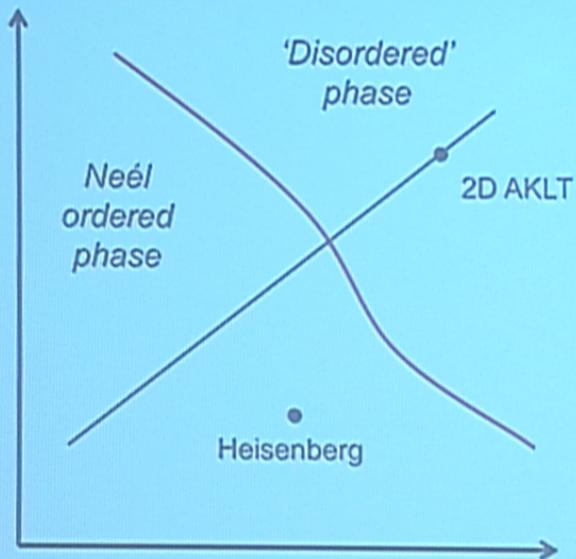
Darmawan, Brent

Quantum computational phase transition

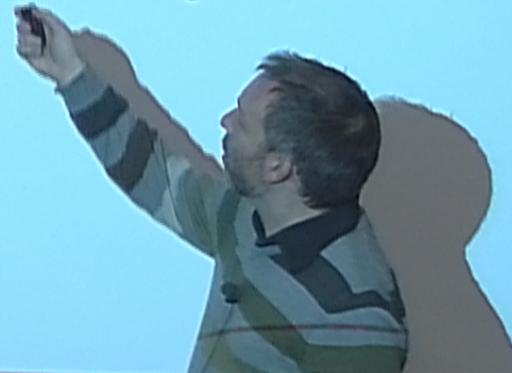


Darmawan, Brennen, Bartlett, arXiv (2011)

2D quantum computational phase



- › 2D lattice of spin-3/2 in a non-Neél antiferromagnetic phase can possess hidden correlations allowing for quantum computation
- › What characterises this order?
- › Is this phase gapped, allowing for computation in the ground state?



Conclusions and future directions

- › Ground-state quantum computing requires a type of 'hidden' long-range order:
 - in 1D systems, it is a symmetry-protected order
 - identical to a type of antiferromagnetic order, in some 1D and 2D systems

- › How is this order characterised in 2D or higher-D systems?
 - Is it related to extensions of symmetry-protected order in 2D?

- › Can this order be robust at non-zero temperature?